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PREFACE

This issue of the Quarterly TRANSACTIONS, which is Part 1 of Volume 50, is dated March 1931 in accordance with a recently adopted change in publication dates. Hereafter the four parts of the TRANSACTIONS will be issued in March, June, September, and December, instead of January, April, July, and October as formerly. This change was made necessary because the publication work in connection with the Winter Convention has increased to an extent which seriously interferes with the publication of the TRANSACTIONS in January.

This volume contains the papers and discussions presented at the Pacific Coast Convention at Portland, Oregon, September 2-5, 1930, the Middle Eastern District Meeting at Philadelphia, Pa., October 13-15, 1930, and at the Southern District Meeting at Louisville, Kentucky, November 19-22, 1930.

A table of contents appears in the front of the book and an authors' index of papers in the back. The complete alphabetical subject and authors' index for the year will be printed in the December Quarterly.

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PREFACE

This volume of QUARTERLY TRANSACTIONS is Part II of Volume 50, issued in June 1931 in accordance with the change of dates of issue explained in Part I of this Volume.

It contains the papers and discussions presented at the Winter Convention of the A. I. E. E. New York, January 26-30, 1931, and in addition, one paper, "Harmonic Generation by Means of Grid-Circuit Distortion," by F. E. Terman, D. E. Chambers, and E. H. Fisher, which was presented at the Pacific Coast Convention, September, 1930. It also contains the Report of the Board of Directors for the fiscal year ending April 30, 1931, and a financial report covering the same period.

A Table of Contents is printed on the front page, and an Index of Authors appears in the back of the volume. The complete subject and authors' annual indexes are printed in Part 4 of the QUARTERLY TRANSACTIONS each year.

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PREFACE

This volume of Quarterly TRANSACTIONS is Part III of Volume 50, issued in September 1931. It contains the papers and discussions presented at the Middle Eastern District Meeting of the A. I. E. E. Pittsburgh, Pa., March 11-13, 1931 and at the North Eastern District Meeting, Rochester, N. Y. April 29-May 2, 1931, and in addition, two contributions, "An Electric Analog of Friction" by H. H. Skilling and "Effects of Electric Shock—II" by W. B. Kouwenhoven and O. R. Langworthy.

An index of authors appears in the back of the volume. The complete subject and authors' annual indexes of Volume 50 will appear in the December Quarterly.

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PREFACE

Beginning 1931, the dates of issue of the Quarterly TRANSACTIONS have been advanced three months, Part I being dated March, Part II June, Part III September, and Part IV December. This volume is the fourth Quarterly for 1931 and contains the papers and discussions presented at the Summer Convention, Asheville, N. C., and the complete reports of the Technical Committees for the current year.

Following the established practise, the last Quarterly of each year contains a complete annual subject and authors' index which in this volume covers the contents of the four 1931 Quarterlies.

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Critique of Ground Wire Theory

BY L. V. BEWLEY*

Associate, A. I. E. E.

Synopsis.—*The paper consists of three parts, I—Induced Potentials, II—Direct Hits, and III—Other Effects. The work of previous investigators is briefly reviewed, and the limitations of their premises pointed out. Under Part I a generalized theory of ideal ground wires is offered, which takes into account the law of cloud discharge, the distribution of bound charge, and the formation of traveling waves. It is found that the protective ratio is independent of these factors. A more extensive theory taking the additional factors of successive reflections and tower resistance into account is then developed. Part II discusses the probability of a line's being hit, and applies a method for computing the effect of successive reflections to the calculation of potentials on the line and ground wires. Curves of these potentials at successive towers, and as functions of tower resistances and of time, are given. Part III discusses the effect of ground wires on attenuation, telephone interference, zero phase sequence reactance, corona, and the reduction in surge impedance due to the introduction of extra ground wires. There are three mathematical appendixes. In Appendix I, Maxwell's electrostatic and electromagnetic coefficients are reviewed, and the theory of traveling waves on any number of parallel wires is developed, including the behavior of these waves at rather general transition points. While this extension to the theory of traveling waves was developed incidental to the study of ground wire theory, it is believed to be of considerable interest and value on its own account. Appendixes II and III are the mathematical analyses corresponding to Parts I and II respectively.*

INTRODUCTION

THE purpose of this paper is to extend and generalize the theory of ground wires, with respect to both induced and direct strokes, and to give a résumé of all the pertinent data which, in the opinion and experience of the author, represent the present status of knowledge on the subject. It is recognized, of course, that the facts available to the author cannot even approximately include all of the worth while information on ground wires, nor is his judgment infallible. However, in light of the present intense interest in ground wires, it seems advisable to present the ideas contained herein. This study is part of a coordinated investigation of lightning which has been under way for a number of years under the general direction of F. W. Peek, Jr.

The paper is divided into three major parts: I, Induced Potentials, II, Direct Hits, and III, Other Effects. The mathematical analysis is confined to the Appendixes, and only such equations are included in the text as are necessary to insure continuity of treatment and clarity of understanding. Incidentally, there is given in Appendix I a treatment of the characteristics and behavior of traveling waves on any number of mutually coupled parallel wires terminating in general impedances, including other outgoing lines.

I. INDUCED POTENTIALS

Review of Previous Investigations. When a charged cloud approaches a transmission line, charges of opposite sign leak over the insulators, or migrate from the line terminals, and appear on the line and ground wires as bound charges fixed in position by the electrostatic field of the cloud, Fig. 1a. The distribution $f(x)$ of any bound charge is proportional to the electric field G, but the actual magnitudes of charge on the several conductors depend upon their size, heights, and arrangements. These geometric factors are uniquely accounted for by Maxwell's coefficients, as described in Appendix I. Now when the field collapses according to some law $F(t)$ of cloud discharge, the bound charges on the line and ground wires are released and form pairs of traveling waves moving in opposite directions away

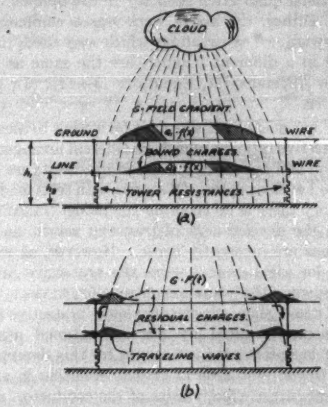

FIG. 1—RELEASE OF BOUND CHARGES ON LINE AND GROUND WIRES

(a) Before cloud discharge
(b) During cloud discharge

from the center of disturbance, as shown in Fig. 1b. The process and mathematical laws by which these waves form and develop have been given in a previous paper.[1] The waves which originated on the ground wires reach the tower where they suffer partial reflections and refractions, the extent of which depends upon the relative values of the surge impedances of the circuits and the tower resistance. However, unless the cloud

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1. For references see bibliography.

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discharge is instantaneous, diminishing residual bound charges remain on the line and ground wires up until the cloud discharge is completed.

The conventional method for calculating the protective ratio (defined as the ratio of the voltage on a line wire with ground wire protection, to the voltage which would exist without ground wire protection) is based on the tacit assumptions that (a) the ground wires are "ideal," that is, perfectly grounded throughout their length, (b) the cloud discharge is instantaneous, (c) traveling waves are not formed, (d) the distribution of bound charge is uniform.

The published theoretical work which has been done with ground wires grounded through resistances[2,3], has also been confined to the case of rectangular bound charges instantaneously released, and subject to the further restrictions of a single ground wire, and a single line wire, although the possibility of introducing an equivalent ground wire and an equivalent line wire was erroneously supposed. Moreover, reflections were limited to a single span. These several simplifying assumptions were introduced in the interests of engineering practicability, and primarily for the purpose of obtaining qualitative results. In the opinion of the present author, the above work was a commendable step forward. The equations which were used, though derived in a different manner, are the same as those given in Appendix III, Equations (3), (4), (5), (6) of this paper.

Peek[4] has made extensive tests on models to ascertain the protective ratio for numerous different arrangements of ground and line wires. Some of his results are given in Table I of this paper. Of course, with tests made in a laboratory there is no room on the model transmission line for the development of traveling waves, and the conditions are inherently static. However, as will be shown, for ideal ground wires the protective ratio is independent of the formation of traveling waves.

Peek's tests show consistently lower protective ratios than those computed by the conventional method. Hunter[5] has attempted to account for this discrepancy as due to the formation of corona, resulting in an enlargement in the effective size of the conductor, and a corresponding reduction of induced potential. His method consists in solving for the charge Q_1 on the ground wire from the equations which obtain before cloud discharge (Equations (1), Appendix II of this paper). Then

$$Q_1 = G \cdot f(r)$$

where G is the field gradient due to the cloud and $f(r)$ is a function of the radii of the conductors. The gradient at the surface of the ground wire due to Q_1 is

$$g = \frac{2\,Q_1}{r_1} = 2\,G\,\frac{f(r)}{r_1}$$

Equating this expression to Peek's equation for the visual corona gradient[4]

$$g_v = 29.8 \left\{ \frac{1 + 0.301}{\sqrt{r_1}} \right\}$$

yields an equation

$$G = 14.9 \left\{ 1 + \frac{.301}{\sqrt{r_1}} \right\} \frac{r_1}{f(r)}$$

from which r_1 may be determined. This value of r_1 is then used in computing the protective ratio by the conventional method. Thus r_1, and with it the protective ratio, depends upon the field gradient G. A number of implied assumptions underlies this work, such as the supposition that corona obeys the same law under transient conditions as under steady state conditions, that it remains constant during the cloud discharge, that positive and negative corona are alike, and that there is no time lag. These limitations were recognized by Hunter. Nevertheless, his calculations agree very well with Peek's tests, and represent the only real effort to take the effects of corona into account. Some of his results are given in Table I, and a set of curves extracted from his paper is reproduced here as Fig. 2.

TABLE I
COMPARISON OF CALCULATIONS AND TESTS

Arrangement		Peek's tests on models	Hunter's method
. . .	(1)	0.52	0.57
0 0 0	(2)	0.44	0.41
1 2 3	(3)	0.52	0.57
. . .	(1)	0.40	0.38
0 0 0	(2)	0.34	0.25
1 2 3	(3)	0.40	0.38
. . .	(1)	0.34	0.31
0 0 0	(2)	0.28	0.24
1 2 3	(3)	0.34	0.31
. . .			
0 0 1	(1)	0.42	0.46
0 0 2	(2)	0.40	0.52
0 0 3	(3)	0.56	0.56
. . .			
0 0 1	(1)	0.33	0.27
0 0 2	(2)	0.38	0.33
0 0 3	(3)	0.44	0.40

McEachron, Hemstreet, and Rudge[6] have studied the ground wire effect with traveling waves, Fig. 4a, and have ingeniously separated the total effect into two parts, first, the reduction in voltage due to the escape to ground of the bound charge on the ground wire; and second, the further reduction in voltage due to the acquisition of a charge of opposite sign by the ground wires. Their results are in excellent agreement with calculations of the protective ratio by the conventional method. Corona did not enter into the picture, as the potentials of the impressed waves were relatively low. They found that tower footing resistance of the order of 75 ohms was practically equivalent to perfect grounding, as far as induced strokes are concerned. The effect of this resistance is easily calculated by Equations (3),

(4), (5), and (6), Appendix III, of this paper. By these equations the voltage wave on the line wire transmitted beyond the grounding point is

$$f_2'' = f_2 = \frac{Z_{12}}{2R + Z_{11}} f_1$$

Using calculated values of $Z_{11} = 532$, $Z_{12} = 107$, $R = 76$, and $f_1 = f_2$ (since the same wave was applied to all three conductors) there is

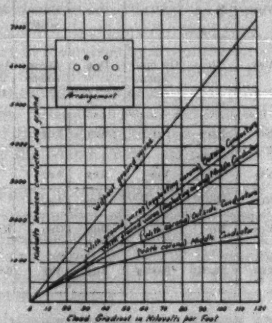

Fig. 2—Hunter's Correction for Corona

$$f_2'' = \frac{2 \times 76 + 532 - 107}{2 \times 76 + 532} f_1 = 0.845 f_1$$

The value actually measured was 0.835. Had the grounding resistance been equal to zero this would have reduced to

$$f_2'' = \frac{532 - 107}{532} f_1 = 0.800 f_1$$

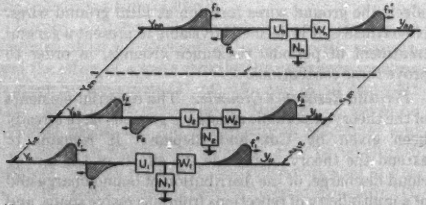

Fig. 3—Waves at Transition Points of N Mutually Coupled Circuits

as compared with a calculated protective ratio (based on the conventional method) of 0.77. The difference is due to the fact that the conventional method for calculating the protective ratio takes into account the difference in voltage between wires at unequal heights, whereas the above calculations with traveling waves were based on the same initial waves on both conductors.

A number of measurements with antennas has been taken in an effort to obtain field data on the protective ratio afforded by ground wires. Smeloff and Price[7] mention one record in which a protective ratio of approximately 50 per cent was indicated. Here again, there is no room for the formation of traveling waves, and the situation is inherently static. The indicated protective ratio is of the order of that calculated by the conventional method,—certainly within the range of accuracy of the measurements.

The author hardly feels justified in mentioning in connection with induced strokes, the statistical data on ground wires that have been accumulated by operating companies, for there is no definite information as to the percentage of outages caused by induced and direct strokes. However, the interesting curves given as Fig. 9 in the paper by Lewis and Foust[8] show some degree of correlation between theory and experience.

From the foregoing review of the work that has been done on induced strokes, it is evident that no single study has explicitly included all of the essential factors

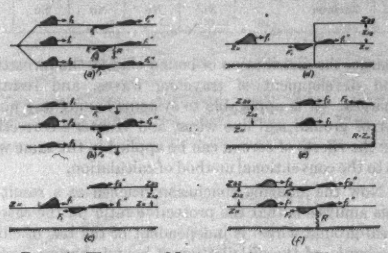

Fig. 4—Waves on Mutually Coupled Wires

involved, nor has any definite effort been made to correlate the different methods of attack on the problem, so as to prove their equivalence. It is not entirely obvious, for example, that the same protective ratio would be found in the case of tests with traveling waves (a strictly dynamic situation) as are calculated by the conventional method based on purely static equilibrium conditions. It is still less obvious that either of these methods gives the same protective ratio as obtained under actual conditions. Table II outlines the assumptions underlying the several methods of attack, and serves to indicate their limitations from the standpoint of rigorous analysis. It is partly the purpose of this paper to correlate these separate investigations by contributing a generalized theory of ideal ground wires, and an extension to the theory of periodic resistance grounds.

Ideal Ground Wires. In Appendix II of this paper, the general theory of ideal ground wires (ground wires perfectly grounded at all points) is worked out. The analysis includes the effects of the law of cloud dis-

TABLE II
INVESTIGATIONS OF GROUND WIRE PROTECTION FOR INDUCED VOLTAGES

	Method	Considered the effect of:											
		Distribution of bound charge	Law of cloud discharge	Residual charges	Formation of traveling waves	Developed traveling waves	Corona	Tower resistance	No of spans	Successive reflections	No. of ground wires	No. of line Wires	Remarks
THEORETICAL	Conventional (Petersen)	No	No	Yes	No	No	No	No	No	No	Any	Any	
	Hunter	No	No	Yes	No	No	Yes	No	No	No	Any	Any	
	Cox & Slepian	No	No	No	No	Yes	No	Yes	No	No	One	Erroneously	Approximate attenuation
	Fortescue, Atherton, & Cox	No	No	No	No	Yes	No	Yes	No	On one span	One	Erroneously	Approx. for the mutual induction
	Ideal ground wires (this paper)	Yes	Yes	Yes	Yes	Yes	Hunter's	No	No	No	Any	Any	
	Periodic resistance grounds (This paper)	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Any	Any	
TESTS	On Models—Peek	No	Yes	Yes	No	No	Yes	No	No	No	Any	Any	
	Antennas	No	Yes	Yes	No	No	Yes	Yes	No	No	One	One	Unsatisfactory
	Traveling Waves—McEachron	No	No	No	No	Yes	Yes	Yes	Partly	Partly	Any	Any	Equal waves on all wires

charge, the distribution of bound charge, the formation and development of traveling waves, and residual charges. It is applicable to systems having any number of ground and line wires, and Hunter's correction for the effects of corona can be applied in the same way as to the conventional method of calculation.

Now the amazing conclusion reached as a result of this analysis is that the protective ratio (in the case of ideal ground wires) is independent of the law of cloud discharge, of the distribution of bound charges, and of traveling waves, and is exactly the same expression as that found by the conventional method of calculation. In other words, the same protective ratio would be found by any method for which ideal ground wire conditions prevail, and these include:

1. The conventional method (and Hunter's correction).
2. Tests on model transmission lines (Peek).
3. Tests with traveling waves (McEachron).
4. Measurements on antennas.

Of course, the actual voltages induced on a line are quite different, depending on the law of cloud discharge, etc., but the *percentage* reduction in voltage to be realized by the employment of ground wires is independent of these factors.

Appendix II, therefore, establishes the validity for obtaining the protective ratio by any of the several different methods. In addition, Appendix II offers a means for computing the actual voltage induced on a line. It may appear academic to talk about ideal ground wire conditions, but as a matter of fact, such conditions substantially obtain at the tower within a fraction of a microsecond after the start of cloud discharge (the time that it takes a wave to travel twice the height of the tower). Out in mid span, one or two microseconds may pass before ideal ground wire conditions prevail. Also, as McEachron, Hemstreet, and Rudge[6] have found experimentally, and as was verified by calculations in this paper, a ground resistance of the order of 75 ohms provides nearly 95 per cent of the protection possible with perfect grounds. And finally, the fronts of waves caused by induced potentials are not abrupt, so that the negative reflections traveling back from the grounding points at the towers arrive before maximum potential occurs and thereby, in effect, the ground wires function as ideal ground wires. Nevertheless, it is deemed advisable to present a general treatment of periodic resistance grounds, in order to prove these contentions.

Periodic Resistance Grounds. The essential elements of the theory of periodic resistance grounds have already been given by Cox and Slepian.[2] It remains to extend the theory to include the effects of the law of cloud discharge, of the distribution of bound charge and of a multiplicity of reflections from successive spans, and to take into account properly the presence of any number of line and ground wires. The generalization will then be complete.

In a previous paper[1] the author derived three different methods (graphical, tabular, and analytic) for calculating the shape of traveling waves induced by lightning in terms of the functions $F(t)$ and $f(x)$ representing the law of cloud discharge and the initial distribution of bound charge respectively. It was there shown that

the shape of the traveling wave is changing during the cloud discharge. This may be called the formation period, to distinguish it from the period following the completion of cloud discharge when the waves are fully developed and suffer no further change in shape except the attenuation and distortion due to losses. However, that part of a forming wave which has moved beyond the region occupied by the bound charge is the same as the corresponding part of the fully developed wave. It is therefore possible to specify the shape of wave issuing from a bound charge distribution as proportional to some function $\phi(t)$ involving $F(t)$ and $f(x)$, and this function may be discovered by any of the three methods given in the bibliography. In particular, the function $f(x)$ may be considered to be only that part of a total bound charge distribution which lies between adjacent towers, and then the simultaneous effect of all spans included by the bound charge can be found by superposition.

In Appendix I of this paper, the theory of traveling waves on any number of mutually coupled parallel wires is worked out, and the behavior of these waves at rather general transition points is specified. The solution obtained applies to Fig. 3 and from it the special cases shown in Fig. 4 have been given as simple examples of its application. Fig. 4f may be taken as a resistance ground on one ground wire protecting one line wire. But it is shown in Appendix III that each line wire behaves independently of the others; and according to Equations (38) to (42) of Appendix I it is possible to lump all the ground wires as a single equivalent ground wire if they are all at the same height above ground, that is, all carrying equi-potential waves. If all ground wires are not at the same potential, then as an approximation, this replacement may be effected by an equivalent ground wire at their average potential. In practical cases the error is negligible, and avoids the complication of including mutual connection links at the transition points of Fig. 3.

Hereby, without loss of essential generality, Fig. 4f contains all of the elements entering into the analysis, and the effect of a wave striking the resistance junction is given by Equations (3), (4), (5), and (6) of Appendix III. Finally, the simultaneous existence of waves on different spans, and the multiplicity of reflections and refractions to which they give rise, may be taken into account through the use of the lattice indicated in Fig. 5. The theory of such lattices is given in Appendix III.

The steps in the complete solution are then as follows:

(A) Replace the actual ground wires by an equivalent ground wire, consistent with the conditions imposed by Equations (38) to (41), Appendix I.

(B) Consider each span between towers separately, and for each span compute the traveling waves on every wire. Then

$f_1 = \phi(t) \,.\, h_1 G$ = *incident wave on ground wire.*

$f_2 = \phi(t) \,.\, h_2 G$ = *incident wave on line wire.*

where h_1 and h_2 are the heights of the wires, G is the field gradient, and $\phi(t)$ is a function involving the bound charge distribution $f(x)$ on the span and the law of cloud discharge $F(t)$. It may be evaluated by any of the graphical, tabular, or analytic methods given in the bibliography.[1] It is simply the function expressing the shape of the wave issuing from the bound charge distribution.

(C) Calculate the reflection and refraction operators for both the line and ground wires as given by Equations (3), (4), (5), and (6) of Appendix III. Then

$F_1 = a f_1$ = *reflected wave on ground wire*

$f_1'' = b f_1$ = *transmitted wave on ground wire*

$F_2 = c f_1$ = *reflected wave on line wire*

$f_2'' = f_2 + c f_1$ = *transmitted wave on line wire.*

(D) Construct a lattice as shown in Fig. 5 of a sufficient number of sections to include the requisite time interval and number of spans, and therefrom

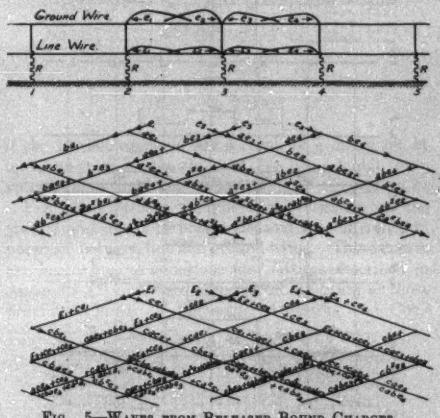

FIG. 5—WAVES FROM RELEASED BOUND CHARGES

Top lattice: Reflections and refractions on ground wire
Bottom lattice: Reflections and refractions on line wire

determine the potentials at all points by superposition.

The most simple case is that of instantaneous cloud discharge and rectangular bound charges. This also gives the maximum departure of the potential at mid-span with respect to that at the tower. Curves calculated on this assumption for a bound charge 2,000 ft. long and 1,000 ft. spans are shown in Fig. 6. They differ from those shown by Fortescue, Atherton, and Cox[8] in being based on different constants and in taking into account all reflections up to the included time interval. The traveling wave on the line wire is 0.550 (18 per cent high), 0.488 (4 per cent high), and 0.472 (1 per cent high) after zero, one, and two microseconds respectively. Potential differences between line and ground wires greater than these values (1.30 maximum for one-half a microsecond) are limited to the two spans which held the bound charge. These excess potentials

diminish in importance as the rate of cloud discharge decreases. Had the ground wires been ideal, the voltage on the line wire would have been 0.467.

II. Direct Hits

Investigations of direct hits on lines fall into two categories. First, there are the statistical studies relating to the probability of a lines being struck and the magnitudes involved, and second, there is the analysis of the resulting disturbances.

In Fig. 7a is shown a transmission line tower situated on a level plane free of brush or other projections. While the idiosyncrasies of a lightning bolt are too erratic for one to predict where it will go, yet the only justifiable assumption is to suppose that on an average

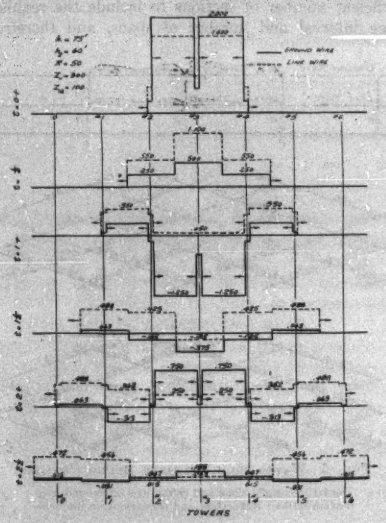

Fig. 6—Approach to Ideal Ground Wire Conditions on the Spans

it will strike to the nearest object. On this basis, the stroke will, by preference, hit the tower when the distance r is less than the height H. In Fig. 7b are plotted a set of curves showing the distance D, corresponding to $r = H$, between the projection of the approaching center of disturbance and the tower, as a function of the cloud height H and the tower height h. If there is absolute certainty that lightning will strike within a zone of width W centered on the transmission line, then the probability that the line will receive the stroke is $2\,D/W$. When it is realized from the curves of Fig. 7b that the susceptible zone $2\,D$ is of the order of 1,000 ft. it is not surprising that lines are hit. If the tower is on the top of a hill or ridge (as is so often the case in order to provide for long spans) then the chance of being hit is greatly magnified. The susceptible distance $2\,D$ can only be ascertained in such a case by drawing the clearance arc as shown in Fig. 7c. The tower is more likely to be struck than the span, because it is higher than the sagging span, and also because the towers usually occupy the high points along the right of way.

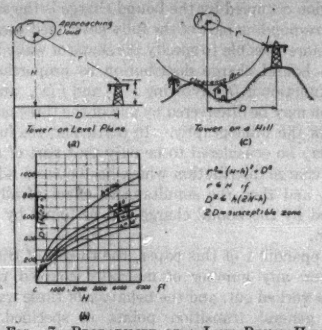

Fig. 7—Probability of a Line Being Hit

It has been proposed to provide line towers with extension masts of sufficient height to insure their being struck instead of any part of the line. The necessary height of such masts is of the order of 200 ft. for line heights of the order of 60 ft. and spans 1,000 ft. long. Still other schemes have been advanced, such as paralleling the transmission line by separate masts supporting a direct hit wire, etc. Of course any scheme that is used must have economic justification.

It is of interest to speculate on the maximum voltage of a direct hit. Lightning waves measured on transmission lines by cathode-ray oscillograph stations have been from 10 to 200 microseconds in length, thus showing that the cloud discharge has persisted long enough to establish equilibrium conditions in the lightning bolt itself (that is, successive reflections, if they actually occur, will have subsided). Thereafter,

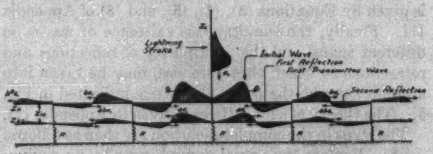

Fig. 8—Reflected and Transmitted Waves Due to Periodic Resistance Grounding

the potential drop along the lightning bolt is probably uniform. Now for a steady impulse lasting 10 microseconds or longer, the flashover voltage in air is from 150 to 200 kv./ft., so that a line wire 60 ft. high on wood poles could not support a voltage much in excess of 10×10^6 volts without a side flash to ground occurring.

If a side flash does take place, then the voltage on the line would be 60 G, where G is the gradient along the stroke, and if uniform, does not greatly exceed 100 kv./ft. Therefore, the maximum voltage on a 60-ft. line is probably of the order of 10×10^6 volts.

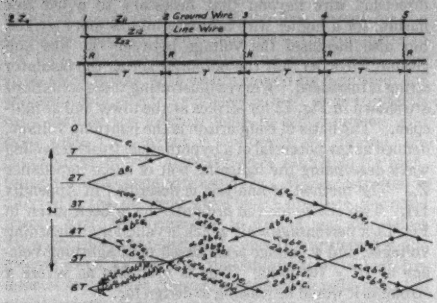

FIG. 9—REFLECTIONS AND REFRACTIONS ON GROUND WIRE. LIGHTNING STROKE AT TOWER

The voltages on the ground wire at the junctions as functions of time are:

$$e_1 = e_1(t) + [a + \alpha a] \cdot e_1(t - 2T) + [(1 + \alpha)(\alpha a^2 + a b^2)] \cdot e_1(t - 4T) + \ldots$$
$$e_2 = b\, e_1(t - T) + [a b + a^2 b + \alpha a b] \cdot e_1(t - 3T) + \ldots$$
$$e_3 = b^2 e_1(t - 2T) + [a b^3 + 2 a^2 b^2 + \alpha a b^3] \cdot e_1(t - 4T) + \ldots$$
$$e_4 = b^3 e_1(t - 3T) + \ldots$$

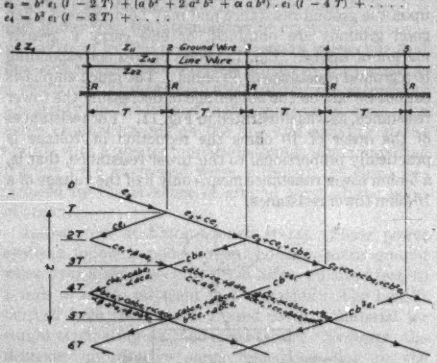

FIG. 10—REFLECTIONS AND REFRACTIONS ON LINE WIRE. LIGHTNING STROKE AT TOWER

The voltages on the line wire at the junctions as functions of time are:

$$E_1 = e_1(t) + [2c + \beta a] \cdot e_1(t - 2T) + [2bc + 2abc + 2\alpha ac + \alpha\beta a^3 + \beta a b^2] \cdot e_1(t - 4T) + \ldots$$
$$E_2 = e_1(t - T) + c\, e_1(t - T) + [c + \beta a + bc + abc + \alpha ac] \cdot e_1(t - 3T) + \ldots$$
$$E_3 = e_1(t - 2T) + [c + bc] \cdot e_1(t - 2T) + [b^2 c + a b^2 c + \alpha abc + a^2 bc + abc + c + \beta a + \alpha ac] \cdot e_1(t - 4T) + \ldots$$
$$E_4 = e_1(t - 3T) + [c + bc + b^2 c] \cdot e_1(t - 3T) + \ldots$$

Currents in the tower of the order of 100,000 amperes have been indicated by the direct hit recorder.[9] If these currents were due to a direct hit at the tower of resistance R, then the voltage across the tower was 100,000 R. By Equation (7) of Appendix III this corresponds to a lightning voltage of

$$e_o = \left\{ \frac{Z_{11}(R + Z_o) + 2 R Z_o}{2 Z_{11}} \right\} 100{,}000$$

which, in the case of two ground wires, gives a range of $e_o = 10 \times 10^6$ to $e_o = 20 \times 10^6$ corresponding to $O < R < 100$ ohms, and an assumed surge impedance of $Z_o = 200$ for the lightning bolt. On the other hand,

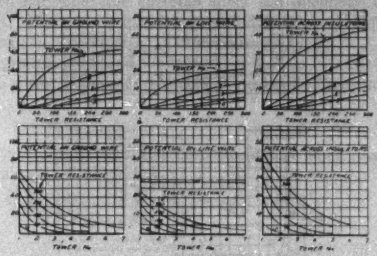

FIG. 11—POTENTIALS AT THE TOWERS IN PER CENT OF THE LIGHTNING VOLTAGE

if the 100,000 amperes was a result of a direct hit at mid span there is a possibility of there being double these voltages in the lightning stroke.

Fig. 11 illustrates the effect of resistance on the ground and line wire potentials, and the difference in potential between line and ground wires. These curves are based on the equations and lattices described in Appendix III and were prepared in imitation of those previously given by Fortescue, Atherton, and Cox.[8]

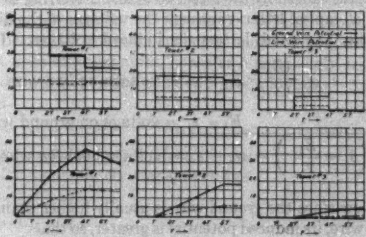

FIG. 12—POTENTIALS AT THE TOWERS AS FUNCTIONS OF TIME

Top: Infinite rectangular applied wave
Bottom: Infinite flat-top wave with 4 T front

Fig. 12 shows the voltages as functions of time at three adjacent towers, and includes the effects of successive reflections from all adjacent towers.

In general, there are four distinct and definite advantages to low ground resistances:

1. Reduced potential on the ground wires at the tower.
2. Reduced potential on the line wires at the tower.

3. Reduced potential between line and ground wires.
4. Limitation of dangerous potentials to fewer spans.

Low ground resistance is obtained by decreasing the tower footing resistance and by the installation of counterpoises. The proper disposition and arrangement of counterpoises appears at present to be a matter of test rather than of calculation.

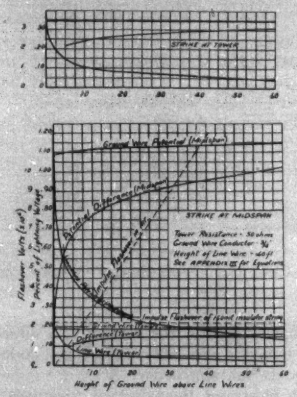

FIG. 13—EFFECT OF INCREASING THE HEIGHT OF THE GROUND WIRE

Induced potentials, co-existing with those due to the direct stroke, alleviate the magnitude of the impressed voltages, for the induced voltages are opposite in sign to those caused by the direct strokes. Referring to Fig. 14, let capital letters represent the voltages of the direct stroke, and small letters the induced voltages. Then

$$E_2 - e_2 = \frac{(Z_{12})}{(Z_{11})} E_1 - e_2 = \text{voltage on line wire}$$

$$E_1 - e_1 = \text{voltage on ground wire}$$

$$E_1\left(1 - \frac{Z_{12}}{Z_{11}}\right) - (e_1 - e_2) = \text{voltage between wires.}$$

Now since the ground wire is usually higher than the line wire, its induced potential is greater, so that

$$e = (e_1 - e_2) = \text{a positive quantity}$$

Therefore, the presence of the induced potential reduces the voltage between wires by e, and to a much greater extent reduces the voltages on both line and ground wires. Direct stroke calculations which neglect the induced charges are consequently pessimistic and err on the safe side.

Peek has advocated the use of one ground wire, called the direct-hit wire, sufficiently higher than the other conductors so that they will fall inside its protective wedge. Other ground wires may be used in conjunction with it to limit the potentials caused by induced strokes. Unless the direct-hit wire is placed sufficiently high such an arrangement does not obviate the possibility of side flashes taking place from the direct-hit wire to the other conductors. Nor is it an unmitigated advantage to increase the height of the direct-hit wire beyond that necessary to avoid side flashes, for a higher wire not only invites more strikes, but also increases the voltage between the line and ground wires, so that the potential across the insulator string is increased. Curves illustrating these conditions are shown in Fig. 13 for strikes at the tower and at midspan. The basis of comparison is the lightning voltage, defined as the potential of a hypothetical freely traveling wave descending the lightning bolt of surge impedance Z_0. The method of analysis is described in Appendix III. Two scales for the ordinates have been given in Fig. 13, a percentage scale which is valid for all lightning voltages, and a voltage scale based on a lightning voltage of 10×10^6 volts. This is probably as severe a stroke as it would be feasible to design for.

In designing a line for a maximum lightning voltage of 10×10^6 it is evident from Fig. 13 that the direct hit wire must be elevated 35 ft. above the line wires at midspan to insure that there will be no side flash to them. The requisite number of insulators at the towers depends upon the ground resistance and may be decreased when good grounds are obtained, or else carry a greater margin of safety. Fig. 13 shows curves corresponding to a ground resistance of 50 ohms. Too much emphasis cannot be laid on the advantage of decreasing the tower resistance, as was illustrated in Fig. 11. For resistances of the order of 10 ohms the reduction in voltage is practically proportional to the tower resistance, that is, a 5-ohm tower resistance means only half the voltage of a 10-ohm tower resistance.

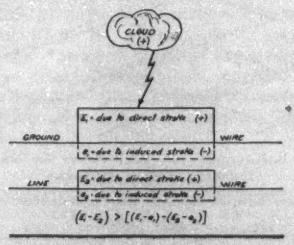

FIG. 14—SUPERPOSITION OF DIRECT STROKE AND INDUCED POTENTIALS

III. OTHER EFFECTS

While ground wires are used primarily as protection against induced and direct strokes, yet they exert a number of interesting subsidiary effects, such as influence on attenuation, telephone interference, corona loss, and zero phase sequence reactance. Some of these effects will be discussed here in a superficial sort of way but with no pretense at completeness.

Effect of Ground Wires on Wave Propagation. The history of a traveling wave is described in terms of three characteristics: (1) velocity of propagation, (2) distortion, and (3) attenuation. The introduction of ground wires does not change the velocity of propagation of waves on a transmission line, except for the negligible influence of that part inside the conductor of the total magnetic field linking the ground wire. For steep short waves, such as are caused by lightning, the transient skin effect is sufficiently high so as to practically exclude the internal fields of self-induction, and even when the current is uniformly distributed, the contribution of the internal field is negligible for high-tension transmission line spacings. Wave distortion is affected in two different ways by the presence of ground wires: first, the losses due to corona and skin effect experienced by that part of the energy transferred to the ground wires; and second, the superposition of high-frequency oscillations caused by successive reflections between towers. If the grounding resistances of the towers are low, then these oscillations are not perceptible, but for high tower resistances they become noticeable in the waves on the line conductors, by mutual induction from the ground wires. In the attenuation tests described by McEachron, Hemstreet, and Rudge,[6] it was found that the attenuation of positive surges was not affected by the presence of ground wires, but that negative surges attenuated less rapidly. They show, as a result of this decreased attenuation, the interesting possibility, in the case of inefficient ground wires, of a surge originating out on the line and arriving at the station with a greater amplitude than it would have had if there were no ground wires, although the surge was reduced by the presence of ground wires at the point of origin.

Introduction of Extra Ground Wires. Some power systems follow a policy of introducing extra ground wires over a short section (from a few hundred feet to a mile or more) adjacent to the station. The benefit derived from these extra ground wires is material for surges originating on the same section. However, the decrease in effective surge impedance due to the introduction of extra ground wires is small, and to reduce an incoming surge by more than a few per cent requires several advantageously placed ground wires. This is the so-called "second effect" of a ground wire, Fig. 4b, and is computed according to Case III, Appendix I. The extra ground wires should extend out from the station sufficiently far, so that surges originating beyond them will have been reduced below the dangerous value by attenuation. High potential lightning waves attenuate very rapidly, approximately to half value in one or two miles, so that the extra ground wires should extend far enough beyond the station to take requisite advantage of such attenuation.

Effect on Telephone Interference. A completely transposed three-phase transmission line with isolated neutral will not induce voltages in an adjacent telephone circuit. But if the neutral is grounded, zero phase sequence currents and the third harmonic current and its multiples are permitted to flow in phase in the line conductors and return through the ground connections, and these currents induce voltages in the neighboring telephone circuit. If there are ground wires present, the currents induced in them by the third harmonic line currents affect the telephone circuit. The amount may be computed by means of Equations (7), Appendix I. Let conductors $(1, 2, \ldots m)$ be ground wires and conductors $(m+1, \ldots, n)$ the line wires. Then if resistance is negligible there must be

$$\phi_1 = \phi_2 = \ldots = \phi_m = 0$$

so that, upon rearrangement, the first m equations of Equation (7) became

$$\left.\begin{aligned} L_{11} i_1 + \ldots\ldots + L_{m1} i_m &= -[L_{(m+1)1} i_{(m+1)} + \ldots\ldots + L_{n1} i_n] \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ L_{1m} i_1 + \ldots\ldots + L_{mm} i_m &= -[L_{(m+1)m} i_{(m+1)} + \ldots\ldots + L_{nm} i_n] \end{aligned}\right\}$$

from which the m ground wire currents may be determined in terms of the $(n-m)$ known line currents. Having found the ground wire currents, the voltage induced in a telephone circuit x is

$$e_x = -\frac{\partial \phi_x}{\partial t} = -\frac{\partial}{\partial t}[L_{1x} i_1 + \ldots\ldots + L_{nx} i_n]$$

This result is not limited to a transposed line, nor to the third harmonic currents, but is general within the limitations of no losses in the conductors. However, the inductance coefficients cannot be calculated on the basis of the zero equipotential surface being at the ground surface. It is usually several thousand feet below the surface, depending upon soil conditions.

As a specific example, consider a completely transposed three-phase line with two ground wires. Then

$$L_{11} i_1 + L_{21} i_2 = -(L_{31} i_3 + L_{41} i_4 + L_{51} i_5)$$
$$L_{12} i_1 + L_{22} i_2 = -(L_{32} i_3 + L_{42} i_4 + L_{52} i_5)$$

But for a completely transposed line

$L_{31} = L_{41} = L_{51}$

$L_{32} = L_{42} = L_{52}$

$I = i_3 + i_4 + i_5 =$ *total 3rd harmonic current in line wires.*

Then

$$i_1 = \frac{L_{23} L_{12} - L_{22} L_{13}}{L_{11} L_{22} - L_{12}^2} I$$

$$i_2 = \frac{L_{12} L_{13} - L_{11} L_{23}}{L_{11} L_{22} - L_{12}^2} I$$

and the voltage induced in the telephone circuit is

$$e_x = -\frac{\partial}{\partial t}[L_{1x} i_1 + L_{2x} i_2 + L_{3x} I]$$

$$= \left[\frac{L_{1x}(L_{22}L_{13} - L_{12}L_{23}) + L_{2x}(L_{11}L_{23} - L_{12}L_{13})}{(L_{11}L_{22} - L_{12}^2)} - L_{3x} \right] \frac{\partial I}{\partial t}$$

Zero Phase Sequence Reactance. The zero phase sequence reactance of a transmission line is of importance in calculations of system stability[10] and ground fault currents. This reactance is considerably affected by the presence of ground wires. Its determination is exactly the same kind of a problem as given in the previous section on telephone interference. Numbering the first m conductors as ground wires and the remaining $(n - m)$ conductors as line wires, there is

$$\phi_1 = \phi_2 = \ldots\ldots\ldots\ldots = \phi_m = 0$$

since the ground wires are at zero potential. For a completely transposed line, the zero phase sequence currents are

$$i_{(m+1)} = \ldots\ldots\ldots\ldots = i_n = i_o$$

and therefore

$$\left.\begin{array}{l} L_{11}i_1 + \ldots + L_{m1}i_m = -[L_{(m+1)1} + \ldots + L_{n1}]\,i_o \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ L_{1m}i_1 + \ldots + L_{mm}i_m = -[L_{(m+1)m} + \ldots + L_{nm}]\,i_o \end{array}\right\}$$

From these m simultaneous equations the m ground wire currents $(i_1 \ldots i_m)$ may be found. Then the flux linkages of the line wires are

$$\left.\begin{array}{l} \phi_{(m+1)} = L_{1(m+1)}i_1 + \ldots\ldots + L_{n(m+1)}i_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \phi_n = L_{1n}i_1 + \ldots\ldots + L_{nn}i_n \end{array}\right\}$$

and the zero phase sequence reactance is then defined as

$$X_o = 2\pi f \quad L_o = 2\pi f \left[\frac{\phi_{(m+1)} + \ldots\ldots + \phi_n}{(n-m)\,i_o} \right]$$

Effect on Corona. The presence of ground wires may influence the gradients on the line conductors due to the normal system voltage, and may thereby change the critical disruptive corona voltage and the corresponding losses. Peek's equation for the corona loss[4] per mile per wire is

$$P = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{s}}\,(e - e_o)^2\,10^{-5}\ \text{kw.}$$

and the critical disruptive corona gradient is

$$g_o = 53.4\,m_o\,\delta = \frac{2Q}{r}$$

where m_o is the roughness factor, δ is the air density factor, and Q is the charge on the conductor of radius r.

Now in the case of n wires of which the ground wires are numbered from No. 1 to No. m inclusive, the system of charges may be solved for from Equation (1), Appendix I in terms of the potentials on the conductors. Since the potentials on the ground wires are all zero, there is

$$\left\{\begin{array}{l} 0 = p_{11}Q_1 + \ldots\ldots\ldots + p_{n1}Q_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0 = p_{1m}Q_1 + \ldots\ldots\ldots + p_{nm}Q_n \\ V_{(m+1)} = p_{1(m+1)}Q_1 + \ldots\ldots + p_{n(m+1)}Q_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ V_n = p_{1n}Q_1 + \ldots\ldots\ldots + p_{nn}Q_n \end{array}\right.$$

The voltages to ground $(V_{(m+1)} \ldots\ldots V_n)$ for a balanced polyphase circuit have the same amplitude V, so that the solution for any Q in the above n simultaneous equations ultimately reduces to

$$Q = V \,.\, \phi\,(p)$$

where $\phi\,(p)$ is the function of the $(p$'s$)$ occurring in the solution. The critical disruptive corona voltage is then that voltage for which

$$g_o = 53.4\,m_o\,\delta = \frac{2Q}{r} = 2V\frac{\phi\,(p)}{r}, \text{ or}$$

$$V = 26.7\,m_o\,\delta\,\frac{r}{\phi\,(p)} = e_o$$

The factor $\sqrt{r/s}$ appearing in the power equation is of empirical origin. In the absence of experimental data as to the best interpretation to place on this factor in the case of more than two wires, it is advisable to use s as the distance to the nearest adjacent conductor.

As an example, consider a three-phase circuit with one ground wire, as in the first arrangement of Table I. Suppose that the line and ground wires are all 0.75 in. diameter, with 10 ft. spacing, and the line conductors 50 ft. above the ground level. Then it is easily verified that

$p_{11} = 16.50$	$p_{12} = 4.06$	$p_{23} = 4.61$
$p_{22} = 16.13$	$p_{13} = 4.80$	$p_{24} = 3.26$
$p_{33} = 16.13$	$p_{14} = 4.06$	$p_{34} = 4.61$
$p_{44} = 16.13$		

and the solution of the four simultaneous equations gives for the charge on the middle line conductors

$$Q_3 = 0.0883\,V$$

Then the critical disruptive corona voltage is

$$V = e_o = 26.7\,m_o\,\delta\,\frac{.375}{0895} = 112\,m_o\,\delta\ \text{kv.}$$

Had the ground wire been neglected it would have been

$$e_o = 113.5\,m_o\,\delta$$

Incidentally, if only adjacent line conductors are taken

$$e_o = 114\,m_o$$

Of course one should not generalize from this single set of calculations, but every important installation should be calculated completely.

Appendix I

Maxwell's Electrostatic Coefficients

Consider a system of n conductors of any shape and arrangement, fixed in position and uncharged. If a

unit charge be placed on conductor x suppose that the potentials acquired by the conductors are $(p_{x1}, p_{x2}, \ldots, p_{xx}, \ldots, p_{xn})$. Had a charge Q_x been placed on conductor x, instead of unit charge, the potentials would have been Q_x times as large, or $(Q_x p_{x1}, \ldots, Q_x p_{xn})$. In this notation the first subscript denotes the conductor on which the charge was placed (in this case x), and the second subscript the conductor taking that potential. It follows by the principle of superposition that the effect of simultaneous charges $(Q_1, Q_2, \ldots, Q_n)$ on all the n conductors is to give rise to the system of potentials

$$\left.\begin{aligned} V_1 &= p_{11} Q_1 + p_{21} Q_2 + \ldots\ldots\ldots + p_{n1} Q_n \\ V_2 &= p_{12} Q_1 + p_{22} Q_2 + \ldots\ldots\ldots + p_{n2} Q_n \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ V_n &= p_{1n} Q_1 + p_{2n} Q_2 + \ldots\ldots\ldots + p_{nn} Q_n \end{aligned}\right\} \quad (1)$$

The above equations give the potentials in terms of the charges and the coefficients p. These linear coefficients of proportionality depend only on the geometrical properties of the conductors, as their size, shape, and position. In a few simple cases, such as parallel cylinders, they can be calculated, but in most cases must be determined experimentally.

Solving Equation (1) for the charge Q_x on conductor x, there is

$$Q_x = K_{x1} V_1 + K_{x2} V_2 + K_{x3} V_3 + \ldots\ldots + K_{xn} V_n \quad (2)$$

where $D K_{xy}$ is the minor of which p_{xy} is the co-factor in the expansion of

$$D = \begin{vmatrix} p_{11} & p_{21} & \ldots\ldots & p_{n1} \\ p_{12} & p_{22} & \ldots\ldots & p_{n2} \\ \ldots & \ldots & \ldots & \ldots \\ p_{1n} & p_{2n} & \ldots\ldots & p_{nn} \end{vmatrix} \quad (3)$$

To show that all of the potential coefficients p are positive, suppose that $Q_1 = +1$ and $Q_2 = Q_3 = \ldots = Q_n = 0$. Then V_1 is the greatest potential in the field, and the potentials of the other conductors must lie intermediate between V_1 and zero, hence all positive. If $V_1 = +1$ and $V_2 = V_3 = \ldots\ldots = V_n = 0$, then it is evident that $K_{11} = Q_{11}$ is positive and K_{12}, K_{23}, etc., are all negative, since the lines from Q_{11} must terminate either at infinity or on the other conductors. Therefore K_{xy} is negative if $x \neq y$. By means of Green's reciprocal theorem (see "Electricity and Magnetism" by J. H. Jeans, p. 92) it may be shown that $p_{xy} = p_{yx}$ and therefore $K_{xy} = K_{yx}$. Recapitulating,

$$\left.\begin{aligned} &p_{xy} = p_{yx} \text{ and } K_{xy} = K_{yx} \\ &p_{xy}, K_{xx} \text{ are positive} \\ &K_{xy} \text{ is negative if } x \neq y \end{aligned}\right\} \quad (4)$$

Now consider a system of parallel cylindrical conductors of sufficient length so that end effects are negligible, and whose spacings are large compared to their radii. If these conductors are over a zero potential plane, as shown in Fig. 15, then the method of images may be applied and from electrostatics there is

$$p_{xx} = 2 \log \frac{2h}{r} \times 9 \times 10^{11} \text{ daraf per cm.} \quad (5)$$

$$p_{xy} = 2 \log \frac{a}{b} \times 9 \times 10^{11} \text{ daraf per cm.} \quad (6)$$

where

r = radius of conductor x
h = height of conductor x above the zero potential plane
a = distance between x and the image of y
b = distance between x and y

Maxwell's Electromagnetic Coefficients

If in a system of n conductors carrying currents, and free from saturation effects, the flux linkages of conductor x due to its own current i_x is $(L_{xx} i_x)$ and the flux linkages of conductor y due to the current i_x is $(L_{xy} i_x)$; then by the principle of superposition the total flux linkages for all conductors are

$$\left.\begin{aligned} \phi_1 &= L_{11} i_1 + L_{21} i_2 + \ldots\ldots L_{n1} i_n \\ \phi_2 &= L_{12} i_1 + L_{22} i_2 + \ldots\ldots L_{n2} i_n \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \phi_n &= L_{1n} i_1 + L_{2n} i_2 + \ldots\ldots L_{nn} i_n \end{aligned}\right\} \quad (7)$$

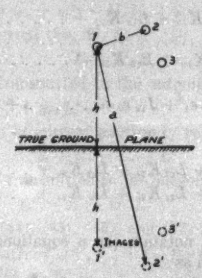

Fig. 15—System of Conductors and Their Images

From energy considerations (see "Electricity and Magnetism" by J. H. Jeans, p. 443) it may be shown that

$$L_{xy} = L_{yx} \quad (8)$$

For the system of parallel wires of Fig. 15

$$L_{xx} = \left(\frac{1}{2} + 2 \log \frac{2h}{r} \right) 10^{-9} \text{ henrys per cm.} \quad (9)$$

$$L_{xy} = \left(2 \log \frac{a}{b} \right) 10^{-9} \text{ henrys per cm.} \quad (10)$$

Self and Mutual Surge Impedances

Equations (2) and (7) apply to a system of parallel wires carrying currents above a zero potential plane, if the losses due to resistance, skin effect, leakage, and corona are ignored. If distance along the circuit is given by the coordinate x, then

$$-\frac{\partial i_1}{\partial x} = \frac{\partial Q_1}{\partial t} = K_{11}\frac{\partial e_1}{\partial t} + K_{12}\frac{\partial e_2}{\partial t} + \ldots\ldots + K_{1n}\frac{\partial e_n}{\partial t} \quad (11)$$

$$-\frac{\partial e_1}{\partial x} = \frac{\partial \phi_1}{\partial t} = L_{11}\frac{\partial i_1}{\partial t} + L_{12}\frac{\partial i_2}{\partial t} + \ldots\ldots + L_{1n}\frac{\partial i_n}{\partial t} \quad (12)$$

Differentiate (11) with respect to t and (12) with respect to x and substitute. Then

$$\begin{aligned}\frac{\partial^2 e_1}{\partial x^2} &= \frac{\partial^2}{\partial t^2}\{L_{11}(K_{11}e_1 + K_{12}e_2 + \ldots\ldots + K_{1n}e_n) \\ &\quad + L_{12}(K_{21}e_1 + K_{22}e_2 + \ldots\ldots + K_{2n}e_n) \\ &\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\quad + L_{1n}(K_{n1}e_1 + K_{n2}e_2 + \ldots\ldots + K_{nn}e_n)\} \\ &= \frac{\partial^2}{\partial t^2}\{(L_{11}K_{11} + L_{12}K_{21} + \ldots\ldots + L_{1n}K_{n1})e_1 \\ &\quad + (L_{11}K_{12} + L_{12}K_{22} + \ldots\ldots + L_{1n}K_{n2})e_2 \\ &\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\quad + (L_{11}K_{1n} + L_{12}K_{2n} + \ldots\ldots + L_{1n}K_{nn})e_n\} \\ &= \frac{\partial^2}{\partial t^2}\{J_{11}e_1 + J_{12}e_2 + \ldots\ldots + J_{1n}e_n\} \quad (13)\end{aligned}$$

where

$$\begin{aligned}J_{rs} &= L_{r1}K_{1s} + L_{r2}K_{2s} + L_{r3}K_{3s} + \ldots\ldots + L_{rn}K_{ns} \\ &= L_{1r}K_{1s} + L_{2r}K_{2s} + L_{3r}K_{3s} + \ldots\ldots + L_{nr}K_{ns}\end{aligned} \quad (14)$$

In operational notation the n equations of type (13) may be written as

$$\left.\begin{aligned}0 &= A_{11}e_1 + J_{12}e_2 + \ldots\ldots + J_{1n}e_n \\ 0 &= J_{21}e_1 + A_{22}e_2 + \ldots\ldots + J_{2n}e_n \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ 0 &= J_{n1}e_1 + J_{n2}e_2 + \ldots\ldots + A_{nn}e_n\end{aligned}\right\} \quad (15)$$

where the operator

$$A_{rr} = \left\{J_{rr} - \frac{\partial^2}{\partial x^2}\int\int dt\right\} \quad (16)$$

In order that (15) may have a solution other than zero there must be

$$\begin{vmatrix} A_{11} & J_{12} & \ldots\ldots & J_{1n} \\ J_{21} & A_{22} & \ldots\ldots & J_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ J_{n1} & J_{n2} & \ldots\ldots & A_{nn}\end{vmatrix} e = 0 \quad (17)$$

Since this determinate is of order n, its expansion yields a polynomial of degree n in the operator

$$\beta = \frac{\partial^2}{\partial x^2}\int\int dt \quad (18)$$

If the coefficients are designated by a's then

$$(a_n\beta^n + a_{n-1}\beta^{n-1} + \ldots\ldots + a_1\beta + a_0)\,e = 0 \quad (19)$$

Now if e is assumed to be a wave, it is

$$e = f(x + vt) \quad (20)$$

and substituting (20) in (19) there is

$$a_n v^{-2n} + a_{n-1}v^{-2(n-1)} + \ldots\ldots + a_1 v^{-2} + a_0 = 0 \quad (21)$$

which is a polynomial of degree n in (v^{-2}), and therefore there are $2n$ values for the velocity of propagation (n positive and n negative) which satisfy the conditions for wave motion. If

$$B_{rr} = (J_{rr} - v^{-2}) \quad (22)$$

then the n values of (v^{-2}) are given by the determinate

$$\begin{vmatrix} B_{11} & J_{12} & \ldots\ldots & J_{1n} \\ J_{21} & B_{22} & \ldots\ldots & J_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ J_{n1} & J_{n2} & \ldots\ldots & B_{nn}\end{vmatrix} = 0 \quad (23)$$

These considerations show that, in general, there can exist simultaneously on each wire of n paralleled wires, n pairs of waves of different velocities of propagation, $(v_1, v_2, \ldots, v_n)$ and each pair consists of a forward and reverse wave. Therefore

$$\begin{aligned}e_1 &= f_{11}(x - v_1 t) - F_{11}(x + v_1 t) + \ldots\ldots \\ &\quad + f_{1n}(x - v_n t) + F_{1n}(x + v_n t)\end{aligned} \quad (24)$$

The current, from Equations (11) and (24), is

$$\begin{aligned}i_1 &= \frac{-\partial}{\partial t}\int(K_{11}e_1 + K_{12}e_2 + \ldots\ldots + K_{1n}e_n)\,dx \\ &= K_{11}\sum v_r(f_{1r} - F_{1r}) + K_{12}\sum v_r(f_{2r} - F_{r2} + \ldots\ldots \\ &\quad + K_{1n}\sum v_r(f_{nr} - F_{nr})\end{aligned} \quad (25)$$

where the summations include all of the waves in the expressions such as (24), for the potentials.

For traveling waves due to lightning, the transient skin effect is so high that the current is confined to a thin skin at the periphery of the conductor. Consequently there is no internal magnetic field, and the (1/2) factor in Equation (9) vanishes (it is due to the internal interlinkages on the assumption of uniform current distribution throughout the cross-section of the conductor). Then Equations (9) and (10) become

$$L_{xx} = 2\log\frac{2h}{r}\times 10^{-9} = \frac{p_{xx}}{c^2}\text{ henrys per cm.} \quad (26)$$

$$L_{xy} = 2\log\frac{a}{b}\times 10^{-9} = \frac{p_{xy}}{c^2}\text{ henrys per cm.} \quad (27)$$

$$c = (3\times 10^{10})\text{ cm. per second} = \text{velocity of light} \quad (28)$$

Substituting these values in Equation (14), there is

$$J_{rs} = c^{-2}(p_{1r}K_{1s} + p_{2r}K_{2s} + \ldots\ldots + p_{nr}K_{ns}) \quad (29)$$

Referring back now to Equation (3), and remembering that $D\,K_{xy}$ is the minor, of which the cofactor is p_{xy}, in the expansion of D, it is evident that Equation (29) is that expansion if the elements of the r and s columns are identical. But in such a case a determinate vanishes. Therefore

$$J_{rs} = \begin{cases} 0 \text{ if } r \neq s \\ c^{-2} \text{ if } r = s \end{cases} \tag{30}$$

Under these conditions Equation (22) becomes

$$B_{rr} = (c^{-2} - v^{-2}) \tag{31}$$

and the determinate (23) degenerates to

$$\begin{vmatrix} B\,0 \ldots\ldots 0 \\ 0\,B \ldots\ldots 0 \\ \ldots\ldots\ldots\ldots \\ 0 \ldots\ldots B \end{vmatrix} = B^n = (c^{-2} - v^{-2})^n = 0 \tag{32}$$

Therefore $v = \pm c$, and all waves have the same velocity—that of light.

Hereby Equations (25) reduce to

$$\left.\begin{aligned} i_1 &= Y_{11}(f_1 - F_1) + Y_{12}(f_2 - F_2) + \ldots\ldots + Y_{1n}(f_n - F_n) \\ i_2 &= Y_{21}(f_1 - F_1) + Y_{22}(f_2 - F_2) + \ldots\ldots + Y_{2n}(f_n - F)_n \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ i_n &= Y_{n1}(f_1 - F_1) + Y_{n2}(f_2 - F_2) + \ldots\ldots + Y_{nn}(f_n - F_n) \end{aligned}\right\} \tag{33}$$

where

$$\left.\begin{aligned} Y_{rr} &= c\,K_{rr} = \text{self surge admittance} \\ Y_{rs} &= c\,K_{rs} = \text{mutual surge admittance} \end{aligned}\right\} \tag{34}$$

Inverting the order of solution which led to Equations (33) there are

$$\left.\begin{aligned} e_1 &= Z_{11}(g_1 - G_1) + Z_{21}(g_2 - G_2) + \ldots\ldots + Z_{n1}(g_n - G_n) \\ e_2 &= Z_{12}(g_1 - G_1) + Z_{22}(g_2 - G_2) + \ldots\ldots + Z_{n2}(g_n - G_n) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ e_n &= Z_{1n}(g_1 - G_1) + Z_{2n}(g_2 - G_2) + \ldots\ldots + Z_{nn}(g_n - G_n) \end{aligned}\right\} \tag{35}$$

where

$$\left.\begin{aligned} Z_{rr} &= \frac{p_{rr}}{c} = 60 \log\left(\frac{2h}{r}\right) = \text{self surge impedance} \\ Z_{rs} &= \frac{p_{rs}}{c} = 60 \log\left(\frac{a}{b}\right) = \text{mutual surge impedance} \end{aligned}\right\} \tag{36}$$

$$\left.\begin{aligned} g &= g\,(x - v\,t) = \text{forward current wave} \\ G &= G\,(x + v\,t) = \text{reverse current wave} \end{aligned}\right\} \tag{37}$$

Suppose that a certain group of conductors, say $(n + 1)$ to n inclusive, are carrying equal potential waves. Then it is possible, and usually convenient, to replace the effects of this group of wires by a single equivalent conductor. Let

$$f_o = f_{(m+1)} = \ldots\ldots = f_n \tag{38}$$

$$i_o = i_{(m+1)} + \ldots\ldots + i_n \tag{39}$$

$$Y_{ro} = Y_{r(m+1)} + \ldots\ldots + Y_{rn} \tag{40}$$

$$Y_{s} = Y_{(m+1)s} + \ldots\ldots + Y_{ns} \tag{41}$$

Then Equation (33) becomes

$$\left.\begin{aligned} i_1 &= Y_{11} f_1 + \ldots\ldots + Y_{1m} f_m + Y_{1o} f_o \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ i_m &= Y_{m1} f_1 + \ldots\ldots + Y_{mm} f_m + Y_{mo} f_o \\ i_o &= Y_1 f_1 + \ldots\ldots + Y_m f_m + Y_o f_o \end{aligned}\right\} \tag{42}$$

Hereby the number of equations has been reduced to $(m + 1)$.

Behavior of Waves at a Transition Point

Suppose that each of the n conductors terminates at a transition point, as shown in Fig. 3, consisting of a lumped series impedance U, a lumped admittance to ground N, and outgoing lines fed through lumped series impedances W. In the general case U, N, W are operators involving circuit constants and time derivatives and integrals. The surge impedances of the outgoing lines are designated by small z, and those of the incoming lines by large Z. When incident waves $(f_1, \ldots, f_n)$ traveling along the incoming lines reach the transition point, waves $(F_1, \ldots, F_n)$ are reflected back onto the incoming lines, and refracted waves $(f_1'', \ldots\ldots, f_n'')$ are transmitted to the outgoing wires, and currents $(I_1, \ldots, I_n)$ flow to ground through the admittances $(N_1, \ldots\ldots, N_n)$.

The total potential on the incoming line at the junction is

$$e_r + e_r' = f_r + F_r \tag{43}$$

and the total current, by Equation (33) is

$$i_r + i_r' = Y_{r1}(f_1 - F_1) + \ldots\ldots + Y_{rn}(f_n - F_n) \tag{44}$$

The potential across the admittance N_r is

$$E_r = (e_r + e_r') - U_r\,(i_r + i_r') \tag{45}$$

so that the current through N_r is

$$I_r = N_r\,E_r \tag{46}$$

The current transmitted to the outgoing line is

$$i_r'' = y_{r1} f_1'' + y_{r2} f_2'' + \ldots\ldots + y_{rn} f_n'' \tag{47}$$

The condition of current continuity requires that

$$i_r + i_r' = I_r + i_r'' \tag{48}$$

The potential transmitted to the outgoing line is

$$e_r'' = E_r - W_r\,i_r'' \tag{49}$$

Substituting (44), (45), (46), and (47) in (48), there is

$$(1 + N_r\,U_r)[Y_{r1}(f_1 - F_1) + \ldots + Y_{rn}(f_n - F_n)] - N_r(f_r + F_r) = (y_{r1} f_1'' + y_{r2} f_2'' + \ldots\ldots + y_{rn} f_n'') \tag{50}$$

Substituting (44), (45), (46), and (47), in (49), there is

$$(f_r + F_r) - U_r\,[Y_{r1}(f_1 - F_1) + \ldots\ldots + Y_{rn}(f_n - F_n)] = f_r'' + W_r\,(y_{r1} f_1'' + \ldots\ldots + y_{rn} f_n'') \tag{51}$$

Since r takes on all values from $r = 1$ to $r = n$ there are, n equations of type (50) and n equations of type (51), so that the $2\,n$ unknowns $(F_1 \ldots F_n, f_1'' \ldots\ldots f_n'')$ may be uniquely determined. A few simple examples will be worked out to show the application of Equations (50) and (51).

Case I. Find operational expressions for the reflected and transmitted waves when there is a single incoming wire and a single outgoing wire. Equations (50) and (51) then reduce to

$$(1 + N_1\,U_1)\,Y_{11}\,(f_1 - F_1) - N_1\,(f_1 + F_1) = y_{11} f_1''$$

$$- U_1\,Y_{11}\,(f_1 - F_1) + (f_1 + F_1) = f_1'' + W_1\,y_{11} f_1''$$

Solving these two equations simultaneously for the reflected wave F_1 and the refracted wave f_1'', substituting $z_{11} = 1/y_{11}$ and $Z_{11} = 1/Y_{11}$, and dropping subscripts, there is

$$F_1 = \frac{(z+W)(1+NU)+U-Z-ZN(z+W)}{(z+W)(1+NU)+U+Z+ZN(z+W)} f_1$$

$$f_1'' = \frac{2z}{(z+W)(1+NU)+U+Z+ZN(z+W)} f_1$$

For a large number of special cases to which these equations apply see *Traveling Waves Due to Lightning* by L. V. Bewley,[1] and *Shunt Resistors for Reactors* by F. H. Kierstead, H. L. Rorden, and L. V. Bewley, (A. I. E. E. TRANS., Vol. 49, July 1930).

Case II. A single wire entering a section consisting of a continuation and an adjacent isolated wire, Fig. 4c. There will be two transmitted waves, and one reflected wave. $U_1 = U_2 = W_1 = W_2 = N_1 = N_2 = 0$. Equations (50) and (51) give

$$\begin{aligned} Y_{11}(f_1 - F_1) &= y_{11} f_1'' + y_{12} f_2'' \\ 0 &= y_{21} f_1'' + y_{22} f_2'' \\ (f_1 + F_1) &= f_1'' \end{aligned}$$

Solving these three simultaneous equations and substituting impedances for admittances, there are

$$f_1'' = \frac{2z_{11}}{z_{11}+Z_{11}} f_1$$

$$f_2'' = \frac{2z_{12}}{z_{11}+Z_{11}} f_1$$

$$F_1 = \frac{z_{11}-Z_{11}}{z_{11}+Z_{11}} f_1$$

Thus the presence of the isolated wire has no effect on either the reflected wave or the directly transmitted wave.

Case III. A single wire entering a section consisting of a continuation, and an adjacent grounded wire, Fig. 4d. The transmitted wave $f_2'' = 0$ since the ground wire is at zero potential, and as there are no lumped impedances $U_1 = U_2 = W_1 = W_2 = N_1 = 0$. Then Equations (50) and (51) give

$$\begin{aligned} Y_{11}(f_1 - F_1) &= y_{11} f_1'' \\ (f_1 + F_1) &= f_1'' \end{aligned}$$

Solving these two simultaneous equations there are

$$f_1'' = \frac{2Y_{11}}{y_{11}+Y_{11}} f_1 = \frac{2(z_{11}z_{22} - z_{12}^2)}{(z_{11}z_{22} - z_{12}^2) + z_{22}Z_{11}} f_1$$

$$F_1 = \frac{Y_{11}-y_{11}}{Y_{11}+y_{11}} f_1 = \frac{(z_{11}z_{22} - z_{12}^2) - z_{22}Z_{11}}{(z_{11}z_{22} - z_{12}^2) + z_{22}Z_{11}} f_1$$

If $z_{11} = Z_{11}$, these equations become

$$f_1'' = \frac{2(z_{11}z_{22} - z_{12}^2)}{2(z_{11}z_{22} - z_{12}^2) + z_{12}^2} f_1$$

$$F_1'' = \frac{-z_{12}^2}{2z_{11}z_{22} - z_{12}^2} f_1$$

so that there is a reduction in the transmitted wave, and a corresponding negative reflection.

Case IV. One of two wires grounded through a resistor, and the other wire left open-circuited, Fig. 4e. There are no transmitted waves, since the wires have terminated. $N_1 = 1/R_1$, $U_1 = U_2 = N_2 = 0$. Then by Equation (50)

$$\begin{aligned} Y_{11}(f_1 - F_1) + Y_{12}(f_2 - F_2) - N_1(f_1 + F_1) &= 0 \\ Y_{21}(f_1 - F_1) + Y_{22}(f_2 - F_2) &= 0 \end{aligned}$$

Solving and substituting impedances for admittances there is

$$F_1 = \frac{R_1 - Z_{11}}{R_1 + Z_{11}} f_1$$

$$F_2 = f_2 - \frac{2Z_{12}}{R_1 + Z_{11}} f_1$$

If $R_1 = Z_{11}$ then

$$F_1 = 0$$

$$F_2 = f_2 - \frac{Z_{12}}{Z_{11}} f_1$$

so that there is no reflected wave in wire No. 1. Had the incident wave in wire No. 2 been due to induction from wire No. 1 it would have been

$$f_2 = \frac{Z_{12}}{Z_{11}} f_1$$

and in that case the reflected wave F_2 also vanishes.

Appendix II

GENERALIZED THEORY OF IDEAL GROUND WIRES

When a charged cloud approaches a transmission line, charges of opposite sign leak over the insulators and appear on the line and ground wires as bound charges. The density of charge at any point on a particular wire depends upon the electrostatic field gradient due to the superimposed effects of the charge on the cloud and the induced charges on the other wires. It is a simple matter to write the characteristic equations for such a system in static equilibrium, in terms of Maxwell's electrostatic coefficients defined in Appendix I.

Now when the cloud discharges according to some time function $F(t)$, the bound charges on the line wires distributed proportional to a function $f(x)$, are released and form pairs of traveling waves moving away from the region of disturbance in opposite directions. The time of cloud discharge is too short to allow any appreciable readjustment to take place by leakage of a charge of opposite sign over the high resistance of the line insulators. However, this is not the case with the ground wires, which are grounded at the towers through relatively low resistances. The present analysis is restricted to the case of ideal ground wires, that is to wires maintained at zero potential throughout their length by perfect grounding.

Consider an overhead system having m ideal ground wires and $(n - m)$ line wires. Number the ground wires from 1 to m inclusive, and the line wires from

$(m+1)$ to n inclusive. Let $p_{kr} = p_{rk}$ be the mutual coefficient between the kth and rth conductors. Before cloud discharge all conductors are at zero potential, by virtue of the charges Q_k which have leaked over the insulators. If G is the field gradient due to the cloud alone, and if h_k is the height above ground of the kth conductor, then the field equations during the conditions of static equilibrium before discharge are:

$$\left.\begin{array}{l} V_1 = 0 = G\,h_1 + p_{11}\,Q_1 + \ldots\ldots\ldots + p_{1n}\,Q_n \\ V_2 = 0 = G\,h_2 + p_{21}\,Q_1 + \ldots\ldots\ldots + p_{2n}\,Q_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ V_n = 0 = G\,h_n + p_{n1}\,Q_1 + \ldots\ldots\ldots + p_{nn}\,Q_n \end{array}\right\} \quad (1)$$

Now suppose that the cloud discharges, so that at any instant t the gradient is given by $G\,.\,F\,(t)$, where $F\,(t)$ is the functional law of cloud discharge, assumed to be uniform over the bound charge distribution $f\,(x)$.

The first term on the right is the potential due to the residual field of the cloud; the second term that due to the residual charges on the line conductors; the third term that due to the stationary charges on the ground wires, and the fourth term that due to the traveling waves. There are thus $2\,n$ unknowns: $(Q_1 \ldots Q_n)$, $(q_1' \ldots q_m')$, $(V_{m+1} \ldots V_n)$ and n equations (1) and n equations (6) for their determination. $(Q_1 \ldots Q_n)$ may be found from Equation (1) above. Substituting for the first two terms of (6) its equivalent from (1) there is

$$-V_k + p_{k1}\,q_1' + \ldots + p_{mk}\,q_m' + a\,p_{k1}\,Q_1 + \ldots + a\,p_{km}\,Q_m + b\,p_{k(m+1)}\,Q_m + \ldots + b\,p_{kn}\,Q_n \quad (7)$$

where

$$a = [\phi(x,t) + \psi(x,t) - F\,(t)] \text{ and } b = [\phi(x,t) + \psi(x,t)] \quad (8)$$

The symbolic determinate therefore is:

$Q_1 \ldots Q_m$	$Q_{m+1} \ldots Q_n$	$q_1' \ldots q_m'$	$V_{m+1} \ldots V_n$	$(k+m)$
$p_{11} \ldots p_{1m}$	$p_{1(m+1)} \ldots p_{n1}$	$0 \ldots 0$	$0 \ldots 0$	$= -G\,h_1$
.....	.....	.....	.....	.....
$p_{1n} \ldots p_{nm}$	$p_{n(m+1)} \ldots p_{nn}$	$0 \ldots 0$	$0 \ldots 0$	$= -G\,h_n$
$a\,p_{11} \ldots a\,p_{1m}$	$b\,p_{1(m+1)} \ldots b\,p_{1n}$	$p_{11} \ldots p_{1m}$	$0 \ldots 0$	$= 0$
.....	.....	.....	.....	.....
$a\,p_{m1} \ldots a\,p_{mm}$	$b\,p_{m(m+1)} \ldots b\,p_{mn}$	$p_{m1} \ldots p_{mm}$	$0 \ldots 0$	$= 0$
$a\,p_{(m+1)1} \ldots a\,p_{(m+1)m}$	$b\,p_{(m+1)(m+1)} \ldots b\,p_{(m+1)m}$	$p_{(m+1)1} \ldots p_{(m+1)m}$	$-1 \ldots 0$	$= 0$
.....	.....	.....	.....	.....
$a\,p_{n1} \ldots a\,p_{nm}$	$b\,p_{n(m+1)} \ldots b\,p_{nm}$	$p_{n1} \ldots p_{nm}$	$0 \ldots -1$	$= 0$

(1) Subtract (a) times the (q') columns from the corresponding (Q) columns.
(2) Subtract (b) times the first (n) rows from the corresponding remaining (n) rows.
(3) Add (b) times the (q) columns to the corresponding (Q) columns.

The bound charges on the line wires will be released proportional to the decrease of the gradient, so that the residual charge on the line conductor at any instant (t) is $Q_k\,F\,(t)$. Pairs of traveling potential and current waves are initiated so that for the freely developed waves (see Equation (35), Appendix I)

$$v_k = Z_{k1}\,i_1 + Z_{k2}\,i_2 + \ldots + Z_{kk}\,i_k + \ldots + Z_{kn}\,i_n \quad (2)$$

Now the shapes of traveling waves on all wires depend only on $f\,(x)$ and $F\,(t)$, and are therefore similar. If I_k is the current corresponding to instantaneous cloud discharge, then there exists a function $\phi\,(x,t)$ involving $f\,(x)$ and $F\,(t)$, such that the current wave at any time (t) and point (x) is given by

$$i_k = I_k\,.\,\phi\,(x,t) \quad (3)$$

Similarly, there is a function $\psi\,(x,t)$ associated with the reverse wave. Moreover, by a comparson of Equations (1) and (35) of Appendix I, there must be

$$p_{kk}\,Q_k = Z_{kk}\,I_k \text{ and } p_{kr}\,Q_k = Z_{kr}\,I_k \quad (4)$$

so that (2) may be written

$$v_k = (p_{k1}\,Q_1 + \ldots\ldots + p_{kk}\,Q_k + \ldots\ldots + p_{kn}\,Q_n)\,[\phi\,(x,t) + \psi\,(x,t)] \quad (5)$$

The total potential at x and t therefore is

$$V_k = h_k\,G\,.\,F\,(t) + (p_{k(m+1)}\,Q_{(m+1)} + \ldots\ldots + p_{kn}\,Q_n)\,F(t) + (p_{k1}\,q_1' + \ldots\ldots + p_{mk}\,q_m') + (p_{k1}\,Q_1 + \ldots\ldots + p_{nk}\,Q_n)\,[\phi\,(x,t) + \psi\,(x,t)] \quad (6)$$

The solution for $V_{(k+m)}$ then takes the form:

$p_{11} \ldots p_{n1}$	$0 \ldots 0$	$0 \ldots -h_1 \ldots 0$		
.....	.....	.....		
$p_{1n} \ldots p_{nn}$	$0 \ldots 0$	$0 \ldots -h_n \ldots 0$		
$0 \ldots 0$	$p_{11} \ldots p_{1m}$	$0 \ldots b\,h_1 \ldots 0$		
.....	.....	.....		
$0 \ldots 0$	$p_{m1} \ldots p_{mn}$	$0 \ldots b\,h_m \ldots 0$		
$0 \ldots 0$	$p_{(m+1)1} \ldots p_{(m+1)m}$	$(-1) \ldots b\,h_{(m+1)} \ldots 0$		
.....	.....	.....		
$0 \ldots 0$	$p_{(m+k)1} \ldots p_{(m+k)n}$	$0 \ldots b\,h_{(m+k)} \ldots 0$		
.....	.....	.....		
$0 \ldots 0$	$p_{n1} \ldots p_{nm}$	$0 \ldots b\,h_n \ldots (-1)$		
			0	
			.....	
Otherwise the same as above.			0	
			0	
			.....	
			0	
			0	
			.....	
			-1	
			.....	
			0	

Let R be a matrix consisting of the first $(n+m)$ rows and columns, and the first $(m+n)$ elements of row $(n+m+k)$ in the above determinate.

Let R_r be that determinate of order $(n+m)$ formed by canceling row (r) in matrix R. Then the expansion of the determinate for $V_{(m+k)}$ in terms of its minors takes the form

$$V_{(m+k)} = \frac{G}{R_{(m+n+1)}} (b\, h_{(m+k)}\, R_{(m+n+1)} + b\, h_m\, R_{(m+n)} + \ldots$$
$$+ b\, h_1\, R_{(n+1)} - h_n\, R_n - \ldots\ldots - h_1\, R_1) \qquad (10)$$

but

$$R_1 = R_2 = \ldots\ldots = R_n = 0 \qquad (11)$$

as may be seen by expanding them in terms of the minors for which the cofactors are the lower right hand elements. Since these elements are in an isolated block, (surrounded by zeros) m elements wide by $(m + 1)$ deep, a point is eventually reached in the expansion, for which the remaining minors contain a solid row of zeros, and therefore vanish.

So Equation (10) finally reduces to the determinate

$$V_{(m+k)} = b\, G \frac{\begin{vmatrix} p_{11} \ldots\ldots p_{m1} & h_1 \\ \ldots\ldots\ldots\ldots & \\ p_{1m} \ldots\ldots p_{mm} & h_m \\ p_{1(m+k)} \ldots p_{m(m+k)} & h_{(m+k)} \end{vmatrix}}{\begin{vmatrix} p_{11} \ldots\ldots p_{m1} \\ \ldots\ldots\ldots\ldots \\ p_{1m} \ldots\ldots p_{mm} \end{vmatrix}} = b\, G\, D \qquad (12)$$

Thus showing that the potential appearing on any line wire $(m + k)$ is independent of the other line wires, and involves only the heights and potential coefficients of that conductor and the ground wires. The potential is directly proportional to the function.

$$b = \phi\,(x, t) + \psi\,(x, t) \qquad (13)$$

related to the distribution of bound charge $f\,(x)$ and the law of cloud discharge $F\,(t)$.

If there were no ground wires the potential would be (putting $h_1 = \ldots\ldots = h_m = 0$ in Equation (12))

$$V_{(m+k)}' = b\, G\, h_{(m+k)} \qquad (14)$$

The protective ratio therefore is

$$P\,.\,R = \frac{V_{(m+k)}}{V_{(m+k)}'} = \frac{D}{h_{(m+k)}} \qquad (15)$$

from which specific cases may be computed. This solution is identical to that given by the conventional method of calculation based on the assumption of purely static conditions Thus the protective ratio is independent of the law of cloud discharge, of the initial distribution of bound charge, and of the traveling waves which are formed; and depends only on the geometric configuration of the system, as specified by Maxwell's coefficients and the heights of the conductors above ground.

Appendix III

Periodic Resistance Grounding of Ground Wires

Consider a transmission line consisting of any number of line and ground wires struck at the tower by a lightning bolt. Suppose that the ground wires are grounded at regular intervals through resistances R (due to the tower and its footing). Following the procedure of Fortescue, Atherton, and Cox, in their paper *Theoretical and Field Investigations of Lightning*, (A. I. E. E. Trans., Vol. 48, April 1929, p. 449), the lightning bolt will be represented by a surge impedance Z_o. Moreover, since all of the ground wires are connected together at the tower, they will carry identical potential waves, and therefore a single equivalent ground wire may be substituted as shown by Equations (38) to (42) in Appendix I. However, since the line wires are not constrained to carry equal potential waves, no such equivalent conductor can be substituted for them. In fact, each line wire acts quite independently of the others. Accordingly, then, call the equivalent ground wire, conductor 1 and number the line wires from 2 to n inclusive. The circuit is indicated in Fig. 8. From the point of strike let the towers be numbered 1, 2, 3, etc. Traveling waves arriving at the towers are reflected and refracted, so that there is a great complexity of superimposed waves involved. The problem of their determination is divided into two parts, first, the derivation of the reflection and refraction operators, and second, the development of a scheme for keeping track of the multiplicity of them.

Referring to Equations (50) and (51) of Appendix I and substituting

$U_r = W_r = 0,\ N_2 = \ldots\ldots = N_n = 0,\ N_1 = 1/R,\ y_{rs} = Y_{rs}$

there results after some obvious rearrangement, the system of equations

$$\left.\begin{array}{l} Y_{11}\,(f_1 - F_1 - f_1'') + \ldots\ldots + Y_{1n}\,(f_n - F_n - f_n'') \\ \qquad\qquad = N_1\,(f_1 + F_1) \\ Y_{21}(f_1 - F_1 - f_1'') + \ldots\ldots + Y_{2n}(f_n - F_n - f_n'') = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Y_{n1}(f_1 - F_1 - f_1'') + \ldots\ldots + Y_{nn}(f_n - F_n - f_n'') = 0 \\ \qquad\qquad f_1 + F_1 = f_1'' \\ \qquad\qquad \ldots\ldots\ldots \\ \qquad\qquad f_n + F_n = f_n'' \end{array}\right\} \qquad (1)$$

Substituting f_r'' from the last n equations into the first n equations and solving, there is

$$2\,F_r = \frac{M_{1r}}{D} N_1\,(f_1 + F_1) = \frac{Z_{r1}}{R}\,(f_1 + F_1) \qquad (2)$$

where M_{1r} is the minor whose cofactor is Y_{1r} in the determinate D formed by the arrangement of Y's in the first n equations of (1). Then by a comparison of Equations (33) and (35) in Appendix I it is evident that $M_{1r}/D = Z_{r1}$. When $(r = 1)$ there is

$$F_1 = \frac{-\,Z_{11}}{2\,R + Z_{11}}\, f_1 = a\, f_1 \qquad (3)$$

and substituting this value back into (2)

$$F_r = \frac{-\,Z_{r1}}{2\,R + Z_{11}}\, f_1 = c\, f_1 \qquad (4)$$

and then from the last n equations of (1)

$$f_1'' = f_1 + F_1 = \frac{2\,R}{2\,R + Z_{11}}\, f_1 = b\, f_1 \qquad (5)$$

$$f_r'' = f_r + F_r = f_r + c\,f_1 \qquad (6)$$

Equations (3), (4), (5), and (6) specify the behavior of waves at all towers except the one struck. These equations are the same (except for a difference in notation) as those derived by Cox and Slepian, although their solution was restricted to the case of a single ground wire and a single line wire. Notice from these equations that the ground wire and each line wire behave as though the other line wires were not present.

At Tower No. 1, where the bolt is assumed to strike, the initial wave impressed on the ground wire, in terms of the lightning voltage e_o, is

$$f_1 = e_1 = \left(\frac{2\,R\,Z_{11}}{Z_{11}\,(R + Z_o) + 2\,R\,Z_o} \right) e_o \qquad (7)$$

and the voltage induced on the other wires is

$$f_r = e_r = \left(\frac{Z_{1r}}{Z_{11}} \right) e_1 \qquad (8)$$

Waves arriving at Tower No. 1 from along the line and ground wires (due to reflections from other towers) are reflected therefrom as

$$F_1 = \left\{ \frac{2\,Z_o\,R - Z_{11}\,(Z_o + R)}{2\,Z_o\,R + Z_{11}\,(Z_o + R)} \right\} f_1 = \alpha\,f_1 \qquad (9)$$

$$F_r = f_r - \left\{ \frac{2\,Z_{1r}\,(R + Z_o)}{2\,Z_o\,R + Z_{11}\,(Z_o + R)} \right\} f_1 = f_r + \beta\,f_1$$

The potential wave passing from the lightning bolt to the ground wire is e_1. This initial wave will eventually give rise to such a multiplicity of reflections and refractions, that it is hopeless to keep track of them without some graphical or tabular system. The author's first attempts in this direction were with rectangular lattices, which proved unsatisfactory. Mr. S. T. Maunder suggested a diamond shaped lattice which not only avoids interference, but furnishes a uniquely beautiful means for observing the time of arrival of all waves at all junctions, and shows at a glance the direction of propagation of the waves. The schemes shown in Figs. 9 and 10 show the application of this lattice to the ground wire problem. In this case, since all reflection and refraction operators are constants, the waves are reflected and transmitted without change of shape. But the lattice is also applicable, in conjunction with operational calculus, to junctions having inductances and capacitances. Indeed, in studying repeated reflections from terminal apparatus subjected to artificial lightning waves, the author has found these lattices indispensable.

Referring to Fig. 9, the vertexes of the lattice represent the towers, or resistance junctions, and fall at their appropriate intervals. At the left is the time scale measured in the units of time T required for a wave to travel the distance of one span. In this scheme it is evident that waves can only "slide down hill." The lightning bolt strikes at Tower No. 1 causing a voltage e_1, Equation (7), to appear on the ground wires, and a voltage e_r, Equation (8), to appear on the line wires. These waves reach Tower No. 2 where they reflect and refract in accordance with Equations (3), (4), (5), and (6). The transmitted waves pass on to Towers No. 2, 3, etc., at each of which there is a reflection.

In general, at any Tower k and time $m\,T$, (where m is integer), a series of waves are arriving simultaneously from the right and left. For each wave on the ground wire striking a junction (other than Tower No. 1) a fraction a is reflected back towards its origin and a fraction b passes through as a transmitted wave. At Tower No. 1 a fraction α of a returning wave is reflected. On the line wires the situation is slightly more complicated. Equation (4) shows that the reflected wave on the line wire depends only on the incident wave on the ground wire, by a fraction c, and Equation (6) shows that the transmitted wave depends on the incident waves of both the line and ground wire. Therefore the lattice for the line wire cannot be constructed until after that for the ground wire is completed.

Thus these lattices adequately account for the vicissitudes of the numerous waves, and the actual distribution as function of time at any tower, may be found by superimposing the various waves displaced by the time lag of the instants of arrival as shown by the time scale to the left of the lattice. Moreover, the superposition is valid for waves of any shape. If all tower resistances are different the method can still be applied, except that the operators a, b, c would be different at every junction. The equations for the potentials at the first few towers, as obtained from the lattice, are given at the foot of Figs. 9 and 10. In this notation $e_1\,(t - 3\,T)$ means a wave having the same shape as the initial wave $e_1 = e_1\,(t)$, but arriving $3\,T$ later, that is, displaced by that amount on the time axis of superposition.

In Fig. 11 are shown, as function of the tower resistance, the potentials on the ground and line wires, and their difference, for the first three towers. Also there are plotted in this figure the potentials at the first three towers as functions of time for a tower resistance of 100 ohms, and an infinite rectangular initial wave. It is evident that the lower the tower resistance, the greater the reduction in all voltages, and the fewer the number of spans included by the danger zone.

The above results apply to the case of a lightning stroke at the tower. When the direct hit strikes at mid-span the situation is more complicated to analyze, but less severe in effect at the tower. On the assumption that there is more than one ground wire present, but that only one ground wire is hit, the waves at mid-span may be computed by Equations (7), (8), and (9) making $R = \infty$, and using the surge impedance of the single ground wire hit, instead of the "equivalent" ground wire. At the tower a new set of equations similar to Equation (1), must be solved. The lattice is also

somewhat different, since the wave travel between the lightning bolt and the adjacent tower is only $T/2$.

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Discussion

J. E. Clem: There are two points in Mr. Bewley's paper which I would like to bring out. Mr. Bewley discusses the use of a ground wire for preventing direct hits on the line and points out that the spacing from the line wires up to the direct hit wire must be in the order of 35 ft. It has been learned during the past few years from the general study of lightning that the ground wires have given a definite improvement in operating records through the reduction in the number of outages. Evidence obtained during the past year apparently has indicated that a large number of outages from lightning is caused by direct hits. These two facts taken together indicate that the ground wire as ordinarily installed can operate in many cases as a direct hit wire and that it may not be necessary in practise to use as great a separation as indicated.

TABLE I

No. ground wires		Surge impedance $\frac{138.2}{3}x$	Protective ratio: Induced potential	Protective ratio: Traveling wave
0	3 ϕ	$A + 2B$	1	1
	1 ϕ	$3A$		
1	3 ϕ	$A + 2B - \frac{3B_{wg}^2}{A_g}$	$1 - \frac{h_g}{h}\frac{B_{wg}}{A_g}$	$\frac{2}{1+\frac{Z_a}{Z_n}}$
	1 ϕ	$3\left(A - \frac{B_{wg}^2}{A_g}\right)$	"	"
2	3 ϕ	$A + 2B - \frac{6B_{wg}^2}{(A_g + B_g)}$	$1 - \frac{h_g}{h}\frac{2B_{wg}}{(A_g + B_g)}$	"
	1 ϕ	$3\left(A - \frac{2B_{wg}^2}{(A_g + B_g)}\right)$	"	"
3	3 ϕ	$A + 2B - \frac{9B_{wg}^2}{(A_g + 2B_g)}$	$1 - \frac{h_g}{h}\frac{3B_{wg}}{(A_g + 2B_g)}$	"
	1 ϕ	$3\left(A - \frac{3B_{wg}^2}{A_g - 2B_g}\right)$	"	"
4	3 ϕ	$A + 2B - \frac{12B_{wg}^2}{(A_g + 3B_g)}$	$1 - \frac{h_g}{h}\frac{4B_{wg}}{(A_g + 3B_g)}$	"
	1 ϕ	$3\left(A - \frac{4B_{wg}^2}{A_g + 3B_g}\right)$	"	"

3 ϕ means calculations are made for the three line wires together, which gives the net surge impedance or the average protective ratio.
1 ϕ means calculations are made for one line wire only.

TABLE II

No. ground wires	Surge impedance 3 ϕ	Surge impedance 1 ϕ	Protective ratio induced potential 3 ϕ	Protective ratio induced potential 1 ϕ	Protective ratio traveling wave 3 ϕ	Protective ratio traveling wave 1 ϕ
0	212.7	467.7	1	1	1	1
0–1	193.8	442.4	0.788	0.696	0.954	0.973
0–2	183.7	432.3	0.584	0.540	0.927	0.961
0–3	177.5	427.8	0.474	0.440	0.910	0.956
0–4	174.6	426.0	0.400	0.372	0.902	0.954
1–2*			0.791		0.974	
1–3			0.642		0.956	
1–4			0.542		0.948	
2–3			0.812		0.983	
2–4			0.753		0.975	
3–4			0.844		0.992	

3 ϕ calculations based on 3 line wires net surge impedance average protective ratio.
1 ϕ calculations made on middle wire.
*Indicates passing from section having one ground wire into section having two ground wires, etc.

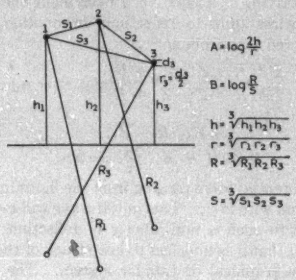

FIG. 1

The practise of increasing the number of ground wires over the section of line adjacent to the station has been extensively advocated during the past few years. The additional ground wires near the station improves the protection on the basis of induced strokes and direct hits. It is surprising, however, what little reduction there will occur in the voltage of a traveling wave when passing into a section of a line having a greater number of ground wires.

I have made calculations for a horizontally spaced line with 1, 2, 3 and 4 ground wires such as illustrated in Figs. 6a, b, c, d. The calculations were made in accordance with the formulas in Table I, and the terms used are defined in Figs. 1, 2, 3, 4 and 5. The derivation of these formulas will be given as an appendix. The results of the calculations are shown in Table II. The calculations were made for three conductors together (3 ϕ) which method gives the average protective ratio, and for a single wire (1 ϕ). For the single wire calculations the center wire was chosen. The protective ratio for induced potentials given in the tabulation is the analytical value and it should be remembered that the protective ratio actually obtained in practise is somewhat better. The protective ratio for a traveling wave is defined as a ratio of the transmitted wave to the incident wave.

In no case is the voltage of a traveling wave reduced more than 10 per cent. But for a case where a line with no ground wires has three or four ground wires added near the station the reduc-

tion is nearly 10 per cent. If, as in the usual case the line already has two wires and two more are added the traveling wave is reduced 2½ per cent and the protection from induced strokes improved 25 per cent. In the laboratory tests to determine the protection of ground wires it was found that the reduction in voltage was considerably more than the calculated value but not

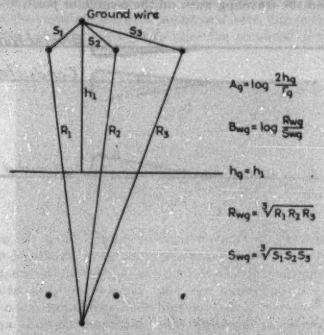

FIG. 2

more than twice as much. If this same condition holds for traveling waves we might expect a reduction of 5 per cent in voltage when passing from a section of line with two ground wires to a section having four ground wires, or of 10 per cent when passing from a section of line having one ground wire to a section having three ground wires. It appears from this that the use

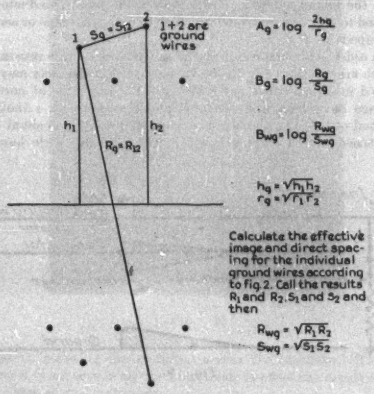

FIG. 3

of additional ground wires adjacent to the station cannot be justified on the basis of the voltage reduction occurring in traveling waves when passing the junction. Further study may, however, indicate that the use of these extra ground wires adjacent to the station is justifiable from the benefits derived through the reduction of direct hits and induced potentials.

APPENDIX

It has been shown elsewhere[1] that the total capacitance to ground in microfarads per mile of three wires in parallel is, approximately

$$C_3 = 3\,C_a = 3\,\frac{0.03883}{A + 2\,B}$$

For definition of A and B see Fig. 1.

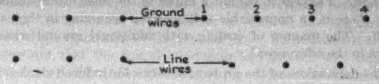

FIG. 4 FIG. 5

Calculate the average image and direct spacing of each ground wire to the line wires according to Fig. 2. Call the results R_1, R_2 etc., S_1, S_2, etc.

$$R_{wg} = \sqrt[2]{R_1 R_2 R_3} \qquad R_{wg} = \sqrt[4]{R_1 R_2 R_3 R_4}$$

$$S_{wg} = \sqrt[2]{S_1 S_2 S_3} \qquad S_{wg} = \sqrt[4]{S_1 S_2 S_3 S_4}$$

The average spacing of the ground wire to themselves is calculated according to Fig. 1

$$R_g = \sqrt[6]{R_{12} R_{13} R_{14} R_{23} R_{34} R_{34}}$$

$$S_g = \sqrt[6]{S_{12} S_{13} S_{14} S_{23} S_{24} S_{34}}$$

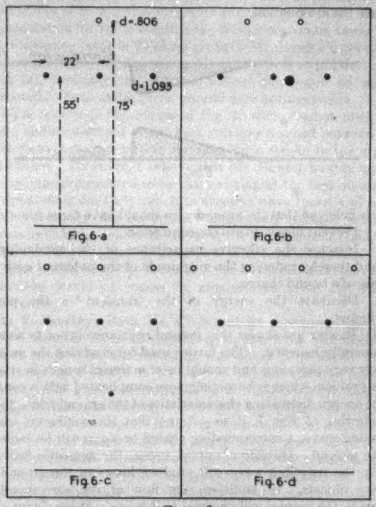

FIG. 6

In the same manner it can be shown that the inductance in millihenrys per mile of three parallel conductors against ground is, approximately

$$L_3 = \frac{0.7411}{3}\,(A + 2\,B)$$

1. *Arcing Grounds*, J. E. Clem. A. I. E. E. TRANS., Vol 49, July, 1930, p. 987.

In this expression the inductance within the conductor material is neglected.

The surge impedance is defined as

$$Z = \sqrt{\frac{L}{C}} = \frac{138.2}{3}\,(A + 2\,B)$$

This expression assumes that the so called neutral plane is at a distance h below the conductor and makes no allowance for any voltage drop in the earth itself. This is the conventional method. I expect to submit a paper in the near future showing that there is an appreciable voltage drop occurring in the earth itself. The manner of dealing with additional ground wires is given in the references.[1]

The derivations of the protective ratios for induced strokes has been given previously.[2]

A. O. Austin: Mr. Bewley has given a very fine mathematical treatment of the ground wire theory which at best is very complicated when reflected waves are taken into consideration. The application of the ground wire is becoming more complicated and it is hoped that Mr. Bewley may continue his analysis for other conditions.

By studying the performance of transmission lines, I came to the conclusion some years ago that the ground wire was of very material benefit in reducing potentials on the transmission line in most cases. There were, however, some cases which indicated that the presence of the ground wire was anything but beneficial in dampening out transients. It is not difficult to make comparisons in which it could be shown that the tendency of the ground would be to lower the natural period and also to reduce the attenuation.

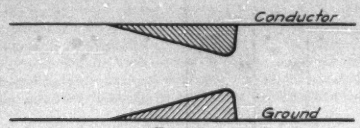

Fig. 7

It is believed that the ground wire should have three functions:

1. Prevent direct hits to the conductor.
2. Increase the effective capacitance of the conductor to ground thereby reducing the magnitude of the potential with the release of a bound charge.
3. Dissipate the energy of the transient in the power conductor.

Mr. Bewley has shown that ground resistance is not as serious as generally believed. The lattice used for studying the ground wire is very ingenious and should be of material benefit in studying a problem which is becoming more complicated with a control of the energy dissipating characteristics of the ground wire.

Referring to Fig. 7, if we assume that the conductor has a traveling wave, a corresponding charge or wave will be induced in the ground. Outside of corona losses, the magnetic field set up by the traveling wave will produce losses in the ground or nearby objects. In addition, the flow of the corresponding charge in the ground will produce $I^2 R$ losses. It is evident that if the resistance in the path of the induced current is zero that the $I^2 R$ losses will be zero. If the conductor had zero resistance as well, the wave could travel on indefinitely without attenuation.

In Fig. 8, a ground wire has been added. The magnitude of the charge on the conductor is reduced over that in Fig. 7 owing to the electrostatic capacitance of the ground wire. The traveling wave or charge induced in the ground is also reduced by an amount equivalent to that taken up by the ground wire.

2. *Discussion*, J. E. Clem, A. I. E. E. Trans., Vol. 49, July 1930, p. 1119.

In general the potential of the bound charge on the ground wire is considerably less than that on the conductor where the induced potential on the ground wire is not sufficient to cause corona, the presence of the ground wire will therefore reduce the potential and corona loss on the power conductor. This results in less attenuation. The ground losses are also less. This is particularly true where the resistance of the ground wire is low and the traveling wave on the conductor positive. If the

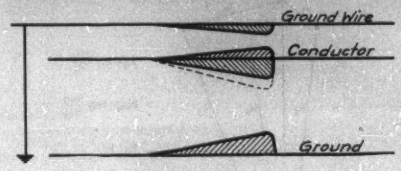

Fig. 8

traveling wave on the conductor is negative and the potential very high or the diameter of the ground wire small and near the power conductor, corona loss may be induced on the ground wire. In this case the ground wire may increase the attenuation. In general, however, it may be said that whether or not a ground wire increases or decreases the attenuation depends upon the magnitude of voltage, the size of the conductor and the polarity. The general effect, however, is to reduce the attenuation which tends to place a higher voltage on the station at a distance than where no ground wire is used.

To obtain the advantage of the ground wire in reducing the potential through an increase in electrostatic absorption, and at the same time using it as a shock absorber or energy dissipator, has led to the sectionalized, or ground wire with controlled impedance, one form of which is shown in Fig. 9. This is the same as Fig. 8 except that the ground wire is sectionalized so that the bound charge opposite that in the power conductor is caused to flow either through the high footing resistance or across an impedance $R\,S$.

In order to prevent loss in the footing resistance or in resistance which may be placed in the downleads, these resistances may be placed in series with a gap which will not discharge at normal voltage but which will discharge under a transient. Another method of reducing the loss at normal frequency is to shunt the resistance with a reactance which will have but little loss at

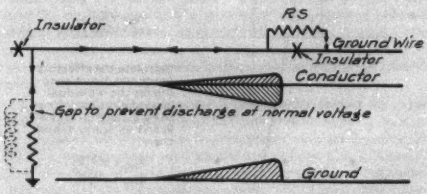

Fig. 9

normal frequency. It is evident that the bound charge upon the ground wire under this condition is made to dissipate energy in the resistances or discharge gaps which may be very material. As the energy dissipated in the ground wire must come from the power conductor, this type of ground wire will dissipate the energy of the traveling wave and greatly increase the attenuation.

Considerable attention has been given to the characteristics of the ground wire so as to make it dissipate the energy of the transient in the conductor and it would seem that this type of

ground wire may be used to advantage in many installations not only to dissipate the transient in the power conductor but to reduce the magnitude of current in the ground wire which may cause inductive interference under normal operating conditions. While this tends to complicate the study of the ground wire, it would seem that there can be little question as to the efficiency of the method.

A high tower footing resistance is not necessarily a disadvantage, as the traveling wave on the ground wire, due to the release of a bound charge, will tend to raise the potential of the tower

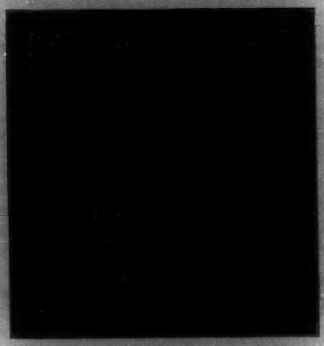

Fig. 10

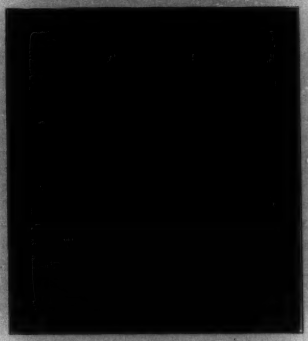

Fig. 11

thereby reducing the potential across the insulator. This, of course, is not true in all cases but tends to offset the disadvantage of a high ground resistance.

It would seem that a high footing resistance might not be detrimental providing the effective capacitance of the conductors to ground is not decreased. It would seem that increasing the effective electrostatic capacity between the conductor and ground is of greater benefit than the reduction in resistance in the tower footing. As the two generally go together, the benefits derived from a high capacitance to ground may be attributed to a low footing resistance.

Fig. 10 supports the theory that the effect of the ground wire in producing attenuation depends upon the magnitude of voltage and polarity. Here it will be noted that the discharge occurs on the positive wave. In Fig. 11 it will be seen that the size of the conductor and potential is such that no discharge takes place. As pointed out, the loss affecting attenuation may take place either on the power conductor or the ground wire. Reflections which materially increase the potential may increase corona losses and attenuation.

To obtain the practical benefits of the ground wire it may be necessary to take into account the effect of ground resistance in raising the potential of the tower as well as the electrostatic absorbing capacitance of the ground wire. In addition it is very important that the size of conductor and ground wire together with the polarity of the transient be given consideration in determining attenuation. It is also necessary to give careful attention to the impedance of the ground wire where the ground wire is sectionalized and absorbing resistances are used.

J. T. Lusignan, Jr.: One of the most thorough contributions to our knowledge of the ground wire theory since Petersen's paper appeared in the *E. T. Z.* of Jan. 1, 1914, is the paper presented by Mr. Bewley. A number of different values for the protection afforded by ground wires against lightning induced voltages, has been given to us by various investigators, some calculated and others measured in the field or laboratory. I am sure that many of us have wondered wherein lay the difference between these values, and which of them could be taken as correct. Mr. Bewley has clearly pointed out the relations between the different methods used, and has shown them to be truly equivalent as far as ideal ground wires are concerned. This then allows us to use with assurance the old conventional method of calculation for static conditions. With a maximum tower footing resistance value of 75 ohms given in Mr. Bewley's paper, the transmission line designer should now be able to pre-determine with fair accuracy the induced voltage protection he can be expected to have under given ground wire arrangements.

With respect to the curves of Fig. 13 some readers might become unduly alarmed at the high voltages reached between conductors and ground wire in midspan for a stroke to the ground wire there. However, I believe that Mr. Bewley merely intends the voltage represented to be that existing at the first instant, as he apparently does not take into account wave travel and reflections. Actually the voltage is considerably reduced after the waves have had a chance to travel to the tower grounds and to be reflected back at opposite polarities. The total time for a 1,000-ft. span would be of the order of a microsecond, and the voltage reduction would of course be greater, the lower the footing resistances.

L. V. Bewley: Both Mr. Clem and Dr. Lusignan have suggested that the 35-ft. elevation of the direct hit wire about the power conductors advocated in the paper is probably higher than necessary except in the case of the most severe lightning stroke. The figure corresponds to a 10,000,000-volt lightning stroke at midspan having a front not exceeding the length of the span, and the impulse flashover characteristic of a needle gap. If the front is longer than the length of the span, there is time for the reflections of opposite polarity from the towers to arrive at midspan and reduce the potential. The amount of this reduction is indicated by the lower curves marked (tower) of Fig. 13 in the paper. The potential difference between the line and ground wires drops from 9,250,000 volts down to 1,500,000 volts at the first reflection, so the danger of side flashing is greatest during the interim preceding the first reflection. Since natural lightning waves having a front of the order of one microsecond have been measured by cathode ray oscillograph stations, it seems reasonable to decide the elevation of the direct hit wire on the basis of the maximum lightning voltage and the one microsecond needle gap spark-over in air, and any lowering of that elevation must be accepted as a departure from 100 per cent protection.

A rational determination of the most *economic* elevation for the direct hit wire must depend upon the probability of occurrence of the most severe strokes. If the great majority of lightning strokes are below 5,000,000 volts and only an occasional one goes up to 10,000,000 volts, then there may be little justification for going to the expense of a high direct hit wire. In any event, to protect a line against a specified lightning voltage four conditions must be met: (1) The direct hit wire must be high enough to prevent side flashes from occurring, based on needle gap characteristics, before the reflections of opposite polarity have arrived. (2) It must be high enough to include the power conductors inside its protective wedge for the lowest clouds. (3) The insulator string must be long enough, based on its own characteristics, so that the potential difference at the tower between line and ground wires is not sufficient to cause a flashover. (4) The clearance of the power conductors at the tower must be great enough, based on needle gap characteristic, to prevent a flashover. These requirements are best met by (a) low line conductors in horizontal plane, (b) short spans, (c) line following the valleys and depressions rather than the ridges, (d) low tower footing resistances. However, these ideals usually conflict with the dollars and cents point of view.

Mr. Clem has given some interesting figures on the effect of introducing extra ground wires, showing that they give very little reduction of an incoming surge. His calculations agree with the statement under the section on *Introduction of Extra Ground Wires* and the simple calculation under *Case III—Appendix I* of the paper. In view of Mr. Clem's contribution I am appending to this discussion the general solution for any number of conductors and any number of ground wires before and after the transition point where the extra ground wires are introduced. My solution does not agree analytically with the expressions given in Mr. Clem's discussion, because no approximations are involved in my general equation. While extra ground wires reduce an incoming surge by only a few per cent, yet that few per cent may prove sufficient so that one or two insulator disks can be removed; thereby conforming to the principle of coordination of system insulation *without changing the factor of safety of the line insulation*. In other words, a line section having extra ground wires and one or two less disks is no more liable to flashover than the fully insulated sections without extra ground wires. This advantage, coupled with the fact that the section next to the station is better protected against both induced and direct strokes may justify the extra ground wires. However, the above argument is worthless if the extra ground wire section is only a few thousand feet long. It must be long enough so that successive reflections will not have a chance to build the voltage up to values that would exist without extra ground wires; and this means that if extra ground wires are used at all that they should extend out from the station for five or six miles. This length also takes care of the requirement mentioned in the paper.

The effect of ground wires on attenuation, discussed by Mr. Austin, has been studied experimentally by McEachron, Hemstreet, and Rudge[3] with artificial traveling waves, and their conclusions seem to fit in fairly well with those of Mr. Austin.

The idea of increasing the dissipative effect of ground wires by including high resistances in the discharge path may operate to increase the attenuation, but delays and decreases the efficiency of the normal function of ground wires as protection against either induced or direct strokes. Mr. Austin's suggestion that a high tower footing resistance is not necessarily a disadvantage is not tenable either from the point of view of the protection of station equipment from induced voltages, or of station and line insulation from direct strokes. Its sole advantage is in reducing the potential across the insulators for induced strokes.

Appendix

Let there be n line wires numbered from 1 to n inclusive. Then Equations (50) and (51) give for the junction where the extra ground wires are introduced

$$\left.\begin{array}{l} Y_{11}(f_1 - F_1) + \ldots + Y_{1n}(f_n - F_n) = y_{11} f_1'' + \ldots + y_{1n} f_n'' \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Y_{n1}(f_1 - F_1) + \ldots + Y_{nn}(f_n - F_n) = y_{n1} f_1'' + \ldots + y_{nn} f_n'' \\ (f_1 + F_1) = f_1'' \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (f_n + F_n) = f_n'' \end{array}\right\}$$

Rearranging, there results

$$\left.\begin{array}{l} (Y_{11} + y_{11}) f_1'' + \ldots + (Y_{1n} + y_{1n}) f_n'' \\ \qquad = 2\,(Y_{11} f_1 + \ldots + Y_{1n} f_n) = a_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (Y_{n1} + y_{n1}) f_1'' + \ldots + (Y_{nn} + y_{nn}) f_n'' \\ \qquad = 2\,(Y_{n1} f_1 + \ldots + Y_{nn} f_n) = a_n \end{array}\right\}$$

Hereby the transmitted wave on conductor r is

$$f_r'' = \frac{\begin{vmatrix} (Y_{11} + y_{11}) & \ldots & a_1 & \ldots & (Y_{1n} + y_{1n}) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ (Y_{n1} + y_{n1}) & \ldots & a_n & \ldots & (Y_{nn} + y_{nn}) \end{vmatrix}}{\begin{vmatrix} (Y_{11} + y_{11}) & \ldots & (Y_{1r} + y_{1r}) & \ldots & (Y_{1n} + y_{1n}) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ (Y_{n1} + y_{n1}) & \ldots & (Y_{nr} + y_{nr}) & \ldots & (Y_{nn} + y_{nn}) \end{vmatrix}}$$

where the Y's are the surge admittances on the incoming side of the junction, and the y's are those on the extra ground wire side of the junction. The ground wires do not enter the equations, except in so far as they change the surge admittances. Hereby the transmitted waves for any combinations of incident waves may be computed.

3. Bibliography 6.

The Influence of Polarity on High-Voltage Discharges

F. O. McMILLAN[1] and E. C. STARR[2]
Member, A. I. E. E. Member, A. I. E. E.

Synopsis.—An experimental investigation of the influence of polarity on high-voltage discharges is reported in this paper. Particular attention is given to those discharges used in high-voltage measurements.

A theory of the formation of Lichtenberg figures is given together with experimental evidence upon which it is based. A polarity indicator utilizing visual Lichtenberg figures is described.

Results of 60-cycle and impulse tests on various types of gaps are given. It is shown that polarity has a distinct effect upon the sparking voltages of all types of gaps. Impulse measurements with grounded sphere-gaps are shown to be subject to serious error unless the polarity effects are taken into consideration.

An explanation of the influence of polarity on spark-over is proposed.

Introduction

HIGH-VOLTAGE discharges in two principal forms are very extensively employed in the measurement of high electrical potentials. For a number of years spark gaps of different types have been in general use and recently extensive use has been made of photographic Lichtenberg figures in high-voltage studies. This paper covers a study of the characteristics of the discharges involved in spark gaps and Lichtenberg figures as affected by electrode polarity.

At the present time the most important use of high-voltage discharges is perhaps in measuring gaps. The spherical-electrode type is the most reliable and is the type generally employed for accurate work. Other varieties such as those employing points and hemispheres are also used. All types of measuring gaps have definite limitations and care must be exercised in their use. Throughout most of the useful range polarity has a very pronounced effect upon the performance of all gaps having dissimilar dielectric flux distribution about the two electrodes. The investigation reported in this paper has to do largely with the influence of polarity on the spark-over voltage of different types of gaps. The ionization phenomena involved in spark-over and in the formation of Lichtenberg figures were also studied.

Lichtenberg figures recorded photographically are valuable in the study of high-voltage transients. The magnitude and character of the voltage wave can be determined with good approximation from the records obtained when certain limitations are recognized. The development of the theory of the formation of Lichtenberg figures proposed in this paper led to the study of other forms of high-voltage discharges and the extension of the theory to explain the observed phenomena.

A Theory of the Formation of Lichtenberg Figures

The Lichtenberg figures in common use for voltage measurements are formed on photographic films or plates by the action of the dielectric flux between a small recording electrode and a plane or large cylinder. The plane or cylinder is insulated with sufficient dielectric material to prevent breakdown at the highest useful voltage which is as a rule approximately thirty kilovolts maximum. The dielectric flux passing from the recording electrode through the air and solid dielectric to the plate electrode is refracted through a large angle at the junction of the air and solid dielectric resulting in a high-voltage gradient on the surface of the solid dielectric, and therefore at the location of the photographic emulsion or recording medium. When the potential difference between electrodes is increased to a certain value, the free electrons in the immediate vicinity of the recording electrode are accelerated to the critical velocity required to produce ionization by collision in the atmosphere under the particular conditions of air density prevailing. These ionization phenomena in the atmosphere surrounding the recording electrode are a normal corona discharge and are easily visible in a darkened klydonograph or surge-voltage recorder. (See discussion of polarity indicator.) The recording of Lichtenberg figures photographically is due to at least two phenomena, the light radiation both visible and invisible resulting from the recombination of the ions associated with the ionization by collision and the electronic bombardment of the silver salts in the photographic film emulsion.

The Negative Figure. The negative Lichtenberg figure is characterized by fine, straight-line striations projecting radially from the recording electrode, (Fig. 1A). This figure is formed by the repelling action of the negatively charged recording electrode on the free electrons in the surrounding dielectric field and on those extracted from the electrode. These electrons are driven radially outward because of the shape of the electric field and bombard the air molecules at the surface of the film as well as the silver salts in the emulsion, producing ionization by collision. Other electrons thus produced also move outward under the influence of the electric field leaving the heavy and relatively immobile positive ions behind. The figure

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increases in size in this manner until the positive-ion space charge reaches a value large enough to reduce the negative field of the electrode below the ionizing potential at the edge of the figure. This condition of equilibrium will be reached quite quickly, resulting in a comparatively small figure, because the free electrons are swept out of the field, reducing the loss of positive ions by re-combination to a relatively small number and because the space position of the positive ions formed is such that a comparatively small space charge will neutralize, at the outer border of the figure, the effect of a large negative electrode potential.

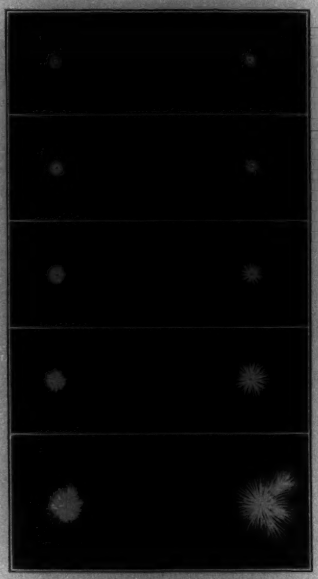


FIG. 1—LICHTENBERG FIGURES

A. 14.5 kv. unidirectional impulse
B. 14.5 kv. quickly forced to *O*
C. 14.5 kv. quickly reversed to 7.25 kv.
D. 14.5 kv. quickly reversed to 14.5 kv.
E. 14.5 kv. quickly reversed to 21.8 kv.

When the electrode potential is decreased the mass of electrons forming a negative space charge at the outer border of the figure is released and drawn back toward the electrode by the positive ions, usually forming a very small positive figure superimposed on the original negative. (Fig. 1A.) The higher the rate of removal of potential on the electrode, the more pronounced this effect becomes. (Fig. 1B.)

The Positive Figure. Coarse striations that branch repeatedly at acute angles with the main striation much like the tributaries of a stream characterize the positive Lichtenberg figures, especially those formed by impulse voltages of comparatively short duration. (Fig. 1A.) This figure is formed by the attraction of the free electrons in the field of the positively charged electrode. The electrons move radially inward toward the positive recording electrode bombarding the air molecules at the surface of the film and the silver salts on the photographic surface producing ionization by collision. As the electrons are swept from the field a positive space charge, composed of the relatively immobile positive ions produced by ionization by collision, is left. This space charge is positive and adds its field to the positive field of the recording electrode causing the figure to grow outward. This growth of the figure continues until the positive voltage gradient at the edge of the figure, due to the combined effect of the potential on the electrode and the positive space charge, is less than the ionizing gradient. The positive figure is therefore always larger than the negative figure for a given voltage because the positive space charge subtracts from the field of the negative electrode and adds to the field of the positive electrode.

The branches of the positive striations are probably a secondary phenomena caused by the combined action of the electrode field and the space charge located at the main body of the striation. Electrons located in the space between striations will then have two forces acting on them, one roughly at right angles to the main striation and one radial with respect to the electrode, therefore they will move toward the electrode along the resultant of these two forces. The action of these forces will cause them to flow into the main striation at an acute angle if they start at sufficient distance from the main electrode to allow the previously formed positive space charge to produce sufficient deviation from the radial path.

As in the case of the negative figure, the positive-figure space charge also forms a small figure, of negative polarity, when the electrode potential is suddenly removed, but in a different manner. The positive space charge holds the area surrounding the electrode at a positive potential while the electrode voltage drops below the space value causing the electrode to become negative with respect to the space and hence electrons flow out along the striations superimposing a small negative figure on the positive. (Figs. 1A and B.)

Investigation of Lichtenberg-Figure Space Charges. To definitely establish the existence of a persistent space charge during and immediately after the formation of Lichtenberg figures a circuit was devised for quickly reversing the potential during the formation of the figures. This circuit and the type of wave produced are shown in Fig. 2. It will be observed that by adjusting the water-tube resistances R_1, R_2 and R_3 and sphere-gaps G_1 and G_2 the magnitude of the initial potential and the amount of reversal applied to the klydonograph can be readily controlled. A series

of four Lichtenberg figures taken with this circuit is shown in Fig. 1, parts *B* to *E*, inclusive. The voltages used ranged from 14.5 kv. maximum, quickly forced to zero, to 14.5 kv. quickly reversed to 21.8 kv.

In all of the Lichtenberg-figure records the initial negative figure was formed on the left, the initial positive figure on the right and figures of the reverse polarity were superimposed on these. Therefore all of the positive figures on the left are superimposed on negative figures and all of the negative figures on the right are superimposed on positive figures. All of the superimposed figures show a very definite change due to the space charge formed by the original figure. For values of reversed potential up to 50 per cent, the superimposed positive figures are entirely confined to the area ionized by the previous negative figure. For values of reversed potential of 50 per cent and greater the positive ionization breaks over the boundary of the previous negative figure. The negative figures superimposed on the positive grow progressively larger as the reversed potential is increased and follow the ionized paths previously formed by the positive striations. Both the

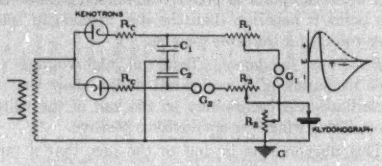


Fig. 2—Circuit for Quickly Reversing the Impulse Potential During the Formation of Lichtenberg Figures

positive and negative figures are altered in size and character by previous ionization. The positive figures are reduced in size until well beyond the boundary of the previous ionization. The negative figures are very much larger than normal.

The Influence of Polarity on Spark-Over Phenomena

Polarity Indicator. The determination of instantaneous polarity in high-voltage circuits has heretofore involved the use of the Duddell type of oscillograph for low-frequency studies and the Dufour cathode-ray oscillograph or the klydonograph for impulse voltage investigations. All of these methods depend upon photographically recording the phenomena. These photographic methods are very accurate and reliable but because of the time and expense involved, limit most investigations to a comparatively few observations. When this investigation of the influence of polarity on high-voltage discharges was started in 1929, it was found that photographic methods for determining the polarity of flashover were so time consuming that it was not practical to continue the study by that method. A very much faster means was needed that would be equally reliable. It seemed reasonable to expect that Lichtenberg figures could be made visible by the use of some florescent screen in the place of the photographic film and some work was done to investigate this possibility. It was found that with a properly constructed instrument the characteristic Lichtenberg figures produced by transients lasting only a few microseconds were easily visible from the ionization of the air surrounding the electrodes without the aid of any florescent material. This instrument is shown in Fig. 3.

Fig. 3—Polarity Indicator

The polarity indicator consists essentially of two pairs of oppositely connected electrodes enclosed in a light-proof viewing hood. Both the positive and negative Lichtenberg figures are seen simultaneously by the observer and hence the polarity of the discharge is definitely established. The observer is therefore able to make polarity determinations in very rapid succession and a large number of observations can be made in a short time. Each of the curves in this paper was determined from a thousand or more individual spark-aver observations and the time element was of considerable importance.

For polarity determinations during 60-cycle tests the instrument was connected as shown in Fig. 4. The

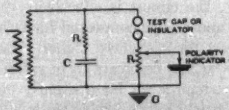


Fig. 4—Circuit for Determining the Polarity of Initial A-C. Spark-Over

capacitance *C* of the high-voltage circuit is charged by the alternating potential and at the instant the test gap breaks down, is discharged through the resistance *R*. This sudden impulse current through the polarity-indicator resistance *R* impresses a high voltage on the indicator. The impulse voltage has the same polarity as the 60-cycle voltage at the instant the gap sparks over and is much larger than the voltage that follows due to the dynamic current.

During the impulse tests the polarity indicator was

connected as shown in Fig. 6. In this case it is not possible to employ a resistance in series with the test gap because the high charging current would produce a voltage drop that would affect the gap spark-over. To avoid this difficulty, the polarity indicator was connected across a portion of the charging resistance R_3. With this connection the instrument has impressed on it an impulse voltage having the same polarity as the generator output. The normal 60-cycle charging current in R_3 does not ordinarily produce a visible

Fig. 5—750,000-Volt Impulse Generator

figure and in all cases it is very much smaller than the impulse figure.

Impulse Generator. The impulse generator used in the tests described in this paper is shown in Fig. 5. The circuit diagram is given in Fig. 6. The three condensers, C_1, C_2, and C_3 are charged in parallel through the graded resistances R_1, R_2, R_3, R_4, and R_5 by means of a high-voltage transformer. At the instant the voltage on C_1 reaches the spark-over value of the 25-centimeter gap G_1, the circuit is effectively closed at that point and C_1 and C_2 are connected in series. The gap G_2, which is set for a slightly higher voltage than G_1, then sparks over and connects C_3 in series with C_1 and C_2. Gap G_3, which is set for a slightly higher voltage than G_2, then sparks over allowing the condensers to discharge through the water-tube resistor R_6. The voltage drop across R_6 is impulsive in character and is applied to the test gap. The charging resistors R_1, R_2, R_3, R_4, and R_5 are of sufficiently high resistance, relative to R_6, to have little effect upon the discharge. If the spacing of the initiating gap G_1 does not exceed approximately 8.5 cm., both positive and negative impulses are obtained. However, as the spacing is increased up to that point when a large total number is taken the ratio of negative to positive impulses, becomes increasingly greater. This follows the characteristic shown in Figs. 12 and 13, *i. e.*, the positive and negative spark-over voltages are not sufficiently different up to approximately 8.5 cm. to cause the 60-cycle spark-overs to be all of one polarity.

In the impulse tests described in this paper the initiating gap G_1 was never spaced greater than 8.5 cm. and hence a number of both positive and negative discharges are obtained from each series of impulses. Furthermore, since the positive and negative spark-over voltages for the 25-cm. initiating gap are essentially the same up to 8.5 cm., the positive and negative impulse voltages are practically identical in magnitude.

Point-to-Plane Gap. The 60-cycle spark-over of the point-to-plane type of gap at all spacings, with the possible exception of very short distances, occurs only when the point is positive with respect to the plane. The spark-over voltage at a given spacing is the lowest of all gaps with the exception of the point to concave-surface types. On continuous voltages at relatively close spacings the negative spark-over voltage is approximately twice the positive and at longer spacings this difference becomes much greater. The continuous-voltage sparking potential of the point-to-plane gap is less when the point is positive and much greater when the point is negative than the d-c. needle-gap spark-over voltage for the same spacing.

Pin-Type Insulators. Tests on high-voltage pin-type insulators of conventional design show that 60-cycle flash-over occurs only on the half of the voltage wave during which the conductor is positive.

This phenomenon is due to the fact that a much higher voltage gradient exists at the conductor than at the pin and intense local ionization at the conductor and tie wire precedes flash-over.

Impulse tests show that pin type insulators flash over at lower voltages when subjected to positive surges on the line than when subjected to negative surges, (Table 1). For the impulse-generator wave used in these tests

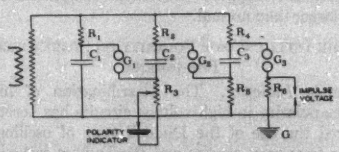


Fig. 6—Impulse Generator Circuit

the positive spark-over voltage of a 66-kv. pin-type insulator was 376 kv. maximum and the negative was 427 kv. maximum. The positive impulse ratio was 1.36 and the negative 1.55. This variation is quite large and shows that the insulation strength of transmission lines insulated with pin-type insulators is quite different for impulse voltages of opposite polarities. These tests further show that values of impulse flash-over voltage and impulse ratio should be defined not only by the voltage wave form used but also by the

polarity of the wave when the voltage gradient is very much higher at one electrode than at the other.

Sphere to Plane. Spark-over tests at 60 cycles were made on a 6.25 cm. sphere-to-plane gap at various spacings up to and including 20 cm. using the polarity indicator.

The tests revealed some very interesting and unexpected spark-over characteristics. These data are given in Table II, and shown graphically in Fig. 7. For sparking distances between 0.5 cm. and 3.0 cm. this gap always sparks over on the half cycle of the voltage wave which makes the sphere negative with respect to the plane and never sparks over when it is positive. For sparking distances between 3.0 and 7.0 cm., the gap may spark over when the sphere is either positive or negative with respect to the plane. In general, near the 3.0-cm. spacing the negative sparkovers predominate. However, the data and curve show a peculiar irregularity in the polarity distribution of spark-over at sparking distances between 4.0 and 5.0 cm. This irregularity when first observed, was thought to be due to chance erratic distribution of spark-overs and it was believed that if sufficient observations were taken, the curve would prove to be a smooth transition from negative to positive spark-overs as the spacing was increased. To check this portion of the spark-over polarity-distribution curve, 1320 individual spark-over observations were made for sparking distances between 4.0 and 6.5 cm. inclusive. This large number of observations checked the original data showing the irregularity in the curve actually exists.

TABLE I

IMPULSE FLASH-OVER DATA FOR PIN TYPE INSULATORS

Standard Mounting

Insulator		Positive				Negative			
Nominal rating kv. r. m. s.	Dry flashover kv. max.	Maximum kv. max.	Minimum kv. max.	Average kv. max.	Impulse ratio	Maximum kv. max.	Minimum kv. max.	Average kv. max.	Impulse ratio
45	202	278	227	253	1.25	354	264	309	1.53
66	276	406	346	376	1.36	435	419	427	1.55

NOTE: Sixty-cycle flash-over on above insulators occurs only when line is positive.
Barometric pressure 755.5 mm.
Temperature, degrees centigrade
Dry bulb 23.2
Wet bulb 16.3

Impulse tests were made on the 6.25 cm. sphere-to-plane gap over the same range of spacings covered by the 60-cycle tests. The data for these tests are given in Table III and are shown graphically by Fig. 8. The impulse tests confirm the 60-cycle tests by showing the spark-over voltage to be lower when the sphere is negative up to approximately 6.25 cm. For spacings greater than 6.25 cm., the positive spark-over occurs at a lower voltage than the negative. It will be noted that for voltages above 155 kv. smooth, average curves have been drawn. However, the experimental points have been checked repeatedly and found to be as indicated. It is therefore probable that the true curve will be found to pass through these points when more completely explored.

Since the impulse generator used gives both positive and negative output waves, depending upon the polarity of the initiating gap at the instant of spark-over, it was necessary to devise a special experimental procedure for determining the impulse data. The impulse generator was adjusted to deliver a definite voltage. The sparking distance of the experimental gap was increased until one impulse in ten of a given polarity failed to spark over. This spacing was termed the minimum gap for that polarity spark-over. The gap was then increased until only one impulse in ten of the chosen polarity caused spark-over. This setting was termed the maximum sparking distance. These two limits represented the spread of the spark-overs at a fixed voltage for the given polarity and their average was taken as the true sparking distance for that voltage and polarity. The mean thus obtained is not the exact

TABLE II

SIXTY-CYCLE SPARK-OVER DATA FOR A 6.25-CENTIMETER SPHERE TO A PLANE

Sparking distance in centimeters	Kilovolts maximum	Per cent of spark-overs	
		Positive	Negative
0.50	12.9	0	100
1.00	24.7	0	100
1.50	35.9	0	100
2.00	43.9	0	100
3.00	64.6	0	100
4.00	75.2	3	97
4.25	77.3	19	81
4.50	79.3	54	46
4.75	81.2	55	45
5.00	83.0	18	82
5.25	84.7	25	75
5.50	86.4	54	46
5.75	87.8	73	27
6.00	89.2	94	6
6.50	91.9	98	2
7.00	95.5	100	0
8.00	99.0	100	0
10.0	106	100	0
15.0	117	100	0
20.0	127	100	0

Barometric pressure 755.6 mm.
Temperature, degrees centigrade
Dry bulb 22.0
Wet bulb 15.5

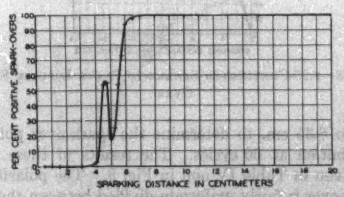


FIG. 7—POLARITY DISTRIBUTION OF 60-CYCLE SPARK-OVERS FOR A 6.25-CENTIMETER SPHERE TO A PLANE

spacing corresponding to the mean voltage, due to the curvature of the spark-over curves, but because of the small differences between maximum and minimum the error is very small. The same procedure was followed for both the positive and negative polarities.

Grounded Sphere-Gaps

2.54-cm. Spheres. Tests were made on a 2.54-cm. sphere-gap over a range of spacings varying from 0.38 cm. to 6.35 cm. to determine the spark-over polarity distribution at 60 cycles. The results of these tests are given in Table IV and in Fig. 9. The polarity of spark-over is fairly uniformly divided between the two polarities for sparking distances between 0.38 cm. and 2.0 cm. From 2.2 cm. to 3.2 cm. the spark-overs all occur when the upper sphere is negative. Between 3.2 and 3.8 cm. there is a very rapid transition from all negative to all positive spark-overs. For all spacings investigated above 3.8 cm. the spark-overs were all positive.

6.25-cm. Spheres. Spark-over tests at 60 cycles were made on a 6.25-cm. sphere-gap spaced from 0.5 to 12.5 cm. The data are given in Table V and Fig. 10. Between 0.5 and 1.5 cm., approximately 10 per cent of the spark-overs were positive. There is a very rapid change in the percentages of positive spark-overs between 1.5 and 3 cm. reaching a maximum of 76 per cent positive at 2.25 cm. This characteristic was carefully checked and definitely established as shown. It will be noted that a similar characteristic existed in the case of the 2.54-cm. sphere-gap. This feature does not mean that there is any great difference between the positive and negative spark-over voltages in this region because in order to obtain spark-overs of both polarities the voltages must be practically identical. Between 3.0 and 9.0-cm. spacing no positive spark-overs could be obtained. A rapid transition from 100 per cent negative to 100 per cent positive occurs between 9.0 and 11.0 cm. Above 11.0 cm. all spark-overs obtained were positive. In portions of the regions where the spark-overs are all of one polarity, or the other, it is probable that a considerable difference exists between the positive and negative spark-over voltages. The impulse tests lend support to this deduction.

Impulse tests on the 6.25-cm. sphere-gap gave the results shown in Table VI and Fig. 11. Up to 1.75-cm. spacing the spark-overs for both polarities occur at practically the same voltages. Between 1.75 and 8.4

TABLE III
IMPULSE SPARK-OVER DATA FOR A 6.25-CENTIMETER SPHERE TO A PLANE

Maximum impulse voltage kilovolts	Sparking distance in centimeters					
	Positive			Negative		
	Maximum	Minimum	Average	Maximum	Minimum	Average
50.9	1.70	1.40	1.55	2.03	1.31	1.67
96.4	4.75	3.00	3.88	4.75	4.60	4.68
110	6.59	4.52	5.56	6.59	5.53	6.06
121	9.14	6.53	7.84	7.00	6.06	6.53
129	11.2	9.10	10.1	7.10	6.44	6.77
155	14.0	12.6	13.3	9.80	8.25	9.03
180	15.8	13.5	14.7	11.0	9.12	10.1
216	19.4	17.8	18.6	14.9	13.0	13.9
241	20.5	19.2	19.9	17.2	14.7	16.0

Barometric pressure 759.4 mm.
Temperature, degrees centigrade
Dry bulb 23.0
Wet bulb 16.0

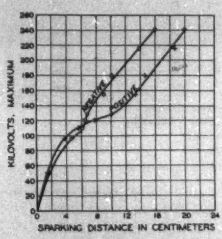


Fig. 8—6.25-Centimeter Sphere-to-Plane Impulse Spark-Over

TABLE IV
SIXTY-CYCLE SPARK-OVER DATA FOR A 2.54-CENTIMETER SPHERE-GAP
Lower Sphere Grounded

Sparking distance in centimeters	Kilovolts maximum	Per cent of spark-overs	
		Positive	Negative
0.381	13.4	32	68
0.635	20.4	30	70
0.952	29.0	10	90
1.27	37.9	58	42
1.59	45.3	56	44
1.91	51.2	39	61
2.22	55.4	0	100
2.54	59.1	0	100
3.18	65.0	0	100
3.49	67.4	60	40
3.81	69.8	100	0
4.45	73.1	100	0
5.08	76.0	100	0
6.35	79.9	100	0

Grounded sphere 20.3 centimeters above ground and fixed in position
Barometric pressure 759.4 mm.
Temperature, degrees centigrade
Dry bulb 22.9
Wet bulb 16.0

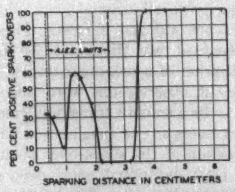


Fig. 9—Polarity Distribution of 60-Cycle Spark-Overs for a 2.54-Centimeter Sphere-Gap

Lower sphere grounded

cm. spark-over occurs at a lower voltage when the upper sphere is negative than when positive. The percentage difference is maximum at 4.0-cm. spacing at which point the positive voltage is 11.5 per cent greater than the negative. At 8.4 cm. the positive and negative become equal and from this spacing to the upper limit of the tests the positive spark-over voltage becomes rapidly less than the negative. It is interesting to note that the negative spark-over curve follows the A. I. E. E. standard sphere-gap data throughout the recommended range, (Table VI).

TABLE V

SIXTY-CYCLE SPARK-OVER DATA FOR A 6.25-CENTIMETER SPHERE-GAP

Lower Sphere Grounded

Sparking distance in centimeters	Kilovolts maximum	Per cent of spark-overs	
		Positive	Negative
0.50	16.8	16	84
1.00	31.7	8	92
1.50	45.5	8	92
1.75	52.3	32	68
2.00	58.3	52	48
2.25	64.9	76	24
2.35	66.2	40	60
2.50	69.4	12	88
3.00	79.2	0	100
4.00	93.5	0	100
5.00	103	0	100
6.00	111	0	100
6.25	113	0	100
7.00	117	0	100
8.00	123	0	100
9.00	127	0	100
9.50	130	32	68
10.0	132	92	8
11.0	136	100	0
12.0	139	100	0
12.5	141	100	0

Grounded sphere 22.4 centimeters above ground and fixed in position
Barometric pressure 759.9 mm.
Temperature, degrees centigrade
Dry bulb 20.9
Wet bulb 14.2

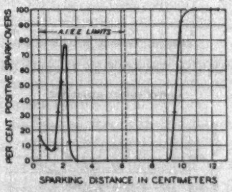


FIG. 10—POLARITY DISTRIBUTION OF 60-CYCLE SPARK-OVERS FOR A 6.25-CENTIMETER SPHERE-GAP

Lower sphere grounded

25-cm. Spheres. Table VII and Fig. 12 give the results of 60-cycle tests on a 25-cm. gap. These tests were limited to a maximum spacing of 10 cm. because the upper limit of the available transformer voltage was 235 kv. maximum. Both positive and negative spark-overs, with the negative predominating, were obtained between 1.0 and 8.0-cm. spacing. At 9.0 and 10.0 cm. no positive spark-overs were obtained.

Impulse spark-over data were taken on the 25-cm. gap up to 38.0 cm. spacing. These data are given in Table VIII and in Figs. 13 and 14. It will be noted in Table VIII that as was found in the case of the 6.25-cm. sphere-gap, the average negative spark-over data conforms with the A. I. E. E. standard over the recommended range of sparking distances. The positive curve coincides with the negative up to approximately 5 cm. From this point the percentage difference becomes increasingly higher up to 25 cm., at which spacing it is a maximum and the positive spark-over voltage is 35 per cent greater than the negative.

Due to the construction of the sphere-gap frame it was not possible to extend these curves above 40-cm. spacing. However, from the trend of the experimental data at the highest spacings, there is reason to believe that the positive and negative curves cross at some spacing not much greater than 40 cm. This extension would make the complete 25-cm. characteristic curves similar to the 6.25-cm. curves shown in Fig. 11. The positive curve was not carried to the 40-cm. spacing

TABLE VI

IMPULSE SPARK-OVER DATA FOR A 6.25-CENTIMETER SPHERE-GAP

Lower Sphere Grounded

Maximum impulse voltage kilovolts	Sparking distance in centimeters						A.I.E.E. Standard
	Positive			Negative			
	Maximum	Minimum	Average	Maximum	Minimum	Average	
51.9	1.78	1.70	1.74	1.80	1.65	1.73	1.73
71.6	2.43	2.35	2.39	2.70	2.50	2.60	2.60
100	3.80	3.60	3.70	4.70	4.50	4.60	4.60
113	4.70	4.60	4.65	6.10	5.80	5.95	6.27
129	9.25	5.50	7.38	9.25	7.00	8.13	
146	13.3	10.0	11.7	9.50	7.75	8.63	

Grounded sphere 22.4 centimeters above ground and fixed in position
Barometric pressure 759.4 mm.
Temperature, degrees centigrade
Dry bulb 23.0
Wet bulb 160.

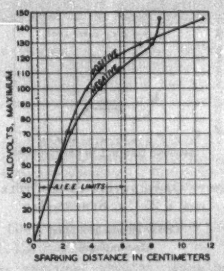


FIG. 11—6.25-CM. SPHERE-GAP IMPULSE SPARK-OVER

Lower sphere grounded

because the increased voltage would have necessitated operating the impulse-generator initiating gap at spacings in excess of those for which a satisfactory number of positive discharges could be obtained.

The wave form of the impulse voltage impressed upon the test gap probably has a pronounced effect upon the spread of the positive and negative spark-over curves. However, it is believed that the wave form of the impulse generator used in this investigation is quite similar to that of the impulse generators used in other laboratories. Calculation shows that the voltage wave rises to a maximum very abruptly and then decreases to half value in approximately 5 microseconds. The impulse ratio obtained on a 25-cm. needle-gap was 1.78 positive and 1.78 negative and on a 66-kv. pin-type insulator was 1.36 positive and 1.55 negative.

TABLE VII
SIXTY-CYCLE SPARK-OVER DATA FOR A 25-CENTIMETER SPHERE-GAP
Lower Sphere Grounded

Sparking distance in centimeters	Kilovolts maximum	Per cent of spark-overs	
		Positive	Negative
1.00	30.8	32.0	68.0
2.00	59.3	32.0	68.0
3.00	85.6	12.0	88.0
4.00	110	8.0	92.0
5.00	132	16.0	84.0
6.00	155	16.0	84.0
7.00	176	16.0	84.0
8.00	197	4.0	96.0
9.00	216	0.0	100
10.0	235	0.0	100

Ungrounded sphere 118 centimeters above ground and fixed in position
Barometric pressure 759.4 mm.
Temperature, degrees centigrade
Dry bulb 22.9
Wet bulb 16.0

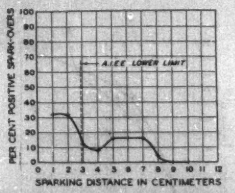


FIG. 12—POLARITY DISTRIBUTION OF 60-CYCLE SPARK-OVERS FOR A 25-CM. SPHERE-GAP

Lower sphere grounded
(Incomplete)

AN EXPLANATION OF THE POLARITY EFFECT IN SPARK-OVER

The dielectric-flux concentration in the point-to-plane gap is many times greater at the point than on the plane. The high-voltage gradient at the point causes local ionization preceding spark-over for all except possibly extremely short sparking distances. When the point is positive, free electrons in the surrounding space are attracted toward the electrode and accelerated to the ionizing velocity in the region where the potential gradient is sufficiently great. These electrons and those liberated by ionization, except those lost by recombination, are absorbed by the electrode. The relatively immobile positive ions resulting from this ionization form a space charge surrounding the point. This space charge adds to the electrode potential and extends the critical ionizing gradient to a greater distance. In this manner the ionization is progressively

TABLE VIII
IMPULSE SPARK-OVER DATA FOR A 25-CENTIMETER SPHERE-GAP
Lower Sphere Grounded

Maximum impulse voltage kilovolts	Sparking distance in centimeters						
	Positive			Negative			A.I.E.E. Standard
	Maximum	Minimum	Average	Maximum	Minimum	Average	
127	5.50	4.50	5.00	5.75			4.75
224	8.75	8.25	8.50	9.25	9.25	9.25	9.45
326	13.0	12.0	12.5	16.5	16.5	16.5	16.5
377	15.3	14.3	14.8	24.5	22.0	23.3	22.2
414	19.0	15.5	17.3	32.0	25.0	28.5	
445	21.0	16.8	18.9	37.0	32.0	34.5	
469	22.0	17.0	19.5	40.0	36.0	38.0	
542	30.5	21.5	26.0				

Ungrounded sphere 118 centimeters above ground and fixed in position
Barometric pressure 759.1 mm.
Temperature, degrees centigrade
Dry bulb 21.4
Wet bulb 15.3

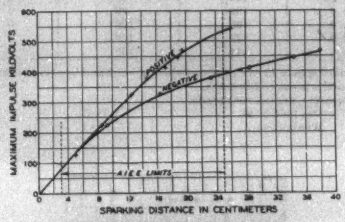


FIG. 13—25-CM. SPHERE-GAP IMPULSE SPARK-OVER

Lower sphere grounded

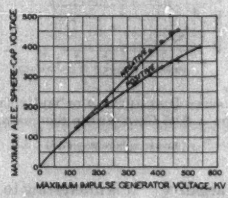


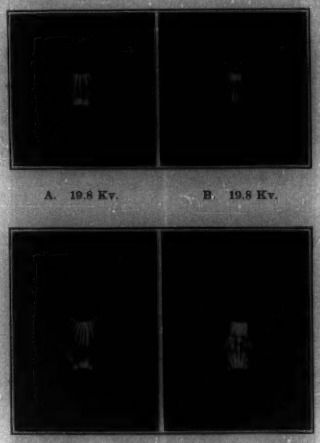
FIG. 14—25-CENTIMETER SPHERE-GAP IMPULSE SPARK-OVER

Lower sphere grounded

extended outward from the point until the gradient at the outer boundary of the ionized region falls below the critical value or, if the electrode potential is sufficiently high, complete spark-over is established. When the point is negative, free electrons in the surrounding space, together with those extracted from the electrode, are repelled outward at high velocity producing ionization by collision in the region which is above the critical gradient. A positive and a negative space charge are thus formed. The field of the positive space charge opposes the field from the electrode which it surrounds and tends to reduce the resultant flux in the area beyond this space charge. Some of the repelled electrons do not migrate entirely across the gap and are held by the positive space charge at the outer boundary of the ionized region. The decrease in flux reduces the distance to which the ionizing gradient extends from the electrode and hence increases the potential necessary to cause complete spark-over. Positive spark-over therefore occurs at much lower voltages than negative for a given electrode spacing. The above phenomena apply in general to all gaps in which ionization occurs at potentials lower than spark-over and in which this ionization is more intense at one electrode than the other. This condition also obtains in sphere-to-plane gaps and grounded sphere-to-sphere gaps at wide spacings and in practically all forms of high-voltage transmission-line insulation.

When a sphere-gap has a voltage impressed upon it, the positive sphere attracts the free electrons in the surrounding space and when the potential is increased to the proper value accelerates them to the ionizing velocity. The free electrons and those liberated by collision, except the ones lost by recombination, are conducted away by the sphere leaving a relatively immobile positive space charge in the surrounding ionized space. This space charge adds to the positive-sphere potential and extends the critical ionizing potential gradient, as well as the effective radius of the sphere. The negative sphere repels the free electrons in the surrounding space and when the critical voltage gradient is reached produces ionization by collision adjacent to the negative electrode. This action results in the formation of both a positive and a negative space charge. The positive space charge is composed of the relatively immobile positive ions left close to the sphere. Part of the electrons expelled from this region are carried entirely across the gap and the remainder, forming the major part of the negative space charge, are bound near the outer edge of the positive space charge.

At close sparking distances this negative space charge is attracted by the positive sphere and carried away leaving the negative-electrode positive space charge in the combined field of three charges. These charges are the potential on the positive sphere, the positive space charge of the positive sphere and the potential on the negative sphere. This resultant field forces the negative-sphere positive space charge back on the negative electrode producing heavy ionization and extracting large numbers of electrons resulting in local breakdown and removal of the positive space charge in the immediate vicinity (Fig. 15B and D, Fig. 17B and C, Fig. 18). The conducting streamer thus formed extends the negative-sphere potential beyond the positive space charge and the breakdown progresses to the positive sphere. At these close spacings the dielectric flux distribution in the sphere-gap between sparking surfaces is practically unaffected by usual ground proximity. Because of this symmetrical flux distribution on the spheres, spark-over takes place from either the grounded or ungrounded sphere at practically the same



A. 19.8 Kv. B. 19.8 Kv.

C. 28.6 Kv. D. 28.6 Kv.

Fig. 15—Lichtenberg Figures of Sphere-Gap Field Slightly Below Spark-Over Voltage, Lower Sphere Grounded

A. Upper sphere positive, diameter spacing
B. Upper sphere negative, diameter spacing
C. Upper sphere positive, 1½ diameter spacing
D. Upper sphere negative, 1½ diameter spacing

voltage and there is no appreciable polarity effect, Figs. 9, 10, 11, 12, and 13.

For sphere spacings greater than approximately $1.6\sqrt{R}$ the influence of ground on the flux distribution causes higher voltage gradients to exist on the ungrounded sphere than exist on the grounded sphere at the sparking surfaces. Therefore ionization starts at a lower voltage on the ungrounded sphere. As shown above when the field is symmetrical local breakdown occurs at the surface of the negative sphere at lower voltage than at the surface of the positive sphere when the spacing is sufficiently close for the removal of the negative space charge. When the field becomes unsymmetrical under the influence of ground, local break-

down and spark-over occur at a lower voltage when the sphere having the more intense field is negative. This condition obtains until the spacing is increased to the point at which the negative space charge neutralizes the negative-sphere positive space charge sufficiently to reduce the gradient near the surface of the sphere, when negative, to that value obtaining at an equal positive voltage. At this spacing, which for the gaps and conditions investigated was considerably over diameter separation, the positive and negative spark-over voltages again become equal. For greater spacings spark-over occurs at a lower voltage when the ungrounded sphere is positive as in the case of the point-to-plane gap, Fig. 16.

A. 19.0 Kv. B. 30.0 Kv.

C. 66.5 Kv. chopped at 58.2 Kv. D. 57.6 Kv.

Fig. 16—Lichtenberg Figures of Sphere-Gap Field

Upper sphere positive.
Lower sphere grounded

Summary

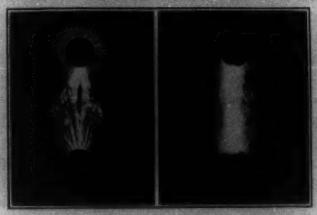
1. Lichtenberg figures are produced by ionization phenomena at the surface of the recording medium and are visible in a darkened instrument.

2. The positive figure is extended in size by the positive space charge surrounding the recording electrode. The negative figure is held to a relatively small size by the positive space charge adjacent to the electrode. The existence of these space charges is clearly demonstrated when figures of opposite polarity are quickly superimposed.

3. Tests with different types of gaps show that polarity has a pronounced effect upon spark-over.

4. The point-to-plane gap at a given spacing sparks over at a much lower voltage when point is positive than when negative, except for possibly very small spacings.

A. 19.5 Kv. B. 43.5 Kv.



C. 66.5 Kv. chopped at 58.2 Kv. D. 62.9 Kv.

Fig. 17—Lichtenberg Figures of Sphere-Gap Field

Upper sphere negative.
Lower sphere grounded

Fig. 18—Impulse Spark-Over on 25-Cm. Sphere-Gap at Diameter Spacing

Upper sphere negative
Lower sphere grounded

This characteristic is typical of all gaps having dissimilar electrode gradients when local breakdown precedes spark-over, and is due to the space charge extending the

field of the higher-gradient electrode when positive and suppressing the field when negative.

5. Pin-type insulators spark over at lower voltage when line is positive than when negative, due to local breakdown occurring first on the line. The impulse ratio is lower positive than negative.

6. The grounded sphere-gap sparks over at the same voltage with ungrounded sphere positive or negative up to spacings of approximately 1.6 $\sqrt{R}$.

7. For the conditions investigated the grounded sphere-gap sparks over at a lower voltage negative than positive at spacings ranging between approximately 1.6 $\sqrt{R}$ and well over diameter separation. This characteristic is due to high local negative gradients caused by the negative-electrode positive space charge.

8. At wide spacings the grounded sphere-gap sparks over at lower voltages positive than negative, (4).

9. Near diameter spacing the grounded sphere-gaps investigated spark over at much lower impulse voltages negative than positive. The shape and duration of the impulse wave probably have a pronounced effect upon this difference.

ACKNOWLEDGMENT

The published works of other investigators have been drawn upon freely in the preparation of this paper and a partial list is given in the bibliography.

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Discussion

Geo. S. Smith: In 1928 Mr. E. J. Wade and I observed a decided polarity effect on certain types of insulators, while determining the arc-over time lag under impulse conditions. The greater portion of our data was obtained by means of the Dufour oscillograph. The impulse generator used gave negative polarity on the high-voltage side while the grounded side was positive.

When using slowly rising wave fronts approaching the time required for a quarter wave length of a 60-cycle wave, voltages averaging 50 per cent greater than the 60-cycle value were required to produce an arc over the insulator. With the insulator inverted and the negative applied to the pin, or with the polarity reversed on the upright insulator, the arc-over voltage was reduced to an average of about 8 per cent above the standard 60-cycle value.

A check on these impulse values was obtained by using direct current and steadily raising the voltage until arcover occurred. The values thus obtained checked the results of the impulse tests very closely.

Arc-over tests were also made using 60-cycle voltage and taking oscillograms on a Dudell type oscillograph of the break-in voltage. The oscillograms showed every arc to occur on the same half of the wave, that is when the cap or line side of the insulator was positive.

It is interesting to note, in the set of curves shown, that for the 15-kv. pin type insulator tested, the difference between the positive and negative arc-over values decreases as the steepness of the impulse wave front is increased. They both approach

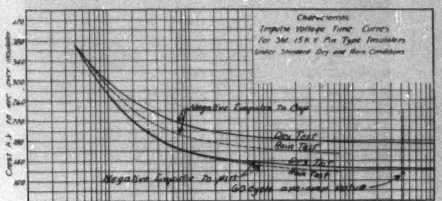

FIG. 1

the same limiting value at very steep wave fronts. Similar tests under standard rain conditions also approached this same limiting value as the curves indicate.

Tests made on suspension type insulators showed little or no effect of polarity on the arc-over values.

While some small differences in results were observed with the pin type insulator inverted as compared with the same polarities applied when erect, the differences were so small, it might be safely concluded that the insulator mounted on a pole as in use is affected very little by the earth beneath. Thus the polarity effect is almost entirely caused by the distribution of the electrostatic lines of flux between the cap or line and tie wires, and the pin with its support.

H. L. Rorden: As far as is known at present, the effect of polarity on spark-overs is not susceptible to a mathematical treatment. It would seem therefore, that voltage differences due to polarity may be attributed to inherent physical properties, the nature of which must be determined empirically. This was recognized by Carroll and Lusignan[1] and later Dr. Lusignan demonstrated spark-over differences due to polarity at power frequencies.[2]

A factor of major importance in the investigation of the effect of polarity on spark-overs is the lack of an accurate voltage measuring device for high potentials. The crest voltmeter

1. *The Space Charge that Surrounds a Conductor in Corona*, by J. S. Carroll and J. T. Lusignan, A. I. E. E. TRANS., January 1928, p. 50.

2. *A Study of High-Voltage Flashovers*, by J. T. Lusignan, A. I. E. E. TRANS., January 1929, p. 246.

probably used most extensively for this purpose,—the sphere-gap,—has proven itself to be only approximately correct. In the extensive investigations of high-voltage phenomena carried on under the direction of F. W. Peek, Jr. surprising disclosures have resulted from comparing impulse spark-over voltages of various electrodes with both polarities. In these investigations, the cathode-ray oscillograph has been unique in obtaining comparative data, in disclosing the approximate variations to be expected in sphere gap measurements, and in the spark-over of other electrodes. If oscillograms could but be used for an accurate voltage calibration, the story would be complete. Oscillograms do, however, give us comparative information.

It has long been known that sphere-gaps, particularly at relatively higher spacings, are not free of time lags. But since in general, sphere-gap calibration curves must depend on voltages at power frequencies, the effect of polarity on their sparkover introduces an error that cannot be so easily detected, and is much more difficult to correct. With the cathode ray oscillograph, and a lightning generator with its polarity controlled, it is found that sphere-gaps generally require a higher voltage with a positive wave to spark over a given spacing, the difference ranging up to 12 per cent, depending on the wave form and the per cent overvoltage held. In agreement with the results

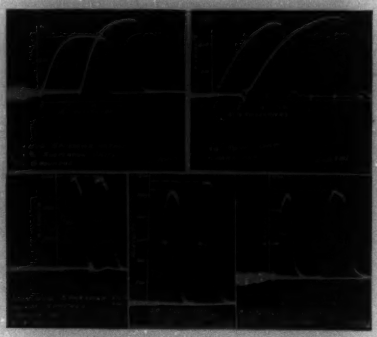

Fig. 2—Spark-overs of Various Electrodes with Both Polarities

obtained by Messrs. McMillan and Starr, is the fact that only at relatively small spacings do the negative polarity spark-overs equal or exceed the voltage of the positive wave, for spheres ranging from 25 to 100 cm. in diameter. However, the larger spheres show less difference at relatively large spacings than do the smaller ones, there being a maximum of 4 per cent difference found in 100 cm. spheres at diameter spacing, for various wave forms.

With non-uniform electrodes the negative polarity wave usually has the higher spark-over value, although it is frequently (but not always) found that a higher energy input to the lightning generator is necessary with positive polarity waves. This higher negative spark-over has been found to run as high as 22 per cent for insulator strings, and may be very much greater for a point-to-plane spark-over. However, merely inverting a string of suspension units may reverse the order of the polarity requiring the higher spark-over voltage. Point gap spark-overs are found to require a higher voltage for the negative wave consistently, with various wave forms, and within the range of 30 to 130 cm. spacing.

The accompanying oscillograms illustrate spark-over differences due to polarity, for the various electrodes and with the various wave forms as shown. Voltage scales are given where oscillographic deflections could be favorably compared with differences found by sphere gap measurements. In the many instances where fair agreement could not be obtained, it was assumed that an appreciable difference existed in the sphere-gap spark-overs.

In the paper presented by Messrs. McMillan and Starr, no direct indication is given of the method used for measuring the impulse voltage. And since it is a measurement of voltage that determines what differences exist due to polarity, this would seem to be an item of major importance. Obviously, the transformer ratio cannot give dependable results, since, as is pointed out, the space charge preceding spark-over is of a different nature for the two polarities, and therefore the energy drain, with its consequent loss of voltage is not constant. Tests and oscillograms show, that particularly with a series-multiple type of generator, this factor assumes huge proportions. Since the sphere gap itself is shown to be in error, we might infer that the Lichtenberg figures are employed as a crest voltmeter. But since they also are susceptible to considerable variation, values thus obtained might involve an appreciable error.

The plotted results of Table VIII, given in Fig. 13, show an increase in spark-over voltage for 25-cm. spheres of the positive over the negative, of 35 per cent at diameter spacing. Such an extreme difference seems incredible, and is not in agreement with oscillographic records. Since these curves depend upon an accurate measurement of voltage, it would be very desirable to know how these voltage measurements were obtained.

F. O. McMillan and E. C. Starr: It is interesting to observe that the small amount of data taken on pin type insulators in this investigation is in agreement with the results obtained in the extensive investigation conducted by Messrs. Wade and Smith. Their work made use of the cathode-ray oscillograph for determining the polarity effects. Mr. Smith summarizes their results in his discussion and they are given in more detail in an article published in the *Electrical World* for August 18, 1928. (See bibliography.)

Mr. Rorden in his discussion states that it would be very desirable to know how the voltages were measured during the investigation. The voltages were measured by the use of sphere gaps using special precautions, as explained below, to avoid the limitations they were found to have, due to polarity effects. It was recognized from the beginning of the investigation that precision methods for measuring high voltages are not available. Even the Dufour type of cathode-ray oscillograph usually depends on sphere-gap measurements and the determination of the ratio of a high-ratio potentiometer for its calibration. Furthermore, the cathode-ray oscillograph when used only for comparative purposes in the determination of polarity effects is not free from polarity errors unless the voltage dividing potentiometer and all of its circuits are free of corona during the measurement.

In the event corona is present on the high-voltage elements of the potentiometer the voltage ratio positive is different from the negative ratio because of the widely differing extents of corona for the two polarities. This difference is dependent on the character and spacing of the electrodes.

The impulse generator used in the investigation reported in the paper was carefully calibrated by means of sphere-gaps. It was observed that a marked correlation existed between the impulsive negative output voltage as indicated by sphere-gap measurement and the maximum 60-cycle voltage of the initiating gap. These data are shown in the accompanying table. It will be noted that the "output factor" which is the ratio of the average negative impulse voltage to the maximum 60-cycle spark-over voltage of the initiating gap is practically constant and equal to 2.62 for all output voltages up to the highest value measurable with the available sphere-gap. The 60-cycle data show that the 25-cm. sphere-gap sparks over on both polarities only up to a sparking distance of 8.5 cm. Beyond this spacing and within the recommended range all spark-overs occur on the negative

half cycle. Therefore, it is evident that sphere-gap calibration data apply to the negative voltage and not to the positive throughout the entire range. Therefore, the negative impulse voltages were used for computing the output factor.

Since the initiating gap spacing never exceeded 8.5 cm. the positive and negative tripping potentials were the same and the stored energies were identical for both polarities. The impulse generator is very compact and practically free from stray discharges which insures the delivered energies and maximum potentials being the same for both polarities. To verify these conditions the spark-over voltages, positive and negative were measured for a well isolated 35-cm. point gap 5.25 times its spacing from any grounded object. As determined by output ratio, the average spark-over voltages, positive and negative, as determined from 100 observations, checked within one per cent. A 25-cm. sphere-gap connected in parallel with the point gap and so spaced that it arced over on approximately 10 per cent of the impulses had no influence on the parallel point-gap spark-over either positive or negative, except when it sparked completely over. These data are significant because they show that the impulse generator was delivering the same maximum voltage positive and negative, and furthermore, the positive and negative wave shapes were essentially the same because the point-gap spark-over voltage depends not only on maximum voltage but also on wave shape. From the results obtained above it appears that the energy drain due to discharges preceding complete spark-over have no more effect on one polarity than the other in the particular lightning generator used.

The phenomenon of polarity effect in sphere-gap spark-over is dependent wholly on the flux distribution on the spheres. Therefore, gaps operating under different conditions such as in the vertical or horizontal position, in the proximity of high-voltage conductors or ground will exhibit different polarity characteristics because of the difference in extraneous fields. Observation has been made of a sphere gap which had practically inverted polarity characteristics because of the proximity of high-voltage conductors and its unusually great distance above ground. It is possible in the case of the 100-cm. sphere-gap test reported by Mr. Rorden that extraneous fields were influencing the polarity effect.

VOLTAGE CALIBRATION OF IMPULSE GENERATOR

Impulse generator initiating gap		Impulse generator output voltage 25-cm. sphere-gap		Impulse generator output factor
Sparking distance centimeters A	60-cycle spark-over voltage kv. maximum B	Average sparking distance cm. negative impulses (Table VIII) C	60-cycle spark-over voltage kv. maximum D	$\frac{D}{B}$
3.00	85.6	9.25	221	2.58
4.63	125.	16.5	327	2.61
5.50	144.	23.3	385	2.67
6.16	158.	28.5	415	2.63
6.68	170	34.5	443	2.61
			Average	2.62

NOTE: The first three values given in column D were taken from the A. I. E. E. sphere-gap spark-over voltage data. The last two values were taken from an empirical spark-over curve.

Corona Loss Measurements on a 220-Kv.
60-Cycle Three-Phase Experimental Line

BY JOSEPH S. CARROLL,* LELAND H. BROWN,† and D. P. DINAPOLI‡
Associate, A. I. E. E. Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*Corona loss measurements were made on seven different conductor specimens. The losses were measured directly, on a three-phase line 700 ft. long, by means of three single-phase high-voltage wattmeters. The object of the test was to obtain data for the choice of conductor to be used on a 220-kv. 60-cycle line. The results show the effect of weathering of conductors, the comparison of conductors of two different diameters, the effect of size of strands and the method of stranding and also the effect on new conductors of the change in surface conditions caused by rain and washing.*

* * * * *

Introduction

THE purpose of these corona loss measurements, made at the Ryan Laboratory, Stanford University, was primarily the comparison of different conductors for use on a 220-kv. transmission line. Among the things that were to be studied was the effect on corona loss of the different methods of stranding, size of strands, the spacing and configuration of conductors, the difference between a conductor 0.91 in.

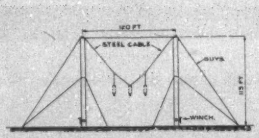

FIG. 1—TRANSVERSE SECTION OF TEST LINE

in diameter and one 1.0 in. in diameter and also the difference, if any, between copper and aluminum conductors.

In order to closely simulate actual operating conditions the tests were to be made on a three-phase line.

Description of Equipment

The test line was 700 ft. long and the conductors were supported between steel masts as shown in Fig. 1 and Fig. 2. With this arrangement the conductors to be tested were laid out on the ground, fastened onto the insulator strings and raised into position by means of winches, one on each mast.

The dimensions involved in fastening the insulator strings on the steel cable were determined mathematically and checked by means of a model.

There were several reasons for starting the test specimen 350 ft. away from the laboratory and continuing it on for two spans of 350 ft. each. The dead ends at the building could be made up once for all and did not have to be lowered for changes in elevation of the line or spacing of the conductors, nor even for a change in configuration from the horizontal to the vertical. Another advantage was that the test line was well in the clear of the building thus eliminating any possibility of stray electric fields in that direction.

The shielded leads from the building to the test specimen were hollow concentric-lay aluminum conductors 1.1 in. in diameter. At the time this cable was made, two No. 12 rubber covered copper wires were placed on the inside of the cable. These two wires were used as the lead from the wattmeter to the test specimen.

The line was energized by means of three 350-kv.-350-kv-a. transformers which were connected delta on the low-voltage side and Y on the high-voltage side with the neutral grounded. The source of power for these transformers was a 1500-kv-a. 60-cycle sine wave motor-generator set. The voltage was controlled by means of a rheostat in the exciter field. The root-mean-square value of the line voltage to ground was obtained by means of the voltmeter coil in the transformers. With the line connected, the reading of the voltmeter across this coil was checked against the voltage from line to ground as measured with a sphere gap. The agreement was within one per cent which is about the limit of accuracy for the sphere gap.

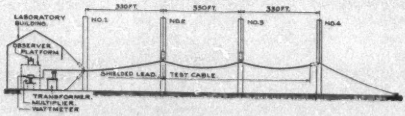

FIG. 2—LONGITUDINAL SECTION OF TEST LINE

Measurements with a harmonic analyzer showed that the third harmonic was the greatest and this was less than one per cent at 260 kv.

The corona loss at 60 cycles was measured directly by means of three high-voltage wattmeters, one connected in on each conductor. The diagram of connections of one of the wattmeters is shown in Fig. 3. Each wattmeter assembly was almost identical with that described in a former paper.[5]

On account of using a sine wave generator and due to

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5. For references see bibliography.

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the reliability of the voltmeter coils in the transformers, the ohmmeter and crest voltmeter described in the paper just referred to were omitted in this set-up. The milliammeter that measured the current through the water resistor wattmeter multiplier was connected in the ground side of the resistor. A three-cylinder piston pump supplied water to all three resistors. The use of salt water for changing the resistance of the multiplier was the same as in the former set-up. The shielding cages containing the wattmeter instruments were mounted directly on top of the upper shields of the water resistors. The meters were read by means of telescopes from platforms directly above.

In all of these tests, temperatures and relative humidities were determined outside near the line, readings being taken immediately before and after each test. The wet and dry bulb thermometers were placed in front of a fan for consistent results.

Shielding Tests

At the beginning of the tests it was necessary to make certain balances and adjustments of the shielding

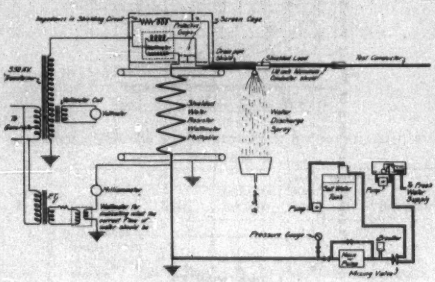

Fig. 3—Wattmeter Connections

circuits. From previous tests, most important of which was the double conductivity test, and also with further check on the present set-up it was found that the water resistor multiplier was, for all practical purposes, correctly shielded.

The shielded lead from the wattmeter to the test conductor was 440 ft. long. The capacitance between this lead and its shield was 0.04 microfarad. This capacitance shunts the field or current coil of the wattmeter and due to the low power factor of these measurements the current through this capacitance would shift the phase angle of the current through the field coil of the wattmeter and cause serious error. This capacitance cannot be eliminated; however, if the potential of the shield is made the same as that of the lead there can be no flow of current through the capacitance. This was accomplished by inserting an impedance in the shielding circuit as shown in the diagram of Fig. 3. The constants of the impedance in the shielding circuit were obtained as follows: The charging current to the test specimen was measured as was also the charging current to the shielding circuit. The inverse ratio of these currents gives the ratio of the impedances in these two circuits. The resistance and inductance of the current coil of the wattmeter are given by the maker. With the above information, coils were designed and made up for the three shielding circuits. One very good check of the proper shielding of the circuits was made by using a wattmeter of a different range and impedance in each of the three phases. Three tests were made and for each test each of the three wattmeters was in a different phase. Therefore, for each phase three sets of readings were obtained each with a different wattmeter. Each change of wattmeters involved a change in the shielding impedances. After slight errors in the shielding had been located and eliminated the measured loss was the same irrespective of the wattmeter arrangement.

Observations were made on the line at night. The voltage at which corona was first seen and also heard on the conductor agreed with the voltage at which the wattmeters began to show a power loss from the line. Corona brushes were observed on some of the insulator units next to the conductor. Proper shielding of the insulator strings eliminated these brushes. Since the losses on one suspension and one dead end string were included with the losses of each conductor it was necessary to determine the magnitude of such insulator losses. The losses were measured on each of the suspension strings that support the conductors at the point where the shielded lead connects to the test specimen. The losses were found to be as follows at a voltage between lines of 295 kv. with 13 units in each string:

Relative humidity	Loss on string No. 1	No. 2	No. 3
45%	3.5 watts	5.0 watts	5.0 watts
76%	60.0 watts	78.0 watts	101.0 watts
81%	132.0 watts	124.0 watts	172.0 watts

From the values in the above table it is obvious that insulator losses can be neglected for fair weather conditions such as existed when the corona loss curves given in this paper were taken. Corona loss data not shown in this paper were taken at the higher humidities, but on account of insulator loss it is difficult to say just how much the loss was increased by the higher humidities. One thing was definitely noted however, and that was that the corona loss was less for the lower temperatures. This is as might be expected because the density of the air increases with a decrease in temperature. In this work time would not permit the studying of corona loss as affected by weather conditions to any great extent, besides the condition of most of the conductors under test was changing too rapidly for such studies. It is hoped that at some time in the future the effect of weather conditions on corona loss may be studied.

Corona Loss on Different Conductors

The first conductor tested was a 0.91-in. diameter rope-lay weathered copper cable. (See appendix for a description of these conductors.) This conductor had been in use on the 220-kv. Pit River line of the Pacific Gas and Electric Co. for several years. A section 2200 ft. long had been taken off the line especially for this

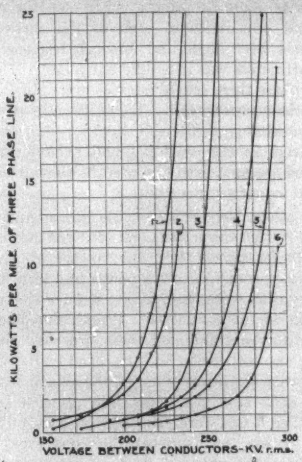

Fig. 4—Corona Loss Curves

CONFIGURATION—20 FT. HORIZONTAL BY 30 FT. MIN. TO GROUND

Curve No.	Conductor	Date	Humidity per cent	Temp. deg. fahr.	Barometer in. hg.	Remarks
2	0.91 in. rope lay copper weathered	Mar.31,1930	63	53	29.85	
1	0.91 in. rope lay copper new	Apr.30,1930	36	72	29.68	Before washing
3	0.91 in. rope lay copper new	May 1, 1930	52.5	67.5	29.90	After washing
4	1.0 in. steel core aluminum weathered	May29,1930	40.5	65	29.91	
6	1.0 in. steel core aluminum unused	May23,1930	35	80	29.94	After washing
5	1.0 in. steel core aluminum unused	May26,1930	57.5	63.5	30.01	After washing

test. As the conductor was removed from the towers it was carefully wrapped with strips of burlap in order to preserve the condition of its surface. In raising the conductor on the test line the burlap was not removed until the conductor had cleared the ground. To aid in the determination of the change in corona loss due to weathering, a new conductor of the same kind was next tested. (See Fig. 4)

It should be noted that for all the loss curves in this paper except those in Fig. 9 and Fig. 10, the corona loss per mile of a three-phase line is plotted as a function of the voltage between lines. The loss per mile of line was chosen in preference to the loss per conductor mile because the loss on the three conductors is not the same; another reason is that the engineer usually thinks of a three-phase line as a unit.

The next tests were on a new two-layer concentric hollow copper cable 0.91 in. in diameter and a new two-layer concentric hollow copper cable 1.0 in. in diameter. While this latter conductor was in place it rained during the night; a few measurements were made during the rain the next day and some immediately after; the

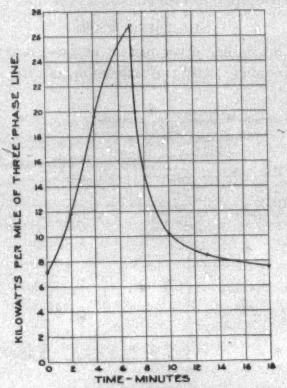

Fig. 5—Corona Loss During Shower at 220 Kv. Between Lines

Conductor— 1.0 in. hollow concentric two layer copper
Date— April 14, 1930
Humidity— 100 per cent
Temperature—58 deg. fahr. (taken in the laboratory)
Barometer— 30.05 in. hg.

losses during the rain were rather high but the rainfall was not steady enough to obtain a good loss curve. The curve in Fig. 5 shows the variation in loss at 220 kv. from the beginning to the end of a short shower. No measurement was made of the rate of rainfall. The loss measurements made on this conductor the day after the rain showed the loss to be much higher than before the rain. (See Curves 1 and 2, Fig. 6) Evidently some change had taken place on the surface of the conductor. The new copper conductors were somewhat greasy, apparently left that way from some process of their manufacture. Since the rain had made such a definite increase in the losses on the new conductor it was decided to see what effect washing it would have. The process of washing was as follows: The line was

lowered and the conductors were supported on wooden horses. The cables were then scrubbed with gasoline, the excess gasoline being wiped off and the rest allowed to evaporate. This was followed by a scrubbing with

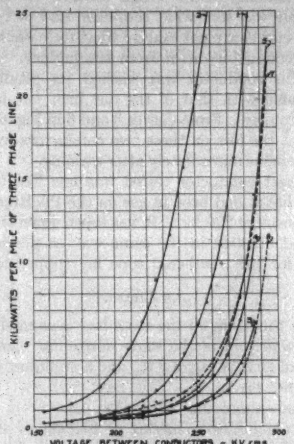

FIG. 6—CORONA LOSS CURVES

CONFIGURATION—20 FT. HORIZONTAL BY 30 FT. MIN. TO GROUND

Curve No.	Conductor	Date	Humidity per cent	Temp. deg. fahr.	Barometer in. hg.	Remarks
1	1.0 in. hollow concentric two layer copper	Apr. 11, 1930	45	69	29.96	Before washing
2	1.0 in. hollow concentric two layer copper	Apr. 15, 1930	59	63.5	30.05	After rain
3	1.0 in. hollow concentric two layer copper	Apr. 23, 1930	52	68	29.90	After washing
4	1.0 in. hollow concentric two layer copper	Apr. 28, 1930	59.5	62.5	30.00	After washing
5	1.0 in. steel core aluminum (unused)	May 21, 1930	26.5	69.5	30.16	Before washing
6	1.0 in. steel core aluminum (unused)	May 23, 1930	35	80	29.94	After washing
7	1.0 in. steel core aluminum (unused)	May 26, 1930	57.5	63.5	30.01	After washing

soap suds which were washed off by means of a cloth and plenty of clean water. To remove any further traces of soap that might be left the conductors were again washed with running water and scrubbing brushes. The line was then raised into place and when dry was tested. The scrubbing brushes used in all this washing process were of the common variety with fiber bristles.

The surprising thing was, that while the rain had left this conductor in a condition such that the losses were higher than those before the rain, the effect of the washing was to reduce the losses to less than those before the rain. (See curves in Fig. 6) The condition of the conductor was carefully inspected before and after washing and the only visible effect that the scrubbing

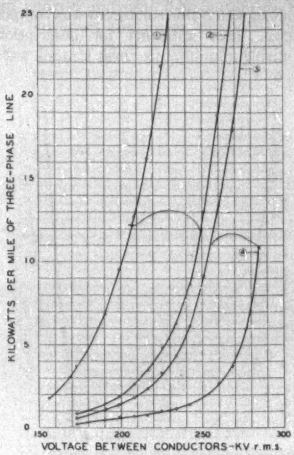

FIG. 7—CORONA LOSS CURVES

CONFIGURATION—20 FT. HORIZONTAL BY 30 FT. MIN. TO GROUND

Curve No.	Conductor	Date	Humidity per cent	Temp. deg. fahr.	Barometer in. hg.	Remarks
1	1.0 in. hollow concentric single layer copper	May 12, 1930	57	64	30.04	Before washing
3	1.0 in. hollow concentric single layer copper	May 14, 1930	60	60	30.02	After washing
2	0.91 in. hollow concentric two layer copper	May 7, 1930	46	60	29.99	Before washing
4	0.91 in. hollow concentric two layer copper	May 8, 1930	33	64	29.87	After washing

seemed to have was to remove some of the stains which had been left by the rain. The strands of this particular conductor were very free from slivers and burrs. A week later when the line was taken down it was noted that the copper had oxidized much more rapidly after washing than before, indicating that the grease had been protecting its surface. Measurements made a few days after the conductor was washed showed a

slight increase in losses above those measured immediately after washing. (See Curve 4 of Fig. 6)

The other two new conductor specimens, the 0.91 rope-lay and 0.91 concentric, that had already been tested and taken down were again put up and tested, lowered and washed in the same manner just described, and then were raised and tested again. The results are shown in the curves of Fig. 4 and Fig. 7. The next conductor to be mounted on the line was a single-layer concentric hollow copper cable 1.0 in. in diameter. The strands of which this conductor was composed were rather large. (See appendix for description.) Apparently during manufacture these strands were scored,

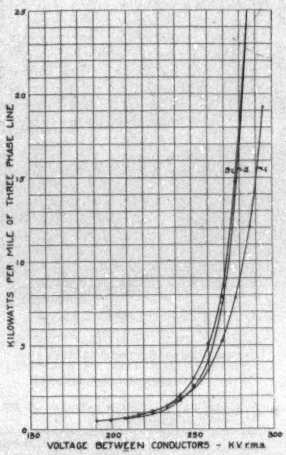

FIG. 8—CORONA LOSS CURVES—DIFFERENT CONFIGURATIONS

CONDUCTOR—1.0 IN. STEEL CORE ALUMINUM (WEATHERED)

Curve No.	Configuration	Date	Humidity per cent	Temp. deg. fahr.	Barometer in. hg.
1	20ft. hor. by 30ft. min. to gr.	June4, 1930	35	77.5	30.00
2	15ft. hor. by 30ft. min. to gr.	June5, 1930	28.5	89	29.96
3	15ft. vert. by 30ft. min. to gr.	June6, 1930	30	83	29.83

leaving slivers and burrs on the finished cable which makes impossible a fair comparison between this conductor and the other one inch copper conductor made up of smaller strands. It is interesting to note that the losses on this conductor before washing were much greater than any of the others. Washing greatly reduced the loss; however, it was still high. See Fig. 7. Just how much of this increase in loss above that of the other one inch concentric copper conductor was due to the larger strands, of which the cable was composed, is still a question, because while the process of washing removed practically all the slivers, the strands themselves were still rather badly burred.

The next specimen tested was a new concentric-lay aluminum conductor 1.0 in. in diameter. This cable was not quite in the same condition as it was when it left the factory. Evidently care had not been taken in the handling before it had reached the laboratory. The cable was somewhat scratched and dirty, so that the loss curves are not exactly what they would be for a similar conductor direct from the factory. However, when this conductor was washed its condition was comparable with that of the one inch two-layer concentric copper. From curves 3 and 6 in Fig. 6 it can be seen that the loss on these two conductors was practically the same immediately after washing.

The next conductor tested was one like the above except that it had been in use on the Pit River line for several years at 220 kv. It was taken down at the same time and handled in the same way as previously described for the weathered rope lay copper conductor.

LINE CHANGES

Up to this time all the tests had been made with all three conductors in the same horizontal plane with a spacing of 20 ft. a sag of 8 ft., and a minimum clearance to ground of 30 ft. The next test was with the weathered aluminum with the conductors in horizontal configuration, having a spacing of 20 ft., at three different elevations beginning with 70 ft. clearance to ground, followed by tests at 30 and 20 ft. clearances. The losses with the line at these three elevations were practically the same, the differences being so small that it seemed unnecessary to show the three curves.

The spacing between conductors was then changed from 20 ft. to 15 ft. with a clearance to ground of 30 ft. The results are shown in curves 1 and 2 in Fig. 8. The conductors were then supported vertically all in the same plane with a 15 ft. spacing and a clearance to ground from the lower conductor of 30 ft. The loss curve is shown in curve 3, Fig. 8.

As can be seen from the curves there is not much difference in loss for the different spacings and configurations. This is as might be expected because the electric field about the conductor is the resultant of the field to ground and the field between conductors. Since the field to ground is at a different phase angle to that between conductors, changing the distance between conductors alters only one component of this field; the same is true with respect to changing the distance of the line to ground. Furthermore the field intensity varies only as the logarithm of the dimensions.

WATTMETER READINGS AND CHARGING CURRENTS

With the three conductors in the same plane the loss on the center one is greater than on either of the outside conductors, because the electric field intensity is greater on the center wire. In this paper no attempt has been made to determine the loss on each conductor separately. For those who are interested in this, the

curves in Fig. 9 and Fig. 10 show what is apparently the loss on each conductor for different configurations. This "apparent loss" was computed from the wattmeter readings in the ordinary manner by multiplying the reading of the wattmeter instrument by the multiplying factor computed for the water resistor wattmeter multiplier. It will be noted that with the three conductors in the same horizontal plane the apparent

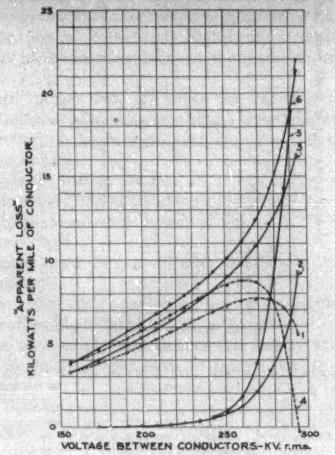

FIG. 9—CURVES, SHOWING "APPARENT LOSS" ON EACH PHASE FOR 20 FT. AND 15 FT. HORIZONTAL SPACINGS

CONDUCTOR—1.0 IN. STEEL CORE ALUMINUM (WEATHERED)

Curve No.	Configuration	Date	Humidity per cent	Temp. deg. fahr.	Barometer in. hg.	Phase
1	20 ft. hor. by 30 ft. min. to gr.	June4, 1930	35	77.5	30.00	1
2						2
3						3
4	15 ft. hor. by 30 ft. min. to gr.	June5, 1930	28.5	89	29.96	1
5						2
6						3

loss for one of the outside conductors is negative, while for the other outside conductor the apparent loss has a large positive value and the loss shown for the center conductor is approximately the actual loss. The algebraic sum of the apparent losses for the three conductors at any particular voltage is the actual loss for the three-phase line at that voltage. This is because the power in this three-phase neutral grounded circuit is measured with three wattmeters. It is interesting to note that the algebraic sum of the apparent loss on the two outside conductors divided by two is almost equal to that shown for the center conductor. If the line could be suspended high enough so that the ground had no effect on it and if the three conductors were equidistant from each other the charging currents would all be equal and 120 deg. apart; the loss on the three conductors would be the same and the three wattmeters would read alike. Of course this condition is far from that met with in practise. The complete vector diagrams and explanations of the circuits used in this work would be almost material enough for another paper. Mr. R. J. C. Wood gives a very good explanation of some of this in his paper listed in the bibliography.[2]

When the conductors were all mounted in the same vertical plane the phase rotation of the line was changed giving an entirely different distribution of wattmeter readings. However, the losses computed from these were practically the same as before so this curve is not

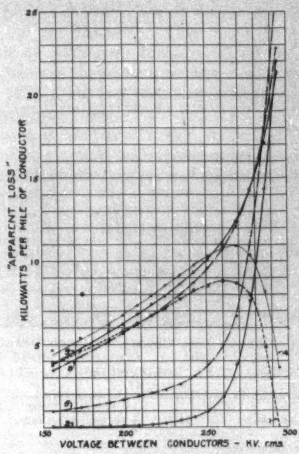

FIG. 10—CURVES SHOWING "APPARENT LOSS" ON EACH PHASE FOR DIFFERENT CONFIGURATIONS

CONDUCTOR—1.0 IN. STEEL CORE ALUMINUM (WEATHERED)

Curve No.	Configuration	Date	Humidity per cent	Temp. deg. fahr.	Barometer in. hg.	Phase
1	15 ft. hor. by 30 ft. min. to gr.	June5, 1930	28.5	89	29.96	1
2						2
3						3
4	15 ft. vert. by 30 ft. min. to gr.	June6, 1930	30	83	29.83	1
5						2
6						3

shown in this paper. For the curves shown in Fig.10 the phase rotation of the line was tested and found to be in the same order as the numbering of the conductors. The curves shown in this paper are all plotted from data taken with this phase rotation.

Corona loss curves taken immediately after each of the five conductors were washed are shown in Fig. 11. It is believed that the surface conditions of the conductors were more nearly alike during these tests and hence these curves offer the fairest comparison of the conductors.

Fig. 12 shows some of the corona loss data plotted on semi-logarithmic cross section paper. The study of the slopes of the different curves is rather interesting.

Charging currents to the line were measured and when plotted against line voltage gave practically

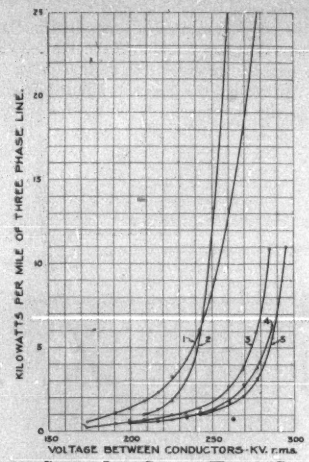

Fig. 11—Corona Loss Curves—Washed Conductors

CONFIGURATION—20 FT. HORIZONTAL BY 30 FT. MIN. TO GROUND

Curve No.	Conductor	Date	Humidity per cent	Temp. deg. fahr.	Barometer in. hg.
1	1.0 in. hollow concentric single layer	May 14, 1930	60	60	30.02
2	0.91 in. rope lay	May 1, 1930	52.5	67.5	29.90
3	0.91 in. hollow concentric two layer copper	May 8, 1930	33	64	29.87
4	1.0 in. hollow concentric two layer copper	Apr. 23, 1930	52	68	29.90
5	1.0 in. steel core aluminum used	May 23, 1930	35	80	29.94

straight line curves. The values of charging currents for different conductor configurations are shown in the following table for a voltage between lines of 220 kv. and a minimum clearance to ground of 30 ft.

Configuration	Spacing	Conductor	Charging current per mile of conductor
Horizontal.....	20 ft.	Outside	667 milliamperes
		Center	717
		Outside	675
Horizontal.....	15 ft.	Outside	694
		Center	754
		Outside	705
Vertical.....	15 ft.	Top	690
		Center	758
		Bottom	709

The power factor computed for the line, when the washed aluminum conductor was used, was less than one-quarter of one per cent at 220 kv.

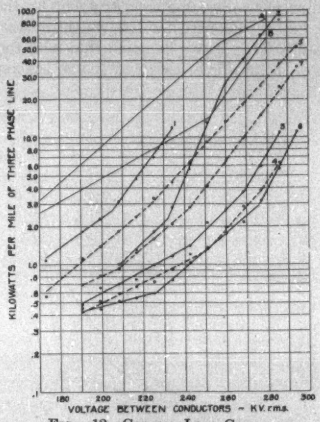

Fig. 12—Corona Loss Curves

CONFIGURATION—20 FT. HORIZONTAL BY 30 FT. MIN. TO GROUND

Curve No.	Conductor	Date	Humidity per cent	Temp. deg. fahr.	Barometer in. hg.	Remarks
1	0.91 in. rope lay copper weathered	Mar. 31, 1930	63	53	29.85	
2	0.91 in. rope lay copper new	May 1, 1930	52.5	67.5	29.90	After washing
3	0.91 in. hollow concentric two layer copper	May 8, 1930	33	64	29.87	After washing
4	1.0 in. hollow concentric two layer copper	Apr. 23, 1930	52	68	29.90	After washing
5	1.0 in. hollow concentric single layer copper	May 14 1930	60	60	30.02	After washing
6	1.0 in. steel core aluminum unused	May 23, 1930	35	80	29.94	After washing
7	1.0 in. steel core aluminum weathered	May 29, 1930	40.5	65	29.91	

Curve	Conductor	Configuration	Remarks
A	0.91 in. rope lay copper	19 ft. horizontal by 30 ft. min. to ground	Data from Wilkins[4]
B	1.0 in. steel core aluminum	19 ft. horizontal by 30 ft. min. to ground	Data from Wilkins

Comparison of this work with that done by others has been purposely omitted on account of space limitation. A bibliography is given at the end of this paper for those not acquainted with such work.

Conclusions

The results of the tests show that the corona loss on new cleaned conductors is the same for copper as it is for aluminum.

The effect of weathering of the conductors, as received from the factory, was such as to decrease corona loss, while the effect of weathering of the new conductors after washing was to increase the loss.

Corona loss over the operating range of voltage of a 220-kv. line is affected only slightly by a considerable change in conductor spacing or configuration, or clearance to ground.

Until further tests are made the use of strands as large as those of the one inch single-layer hollow copper conductor, on a one inch conductor, for 220 kv. will be questionable.

While the loss on the washed 0.91-in. hollow concencentric copper conductor is very low at 220 kv. the use of such a cable is not recommended for a 220-kv. line because there is not sufficient margin to allow for the increased loss at higher altitudes and for the accumulation of dirt.

The losses on a rope-lay conductor are greater than on a concentric-lay cable of the same diameter.

The amount of decrease in corona loss by washing a new conductor would seem to justify the removal of the grease on the conductor at the factory rather than wait for the weather to do this at a considerable loss of power.

Acknowledgments

The authors are indebted to Dr. Harris J. Ryan for his helpful advice and suggestions.

The tests described in this paper were made possible through the cooperation of the Pacific Gas and Electric Company.

The hollow aluminum conductor used for the shielded wattmeter lead was donated to the laboratory by the Aluminum Company of America.

The three specimens of hollow copper conductor used in these tests were contributed by the Anaconda Wire and Cable Company and are to be left at the laboratory.

Appendix

Description of Conductors

Rope-Lay Conductor 0.91 in. Diameter. This conductor is made up of seven large strands, each of which is made up of seven wires 0.101 in. in diameter with a left hand lay and a pitch of 5 in. The seven large strands have a right hand lay and a pitch of 15 in. There were two specimens of the above conductor, one new and one which had been in use on the 220-kv. Pit River line of the Pacific Gas and Electric Co. for several years.

Two-Layer Concentric-Lay Hollow Copper Conductor, 0.91 in. Diameter. There were 24 wires in the outer layer 0.101 in. in diameter. The pitch of this layer was approximately 12 in. The inner layer was made up of 14 wires 0.125 in. in diameter. The core consisted of two rubber covered pilot wires. (This conductor was made up especially for these tests and is not a commercial cable.)

Two-Layer Concentric-Lay Hollow Copper Conductor, 1.0 in. Diameter. This conductor was composed of 50 wires 0.097 in. in diameter. The 28 wires in the outer layer had a pitch of approximately 11 in. The core of the cable was a twisted copper *I*-beam.

Single-Layer Concentric-Lay Hollow Copper Conductor, 1.0 in. Diameter. This one layer was made up of twelve wires 0.196 in. in diameter and had a pitch of approximately 8.5 in. The core was a twisted copper *I*-beam.

Two-Layer Concentric-Lay Steel Core Aluminum Conductor, 1.0 in. Diameter. This conductor was composed of 42 aluminum wires 0.112 in. in diameter. The 24 wires in the outer layer had a pitch of approximately 11 in. The core consisted of 19 strands of 0.111 in. diameter steel wire.

The area of the cross section of each of the above described conductors, with the exception of the 0.91 in. concentric-lay hollow copper was approximately 500,000 circular mills.

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Discussion

L. V. Bewley: A great amount of work has been done in the past on corona loss at normal power frequencies. We are now entering an era where it is of increasing importance to know something about the laws of corona under transient, and particularly traveling wave conditions. This requirement is occasioned by the study of abnormal surges on transmission lines, and means of protection against them. Corona losses are responsible for the rapid attenuation and distortion of high-potential traveling waves. The attenuation is known to be extremely sensitive to weather conditions, wave shape, polarity and voltage, and varies over a wide range, even for the same line. However, these separate effects have not been evaluated, nor has the law of corona been established for traveling waves. Therefore, from the point of view of transmission system protection, it would seem advisable for the experimenters to pay some attention to *transient* corona loss. A knowledge of the law which governs these losses may point the way to at least the partial control of dangerous traveling waves.

A. O. Austin: Anyone who has not made tests to determine corona loss can hardly appreciate the amount of time, effort, and expense required to obtain the information disclosed in the paper.

The various factors affecting corona are too complicated to be

covered by a simple formula except for a very limited range, hence we must depend upon measurements for losses. At the present time there is need for information on the larger conductors covering a considerable difference in the size of wire. Laws which may hold for a small conductor may be misleading for the large sizes of conductor and it is hoped that the work at the Ryan Laboratory will be continued.

Owing to the high reactive drop at 60 cycles, it is advisable to use higher voltages for many installations in order that the capacity per circuit in kilowatts may be increased and the cost of transmission reduced.

Information such as that included in the paper is, therefore, very valuable in making it possible to work within narrow limits.

Corona is not only a loss in power but in recent years has been a source of radio interference which must be carefully watched. In fact radio interference may become a factor in determining the size of a conductor on many lines rather than loss from corona. This is particularly true where the amount of power transmitted will not require a large conductor, such as in taps from large trunk lines. There are many transmission lines in operation in which corona is a source of radio disturbance. Where these lines were installed in advance of radio reception, complaints have not been serious. Where, however, a new line is located in a moderately or thickly populated district, serious complaints are likely to result if the conductor size is too small.

It is possible that a conductor having a rather high corona loss may not be as serious a source of radio interference as a larger conductor with less corona loss. In the large conductor, the corona or discharge starts only after there is considerable potential on the conductor, hence the radio disturbance may be more severe than for a smaller conductor which discharges initially at a relatively low voltage on the wave. For a large range of conductor sizes, however, it would appear that the magnitude of the radio disturbance curves will be approximately the same as that for corona losses shown in the paper. Fig. 1 shows such a series of tests run on several different sizes of conductor.

In order to show that the disturbance is due to a discharge on the positive wave, a series of tests was run under the conditions in Fig. 1 by using one large conductor which would be well above the corona point. Fig. 2, A, B, and C show the disturbances as indicated by oscillograms.

It is possible that some method may be devised which will make it possible to determine the corona loss by a radio interference method as a means of supplementing work similar to that carried out in the Ryan laboratory.

The oscillograms show that the radio interference from a coated conductor Fig. 3 has been greatly reduced. Since a slightly increased dielectric strength in the air will greatly increase the corona point, it is evident that a coating having a slight increase in dielectric strength over that of the air may be very beneficial in reducing corona loss or radio interference. Figs. 2 and 3 show the comparison between a plain and a coated conductor in which it will be seen that radio disturbance and corona loss have been materially controlled. Fig. 4 shows two types of coated conductor one with control or projecting strands and the other with all strands coated with insulating paint or varnish.

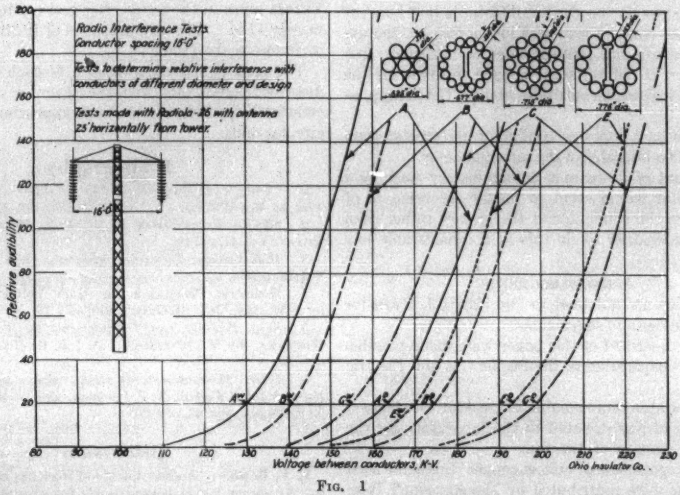

FIG. 1

It is possible that conductors of this kind may find a use in clearing up radio trouble or for operating at very high voltages or at high altitudes, as means have been worked out for coating conductors in place.

Longer transmissions at higher voltages make information as to corona losses far more important than in the past and it is hoped that the work at the Ryan Laboratory will be extended as the information is of great economic importance to the transmission industry.

R. S. Daniels: In November, 1929, we put in operation, 65 miles of 130-kv. line, and almost immediately experienced serious radio trouble. This trouble disappeared considerably in about three months from no reason that could be accounted for, other than that it was due to corona which in the meantime had decreased.

The noise was readily detected from the ground by ear, and was about the same in the middle of a span as at the supports. This was checked on a span over 2000 ft. long, where the observer was about the same distance from the conductor when in the center of the span as when at the foot of the towers.

The conductor was Anaconda composite, stranded cable 2/0 conductivity, with a three-strand bronze core. At one end the line joined an old line of 2/0 copper, operating at the same voltage. Both lines were similarly insulated and constructed on

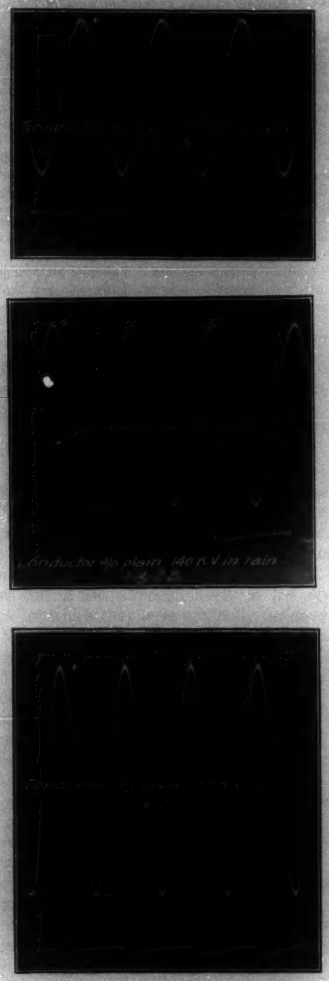

Fig. 2

wood "H" frames, and no similar noise was observed under the old line.

E. Van Atta: It has been shown in this paper that corona losses were increased due to rain and that these losses remained

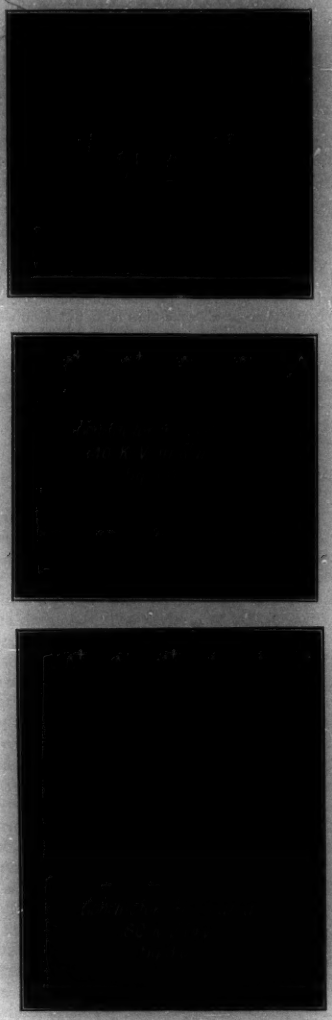

Fig. 3

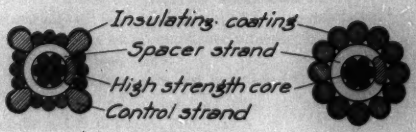

Fig. 4

high after the conductors had dried off. This fact is rather mystifying.

J. B. Whitehead: Corona begins on a smooth round wire at a definite value of surface voltage gradient, dependent on the pressure and temperature of the air. Corona energy loss is due principally to the frictional motion of gaseous ions formed in the corona through the air towards the opposite conductor. The magnitude of the loss depends on the number of ions generated, their size, the distance they travel, their specific mobilities, and on the temperature, pressure, and moisture content of the air.

The lack of uniformity and of definite indications of the results given in the paper by Messrs. Carroll, Brown, and Dinapoli is due to the impossibility of controlling or even measuring all of the factors which enter. The surface conditions of conductors are irregular, unknown, and uncontrolled. Corona begins at local spots and increases irregularly, the sizes and mobilities of the ions vary with the atmospheric conditions in an unknown way, and the length of travel of space charge is uncertain in the unsymmetrical arrangement of the conductors.

The same uncertainty has characterized all measurements of corona loss on lines in the open since those of Mershon, who first emphasized the unknown influence of atmospheric conditions. Peek has attempted to take account of some of the unknown variables by the introduction of empirical constants into his formulas, but they do not cover all the factors entering, and the results of Ryan, Harding and others show only approximate agreement.

The results of the present paper will have a certain value as the first measurements at 220 kv. It is regrettable, however, that so elaborate and carefully developed a measuring equipment should yield such indefinite results. In the writer's opinion the greatest value is in the indication that corona loss can never be definitely computed in advance until a greatly simplified conductor surface is adopted, and the laws of the influence of atmospheric conditions are uncovered by controlled experimentation.

Development of the Porcelain Insulator

BY K. A. HAWLEY*
Member, A. I. E. E.

Synopsis.—*Porcelain insulators have been manufactured and used for the transmission of high-tension electric power for forty years. The first designs were of the single-piece and multipart cemented pin type. Necessity for higher safety factors against flash-over and increase in operating voltages demanded a rapid increase in the size of the insulators. This reached an economic limit at the operating voltage of 66 kv. The suspension unit overcame this temporary check of increased operating voltage.*

Further study of the electrostatic capacitance of the various parts and consequent voltage distribution, made marked refinements in the pin type insulator possible. During this time the single-piece porcelain suspension unit took practically its present form.

Early improvements were the provision of proper expansion joints and the separation of the lip of the cap from the porcelain hood.

Gradual improvements have since been made resulting in a great increase in mechanical strength. These changes have been principally in hardware design. By experiment and analysis the shapes of the cap and pin have been determined to give a uniform distribution of load from the pin to the cap. Constant check tests by the quick pull and time loading methods have shown that the suspension insulator with properly designed hardware and a suitable coating on the cap to prevent the cement from adhering to the metal, has a high strength associated with electrical reliability.

Ceramic research and exact manufacturing control has made possible the production of non-absorbent, thoroughly vitrified porcelain of consistent strength. This has centered largely about the proper firing of the clay.

Recent experiments upon the properties of the combination of porcelain and glaze has eliminated surface stress and consequently assured stronger, longer lived porcelain. Still greater uniformity has been gained by glazing the sanded surfaces.

The elimination of the abutting joint and the proper design of the cemented joint has stopped expansion troubles. Proper use of Portland cement has resulted in insulators able to withstand drastic temperature changes without harm. A recent improvement in the pin type insulator is the metal threaded pin hole. This has lessened manufacturing and construction difficulties and in addition, due to the exact fit of the insulator on the pin, overcomes hidden corona and the consequent radio interference.

* * * * *

PORCELAIN insulators for the transmission of high-tension electric power have been made and used for a period of approximately forty years. Progress in their design and manufacture can be divided into ten year periods.

The last decade of the nineteenth century marked practically the beginning of electric transmission in America and also the beginning of the continuous manufacture of porcelain insulators. Insulators of this decade were all of the pin type, both single-piece and multipart cemented types. Many of these old insulators are still in use, although rarely at the operating voltages for which they were originally designed. The safety factor against flash-over was rarely sufficient for practical requirements so where the voltage has remained the same larger insulators have in most cases been substituted.

During the first ten years of the twentieth century pin type insulators advanced rapidly in size. Even at this early date it has reached its maximum economical size. For 66,000-volt transmission, pin type insulators proved to be quite satisfactory, but since the cost of the insulator varies as a power of its rating larger insulators of this type were not practical from a cost standpoint. The suspension insulator was the logical outcome of this temporary check. Comparison of the present pin type insulators shows that the cost is proportional to the 2.6 power of its flash-over value. The suspension insulator, however, offers a resistance to flashover in direct proportion to the string cost. Plotting separate cost curves for equal arc-over values show that the curves cross at approximately the 66,000-volt operating range. This corresponds very closely to the usual practise and for the higher voltages the suspension insulator has almost invariably been used.

During the next decade from 1910 to 1920, marked refinements in the pin type insulator were made. The earlier insulators were little more than porcelain cups cemented together with but a slight understanding of voltage division.[1] The division of voltage is inversely proportional to the electrostatic capacities of the respective parts. These in turn are proportional to the adjacent conducting surfaces upon the porcelain, the cement being a conductor. As these surfaces were generally small on the central and top shells, these two parts carried most of the voltage. By careful study this condition was rectified and the correct balancing of the pin type insulator was one of the decided advances of this period.

During this decade the suspension insulator took practically its present form. The unit using two pieces of porcelain, while in many ways satisfactory, necessitated a larger head diameter and a correspondingly large size of metal cap. The single porcelain unit removed these unsatisfactory features and was accordingly almost universally adopted.

Experience promptly pointed out necessary refinements. Proper expansion joints were provided and the cap which originally had been allowed to rest on the porcelain was lifted, so that under all conditions it stayed clear of the hood. (Fig. 1.) With these improvements practically all field failures stopped. Insulators even of the largest size have established a perfect life record over a period of fifteen years.

During the ten years just completed there has been a

*Chief Engineer, Locke Insulator Corporation, Baltimore, Md.

Presented at the Pacific Coast Convention of the A. I. E. E., Portland, Oregon, Sept. 2-5, 1930.

1. *Insulator Depreciation and Effect on Operation*, A. O. Austin, A. I. E. E. TRANS., 1914, p. 1731.

30-148

decided improvement in the suspension insulator. This improvement is not the result of any radical changes but has been rather a matter of refinements of design and manufacture. Slight changes in hardware shape, for example, have resulted in a more uniform distribution of the load delivered to the shell with a consequent greater uniformity and higher average mechanical strength. The desirability of such improvements is constantly evident. Unexpected loads greatly in excess

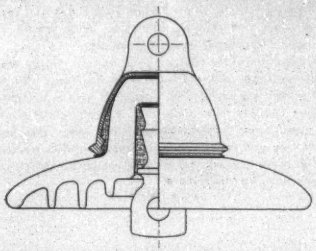

FIG. 1—EARLY 10-INCH SUSPENSION UNIT WITH EXPANSION JOINT AT EDGE OF CAP

of maximum calculated loads have been encountered and the strength of the insulator has been exceeded.[2] Three such failures upon insulators of the old type have occurred in the past two years in widely separated localities.

It was at one time thought that the strength of the insulator was limited by the strength of the standard cement used, breakage resulting in loading tests from crushing and shearing of the cement and resulting in unfavorable loads on the porcelain. This is not so. A

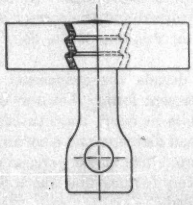

FIG. 2—EYE BOLT CEMENTED INTO STEEL RING FOR A TENSION TEST OF CEMENT STRENGTH

standard pin when cemented into a metal ring held consistently loads as high as twice the breaking strength of the insulator.

Following this a cup of porcelain was prepared representing the head of an insulator. Into this a standard pin was cemented in the usual manner and the cup was then supported on its lower edge on a metal ring. When pulled in this manner the standard pin was broken before it could be pulled from the cup. There was no evidence of bursting the cup unsupported except at its lower edge.

Following the many efforts to improve the pin shape, attention was directed to the cap. An analysis based upon directions of stress from cap to pin indicated possibilities of improvement. Changes in caps were made by filling the cap with babbitt and turning to the desired shape and as high as fifty per cent increase in combined electrical and mechanical strength resulted.

These caps when copied in malleable iron, however, showed no such improvement in performance. A close scrutiny of the previous tests on the caps containing the babbitt showed that under strain a slight slipping took place between the cap and the cement. Evidently the malleable iron caps must also be made to yield upon the cement. This research led to the painting of the inside surfaces of the cap with a suitable coating to prevent the

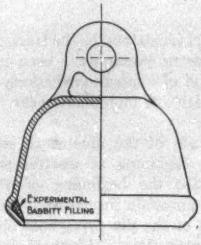

FIG. 3—CAP WITH BEARING SURFACE FILLED WITH BABBITT TO DESIRED ANGLE

cement from adhering to the metal. When this was done the desired results were obtained.

It was soon recognized that strength in quick pull-off was not always an indication of reliability. High strength against pulling apart must be associated with electrical reliability. The comparison of various available standard rated insulators brought interesting results. The insulator that had been heralded as the strongest showed on time test the earliest electrical failure. The following table shows this in detail.

The last two columns in the table show the results of the previous research. The provision of a yielding abutment to the arch of the insulator with no changes otherwise had greatly strengthened the unit. By no known test could any hazard be shown.

Since then, constant checks and rechecks with slight changes in shapes have further improved the insulator. All such changes have been tried first by the quick pull (A. I. E. E. Standard 41-305; two thousand pounds increase per minute) and by time loadings. Such time tests are made in out-of-door frames subject to con-

2. *Transmission Line Construction in Crossing Mountain Ranges*, M. T. Crawford, A. I. E. E. TRANS., 1923, p. 970.

TIME LOADING TESTS UPON STANDARD SUSPENSION INSULATORS

Load lb.	Insulator A		Insulator B		Insulator C		Insulator D		Insulator D revised	
	Hours held	Result	Hours held	Result	Hours held	Result	Hours held	Result	Hours held	Result
	String of 6		String of 6		String of 6		String of 12		String of 6	
6,000	150	O. K.	216	O. K.	96 72	2 *E* 3 *E*	168	O. K.	192	O. K.
7,000	168	O. K.	15	*M*	144 24	3 *E* 4 *E*	15	*M*	168	O. K.
8,000	48	*M*			96	4 *E*			168	O. K.
9,000					240	4 *E*			168	O. K.
10,000					168	5 *E*			24	*M*
11,000					168	5 *E*			24	*M*

E—Electrical failure. Numeral indicates total number failed.
M—Complete mechanical failure of one unit in string.

siderable vibration and all changes of the weather at both Victor, New York and Baltimore, Maryland.

FIG. 4—OUTDOOR FRAME AT VICTOR

Outdoor loading frame for suspensions, levers and weights, used. Standard strength units carrying 5000 lb. and intermediate carrying 12,000 lb. since September 1927. All still sound.

In addition to this, periodical check tests are regular routine. Sample check test report follows:

PERIODICAL CHECK TEST

A. Puncture (A. I. E. E. Standards 41; 153) (1) 145,000; (2) 140,000; (3) 152,000 volts.

B. Mechanical Ultimate and Electrical Test. (A. I. E. E. Standards 41: 154). (1) 16,000; (2) 15,300; (3) 16,000 lb.

C. Combined thermal—tension test.
Water temperatures, 205 deg. fahr. and 39 deg. fahr.
Ten minutes in each alternately.
Initial load 4,000 lb., increase 1,000 lb. per cycle.
Two units tested. First failure at 12,000 lb.

D. Time loading tests. Out-of-door any weather.
Initial load 8,000 lb., increase 1,000 lb. each working day.
Arced over before each increment of load.
Two units tested, first failure 17,000 lb.

Efforts have been made to distribute the load upon the outside of the porcelain more uniformly by multiple stepped caps. No advantage in any test was observed and in many cases decidedly unsatisfactory results were forthcoming. The thin metal of the cap apparently yielded so that the upper step would carry more than its share of the load. Annular ridges within the caps are of the same character and are generally worse than useless.

Similar tests upon the pin, however, have always shown advantages from the multiple stepped pin, provided such steps are above the lower edge of the insulator cap. In this case, of course, the mass of the pin is sufficient to prevent any yielding.

Most of the early porcelains were thoroughly vitrified, but only too often with an excess of flux (feldspar). The comparatively thin sections allowed quick firing and rapid cooling. Such over-fluxing and rapid cooling can only result in ware that is lacking in strength, and there is little doubt that differences in performance of individual specimens of the same type and lot of the old insulators, were largely the result of variation in strength.

Over firing to the beginning of volatilizing of some of the ingredients (ceramically known as "bloat") has rarely been the cause of trouble. In fact, experience with such ware suggests that this condition may possibly be desirable rather than hazardous.

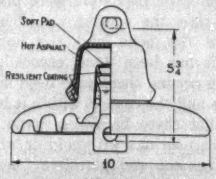

FIG. 5—MODERN CEMENTED SUSPENSION INSULATOR SHOWING PROVISION FOR EXPANSION AND YIELDING JOINTS

In some cases, under firing may have resulted in a short lived semi-vitrious ware which would be a cause of trouble. With the exact manufacturing con-

trol exercised today such conditions no longer exist and need not be considered in this discussion. The best porcelain today is recognized to be one of greater sturdiness, less flux, thorough vitrification, and careful cooling. Heavy sections have been demanded to withstand external violence, power arcs, punctures, and various forms of mischievousness. Lesser flux means greater mechanical and electrical strength, but with this reduction in flux in the thicker sections far greater care must be exercised in firing. Improper firing will invariably result in internal stress in the porcelain which seriously affects both the strength and life of the ware. Today constant thermal cycle checks are employed (A. I. E. E. Standard 41-250; 350). Such tests are largely a measure of the care used in kiln cooling. Regular porosity checks are made to insure thorough vitrification and proper coordination between all details of firing.

Recently greater strides have been made in the combination of porcelain and glaze. If the glaze covering the porcelain does not "fit" due to its coefficient of expansion being different from the porcelain it will be under stress. When this stress is above the elastic limit of the glaze crazing or crackling of the surface will be apparent. In the majority of cases, however, there are no visible cracks and yet the strains are there. Add an external load either of a mechanical nature or due to thermal changes to those already inherent in the surface of the dielectric, and small cracks will develop which will form the basis for a progressive failure of the porcelain.

Making and breaking in the testing machine comparative rods with various glazes pointed the way to the elimination of surface stress and the consequent development of stronger, longer lived porcelain. As an even more recent development, further strengths and greater uniformity have been gained by glazing the sanded surfaces.[3]

The abutting joint in the insulator assembly wherever found has been a source of trouble. As already explained, the separation of the cap of the suspension insulator from the horizontal hood has stopped losses in that type. Similar breakages occurred when the caps of switch type insulators were placed upon the porcelain so that the cement would bear upon the fillet between the head and the horizontal hood. Wherever this has been done, especially in climates where freezing occurs, there have been insulator losses. When the cap and the cement within it have been kept above this fillet these breakages have not occurred.

When the cap shrinks with cold or when free water between the cement and porcelain freezes there is a radial pressure against the fillet with an upward component. This reaction becomes a tension stress upon the porcelain—a stress which it is least able to carry.

If the cement is above the fillet the stress is entirely horizontal and becomes purely a compression force upon the porcelain. Against such stress porcelain is one of the strongest materials known.

Another form of the abutting joint is that which was used largely in pin type designs in the past. In this case the joint was between two porcelain parts. This, commonly known as the closed joint, was used chiefly for two purposes. First, such an insulator would be free from corona display at relatively high voltages, and second, the self-alining features of the closed joint offered a decided advantage in ease of factory assembly. The cause of the breakages experienced with this type of insulator was the same as with the suspension or switch type already described. The outer piece, however, in this case was generally the weaker and was usually the first to break. The edges of the upper porcelain part at the joint were frequently thin and were consequently weakened by over-firing because of this thinness.

Portland cement has been blamed for many insulator failures. It is certain that the chief complaint is not against the cement but against its improper use. It has been found that cement which will successfully pass the A. S. T. M. Soundness Test (A. S. T. M. Standard Specifications and Test for Portland Cement C9-26) is fit for insulator work. As a result special stress is laid on soundness, and every lot of cement before it is released for production is carefully checked as to this characteristic. In connection with this research, cement specimens were alternately soaked in water and frozen to − 20 deg. fahr. There was a slight contraction in the specimen corresponding to such a temperature drop. There was no increase in length which would correspond to a volume increase by formation of ice crystals. This confirms statements made by cement manufacturers that water within the colloidal structure of the cement does not freeze. Any harm done by freezing must be from free water within open voids in the cement.

Let us assume that the cement and central porcelain parts completely fill a porcelain ring outside of it. A ring of porcelain placed about an unyielding central part must shrink according to the following formula to destroy itself.

Let:—

E = Modulus of elasticity
C = Coefficient of expansion
S = Strength per unit area
t = Temperature difference

Then,

$$t = \frac{S}{E\ C} \text{ for shrinkage to cause porcelain breakage}$$

Assuming,

E = 7,500,000 lb. per sq. in.
C = 0.0000025 (fahr.)
S = 5,000 lb. per sq. in. (conservative)

then,

t = 267 deg. fahr.

3. D. H. Rowland, *G. E. Review*, March 1929 and June 1930.

Examination of the open type of cement joints shows that insulator failures have almost always occurred in those insulators with wide cement joints or with great cement masses, or with overfilled tapered joints.

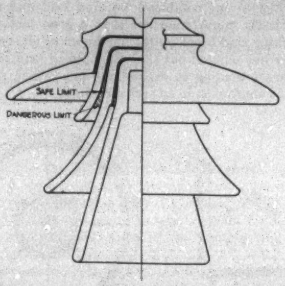

FIG. 6—SAFE AND DANGEROUS LIMITS IN CEMENTED JOINTS

All cement when drying shrinks. Mixtures which contain the smallest percentage of water are those which show the least shrinkage when drying. Such mixtures are also stronger. The use of ⅓ sand in the mix will cut down the shrinkage exactly ⅓ and make the material much more inert. When the cement shrinks against the parts within there is a tendency towards cracking of the cement. This cracking is reduced to a minimum, if not prevented entirely, if the cement joints are narrow, if the cement is dense and strong, and if it is firmly anchored in place by sanded surfaces upon both sides of it. Where insulators with these features have been found there have been no breakages.

On the other hand, where there has been an excessive

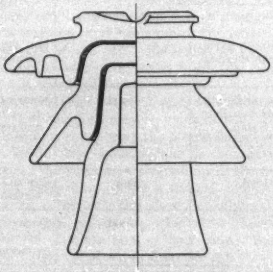

FIG. 7—PIN TYPE INSULATOR
Cement joints properly filled and proportioned

cracking of this cement or where water can find a passageway between the cement and porcelain the freezing of that water and the consequent increase in volume is frequently accompanied by a failure of porcelain. Such breakages have been observed to occur immediately adjacent to a crack in the cement as a result of localized pressure.

An examination of insulators made over a long period of time shows that they can be separated into groups, one of which may be expected to fail and the other not. This dividing line has been sufficiently sharp and close that a reasonable insulator life may now be confidently predicted. The modern insulator is made with cement joints as narrow as manufacturing tolerances will allow, while the cement used is dense and contains a minimum amount of water in the mix. The joints are not overfilled and the cement is fully set and expanded by curing at a suitable temperature in a saturated atmosphere.

One of the recent improvements in the pin type insulators has been the metal threaded pin hole. It is relatively difficult to manufacture porcelains with the thick sections demanded by modern practise. These thick sections even under the best controlled conditions do not shrink uniformly. This is chiefly due to the condition of the plaster mold. The amount of moisture in the mold varies during different hours in the day due to week end and other interruptions.

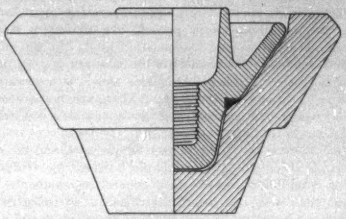

FIG. 8—INSULATOR PART DRYING IN MOLD. SHOULDER DRIED QUICKLY BY CONTACT

While the piece is resting within the mold it partially dries, shrinks, and rests upon its shoulder. This shoulder which is opposite the outer end of the pin hole dries rapidly ahead of other sections. It takes a quick shell like set, and later on restrains the clay immediately under it from shrinking at the normal rate of the other parts of the piece. This results in a distortion of the pin hole which cannot be accurately foretold and compensated for. Realization of this led to the adoption of the metal thread. This thread is now quite uniformly used in the pin type insulators of the higher voltages. Not only does it simplify construction and save time due to the exactness of its fit upon the pin, but in addition it is of special value in overcoming a hidden source of corona which might cause hidden stress and radio interference.

While we believe that the developments in insulator art have been worthy ones, we are by no means drifting into a self-satisfied condition. Organized research is constant and consistent and all indications are that insulators will not be the restraining factors in any future developments no matter how rapid those developments may come.

Discussion

S. Withington: The increasing reliability of insulators for medium and high voltages is a source of a great deal of gratification to all who are interested in maintaining the integrity of power supply, and there is no doubt that there has been satisfactory advance in this direction, as those associated with transmission and distribution of power before the war will agree. It is perhaps an overstatement, however, to say as Mr. Hawley does, that "insulators even of the largest size have established a perfect life record over a period of fifteen years."

It is true that the initial cost of insulators is relatively small when compared with the total construction expenses involved in a transmission line, and it is also true that this cost is relatively unimportant where continuity of service and the cost of replacement are considered. Nevertheless, the fact remains that the initial cost of standard types of porcelain insulators has risen in the past few years more than the normal indexes would seem to justify, and it is to be hoped that when the expenses of development to which Mr. Hawley briefly refers have been amortized there will be a marked decrease in cost of the consumer.

It would be of interest if Mr. Hawley were to expand his reference to the metal threaded hole for pin insulators. This is a development which is especially important in connection with the curing of cement at suitable temperatures to which Mr. Hawley refers.

Steam railroad electrification presents an important field in the use of insulators. In addition to the problems presented by the smoke from steam locomotives operating over electrified tracks, the requirements in general are similar to those of standard distribution and transmission lines, but the support, and particularly the dead-ending of the catenary system, produce somewhat heavier mechanical stresses and it is necessary that insulators be designed accordingly. There are numerous occasions also where on account of close clearance and for other reasons special forms of insulators are required. Pedestal type insulators are often necessary at low bridges, tunnels, etc., and the closest attention of the manufacturers should be given to satisfactorily adapting insulator design to these general requirements. It is probable that with increasing electrification of railroads more attention can be given to this phase of the subject and a satisfactory solution reached for the various problems involved.

A. O. Austin: The general characteristics of insulators having different types of mechanical construction are shown in Table I.

In the past losses due to abnormally high working loads have been negligible although losses due to differential thermal expansion or to concentrated loads have been very high, necessitating reinsulation in many cases.

Where long life and reliability are desired, low stress in the dielectric, due to the combined working load and differential expansion, is far more important than a good factor of safety for the working load based on the maximum or ultimate on test. Mr. Hawley has pointed out that the applied load produces an outward thrust tending to expand the cap, which may be regarded as the abutments in an arch structure.

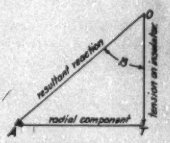

Fig. 1

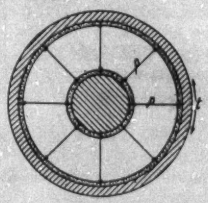

Fig. 2

Figs. 1 and 2 show the general relation of stresses produced by the working load.

Let $O\,T = L$ = load applied to insulator.

Then $O\,A$ = force in magnitude and direction, between the pin and bead or rim of cap, due to load $O\,T$.

$A\,T = P$ is the horizontal component of $O\,A$ or the total radial stress set up in cap and porcelain.

Then $P = L \tan \beta$

If D is cap diameter at bead or rim, $\pi\,D$ is the circumference and

$$p = \frac{P}{\pi\,D} = \frac{L \tan \beta}{\pi\,D}$$ = radial stress per inch of cap circumference.

TABLE I

Style	Construction	Characteristics for working loads			High test loads	Inherent limitations
		Light loads	Medium loads	Heavy loads		
1	Porcelain and metal fixed	Good	Fair	Heavy metal parts with high thermal stress	Fair	Low permissible temperature range and stress in metal
2	Compensated by slipping or wedging	Good if slipping does not take place due to thermal expansion of cap or contraction of pin	Good if slipping does not take place followed by reduction in load or temperature	Good for moderate range of temperature and load	Good	Shearing stresses due to variation in loads and temperature
3	Resilient metal and joint	Good	Good	Good	Fair	Cap and porcelain distortion
4	Restoring compensated	Good	Good	Good	Good	Shear due to small bearing area and concentrated load
5	Resilient and compensated	Good	Good	Good	Good	Cost
6	Link	Good	Fair	Poor	Poor	Shear due to small bearing area and to inherent form
7	Porcelain in tension	Good	Good	Good	Good	Necessity of multiple strings to offset mechanical hazard

If t = tangential stress in bead of cap

$$t = pr = \frac{p\ D}{2} = \frac{L \tan\beta\ D}{2\pi\ D} = \frac{L \tan\beta}{2\pi}$$

The circumferential stretch in the cap bead for load L will then be

$$\Delta C = \frac{t\pi\ D}{A\ E} = \frac{L \tan\beta}{2\pi} \cdot \frac{\pi\ D}{A\ E} = \frac{L \tan\beta\ D}{2\ A\ E}$$

Where A = cross sectional area of cap bead and
E = modulus of elasticity.

From this it will be seen that dividing by π gives the diametrical distortion ΔD.

$$\text{Hence } \Delta D = \frac{L\ D \tan\beta}{2\pi\ A\ E}$$

By applying the proper values to the above equation and setting up a similar equation for the deformation of the porcelain, the radial distortion may be computed. If we assume that this amounts to approximately 0.008 in. for both cap and porcelain, and that the slope of the bearing ring of the cap $\angle\ Y\ O\ T$, is such that the tangent of $\angle\ Y\ O\ T$ = 0.5, the amount of slip between the cement and the bearing surface of the cap which will compensate for the distortion, can be readily computed, or the

Longitudinal slip = 0.008/0.5 = 0.016 in.

While the slipping readily compensates for the distortion, giving high test values, reference to Fig. 3 shows that with the reduction of load a heavy unbalanced radial component may result which will tend to cause shearing of the porcelain. The coating of the surfaces with asphalt, graphite or metal foil will tend to control the coefficient of friction. The chief difficulty is that the coefficient of friction is likely to change with time.

While a few of the very high ultimate insulators were made some years ago in which the cap slipped to give the highest ultimates, it is believed that the initial stress set up by the temperature of assembly was such that slipping did not take place under maximum working load. The last of the above equations shows that the increase in diameter of the cap due to a given load will vary directly as the diameter and inversely as the effective cross section of the area and the modulus of elasticity. This has long been recognized, as many of the very heavy insulators used for long spans and catenary work have had reinforcing ribs to reduce cap distortion. Heavy cap construction, however, may result in high shearing stresses with low temperatures and light loads unless the pressure is tapered off near the edge of the cap. Reference to Fig. 3 shows the general relation of stress components for the insulator shown in Fig. 5 of Mr. Hawley's paper.

With a heavy plastic material such as asphalt, the angle L during setting up the load may be much smaller than the angle θ which is effective in restoring the porcelain and cap to normal condition. While this arrangement is beneficial in giving high test value, the shearing stress set up in the porcelain with light working loads may create a hazard. Since porcelain is relatively weak in tension and shear, unbalanced radial components which will set up shearing stresses are dangerous, regardless of the ultimate as they may cause "dough-nutting" which is due to the radial stress from the cap shearing the head of the insulator from the flange at low temperature or under a light load following a heavy load.

Fig. 2 of Mr. Hawley's paper is hardly applicable to insulator design as the large section of metal in the ring surrounding the pin reduces distortion to a negligible quantity. The very short cement strut between pin and ring will have but slight distortion compared to the very long strut made up of cement and porcelain in the insulator. Furthermore, a crack in the cement due to a displacement might not cause failure whereas a crack in the insulator due to distortion would result in dielectric failure with the possibility of mechanical failure due to an explosion from discharge through the fault.

In many of the earlier insulators equipped with sanded surfaces, the sanding was covered with glaze, the sand grains forming pyramids with the surface. At high mechanical loads there is a tendency for this surface to slip and compensate for distortion. It would seem that it is this slipping which increases the mechanical ultimate on test. This, however, is gained at the expense of resiliency in the joint.

The mechanical characteristics which may be developed in the porcelain depend to a large extent upon the method of making the test and the size of samples, the small samples giving abnormally high values which cannot be developed in the larger sections necessary in practical construction. A glaze which will tend to cause crazing will reduce the mechanical ultimate; whereas a glaze which will tend to cause shivering may increase the mechanical ultimate for some conditions. For best conditions, the linear coefficient of expansion of the glaze and body should be the same. It is believed that the latter condition will produce the best results as there are many insu-

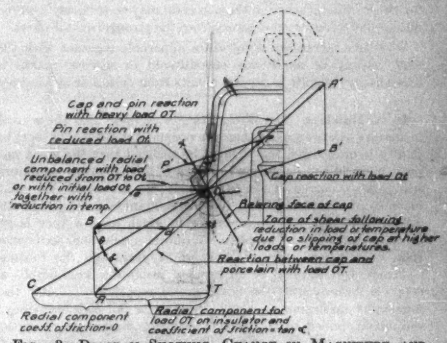

FIG. 3—DIAGRAM SHOWING CHANGE IN MAGNITUDE AND DIRECTION OF STRESSES IN INSULATOR WHERE DISTORTION IS COMPENSATED BY WEDGING AND SLIPPING

lators which have been in service for some years in which there is no evidence of failure. The first of the insulators of the type shown in Fig. 7 were installed in 1914 and have shown no evidence of cracking.

The limit of 267 deg. given by Mr. Hawley applies to porcelain only. Even in this case the figure must be materially reduced for concentrations of stress owing to the shape of the parts. Where porcelain and metal are used together the limits in temperature may be comparatively low. To offset this limitation the resilient joint is exceedingly valuable for either the pin type or suspension insulator.

If the conditions in Fig. 8 cause trouble, they can be largely remedied by coating the mold so as to prevent absorption of water at the shoulder, or by trimming the case hardened surface while the insulator part is soft.

If thin metal thimbles are used for threads there is always danger that the screwing of the insulator on the pin will produce a dangerous longitudinal or radial stress which will cause destruction of the insulator. It is, therefore, advisable to use heavy thimbles which will prevent damage to the insulator. With this arrangement, however, the insulator is subject to a thermal stress which is lacking in the case of the insulator having the thread made in the porcelain. The longest lived

insulators are apparently those which have threaded porcelain pin holes mounted on lead tipped pins, of proper design.

While the correction for distortion in the suspension insulator pointed out by Mr. Hawley is highly desirable for heavy loads, it is even more essential that the stress return to normal condition with the relief or the reduction of the load. Coatings which flow are not resilient and any unevenness in a coating with rigid parts adjacent may cause a concentration of stress and destruction. While a compensating or resilient structure for the cap or outer cement joint can be used to advantage, the cost of this type of construction is such that with the present high standard of insulators it has been impossible to commercialize the design embodying this feature. Reducing the coefficient of friction of the surface to zero will make the compensated type of construction safe over a much wider range of working conditions. It would seem, however, that this type of construction must be brought about by a roller bearing surface of joint or one equipped with tilting struts.

In the past, insulators having high ultimates have always shown the shortest life. It is, therefore, a serious mistake to use an insulator having a higher ultimate than is necessary for the maximum working load as the effective factor of safety for the maximum stress in the porcelain may be seriously lowered due to increased thermal stress from the stronger metal parts.

The time lag before defects are apparent, together with the wide variety of conditions encountered in service, makes it exceedingly difficult to predict results from design or accelerated tests.

K. A. Hawley: Mr. Withington's statement that the cost of insulators has risen more than the normal indexes would seem to justify is quite true, but it must be realized that the great improvement in the design and quality of high-voltage insulators which has occurred in the last decade has necessitated a great increase in overhead cost. The larger manufacturers maintain million-volt high-voltage laboratories which have a very large carrying capacity. The plant routine is much more extensive than ever before. For example, some years ago the routine electrical test applied to suspension insulators was simply a 60-cycle flash-over on the unassembled porcelains. Today the porcelains are flashed over at both 60 cycle and high frequency, assembled, pulled to 40 per cent of their guaranteed strength, and then flashed over again. The other routine processes of inspection and check throughout the plant have been extended in a manner similar to the electrical tests in order to produce units of the reliability which is now demanded.

Insulators are required to be nearly perfect, surface imperfections being no longer tolerated. This perfection has been reached only at a marked increase in cost which must necessarily be carried by the customer.

Mr. Austin's statement that the linear coefficient of expansion of the glaze and body must be the same does not check up with our test results. It is impossible to test a glaze and a body with respect to the coefficient of expansion separately and then to put them together and to expect thus to get the best results. The intimate contact between the porcelain and glaze causes some of the constituents of the glaze to run into the body and vice versa. We have definitely found that a glaze which will give the highest mechanical strength to a given porcelain will give the best results in insulators.

The use of metal thimbles in the pinholes of pin type insulators cannot possibly be more of a hazard to the insulator when screwing it on to the pin than would be the case with the ordinary plain pinhole and lead-covered pin. The thickness of cement between the thimble and the porcelain will go a long way towards distributing any stress which may be produced, and in preventing a concentration on the porcelain itself which might produce cracking.

It is impossible to agree with Mr. Austin's statement that insulators having high ultimate strength have always shown the shortest life. It is very true that an insulator may be designed which will give a very high ultimate strength and a low mechanical and electrical strength, but if the design is correct these two points will be very close together and it may be expected that such insulators will give a much longer life in service than those of low mechanical strength, other things being equal. The thousands upon thousands of high-strength insulators that have been giving perfect service for years substantiate this.

Due to the heavier line construction which is becoming more and more prevalent, insulator designs that will be the most effective from a mechanical standpoint will be the most economical from a cost standpoint. It is hard to reconcile Mr. Austin's statement regarding insulators of high ultimate strengths when it is realized that the units which are being made today by a large majority of the manufacturers have mechanical strengths practically 50 per cent greater than units of the same size which were made a few years ago. This increase in strength has been brought about by better attention to small details and a better understanding of ceramics. It is believed that the modern units will give service in the field of a character which has never been surpassed. It is only fair to give to the operating companies the economic advantages brought about by constant research work in the mechanical and electrical design of porcelain insulators.

Steam Power Development
of the Pacific Gas & Electric Company

BY RICHARD C. POWELL[1]
Associate, A. I. E. E.

Synopsis.—*Improvements in steam power generation and economic changes in the fuel supply have caused increasing interest in steam power on the Pacific Coast.*

After a brief history of the development of steam power on the system of the Pacific Gas & Electric Company, the author discusses some of the fundamental factors entering into the problem of providing additional steam plant capacity for this company in accord with the changed economic conditions.

The author describes recent work completed and under construction and gives some of the economic results obtained and expected.

INTRODUCTION

As a result of recent progress in the art of electric generation by steam power, most of the utilities on the Pacific Coast are either engaged on or planning power developments in which steam is predominant or occupies a very prominent position. Consequently, the economics and design of steam plants formerly of little interest to most Pacific Coast utility executives and engineers, are now of increasing importance.

In this paper will be discussed some of the fundamental factors entering into the problem of increasing the steam electric generating capacity on the system of the Pacific Gas & Electric Company and a brief description will be given of work recently completed and under construction. This will be preceded by a brief history of the steam power development of this company and a short description of the design features of the older plants together with service and operating requirements. This will serve as a background to the general picture and enable one readily to orient himself with respect to the various features involved.

HISTORY

The electric system of the Pacific Gas & Electric Company and its subsidiaries as it now stands, is essentially an interconnected system supplying service to almost the whole of Northern and Central California, an area roughly 425 miles long and 150 miles wide with a peak demand of about 850,000 kw. and an annual generation of about 4,500,000,000 kw-hr. Fig. 1 is a map of the generating and transmission system. Out of a total of 1,100,000 kw. in generating capacity, 300,000 kw. or roughly 30 per cent is in steam. It has reached its present size by consolidation and extension covering a period of over 40 years. The nuclei for this growth were the independent systems which had been started in San Francisco, Oakland, Sacramento, and San Jose, all supplied from generators driven by steam engines. Thus, originally the output was entirely steam power.

The development of high-voltage transmission about 1900 enabled water power to be brought to these and other cities at a cost much lower than steam power, which was then unable to hold its position and was rapidly shoved into the background. Steam power would undoubtedly have been almost entirely supplanted but for two factors, *viz.*, first, the unreliability of those early lines, and, second, the entire lack of sites for, or prohibitive cost of reservoirs to provide the necessary storage to enable the water power plants to carry the load during the dry seasons when the stream flows fell greatly below normal.

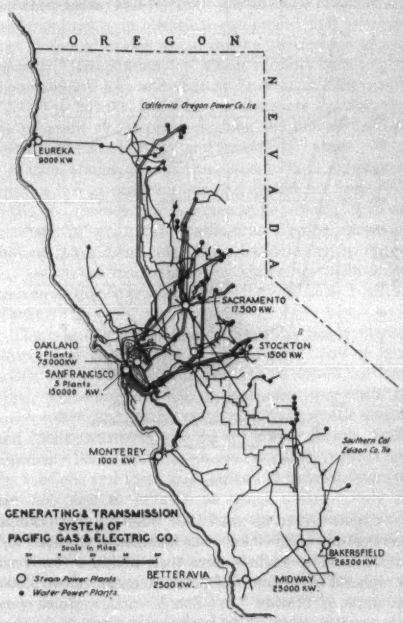

FIG. 1

During this period of great water power development supplementary steam power was necessary, and hence the growth of steam and water power followed somewhat roughly together. Up until about 1924 the ratio

1. Pacific Gas & Electric Co., San Francisco, Calif.

Presented at the Pacific Coast Convention of the A. I. E. E., Portland, Oregon, Sept. 2-5, 1930.

30-147

of total steam plant capacity to system peak demand for the Pacific Gas & Electric Company system varied rather consistently from about 40 to 60 per cent and the ratio of kw-hr. generated by steam to the total, ranged from about 15 to 30 per cent. For the years 1924 to 1929 inclusive, these data are shown by the curves in Fig. 2. During these years the ratio for kw-hr. output swung through a very wide range, the high point being slightly higher and the low very much lower than for any previous year. The high point of steam generation was for the excessively dry year of 1924 and the low point was in 1927 when there was an excess of water power due to the completion of several water power projects in Northern California.

The increase in steam power capacity was obtained by the gradual process of adding one unit at a time,

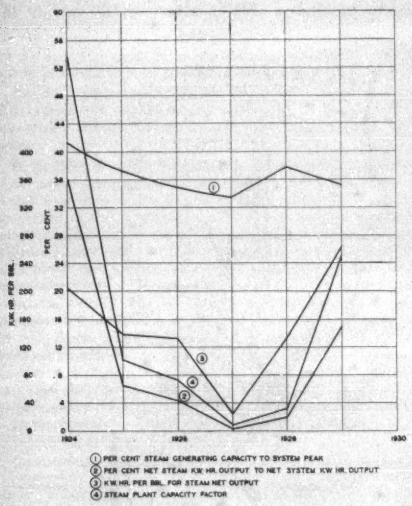

FIG. 2

first engine driven up until 1907 when the first turbine, a vertical one of 9,000 kw., was installed at Station "C," Oakland. The engines were then within a few years all replaced by turbines with the exception of two 3,500-kw. units at Station "A," San Francisco, which were just recently removed to make room for a 50,000-kw. turbine set. At the beginning of 1928, there were seven steam plants on the system ranging in capacity from 1,000 to 64,000 kw. The largest turbine unit was 15,000 kw., the largest boiler 8,900 sq. ft., and the highest steam pressure and temperature were 250 lb. and 540 deg. fahr. respectively.

The requirements for the steam plants during the period under review were: (1) low investment cost since the load factor on steam over a period of years was not high, (2) high reliability, since steam was always the pinch hitter during seasons of low water and at times of sudden failure of a line, flume, or some other part of the water power and transmission system, and (3) the ability to pick up full load quickly from a very light load condition. On the whole, these plants met the above requirements very satisfactorily. They have never seriously failed to meet the service demands made upon them in the way of picking up load in the event of line or other failures on the water power system although no especial features of design were employed to enable the equipment to respond rapidly to a sudden increase of load. Dependence was placed entirely upon keeping sufficient turbine capacity on the line at very light load, that is, from just floating to 5 per cent of rating, with corresponding boiler capacity either steaming or with pressure at or near that of the line, and the ability with oil firing to get boilers to full steaming capacity in a very short time. Experience has proved that under the conditions just outlined, full load could be picked up with little or no disturbance to service. Usually the disturbance was for only the momentary time required for the operator to separate the load to be protected together with its steam standby from the transmission system in trouble.

The efficiency of these plants was rather low, averaging about 30,000 B. t. u. per kw-hr. and the operating and maintenance costs were rather high during dry years when the plants were forced for considerable lengths of time, but since the excessively dry years occurred on an average of only one in seven, the average maintenance costs over a seven year cycle were not excessive for plants of their time.

At the beginning of 1928, the investment per kw. varied among the plants from about $60 to $80; the steam pressures and temperatures were low, the design was simple with no economizers, fans, or automatic devices worth mentioning. The auxiliaries were mixed steam and electric in order to maintain a sort of heat balance at or near full load and to draw most or all of the power for auxiliaries from the water power system during light load times when there was an excess of water.

PLANS FOR STEAM POWER DEVELOPMENT

About four years ago it became apparent that the economic relation between steam and water power had become reversed. The reasons for this change were:

1. The generally increasing cost of additional water power due to the best sites having been developed and, for sites already developed, the rather prolonged series of years of subnormal water supply which indicated that the supply was not so great as formerly supposed.

2. The rather pronounced lowering of steam power costs, due to improved efficiency, lowering investment, operating, and maintenance costs, and for California, at least, the oversupply of oil with consequent reduction in price, and finally the tremendous production of natural gas and its supply to Northern California.

A comprehensive plan was developed which would provide not only for the immediate needs, but also for at least fifteen years hence in the matter of the acquisition of steam plant sites and transmission line rights-of-way for lines necessary to connect such sites with the general system. The main objective involving power development is, of course, always the economic one to provide power at the lowest possible cost. Furthermore, for the Pacific Gas & Electric Company, the new units should perform at least as well as the existing ones in the ability to pick up load quickly. In working up this plan consideration was given to all the elements involved which included:

1. Sizes of units and when and where to be installed.
2. Costs for investment, maintenance, operation, and fuel.
3. Obsolescence.
4. Operating reliability and flexibility.

A careful study was made of all the factors entering into the above elements. These factors included such items as:

1. New plants *versus* rebuilding of the old plants.
2. Sizes of turbine and boiler units.
3. Steam pressure and temperature.
4. Extraction heaters, economizers, air heaters, and furnace water walls.

Most, if not all large systems have a need for two types of generating units, the first highly efficient for so-called base load operation, and the second for peak load service and reserve. The Pacific Gas & Electric Company also requires a third type which is required only in excessively dry years, the occurrence of which are infrequent and uncertain. That any given year will be very dry is not known until about February. It is very difficult if not impossible to provide against such deficiency of water, unless such abnormal deficiencies are recognized as liable to occur and provided for at all times. Examples of where failure to provide for this contingency have resulted in power shortage are well known. On the Pacific Coast such cases occurred in California in 1924 and in the Northwest in 1929. The Pacific Gas & Electric Company has never found it necessary to cut off load on account of a dry year except during the war when it was required to curtail new construction on account of war conditions.

On account of the available space conditions at Station "C" and the nearer location of Oakland to the water power supply, it was determined that increased capacity at Station "C," at least to the extent of 75,000 kw. would conform to the second type of plant just mentioned and that Station "A" would provide the first type. To provide for the third type of capacity, it was decided to retain the North Beach plant in San Francisco and the Station "B" plant in Sacramento with a combined capacity of 35,000 kw., and these during normal years would probably be shut down.

The problem of how best to provide peak and reserve capacity is not always clear as is evidenced by the discussions which appear from time to time, and include proposals such as steam plants with accumulators and pumped storage for water plants. It is believed that the most practicable way to obtain such capacity in steam is to provide overload capacity for each unit. The rating of both turbines and boilers is entirely arbitrary. The maximum output of a turbine is determined by the maximum steam flow for a unit built to a given frame. A boiler can be forced to a very high output. The ratings may be set according to the efficiencies desired for both turbines and boilers and the maintenance cost for boilers which will increase at high outputs.

Cost studies showed that it would be more economical to rebuild and enlarge Station "A" in San Francisco and Station "C" in Oakland than to provide the same additional capacity at new sites. Both of these sites are admirably located with respect to both cooling water and their loads, and for both plants there are substantial investments in electrical equipment and outgoing duct lines and cables which can be utilized for greater capacity. The Station "A" site is suitable for a total of about 260,000 kw. and that at Station "C" for about 175,000 to 200,000 kw. It was therefore decided that the program would proceed along the line of first developing the Station "A" and Station "C" sites.

It is, of course, always the desire to hold investment costs as low as possible, but this should not be carried to the point where maintenance and operating costs are adversely affected. It was fixed as a policy, however, that no investment cost would be increased to reduce any estimated maintenance or operating costs unless such reductions were sufficiently large to appear certain of occurring.

The size of turbine and boiler units has a marked effect upon costs. This is particularly true of large boiler units. Investment per kw. is lower for large units, efficiencies are higher, and operating costs are much lower, and the investment for automatic control and instruments is decidedly lower. Large units, especially boiler units, are more flexible as regards load fluctuations and there is no reason to believe that they are not just as reliable as small units. Costs for maintenance and periodic inspection are less for large turbine units than for small ones and should be no higher for large boiler units, and maintenance costs for automatic control equipment is much less, since the cost and amount of such equipment is almost entirely dependent upon the number of units and not upon the size.

In the matter of pressure and temperature, it was recognized that the trend was very definitely to higher pressures and temperatures. Careful estimates made in 1926 showed that a plant operating at 450 lb. pressure should cost no more than one operating at 250 lb. although the use of 650 lb. with reheating would cost more. No very careful estimates at that time were made for 1250 lb. since that pressure was then regarded as still in the development stage. The use of pressures

and temperatures as high as consistent with reliable operation will, of course, have a favorable influence upon obsolescence.

Extraction heaters, economizers, and air heaters, although adding complication as compared with the old plants did not seem to affect reliability adversely and their use was considered entirely a matter of economics. With reference to furnace water walls, an installation made in 1925 in one boiler as a trial had definitely showed that their use would not only reduce furnace maintenance, one of the largest items of boiler room maintenance, but would greatly improve reliability. They were necessary if boilers were to be operated at high rating with mechanical atomizing oil burners, since with such burners furnace temperatures of the order of 3200 deg. fahr. are encountered.

To obtain increased steam power capacity, the first in-installation was to be made at Station "C," to be followed with the next increase at Station "A," San Francisco. From the studies made in 1926, it was concluded that design for Station "C" should be based upon 450-lb. drum pressure, approximately 750 deg. fahr. total steam temperature, large boilers with water walls, the question of extraction heaters, economizers, and air heaters to be a matter of economics and layout in the space available. It was further concluded to install automatic control.

Installation at Station "C," Oakland

The installation for a 37,500-kw. unit at Station "C" was started in 1927 and put into operation in October, 1928. The size of turbine was determined by the then existing turbine room which permitted with a small extension the erection of two 37,500-kw. turbines in the space occupied by two vertical units, one a 9,000 kw. and the other a 12,000 kw. In line with the type of capacity desired, *viz.*, the second type mentioned above, and on account of space limitations, a 37,500-kw., 9-stage impulse multi-valve turbine with guaranteed overload capacity of 42,000 kw. was selected. This was quite a departure from established practise. As compared with the conventional unit of this size, it has better efficiency up to about 30,000 kw. and by extracting heavily its efficiency is not greatly less for higher loads, and the cost for the unit, foundation, and building was about $1.50 per kw. less. For the load conditions it will have to meet during its life, it should be considerably more efficient.

Straight tube sectional header boilers were decided upon because experience on the whole had been better with this type than with the bent tube type. The size was selected by making the width the maximum that could be obtained for a riveted drum, and the number of tubes high was made such that without an economizer, the preheated air temperature would not exceed 500 deg. fahr. With 24-ft. tubes, this resulted in a very economical unit of 35,453 sq. ft. with a maximum guaranteed output of 425,000 lb. per hr. having about 84 per cent efficiency at maximum output with an air heater of 51,232 sq. ft. Two such units were installed, one to serve as a spare for the first and future turbines. A maximum steam temperature of 730 deg. fahr. was selected as the highest then considered desirable without the use of alloy tubes.

Approximately 25 per cent of the throttle flow is extracted for feed water heating with a final temperature of 390 deg. fahr. A single pass condenser of 28,000 sq. ft. was installed and although the capacity of the plant was increased from 33,500 kw. to 68,000 kw., no additional pump capacity for circulating water was required.

Automatic equipment was provided for combustion control, condenser hotwell level, feed water pressure, and drum water level.

Auxiliary power is supplied from two three-phase transformers connected to the main station buses. Most of the motors, including one 500-hp. synchronous motor, are started on full voltage.

This brief description will give an idea of the type of design and character of the equipment. The over-all cost of the plant including land, building, etc., after the installation of this unit and increase to 68,000 kw. capacity was very closely the same per kw. as for the original 33,500-kw. plant, although the efficiency was about doubled and the operating and maintenance costs reduced to about half on a kw. basis. The turbine has a maximum output of 43,750 kw. and each boiler has put out 435,000 lb. per hr. The boiler room installation is quite compact and occupies 7.7 sq. ft. of ground area and 586 cu. ft. of building volume per 1,000 lb. per hr. output. This includes the boilers, air heaters, fans, and fuel oil pumps and heaters.

The new unit was put into service in October 1928 and shortly thereafter on account of very cold weather in the mountains with consequent reduction of water supply, was required to carry practically a continuous load of 43,750 kw. until some time in February. It was therefore not until the spring of 1929 that the turbine or a boiler could be taken out of service for the necessary adjustments of automatics, etc., incident to the starting up of a new unit. Leaving out of account the time when equipment could have been operated, but was out for adjustment of automatic equipment, the turbine during 1928 was available over 99 per cent of the year and both boilers about 96 per cent. The turbine output was on a 65 per cent capacity factor and the net output was just at the rate of 14,000 B. t. u. per kw-hr. The auxiliary power was 3 per cent. It is to be noted that the unit was loaded very unfavorably since for practically all of the time the load was either very light (500 to 1000 kw.) or with heavy overload at 43,750 kw. and for very little or none of the time at the most efficient point of about 30,000 kw.

Curve 3, Fig. 2, shows the average kw-hr. per bbl. of oil for all the plants for the years 1924 to 1929 inclusive. The effect of the Station "C" installation is easily seen

by referring to curve 4. In 1924 operating on 54 per cent capacity factor for the total system steam capacity, the net output was at the rate of 205 kw-hr. per bbl., but in 1929 although the capacity factor was only 25 per cent, the net kw-hr. per bbl. was raised to 260. Had the capacity factor been 54 per cent, the kw-hr. per bbl. would have been about 280, which represents a saving of 36 per cent in fuel.

Installation at Station "A," San Francisco

In 1928 studies were made for the rebuilding of Station "A" and included plans and estimates for 450, 750, and 1350 lb. drum pressures. The use of any pressure between 450 and 1350 lb. was shown to be uneconomical and it appeared that 1350 lb. might cost $3.00 per kw. more than 450 lb. However, there was a very pronounced trend toward lowering costs for 1350 lb. equipment and an increase in the size of boilers which manufacturers were willing to build. All items considered, it was decided that the use of 1250 lb. throttle pressure and 750 deg. fahr. total temperature would be the most economical and that the plant could be rebuilt for 1350 lb. cheaper than for some lower pressure. Even at some higher cost for the equipment, the ultimate plant would be cheaper per kw. because, first, more capacity could be installed in the existing buildings, and second, the circulating water system required rebuilding and with the much less quantity of water needed this item was much less for the 1,350 lb. It was believed that the cost of 1,350-lb. equipment would go down as production increased, and that within a few years it would be possible to build a plant for this pressure as cheap as for some lower pressure. This view has proved correct for it is now definitely known that the costs for 1350 lb. are no greater than for 450 lb.

The old building space permits of the installation of 4 turbine units, each of about 60,000 kw. output, and 6 boiler units, each of 500,000 lb. per hr. maximum output. The installation now under construction consists of 2 turbine and 3 boiler units. The turbine units are each compound rated at 50,000 kw. with maximum output of 65,000 kw. at unity power factor. At a sacrifice of 10 B. t. u. per kw-hr. chargeable to the turbine room at the most efficient load point the maximum turbine output for the same frame was increased from 58,000 kw. to 65,000 kw. That is, for slight dedecrease in efficiency, considerable overload capacity was obtained. The arrangement of high- and low-pressure turbines is the so-called vertical compound with the high-pressure turbine and generator mounted on the low-pressure generator. This arrangement saves space and cost of foundations and simplifies somewhat the generator air cooler arrangement and the piping for cooling water.

The exhaust from the high-pressure element of each unit is reheated to 750 deg. fahr. before passing to the low-pressure element, first in a live steam reheater, the heating steam being saturated steam at 1,350 lb. pressure, and, second, in a convection reheater which is a part of the boiler unit.

By this combination of reheaters the temperature of the reheated steam is maintained approximately constant over a wide load range.

All of the steam for a turbine unit is reheated in a single boiler with no cross-connections on reheaters so that if a reheat boiler unit is out of service, the low pressure turbine will be run without reheat. Thus, two of the boilers now being installed are reheat boilers and the third is a so-called standard unit, that is, without reheat. The standard boiler provides some spare capacity and also the additional steam required for each turbine unit at loads when more than 500,000 lb. per hr. is required. The complication of piping for a reheat cycle is not so great as commonly assumed, particularly when all the reheating for a turbine unit is done in a single boiler, and is offset to a considerable extent by the fact that much smaller pipe sizes are required for 1250 lb. than for say 400 lb.

For the type of boiler unit which has been used in most of the 1250-lb. plants, the cost on a kw. basis is very little more than for a 400-lb. plant. Although the so-called boiler part, that is, drum, headers, and tubes, is costly, it is relatively small and most of the complete boiler unit surface is in other elements, that is, superheater, resuperheater, economizer, and air heater. Table I gives the percentages for the various surfaces.

TABLE I

	Per cent of heating surface			
	Reheat boilers		Standard boiler	
	Steam and water absorbing surface	Steam, water, and air absorbing surface	Steam and water absorbing surface	Steam, water, and air absorbing surface
Boiler	24.1	11.2	26.5	12.3
Water walls	3.6	1.7	4.0	1.9
Superheater	14.2	6.6	17.6	8.2
Reheater	18.0	8.3		
Economizer	40.1	18.5	51.9	24.0
Air heater		53.7		53.6

The Station "A" boilers have forged steel drums 52 ft. long, 5 ft. outside diameter with 4 in. walls and weigh about 70 tons each. The tubes are arranged 62 wide and 8 high.

The area and space occupied by the boiler and fan rooms are per 1000 lb. per hr. output, 9.35 sq. ft. and 865 cu. ft. respectively. In comparison with similar figures for Station "C" on a kw. basis, the area for Station "A" is 90 per cent and the volume 1.09 per cent of those for Station "C." At Station "A" the space is that of an existing building with height increased to provide for the fans and the cubic space could be reduced for a new plant. It appears that on the whole a high-pressure plant will require at least no greater space than a plant for lower pressure.

The remainder of the equipment such as fans, feed

water heaters, pumps, etc., are similar to that for lower pressure plants and requires no detailed description here.

The reconstruction of Station "A" is an example of the recent progress in the art of generating electricity by steam power, and shows how the great improvement in fuel economy has been effected without increase in capital. In fact for Station "A" there will be a marked reduction in investment. With the completion of the present construction, the capacity will have been increased from 64,000 kw. to about 160,000 kw., and it is expected that the over-all investment per kw. will be reduced by about $15.00 per kw. and the fuel, operating and maintenace costs per kw. reduced to one-half. With the completion to the ultimate capacity of about 260,000 kw. a further reduction of $5.00 per kw. is expected or a total of $20.00 per kw. The cost of the new Station "A" will be comparable with steam plant costs in Great Britain and Europe. The fuel consumption for the 1,250-lb. units should be at the rate per net kw-hr. of 12,000 B. t. u. or better.

Since plants for operation at 1,250-lb. pressure cost no more than for some lower pressure, undoubtedly in the near future there will be a number of plants at this pressure in operation on the Pacific Coast. Higher pressures and temperatures are to be expected in time, but with the present fuel situation, particularly in California, it is not anticipated that pressures higher than 1250 lb. and temperatures above 800 to 850 deg. fahr. will prove profitable for some years.

Summary

The author has outlined the development of steam power in the system of the Pacific Gas & Electric Company and attempted to discuss briefly those factors which are of most importance in the economical design of steam plants with especial reference to Pacific Coast conditions.

Grounding Banks of Transformers with Neutral Impedances

and the Resultant Transient Conditions in the Windings

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Associate, A. I. E. E. Member, A. I. E. E.

Synopsis.—*The question of grounding transformer bank neutrals through different impedances has recently arisen due to the desire to limit system single-phase and two-phase short-circuit currents. This limitation has been required to ease the duty on circuit breakers and assist in maintaining system stability. The use of resistance, inductance, and various combinations has been studied as to the effect on the short-circuit current and particularly upon lightning transients within the transformer windings and at the neutral. It was found that the use of resistance only may be undesirable on account of the high voltage at the neutral in limiting the short-circuit current. The use of inductance only may result in high voltages within the transformer and at the neutral due to lightning transients, which necessitates that the transformer be fully insulated throughout. Methods using parallel paths with the inductance, these parallel paths being designed primarily to reduce the lightning transients at the neutral, have been studied and found to limit the transients within the transformers to values approximating those for solidly grounded neutral which permit the grading of the transformer insulation. The method to be selected depends upon the individual case, but generally the use of the valve type lightning arrester is the simplest to apply.*

* * * * *

I. Introduction

MODERN methods of power transmission involve the transformation of energy with large high-voltage transformers, which usually have one or more windings connected in star and until recently solidly grounded at the neutral. With the increase in size of transformers and generating equipment, trouble has developed due to excessive currents under single- and two-phase short circuits. To relieve these conditions and to improve the stability of the system generally, the practise of inserting impedance between the neutral and ground of some of the transformer banks has been adopted.

Either a resistance or an inductance can be used for the impedance. Its purpose is primarily to limit the short-circuit neutral current. Having a knowledge of the circuits involved, the necessary impedance can be determined to limit the current to a safe value. With impedance in the neutral, the stresses within the transformer winding when subjected to lightning surges require similar study to that made for transformers with solidly grounded neutral. This is particularly the case since the impedance is selected with no thought of its performance under impulse voltages but only its performance under normal frequency short-circuit conditions. It is the purpose of this paper to study the characteristics of neutral impedances under lightning surge conditions and suggest means for improving their performance.

II. Factors Influencing the Lightning Transient

The effect of voltage surges on transformers with solidly grounded neutral has been previously described.[1] The general factors influencing the transient phenomena are well known, but a brief résumé will be given here.

It was shown in the paper mentioned that the worst stresses within the transformer windings were those due to steep front short waves and steep front long waves. The former imposes the greatest stress upon the winding near the entrance terminal by virtue of its greater magnitude. With a short wave the stresses are proportional to the distribution of voltage determined by the capacitance relationship of the winding. This has been referred to as the initial distribution. With steep front long waves the initial distribution is essentially the same except for absolute magnitude, but the amplitude of voltage at internal points is greater due to oscillations within the winding. The magnitude of lightning voltages which can be propagated on transmission lines is limited by the line insulation and is lower for long than for short waves. A survey of laboratory and field tests on transmission lines and transmission line insulation gives a good indication of the characteristics of lightning waves expected in service. The curve, Fig. 1, estimated on the basis of some of these data, illustrates the variations in the maximum voltage expected on normally insulated lines with length of the traveling wave. Oscillographic records of lightning waves taken in the field, indicate that the greatest duration of voltage probably would not exceed 60 microseconds to one-half value, and the most rapid rate of rise was of the order of one microsecond to maximum value.

Waves having the characteristics of the two above would impose the most severe stresses upon trans-

*Both of the Westinghouse Elec. & Mfg. Co., Sharon, Pa.

Presented at the Pacific Coast Convention of the A. I. E. E., Portland, Oregon, September 2-5, 1930.

1. For references see bibliography.

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formers grounded through neutral impedance. It has been previously pointed out[2] that for short surges only a small amount of energy penetrates the winding as far as the neutral and consequently the increase in voltage at the grounding impedance will be very small. From the standpoint of stress at the neutral impedance and the interior of the transformer winding, it will be necessary to consider only the 60-microsecond surge. Therefore,

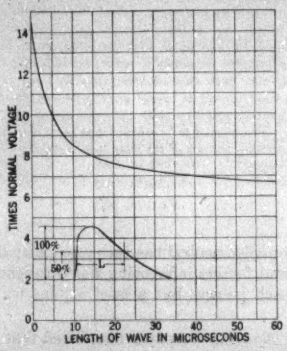

Fig. 1—Variation in Amplitude with Length of Lightning Surges on Normally Insulated Transmission Lines

in performing the tests described below a wave rising to maximum in approximately one microsecond and of 60 microseconds duration to half value was used.

Another important factor influencing the amplitude of voltage in the transformers and at the neutral is the magnitude of voltage occurring simultaneously on the three separate phases. A traveling wave on one line induces waves of the same polarity in the other two lines, but the induced waves are usually of lower amplitude, depending upon the space configuration of the lines.[3] If simultaneous waves of equal amplitude are impressed on each of the three phases, the voltage at the

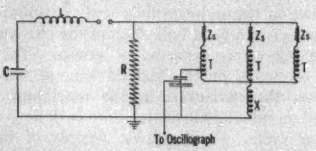

Fig. 2—Schematic Diagram of Testing Circuit

neutral would rise to three times the amplitude that it would if the wave was impressed upon one phase only. In making this investigation the worst condition of three equal surges is assumed.

III. Test Circuits and Method of Testing

Impulses were generated by discharging banks of condensers through a resistance and impressing the voltage drop upon the transformer terminals. The windings of the transformers under test were connected together at the neutral and grounded through the impedance under consideration. The connections from the surge generator to the high-voltage terminals of these windings were made as short as practicable and terminated in lumped surge impedances of approximately 450 ohms. The remaining windings of the transformers were connected to ground through similar surge impedances. Fig. 2.

A surge was impressed upon the transformers and the potential at various points in the winding and across the neutral impedance was measured. The maximum value of the voltage was determined by means of a sphere spark-gap and the voltage-time relation recorded for at least one complete cycle with a cathode ray oscillograph. A capacitance potentiometer was used to reduce the voltage for measurement with the oscillo-

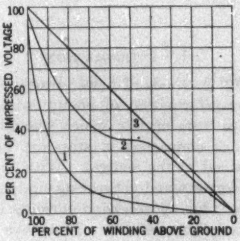

Fig. 3—Voltage-to-Ground Produced in a Transformer Winding with Neutral Solidly Grounded by a 60-Microsecond Surge

1. Initial voltage distribution
2. Maximum voltage by oscillation
3. Final or steady state distribution

graph. This potentiometer was of very low capacitance so that the constants of the circuit were affected very little.

The transformers used in making this study were 25,000-kv-a., 220-kv. power transformers of shell type construction, and of recent design.

The neutral reactors were air core and the neutral resistors were practically non-inductive.

IV. Transformers with Solidly Grounded Neutral

To form a basis for comparing the effects of various neutral devices, the reaction of the transformers with the neutral solidly grounded was determined. The transient conditions involved in this problem have been analyzed elsewhere[1] and will not be considered here.

The initial distribution of voltage with the steep front wave is shown as curve 1, Fig. 3. The maximum voltage to ground occurring at internal points in the winding by oscillation is represented by curve 2 of the same figure. These two boundary curves define the envelope of oscillation of the internal voltage transient.

It is to be noted that the maximum voltage does not exceed the uniform distribution curve.

V. Transformer with Neutral Insulated

The initial distribution, Fig. 4, is practically the same irrespective of whether the transformer is solidly grounded, grounded through an impedance, or completely isolated. This is expected from the initial

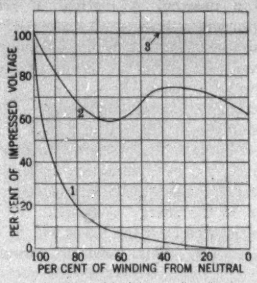

Fig. 4—Voltage to Ground in a Transformer Winding with Isolated Neutral

1. Initial voltage distribution
2. Maximum voltage by oscillation
3. Final distribution or axis of oscillation

distribution curve of Fig. 3 which shows that but a small percentage of the voltage exists in the portion of the winding near the neutral. Consequently in considering the transient effect with various neutral devices the same initial conditions hold within practical limits.

The final distribution is quite different from that with the neutral grounded. When the neutral is isolated and a surge is impressed, the winding tends to assume the same potential throughout. In reaching this potential, it oscillates about the potential on the line terminal. The amplitude of the oscillation at the neutral depends upon the magnitude of the terminal voltage and the relation of the length of the surge to the natural period of oscillation. Under lightning surges the voltage rises to relatively high values. In the experimental case, the maximum amplitude in per cent of the impressed voltages are given in curve 2, Fig. 4.

It is to be noted that the voltage does not attain values in excess of the applied voltage. This is explained, as mentioned above, by the fact that curve 3 representing the initial axis of oscillation decreases as the voltage on the transformer decreases. Since the rate of decrease of the latter is rapid compared to the period of oscillation of the transformer, the oscillations do not develop to the extent that they would if limited only by inherent damping, as would be the case with an infinitely long wave.

VI. Transformer with Neutral Grounded through Resistance

When the transformer neutral is grounded through a non-inductive resistance, the axis about which the winding tends to oscillate is a straight line extending from the line to the neutral end of the winding. The voltage at the neutral will be raised to a value depending upon the value of neutral resistance, the relative effective impedance of the transformer winding, and the length of the traveling wave. In the limiting cases of

Fig. 5—Oscillograms Showing the Variation of Internal Oscillations in Transformer Windings with Different Values of Neutral Resistance

A—69 per cent of winding from neutral
B—36 per cent of winding from neutral
C—Neutral
1 – R = infinite (neutral isolated)
2 – R = 100,000 ohms
3 – R = 25,000 ohms
4 – R = 5,000 ohms
5 – R = 450 ohms
6 – R = 0 (neutral solidly grounded)

zero resistance and infinite resistance, the axes coincide with those for a solidly grounded transformer and for an isolated neutral transformer, curves 3, Figs. 3 and 4. For finite resistances the axis lies between these limits.

When the transformers are subjected to a lightning surge, the voltage at the neutral and the voltage throughout the winding rises to a value in excess of that

attained when the neutral is solidly grounded, depending upon the value of the neutral resistance and the characteristics of the applied surge. With the 60-microsecond test wave, the voltage at the neutral and at points in the winding 36 per cent and 69 per cent from the neutral are shown in the oscillograms, Fig. 5, for various values of resistance between the two limiting conditions. The axis of oscillation rises and the magnitude of voltage increases with increasing resistance. With the transformers grounded through 450 ohms resistance, the rise in voltage at the neutral was approximately 4.5 per cent. The increase in voltage at the two points in the winding, mentioned above, over that for a solidly grounded neutral was small, only about 2 to 3 per cent. With larger values of grounding resistance, the increase in voltage becomes more pronounced, particularly in the portion of the winding near the neutral.

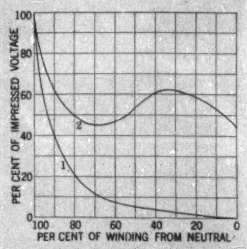

Fig. 6—Voltage-to-Ground in 25,000-Kv-a. 220-Kv. Shell Type Transformer Windings with Neutral Grounded Through a 315-Millihenry Reactor

1. Initial distribution of voltage
2. Maximum voltage by oscillation

VII. Transformer Neutral Grounded through Inductance

In the case of the neutral grounded through inductance as in the previous cases, the initial distribution is the same as for solidly grounded neutral. The oscillatory voltage occurring across the reactor is a function of the relative inductance of the transformer and reactor, and the relative length of the traveling wave. The higher the ratio of reactor inductance to transformer inductance, the greater will be the rise of voltage at the neutral. During the transient existing in the transformer winding, the reactor enters into oscillation with the transformer winding. This results in an increase in voltage stress at the neutral and throughout the winding. This is illustrated in the curves of Fig. 6 and Fig. 3. In the case illustrated, the bank of 25,000-kv-a. transformers was grounded through a reactor of 315 millihenrys inductance. The self inductance of the transformer winding was calculated to be 850 millihenrys. The reactor was designed so that the dynamic rise of voltage was approximately one-half normal line voltage, and the kv-a. capacity as compared to that of one transformer was 144 per cent.

A reactor is a desirable means for grounding the neutral, since its reactive drop adds directly to the impedance voltage of the transformer, reduces the shock of short circuit on generating equipment, has less watts loss, and can generally be designed more economically than a resistor. A method of reducing the excessive transient voltage at the neutral is, therefore, highly desirable. Several ways of improving this feature have been proposed. Some of these are described below.

A. Reactor Shunted with Resistance. If the neutral reactor is shunted by resistance, a part of the energy is conducted directly to ground and a damping effect imposed upon the oscillations in the reactor circuit. The voltage at the reactor will be reduced by an amount proportional to the energy by-passed. Obviously the lower the resistance used the lower will be the voltage across the reactor. This, however, cannot economically be carried to any optional limit, but the influence of the resistor on the characteristics of the neutral impedance must be considered.

Fig. 7—Oscillogram Showing the Effect Upon the Transient Neutral Voltage of Shunting the Reactor with Different Values of Resistance

1 – R = Infinite (neutral grounded through reactor only)
2 – R = 25,000 ohms
3 – R = 11,000 ohms
4 – R = 6,000 ohms
5 – R = 1,200 ohms
6 – R = 580 ohms

The effect of shunting resistance upon the voltage across the reactor is shown in Fig. 7. In support of the statement above, it is seen that the amplitude of the voltage decreases with decreasing shunting resistance. In order to limit the voltage to a low value the resistance

would have to be such that its losses would be high under short circuit and its proportions relatively large.

B. Reactor Shunted by Spark-Gap. The second method of preventing excessive voltage at the neutral is to shunt the reactor with a spark-gap. This method permits the voltage to rise to some safe predetermined value and then reduces it to zero. The voltage at which the gap would be set is determined by the insulation of

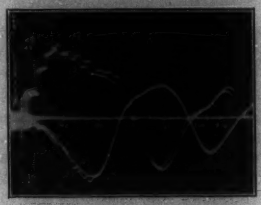

FIG. 8—OSCILLOGRAM SHOWING THE EFFECT UPON THE INTERNAL TRANSFORMER WINDING TRANSIENT OF SHUNTING NEUTRAL REACTOR WITH SAFETY GAP

A —Voltage at 69 per cent of winding from neutral
B —Voltage at 36 per cent of winding from neutral
1,3—Neutral grounded through reactor shunted with safety gap
2,4—Neutral solidly grounded

the reactor. The super-imposed transient accompanying the gap discharge has no damaging effect upon circuits of as long a time constant as that of the transformer winding. The effect of this transient is shown by the oscillograms in Fig. 8, where the resulting voltages at 36 or 69 per cent from neutral are compared with those with the neutral solidly grounded. The increase in voltage is 7 to 9 per cent over that with solidly grounded neutral or about 3 per cent of the impressed voltage.

The principal objection to this device is the liability of the same impulse that causes the gap to discharge also causing a line flash-over or a line to neutral fault, thereby maintaining the arc across the gap with dynamic current and shunting out the reactor at the time when its service is required.

C. Reactor Shunted by Lightning Arrester. Proceeding as in the case of the shunting spark-gap, the proper lightning arrester for protecting the insulation of the reactor was selected. An arrester of the valve type is ideal for this application. The properties of this arrester are well known.[4] Its impedance is practically infinite up to the discharge voltage where it is instantly reduced to a relatively low value. It has a definite cut-off voltage which assures that none of the dynamic current will be by-passed through it. In applying the arrester it is necessary only to fix the cut-off voltage slightly in excess of the dynamic rise of voltage. By so doing, the maximum discharge voltage can be limited to approximately 2.5 times the dynamic voltage even when discharging large currents.

The effect of the arrester discharging on the transient voltage in the transformer winding is shown in Fig. 9. This illustration is a comparison of the voltages with solidly grounded neutral with those when an arrester bridges the reactor. The increase in voltage is of the order of 2 to 3 per cent over that with the neutral solidly grounded.

This method of reducing the neutral voltage has other advantages in addition to those mentioned. It employs a standard protective device which requires no special consideration or calculation for its application.

D. Reactor Shunted with Resonant Circuit. A series capacitance and inductance circuit was connected in parallel with the neutral reactor and the constants proportioned so as to be in resonance at the frequency of the transient voltage existing at the reactor terminal. This circuit offers a very low impedance path to the surge current and shunts it directly to ground, but has a very high impedance at normal operating frequencies. With the surge current determined largely by the impedance of the transformer winding, the two elements are proportioned so as to give a minimum rise in voltage at the neutral and keep the drop across each within safe limits.

FIG. 9—OSCILLOGRAMS SHOWING THE EFFECT ON THE INTERNAL TRANSFORMER WINDING TRANSIENT OF SHUNTING NEUTRAL REACTOR WITH LIGHTNING ARRESTER

A —Voltage at 69 per cent of winding from neutral
B —Voltage at 36 per cent of winding from neutral
1,3—Neutral grounded through reactor shunted with lightning arrester
2,4—Neutral solidly grounded

In the experimental application the circuit was proportioned so that the rise in voltage at the neutral due to lightning would not exceed twice the dynamic voltage under the most severe conditions. Fig. 10 shows oscillograms of the voltage at two internal points in the winding and at the neutral. In oscillogram *A* the voltages resulting from the reactor being shunted with the resonant circuit are compared with the voltages with the neutral solidly grounded. The maximum amplitudes of the voltage at each point are practically

equal for the two conditions, and the shape of the volt-time curves are practically identical, except that with the reactor shunted by the resonant circuit the axis of oscillation is shifted upwards on account of the drop through the circuit. In oscillogram *C* the voltage at the neutral with the reactor alone and with it shunted with the resonant circuit are compared. The amplitude of the voltage in the latter case is reduced to approximately one-sixth of that when the neutral is grounded through the inductance only.

Conclusions

a. Transformers with isolated neutral are subject to transient voltages of a high order from lightning surges throughout the winding and at the neutral and require full insulation to ground for the entire winding.

Fig. 10—Oscillograms Showing the Effect Upon the Internal Transformer Winding Transient of Shunting Neutral Reactor with Auxiliary Resonant Circuit

A —Voltage at 69 per cent of winding from neutral
B —Voltage at 36 per cent of winding from neutral
C —Voltage at neutral
1, 3 & 6—Neutral grounded through reactor shunted with resonant circuit
2, 4—Neutral solidly grounded
5—Neutral grounded through reactor only

b. If the transformer has the neutral grounded through a resistance of not over 400 to 500 ohms, the internal transient voltage stresses exceed those for solidly grounded neutral by only a small percentage. Therefore, in general, the same rules regarding the grading of the insulation to ground apply as for solidly grounded neutral taking due account of the dynamic rise in voltage due to short circuit.

c. The drop in voltage across a neutral resistance is not in phase with the transformer impedance drop, a larger value of resistance is required to limit the current to a given value and consequently a higher voltage appears at the neutral than when the neutral is grounded through an inductance.

d. Transformers with the neutral grounded through a reactor may be subjected to high oscillatory voltages due to lightning and unless some means are provided for reducing this voltage the transformers must have full insulation.

e. The transient voltage as a result of lightning surges in transformer windings grounded through a reactor can be reduced to within a small percentage of that with solidly grounded neutral and the voltage at the neutral maintained at a safe value by shunting the reactor with resistance, with a safety gap or with a lightning arrester or with an auxiliary resonant circuit.

f. In order to obtain the desired characteristics with a shunting resistance, the resistance has to absorb a relatively large amount of energy which may result in its having proportions comparable to that of the reactor.

g. The use of safety gaps appears undesirable on account of the possibility of maintaining the transient arc when the system is operating under short-circuit conditions.

h. Lightning arresters offer advantages in that they have low impedance to lightning transients of high amplitude and practically infinite impedance to the short-circuit currents, are a standard piece of apparatus, relatively small in size and easy to apply, requiring only a knowledge of the short-circuit voltage drop across the reactor.

i. An auxiliary resonant circuit offers an effective means of reducing and maintaining the transient to a relatively low value, approximately that with solidly grounded neutral, and at the same time offer a high impedance to normal frequency current.

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Discussion

A. W. Copley: This paper is valuable in that it gives the results of actual tests on a commercial bank of transformers with various kinds of grounding devices inserted between neutral and ground and with surges of known value impressed across them. The interest in the paper does not center about the use of terms or about the theories involved but rather about the practical results of the tests from the standpoint of the operating man.

Many years ago the ungrounded neutral transmission system was not uncommon. Soon after the advantages of the grounded neutral system were developed most transmission systems had their neutrals solidly grounded. The one disadvantage of the solidly grounded neutral did not appear until systems grew to such a size that it became important. This is the large flow of ground current occasioned by fault to ground on the transmission system. As this feature developed means were sought to reduce the ground current at the same time retaining the advantages of the neutral ground. Resistances were placed in the neutral, then other means, such as reactance, Peterson earth coils and other resonant circuits were used. The effect of these devices on neutral ground current and on the distribution of normal frequency voltage has been naturally well understood. The effect which might be produced by lightning surges has, however, been open to question and the tests which have been reported by Messrs. Vogel and Hodnette give very valuable evidence as to what takes place under these surge conditions. The conclusions which they have reached appear to be logical when a study of the results is made.

The neutral grounded through reactance shunted by a resonant circuit apparently gives good results. The neutral grounded through reactance shunted by a lightning arrester also gives results which appeal to the operating engineer.

L. V. Bewley: The test circuit used in the paper (Fig. 2), shows some lumped resistances Z_s of approximately 450 ohms, which the authors refer to as "lumped surge impedances." The implication is that these resistances behave as true surge impedances, and that the circuit is therefore the equivalent of an actual transmission line. It is the purpose of this discussion to show to what extent their test circuit fails to duplicate the functioning of an actual line. If this fact was understood by the authors they should have called the resistance a *resistance* and not a "lumped surge impedance," as otherwise the terminology is rather misleading.

If any generalized impedance network $Z(p)$, where $p = d/dt$ is the time derivative operator of operational calculus, be connected to the terminal of a transmission line of surge impedance $z = \sqrt{L/C}$, and an incident wave $e = f(t)$ is sent down the line where t is counted from the instant that e arrives at the network, then the total potential at the terminals of the network is

$$e_o = \frac{2\,Z(p)}{Z(p) + z}\,e \tag{1}$$

Therefore the criterion that a test circuit shall duplicate the functioning of a transmission line, is that the total voltage across the test piece shall be expressible in the form of Equation (1), in which e is independent of the connected impedance network.

Solving the circuit of Fig. 2 in the authors' paper for the voltage across the test piece $Z(p)$, (in this case the transformer) there is (calling $Z_s = 3\,z'$)

$$e_o = \frac{2\,Z(p)}{(R + z') + Z(p)} \left\{ \frac{R}{2\,L}\,\frac{p\,E}{p^2 + \dfrac{R}{L}\,\dfrac{z' + Z(p)}{R + z' + Z(p)}\,p + \dfrac{1}{L\,C}} \right\} \tag{2}$$

where E is the voltage to which the condensers were charged.

Now putting

$$z = (R + z') \tag{3}$$

and

$$e = \frac{R}{2\,L} \left\{ \frac{p\,E}{p^2 + \dfrac{R}{L}\,\dfrac{z' + Z(p)}{z + Z(p)}\,p + \dfrac{1}{L\,C}} \right\} \tag{4}$$

Equation (2) superficially takes the same form as Equation (1), *but* e is not independent of $Z(p)$ and therefore the circuit is not a true representation of a traveling wave condition.

However, if the impedance $Z(p)$ is at all instants considerably greater than z, then approximately

$$e = \frac{R}{2\,L} \left\{ \frac{p\,E}{p^2 + \dfrac{R}{L}\,p + \dfrac{1}{L\,C}} \right\}$$

$$= \frac{n + m}{n - m}\,(\epsilon^{mt} - \epsilon^{nt})\,\frac{E}{2} \tag{5}$$

where

$$\left.\begin{aligned} n &= \frac{R}{L} + \sqrt{\frac{R^2}{L^2} - \frac{4}{L\,C}} \\ m &= \frac{R}{L} - \sqrt{\frac{R^2}{L^2} - \frac{4}{L\,C}} \end{aligned}\right\} \tag{6}$$

But to make use of this possibility necessitates making R so small as to decidedly reduce the efficiency of the impulse generator.

The authors make a distinction between a resistance having a constant characteristic (which they call a *resistance*) and a resistance having a variable characteristic (which they call a *lightning arrester*). Apparently this distinction has accounted for many years of misunderstanding over the relative merits of shunting ordinary power limiting reactors with resistors. It is therefore worth mentioning that the resistors which have always been used by the General Electric Co. for shunting power limiting reactors have been "lightning arresters" according to the Vogel-Hodnette definition. Reference may be made to *Shunt Resistors for Reactors* by F. H. Kierstead, H. L. Rorden and L. V. Bewley, A. I. E. E. Trans., Vol. 49, p. 1161, and to the discussion of that paper by F. J. Vogel.

K. K. Palueff: Since January 1929, the attention of the Institute has been called to the phenomena of transient voltages within transformer windings by five papers and considerable discussion.

There are three principal subjects of the above study—the core type transformer, the shell type transformer, and the nonresonating type of transformer. The behavior of these transformers, with neutral isolated or grounded either directly or through various impedances, was described in the above papers.

The two groups of contributors at the beginning entertained practically diametrically opposite views on the phenomena which take place within the winding of an ordinary transformer, particularly of the shell type. It is gratifying to find that the differences in opinion are gradually disappearing.

We were confident that the initial voltage distribution produced by a steep wave front causes an extremely high-voltage concentration at the line end of windings having either wide coils and a short stack or narrow coils and a long stack.

Mr. Hodnette's group was confident that in a short stack of wide coils, which is typical of shell type transformers, the initial or electrostatic voltage distribution is practically uniform, and that such uniform distribution is obtained by properly proportioning the windings [1] and by the slanting of the front of the inci-

1. *Effect of Surges on Transformer Windings*, by J. K. Hodnette, A. I. E. E. Trans. Jan. 1930, p. 72.

dent wave by means of the electrostatic capacitance of the condenser type bushing.[2] That the bushing has an absolutely negligible effect on the transient voltages in a transformer is shown in the TRANSACTIONS for July 1929 on pp. 699 and 700.

In my discussion of Mr. Hodnette's paper of a year ago, I suggested that further study of the phenomenon would bring us to complete agreement on this subject. I am delighted to find that Mr. Hodnette's latest tests have confirmed this suggestion. This can be seen from Fig. 1 of the present discussion.

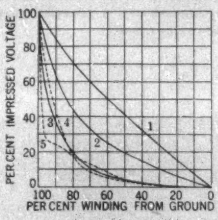

FIG. 1—INITIAL VOLTAGE DISTRIBUTION

Hodnette's Data
1—Original
2—Second
3—Latest
Palueff's Data
4—Order of magnitude, core and shell type transformers
5—Test on shell type transformer

Curve 1 represents Mr. Hodnette's original conception of initial voltage distribution in shell type transformers. It is taken from Fig. 7 of his paper presented in August 1929. Curve 4 represents our conception of the order of magnitude of the initial voltage distribution of the core and shell type transformer in general, and curve 5 shows the results of our test on a shell type transformer. These curves are taken from my papers of January and May 1929. Curve 2 is copied from Mr. Hodnette's discussion of my paper of last May, and represents the result of his test on a 30,000-kv-a. shell type transformer.

Curve 3 is a copy of the initial voltage distribution found by Mr. Hodnette in a 25,000-kv-a. shell type transformer, and published in the present paper as curve 1 of Figs. 3, 4, and 6. The above indicates that we have gradually come to a fairly good agreement as to the initial voltage distribution in shell type transformers. I say, "fairly good agreement," because in shell type transformers, due to the great width of the coil, the initial voltage distribution cannot be represented by a smooth curve as is done by Mr. Hodnette but actually has a saw-tooth shape, which accentuates the severity of the voltage concentration at the first two or three coils at the line terminal, as illustrated on p. 77 of my paper in the January 1930 TRANSACTIONS.

VOLTAGE TO GROUND—GRADED INSULATION

Another point of disagreement between the two groups of investigators is the maximum voltage to ground that can be created in solidly grounded or isolated-neutral shell type transformers when subjected to a lightning or switching wave.

We stated that the envelope of maximum voltages to ground in both core and shell type transformers is about the same and can be represented as shown here by curve 1 of Fig. 2. The other group of investigators at first expressed an opinion that a single wave cannot cause oscillations in shell type transformers. Later, Mr. Hodnette showed that the shell type transformer, even with solidly grounded neutral, oscillates in very much the same way as we have shown. He expressed a belief, however, that these oscillations do not cause the voltage to rise above a uniform voltage distribution (curves 2 and 3 of Fig. 2 of this discussion). This was in accord with his opinion that no lightning waves longer than 60 microseconds should be considered of importance, and that 5,000 cycles was the maximum natural frequency for shell type transformers. If these suppositions were correct, then it could be shown by means of the laws published by the writer in January 1929, that the voltage due to oscillation cannot rise above the straight line, and for this reason he felt confident that grading of the insulation between the high-voltage winding and the other parts of the transformer is permissible. Mr. Hodnette therefore gave curve 2 of Fig. 2 (this discussion) as the maximum possible voltage to ground in a shell type transformer that can be produced by a 60-microsecond wave. This was obtained by him from scaling the oscillograms in his paper. I have shown that more precise measurements of his oscillograms give curve 4 as a more accurate representation of the voltages found in his transformer. Besides, the wave he termed "60 microseconds," as recorded in his oscillograms, was actually only 47 microseconds. Furthermore, I expressed confidence that 5,000 cycles is not the upper limit of natural frequency of a shell type transformer, and that with a higher natural frequency the voltage will rise up to the limit shown by curve 1 of Fig. 2 (this discussion). I am glad that Mr. Hodnette has found a shell type transformer with natural frequency of 12,500.[3]

Mr. Hodnette must certainly agree that at such a natural frequency a 60-microsecond lightning wave will cause a voltage to ground as shown by curve 1 of Fig. 2. This is an inevitable conclusion from the above mentioned law which Mr. Hodnette evi-

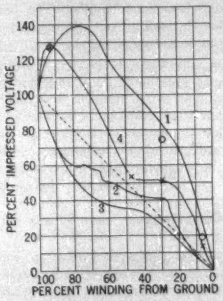

FIG. 2—MAXIMUM VOLTAGE TO GROUND DUE TO A SINGLE LONG TRAVELING WAVE

Paleuff's Data
1—Theoretical maximum. Points shown (circles) were scaled from Hodnette's oscillograms, with 250-microsecond wave in his Sept. 1929 paper
Hodnette's Data
2—Hodnette's first hypothesis (60-microsecond wave) from his Sept. 1929 paper
3—Hodnette's latest, from Fig. 3 of his present paper
4—No. 2 corrected according to Hodnette's present paper with 60- (actually 47-) microsecond wave shown in his Sept. 1929 paper

dently accepted, since he used it as an argument in favor of curve 3, Fig. 2. The law states that the voltage to ground will reach the above mentioned curve if a transformer terminal voltage is maintained at a given value for at least half a cycle of the natural frequency of a transformer. Very simple calculations show that should the terminal voltage, instead of being maintained during half of the period, decay to half of its value during that time, practically the same internal voltage will still be

2. Discussion by J. F. Peters, A. I. E. E. TRANS., Oct. 1928, p. 1014.

3. Discussion by J. K. Hodnette, A. I. E. E. TRANS., January, 1930, p. 81.

reached, in case the oscillation contains higher harmonics. The oscillograms and the theory show that the oscillation of a shell type transformer, like that of a core type transformer, contains a great many harmonics.

The above considerations lead us to the conclusion that the grading of major insulation in solidly grounded transformers is a dangerous practise. The original conception of the phenomena by Mr. Hodnette led him to a diametrically opposite conclusion, as we see from curve 2 of Fig. 2. The actual observation of shell type transformer oscillations contradict his idea as was just shown. In addition, even some of Mr. Hodnette's associates do not agree that a wave longer than 60 microseconds is required to produce voltage above uniform voltage distribution in shell type transformers.

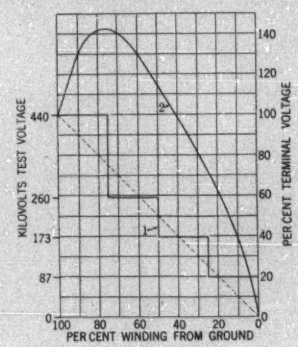

Fig. 3—Comparison of Voltage to Ground and Major Insulation Strength in 220-Kv. Transformer that Failed During Lightning

1—Graded insulation strength
2—Maximum lightning voltage to ground

For example, J. F. Peters states[4] "When the impulse is maintained for a considerable time, 10 to 20 microseconds, the voltage will penetrate into the winding and may overshoot the values beyond a uniform distribution. The amount the voltage overshoots depends to a considerable extent upon the initial distribution which in turn is a function of the physical proportions of the design." It should be noted that curve 2 of Figs. 3, 4, and 6 of Mr. Hodnette's paper show that the initial voltage distribution in a shell type transformer is extremely non-uniform. Furthermore, the failure of a 220-kv. transformer with graded insulation and solidly grounded neutral described by Mr. Floyd substantiates our theory. This transformer failed during lightning, in spite of the fact that the line insulation was coordinated with the insulation of the line group of the transformer, as was shown by Mr. Floyd and by my discussion of Mr. Floyd's paper. Fig. 3 illustrates the relation between the dielectric strength of that transformer as stated by Mr. Floyd and the voltage which should be expected in that transformer when subjected to lightning.

The grading of major insulation is permissible only in case the transformer is of the non-resonating type, as we have shown analytically, as well as experimentally, in the three papers presented to the Institute since 1929. The experimental results were obtained on commercial transformers with artificial lighting voltages of several hundred thousand volts. A short time ago, we were able to demonstrate that our experimental and theoretical conclusions apply equally well to lightning voltages of magnitude met in service. One of the 13,000-kv-a. 220,000-volt non-resonating transformers, built for the New England Power Company has been subjected to an artificial lightning test. This test consisted of repeated applications of voltage from one million two hundred thousand volts up to three million volts. The transformer was connected to the lightning generator by means of a few hundred feet of wire. In shunt with the transformer, fourteen 5¾-in. insulator disks were connected. Voltage waves of one-half microsecond front and of varied length (from 20 up to more than 120 microseconds) were applied, of values just below that necessary to arc-over the insulator strings, thus subjecting the transformer to the highest full lightning wave that could be experienced in service. Also, waves far in excess of the arc-over value of the strings were applied but of course were reduced to arc-over value by flash-over of the strings. Cathode ray oscillograms were taken of a great many of these waves. These lightning tests were preceded and followed by complete A. I. E. E. acceptance tests, which included an induced voltage insulation test of 460 kv. The transformer passed all these tests successfully, and a thorough examination of the windings and insulation of the transformer did not reveal any sign of damage.

It should be noted that in accordance with proposed A. I. E. E. recommendations for coordination of transformer and line insulation, the above induced voltage test corresponds to line insulation of 14 disks.

Fig. 4—Artificial Lightning Test on 13,000-Kv-a. 220,000-Volt Transformer

From the above it appears that Mr. Hodnette's group and ourselves will also reach complete agreement on the question of graded insulation in the near future, as we did on the electrostatic voltage distribution; because it seems beyond any doubt that our minds already run along the same channel. This may be best illustrated by parallel quotations from Messrs. Vogel and Hodnette's present paper and my paper on the same subject of last May, as well as my other contributions.

Vogel-Hodnette—(Page 63)

"The initial voltage distribution, Fig. 4 is practically the same irrespective of whether the transformer is solidly grounded, grounded through an impedance or completely isolated."

Palueff—(Page 1181, A. I. E. E. Trans., July 1930)

"In ordinary power transformers, either of shell or of core type, the initial voltage distribution produced by a steep front of a

4. Discussion, A. I. E. E. Trans., July 1929, p. 1008.

traveling wave is practically the same whether the neutral is solidly grounded or isolated."

(Page 1186)

"With a transformer grounded through an inductance the initial voltage distribution will be the same as in case of a solidly grounded transformer. . . ."

Vogel-Hodnette—(Page 63)

"This is expected from the initial distribution curve of Fig. 3 which shows that but a small percentage of the voltage exists in the portion of the winding near the neutral."

Palueff—(Page 1181)

"This occurs because at the first instant voltage concentrates across the line end of the winding, and the drop across a considerable part of the winding near the neutral end is only a few per cent of the total applied voltage."

Vogel-Hodnette—(Page 63)

"The final distribution is quite different from that with the neutral grounded. When the neutral is isolated and a surge is impressed, the winding tends to assume the same potential throughout."

Palueff—(Page 1181)

"The final voltage distribution in a transformer with isolated neutral, however, is radically different from that of a transformer with grounded neutral."

". . . Therefore all points of an isolated winding finally acquire a potential above ground equal to the terminal voltage . . . The line of final voltage distribution will serve as a line of equilibrium or axis for the oscillation."

Vogel-Hodnette—(Page 63)

"The amplitude of the oscillation at the neutral depends upon the magnitude of the terminal voltage and the relation of the length of the surge to the natural period of oscillation."

Palueff—(Page 1001, Trans., July 1929)

"The amplitude of oscillation depends upon voltage of applied wave, the steepness of the initial (or electrostatic) voltage distribution, the steepness of the front of the traveling wave and the length of the tail of the wave."

(In addition to this a numerical relation was established between the above factors and natural period of a transformer oscillation and was presented principally in the form of a set of curves in my various papers.)

Vogel-Hodnette—(Page 63)

"When the transformer neutral is grounded through a non-inductive resistance, the axis about which the winding tends to oscillate is a straight line extending from the line to the neutral end of the winding. The voltage at the neutral will be raised to a value depending upon the value of neutral resistance, the relative effective impedance of the transformer winding and the length of the traveling wave."

Palueff—

The law stated in the first sentence is shown in my paper in the form of diagram V (Fig. 3, p. 1181 of July 1930 Trans.).

The law stated in the second sentence is stated also in the form of diagrams, (Figs. 10, 11, and 12, pp. 1184 and 1185).

Vogel-Hodnette—(Page 64)

"With larger values of grounding resistance, the increase in voltage becomes more pronounced, particularly in the portion of the winding near the neutral."

Palueff—(Page 1185 Trans., July 1930)

"The effect (the increase of the voltage due to the increase of resistance in the neutral) is greatest at the neutral point; the effect near line end is small."

Vogel-Hodnette—(Page 64)

"The oscillatory voltage occurring across the reactor is a function of the relative inductance of the transformer and reactor, and the relative length of the traveling wave. The higher the ratio of the reactor inductance to transformer inductance, the greater will be the rise of voltage at the neutral."

Palueff—

The same is stated in my paper on page 1186 (Trans., July 1930) in the form of an equation, and by Fig. 14, which gives the numerical relation between the maximum voltage from neutral to ground as a function of the ratio of the neutral inductance and transformer inductance.

The same sort of similarity exists between my first two papers (Jan. and May 1929) and Mr. Hodnette's first paper (Sept. 1929).

Some further similarity could be established throughout the rest of the present paper, but the above is believed to be sufficient to show that fundamentally we are in agreement and that it is just a matter of further tests by Messrs. Vogel and Hodnette to bring their experimental results into as complete accord with ours as are their theoretical conceptions of the phenomena at present.

The "conclusions" arrived at in the present paper, except for the statement regarding the advisability of using graded insulation, appear to be in complete accord with our conclusions. The lightning arrester used at the neutral is the only one variety of the impedor, as I explained in my closing discussion of last May, Trans., July 1930, p. 1196. This was applied by us in practise quite some time ago.

Grounding the neutral through an auxiliary resonating circuit, suggested in the last paragraph of the conclusions, is objectionable for the following reasons. First, because of the possibility of excessive voltages due to resonance in the circuit in case of a damped oscillation being applied to the transformer. Second, the natural period of oscillation of the neutral changes with the mode of transient voltage excitation of the three units of the transformer bank. Therefore, a circuit at the neutral selected to resonate for one condition of excitation will not resonate when the excitation changes, and therefore will become ineffective as a voltage reducing device.

It is regrettable that the authors have not published the oscillogram of the transformer terminal voltage wave that produced the internal oscillations shown in the paper. It is also unfortunate that only two points of the winding in the case of a grounded neutral and three points of the winding in the case of an isolated neutral were investigated, as such a number of points is not sufficient to allow drawing curves such as curve 2 of Figs. 3 and 4 of the paper with proper accuracy. As I pointed out in previous discussions, when tests are made on windings with wide coils it is particularly important that the voltages are measured not only on the outside edges of the coils but also on the inside, since the measurement on the inside edges discloses decidedly higher stresses than those found on the outside. Of course I appreciate that in a completely assembled shell type transformer the inside edges are not accessible unless special means are provided.

E. L. White: It appears that the most desirable form of impedance is a reactor protected by a sphere gap. I should like to ask Mr. Vogel whether or not experiments were made with resistances cut in series with the gap to limit the flow of dynamic current.

F. J. Vogel: Mr. L. V. Bewley's discussion is particularly interesting in that part referring to the surge generator. Strictly speaking the lumped line surge impedance Z_s is a resistance, but in the tests made, with a few exceptions, the assumption that the test circuit used would act similarly to a transmission line circuit was justified. An endeavor to show the reasons for this is given as follows:

First, the wave of the voltage across the resistance R was within a few per cent of being identical whether the transformer was connected or not. Under these circumstances the sole impedance between the surge generated and the transformer was in effect the 450-ohm resistance. This fact states definitely that the surge generated is independent of the connected transformer impedance and, therefore, the surge generator circuit used corresponds in effect to a transmission line connected to the transformer.

Second, the fundamental equation for the voltage across the transformer or load, with reference to the surge generator circuit, is

$$e_o = \left[\frac{Z(p)}{z' + Z(p)}\right]\left[\frac{\dfrac{R(z' + Z(p))}{R + z' + Z(p)}}{\dfrac{1}{c\,p} + L_p + \dfrac{R(z' + Z(p))}{R + z' + Z(p)}}\right] E \quad \text{(A)}$$

No doubt Mr. Bewley arrived at his Equation (2) from the equation above. The first term represents the proportional part of the voltage across the resistance R that is across the transformer. The second term represents the voltage across the resistance R and therefore is the important term. Obviously this second term can be expressed as

$$\frac{\left(\dfrac{1}{c\,p} + L_p + R\right)\dfrac{z' + Z(p)}{R + z' + Z(p)}}{\dfrac{1}{c\,p} + L_p + \dfrac{R(z' + Z(p))}{R + z' + Z(p)}} \quad \frac{R}{\dfrac{1}{c\,p} + L\,p + R} \quad \text{(B)}$$

With reference to the indicial impedance $Z(p)$, $L\,p$ is negligible; furthermore the capacity of the surge generator is made large so that relative to both R and $Z(p)$ the term $\dfrac{1}{c\,p}$ is of secondary importance. The resistance R used is generally considerably greater than z'. These relations state the physical fact that the load network current flows through the surge generator capacity with a relatively small impedance drop. The function of the resistance R is to discharge the surge generator capacity at a rate according to the surge that is desired to be generated. The second term in Equation (A) may then be approximately given as

$$\left[\frac{R}{-\dfrac{1}{c\,p} + L\,p + R}\right].$$

This corresponds to the voltage generated without a load on the surge generator. We can now see the similarity between Equation (A) simplified and the fundamental equation for a transmission line as given in Equation (1) by Mr. Bewley.

$$e_o = \left[\frac{2\,Z(p)}{z' + Z(p)}\right]\left[\frac{R}{\dfrac{1}{c\,p} + L\,p + R}\right]\frac{E}{2} \quad \text{(C)}$$

$$e_o = \frac{2\,Z(p)}{z' + Z(p)}\,e \quad \text{(D)}$$

The latter term, e, then represents the surge voltage on the transmission line, and in combination with the first term combines to form the fundamental equation for the transmission line.

Referring to Mr. Bewley's comments with regard to the resistance units used for shunting reactors, we believe that previous to the paper mentioned, no description of these resistances was offered. It is also to be noted that thyrite is a fairly recent development and is also used as lightning arrester material. It is also believed that resistances, whether of this type or not, were advocated for many years.

J. K. Hodnette: The subject matter given in Mr. Palueff's discussion has largely been covered elsewhere. It deals with impulse transients as a whole, rather than the particular subject of this paper. To prevent repetition it will not be developed to any extent here. Reference is made to discussions in the TRANSACTIONS, January 1930, p. 75, and July 1930, p. 1190. The authors' opinions on the matter have not changed from those expressed in these discussions, although Mr. Palueff interprets them to the contrary.

The positions expressed by him as being diametrically opposite with respect to the views on the phenomena, in reality are and have been different, principally, only in the hypothesis taken at the outset. That is, the characteristics of the impressed surge. In our experiments we have used a surge of characteristics dictated by field tests on natural lightning as being the worst, with respect to the stresses produced, that is ever expected to exist under service conditions. This wave is one that rises to maximum in one microsecond and decreases to half value in 60 microseconds. To establish the logic of this hypothesis it is necessary only to refer to the papers on lightning investigation published in the TRANSACTIONS during the past two years.

Mr. Palueff, on the other hand, has adopted a surge of great if not infinite length. Since no such wave can exist in practise, there appears to be little justification for such a hypothesis. Mr. Palueff apparently recognizes this fact and in his papers states that for finite waves the stresses are not so great, but gives no direct data as to how great they are.

If Mr. Palueff is interested in bringing about an agreement between his data and those published by ourselves and other experimenters, he may readily do so by adopting the above hypothesis. As previously pointed out, he cannot hope to

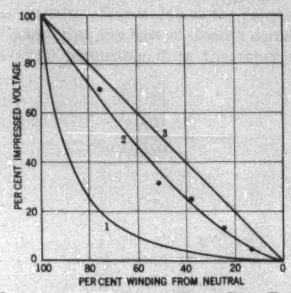

FIG. 5—VOLTAGE TO GROUND IN A SHELL TYPE TRANSFORMER PRODUCED BY A 100-MICROSECOND SURGE

1. Initial voltage distribution
2. Maximum voltage by oscillator
3. Final or steady state of distribution

arrive at conclusions or results representative of actual existing conditions if his hypothesis is not consistent with these conditions.

We object to Mr. Palueff's misinterpretation of our test results. Reference to the discussions mentioned above and the papers discussed will clear these points. This applies to his Fig. 1 showing the evolution of the initial voltage distribution from practically a straight line to that illustrated as curve 3 which agrees with his conception of what the initial distribution should be. There has been no evolution of opinion on this subject. Test data only have been presented, representing a number of different transformers. Mr. Palueff has selected three curves from these and attempted to convey the idea of a changing opinion on our part.

With reference to the natural frequency of transformers, we are quoted as stating that 5,000 cycles was the maximum natural period of shell type transformers. No such opinion can be found in any of our publications. In addition, Mr. Palueff speculates that a 60-microsecond wave would produce a voltage to ground as shown on curve 1 of his Fig. 2 in a transformer having a natural period of 12,500 cycles. Actually the voltage is that shown in Fig. 5, of this discussion, which does not alter but confirm our conclusions.

Whether or not the grading of insulation is a safe practise is

best answered by the service record of the transformers, for that is the true criterion of the sufficiency of a design. Our conclusion in this respect is forcefully illustrated by the fact that there is no record of failure to ground of any transformer built by the company with which the authors are associated in the internal or graded groups. Evidently Mr. Palueff does not understand this system of grading insulation, nor is he in possession of the facts relative to the failure of the transformer reported by Mr. Floyd.

We cannot agree with Mr. Palueff that shunting the grounding reactor with a resonant circuit is objectionable. Irrespective of how an oscillation is produced at the transformer neutral the resonant circuit offers a low impedance path to ground. The tuning of this circuit can be made sufficiently broad to accommodate any variation in the frequency produced by various modes of excitation, which our tests indicate to be small.

Regarding Mr. White's question, no tests were made with resistance in series with the discharge gap across the reactor, as the characteristics can be readily obtained by combining the data for the shunting gap alone with that for the shunting resistance. The former apply up to the time the gap discharges, after which the latter applys since the gap resistance is negligible where the current is relatively large. The objections to such a scheme are the same as for a shunting resistor, that is, it would have to have sufficient thermal capacity to carry its share of the short-circuit current until the fault is removed. This would require a fairly large resistor. Under ideal conditions this scheme would approach the characteristics of a shunting lightning arrester.

New Trends in Mercury Arc Rectifier Developments

BY OTHMAR K. MARTI[1]
Member, A. I. E. E.

Synopsis.—Research investigations conducted by engineers as well as physicists during the last few years have resulted in notable improvements in the mercury arc rectifier, in particular with reference to backfire protection and voltage regulation as well as improved manufacturing methods. These improvements made it possible to build rectifiers of very large current capacities as well as for very high voltages. The basic principle of rectification is briefly reviewed in order to explain the results obtained with the aid of these improvements. Methods of testing rectifiers have also been improved on and new ones developed. Standard parts are now used for different rectifiers up to the largest capacities. An account is given of notable recent rectifier installations for city subway service, for a portable street-railway substation, radio transmission, electrolytic zinc refining, and for Edison systems.

WHEN the steel-enclosed mercury arc rectifier became available to users of direct current, it entered into keen competition with rotary converters and motor generators and presents over the latter a number of advantages now well-known to operating engineers. In the last few years the use of rectifiers has been extended to fields which may be designated as their own in that they comprise applications for which rotary converters or motor generators are either costly and inefficient or not obtainable. One outstanding example is the case of 3000-volt direct current traction where two 1500-volt machines have to be used in series on account of the limitations of commutators, resulting in very large conversion losses and a complicated operating procedure. Another promising field for rectifiers is the electro-chemical industry, where certain processes require voltages as high as 12,000 volts, for which no high-voltage source of direct current of the required capacity was available until the advent of the high-capacity mercury arc rectifier. The steel-enclosed rectifier is also used at present for radio transmission, where a single high-voltage unit replaces numerous vacuum tubes or complicated and delicate mechanical converting equipment.

It is not generally known that the modern steel-enclosed rectifier is a product of the skill not only of the engineer but largely also of the physicist and, to a lesser extent, also of the chemist. Very little has so far been published about the work done by physicists on the mercury arc and comparatively scant literature is available to power engineers, traction engineers, and chemical engineers, in spite of the fact that during recent years they have been using rectifiers in increasing numbers in all parts of the world. It is not intended to give here a complete account of the theory of rectification, but a brief explanation of the valve action of the rectifiers will be given in order to explain some of the latest developments.

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Presented at the Pacific Coast Convention of the A. I. E. E., Portland, Oregon, September 2-5, 1930.

Theory and Design

In order to explain simply the principle of the rectifier, a single-phase two-anode rectifier set is usually considered. Such a rectifier is shown schematically in Fig. 1. Each anode carries the whole current for the half cycle during which it is positive with respect to the cathode, whereas for the other half of the cycle it carries no current. This is due to the property of the anode of preventing the flow of electric current in one direction while permitting it in the other direction.

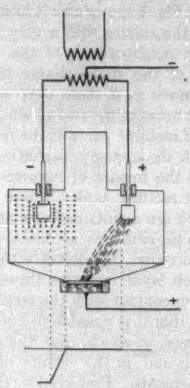

Fig. 1—Single-Phase Two-Anode Rectifier Set and Diagram of Voltage Between Anodes

This alternation of presence and absence of current on each anode is repeated once for every cycle so that, for a 60-cycle alternating-current supply, it is repeated 60 times per second. During this operation, the voltage between the two anodes is equal to the whole voltage of the transformer secondary, whose peak value is over twice the d-c. voltage. In the case of a 13,000-volt radio transmission rectifier, described below, the mercury vapor between the two anodes must, therefore, be able to withstand over 26,000 volts without breaking down.

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The physical phenomena which produce this valve action are the whole secret of the operation of the rectifier and extensive investigations were carried out by physicists in cooperation with engineers in order to elucidate this action. It is not immediately apparent why the current flows between the anodes and the cathode, under the action of a potential difference of only about 25 volts, instead of between the anodes to which the potential difference applied may be 1000 times as great. The passage of current is due to the motion of positive ions and negative electrons traveling freely between the cathode and whichever anode is at a positive potential. The other anode, which is at that time negative with respect to the cathode, is at too low a temperature for emitting electrons. Since it is charged negatively it continually repels the free electrons moving about inside the cylinder, and at the same time attracts the positively charged mercury ions which therefore concentrate around the anode and form all around it what is called a positive space charge which will insulate it for considerable voltages. The actual thickness of this layer, represented by plus signs in Fig. 1 varies in proportion to the voltage. The voltage diagram below Fig. 1 shows how it localizes the voltage drop between the anodes with a very high gradient in the immediate neighborhood of the negative anode, while outside of the space charge the electrons and positive ions move as if there were no potential difference present between the two anodes.

All insulated metallic parts of the rectifier which are in contact with the mercury vapor, receive a negative charge due to the impact of numerous free electrons present in the rectifier tank. Those parts, and also any parts which are maintained at a negative potential with respect to the cathodes, act like the negative anode in that they surround themselves with a layer of positive ions, which layer insulates those parts and protects them from contact with the rectifying arc. It is for this reason that it is possible to build rectifiers with a metal enclosure, in which, also, the arc is guided by metallic funnels and, in the vicinity of the anodes, by metallic anode shields. Protected by this space charge, the different metallic parts act as if they were made of insulating material.

If for any reason the space charge surrounding the negative anode is broken down, the ions and electrons will then circulate between the two anodes, impelled by a potential difference which is no longer 25 volts but 26,000 volts. The secondary of the transformer is actually short-circuited by the arc and a heavy short-circuit current circulates between the anodes. This phenomenon is called an arc back or back-fire. The causes of the breakdown of the space charge around the negative anode are not fully understood. It is, however, well established that a back-fire may be caused by poor vacuum, condensation of liquid mercury on the anode or by impurities of the anode material. The anodes are usually made of very pure iron or graphite, which materials do not emit electrons at the operating temperature of the anode. Foreign particles on the surface of the anodes may, however, become incandescent and emit electrons. Condensed mercury, which emits electrons at a relatively low temperature will also produce a breakdown of the space charge. If the short-circuit current following the breakdown volatilizes the foreign particle, the space charge is reestablished during the next half cycle, and the valve action is restored, which phenomenon is called a silent back-fire. If the breakdown is maintained, however, the protective equipment with which the rectifier is provided will disconnect it from the a-c. line, the valve action is immediately restored, in less time than the breaker requires for reclosing, and the rectifier is again ready to take load.

Engineering Developments

The danger of breakdown of the space charge increases with the capacity of the rectifier so that special means had to be devised for insuring continuity of the valve action of the rectifier. The most important improvement along this line was obtained by means of screens introduced into the arc path in the vicinity of the anodes. These screens may be made of iron or graphite in the shape of concentric rings or wire meshes, and may be solidly connected to the anode shields or insulated from them, or else energized by an outside source of electric potential. When designing such screens, care must be taken that they do not increase the voltage drop in the arc. Following is a brief explanation of the action of these screens:

When an anode is carrying current, its screen takes its potential from the arc and is, therefore, at a lower potential than the anode. When the anodes cease to carry current, the screen retains its potential for a short time, during which it is negative with respect to the anode. It therefore attracts the positive ions and maintains the space charge at the instant when the anode ceases to be positive, at which time the tendency to back-fire is most pronounced.

Since the introduction of these screens, it has become possible to build rectifiers for very high current capacities per anode at medium high voltage and also rectifiers for very high voltages, while keeping their dimensions within reasonable limits. The progress made in this respect is evident when one considers that rectifiers for traction purposes now in operation are rated as high as 3,000 kw. nominal rating at 600-650 volts and 2,000 kw. at 3,000 volts; cylinders used for electro-chemical purposes range from 7,200 kw. at 500 volts to 2,000 kw. at 8,000 volts; for radio transmission, only 400 kw. at 13,000 volts has so far been required.

With the broadening of the applications of rectifiers, a number of problems not directly connected with the operation of the rectifiers themselves had to be considered. One problem which gave some concern to operating engineers a few years ago is the problem of

interference with commutation circuits due to the ripple in the direct current delivered by the rectifier. This problem was carefully studied at the outset and may now be said to be completely solved. As may be seen from the technical literature, equipment was immediately developed to eliminate any case of telephone interference that may occur so that no troubles need be feared from this source any longer.

In order to reduce the effect of back-fires on the rectifier circuit, large capacity rectifiers are equipped not only with quick-acting d-c. breakers but also with high-speed a-c. breakers. Oscillographic records taken in service show that with modern a-c. breakers back-fires can be interrupted in less than 8 cycles after the trip coil of the breaker is energized. The rectifier is put back into service immediately after the interruption.

Another recent development of great value is the regulation of the d-c. voltage of the rectifier by controlling the electric field inside the rectifier. This is obtained by controlling the point of the cycle at which the anodes pick up current in rotation.* This control is obtained by means of the energized screens mentioned above. In normal operation the arc is transferred from one anode to the next when the voltages applied to the two anodes are equal, or, in other words, at the point of intersection of their voltage waves. By retarding the point at which each anode picks up current, instead of using only the peak of the voltage wave for each anode, the firing occurs partly on the peak and partly on the sloping part of the voltage wave. The average d-c. voltage of the rectifier is therefore reduced and it is possible to vary it over wide limits without changing the value of the applied a-c. voltage. This will be made the subject of a later paper.

In the case of rectifiers provided with interphase transformers it is also often desirable to eliminate the abrupt rise of the voltage curve at no load. This voltage rise is present because, when the load decreases below a certain value, the third-harmonic flux in the interphase transformer decreases below its saturation value, thereby making the interphase transformer ineffective. This is easily remedied by providing the interphase transformer with an exciting winding permanently connected to a bank of very small auxiliary transformers connected so as to give a voltage which is a third-harmonic of the fundamental wave.

Not only in the design of rectifiers has considerable progress been made, but also in the measuring methods used with them. This was made necessary by the increased capacities of rectifier cylinders. In the past, the losses have been measured by means of a wattmeter having its current coil inserted in one anode lead and its voltage coil connected between the same anode and the cathode. The voltage drop in each anode can also be determined directly by taking an oscillogram and computing the average of the oscillographic curve. Since, in a rectifier, the losses are independent of the output voltage, they become large compared to the output at very low voltages, and it is possible to measure them accurately by means of the input-output method. The output is easily determined by means of d-c. instruments while the input is determined by measuring the power output of each phase of the transformer secondary, which requires six wattmeters for a six-phase connection. With the very large anode currents encountered at present, it becomes necessary to connect the wattmeter current coil to the anode leads by means of current transformers. Since the anode currents are uni-directional the cores of the current transformers become saturated and their accuracy is impaired. This inconvenience can be avoided and the number of wattmeters reduced to three, by the following novel method:

Each wattmeter current coil is connected to a current transformer having two primaries inserted in opposition into the leads of two anodes working half a cycle apart.

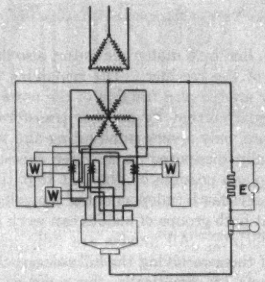

Fig. 2—Measurement of Rectifier Losses—Diagram of Connections

The wattmeter voltage coil is connected to one of these anodes and ground. The wattmeter thus measures the output of two opposite phases of the transformer, and the current transformer, alternately receiving positive and negative current impulses, is no longer saturated so that accurate measurements can be made. A diagram of connections is shown on Fig. 2. By this method half of the secondary copper losses of the transformer are included in the measurement, but the readings can be corrected accordingly.

In the manufacture of rectifiers, load tests are more essential than in the manufacture of other electrical equipment, as there are no other ways for predetermining the performance of rectifiers under load, and for checking whether their construction has been conducted with the requisite care. For a load test at normal voltage and normal output, the rectifier is fed by a transformer having the same rating. In the last few years, due to the rapid increase in capacity of rectifiers, a test at full load or overload, run simultaneously on several

*Mercury Arc Power Rectifiers, by Marti and Winograd, McGraw-Hill, Chapter XII.

rectifiers, will tie up an excessive amount of testing equipment. The capacity of the required transformers can, however, be reduced almost one-half by the following ingenious method:

As shown in Fig. 3, half of the anodes of the rectifier are fed at normal voltage and normal current by a transformer which therefore must have only half the capacity of the rectifier. The d-c. output is fed back

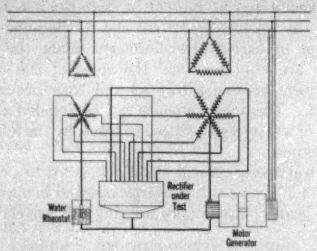

Fig. 3—Load Test of Rectifier—Diagram of Connections

into the a-c. line by a motor generator, also having half the rating of the rectifier. The remaining anodes of the rectifier are fed at a fraction of the normal voltage and at normal current by another transformer which has, therefore, only a very small rating and whose output is usually dissipated in a water rheostat. This method is made possible by the fact that the voltage drop in the rectifier is independent of the voltage on the load, so that both groups of anodes can work in parallel without difficulty. All anodes are fully loaded and, with half of them receiving the full voltage, the operating conditions are practically the same as when all anodes are fed at normal voltage. When the test has run for half the required time, the connections between the groups of anodes and the two transformers are interchanged so that each group of anodes will have received full voltage during one-half of the run.

Standardization of Rectifiers

Fig. 4 shows schematic cross-sections on a comparative scale of one series of rectifiers of the Brown Boveri design having respectively, 6, 12, 18, and 24 anodes. It may be seen that the general outline of all types is the same; this basic construction was adopted over 15 years ago and has proved satisfactory ever since without any major changes. Each rectifier comprises a water-jacketed cylinder covered by an anode plate which carries a condensing dome. The anodes are carried by the plate, from which they are insulated. The capacity of the rectifier determines the number of anodes required, which in turn determines the dimensions of the anode plate. The anodes are sealed in their insulators by means of mercury seals provided with gages giving a permanent check of the condition of the seals. In order that the cathode may easily be removed without lifting the cylinder, the latter is provided with feet of adequate height. Each welding seam of the working cylinder is covered by a channel welded on the cylinder so that, by putting the channel under air pressure, it is possible to test each seam separately without closing and evacuating the whole cylinder.

The types of rectifiers shown on Fig. 4 are standard and, in their design, care was taken to use as many identical parts as possible so as to reduce the number of spare parts necessary for different sizes. Among those parts are the anodes, the anode insulators, ignition anodes, excitation anodes, cathode plates, and the component parts of the seals for their various joints.

The evacuating equipment, which consists of a rotary pump and a high vacuum pump, is the same for all types of rectifiers, two sets of pumps in parallel being used for the larger types of cylinders. In order to simplify erection and reduce the space required for the rectifier, the pumps are now mounted directly on the cylinder.

The excitation and ignition transformers are combined in one compact unit which is the same for all sizes of rectifiers.

Expansion of Fields of Application of Mercury Arc Rectifiers

The majority of high-capacity rectifiers put in service recently are used for traction purposes, for which the experience of a number of years has shown that they are especially suitable. Advantage has often been

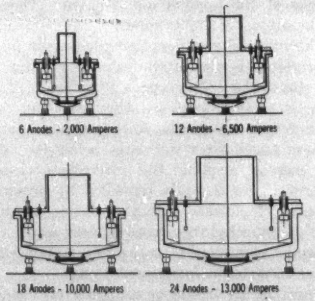

Fig. 4—Comparative Cross-Sections of a Series of Brown Boveri Rectifiers

taken of the possibility of connecting a 25-cycle rectifier set to either a 25-cycle or a 60-cycle supply, as necessary in some installations, or switching it from one supply to the other, should either supply fail. Several rectifiers operating in parallel can also be fed from different a-c. sources, and even from sources of different frequencies, without any of the difficulties encountered with other converting equipment.

A new field of application for large-capacity mercury arc rectifiers was opened with the putting in operation early in 1930 of two 2500-kw., 650-volt, nominally rated rectifiers in a substation supplying power to the Philadelphia City Transit subways. The installation is shown in Fig. 5 and, as can be seen, space has been provided for two additional units. The load conditions which have to be met in this kind of service are best illustrated in the portion of a load chart shown in

Fig. 5—View of Two, 2500-Kw., 630-Volt Nominally Rated Rectifiers in Substation of Philadelphia City Transit Department

Fig. 6. This shows the suddenness with which peak loads are imposed on the converting equipment supplying rapid transit systems. In this installation the voltage rise at no load is suppressed in the manner explained previously, under the heading Engineering Developments.

The mercury arc rectifier has a most promising application in railway portable substations. Fig. 7 shows such a substation for 600-kw., 575 volts in use on the

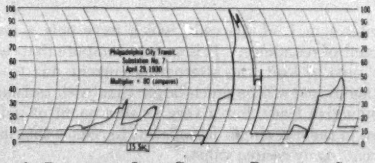

Fig. 6—Portion of Load Chart of Rectifiers Shown in Fig. 5

Calgary Street Railway system. In this substation, the rectifier and auxiliaries, together with switching equipment, are protected by housings, whereas the transformer and reactor are left outdoors. Portable substations being mounted on springs, the capacity of rotating equipment which could be installed in them is very limited as machines of large capacity would produce excessive vibration in a car of standard design. The mercury arc rectifier, on the other hand, being a static piece of apparatus, the only limitations to its use will be the weight and the size of the transformer. It is comparatively easy to install a complete rectifier substation for 3000 kw. at a d-c. voltage of 600, 1500 or 3000 volts on a standard flat car, which is far beyond the present requirements for portable substations. The mercury arc rectifier is specially desirable for this application in view of the ease with which it can be remote-controlled or made fully automatic.

The limitations of rotating equipment have retarded the development of portable substations, but with rectifiers the advantages become so numerous that it may be predicted that their use will be very extensive in the future. Portable substations can be erected complete on the manufacturer's premises and delivered ready to be connected to the line, therefore doing away with all erection expenses. In case of a failure of the equipment, the complete substation can be hauled into the car barn where all repair facilities can be concentrated so that

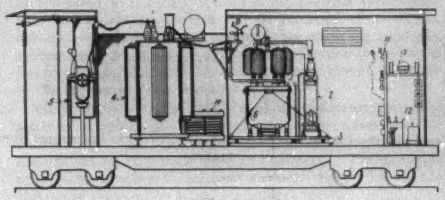

Fig. 7—Elevation of 600-Kw., 575-Volt Automatic Portable Rectifier Substation of City of Calgary

1. Rectifier cylinder
2. Vacuum pump
3. Frame for rectifier and vacuum pump
4. Transformer
5. Oil circuit breaker
6. Guy wires for rectifier cylinder
9. Lightning arresters
10. Air-core reactor for filter equipment
11. Switchboard
12. Ignition-excitation set
13. Filter equipment
14. Protective rail

repairs can be made rapidly and economically. The portable railway substation therefore presents a number of advantages over the stationary type and it may not be impossible that some day they will completely supersede the latter, at least for street railway service and especially for electric trackless trolley buses where the lines could be re-routed with no further expense than that necessary for removing and re-erecting the trolley wires.

For installations with very large d-c. outputs, it may be economical to install the rectifiers outdoors, and, as may be seen from Fig. 8, rectifiers lend themselves readily to very compact layouts. The cylinders are covered by sheet metal housings, and are directly connected to their transformers. Rectifiers and transformers can be rolled into position on a car running on adjacent track siding. The d-c. feeders can be brought out overhead or through cables from the switch house

located in the center of the substation. The a-c. feeder layout is in accordance with standard practise, and requires no further explanation.

A very interesting application of high-voltage rectifiers was found in radio transmitting stations, where

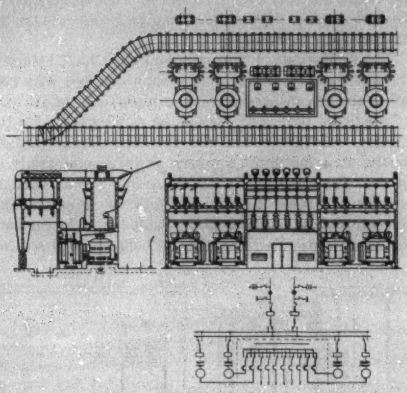

Fig. 8—Project of Layout of Outdoor Rectifier Substation

voltages ranging from 10,000 to 30,000 volts are required. For such stations of considerable power, the mercury arc rectifier is especially well adapted as it can be put in service instantly, resists short-circuits which

Fig. 9—View of 400-Kw., 13,000-Volt Rectifier for Marconi Wireless Telegraph Co., Ltd.

are disastrous to other types of converters and has a high efficiency which will effect a considerable saving compared to thermionic tubes. In addition, the life of the rectifier is indefinite, whereas the tubes have a limited life. Fig. 9 shows a 400-kw., 13,000-volt rectifier installed by the Marconi Wireless Telegraph Company, Ltd., at Chelmsford, England. It can be seen in this figure that the anodes are equipped with over-size insulators; furthermore, that the cooling water is supplied through long rubber hoses wound on insulators shown in the foreground, in order to prevent excessive leakage currents to ground. Another unit, of 270 kw., will shortly be installed by the same company at Lucerne, Switzerland. This application was described because it shows that mercury arc rectifiers can, without difficulty, be built for heavy currents and high d-c. voltages.

Considerable interest was aroused by the installation of six rectifiers for 5,000 amperes each at 460-560 volts by the Consolidated Mining & Smelting Company, Trail, B. C., as shown in Fig. 10. These rectifiers are used for electrolytically refining zinc and are the largest installed on the American continent. In this type of installation, the load is constituted mainly by the

Fig. 10—Rectifier Installation of Six, 5000-Amperes, 460/560-Volt Rectifiers for Consolidated Mining & Smelting Co., Trail, B. C., Canada

counter-e. m. f. of the cells, which have only a small amount of resistance. It is easy to compute that, with this type of load, the alternating current due to the voltage ripple will be much larger than with resistance of inductive loads, but a very small amount of inductance in the d-c. circuit is sufficient to reduce the ripple current to any desired value.

In hydrogen plants, one advantage of the rectifier is that it is not subject to reversals of polarity, which are a source of disastrous explosions. Moreover, in the case of large electro-chemical installations operating continuously 24 hr. a day, the absence of brush wear alone constitutes an appreciable advantage for the steel-enclosed mercury arc rectifiers.

Very recently a number of rectifiers has been installed for supplying power alternately to a 250-volt, 3-wire, d-c. Edison system and a 600-volt railway system. As the mercury arc rectifiers work at any commercial voltage these abnormal operating conditions

are being taken care of by merely changing the transformer connections.

Conclusion

This brief study of recent rectifier developments shows that the rectifier can be used to advantage for any power application requiring conversion from alternating to direct current. It has made available for existing applications reliable units of higher capacity and has permitted the development of applications hitherto retarded by the lack of a suitable converter. Due to the ease of its generation and conversion, alternating current has taken a long lead over direct current, and is being used for applications where direct current would be preferable. Since the advent of the rectifier, however, the use of direct current has received a considerable impetus, and it will no doubt gain additional momentum in the near future.

Discussion

S. Withington: Mr. Marti's paper constitutes an appropriate addition to the literature of experience and description which is accumulating in connection with the mercury arc rectifier.

One of the more important fields for which rectifiers or converters are required is in electric traction. It is of interest to note that of the two examples of 3,000-volt d-c. steam railroad electrification which have been placed in operation during the present summer, one is equipped entirely with motor-generators and the other entirely with static rectifiers. The relative performance of these two types of equipment will be observed with a great deal of interest, as the duties are nearly identical and the inauguration of operation practically contemporaneous.

The subject of rating of mercury arc rectifiers should be settled as soon as possible. The characteristics of rectifiers are in some respects fundamentally different from those of rotary converters or motor-generator sets, and it is not logical to base the rating upon such types of electrical apparatus. It is to be hoped that the joint committee which has undertaken the standardization of mercury arc rectifier ratings will arrive at an early conclusion and that this conclusion will be based upon actual characteristics and not upon analogy.

One of the advantages of static rectifier equipment as compared with rotary converters or motor-generators which Mr. Marti might well have mentioned is in the restoration of service in the event of an interruption to the source of power. In the case of the rectifier the d-c. power may be completely restored simultaneously with the restoration of the supply, while with rotating equipment the entire requisite machine capacity must be started and connected to the supply before the d-c. load can be carried. This difference may be of considerable importance in the case of comprehensive d-c. networks. Some further discussion of this would be of interest.

Mr. Marti has called attention to the adaptability of rectifiers as compared with rotary apparatus for portable substations. There is one limitation which may be important under certain circumstances, unless proper provisions are made. A supply of cooling water is not always available at a location (especially a temporary one) which may be convenient, even though the water cooling system may be a closed one.

The reference which Mr. Marti made to outdoor application of rectifiers is significant. The tendency towards adaptation for outdoor installation of all sorts of electrical apparatus is evident and the saving thus made possible is obvious. The chief difficulty is maintenance in inclement weather. It would seem that this can be overcome by proper temporary shelter.

Development of a Relay Protective System
On the Lines of the Southern California Edison Company, Ltd.

BY E. R. STAUFFACHER[1]

Member, A. I. E. E.

Synopsis.—*A record of the application of protective relays on the system of the Southern California Edison Company, Ltd., is presented. The history of this development of the protective relays in the 220-kv. transmission line, together with the results of the application of protective relays is recorded. In view of the limited application of carrier current protection, considerable discussion is presented covering its application on a section of the 220-kv. transmission system. Allied considerations pertaining to system stability on the 220-kv. system are discussed, and the rearrangement of the 66-kv. system as a means of improving system stability is also presented. In the interest of improving service, certain changes on the lower voltage 16-kv. and 11-kv. systems have been made.*

Introduction

THE last decade has witnessed notable developments in steam generating units, the extension of high-voltage transmission together with the application of increased sizes of transformers, oil circuit breakers and synchronous condensers. The plans for the operation of extended high-voltage transmission networks have become a reality and the reliability of transmission over long distances now compares favorably with the reliability of transmission over much shorter distances a few years ago. The development and application of protective relays has made it possible to maintain continuity of service under conditions which would have been impossible ten years ago. In addition to providing uninterrupted service, protective relays permit a more efficient use of transmission lines by utilizing several lines in parallel with a consequent reduction in $I^2 R$ losses even though one line might be of sufficient capacity to handle the load.

The serious application of protective relays to the system of the Southern California Edison Company, Ltd., dates from approximately ten years ago. In 1920 a program of installing modern type induction relays was inaugurated on the 66-kv. lines. A few years later when the Big Creek 150-kv. lines were modified for 220-kv. operation protective relays were installed although these lines up to this time had been operated without automatic devices of this nature. Since this time the use of modern induction type relays has extended throughout the system with the exception of the 2.3-kv. and 4-kv. lines which only require plunger type relays. At present there are in use approximately 9,300 protective relays on the system.

Previous Work

In 1923 the Big Creek transmission system was changed from 150-kv. to 220-kv. operation. Previous to that time no protective relays for automatically isolating defective sections of the lines had been used. The practise up to 1923 had been to lower the voltage on the transmission lines manually by introducing resistance in the fields of the various generators and synchronous condensers. By means of a "trouble" rheostat the operators throughout the transmission system lowered the voltage until the arc broke, and when ground currents ceased to flow, the voltage was restored to normal by hand. This served to remove the short circuit from the system, but necessarily resulted in a service interruption. At the time the lines were changed to 220-kv. operation it was realized that this system of removing short circuits could no longer be tolerated, and the decision was made to install protective relays. The transmission system at that time consisted of only two lines from the Big Creek group of generating plants to the Eagle Rock and Laguna Bell terminal substations, with a substation known as Vestal and a switching station known as Magunden located in the San Joaquin Valley and approximately in the middle portion of the transmission system. Induction type current balanced relays were relied upon for the automatic isolation of faults. The connections were made so that two lines at a given generating plant or substation were balanced against each other and under automatic protection, but if it was necessary to remove one line from service the remaining line was made solid or non-automatic. Phase relays only were installed. Provision was made at the generating plants and substations to automatically insert resistance in the field of each of the machines in service and thus lower the voltage if by chance a fault should occur on a section where only one line, that is, the line which was not provided with protective relays, was in service in a given section. This scheme of relay protection was a great improvement over the past non-automatic scheme, but a few years' service showed that there were certain improvements which could be made to advantage.

The first change was the addition of current balanced residual or ground relays similar to the phase relays already installed, but designed to operate at a lower value of current. This modification improved the relay operation considerably as it speeded up operation at the time of the heavier ground fault currents and eliminated a faulty section of line when a comparatively light ground current would flow. Under the previous

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conditions of using phase relays only there would not have been sufficient current to cause the phase relays to operate. With this relay set-up the larger percentage of faults was cleared without interruption to service. However, a few of the short circuits were either slow in clearing or occurred at the time of a heavy load on the system, resulting in the two ends of the system going out of step even after proper elimination of the faulty section of the transmission line.

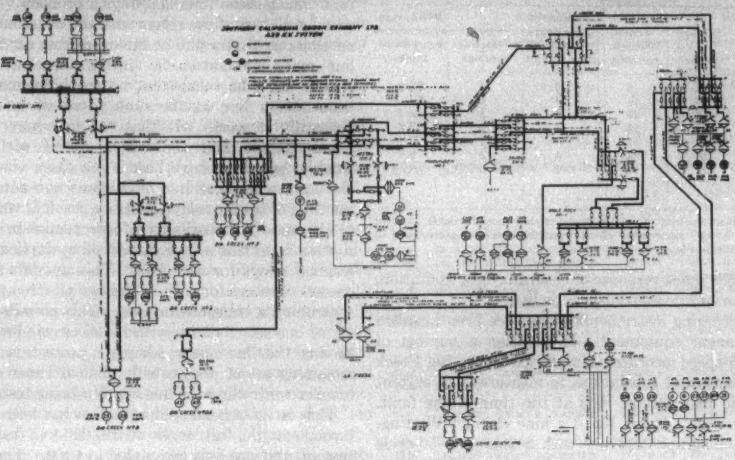

FIG. 1—SINGLE-LINE DIAGRAM SHOWING THE 220-KV. TRANSMISSION SYSTEM OF THE SOUTHERN CALIFORNIA EDISON CO., LTD.

In 1925 arrangements were made with the Westinghouse Company to install special recording instruments on the transmission system. The results of the first eighteen months of investigation have been published already[2] and need not be repeated here. Since 1927 there has been a considerable extension of the 220-kv. transmission system. The development of the protective relay system and the results of its use will be recorded in this paper.

The present 220-kv. system extends from the hydro plants in the Big Creek territory to the steam generating plant at Long Beach—a distance of about 280 miles. Fig. 1 is a single line diagram of this system showing the connections and relation of the various generating plants and substations. Table I presents the synchronous capacity of the rotating equipment at the various generating plants and substations.

220-Kv. Records of Faults

A careful record of the flashovers and other causes which might cause interruptions together with the protective relay operations has been kept from the beginning of the period of automatic elimination of faults on the 220-kv. transmission system. Table II gives this information up to the present time. In this table will be noted the number of transmission faults and the faults cleared correctly together with the number of times the protective relays were not able to clear faults. The relay scheme during the past was such that the removal of one line of a section for maintenance left the remaining line non-automatic. Consequently a fault on this remaining line could not be cleared except by lowering the transmission line voltage with a resulting interruption to service. Another situation was soon apparent. With only two 220-kv. transmission lines available and with the generating stations located at one end of the system and the load at the other, as was the case up to a few years ago, instability would often result at the time of heavy loads even though the relays

TABLE I

Name of station	Number of units	Total capacity name plate rating
Generators		
Big Creek No. 1 hydro generating plant	4	80,500 kv-a.
Big Creek No. 2 hydro generating plant	4	70,000 "
Big Creek No. 2A hydro generating plant	2	90,000 "
Big Creek No. 3 hydro generating plant	3	84,000 "
Big Creek No. 8 hydro generating plant	2	60,000 "
Long Beach No. 3 steam plant	2	200,000 "
Synchronous Condensers		
Eagle Rock substation	4	90,000 kv-a.
Laguna Bell substation	4	140,000 "
Lightipe substation	2	100,000 "
50-60 Cycle Frequency Changers		
Rector	1	30,000 kv-a.
Vestal	2	30,000 "

Total synchronous capacity—974,500 kv-a.

2. *Transients Due to Short Circuits*, by Wood, Hunt and Griscom, A. I. E. E. TRANS., Vol. 47, p. 68.

operated correctly to clear a fault. This situation led to the installation of oscillographs and curve drawing instruments at a number of the generating plants and substations so that the transient conditions resulting from a fault could be recorded. A study of these records indicated the necessity for clearing the faults in a shorter time, limiting the ground current to as small a value as

TABLE II
220-KV. INTERRUPTIONS

Year	Number of faults	Faults cleared correctly	Number of times relays not in at time of fault	*System inter-ruptions	Percentage system inter-ruptions to faults
1924	22	8	13	14	63.8
1925	25	19	4	3	12
1926	36	27	1	8	22.2
1927	24	13	4	9	37.5
1928	26	18	5	7	27
1929	30	25	0	2	6.7
1930 Through June	20	18	0	0	0

*Due to load, severity of trouble, speed of clearance of trouble, location of trouble, system connections which determine source of power, and to protective relays being out of service.

practicable, quick response of generator field excitation, and for an additional transmission line.

The recording instruments have since been installed as permanent equipment. In addition a number of Hall high-speed recorders is in service at the Big Creek hydro plants and substations to measure fault current to ground and voltage dips at the time of the fault. At Long Beach Steam No. 3 a nine element Westinghouse oscillograph together with a standard 50-cycle tuning fork is permanently installed to record transient disturbances.

Protective Relay Installations

The records which have been kept of the flash-overs and other troubles on the 220-kv. transmission system show that at no time has there been any transient fault which has not been accompanied by a flow of some ground current. The wide spacing of the conductors at this high voltage no doubt is responsible for the fact that the likelihood of phase-to-phase faults without a fault current to ground is rather remote. As a result of this situation, the practise of the past few years has been to abandon the use of phase relays entirely, and to depend upon ground or residual relays only for protection. Where two or more lines are connected to a generating plant or substation, these lines are protected by induction type current balanced relays connected in the residual connection of the bushing type current transformers mounted in the high-tension oil circuit breakers. The relay scheme for single-line operation was changed so that protection was provided for a single line by means of residual directional relays actuated from the residual connection of the current transformers in the oil circuit breakers and from current transformers connected in the neutral or the residual connection of the station transformers. The limitations of installation of the neutral or ground end of the 220-kv. transformers necessitated that current transformers be connected in each of the single phase units, making up a three-phase bank and the residual connection from these current transformers in place of the usual method of using one current transformer connected in the neutral of a bank of three transformers, as is generally the case when lower voltage transformers are under consideration.

The question of testing protective relays on important transmission lines is always a problem. To make an over-all test of the relays with the oil circuit breaker necessitates that a line be removed from service or the line made non-automatic during the test, and the chance must be taken that no trouble occurs on the transmission line at the time a test is being made. Generally it works out that the line may be made non-automatic and the relays tested without any trouble, but there have been a few cases when trouble did occur when a section of line was non-automatic at the time of testing relays, and as a result it was decided to eliminate the possibility of a transmission line interruption from this source by installing duplicate sets of residual relays for each line. This amounts to having one set of relays for each of the two oil circuit breakers controlling a transmission line. This permits the testing of one set of relays with its oil circuit breaker and leaving the line under adequate protection with the remaining set of relays, with this particular oil circuit breaker controlling the line which is being tested.

This latest scheme of protection has been installed throughout the full length of the 220-kv. transmission system, and was only completed in 1930. The two sets of relays are designated as set *A* and set *B* at the various generating plants and substations. The connections to the relays are such that normally, with all lines in, dependence is placed upon current balanced residual relays. For single-line protection residual directional current relays or simple over-current relays are used. By means of the type JD relay, which is an accurate disk type of d-c. timing relay or the type PQ-26 relay which is a plunger type, provision is made to assure operation of only the oil circuit breakers on the line in trouble at the time of a fault. This is accomplished by utilizing the timing relay to introduce a few seconds interval of time between the opening of the oil circuit breakers on the line in trouble, and the making of the remaining line automatic for single-line protection.

The 220-kv. Vincent line from Gould substation to Magunden substation, and from Magunden substation to Power House Big Creek No. 3, has slightly different characteristics from the two original Big Creek lines and does not connect with all of the substations between Big Creek No. 3 and Gould, as is the case with the two original lines. As a result, the Vincent line has a somewhat different current distribution than the two other lines at the time of fault on certain portions of the system, and it has always been somewhat of a problem to provide it with proper protection when depending upon

current balanced relays which were satisfactory for the two original lines. The two terminals of one of the two sections of this line are connected to switching stations which are not provided with any means of obtaining potential from the 220-kv. transmission line, and for this reason it was not possible to provide protection with power directional relays. As a result of this condition, it was decided to investigate the possibilities of utilizing the carrier current pilot protection scheme of protection originally described by Fitzgerald,[3] and which offered certain advantages not readily obtainable with other forms of protection on high voltage and long sections of a transmission line.

The carrier current pilot scheme of protection has been in service for approximately a year on the Gould-Magunden section of the 220-kv. Vincent transmission line. It is similar to the other forms of protection on the 220-kv. system in that it provides for line-to-ground short circuit protection only. It may be compared to the pilot wire relay circuit where the pilot circuit is completed by means of a physical conductor between the two stations at the end of the transmission line section. The same net results are obtained, but the exchange of relay operating current does not occur with the carrier current system of pilot protection. It operates in such a manner as to compare the direction of the instantaneous residual currents at the two ends of the section of line affected. If the instantaneous residual current is in phase, that is, in the same direction, indicating that a fault is external to the section protected by the carrier current pilot protection, the equipment operates to lock out and prevent the induction type over-current relays from tripping the automatic oil circuit breakers. However, if the fault is in the section of line under carrier current pilot protection, current will flow towards the fault from the energy sources at each end of the section and the currents at the two ends will be approximately 180 deg. out of phase with each other. Under these conditions of the instantaneous residual currents flowing in opposite directions, a fault within this section is indicated and the carrier current receiver at each end is made inoperative automatically, which permits the induction type over-current relays to function at both ends and trip the oil circuit breakers, thus clearing the fault in this section from the system. A detailed description of this installation has been published elsewhere,[4] so it will not be necessary to repeat it here. Table III gives the results obtained with this protective equipment. When it is considered that it is an innovation in protective schemes, and that a great many problems had to be worked out before it was manufactured and installed the results have been quite satisfactory. At each of the two switching stations the section of the 220-kv. line which is protected terminates in a set of two oil circuit breakers which connect to the two high-voltage buses at each station. Coupling between the 220-kv. Vincent transmission line and the carrier current pilot protection is obtained by means of oil-filled coupling capacitors connected to two of the three-phase wires,

3. *A Carrier-Current Pilot System of Transmission Line Protection*, by A. S. Fitzgerald, A. I. E. E. Trans., Vol. 47, p. 22.

4. "Carrier Current Replaces Relay Pilot Wire," E. R. Stauffacher and F. B. Doolittle, *Electrical World*, Vol. 95, No. 12, p. 580.

TABLE III

OPERATION OF CARRIER CURRENT PILOT PROTECTION ON THE VINCENT 220-KV. LINE—95 MILE—MAGUNDEN TO GOULD SECTION

Date	Location of fault	Operation of carrier current pilot protection	Remarks
July 3, 1929	Exterior to section	Incorrect	Insulator string parted, grounding conductor. Oil circuit breaker at Gould tripped out on Vincent Line as the result of incorrect current transformer connection at Gould
July 3, 1929	Exterior to section	Incorrect	Same trouble as above. Switch opened at Magunden when defective line accidentally energized
Aug. 9, 1929	Exterior to section	Correct	Tree blown into line
Aug. 13, 1929	Within section	Correct	Lightning
Aug. 14, 1929	Exterior to section	Correct	Station 220-kv. transformer breakdown
Aug. 19, 1929	Exterior to section	Correct	Brush fire under line
Aug. 20, 1929	Exterior to section	Correct	Brush fire under line
Aug. 25, 1929	Exterior to section	Correct	Breakdown of 220-kv oil circuit breaker
Oct. 27, 1929	Exterior to section	Correct	Bird trouble on line
Nov. 17, 1929	Exterior to section	Correct	Bird trouble on line
Dec. 25, 1929	Exterior to section	Correct	Bird trouble on line
Dec. 31, 1929	Exterior to section	Correct	Wire thrown over 220-kv. line
Jan. 7, 1930	Exterior to section	Correct	Unknown—heavy wind at time of trouble
Jan. 13, 1930	Exterior to section	Correct	Bird trouble on line
Feb. 1, 1930	Exterior to section	Correct	Bird trouble on line
March 14, 1930	Within section	Correct	Sleet storm along line
March 14, 1930	Exterior to section	Correct	Overhead ground wire parted.—Storming
March 14, 1930	Within section	Correct	Sleet storm along line
March 14, 1930	Within section	Correct	Sleet storm along line
March 14, 1930	Within section	Correct	Sleet storm along line
March 14, 1930	Within section	Correct	Sleet storm along line
March 14, 1930	Within section	Correct	Sleet storm along line
March 15, 1930	Within section	Correct	Sleet storm along line (The eight cases of trouble above were evidently caused by sleet loading the conductors and the overhead ground wires resulting in contact between them)
March 25, 1930	Exterior to section	Correct	Wire in contact with conductors
April 13, 1930	Exterior to section	Correct	Flashover on bushings of 220-kv. station transformer
April 16, 1930	Exterior to section	Correct	Bird trouble on line
April 23, 1930	Exterior to section	Correct	220-kv. oil circuit breaker trouble, flashing to ground
April 23, 1930	Exterior to section	Correct	Bird trouble on line

SUMMARY

Total cases of trouble since carrier current pilot protection installed	28
Cases of trouble exterior to section	20
Cases of trouble within section	8
Correct operations	26
Incorrect operations	2

each connected from line to ground, which gives what is termed "interphase" coupling. This coupling is made to the same phase wires at each end of the section protected, and the same set of capacitors is utilized for transmission of the carrier current pilot protection carrier frequency and the carrier current telephone system carrier frequency over the same transmission line. Between the coupling capacitors and the 220-kv. oil circuit breakers at each station are installed carrier current frequency traps built quite similar to a large choke coil and having a capacity of 800 amperes continuous duty, and an inductance of approximately 70 microhenrys. Due to the double use of the capacitors, it is necessary to filter the protection carrier frequency and the communication carrier frequency into their proper channels.

The relatively heavy volt-ampere burden placed on the bushing type current transformers by the carrier equipment and the necessity of exciting the carrier current equipment from the secondaries of these bushing type current transformers requires that the fault current to operate the equipment must be 320 amperes or more. This rather coarse setting of 320 amperes has been satisfactory, but it would be desirable to obtain a setting of from 200 to 250 amperes if practicable. Studies and experiments are now under way by the manufacturers which give promise of obtaining certain characteristics which will permit the lower setting to be used. The over-current induction type relay which is used is so designed as to operate at an unusually high speed for this type of relay. In order to obtain the greatest speed of operation of the carrier current pilot protection consistent with a reasonable life of the vacuum tubes, the filaments of all of the tubes are operated at approximately full voltage, but with no excitation on the plates. The experience so far indicates that it is necessary to make a transmission check of the equipment at frequent intervals, and provision has been made to make a simple transmission check at each station every two weeks. This check requires about one hour to make, and has been simplified so that it is now a part of the regular duties of the station crew in each station.

Allied Considerations

The successful application of protective relays on a transmission system is so closely linked with the static and transient stability of the system that consideration must be given to these features whenever relays are considered.[5] Stability is the outstanding feature of long high-voltage transmission lines which determines the amount of power which can be commercially handled, and it is obvious that the stability of a transmission system is determined by the longest section of line which may be suddenly isolated from the system to clear a fault. The static stability limit can be readily calculated so that it is known that if the power demand is such that the reduced transmission line capacity will not hold the generating and receiving ends of the line in synchronism, instability will result. The determination of the transient stability limit does not readily permit a definite solution as the factors to be considered cannot always be obtained with the accuracy which is desirable. However, there is a number of outstanding features which can be considered and applied to a transmission system which will help greatly in maintaining stability under conditions of heavy short-circuit currents.

First, a prompt clearance of a fault from the system is without a doubt the most important consideration. It is agreed now that the main factor in reducing disturbance and preventing loss of synchronism between the generating and receiving ends of a transmission line is the rapidity of operation of protective relays and oil circuit breakers. During the past few years a great deal has been accomplished along these lines. Protective relays are now available which will operate in a few cycles, and the manufacturers can now supply oil circuit breakers guaranteed to operate in periods of between eight and twelve cycles. Unfortunately such equipment has not been available for any great length of time, and the cost of replacing or rebuilding present 220-kv. oil circuit breakers is so high that it is not justified except as a last resort. On the system of the Southern California Edison Company, Ltd., the recent oil circuit breakers purchased have been of the modern high-speed type, and all future oil circuit breakers no doubt will take advantage of this feature of protection to gain greater assurance of stability under times of system disturbance.

Second, the opinion still holds that the design of generators and synchronous condensers from the standpoint of short-circuit ratio is important. A high short-circuit ratio, somewhere between 1.5 to 2, is quite desirable as it results in certain characteristics of machines which tend to increase the system stability. The generators which have been installed at Big Creek and Long Beach Steam Plants during the past five years have been designed with as high a short circuit ratio as practicable so as to take advantage of this feature of maintaining system stability.

Third, quick response or super-excitation provided for the purpose of building up a voltage quickly across the field winding of a synchronous machine by providing additional magneto-motor force to neutralize the demagnetizing action of the short-circuit current in the armature winding, is not considered quite as important now in relation to other features of system stability as it was a few years ago. However, it is felt that if this quick response excitation can be applied at the time the newer and larger generators are installed, and to certain of the older and larger sized generators, that it is justified and assists materially in maintaining transient stability.

Fourth, on the system of the Southern California Edison Company, Ltd., experience has shown that in the

5. "Extra High Voltage Transmission with Special Reference to American Practice," J. P. Jollyman and E. R. Stauffacher, World Power Conference, Tokyo, 1929, Paper No. 433.

case of transmission line troubles on high-voltage lines, faults from phase to ground predominate, and due to the construction of 220-kv. transmission lines, this is probably the case with most systems. The power consumed in the fault causes a heavy power demand upon the transmission system as a result of the flow of ground current. Accordingly, it has been decided to take steps to limit the flow of ground current and the latest transformer banks connected to the 220-kv. system in the metropolitan area adjacent to Los Angeles have been designed to operate with some device in the neutral to limit the flow of current. At La Fresa Substation, current limiting reactors rated at 600 amperes—60,000 volts—50 cycle have been installed. A reactor is installed in the neutral of each of the two 75,000-kv-a., 220-kv. star to 66-kv. delta transformer banks and the transformers are so designed that the 220-kv. neutral end is provided with 73-kv. insulation. Around each of the reactors from terminal to ground is mounted a sphere-gap set to flash over at 100 kv. The reactors are enclosed in a tank and oil immersed. The 120,000-kv-a., 16-kv. delta to 220-kv. Y transformer bank recently installed at Long Beach Steam Plant No. 3 is designed with 73-kv. neutral insulation and operates with a neutral grounding device designed for use with a non-resonating transformer and rated at 4 amperes, 50 cycles, 73,000 volts. These devices for limiting the ground current have been in service for several months, and so far have performed satisfactorily. They are considered the first step in the program of limiting the flow of ground current on the 220-kv. system, and no doubt will be used further as other transformer banks are installed.

Fifth, by means of momentarily decreasing the output of the generators at the sending end of a long transmission line at the time of a fault, it appears that the tendency for the generators to get out of step with the load at the receiving end could be decreased considerably as the result of reducing the load which is being transmitted. A device which will be actuated by ground current and which will operate upon the governors of the water wheels at the generating plants is under consideration. Generators at the steam plant adjacent to the receiving end should be able to pick up momentarily the greater portion of the load which is dropped, and as a result the system frequency should not suffer to any extent.

Sixth, in applying transient stability calculations, the characteristics of the generators, synchronous condensers, transformers and lines can be determined, but there is little information available covering the characteristics of a mixed load of a large city. There is need for further knowledge concerning the stability characteristics of a load supplied by a transmission system. The combination of industrial and railway power, together with lighting and heating service, and the question as to how much such a load affects the power system at the time of a fault on the transmission line, is one which deserves considerable study, and when the information is available concerning it, should assist materially in making transmission line stability calculations.

Seventh, it is apparent that the ground impedance of transmission lines at the time of faults along the line is a factor which must be considered carefully at the time of studying relays or predicting inductive interference phenomena, and the available knowledge concerning this ground impedance throughout various portions of the country is not sufficient. To gather more information concerning this, a few tests have been made on the system of the Southern California Edison Company, Ltd.,—the latest being made on the Lighthipe-La Fresa 220-kv. lines. It was found that the measured zero phase sequence reactance per phase of the Lighthipe-La Fresa 220-kv. lines when considering one line alone amounted to 1.64 ohms per mile. This line was 9½ miles in length. When measuring with both lines,—out one line and back the other, a total distance of 19 miles, the zero phase sequence reactance per phase was found to be 1.59 ohms per mile. With two lines in parallel, a total distance of 9½ miles, it was found to be 1.30 ohms per mile. It was found that if all three conductors of one line be used as an artificial ground wire for the other line, they will together carry about 50 per cent of the total return current. From this it appears that no matter how elaborate an arrangement of ground wires is installed, it is probable that the earth will carry at least one-half of the ground current at the time of a fault. The 2/0 HTBB overhead ground wire installed on the Lighthipe-La Fresa line carried from 20 to 30 per cent of the total return current. This ground wire current was not uniform along the ground wire, but decreased from the source end towards the grounded point,—the value at the grounded point being about 70 per cent of that at the source end.

66-Kv. System

The 66-kv. network has been rearranged considerably during the past few years, particularly from the standpoint of routine operation and protection. Previous to about four years ago, the 220-kv. to 66-kv. transmission receiving stations were interconnected quite closely on the 66-kv. side with the 66-kv. distribution stations looped into these connecting lines. The general practise was to have two or more incoming 66-kv. lines to each substation. This resulted in a closely coupled 66-kv. network between the various receiving stations and the condition that a fault on any of the 66-kv. lines affected all of the 220-kv. receiving stations. The magnitude of the short-circuit currents was increasing rapidly, and it was evident that some change would be required if we could hope to hold these fault currents down to a reasonable value. Reliance was placed upon directional power protective relays to isolate faults, but due to the necessity of progressive timing for selective operation, the possibility of quickly isolating extremely heavy short circuits adjacent to the 220-kv. receiving stations became more difficult. These receiving sta-

tions being the source of supply for the 66-kv. network, of necessity had to have the longest time setting when any form of protective relays embodying progressive time setting was used. Accordingly, in 1926 it was decided to begin changing the 66-kv. system from a loop system wherein the 66-kv. distribution stations were supplied from adjacent receiving stations, to a radial system wherein each of the 66-kv. distribution stations was to be supplied by two or more 66-kv. lines radiating from one of the receiving stations. Furthermore, every effort was to be made to limit the phase-to-phase or phase-to-ground 66-kv. short-circuit current to approximately 1,000,000 kv-a. with an occasional exceptional upper limit of 1,500,000 kv-a.

With the radial scheme of operation where two or more lines radiated from the major receiving station to a 66-kv. distribution station, it was no longer necessary to rely upon power directional relays progressively timed, so advantage was taken of the characteristics of current balanced relays wherein the currents in corresponding phases of two or more lines were balanced against each other. This permitted a more prompt clearing of 66-kv. faults with a consequent improvement in system operation. This changing of the transmission system has been going on steadily since 1926, and at present is fairly complete in the central portion of the 66-kv. 50-cycle system. Reference to Fig. 2 will show the system at present, and it will be noted that out of Eagle Rock, Laguna Bell, Lighthipe, Long Beach No. 2, and La Fresa substations pairs of radiating lines are quite general. In a number of cases the 66-kv. substations can be supplied from two of the major receiving stations, and at these sectionalizing points one set of oil circuit breakers is opened or the bus is split so that under normal operating conditions the station is supplied from only one receiving station. This change in transmission line connections and in the method of operating the 66-kv. network has resulted in a marked improvement in service and a decided limitation of a 66-kv. disturbance to a comparatively small area. The decrease in the short-circuit current and particularly the increased speed of clearing short circuits due to the use of current balance relays has resulted in a marked decrease in the tendency towards system instability at the time of 66-kv. faults. As an adjunct to the current balanced relays, current balanced residual or ground relays are installed when necessary, and provision is made to maintain protection by means of over-current or power directional phase or residual relays for single-line protection.

16-Kv. and 11-Kv. Systems

Heretofore the usual method of providing for supply to 16-kv. or 11-kv. substations was by means of a separate feeder or a tap off of a feeder radiating from a 66-kv. substation. During the past two years steps have been taken to provide two or more sources of supply to these lower voltage substations, using either a scheme of loop operation between two 66-kv. substations, or the preferred and emergency supply method of operation. Recent analysis of the status of these lower voltage substations supplied from the major 66-kv. stations shows that at present there are thirty-five distribution substations now being operated on the loop system, of which twenty-four stations are operated on the 16-kv. system and eleven on the 11-kv. system. The maximum transformer installation is 12,000-kv-a. and the minimum is 750 kv-a., and the total transformer capacity aggregates 139,150 kv-a. There are ten distribution substations now supplied by the preferred

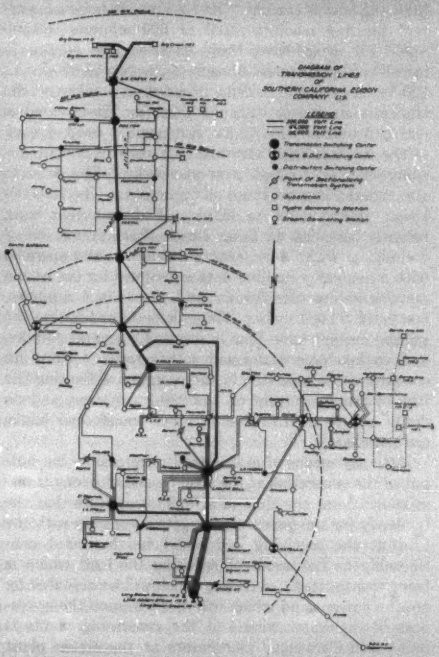

Fig. 2—Diagram of Transmission Lines of the Southern California Edison Co., Ltd.

Showing general scheme of 66-kv. parallel line supply to substations adjacent to major receiving stations

and emergency operation scheme. The maximum installation has a capacity of 3,000 kv-a. and the minimum 750 kv-a., and the total capacity of these substations amounts to 18,750 kv-a.

An analysis of the cases of trouble experienced on lines supplying the stations operating on the low-voltage loop connections shows that during the first six months of 1930, 38 cases of trouble were experienced, and 31 out of the 38 cases were cleared correctly. In analyzing the seven cases which did not operate correctly, it was found that two were caused by mechanical trouble with

the oil circuit breakers, two by incorrect relay wiring, one by insufficient ground current, one by the direct current trip circuit being de-energized, and one unknown. The above record shows that there is some room for improvement, but the results were so much more satisfactory than what would have been experienced had the substations been supplied from a single radial line as had been the practise in the past, that it is felt that loop operation of low-voltage distribution substations is successful and should prove a very prominent factor in minimizing service interruptions.

An analysis of the experience with the preferred and emergency scheme of operation shows that during the first six months of 1930 there have been five cases where the supply had to be changed over, and the only case of trouble experienced so far has been due to a minor mechanical adjustment causing one of the stations to fail to function properly. The mechanism of the preferred and emergency service scheme is so adjusted that the station supply is interrupted for such a short time that the interruption is not noticeable, for tests show that the time required to change from a deenergized line or the line in trouble, to the energized or emergency line, is approximately 18 cycles on a 50-cycle system. This small interval of interruption is barely enough to cause a perceptible voltage dip.

Operating Experience

Careful records of the performance of protective relays and automatic oil circuit breakers have been kept for the past several years. Table IV is a tabulation of

TABLE IV
OPERATION OF PROTECTIVE EQUIPMENT
220 Kv. to 11 Kv. Inclusive

	Year				
	1925	1926	1927	1928	1929
Cases of trouble	1493	2185	1799	1415	1303
Automatic switch operations	2368	3231	2831	2085	1923
Unnecessary switch operations	278	316	287	151	70
Failure of switches to operate	50	47	48	25	33

the experience through 1929. It will be noted that the trend is towards fewer cases of trouble and consequently, in general, fewer automatic switch operations. There has been a decided decrease in the unnecessary switch operations which shows the result of wider application of protective relays together with more knowledge of their characteristics and limitations. Intensive studies of the magnitude and distribution of phase-to-phase and phase-to-ground short circuit currents have been of immense value in determining proper relay settings for various types and locations of faults. The decrease in "failure of switches to operate" has not been so marked. These failures are in general the result of some form of trouble on the tripping side of the protective relays. A tripping coil may become defective, an auxiliary switch in the tripping circuit may not be making proper contact, mechanical faults may develop in the oil circuit breaker operating mechanism, or the control wiring may be defective. A program of inspection, testing and careful maintenance of the oil circuit breakers, protective relays, and tripping circuits is the only means of lessening these "failures to operate."

Protective Equipment—General

In addition to protective relays and automatic oil circuit breakers there is certain allied protective equipment which must be considered as part of the necessary apparatus to provide against abnormal conditions. Current limiting reactors, lightning arresters, ground detectors, and grounding banks fall in this class. Table V shows the amount of protective equipment in service from year to year since 1925. It will be noted that the amount of equipment in service has, in general, increased steadily each year. This is to be expected as

TABLE V
PROTECTIVE EQUIPMENT IN SERVICE

	Year				
	1925	1926	1927	1928	1929
Protective relays	3562	6631	7600	7844	9180
Automatic oil circuit breakers	1961	2129	2464	2707	3043
Lightning arresters	592	685	1054	1243	1268
Current limiting reactors	63	68	84	112	155
Feeder ground detectors	89	88	94	87	77
Feeder grounding banks	..	..	..	..	9

it is consistent with the growth of the system. The exception is in the case of the number of feeder ground detectors and is the result of changing the transformer connections from isolated delta to grounded Y for the feeder supply in a number of the distribution substations. With grounded Y supply there is not the necessity for ground detectors as in the case with the other form of connection.

Remote Alarms

In unattended distribution substations located in rural districts a marked improvement in decreasing the length of a feeder interruption has been accomplished by installing remote alarms to the nearest attended substation. Where available, the existing private telephone lines between the attended and unattended stations are used to transmit the signal. Auxiliary switches on the feeder oil circuit breakers in the unattended substation are connected so that the opening of the oil circuit breaker closes the contact from the two telephone wires to ground, thence over the telephone wires to a source of low-voltage direct current and a relay with a bell alarm located in the attended substation. After this signal is received the station attendant can connect with the district headquarters and a trouble man is sent out to determine the cause of the interruption.

In a number of cases the unattended substations do not have telephone connections. To obtain an alarm without the necessity of building telephone lines it was decided to take advantage of another application of

vacuum tubes and to install carrier current alarm equipment. Recently orders have been placed for seven transmitters, five single and one double receivers. In general the alarm scheme is the same as the one utilizing telephone wires. There will be installed at each unattended substation a carrier current transmitter which will operate from the auxiliary switches on the oil circuit breakers. A receiver to receive the signal and operate the alarm will be installed in the attended station.

Conclusions

(a) In the interest of assuring a more reliable performance of the protective relays on the 220-kv. transmission system under conditions of testing and overhauling the protective equipment as well as during normal operating conditions, duplicate sets of relays are installed. This permits the protection to be retained at all times.

(b) The use of carrier current pilot protection on one section of the Vincent 220-kv. transmission line has been satisfactory and has shown the value of this form of protection under conditions which would not permit the use of other forms of protective relays.

(c) The use of current balanced relays and radial lines between the 220-kv. major receiving stations and the 66-kv. distribution stations has been an improvement over the previous practise of looping through the 66-kv. stations between adjacent 220-kv. stations. The elimination of the necessity of depending upon power directional relays progressively timed has speeded up the time required to clear faults.

(d) The practise of looping 16-kv. and 11-kv. distribution stations between adjacent 66-kv. stations in place of the previous practise of using single radial lines has resulted in a marked improvement in service.

(e) The use of a remote alarm between unattended and attended stations to indicate when an oil circuit breaker trips out at the unattended stations has lessened the time of outage considerably in rural districts. Private telephone lines or carrier current equipment coupled to the transmission lines permits the installation of this remote alarm at a reasonable expenditure.

Acknowledgment

The author wishes to acknowledge the assistance of Miss M. Macferran, Messrs. C. H. Hagey, and F. L. Eley, all of the Southern California Edison Company, Ltd., in compiling the data on the ground wire tests, the 220-kv. relay performance and the preparation of the diagrams.

The Communication System of The Southern California Edison Company, Ltd.

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Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*This paper gives a general description of the Southern California Edison Company's communication system which not only meets the requirements of load dispatching but also serves to bring about close cooperation between the outlying district forces and their directing heads at division headquarters and in the general office. Decentralized dispatching which makes possible the restoration of service after an interruption without the immediate need for communication is touched upon. The detail of the development and construction by the Company of the "Serjdetour" arrester is made public for the first time. This piece of equipment has made possible a protective system which not only prevents damage to valuable communication equipment in stations for the extreme case of actual contact between telephone lines and power lines but also preserves continuity of communication service after power system disturbances which create surges of considerable magnitude on inductively exposed telephone lines. A brief outline is presented of the standardized assemblies of telephone equipment into units which are adaptable to wide variations in requirements by the use of different combinations and quantities of standard units. The communication building where the telephone system centers is described, together with the methods of handling traffic through this point. A brief description of the testing equipmens for initial tests on new lines and maintenance of existing facilitiet is given together with test results on a typical line 279 miles long.*

Introduction

In general the communication system of a power company is not given the consideration it deserves; in fact, its existence is usually through necessity alone. In the case of a power company operating a transmission system covering hundreds of miles and extending to hydroelectric plants located in sections remote from population centers, it is very unlikely that commercial communication facilities will be available, and therefore the power company builds telephone lines to the points where service is needed. The tendency has been to construct these lines as cheaply as possible.

In most cases the lines are built jointly with power transmission lines where they are exposed to inductive interference to a maximum degree. Although commercial communication lines are constructed with as great separation from power circuits as practicable they are coordinated to minimize any interference, while power company communication lines with their maximum exposure to inductive interference are often given little attention in this regard. Realizing that such a communication system could not provide high grade service and that but a relatively small additional expenditure for proper construction and coordination would greatly enhance the value of the system, the Southern California Edison Company, in 1919, started toward this end by transposing the old 50,000-volt Borel line and the associated telephone circuit, which made it possible to carry on a satisfactory telephone conversation over the entire length of a line which previously could be used with difficulty only by relaying through an intermediate operator. This improvement was made necessary by the volume of traffic which had to be handled over the line in connection with the construction of the Kern River No. 3 power-house. As an outgrowth of the success in this first effort to improve the quality of communication, many very good lines, primarily for dispatching purposes, resulted in the following years. With this improvement an insistent demand was created for more telephone connections for various other services such as district offices, stores, etc., so that they might be used for company business when not required for operating routine.

It may be possible to operate the power system without an extensive communication system, but the continually growing complications of the power network and the progressive steps of other branches of the power company organization require closer cooperation between the branches, and this is accomplished through the medium of efficient telephone communication.

From such beginning the communication system of the Southern California Edison Company has grown until now it renders complete telephone service to all departments within the company, serving an area of 55,000 square miles through three 100-unit private automatic exchanges, a modern three-position tollboard and three outlying exchange centers. It involves 6,000 circuit miles of toll line, 400 circuit miles of cable and 1,400 telephones representing a capital investment of over two and one-half million dollars in communication plant. Fig. 1 illustrates the extent of the communication lines in conjunction with the power network.

While it is necessary for many reasons that the Southern California Edison Company provide its own communication facilities, it also makes use of the commercial facilities to supplement its own wherever they are available. Commercial telephones are located in many substations and in all district offices and stores so that in effect an independent duplicate telephone system is available for emergency use by long distance calls to many strategic points on the system.

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Decentralized Dispatching

The first and most important function of the Southern California Edison Company's communication system is to provide direct connections between the supervisory dispatching center at Alhambra and the sub-dispatchers located at important switching centers; and of nearly equal importance is the necessity of a like service from the switching centers to the stations whose operation they direct.[1] Under the decentralized dispatching system now in use by this company, switching centers are located at the major distributing points which are authorized to maintain service on all lines to stations within their jurisdiction. Eighty per cent of all the outlying stations are provided with one or more power lines radiating directly from the switching center, so that in the event of an interruption, service can be restored by the switching center without the need of communication. The remaining twenty per cent of the stations must be supplied with power from the switching center through an intermediate station and direct communication with outlying stations is often required to reestablish service. However, it is also desirable that communication be available to all stations after an interruption so that the system may be promptly restored to normal operating condition.

Protection

The status of wire telephone facilities as to serviceability immediately after power system trouble depends on the design and proper operation of the telephone protective equipment. Since commercial communication facilities are generally separated from power systems by as much distance as is practicable, they have not felt the need of high current capacity protective equipment, and hence this has not been obtainable from the manufacturers of telephone supplies. The chief requirements of protective apparatus for power system telephone lines are: first, that this equipment shall not be blown to pieces by the discharge resulting from direct contact between a telephone line and power line, and second, that it shall be capable of carrying the currents occasioned by induced surges arising from power system troubles without becoming internally short-circuited. As these requirements are not met by any commercially available protective gaps, it was

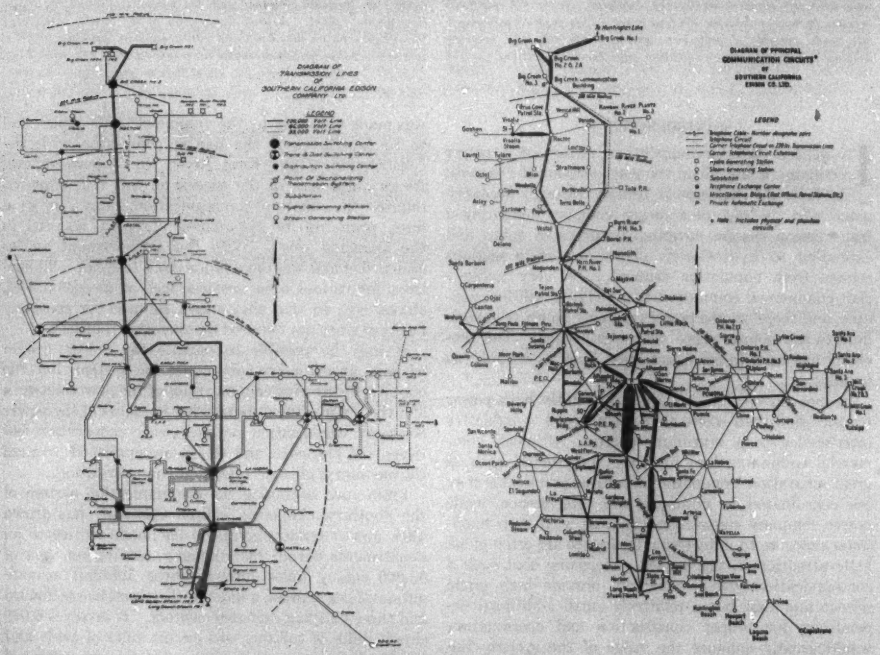


Fig. 1—Maps Showing the Major Transmission Network and the Important Communication Circuits

1. Decentralized Dispatching, by H. W. Tice, *Electrical West*, Vol. 63, No. 6, December 29, 1929.

necessary to develop a device to fill the need. After a great deal of experimentation there resulted a device which has been named the "serjdetour."

The serjdetour is capable of sustaining the discharge occasioned by contact between a telephone line and power line with no damage other than the welding together of the electrodes. When this occurs, it is only necessary to loosen four screws, remove the pitted electrodes, and substitute new ones. The device is then ready for service again. The pitted electrodes can readily be resurfaced in the shop.

While direct contacts between telephone and power lines occur occasionally the great majority of disturbances consist simply of induced voltages to ground or between wires due to exposure to power circuits under fault conditions. The discharge currents resulting from such induced voltages are much more moderate in value than those resulting from direct crosses, and the serjdetour is capable of handling these without becoming internally short-circuited.

This is an extremely valuable attribute, as it means continuity of communication service despite power system troubles. It has been determined by experiments that the current in our telephone lines during a power system fault will seldom exceed 25 amperes; so in many installations 25-ampere fuses are placed in the line. Then for the great majority of disturbances the fuses will not be affected. The serjdetour will carry the discharge for the duration of the power system trouble without becoming short-circuited. In the few cases where the current exceeds 25 amperes the fuses will blow before there is any danger of the serjdetour gap becoming short-circuited except in the case of a

Fig. 2—The Component Parts of a Serjdetour Gap

Showing the Pyrex spacing ring, the silver tipped copper electrode, and the electrode mounted in its heat, absorbing block

direct cross, where it would not be reasonable to expect the gap to remain open. There is ample margin of safety here, as a serjdetour set to break down at 1,700 volts to ground will carry 50 amperes for several minutes without becoming short-circuited. If set to break down at 600 volts it will carry 20 amperes for a like time.

The construction of the Serjdetour is extremely simple. In its most commonly used form each unit consists of a group of three gaps, designed to be connected from each wire to ground and between wires, respectively. Each gap consists of a pair of cylindrical copper electrodes clamped in three-inch steel cubes. (See Fig. 2). The faces of these cubes are machined, and are held parallel by a Pyrex ring separator. This brings the faces of the electrodes into perfect parallelism,

Fig. 3—Cast Silver Electrode Showing Radial Cleavage Lines Caused by Heat and Beads Formed by the Concentration of the Arc on the Edges of these Lines

and the spacing between them is readily adjustable. The electrodes are tipped with pure rolled silver one-quarter inch thick, so that it is the silver which actually forms the arcing surfaces. The silver faces are machined with the greatest care to be perfectly flat, except for a slight beveling at the edges. They are given a mirror-like polish by the use of rouge. The success of the arrester action depends on the perfect parallelism of the faces and the smoothness of their surfaces. When these conditions are fulfilled the arc spreads uniformly over the entire surface of each electrode face, and so has no tendency to concentrate at one place and build up a projection. It is also essential that the silver faces be backed by the large copper electrodes and the massive metal blocks, as this insures that the heat is conducted away from the surface and absorbed in the blocks, thus avoiding undue melting of the silver.

The selection of polished rolled silver of great purity for the faces of the electrodes was the result of exhaustive experimentation and some study of metallurgy. Copper, zinc, tungsten, platinum, gold-silver alloy, hardy metal, and many other materials were tried. It was found that pits and beads were formed on the surfaces of electrode tips of these materials when subjected to an arc of 25 amperes. As the gap necessary for the required protective action is only 0.016 in., it was easily short-circuited by these beads. Of all the materials studied, the Hardy metal appeared to be much the best until tests were made on cast silver bearing a government stamp ".99997 Fine." This proved to be much superior in performance, although it did show a tendency to short-circuit under very heavy

currents. Microscopic investigation showed that the crystalline structure of the cast silver was such that under extreme heat radial cleavage lines were formed in the electrode faces. Minute cracks opened and presented sharp edges along which the arc concentrated, resulting in the melting of the silver and the formation of beads. (Fig. 3). The substitution of rolled silver for cast eliminated this difficulty and gave a much improved performance. (Fig. 4). Further refinements were the polishing of the surfaces and the beveling of the edges, in order to avoid the slightest tendency of the arc to concentrate at any point.

Fig. 4—Rolled Silver Electrode Subjected to the Same Test Given to the Electrode in Fig. 3

The purity of the silver is a prime factor in the success of the serjdetour. Under the action of an arc, impurities seem to "boil up," causing projections which short-circuit the gap.

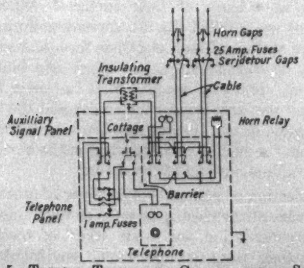


Fig. 5—Typical Telephone Circuit for a Substation at an Intermediate Point on an Important Dispatching Line

The serjdetour is, of course, not in itself a complete protective measure, but is the foundation unit upon which the protection system is built. The duty which this unit must perform is illustrated by consideration of Fig. 5, showing a typical telephone circuit for a substation at an intermediate point on an important dispatching line. For convenience in sectionalizing for test from a central wire chief's office, the line is looped through the switching panel in the station instead of merely being tapped. Continuity of service is a prime requisite; therefore no small fuses or protective gaps are inserted between the terminal pole and the switching panel. The serjdetour and its associated 25-ampere, 25,000-volt arc-quenching fuses are mounted on the terminal pole. The telephone instrument is insulated for potential from line to ground of 25,000 volts by the insulating transformer installed between the line switching panel and the telephone panel. It is also protected from destructive between-wire voltage by duplicate sets of one ampere fuses and gaps, either set being selected by the fuse changing switch on the telephone panel.

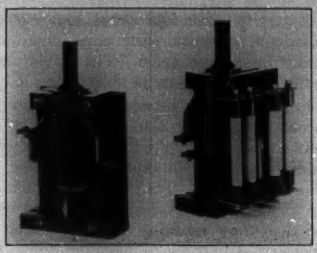


Fig. 6—Southern California Edison Company's Key Type Switch

Arranged for selecting either of two sets of fuses

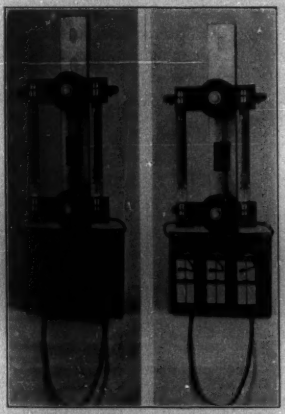


Fig. 7—Southern California Edison Company's Serjdetour Protective Unit

For use on exposed telephone lines

Since the switches on the line switching panel are at

line potential, protection of the operator requires that live parts be covered and that operation of the switches be through a handle of ample insulating value. The switches Fig. 6, are double-pole, double-throw key type, and are operated by a long bakelite handle extending through the metal cover enclosing the entire grounded metal panel upon which they are mounted. The silver contacts have a wiping action to insure low resistance connections necessary to maintain balance, which is of great importance, in lines exposed to inductive interference. The lead-in cable from the terminal pole feeds through the back of the panel so that no part above ground potential is exposed.

The equipment on the line switching panel and the rubber insulated lead covered lead-in cable will withstand a potential of 1,700 volts to ground while the bells, relays and insulating transformers have been tested and found capable of withstanding 600 volts between wires. Thus it is required that the

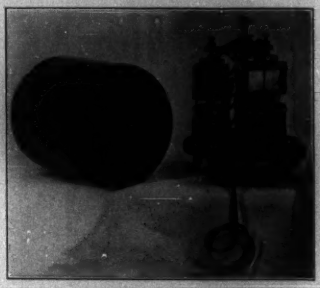


Fig. 8—Southern California Edison Company's Serjdetour Protector for Use with 14-Ampere Fuses on Unexposed Telephone Lines

serjdetour positively limit the voltage to ground to 1,700 volts and the between-wire voltage to 600 volts. To meet these requirements the serjdetour combination shown in Fig. 7 has been assembled. This consists of two gaps set for 1,700 volts and one gap set for 600 volts. The unit is combined with the 25-ampere, 25,000-volt arc-quenching fuses to form a protector unit for installations such as that just described.

In case of contact between the telephone line and the power line the 25-ampere, 25,000-volt fuses on the line side of the serjdetour will open and an arc will start at the horn gap arrester located on an adjacent pole, thus eliminating any tendency for flashovers on the terminal pole. An independent ground for the horn gaps is used in places where it is impracticable to connect to the substation ground network; because, should a ground connection be common to the horn gaps and the serjdetour and not connected to the station ground, current might flow across the horn gaps, the common ground connection, across the serjdetour gaps and thence through inside telephone equipment to the station ground after the fuses had opened. It is to be expected that the serjdetour gaps may be short-circuited in cases of direct contact between telephone and power lines; but the arrester still protects valuable inside equipment and the gaps may be cleared when the fuses are replaced.

For protection of equipment connected to telephone lines not paralleling power lines an arrester requiring less pole-top hardware is shown in Fig. 8, and is used in conjunction with 14-ampere telephone fuses. All three gaps in this arrester are set for 600 volts breakdown. The cables from the terminal poles to equipment panels in these cases may be paper insulated, as tests have shown that the 600-volt gap is adequate protection for the paper-insulated cable.

It may be of interest to cite an actual case wherein a boy threw a wire in such a way as to make contact between a 66,000-volt power line and a telephone line at a point 50 ft. from a terminal pole. No injury whatever was done to the cable or to the equipment inside the substation. Although the discharge current to ground through the serjdetour before the fuses opened was between 3,500 and 4,000 amperes, the device was uninjured except for the short-circuiting of the electrodes. The replacement of these electrodes and the two fuses restored the station to normal so far as the equipment was concerned, although the No. 8 copper telephone line wire fused a short distance from the arrester terminal.

During this same case of trouble, the fuses on the other end of the 15-mile telephone line were blown, but the serjdetour gaps were not short-circuited. Thus was illustrated the ability of the arrester to carry a current heavy enough to blow the fuses without short-circuiting the gaps.

The case described above is only one of a number of such major contacts handled by the serjdetour, in which no damage was done to valuable inside equipment; in fact, there has not been a single instance where equipment has been damaged since the serjdetour was adopted and connected to proper grounds.

It has been shown that the connections and the current carrying parts of the serjdetour must be of sufficient size to handle currents of large value. This idea of course, must be carried out in the entire ground system. Terminal poles frequently have as many as five or six terminating lines with the associated protective devices mounted thereon. The ground wire to these protectors consists of a No. 00 stranded copper cable. Precautions are taken to obtain the very best ground connection. Wherever possible, a tie is made with the station ground network. Connections are made by mechanical means rather than solder lugs. The horn gaps are given the same large capacity grounding treatment.

Carrier Current

On the 220,000-volt backbone of the system every

possible step must be taken to preserve continuity of communication. Owing to the mountainous nature of the country which the lines traverse, there is always the possibility of the telephone lines being damaged by sleet storms, brush fires, etc. Therefore it was considered advisable to install a carrier current telephone system. This extends from the Big Creek No. 3 hydro plant by way of the 220,000-volt Big Creek and Vincent transmission lines to Gould switching station near Los Angeles, with a wire extension to the supervisory dispatcher's office. An intermediate carrier current telephone station is provided at Magunden switching center.

The carrier current telephone system is of the single frequency duplex type coupled interphase to the 220,000-volt system by means of oil insulated tank type capacitors. Coupling is made to one of the buses and also direct to the Vincent transmission line at each station. This provides for clearing any bus or line for maintenance without interrupting communication since two complete carrier channels are available.[2] The coupling to the Vincent transmission line also provides a carrier frequency channel for the carrier current pilot protective system now in use on the 96-mile section of line between Gould and Magunden, an intermediate switching station. The carrier protective system is also contemplated for use on the section between Magunden and Big Creek No. 3. While the quality of voice transmission on the carrier telephone has been rather poor due to the very unequal attenuation of carrier frequencies included in the side bands, it has proved its worth on several occasions when after storm damage to the wire lines it was the only means of communication between the dispatcher and Big Creek. Careful investigation conducted with the cooperation of the manufacturer revealed that the very irregular attenuation of carrier frequencies differing by increments as small as 50 cycles, was due to reflection of the carrier from lines connected to the 220,000-volt system south of Gould and north of Big Creek No. 3. In order to correct this condition carrier frequency traps have been ordered and will be installed at Gould and Big Creek No. 3 to isolate the carrier communication channels from other parts of the system. It is expected that the installation of these traps will not only improve the quality of voice transmission but will also increase the ratio of signal to noise and reduce the effect of line switching conditions on the signal level.

Station Equipment

Due to the multiplicity of overhead power lines concentrating at substations, telephone circuits are completed from terminal poles to inside equipment panels through underground lead-covered cables. It has not been possible to obtain quadded cables with sufficient insulation for this use. Reduction of cross-talk to a satisfactory minimum, in the No. 14 rubber insulated lead-covered cable which is used, required that it be balanced and spliced at frequent intervals. Careful testing and splicing at the time of installation has produced highly satisfactory phantom groups in runs as long as 1,500 ft.

Four general types of station equipment fulfil the requirements on the Southern California Edison system. The choice between types is determined by the amount of telephone traffic to be handled at each location.

The first and simplest type of equipment consists of a telephone enclosed in a weatherproof wooden box arranged for attachment to a pole. A switch is provided which automatically disconnects the telephone from the line when the door to the box is closed. One-ampere fuses and a between wire gap protect the instrument. A container holding spare fuses is provided. An insulating platform is attached to the pole

Fig. 9—Typical Installation of a Telephone Panel and One Auxiliary Signal Panel

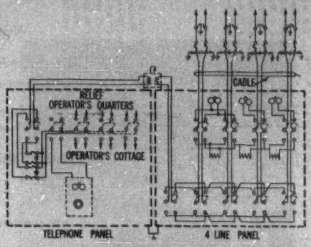
to protect the user from voltage to ground. Pole box installations are made at outdoor unattended substations and at occasional pole-top switches on the lines from which points it is necessary for an operator to communicate with the switching center.

The second type of installation (Fig. 9) consists of a steel panel on which is mounted a telephone and five enclosed dead front switches. One switch is a fuse changing switch, the others being wired for an additional line, extension telephone or signal horn as a particular installation may require. A transformer insulates the telephone and local circuits from the incoming lines. One or more auxiliary signal panels equipped with bells and relays for horn signals may be added as required. Unattended substations, substations having but one operator who lives on the property, transmission patrol stations and other points using the telephone infre-

2. Carrier Current Pilot Protection, by E. R. Stauffacher and F. B. Doolittle, *Electrical World*, March 22, 1930.

quently and having buildings available, are provided with such equipment.

Installations of this class may be extended to provide for terminating or looping more than two lines by the addition of one or more four-line panels depending upon the number of lines to be accommodated. The talking



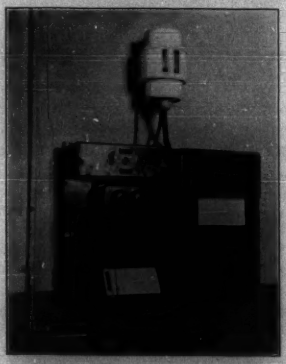


FIG. 10—TYPICAL INSTALLATION OF A FOUR-LINE PANEL AND CIRCUIT DIAGRAM

Showing one line looped through and two lines terminating

buses on the various four-line panels are paralleled and extended through the insulating transformer to the telephone panel previously described. Where four-line panels are installed no auxiliary signal and relay panels are used, as provisions for bells and relays are made in the four-line panels. An example of this is shown in Fig. 10.

The third general type of installation is similar to the second, but because of more constant use of the telephone facilities, the lines are run from the four-line panels to dead front switches mounted on a desk. The desk, designated as type *C*, makes use of a turret equipped with eight line switches, a fuse-changing switch and a switch which connects the operator's telephone to either of two buses. A type *B* desk, similar to type *C*, but accommodating fifteen lines and providing four buses with the necessary switches mounted on a sloping metal panel built into the desk, is also standard equipment. The insulating transformer connected between the telephone and any of the buses is mounted in the desk of either type. These desks fulfil the requirements of the larger substations and switching centers using the telephone frequently but not required to do much telephone line switching.

The fourth and most elaborate standardized installation makes use of a desk type switchboard having a slanting key shelf accommodating twenty-seven lines and providing four buses for switching and two for talking. Each line position is equipped with three standard telephone keys, a white lamp which flashes the code rings and a red lamp which remains lighted for ten seconds after the termination of a ring to assist the operator in locating the line when he has heard his code call but did not see the white light. Bells mounted in the back of the desk give audible signals on the magneto lines, and the usual night alarm buzzers are used on the common battery lines. Each talking bus is equipped with an operator's telephone connection which includes an acoustic shock suppressor in the receiver circuit. The acoustic shock suppressor, developed by the company, consists of a vacuum tube amplifier circuit using low plate voltage so that the output of the amplifier is limited by saturation of plate current below such value as would produce acoustic shock on reaching the receiver.

FIG. 11—TYPE A TELEPHONE DESK SHOWING KEYS AND LAMP SIGNAL ARRANGEMENT

The annunciator panel to the left of the key shelf is equipped with 30 positions, with space to the right for additional positions when required

This desk, known as type *A*, Fig. 11, provides an annunciator panel in addition to the telephone panel. On this panel are mounted lamps which give indications of power switching operations, transformer temperatures, etc. Both panels are built in and are completely wired to terminals in the back of the desk, Fig. 12.

From the terminals on the desk all the circuits are carried in cable to a cross-connecting box located in the rear of the equipment panels from which the various circuits may be jumpered to magneto line panels, common battery line panels, annunciator panels, relay panels, etc.

Unit type standard designs of equipment panels are available as auxiliary for this class of installation. They are mounted as required on steel racks which may be either set into a partition between two rooms or erected as self-supporting units, the back and sides of which are protected by a steel grill-work. A number of standard panels are listed and their purposes indicated as follows:

1. Power panels, on which are mounted series lamps for ringing power, alarm equipped fuses on the battery supply, auxiliary relays for alarms and relays for grouping the audible signals for several lines on one bell.

2. Auxiliary fuse panels, which are used when additional fused battery circuits are needed for annunciators, etc.

Fig. 12—Type A Desk Showing Key Shelf Raised and Back Panels Removed

3. Magneto panels No. 1, on which are mounted alternating current line relays and their auxiliary relays for code lights and signals.

4. Magneto panels No. 2, on which are mounted pendulum and slow release relays for operating the red holding lights.

5. Common battery panels, are equipped with repeat coils, condensers and relays to accommodate five common battery lines to offices and cottages on the property.

6. Bell panels are equipped with six bells per unit.

7. Switch panels, which accommodate ten back-mounted dead front switches.

8. Telephone panels, on which are magneto telephones for testing and emergency use.

9. Fuse-change panels are equipped with duplicate sets of fuses, selected by switches, accommodating three lines. Three repeat coils are provided on these panels for connection of phantom groups.

10. Test jack panels are equipped with 15 pairs of jacks for testing and patching lines.

11. Test jack and patch panels, which are provided with three strips of twenty jacks for phantom patching and equipment testing.

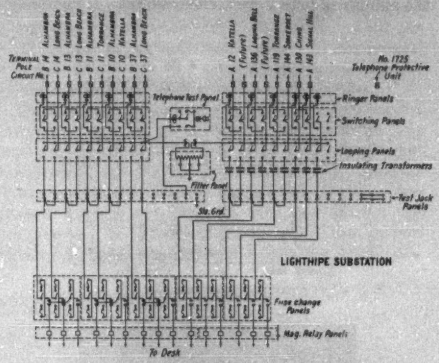


Fig. 13—Single-Line Diagram of a Typical Large Switching Center

12. Low pass filter panels, which are equipped with 40-cycle cut-off low pass filters for use when making bridge tests on lines when induced voltage to ground is present.

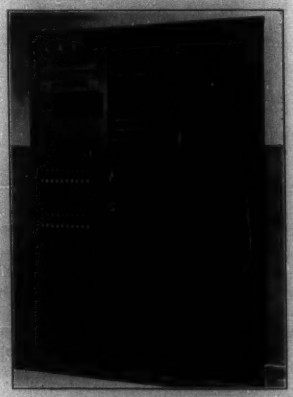


Fig. 14—Front View Showing the Unit Type Equipment Panels with Some of the Relay Covers Removed

13. Miscellaneous blank panels are used to fill up excess space in the racks.

A single-line wiring diagram (Fig. 13) and illustrations (Figs. 14 and 15) of a typical installation of this equipment are shown. It may be observed that

insulating transformers are provided for all the exposed lines, which permits their safe interconnection with unexposed and common battery lines.

The flexibility permitted by assembly of these standard units to fit various requirements makes this equipment adaptable to the needs of the largest substations and switching centers. With the substitution of a cordboard for the desk, to permit more cross connecting, the same equipment is used in exchange centers. In cordboards the code lamps and holding lamps are associated with their line jacks. An acoustic shock suppressor is provided for the operator's protection. All equipment other than the cord circuits and signals is located on the equipment panels.

FIG. 15—REAR VIEW OF THE EQUIPMENT PANELS SHOWING THE METHOD OF JUMPERING FOR INTERCONNECTION OF THE VARIOUS UNITS

EXCHANGE CENTERS

The communication facilities required for other than dispatching the immediate operation of the power system are provided in a manner similar to that of commercial toll service. The term "toll" as used hereafter applies to service between different localities, that may be handled on a delay basis. For operating and commercial purposes, the Southern California Edison system is divided into six divisions and thirty-two districts. The communication facilities for the Eastern Division (Fig. 16) will be described as typical of a division's requirements. The division manager and the division superintendents of distribution, substation operation and transmission, and their clerical assistants, are located in adjoining offices at division headquarters.

In the outer office, where the operator may also serve as information girl for the division personnel, a telephone exchange board is located. Nine magneto party lines radiate from this switchboard, to the district commercial offices, superintendents' offices, stores, and substations. Ten common battery lines provide service to the local telephones and five toll trunks connect this exchange center with the main communication center at Alhambra.

The reason for the high per cent of trunking is that some departments, such as the general store, transportation and medical, have no division heads and require communication through the exchange centers to their individual district men.

At commercial offices and stores several desk telephones are provided rather than a single booth because the circuit time saved in getting the called party promptly, justifies the cost of the additional instruments.

COMMUNICATION BUILDING

The impracticability of maintaining many overhead telephone circuits to the general office building in downtown Los Angeles, and the risk of loosing all the circuits at once should anything happen to an underground cable which might contain them, as well as the higher transmission loss in such a cable, were the factors whose consideration lead to locating the communication center and supervisory dispatcher's office at Alhambra, a point accessible to the overhead lines, and some 12 miles from the general office. To properly house the facilities for this purpose a modern fireproof structure was built and is known as the Communication Building. The space on the ground floor of the building is occupied by facilities of the Communication Department, consisting of equipment panels, the wire chief's testboard, the power switchboard,

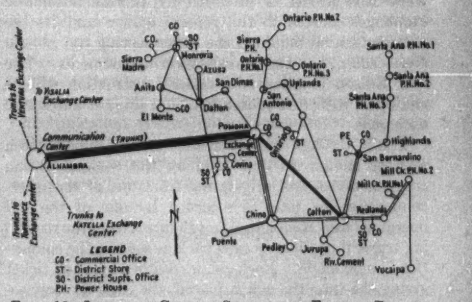


FIG. 16—LAYOUT OF CIRCUITS SERVING THE EASTERN DIVISION

motor-generator sets and storage batteries, the private automatic exchange serving the company's plant on adjacent property, as well as office space for the department's personnel, storage space for equipment and supplies, and the communication laboratory. On the second floor of the building are located the supervisory dispatcher's office and the main toll switchboard together with kitchenettes for the personnel of each and a

comfortably furnished rest room for the toll operators.

The telephone circuits terminating at the communication building are protected and carried in underground cables to the equipment panels just as in any large substation installation. From these panels they are distributed to the dispatchers' desks, the toll board and the wire chief's board. Many of the long lines are party lines and ringing on them is so frequent that it would be confusing to the dispatchers and toll operators if they received all the signals. To avoid this confusion, a selective device operated by the ringing current and consisting of relays and a rotary line switch discriminates between the code rings and brings in the line lamp on only the particular board desired. The result is that in so far as the dispatchers and operators are concerned, the lines do not appear as party lines, their lamps lighting only when they are wanted and remaining on until they are answered.

Three 25 line key-operated desk switchboards provide a means of establishing direct connections from the supervisory dispatchers to the switching centers. These desks are similar to those described for the largest substations but have an additional feature which is a toll line selector. By dialing the circuit number on this selector, all of the lines terminating at the communication building are available to the dispatchers. In effect each dispatcher has a 100-line switchboard within a very small space.

The three-position toll switchboard provides a connecting link from the intercommunicating systems serving the general office, general garage, and Alhambra properties, to the toll lines. A manual exchange handles the intercommunication in the present general office while this service is provided by private automatic exchanges located in the general garage and in the communication building, the latter serving the general store, shop, test and transmission departments. The new Edison building now under construction will be equipped with a four hundred line private automatic exchange.

Traffic through the toll switchboard is handled in a manner similar to that used by the communication companies. A call originating from one of the intercommunicating lines is recorded by one of the toll operators who completes the call direct from the recording trunk if possible or calls back as soon as the connection can be made. Each call is recorded on a toll ticket giving the time the call is placed, the time the connection is established, the duration of the conversation and the trunk and circuit numbers involved. Space on the back of the ticket is used to record the cause of any delay encountered in completing the call. These data are used for traffic studies and sometimes to call the attention of employees to negligent or careless use of the communication facilities which, if continued, would result in waste of valuable circuit time.

The wire chief's board is located adjacent to the equipment panels, Fig. 17, making it convenient for the wire chief to do any line switching or patching which may become necessary. The usual testing facilities consisting of a standard voltmeter and bridge, together with some special testing equipment, are provided on the two-section wire chief's board. Since the toll board is operated only during working days, ten cord circuits and a switching arrangement to bring in line lamps on the wire chief's board are provided so that the wire chief may maintain 24-hour toll service without the necessity of toll operators during periods of light traffic. All lines, including phantoms, are looped through in, out, and multiple jacks. Associated with each set of jacks are four lamps, being, respectively, the code lamp indicating the code signal rung, the selector lamp showing the operation of the selector when the dispatcher or toll operator is to answer, the busy lamp, and the monitor lamp which shows the position of the automatic monitor. The automatic monitor is a device which connects the wire chief's head phone consecutively to each line for an

Fig. 17—The Wire Chief's Test Board and the Conveniently Located Equipment Panels

interval of one second or three seconds. It may be stopped, started, make one complete cycle of one hundred lines, repeat, or the interval changed by the operation of three keys. Use of this equipment permits the wire chief to write up trouble reports or do other clerical work and at the same time to subconsciously note anything unusual that may take place on the lines.

The wire chief is to the communication system what the load dispatcher is to the power system. His first duty is to maintain continuity of service on important dispatching lines which he does by switching and patching with less important lines whenever this is possible and there is trouble on a dispatching line. He is responsible for maintaining normal condition and continuity of communication channels over the entire system. This he does with the aid of two sub-wire chiefs, one being located at Big Creek and responsible for the communication facilities at the Big Creek power plants, the other being located at Vestal substation at

which point is located the communication center for the company's system in the San Joaquin Valley. The men responsible for these portions of the system make a daily report of the status of communication plant in their respective territories which combined with similar information obtained by the wire chief himself for his territory together with records of the previous day's troubles form a daily report to the superintendent of communication. In addition to these daily checks of all communication channels, the wire chief makes periodic routine tests of a quantitative nature to detect any deterioration affecting communication. Where minor routine maintenance will bring the facilities back to normal, it is performed under the wire chief's direction, otherwise it is reported to the superintendent of communication who receives authority from the communication committee at one of its weekly meetings to make an application for an expenditure to handle major reconstruction. The superintendent of communication is responsible to the manager of operation for the personnel, operation and maintenance of the communication system. The manager of operation is chairman of the communication committee which does the planning of future additions to the communication system. The requirements are based upon the wire chief's reports together with traffic studies compiled from data furnished by the chief toll operator, and extensions of the power system.

Line Testing and Results

When a new telephone line is constructed, it is tested in its entirety and in sections before it is accepted for operation. Any changes or corrections for improvement are made before it is put in service. A folder showing the route of the line, pole numbers, distance between poles, type of construction, size of wire and transposition scheme together with a report of the initial tests is made up at this time and filed where it is available to the wire chief thus providing records for reference in maintaining lines up to standard.

For the purpose of making the initial tests as well as for use in maintaining and improving existing facilities, a test truck is maintained by the communication department. All of the instruments necessary for complete line testing are permanently mounted in this truck and are wired to convenient terminals from which jumpers are run to the lines. The equipment for properly terminating the far end of the line under test is mounted and permanently wired in a box of such size as to be portable in an ordinary passenger automobile.

The following tabulations give the results of an overall test on a four-wire telephone line between Alhambra and Big Creek, a distance of 279 miles. The line follows the general route of a 220,000-volt transmission line. It is constructed of No. 8 copper wire in the light loading sections and No. 6 copper wire in the heavy loading sections. It loops through three major switching stations and four patrol stations.

Insulation resistance	365 megohms per mile
Resistance unbalance Circuit No. 1	1.3 ohms
Resistance unbalance Circuit No. 2	0.8 ohms
Resistance unbalance phantom	0.3 ohms
Cross talk side to side	220 cross talk units
Cross talk circuit No. 1 to phantom	500 cross talk units
Cross talk circuit No. 2 to phantom	800 cross talk units
Noise, metallic circuit No. 1	250 noise units
Noise, metallic circuit No. 2	150 noise units
Noise, metallic phantom	150 noise units
The transmission equivalents including equipment are:	
Circuit No. 1	15.85 decibels
Circuit No. 2	15.88 decibels
Phantom	13.15 decibels

This is representative of lines of this class and length, however other more recently constructed lines have better characteristics.

Conclusion

By proper attention to coordination and maintenance of good balance the communication lines of a power company may be highly satisfactory in spite of their severe exposure to inductive interference because of their proximity to high-voltage lines. Their serviceability immediately following power system disturbances involving high induced voltages on the telephone lines depends upon the effectiveness of the protective system. The serjdetour arrester and its associated equipment provides a protective scheme which will maintain continuity of service despite power surges of magnitudes ordinarily encountered and will protect inside equipment from damage in the extreme case of contact between telephone lines and power lines. A comparatively few assemblies of equipment into standard units provide economical material which is adaptable to installations of widely varying requirements and magnitudes. Original testing and maintenance of lines to the original standards by subsequent routine tests keep the facilities in first-class condition. Several years' experience with a constantly improving communication system have demonstrated that good telephone communication promotes close cooperation between departments of the company to the end that better service may be rendered to consumers.

Discussion

E. L. White: While my own Company operates in the north near Puget Sound, and the Southern California Edison Company operates far south of here, I note that we have many similar problems. In particular, the question of adequate protection seems to be common. After much experimenting, the Southern California Company has arrived at the serjdetour, which appears to be more than adequate for the duty it has to perform. In our own case, we have modified the standard horn gap and equipped it with a small gap between wires, in addition to the two gaps already existing between the two wires and ground. Our normal gap setting is approximately 1/32 in. on all three gaps. Closer settings are inadvisable because of possible trouble from dust, lint, and other materials.

R. D. Evans: An interesting feature of the paper is that concerned with the measures for protecting power-line com-

munication systems from induction. It may be of interest in this connection to describe briefly some other protective systems which have been in successful use.

Supervisory control lines are subject to induction in much the same way as the power company telephone lines described. An important requirement of the protective apparatus is that of low maintenance on repeated discharge of relatively high-current capacity. In connection with our supervisory control lines, we have been using successfully a tube type of protector which is capable of discharging 50 amperes for two seconds repeatedly without injury to the electrodes and without change in characteristics for subsequent operation.

In some applications supervisory control circuits are subjected to relatively high sustained fundamental frequency induction of the magnitude of say 1,000 volts. A particular case was solved by the use of drainage in connection with tube type protectors. This supervisory control circuit being operated with direct current signaling made it necessary to introduce a resistance between the lines and ground in order to prevent by-passing too much operating current from the relay. The tube protectors, of course, are not operated except in the event of abnormal induction as under short-circuit conditions in the neighboring power supply circuit. This drainage system has been in successful operation for some time and has provided a simple and practical solution of the relatively severe induction condition on supervisory control circuits.

R. B. Ashbrook and F. B. Doolittle: Referring to Mr. White's discussion, it is our experience that the use of horn gaps alone for telephone protection is insufficient. In the case of direct contact between telephone circuits and high-voltage power lines the horn gap does not serve as a definite voltage limiting device but allows the voltage to rise as the arc lengthens due to traveling upward toward the points of the horns. Consequently it does not protect telephone cable and inside equipment from damage under this condition.

The use of horn gaps as a part of the protection scheme of the Southern California Edison Company is to provide a path for the discharge current after the 25-ampere, 25,000-volt fuses associated with the serjdetour have opened, thus eliminating any tendency for flash-over at the fuse mountings on the terminal pole. The horn gaps are mounted on an adjacent pole ahead of the fuses.

The Pennsylvania Railroad Electrification

BY J. V. B. DUER[1]
Fellow, A. I. E. E.

Synopsis.—This paper covers the electrification program of the Pennsylvania Railroad and consists of a brief explanation of the reasons for the decision to embark on this electrification program, a review of the operating experiences leading up to the present designs used on the railroad, of catenary and transmission circuits, substation layout and types of equipment, as well as a description of the progressive steps of electric locomotive design which preceded the development of the electric locomotives to be used in this program, and concludes with a brief resumé of the points that should be given attention in applying an electrification to a stretch of railroad.

* * * * *

ON October 31, 1928, General W. W. Atterbury, President of the Pennsylvania Railroad, announced that the Board of Directors had authorized a program of electrification, over a period of years, of the entire road train service, freight and passenger, between New York and Wilmington, Delaware, as well as the electrification of the grades between the Susquehanna, Schuylkill, and Delaware River Valleys, and the Eastern Terminal of the Railroad; a project covering a passenger and freight service of 325 mi. of line and 1300 mi. of track and extending from Hell Gate Bridge in New York, where connection is made with New England, west and south to Wilmington and west on the main line in the direction of Harrisburg.

The authorization of the Board of Directors to inaugurate this electrification work followed exhaustive studies of the whole industrial and transportation situations in the eastern part of the country, including the terminal developments already under way or projected for Philadelphia and Newark. While this analysis was worked out in detail, on the basis of the traffic estimated for the year 1935, the probability was not lost sight of that by 1950, the metropolitan area around New York would extend to New Brunswick on the west and well out on Long Island on the east and contain 30,000,000 people, and that there would be similar developments in other cities.

The system adopted is such that by the simple addition of increased power and increased rolling stock, a movement of any magnitude which it is possible to transport over the existing tracks and at a speed within the bounds necessary for safe operation may be handled as the demands of the traffic may from time to time require. The immediate factors which influenced the decision to proceed with the electrification were as follows:

1. The greater economy of electric traction as compared with steam operation in dense traffic territory.
2. The growth of the southern passenger business.
3. The increasing density of both freight and passenger business on our eastern lines and the probability that in the future more rapid movement would be required.
4. The desirability of utilizing the advantages of electric traction in connection with the construction of the new passenger terminals at Philadelphia and Newark.
5. The desirability of building a locomotive that would meet the requirements from the standpoint of weight of train, speed, and reliability which it is believed will have to be met in this territory in the next twenty years.
6. The probability that the project could be completed with a less total expenditure, all matters considered, than if started at a later date.

Of this program, the electrification of the New York Division, from Philadelphia to Trenton, has been completed and electrical suburban service inaugurated; electrification of the Schuylkill Division from Philadelphia to Norristown has been completed and electrical suburban service inaugurated; and at the present time electrification work is progressing from Sunnyside Yard and Jersey City to Manhattan Transfer and New Brunswick as an initial step in the operation of trains by single-phase locomotives from New York to Philadelphia, and for the operation of our suburban service between Jersey City and New Brunswick with single-phase multiple unit trains.

The announcement of this electrification program is the sequel to an interesting story of operating experience, of trial of electrical equipment, and of design and experimental work which started in 1905 when the Long Island Railroad was electrified, and which extends down to the present time when our electrification program is well under way and which covers experiments with d-c. electric locomotives and a complete trial of the single-phase system.

During the course of these experiments, an especially equipped section of the Long Island Railroad was used to develop the possibilities of this system, which, while not used for initial operation in the New York tunnels, was adopted shortly thereafter for the electrification of the suburban lines around Broad Street, Philadelphia. It has now been selected as our standard system for use in the electrification program upon which this railroad has embarked.

To be prepared for an extensive electrification, it was necessary to develop single-phase passenger and freight electric locomotive designs, as well as multiple

1. Elec. Engr., Pennsylvania Railroad Co., Philadelphia, Pa.

Presented at the Pacific Coast Convention of the A. I. E. E., Portland, Oregon, September 2-5, 1930.

30-153

unit car designs, and accordingly, in 1917 a constant speed, split-phase electric locomotive, (railroad classification FF-1,) was designed, built and tried out in service. The experience with this locomotive led to the development of a commutator motor type locomotive (railroad classification L-5) of somewhat less horsepower than the constant-speed locomotive above referred to and having the variable-speed characteristics which experience seemed to teach were more suitable for a railroad handling a dense passenger and freight traffic.

Locomotives of this design were built and placed in service on alternating current in the Philadelphia territory, and on direct current in the New York Terminal. They proved satisfactory and have given good service, and to the best of our knowledge, were the first electric locomotives built in this country in which a single design of mechanical chassis was used for the installation of the electrical equipment supplied by three different manufacturers, which parts, while not interchangeable with each other, produce a locomotive of practically identical transportation characteristics and of the same mechanical design.

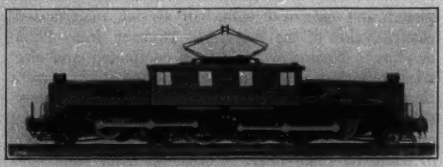


Fig. 1—Four Driving Axle General Utility Locomotive
Railroad Classification L-5

In these locomotives, by change of gears, it is possible to have either a passenger locomotive for high speed or a freight locomotive for high tractive effort, and by change of type of control with which they are equipped, it is still further possible to utilize them on 600-volt d-c. circuits, or on 11,000-volt a-c. circuits. In other words, by minor modifications in construction, they were designed as a general utility locomotive for use either in passenger or freight service on alternating or direct current.

The study of electric locomotive design has been continuously directed toward the production of a simpler, more easily maintained, and more reliable locomotive. Shortly after the L-5 locomotive was built and placed in service, developments in the design of single-phase motors indicated that a still simpler and sturdier locomotive could be produced. The progress in single-phase motor design has made possible motors of sufficient capacity to handle weights on drivers permitted on the railroad which could be placed between the driving wheels of the locomotive, thus eliminating the necessity for jack shafts and side rods.

It was thought desirable to design some locomotives having these general characteristics and, accordingly, the construction of ten passenger and two freight locomotives was authorized. These locomotives are of three types:

1. A two driving axle passenger locomotive having a four-wheel truck on either end (Railroad classification O-1).

Fig. 2—Two Driving Axle Passenger Locomotive
Railroad Classification O-1

2. A three driving axle passenger locomotive having a four-wheel truck on either end (Railroad classification P-5).

3. A four driving axle freight locomotive having a two-wheel truck on either end (Railroad classification L-6).

The passenger locomotives have twin motors of 1,060 hp. each, mounted above each driving axle and driving the wheels with gears and pinions through the medium of the well-known link type drive. The freight locomotive has axle mounted motors of 530 hp. each, driving the wheels through gears and pinions of the same general type of construction as in a street car.

The motors of all the locomotives are identical, the

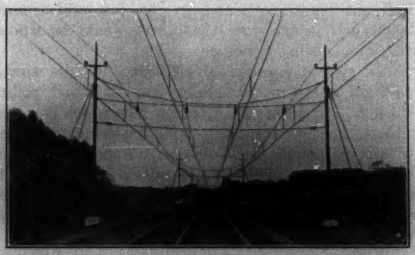


Fig. 3—Main Line Catenary and Transmission Line Construction
On the Philadelphia Terminal Division between Philadelphia and Paoli

twin motors for the passenger engines being made up of two of the motors of the freight engines. All of the locomotives have roller bearings throughout—in the trucks, in the driving axle journals, and in the motor armature bearings,—the only plain bearings being those on axle to support the motors of the freight engine, and the quill bearings which support the quill in the frame of the twin motors on the passen-

ger engines. The electrical apparatus on the locomotives is interchangeable to a very great extent, the auxiliaries and contactors are identical, and the transformers of the same design though of different capacities. These three types will be the standard until some further advance either in the art or in operating experience indicates further improvement in their design.

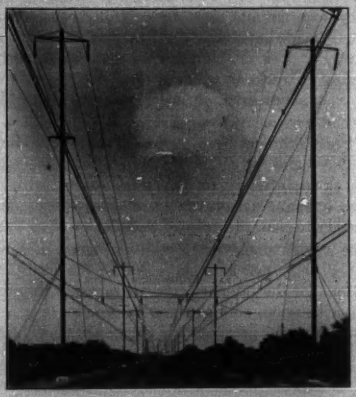


Fig. 4—Main Line Catenary and Transmission Line Construction

On the New York Division between Trenton and Philadelphia

It might be interesting to note that locomotives of the L-5 and O-1 types were placed on the Locomotive Test Plant at Altoona and were given a thorough period of test to develop the complete operating characteristics before being put into the service. Two of the O-1 type have been completed and placed in service.

While the design and operating experience were in progress on the locomotives, the railroad was active also in developmental work in connection with the circuits for supplying the trains with current.

The initial installation in 1914 provided for 44,000-volt transmission circuits, indoor substations and oil circuit breaker equipment of relatively slow speed. A large part of the overhead catenary construction was of steel and was subject to frequent painting to keep it in condition for service. A brief summary of what has been done to make this layout of substations, transmission lines, and catenary construction, more adaptable to railroad operation, as well as to reduce maintenance costs, is as follows:

On the more recent electrifications, 132,000-volt transmission is used instead of 44,000, thus providing capacity for the transmission of current from one end of a division to another and insuring against shut down due to the loss of any one source of energy.

Substations are now designed as outdoor stations, thus eliminating the major portion of the building with its attendant first cost and cost of maintenance.

Automatic circuit breakers are not used on the 132-kv. circuits, except at junction points where the circuits of one division must, under certain conditions, be automatically separated from those of another.

The trolley circuit breakers on the original installations operated in 12 cycles, including the relay action, and ruptured 30,000 amperes successfully. The modern trolley breakers, however, must operate in one cycle, including the relay action, and rupture 50,000 amperes. One of the electric companies developed an air-break trolley circuit breaker not requiring the use of oil and capable of rupturing currents of the same magnitude and in the same time as the latest oil trolley circuit breakers. We have purchased and installed many of these air breakers and they are giving successful service.

Experience with overhead catenary construction led us to believe that continuity of service secured by the use of non-corrosive materials was of sufficient value in operating reliability to warrant the use of these materials and, accordingly, all of our catenary construction, except parts of material bulk, is of bronze or copper and such bulky pieces are galvanized malleable castings. By this means, painting and other maintenance attention to the overhead catenary system is reduced to a minimum and the continuity of use of track is raised to a maximum, this being of prime importance on a busy railroad.

There has been installed and is being tried out, a length of track with the overhead catenary riveted

Fig. 5—Main Line 44-Kv. Substation at Bryn Mawr

Capacity four 2000 kv-a. transformers and necessary switching equipment

instead of bolted together as in the past. This construction gives every evidence of being successful and if this is the case, it will even further materially reduce the amount of attention which the overhead catenary system will require and still further increase the utility of the track beneath it.

A new type of rail bond similar to the well-known signal bonds,—that is, a stranded cable welded or compressed at its ends into plug terminals driven into the rail by a hammer,—has been developed and is used on the Trenton and Norristown electrifications. The use of this bond reduces the initial cost of bonding materially and will, it is believed, reduce bonding maintenance to a minimum.

In conclusion, it may be interesting to indicate some of the questions involved in the actual application of electrification to a railroad after these various points have been considered.

First, an adequate and economical source of power supply must be provided either by providing for the purchase of current, or by designing and building a railroad power plant. This question may be settled largely on economic grounds, as railroad electrifications are now being operated successfully by power supplied from plants designed and built by the railroad companies and that purchased from electric companies.

Fig. 6—Main Line 132-Kv. Substation at Edgely

Capacity four 4500-kv-a. transformers and necessary switching equipment

Next, it is important to review the real estate situation along the section of railroad to be electrified, so that proper provision may be made for pole location, overhanging wires, and other questions of this character which may affect the placing of supporting structures and transmission circuits along the railroad right-of-way.

Substations are important and sometimes difficult to locate. They must not only be properly placed from the standpoint of voltage regulation, but must also be near a tower so that the apparatus in them may be readily controlled, and at a place where real estate is available or procurable.

A properly systematized working schedule should be prepared showing the beginning and completion of all designs and the dates for placing orders for material and for completion of the work. Items of work which should be completed before the electrification is started, such as the placing of communication circuits and signal power lines underground where this is necessary must be cared for. If this is not necessary, a careful check should be made to see that the poles on which these wires are supported do not interfere with the poles on which the electrification wires are supported and before electrification is started such changes must be made as are necessary to remove physical or electrical interference between them.

Provision of adequate overhead clearance of the trolley wire circuits must be carefully considered and changes made involving the lowering of tracks under highway bridges, or the raising of the bridges, as well as lowering of tracks in tunnels, or at other points where close clearances exist, in order to make room for the trolley wire.

Wires of other ownership crossing the railroad must be considered and the railroad wires erected at such elevations as to interfere to the least extent with these others, or changes made in such crossing wires as may be necessary to clear the railroad circuits.

The signals along the right-of-way must be carefully checked, that their location may coincide with the pole locations for the catenary construction and the changes in these locations confined to a minimum.

The character of signal circuits must also be checked and if necessary changed to properly coordinate them with the traction circuits.

After this preliminary work has been done, the pole foundations may be laid out, the poles designed and purchased, the substation sites prepared and the substation equipment purchased, arranging the dates of delivery of equipment and material in such a way as to meet as nearly as possible the actual erection schedule, thus holding unnecessary interest charges to a minimum.

In this connection, the great advantage of standardizing designs should be pointed out. As an example, if pole spacings can be kept standard, the poles themselves may be standardized, the number of sizes and length of pole reduced to a minimum, the number of sets of catenary hangers also reduced to a minimum and the ease of production and erection materially assisted.

By standardizing pole spacings in terms of catenary hanger spacings, as well as by other means, it has been possible in our electrification work to reduce the number of types of poles used for a given section of track, say of 50 mi., from 150 different types of poles to 15 different types, and from several hundred different sets of catenary hanger lengths to two sets for tangent track and two sets for track of each radius of curvature used.

In addition to the actual application of the electrical circuits and substations to the railroad, the necessary multiple unit cars and locomotives must be purchased upon a schedule which will meet the completion dates of the electrification with as little overlap as possible

and yet with sufficient overlap for safety, again reducing idle investment to a minimum.

This is a necessarily brief description of the electrification program of the Pennsylvania Railroad and the preparation which led up to it, but the work done will show results with the inauguration of the through electrification and will result it is believed in a service of maximum economy and reliability, with comfort to the traveling public.

Discussion

S. Withington: One of the outstanding thoughts which arises in the minds of the readers of Mr. Duer's paper is the large amount of experimental and development work which the Pennsylvania Railroad has accomplished. The magnitude of the installation, by far the most extensive in this country, has justified a great deal of experimentation, and railroads electrifying in the future will obtain much benefit from the investigations carried on by the Pennsylvania engineers. It is of especial interest to note that with its broad background of experience the Railroad's choice should have been the single-phase system of distribution and utilization of power.

One of the outstanding efforts of the Pennsylvania engineers has been the standardization of electric locomotives. This should result in a very considerable saving of expense on account of quantity production, and this saving will be more important as other railroads take advantage of the development, and standardization becomes more general. The economies in quantity production of electric locomotives in many respects are quite analogous to those so thoroughly demonstrated in the automobile industry.

With respect to Mr. Duer's references to substation design, a glance at the illustration indicates the importance which the Pennsylvania engineers attach to a generous allowance of space about oil circuit breakers, transformers and buses. Such spacing undoubtedly brings about maximum reliability and tends to localize any failures which may occur. If the substations can be located where land is relatively inexpensive, the logic of this arrangement is especially obvious.

Mr. Duer suggests that substations should be located near a signal tower so that the circuit breakers may be readily controlled. On account of the growing practicability of remote-control facilities this is not now as important as it was formerly, but it is of primary importance to locate the trolley wire sectionalizing points and circuit breakers where there are crossovers between tracks so that in the event of abnormal conditions, trains may be crossed from one track to another to run by the sections where there may be trouble.

Regarding the use of non-corrosive materials, this is a notable development of electrification installations in recent years, and is an excellent example of the kind of lessons which are being learned through experience.

The rail bonds adopted for the Pennsylvania electrification are a somewhat radical departure from previous standards, and it will be interesting to note the performance on this very heavy section of route.

Mr. Duer's reference to the source of power is of interest. He points out that the question of decision as between purchase and manufacture of power is largely economic. All railroads which contemplate electrification are of course faced with this important question very early. The development of larger and more efficient central stations and the economies of interconnection of power systems are paralleled by the growing efficiencies of smaller plants, and both the railroads and the power companies must figure very closely to arrive at a satisfactory solution. One of the important items for consideration in figuring costs of power may be the high rate which power companies sometimes assume as interest on the capital represented by their power plants and associated facilities, which is often considerably more than the railroads can justify for their own facilities. A relatively small difference in these assumed percentage figures may be sufficient to swing the decision one way or the other.

In connection with preparations which are necessary for electrification, Mr. Duer has indicated a number of points which must be considered, but he has not mentioned the desirability of completing any general improvements such as the elimination of grade crossings, straightening tracks or lengthening curves. Changes of this nature should be completed, so far as they can be foreseen, before the installation of catenary construction, because the difficulties of any changes which are made after electrification has been completed are of course considerably greater than before.

The standardization of detail parts for catenary construction is extremely important, and general agreement in such design among electrified railroads is one of the items which will advance electrification as much as any other single question.

T. A. Purton: In general, what special measures were required to prevent telephone and communication interference?

K. V. B. Duer: In reply to Mr. Purton's question, it may be said that no special measures were required to prevent telephone and communication interference. Electric-traction circuits were very carefully designed and laid out for the best results from an electric-traction standpoint, and operating experience so far has indicated that no modifications in these circuits were necessary to take care of telephone and communication interference.

Electricity's Part in Open Cut Copper Mining

BY R. J. CORFIELD[1]
Associate, A. I. E. E.

Synopsis.—*This paper is intended to describe the electrification of the world's largest open cut copper mine, and also covers briefly main line transportation and miscellaneous uses of electrical energy at the concentrating plants, that are located approximately 18 miles from the mine.*

This electrification project was an economical and progressive step in copper mining and developed some very interesting engineering problems.

Electric shovels, both alternating and direct current, together with the special type of electrical locomotive required for this particular service, are discussed.

The rather elaborate trolley and feeder system required to transmit power to a fleet of 23 electric shovels and 39 electric locomotives, scattered over a 725-acre area, is described.

The main power system, together with mill, railway and shovel substations, is briefly described.

Some operating statistics are given, to assist in visualizing the magnitude of the operations.

THE Utah Copper Mine is in reality a mountain of low grade copper ore, covered by an overburden of decomposed rock, varying in depth from 75 to 200 ft. The clearly defined levels, or benches, as shown in Fig. 1 vary in height from 70 to 130 ft. and have a slope of approximately 45 degrees. Some of the levels are very narrow at the ends and gradually widen towards the center. The ore, after being blasted from the banks and loaded, is transported to the main assembly yard over a series of switchbacks, built in most cases with an average grade of 4 per cent.

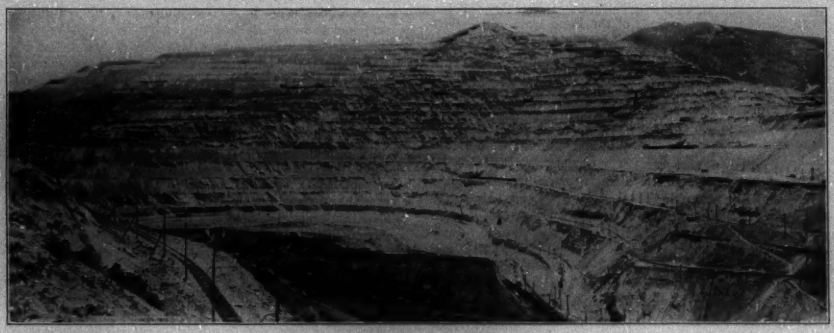


FIG. 1—GENERAL VIEW OF MINE

Fig. 2 is a map showing a part of the haulage system. Both the ore and waste break rather easily and the blasted material varies from gravel size to very large boulders which, if too large to go in the dipper, have to be separately blasted before loading. The material in its blasted condition, ready for loading, weighs approximately 4160 lb. per cu. yd.

TYPES OF LOADING EQUIPMENT

The first steam shovel was put in operation in 1906, and more were added as conditions warranted. The original shovels were equipped with railway type trucks and 2½ cu. yd. dippers. These dippers were later replaced by 3¼-yd. dippers and some caterpillars were installed. Later all steam shovels were equipped with 4½-yd. dippers and caterpillar tractors.

The first electric shovel appeared in 1922 at which time two, Marion Model 92, were purchased, both being equipped with caterpillar tractors and 4½-yd. dippers.

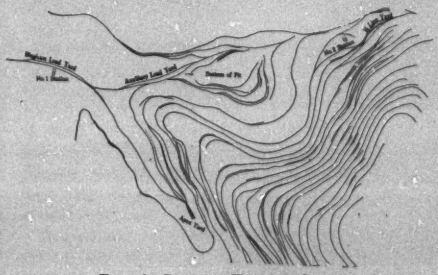


FIG. 2—PART OF HAULAGE SYSTEM

1. Asst. Elec. Eng., Utah Copper Co., Garfield, Utah.

Presented at the Pacific Coast Convention of the A. I. E. E., Portland, Oregon, September 2-5, 1930.

30-154

One of these shovels was driven by d-c. motors while the other was equipped with a-c. motors. Rather extensive tests, both electrical and capacity, were made and both of these shovels proved to be far superior to steam shovels, considerable increase in economy being shown,

Fig. 3—Electric Shovel

together with a great many operating advantages that could not be capitalized. In 1923, eight additional shovel equipments, using a-c. motors were ordered, and eight shovels were converted from steam to electric drive. After these shovels had been in operation sufficient time to prove their superior operating economies, other shovel equipments were ordered until at the present time there are 23 electrics operating and no steamers. Nine of these shovels are equipped with a-c. motors, with rheostatic control, and fourteen with d-c. motors using the Ward-Leonard system of control, modified to meet the particular kind of service, which is

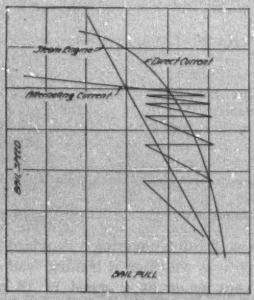


Fig. 4—Comparative Shovel Characteristics

very severe and requires the most sturdy motors and control equipment. Fig. 3 is a view of a modern electric shovel.

Shovel Characteristics

The electric shovel was primarily designed to replace the steam shovel which, with the exception of its poor economy and high maintenance, has an ideal control characteristic, and an electric shovel should therefore be designed with a type of control that will duplicate the steam characteristic as nearly as possible. A curve showing the comparative characteristics of steam, alternating current, and direct current drive, is shown in Fig. 4, and it should be noted that the Ward-Leonard system of control approaches very closely to the steam characteristic, giving the necessary smoothness and flexibility.

Three types of d-c. motors are available for shovel drive, *i. e.*, shunt, compound, or series wound.

Which of these motors is best adapted for shovel drive depends on the nature of the material handled

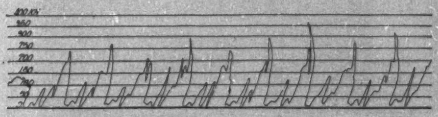


Fig. 5—Power Input Curve, D-C. Equipment

and speed requirements. In general, it may be said that the separately excited, differentially compound wound generator, supplying power to series wound hoist motors and shunt wound swing and thrust motors is best suited for general application to railway type shovels. Typical graphic charts showing the power required, peaks, and general character of the duty cycle, are shown in Fig. 5, for the d-c. equipment, while Fig. 6 shows the corresponding curves for the a-c. equipment. These curves are intended to be typical and not comparative, as a number of almost impossible conditions must be realized before comparative curves can be made, *i. e.*, character of

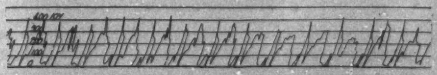


Fig. 6—Power Input Curve, A-C. Equipment

material, personnel of crew, and condition of equipment must all be equal. These curves indicate the severity of the duty cycle and show clearly the necessity for using the most substantial type of electrical equipment available. It should also be noted at this point that the electrification of shovels has been directly responsible for an improved mechanical shovel, as the torques obtained from the electric motors are greater than from steam engines, making it necessary to strengthen the underframing, gearing and other structural parts of the shovel. This increased strength and sturdiness means fewer mechanical delays and is, therefore, a contributing factor in electrification economies.

Electrification has also fostered a close relationship between the shovel manufacturer and the electrical equipment manufacturer, with the result that the electric shovel in its present form is a highly co-ordinated and well arranged piece of equipment. The shovel operator has also had an important part in this development in suggesting certain changes in both mechanical and electrical equipment. Tests, extending over a period of years, are necessary before practical conclusions can be drawn. During this period, power costs, tonnage loaded, repairs and operating time must be tabulated. Such a tabulation, showing comparative operating costs of electric and steam shovels, is shown in Table I. About one-half of the saving shown is due to electrification, while the other half is due to the substitution of caterpillar traction for railway trucks. It should be noted that the tabulation does not include fixed charges on either type of equipment, and while it is hardly expected that the electric drive will ever be as low as steam in first cost, economies and other advantages outweigh the advantage of steam in this respect.

TABLE I—PROPORTIONATE COST PER TON LOADED

Steam		Electric	
Engineer	100	Engineer	100
Craneman	100	Craneman	100
Fireman	100	Electrician	51.1
Coalman	100	Electrician's Helper	51.8
Pitman	100	Pitman	100
Watchman	100	Watchman	0
Total Operating Labor	100	Total Operating Labor	76.2
Coal and Water	100	Power	17.5
Packing and Oil	100	Packing and Oil	34.0
Total Operating Material	100	Total Operating Material	20.4
Repairs	100	Repairs	21.5
Total Cost Per Ton	100	Total Cost Per Ton	37.2

LOADING CAPACITY

The electric shovel is capable of loading more ore per shift than its steam competitor, due to the fact that it is practically impossible to keep up uniform steam pressure. Delays due to leaking steam and water lines, choked boilers, etc., also contribute. Individual cycles timed by stop watch show the steam shovel to be just as fast and in some cases faster than the electric, but the fast cycle cannot be repeated over the days work, for the reasons outlined above. On the other hand, the present electric shovel is a highly developed and simplified machine, and while electrical delays occur, they are usually due to control equipment and can be repaired very quickly, and in a great many cases are of such a nature as not seriously to affect the operation of the shovel, and can be left until the shift is completed. The loading capacity of the electric is roughly 14 tons per minute, and if cars were available and there were no delays, it would be possible to load 6,720 tons per shovel shift, or a total tonnage of 154,560 tons with 23 shovels. However, as the operating time factor averages about 80 per cent, and as cars are not always available, this tonnage is reduced to about 105,000 tons maximum per day.

The ore tonnage varies from 35,000 to 50,000 tons per day, while the waste tonnage varies with the mine development. The ore, after being loaded, must be transported to various assembly yards, while the waste must be dumped in adjacent ravines. This work is performed by 39 electric locomotives, designed especially for the service. Due to the complexity of trackage, it was not considered practical to string trolley over all tracks and there were certain places, particularly on the benches, where trolley could not be placed over the center of the track. To obtain maximum operating range, the locomotives were equipped with a standard pantograph, two side arm collectors and a cable reel and, in some cases, a 680-ampere hour storage battery was used in conjunction with the regular collecting devices. Fig. 7 is a view of a typical electric locomotive, while Table II is a condensed specification covering a standard locomotive. These locomotives will haul a trailing load of 250 tons up a 4 per cent grade at a speed of 10 mi. per hr., with a power input of approximately 825 kw. Motors are totally enclosed and have a one hour rating of 280 amperes, giving a tractive effort of 35,000 lb. The continuous rating is 120 amperes, corresponding to a tractive effort of 11,200 lb.

FIG. 7—TYPICAL ELECTRIC LOCOMOTIVE

TABLE II—LOCOMOTIVE SPECIFICATIONS

Rigid Wheel Base	8 ft.–0 in.
Total Wheel Base	25 ft.–2 in.
Wheel Diameter	44 in.
Length Inside Knuckles	36 ft.–6 in.
Weight	154,000 lb.
Compressor Capacity	150 cu. ft.
Motor Rating (One Hour)	261 Hp. at 105 deg. cent.
Gear Ratio	85 : 18
Rail Clearance	4¾ in.

TROLLEY SYSTEM

The design of an adequate trolley system to serve the

locomotives required considerable study, as there were many limitations imposed by the operating department. The first problem was that all poles had to be placed on the bank side of the track, in order that ditching and spreading operations could be carried on. This problem was solved by using extra long bracket arms and placing the pull-over backbone on the end, as indicated in Fig. 8. A large number of waste fills, varying in height from

Fig. 8—Trolley Construction on Switchbacks

100 to 250 ft., had to be electrified, and as these fills will settle approximately 10 per cent of their height, it was necessary to use a construction as shown in Fig. 9, where the pole is made a part of the track structure by means of extended ties. A number of bridges, on curves varying from 12 to 24 deg., was taken care of by a light "I" beam structure as shown in Fig. 10. On the benches, the track is moved every 25 days and some kind of portable structure was required. It was also

Fig. 9—Trolley Construction on Waste Fills

necessary to have a portable shovel line on the bench. A light steel tower, as shown in Fig. 11, was designed to take care of both lines. These towers are 25 ft. high and weigh approximately 900 lb. They are provided with heavy skids on the bottom for sliding, or may be picked up by a locomotive crane. Despite the fact that these structures are within the blasting zone, very little trouble has been experienced. All overhead construction is of the direct suspension type, with the exception of the Bingham Yard, which is catenary.

Main Line Transportation

The ore is taken from the assembly yards at the mine and transported 18 miles to the milling plants for concentration. This work is done by seven Mallet locomotives, weighing 316 tons each. The main line

Fig. 10—Trolley Construction on Bridges

railroad is single track, with six passing sidings, the Bingham end passing through four tunnels, bored into solid rock, and having an aggregate length of approximately one mile. For years this railroad operated without any signaling devices, and the train crews were directed by train orders from the dispatcher, and while

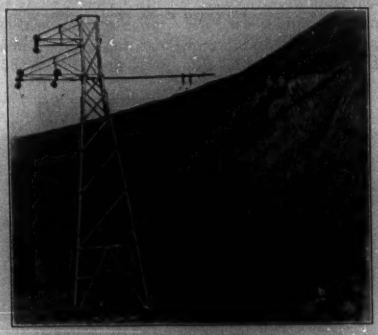


Fig. 11—Trolley Construction on Benches

these dispatchers were very proficient in making train meets, it was known that more efficient operation could be realized with the road properly signaled. Accordingly, for the purpose of study, the tunnel section was protected with automatic signals and two switches were equipped with remote control. The value of signals and remote-controlled switches was soon recognized and resulted in complete signaling of the road, as well as

the Magna Yards. Train orders were, however, still necessary as no superiority could be established between uphill and downhill trains.

In 1925, a system of centralized traffic control was developed whereby the dispatcher controlled the siding signals, thus giving him complete control of all trains without the necessity of train orders. This system was installed on the railroad in 1929, and has resulted in a

Fig. 12—Signals at Typical Siding

marked saving in running time in addition to the natural safety features inherent in signal systems. Fig. 12 shows signals at a typical siding, while Fig. 13 shows the dispatcher at work.

At the mill yards, electric locomotives break up the 50-car train from the mine into 10 car cuts, ready to be dumped. This service was formerly performed by a steam switcher, and due to the rather erratic demands at the car dumpers, this engine was on spot about 50 per

Fig. 13—Train Dispatcher at Work

cent of the time, resulting in a high dumping cost per car. The ore, after being dumped, is given a primary crushing, a secondary crushing, preliminary and final grinding, to reduce to minus 100 mesh. From these mills the slime is conveyed to the flotation plant where the ore and gangue are separated. After being dewatered, the concentrate is ready for shipment to the smelter for preliminary refining. The mill work is done by motors varying in size from one-quarter to 850 hp. most of them being of the standard, squirrel-cage, induction type. Synchronous motors are used wherever possible, and all new drives are carefully considered with the idea of using synchronous instead of induction motors. All motors are started across the line at 440 volts, and in the majority of cases with contactors designed and built by the company's forces. Miscellaneous uses about the plants include electric rivet heaters, hot plates, water and acid heating, pipe annealing, floodlighting, etc. All new motors are purchased with anti-friction bearings, and the conversion of sleeve to anti-friction bearings has been going on for several years, the end bells and caps being designed by the engineering department and these parts cast and finished in the company's foundry and machine shops.

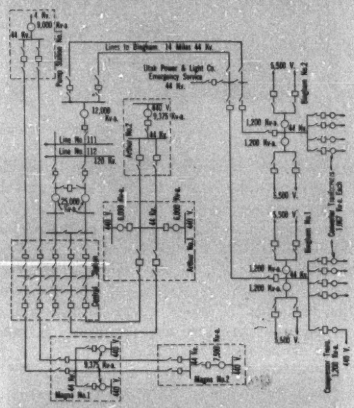


Fig. 14—Line Diagram of Power System

Power Supply

Power for these operations is furnished by the Utah Power & Light Company and is received at Magna Central Station at 120,000 volts, where it is stepped down to 44,000 volts and distributed to seven substations. Fig. 14 is a single line diagram of the system. This central station has an installed capacity of 62,000 kv-a., divided into three transformer banks, two 25,000-kv-a. banks for the mills, and one 12,000-kv-a. bank for the mine. All 120,000-volt apparatus is installed outdoors, while the 44,000-volt switches and control equipment is indoors. Figs. 15, 16, and 17 are views of this station.

As continuity of service is a very important consideration in an industry of this kind, duplicate lines are carried to each substation and ring buses are used to gain switching flexibility. Each mill substation is provided with a spare transformer, so arranged that it may be used to spare any one of the bank with a mini-

mum delay. Power at 440 volts is transmitted through three conductor cables direct to the motors, or to suitable buses, each circuit being protected by an oil circuit breaker and suitable relays. A typical mill substation is shown in Fig. 18. Power is transmitted to Bingham over a two-circuit tower line and distributed to two, composite, shovel and railway substations. (Fig. 19.) Each shovel substation, which is of the outdoor type, contains two banks of transformers, rated at 1200 kv-a. each, the secondary voltage being 5500 volts.

This energy is carried over two independent pole lines arranged to completely encircle the mining area, and built to approach as closely as possible the ends of the benches, where they are tapped and run through a portable switch house containing an automatic oil circuit breaker. From this point they are carried across the bench on the portable towers before mentioned. With this system it is possible to feed the shovel from

Fig. 15—General View of Central Station

Fig. 16—Control Room

Fig. 18—Typical Mill Substation

Fig. 17—Interior of Station

Fig. 19—General View of Shovel and Railway Stations

any one of four secondary trunk lines. Such a distribution system may seem rather elaborate, but when it is considered that a large fleet of shovels would be delayed in case of line failure, such a system is warranted.

The railway substations are of the semi-outdoor type and are full automatic. The 44,000-volt oil circuit breakers and converter transformers are outside, while the converters and auxiliaries are inside. Each station (Fig. 20) contains four, 1,000-kw., 750-volt, 1,200 rev. per. min., shunt wound converters, all tied to a common bus from which eight feeder circuits are taken. As the speed of the locomotives is directly proportional to the trolley voltage, it is important that adequate feeders be provided, and also that the trolley system be properly sectionalized so that an interruption on one feeder does not affect a large number of locomotives. The feeder system at Bingham is so arranged that six locomotives, operating on three levels, are supplied from two sources over two separate feeders. As faults are most likely to occur on the benches, this arrangement

Fig. 20—Interior of Automatic Railway Station

permits operation up to the bench where sectional switches are provided for switching to a good feeder. The magnitude of these operations may be more easily visualized by consulting the following statistics, representative of one year's operation.

Tons of ore loaded	16,556,070
Tons of waste loaded	14,400,170
Total tons of material loaded	30,956,240
Tons of ore hauled by electric locomotives	16,031,728
Tons of waste hauled by electric locomotives	5,465,939
Total tons of material hauled by electric locomotives	21,497,664
Total kilowatt hours used by electric shovels	6,190,136
Total kilowatt hours used by electric locomotives	7,536,604
Total kilowatt hours used by Utah Copper Company	323,454,428
Average yearly load	36924 kw.
Maximum load for year	45155 kw.
Annual load factor	81.8%

Discussion

L. W. Birch: During 1924 the Utah Copper Company replaced two steam switching locomotives in the Magna-Arthur crusher yards, with two 75-ton electric locomotives. These locomotives were equivalent in weight to the replaced steam locomotives. The crusher yards include seven miles of track and are used for handling ore cars at the two mills where the ore is crushed previous to its treatment in the flotation plants. Approximately 600 cars of ore have been handled per day through these yards during the past six years.

Fig. 1—Catenary at Bingham Mines 1930

The distribution and collection system on this electrification consists of light catenary with normal tangent spacing of 150 ft. A 7/16-in. steel messenger wire, supporting a 4/0 grooved hard-drawn copper contact wire, was used throughout the system. Flexible steel catenary hangers connect contact wire and messenger wire. This system was insulated for 600 volts d-c. and is operating at that voltage at the present time, energy being obtained from a central substation located at the company's large distribution substation at Magna, Utah.

A recent check of this system has indicated very little maintenance and extremely minute wear on the contact wire. Initial costs in connection with the insulation were reasonable and were

Fig. 2—Catenary at Bingham Mines 1930

completely written off through the savings obtained by elimination of steam after the first eighteen months of operation. Through the favorable operation of this section, it was decided to build catenary in the yards at the Bingham mines. This is mentioned in Mr. Corfield's paper and is illustrated in two of the attached illustrations. This light type of catenary is suspended mostly by steel structures, the structures being of the bolted type and erected on the ground.

In designing and constructing the overhead system at the Bingham mines, considerable thought was given to the flexibility of the overhead system. Wherever possible the flexible overhead system was constructed in lieu of the rigid system. Photographs

indicated as Figs. 8, 9, and 10 in Mr. Corfield's paper, illustrate how flexibility was built into this system. For instance, the contact wire in Figs. 9 and 10 is suspended by a piece of flexible cable passing over a porcelain insulator and attached to the contact wire by half strain ears. A flexibility is obtained in this way which reduces trolley wire wear and breakage at the suspension point.

Fig. 3—Magna-Arthur Crusher Yards Electrified 1924

Fig. 4—Another View of Yards Electrified 1924

This flexible arrangement was also installed on some of the original stripping tracks on the benches. Owing to complications in moving the portable towers and to the occasional breakage due to blasting, it was thought that a more simplified system, even though it included rigid suspension points, would be advisable. As an outcome the portable structures on the benches are now equipped with a standard type of insulated mine hanger to which is attached a standard type of mechanical mine clamp. This mine clamp has an opening above the jaws into which the trolley wire can be inserted and held loosely while the towers are being moved. In this manner the shifting of track and towers is accomplished without much work, and the contact wire can be held in a position above ground without injury. After the towers are moved, the contact wire is taken out of this opening in the clamp and placed in the regular jaws of the clamps, the wire having been tensioned previous to the clamping.

Hydraulic and Electrical Possibilities
of High-Speed, Low-Head Developments

BY GEORGE A. JESSOP[1] and C. A. POWEL[2]

Member, A. I. E. E. Member, A. I. E. E.

Synopsis.—The paper deals with the change brought about in hydroelectric practise by the introduction of the high-speed low-head propeller type of turbine. For a given head, up to about 70 ft., it is possible to run the units at a much higher speed, thus reducing materially the cost of the generator. The maximum efficiency of the unit is maintained and the part load efficiency greatly improved by the use of adjustable blade turbines, so that there is also a material gain where variable load or flow is to be handled. The fundamentals underlying the design of an adjustable blade propeller type turbine are given and an illustrative study made of a three-unit station, to show the gain in output obtainable from this type of unit.

BECAUSE of economy in first costs, a great deal of effort has been expended, for the past twenty years, to secure higher speed hydroelectric units. The principal reduction in first cost has been due to the lower price of the generator equipment as the speed increases, the hydraulic turbine costs decreasing to a less extent. Until recently the increase in speed has resulted in reduction of efficiency, particularly at part loads, thus to a large extent offsetting the advantages of reduced investment. The improvement in generator efficiency with increased speed has only partially offset the reduced turbine efficiencies. Turbine designers have now succeeded in maintaining the maximum, or peak, efficiency at a given figure over a very wide range of specific speed, and by means of the Kaplan, or automatically adjustable blade runner, have improved the part load efficiency far above that formerly obtained even with slow-speed Francis turbines.

The hydraulic turbine horsepower and efficiency data and information contained herein have been obtained from tests made in the field and in specially designed and constructed testing flumes. The relationship between head, power and speed given in Fig. 4 has been checked by numerous actual installations, and for the axial flow turbines, further investigations have been made recently by means of cavitation tests. In determining specific speed the well known formula

$$N_s = \frac{R\,P\,M\,\sqrt{H\,P}}{H^{\frac{5}{4}}}$$

has been used. For slide rule computation this is, perhaps, more conveniently used in the form $N_s = \frac{R\,P\,M}{H}\sqrt{\frac{H\,P}{\sqrt{H}}}$. The specific speed of a runner is the rev. per min. at which a homologous runner will operate under one-ft. head when it is so reduced in size as to develop one hp. under one-ft. head.

Fig. 1 shows the relationship between specific speed and maximum efficiency. Generally speaking, Francis turbines are used up to a specific speed of 110 and the adjustable and fixed blade propellers for specific speeds above this figure. It will be seen that very high specific speeds can be used before any material reduction in maximum efficiency is suffered.

Fig. 2 shows a comparison between the peak and part load efficiency of turbines with specific speeds ranging from 25 to 150. The turbines with specific speeds of 25, 50, 75 and 105 are Francis turbines. Of the two turbines with a specific speed of 150, one is of the fixed blade and the other of the adjustable blade axial flow type. It is interesting to note how

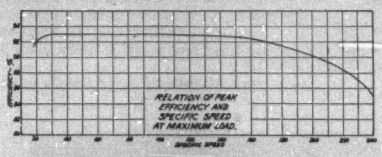


FIG. 1—RELATIONSHIP BETWEEN SPECIFIC SPEED AND MAXIMUM EFFICIENCY

the part load efficiency drops off very rapidly as the specific speed is increased—a defect which can be overcome by the use of the adjustable blade turbine. The notable feature of this latter type is the fact that the efficiency over the entire range of output is very high, exceeding even that of the lowest speed Francis turbine at low loads.

Fig. 3 shows how the efficiency of the generator varies with the load. It will be noted that the efficiency increases with the size of the unit and also with speed. The part load efficiency increases less rapidly than the full load efficiency, because of the more marked effect of the windage losses at the higher speeds. Data similar to that given by Figs. 2 and 3 must be combined to obtain the performance of the complete unit.

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Presented at the Middle Eastern District Meeting No. 2, of the A. I. E. E., Philadelphia, Pa., October 13-15, 1930.

Fig. 4 makes available graphically a method for determining the allowable relationship between head, horsepower, and speed. The head curves represented by full lines are to be used for Francis turbines, and those represented by dotted lines for axial flow turbines. At the upper left hand, part of the chart lines have been drawn indicating approximately the limits

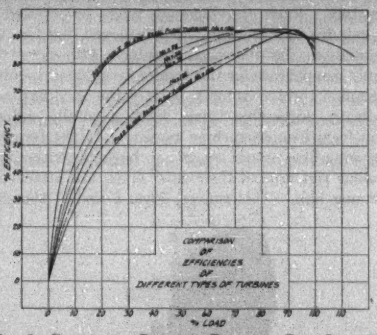

Fig. 2—Comparison Between Peak and Part Load Efficiency of Turbines with Specific Speeds Ranging from 25 to 150

of capacity corresponding with the different heads. It is believed this information will be very useful for preliminary engineering work in connection with the designing and estimating of hydroelectric developments and the selection of suitable generator speeds. The limitations of specific speed—that is, the combination of horsepower, head, and speed—set by this chart

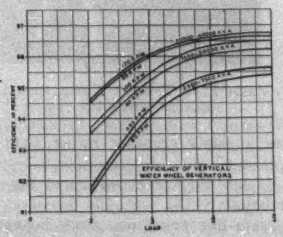

Fig. 3—Shows Variations in Generator Efficiency with Load and Speed (Three-Phase 80 Per Cent Power Factor, 60 Cycles)

are the results of many years of experience and may be regarded as good practise at the present state of the art. If plants are designed within these limits and with the proper relationship between the center line of the unit and tailwater, best results in output, economy and long life are secured.

The field of application of the fixed and adjustable blade axial flow turbine lies in relatively low-head developments, approximately 70 ft. or less. One of their great advantages is that they permit of considerably higher speeds than the older Francis turbine, so that today it is permissible to speak of "high-speed low-head" units.

The effect of speed on the first cost of the turbine is comparatively slight, but there is a definite trend towards reduction in cost with increase in speed. The rev. per min. of a fixed blade axial flow turbine may be about 70 to 80 per cent higher than that of a medium speed Francis turbine, and, very approximately, the reduction in cost about 5 to 10 per cent. The reason for the small reduction in cost lies in the fact that the amount of water to be handled is the same in either case and the same water control mechanism is required surrounding the runner. The small saving is the result of handling the water at somewhat higher velocities

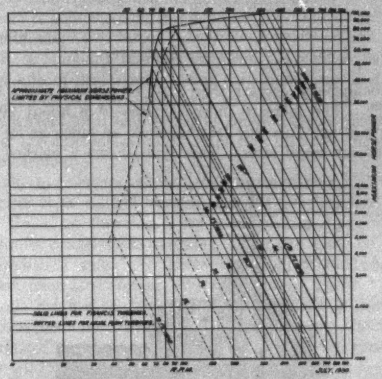

Fig. 4—Relationship Between Head, Horsepower and Speed of Francis and Axial Flow Turbines

and, therefore, with somewhat less cross-sectional areas, and greater simplicity of runner design.

From the generator designers' point of view, there is always an advantage in increasing the speed. With increased speed, the cost of the generator goes down and the efficiency goes up. Fig. 5 shows how the cost of water-wheel generators varies with capacity and speed. The curves are drawn upon the basis of normal overspeeds, that is, 100 per cent for generators up to 200 rev. per min. and 85 per cent above that speed. When the unit speed can be selected with a view of obtaining suitable efficiency at or n ar the maximum head, the overspeed of axial flow turbines is usually double the normal speed. For plants having a head that varies through a large range when it may be desirable to select a speed suitable for a head considerably under the maximum, the overspeed may be more than double normal, and special provision must be

made in the generator design. For the higher ranges of heads suitable for axial flow turbines, when it is desirable to set the turbine as high as possible with respect to tailwater, the overspeed is frequently reduced by cavitation.

The cost of an automatically adjustable turbine is

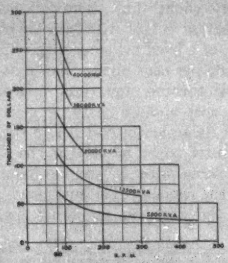

Fig. 5—Shows Variations in Generator Costs with Capacity and Speed

substantially greater than that of a fixed blade turbine. The increase in cost varies from about 20 to 25 per cent for very large sizes to as much as 50 per cent for small sizes. This type shows a marked in-

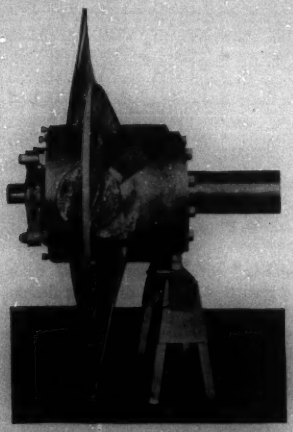

Fig. 6—Axial Flow Turbine Wheel with Adjustable Blades—Closed Position

crease in kilowatt-hour output and, therefore, in earnings. Since the cost of the turbine is, in almost every development, but a small fraction of the whole cost, the total investment is increased only a small percentage.

There are other factors to be considered, such as excavation and power house cost. Turbines of high specific speed, in general, will be set at a lower elevation than those of low specific speed, thus tending to increase excavation cost. This tendency is counter-balanced to some extent, because the water flowing through high specific speed turbines is handled at slightly higher velocities and the length of the power house may be decreased, resulting in a saving in superstructure. Each development must be worked up as an individual problem. It is believed that for every installation the increased earnings will more than amply compensate for the greater investment involved.

Kaplan turbine runners, both of the fixed and automatically adjustable types, have a totally different form from Francis turbine runners. In the former, a few wing-like runner vanes, or buckets, without the familiar rim around their outer ends, are attached to a strong hub. This runner, Figs. 6 and 7, therefore,

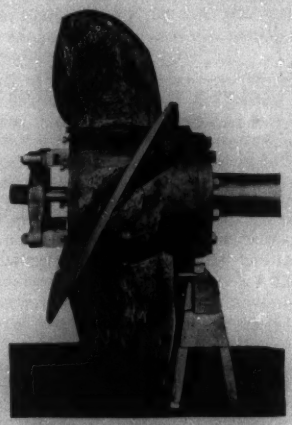

Fig. 7—Axial Flow Turbine Wheel with Adjustable Blades—Open Position

resembles a ship's propeller rather than a Francis turbine.

The Kaplan turbine shows three important new characteristics:

1. A vane-free transition space between the discharge area of the wicket gates and the runner entrance. The water leaving the gates in a whirling inward direction is deflected in an axial direction in this transition space and flows axially through the runner.

2. Wing shaped runner vanes, so spaced and proportioned that two adjacent vanes form a nozzle or double guidance for part of their radial extent and single guidance for the remainder.

3. Adjustable runner vanes.

Prof. Kaplan was the first to adjust the runner vanes and the wicket gates simultaneously. In the closed po-

sition, Fig. 6, the whole surface of the vanes, or buckets, lies approximately at right angles to the shaft axis. In this position the discharge is a minimum and the runner develops minimum power. With increasing wicket gate opening (increasing load) the vanes are opened so that the area of the water passage increases, Fig. 7. Each position of the runner vanes corresponds to a definite position of the wicket gates, and the correct relationship of these positions must be maintained if the efficiency at all rates of discharge is to be a maximum.

The simultaneous adjusting of the runner blades and wicket gates maintains the best conditions of flow throughout the entire range of output and results in the obtaining of exceptionally high part load efficiencies, thus securing very flat efficiency-load curves. Since the blade and gate angles and openings always

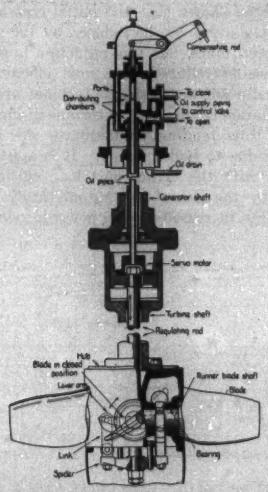

Fig. 8—Hub Casting of Axial Flow Turbine Wheel

have the correct relationship to each other, the adjustable blade turbine operates with maximum smoothness and with minimum vibration and pitting.

The automatically adjustable blade turbine casing is constructed like the usual wicket gate casing except that the top plate contains an inverted curved cone-like projection which forms the inner surface of the transition space.

The construction of the runner hub which carries the movable blades requires special attention due to the relatively small space available. All of the operating connections for moving the vanes must be contained inside the single piece hub casting (see Fig. 8, lower view) and must be of such form that they will stand up indefinitely under the most severe service requirements without any adjustments. By very carefully utilizing every cubic inch of space, a design has been developed which has proved entirely satisfactory and reliable. The stresses must be held within very conservative limits to guarantee freedom from excessive strains and deformation and to reduce the possibility of wear to the absolute minimum. The moving parts of a Smith-Kaplan automatically adjustable blade runner, consisting of the regulating rod, the sliding cross and the link connection to the crank on the runner vane trunion, are clearly shown.

The bearings for the runner blade stems and the links are bushed with bronze to resist wear, and special provision is made to secure the blades against lateral displacement due to the large centrifugal forces developed.

In order to provide constant lubrication for the moving parts, the hub is filled with a heavy body oil. Each runner-vane trunion is packed against leakage where it goes through the hub body, so that oil loss is eliminated.

It is standard practise to make the runner vanes and hub of cast steel. Great care should be used to finish the vanes to templets, so that they will have the correct form and degree of smoothness required to offer the least disturbance to the flow of water through the turbine.

The movements of the gate control system and the runner-vane control system must be synchronized at all times, so that for every gate position there is a definite runner-vane position. This can only be accomplished when the gates and runner each have their own servo-motor and distributing valve. The regulating rod in the hollow turbine shaft is moved axially by a servo-motor (Fig. 8, center view) which is arranged between the flanges of the turbine and generator shafts and is rigidly connected to them, so that it revolves with the shaft. The servo-motor piston is directly connected to the upper end of the regulating rod which communicates the piston movements to the runner vanes. The servo-motor is operated by oil supplied under pressure from above (Fig. 8, upper view), through two pipes arranged concentrically within the hollow generator shaft.

The inner pipe carries oil to the lower side of the piston and is rigidly connected to it, moving upward and downward with the piston. The governor compensating mechanism is connected to the upper end of this pipe. The two pipes to the servo-motor terminate in two distributing chambers arranged one on top of the other, and mounted on top of the generator. These chambers receive the oil from the governor and collectively form the oil supply head.

Fig. 9 shows schematically the regulating mechanism. The governor itself is the usual speed governor which operates the wicket gates either through a vertical gate shaft or through servo-motors. In addition, this governor has a second distributing valve controlling

the flow of oil to the runner-vane servo-motor in the turbine shaft. A cam is connected to the gate operating mechanism and to the runner-vane compensating connection in such a way that the gates and runner vanes always move in the definite relationship required to give the maximum efficiency for every variation in load.

The correct shape of the cam on a given turbine varies with the head, the speed of the turbine remaining

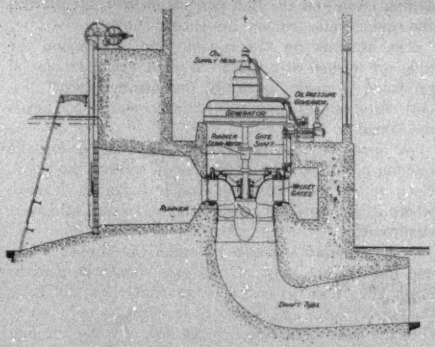

Fig. 9—Schematic View of Regulating Mechanism

constant. With a moderate range in head one cam will maintain such gate and bucket relationship as to produce the best efficiency, or very close to it, throughout the entire range of horsepower. When the variation in head is large, a series of cams may be used or a continuous warped surface cam can be designed and operated so that its position is moved as the head changes.

There are two types of generators suitable for use with this type of turbine; the more general two-bearing design, in which the thrust bearing and one guide bearing are located above the rotor with the second guide bearing below the rotor; and the umbrella design, in which the single combination thrust and guide bearing is located below the rotor. In the matter of cost and efficiency, there is practically no difference between the two designs, but the umbrella design has a number of advantages. It requires less head room under a crane and is easier to dismantle. Its single bearing runs in a bath of oil so that all oil piping is eliminated. It lends itself readily to almost any scheme of ventilation. The air may be taken directly from above or below the rotor, or the machine may be totally enclosed and a recirculatory scheme used with the coolers arranged conveniently around the periphery of the stator. If a scheme of excitation other than by means of a direct-connected exciter is used, the upper bracket may be omitted entirely. This may prove an advantage where it is desired to eliminate the building super-structure, and consideration should, perhaps, be given to motor-driven exciters. Where a dependable auxiliary source of power is available from which such exciters may be supplied, the problem is simple, but in most hydro-stations an auxiliary source is either not present or too small to handle all the excitation required. The motor-generator exciters may be supplied from the main units through suitable transformers, and in this case it might be advisable to provide them with a flywheel to maintain their output during faults. Some arrangement would also be necessary for starting up the dead station.

The field of application of the umbrella generator is for the large low-head units, and because of its inherent advantages it is used wherever possible. Fig. 10 shows a section through a typical modern umbrella generator.

It has been shown that the initial cost of a hydroelectric installation can be reduced by the use of high specific speed units, but this reduction is obtained at the expense of reduced part load efficiency, unless automatically adjustable blade turbines are installed. The installation of this type of turbine in turn increases the cost, thus tending to minimize the investment advantage due to higher speed. It is apparent, however, that with the higher part load efficiency to be obtained by the use of the automatically adjustable blade turbine, increased earning power is to be expected.

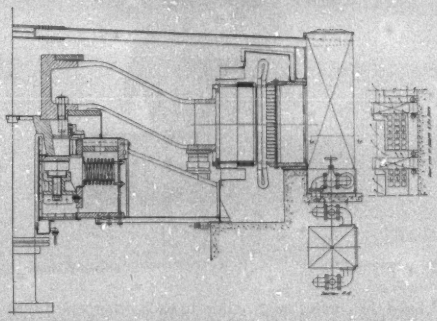

Fig. 10—Section Through Typical Umbrella Type Generator

In order to study the effect of the characteristics of the various types of hydroelectric units on the yearly output, a kilowatt-hour output study has been prepared. A duration curve has been selected, which shows the cu. ft. sec. available during the year, and the head corresponding to the flow. It is to be expected that in the normal plant the head will decrease as the quantity of water increases. The duration curves are such as might be expected on a river having a fairly variable flow. A three-unit development has been chosen for purposes of illustra-

tion. A comparison has been made between three medium speed Francis units, three fixed blade axial flow units and three automatically adjustable blade axial flow units. The rated capacity of each unit is 28,500 kw. at extreme full load under 53-ft. head. The adjustable blade turbine has a surplus of 15 per cent above the maximum capacity of both the Francis and fixed blade turbines. Referring to Fig. 2, it will be seen that the size of the adjustable blade turbine can be selected to have this surplus capacity and still have the part load efficiency very much better than that of the other types. Even with their surplus capacity, the physical dimensions of the adjustable blade turbine are smaller than for the fixed blade turbines. Generally speaking, therefore, it is possible to operate an adjustable unit at a somewhat higher

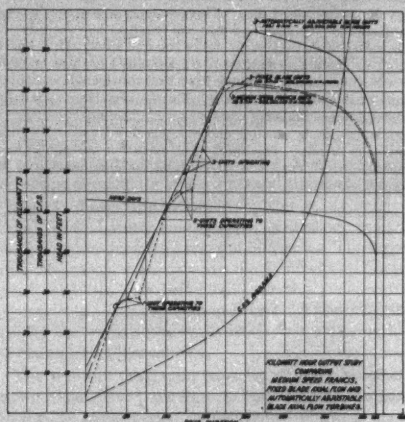

Fig. 11—Output Curves of Three-Unit Station Equipped with Different Types of Water-wheels

speed than a fixed blade unit. Referring to Fig. 11, it will be noted that for the turbines used for the study, the adjustable unit is operated at 109.1 rev. per min., the fixed blade unit at 100 rev. per min. and the medium speed Francis unit at 58 rev. per min. The generator cost for the automatically adjustable units will, therefore, be somewhat less than for the fixed blade units.

The kilowatt output has been worked up from the head and cu-ft-sec. available, starting with one unit running and changing to two units as soon as additional kilowatts can be obtained by so operating. The operation is then continued with two units until increased output can be obtained by the use of three units. In making these curves it has been assumed that each unit, if desirable from the standpoint of economy in the use of water, can be operated to maximum load and that there is no necessity for maintaining a surplus for speed regulation.

It will be seen that in periods of low flow the automatically adjustable blade unit shows increased power output over the other types. As flood conditions are approached the greater capacity from the adjustable blade units is particularly marked. This increase is largely due to the fact that the size of these units was selected to secure a curve similar to that shown by Fig. 2.

It is of particular interest to note that the adjustable units produce 11 per cent more kilowatt hours than the fixed blade units and that the Francis and the fixed units have about the same yearly output.

If it is necessary to maintain a certain operating reserve, the adjustable blade units will prove to be even more advantageous than shown, because it will be necessary to operate the Francis and fixed blade units at even lower loads and, therefore, at lower efficiencies, for part of the low flow period.

The head and cu-ft-sec. duration curves are different for each development and, therefore, it is necessary to make a special study for each case. The illustration here offered shows a material advantage for hydroelectric units with automatically adjustable blade axial flow turbines, and it is believed that for most plants having a head of 70 ft. or under a comparison will show a similar increase in output.

Discussion

R. E. B. Sharpe: The advantages of the Kaplan type of turbine may be divided into two distinct headings:

First, in the higher part load efficiencies secured, and second, in the increased power possible under low heads and high-flow conditions often obtaining with consequent increased minimum capacity.

Referring to Fig. 4, the authors are somewhat conservative in the approximate maximum horsepower value as limited by physical dimensions. For the Francis installations for heads between 30 and 50 ft., it is believed that no difficulties would be encountered in constructing turbines of larger dimensions and greater power than indicated. For a head of 50 ft., for instance, a limiting power of 32,000 hp. at a speed of 67 rev. per min. is shown. The dimensions of the runner for this turbine correspond very closely with those of turbines actually being constructed both in this country and Canada. It is believed that for these conditions the maximum unit capacity may safely be increased to as much as 40,000 to 42,000 hp.

For the axial flow turbines, including both the fixed blade propeller runners and the movable blade Kaplan runners, it is believed that even a greater increase in maximum values is allowable. The maximum runner diameters which the authors have allowed for are apparently in the neighborhood of 290 in. The writer understands that Kaplan turbines are now being constructed for the Soviet government which will have a runner diameter of 30 ft. or 360 in. This information is rather meager, however, and perhaps the authors have more definite information as to this installation.

In classifying three factors as being Kaplan contributions, the authors, I think, ignore significant American contributions in this field. In the first two factors set forth, the work of Messrs. Moody and Nagler should be recognized. Particularly in connection with the second factor should the early work of Moody be mentioned. The Moody idea is the "double guid-

ance" feature with very liberal blade area as was exemplified in the Manitoba installation. This installation is recognized both in Europe and in this country as being the first actual proof that the propeller type of turbine can successfully be applied to heads as high as 56 ft.

The third factor, namely the adjustable runner vanes, is a Kaplan contribution.

Notwithstanding the Kaplan advantages, I cannot agree that, "for every installation the increased earnings will more han amply compensate for the greater investment involved." For instance, consider the St. Lawrence River developments involving a large amount of power with necessarily a large amount of units and with more or less steady flow and correspondingly small variation in head. Under such circumstances the advantage of the Kaplan type of turbine as compared to the fixed vane propeller type is not apparent. The steady flow and head conditions tend to wipe out the advantages of the Kaplan turbine and the lower costs and greater simplicity of the fixed vane type of propeller installation would, in all probability, result in the installation of that type of unit. The fixed vane type of turbine has also some advantage over the Kaplan type of turbine, in the matter of runaway speed, as a higher runaway speed is possible with the Kaplan type than with the fixed vane propeller type, with consequently a differential in generator costs.

G. A. Jessop and C. A. Powel: Mr. R. E. B. Sharpe's interesting discussion brings out several valuable points. In making up Fig. 4, the authors, as stated in the paper, intended to indicate the approximate limits of capacity. It is true that it is possible to exceed the limits as set. It is believed, however, that when going beyond the chart limits, hydroelectric engineers should consult the generator and turbine manufacturers before making definite plans or estimates. This will eliminate the danger of getting into a field of special design and development, thus increasing the costs beyond what might normally be expected.

It can also be stated that for the high-speed axial flow type of turbine, the specific speed as given by the chart may sometimes be somewhat increased. This would apply to plants where efficiency is not of paramount importance, but where first cost is the determining factor. In general, if the specific speeds as given are exceeded, it will be necessary to set the turbine somewhat lower than usual with respect to tailwater. It is recommended, however, that if it is considered desirable to use excessive specific speeds, the manufacturer be consulted and the entire setting be studied very carefully.

Referring to the economics of a Kaplan adjustable runner installation, as pointed out, it is necessary to make a study for each set of conditions. The cost of the entire development will be increased only a very few per cent by the use of automatically adjustable blade runners rather than fixed blade runners. It follows that a few per cent increase in kilowatt hour output will pay for the increased cost. There are only a very few developments where the head and flow are found to be so steady that adjustable blade runners are not of material advantage. For the small installation, in addition to a practically fixed head and flow, it will be necessary to have a fixed load to wipe out the advantages of the adjustable runner. For the very large installation having a large number of units, a variable load might be taken at good efficiency by starting and shutting down units as the load comes on or off. This method of operation has many disadvantages which can, to a large extent, be eliminated with the Kaplan turbine.

Trend in Design and Capacity
of Large Hydroelectric Generators

BY M. C. OLSON*
Associate, A. I. E. E.

Synopsis.—This paper describes the recent trend in design of large water-wheel driven generators. New features of mechanical construction made possible by fabricated designs are particularly emphasized, and illustrations of some recent machines are given. The 77,500-kv-a. generators for the Dnieprostroy project in Russia, which will be the world's largest hydroelectric generators, are described. In conclusion, some of the future possibilities, such as outdoor machines and multi-speed designs, are discussed.

WITHIN the last few years many changes have been introduced in the mechanical and electrical design of large hydroelectric generators. The purpose of this paper is to present a few of the salient features of some recent installations, and to indicate the trend of progress in design of this class of machines, as indicated by the experience of one manufacturer.

As interconnected systems are continually growing in size and the load carried by an individual generating unit becomes a smaller and smaller proportion of the total load, it is increasingly feasible and desirable to operate some units of a station or some stations of a system at the most efficient load. With this condition of operation with higher load factor, there is an ever increasing demand for larger units and higher efficiencies.

Fig. 1 shows the trend in maximum capacity of large, vertical water-wheel-driven generators, built by one company, during a period of 30 years. The largest units shown on this curve are the 77,500-kv-a. generators, which are now being built for the Dnieprostroy Hydroelectric Development of Russia. These machines will be the world's largest hydroelectric units, both as to rating and physical size.

The trend in maximum capacity of large horizontal water-wheel-driven generators, built by this company, during the same period as mentioned above is shown in Fig. 2. The largest unit shown on this curve is the 55,000-kv-a., 360-rev. per min., generator, now being built for the Sao Paulo Tramway Light and Power Co., Brazil. This generator will be driven by double overhung impulse water-wheels.

During the past six years, on machines above 5,000-kv-a., the horizontal water-wheel-driven generators, in kv-a., were 14 per cent of the total water-wheel-driven generators. In 1924, the horizontal water-wheel driven generators[8] were almost in the same ratio or 13.5 per cent. This indicates a fairly stable ratio.

One can appreciate the growth of water power[1] in the United States, from Fig. 3 which shows a curve of total water power developed in horsepower starting from the year 1870, up to 1930. Approximately 36 per cent of the total power generated by public utility power plants in 1929, was produced from water power.

According to the U. S. Geological Survey, water power has generated more than one-third of the total

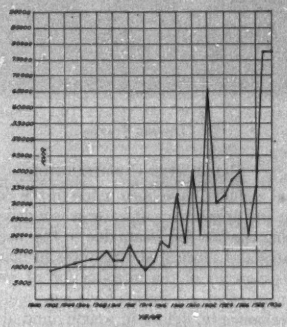

FIG. 1—TREND IN CAPACITY OF LARGE VERTICAL HYDROELECTRIC UNITS

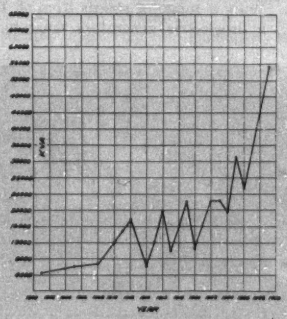

FIG. 2—TREND IN CAPACITY OF LARGE HORIZONTAL HYDROELECTRIC UNITS

electricity produced every year since 1919. In the year 1928, 40.6 per cent of the total power generated was produced by water power. The total water power in the United States available 50 per cent of the time

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Presented at the Middle Eastern District Meeting No. 2, Philadelphia, Pa., October 13-15, 1930.

8. For references see Bibliography.

is 59,000,000 hp., so that at the present time, only 23 per cent is developed. If it were feasible to develop all the water power sources of the United States, the total capacity of water-wheels that would be installed is placed at 85,000,000 hp.

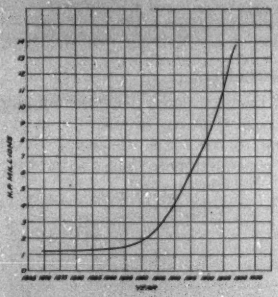

Fig. 3—Growth of Water Power Development in the United States

Fabricated Construction

Radical changes have been made in the design, construction and appearance of water-wheel-driven generators during the last few years due to the use of rolled steel plates and fabricated structures in place of castings. As a result of the introduction of welded steel plate construction, generators have been made more reliable mechanically, lighter in weight, and much more adaptable for special ventilating or constructional requirements. Also, shorter deliveries have been made possible. With a range of speeds from 60 to 720 rev. per min., hydroelectric generators must have widely

Fig. 4—Conventional Design when Castings were Used
32,500-kv-a., 100-rev. per min., 12,000-volt generators

varying designs, which can be more easily made with fabricated than with the older cast construction.

The Muscle Shoals generators Fig. 4 are typical of the latest period of cast frame design. On the Conowingo 40,000-kv-a. generators,[2] the mechanical parts except the upper bearing brackets, which consisted of a cast central hub and arms were fabricated.

One of the first large generators with completely fabricated construction was the 33,000-kv-a., 125-rev. per min., 6,600-volt generator, for the Brazilian Hydro-Electric Company. This machine illustrated in Fig. 5 has practically no castings.

The upper bearing bracket consists of a fabricated central hub to which are bolted radial steel arms. The hub consists of top and bottom plates between which

Fig. 5—33,000-Kv-A. 125-Rev. per Min. 6,600-Volt-50-Cycle Generator with Practically No Castings

Fig. 6—Stator Frame for 47,000-Kv-a., 81.8-Rev. per Min., 13,800-Volt Vertical Generator for Spiers Falls, N. Y.

are welded radial steel ribs. The fabricated arms are attached to the central hub by means of reamed bolts and also by plates at the top and bottom of each arm. A steel ventilating hood completely surrounds the frame.

Features of Stator Design

Welding and the use of rolled steel plate have progressed to such an extent that it is now possible to make almost any size of frame and by designing it in sufficient sections to ship it to the installation without exceeding permissible weights or clearances.

Fig. 6 shows the stator frame for the 47,000-kv-a.,

vertical water-wheel-driven generator for Spiers Falls. This frame has an outside diameter of approximately 37 ft. and a height of 7 ft., is made in six sections and weighs 100,000 lb. The numerous supports between the ribs make a very rigid construction. Being made of steel, it is much lighter and stronger than a cast frame. The ventilating openings in a design of this kind can be conveniently placed at any desired location. The punchings and windings will be assembled in this frame at destination.

The novel frame design of the 26,667-kv-a. generators, for the Central Maine Power Company, shown in Fig. 7 illustrates the flexibility of design characteristic of fabricated construction. The frame is of conventional circular shape when viewed from the outside, but the air gap circle is eccentric with the outside, thus giving a volute form of ventilating air space behind the laminations. The design permits the collection of air from the stator ducts around the entire periphery, and its complete discharge from one side of the frame, thus simplifying station duct work and construction. The design has obvious advantages over the use of a standard concentric frame with external eccentric ventilating hood.

Fig. 7—Eccentric Stator Frames

For 26,667-kv-a. 138½-rev. per min. 13,800-volt vertical generators

Features of Rotor Design

Fig. 8 shows the rotor spider and Fig. 9 the assembled rotor of the 33,000-kv-a. generator. The center spider is a fabricated structure consisting of I beams bolted between circular plates. The rim is made of segmental punched laminations mounted on the center spider. The segments are punched for pole-piece dovetail, rim key and for through bolts for clamping the core. At the end of the rotor-spider arms are welded vertical ribs, slotted for receiving the rim keys.

In this design, the rim is not anchored to the rotor spider, but the torque is transmitted through axial keys, which fit into notches in the rim punchings and slots on the spider arm ribs. This leaves the rim free to expand under centrifugal stresses without transferring any stress to the spider, and thus makes the design of the rim and the spider independent and their stresses definite. When rim and arms are rigidly fastened together, the radial arm tension and the peripheral rim tension divide the total stresses in inverse ratio to their elastic deformations. The uncertainties thus introduced are avoided in this new design. A similar free rim construction was also used in the Conowingo generators.[3]

Fig. 8—Rotor Spider

For 33,000-kv-a. vertical generator

Numerous square holes are punched around the periphery of the rim for the insertion of coil springs which support the field coils and keep them tight against any possibility of shrinkage of insulation.

Fig. 10 shows a 10,000-kv-a., 180-rev. per min., 13,800-volt vertical water-wheel-driven generator rotor with poles completely assembled. A new type of braking surface consisting of machined segmental plates attached to the rotor-spider arms by means of tapped bolts is clearly shown. The air or oil operated brakes act against this surface to stop the rotor in case of gate leakage.

Fig. 9—Fabricated Rotor with Laminated Rim

For 33,000-kv-a. vertical generator

A sectional view of this generator is shown in Fig. 11. It has a direct-connected exciter, two guide bearings and

a thrust bearing and is constructed with welded steel plates and structural steel parts throughout.

The largest fabricated rotor spider up to the present time is shown in Fig. 12. This rotor spider is in an inverted position in the process of machining the segmental braking plates. The outside diameter of these plates is nearly 29 ft. It is only exceeded in size by the rotors of the 77,500-kv-a. generators that are being built for the Dnieper River Project.

On account of the thrust bearing being located beneath the rotor spider, the arms are designed with a downward curve so as to make the center line of the laminated rim considerably below the center line of the hub. The web plates of these arms are flame cut to the desired shape. The top and bottom plates are formed to this shape and welded to the web plate. Ribs or stiffening plates are welded to each side of the web plate to give additional rigidity. The eleven arms of

Fig. 10—Assembled Rotor Showing Segmental Braking Plates

For 10,000-kv-a. 180-rev. per min. 13,800-volt vertical hydroelectric generator

the rotor are bolted to thick steel plates at the top and bottom of the arms by means of body bound bolts. The plates are attached to the steel spider hub by body bound stud bolts.

In order to lessen the windage on this large rotor, the top and bottom of the arms will be partially covered by a sheet steel covering. The maximum overspeed of the water-wheel on this unit is 192 per cent or 157 rev. per min.

Ventilation

Enclosed System. So many advantages are obtained by the use of the enclosed system of ventilation that there is an increasing tendency to adopt this method.

This system allows only clean, cool air to be circulated through the machine, thus insuring a clean machine and eliminating shut down for cleaning—therefore, reducing the maintenance cost. This arrangement requires the minimum amount of duct work, lessens fire hazards, and makes a more quiet machine, thus improving operation room conditions.

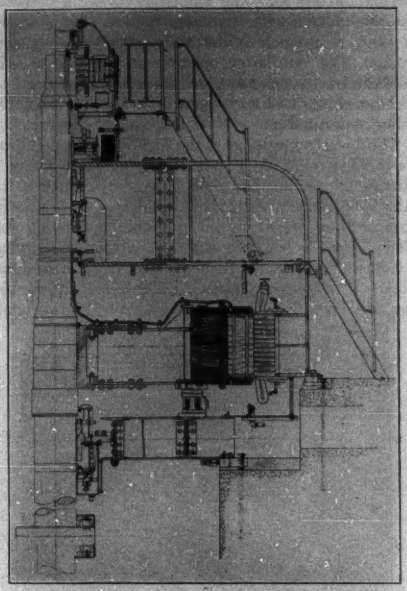

Fig. 11—Sectional View of the 10,000-Kv-a., 180-Rev. per Min., 13,800-Volt Vertical Generator

Fig. 12—Rotor Spider in an Inverted Position in the Process of Machining the Segmental Braking Plates

For the 47,000-kv-a. generator

The 47,000-kv-a. generator, shown in Fig. 13, has 12 surface air coolers located in the ventilating hood outside the stator frame. Two temperature detectors

90 deg. apart are located in the incoming air to the rotor so that its temperature can be read at the switchboard.

Fig. 14, shows the Lake Chelan plant of the Washington Water Power Company, arranged with enclosed

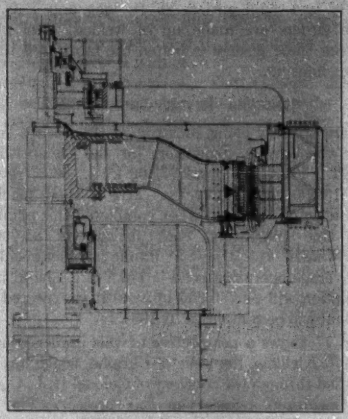

Fig. 13—Construction of 47,000-Kv-a., 13,800-Volt, 60-Cycle, 81.8-Rev. per Min., Overhung Type Vertical Water-wheel Driven Generator

system of ventilation. In this station, there are two 30,000-kv-a., 300-rev. per min., 11,000-volt generators. The eight coolers for cooling the recirculating air are located in the ventilating hood and are arranged vertically so that they may be easily installed and cleaned.

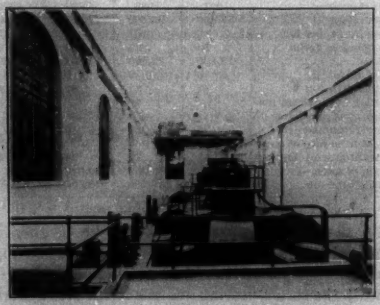

Fig. 14—Lake Chelan Plant of the Washington Water Power Co. Showing Enclosed System of Ventilation

The air comes out at eight places in the stator frame and passes circumferentially through the coolers to the pit underneath the machine and also to the space above the rotor between the bearing bracket arms.

Temperature Rise Before and After Cleaning. If machines are not kept clean, and air ducts are partly or wholly closed and other openings restricted, there certainly will be considerably higher temperatures when this condition obtains.

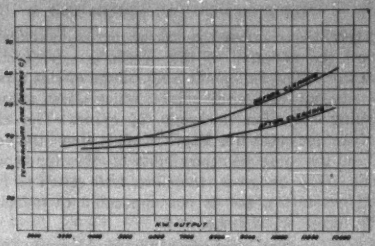

Fig. 15—Curve Showing Temperature Rise Before and After Cleaning

As a matter of interest, Fig. 15, shows curves of temperature rise before and after cleaning on an 11,000-kw., 0.9 power-factor, (12,222-kv-a.), 514-rev. per min., 6600-volt vertical water-wheel-driven generator. At full load the temperature rise is 11 deg. cent. lower after cleaning the machine.

Overspeed Requirements

Rotors of water-wheel-driven generators are designed to withstand the maximum runaway speed of the water-wheels which is usually from 80 to 100 per cent over normal speed.

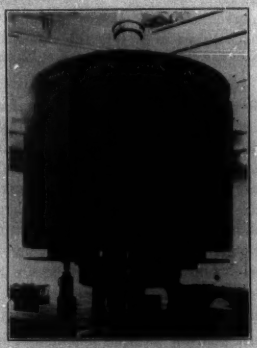

Fig. 16—Rotor of 35,000-Kv-a., 375/450-Rev. per Min. Generator in High-Speed Testing Pit

Occasionally, it is required that rotors be tested at the overspeed before leaving the factory, but with improved forms of fabricated and welded construction this requirement is being less and less encountered.

Fig. 16, shows the rotor of the 35,000-kv-a., 375/450-

rev. per min. generator, in the high-speed testing pit. It was run at 85 per cent above the 450-rev. per min., 60-cycle speed, or 833 rev. per min. for one minute. The peripheral speed of the rotor at the air gap at this overspeed is 28,500 ft. per minute.

Overhung Vertical Generators

In recent years, there has been an increasing tendency toward the adoption of the overhung type vertical generator. The outstanding advantage of this type is the reduction of over-all height, which allows a corresponding reduction in height of crane. The chief disadvantage is the free length of shaft above the bearing, and the consequent possibility of vibration at high speeds. The design is, therefore, well suited to low-speed generators, but is not applicable to machines in the highest speed range.

Some of the first machines of this kind manufactured by the company with which the author is associated, were built seven years ago. They are rated 8,200 kv-a., 150 rev. per min., 6,600 volts.

Fig. 17—Perspective of the Dnieper River Project at Kichkas Republic of Ukraine, Russia

Since that time, a large number of machines of this type has been built. The largest in physical dimensions, which is now being built for the New York Power and Light Co., Spiers Falls, N. Y., is rated 47,000 kv-a., 81.8 rev. per min., 13,800 volts. This generator will be driven by a Francis type water-wheel rated 57,000 hp. It will require 6,200 sec-ft. to operate it at full load, or the entire flow of the Hudson River, during low-water period. The net head is 81 ft.

A sectional view of this unit is shown in Fig. 13, which forms an interesting contrast with Fig. 11.

The thrust bearing for this machine is located just below the rotor spider. It is the largest combined guide and thrust bearing ever manufactured, is 84 in. outside diameter, and will carry a total weight of 1,365,000 lb., of which 815,000 lb. is the combined weight of water-wheel runner and shaft and hydraulic thrust. The guide bearing is 52 in. diameter by 26 in. in length. The stationary ring, rotating ring, and guide bearing are all split to facilitate inspection.

Thermometers with bell alarm contacts are provided for the thrust and guide bearings, and an oil level gage with an alarm contact, to indicate the height of oil in the bearing, is also furnished.

The bracket for supporting the thrust bearing consists of two main girders 26 ft. long and 7 ft. high These girders are made up of welded, steel plates. Between these girders is bolted the fabricated thrust-bearing housing.

Dnieper River Generators

There are now being constructed the world's largest water-wheel-driven generators for the Dnieper River Project. These are rated 77,500 kv-a., 88.25 rev. per min., 13,800 volts, 50 cycles. Five of these machines are now being built and when completed this plant will have nine generators, each driven by 84,000-hp. water-wheels—the largest hydraulic turbines ever built. The average operating head is 116.5 ft. The ultimate installation will be 750,000 hp. and it is expected that the plant will be completed by 1934.

Fig. 17 shows a perspective of this immense installation at Kichkas, Republic of Ukraine, which has been furnished through the courtesy of Colonel H. L. Cooper, the consulting engineer on this project. The construction of locks and the raising of the water level of the Dnieper River will make it navigable for 1,500 miles.

An idea of the proportions of these generators is given by the following comparison of their principal weights and dimensions with those of the largest American and European[4] machines heretofore built.

	Dnepro-stroy	Niagara	Spiers Falls	Ryburg-Schwor-stadt
Date completed	1931	1923	1930	1929
Kv-a. rating	77,500	65,000	47,000	35,000
Speed in rev. per min	88.2	107	81.8	75
Frequency in cycles	50	25	60	50
Voltage	13,800	12,000	13,800	10,500
Power factor	0.8	0.8	0.8	0.7
Outside diameter—ft	41	31	37	44
Over-all height from bottom of stator frame to top of machine—ft	26	27	16	..
Height from face of coupling to top of machine—ft	41	27	24	32
Weight of rotor—lb	978,000	785,000	540,000	..
Total weight—lb	1,740,000	1,517,000	976,000	1,210,000

These Dnieprostroy machines will be of the conventional type with thrust bearings located in the upper bearing brackets. There are two guide bearings, one beneath the thrust bearing and one in the lower bearing bracket.

An auxiliary generator rated 750 kv-a., 68 poles, 2300 volts, is mounted above the thrust bearing on the main generator shaft and a direct-connected exciter for the auxiliary generator is mounted above it. As on machines of smaller sizes, these units are also made of fabricated and welded parts.

The design of the upper thrust deck is similar to that shown in Fig. 5, having 12 arms, and weighing 250,000 lb. The lower bearing bracket is made of structural steel girders 36 in. high.

The rotor has twelve fabricated steel arms, each bolted between two heavy steel plates by means of tapered bolts. The steel hub for the rotor spider weighs 62,700 lb., is nearly 8 ft. in diameter over the flanges, and is over 6 ft. high.

Largest Thrust Bearings. The General Electric thrust bearings used on these machines are the largest ever designed, having an outside diameter of 92 in. and an inside diameter of 31 in. The total weight carried on a bearing is 1,900,000 lb., including the weight of water-wheel, water-wheel shaft, and hydraulic thrust.

Shaft. An idea of the size of this machine can be obtained from the dimensions of the shaft, which is 40 in. in diameter. The diameter over the coupling flange is 70 in., the total length of shaft is 36 ft., and its weight is 136,000 lb. To assure proper alinement of shafts on these large units, the water-wheel shaft is lined up on centers with the generator shaft. Coupling bolts are fitted and face of collar trued up.

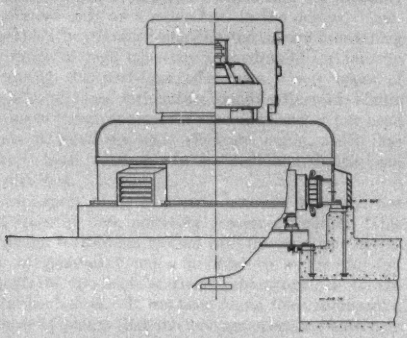

Fig. 18—Proposed Construction for an Outdoor Water-wheel Driven Generator

Future Possibilities

It is interesting to speculate on the possibilities which the future holds in store.

In respect to capacity, it is clear that the limit has by no means been reached. There is no reason why generators of 150,000-kv-a. rating, at speeds of 150 rev. per min. or less, can not be built if they are needed, so that it is extremely probable that the present trend toward larger capacities will continue.

In respect to voltage, it is feasible to build generators of normal design for 24,000 volts and higher, though there is no present demand for voltages much in excess of 15,000 volts. At the transmission voltages used for large stations, the use of transformers is imperative at the present time, and so the choice of generator voltage is based on considerations of station equipment rather than transmission. In large stations it may be found economical to generate at 22,000 volts, but it seems more probable that lower voltages will continue to be used, while double windings[5] and unit operation of transformers and generators will be brought in to reduce the switching requirements and improve stability.

The generator speeds required with present hydro-electric practise, except in very high-head developments, seldom bring in any mechanical limitations, so that the speeds of future water-wheel generators will be limited by considerations of turbine design. The recent developments of the propeller and Kaplan turbines,[6] making it possible to use higher speed wheels in low-head plants, will create a trend toward higher speeds, and will make feasible the development of very low-head plants.

An interesting possibility of the future is the use of multi-speed generators to permit efficient use of the water under varying conditions. Propeller type turbines with adjustable pitch blade, can be set to give the greatest efficiency under the actual operating conditions. By designing the generator for pole changing, however, a similar effect can be obtained electrically. Two-to-one pole changing is well known, but it gives too great a speed change for normal conditions. Recently a three-to-two speed salient-pole synchronous motor has been built which gave satisfactory performance at both speeds, without great sacrifice of characteristics at either speed. The same design can be applied to a water-wheel-driven generator, making it possible to develop power at a fixed frequency and voltage with either full or two-thirds speed.

The overhung type of vertical water-wheel-driven generators will likely increase in popularity.

Fabricated construction will continue, without doubt, until all important water-wheel generators are so made. With increasing appreciation of the flexibility of design made possible by this development, it is probable that there will be a strong tendency to adapt the generator frame design to the power house. The final result of this trend may well be the adoption of outdoor generator designs, of which several are under consideration. Fig. 18, illustrates a proposed form of outdoor construction for a water-wheel generator.

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(b) "Recent Development in Hydro-Electric Generators," F. D. Newbury, Annual Hydro-Electric Conference, March 1925.

(c) "Umbrella Type Waterwheel Generators," M. W. Smith, *Elec. Journal*, June 1929, pp. 253-254.

Automatic Operator for Economy Control

Applied to Hydroelectric Generator Stations

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Non-member

Synopsis.—*This paper deals with the great importance of economic division of load between various units in a hydroelectric plant and the proper combination of units for any given output allowing for the proper reserve required. It points out that in the past, the attempt to achieve ideal economy of operation has been handicapped by the human element. The duties of the average operating force are usually such that the constant maintaining of accurate loading schedules for best economy is a physical impossibility when the usual manual operation is employed.*

The solution for this problem has been found in the development of the "automatic operator" designed specially for each plant through the combination of the preparation of ideal schedules, of economy of operation adapted to the particular system requirements and the installation of special automatic control equipment which permits the rigid enforcement of these ideal schedules.

The actual application of the automatic operator and the results obtained in the Norwood Plant of the Carolina Power & Light Company and the Morony Development of the Montana Power Company are fully described.

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THE improvements in hydraulic turbine and hydraulic plant design have gradually reached a point where further increases in maximum over-all efficiency can be of very small degree and will probably be effected only as a refinement in the design. Considerable study is now being directed toward broadening the range of high efficiency of hydraulic turbines by increasing the part-load efficiencies. New types of propeller turbines with automatically adjustable blades are one of the great steps forward and bring to low-head plants a relatively flat efficiency curve which has hitherto been found only in the medium and high-head installations.

One aspect of hydroelectric practise, however, has just recently been receiving greater attention. This particular feature has to deal with the economic operation of generating units in order to secure the full benefits of the high efficiency characteristics of the units themselves. It has been found from studies of a number of plants that the average operating efficiency rarely equals the maximum over-all efficiency of the station. In one of the recent large hydroelectric plants the over-all efficiency of the station was found from test to be approximately 90 per cent. Detailed analyses showed that the maximum as attained in daily service was only on the order of 80 per cent. The margin between these two figures was traced directly to inefficient loading of the plant and to inefficient load distribution between the various units in service.

Other factors, such as the unnecessary lowering of the maximum operating head and excessive draw-down on small ponds and storage reservoirs, greatly impair a high over-all efficiency of station operation. Attention directed to these factors indicates a very fruitful source of study with the result that improvements in operating practise can be effected and losses recovered in proportions that have hitherto been unsuspected.

Hydroelectric power plant operation, particularly when interconnected with large power systems with a fair proportion of steam generating plants available, must be considered in combination with these other plants so that the system as a whole will reflect the benefits of economies which can be made in hydroelectric plant operation.

Considering only the hydroelectric plants in this paper, the operation of such stations must be considered with reference to the capacity of the station, the stream flow available, the market for power and the operating characteristics of the system to which this power is being supplied. Assuming that the plant has already been designed and installed, the general problems of operation in so far as they are considered with respect to economy, may be classified as follows:

1. Economic load distribution between units for any given station load and combination of generating units.
2. The economic output of the station corresponding to the points of maximum efficiency for the various combination of units.
3. The maintenance of the economic number of units or combination of units in service for any given load on the station.
4. The maintenance of maximum operating head and conservation of flow.
5. The control of pond levels with respect to flow when surplus water is available.

These five classifications deal principally with the hydraulic characteristics of the plant itself. Two other factors, dealing with the relation of this particular plant to system operation, are as follows:

6. The maintenance of sufficient system reserve capacity to provide for system requirements, to be operated in parallel with the system, and carried at a minimum cost. This may be divided into two types; reserve for immediate availability and reserve for delayed availability.
7. The maintenance of a fixed draft of water from storage reservoirs or through run-of-river plants, to provide for the proper water balance on a given stream

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where more than one station is installed, in order to provide for plants lower down on the river or to absorb the discharge from plants above the one under consideration.

There are two other considerations which may be classed as those which affect the type of service rendered by the power company, namely:

8. Automatic frequency control to maintain constant frequency with the possible refinement of maintaining constant average frequency where electric timekeeping is a factor.

9. Automatic power-factor correction to improve line and distribution voltages and power factor within the limits of the machine capacities and by utilizing from time to time surplus machines as synchronous condensers.

Each of these items represents a field of study in itself and with the proper coordination of these elements, or as many as would be applicable or desirable in a given plant, the over-all operating efficiency of any given station can usually be improved and in addition, reflect an improvement in the over-all operation of the system.

Practically all of the considerations as outlined above have been given considerable study as individual problems and as such are fairly well known and fairly well appreciated by designing and operating engineers. A number of systems set very high standards of operation by preparing economic loading schedules and endeavoring to have the plants operated in accordance with these schedules. There has been a number of articles published in the technical press, the most comprehensive of which was that presented by Mr. Frank H. Rogers and Mr. Lewis F. Moody, at the Third Hydroelectric Conference held in 1925, on the subject of "Interrelation of Operation and Design of Hydraulic Turbines." This paper deals principally with the first problem, namely, the economic load distribution between units, but other elements are considered including the design factor of determining the proper combination of units to install in a given plant, based upon the normal river flow and the load factor under which the plant would operate.

System Analyses

A brief outline of the scope of an analysis of operating economy on any generating system will be of interest. The first step in any comprehensive study of system and plant economy should be an accurate field test of each type of unit in service in a particular plant and preferably, each unit even though they are of the same type. These tests establish the characteristics of the machines under actual service conditions and very often bring to light unsuspected losses resulting from many different causes. An entire paper could be written dealing with this subject alone. Greater utilization of field testing results in improved standards of operation and increased efficiency of utilization of the plant itself. Field tests today can be made very cheaply and simply by either the Allen method or Gibson method in practically all plants, and where these methods can not be applied, it is usually possible to employ current meters with more or less accuracy, depending upon the type of meter used and the methods employed in the flow measurement.

Assuming then, that the first step has been taken, namely, the accurate determination of the unit operating characteristics; schedules of economic load distribution between the units can be prepared, which determine the proper loading of the units and the proper combination of units for any given output on the generating station. With existing plants where operating records are available, it is usually possible with a brief analysis of the operating records taken from the daily log sheets, to determine the relative operating efficiency of the station.

In cases where the accurate test data are not available, it is possible that a combination of the manufacturer's test curves and the manufacturer's expected curves combined with power-gate opening tests on the actual installation will provide sufficient information to prepare operating charts for units having similar characteristics.

For units of dissimilar characteristics, which of course include units of different specific speeds and capacities, and occasionally, units having the same capacities and speeds, but built by different manufacturers, it is necessary to make field efficiency tests, since the information available by approximate methods is not sufficiently accurate to determine the relative loading of these dissimilar units.

The over-all performance of the station is the efficiency relation which is of most value for power plant operation. The combined efficiency of the turbine and generator is not sufficient for this work since it does not include penstock losses, intake losses and other similar factors which have a decided bearing on the station performance. Allowances should be made for excitation and for auxiliary power so that a complete operating efficiency is represented.

With the station and unit characteristics determined, an over-all schedule for operation can be prepared which, for each load on the station, will specify the number of units required to carry the load; the proper combination of units which should be in operation if they are of unlike characteristics; the proper loading for each unit in order to maintain the best over-all efficiency.

Based upon an estimated percentage of adherence to this schedule, the question of load factor, load demand and the rate of draw-down of the forebay can be studied, in their effect on the maintenance of the maximum head. In cases where appreciable pondage or storage is available, a nice balance can be worked out between the permissible rate of draw-down during the demand hours and the building up of the head water level at night.

With low and medium head plants, particularly those which form a series of development without having the head overlap between plants, it is of great importance to

maintain the maximum operating head and prevent the loss in output and efficiency. In large systems where the output of any single plant does not form a large proportion of the total capacity, the operation of the individual plants are subject to a fair amount of flexibility which will permit the run-of-river plants to operate at maximum efficiency and also to maintain maximum head. In low-head plants, particularly those with heads below 30 ft., a very slight change in level in the forebay or the tailrace results in an appreciable loss in power. Operation at an inefficient point increases the draft on the reservoir and also discharges a surplus of flow downstream which, in turn, increases the tailwater level and thus doubly decreases the head and still further aggravates the losses incurred. It may be seen, therefore, that the closest attention should be directed toward efficient operation in low-head plants, considering the three factors involved namely, the economic load distribution between units, the economic output of the station and the maintenance of the maximum operating head.

In such plants still another factor is involved in periods of excess flow namely, the controlling of the forebay level through the spillway gates so that the maximum head is obtained during the high water season particularly in the intermediate zones where the excess flow has not reached the proportions of a flood. In such times the operation of the plant should still give due consideration to efficiency and should draw as much flow as possible through the units on the basis determined by the economics of operation of the particular units involved. It may even be desirable to spill a certain amount of water over the dam rather than operate the machines at full gate. This would be true in the case where the discharge from the plant is taken through a tailrace canal and excess flow reduces the head on the turbine and the loss in the conduit more than would be the case if the water were spilled over the dam into the main channel of the river. In certain cases, the question of the rate and time for spilling surplus water and its effect on over-all performance may result in increased output after the intermediate zone is passed.

System Reserve

The factor of system reserve capacity is one which should receive a great deal of attention and careful study to determine a reasonable margin for reserve to adequately protect the system against any shortage of power during sudden demand periods and at the same time avoid excessive waste of water due to carrying too much reserve in parallel with the system. This phase of operation introduces a still further burden on the operation staff due to the requirement of shutting down and starting up units at the proper time. It also introduces a serious responsibility on the part of the system load dispatchers, due to the necessity of maintaining sufficient capacity in service to care for the normal operating emergencies which would interfere seriously with rendering service, if such reserve capacity were not immediately available. The operation of any generating station or any generating equipment for reserve purposes requires the intimate knowledge gained from contact with the particular system in determining the proper amount and location of the system reserve.

Once these general requirements are established, the reserve can be allocated to plants best adapted to carry it and further studies made to decide the most economical manner in which this reserve can be maintained. All of these studies would usually divide the reserve into two classes, namely, that for immediate availability and second, for so-called delayed availability. Studies can be made of actual cost of carrying reserve capacity in any one plant or with any combination of units. On the same basis additional studies will indicate the desirability of operating units as synchronous condensers or merely as reserve capacity floating on the line. At most plants some saving can be secured by motoring the units from the line with the wheel gates tightly closed and the runner vented so that it is operating in air rather than in water. The difference between the cost of maintaining the reserve by these two methods and of having surplus units carrying light loads represents a very marked saving which can be effected if the element of availability can be secured.

The last element of economy to be considered is the relation of the output of one hydroelectric station with respect to other plants on the same stream, which is intimately tied up with local conditions, such as the reservoir capacity at the plant furthest up the stream, the distances between plants, the type and amount of pondage available and the characteristics of the stations themselves, with particular reference to the load factors for which they were designed and their relative discharge capacities. Each operating economy has its own peculiar problems affected by such local conditions and except for the value as relative information, the solution of this operating problem on any one system usually is not applicable directly to other systems.

It is of course self-evident that all of these factors have been given and are still being given a great deal of attention by the operating companies and by the operating engineers. The one element, however, which has seriously hampered the achievement of ideal economy of operation has been the necessity of depending entirely upon the human element.

The tendency to reduce operating forces to a minimum in order to make corresponding savings in operating costs, has greatly increased the duties and the responsibilities of the individual operators with the result that the operator's attention has been forcibly directed to the prime requirement of maintaining continuous service and the question of economy has been made a second consideration. The burden of maintaining accurate loading schedules and in fact, of giving careful attention to each of the factors outlined at the beginning of this article, will pass beyond the realm of possibility for manual operation unless con-

siderable increases in the operating staffs are made, combined with rigid supervision.

Automatic Control of Economic Load Division

With the development of automatic control equipment the whole aspect of these operating problems changes, since the human element no longer becomes the limiting factor. The introduction of the so-called "automatic operator," the development of which has been sponsored by I. P. Morris & De La Vergne, Inc., represents the application of a new principle to operating practise. By analyzing the economies of operation, preparing ideal schedules adapted to the particular system requirements, the application of specially designed automatic control equipment permits the rigid enforcement of these ideal schedules and eliminates the losses which result from the inability of operators to follow these schedules continuously.

One of the first opportunities to apply this new principle of operation was found at the Norwood plant of the Carolina Power & Light Company. Descriptions of this plant's operation have recently been published.[1] The problem presented in this case was somewhat different from the usual operating problem in that the three units in the station with an aggregate capacity of of 87,000 hp., consisted of two high-speed, large capacity units with relatively steep efficiency curves, and one unit of the slow-speed type with somewhat

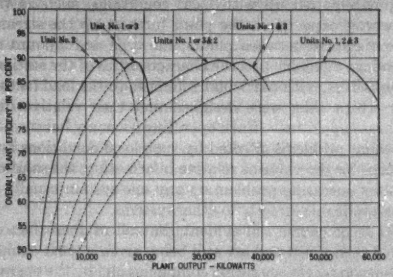

Fig. 1—Over-All Efficiency Curve. Norwood Development

smaller capacity, but having a relatively flat efficiency curve when compared with the other two units. By installing units of these different capacities, it was possible to secure a higher average efficiency for the plant over its entire range of output than would be possible if all three units were of the same high-speed characteristics.

The economic loading schedules for the proper distribution of load between units to maintain the maximum combined efficiency indicated a rather complex problem of operation. It was easily demonstrated that a slight departure from the proper loading schedule would result in an appreciable loss in efficiency. This plant therefore represented an ideal application for automatic load control equipment and after intensive studies and investigations upon the part of the staff of I. P. Morris & De La Vergne, Inc., it was felt that the problem could be solved very simply through the application of certain existing instruments which heretofore had not been used for this purpose but which could, with minor changes be adapted to fulfil the requirements at Norwood.

The equipment as actually installed, consists principally of a multiple bridge circuit, one side of each bridge consisting of a manually adjusted standard potential

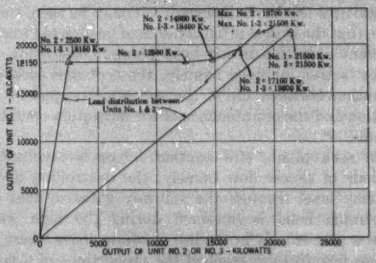

Fig. 2—Curves Showing Ideal Loading Schedule. Norwood Development

and the other side, a potential in proportion to the unit output. By basing the standard potential relations upon the ideal schedule for load division between units, the balance would be secured, at which point the load on the various individual units would be brought to the standard as manually set, which in turn was in accordance with the desired load distribution between units. By this relatively simple scheme, the loading schedules could be maintained through the operation of a single dial and the economic load distribution effected between any combination of units in service. A number of additional attachments were incorporated in the control in order to provide more flexible operation, but these did not introduce any change in the basic principle upon which the control is designed.

Fig. 1, showing the over-all efficiency curve for the Norwood Development and Fig. 2, showing the ideal loading schedule for one unit with respect to the other, illustrate what can be accomplished by the application of automatic control to relieve the operators of the burdensome routine and at the same time, maintain the highest possible operating efficiencies on the station. Fig. 3 is the same as Fig. 1 except that points of actual operation, as taken from the log sheets, have been plotted. The circles show the results of manual operation while the triangles represent the operation after the automatic control was in service. The general curve drawn through the points of efficiency on manual

1. "Automatic Operator a Success," F. M. Nash, *Electrical World*, Aug. 25, 1930.

control, shows the trend and is based on the results obtained from a study of the integrated averages of kilowatt hours generated before and after the control was installed.

The approximate figures indicate that the production efficiency of the Norwood plant, in so far as its relation with the ideal operating curve is concerned, was approximately 90 per cent on manual control. After the development of the automatic operator and its installation at Norwood, the losses were materially reduced and

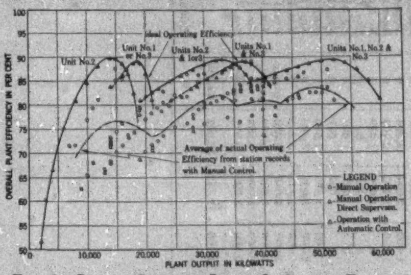

Fig. 3—Curves Showing Ideal Operating Efficiency as Compared with Actual Operating Efficiency Under Manual Control and Automatic Control

the average operating efficiency was in excess of 96 per cent and was later increased to approximately 98 per cent for appreciable periods of time. The difference between the ideal operation, which would be represented as operation along the envelope curve in Fig. 1 and Fig. 3, and the actual results secured at Norwood, include all losses due to having the incorrect number of units in service as well as the losses introduced by maintaining system reserve capacity or minimum loads on the station for reserve purposes.

The actual loss incurred by incorrect distribution of the load between the units which were in service was found to be materially less than 1 per cent, and thus the operating efficiency of the Norwood Station, in so far as load division was concerned, has been consistently better than 99 per cent.

These figures have been taken from actual station records which are prepared daily in the general form as shown in Tables I and II. A detailed analysis of these efficiency log sheets enables the operating engineers to determine the reasons for production losses and permits them to direct attention upon the elimination of these losses with accurate records upon which to base their analyses.

The efficiency logs as shown in Tables I and II list two losses, namely, the distribution loss and the unit loss. The distribution loss concerns only the question of economic load distribution between units and the economic combination of units in service. The unit loss represents the loss incurred due to the operation at points other than at the maximum efficiency of the unit or combination of units. Thus, if the units are operated with the proper load distribution and are also operated at some point of maximum efficiency on the envelope curve in Fig. 1, the distribution loss would be zero, and the unit loss would also be zero, giving a production efficiency of 100 per cent. In cases, however, where the load dispatched to Norwood is at some other point on the over-all efficiency curve, then even if the load is economically distributed between the units in service and the corresponding distribution loss is zero, the unit loss will be introduced due to operating the plant at a lower efficiency than its maximum. The difference in discharge or possible output between the actual point of operation and point of maximum efficiency has been established as this unit loss. In each of the log sheets, both these losses are given and a notation is made of the distribution efficiency as well as the unit efficiency, the product of the two being the production efficiency.

In order to simplify the work of the load dispatcher, the curve shown as Fig. 4 was prepared representing the station output with respect to station discharge, upon which has been superimposed radial lines representing uniform increments of the kilowatts obtainable for each cu-ft-per-sec. of water through the station under the rated head at which the plant normally operates. These radial lines intersect the power-discharge curves at various points and both the load dispatchers and station operators can see at a glance the zones of desirable operation, and where system conditions permit, the

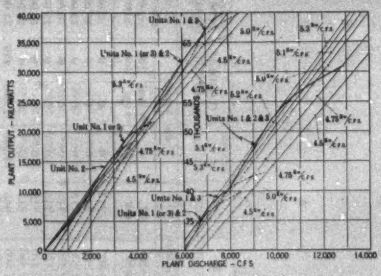

Fig. 4—Curves Showing Station Output vs. Station Discharge—Norwood Development

Lines of kilowatts per second-feet included

maximum efficiency loads are dispatched to the Norwood Station.

By means of these curves and log sheets, when combined with the assurance that whatever load is dispatched to the Norwood Station will be carried in the most economic manner possible, the system operators and load dispatcher were provided with a more efficient tool with which to work, with the result that the operating efficiencies at Norwood have reflected exceptionally large savings to the power company over and above the

TABLE I
CAROLINA POWER & LIGHT COMPANY
DAILY LOAD EFFICIENCY DATA

Plant Norwood — 24 hours ending August 19, 1929

Time	No. 1 unit			No. 2 unit			No. 3 unit			Total kw. generated	Total water used	Possible kw.	Production eff. kw. losses			By-pass discharge sec. ft.	Remarks
	Kw.	Water used	Eff. loss	Kw.	Water used	Eff. loss	Kw	Water used	Eff. loss				distr.	unit	total		
12 m.n.	0	0	0	12,000	2,300	213	0	0	0	12,000	2,300	12,000	0	213	213	0	No. 2 unit—night load—LD
1 a.m.	0	0	0	12,000	2,300	213	0	0	0	12,000	2,300	12,000	0	213	213	0	No. 2 unit—night load—LD
2 a.m.	0	0	0	12,000	2,300	213	0	0	0	12,000	2,300	12,000	0	213	213	3,060	No. 2 unit—night load—LD
3 a.m.	0	0	0	13,000	2,470	115	0	0	0	13,000	2,470	13,000	0	115	115	3,060	No. 2 unit—night load—LD
4 a.m.	0	0	0	13,000	2,470	115	0	0	0	13,000	2,470	13,000	0	115	115	3,060	No. 2 unit—night load—LD
5 a.m.	0	0	0	13,000	2,470	115	0	0	0	13,000	2,470	13,000	0	115	115	3,060	No. 2 unit—night load—LD
6 a.m.	0	0	0	10,000	1,980	513	0	0	0	10,000	1,980	10,000	0	513	513	0	6:10 a.m. No. 1 + 2 units on increasing load
7 a.m.	13,000	2,640	939	7,000	1,500	965	13,000	2,640	939	33,000	6,780	35,800	2,800	2,843	5,643	0	6:30 a. m. No. 3 on reserve for day load
8 a.m.	16,500	3,220	501	7,000	1,500	965	16,500	3,220	501	40,000	7,940	40,600	600	1,967	2,567	0	Day load increasing
9 a.m.	18,000	3,420	57	8,000	1,670	967	18,000	3,420	57	44,000	8,510	44,000	0	981	981	0	Day load normal
10 a.m.	18,000	3,420	57	9,000	1,830	717	18,000	3,420	57	45,000	8,670	45,000	0	831	831	0	Day load normal
11 a.m.	18,000	3,420	57	12,000	2,300	213	18,000	3,420	57	48,000	9,140	48,000	0	327	327	0	Day load normal
12 m.	11,000	2,330	1,302	0	0	0	11,000	2,330	1,302	22,000	4,660	23,400	1,400	2,604	4,004	0	Noon drop No. 2 retired for economy
1 p.m.	16,500	3,220	501	0	0	0	16,500	3,220	501	33,000	6,440	34,100	1,100	1,002	2,102	0	Reserve for increasing load
2 p.m.	18,000	3,420	57	7,000	1,500	965	18,000	3,420	57	43,000	8,340	43,000	0	1,079	1,079	0	No. 2 on for additional reserve
3 p.m.	18,500	3,500	0	15,000	2,820	0	18,500	3,500	0	52,000	9,820	52,000	0	0	0	0	Nos. 1, 2 and 3 on max. produc. effi.
4 p.m.	18,000	3,420	57	10,000	1,980	513	18,000	3,420	57	46,000	8,820	46,000	0	627	627	0	No. 2 held for system reserve
5 p.m.	18,000	3,420	57	10,000	1,980	513	18,000	3,420	57	46,000	8,820	46,000	0	627	627	0	No. 2 held for system reserve
6 p.m.	17,000	3,300	424	7,000	1,500	965	0	0	0	24,000	4,800	24,200	200	1,389	1,589	0	No. 3 retired for economy
7 p.m.	7,000	1,730	2,134	7,000	1,500	965	0	0	0	14,000	3,230	16,800	2,800	3,099	5,899	0	No. 1 and 2 on night load
8 p.m.	12,000	2,490	1,147	7,000	1,500	965	0	0	0	19,000	3,910	20,200	1,200	2,112	3,312	0	Nos. 1 and 2 on night load
9 p.m.	9,000	2,030	1,718	7,000	1,500	965	0	0	0	16,000	3,530	18,600	2,600	2,683	5,283	0	Nos. 1 and 2 system reserve stormy
10 p.m.	7,000	1,730	2,134	7,000	1,500	965	0	0	0	14,000	3,230	16,800	2,800	3,099	5,899	0	Nos. 1 and 2—system reserve—stormy
11 p.m.	7,000	1,730	2,134	7,000	1,500	965	0	0	0	14,000	3,230	16,800	2,800	3,099	5,899	0	Nos. 1 and 2—system reserve—stormy
12 p.m.	7,000	1,730	2,134	7,000	1,500	965	0	0	0	14,000	3,230	16,800	2,800	3,099	5,899	0	Nos. 1 and 2—system reserve—stormy
Total kw.	249,500	50,170	15,410	207,000	41,570	13,757	183,500	35,430	3,585	640,000	127,170	661,100	21,100	32,752	53,852	12,240	
Total kw-h.	250,000			220,000			180,000			650,000							Production effici. = 92.3

TABLE II
CAROLINA POWER & LIGHT COMPANY
DAILY LOAD EFFICIENCY DATA

Plant Norwood — 24 hours ending August 6, 1929

Time	No. 1 unit			No. 2 unit			No. 3 unit			Totalkw. generated	Total water used	Possible kw.	Production eff. kw. losses			By-pass discharge sec. ft.	Remarks
	Kw.	Water used	Eff. loss	Kw.	Water used	Eff. loss	Kw.	Water used	Eff loss				distr.	unit	total		
12 m.n.	0	0	0	17,000	3,310	576	0	0	0	17,000	3,310	17,000	0	576	576	0	Light load most eff. on No. 2
1 a.m.	0	0	0	15,000	2,820	0	0	0	0	15,000	2,820	15,000	0	0	0	0	Light load most eff. on No. 2
2 a.m.	0	0	0	16,000	3,030	89	0	0	0	16,000	3,030	16,000	0	89	89	0	Light load most eff. on No. 2
3 a.m.	0	0	0	16,000	3,030	89	0	0	0	16,000	3,030	16,000	0	89	89	0	Light load most eff. on No. 2
4 a.m.	0	0	0	16,000	3,030	89	0	0	0	16,000	3,030	16,000	0	89	89	0	Light load most eff. on No. 2
5 a.m.	0	0	0	16,000	3,030	89	0	0	0	16,000	3,030	16,000	0	89	89	0	Light load most eff. on No. 2
6 a.m.	0	0	0	15,000	2,820	0	0	0	0	15,000	2,820	15,000	0	0	0	0	Light load most eff. on No. 2
7 a.m.	18,500	3,500	0	11,000	2,150	416	18,500	3,500	0	48,000	9,150	48,000	0	416	416	0	Increasing load Nos. 1 and 3 on max. effi.
8 a.m.	18,500	3,500	0	13,000	2,470	115	18,500	3,500	0	50,000	9,470	50,000	0	115	115	0	Increasing load Nos. 1 and 3 on max. effi.
9 a.m.	19,000	3,600	0	15,000	2,820	0	19,000	3,600	0	53,000	10,020	53,000	0	0	0	0	100 % product. effi. on all units
10 a.m.	19,000	3,600	0	15,000	2,820	0	19,000	3,600	0	53,000	10,020	53,000	0	0	0	0	100 % product. effi. on all units
11 a.m.	19,000	3,600	0	15,000	2,820	0	19,000	3,600	0	53,000	10,020	53,000	0	0	0	0	100 % product. effi. on all units
12 m.	19,000	3,600	0	15,000	2,820	0	19,000	3,600	0	53,000	10,020	53,000	0	0	0	0	100 % product. effi. on all units
1 p.m.	0	0	0	15,000	2,820	0	19,000	3,600	0	34,000	6,420	34,000	0	0	0	0	Noon drop. No. 1 retired for economy
2 p.m.	19,000	3,600	0	15,000	2,820	0	19,000	3,600	0	53,000	10,020	53,000	0	0	0	0	100 % product effi. on all units
3 p.m.	19,000	3,600	0	15,000	2,820	0	19,000	3,600	0	53,000	10,020	53,000	0	0	0	0	100 % product effi. on all units
4 p.m.	19,000	3,600	0	15,000	c 2,820	0	19,000	3,600	0	53,000	10,020	53,000	0	0	0	0	100 % product effi. on all units
5 p.m.	19,000	3,600	0	0	0	0	19,000	3,600	0	38,000	7,200	38,000	0	0	0	0	100 % product effi. on units Nos. 1 and 3
6 p.m.	19,000	3,600	0	0	0	0	19,000	3,600	0	38,000	7,200	38,000	0	0	0	0	100 % product effi. on units Nos. 1 and 3
7 p.m.	15,000	2,980	734	0	0	0	0	0	0	15,000	2,980	15,800	800	734	1,534	0	Inefficient load on
8 p.m.	19,000	3,600	0	0	0	0	0	0	0	19,000	3,600	19,000	0	0	0	0	Max. effici. load on No. 1
9 p.m.	19,000	3,600	0	15,000	2,820	0	0	0	0	34,000	6,420	34,000	0	0	0	0	Max. effici. night load on Nos. 1 and 2
10 p.m.	0	0	0	15,000	2,820	0	0	0	0	15,000	2,820	15,000	0	0	0	0	Load dropping—on No. 2 for effici.
11 p.m.	0	0	0	12,000	2,300	213	0	0	0	12,000	2,300	12,000	0	213	213	0	Load dropping—most effici. on No. 2
12 p.m.	0	0	0	10,000	1,980	513	0	0	0	10,000	1,880	10,000	0	513	513	0	Load dropping most effici. on No. 2
Total kw.	261,000	49,580	734	290,000	54,860	1,613	227,000	43,000	0	778,000	147,440	778,000	800	2,347	3,147	0	Load dropping most effici. on No. 2
Total kw-h.	270,000			300,000			210,000			780,000							Production effici. = 99.6

very best that could possibly be obtained with manual operation.

Automatic Control of Economic Number of Units in Service

The automatic control of the Norwood Station was only the starting point for the application of automatic economy control to power generating stations. In each of the new plants undertaken, new problems arose which have so far been successfully cared for by the application of automatic control equipment.

One of the most interesting developments which has recently completed the first few months of successful

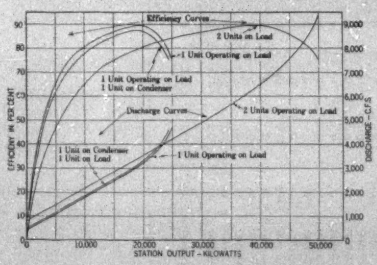

Fig. 5—Automatic Operator for Montana Power Co., Morony Development

Station curves showing performance with one and two units in operation

operation, has been the Morony Development of the Montana Power Company. In line with the general desire to improve operation, and finding that the Morony plant was best adapted for this work, automatic frequency control equipment was installed as part of the hydraulic turbine contract. This station, however, had two units of the same characteristics and it was estimated that the requirement for maintaining frequency on the Montana system would call for the continuous operation of two units through a range of output from one-half to three-quarters of their full-load rating. As the load on that system is subject to variations which are not easily predicted, the question immediately arose in regard to the loss of efficiency due to operating two units below one-half of their individual capacities where the load could be more efficiently carried on a single unit.

With the application of automatic frequency control to this plant, a further step was made in the development of automatic economy control by arranging special equipment which would automatically transfer one unit from operating on load to operation as a synchronous condenser when the point had been reached where one unit alone could carry the particular load of the station more efficiently than two units. Fig. 5 shows the curve of over-all plant efficiency with respect to plant output for both one and two units. It may be seen very readily that below an output of about 23,000 kw., it is more efficient to run a single unit than it would be to operate both units. The question immediately arose as to whether it would be desirable to signal the operator at this point and have him shut down one unit and carry the load on the remaining unit. However, as the immediate availability of this surplus unit was of great value in supplying sudden demands for load, it was found that instead of shutting down one unit, it could very conveniently be transferred automatically to synchronous condenser operation. The first unit was arranged to pick up whatever load was required as well as to supply the power required to drive the other unit as a synchronous condenser. By the use of very efficient automatic air vents, the power required to drive the unit as a synchronous condenser was held to very small limits and from actual tests, the curve shown for one unit operating on load with one unit as a condenser gives a loss of less than 2 per cent below the efficiency of one unit operating alone. Provision was also made for this reserve unit to come back on load automatically when it was required either at a very slow rate for normal service, or at a very rapid rate if it was required for emergency conditions.

Fig. 6 represents the economic analyses of this station, with the savings effected by this control plotted with respect to station output. Approximately 2,400 kw. are saved by the operation of one unit on condenser and one unit on load based upon the utilization of the same flow as would be required for two units carrying the same load. This saving in kilowatts can be translated into money, and for this purpose, an arbitrary

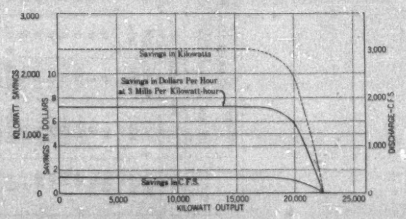

Fig. 6—Automatic Operator for Montana Power & Light Co.—Morony Development

Station curves showing savings in kilowatts, dollars and discharge against kilowatt output for automatic operation

figure of three mills per kw. hr. was used. According to the latest studies, this represents slightly over $7.00 per hr. saving with this very low power rate for each hr, that the plant is operated below an output of 18,000 kw. Since storage facilities are available, these same savings can be translated into savings in discharge, which is the third curve shown on this figure.

The possibilities of extending this type of control to other applications is virtually unlimited at present, and installations are now under way for similar equipment in plants containing as many as four units. In

this case, the savings increase materially above those shown for the Morony Development in Fig. 6, since the possibilities of saving are proportionately larger.

This feature introduces another phase of operation, namely, the economic carrying of system reserve capacity through the operation of one or more units in the station as synchronous condensers combined with automatic control equipment to make them immediately available if they are required for service either as the result of a normal increase in load occurring at the particular time, or due to system disturbances requiring additional generating capacity at once, in which case the rate at which these reserve units will pick up the load will be very much more rapid. It is thus possible to allocate the system reserve to hydroelectric plants in which surplus capacity is available and by the proper use of automatic control equipment have this reserve capacity automatically available without the necessity of any operation on the part of either the load dispatchers or of the operators themselves.

The actual performance of this automatic frequency control equipment, combined with the elements of automatic economy control to effect both the economic load distribution and the maintenance of the economic combination of units in service, has been very gratifying indeed. The control at Morony has been in service for several months and has been very effective indeed in stabilizing the frequency under rather severe service conditions. The type of load on this system fluctuates greatly, due to its inherent characteristics, and to the self-regulating characteristics of the system itself being relatively small. The automatic transfer from load to condenser and from condenser to load of the reserve unit is effected very easily and without any attention being required on the part of the operator. A slightly revised operating procedure based upon the new conditions, was inaugurated by the load dispatchers and the water balance between the various plants was found to be more easily maintained than with the previous condition of hand regulation of the speed.

Combined Operation of Series of Plants

These same principles which have been outlined in describing the application of automatic economy control to individual plants, are readily adaptable to the operation of groups of plants. The inter-relation of the loading on different stations, particularly with regard to a series of plants on a single river, has the effect of making the series of plants perform as the equivalent of a single over-all development. The actual operation of such a system when combined with other devices to make use of pondage and peak load requirements, offers a very effective solution for the difficulties in operating such stations. There are several different methods which can be employed to effect this type of operation, each of which may be more desirable, depending upon the local requirements, the distances between the plants, and the general conditions existing in the particular application under consideration.

Combined Operation of Steam and Hydroelectric Plants

The automatic economy control applied to groups of hydroelectric plants can also be applied very readily to a combination of steam and hydroelectric plants whereby the hydro plant can maintain a base load during high flow seasons, leaving the steam plant to take the fluctuations in load, while for periods where surplus water is not available, the hydroelectric plant can more readily supply the fluctuating load leaving the steam plant to operate on fixed output.

This general scheme of operation has been suggested in a number of papers, but by the use of automatic economy control it can be made a reality and most of the benefits from the combined operation of such stations can actually be secured.

The application of automatic economy control to electric power generation requires an intimate knowledge of system characteristics as well as the prime mover characteristics. The success of automatic economy control installations can only reach the highest degree through complete cooperation of the manufacturer with the operating companies and by maintaining an impartial attitude in regard to the equipment to be used, its application and its limits. The engineering of design and application must be carried out with these principles in view in order to secure the ultimate aim of this equipment, namely, the highest efficiency of power generation possible with the installed equipment.

Discussion

H. A. Von Eiff: Referring to the section covering condenser operation, it is interesting to state that we have found a large gain in economy at the Holtwood plant by operating one unit as a synchronous condenser on both the 25- and 60-cycle systems.

The Holtwood plant prior to 1924 had eight 25-cycle units, seven of which were double-runner units and one a single-runner unit. The single-runner unit was installed in 1914 and in 1921, when the flow of the Susquehanna river was at a low stage, this unit was used as a synchronous condenser solely for power-factor correction to permit the plant to shut down. Since 1921 when the flow necessitates a complete shut down this unit is used for condenser operation and has operated very satisfactorily. It requires approximately 650 kw. or 5.4 per cent of the rated capacity to operate this unit as a condenser with gates closed and vacuum broken and this loss is small compared to the gain in economy and peak service.

A study was made to determine the gain in output by operating one unit as a condenser which would be available for spare capacity and one unit carrying load when the load on the hydro plant could be carried on one unit but the capacity of two units would be needed for spare. This study showed a saving of 150 to 300 cu. ft. per sec., depending on the head and the load carried, or a gain of approximately 500 to 1,000 kw. This operation is entirely by manual control and the governor on the condenser unit is adjusted so that the unit will pick up load if the frequency drops.

In 1934 two 60-cycle single-runner units were installed in the Holtwood plant and the gain in economy from operating a 25-cycle unit as a condenser led to making a study of synchronous condenser operation on one of the two 60-cycle units for loads of one-unit capacity but requiring two units for spare capacity. The operation proved satisfactory when the tailrace elevation was below the runner and the saving from condenser operation

on these two units varied from 500 to 700 cu. ft. per sec. and from approximately 1,500 to 2,300 kw., or from 13 to 27 per cent of the load, depending on the head and load carried. The large gain in economy on these units compared to the 25-cycle units is due to the fact that they have higher specific speed, or lower part gate efficiencies. It is also interesting to note that the energy required to operate these units as condensers with gates closed and vacuum broken amounts to approximately 400 kw. or 3.3 per cent of rated capacity.

Carroll F. Merriam: A most valuable point brought out by this paper is the fact that although hydro units have approached the ultimate in maximum efficiency, it does not indicate that the ultimate in economy in the use of water has by any means been reached. It is often heard that because hydro units have now been built with efficiencies so near 100 per cent, little improvement can be expected in hydro development. The list of nine general problems included in this paper is in itself sufficient evidence that maximum efficiency is not the sole measure of the success of the hydraulic turbine designer.

In addition to these nine, there are two others which come readily to mind and which offer fruitful fields for study where favorable conditions exist. The first is the possibility of making use of flood waters by means of head increasers such as the ejector turbine, and the backwater suppresser. It is entirely possible that when the principles of the ejector are better understood there will be found many applications for these means of getting increased capacity during times of flood. Another problem which may at first appear paradoxical is the economy of spilling water over a fixed crest dam at times of insufficient flow. When conditions are particularly propitious, that is, with a sharp peak during the early hours of hydro generation, a short spillway, and especially if in order to accommodate the steam plants, the pond is drawn down weekly so that the maximum elevation is reached but once a week, there can be shown an appreciable saving by securing higher average head even though the pond is allowed to overflow a few tenths of a foot.

The importance of test data should receive emphasis since the incidental information gathered at the time of an acceptance test may easily pay for the entire cost. The curves derived from the tests on the units at the Holtwood plant of the Pennsylvania Water & Power Company, are frequently used as the basis of economic studies leading to improved operation. Attention should also be given to supplementing the official tests with comparative tests made by an index method so that the range of conditions covered by the tests in which the water measurements are carefully made may be extended to cover extraordinary values of head which could not be obtained at the time of the test itself. There are several methods that are now being developed for making comparative measurements of water, which methods can be calibrated at the time of acceptance tests, and used for many purposes afterwards by the plant operators or test department.

The author's statement that it may even be desirable to spill water over the dam rather than operate the machines at full gate, suggests that we look beyond the "station over-all performance" which the author states is "the efficiency relation which is of most value for power plant operation" and consider the economy of the project as a whole. This is particularly true where there is an inadequate tailrace. In this connection it should be kept in mind that excess water passing through a turbine not only lowers the head on the turbine itself but also that of the other turbines in the same plant. An example of this was found at Holtwood where by reducing the output of two turbines below their own points of maximum output, the send out of the station as a whole was increased about 400 kw.

In regard to maintaining maximum head on a single hydro plant feeding into a large system, it is often considered the best economy in times of insufficient flow, to give the steam station a flat schedule and take all of the swings on the hydro. Any inequality in the hydro load during the day will draw down the pond and cause the plant to suffer a consequent loss. To determine the economy of such practise involves a knowledge of steam plant conditions and the cost of carrying the swinging load by steam. This is exceedingly complicated, and although it is doubtful if it should prove economical to carry steady load on a run of river plant, nevertheless system operators should not be unmindful of the loss in economy. It is entirely possible that some compromise would be found to give the best results in the end.

An interesting phase of interconnected plant economics, not mentioned in the paper, is where two plants are close together on the same river and have overlapping head. Let us assume that the upper of the two plants has a pond about three times as large as the lower, and furthermore, that the lower has a poor tailrace whereas the upper discharges directly into the pond of the lower with very slight loss of head between the two plants. At time of low flow there are three distinct methods of operation, to say nothing of many combinations which can be overlooked for the present. The first is to carry base load on the upper plant which means that the average forebay will be maintained at as high level as possible. The second is that both plants would share the load in such a manner that both discharge the same amount of water at all times. This means that the lower pond may be maintained always full and that the loss in head between the plants will be a minimum. The third method is to have the lower plant carry the base in which case the average tailwater will be lowered on account of a more steady discharge during the hours of hydro operation.

It is a very nice problem to determine the most economical method and although it appears from knowledge of the situation that the lower plant should carry the base, the final solution will probably be some form of compromise.

It will be noted that the estimates given of the savings realized by the use of the automatic operator are based upon hourly integrated loads. It should be pointed out that this is at least conservative since the difference between using integrated values and instantaneous values taken at random would tend to make the savings appear to be *less* than they really are. The reason is that any variation from the average values during the hour under manual control will tend to give actually less efficiency than that corresponding to the average loading for the hour. Under automatic control the efficiency is maintained at the theoretical maximum by supervision every two seconds. On the other hand, it does not stand to reason because Norwood showed a difference of about 10 per cent between the efficiency under manual operation and the theoretical maximum, that this is typical of hydro plants in general. The conditions at this plant, having units of different characteristics, are such that it would be particularly hard for the operators to appreciate the value of holding their units to a fixed loading schedule. In a plant where all the units are alike it is much easier for the operators to be educated to maintain equal loadings on all units. Although it is difficult to train operators to pay strict attention to unit loading, it is hard to believe that the resulting loss will ordinarily amount to as much as ten per cent.

Reference is made to the fundamental principles of economic loading as presented in a paper by Rogers and Moody at the Third Hydroelectric Conference in 1925, but it is not stated clearly how these principles have been applied in the case of the problem at hand. A very simple method of constructing economic loading schedules has been employed by the Pennsylvania Water & Power Company and it would probably be of interest to the hydroelectric industry to know how easily such schedules can be laid out from given efficiency curves.

The author has also omitted to mention the course of action to be taken when the variation in head on the plant is sufficient to be considered.

It is undoubtedly a considerable advance in power plant design to have units of dissimilar characteristics, but considerable care must be taken in selecting the units in order to properly balance all of the factors and thus avoid over emphasis upon any one element. For example, if high combined efficiency were the only consideration, it would have been better at Norwood to have made No. 2 unit somewhat smaller. It can be seen by reference to Fig. 1 that the "Unit 2" curve would have been shifted somewhat to the left, "Unit No. 1 or 3" curve would have remained where it is, but "Unit No. 1 or 3 with Unit No. 2" curve would also be shifted to the left to fill in the deep gap, or inefficient zone, between. On the other hand, the cost per horsepower would certainly have been increased and the capacity of the plant at time of high water would have been reduced.

Damper Windings for Water-Wheel Generators

BY C. F. WAGNER*
Member, A. I. E. E.

Synopsis.—The severity of unsymmetrical system faults is affected by the negative sequence impedance of the connected machines. These impedances in turn depend largely upon the character of damper windings. The paper discusses the effect of amortisseur or damper windings upon both the real and reactive components of the negative sequence impedance. Machines without damper windings possess the highest negative sequence reactance and from this viewpoint are the most desirable, but machines with high-resistance damper windings possess the highest negative sequence resistance. Calculations show that combining the effect of resistance and reactance, the high-resistance damper is more desirable.

In the event of system oscillations low-resistance copper damper windings produce the greatest damping of the mechanical movement. However, this effect is unimportant during and following a system fault except in the exceptional, rather rare case in which the system is so constituted that pull-out takes place as a result of compound oscillations following a disturbance.

Consideration is also given to a special type of double deck damper winding.

* * * * *

Introduction

AMORTISSEUR or damper windings have been installed in water-wheel generators in the past only when particular requirements, such as increased starting torque for automatic operation, demanded. In general, water-wheel generators in manually controlled power stations were not equipped with damper windings. The recent emphasis placed upon stability problems and power limits has raised the question as to whether or not some form of damper winding was desirable from stability considerations alone. Some have argued that inasmuch as stability problems involve rotor oscillations and since it is known that the installation of damper windings facilitates the operation of engine-driven generators, it follows that dampers should ameliorate instability conditions. The problem cannot be reduced to such a simple simile. It is the purpose of the present article to discuss some of the factors affecting the choice of damper windings for water-wheel generators.

I. Negative Sequence Impedance

It has been shown by Evans and Wagner[1] that any unbalanced fault in a symmetrical† power system can be represented, through the application of the "method of symmetrical components," by means of a symmetrical set of impedances connected in shunt at the point of fault; thus converting the unbalanced fault to a symmetrical impedance load which is more amenable to solution. For the simple system shown in Fig. 1A consisting of a generator and a transmission line, with transformers at each end, feeding into a system, the equivalent symmetrical system for a line-to-line fault on the high tension bus at the generating end is shown in Fig. 1B. The single line diagram of Fig. 1C shows the synthesis of the shunt impedances, which are all negative sequence impedances. For the transformers and line the negative sequence impedances are the same as for the positive sequence, but for generators they are different. It follows, therefore, that by the aid of this conception and a knowledge of the negative sequence impedance of the generator, the synchronizing power between the voltages at the two ends of the system can be determined by a solution of the equivalent network. Similar relations are true for other kinds of faults. These may involve the zero sequence impedances of the system, but for delta-star transformer connections the zero sequence impedance of the generator does not enter the problem.

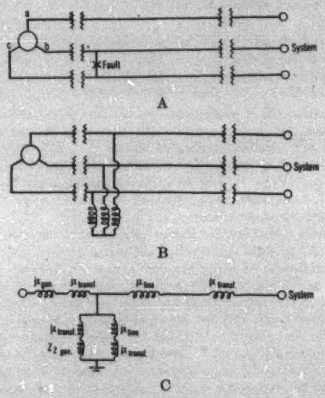

Fig. 1—Reduction of Unbalanced System Condition Caused by Line-to-Line Fault to its Equivalent Single-Line Equivalent and Showing the Synthesis of the Equivalent Shunt Impedance

A. Line-to-line fault on high-tension bus at generating end.
B. Symmetrical equivalent of (A)
C. Single line equivalent of (B)

What is the nature of this "negative sequence

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1. For references see Bibliography.

†Symmetrical is used here to describe a system that has the same characteristics and constants viewed from any phase.

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impedance?" The method of symmetrical components involves the resolution of three unsymmetrical currents or voltages into, first a set of three symmetrical currents or voltages whose instantaneous values reach their maxima in the order *a b c* (positive sequence); second, a set of symmetrical currents or voltages whose values reach a maximum in the order *a c b* (negative sequence); and third, a set of three currents or voltages which are in phase (zero sequence). Now while all of these vectors rotate in the same direction, when applied to a phase wound machine, the positive sequence produces a field rotating synchronously in the same direction as the rotor; the negative sequence a field rotating synchronously in the opposite direction, and the zero sequence a more complicated field, which is determined by differential leakages since the fundamental space distribution of the three phases just cancel one another. The negative sequence impedance of a machine is the impedance offered to the flow of negative sequence current, that is, it is the negative sequence voltage across the machine when one unit

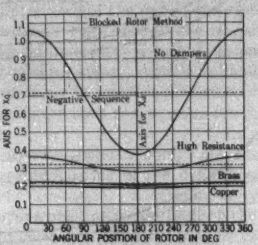

FIG. 2—RELATION BETWEEN SUBTRANSIENT AND NEGATIVE SEQUENCE REACTANCE

of negative sequence current flows through the machine. Distortion arising from saliency effects results in somewhat different values depending upon whether sinusoidal voltage is applied or sinusoidal current is circulated.

a. Negative Sequence Reactance. As stated previously, when negative sequence voltage is applied to the armature, a field is set up which rotates with synchronous velocity in a direction opposite to that of the rotor. This field sets up currents of double system frequency in the rotor. Now the subtransient reactances may be measured by blocking the rotor, with the field winding short-circuited, and applying a single-phase alternating voltage across two terminals of the armature. The reactance per phase measured in this manner varies with the position of the rotor. If the machine is not equipped with damper windings the variation is very great for when the axis of the rotor coincides with the axis of the pulsating field, the field winding constitutes a short-circuited secondary, producing a low impedance, but when the two axes are in quadrature, the impedance is merely that determined by the exciting current and is quite high. The upper curve in Fig. 2 shows these subtransient reactances plotted as a function of the angular position of the rotor. For the negative sequence measurement a similar phenomena is involved except that the rotor is rotating with double frequency with relation to the field set up by the applied voltage and is taking successively all the possible positions used in determining the subtransient reactance. One would expect, therefore, the negative sequence reactance to be some kind of a mean between the maximum value of subtransient reactance, x_q'', and the minimum value, x_d''. Park and Robertson[2] give for this value, when the circuit has a large value of reactance in series

$$x_2 = \frac{1}{2}(x_d'' + x_q'') \qquad (1)$$

The straight lines in Fig. 2 show the test results obtained for the imaginary component of Z_2, that is, the negative sequence reactance.

When low-resistance damper windings are added x_d'' is reduced somewhat because the damper winding is more closely associated with the air gap flux than the field winding and does not permit so much leakage but x_q'' is reduced greatly because the damper winding constitutes a short-circuited secondary with a relatively small leakage, whereas without a damper the reactance is determined by the exciting current. The reactances in the two axes, x_d'' and x_q'', are very nearly equal as is shown by the curve for this case in Fig. 2. It follows therefore that the negative sequence reactance is also very nearly equal to these values.

As the resistance of the damper winding is increased, little effect is at first discernible upon the reactance component of Z_2 because the resistance drop in the rotor circuit is in quadrature to the leakage reactance drop.

Interpreted in terms of the equivalent circuit of Fig. 1, the higher the negative sequence reactance, the greater will be the shunt impedance and the synchronizing power during the fault. This results in increased power limits.

b. Negative Sequence Resistance. The power associated with the negative sequence current may be expressed as a resistance times the square of the current. This resistance will be designated the "negative sequence resistance." For a machine with no dampers the only source of loss is in the armature and field resistances, eddy currents and iron loss. The copper loss in the armature and field is very small as is also the iron and eddy loss in the armature, but the iron and eddy loss in the rotor may reach quite high values. Copper damper windings provide a lower impedance path for the eddy currents and hinder the penetration of flux into the pole structure. The relatively low resistance of this path results in a smaller negative sequence resistance. For higher resistance

dampers, the negative sequence resistance increases to a point beyond which the larger resistance diminishes the current in the rotor circuits sufficiently to decrease the loss.

A large negative sequence resistance enables more power to be absorbed during the fault so that the decrease in power output is not as great, resulting in less acceleration and a greater stability limit. These considerations, manifestly, do not apply to synchronous condensers; because since they are usually connected to the receiving end of the line and usually retard during a fault, a high-resistance damper merely accentuates this effect and reduces the stability limit.

The following test results of negative sequence impedance illustrate the effect of the damper material for a 5,000 kv-a. condenser.

Material	Negative sequence impedance in per unit of machine rating
Copper	$0.026 + j0.195$
Brass	$0.045 + j0.195$
High resistance	$0.12 + j0.20$
No dampers (But with damper slots)	$0.045 + j0.75$

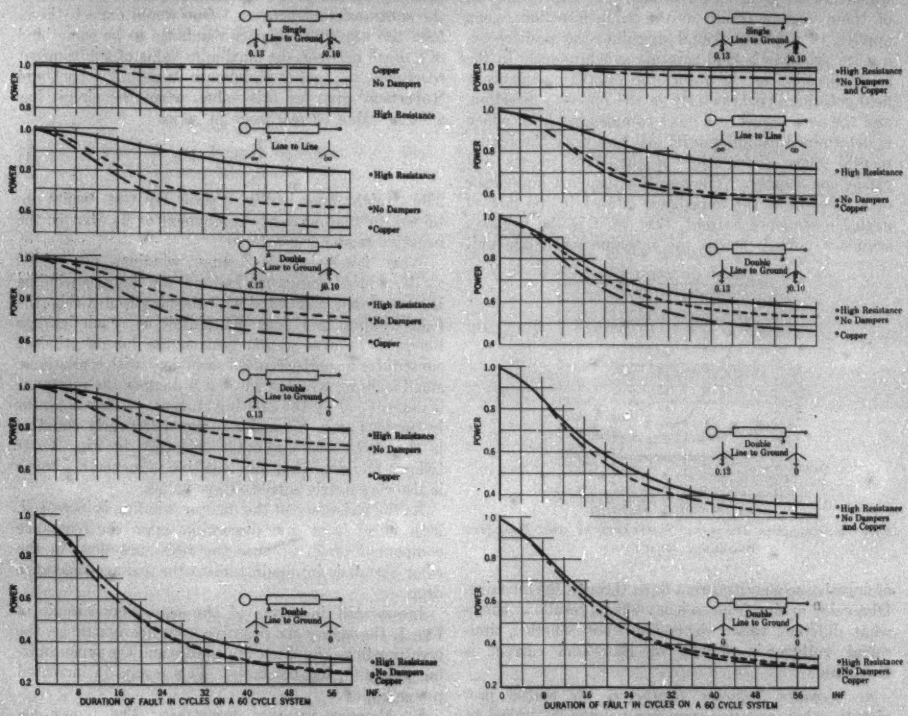

FIG. 3—STABILITY LIMITS WITH DIFFERENT KINDS OF FAULTS AND DAMPER WINDINGS AS A FUNCTION OF THE DURATION OF THE FAULT

A. Fault on high-voltage side of sending-end transformer

B. Fault on high-voltage side of receiving-end transformer

The above values of impedance were obtained by the method described in the discussion of negative sequence resistance in Appendix II.

There remains yet to evaluate the effect of these different impedances upon the power limits. This will, of course, depend upon the particular kind of fault, the location and the duration of the fault under consideration. Fig. 3 shows the maximum power that can be

transmitted on the given system for different kinds of faults, including the effect of the subsequent switching of the faulted line. It will be observed that the high-resistance dampers give the highest limits for all cases except for the single line-to-ground fault at the generator end, and for most cases the difference is quite considerable. The improvement made possible with the high-resistance damper may be made more evident by expressing the limits as a per cent of the minimum limit.

The above values of negative sequence resistance do not represent the ultimate that may be obtained by increasing the resistance of the damper bars. Further increase is possible with some additional gain in stability limit. However, as the resistance is still further increased, the reactance also increases because, in the equivalent circuit[3] for the machine, the field winding parallels the damper circuit; and for very high resistance the reactance approaches that of a machine with no dampers. In addition, in the equivalent circuit for the system, for a line-to-line fault on the high-tension bus at the generator end, the negative-sequence impedance of the machine and transformer is paralleled by the negative-sequence impedance of the rest of the system. As the negative-sequence resistance of the machine increases more of the total negative-sequence current is diverted to the rest of the system. A point is finally reached at which the decrease in negative-sequence current through the machine more than compensates for the increase in resistance.

As the negative-sequence energy absorbed in the machine increases, heating of the damper windings may constitute a limitation. A terminal-to-terminal fault on the machine or a line-to-line fault on the high-tension bus will produce a temperature rise of about 80 deg. cent. per second for a machine with 0.12 per unit negative sequence resistance, assuming all the energy is absorbed thermally in the damper bars themselves and making no allowance for thermal conduction or radiation to the surrounding laminations. With even these assumptions, no dangerous temperatures should result if the fault held on for three to five seconds. For other types of faults, such as single or double faults to ground, the negative sequence current through the machine is smaller and consequently the heating is also much smaller.

The normal current unbalances met are of no consequence in determining dangerous operating temperatures. A 10 per cent negative sequence current which would result if the currents in the three phases are 1.15–1.0–1.0–or 1.0–1.1–1.2 produces but ¼ per cent loss in the dampers for the 0.12 per unit negative sequence resistance machine. Such a machine, of course, does not lend itself to single phase operation so readily as one with a copper damper.

Alteration in Generator Torque During and Following a System Fault due to Currents in the Damper Windings

At times of any symmetrical change in machine conditions such as, short circuits or change in operating angle, transients are set up in both the field and armature. For all practical purposes these transients may be reduced to (1) a unidirectional component in the field with its associated fundamental frequency transient in the armature, and (2) a fundamental frequency transient in the field and a unidirectional and double frequency transient in the armature. It can be shown that while the latter components are important in determining the maximum mechanical stress between turns or the maximum impulsive torque exerted on the shaft, for three-phase short circuits on the terminals of the machine, they are unimportant, because of their rapid decrement for other types of faults, in their net effect upon the angular oscillations following a system disturbance. The latter components will, therefore, be neglected and consideration given to the former. As previously stated, unbalanced faults can be reduced to equivalent symmetrical circuits, so that the analysis pertaining to transients in symmetrical circuits may be applied to the resultant positive sequence network.

The Blondel two-reaction method of analysis of salient pole machines, extended to include the effect of amortisseur torque will be used in this study. The nomenclature by Doherty and Nickle[4] in their excellent analysis of synchronous machines will be used as far as possible. Armature resistance will be neglected.

At times of change in load occasioned by sudden change in circuit condition or rotor oscillations, the flux linkages with the field winding (assuming no damper windings) tend to remain constant and since the present investigation has to do with damper windings, the exciter voltage shall be assumed to vary instantaneously in such manner as to make the assumption of constant flux linkages with the field winding rigorously true. This corresponds to the assumption of constant e_d', where e_d' is the "transient internal voltage in the direct axis." To obtain this voltage in terms of the terminal voltage and drops within the machine for a machine without damper windings (or a machine with damper windings for steady state conditions) it is necessary to use the transient reactance x_d' of the machine.

Because of the close proximity of the damper windings to the air gap, the damper windings tend to prevent any change in the air gap flux. The extent to which this fails to hold depends upon the leakage flux associated with the damper windings and is therefore affected by the nature of the magnetic circuit surrounding the individual bars. Buried damper bars surrounded by a complete iron circuit possess considerable leakage and would not be nearly as effective in maintaining the air gap flux as bars with an intervening air gap. This effect may be evaluated by the introduction of a new reactance x_d'', the subtransient reactance. Using this reactance instead of the synchronous reactance a new voltage, e_d'', which we shall term the "subtransient voltage in the direct axis" is obtained. It is the voltage associated with flux linkages with the damper windings in the di-

rect axis. In the event of any sudden change it is this quantity which remains constant.

The time constant of the transient due to the current in the damper winding is in general quite small so that after a very short time the flux in the air gap changes to that corresponding to zero current in the damper winding and constant flux linkages with the main field winding. For our present discussion this is the steady state condition because the original assumption premised such a variation in exciter voltage as to maintain constant flux linkages with the field winding. The steady state value which e_d'' tends to approach may then be obtained by determining i_d for e_d' constant, using for this purpose the corresponding constants of the machine, namely, x_q and x_d'. The steady state value of e_d'' is then

$$e_d''_{(steady\ state)} = e_d' - (x_d' - x_d'')\, i_d \qquad (2)$$

or substituting i_d from Eq. (17) in the appendix

$$e_d''_{(steady\ state)} = e_d' - (x_d' - x_d'')\,(K_3 e_d' - K_4 E \cos\theta) \qquad (3)$$

As in any problem involving a driving force, a variable, and a simple lag of the variable behind the driving force, the rate of change of e_d'' is

$$\frac{d\,e_d''}{d\,t} = -\frac{e_d'' - e_d''_{(steady\ state)}}{T_d''} \qquad (4)$$

so that given the instantaneous value of e_d'', the time constant of the damper winding in the direct axis, T_d'', and the value of e_d'' would attain if θ were maintained constant at its instantaneous value, the value of $\frac{d\,e_d''}{d\,t}$ is determined. It will be observed from the above formula and values of K_3 and K_4 from the appendix that since no constants for the quadrature axis are present, the phenomena in the direct axis is independent of the fluxes in the quadrature axis.

Just as the fictitious voltages e_d' and e_d'' were utilized to designate the generated voltages associated with certain flux relations in the direct axis, e_q'' will be used to designate the voltage corresponding to the flux linkages with the damper winding in the quadrature axis. The steady state value of this voltage is from the appendix

$$e_q''_{(steady\ state)} = -K_1 (x_q - x_q'')\, E \sin\theta \qquad (5)$$

The voltage, e_q'', is approximately equal to the voltage produced by the quadrature flux. It is restrained from changing, instantly, by currents induced in the damper windings. The rate at which it changes is

$$\frac{d\,e_q''}{d\,t} = -\frac{e_q'' - e_q''_{(steady\ state)}}{T_q''} \qquad (6)$$

To evaluate the effect of these damper currents, a line-to-line fault was assumed applied for 0.2 second to the high-tension bus at the generating end of the 60-cycle system shown schematically in Fig. 4. The power-angle diagram for $e_d' = 1$ (assuming no damper currents) is shown for the three-circuit conditions involved—two lines in service, before the fault; the fault on the system; and the fault removed, one line in service. Then for an initial power of 0.8, the angle-time curve was obtained. The assumption was then made that the damper currents would not change the angle-time curve appreciably—an assumption that the results will be seen to justify. Then knowing the variation of the angle with time, $e_d''_{(steady\ state)}$ and $e_q''_{(steady\ state)}$ can also be determined, from which by a step-by-step process, using the time constants for a copper damper, the instantaneous values of e_d'' and e_q'' can be calculated. Converted into power the increment in power is shown by the shaded areas. It will be observed that during the fault condition the generator output decreases but following the clearing of the fault the output increases. The net result is quite small for, expressing the effect in terms of the actual increment in

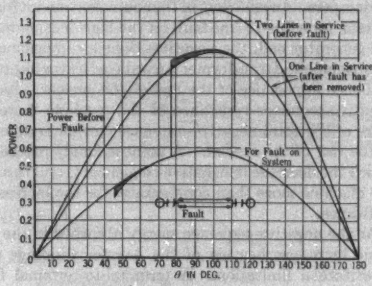

FIG. 4—INCREMENT IN POWER DURING AND FOLLOWING A FAULT DUE TO INDUCED CURRENTS IN THE DAMPER BARS FOR A TYPICAL MACHINE ASSUMING COPPER DAMPERS

Line reactance = j 0.3 per line
Transformer reactance = j 0.1
$x_d = 1.0$ $x_q = 0.6$
$x_d' = 0.4$ $x_q'' = 0.35$
$x_d'' = 0.3$
$T_{do} = 5.0$ $T_{do}'' = 0.04$ $T_{qo}'' = 0.12$
$e_d' = 1.0$ $E = 1.0$
Stored energy per kv-a. = 2.6 kw-sec.

power limit, it amounts to slightly less than ½ per cent, which in view of the other effects of damper windings is negligible. Brass or higher resistivity material would be correspondingly smaller.

HUNTING

The phenomena associated with hunting[7] may be visualized by assuming the rotor to oscillate about the mean operating angle θ. As the angle increases the demagnetizing component tends to increase and thus decrease the flux in the direct axis. However, due to the induced currents in the field and damper windings the flux and the internal voltages corresponding therewith lag behind the values which they finally would attain if the angle remained constant at any particular value. During the oscillation, therefore, instead of moving along the same locus with increasing and de-

creasing displacement, the instantaneous value of internal voltage forms a loop. The damper winding serves a similar function in the quadrature axis as the field and damper winding do in the direct axis, resulting in a similar voltage loop for the quadrature transient voltage. The voltage loops so formed can be readily transformed into a similar power loop, the area of which represents the energy absorbed per cycle of system oscillation. The results of such calculations are plotted in Fig. 5 which shows the energy absorbed per cycle of

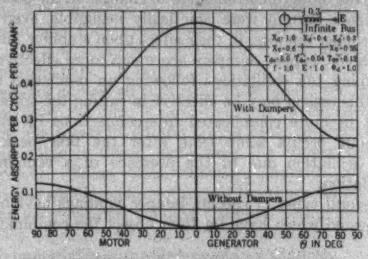

FIG. 5—ENERGY ABSORBED PER CYCLE OF SYSTEM OSCILLATION FOR SMALL OSCILLATIONS FOR TYPICAL MACHINE WITH AND WITHOUT COPPER DAMPER WINDINGS

system oscillation (per radian)^2 for small oscillations. The generators were assumed to be typical with and without copper dampers and tied to an infinite bus through a reactance of 0.3 per unit. Machines without damper windings can be influenced only by the variation in the demagnetizing component of current because the flux in the quadrature axis can change instantaneously. For small values of θ the demagnetizing current does not change appreciably for small variations in θ so that one may expect the energy absorbed to be small, but as θ increases the variations and hence the energy absorbed becomes greater. With damper windings, currents are induced in both the field and damper windings in the direct axis and in the damper windings alone in the quadrature axis.

The ordinates of these curves divided by the square of the amplitude of oscillation are a measure of the energy absorbed per cycle of oscillation, and represent the variation in energy input necessary to sustain oscillations of the given amplitude. For other variations in input the amplitude of the oscillations vary as the square root of the energy. To form a more concrete conception of the variations in power assume an operating angle of 30 deg., the machine equipped with copper dampers, and a sinusoidal oscillation of 10 deg. in amplitude. The power-angle diagram for the system under consideration and for which the curves in Fig. 5 were obtained is plotted in Fig. 6 along with the power loop which is shown dotted. The area of the power loop is $\pi \Delta P_m \Delta \theta_m$ in which ΔP_m is the maximum deviation of the actual locus from the power-angle curve for the mean operating angle. It may be seen that the energy per cycle per (radian)^2 for this particular case is 0.48, so that the energy per cycle for 10 deg. is 0.0146. Equating this to the above, $\Delta P_m = 0.0267$.

These data in themselves are not sufficient to form a conclusion as to the desirability of installing damper windings for the purpose of suppressing hunting, but merely show that cyclic variations in torque, from the mean, of the order of 2.5 per cent may arise in the prime mover and be absorbed in the generator for an angular oscillation of 10 electrical degrees and a frequency equal to unity. To complete the story it is necessary to know whether such torques are usually present. It is known that the torque characteristics of hydraulic and steam turbines are very uniform and that alternators driven by such prime movers do not usually experience hunting troubles. Improper governor adjustment may, however, give rise to such phenomena. In such a case, the cause should be sought at the source of the trouble rather than in the dampers.

The curves of energy absorbed are also of assistance in estimating the decrement of oscillations following a disturbance. Assume that the operating angle following a certain disturbance is 30 deg. (see Fig. 6) and that at some instant the peak of the oscillation has been reached at the point A at 40 deg. The relative velocity at this instant is zero. A measure of the stored energy of oscillation at this instant is the area of the shaded

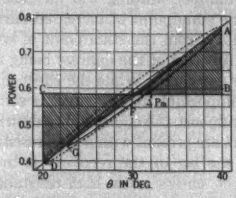

FIG. 6—ILLUSTRATING MAXIMUM DEVIATION OF POWER FROM MEAN FOR SINUSOIDAL OSCILLATIONS AND DECREMENT OF FREE OSCILLATIONS DUE TO ENERGY ABSORBED

portion bounded by $O\,A\,B$, which is $\frac{1}{2} \times 0.187 \times \frac{10 \times \pi}{180} = 0.0163$. If there were no lag of internal flux behind the steady state value, the locus of the operating point coincides with the line $O\,A\,D$, and the angle decreases until the area $O\,C\,B$ equals the area $O\,A\,B$, but actually by the time the point of maximum negative oscillation is reached the energy of a half-cycle oscillation, namely $\frac{1}{2} \times (0.0146) = 0.0073$, has been absorbed. The remaining energy which is again all in the form of potential energy is $0.0163 - 0.0073 = 0.0090$ and the amplitude of oscillation $10\sqrt{\frac{0.0090}{0.0146}} = 7.42$ degrees. The amplitude of subsequent successive half cycles bear the same relation to each other,

producing the locus shown, resulting in a time constant for the decrement of the oscillation of 1.7 seconds. It should be recognized that this is not strictly correct as the damping was assumed to be proportional to the area of the dotted loop whereas in reality it should be proportional to the somewhat smaller loop. As the operating angle increases the decrement decreases until at around 90 deg. it is but half of the value at 30 deg. (see Fig. 5). For a machine with brass dampers operating under the same conditions the time constant is 10 seconds. With brass, therefore, the duration of the oscillation would be six times as long.

The property of high damping is of no particular value for such simple systems that can be reduced to an equivalent generator and motor tied together through a transmission line. It only means that the oscillations do not continue so long. However, in more complicated systems involving compound oscillations following system disturbances, it is possible that pull-out of an individual unit will not take place until after several oscillations of the fundamental oscillation, in which case the damping afforded by the copper damper should be of considerable value. This phenomena must not be confused with the oscillations that accompany improper governor adjustments.

Breaker Duty and Quick Response Excitation

A copper damper winding will increase the breaker duty from the standpoint of increased current but will reduce the duty from the standpoint of lower recovery voltage, the function of the damper being to retard the change in air gap flux. While the effect upon the voltage is more important neither effect is sufficiently pronounced to influence the choice of the desirable damper. Similar effects are present with a high resistance damper except that the lag of the recovery is much smaller. Another effect enters with high-resistance dampers. Because of the large energy component of current the relative phase position of current and voltage shifts so that the voltage is no longer at its maximum as the current passes through zero and interrupts the circuit, thus reducing the recovery voltage. It may be argued that this condition does not hold for a three-phase fault but in opening the three-phase fault the breaker must pass through the phase-to-phase condition. On the whole these effects are too small to consider in applying a breaker.

The question also arises as to the effect of the damper winding on quick response excitation systems. The amount of copper associated with the damper winding is always small in comparison with that associated with the main winding and for that reason is relatively unimportant in connection with quick response. The actual tendency, of course, is to hinder any change of flux linked with the damper winding so that when the function of quick response system is to just maintain the flux, the dampers are an aid but, when the function of excitation is to actually increase the flux, the dampers are a hindrance. These effects become less important as the damper resistance increases.

Double Deck and Other Special Types of Damper Windings

In reviewing the properties of the different kinds of dampers, it will be recalled that from the standpoint of power limits as affected by the negative sequence impedance the order of preference is high-resistance damper, no damper and copper damper. It was also shown that the induced currents due to changes in the positive sequence current were practically negligible, even for the copper damper, in their effect upon stability limits. However, copper dampers are much more effective in damping the oscillations following an oscillation and for this reason may be effective in raising the stability limit in more complicated interconnections in which pull-out occurs some time after the first peak of the oscillation due to the presence of a compound oscillation.

Several types of special damper windings have been suggested which have for their object the combination

Fig. 7—Double-Deck Damper Construction

of the advantages of the high-resistance damper from the standpoint of negative sequence loss and the copper damper from the standpoint of damping action. These dampers sought to take advantage of the fact that the rotor currents determining the negative sequence are of double system frequency whereas the currents determining the damping action proper are of the frequency of the electro-mechanical oscillations of the system. In effect a damper was sought having a large skin effect ratio at double system frequency. What appears to be the most promising of this type of damper is shown in Fig. 7 which consists essentially of two distinct sets of windings, the lower one of copper, completely buried in the rotor iron, and the upper one of high-resistance material with an air gap inserted in its magnetic circuit. At double system frequency the reactance of the buried bars is so great that most of the current will circulate in the upper high-resistance bars producing a high-negative sequence resistance. At the low frequencies encountered in system oscillations, (about 1 to 1.5 cycles per second) the reactive drop is very small, so that the resistance is controlling in determining the current distribution between bars. In this case most of the current passes through the low-resistance copper, the presence of high-resistance bars thus detracting little from the effectiveness of the copper bars.

The extent to which these conditions hold is dependent upon the saturation of the iron surrounding the copper bars. Tests have been made on different proportions of iron circuit, a typical result of which is shown in Fig. 8. The ratio of currents in the high resistance and copper bars was determined for 0, 17.8, 25, 39, and 60 cycles, for currents ranging from 600 to 2,000 amperes, the latter being the order of magnitude expected for line-to-line faults on the high-tension bus. The results so obtained were extrapolated to 120 cycles and 3,000 amperes by plotting on logarithmic paper, resulting in the curves shown. The curve for 120 cycles shows how saturation affects the current distribution between bars. However, it is not until about 1,000 amperes is reached that saturation affects the loss importantly. A ratio of three-to-one in currents decreases the loss to about 90 and 50 per cent, respectively, of the value it would have were the copper bars not present. These tests, therefore indicate, that the double winding does not impair the characteristics that the low-resistance winding possessed acting alone, but does, especially at high currents, somewhat impair the characteristics of the high-resistance winding.

Structurally a double damper of this character offers some difficulty. Since practically all of the energy is absorbed by the heat capacity of the high-resistance bars, the rise in temperature of these bars is much greater than that of the copper bars. The difference in expansion of the hot and cold bars requires that the ends of the two sets of bars be not fastened together.

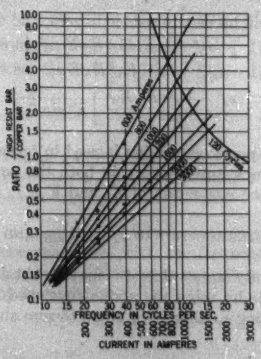

Fig. 8—Showing Ratio of Current Distribution Between High Resistance and Copper Bars as a Function of Frequency and Current

One solution of this difficulty is the use of two separate end rings. In tests made on a synchronous condenser it was found that insulating the end rings from the bar connectors had little effect upon the negative sequence resistance. Taking advantage of this fact the high-resistance bars may be welded to a connector laid flat against the side of the pole piece instead of in the conventional manner, the connections between poles being omitted.

Conclusions

The choice of the most appropriate damper winding is dependent upon the emphasis to be placed upon the different characteristics. The desirable property of rapid damping, associated with copper dampers, should

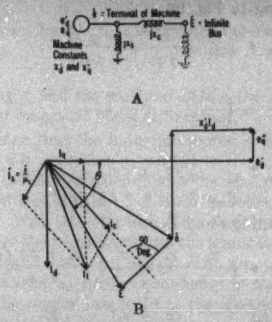

Fig. 9—Vector Diagram for Synchronous Machine with Series and Shunt Loading

be of value only in special cases in which there is danger of compound oscillations of such character that pull-out occurs on the second or third oscillation. Only rarely has such a case come to the attention of the author and his associates. When pull-out does take place it usually occurs on the first swing or not at all. High-resistance dampers should be of value regardless of the nature of the oscillations and from this viewpoint are more desirable. Double-deck or similar dampers are somewhat of a compromise, conferring to the machine some damping characteristics (but not as much as though both windings were of copper) and at the same time increasing the stability limit because of the increase in power absorbed by the damper (but not as much as for a higher resistivity material occupying the same space and using the same temperature; a loss must result either in stability limit or in temperature tolerance). These conclusions are premised upon a given pole shoe. Of course, more effective dampers can always be installed if more space be made available, by increasing the distance from field coil to the outer edge of the pole shoe. This necessarily increases the diameter of machine and results in a greater increase in cost than is justifiable by the improvement in stability.

Appendix I

Effect of Damper Currents During and Following Faults

Symmetrical or unsymmetrical faults on simple two-machine systems can always be reduced to an equivalent circuit of the form shown in Fig. 9A in which

x_s is a shunt reactance across the terminals of the machine and x_c a series reactance. Assuming an infinite bus at the receiving end of voltage E, the shunt admittance, shown dotted, will not enter the problem. The power output of the machine in terms of the internal voltages in the two axes e_d' and e_q'' and the corresponding reactances x_d' and x_q'' may be obtained as follows.

Referring to Fig. 9B

$$i_q - j\,i_d = \check{i}_s + \check{i}_c = \frac{\check{e}}{j\,x_s} + \frac{\check{e} - \check{E}}{j\,x_c}$$

$$= \left(\frac{1}{j\,x_s} + \frac{1}{j\,x_c}\right)\check{e} - \frac{\check{E}}{j\,x_s} \qquad (7)$$

But

$$\check{e} = e_d' - x_d'\,i_d + j\,(-\,x_q''\,i_q + e_q'') \qquad (8)$$

$$\check{E} = E\cos\theta - j\,E\sin\theta \qquad (9)$$

Substituting e and $\check{E}$

$$i_q - j\,i_d = -\,j\,\frac{x_s + x_c}{x_s\,x_c}\,e_d' + j\,\frac{(x_s + x_c)\,x_d'}{x_s\,x_c}\,i_d$$

$$-\,\frac{(x_s + x_c)}{x_s\,x_c}\,x_q''\,i_q + \frac{(x_s + x_c)}{x_s\,x_c}\,e_q'' + j\,\frac{E}{x_c}\cos\theta + \frac{E}{x_c}\sin\theta$$

Equating reals and imaginaries

$$i_q\left[1 + \frac{(x_s + x_c)\,x_q''}{x_s\,x_c}\right] = \frac{E}{x_c}\sin\theta + \left(\frac{x_s + x_c}{x_s\,x_c}\right)e_q''$$

or

$$i_q = \frac{x_s}{x_s\,x_c + x_s\,x_q'' + x_c\,x_q''}\,E\sin\theta$$

$$+\,\frac{(x_s + x_c)}{x_s\,x_c + x_s\,x_q'' + x_c\,x_q''}\,e_q'' \qquad (10)$$

and

$$i_d\left[1 + \frac{(x_s + x_c)\,x_d'}{x_s\,x_c}\right] = \frac{x_s + x_c}{x_s\,x_c}\,e_d' - \frac{E}{x_c}\cos\theta$$

or

$$i_d = \frac{x_s + x_c}{x_s\,x_c + x_c\,x_d' + x_d'\,x_s}\,e_d'$$

$$-\,\frac{x_s}{x_s\,x_c + x_c\,x_d' + x_d'\,x_s}\,E\cos\theta \qquad (11)$$

Letting

$$K_1 = \frac{x_s}{x_s\,x_c + x_c\,x_q'' + x_q''\,x_s} \qquad (12)$$

$$K_2 = \frac{x_s + x_c}{x_s\,x_c + x_c\,x_q'' + x_q''\,x_s} \qquad (13)$$

$$K_3 = \frac{x_s + x_c}{x_s\,x_c + x_c\,x_d' + x_d'\,x_s} \qquad (14)$$

$$K_4 = \frac{x_s}{x_s\,x_c + x_c\,x_d' + x_d'\,x_s} \qquad (15)$$

then

$$i_q = K_1\,E\sin\theta + K_2\,e_q'' \qquad (16)$$

$$i_d = K_3\,e_d' - K_4\,E\cos\theta \qquad (17)$$

Power Output of Machine = P

$$P = (e_d' - x_d'\,i_d)\,i_q + (-\,e_q'' + x_q''\,i_q)\,i_d$$

$$= e_d'\,i_q - e_q''\,i_d + (x_q'' - x_d')\,i_d\,i_q$$

Substituting values of i_d and i_q

$$P = K_1\,e_d'\,E\sin\theta + K_2\,e_d'\,e_q'' + K_3\,e_q''\,e_d' + K_4\,e_q''\,E\cos\theta$$
$$+\,(x_q'' - x_d')\,K_1\,K_3\,E\,e_d'\sin\theta - (x_q'' - x_d')\,K_3\,K_4\,E\,e_q''\cos\theta$$
$$+\,(x_q'' - x_d')\,K_2\,K_3\,e_q''\,e_d' - (x_q'' - x_d')\,K_1\,K_4\,E^2\sin\theta\cos\theta$$
$$= [-\,K_3\,e_d' + K_2\,e_d' + (x_q'' - x_d')\,K_2\,K_3\,e_d']\,e_q''$$
$$+\,[K_1\,e_d'\,E + (x_q'' - x_d')\,K_1\,K_3\,E\,e_d']\sin\theta$$
$$-\,[(x_q'' - x_d')\,K_2\,K_4\,E - K_4\,E]\,e_q''\cos\theta$$

Substituting the values of K_2 and K_3 in the coefficient

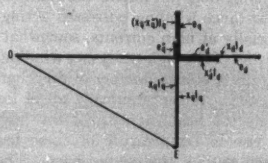

Fig. 10—Vector Diagram for Synchronous Machine Showing Relation Between Transient and Subtransient Reactances

of e_q'', it will be seen to reduce to zero, and there results

$$P = [1 + (x_q'' - x_d')\,K_3]\,K_1\,E\,e_d'\sin\theta$$
$$+\,[1 - (x_q'' - x_d')\,K_2]\,K_4\,E\,e_q''\cos\theta$$
$$-\left[\frac{(x_q'' - x_d')\,K_1\,K_4}{2}\right]E^2\sin 2\,\theta \qquad (18)$$

When the power expression is used in terms of e_d'', x_d'' must be substituted for x_d' in both the power expression and the evaluation of K_3 and K_4.

Determination of e_q'' (steady state) and $\frac{d\,e_q''}{d\,t}$

The steady state value of i_q may be obtained for any given angle by using the above developed formula for i_q and letting e_q'' therein be equal to zero and $x_q'' = x_q$, therefore

$$i_q = \frac{x_s}{x_s\,x_c + x_c\,x_q + x_q\,x_s}\,E\sin\theta \qquad (19)$$

For steady state conditions, from Fig. 10

$$e_q - e_q'' = (x_q - x_q'')\,i_q$$

and since $e_q = 0$, then

$$e_q''\,(\text{steady state}) = -\,(x_q - x_q'')\,i_q = \frac{-\,(x_q - x_q'')\,x_s}{x_s\,x_c + x_c\,x_q + x_q\,x_s}\,E\sin\theta \qquad (20)$$

Substituting $e_q''_{(steady\ state)}$ in Equation (6)

$$\frac{d\,e_q''}{d\,t} = \frac{(x_q - x_q'')\,x_s}{x_s\,x_c + x_c\,x_q + x_q\,x_s}\,E \sin\theta - e_q'' \tag{21}$$

Substituting $e_q''_{(steady\ state)}$ in Equation (6)

$$\frac{d\,e_q''}{d\,t} = \frac{1}{T_q''}\left[\frac{(x_q - x_q'')\,x_s}{x_s\,x_c + x_c\,x_q + x_q\,x_s}\,E \sin\theta - e_q''\right] \tag{22}$$

Appendix II

Negative Sequence Resistance

The nature of the negative sequence resistance is best visualized by analyzing the phenomena occurring in induction motors. Fig. 11 shows the usual equivalent circuit* of an induction motor in which

r_s = Stator resistance
x_s = Stator leakage reactance at rated frequency.
r_r = Rotor resistance.
x_r = Rotor leakage reactance at rated frequency.
Z_m = Shunt impedance to include the effect of magnetizing current and no load losses.
E_s = Applied voltage.
I_s = Stator current.
I_r = Rotor current.
s = Slip.

All of the above quantities are assumed to be given in per unit quantities.

The justification for this diagram may be seen briefly

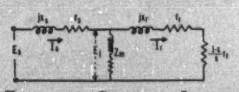

Fig. 11—Equivalent Circuit of Induction Motor

as follows: The air gap flux due to the currents I_s and I_r induce the voltage E_i in the stator and $s\,E_i$ in the rotor. In the rotor the impedance drop is

$$r_r\,I_r + j\,s\,x_r\,I_r$$

since the reactance varies with the frequency of the currents in the rotor. The rotor current is therefore determined by the equation:

$$s\,E_i = r_r\,I_r + j\,s\,x_r\,I_r$$

or

$$E_i = \frac{r_r}{s}\,I_r + j\,x_r\,I_r \tag{23}$$

It follows from this equation that the rotor circuit can be completely represented by placing a circuit of impedance $\frac{r_r}{s} + j\,x_r$ across the voltage E_i. The total power absorbed by $\frac{r_r}{s}$ must be the sum of the rotor losses and the useful shaft power, so that, resolving $\frac{r_r}{s}$ into the resistances r_r and $\frac{1-s}{s}\,r_r$ the power absorbed by r_r represents the rotor copper loss and the power absorbed by $\frac{1-s}{s}\,r_r$ represents the useful shaft power.

Neglecting r_s and the real part of Z_m, the only real power is that concerned in the rotor circuit.

Now assume that the induction motor is loaded by means of a d-c. generator connected to the shaft. At small slips the electrical input into the stator is equal to the copper loss, that is, the $I_r^2\,r_r$ of the rotor, plus the shaft load. With the rotor locked the shaft load is zero and the total electrical input into the stator is equal to the rotor copper loss. At 200 per cent slip, *i. e.*, with the rotor rotating at synchronous speed in the reverse direction, the copper loss is $I_r^2\,r_r$, the electrical input into the stator is $\frac{I_r^2\,r_r}{2}$ and the shaft load $\frac{1-2}{2}\,r_r\,I_r^2$ or $-\frac{I_r^2\,r_r}{2}$. A negative shaft load signifies that the d-c. machine instead of functioning as a generator is now a motor. Physically that is just what would be expected, for, as the slip increases from zero the shaft power increases to a maximum and then decreases to zero for 100 per cent slip. A further increase in slip necessitates motion in the opposite direction, which requires a driving torque. It will be observed, therefore, that at 200 per cent slip, the electrical input into the stator is equal to the mechanical input through the shaft, half of the copper loss is supplied from the stator and half through the shaft. This is the condition obtaining with respect to the negative sequence in which the rotor is rotating at a slip of 200 per cent relative to the synchronously rotating negative sequence field in the stator. Half of the machine loss associated with the negative sequence current is supplied from the stator and half by shaft torque through the rotor.

The factor of fundamental importance is the power supplied by the stator and through the shaft, which can always be determined by solving the equivalent circuit involving the stator and rotor constants and the magnetizing current constants. A more convenient device, since s is constant and equal to 2 for the negative sequence, is to reduce the equivalent network to a simple series impedance as shown in Fig. 12. The components of this impedance will be called the negative sequence resistance R_2 and negative sequence reactance X_2. The values of these constants are also

*For more detailed description of this circuit see any standard text book such as Ralph R. Lawrence's, "Principles of Alternating Current Machinery."

given in Fig. 12. The current flowing through the negative sequence impedance is the current flowing through the stator of the machine and the power loss in R_2 is equal to the loss supplied from the stator of the machine and the loss supplied through the rotor.

The total electrical effect of the negative sequence resistance is obtained by inserting the negative sequence resistance in the negative sequence network for the system in the usual manner and solving the network in

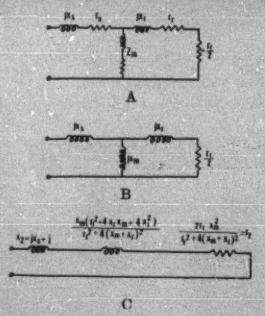

FIG. 12—DEVELOPMENT OF NEGATIVE SEQUENCE RESISTANCE AND REACTANCE FROM EQUIVALENT CIRCUIT OF INDUCTION MOTOR

a. Negative sequence diagram for induction motor
b. Neglecting armature and no load losses
c. Simplified network—negative sequence resistance and reactance

the usual manner. All three of the sequence currents are thus affected by a change in the negative sequence resistance. The total electrical output of a generator is equal to the total terminal power output plus the losses in the machine. However, since the negative and zero sequence power outputs are merely the negative of their losses the contribution to the electrical output by the negative and zero sequences is zero. The total electrical output is therefore that due to the positive sequence and to include the positive sequence armature resistance loss it is only necessary to use the positive sequence internal voltage in the calculations. Or viewed differently, since there are no internal generated voltages of the negative and zero sequence, the corresponding internal power must be zero. In addition to this electrical output which produces a torque tending to decelerate the rotor, there also exists the negative sequence shaft power supplied through rotor. It was shown that the value of this power tending to decelerate the rotor is numerically equal to the negative sequence power supplied to the stator which is equal to the loss absorbed by the negative sequence resistance. Therefore, the total decelerating power is equal to the positive sequence power output plus the loss in the negative sequence resistance.

The assumption was made that the stator resistance and the losses in magnetizing branch were neglected. The justification for this assumption lies in the fact that they are usually small so that their effect on the net result is unimportant and in addition, in the comparative analysis under consideration, the difference will be completely submerged. For greater refinements, the stator resistance and the losses in the magnetizing branch can be taken into consideration by substituting them in the equivalent circuit and reducing that circuit to a simple series resistance and reactance, wherein the resistance becomes the negative sequence resistance and the reactance the negative sequence reactance. The ratio of the negative sequence shaft power to the negative sequence stator power is then equal to the ratio of the power loss in $\frac{r_r}{2}$ for unit negative sequence current in the stator to r_2. This ratio can be obtained very easily by test by measuring the shaft torque and the negative sequence input when negative sequence voltages only are applied to the stator.

While this analysis has premised induction motor construction, the conclusions may also be applied to multicircuit rotors.

The significance of these considerations may be illustrated by a description of the method utilized to deter-

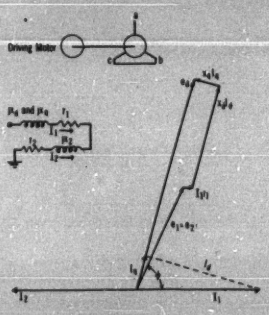

FIG. 13—NEGATIVE SEQUENCE RESISTANCE OF SYNCHRONOUS MACHINES

mine the negative sequence resistance. For this purpose two terminals of the machine under test were short-circuited and the machine driven at rated frequency by means of a d-c. motor. The equivalent circuit and vector diagram for this connection are shown in Fig. 13. The positive sequence reactions are resolved into the two axes—the direct and quadrature. The positive sequence power at the terminals is equal to the product of e_1 and I_1 and the cosine of the angle ϕ. It will be observed that this power is positive. However, the negative sequence power output is equal to e_2, I_2 and the cosine of the angle between e_2 and I_2, and since $I_2 = -I_1$, and $e_1 = e_2$, the negative sequence power output is the negative of the positive sequence power output, which, of course, must follow since the output of ma-

chine is zero. A negative output is equivalent to a positive input. This input is equal to $r_2 I_2^2$. Therefore, the positive sequence terminal output is $r_2 I_2^2$ and the total shaft load due to the positive sequence is equal to $r_2 I_2^2 + r_1 I_1^2$. It has been shown that the negative sequence shaft load (neglecting the armature resistance) is equal to the negative sequence input or $r_2 I_2^2$. Including the armature resistance but still neglecting the loss in the magnetizing branch and harmonics the negative sequence shaft load, since $I_2 = I_1$, is $(r_2 - r_1) I_2^2$. Therefore, the total tool input into the a-c. machine is equal to $r_2 I_2^2 + r_1 I_1^2 + (r_2 - r_1) I_2^2$ or $2 r_1 I_2^2$ and

$$r_2 = \frac{\text{Tool input}}{2 I_2^2} \tag{24}$$

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Discussion

W. F. Skeats: The author has pointed out that a damper winding has the effect of increasing the breaker duty from the point of view of increased current, but of decreasing the duty from the point of view of recovery voltage. I am in agreement with these statements.

It may be added that a high-resistance damper increases the current less and, if the breaker operates in about one-tenth of a second, decreases the recovery voltage more than does the low-resistance damper.

However, an additional and more pronounced effect upon breaker operation is found in the case of three-phase short circuits. Here the current is limited by the direct-axis subtransient reactance of the machine, whereas the recovery voltage of the first phase to clear is determined by the quadrature-axis subtransient reactance. This is because at the time of clearing, the phase clearing first links the quadrature axis of the machine. As a result, the recovery voltage of the phase clearing first suffers a factor equal to the ratio of quadrature to direct subtransient reactance, which may be as high as 2.5, on machines without amortisseur windings, and is only 1.1 or 1.2 on machines with amortisseur windings.

That this effect is important is strikingly demonstrated by a series of tests made on the system of the Northern States Power Company in September, 1925. The same plain-break oil circuit breaker was tested with power supplied from the Riverside steam station through step-up and step-down transformers and a 43-mile, 110-kv. transmission line and with power supplied directly from three water-wheel generators at Wissota. The results of these tests are shown in Table I. It will be noted that in spite of the slightly lower voltage at Wissota, the duty, as measured by breaker distress, was several times as great. Undoubtedly the existence of the long transmission line between the generator and the short-circuit point was partly responsible for the difference in breaker performance, but it is my opinion that the use of water-wheel-driven generators also had a decided influence.

This effect appears in three-phase short circuits and in two-phase-to-ground short circuits with very low zero phase sequence reactance. While these are not the most common types of short circuit, they are likely to be the most severe, and are therefore of importance.

This phenomenon will be influenced only to a very slight extent by the resistance of the amortisseur winding.

TABLE I

Source of power	Riverside	Wissota
Number of tests made	2	5
R. m. s. line-to-line volts before short circuit	15,000	13,200
Average initial r. m. s. current in the arc	1,400	1,420
Inches of arc	1.2	4.8
Half cycles of arc	3.5	10.0
Maximum pressure, lb./sq. in.	6.5	36.0

P. L. Alger: This paper presents a most interesting analysis of the stabilizing effects of amortisseur windings, which brings out very clearly the advantages of high-resistance amortisseur windings under prolonged fault conditions. Our studies of this subject have led us to conclusions in general accord with Mr. Wagner's, though more favorable to low-resistance amortisseurs

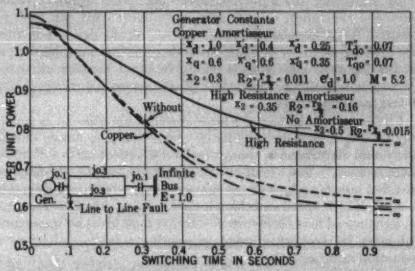

Fig. 1—Power That Can be Carried Through the First Swing with a Line-to-Line Fault by a Water-Wheel Generator with and without Amortisseur Windings

than his. The accompanying figure gives our results comparable with those for a line-to-line fault given in Fig. 3A of the paper. They assume the most favorable design of amortisseur in each case, and also assume infinite inertia at the receiving end.

An amortisseur winding produces three principal effects on stability, due respectively to its damping and braking torques, and to the increased fault current, resulting from the reduced negative phase sequence reactance.

The damping torque opposes all speed oscillations, and so is always beneficial. To secure an increased stability limit by this means, it is essential to make the damping torque large, while the generator is swinging from its full load angle toward the pull out angle, or in the neighborhood of 90 deg. displacement. This can be secured by putting maximum copper in the direct axis of the amortisseur. The amortisseur winding whose damping torque angle curve is shown in Fig. 5 of the paper has three times as much resistance in the direct axis as in the quadrature axis ($T_{qo}'' = 3\,T_{do}''$), and it is, therefore, not very effective, as indicated by its relatively low damping at the important 90 deg. point.

By thus designing a low-resistance amortisseur for maximum conductivity in the direct axis, an appreciable gain in stability limit can be secured from damping. If the fault was cleared instantaneously, this would be the only effect of the amortisseur, and so the initial point on the power time curve of the copper amortisseur is the highest of all, as shown in the figure. Besides the direct stability gain shown, a low-resistance amortisseur is beneficial during all system disturbances, in synchronizing, in damping out oscillations due to high-line resistance or faulty regulator action, and in holding machines in the same station together.

The increased fault current resulting from the reduced negative phase sequence reactance lowers the sending end voltage, and hence the power transferred, during the period the fault is on.

This effect can be minimized by designing the amortisseur with open end rings between poles, thus increasing the quadrature axis subtransient reactance without materially affecting the useful damping torque.

Also, if an unsymmetrical fault stays on an appreciable time, the negative phase sequence currents induced in the amortisseur produce resistance losses, and consequent braking torques on both generating and receiving end machines. Usually, the braking torque on the generator is about $^2/_3$ of the whole, so that with a high receiving end inertia compared to the generator inertia, the receiving end is not slowed down appreciably. However, the assumption of infinite receiving end inertia gives the maximum possible gain for the braking torque, and the actual gains in practise will be much less than those shown, if the receiving end inertia is not large.

By designing the amortisseur for maximum negative phase sequence resistance, a large gain in stability limit can thus be secured, for prolonged switching times. As the figure indicates, the braking torque of the high-resistance and the damping torque of the low-resistance amortisseur give equal benefits at about 0.05 seconds switching time, the high-resistance amortisseur being better for longer times.

I believe that the use of a low-resistance amortisseur is generally desirable, on account of its favorable damping action during all system disturbances, though its over-all effect on the stability limit is small. The use of a high-resistance amortisseur may be justified occasionally, but the actual gain in stability obtainable is small, if high-speed switching is used, while a risk of overheating on unbalanced loads is incurred. Either type gives an important reduction in recovery voltage on clearing a fault, and hence a reduction in switch duty.

C. F. Wagner: Mr. Skeats discusses the effect of damper windings upon recovery voltage having in mind in particular the comparison of high- and low-resistance dampers. His statements point to the conclusion that the difference in recovery voltage as influenced by the resistance of the damper winding is negligibly small. The case which shows up the low-resistance damper winding to best advantage in this regard is the three-phase short circuit but even in this case the effect of the damper winding resistance is negligibly small. As evidence of this fact we might refer to the criterion of recovery voltage suggested in Messrs. Park and Skeats' paper in which it was stated that the recovery voltage is dependent upon the value of x_q''. For the high-resistance damper discussed in my paper this value is not much different from that of the low-resistance damper winding. It follows therefore that for the range of damper winding resistance under consideration the resistance has little effect upon the recovery voltage.

Mr. Alger questions the choice of the direct axis time constant arguing that since $T_{do}'' = 3\,T_{qd}''$ the distribution of damper copper must be such that there is three times the copper effective in the quadrature axis as in the direct axis. Time constants, however, are not only dependent upon resistance but also upon inductance which accounts for the difference. Subsequent to the presentation of the paper I recalculated the effect of the induced currents in the damper bars for the larger values of time constants.

Results of these calculations for fault conditions again show that the unidirectional induced currents in the field winding, even with copper dampers, are extremely small in influencing the stability limits for the reasons elucidated in the paper. For high-resistance damper windings this effect is altogether negligible.

This discussion of effects of the order of one or two per cent calls for a review of the original assumptions. It will be recalled that the double frequency and the d-c. transient components of current in the armature were neglected. The former is always extremely small in its effect upon the power limits but the latter under certain cases may well reach values of the order of several per cent. The d-c. transient in the armature produces a stationary field which produces currents of system frequency in the damper windings. This phenomenon is very similar to that produced by the negative sequence currents in the armature except that the negative sequence currents produce currents of double frequency in the damper windings. The loss associated with the d-c. component of armature current is therefore proportional to the negative sequence resistance and is much greater with high-resistance damper windings than with low-resistance damper windings.

Including the effect of the d-c. transient in the armature as well as the d-c. transient in the damper windings, the detailed calculations show that the power limit curve plotted as a function of the time of fault duration is always higher for the high-resistance damper winding than for the low-resistance damper winding. Mr. Alger's argument that the two curves cross at 0.05-second switching time is therefore fallacious. I do not wish to emphasize by this discussion the importance of these small differences except to point out that, whereas Mr. Alger accredits their small beneficial effect to the copper dampers, their net effect when the d-c. component in the armature is considered is in favor of the high-resistance damper. These results were obtained by assuming an even larger value of T_{qd}'' than suggested by Mr. Alger.

Mr. Alger also suggests designing the damper winding with open end rings between poles in an effort to minimize the negative sequence current. My associates and I have given this some consideration. Tests were made on the 5,000-kv-a. condenser previously referred to with the end rings insulated. Whereas z_q'' with the high-resistance damper and with the end ring connected was 0.37, with the end ring insulated z_q'' was 0.50. These values compare with z_q'' without dampers of 1.06. It is true that the open end ring produces a somewhat higher value negative sequence impedance but it can only do so by increasing z_q''. Using the argument of Mr. Alger's associates, Mr. Park and Mr. Skeats, a high value of z_q'' results in a higher recovery voltage on opening the circuit and increases the duty on circuit breakers.

To recapitulate, the larger values of time constants T_{do}'' and T_{qo}'' suggested by Mr. Alger's discussion do not produce any appreciable increase in power limits and although these effects are small when the effect of the d-c. armature transient is also included in the analysis, the high-resistance damper increases the power limits above those shown in Fig. 3 by an amount greater than other types of dampers. High-resistance dampers should show up better, relatively, than these curves indicate. Open end rings would only be effective at the expense of increasing the recovery voltage.

Outdoor Switching Equipment
at Northwest Station Commonwealth Edison Company

BY W. F. SIMS[1] and C. G. AXELL[2]

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Synopsis.—*This paper describes the switching equipment and its arrangement in an outdoor switching center at 132, 66, and 12 kv., in connection with an installation of three-winding transformers. This installation is a junction point on an interconnected system, from which energy is distributed at the lower voltage.*

The development of the original 12-kv. installation with indoor type of equipment enclosed in metal housings and concrete cells is described. The reasons leading to its extension with metal-clad oil-filled gear are also discussed.

A feature of this metal-clad oil-filled installation is that it is laid out on an isolated phase basis with wide separation between phases, which is the first time that this arrangement has been attempted with this type of equipment.

Included in the design are complete facilities for grounding and testing of the equipment, which are fully interlocked to insure maximum safety in their use.

A brief discussion of costs as compared with indoor installations is also given.

General

When the installation of the first 132-kv. underground cable to the Northwest Station of the Commonwealth Edison Company was planned, as a part of the interconnection of systems in the Chicago area, the design of suitable terminal arrangements became an important problem.

It was necessary to make provision not only for the 132-kv. equipment, but also for the connections of the transformers to the 12-kv. buses in the switchhouse and for additional 12-kv. feeders to serve the growing demand in this district. Consideration of the requirements of the development of the 66-kv. inter-station tie line system which would connect with both the 132- and 12-kv. systems, also had to be taken into account.

Studies of the problems involved resulted in the development of an outdoor installation with equipment to serve three different voltages, namely 132, 66, and 12 kv., of sufficient size to meet the present demands and to provide space for additional equipment at all three voltages for future needs. The terminal was designed on the isolated-phase basis with double buses on both the 66 and 12-kv. portions. Single-phase three winding transformers are centrally located between the 132- and the 12-kv. switch yards with the 66-kv. switch yard adjoining the 132-kv. installation. The terminal is located about 150 ft. east of the Northwest Station switchhouse, on the opposite side of a public street.

Fig. 1 shows the arrangement of this terminal with the present equipment, together with the space available for future extension.

The general diagram of connections is shown in Fig. 2.

At present the terminal connects to two 132-kv. lines from the Waukegan Station, two 66-kv. lines from the Crawford Avenue Station and to sixteen 12-kv. lines to substations in the Northwest Station zone, with tie line connections to the indoor 12-kv. buses in the switchhouse. All of these lines are underground.

The circuit breaker control and the metering equipment are mounted on panels in the operating gallery of Northwest Station. The layout of the control wiring from the terminal to the gallery required careful attention. All of this wiring is in underground conduit and so located as to secure the best possible segregation between the wiring for different sections.

The control for the motor-operated disconnective switches is installed in weather-proof metal boxes, mounted on the structure, and in the case of the 132- and 66-kv. installations is so located that the operator can observe the operation of the switches.

132-Kv. Installation

In the selection of the breakers, calculations showed that they might be called upon to interrupt the circuit, under certain conditions of system operation, with voltages as high as 145 kv. Therefore circuit breakers of the 154 kv. class were chosen for this service. On the basis of calculated three-phase instantaneous short-circuit conditions a rated interrupting rating of 1,500,000 kv-a. at 154 kv. was specified.

Each breaker consists of three outdoor single-pole 600-ampere wheel mounted tanks, each provided with a separate solenoid operated mechanism, both mechanically and electrically trip free. The three poles are only electrically interlocked, with the operating coils connected in parallel.

Bushing type multi-ratio current transformers are mounted in the tanks for the recording and indicating switchboard instruments and for operating the relaying system.

The breaker poles are installed between concrete walls with the poles of like phase grouped together, the center to center spacing being 16 ft., 8 in.

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Oil drain and supply piping has been installed with arrangements for portable flexible connections to the switch tanks. This piping is connected to underground tanks which, with the necessary pumping and filtering equipment, are located convenient to a nearby railroad track.

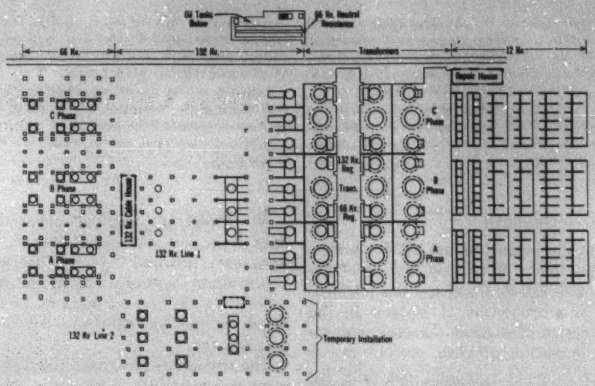

FIG. 1—GENERAL ARRANGEMENT OF OUTDOOR TERMINAL

Northwest Station

All disconnective switches are made up of three single-pole motor-operated elements, connected in parallel for simultaneous operation and a minimum gap of 53 in. between live parts when in the open position.

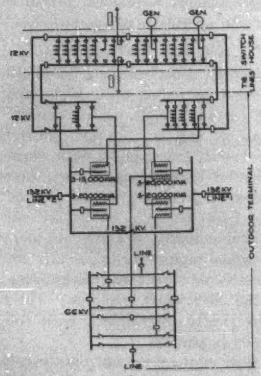

FIG. 2—GENERAL DIAGRAM OF 132-, 66- AND 12-KV. CONNECTIONS

Northwest Station

Motor-operated grounding switches are installed for grounding the lines and transformers, with the control circuits so arranged that each phase will be grounded separately. These are both mechanically and electrically interlocked with the corresponding disconnective switches so that neither can be closed unless the other is fully open. A low-voltage knife switch is installed on the ground side of each line grounding switch, so that the latter may be disconnected from ground and used for applying low voltage tests to the cables.

66-Kv. Installation

In the layout of the 66-kv. switch yard, it was necessary to provide for considerable flexibility in so far as bus sectionalizing is concerned, so that in the ultimate installation the proper distribution of transmission lines and transformers on the bus sections may be secured.

As shown in Fig. 3, the switch yard is divided into three zones each 54 ft. wide and containing the equipment for one phase.

The layout of the 66-kv. buses is on the isolated phase basis with a main and an emergency bus. Each bus is divided into two sections so as to conform to the general scheme of sectionalizing of the system.

The supporting structure is made up of simple columns and cross connections using standard beams and channels. For each phase the structure is in three parts, with solidly grounded double metal screen barriers between adjacent phases for the entire length of the structure, in order to prevent any possible flashover between phases. The central part, which is insulated from ground, supports the buses and connections and is not tied in with the two outer parts. These two parts are both grounded. On one part are mounted the 66-kv. cable potheads, potential transformers and disconnective switches for the transformer connections and the other supports similar equipment for the 66-kv. lines.

The ungrounded center section of this steelwork is used as a fault bus for the protection of the 66-kv. buses and disconnective switches connected thereto.

As in the 132-kv. installation three single-pole separately operated, electrically interlocked breakers are used for each three-phase unit. The operating coils are in parallel and no mechanical interlock is used. These breakers have a rated interrupting capacity of 1,000,000 kv-a. at 66 kv.

Due to the use of a 30-ohm neutral resistor, the possible voltage to which the breakers may be subjected under fault conditions, with the regulating transformers in the position of maximum boost, will be of the order of 76 kv. This determined the selection of 88-kv. breakers, which is the nearest manufacturer's rating above this value.

supports are required for the buses as the selector disconnective switch insulators serve for that purpose, the disconnective switches being installed horizontally on the top of the structure.

All disconnective switches are made up of three single-pole motor-operated elements with the motors connected in parallel. Those for the lines and transformers are equipped with a motor-operated grounding blade, insulated for 7,500 volts and are individually operated. These are mechanically and electrically interlocked with the main blades. Provision is made for connecting low-voltage test circuits to the line cables through the grounding blades. The motor-operated grounding blades on the bus section switches which are used for grounding the buses, are also similarly fully interlocked.

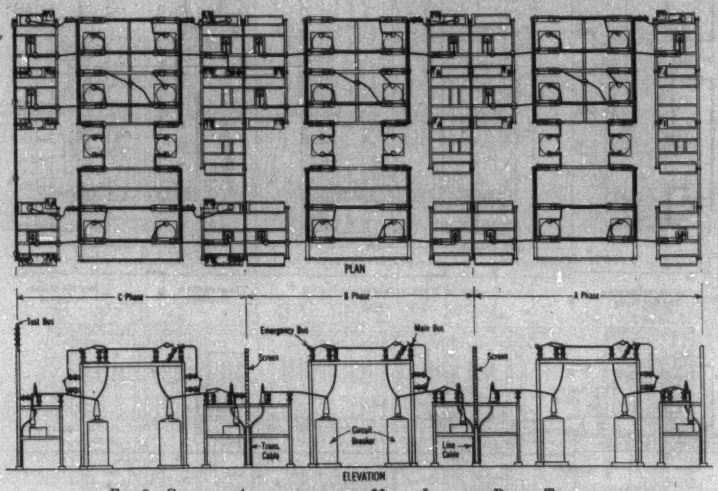

FIG. 3—GENERAL ARRANGEMENT OF 66-KV. ISOLATED PHASE TERMINAL

Northwest Station

Each breaker pole is individually frame mounted with tank lifting devices for lowering the oil pots for inspection and repair. The line and transformer breakers have multi-ratio bushing current transformers for relays and meters. Only one three-phase oil circuit breaker is installed for each transformer and for each line, the connections between these breakers and the buses being made through two sets of selector disconnective switches.

These single-pole breakers are arranged in two rows for each phase, located between the main and emergency buses. This makes a very simple layout for the connections between the breakers and the disconnective switches and allows ample space for the removal of the tanks. With this arrangement no extra insulator

12-Kv. Metal Housed Installation

There was no space available in the switchhouse for the additional 12-kv. switching equipment required. On account of space limitation, an extension of the building on the isolated phase basis could not be worked out satisfactorily and sufficient room was not available to provide for expected future requirements. These conditions together with the rising costs of building construction, led to a study of the possibility of an outdoor 12-kv. installation, in connection with the high-voltage transmission terminal where sufficient space was available to provide for the probable ultimate needs. Preliminary studies indicated that a satisfactory layout could be made on an isolated phase basis, with breakers of adequate rupturing capacity,

which could be located within a short distance of the power transformers.

The original idea was to make a conventional type of outdoor installation using standard outdoor equipment. Estimates of cost indicated a probable saving as compared with the indoor installation including the cost of building. As a higher degree of reliability of service was required than could be realized with a conventional grouped phase-open bus type of outdoor structure, a modification in design was brought about. This developed into an isolated phase installation with the breaker equipment mounted in steel housings installed in concrete compartments. The estimated cost of this design as developed, including the additional breakers required for tie lines, approximated that for a standard isolated phase indoor installation with the building cost included. However, the apparent possibilities of the outdoor scheme and the expectation that the future development of the project would tend to reduce these costs, together with the fact that an extension of the present switchhouse would include the cost of additional space for future installation, led to the decision to make a trial installation of 12-kv. equipment on this basis.

The first installation, which used a standard type of indoor breaker installed in a metal housing, provided for two 60,000-kv-a. banks of transformers, four 12-kv. distribution lines and two tie lines to the indoor 12-kv. buses. The scheme of connections follows the usual layout in all of the generating stations of the company. This provides for two main buses with line buses accommodating four distribution lines each, connected through group breakers to the main buses, with circuit breakers for connecting the transformers to either of the main buses. Six hundred ampere breakers are used for the lines, 2,000-ampere for the group switches and 3,000-ampere for the transformers and tie lines.

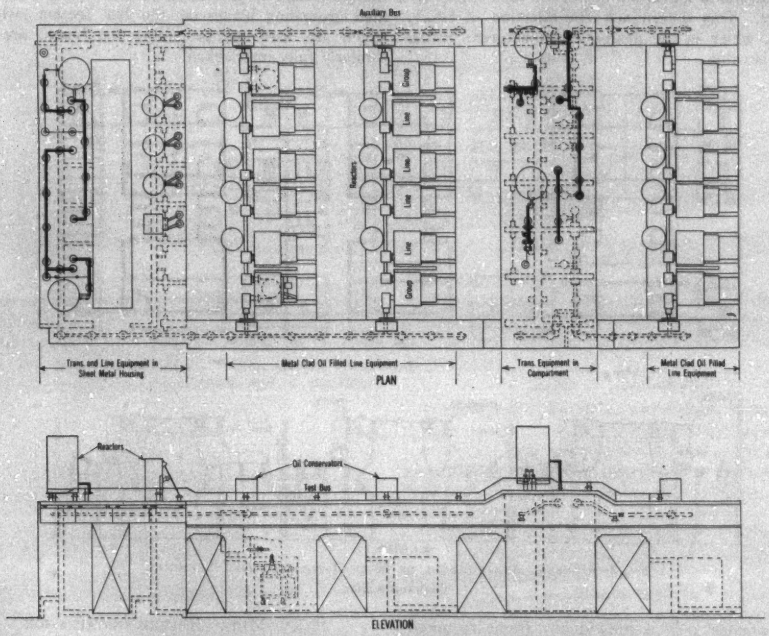

FIG. 4—GENERAL ARRANGEMENT SHOWING ONE PHASE OF 12-KV. ISOLATED PHASE INSTALLATION

Northwest Station

The equipment for each phase occupies a space of 40 ft. in length with 10-ft. aisles between phases. This corresponds to the phase segregation in the 66-kv. switch yard. Fig. 4 shows the general arrangement.

There are two concrete structures for each phase 7 ft. 8 in. wide and 15 ft. high with an operating space of 5 ft. 7 in., between structures. In each structure there is a longitudinal wall to separate the mechanisms and the testing and grounding equipment from the breaker tanks. Concrete barriers on the breaker side of the division wall, provide cells for seven single-pole

breakers in each structure. There are no barriers on the mechanism side of the division wall.

The breaker tanks and the disconnective switches are mounted in sheet metal housings ⅛ in. in thickness, which are installed in the concrete cells and are insulated from the concrete by porcelain insulators good for 7,500 volts, in order that the housings may serve as an integral part of the fault bus system. Access to the breakers is given through a double hinged door in the lower part of the housing with a small single door in the upper portion to give access to the disconnective switches. Weather proof strips cover the space between the housings and the concrete. The compartments on the opposite side of the longitudinal wall are also enclosed by weather proof metal doors, and house the operating mechanisms for the circuit breakers and the motor-operated disconnective switches. The test and ground buses with the necessary selector knife switches for grounding and testing, are also installed in these compartments. The mechanisms for the oil circuit breakers and the disconnective switches are mechanically interlocked to prevent improper operation, and the selector knife switches for grounding and testing are interlocked with the disconnective switches. To prevent condensation, space heaters are installed in all compartments.

The main buses, which are designed for 6,000 amperes, are installed in compartments carried across the ends of the structure near the top. They are made up of eight copper bars of 4 in. by ¼ in., arranged to form a hollow square section and are supported on pin type insulators having a dry 60-cycle flashover of 70 kv. and a tensile strength of 12,000 lb.

Reactors of the outdoor type for the lines and transformers are mounted on the top slab of the structure. The connections to these reactors are carried through this slab in roof type bushings.

Cables and control conduit and wiring are carried through an open space below the ground level. Terminal boards for the control and instrument wiring are mounted in weather proof metal boxes at the ends of the structure.

Circuit breakers of the indoor type having a rated interrupting duty of 800,000 kv-a. at 12 kv. and insulated for 25,000 volts, were selected. This voltage rating, which was also used for the bus and disconnective switch insulators, was chosen in order to obtain a high margin of safety. This was deemed necessary as there was no assurance of entire absence of moisture in the housings and as this was the first outdoor 12-kv. installation on the system, especial attention was given to the insulation.

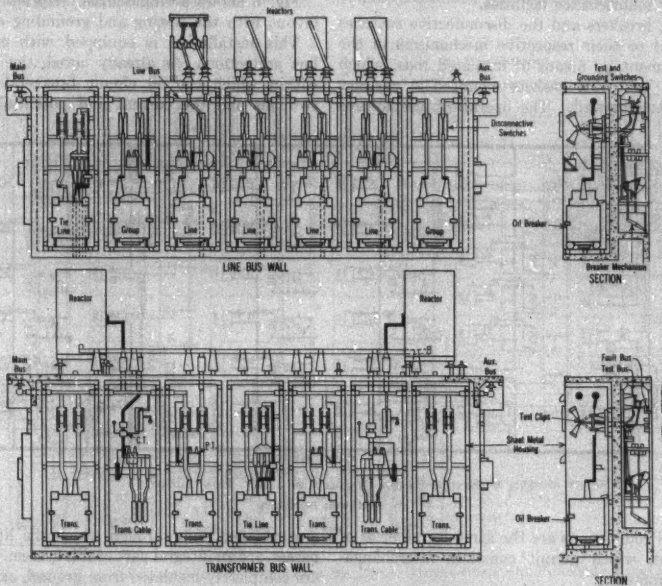

FIG. 5—ELEVATIONS AND SECTIONS OF ONE PHASE OF 12-KV. INSTALLATION IN SHEET METAL HOUSINGS

Northwest Station

Each three-phase breaker consists of three single-pole units installed on 50-ft. centers with separate mechanically trip free mechanisms, the solenoids of which are connected in parallel for simultaneous operation. An operating relay controls the operation of each pole.

Each pole is truck mounted, with roller bearing wheels and provision is made for securely holding the trucks in position on the rails in the housing. The breaker tanks may be withdrawn from the housings on to a transfer truck having a motor-operated winch. A repair house located adjacent to this installation, equipped with a hoist and oil piping connections affords the necessary maintenance facilities.

The circuit breakers and the disconnective switches are connected to their respective mechanisms in the rear compartment by means of insulated rods, which pass through gas proof gaskets mounted in castings set in the concrete wall. The operating mechanisms with rods and connections are the same for all sizes of breakers used, which permit complete interchangeability of equipment.

Bolted connections are used between the breaker poles and the motor-operated disconnective switches, which are installed in the same housings, directly above the breakers. These disconnective switches have the same carrying capacity as the breakers to which they are connected and are operated as a two-pole unit. Insulated operating rods connect these switches to the motor mechanisms, through gas tight gaskets.

The design of this installation included complete facilities for the grounding and testing of all 12-kv. buses, transformer leads, tie lines, distribution lines and circuit breakers. By means of lever-operated knife switches in connection with the disconnective switches the desired equipment can be connected for either grounding or testing. For this operation the disconnective switch can be changed from motor to hand operation, when in the open position, for further movement to the test or ground position. This operation is completely interlocked to prevent improper operation.

The testing and grounding switches together with their operating rods and interlocks are alike for all circuit breakers and form a unit assembly. This feature greatly facilitated their installation.

The ground bus is insulated from the structure and is connected to ground through the fault bus, so that in the case of accidental grounding of a live bus, the system will be cleared by the operation of the fault bus relays.

Fig. 6 shows a single-phase diagram of the 12-kv. layout with the testing and grounding connections.

This installation is equipped with complete fault bus protection. As already noted, the steel housings within which the oil circuit breakers, disconnective switches and the current and potential transformers are mounted, and through which the line bus copper passes, are an integral part of this system. All housings are completely insulated from ground, excepting where predetermined ground connections were made, so that there will be no parallel path for fault current to flow to ground. For this reason, insulated sections were installed in the conduit for all wiring entering the housings. The sheet metal covers of the main bus compartments also form a part of the fault bus, which is so sectionalized that in the event of a fault, a minimum of equipment will be taken out of service.

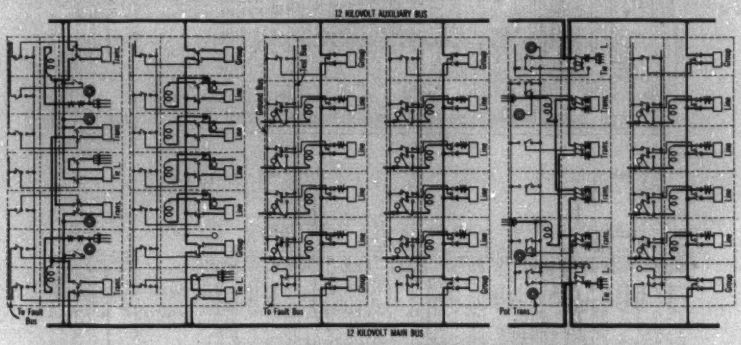

Fig. 6—Single-Phase Diagram of 12-kv. Connections
Northwest Station

12-Kv. Metal-Clad Oil-Filled Installation

Since the installation of the equipment described above was made, the developments which had taken place in the design of metal-clad oil-filled switch gear led to the consideration of this type of equipment when the need for additional 12-kv. facilities became apparent. Up to this time no such installation had been

made on the isolated phase arrangement, but it was thought that such an arrangement could be worked out and installed at a cost not materially greater than that for the previous outdoor installation.

With the active cooperation of the manufacturer's engineers an intensive study was made and an isolated phase design that met the space limitation established by the previous installation was finally developed. In working out this design, efforts were made to avoid unnecessary complications and a relatively simple arrangement was secured.

This project differs from other metal-clad installations in several particulars. It is the first installation of this type built with the isolated phase arrangement and the breakers are withdrawn horizontally instead of being lowered. This feature greatly simplifies the design. The oil-filled line buses are supported on a concrete wall, which also supports the line reactors and the grounding and testing equipment. Owing to the difficulty of designing metal-clad oil-filled buses for over 3,000 amperes capacity, the main buses are similar to and a continuation of the buses in the installation previously described. These differences in no way reduce the safety features of the conventional metal-clad gear. In addition there is included in this project an installation of four circuit breakers for transformer connections installed in open concrete compartments with open type copper bar connections to the main buses.

The oil-filled equipment has been installed as a continuation of the original 12-kv. installation and the general layout is along similar lines. The main buses are carried in enclosed concrete compartments 12 ft. above the ground, supported on concrete walls, and are in line with the buses of the first section. Openings through these walls give access to the operating space in front of the breakers. Transverse concrete walls 7 ft. 8 in. in height, for supporting the oil-filled line buses and equipment, are installed across the space between the main bus walls. Concrete barriers forming six compartments have been built at the rear of these walls, in which the 12-kv. cable terminals, the oil-filled cabinets containing the testing and grounding switches, and the potential transformers are installed. No concrete barriers have been installed between the oil circuit breaker tanks, except between the group breakers, and the adjacent line breakers, for segregation of fault bus operation. An open space below the ground level provides space for conduit and cable runs. Fig. 7 shows a section and elevations of this arrangement.

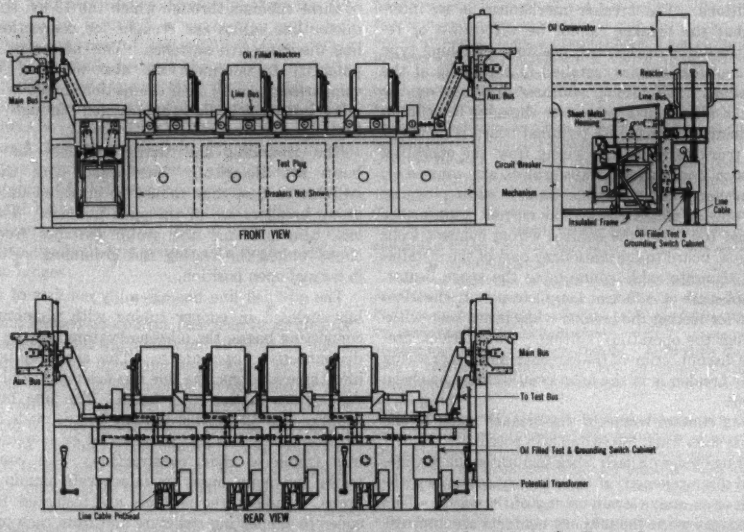

FIG. 7—SECTION AND ELEVATIONS OF ONE PHASE OF 12-KV. METAL-CLAD, OIL-FILLED INSTALLATION

Northwest Station

In contrast with the original 12-kv. outdoor installation, outdoor type of equipment is used. The breakers selected have a rated interrupting duty of 900,000 kv-a. at 12 kv. and are equipped with bushings having a 60-cycle wet flash-over value of 60 kv. Each three-phase breaker consists of three truck mounted single-pole units installed on 50-ft. centers, each equipped with

a solenoid operated mechanism which is both mechanically and electrically trip free. The mechanisms are mounted in sheet metal housings supported on the truck with space heaters in the housing to prevent condensation.

The breaker and wheel mounted truck assembly is of the horizontal draw out type installed on rails which are an integral part of a rigid frame work, insulated from ground, on the upper part of which is supported a sheet metal housing enclosing the stationary bushings and contacts with which the contacts on the bushings of the circuit breaker are engaged. This housing is equipped with automatic shutters which are operated by the withdrawal or replacement of the breaker truck and protect the live parts from accidental contact under all conditions. The breaker mechanism is so interlocked that the breaker cannot be withdrawn or replaced unless it is in the open position. Bushing type current transformers are installed in the tanks of the line breakers. Self alining contacts are used on the bushings of all circuit breakers for engaging the clip on the stationary contacts. A terminal board is mounted on each truck to which the wiring from the operating mechanisms and current transformers are connected. Two multi-conductor flexible cables with plugging devices, one of which carries the current transformer wiring and the other the control wiring connect from the terminal board to the stationary part of the installation. A separate cable connects to the space heater. These cables are of sufficient length to permit electrical operation for testing the breaker when it has been withdrawn from the operating position. An interlock prevents the disconnection of the current transformer wiring unless the breaker is in the open position, and has been withdrawn.

A sliding contact connects the breaker truck to the fault bus system when the switch is in position.

A levering device on each truck facilitates the engagement and disengagement of the primary contacts without shock or excessive strain on any of the parts. This device engages when the primary contacts are approximately 4½ in. apart. It also securely latches the truck in the operating position by means of a toggle action. The breakers are withdrawn and handled by the truck previously described.

The scheme of connections of the knife blade switches for testing and grounding is similar to that previously described in the original 12-kv. installation excepting for necessary modifications due to the absence of separate disconnective switches at the circuit breakers. These switches are enclosed in oil-filled metal cabinets, mounted on the rear of the structure, behind the oil breakers. A disconnective switch to which the line cable is connected, is also mounted in each cabinet in connection with the line breakers. The grounding and testing switches are operated by means of two shafts, with a separate shaft for operating the line disconnective switch, passing through oil tight bushings, with indicating devices to show the position of the switches. This equipment is fully interlocked so that these switches cannot be moved while the oil circuit breaker is open and the breaker cannot be closed unless these switches are in the proper position. The interlock also prevents their operation unless the line disconnective switch is open.

A test connection is brought out from the test cabinet through an oil tight bushing, to the breaker side of the concrete wall, to which a flexible cable may be connected for applying test pressure to the breaker when it has been withdrawn from the operating position. This test contact is covered by a hinged door which cannot be opened unless the breaker has been withdrawn. Cable potheads are bolted to the bottom of these cabinets through which the 12-kv. single conductor line cables are brought for connection to the line disconnective switches. The necessary oil-filled potential transformers are also mounted in these compartments with their connections made in the cabinets into which they are carried through oil tight bushings.

The grounding and testing switches for the line buses are electrically interlocked with the group switches so that they cannot be operated unless both group breakers are in the open position. This interlock also prevents the group switches from being closed unless the testing and grounding switches are in normal open position.

The oil-filled line bus assembly consists of a copper bus enclosed in copper tubing with aluminum alloy connecting boxes, the oil conservators and the primary disconnective bus contacts. This assembly is divided into three sections, the line bus assembly, and the two connections to the main buses. The line bus which supplies four 12-kv. distribution lines is made as a unit with flexible joints and is mounted on a welded angle iron frame of rigid construction. The connections to the two main buses are in separate aluminum alloy boxes. These three sections are insulated from the concrete supporting structure and from each other and form parts of the fault bus system. The entire bus system is filled with oil on which a constant pressure is maintained by the conservator tanks located at each end. The oil is carried to the system through connecting pipes with suitable valves and insulating couplings.

The connection boxes which are of heat-treated cast aluminum alloy form the supports for the line bus and bus connections.

The line bus is made up of sections of hard drawn copper tubing 3½ in. in diameter, with a carrying capacity of 2,000 amperes, of the proper lengths required to span the distance between two adjacent switch units. It is insulated with machine wrapped micarta paper impregnated with varnish and has flexible connections to the disconnective bushings. The bus is supported in micarta collars grooved to permit circu-

lation of oil which center the bus in the tubing. Micarta stops are provided to limit the travel of the bus in one direction if there is a tendency to crawl due to expansion. All connections from the line bus are carried out of the connecting boxes through oil tight porcelain bushings.

The conservator tanks are equipped with a gage glass, a low oil level alarm and a dry air breathing device.

The current limiting reactors for the 12-kv. lines which are of 450-ampere capacity and ½-ohm reactance are of the air core, metal clad, oil immersed type, with connections brought out at the bottom of the tank. Each reactor tank serves as an oil conservator for the testing and grounding switch cabinet.

Reference has been made to circuit breakers for two transformer banks installed in connection with this project which are not of the oil-filled type. This arrangement was made because the carrying capacity required in their connections is such that a satisfactory design of oil-filled equipment could not be worked out, as was also the case with the main buses. These breakers and their disconnective switches are of the outdoor type installed in concrete compartments similar to the original 12-kv. installation with the exception of the metal houses which are omitted. The switch compartments are enclosed by hinged ventilated doors connected to the fault bus system with flexible shunts around the hinges.

Conclusion

The metal housed equipment has been in service for about three years and the metal-clad oil-filled for about eight months. The operating experience with both installations has so far been entirely satisfactory and with the possible exception of maintenance difficulties under severe weather conditions, is fully as convenient to operate as an indoor installation. The degree of safety both to life and service that can be secured in the outdoor design is as high as that in an indoor installation. The distance by which adjacent phases may be separated is such that the liability of trouble spreading is much less than would be the case in a similar installation located in a building. Any arc or explosion that might occur would not be confined in an enclosed space and the amount of damage that might result should be reduced to a minimum.

Preliminary estimates had indicated the possibility that a 12-kv. outdoor installation should cost less than one indoors, with the cost of building included. Exact comparisons are difficult to make because of varying conditions on different projects. However, an analysis of actual cost figures of both kinds of installations with allowances made to place their costs on a comparable basis showed that there was very little difference in the final cost. The metal housed type cost slightly less and the metal-clad oil-filled type slighty more than a standard indoor isolated phase installation.

As these outdoor installations were along new lines, considerable developmental work was required which necessarily had an effect upon the cost. Future installations of this character, should show a tendency toward lower costs, provided that a reasonable degree of standardization and duplication of equipment can be secured.

Discussion

W. W. Edson: Installations of this type having the three phases separately operated and not mechanically interlocked should in some cases be considered with respect to the type of line relays used. Thus, for parallel lines, if one of the incoming breakers should not be uniform in closing, there will be a short interval of time in which one of the phases is open, thereby causing an unbalanced condition in the lines. A sensitive high-speed relay either in the ground residual circuit or in a cross-connection between the phases of the parallel lines might operate and improperly trip one or both lines.

R. A. Hentz: The paper indicates that while there are two 12-kv. buses, the selection of which is through oil circuit breakers, this is not the case of the 66-kv. or the 132-kv. sections. In the 66-kv. section the two buses are selected by disconnecting switches with one oil circuit breaker per line or transformer bank, and in the 132-kv. section there is but a single bus sectionalized with a disconnecting switch.

From an inspection of Fig. 3 there does not seem to be any provision for a second oil switch in the 66-kv. section and from Fig. 1 there appears to be no means of double busing in the 132-kv. section, either by two oil circuit breakers or one oil circuit breaker and two sets of disconnectors. Apparently the authors believe the extra flexibility obtained by double busing is not worth the cost. I would be interested to hear further comment on this point.

W. F. Sims and C. G. Axel: The bus arrangement in the 66-kv. transmission terminal provides for a main bus for regular operation and an emergency bus for use only during abnormal conditions. The main bus is divided into two principal sections, which division runs through the entire system. The installation of selector oil circuit breakers between the main and emergency buses for each 66-kv. line and transformer bank was not considered to be justified in view of the materially increased cost of such an arrangement. As will be noted from the bus diagram, it is possible to connect these two buses through the tie switch after which it is possible to transfer a line or transformer bank by manipulation of the disconnective switches, which gives sufficient flexibility to meet the conditions of service and saves the cost of the additional breakers.

The 132-kv. installation is a part of a general interconnected system with sufficient reserve on the 66-kv. supply to take care of an outage on the 132-kv. bus, as both systems supply the same banks of three-winding transformers which feed into the 12-kv. distributing buses. For this reason the expense of a double bus arrangement on the 132-kv. installation was not considered justified.

A New System of Speed Control
For A-C Motors

BY A. M. ROSSMAN[1]
Fellow, A. I. E. E.

Synopsis:—The drive unit of this system consists of a constant speed a-c. motor supplemented by an adjustable speed d-c. machine of much smaller size. Both rotor and frame of the a-c. motor are mounted on bearings. The d-c. machine is mechanically connected to the frame of the a-c. motor so that the d-c. machine may drive or be driven by the frame. The d-c. machine is electrically connected through a motor-generator set of equivalent rating to the source of alternating current energy. The shaft speed of the a-c. motor is increased above the fixed speed by rotating the frame of the a-c. motor in the same direction as the rotor. The shaft speed is decreased by rotating the frame in the opposite direction. The direction of rotation and the speed are governed by adjusting the voltage impressed on the armature of the d-c. drive machine by the generator of the motor-generator set. When the unit is used to drive fans, the speed range is obtained by a combination of armature voltage control and field control of the d-c. drive machine. This permits a still further reduction in the rating of the d-c. drive machine so that for comparatively wide ranges of speed it forms but a small percentage of the total drive unit rating.

Twenty-four units of this type, aggregating 7,020 hp., are now being built for Powerton Power Station for driving forced and induced draft fans. This system of fan drive is being installed in preference to the two-speed squirrel-cage type induction motor system previously used, because it shows large savings in energy, costs little more, and provides a simple method of fan control which permits the adoption of a simplified system of automatic combustion control. Energy savings and investment costs are given in detail. The versatility of the system is further illustrated by a description of a 2,500-hp. unit of this type designed to drive a high-pressure reciprocating boiler feed pump.

During the first seven months of 1930, orders were placed for 35 of these units ranging in size from 166 hp. to 2,500 hp., aggregating 16,000-hp. for installation in four different power stations.

* * * * *

Description of the Motor and Speed Control Equipment

THE drive unit of this system consists of a constant speed a-c. motor of either the synchronous or induction type, supplemented by an adjustable speed d-c. machine of much smaller size. The frame of the a-c. motor is mounted on bearings so that the frame as well as the rotor may rotate. The d-c. machine which is shunt wound but separately excited is mechanically connected to the frame of the a-c. motor so that it may drive or be driven by the frame. The d-c. machine is electrically connected through a motor-generator set of equivalent rating to the source of alternating current energy. Fig. 1 shows a typical diagram of the machines with their electrical connections. Fig. 2 shows a typical arrangement of the drive unit.

Method of Operation

The speed of the driven machine is increased above the fixed speed of the a-c. motor, which will be called the base speed, by causing the d-c. machine acting as a motor to drive the frame of the a-c. motor in the same direction as the rotor. The speed of the shaft is then the sum of the base speed plus the frame speed. The speed of the shaft is decreased below the base speed by causing the frame of the a-c. motor acting as a motor to drive the d-c. machine as a generator in the direction opposite to that of the rotor. The shaft speed is then the difference between the base speed and the frame speed. The d-c. machine then delivers energy through the motor-generator set back into the a-c. system.

The direction of rotation and the speed of the d-c. drive machine are governed by holding constant excitation on its field and adjusting the voltage impressed on its armature terminals by the generator of the motor-generator set. The range of d-c. voltage is from

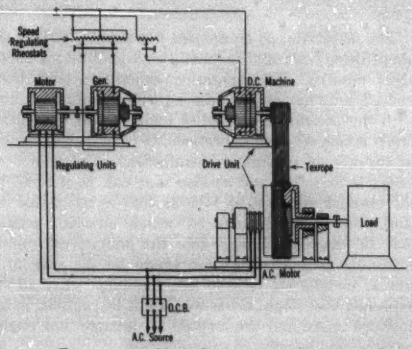

Fig. 1—Diagram of New System of Adjustable-Speed Drive

maximum positive, through zero, to maximum negative. This voltage is controlled by a rheostat inserted in the field circuit of the generator. The armature current does not reverse when the voltage is reversed.

Extension of the Speed Range

Where this system is used to drive fans (or centrifugal pumps), machines in which the power is a func-

1. Research Engineer, Sargent & Lundy, Inc., Chicago, Illinois.

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30-159

tion of the cube of the speed, the speed range may be extended beyond that attainable by armature voltage control, by weakening the field of the d-c. drive machine. The field strength is controlled by a rheostat inserted in the field circuit. The use of field control of the d-c. drive machine to furnish a part of the speed range decreases the spread between the rated speed of the

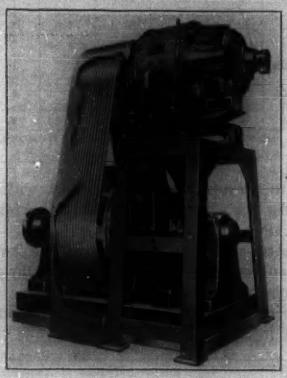

Fig. 2—430-Hp. 2,200-Volt 60-Cycle Three-Phase 1088/444-Rev. Per Min. Variable-Speed Drive Unit

a-c. motor and the maximum operating speed of the fan. As the apportioning of the rating of the drive unit between a-c. motor and d-c. machine is determined by the percentage of speed which each machine contributes to the maximum operating speed (which is coincident with maximum load) of the fan, a decrease in the spread means a corresponding decrease in the percentage of rating of d-c. machine in the drive unit

TABLE I

Proportioning of the drive unit between a-c. motor and d-c. machine under various combinations of armature voltage control and weakened field control of the d-c. drive machine to give a speed range of 100 to 50 per cent.

Ratio max. speed / rated speed	D-c. machine: No. of units of speed range accomplished by— Armature voltage control: Positive	Negative	Weakening the field	Rating of: A-c. motor	D-c. machine
1/1	0	1	0	100	50
1/1	1	0	0	50	50
1/1	1	1	0	75	25
2/1	1	1	1	83.3	16.6
3/1	1	1	2	87.5	12.5
4/1	1	1	3	90	10

Note: Full torque is available with full field and armature voltage control; torque should be reduced in direct proportion to the amount of field weakening.

and an equivalent decrease in the size of the speed regulating motor-generator set. These relations are shown in Table 1. The last three rows of figures show the apportioning of the drive unit rating between a-c. motor and d-c. machine corresponding to three different degrees of field weakening of the d-c. drive machine.

As the d-c. motor designed to give a 3/1 speed range by field control is a recognized standard machine, it has been chosen as the basis for determining maximum fan speeds corresponding to various speeds of the a-c. motor for those ranges of fan speed which exceed 100 to 57½ per cent. For ranges of fan speed less than 100 to 57½ per cent, the amount of field weakening must be reduced to prevent overloading of the d-c. drive machine.

Tabulations of Fan Speeds and Motor Ratings

Table II shows the maximum operating speed of the fan or pump corresponding to the two variables which determine it, *viz:*

(1) The speed of the induction motor;

(2) The speed range of the load.

TABLE II

The following data apply to fans and centrifugal pumps, machines in which the power varies as the cube of the speed.

The table shows the maximum speed of the fan or pump corresponding to the two variables which govern it, viz:

1. Speed of squirrel-cage type induction motor.
2. The speed range of the fan or pump.

The table also shows for each speed range the relative sizes of induction motor and d-c. machine which together make up the drive unit.

The speed range is obtained by a combination of armature voltage control and field weakening of the d-c. drive machine. For ranges greater than 100 to 57.5 per cent. one-half the total range is obtained by each method. For ranges less than 100 to 57.5 per cent the proportion obtained by field weakening must be decreased to prevent overloading of the d-c. drive machine. The method of determining the limiting value of field weakening and the method of correcting for slip corresponding to variatons of load are given on the next table.

Speed range—100 per cent to.....(per cent)		95	90	85	80	70	60	50	40	30	20	10
Speed reduction in per cent of max. (per cent)		5	10	15	20	30	40	50	60	70	80	90
Relative ratings of machines of the drive unit.....	A-c.	97.4	95.1	93.2	91.6	89.5	88.7	86.7	84.1	81.6	79	76.5
	D-c.	2.6	4.9	6.8	8.4	10.5	11.3	13.3	15.9	18.4	21	23.5

Induction motor: Syn. speed	Full-load speed	Maximum operating speed of fan or pump										
1800	1750	1800	1840	1880	1920	1960	1975	2025	2080	2150	2210	2300
1200	1160	1190	1220	1245	1270	1300	1310	1340	1380	1425	1470	1520
900	870	890	915	935	955	975	985	1005	1030	1065	1100	1135
720	690	710	725	740	755	775	780	800	820	850	875	905
600	575	590	605	615	630	645	650	665	685	705	725	750
514	490	505	515	525	535	550	560	570	585	605	620	640
450	430	445	455	465	475	485	490	500	515	530	550	565

TABLE III

The following data apply to fans and centrifugal pumps, machines in which the power varies as the cube of the speed.

The table shows the method of determining and the figures for—

(1) The slip of the induction motor at various loads. Full load slip is assumed to be 4 per cent for all motors.

(2) The amount of field weakening permissible without overloading the d-c. drive machine.

(3) The speed range obtainable with the d-c. machine.

(4) The relative ratings of the d-c. machine and the a-c. motor of the drive unit.

The letters used in the formulas are the serial letters in the first column of the table. In applying the formula substitute for the letter the figure in the corresponding speed range column.

a. Speed range 100% to ... %	95	90	85	80	70	60	50	40	30	20	10
b. Speed reduction in % of max. speed ... %	5	10	15	20	30	40	50	60	70	80	90
c. Torque at min. speed ($=a^2/100$) ... %	90	81	72	64	49	36	25	16	9	4	1
d. Correction for slip [$=4\ (1-c/100)$] ... %	0.4	0.8	1.1	1.4	2	2.6	3	3.4	3.6	3.8	4
e. Speed range by d-c. machine ($=b+d$) .. %	5.4	10.8	16.1	21.4	32	42.6	53	63.4	73.6	83.8	94
f. Speed range by armature control ... Units	2	2	2	2	2	2	2	2	2	2	2
g. Speed range by field control ($=100/e-1$) Units	0.11	0.23	0.39	0.56	1.05	1.78	..	..	..	..	..
g. Speed range by field control ($=2$) .. Units	..	..	..	..	..	..	2	2	2	2	2
h. Speed range total ($=f+g$) ... Units	2.11	2.23	2.39	2.56	3.05	3.78	4	4	4	4	4
k. Rating of d-c. machine ($=e/h$) ... %	2.6	4.9	6.8	8.4	10.5	11.3	13.3	15.9	18.4	21	23.5
m. Rating of induction motor ($=100-k$) . %	97.4	95.1	93.2	91.6	89.5	88.7	86.7	84.1	81.6	79	76.5

The maximum operating speed of a fan or centrifugal pump operating on this system is determined by dividing the full load speed of the induction motor by the figure on line "*m*" in the corresponding speed range column.

This table also shows the relative ratings of induction motor and d-c. machine corresponding to each range of speed.

Attention is directed to the wide range of speed that can be obtained from a comparatively small percentage of d-c. machine rating in the drive unit. For example, a speed range of 100 to 50 per cent is obtained with a drive unit in which the rating of the d-c. machine is but 13.3 per cent of the total. If the a-c. motor were of the synchronous type, the rating of the d-c. machine would be but 12.5 per cent of the total.

Table III gives the list of constants used in computing the figures of Table II and the formulas for deriving these constants.

Extension of the speed range by field control is not practicable where horsepower varies directly with the speed; the full load speed of the a-c. motor then becomes the mid-point between maximum shaft speed and minimum shaft speed.

Equipment in the Powerton Power Station

An extensive installation of this system of speed control is now being made in the power station of the Super Power Company of Illinois at Powerton (near Pekin) Illinois. Here the forced and induced draft fans of six boilers are driven by motors which operate on this system. Each boiler has two forced draft fans and two induced draft fans. Each fan has its own independent drive unit. The a-c. motors of the drive units are of the squirrel-cage induction type.

The frame and the rotor are supported independently, each having its own pair of pedestal bearings. Hollow stub shafts, cast integrally with the end shield, support the rotating frame on the inner pair of bearings. The rotor is carried on a shaft which passes through the hollow stub shafts of the frame and is supported on the outer pair of bearings. All bearings are of the sleeve type. One of the hollow stub shafts is made sufficiently long to mount the three collector rings between the end shield and its bearing support. The collector rings and brush rigging are enclosed in a steel housing to prevent accidental contact with the live parts. Fig. 2 is a photograph of the Powerton induced draft fan drive unit.

One motor-generator set controls the two forced draft fans of each boiler, while a second motor-generator set controls the two induced draft fans.

The principal data pertaining to this installation are given in the following tabulation:

	Forced draft fans	Induced draft fans
Number of fans	12	12
Number of drive units	12	12
Maximum fan speed	1,004 r. p. m.	1,088 r. p. m.
Minimum fan speed	452 r. p. m.	435 r. p. m.
Range of fan speed	100% to 45%	100% to 40%
Constant-speed component (a-c. power) at maximum fan speed	141 hp. @860 r. p.m.	330 hp. @860 r. p.m.
Adjustable-speed component (d-c. power) at maximum fan speed	25 hp. @ 144 r. p. m.	89 hp. @ 228 r. p.m.
Total power at maximum fan speed	166 hp.@1004 r.p.m.	419 hp. @1,088 r.p.m
Speed of d-c. machine at maximum fan speed	Plus 625 r. p. m.	Plus 625 r. p. m.
Speed of d-c. machine (operating as a generator) at min. fan speed	Minus 1,875 r. p. m.	Minus 1,250 r. p. m.
Ratio of ratings—d-c. machine/a-c. motor	14.5/85.5	21/79
Number of speed regulating motor-generator sets	6	6
Rating of each speed regulating motor-generator set	40 kw.	150 kw.
Total rating of all drive units	7020 hp.	
Total rating of all speed regulating motor-generator sets	1140 kw.	

The a-c. motors are designed to operate at 2,200 volts, three phase.

The curves of Fig. 3 show the division of load between the a-c. motor and the d-c. machine over the speed range of the forced draft fans. Fig. 4 shows similar curves for the induced draft fans. These curves also show how the total speed range is apportioned between armature voltage control and field control of the d-c. drive machines. On the forced draft fans one-half the speed range is obtained by armature voltage control and one-half by field control; on the induced

draft fans two-thirds is obtained by armature voltage control and one-third by field control. The relatively small proportion of d-c. machines is reflected in the rating of the conversion equipment which, as shown

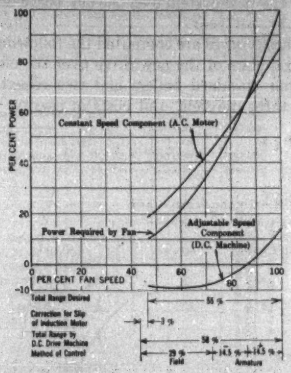

FIG. 3—POWER CURVES FOR FORCED DRAFT FAN

Maximum speed of fan	1,004 rev. per min.
Minimum speed of fan	452 rev. per min.
Rating of drive unit	166 hp.
Rated speed of a-c. motor	860 rev. per min.
Ratio of rating $\frac{\text{d-c. machine}}{\text{a-c. motor}}$	$\frac{14.5}{85.5}$

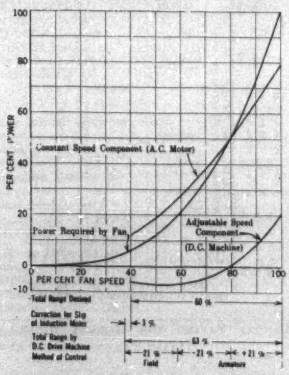

FIG. 4—POWER CURVES FOR INDUCED DRAFT FAN

Maximum speed of fan	1 088 rev. per min.
Minimum speed of fan	435 rev. per min.
Rating of drive unit	419 hp.
Rated speed of a-c. motor	860 rev. per min.
Ratio of rating $\frac{\text{d-c. machine}}{\text{a-c. motor}}$	$\frac{21}{79}$

by the above table, is but 1,140 kw. in motor-generator set capacity to control 7,020-hp. in motor capacity. An induced draft fan having a maximum speed of 1,004 rev. per min. would have still further reduced the rating of the motor-generator set capacity, but as the fans and building steel were on order for some time before it was decided to install this system of fan drive, the change was not regarded as practical.

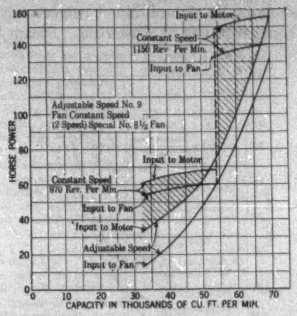

FIG. 5—COMPARATIVE PERFORMANCE CURVES FOR FORCED DRAFT FAN

REASONS FOR INSTALLING THIS SYSTEM AT POWERTON

The reasons for installing this system of driving the fans at Powerton are as follows:

(1) The simplicity and smoothness of the method of speed control.

(2) The energy saving due to the high efficiency of the method of control.

(3) The fact that the new system can be installed at nearly the same cost as the system previously used.

(4) The ease with which the system may be adapted to automatic combustion control.

(5) The successful experience with two 300-hp. units of this type installed at State Line Power Station.

BASIS OF COMPARING THE NEW SYSTEM WITH OTHER SYSTEMS

Various types of motors have been used for driving

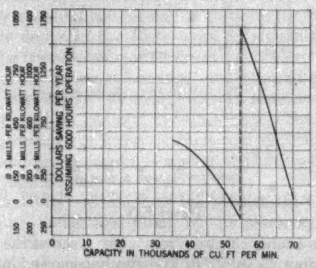

FIG. 6—ANNUAL SAVING FOR STANDARD FORCED DRAFT FAN

forced and induced draft fans. The motors most commonly used are of the induction type, in both squirrel-cage and slip-ring forms. As the draft fans on the boilers previously installed at Powerton are driven

by two-speed squirrel-cage type induction motors, the comparisons in this paper are made between the new system and that system.

Comparisons of the two systems are based on the installation of two 105,000-kw. turbo-generator units designated units No. 3 and No. 4 respectively, which are now being installed at Powerton. Steam for each

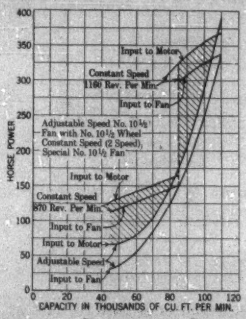

FIG. 7—COMPARATIVE PERFORMANCE CURVES FOR INDUCED DRAFT FAN

unit is furnished by two boilers of standard type and one boiler of the reheat type. The load curve for units No. 3 and No. 4 is assumed to have the same shape as the load curve for present units No. 1 and No. 2 (each rated 52,500 kw.) but increased in direct proportion to the increase in generating capacity.

ENERGY SAVING OF THE NEW SYSTEM AT POWERTON

An analysis of the application of this system to the fans of the six new Powerton boilers shows a distinct

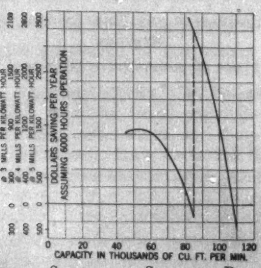

FIG. 8—ANNUAL SAVING FOR STANDARD BOILER INDUCED DRAFT FAN

saving in energy consumption. Fig. 5 shows the horsepower input to each fan and the horsepower input to each motor at various fan capacities for the two different systems of driving the forced draft fans. Fig. 6 shows for a standard boiler the saving in dollars of the new system over the old at various fan capacities based on operating 6,000 hr. per year.

Similar curves for the induced draft fan are shown in Figs. 7 and 8.

Figs. 9 and 10 show for the forced and induced draft fans respectively of a reheat boiler the saving in dollars of the new system over the old, based on operating 8,000 hr. per year.

From these curves are computed the following annual savings in dollars of the new drive over the old for the fans of four standard boilers and two reheat boilers, based on three different unit costs of energy:

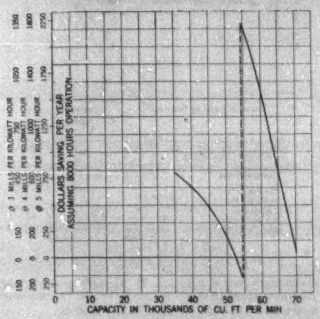

FIG. 9—ANNUAL SAVING FOR REHEAT BOILER FORCED DRAFT FAN

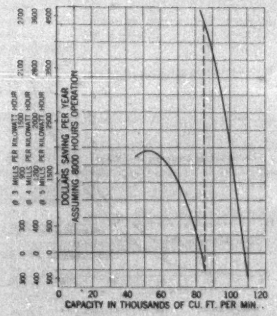

FIG. 10—ANNUAL SAVING FOR REHEAT BOILER INDUCED DRAFT FAN

	Energy cost per kw-hr.		
	3 Mills	4 Mills	5 Mills
Annual saving for 12 forced draft fans	$ 4,535	$ 6,047	$ 7,559
Annual saving for 12 induced draft fans	8,780	11,707	14,634
Total saving for six boilers	$13,315	$17,754	$22,193

The detailed analysis of these figures is given in Table IV.

TABLE IV

ESTIMATED ENERGY SAVINGS OF NEW SYSTEM OF FAN DRIVE OVER TWO-SPEED INDUCTION MOTOR DRIVES FOR BOILER DRAFT FANS OF POWERTON UNITS 3 AND 4

Savings are figured at 3 mills, 4 mills and 5 mills, per kw-hr. respectively. Detail figures used in the following table are based on 3 mills.

Turbine and boiler loads are based on average daily load curves of Units No. 1 and 2 pro-rated for the increased capacity

	The 24-hr. load is divided into 3 periods, as follows:		
Units No. 3 and 4:			
Hr. per day.....Hr.	11	8	5
Turbine load (2 turbines) Kw.	210,000	108,000	150,000
Steam requirements. ...lb./hr.	1,910,000	980,000	1,350,000
No. of boilers in operation.....	6	4	6
Steam from ea. reheat blr. lb./hr.	290,000	150,000	210,000
Steam from ea. std. blr. ..lb./hr.	330,000	340,000	233,000
	Fan outputs and savings:		
Std. boiler forced draft fan:			
Cu. ft./min. of air per fan.....	46,500	48,000	33,000
Annual saving per fan in each daily period.....$	78	43	76
Annual saving all fans in each daily period.....$	605*	170†	610*
Total annual saving all fans.....$1,385			
Std. boiler induced draft fan:			
Cu. ft./min. of gas per fan.....	79,000	82,000	56,000
Annual saving per fan in each daily period.....$	90	35	175
Annual saving all fans in each daily period.....$	720*	140†	1,400*
Total annual saving all fans.....$2,260			
Reheat boiler forced draft fan:			
Cu. ft./min. of air per fan.....	56,000	35,000	44,000
Annual saving per fan in each daily period.....$	565	160	63
Annual saving all fans in each daily period.....$	2,260†	640†	250†
Total annual saving all fans.....$3,150			
Reheat boiler induced draft fan:			
Cu. ft./min. of gas per fan.....	89,000	55,000	70,000
Annual saving per fan in each daily period.....$	1,080	385	165
Annual saving all fans in each daily period.....$	4,320†	1,540†	660†
Total annual saving all fans.....$6,520			
Grand total annual saving all fans of units No. 3 and 4—			
@ 3 Mills per kw-hr..$13,315			
@ 4 Mills per kw-hr.. 17,754			
@ 5 Mills per kw-hr.. 22,193			

*Indicates 8 fans in service.
†Indicates 4 fans in service.

Cost of the New System at Powerton

Energy savings of this magnitude usually call for a considerable additional capital expenditure. In this instance, such is not the case. Estimates show that the savings in miscellaneous accessory equipment required by the old system but not by the new nearly offset the higher cost of the drive units and motor-generator sets of the new system.

A detailed analysis of the estimated costs is given in Table V.

TABLE V

ESTIMATED COST PER BOILER OF FAN DRIVE AND DRAFT CONTROL EQUIPMENTS FOR THREE DIFFERENT SYSTEMS FOR POWERTON UNITS 3 AND 4

	Existing systems		New system
	Damper system (2-speed motors)	Inlet vane system (2-speed motors)	Adjustable speed drive*
Fan motors:			
2—Induced draft fan motors	$ 5,500	$ 5,500	$13,400
2—Forced draft fan motors.	3,300	3,300	8,300
—Supports and enclosures.	..	..	400
—Installation.....	800	800	2,000
—Circuit breakers and control, installed.....	5,000	5,000	6,650
1—Exciter motor generator set.....	..	..	550
—Speed selector switches for four fans.....	6,000	6,000	..
—Installation.....	1,100	1,100	..
—Power and control cable and conduit, installed.	2,000	2,000	3,000
	$23,700	$23,700	$34,300
Vanes and dampers:			
2—Sets vanes or dampers with mech. (ind. dft.).	$ 600	$ 2,000	$ 600
2—Sets vanes or dampers with mech. (fcd. dft.).	600	1,300	600
—Installation.....	300	1,050	300
2—Damper drive units.....	4,250	2,400	..
—Installation.....	360	360	..
—Cable and conduit, inst.	2,000	2,000	..
	$ 8,110	$ 9,110	$ 1,500
Fan speed change accessories:			
4—Centrifugal relay switch equipments.....	$ 425	$ 425	..
2—Centrifugal relay switch equip. (for interlock)..	215	215	..
—Installation.....	760	760	..
—Wire and cable, inst.....	800	800	..
	$ 2,200	$ 2,200	
Total cost per boiler.....	$34,010	$35,010	$35,800

*Based on machine ratings given in Table II.

Other Installations

This system of driving forced and induced draft fans is also being installed in two other power stations, *viz:* the Philo Power Station of the Ohio Power Company, and the Sheboygan Power Station of the Wisconsin Power and Light Company.

The bearing construction of the a-c. motors of the Sheboygan units will be similar to those for Powerton but the design of the Philo units differs somewhat in that the frame is supported directly on the rotor shaft through anti-friction bearings, and the motor shaft, which thus carries the weight of both rotor and frame, is supported on pedestal type bearings. The method of supporting the collector rings and of housing the rings and brush rigging is similar to that used on the Powerton machines.

An analysis was made of the problem of driving a centrifugal boiler feed pump by the new system compared with using induction motors of both squirrel-cage and slip-ring types. For the motors of the adjustable speed type, the speed range was assumed to be 100 to 85 per cent. The analysis gave the following results:

While the investment costs of the slip-ring and squirrel-cage motor drives were both lower than the

cost of the new system, the total annual cost, which includes both investment and operating costs, favors the new system. It was therefore decided to install this system of drive on one boiler feed pump at Sheboygan.

The brief summary of data relating to these units is given below:

PHILO POWER STATION
(Two Pulverized Fuel Boilers)

	Forced draft fans	Induced draft fans
Number of fans	4	2
Number of drive units	2	2
Maximum fan speed	1340-r. p. m.	1005-r. p. m.
Minimum fan speed	670-r. p. m.	502-r. p. m.
Range of fan speed	100% to 50%	100% to 50%
Constant speed component (a-c. power) at maximum fan speed	244-hp. @ 1160-r. p. m.	648-hp. @ 870-r. p. m.
Adjustable speed component (d-c. power) at maximum fan speed	38-hp. @ 180-r. p. m.	100-hp. @ 135-r. p. m.
Total power at maximum fan speed	282-hp. @ 1340 r. p. m.	748-hp. @ 1005-r. p. m.
Speed of d-c. machine at maximum fan speed	Plus 568-r. p. m.	Plus 340-r. p. m.
Speed of d-c. machine (operating as a generator) at minimum fan speed	Minus 1704-r. p. m.	Minus 1020-r. p. m.
Ratio of ratings d-c. machine/a-c. motor	13.5/86.5	13.5/86.5
Number of speed regulating motor-generator sets	2	2
Rating of each speed regulating motor generator set	30-kw.	85-kw.
Total rating of all drive units		2160-hp.
Total rating of all speed regulating motor-generator sets		230-kw.

SHEBOYGAN POWER STATION
(Two Pulverized Fuel Boilers)

	Forced draft fans	Induced draft fans	Boiler feed pump
Number of drive units	2	2	1
Maximum load speed-r. p. m.	1360	1005	1880
Minimum load speed-r. p. m.	680	502	1692
Range of load speed	100% to 50%	100% to 50%	100% to 90%
Constant speed component (a-c. power) at maximum load speed	157-hp. @ 1165-r. p. m.	339-hp. @ 870-r. p. m.	605-hp. @ 1770-r. p. m.
Adjustable speed component (d-c. power) at maximum load speed	26-hp. @ 195-r. p. m.	52-hp. @ 135-r. p. m.	45-hp. @ 110-r. p. m.
Total power at maximum load speed	183-hp. @ 1360-r. p. m.	391-hp. @ 1005-r. p. m.	650-hp. @ 1880-r. p. m.
Speed of d-c. machine at maximum load speed	Plus 500-r. p. m.	Plus 400-r. p. m.	Plus 535-r. p. m.
Speed of d-c. machine (operating as a generator) at minimum load speed	Minus 1500-r. p. m.	Minus 1200-r. p. m.	Minus 535-r. p. m.
Ratio of ratings d-c. machine/a-c. motor	14.2/85.8	13.3/86.7	6.9/93.1
Number of speed regulating motor generator sets	2	2	1
Rating of each speed regulating motor generator set	30-kw.	50-kw.	40-kw.
Total rating of all drive units			1798-hp.
Total rating of all speed regulating motor generator sets			200-kw.

The a-c. motors are designed to operate at 2,200 volts three-phase.

Another application of this system to the driving of power station auxiliaries differs radically from the ones previously described and it illustrates the versatility of the system. It consists of a 2,500-hp. drive unit geared to a reciprocating pump which delivers boiler feed water under a pressure of 1,500-lb. per sq. in. This application calls for constant torque at adjustable speed. The design of the drive unit differs from the Powerton fan drive units in that the armature

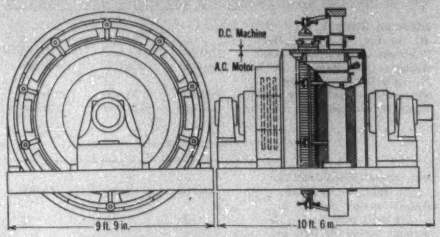

FIG. 11—2,500-HP. DRIVE UNIT

of the d-c. machine is assembled on the frame of the a-c. motor. The d-c. machine is thus concentric with the a-c. motor. The outline and dimensions of this drive unit are shown in Fig. 11. Two machines of this type are now under construction. The principal data pertaining to this application are given in the following table:

Max. shaft speed	708 rev. per min.
Min. shaft speed	282 rev. per min.
Range of shaft speed	100 to 40 per cent
Constant-speed component (a-c. power) at max. shaft speed	1750 hp. @ 495 rev. per min.
Adjustable speed component (d-c. power) at max. shaft speed	750 hp. @ 213 rev. per min.
Total power at max. shaft speed	2500 hp. @ 708 rev. per min.
Ratio of ratings—d-c. machine/a-c. motor	30/70
Rating of speed regulating motor-generator set for each drive unit	750 kw.
Over-all efficiency of drive unit and motor-generator set—	
At maximum speed	88.4 per cent
At middle speed	88.6 per cent
At minimum speed	78.3 per cent

The a-c. motors are designed for 2,200 volts, three phase.

AUTOMATIC COMBUSTION CONTROL

Present systems of automatic combustion control are complicated because of the complications of the draft control. In a system which employs two-speed squirrel cage motors for driving the fans a change in load which necessitates a change in motor speeds calls for a switching operation, the operation of dampers both before and after the switching operation and, when dropping from the higher to the lower speed, the proper timing of the switching operation to prevent undue mechanical stresses on the equipment. All of these operations must necessarily be controlled by the automatic combustion control system.

A new and simple system of automatic combustion control has been developed to take advantage of the simplified new system of fan control. The combustion control functions by simultaneously increasing or decreasing, in the proper ratio, the speeds of the forced draft fans, the induced draft fans, and in the case of pulverized fuel plants, the coal feeders, to meet a change in the demand for steam. While these adjustments are being made on the group as a whole to regulate for steam demand, secondary adjustments are made simultaneously on the individual controls to maintain the proper furnace pressure and the proper ratio of air to fuel. Three pilot motor-operated field rheostats make all the necessary adjustments on each boiler. Each fan control rheostat is driven by two pilot motors through a differential gear. One motor is controlled by the master pressure gage, while the other is controlled by either a furnace pressure gage or a fuel/air ratio control. These secondary corrections are superimposed on the master control adjustments through the differential gears.

This brief description gives some idea of the possibilities for simplification of the automatic combustion control equipment. A complete description of the system would be too long to include in this paper. A wiring diagram which shows the functioning of the principal control elements is given in Fig. 12.

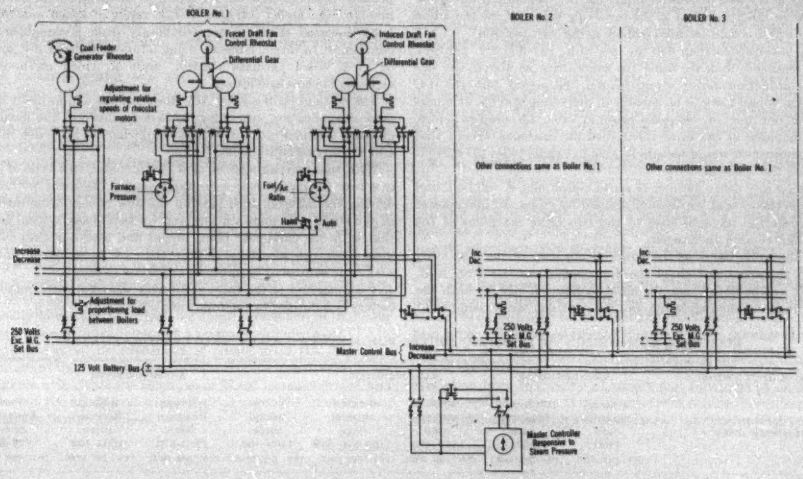

FIG. 12—WIRING DIAGRAM OF SIMPLIFIED SYSTEM OF AUTOMATIC COMBUSTION CONTROL

ACKNOWLEDGMENT

In conclusion, the author wishes to acknowledge his indebtedness to Mr. R. B. Williamson of the Allis-Chalmers Manufacturing Company for the interest he has shown in this system through the various stages of its development and for the assistance he has rendered in bringing it to its present stage of development.

Discussion

Fraser Jeffrey: To derive any benefit such as a gain in efficiency from a variable speed drive as the system described by Mr. Rossman, or a Ward Leonard system, or the Scherbius system, etc., it is necessary to operate at reduced speeds for reasonable lengths of time otherwise a regular single-speed slip-ring motor operating at, or nearly at its full speed will be more efficient than any of these multi-unit variable speed systems. Therefore, the necessity for adjustable speed is of prime importance before the selection of the apparatus can be intelligently made.

We might think of the drive just described as being essentially a constant torque drive best suited for operating loads of this nature but it can also be advantageously used for variable torque drives such as fans or centrifugal pumps. It has the further advantage of maintaining constant speed regardless of load and with this characteristic should find a much wider field of application than it has up to the present time. It is not necessarily confined to equipment of relatively small capacity and it is of interest to note that units of this type are now being built which have a rating of 2,500 hp. with a two and one-half to one speed range on a constant torque load. The capacity and speed range desired, dictates largely whether the armature of the d-c. machine should be rigidly connected to the revolving stator of the a-c. machine, or flexibly connected to it by means of texropes capable of operating on short centers at a fairly large speed reduction thus allowing the use of a standard d-c. machine.

Outside of the fact that the Rossman system is highly efficient over its entire speed range because the converted power is only that relatively small amount represented by the speed range and not the entire power, the use of practically standard types of machines for the auxiliaries and, with the exception of mechani-

cal features, the same for the a-c. machine of the main drive units, tends toward simplicity, low maintenance costs and the use of apparatus with which the operating engineer is most familiar.

The Rossman system has some points pertaining especially to the relationship of power requirements with respect to speed range and kind of load that are interesting.

While the relative capacities of the d-c. machine and the induction motor of the drive unit are computed from a socalled "base speed," which is the full load speed of the induction motor, theoretically the size of these machines will always remain unchanged regardless of what the base speed is in its relationship to the limits of the speed range desired, provided the power required at the high speed remains the same. For instance, assume a constant torque condition of load requiring 600 hp. at a maximum speed of 720 rev. per min., a base speed, neglecting slip of 600 rev. per min. and, that a speed range of 120 rev. per min. plus or minus is required. Then for either the maximum speed of 720 rev. per min. or, the minimum speed of 480 rev. per min., the d-c. machine would have to be capable of delivering 100 hp. at a speed of plus or minus 120 rev. per min. if the armature were direct coupled to the revolving stator of the induction motor which would be rated 500 hp. at 600 rev. per min.

If a lower speed than 480 rev. per min., say 360 rev. per min. were required, then by using the same drive as above, the size of the induction motor would remain unchanged, but, the d-c. machine would have to be capable of delivering 200 hp. at minus 240 rev. per min., or, twice the power at twice the speed as before, or, a machine of the same theoretical physical size, except that the commutator would have to be able to handle twice the current at twice the speed as before.

Taking any other number of speed conditions, it will be found that the theoretical horsepower divided by speed, or, horsepower per revolution physical sizes of the two drive machines of the main unit remain unchanged.

On the other hand, as the power or capacity of the regulating unit is the same as that of the d-c. machine of the drive unit, then it will be noted that the most economical size for both the drive unit and the regulating unit for constant torque conditions will be obtained when the base speed is half way between the maximum and minimum speeds.

For a variable torque load, however, this does not hold true, as it is then desirable to have the base speed closer to the high speed, so that the plus capacity of the d-c. machine of the drive unit will be as small as possible consistent with the mechanical limitations that might be imposed on either the induction motor stator or the d-c. machine of the drive unit when rotating negatively at maximum speed. For fairly large speed variations such as two to one, or three to one, the capacity of the d-c. machine of the drive unit might be set as some value of load approximately the same or somewhat less than that required by the fan at its lowest speed and then the size of the regulating unit will also be as small as possible. Thus, if we have a two to one variable speed, variable torque fan load requiring 600 hp. at 720 rev. per min. for the maximum speed, and if it were possible to select a base speed of 630 rev. per min., then the size of the induction motor would be 525 hp. at 630 rev. per min. The size of the d-c. machine for the high speed for 720 rev. per min. would be 75 hp. at plus 90 rev. per min., which is the exact load required (75 hp. at minus 270 rev. per min.) to drive the fan at the low speed of 360 rev. per min.

In actual practise it is necessary to adhere to "base speeds" that correspond to a particular synchronous speed, and to consider the mechanical limitations of the revolving stator of the a-c. machine and also the d-c. machine of the drive unit.

C. M. Gilt: There is no question in any one's mind that there is a large field for a more satisfactory method of obtaining speed adjustment in a-c. motors. Particularly in the larger size motors, present methods are both expensive in installation cost, and involve high losses. Where a wide range of speed control has been essential it has quite commonly been accomplished by conversion to direct current, using adjustable speed d-c. motors or even Ward Leonard control, both of which methods are expensive in cost and losses.

The method proposed by Mr. Rossman has undoubtedly a real field of application, providing, as it does, much of the flexibility of Ward Leonard control, without requiring the large d-c. motors and converting equipment.

With these advantages in view, we included this type of drive in a very intensive study comparing several electrical methods of driving the induced draft fans on the new boilers to be installed in our Hudson Avenue station, and we were disappointed to find that with our particular plant layout and load duration curves, total costs ran somewhat higher than on squirrel-cage motors with vane control or on slip-ring induction motors.

A comparison of the total costs with their various component factors is indicated in the following table:

ECONOMIC COMPARISON OF ELECTRIC DRIVES FOR SIXTEEN 1,000-HP. INDUCED DRAFT FANS

Method of draft control =	Constant-speed		Vane regulation		Adjustable speed			
Type of electric drive =	1-Speed squirrel cage 1160 rev. per min.	2-Speed squirrel cage 1160/580 rev. per min.	2-Speed squirrel cage 1160/870 rev. per min.	3-Speed squirrel cage 1160/870/580 rev. per min.	19-Point wound rotor 1160–400 rev. per min.	1-Range Rossman drive 1340–670 rev. per min.	2-Range Rossman drive 1005–100 rev. per min.	Ward Leonard drive 870–50 rev. per min.
Electrical installation	23.9	28.0	27.5	30.8	41.5	49.5	57.6	84.0
Regulating damper installation						0.5		
Vane installation	2.6	2.6	2.6	2.6				
Space evaluation	2.3	3.1	2.8	3.9	5.5	4.3	5.4	4.1
Capacity charge	47.5	47.7	43.2	43.1	51.3	44.7	43.7	61.5
Energy charge*	23.3	10.2	14.2	9.9	17.6	14.6	8.4	26.5
Electrical maintenance*	1.2	1.5	1.4	1.7	2.7	6.9	7.5	4.2
Fan maintenance*	9.6	9.6	9.6	5.9	5.4	5.4	4.7	4.7
Vane maintenance*	8.2	3.3	3.3	2.1				
Total	113.6	106.0	104.6	100.0	124.0	125.9	127.3	185.0

*Capitalized on the basis of a 12½ per cent carrying charge.

This study was based upon sixteen 1000-hp. driving units each of which in the case of the Rossman drive consisted of an 850-hp. squirrel-cage a-c. motor and an 150-hp. d-c. adjustable speed motor. Two methods of obtaining draft control at speeds less than 50 per cent were considered, first by damping, second, by using a two-speed squirrel-cage motor with the adjustable-speed d-c. motor superimposed. Previous studies on different size fans using a single-speed a-c. motor with a large enough d-c. motor to reduce the speed to 15 per cent of maximum showed a

higher cost due to the large d-c. motor and motor-generator sets required.

In our installation, expensive building changes would be required for either the slip-ring, Rossman or Ward Leonard drives. In the above tabulation, these building changes are not included but the required space is evaluated at $0.75 per cu. ft., and is found by increasing each dimension of a piece of equipment by two ft. and then doubling this volume to provide adequate space for handling.

Energy was estimated at $0.003 per kw-hr. A life of 28 years was assumed with a usage increasing during the first three years of life and then a gradually decreasing hp-hr. usage per year. The load factor and method of operation in which all boilers operate at reduced ratings rather than banking some during the daily light load periods require many hours of service at light loads, in which the total energy consumption, regardless of motor efficiency is small.

The capacity charge was based on a fan output which loaded the motors to about 75 per cent of their full load rating as it was found that the maximum demand of the station, with 10 per cent of the boilers out, would place this load on the motors, the additional capacity being installed to provide for emergency conditions of fan or boiler outage. The capacity and energy charges for the Ward Leonard control may appear surprisingly high. They may be accounted for in the low over-all efficiency of the motor-generator sets and d-c. motor drives expecially at the light loads under which these sets operate during the greater part of the time.

Installation costs include engineering, drafting, contingencies and overhead, with somewhat greater margin in engineering and drafting allowed for the Rossman drive due to lack of experience with such an installation.

The electrical maintenance charged against Rossman drive may appear high, but it was believed that the maintenance (including blowing out, cleaning inspecting, repairing, etc.) for the d-c. portion of the Rossman drive, would be more than that of a squirrel-cage motor, in addition, there is the maintenance of the motor-generator set, the high-voltage a-c. rings, switches, rope drive, etc. Fan maintenance, including fan reblading is materially lower for the adjustable speed drives due to the lower average speed with the consequent decrease in cinder wear.

In these figures, no evaluation is placed upon the intangible advantage of the smoothness and wide range of speed adjustment obtainable with either the Rossman or Ward Leonard drives, and its effect on fire conditions, a factor which cannot be ignored when the tangible factors are close.

It is apparent from the figures in the above tabulation that the capacity charges for the squirrel-cage motors and the Rossman drive are approximately equal, and while there is some saving in energy for the Rossman drive over any of the other methods, this saving is not great enough to offset the higher installation costs.

It is quite evident, however, that with a load duration curve such as would apply if the load factor on the station were considerably better, the energy item would assume such proportions as to throw the balance of over-all cost in favor of the Rossman drive.

R. R. Sheely: The fact that Mr. Rossman uses the d-c. regulating machine as a motor through part of the speed range and as a generator through the remainder of the speed range permits the use of a comparatively small d-c. machine. This also accounts for the comparatively high efficiency of the scheme as compared to adjustable speed drives such as the Kramer or Ward Leonard systems since the major portion of the energy is transferred directly from the a-c. bus to the mechanical load by means of the special a-c. motor.

The special a-c. machine is the most undesirable unit of the system. Slip rings operating continuously at voltages commonly used for power house auxiliaries will require frequent inspection and cleaning or trouble will be sure to result. The development charges will also be high since practically every installation will be sufficiently different from previous jobs to prevent the use of duplicate parts. It appears, however, that the advantages to be gained from the simplified combustion control system when using this type of drive will overbalance the disadvantages of the introduction of complicated electrical machines.

R. A. Hentz: The author stated that the motor-generator set under certain conditions acts as a motor and in other cases as a generator. Can this motor be either synchronous or induction and what is usually used? I presume, as a stator is excited from the source of supply, an induction motor could be used to operate as an induction generator when delivering power back to the system.

A. M. Rossman: Answering Mr. Hentz' question,—the a-c. machine of the motor-generator set or the a-c. motor in the case of the drive unit may be of either the synchronous or the induction type. In the equipments that have been purchased up to date the a-c. machines have all been of the induction type. When delivering electrical energy the induction motor receives excitation from the source of supply and functions as an induction generator. Some of our more recent studies, involving large sized units, are based on a-c. machines of the synchronous type.

Transformers with Load Ratio Control

BY ARTHUR PALME[1]

Member, A. I. E. E.

Synopsis.—*Voltage regulation on transformers under sustained load has made rapid strides in the last eight years and found very wide application in extended power systems and on industrial loads. Originally it was found economical to equip only very large transformers for load ratio control, using apparatus designed for high currents.*

This paper describes, and compares with earlier developments, a novel equipment of low current rating, that has been devised especially to control small blocks of power, thereby greatly widening the economic field of application of load ratio control. The electro-mechanical features of this equipment are stressed particularly.

* * * * *

FROM a small beginning in 1923, changing of ratio on transformers under load, or "load ratio control," developed rapidly until it has now become quite indispensable, without which many a modern transmission system and network would be under a decided handicap. Reducing the amount of reactive kv-a. flowing between power stations, it increases the efficiency and the maximum output of systems; it allows a flexible tie-in between systems; and it finally offers a simple and economic method of regulating voltage for industrial loads.

There have been numerous publications on this subject, and it is assumed that the reader is already familiar with the modern practise of designing a transformer winding with taps, so arranged as to permit changing ratio under sustained load. Classified according to basic principle, the following three methods are being used today:

1. A single winding with a tap for each voltage, two selector switches and two current interrupting devices. (See diagram 1A of Fig. 1).
2. A single winding with N taps for $(2N-1)$ operating positions, two selector switches and two current interrupting devices. (See diagram 1B of Fig. 1).
3. A single winding with N taps for $(2N-1)$ operating positions, with a current interrupting device for each tap. (See diagram 1C of Fig. 1).

Method (1) will give uniform steps in ratio under all conditions of load and power factor. Since the reactor is short-circuited on all operating positions, it does not increase the no-load loss on any ratio, nor does it add any reactive kv-a. to the system.

Method (2) has additional no-load losses and reactive kv-a. on alternate positions, and requires a reactor twice the size of the one needed for Method (1) which introduces an appreciable transient drop in voltage during the process of changing taps.

Method (3) also gives additional no-load losses and reactive kv-a. on alternate positions, and furthermore gives incorrect or unequal ratio steps when loaded at less than unity power factor. This inequality of steps makes it necessary, therefore, to provide a greater number of steps than for Method (1) for a specified voltage range.

The shortcomings of Method (3) can be alleviated by providing an additional current interrupting device, which short-circuits the reactor on alternate operating positions.

The advantages of Method (1), explained above, plus the fact that it lends itself readily to the standardization of parts for use on a variety of ratings, were the deciding factors leading to the development of the apparatus herein described.

During the past few years the rapid growth in the number of load ratio control transformers has enabled the designer to obtain valuable operating experience and to formulate exact requirements for the essential parts of these equipments.

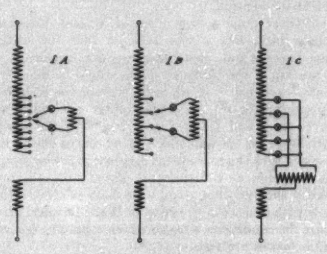

FIG. 1—THREE METHODS OF LOAD RATIO CONTROL, EACH FOR NINE VOLTAGES

The cost of the necessary electrical and mechanical accessories to permit changing of taps under load, is an appreciable percentage of the over-all cost of the transformer. With a given and standardized mechanical equipment, the cost of the load ratio control apparatus proper does not change materially with the amount of power it has to handle. This means that the smaller the kv-a. rating of the machine to which it is applied, its cost in per cent of that of the whole transformer will obviously be higher. Considering, however, the many operating advantages

1. General Transformer Engg. Dept., General Electric Company, Pittsfield, Mass.

Presented at the Middle Eastern District Meeting No. 2, of the A. I. E. E., Philadelphia, Pa., October 13-15, 1930.

which the installation of such a machine affords, this increase seems sound and justified, provided the output of the transformer is not too small. Experience and past demand indicated that the majority of transformers which were equipped with load ratio control were of such size that the current to be handled by the selector switches and interrupting device was between 500 and 1,000 amperes. Consequently, a line of apparatus was developed suitable for this range of current and for circuit voltages ranging from 15 kv. to 73 kv.

The wide use of load ratio control transformers in this country can perhaps best be appreciated when it is realized that the total regulated transformer capacity now installed is more than five million kv-a. It was perhaps a happy coincidence that load ratio control was developed just about the time that the use of long transmission systems, networks, and tie-ins began to spread rapidly over the country.

Having thus established a firm foundation for

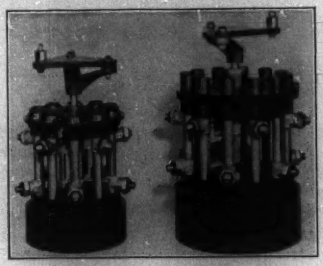

Fig. 2—Ratio Adjusters up to 1,000 Amperes for 9 and 11 Positions, Respectively

heavy current apparatus, the demand for a similar equipment, suitable for much smaller currents, and therefore smaller in size and lower in cost, became acute. With a load ratio control apparatus of such a type, the cost per cent of the switching equipment for relatively small transformers can be maintained at about the same economically sound ratio as for large units.

Following this plausible route of reasoning, the existing heavy current type of load ratio control equipment has recently been augmented by a new line of apparatus for voltages up to 15,000 volts and currents not in excess of 400 amperes. Experience in the field, covering a period of several years with the heavy current equipment, justified the use of the same method of operation for the small type. In other words, diagram 1A of Fig. 1 applies unchanged to either of the two types; the difference being entirely in the mechanical construction. A short description will best serve to illustrate the differences.

A. Heavy Current Type

For current ratings up to 1,000 amperes, the multipoint selector switch, or ratio adjuster is a very substantially built device consisting of two molded compound heads, into which are fastened securely 10 or 12 round copper rods. (See Fig. 2.) To insure additional electric strength between adjacent rods, a short collar of "herkolite" surrounds each rod where it goes through the top and bottom head. In the center of this cage-like structure moves a set of copper contact fingers, each under the pressure of a strong helical compression spring. A combination of crankshaft and cam arrangement causes this set of fingers to slide on and off each rod as the center shaft is revolved, but maintaining at the same time permanent contact with the center which is connected to one of the 10 or 12 rods. To 9 or 11 of these rods are carried cables of suitable size from the taps in the transformer winding. The wiping action of the fingers on each rod immediately before making final contact, and the mechanical sturdiness of the whole adjuster (the 10-point type weighs 82 lb., the 12-point type weighs 116 lb.) are important features of these switches. Operating always under oil, the lubricating problem is easily solved. Mechanical life tests on these adjusters showed no visible wear after half a million operations.

For a single-phase transformer two of these adjusters are required; for a three-phase transformer two three-phase stacks are necessary. According to the cycle of operation, provision must be made to operate these adjusters intermittently, that is, on a single-phase transformer one of them has to be reset at a time. On a three-phase transformer one three-phase stack must be reset at a time. This is accomplished by a specially designed *intermittent gear*, consisting essentially of a gear sector which when turned, engages alternately with a small pinion at the right and one at the left; the two pinions transmitting their motion to the operating shafts of the ratio adjusters. Gear and adjusters are mounted as a rule, inside the transformer tank and form a self-contained unit with core and coils as shown for a single-phase example in Fig. 3, with the main driving shaft brought out oil-tightly through the wall of the transformer tank.

Consistent with a maximum current rating of 1,000 amperes, a heavy duty current interrupting device is fastened to the outside of the transformer tank. A special type of *oil-immersed contactor* was developed for this purpose. The very high kv-a. rupturing capacity to which modern circuit-breaker designers justly point with pride, is not required for the duty imposed upon breakers in circuits as in Fig. 1. Current carrying ability, good insulation to ground, short-circuit strength and long mechanical life are of sole importance. Life tests have indicated that for very frequent operation such as encountered in load ratio control, as compared with the only occasional opening of a circuit breaker on

a line, a much longer life of arcing contacts results if they separate horizontally forward rather than downward, because the arc has less chance to burn the contacts.

These considerations have led to the design of a single break, forward motion, double contact type of

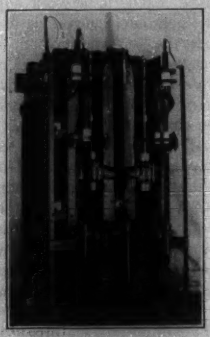

FIG. 3—CORE AND COILS FOR SINGLE-PHASE TRANSFORMER, EQUIPPED FOR LOAD RATIO CONTROL

Transformer rated 12,500/18,750 kv-a., 69,000 to 13,800 volts, with load ratio control of the 73-kv. type in the high-voltage winding

contactor; one pair of contacts to carry the current under normal operation, and one pair of arcing contacts. The contactor unit is mounted on two heavy type petticoated porcelain through-bushings, which

FIG. 4—SINGLE-PHASE CONTACTOR PANEL FOR 37-KV. CIRCUIT AND UP TO 1,000 AMPERES

Oil tank removed

are bolted to a thick steel plate. (See Fig. 4.) For single-phase transformers such a steel panel carries two contactors; for a three-phase machine, six contactors. A scroll cam and two steel rollers moving in its groove, together with two sets of toggle levers, translate the rotating motion of a vertical drive shaft into intermittent opening and closing of the contactors. While the steepness of the groove in the scroll cam determines the *closing* speed of the contactors, their *opening* is made trip-free and very fast by the roller upsetting the toggle. The whole arrangement is mechanically very simple. Depending upon the circuit voltage, these contactors can be mounted on porcelain bushings for 15-, 37- or 73-kv. operating voltage. Contactors of this type are capable of rupturing 1,000 amperes at least 7,500 times with one pair of arcing tips, representing an average useful life of over two years, at the end of which time the two pairs of tips per phase can be readily and cheaply renewed. Enclosing the contactors and bolted oil-tightly against the main steel panel, is an oil tank. For the three-phase type, insulating flash barriers are placed between phases.

For the remote operation of these mechanisms is provided a powerful—

Motor Drive, located underneath the contactor

FIG. 5—EXTERNAL VIEW OF SINGLE-PHASE TRANSFORMER WITH LOAD RATIO CONTROL

Transformer rated 20,000 kv-a., 132,000 grounded Y to 22,000 volts

tank. On transformers in short tanks, the drive is directly underneath, while on tall tanks the drive is lowered near the trucks so as to be conveniently accessible for inspection or emergency hand operation. In addition to the customary accessories such as a motor reversing relay, a positioning controller, a limit switch, a dial for local and remote position indication, etc., the drive also contains an automatically operated clutch which disengages the motor if an attempt should be made to run it beyond limiting positions. Worthy of note also, is the introduction of electrodynamic braking of the motor, which insures prompt stopping without the necessary maintenance of brake bands. A weatherproof and dustproof housing encloses the motor drive.

Fig. 5 shows the external appearance of a complete, heavy current type load ratio control transformer. This is one unit of a 60,000-kv-a. three-phase bank,

rated 132,000 volts grounded Y to 22,000 volts, with control at the neutral of the 132-kv. windings. Nine operating positions are provided between the no-load ratio limits of 116,000 and 148,000 volts.

B. Low-Current Type

The main difference between the heavy-current type and the new equipment for lower current and voltage lies in the simplified type of selector switches (ratio

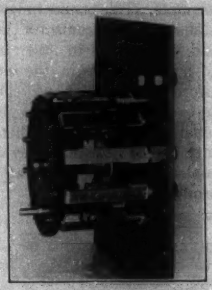

Fig. 6—Twin Ratio Adjuster for Nine Operating Positions and up to 400 Amperes

adjusters) in a smaller type of oil-immersed contactors, and a smaller motor drive.

1. Ratio Adjusters. To comply with the diagram 1A of Fig. 1, two ratio adjusters or selector switches are required, the individual points of which are connected to one set of transformer taps. To insure the proper mechanical sequence of operation, provision has to be made to move these two sets of switches one at a time. In the large type of apparatus an intermittent gear is used to accomplish this. In the smaller sizes it has been found possible to combine two ratio adjusters and the intermittent gear into one piece of apparatus called "twin ratio adjuster." (See Fig. 6.) The construction comprises essentially two dial type switches, each operated by a Geneva stop gear and a driving pawl. When turning the main driving shaft one complete revolution, each dial switch will be moved a definite angular distance from one tap contact to the next.

Formerly known dial switches were either of the solid contact and movable brush type, or of the stationary clip and moving rigid knife type. Both of these constructions, even when made very accurately, are difficult to maintain in perfect contacting condition. The new dial switches have stationary knife contacts and movable but *self-alining* clip contacts. Pressure on the clips is maintained by reliable helical compression springs. The stationary contacts are copper bars to which are welded suitably located copper blades. Nine of these copper bars are held in a cage-like fashion between two heavy compound end-plates. The moving clip contacts are attached to two molded compound Geneva gears, each with nine scallops, corresponding to the nine fixed contact bars.

A twin ratio adjuster of this type represents therefore, a very compact unit, giving the two sets of contact fingers and the desired intermittent motion in a mechanically simple way. The wiping nature of the contact assures excellent current carrying ability, which further improves with use. Electro-dynamic stresses in case of a short circuit are entirely eliminated, since the contact clip grips the stationary blade from both sides. Tests with over 25 times normal current (more than 10,000 amperes) showed neither welding nor sputtering. One such unit is required for a single-phase transformer; three of them for a three-phase machine. In the latter case, the three twins are gang-mounted in a straight line, with insulating couplings between them. It is customary to use the rear plate of the twin adjuster as an oil-tight window, fitting over an opening in the upper part of the transformer tank so that the adjuster is mounted outside of the main tank in its own oil tank.

2. Contactors. The current carrying and current interrupting duty of these new, smaller equipments, being limited to 400 amperes, makes it possible to simplify the design of the contactors. (See Fig. 7.) A great many laboratory tests were made, to investigate the possibility of carrying and arcing on the same pair of contacts, with very satisfactory results. Even at double the rated current (800 amperes) thermo-couples fastened to well-worn tips, recorded after several hours no undue temperature rise of the contacts. Consequently, the new light-duty contactor was made with only a single pair of contacts per pole.

Fig. 7—Single-Phase Contactor up to 400 Amperes

Removed from oil tank

Further research indicated that the position of the contacts has a decided influence on the life of the arcing tips, as explained for the heavy current type. Therefore, the design of these contactors is also made so as to open forward in a nearly horizontal plane. A further simplification was made possible by mounting the con-

tactors on an insulating panel, without the use of porcelains, and by using insulating arms on the contactors.

A circular metal scroll cam, located between the two contactors, accomplishes enforced opening and closing of the two contactors in proper and definite time

Fig. 8—Assembly of Twin Ratio Adjuster and Contactor for Single-Phase Load Ratio Control

Removed from its oil tank

sequence with the twin ratio adjuster, which is driven by the same shaft.

The contactors, with their simple cam mechanism, are located in a small cubical steel tank, which contains also the twin ratio adjuster already described. The arcing of the contactors gradually carbonizes the oil, which necessitates taking two precautions:

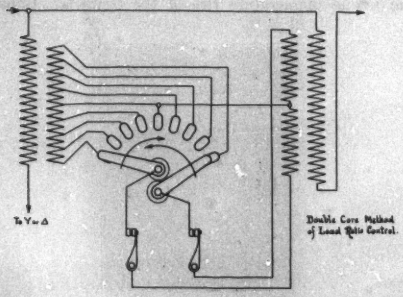

Fig. 9—Single-Phase Diagram for Double-Core Method of Load Ratio Control

1. All insulating surfaces are in a vertical position to prevent carbon depositions.

2. The oil in the arcing compartment is prevented from mixing with the oil in the ratio adjuster compartment.

Fig. 8 shows a view of the assembled ratio adjuster and contactor panel taken out of their oil tank.

For certain transformer connections, particularly for double-core transformation on very high-voltage regulating transformers shown in diagram Fig. 9 it becomes necessary to move the two contact arms of a twin ratio adjuster in opposite directions. This is accomplished readily by the introduction of a pair of reversing bevel gears.

3. Motor Drive. Depending upon local circumstances, various methods of remote motor control are applicable to these small load ratio control equipments. A bank of three single-phase transformers can be operated by *one* motor drive with interconnecting shafts between the three units. This results in lowest cost, great simplicity, and absolute synchronism of the three phases. Universals and slip-joints in the interconnecting shafting make misalinements in the set-up of the individual transformer units, of no consequence. It is also possible to provide each single-phase unit with its own individual motor drive, all of them connected to a common control switch, so that they all pick up together and complete their cycle of operation as governed by a controller on each unit.

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Discussion

L. F. Blume: The development of load ratio control apparatus for application to large high-voltage power transformers has been fully justified by their successful operation over a number of years, and by the increasing demand for them on large interconnected systems. The fact that in this field the inherent cost of the mechanical parts was a rather small fraction of the transformer cost warranted the development of the mechanism on the most conservative lines, in order to obtain complete assurance of reliable operation and also the best possible electrical characteristics. This point of view produced a piece of apparatus, which, on account of the inherent cost of the mechanical structure, became uneconomical for application to smaller transformer ratings.

The smaller outfit which is described in the paper under the heading of "Low Current Type," was developed primarily to meet the demand for a less expensive equipment for application to moderate size transformers. It is obvious that it would have been a step backward to obtain a less expensive outfit by sacrificing either reliability of operation or the many desirable electrical features of the larger size. Such features as the following are all valuable in the smaller as well as in the larger applications.

1. Uniform voltage steps throughout the range of control at all loads and power factors.
2. Elimination of circulating current on all operating positions.
3. Minimum losses and costs obtained for current limiting reactor.
4. Flexibility in application.
5. Two multiple contacts on all operating positions.

Cost reduction was brought about without sacrificing any of the above features, by an intensive study of the mechanical design, having in mind the simplification of the mechanical structure and taking advantage of the accumulated knowledge of the performance of load ratio control in test and in the field during the last five years.

John S. Lennox: We have found that important advantages in manufacture and installation have resulted from assembling the ratio adjusters and contactors in the manner described.

Until the development of the "twin ratio adjuster" this unit assembly mounted entirely separate from the core and coils of the transformer was not practical without sacrificing the important advantages of method (1) as listed in the paper. Furthermore, it is accomplished in this design with a minimum length of tap leads, because the connections to the ratio adjuster are made on the inside of the transformer tank.

The sturdy construction, heavy contact pressures and complete mechanical interlocking that have resulted in uniformly successful operation of the heavy current equipments have been retained.

This design, which will be known as type LR-12, has proven so satisfactory in manufacture, application and operation that its use is being extended to other ratings. A 1000-ampere, 15,000-volt equipment has already been developed, only those changes being made as are required for the higher current rating.

W. W. Edson: The advantage of load ratio control soon won its adoption to many transformers, but, as so often occurs with new apparatus, mechanical defects of the equipment itself have frequently tended to discredit such installations.

A single mechanical failure may cause a disastrous electrical fault or may produce an adverse reaction against tap-changing equipment in general. An installation for which the customer is paying thousands of dollars may be marred by the use of a cheap and unreliable contactor or by attempting to use a house lighting switch as a limit-switch. Frequently the gears are fastened to a shaft by inadequate pins instead of keys, thus introducing the possibility of the equipment being left on different phases. Operating cams or other details made of insulating material may not have been properly dried and the entrapped moisture might cause an electrical fault.

Accessibility for maintenance, inspection and repairs, visibility of the operation indicator, ease in manual operation, etc., are requirements which must be given careful consideration in designing new equipment such as this.

The final inspection and tests should be made after the tap-changing equipment has been installed on the transformer to insure there has not been an unexpected lowering of the voltage rating and to prevent any possibility of the three phases being out of step with each other.

Forces in Turbine Generator Stator Windings†

BY J. F. CALVERT*
Associate A. I. E. E.

Synopsis.—*Experience has shown that the forces on turbine generator armature windings during short-circuit conditions are large. The windings may be seriously damaged if adequate consideration is not given to the design of the coil bracing and support.*

In this paper the forces are calculated for a particular case by the use of images, by the change of flux interlinkages, and by the $\int H^2 d s$ *over some surface in air which surrounds the conductor under consideration. A new and relatively simple proof is given for the validity of the latter method. The limitations and desirable applications of each method are considered.*

Forces on conductors in armature slots are computed when saturation is considered, and when neglected. Approximate formulas are derived for the forces on coil sides within one layer of the conical end winding, and for the straight coil extensions from the slots. A numerical example is given in each of those cases. The applicability of certain of these formulas to the phase connecting rings is pointed out. The force on the entire conical surface of the end winding is discussed qualitatively.

Certain desirable features of end winding bracing are mentioned.

In most of these cases, the instant considered is that following a line-to-neutral short circuit when the maximum possible instantaneous current is flowing.

* * * * *

I. General Discussion of Forces during Short Circuit

A. General Problem. It is the purpose of this paper to consider the maximum forces exerted in turbine generators on the various parts of diamond coil stator windings. Conditions at the instant when the maximum possible instantaneous current flows in the armature winding on a line-to-neutral short circuit are given the most attention. A great deal of work has been done by previous investigators to establish methods for pre-determining the magnitudes of the currents, which result from instantaneous short circuits. The various cases, such as line to line short circuits, line to neutral short circuits, etc., have been considered in great detail. Much work has been published on the calculation of forces on bus structures and switches. However, it is believed that very little, if any, work has been published concerning the forces exerted on the windings of rotating machines during transient conditions.

A very brief consideration of the magnitudes of the short circuit currents and the proximity of these in the coil sides will quickly convince one of the importance of these forces. Peak values of 50,000 amperes in parallel conductors with a spacing of about 1¾ in. from center line to center line of the coil sides is quite a reasonable possibility in well designed machines of only moderately large ratings.

Three things are necessary in the study:

1. The determination of short-circuit currents,
2. The determination of the flux fields, and
3. Methods of calculating the forces exerted on the conductors after the currents and the flux fields are known.

The peak value of the armature current per coil side can be determined from the works of various authors.[1,2] In this paper, it will be assumed that the armature currents can be computed with reasonable accuracy, (see Appendix A). The other currents, or perhaps it should be said magnetomotive forces, will be determined through the very useful principle applied by the same authors, namely, that of the tendency for currents to flow in such directions as to maintain constant flux interlinkages in all the closed electric circuits in the metallic parts of the machine.

The flux distributions will be established by means of the relation $\int H' d l' = 0.4 \pi I$ (see Appendix B). The usefulness and applicability of this principle has been thoroughly demonstrated by the quite accurate solutions of a number of the more important two dimensional flux fields in electrical machines.[2,3,12]

The forces will be calculated in this paper for two dimensional fields by one of three methods; each of these will be demonstrated later in detail. The first, is based on the use of images, and that the force exerted between two parallel conductors is proportional to the product of the currents, to the length of the conductors, and inversely proportional to the distance between the conductors.[4,5]

The second method is one which is very commonly employed. It is based on the rate of change in interlinkage (and, hence, of stored energy) with the displacement of a conductor while the currents are maintained constant. The third method is based on the fact that the force on any current carrying conductor existing in an electric field can be determined from $\int H^2 d s$, taken over some surface surrounding the conductor.[6]

In one instance the force will be computed by each of these three methods to show that for the same boundary conditions, all give the same results. In other cases, it may then be assumed that the most convenient method is the best. However, the third method mentioned above will appear to be the most promising where saturation must be considered. This is very

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commonly the case, because the enormous currents necessary to produce forces of real consequence are often so situated that they do saturate some of the adjacent iron parts.

B. Division of Problem. The problem is arranged in parts according to the most convenient divisions of the flux fields. These will be taken up as follows:

1. Forces on conductors in slots.
2. Forces within either layer of the conical surface of the end winding exerted on the sides of the coils.
3. Forces on the straight coil extension from the slot.
4. Forces on the phase connecting rings.
5. Forces on the whole conical surface of the end winding.

II. Detailed Discussion of Forces during Short Circuit

A. Forces on Conductors in Slots

In the study of the forces acting on the embedded portions of the armature conductors, there are three main problems to be considered, *i. e.*:

1. Is the magnitude of the compressive force on the coil sides great enough to damage or rupture the insulation?

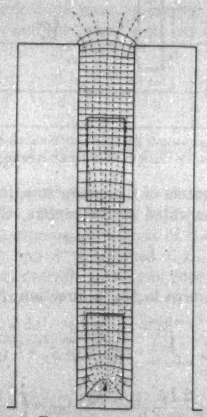


Fig. 1—Currents Equal and in the Same Direction

Iron of infinite permeability and resistivity

2. Are there repulsive forces between coil sides which might send a coil into the air gap (carrying the stator wedge with it)?
3. Are there forces which may rend apart the strands of one coil side?

As previously mentioned, it is desirable in some one instance to use all three of the methods for calculating the forces, *i. e.*, those involving—(1) images, (2) change of interlinkage with displacement, and (3) integral of H^2 around each conductor. This will be done for the case of embedded armature conductors when saturation is not considered.

Figs. 1, 2, and 3 illustrate the flux distributions in rectangular slots for three different current values. (These are taken from an article on graphical field mapping,[3,11] where they were drawn in accordance with the principle outlined in Appendix B of this paper). Substantially the same results were obtained in a later

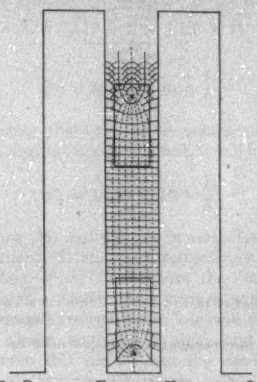


Fig. 2—Currents Equal in Value but Opposite in Direction

Iron of infinite permeability and resistivity

paper by Messrs. Robertson and Terry, who made a very careful mathematical analysis.[7]

Fig. 1 represents the current conditions on a single-phase line-to-neutral short circuit in machines with an armature throw greater than $\frac{2}{3}$, or a single-phase line-to-line short circuit with any commercially practicable coil throw. Fig. 2 represents some instantaneous condition during short circuit on a machine with a chorded winding, while Fig. 3 corresponds to some other instantaneous relation. These figures illustrate possible flux and current relations for a number of short-circuit conditions if saturation can be neglected. The effect of saturation will be considered later.

Calculation by the Use of Images. By comparing Figs. 1 and 4, it will be seen that the influence of the iron boundaries of the former are replaced by an infinite series of conductors in the latter.[8] (In Appendix C are listed the more usual boundary conditions found in electrical apparatus which can be represented or replaced by appropriate series of images.) It will be found that the corresponding conductors, shown in Figs. 1 and 4, not only carry exactly the same currents, but exist in flux fields of identically the same form and densities. It follows that the forces exerted on the conductors must be the same in the two cases. With

some modifications, the forces will be calculated from the conditions shown in Fig. 4.

Methods are available for considering the shape, dimensions, and separation of rectangular conductors

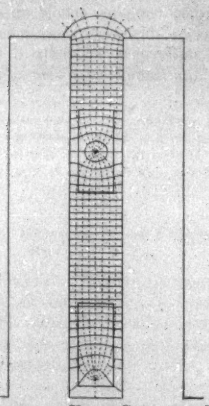


FIG. 3—CURRENT IN THE UPPER CONDUCTOR TWICE THAT OF THE LOWER AND IN THE OPPOSITE DIRECTION

Iron of infinite permeability and resistivity

in computing forces, if adjacent sides of the conductors are parallel.[4,5] However, there do not seem to be any methods worked out for the case of conductors placed in other positions relative to one another. It will be assumed for the present that the current in each conductor in Fig. 4 is concentrated at the center.

Then the force between two parallel line conductors in Fig. 4 will be

$$F = \frac{2}{445 \times (10)^5} \times \frac{I_2 I_n}{a} \qquad (1)^4$$

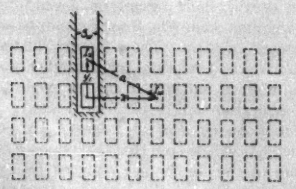


FIG. 4—REPLACEMENT OF THE SLOT SIDES OF FIG. 1 BY AN INFINITE SERIES OF CONDUCTORS, TO OBTAIN THE IDENTICAL FLUX FIELD

where F = force in lb. per linear in. of conductor.
I_2 = current in one conductor.
I_n = current in the other conductor.
a = distance between the conductors in inches.

Then in Fig. 4, the total force on the upper conductor is

$$F = \sum_{n=-\infty}^{n=+\infty} \frac{2}{445\,(10)^5} \frac{I_2 I_n}{a} \qquad (2)$$

also $$a = \sqrt{y^2 + x^2}$$

From symmetry, it can be seen that the forces in the x direction cancel, because corresponding to every pull to the right there is a duplicate one to the left.

Then the y component of force is

$$F_y = \sum_{n=-\infty}^{n=+\infty} \frac{2}{445\,(10)^5} \frac{I_2 I_n}{\sqrt{y^2 + x_n^2}} \frac{y}{\sqrt{y^2 + x_n^2}} \qquad (3)$$

$$F_y = \sum_{n=-\infty}^{n=+\infty} \frac{2}{445\,(10)^5} \frac{I_2 I_n y}{y^2 + x_n^2} \qquad (4)$$

To avoid the labor of making this summation to values where the contributions to F_y are negligible, a further simplification may be made. It will be observed that $I_n = I_2$. If each value of I_n is assumed to be spread out evenly so that the group form a sheet

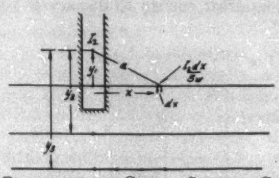


FIG. 5—REPLACEMENT OF CERTAIN SERIES OF CONDUCTORS IN FIG. 4 TO AID IN CALCULATING THE APPROXIMATE FORCES

through the centers of the conductors, instead of being assumed concentrated at the centers, then

$$I_n = I_2 \frac{d\,x}{S_w} \qquad (5)$$

and the total force in the y direction, from Fig. 5, is

$$F_y = \sum_{y=y_1}^{y=y_2} \int_{x=-\infty}^{x=+\infty} \frac{2\,I_2^2}{445\,(10)^5\,S_w} \frac{y\,(d\,x)}{(y^2 + x^2)} \qquad (6)$$

$$F_y = \frac{2\,I_2^2}{445\,(10)^5\,S_w} \sum_{y=y_1}^{y=y_2} \tan^{-1}\left(\frac{x}{y}\right)\Bigg]_{x=-\infty}^{x=+\infty} \qquad (7)$$

$$F_y = 14.1 \times 10^{-6} \frac{I_2^2}{S_w} \text{ multiplied} \qquad (8)$$

by [Number of layers below the conductor minus the number of layers above the conductor]

Where the layers are both imaged and real, as shown in Fig. 5.

In Fig. 1, where the currents are equal and in the same direction,

For the upper conductor,

$$_2F_y = 42.3 \times 10^{-8} \times \frac{I_2^2}{S_w} \tag{9}$$

For the lower conductor,

$$_1F_y = 14.1 \times 10^{-8} \times \frac{I_2^2}{S_w} \tag{10}$$

These forces are toward the base of the slot.

Calculation of the Force from the Rate of Change of Interlinkage. This method is very commonly employed. In the following case, it is based on Maxwell's equations for the stored magnetic energy in ergs in two electric circuits, *i. e.*

$$100\, W' = \left(\frac{I_1^2\, L_1'}{2} + \frac{I_2^2\, L_2'}{2} + I_1\, I_2\, M'\right) \tag{11}$$

The single-phase short circuit will be considered. Then

$$I_1 = I_2 \tag{12}$$

If, for the instant when the current is at the peak value, a conductor is allowed to move by a small amount, then

$$d\, W_y' = F_y'\, (d\, y') \tag{13}$$

or

$$d\, W_x' = F_x'\, (d\, x') \tag{14}$$

But the work done must be at the expense of the stored energy, and the forces in lb. per in. of conductor are

$$F_y = 8.85\, \frac{I_2^2}{2} \left(\frac{d\, L_1}{d\, y} + \frac{d\, L_2}{d\, y} + 2\, \frac{d\, M}{d\, y}\right) \tag{15}$$

$$F_x = 8.85\, \frac{I_2^2}{2} \left(\frac{d\, L_1}{d\, x} + \frac{d\, L_2}{d\, x} + 2\, \frac{d\, M}{d\, x}\right) \tag{16}$$

As before, the x components of the force will be considered first. The values of L_1, L_2 and M are single valued and continuous functions of the displacement. In Figs. 1, 2, and 3, the values of L_1, L_2 and M may either increase, decrease or remain constant, but it will be the same for a motion $+\Delta x$ as for $-\Delta x$. Then $\frac{d\, L_1}{d\, x} = 0$, $\frac{d\, L_2}{d\, x} = 0$, and $\frac{d\, M}{d\, x} = 0$, individually, so that

$$F_x = 0 \tag{17}$$

(as was found before from the solution by the aid of images).

The forces in the y direction will now be considered. Robertson and Terry[7] made calculations which showed the usual assumption was a good one, namely, that the flux went straight across the slot. Proceeding on this basis, suppose that the upper conductor is moved downward a distance $d\, y$.

Then

$$\frac{d\, L_1}{d\, y} = \frac{d}{d\, y}\left[\frac{N_1\, \phi_1}{I_1}\, (10^{-8})\right] = 0 \tag{18}$$

$$\frac{d\, L_2}{d\, y} = \frac{d}{d\, y}\left[\frac{N_2\, \phi_2}{I_2}\, (10^{-8})\right] \tag{19}$$

$$\frac{d\, L_2}{d\, y} = \frac{3.19}{S_w}\, (10^{-8}) \tag{20}$$

$$\frac{d\, M}{d\, y} = \frac{d}{d\, y}\left[\frac{N_1\, \phi_m}{I_2}\, (10^{-8})\right] = \frac{319\, (10^{-8})}{S_w} \tag{21}$$

Substituting equations (18), (20), and (21) in (15)

$$_2F_y = 8.85\, [(3.19 + 2 \times 3.19) \times 10^{-8}]\, \frac{I_2^2}{2} \tag{23}$$

$$_2F_y = 42.45 \times 10^{-8}\, \frac{I_2^2}{S_w} \tag{24}$$

By supposing that the lower conductor is moved downward, and the upper held fast, it will be found that

$$_1F_y = 14.15 \times 10^{-8}\, \frac{I_2^2}{S_w} \tag{25}$$

Equations (24) and (25) are in very close agreement with equations (9) and (10), respectively.

Calculation of the Forces from the Integral of H^2 Around the Conductor. Any surface will be chosen which surrounds a conductor. It will be imagined that the chosen boundary is rigid, and that forces exist between this surface and the conductor. The forces on the surface will act as follows: there will be tension along the flux tubes producing a force in dynes per unit of area.

$$F' = \frac{(H')^2}{8\, \pi} \tag{26}$$

and there will be pressure per unit of area exerted perpendicular to the flux surfaces (designated by a negative sign).

$$F' = -\frac{(H')^2}{8\, \pi} \tag{27}$$

(H' = lines per sq. cm.)

The force on the conductor will be equal and opposite to that acting on the surface. (What is believed to be a new proof of this principle is given in Appendix D. It is a proof which depends only on the more commonly used laws of electricity and magnetism, and which requires no detailed mathematical study).

Converted into units of lb. per sq. in. and lines per sq. in., equations (26) and (27) become

$$F = +\, 0.0139 \left(\frac{H}{1000}\right)^2 \tag{28}$$

and

$$F = -\, 0.0139 \left(\frac{H}{1000}\right)^2 \tag{29}$$

respectively.

Applying equation (**29**) to the conditions indicated by Fig. (1), it is found that between conductors $H_1 = 3.19 \frac{I_2}{S_w}$ and above the upper conductor $H_2 = 2 \times 3.19 \frac{I_2}{S_w}$, and the same results are found for ${}_2F_y$ and ${}_1F_y$ as are given by equations (**24**) and (**25**), respectively. Since the pulls on the two sides of the slot are equal and opposite, $F_x = 0$ (as was found before).

Forces Tending to Drive a Conductor out of the Slot. In any case where the winding pitch is between ⅔ and full pitch, a three-phase short circuit may be expected to produce a flux distribution at one instant somewhat like that shown in Fig. 2, where the currents are equal but opposite in direction.

It can be shown by any of the three methods developed that for Fig. 2 (when saturation is neglected)

$$ {}_2F_y = -14.15 \times 10^{-8} \frac{I_2^2}{S_w} \qquad (30) $$

The minus sign indicates a force tending to drive the upper conductor out of the slot. Also

$$ {}_1F_y = +14.15 \times 10^{-8} \frac{I_2^2}{S_w} \qquad (31) $$

and acts toward the bottom of the slot.

It will be found that the force ${}_2F_y$ will be considerably less than the maximum value existing on a single-phase short circuit. However, it is still important, because the wedge must carry the load. This force should be given consideration, particularly in machines

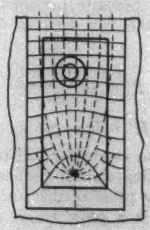


Fig. 6—Section of Fig. 1 with a Small Part of the Conductor Electrically in Parallel but Insulated from the Remainder

which can have high currents per slot, and which have wide slots.

If the coil were slightly loose so that the wedge could receive a series of impact blows (which would arrive at twice the normal electrical frequency of the machine), the possibility of failure would be considerably greater.

Forces on Individual Strands. It is intended, next, to determine whether there are any electromagnetic forces tending to separate the strands of any armature conductor. It will be assumed, first, that a small cylindrical hole is cut through a coil, as shown in Fig. 6. In this hole there is one small conductor, which is electrically in parallel with the main conductor, but the two are not in electrical contact at the section shown. The flux density at this point will be just the same as if the insulating space surrounding this strand were of conducting material,

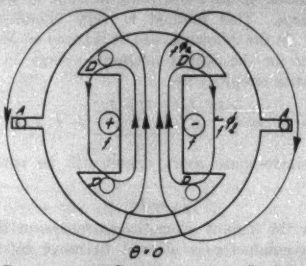


Fig. 7—Diagrammatic Sketch for the Instant before Short Circuit

for this space may be assumed to be very small. By making the hole extremely small, the direction of the force is more easily perceived. The flux density H' (due to the main conductor) must be single valued, so that as the isolated conductor is made smaller the main field on its two sides becomes more and more nearly parallel. The force acting on this conductor is

$$ \Delta F' = H' \frac{(\Delta I)}{10} \qquad (32) $$

and is in the direction of the weaker resultant field. This is along the ampere turn line toward the m. m. f. center. Hence, all parts of the conductor are compressed toward the m. m. f. center. The strands further out are in stronger fields. They are acted on with greater force so that there are no effects due to the inertia of the strands which may tend to separate them. From an examination of Figs. 1, 2, and 3, it will be seen that there probably are no combinations of current values possible in armature conductors, which will result in electromagnetic forces, tending to disrupt the coil sides by the separation of the strands. Any forces which separate the strands must be brought about by pulsating forces which press the whole conductor against its supports (bottom of the slot and the wedge) with such violence that the strands are shaken apart.

Reduction of Forces Due to Saturation. Saturation of the armature teeth will greatly reduce the forces. In order to consider this feature, a brief survey of the general flux distribution is necessary.

The particular condition which will be considered is that existing at the instant one-half cycle after a single-

phase line-to-neutral short circuit has occurred. Figs. 7 and 8 show the resultant flux immediately before the short circuit, and one-half cycle later, respectively.

It is assumed in Fig. 7 that the armature, A, is open-circuited, but that the field winding, f, is excited so as to maintain normal voltage at the generator terminals. No currents of fundamental frequency are flowing in the damping circuits D. The armature is short-circuited at the instant ($\theta = 0$) when the flux "trapped" in the electric circuits, A, D and f are as shown.

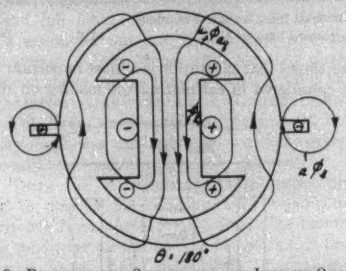


Fig. 8—Diagrammatic Sketch for the Instant One-Half Cycle after the Short Circuit Occurred

It is assumed that in each circuit current will flow which tends to maintain constant flux linkages in its own particular circuit.

Then when the rotor has turned 180 electrical deg., the resultant flux will be as shown diagrammatically in Fig. 8. The sum of the gap and field leakage flux ($_1\phi_o + {}_1\phi_i$) must remain practically up to its initial value, because this constitutes the flux linkages of the rotor winding. However, nearly all of the gap flux must pass inside the armature conductors, because the latter must have a net interlinkage opposite to that which the gap flux, alone, would produce if it returned through the stator core. The armature must have sufficient leakage flux to maintain its interlinkages up to the initial value, and in the initial direction. If the stator and rotor had precisely equal leakage paths, and constant interlinkages were assumed, the m. m. f. of each member would be practically equal. Oscillographic records show that the currents of the field winding, f, will not produce an m. m. f. anywhere nearly comparable with that of the armature winding during the first cycle after the short circuit occurs. As the mutual reactance of the rotor and stator circuits is high, there must be a very large current flowing on the surface of the rotor. The leakage path for this current has a greater reluctance than that of the armature. From Fig. 7 the initial flux linkages of the rotor are greater than those of the stator. Therefore, in order to maintain these very nearly the same in Fig. 8 as in Fig. 7, the total rotor m. m. f. must be greater than that of the stator. This means that at a very great distance from all circuits the total m. m. f. of the rotor will prevail. Hence, there is a very slight amount of air gap flux threading both the armature and rotor circuits in a direction opposite to that of the armature leakage flux.

The total armature leakage flux probably will be somewhat greater than the no load air gap flux, because of the difference in distribution of the two fields. A less percentage of the leakage flux links all the winding. In addition to this, there must be enough armature flux to offset the linkage of the small amount of air gap flux which links both windings. The armature leakage flux will require relatively few ampere turns for the core compared with the number required for the teeth and slot.

The retaining ring leakage flux will be prevented from increasing greatly during the first half cycle by eddy currents and also by saturation. Eddy currents in the sides of the rotor teeth will oppose increase in rotor slot leakage flux. Consequently, the air gap flux can decrease only slightly during the first half cycle. This means that the total rotor magnetomotive force due to the winding currents and rotor surface currents will exceed the total m. m. f. of the stator by an amount approximately equal to the no load normal voltage m. m. f.

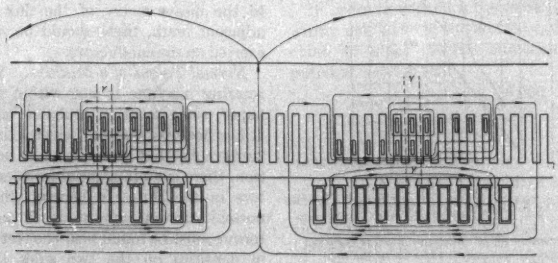


Fig. 9—Approximate Directions of the Flux in the Gap Teeth and Core

If it were intended to map the complete flux field, cer-

tain steps would be necessary. Determine the magnitude of the armature current as described in Appendix A, or by any suitable method. Give to the rotor a m. m. f. greater than that of the stator by an amount approximately equal to the no load gap ampere turns. Determine the rotor winding ampere turns relative to the stator from tests on the nearest similar machine. Locate the remaining rotor current on the rotor surface opposite the stator windings. Draw the flux field for the non-magnetic parts by assuming no saturation. Complete the field in the iron on the assumption of slight saturation and uniform permeability. Calculate the ampere turns for the iron parts and attempt to redistribute the ampere turn lines to take care of this.

It is only intended here to consider the effects of saturation in a single slot pitch.

Fig. 9 shows the approximate direction of the flux field in the gap teeth and core on a single-phase line-to-neutral short circuit for the instant shown in Fig. 8.

The effect of saturation will be considered in the zones *Y-Y* of Fig. 9. The flux in this zone will travel nearly tangentially across the slot and teeth. Except for a negligible number of ampere turns required for the core, the ampere turn lines from the stator conductors in this slot will proceed almost directly toward the rotor currents. These will spread enough to supply the requirements of half of each adjacent tooth as well as the slot itself. The values which will be used are approximately those which existed in an actual machine. There will be 50,000 amperes per coil side, or 100,000 ampere turns for the slot. The densities in the core are such as to require less than 1000 ampere turns in the iron back of the slot pitch, and hence can be neglected. Fig. 10 shows the densities in the slot with each conductor carrying 50,000 amperes, when the saturation of the teeth is neglected, and when it is considered.

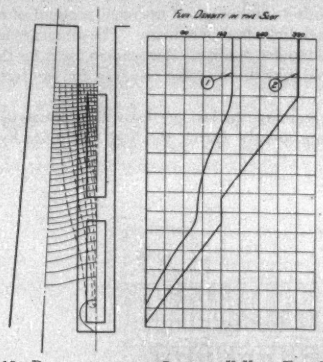


Fig. 10—Densities in the Slot at *Y-Y* of Fig. 9 Curves Show Slot Densities for 50,000 Amperes per Conductor

(1) when saturation is considered, (2) when saturation is neglected and it is assumed that the flux goes straight across the slot

By equations (**28**) and (**29**) the forces are as follows:

	Force in lb. per in. of length	
	Saturation neglected	Saturation considered
Electromagnetic force on upper conductor (toward base of slot).....	1062	695
Electromagnetic force on lower conductor (toward base of slot).....	354	210
Sum = total force on lower conductor (toward base of slot).....	1416	905

This shows that saturation is a most important factor to be considered in calculating the loading on the coil insulation in the slot. In Fig. 9 it is seen that for certain other conductors the flux will travel nearly radially in one tooth which bounds the slot, and it would be expected that saturation would have an even greater effect in reducing the force on these conductors. Due to the dissymmetry of the flux distributions in the adjacent teeth, there should be some tangential force exerted on the conductors.

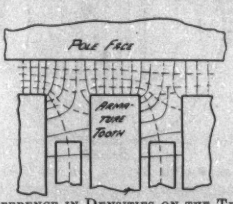


Fig. 11—Difference in Densities on the Two Sides of a Tooth under Load Conditions

Normal Torque of a Machine. While the torque of a rotating machine is not within the province of this paper, some mention probably should be made of it. It has been pointed out several times in the preceding paragraphs that when there is no appreciable saturation, there is no tangential force exerted on the conductors in the slot. In general, this approximates the normal running conditions of many machines. The torque must be found near the teeth tip in the difference in densities on the two sides (see Fig. 11). With saturation, the flux will not enter circumferential surfaces in a radial direction and some tangential pull will result from the corresponding components, at these surfaces. In the old surface wound machines the torque was carried almost entirely by the armature conductors.* It should be noted that when the field map has been drawn, equations (**28**) and (**29**) can be used to determine the forces exerted on the iron and copper

*This is not a new idea, but has been studied in part both theoretically and experimentally. See bibliography reference 13.

Prof. A. D. Moore has also used for some time in his lectures these general facts concerning torques.

parts in any portion of the field. This applies not only to rotating machines, but to contactors, and to any other apparatus in which the flux distribution can be determined.

Conclusions concerning the forces on conductors in armature slots.

1. The force pulsating at twice normal frequency and pressing the coils toward the bottom of the slot on a single-phase short circuit may have a value of 695 lb. per in. of conductor (or greater).

2. On three-phase short circuits in machines of less than full pitch, there will be forces tending to drive a conductor out into the air gap, carrying the wedge with it. These forces, while less than those just mentioned above, can be very serious if a coil is loose so that the forces on the wedge arrive as impact blows.

3. The forces exerted toward the sides of the slot are very much less, and only exist when the saturation in two adjacent teeth is appreciably different.

4. There are no electromagnetic forces directly acting to disrupt the conductors by separating the strands one from another. However, the hammering of the conductor on the bottom of the slot, if the conductor is loose, may cause some internal motion. This might be capable of wearing out the insulation between the strands very quickly.

B. Forces Within the Conical Surface of the End Winding

Fig. 12 indicates the approximate paths of the flux linking the armature end winding on a single-phase line-to-neutral short circuit. All currents in the parallel conductors are in the same direction, so that

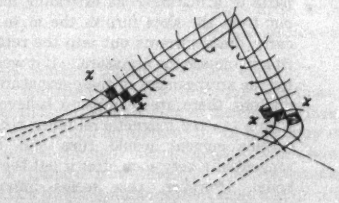


Fig. 12—Approximate Paths of the Flux Linking the Diamond Coil Armature End Winding

these coil sides tend to pull together. The maximum force will be exerted on the inside conductor at sections X-X, because the greatest concentration of flux is in this elbow of the phase group. Toward the outer or diamond ends of the coils there will be a force between the coil sides in the two layers tending to open out this angle. That is the coil ends have a tendency to bow out to a sort of semi-circular shape. The flux distribution around the semi-straight portion of the coil, between the inner elbow (X-X) and the diamond (outer) end, has much the same shape as if it were due to a group of long straight conductors in air, far removed from other metallic parts. By making just that assumption, the force on any conductor may be computed by the successive use of equation (**1**). Thus,

$$F = \frac{2}{445 \times 10^8} \frac{I^2}{a}\left(1+\tfrac{1}{2}+\tfrac{1}{3}+\ldots+\frac{1}{N-1}\right) \quad (33)$$

Where:

I = maximum armature current per coil side.

a = spacing center to center between coil sides in one layer of the conical surface of the end winding.

N = number of coils per phase group.

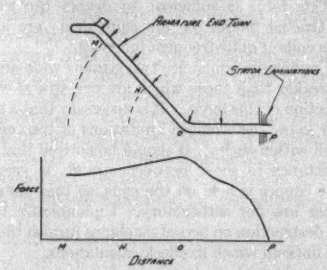


Fig. 13—Approximate Force against the Side of the Coil

As before, the correction factors for the shape of the conductors are neglected. These factors will reduce the force to some extent. There will be a different correction factor for the force between each pair of conductors, depending upon whether their spacings are a, $2\,a$, $3\,a$, or $(N-1)\,a$, etc.

In order to calculate the force per inch of conductor within one layer of the conical end winding of the machine for which the last numerical example was computed, it is noted that

$$I = 50{,}000$$
$$a = 1.79$$
$$N = 7$$

Then from equation (**33**)

$$F = 154 \text{ lb. per in. of conductor.}$$

Fig. 13 shows the probable sort of force distribution in the plane of the windings based on the conditions indicated in Fig. 12.

Fig. 12 shows that greater densities exist at the inner bend of the coil. The force must fall off toward the outer end of the coil for there are less and less coil sides outside of the inner one. Then as the outer bend of the coil is reached the repulsion from the coil sides in the other layer of the phase group will tend to increase the force.

Conclusions concerning the forces on the conductors within the conical surface of the end winding.

1. The forces on a single-phase short circuit tend to pull all the conductors together within one layer of the conical end winding.

2. The force is greatest on the inner coil sides of the group at the bend (or elbow) of the coil nearest the frame. It decreases from there toward the outer bend of the diamond coil.

3. In between these two bends in the coil the force may be 150 lb. per in. of conductor, or greater, on the assumption that the force may be computed as would be done for long straight conductors in air. The fact that the space within the phase group is limited indicates that the force should be greater than calculated, but if the correction for the coil shape is neglected, this is compensated for to a certain degree.

4. This force is sufficient to distort the winding unless bracing blocks (or the equivalent) are located between coils at quite frequent intervals.

5. Conductors which are two strands wide are very much weaker than those which are one strand wide in the direction of this force. It is necessary that a manufacturer know the strength limitations of the coils and stay well within them. It should be noticed that while the resistance to shear between strands is very low, that the copper is held on the ends, so that tests on a short bar are not satisfactory. Undoubtedly testing coils to destruction on actual machines furnish the most reliable data on which to check calculations.

C. Forces on the Coil Extensions from the Frame

Tests on actual machines have shown that the straight parts of the coils extending from the slots over the retaining rings are almost as liable to failure as are the coil sides within either of the two layers of the conical end winding.

The conditions illustrated in conjunction with Figs. 7 and 8 will be used for the basis of the following calculations. It was concluded that at the instant indicated

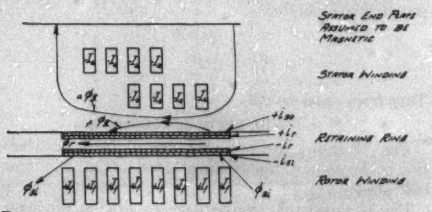


Fig. 14—Diagrammatic Sketch of the Currents Flowing in the End Zones Just Outside the Frame

by Fig. 8, the rotor m. m. f. was equal to that of the stator plus an amount approximately equal to the field m. m. f. at the no load normal voltage condition. This applied to the main magnetic circuit. A detailed study will be undertaken now in an attempt to picture the current and flux distributions just outside the ends of the "active" armature iron.

Figs. 14 and 15 indicate, in a diagrammatic way, the currents which must flow in this zone.

The currents indicated are as follows:

I_a = the armature current per coil side (as defined in Appendix A).

I_f = the current in the field winding for the instant shown in Fig. 8.

i_{so} = the rotor surface current on the outside of the rotor. This is practically the continuation of the damping currents of the rotor body out into the retaining ring. The straight

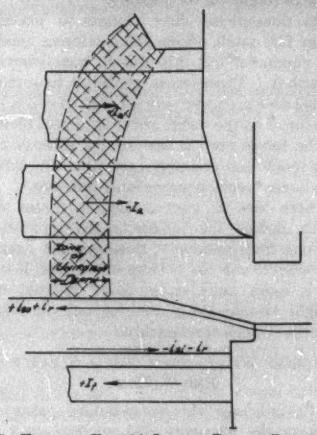


Fig. 15—View of Fig. 14 In the Radial Plane at the Centerline between Poles

parts of armature coils extending directly out from the slots furnish the m. m. f. to carry these currents out into the retaining ring to oppose the armature. If it were not for the arrangement of these armature conductors, there are reasons to believe that with a tapered magnetic retaining ring this surface current would turn and travel tangentially very soon after it left the rotor body. However, this seems impossible when the armature winding is so near, and this belief has been very forcefully upheld in a test to destruction. It is assumed, then, that this rotor surface current i_{so} travels axially across the outside surface of the tapered section of the retaining ring.

i_r = the transient current flowing in the retaining ring to oppose any change in the retaining ring leakage flux.

ϕ_r = the retaining ring leakage flux.

i_{si} = the retaining ring surface current on the inside surface which opposes any change in the flux entering or leaving the ring from the inside.

With the currents defined, it will now be possible to

consider the ampere turn surfaces. These emanate from the rotor field winding and pass through the retaining ring. Most of them terminate on the armature conductors which carry current in the opposite direction to the field windings. A few of these ampere turn surfaces go through the air gap and through part of the stator iron to terminate in the field windings on the other side of the pole.

The effects of the rotor surface currents must be considered. The current on the inner surface of the retaining rings, i_{ri}, will be neglected. This is probably the wrong thing to do, but it is desired to be on the safe side in the force calculations. It is possible, and it even seems probable, that this current might nearly cancel the increase in m. m. f. in the end zone, due to the increase in field current, but this is neglected.

The net effect of the current i_r is practically zero. This component is shown by two current sheets. One is in the inner surface of the ring and one in the outer. These flow in opposite directions. What the inner sheet subtracts from the m. m. f. between a given pair of ampere turn surfaces, the outer adds in again.

ring. The real current sheet, i_{ro}, will travel nearly axially outward under the straight part of the armature coil extensions, because it flows as a sort of image of the armature current.

Oscillographic records of armature and field winding currents show that the armature m. m. f. is much the greater of the two. Yet the constant interlinkage theory demands that the total rotor m. m. f. be slightly greater than the stator. This means that the m. m. f. of the rotor surface currents, i_{io}, is much greater than that of the rotor winding I_f at the instant shown in Figs. 8, 14, and 15. This real surface current, i_{ro}, must be as much larger than the imaginary current sheet used to replace the saturation of the ring. The resultant current sheet of the real and imaginary one must have almost axial directions of flow under the armature coils. The strength of the stator m. m. f. may actually be the greater at a short distance out in the tapered section of the ring if the effect of the current sheet, i_{ri}, on the inside of the ring is considered.

The purpose of the foregoing discussion of current and flux in the end zone near the tapered section of the re-

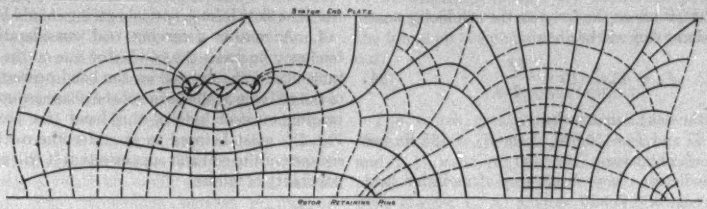


Fig. 16—Flux Field, in the End Zone Just Outside the End Bells

Very small armature conductors are used for simplicity in mapping

There remains the rotor surface current, i_{ro}, and a certain contribution from the rotor windings. The ampere turn surfaces arriving at the outer surface of the retaining ring can be replaced by an imaginary current sheet of just the right current density, which flows on a ring of infinite permeability. (The replacement of saturation by the suitable current sheet and unsaturated iron has been demonstrated by Th. Lehmann in some of his excellent papers concerning graphical field mapping.[9,10,11]) This imaginary current sheet may now be superposed on the real (transient) current sheet, i_{ro}, flowing on the outside surface of the ring.

The directions of these two component current sheets are probably not the same.

The saturation in the ring is greatest in the tapered section. Consequently, the leakage flux in the ring will travel nearly axially across this section. A large proportion of the ampere turn surfaces maintaining this leakage flux will lie in the ring in fairly near circumferential directions. The imaginary current sheet used to replace this saturation must have the same directions as these ampere turn surfaces inside the

taining ring has been an attempt to get a graphic picture of the actual conditions. Some approximations will be made now to compute the forces on the armature windings.

A resultant or equivalent current sheet will be assumed to exist on the rotor retaining ring, and flowing axially outward from the rotor slots. It will have a uniform current density and a total m. m. f. equal to that of the armature plus the no load normal voltage rotor m. m. f. The flux to be considered will be assumed to remain within the zone of uniform width indicated in Fig. 15. (The direction of the flux is approximately perpendicular to the plane of that drawing.) Saturation in the stator end plate is neglected, because it is a magnetic parallel on the stator core. The magnitude of the eddy currents in this plate cannot be estimated satisfactorily. They will be neglected for the same reason that the rotor m. m. f. was assumed at its maximum possible value, *i. e.*, to be on the safe side. On this basis, Fig. 16 was drawn (using small armature conductors), and Fig. 17 drawn from Fig. 16. The forces on the conductors may be computed from the

integral of H^2 taken around each one. The way the flux encircles the group of coils indicates appreciable tangential forces on the outer ones of the group. However, the coils may be well braced against this by roped-in blocking. The highest radial forces occur on the coil sides in the center of the group. A very easy means of approximating this force becomes evident from the work done concerning the conductors embedded in slots. It is as though the conductors lay in slots of width equal to the slot pitch. See section Y'-Y', Fig. 17. From

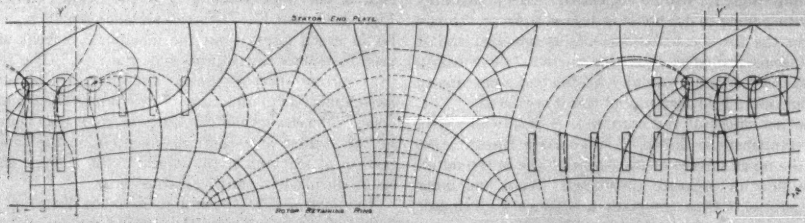


Fig. 17—Fig. 16 Redrawn for Armature Conductors of Normal Size

equation (**24**), it seems the force on the conductor nearest the retaining ring will be about

$$ {}_2F_v = 42.45\ (10^{-8})\ \frac{I_2^2}{q} \qquad (34) $$

where $q =$ the slot pitch in inches.

If $q = 2.185$ and $I_2 = 50{,}000$, then ${}_2F_v = 485$ lb. per running in. of conductor.

Fig. 18 indicates the probable sort of variation load against the edge of the coil.

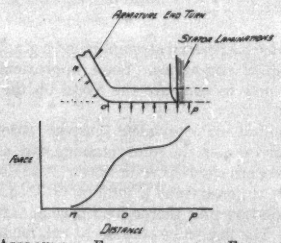


Fig. 18—Approximate Force against the Edge of the Coil

Conclusions concerning the forces on the coil extensions from the frame.

1. There are fairly large tangential forces on the outer conductors of the phase group, similar to those within each layer of the conical end winding. However, these forces operate against the edges of the strands composing the coil sides. Bracing blocks are always inserted, and these can be made to give adequate support against such forces.

2. The forces acting radially outward are of appreciable magnitude. These act against the coil in the direction in which the individual strands are the weakest, but there are many of them in the coil side. Tension and compression along these strands operate to prevent failure. The force falls off with distance from the slot toward the first bend or elbow of the coil, and very rapidly beyond the bend.

3. Experience indicates that this is a less likely place for failure to occur than within a layer of the conical surface of the end winding, but if the straight extension from the slots is long, special bracing should be supplied.

4. A matter deserving real consideration is the tendency to twist the conductor due to the radial and tangential force exerted on the bend nearest the frame of the coil side at the inside of the phase group. Careful bracing on each side of this bend (see section X-X, Fig. 12) must be used to so stiffen the mechanical resistance of the coil and supports that it will remove this possibility of failure.

D. Forces on the Phase Connecting Rings

Rectangular copper straps are bent into semi-circular arcs, and used to connect the terminals of the machine to the ends of the various phase groups. These phase connecting rings are considerably closer to each other than to any heavy iron parts. They are located in a relatively weak part of the field of the armature end windings. The radius of curvature of the rings is rather large. Therefore, it seems a reasonable approximation to assume that the forces on the phase connecting rings will be about the same as if they were straight conductors in air well removed from other metallic parts.

The forces can be calculated by equation (**1**). However, the directions of the currents must be determined from the conditions of the short circuit. The usual forms of bracing, blocking, and clamping are well known. On the whole, they are easy to apply, but special attention must be given to the ends of the phase connecting rings where they join the coil ends, in the machines in which the loading is large, for this is an inherently weak place in one turn coils.

E. Forces on the Whole Phase Group in the Conical End Winding

A quantitative solution has not been worked out

for estimating these forces. Probably their directions are almost as important. The currents on the retaining ring (the real transient current and the imaginary current used to replace the saturation of the ring) will tend to produce an approximately outward force on the phase group. The coils are rather centrally located between the iron parts of the frame and end-bell, so that the effect of these latter parts should tend to cancel out.

It would be very desirable to have a definite, but not too great force, pressing the coils outward toward coil supports which are mounted on the frame. Such an arrangement would eliminate any need for bolts through the winding to hold it back toward the frame. There are two ways to accomplish this. Laminated iron near the stator end plate to furnish a strong attraction would achieve this result, but it is commercially a rather impractical measure. A good conductor on the inner end-bell would carry eddy currents which would offer a force of repulsion to the armature winding and tend to force it back toward the frame. This is a practical expedient. An aluminum diffuser for a double entrance fan will serve this purpose. However, it is possible such a thing might be very badly done. If the entire inner end-bell were made of aluminum or copper, it would probably have high losses, and possibly excessive heating under normal load conditions. The reason would be that the path of the armature leakage flux would be too greatly restricted. This damper would have to render a magnetomotive force almost equal to that of the armature. The smaller damper will carry only a relatively small current and loss at normal load. The tremendous increase in flux on short circuit will give sufficient current to produce a definite force in the desired direction.

In addition to the forces just discussed where the resultant lies in a radial plane, there must be a tangential force acting on the phase group in the direction opposite to that of rotation. This torque will be due to the losses occurring in the end-bell, end plate, etc. It is as though these iron parts were the secondary, and the stator winding the primary of a surface wound induction motor, so that the burden of the torque falls on the armature conductors.

Some further considerations of the forces are given in Appendix E for certain limiting cases.

Summary

Fairly detailed conclusions have been given at the end of each section in the paper, so only a brief summary will be made here.

Three methods of calculating the force exerted on current carrying conductors have been demonstrated. The method involving the use of images can only be used when there is no appreciable degree of saturation. (The most common configurations which can be imaged are summarized in the Appendix C). The method involving the change of flux interlinkages is most useful where a reasonably simple mathematical expression can be obtained; but if the method involves the difference of two large values of interlinkages by the use of graphical field maps, it is not satisfactory. The method which uses the $\int H^2 d s$ over any surface enclosing a conductor is the most generally applicable. It frequently involves the plotting of one rather difficult field map, but in such a case there is probably no way of avoiding this labor. (A simple proof is given in Appendix D for the basic principle of this last method for the calculation of the forces.)

The forces exerted on the embedded armature conductors during short circuit are computed and the great influence of saturation is illustrated. It is shown that there are no electromagnetic forces tending to disrupt the coil sides by rending apart the strands. A graphical picture is given for the normal torque of a machine. The forces between coil sides in one layer of the conical end winding are computed. The forces on the straight coil extensions from the frame are approximated. The forces on the whole conical surface are discussed, and methods of causing the force to be definitely toward the frame are pointed out. The applicability of simple formula for approximating the forces on the phase connecting rings is also pointed out.

Acknowledgments

The writer greatly appreciates the assistance of Messrs. M. G. Leonard, A. M. Harrison, M. R. Lory, and R. D. Reed, for their assistance in making the field maps, and also appreciates the helpful discussions with Messrs. L. A. Kilgore and C. M. Laffoon.*

Appendix A

Armature Short-Circuit Current per Coil Side

The maximum possible peak current per coil side which exists at the end of the first half cycle after an instantaneous line-to-neutral short circuit is often computed by the approximate formula

$$I = \frac{I_{rms} \times 1.5 \times 2\sqrt{2} \times .9}{X_d''} \tag{35}$$

$$I = 3.82 \frac{I_{rms}}{X_d''} \tag{36}$$

where,

I = maximum short-circuit current per coil side,

I_{rms} = r. m. s. value of normal load current per coil side.

X_d'' = subtransient reactance expressed as a decimal fraction.

*All of the Westinghouse Company.

Appendix B

Basis of Graphical Field Mapping

The basis for graphically mapping two dimensional flux fields, which is used in this article, has been published several times,[3,8,12] but is given here in an abbreviated form for the sake of completeness.

Fig. 19 illustrates the flux field of a salient pole machine at the no-load normal voltage condition.

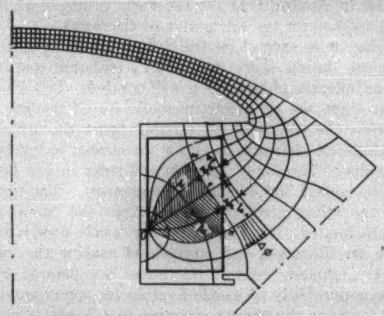


Fig. 19—Flux Field of a Salient-Pole Machine at the No-Load Normal Voltage Condition

It will be assumed correct, and the relations necessary that it be the true field will be illustrated (uniqueness is assumed). If a unit pole is moved around the path $o\,p\,n\,o$ linking the current I_2 and area A_2 the work done is $\int H'\,d\,l' = 0.4\,\pi\,I_2$. This is all done along the flux line p-n of length l_1'. If the elemental tube of flux $\Delta\,\phi$ considered, has a mean width $(\Delta\,d_2')$ and mean density H_2' in this length l_2', then

$$H_2'\,l_2' = 0.4\,\pi\,I_2 \tag{37}$$

$$\frac{(\Delta\,\phi)\,l_2'}{\Delta\,d_2'} = 0.4\,\pi\,I_2 \tag{38}$$

Similarly for the path

$$\frac{(\Delta\,\phi)\,l_1'}{\Delta\,d_1'} = 0.4\,\pi\,I_1 \tag{39}$$

From the last two equations

$$\frac{l_1'/\Delta\,d_1'}{l_2'/\Delta\,d_2'} = \frac{R_1}{R_2} = \frac{I_1}{I_2} = \frac{A_1}{A_2} \tag{40}$$

where R is the reluctance and uniform current density is assumed so that I is proportional to A. The last series of relations is sufficient for graphically mapping any two dimensional field in the non-magnetic parts, if the currents are located and the saturation known approximately.

Appendix C

Images[6,8]

The most common two dimensional field in which the boundaries may be replaced by the use of other conductors are those of

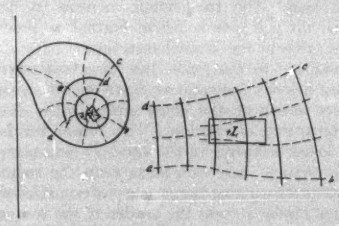
1. Conductors between two planes which meet at a certain angle. The angle expressed in degrees when multiplied by some integer must give 180 deg.
2. Conductors between parallel planes.
3. Conductors in rectangular slots.
4. Conductors in completely closed rectangles.

In the first three cases the plane sides should have either infinite permeability (and if an alternating field is considered infinite resistivity, or unit permeability and zero resistivity). In the fourth case if the boundary is of infinite permeability there must be as much current in one direction as the other so that no ampere turn lines penetrate the boundary. If the surrounding surface is of zero resistivity, it is not necessary that the sum of the currents in the enclosed conductors be zero for the surface will carry currents to receive any excess ampere turns from the conductors.

Appendix D

Calculation of Force from $\int H^2\,d\,s$

It is apparent that every element of current tends to establish a field of force around it in such a manner that the flux lines form closed loops. Consequently at certain conductors in any field the ampere turn lines, or lines of no work, must converge at points of no flux density. These points will be called m. m. f. centers. However, not all current carrying conductors may sustain a point of zero density. The centers which they might sustain if excited alone are submerged in



A B

Fig. 20—a. A Conductor which Maintains a M. M. F. Center

b. A Conductor which does not Maintain a M. M. F. Center

the main field and only swirls or waves may be left to indicate the tendency (See the upper conductor in Fig. 1). The forces exerted will be considered for these two cases, namely, (1) when the conductor contains an m. m. f. center and, (2), when it does not.

These two possibilities are illustrated in Figs. 20A and 20B, respectively. Figs. 21A and 21B show exactly the same conductors existing in exactly similar fields, but with different boundary conditions. These are

strictly two dimensional problems. Unit depth perpendicular to the drawing will be assumed. In Fig. 21A two current sheets, $a\,b\,c$ and $d\,e\,f$, have replaced the flux lines of Fig. 20A, which were in these positions. The current in these sheets travels parallel to that of the conductor, but in the opposite direction. The current densities are such that the ampere turn lines leaving the conductor may terminate in these sheets at precisely the points where they formerly crossed a flux line.

In Fig. 21A the lines $c\,d$ and $f\,a$ form boundaries for the iron of infinite permeability and infinite resistivity and are so arranged that the flux lines link the rectangular conductor in the same direction as before. In Fig. 20A, these were portions of ampere turn lines. In both cases the flux crosses these lines perpendicularly. Thus it is seen that the boundaries of flux tubes, which appear in these two dimensional fields, may be replaced by sheets of current which travel perpendicular to the plane of the drawing, and which have such current density and sign that the ampere turn lines arriving there may terminate in the current sheet. Ampere turn lines, which are lines perpendicular

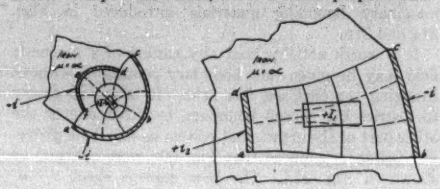


Fig. 21—A. Fig. 20A with Special Boundary Conditions to Isolate the Conductor

B.—Fig. 20B with Special Conditions to Isolate the Conductor

to flux may be replaced by surfaces of imaginary iron bodies that have infinite permeability (and for a-c. fields, infinite resistivity).

Fig. 20B represents a current carrying conductor without an m. m. f. center. It has been arranged in Fig. 21B with a current sheet of like sign at $a - d$, and one of opposite sign at $b - c$. The latter is numerically equal to the sum of the other two currents, but of opposite sign. The ampere turn lines $d - c$ and $a - b$ of Fig. 20B are iron surfaces in Fig. 21B. Again the magnetic circuit in the iron is so arranged that the flux lines link the original conductor in the same manner as before.

The problem is to determine the forces on the rectangular conductors. These are, obviously, the same in Fig. 21A as in Fig. 20A, and the same in Fig. 21B as in Fig. 20B. In Figs. 21A and 21B the forces on the rectangular conductor must be equal and opposite to the integral of the forces taken over the boundary materials in each case. It will be found easier to make the computations on the boundary.

The force on a small area of the material carrying the current sheet ($a\,b\,c$ Fig. 21A) will be considered first. This element of the conducting sheet will be chosen so small that it can be considered as plane surface. Two views of it are shown in Figs. 22A and 22B.

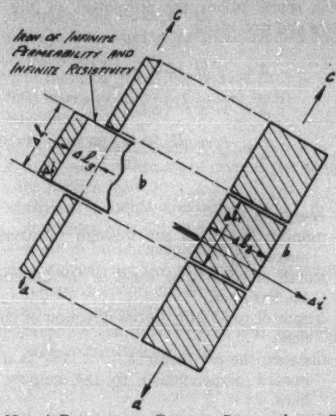


Fig. 22A—A Part of the Current Sheet a-b-c of Fig. 21A

In Fig. 22B the ampere turn lines are shown arriving from the rectangular conductor and terminating in the sheet. When the small conductor is moved (Fig. 22A) it will be supposed that there is a rectangular iron block behind it whose surfaces coincide with the

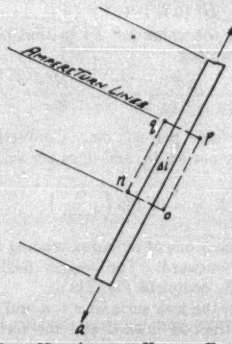


Fig. 22B—Another View of Fig. 22A

ampere turn lines. Work is done on the small conductor, but not on the iron block, because the latter moves perpendicular to the flux and has the same intensities at one surface as on the opposite. The change in stored energy in the whole field is

$$\Delta w' = -\frac{1}{20}\Delta i\,(\Delta\phi) \tag{41}$$

Where Δi is the current in the small element and $\Delta\phi$ the flux which passes through the iron block. This flux has the same magnitude as that which occupied this space before the movement of the small conductor and iron block.

but

$$\Delta\phi = H'(\Delta l_3')(\Delta l_2') \tag{42}$$

and

$$\Delta w' = \Delta F'(\Delta l_3') \tag{43}$$

$$\Delta F' = -\frac{H'(\Delta l_2')(\Delta i)}{20} \tag{44}$$

where

w' = energy in ergs

$\Delta\phi$ = number flux lines cut through (measured in maxwells.)

$\Delta l_1'$ = length of conducting sheet in the direction of the flux lines.

$\Delta l_2'$ = length of conductor in the direction of current flow.

$\Delta l_3'$ = distance the conductor $\Delta l_1' \times \Delta l_2'$ is moved inward perpendicular to the ampere turn lines.

H' = field intensity near the conductor.

Δi = current flowing in the element of the sheet.

$\Delta F'$ = force acting on the small conductor (measured in dynes).

If a unit pole is moved around the path $o\,p\,q\,n$ in Fig. 22B linking the current Δi, the work done is all along the line $q\,r$ or $\Delta l_1'$ because in this figure H' is zero outside the current sheet.

$$H'(\Delta l_1') = .4\,\pi\,(\Delta i) \tag{45}$$

Substituting the value for Δi in that obtained for $\Delta F'$ and rearranging

$$\frac{\Delta F'}{(\Delta l_1')(\Delta l_2')} = -\frac{(H')^2}{8\,\pi} \tag{46}$$

which is the force per sq. cm. Converting this to the force in lb. per sq. in. and lines per sq. in.

$$F = -\,0.0139\left(\frac{H}{1000}\right)^2 \tag{47}$$

The above force is one of repulsion tending to force the current sheet outward. The weaker field is on the outside (see the complete Fig. 21A).

The force on the iron surfaces $c - d$ and $f - a$ must be ones of attraction inward and the force per unit area is

$$F = +\,0.0139\left(\frac{H}{1000}\right)^2 \tag{48}$$

(Because this is commonly proved and given in handbooks for use in magnet calculations no proof is given here).

The same arguments as given above for the conductor which contained an m. m. f. center (Fig. 21A) may be applied to the conductor which had no m. m. f. center (Fig. 21B).

It must be remembered that the force on the rectangular conductors which is really sought is equal and opposite to the integral of the force taken over the

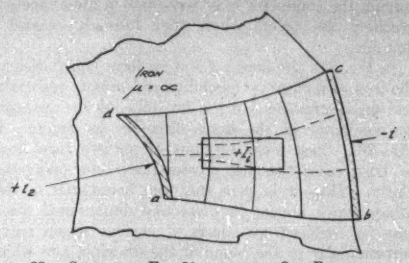


FIG. 23A—SIMILAR TO FIG. 21B BUT WITH ONE BOUNDARY NOT ALONG EITHER PART OF THE ORTHOGONAL SYSTEM

imaginary bounding materials introduced in Figs. 21A and 21B.

It is worth noticing that any section of a magnetic field may be given new boundary conditions without changing the enclosed flux field. It is not necessary to restrict the chosen boundary to either one or the other part of the orthogonal system as was done above. For instance, suppose that Fig. 20B were changed as shown in Fig. 23A. The new current sheet $d - a'$

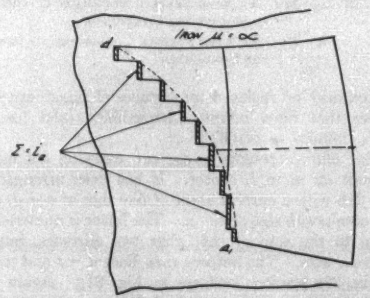


FIG. 23B—THE NEW BOUNDARY $d - a'$ OF FIG. 23A RESOLVED INTO THE CUSTOMARY ORTHOGONAL COMPONENTS

has just the right density and the iron body is brought up behind it.

It might be imagined that this new boundary is composed of a large number of steps (Fig. 23B), the surfaces of which lie either parallel to the flux or to the ampere turn surfaces. To obtain the resultant force on a small section, it is only necessary to compute

the two components of force indicated in the latter figure.

Appendix E

Limiting Conditions

A serious attempt was made to gain a view of the forces exerted on the entire phase group by mapping fields for certain limiting conditions. The distribution of the ampere turn surfaces on the retaining ring were

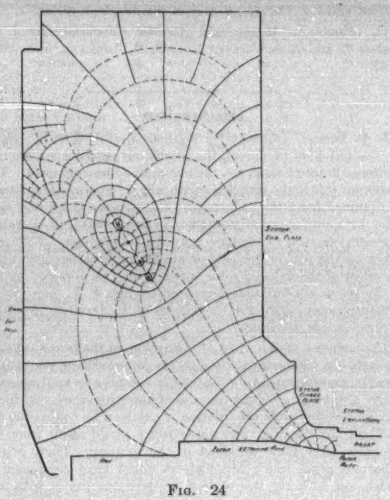


Fig. 24

approximated at the center line between poles. The two layers of armature conductors in this radial plane were plotted as one layer since only the force on the entire group was desired. The fields were plotted as though they were two dimensional, because this had given fairly good results when compared with compass readings on stationary flux fields in this end zone. It was recognized that the stator and end-bell were neither perfect magnetic nor perfect damper surfaces; but the field was plotted (Fig. 24) as though it were the first, and then (Fig. 25) as though it were the second. These both indicate that the winding at its outer edge would have a force component toward the frame.

Aside from approximations necessary to plot these fields, it was felt that the omission entirely of the power component of the damping currents on the bell and frame might be a fallacy which would completely nullify the value of any quantitative results. However, this method will probably have merits in parts of machines where the currents travel either only axially or only tangentially, and these two figures are submitted for this reason, and because they do give a partial picture of conditions in the end zones.

List of Symbols

Flux, Field Intensity, and Reluctance

ϕ = flux in maxwells
${}_a\phi_l$ = armature leakage flux per pole
${}_f\phi_l$ = field leakage flux per pole
${}_a\phi_g$ = air gap flux per pole
ϕ_r = retaining ring leakage flux
ϕ_{si} = leakage flux entering the retaining ring from the under side
H' = field intensity (lines per square cm.)
H = field intensity (lines per square inch)
R = reluctance

Currents (all in practical amperes)

I_a = r. m. s. value of armature current per conductor at normal load
I_2 = maximum possible armature current in upper conductor in a slot on short circuit
I_m = current in an imaged conductor
I_f = field current per slot at some particular instant following a short circuit
i = current in some current sheet
i_{so} = current on the rotor surface and retaining ring surface near the air gap

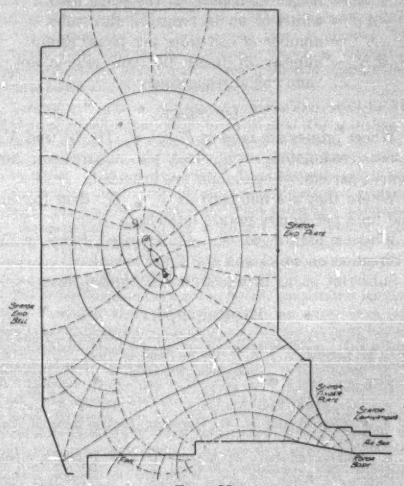


Fig. 25

i_r = component of retaining ring current flowing to oppose change in ϕ_r
i_{si} = retaining ring current on the inside surfaces flowing to oppose change in ϕ_{si}

Inductances and Reactances

(All inductances in practical henrys per in. of length along the conductor)

L_1 & L_2 = self inductances of circuits 1 and 2, respectively.
M = mutual inductance of the same circuits.
X_d'' = subtransient reactance expressed as a decimal.

FORCE, ENERGY, AND POWER

F' = force in dynes per cm. of length and also per sq. cm.
F = force in lb. per in. of length and also per sq. in.
w' = energy ergs.

AXES AND DIMENSIONS

x = distance measured across a slot, or across a conductor in the end winding in the same direction with respect to the strands.
y = distance measured radially inward in a slot, or in the same direction with respect to a conductor at any points.
a = distance between conductors.
s = a surface in air surrounding a conductor.
A_1 & A_2 = cross-sectional areas taken in current carrying conductors.
S_w = slot width.
q = slot pitch.
l = length as defined at the place used.
Δd = width of an incremental flux tube.
N = number of coil sides per phase group.
N_1 & N_2 = number of turns per coil side-in coil 1 and coil 2, respectively.

GENERAL

Where primes are used as F', y', w', H', L', and M', dynes, centimeters, ergs, lines per square cm. and henrys per cm. of conductor are intended.

Where they are not used as F, y, H, then lb., in., and lines per square in. are intended.

Where subscripts as ${}_1F_y$, ${}_2F_y$, are used, forces in the y direction on coil 1 and coil 2 are meant.

Subscript m, as in ϕ_m, indicates mutual flux.

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Discussion

J. A. Terry: Of the three methods employed to determine the magnitude of the forces, the second one, based upon the rate of change in interlinkages, is the one most commonly used. The reason for this is the simplicity with which the desired result can be obtained from the inductance of a system, which of course must be quite accurately known in order that the operating characteristics of a machine may be understood. Thus the forces on the end projections of alternator windings neglecting the effect of the induced rotor currents may be obtained by use of the reactance formulas presented by Mr. P. L. Alger in an Institute paper (1928) *The Calculation of the Armature Reactance of Synchronous Machines*.

The inclusion of the effect of saturation on the forces between the coils is a good contribution to the knowledge of the subject since, in general, saturation materially complicates analyses, and nearly always requires the free use of approximations which can only be justified after the solution is obtained by comparing the resulting flux densities with those which would result from the magnetomotive force obtained in the solution. There is one important point which Mr. Calvert might have mentioned in connection with the comparison between the forces with and without saturation. The comparison has shown that, on the basis of the same slot current in both cases there is a total of 1416 lb. per in. length, neglecting saturation, and 905 lb. per in. length including saturation. As a matter of fact saturation materially reduces the reactance of the machine and thus increases the current flowing during a short circuit so that the comparison on the basis of the same current in both cases does not present the true effect of saturation. Tests have shown that the commercial turbine alternators saturation may decrease the reactance which limits the short-circuit currents to 0.6 of the unsaturated value. If this new current were then substituted in the force expressions the force would be $(1.6)^2 \times 905$ or 2300 lb. However, the saturation for such a current would be greater than for the assumed current so this force is fortunately not correct. It appears offhand that, depending upon the location and degree of saturation, the forces in case of a short circuit might be either greater or less than in the unsaturated case.

In Appendix A a more accurate formula for the line-to-neutral short-circuit current which consists of replacing x_d'' by $\frac{x_d'' + x_1 + x_o}{3}$ might just as well have been given; x_1 and x_o are the negative- and zero- sequence reactances respectively. This will give a slightly higher short-circuit current in most commercial machines.

Since the repulsive forces between the coil sides of a slot is of interest in the design of slot wedges it is of interest to study these for some typical cases. Consider that the currents in the upper and lower coil sides are numerically equal but out of phase by an

angle θ in time. The force between the two coil sides will be proportional to $I^2 \left[\sin^2 w t - 2 \cos^2 \frac{\theta}{2} \sin \left(2 w t + \frac{\theta}{2} \right) \right]$

or a maximum when

$$\sin 2 w t = 2 \cos \frac{\theta}{2} \sin \left(2 w t + \frac{\theta}{2} \right)$$

The following tabulation gives the relative magnitude of the forces and the instants at which they are greatest:

Condition	θ deg.	Max. repulsive force occurs when $w t$ is	Relative values of max. repulsive force are	Relative values of max. attractive force are
Full pitch	0	0 deg.	0	3
Three phase, 2/3 pitch	60	−47.6 deg.	0.407	
Two phase, 1/2 pitch	90	−62.6 deg.	0.605	
Three phase, 1/3 pitch	120	−75 deg.	0.864	
0 pitch	180	−90 deg.	1.00	0

Calculations for a number of slow speed machines have shown that the slot wedges generally used are many times as strong as would be required for repulsive magnetic forces only.

B. L. Robertson: The subject of mechanical forces between electric circuits has been treated in a theoretical manner in several papers which have previously been presented, and the study has also been made of the short-circuit forces existing in reactors and transformers. However, the subject of mechanical forces between coil sides of rotating apparatus when abnormal currents are flowing has received little, if any, attention. The present paper therefore becomes valuable because of its attack on this problem, and because the treatment gives quantitative results.

Of general comment is the fact that flux plotting, or magnetic field mapping, has been resorted to throughout the paper. It forms the only basis of solution to the question of forces when one goes just beyond the realm of very simple configurations of iron and copper. This statement readily becomes apparent as soon as the case of saturation enters the picture, or in treating the regions about the coil ends and the end bells. Magnetic field mapping, done either graphically or analytically, has become of practical importance and this is one more illustration of that fact. The industries, however, are the only ones who have as yet done much with it. It is treated to any extent only in a few of the engineering schools of this country, and it is feared that in many institutions it is not even given passing mention.

In such problems as given in the paper, an average condition of operation cannot well be assumed. Hence it is necessary to determine the maximum forces which may exist. Nature is sometimes just so perverse that if the worst condition were not accounted for in design, it would surely be met in operation. The maximum force calculated is 1,416 lb. per in. length of conductor and has been obtained for that case involving all unfavorable influences. At such high values of current as those with which the paper treats, or really for all those cases in which the short-circuit current is many times the normal value, it is shown that saturation is appreciable and should not be neglected. The inclusion of it in the analytical work yields results not quite so pessimistic, as pointed out, when only one value of current is considered.

Because of the fact that saturation is so pronounced, it clearly indicates the importance of knowing in a quantitative way just what it does under other circumstances in which it may have a significant magnitude. Saturation, of course, is always recognized as influencing the results obtained on the basis of no saturation, but is seldom assigned any value. The paper illustrates one case in which its effect is calculable, and as mentioned above, the result has come through field mapping.

It is interesting to note that the interpretation of equation (8) of the paper shows that the value of y is immaterial just so long as it is finite. The radial forces toward the tops or bottoms of the slots are not dependent upon conductor spacing in a vertical direction. This fact could also be reasoned out from Fig. 4 with its symmetry of current carrying conductors and their images.

It is an experimental fact that two conductors carrying current when placed parallel with each other exert such a force action upon each other that they tend to draw together unless constrained. The local flux fields between the conductors are thought of as wiping each other out, leaving a field which encircles both conductors as a unit. The same fact may be shown to be true with many conductors carrying current in the same direction and placed mutually parallel.

This is exactly the situation met in any coil side, and since the condition does not exist that the current in some of the strands is reversed to that in others (skin effect excluded), the resultant force is always one of contraction upon the bundle of conductors. It makes no difference as to the direction of the current flow. For any coil side taken singly, the force will always be toward the bottom of the slot, as can be deduced from the treatment of images, since the conductors and images act as a larger bundle of conductors with currents flowing in the same direction.

The physical fact just stated is experimental proof of conclusion (4), concerning the forces on conductors in armature slots, that the coil sides have no disrupting forces within them. It can also be used to verify the results, shown by Equations (9) and (10), that the force on the upper coil side is greater than that on the lower one. Conductors situated farther from the center of the coil are in a stronger field, and assuming equal current densities in the strands, the force action upon them is greater. The observed fact that parallel conductors carrying currents in opposite directions have a mutual force tending to separate them leads to conclusion (3) of that same group.

I would like to point out that the calculations made by Mr. Terry and myself in our paper on *Analytical Determination of Magnetic Fields* show that the assumption that flux went straight across the slot applies only to total slot flux, or to slot inductance. It is not true when near the kernel, and if that region is to be investigated, the approximate method of assuming straight flux lines cannot be used. That paper also determined by the analytical method that a m. m. f.-center does not exist in every current carrying conductor. This had previously been shown by Calvert and Harrison in a graphical treatment, and is stated in Appendix D of the above paper.

The paper furnishes the start on further quantitative information concerning short-circuit forces on armature coils, and results of the investigation should lead to better or more economical methods of coil bracing, spacing, and wedging. The general treatment also has an application to revolving armature machines in which centrifugal force on the coils is fairly great and must be considered together with the forces due to currents.

L. A. Kilgore: Due to their inherent low reactance turbine generators have always been subjected to strong forces in the end windings on short circuit. As a result of experience methods of very securely bracing the end windings of these machines have been devised. However, the development of an accurate thorough method of calculating these forces is very timely, because the increased ratings and special designs which are now being considered will make demands on the bracing of end windings exceeding the limits of any past experience.

Salient-pole machines in general do not present as difficult a problem as turbine generators, due to their inherently higher reactance. A large part of the method developed here can be applied directly to salient-pole machines. The forces on conductors in the slot and within the conical surface of the end winding are calculated in the same way for both salient-pole and turbine generators.

The forces on the straight section of the end winding are

relatively lower in salient-pole machines because even though the damper winding may project out part way under the end winding, there are no bars between poles where they would be most effective in producing force on the straight section of the end winding.

J. F. Calvert: In reply to the discussion by Mr. Terry: It is true that the change of inductance with displacement is one of the most commonly used methods for computing forces. However, not only the total inductance must be known, but also the rate of change of inductance with displacement. Thus, in the diamond portion of the end winding, it is hardly possible to use Mr. Alger's reactance formulas for the following reasons:

While these formulas probably give sufficiently accurate values for the total flux linkage in this zone, they are built on the assumption that the coils lie in a plane, and that the resultant flux field is symmetrically distributed with respect to this plane. The same argument may be applied here as was used in the paper to compute the tangential force on the conductors in slots. If the coil side is moved a small distance in either of the directions in which the flux field is symmetrical with respect to its initial position, the same change of interlinkage occurs with displacement in one direction as occurs with an equal displacement in the other direction. As the interlinkage of the coil is a single valued and continuous function of the position of the coil, then the rate of change of interlinkage at the initial position is zero. Hence, the force is zero. However, this is not true, as has been amply demonstrated on tests to destruction.

This simply means that while the total interlinkage as given by these reactance formulas may be very nearly correct, and give good results for this purpose to which they were designed, they do not assume the strictly correct distribution of flux, and hence will hardly furnish a means for correctly computing the forces.

In computing the forces on the portion of the armature coils which projects straight out over the retaining rings, the induced rotor currents cannot be neglected. This is particularly true if a good metallic contact exists between the retaining ring and the rotor body. In this case, the rotor surface currents are among the most important factors.

It is stated in the paper that the currents used to compute the forces on conductors in slots (see caption of Fig. 10) are approximately those measured in actual test. Hence, the change in reactance is not involved. It is only necessary to consider the reduction in flux due to the saturation in the iron, as is done in the paper.

The author is in agreement that the more theoretically correct expression for reactance might just as well have been used in Appendix A. Of course, in any case, the proper consideration of saturation is a most important factor. The effects of saturation on a single-phase line-to-neutral short circuit are different from those encountered on a single-phase line-to-line short circuit. The reactance used must be adjusted to take care of this. The most nearly correct value of current is necessary in computing forces.

The instants at which the forces are maximum due to out of phase currents is an interesting contribution. However, slow speed machines are not the ones in which to expect appreciable forces on the wedges. It is in the low-reactance high current per slot (or, in general, high-speed) generators in which this force should be given consideration.

The author is in practical agreement with the discussions of Mr. Robertson and Mr. Kilgore, and is indebted to them and to Mr. Terry for additions to the subject matter of the paper contributed through their discussions.

75-Kv. Submarine Cable for Deepwater Station

BY R. W. WILBRAHAM[1]

Member, A. I. E. E.

Synopsis.—This paper describes the problems attending the laying of eight 75-kv. submarine cables across the Delaware River in the vicinity of Wilmington.

To insure against injury the cables were laid in a backfilled trench, the depth of which was determined from experiments.

By terminating the cable on platforms just inside the pierhead lines it was possible to use a cable of 4,050 ft. (maximum length one of the accepted manufacturers could make) as compared with a river width of 5,100 ft.

To avoid excessive heating of that portion of the cable out of water at the cable platforms, the steel armor was replaced by one of non-magnetic material so designed to avoid corrosion and electrolysis.

The problem of laying the limited lengths of cable in the trench with minimum deviation was satisfactorily met with specially developed methods.

The construction work was completed in five months under winter conditions and heavy river traffic.

* * * * *

THE location of the Deepwater Generating Station on the New Jersey side of the Delaware River, opposite Wilmington and four miles south of Pennsgrove, made it necessary to transmit across the river a portion of the energy to supply the Wilmington and Philadelphia districts.

An overhead wire crossing was proposed, but refused by the United States Government as a potential hazard to aerial navigation. This made it necessary to consider a submarine cable and a study of this problem developed a number of controlling factors as follows:

1. It was desirable to use 66 kv. cable, but the longest piece obtainable was 3,600 ft. as compared with a width of 5,100 ft. of the nearby narrowest part of the river.

2. Pierhead lines had been established on both sides of the river thus extending riparian rights of land owners beyond the shore line, with consequent rights to dredge to a 35-ft. depth or erect structures thereon.

3. All unused land at this narrowest part of the river on the Delaware side was owned by one person who refused to sell and, in Delaware, a power company has not the right of eminent domain. This very greatly limited the possible location for a cable crossing.

4. This part of the river, below Philadelphia, League Island Navy Yard, Chester and Wilmington, has considerable traffic. It is also a natural deep water basin in which ships anchor in fogs or storms; frequently dragging their anchors as attested by interruptions to cables already laid on the bottom.

The solution of the right-of-way problem, and therefore the location of the crossing, was happily reached by the cooperation of the Reading Company which owned the riparian and upland rights on the Delaware side. Rights-of-way on this property were granted by the Reading Company and further inland by the Pennsylvania Railroad for aerial lines (Figs. 1 and 2).

CABLE

Analysis of the cable problem indicated the probability of successful operation of a circuit at 66,000 volts, which was desirable from the point of view of construction, operation, and economy.

The first and most important step was to obtain an agreement by the manufacturers to increase the size of their factory equipment in order to make a cable of sufficient length. By terminating the cable on platforms just inside the pierhead lines it was possible to use a cable of 4,050 ft. which was the maximum length one of the accepted manufacturers could make.

The selected cable was rated at 75 kv. (between conductors), to be used on a nominal 66-kv., three-phase, 60-cycle system with solidly grounded neutral. Eight

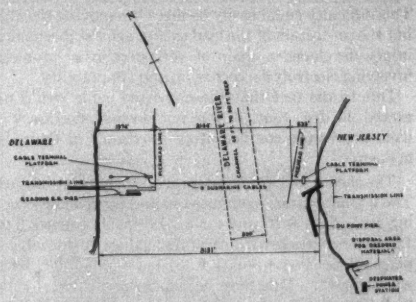

FIG. 1—PLAN SUBMARINE CABLES

single conductor cables were so arranged as to make up two three-phase circuits with a spare conductor for each.

The cable is of the single conductor type, paper insulated, electrostatically shielded, lead covered and steel armored. The conductor is of the standard stranded type (not hollow core), 750,000 cir. mils in area. The insulation is 54/64 in. thick, over which is applied a perforated shielding tape. The lead is 5/32 in. thick, covered with two layers of asphalt saturated jute and one layer of No. 4 galvanized steel wire armor, giving an over-all diameter of 3¾ in. and a weight of 21 lb. per ft.

The cable was made according to the specifications of

1. Electrical Engineer, United Engineers & Constructors, Inc., Philadelphia, Pa.

Presented at the Middle Eastern District Meeting No. 2, of the A. I. E. E., Philadelphia, Pa., October 13-15, 1930.

the Association of Edison Illuminating Companies with certain minor modifications.

Each circuit was designed to carry 60,000 kv-a. continuously (525 amperes per conductor) with a resulting conductor temperature not to exceed 60 deg. cent. This rating was determined on an "in air" basis without steel armor, because it operates in air for an appreciable distance above the water at terminal points.

A comparison of the difference in the losses and first costs for bronze and steel armor was decidedly in favor of the steel and, accordingly, steel was used. There

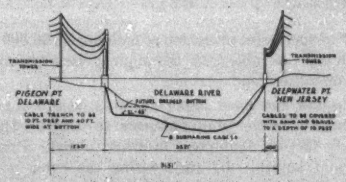

FIG. 2—SECTION SUBMARINE CABLE

would be increased heating from the steel but this was inconsequential for the major part of the cable which was immersed. However, for the short section out of the water, between the river level and the cable potheads, additional heating due to steel was prohibitive. This difficulty could easily be met by removing the steel but the protection of the lead sheath against mechanical injury developed a chain of very perplexing problems involving electrolysis, corrosion, and adaptability.

Due to the fact that the immersed cable would not exceed 40 deg. cent. it was not deemed necessary to include pressure compensating provisions in the cable makeup.

The sheath and armor losses are of particular interest. The manufacturers' computed values for spacings varying from 2 to 6 ft. and for cables in air ranged from 12.5 to 15 watts per ft. The test values of the cable in place on an average of 4 ft. spacing amounted to 10 watts per ft. This may be explained in part by the fact that the submerged cables are subjected to an envelope of brackish conducting water, the effects of which could not be taken into account in the calculations. The dielectric loss was very small, being about one-third watt per ft. at 75 kv. and a copper temperature of 40 deg. cent.

Due to the importance of the installation and the question of the respective merits of the various products, one circuit and spare cable was purchased from each of two manufacturers. The cable was manufactured in eight lengths (each length complete without splices) and wound on reels, having a total weight of 46 tons each. Each piece was 4,050 ft. long from which approximately 125 ft. was taken for test purposes, leaving a net length of 3,925 ft.

The cables were terminated in potheads of the 110-kv. outdoor porcelain type, oil filled design, so constructed as to provide ample expansion of the copper conductor apart from any other portion of the cable.

The standard pothead design was slightly modified by replacing the usual glass oil reservoir at the top by one of all metal oil tight construction. A six gallon oil reservoir under a pressure of seven lb. was connected to the cable immediately below the pothead to insure against the running of the cable compound and voiding the insulation, particularly the portion emerging from the river.

These cable potheads were mounted on the terminal platforms (Fig. 3) which also supported the transmission line dead-end towers, a structure for the operation of the spare cables and bus, a small gantry crane and a deck house for telephone and repair equipment.

MECHANICAL PROTECTION OF THE CABLE

Cables previously laid near the proposed location were subjected to excessive service interruptions due to vessel anchors dragging across them. For this reason it seemed advisable either to lay the proposed cables on the river bed, and provide protection against dragging anchors, or to bury them under the river bed to a depth where they would not be disturbed.

Marine authorities were consulted as to the action and penetration of dragging anchors, but not even fairly accurate information was available. Therefore, it was decided to conduct a series of experiments, based on two methods of protection:

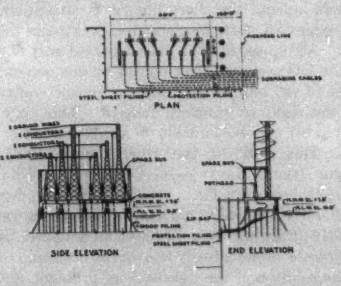

FIG. 3—CABLE TERMINAL PLATFORM

1. Placing the cable in an excavated and backfilled trench at a depth greater than anchor penetration.
2. Cables laid directly on the bottom between two rows of steel sheet piling flush with the river bottom.

Anchors of the stockless type as used on various sizes of vessels were used strictly in accord with marine practise.

Tests in deep water by soundings and in shallow water by observation clearly indicated that an anchor, either dropped or dragged, will not penetrate deeper than its flukes. It was found that sheet piling placed in rows to protect the cables from the anchors was not

effective since the flukes, after engaging the sheet piling, would slip and jump the piling into the cable area and "hook" the cable. These experiments showed that the dragging of anchors was a real menace to the cables; that anchors, except possibly under extraordinary conditions, could not penetrate greater than their flukes or 8½ ft. for the largest anchors normally used, and that the placing of sheet piling in rows was not effective. Therefore, it was decided to install the cables in a

Fig. 4—Spare Cable Support and Slots

trench 10 ft. deep and 40 ft. wide at the bottom and to backfill it to approximately the original level.

Simultaneous with the above experiments, test borings in the river bottom were obtained which showed very unsatisfactory conditions. Three-fourths of the river width from the Delaware side had a plastic mud bottom. This mud was so soft that at times the 4 in. bore casing hung free in the chains 25 ft. below the river bottom. The remainder of the river bottom is firm sand and gravel. This exploration gave rise to the question of whether or not a trench could be dredged and how long it could be maintained open for cable laying. Therefore, further experiments were conducted as follows:

1. Dredging test trenches in mud and gravel sections of the river to determine the time that the trench could be maintained open for cable laying.

2. Dredging test trench in the river bottom between rows of sheet piling to ascertain the effectiveness of piling in retaining the slopes and preventing resilting.

3. Handling of the cable laying or derrick boat across the river for determining alinement, method, and accuracy of travel.

Contrary to the conclusions drawn from the test borings, the results indicated that there would be no difficulty in excavating a trench in the mud with a side slope of one and one-half to one and showed practically no evidence of resilting. However, the velocity of the river eroded the sides of the trench into a very long slope, thus tending to form a new river bottom, which, if allowed, would increase the backfill required or decrease the protection. On the New Jersey side the sand and gravel trench sides held up, but "silted in" about as rapidly as the erosion of the side walls in the mud section. Soundings in the test trenches over a short period indicated that the trench in both the mud and gravel would not maintain, in a satisfactory condition, for more than approximately 45 to 50 days, and that unusual efforts would be required to remove and dispose of 205,000 cu. yd. of excavation, and lay the cable in that period.

Cable Terminal Platforms

Since the cable was of insufficient length to reach entirely across the river, it was necessary to build concrete island platforms just inside the pierhead line to carry the cable potheads and other necessary equipment.

From the beginning it was recognized that this structure would have to be constructed with open slots from and through the piling, straight up to the top of the concrete deck, so that the cable could be "rolled" or placed in position without the necessity of "threading" under the piling and up through the platform, shown at *A* on Figs. 3 and 4.

On the Delaware side the space for the cable platform was not only very limited but accompanied with perplexing conditions. The Reading Company coal load-

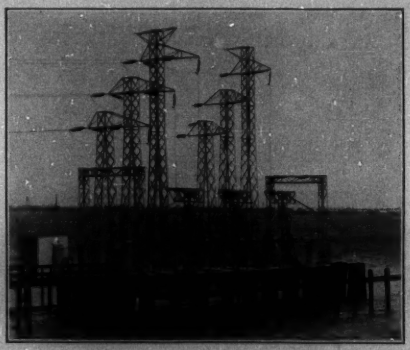

Fig. 5—Cable Terminal Platform

ing and freight car ferry piers were immediately adjacent, and there was every possibility that they would dredge around the cable platforms to provide a 35-ft. ship basin, which would undermine the platform foundations unless special precautions were taken. Protection was provided by steel sheet piling, flush with the present bottom, acting to confine the foundations. Wood piling, 8 ft. on centers, was driven parallel with the sheet piling to act as a guide or warning for future construction operations of the Reading Company (Figs. 3 and 5).

The foundations are of wood piling and timber work, on which the platform proper is carried in the form of a slotted concrete box, setting over corresponding slots in the piling and timber work (Fig. 5). Each platform has four compartments, completely "closed off" with concrete walls, providing structural strength and safety against fire. Each compartment is provided with two slots for two cables and after the cable was installed, the slot was closed with stop logs of such design as to keep out heavy floating debris, but at the same time to provide partial automatic cleansing of the water in the compartment with the changing tides. The platforms are fenced and protected by piling.

The spare cables can be used in any circuit by having each spare cable permanently connected to a spare bus

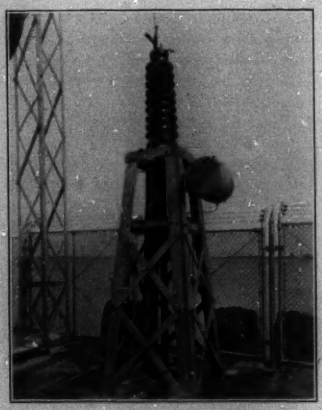

Fig. 6—Cable Terminal and Support

and providing removable connectors in the vertical drops to the remaining six cables. In the event of failure of any one cable, its connector in the drop is removed and the upper part of the drop swung over to either spare bus.

The porcelain cable potheads are carried on a tapering four legged latticed steel structure which is fastened to, but completely removable from, the concrete deck (Fig. 6). It is so designed that the latticed work, which normally acts as a protective barrier, can be completely removed, providing complete accessibility to the cable.

A small traveling cantilever type crane is provided to handle the porcelain potheads or other work in connection therewith, and is so designed that it cannot be moved from the storage point to the pothead space without placing the rotating boom in such a position as to avoid conflict with live risers of other cables.

To comply with the heating conditions of the specifications, the armor was removed from that part of the cable in the air. The steel armor is terminated one foot below low water and lapped around three large cast iron flanges bolted together, which are suspended from the underside of the concrete platform by means of two chains and turnbuckles (Fig. 4), which provide an effective manner of dead ending the steel. To prevent tide action and ship wash from moving the cables where they slope into the cable platform a mat of "rip rap" was laid on the cables from inside the structure to a reasonable distance outside the structure as indicated on Fig. 3.

The exposed cable above the armor is partly in air and water according to the tides, which required all the cable under the platform to be protected against damage from ice and debris. This matter of protection gave rise to an electrolysis and corrosion problem, which is discussed later.

As originally planned, the dead weight of the cable between the armor clamping flanges and the bottom of the pothead (22 ft.) was to have been relieved from the pothead by means of double ear woven wire cable grips, approximately 4 ft. long, suspended inside the pothead support and immediately under the wiping sleeve of the pothead (Fig. 6), but the cable was laid with such accuracy that approximately 190 ft. remained on each side. It was planned to utilize as much as possible of this extra length by providing loops in the cable in the compartments under the deck of the platforms. This was accomplished rather easily, as shown in Fig. 4, by laying the spare cable in two cast iron troughs provided with treated wood shoes, to prevent battery action. This removed all of the dead weight between the armor clamping rings and the underside of the platform, leaving but 9 ft. of cable to the potheads. The use of the cable grips on this short length might be questioned, but they were installed to provide against slippage of the cable in the troughs and resulting possible stresses on the potheads.

Cable grips have been mostly used heretofore for construction purposes only and experimental work was conducted to determine the holding power and distribution of pressure for various lengths of grips. It was found that, with proper selection and weave of material, together with the length and separation of the eyes, that the forces equal to the ultimate strength of the grip could be distributed over the entire length of the grip without injury to the lead sheath.

Electrolysis Protection

The removal of the steel armor from the cable necessitated the installation of non-magnetic protective armor after the cable was laid and placed in the slots of the platform. This was particularly difficult. Materials had to be selected or placed in such a manner so as not to be affected by the brackish and corrosive river water or set up electrolytic action, and be of such mechanical composition and arrangement that it could be installed in the restricted slot space of 18 in. between the piles and

immediately adjacent to the armor clamping rings, one foot below mean low water. Many methods of accomplishing this were considered and experimented upon, but were found to be weak or practically impossible.

The winding of steel or bronze tape or wire was out of the question as was demonstrated by experiments. Such metals in the presence of river water would set up battery action which, in the case of bronze, would attack the lead and, it was estimated, completely destroy it in approximately seven years. In the case of the steel, the lead would not be attacked but the steel would be subject to electrolysis and the corrosion of the river water, giving it a life not in excess of four to five years. Many other methods of wrapping the lead cable with a waterproof material on which bronze armor could be wrapped was not only questionable as to protective value, but was next to impossible to install due to the limited space.

Finally a method was devised that avoided all of the other difficulties and was extremely simple in every way. This consisted of slipping over the cable a commercial, flexible, liquid tight bronze hose, to each end of which was fastened a coupling. This flexible hose was fastened to the armor clamping rings at the bottom

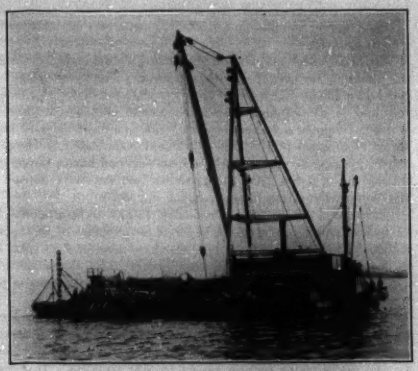

Fig. 7—Cable Laying Barge

and stopped at a point just below the top of the concrete deck. The couplings at each end were made tight against the lead sheath over a lapping of asbestos tape. The hose, being larger than the cable, was filled through a grease fitting in the lower coupling with a neutral grease and oil until it escaped from a relief valve in the top coupling. This has proved to be very successful. The hose is strong and affords every protection against ice and debris. The neutral oil and grease prevents the bronze from coming in contact with the lead and setting up battery action and most effectively keeps out the river water, thus preventing electrolysis.

Construction and Installation

The construction work was divided into four operations: dredging, laying cables, backfilling and cable platforms.

The cables were laid as soon as possible after trenching was completed in order to avoid the results of sloughing and resilting of the trench, which was considerably in advance of the completion of the cable platforms.

Trenching was carried on with five dredges operating

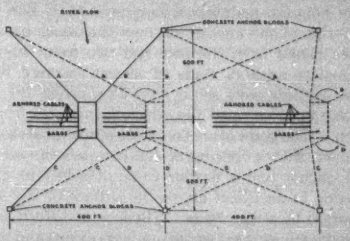

Fig. 8—Cable Barge Propelling Itself across River

day and night. The trench was approximately 40 ft. wide by 10 ft. deep at a maximum depth of 62 ft. below mean low water and an average of 45 ft. The dredged material was barged down the river to a point in front of the new Deepwater Power Plant where a suction dredge rehandled the material and filled up the low land around the plant. Material to the extent of 205,000 cu. yd. was removed and disposed of in 43 days.

Considerable thought had been given to the handling of the cable laying or derrick boat, particularly to keeping it in as straight a line as possible. In considering this problem it is well to have an idea of the magnitude of this particular part.

The river at ebb tide has a velocity of about 2½ miles per hour, the wind and sleet storms in the fall and winter are severe and at all times the river traffic was annoying and uncomfortably close. It was felt that it was best to lay four cables at a time which gave approximately a 250-ton, reel and attendant equipment load. A derrick of ample capacity had to be available to reclaim cable in the event of the boat getting off course beyond the trench, or the occurrence of an accident. There was but a difference of 209 ft. between the amount on the reels and the actual computed length of cable required, without allowances for deviation. Consequently, the question of keeping the cable boat on a straight course was of outstanding importance.

Experiments were conducted with the equipment intended to be used and it was found that the laying of cables could be best accomplished by placing heavy concrete anchors in the river bed on each side of the cable laying boat, and by means of anchor lines, have the boat pull itself equally between the anchors with

relative safety. The total horizontal deviations in these trials was found to require about 150 ft. of cable, thus leaving the very small spare amount of 59 ft. for unforeseen conditions.

The cables were laid four at a time, two trips being required to complete the operation. The cable reels were mounted on the bow end of a large derrick boat (Fig. 7) which was towed to one end of the trench, immediately adjacent to the cable terminal platform, where the ends of the cables were taken off and placed in position on a temporary rack.

The boat was maintained lengthwise to the flow of the river (Fig. 8) and kept in line over the trench by means of two cables from the bow and two from the stern, fastened to anchors made of 30-ton concrete blocks. These anchors were set in two lines across the river, one line upstream and the other downstream from the trench. The rows of anchors were approximately 1,200 ft. apart, with a spacing of 400 ft. apart in each row. Each anchor had a flexible cable attached to it, terminating in an eyelet above the surface of the water, supported by a spar buoy. The bow and stern lines of the derrick boat were wound on large drums and so connected to an engine drive on the boat that all four drums could be operated in any combination, in synchronism or separately. Thus the derrick boat pulled itself across the river and maintained itself on the established line by the use of ranges and check methods developed especially for this service. As the boat progressed across the river, the bow and stern lines were shifted from one anchor to another as shown in Fig. 8.

As the cables were being laid they were frequently checked by a diver and later by soundings which showed the cables to be 4 ft. apart and practically balanced across the center line of the trench, with a maximum deviation of any cable from a straight line not exceeding 6 ft. The experimental or trial run of this method indicated 150 ft. required for deviations, but in the actual laying only 60 ft. was used.

This was the most important element in the satisfactory completion of the job.

Immediately following the laying, the concrete anchors were removed and the trench backfilled with sand and gravel.

The construction of the cable terminals did not involve difficulties except that, due to the fact that the cables were placed on a temporary structure long before the concrete was poured, more than usual care had to be exercised to prevent injury to them.

The engineering features and the right-of-way were started in the spring of 1929, but nothing conclusive could be done until the right-of-way was determined in August when the work was given a definite beginning and prosecuted vigorously in the face of approaching winter.

The dredging was started October 7, 1929 and finished November 25th. The cable laying was started on November 27th; the first run finished in 5 days and the second in 3½ days. The backfilling and cable platforms were finished on February 14th and March 7, 1930, respectively, and the cables placed in service on March 9, 1930, a total elapsed construction time of 131 days.

This project was constructed for the Delaware Power & Light Company, subsidiary of the United Gas Improvement Company, by United Engineers & Constructors, Inc., and is operated jointly with the Philadelphia Electric Company.

Discussion

R. J. Wiseman: Some facts pertaining to the manufacture of the cables, the construction of which is given in the paper, on account of being the longest lengths of 75-kv. cable ever made in the world and the first extra high-voltage submarine cable are given in the following discussion.

A jute serving which held the armor wires snugly in place while installing was wrapped for 50 ft. on each end. We believe that submarine cables should have a jute serving throughout their entire length. Also, we thoroughly slushed the armor wires during the armoring process in order to reduce armor losses.

The order called for four 4,050-ft. lengths of cable. Four 4,115-ft. lengths were manufactured. The tank equipment used for drying and impregnating was especially designed to take care of extra long lengths of cable. The impregnating reel had a 17-ft. traverse and weighed 14,000 lb. It is capable of handling 6,000 ft. of cable of this size in one piece. Our standard impregnating reel will handle a 1,700-ft. length of this cable which weighs 3,800 lb.

The cables were shipped on the impregnating reels, so designing them to be used for both purposes. The total weight of reel and cable as shipped was 48 tons. Special shafts and pedestals had to be designed and built to take care of this weight. A special yoke which weighs three tons was designed and built to lift the loaded reel. This equipment was loaned to the purchaser in order to transfer cables from railroad car to barge and setting up on barge. On shipment we very carefully housed in the reel to prevent any damage during shipment.

Each reel of cable took 54 hr. to dry, 130 hr. to impregnate, 20 hr. to lead and 32 hr. to armor. There were 39,000 lb. of copper, 56,000 lb. of paper and oil, 116,000 lb. of lead and 115,000 lb. of armor wire required for this order. The dielectric loss averaged 0.35 watts per ft. at 40 deg. cent. and 0.50 watts per ft. at 60 deg. cent.

Tests on the installed cable show that with a 560-ampere load, the temperature rise was 29 deg. cent. On this basis and assuming an ambient temperature of 15 deg. cent., the permissible load for the under-water section would be 700 amperes. However, the maximum load on account of terminal conditions is no more than about 560 amperes.

The armor and sheath losses were estimated at about 14 watts per ft., but actual tests showed for a 560-ampere load, about 10 watts per ft.

The cables have come up to our expectations in every way and we have full confidence in them to perform as intended. We feel that both the United Engineers & Constructors and the Philadelphia Electric Company are to be congratulated in their very thorough study of the whole problem, the painstaking care exercised during installation and the courage to be a pioneer in extra high-voltage submarine cables. We are very thankful to them for the opportunity to furnish one circuit.

R. W. Atkinson: The measurements of effective resistance upon these cables have resulted in a very satisfactory check on an exceedingly simple method of calculation of sheath and armor losses for this type of cable. Preliminary calculations showed

that the combined current in the lead sheath and armor would approximate rather closely to the current flowing in the conductor and that wide variations in assumptions would affect only slightly the calculated amount of this circulating current. Further calculations made to determine the probable amount of the magnetic losses with both armor and sheath short-circuited, and also of the loss produced by circumferential flow of current in the lead sheath caused by the spiralling of the armor wires, showed these to be negligible. Determination of losses was therefore made on the following basis. The combined sheath and armor current was assumed to be equal to the conductor current and assumed to be divided between the armor and sheath in proportion to the relative a-c. conductance. Based on a conductor current of 525 amperes, calculations by this method gave a total sheath and armor loss of 12 watts per ft. which was the value given by our company. The agreement with the measured value of 10 watts per ft. given in Mr. Wilbraham's paper is surprisingly close.

This installation marks a distinct advance in the use of submarine power cables. The proposition had been under consideration for a number of years and the general impression had been that the installation of such cables would be extremely difficult if not actually hazardous. The preparations for the job as described in Mr. Wilbraham's paper, however, were so complete and thoroughgoing, that the solutions of all the problems involved were ready in advance and the actual installation was accomplished with no greater difficulty than attends normal submarine installation.

From the manufacturing standpoint, most of the procedure was exactly the same as for the usual lengths, the main difference being the necessity for providing the necessary huge reels and equipment for handling them. The testing of such a cable, however, presented some unusual and interesting features. The charging current on these lengths at 169,000, the voltage required by standard specifications, is approximately 16 amperes, whereas testing transformers within this voltage range ordinarily have one-ampere secondaries. Thus, the supply of this very high charging current was beyond the range of any ordinary testing equipment even where augmented by the use of several transformers in parallel. There was discussion of testing the cables at a modified or compromise test voltage. It was, however, appreciated that since the result of failures on submarine cables are very much more serious than on underground, any reasons justifying the testing of underground cable at standard test voltages become doubly important for submarine cable. Furthermore, the very fact that even though these cables were different from other cables of regular manufacture only from the standpoint of their unusual length, the mere fact of this one difference from the standard and normal factory procedure made it imperative that no precaution that would normally be considered important for underground cable should be waived for these especially important cables. Since the special apparatus necessary to supply the charging current could not be obtained in time in any other way, this was designed and built by the Research Laboratory and standard tests were made according to the A. E. I. C. Specifications. Thus, in addition to emphasizing the usual careful factory inspections, no part of the assurance obtained by testing on the finished cable normally obtainable on short sections was omitted with these important submarine cables.

J. W. Sylvester: There is little that can be added to the paper except to emphasize several of the unusual problems which presented themselves in connection with its manufacture and its installation.

The manufacturers' problems were, first; to build a cable rated for 75 kv. in such long lengths that it required numerous changes in their factory equipment, second; to provide the electrical testing equipment which would permit the testing of these cables after completion. These problems were solved by the manufacturers as the tests of various kinds which were applied during and subsequent to the manufacture of the cable clearly showed.

The construction company's problems were many, but the two which were outstanding were, first; to excavate a trench across the river at a very unusual depth for river crossings in which it was intended to bury the cables, second; to lay these cables in the trench and in as straight a line as possible.

As so well described, both of these problems were solved in a very satisfactory manner and both the manufacturers and the construction company are to be complimented upon having planned and executed their respective responsibilities so thoroughly.

This installation was the outstanding cable event of 1930. Judging by the extremely careful manner in which every process in the manufacturing of the cable and its installation was carried out, as it was the writer's privilege to view them, these cables should render the service expected of them for many years to come. They are well buried beneath the bed of the river and we believe are out of the reach of ships' anchors or any other disturbing cause likely to result in mechanical injury, which from past experiences has been the bane of most submarine cable installations.

Circuit Breaker Recovery Voltages

Magnitudes and Rates of Rise

BY ROBERT H. PARK* and WILFRED F. SKEATS†

Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*This paper shows the conditions affecting the magnitude and rate of rise of recovery voltage at the terminals of an oil circuit breaker upon interruption of a short circuit, and explains their effect in quantitative terms.*

Factors are presented whose numerical value takes into account the number of phases involved in the short circuit and the ground connections of short circuit and generator, the decrement of short-circuit current previous to interruption, and the effect of unequal reactances in the direct and quadrature axes of synchronous machines. It is also indicated how to take into account the effect of displacement and of initial load current, and the effect of saturation is discussed very briefly. Magnetic oscillograms are presented which verify the most important of the points brought out.

The most common locations and a rough idea of the magnitude of the capacitances affecting the rate of rise of recovery voltage for various types of short circuit are indicated. Calculations of the recovery voltage curve for several representative cases, at both low and high voltages, are presented, and in some cases cathode ray oscillograms are presented for comparison with the calculated curves.

While no systematic data appear to be available at present regarding the effect of the rate of recovery voltage rise upon circuit breaker operation, several tests showing a very pronounced effect, some of them made by the authors and some by others, are reported briefly.

A discussion of the method by which overvoltages are built up during the interruption of transmission line charging currents is presented at the end of the paper.

* * * * *

Introduction

While the duty on circuit breakers is commonly expressed in terms of the circuit voltage previous to short circuit, and the magnitude of the current in the arc, it is a matter of experience that in practise other circuit characteristics may affect the duty to an important extent.

For instance, tests made on the system of the Northern States Power Company, at St. Paul, Minn., during the month of September, 1925, brought to light a case of marked difference in the difficulty of opening the circuit, depending on the source of power employed.

This difference was between three-phase tests with breaker neutral ungrounded when connected through a 43-mile, 110-kv. line to an 18,750-kv-a., turbine generator at the Riverside Station; and similar tests when directly connected to three 6,600-kv-a. water-wheel generators at the Wissota Station.

Referring to Table I it will be seen that in spite of the slightly lower voltage, much greater difficulty was encountered when directly connected to the bus at Wissota.

Table I

Source of power	Riverside	Wissota
Number of tests made	2	5
R. m. s. line-to-line volts before short circuit	15,000	13,200
Average initial r. m. s. current in the arc	1,400	1,420
Inches of arc	1.2	4.8
Half cycles of arc	3.5	10.0
Maximum pressure, lb./sq. in.	6.5	36.0

Again, tests recently conducted on a 110-kv. (63.5-kv. to ground) explosion chamber breaker showed a marked difference between tests with the standard test circuit of the company with which the authors are associated, and a modification of this circuit in which a shunt resistance of 1,000 ohms was connected across the circuit breaker terminals.

In these tests it was found that without the resistor, the breaker would not clear the circuit at 88 kv. to ground across one pole, while with the resistor 132 kv. to ground across one pole was cleared. Actually the arc length required at 132 kv. with the resistor was less than that required without the resistor at 66 kv.

Also similar effects have been observed to a lesser degree in many other cases.

As a result of studies made at the time of the St. Paul tests, it became evident that these effects could be explained in terms of the brief "kicks" which occur in the recovery voltage and the time delay required for their establishment.

Table II shows data prepared in connection with this study.

Table II

Source of power	Riverside	Wissota
Phase voltage before short circuit	12,240	10,780
Phase voltage one cycle after recovery	8,200	7,500
Recovery voltage of phase which opened first		
(*a*) "abrupt"‡ rise	2,800	13,500
(*b*) peak	14,600	18,000
Time delay from *a* to *b* in milliseconds (as measured on magnetic oscillogram)	1.0	0.8
Peak recovery voltage of phases which opened later	9,100	7,800

It will be seen that at Wissota the kicks were of greater amplitude and were established in a shorter period. The conclusions arrived at in this study were briefly stated by Mr. J. D. Hilliard in his discussion

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‡The only oscillograms available in these tests were of the magnetic type and hence any very high frequency voltages which may have been present would not be observed. Probably such voltages were present in the form of an overshoot to approximately double the "abrupt" value.

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of the paper presented in 1927 by Sporn and St. Clair. Since that time Slepian and Biermanns have further emphasized the significance of these factors. Thus the phenomena have been demonstrated and there has been agreement as to the basic conceptions necessary to understand them.

On the other hand, no very comprehensive analysis has been presented, to show how the magnitude and time delay of these kicks of recovery voltage are affected by the various types of apparatus which are found on transmission systems. This paper attempts to present such an analysis with numerical calculations for several of the simpler cases, and magnetic and cathode-ray oscillograms confirming these calculations.

The paper is divided into two parts of which Part I summarizes the conclusions and Part II gives the theoretical work on which the conclusions of Part I are based.

Part I

Summary of Conclusions

The phenomena of recovery voltage may be divided into two parts, low frequency effects and high frequency effects. The low frequency effects comprise those arising from the type of short circuit, decrement, displacement, and flux distribution in rotating machinery, and include most of the factors determining the magnitude of recovery voltage. The high frequency effects are attributable to the capacitances of the various parts of the system to ground. They are responsible for an overshoot, often to double the value predicted from the low frequency considerations, and include all of the factors determining the time required to reach the maximum value. It is convenient to consider these two types of phenomena separately. The low frequency phenomena will be treated first.

Low Frequency or Magnitude Phenomena

A. Nature of the Impedance Limiting the Short-Circuit Current

In a short circuit in which the current is limited principally by resistance, the recovery voltage wave starts near the zero point of the cycle, so that the instantaneous value for the first-thousandth of a second is low. When the current is limited principally by reactance, however, unless the current wave is displaced, the recovery voltage wave starts at its crest value. A reactive short circuit is therefore much more difficult to interrupt than a resistive short circuit of the same current and voltage. Unfortunately practically all serious short circuits are almost purely reactive.

B. Number of Phases Involved and Ground Connection of System; Effects of Stationary Apparatus, No Load

1. Single-Phase Short Circuits. Where the short-circuit impedance of stationary apparatus is much greater than that of rotating machinery, for single-phase short circuits, either line-to-ground or line-to-line, the voltage after interruption is the same, except for high frequency effects, as the voltage before establishment of the short circuit. The oscillogram of Fig. 1 shows both voltage and current for a line-to-line short circuit on a 13,200-volt, 26,700-kv-a. generator with 1.5 ohms external reactance in series per line. Voltage both before and after the short circuit are shown and these will be seen to be identical.

2. Three-Phase Short Circuits. The general expression for the recovery voltage of the first phase to clear of a three-phase short circuit, where the effect of stationary apparatus predominates, in terms of symmetrical phase sequence components, is

$$e = \frac{3\,E\,Z_0\,Z_2}{Z_1\,Z_2 + Z_0\,Z_1 + Z_0\,Z_2}$$

where E is the phase-to-neutral voltage existing before short circuit.

If either the generator or the short circuit is ungrounded, Z_0 is equal to infinity, and in stationary apparatus $Z_1 = Z_2$. Making these substitutions, for

Fig. 1—Oscillographic Record of a Line-to-Line Short Circuit on a 13,200-Volt, 26,700-Kv-A. Generator with 1.5 Ohms External Reactance in Series per Line

the first phase to clear of an ungrounded three-phase short circuit, or of any three-phase short circuit on an ungrounded system,

$$e = \frac{3}{2}\,E$$

Fig. 2 shows an oscillogram taken on the first phase to clear of a three-phase short circuit at 13,200 volts on a 100,000-kv-a. alternator, ungrounded, with reactance in series per phase equal to about ten times generator subtransient reactance.[3] The ratio of the first peak of recovery voltage to the crest voltage existing before short circuit (as determined from a voltmeter) is 1.47.

If both the system and the short circuit are solidly grounded, Z_0 may have a low value and the recovery voltage of the first phase to clear may be very low, or, on the other hand, Z_0 may be equal to or greater than Z_1, and the recovery voltage equal to or greater than the voltage existing before short circuit.

In systems grounded through a neutral impedance, the neutral impedance is usually high enough so that

3. See Bibliography.

the recovery voltage is practically that obtained on an ungrounded system.

If the neutral is grounded through a reactor, this peak voltage occurs immediately (except for "high frequency" delay) upon interruption. If the neutral impedance is a resistor, however, the initial value of recovery voltage is the same as if the neutral were solidly grounded. The voltage then rises more slowly to its peak value, which occurs usually about 10 or 20 deg. after the interruption. A neutral resistor thus possesses a slight advantage over a neutral reactor from this point of view.

FIG. 2—OSCILLOGRAPHIC RECORD OF CURRENT AND VOLTAGE OF THE FIRST PHASE TO CLEAR OF A THREE-PHASE SHORT CIRCUIT AT 13,200 VOLTS ON A 100,000-KV-A. ALTERNATOR, UNGROUNDED

External reactance per phase ten times generator subtransient reactance

3. Two-phase-to-ground Short Circuits on an Impedance Grounded System. For the first phase to clear of a two-phase-to-ground short circuit, the recovery voltage varies from a theoretical minimum of half line voltage, with negligible zero-phase sequence impedance, to line voltage, with very high neutral impedance, if the neutral impedance has the same phase angle as the line impedance. If the neutral impedance phase angle differs from that of the line impedance, the recovery voltage for one phase is somewhat decreased, while that for the other is increased.

C. Effects Arising From Rotating Machinery at No Load

Fig. 3 represents the decrement of the a-c. component of current through the breaker during a severe three-phase fault. In this figure,

i'' = initial inrush current

i = current at the time of clearing.

The recovery voltage for the first phase to clear of a three-phase short circuit at the machine terminals, if either the machine or the short circuit is ungrounded or has a high ground impedance, is

$$e = 1.5\, x_q''\, i$$

where x_q'' = the quadrature subtransient reactance of the machine.

For a three-phase short circuit at the machine terminals at no load,

$$i'' = \frac{E}{x_d''}$$

where E = the leg voltage of the machine before short circuit, and x_d'' = the direct subtransient reactance of the machine.

Thus the recovery voltage formula may be written

$$e = 1.5\, E\, \frac{i}{i''}\, \frac{x_q''}{x_d''}$$

If the short circuit is not at the machine terminals the quantities x_q'' and x_d'' should be replaced by quantities s_q'' and s_d'' where s_q'' and s_d'' are the reactances of the system using, respectively, quadrature and direct subtransient reactances as the reactances of the generators.

The recovery voltage for the first phase to clear of a three-phase short circuit may therefore be considered as the leg voltage of the system before short circuit, modified by three factors:

1. A factor k_g, which depends upon ground connections.
2. A decrement or "change in excitation" factor, k_i.
3. A quadrature reactance factor, k_q.

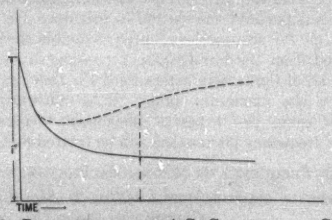

FIG. 3—DECREMENT OF THE A-C. COMPONENT OF CURRENT DURING A SEVERE FAULT

i'' = initial inrush current
i = current at time of clearing
Full line refers to hand-regulated machines, and dotted line to machine with high-speed excitation

The values of k_g, k_i, and k_q for all types of short circuit are given below:

1. $k_g = 1.0$ for one-line-to-ground short circuits.

$$k_g = \frac{2\sqrt{3}\,(s_q''^2 + s_q''\,x_0 + x_0^2)^{3/2}}{s_d''(2\,s_q'' + x_0)^2 + x_0(2\,s_q'' + x_0)(s_q'' + 2\,x_0)}$$

for two-line-to-ground short circuits.

$k_g = 1.73$ for line-to-line short circuits or (approximately) for the first phase to clear of two-line-to-ground short circuits on an impedance-grounded or ungrounded system. (In the case of a line-to-line short circuit on any system or a two-line-to-

ground short circuit on an ungrounded system, the voltage usually appears across two poles of the breaker in series.)

$$k_g = \frac{3\,x_0}{s_q'' + 2\,x_0} \text{ for three-phase short circuits.}$$

$k_g = 1.5$ for three-phase ungrounded short circuits or for three-phase grounded short circuits on an ungrounded system.

2. $k_s = \frac{i}{i''}$ for all short circuits.

3. $k_g = 1.0$ for all single-phase short circuits.
$k_g = 1.0$ for two-line-to-ground short circuits.

$$k_g = \frac{s_q''}{s_d''} \text{ for all three-phase short circuits.}$$

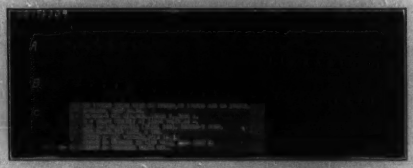

Fig. 4—Oscillographic Record of Current and Voltage of the First Phase to Clear of a Three-Phase Short Circuit at 13,200 Volts

On a 100,000-kv-a. generator, without an amortisseur winding, operating with the neutral ungrounded. No external reactance

Average values of $\frac{x_q''}{x_d''}$ for machines of various types are given by Table III.

Table III
Quadrature Reactance Factors

Types of machine	$\frac{x_q''}{x_d''}$
Turbo-alternators	
a. Laminated rotor	2.5
b. Solid rotor	1.4
Salient-pole machines	
a. Without amortisseur	2.3
b. With amortisseur	1.1
Induction motors	1.0

Effect of Amortisseur Winding

Table III shows very strikingly the effect of an amortisseur winding in reducing the quadrature reactance factor of a salient-pole machine, and thus in reducing the recovery voltage of the first phase to clear in short circuits involving more than one phase.

This table indicates a reduction of a little more than 50 per cent in the recovery voltage of the first phase to clear of a three-phase ungrounded short circuit. Actually the reduction may be even more than is indicated by Table III, for the decrement factor is likely to be lower with an amortisseur winding than without. Saturation may modify the effects slightly.

Thus an amortisseur winding brings about a substantial reduction in the duty of a circuit breaker opening a three-phase ungrounded short circuit close to the terminals of a salient-pole machine.

Figs. 4 and 5 show oscillograms of two short-circuit tests on the same machine, that of Fig. 4 being taken before the amortisseur winding was installed on the machine and that of Fig. 5 afterwards. The first peak of recovery voltage is much higher in Fig. 4 than in Fig. 5. A brief quantitative analysis of these films is given in Part II.

D. Effect of Displacement

The existence of a large d-c. component in a short-circuit current may cause the recovery voltage wave to start near the zero point instead of at its crest. The instantaneous value of recovery voltage is thus considerably reduced.

Fig. 6 shows an oscillogram of a highly displaced short circuit on a 26,700-kv-a. alternator. It may be seen that the initial instantaneous value of recovery voltage is reduced to less than 50 per cent.

E. Effect of Initial Load Current

The recovery voltage depends upon the current through the switch. This may include load current as well as fault current. The formulas given under part *C* may still be used in this case, if i is taken as the total reactive current through the switch.

Fig. 5—Oscillographic Record of Current and Voltage of the First Phase to Clear of a Three-Phase Short Circuit at 14,500 Volts on the Same Generator as Fig. 4, but with an Amortisseur Winding, Operating with the Neutral Ungrounded and with no External Reactance

F. Effect of Saturation

Saturation tends to reduce any overvoltage requiring an increase in the flux of the generator. If, therefore, the net effect of initial load current and the decrement factor tends to increase the recovery voltage, their effect will be modified by saturation.

The presence of flux at a density near saturation in the teeth and the end of the pole operates to cut down the permeability of these parts to a change in total flux in any direction. Quadrature axis flux has a greater length of path through the end of the pole than has

direct axis flux. Thus saturation probably tends to decrease the value of both quadrature reactance and the quadrature reactance factor.

The reduction in the quadrature reactance factor is apparently about 10 per cent for the 100,000-kv-a. testing generator in the plant of the company with which the authors have been associated.

High Frequency or "Rate of Rise" Phenomena

Qualitative Discussion of Important Types of Apparatus

Practically all of the high frequency phenomena attendant upon the build-up of recovery voltage are attributable to the capacitance to ground of apparatus between the circuit breaker terminals and the principal part or parts of the reactance limiting the short-circuit current.

There are six types of apparatus which are of importance from this point of view:

1. Transmission lines and cables.
2. High voltage bushings.
3. Station bus structure.
4. Transformers.
5. Current limiting reactors.
6. Rotating machines.

1. Transmission lines and cables are well known to be distributed circuits capable of supporting traveling waves. They act substantially as resistances until a wave has traveled from one end to the other and back, which, with a line of even moderate length, requires a time very much longer than is taken to reach crest voltage when no transmission lines are connected to the bus. The resistance corresponding to a transmission line is 300 or 400 ohms; that corresponding to a cable is 20 to 50 ohms.

2. High voltage bushings have a capacitance of 0.0001 to 0.0003 μf. They are of importance only when a number of them is connected to the circuit and there is very little capacitance of any other kind.

3. The capacitance of station bus structures varies widely according to voltage and kv-a. ratings. Its limits are probably about 0.001 μf and 0.02 μf. It is of importance only when no transmission line or cable is connected.

4. The effective capacitance to ground of the high voltage winding of a transformer varies from about 0.001 μf. to 0.002 μf., the higher values occurring with shielded transformers.

5. Current limiting reactors have a capacitance to ground of about 0.0001 μf., so that unless some apparatus of appreciable capacitance to ground is connected between the reactor and the circuit breaker, the frequency of recovery voltage may be as high as several hundred thousand cycles.

6. A rotating machine has a capacitance usually between one-tenth and one microfarad between the copper of its windings and the iron of its stator laminations. This capacitance determines the frequency and rate of rise of recovery voltage in the case of a dead short circuit on the machine.

Effect of Arc Voltage Just Previous to Interruption.

The form of the recovery voltage curve is usually an oscillation of one form or another starting from the value of arc voltage immediately before interruption, and having as its final center the voltage determined from low frequency considerations. In most of the calculations connected with this paper the arc voltage immediately before interruption is assumed zero. If this voltage is much different from zero, the initial amplitude of the oscillation is increased and the first peak of the high frequency oscillation is higher than otherwise. This effect is shown in the calculated curve of Fig. 12, in which the dotted line corresponds to the first half cycle at high frequency, as calculated on the basis of zero arc voltage immediately before interruption, and the full line corresponds to the curve actually obtained.

Fig. 6—Oscillographic Record of Current and Voltage for a Highly Displaced Short Circuit on a 26,700-Kv-A. Alternator

Recovery Voltage Rates on High-Voltage Systems

The extreme values of frequency and rate of rise of recovery voltage at the high side of a transformer are experienced when there is no apparatus of appreciable capacitance (say 0.0002 μf. or more) on either the high or low side of the transformer.

Under this condition, assuming an effective capacitance to ground of 0.001 μf. per transformer winding, the frequency and rate of rise are given by the equations.

$$f = \frac{3200}{\text{kv.}} \sqrt{\frac{\text{kv-a.}}{n\, k_q}} \text{ cycles per second}$$

and

$$r = 20\, k_e\, k_s \sqrt{\frac{\text{kv-a.} \times k_q}{n}} \text{ volts per microsecond.}$$

where

Kv-a. denotes the initial short circuit kv-a. per phase (neglecting the d-c. component of current),

Kv. denotes the leg voltage of the system before short circuit, in kilovolts,

and n denotes the number of transformers in parallel.

It should be appreciated that these formulas represent extreme conditions, and that even with only a moderate amount of bus structure and no transmission

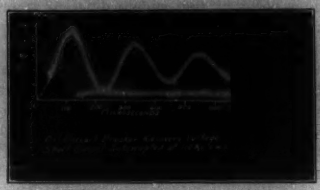

FIG. 7—CATHODE-RAY AND MAGNETIC OSCILLOGRAPHIC RECORDS OF THE RECOVERY VOLTAGE OF A 231,000 KV-A. SINGLE-PHASE SHORT CIRCUIT AT 110 KV., AT TRANSFORMER TERMINALS WITH NO TAP LINES

lines connected between circuit breaker and transformer, values lower than one-quarter of those given by the formulas may exist.

Fig. 7 shows cathode ray and magnetic oscillographic records of the recovery voltage following a single-phase 231,000-kv-a. short circuit (d-c. component neglected) for which $k_4 = 0.74$, on the 110-kv. connection of a cir-

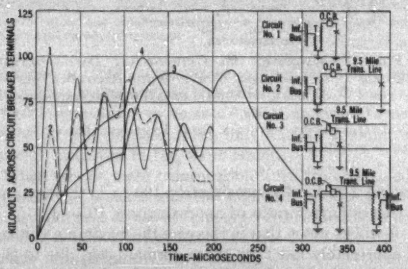

FIG. 8—CALCULATED RECOVERY VOLTAGE CURVES FOLLOWING A SINGLE-PHASE LINE-TO-GROUND SHORT-CIRCUIT AT POINTS MARKED X ON CIRCUIT DIAGRAMS

System voltage: 66,000
Capacity of transformer in each case: 20,000 kv-a. per phase
Transformer reactance: 10 per cent
Transmission line surge impedance: 400 ohms
Length of transmission line: 9.5 mi.

cuit breaker testing plant, two transformers being used in parallel. The frequency of the oscillation is 4,600 cycles per second, and the rate of recovery voltage rise about 2,400 volts per microsecond.

The values given by the formulas for extreme conditions are:

$$f = 9{,}800 \text{ cycles per second}$$

and $$r = 5{,}000 \text{ volts per microsecond.}$$

The discrepancy is ascribed to two factors:

1. Several hundred feet of line were required to parallel the transformers and connect to the circuit breaker.

2. Each transformer had six taps and therefore six bushings connected to the high side winding.

The beneficial effect of a capacitance to ground connected between the high side of a transformer and the circuit breaker is strikingly presented in Fig. 8, where calculated recovery voltages at the high side of a 20,000-kv-a., 38.1-kv. line-to-ground transformer following a single-phase line-to-ground short circuit are shown for the four circuits illustrated, the low-tension winding of the transformer being assumed connected to an infinite bus in each case.

It will be noted that the rates of rise of recovery voltage in curves Nos. 3 and 4 are very much less than those in curves Nos. 1 and 2. The change in the rate of rise of recovery voltage would be even more marked than is shown by Fig. 8 if—

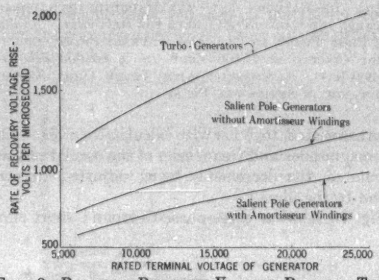

FIG. 9—RATES OF RECOVERY VOLTAGE RISE FOR THREE-PHASE UNGROUNDED SHORT CIRCUITS AT THE GENERATOR TERMINALS

1. The system voltage were higher.

2. The kv-a. capacity of the transformer were less.

3. More than one transmission line were connected between S_1 and T_1.

or

4. The transmission line L_2 were replaced by a cable.

It would be less marked if a bus of finite reactance were assumed on the low-tension side of the transformer.

The length of line L_2 has no effect on the rate of rise of recovery voltage except in changing the time when reflections return from the far end. The time when these reflections appear is approximately 10.6 microseconds per mile of line after the interruption. Thus with the 9.5-mile lines of Fig. 8, reflections return at $t = 100$ microseconds.

Recovery Voltage Rates on Low Voltage Systems

On the basis of average values for the design constants of a rotating machine, the rate of recovery voltage rise for the first phase to clear of an ungrounded

three-phase short circuit is given with reasonable accuracy by the curves of Fig. 9.

Figs. 10, 11, and 12 show cathode ray and magnetic oscillographic records of recovery voltages on the low-tension (14,500-volt) system of the circuit breaker testing plant of the company with which the authors have

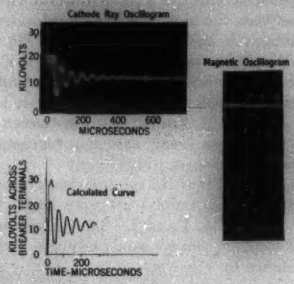

FIG. 10—CATHODE RAY- AND MAGNETIC OSCILLOGRAPHIC RECORDS AND CALCULATED CURVE FOR RECOVERY VOLTAGE OF THE FIRST PHASE TO CLEAR OF A THREE-PHASE GROUNDED SHORT CIRCUIT AT 14,500 VOLTS ON A SOLIDLY GROUNDED 100,000-KV-A. ALTERNATOR WITH THREE OHMS EXTERNAL REACTANCE IN SERIES PER PHASE

been associated, together with calculated curves. (Only the magnitudes and frequencies of the oscillations were calculated, the decrements being adjusted to correspond to the films.)

Fig. 10 shows a three-phase-to-ground short circuit

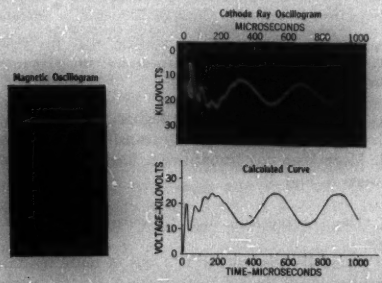

FIG. 11—CATHODE-RAY AND MAGNETIC OSCILLOGRAPHIC RECORDS AND CALCULATED CURVE FOR RECOVERY VOLTAGE OF THE FIRST PHASE TO CLEAR OF A THREE-PHASE GROUNDED SHORT CIRCUIT AT 14,500 VOLTS ON AN UNGROUNDED 100,000-KV-A. ALTERNATOR WITH THREE OHMS EXTERNAL REACTANCE IN SERIES PER PHASE

on a solidly grounded generator with three ohms external reactance in series per phase.

Fig. 11 shows the phenomena occurring on the first phase to clear of a three-phase-to-ground short circuit on an ungrounded generator with three ohms external reactance in series per phase.

Fig. 12 shows the recovery voltage of the first phase to clear of a two-phase-to-ground short circuit on an ungrounded generator with 6.1 ohms external reactance in each line. In the calculated curve of Fig. 12, the starting point was taken equal to the arc drop just prior to clearing, as shown on the magnetic oscillogram.

It will be noted that in both Fig. 11 and Fig. 12, oscillations occur at two widely different frequencies. This happens because the generator neutral is raised above ground potential during such a short circuit, and the two frequencies result (1) from oscillation of the bus capacitance to ground with the inductance in the line clearing first, and (2) from oscillation of the copper-to-iron capacitance of the generator with the inductance in the line remaining grounded. The two inductances having the same value and the capacitances

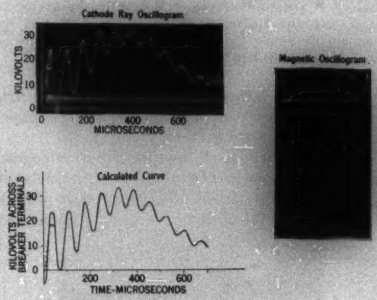

FIG. 12—MAGNETIC AND CATHODE-RAY OSCILLOGRAPHIC RECORDS AND CALCULATED CURVE OF THE RECOVERY VOLTAGE OF THE FIRST PHASE TO CLEAR OF A TWO-PHASE-TO-GROUND SHORT CIRCUIT AT 14,500 VOLTS ON AN UNGROUNDED 100,000-KV-A. GENERATOR WITH SIX OHMS EXTERNAL REACTANCE IN SERIES PER PHASE

having a ratio of approximately 100 to 1, the two frequencies have a ratio of approximately 10 to 1.

It will be noted that in this case the maximum voltage is appreciably less than twice normal peak, due to the fact that the high frequency oscillation is almost completely damped out by the time the low frequency oscillation has reached its first peak. This may occur in any circuit in which the capacitance and reactance are so distributed as to give rise to oscillations of widely different frequencies and of about the same amplitude. Unless this is the case, however, or unless special damping is provided, an overshoot very nearly to twice normal, plus the negative arc voltage peak just prior to clearing, is almost sure to be experienced.

A comparison of the cathode ray and magnetic records of the recovery voltages in Figs. 7, 10, 11, and 12 shows very clearly that the magnetic record can not be relied upon to show correctly the high frequency components of the recovery voltage curve and that the circuits

having the highest recovery voltage rates may show the most innocent films, while circuits having a comparatively low rate of recovery voltage will show the oscillations correctly on the film, and thus give the impression of being very difficult to interrupt.

Effect of Parallel Resistance on the Rate of Rise of Recovery Voltage.

Resistance connected across the terminals of a circuit breaker tends to prevent overshooting and to cut down the rate of rise of recovery voltage. A resistance of 1,000 ohms so connected materially reduces the rate of rise of recovery voltage on a severe 220-kv.

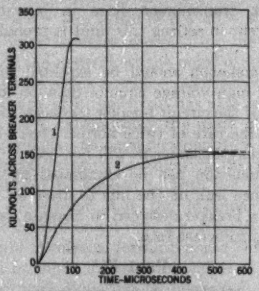

Fig. 13—Recovery Voltage Curves for Tests on Experimental Explosion Chamber Breaker

Curve No. 1 is for standard 110-kv. test circuit; Curve No. 2 is for standard 110-kv. test circuit modified by the connection of a 1,000-ohm resistor across the breaker terminals

short circuit, with no transmission lines connected between the breaker and the high side of the transformer.

On lower voltage systems a lower resistance is required for this purpose unless only a small amount of power is involved in the short circuit.

When one or more transmission lines are connected between the breaker and the high side of the transformer, the rate of rise of recovery voltage is already so low that a very low resistance is required to effect any further reduction.

Effect of Nature of Recovery Voltage on Breaker Operation

Mention was made in the introduction of two cases in which the nature of the recovery voltage curve had a very pronounced influence upon breaker operation. A few more details regarding these tests are given in this section, as well as brief reports of other data of this character.

As regards the tests on the Northern States Power Company system, power from the Riverside Station was supplied through step-up and step-down transformers with 43 miles of 110-kv. transmission line intervening. Thus stationary apparatus played a large part in the limitation of short-circuit current, and the quadrature reactance factor was low. Also, the circuit had sufficient capacitance to reduce its frequency to about 300 cycles. At Wissota, on the other hand, power was supplied directly from water-wheel generators, which have a high quadrature reactance factor (2.0 or greater), and the frequency of recovery voltage was probably about 5,000 or 6,000 cycles. The distress evidenced by the breakers was much greater at Wissota than at Riverside, as is shown by Table I.

In connection with the high voltage tests, apart from reflections, the resistance connected (about 1,000 ohms) had approximately the correct value to duplicate the effect of a single transmission line connected as in diagrams Nos. 3 and 4 of Fig. 8 upon the recovery voltage of a single-phase short circuit of three times the kv-a. available in the test, or approximately 600,000 kv-a., single-phase.

On the standard test circuit, the breaker cleared on the 66-kv. connection, requiring from 58 to 73 per cent of its total contact separation, and failed to clear on the 88-kv. and 110-kv. connections. With the resistance connected across the breaker terminals, the breaker cleared consistently, not only on the 88-kv. and 110-kv. connections, but also on the 132-kv. connection, the maximum arc length being only 58 per cent of the total stroke.

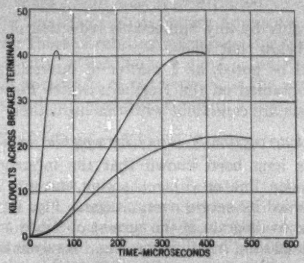

Fig. 14—Calculated Recovery Voltage Curves for Tests on 15,000-Volt Breaker

Recovery voltage curves for the two conditions are shown in Fig. 13, that for the standard circuit being taken from the cathode ray oscillogram of Fig. 7, and the modification due to the connection of the resistor being calculated.

A decrement factor of 1.0 is assumed for both curves.

A short series of tests has been made under the direction of the authors of this paper upon a plain-break oil circuit breaker at 14,500 volts and from 800 to 1,200 amperes. The circuit had an oscillating frequency of about 8,000 cycles for the first group of tests. For the second group of tests the oscillating frequency was reduced to 1,300 cycles by the addition of a shunt capacitor, while for the third group, the 1,300-cycle circuit was very nearly critically damped by resistance connected across the breaker terminals. Calculated recovery voltage curves for the three groups of tests are shown in Fig. 14. It was found that the arc length

in the second group of tests was approximately 10 per cent shorter than in the first group, while in the third group, the reduction was 30 per cent.

Biermanns has reported a series of tests on a plain-break breaker at 7,500 volts and about 20,000 amperes at recovery voltage frequencies of 100,000 cycles, 1,300 cycles, and 500 cycles. At 1,300 cycles, he finds the arc lengths only about five per cent less than at 100,000 cycles, but at 500 cycles, the arc length is reduced to 50 per cent of its value at 1,300 cycles. This would indicate that the operation of this type of breaker is independent of recovery voltage frequency when this frequency is above about 1,000 cycles, but is improved

Fig. 15—Oscillographic Record of the Opening of a 220-Kv. Transmission Line Charging Current

considerably by any appreciable reduction of this frequency below that value.

It will be noted, as a matter of interest, that the results obtained on the Northern States Power Company tests are consistent with this deduction.

Interruption of Line Charging Currents

It has long been known that the interruption of transmission line or cable charging currents may be accompanied by severe overvoltages. Fig. 15 shows a magnetic oscillogram of the current obtained on such a test on a 220-kv. transmission line. Analysis indicates that the voltage appearing across the switch just previous to a current surge should be roughly equal to the product of line surge impedance and the peak current in the surge. On this basis, this oscillogram indicates a voltage equal to several times normal peak. The fact that such voltage actually occurs is borne out by surge recorder data.

This phenomenon occurs as follows. The arc is extinguished at the time when the current is zero and the transmission line completely charged to one polarity. One half-cycle later the transformer terminal voltage has reversed its polarity, but the voltage of the transmission line has remained unchanged. Double leg voltage, therefore, exists across the switch. This voltage may be sufficient to break down the gap, or the gap may be broken down while this voltage is building up. If this occurs, the gap conducts current while a wave travels down the line, is reflected at the end, and travels back, reversing the potential of the line with respect to the transformer and charging the line up to a potential which may be as high as three times normal. When this has occurred, the current in the breaker drops to zero, so that the arc is again interrupted. The transformer side of the breaker then changes polarity during the next half-cycle, while the transmission line holds its charge and potential. This time the voltage across the gap may reach four times normal peak, and thus the gap may again be broken down. The oscillation is then repeated, again reversing the polarity of the line and leaving it at a potential which may be as high as five times normal. On a long transmission line with no leakage, there is no theoretical limit to the voltage which may build up in this manner, and voltages as high as 5.5 times normal have been recorded on transmission lines in operation. The principal agent in setting this limit is probably corona loss.

This phenomenon should be expected to be more common on high voltage circuits than on low-voltage circuits, for if the circuit breaker increases its dielectric strength at the rate of twice normal crest voltage per half cycle, the phenomenon will not occur.

The phenomenon can be eliminated by putting a high resistance leak on the line to draw off its charge or by using a breaker designed to increase its dielectric strength at a sufficient rate.

Part II

Theory and Calculations

In this part, as in Part I, the low frequency and high frequency phenomena are considered separately, the low frequency phenomena first, and the high frequency phenomena afterward.

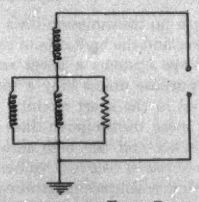

Fig. 16—Circuit for the First Phase to Clear of a Three-Phase Grounded Short Circuit on a Generator with the Neutral Grounded through a Resistance

In both cases, the calculations are made by means of the principle of superposition. This principle is stated as follows: In any network unappreciably affected by saturation and hysteresis, if the mechanical displacements of moving parts during the period under consideration are unaffected by the disturbance, the changes in voltages and currents throughout the network due to a change in voltage, Δe, or current, Δi, applied at any point are equal, at any instant, to the voltages and currents flowing as a result of the application at that point of a voltage, Δe, or current, Δi, when no other voltage or current is applied to the network. This is true of both transient and sustained

conditions. Using this principle to determine the voltage arising across a circuit breaker upon the final interruption of current, we have, just before the interruption

$$i_1 = I_0 [\sin (w t + \phi) - \sin \phi] \quad (1)$$

and $$e_1 = 0 \quad (2)$$

After the interruption, $$i_2 = 0 \quad (3)$$

hence, for the change in current, we have

$$\Delta i = i_2 - i_1 = 0 - I_0 [\sin (w t + \phi) - \sin \phi] \quad (4)$$

The change in voltage at the time of interruption, Δe, and the voltage, e_2, appearing after the interruption, are, therefore, given by

$$e_2 = 0 + \Delta e = 0 - I_0 [\sin (w t + \phi) - \sin \phi] Z \quad (5)$$

where Z is an operator representing the effective impedance of the network at the point of interruption.

In a simple series circuit containing only resistance and reactance, or if the circuit has parallel branches all of which have substantially the same phase angle, Z may be calculated as a complex quantity. In applying this complex quantity to Equation (5), the real component should be applied to both d-c. and a-c. components of current, but the imaginary component should be applied only to the a-c. component of current and should involve a forward phase shift of 90 deg. Thus, for example, if, when the values of the variables are substituted, Equation (5) becomes

$$\Delta e = - 10 [\sin (w t + 30^0) - \sin 30^0] (1 + j 5) \quad (5a)$$

by using the correct application of the operator $(1 + j 5)$, we obtain

$$\Delta e = -10 [\sin(w t + 30^0) - \sin 30^0] - 50 \cos (w t + 30^0) \quad (5b)$$

In a circuit having parallel branches of different phase angle, the voltage rapidly attained may be determined by considering all resistances as short circuits. The peak value of voltage is very nearly that given by Equation (5). The curve joining the two is a combination of logarithmics, one for each parallel circuit. The time constants of these logarithmics depend upon the relative values of resistance and inductance in the circuits to which they apply.

Low Frequency Phenomena

Effect of Type of Short Circuit and Circuit Conditions

For a given value of I_0, then, the recovery voltage depends upon the value of ϕ and the nature and magnitude of Z; ϕ depends purely upon the displacement of the current wave, and its effect can be more conveniently discussed after the effect of circuit conditions.

a. Resistance vs. Reactance. It has long been known that a current flowing in a resistive circuit is much easier to interrupt than the same current flowing in a reactive circuit, with the same r. m. s. value of re-established voltage. This is to be expected from the fact that a negative sine wave of current impressed upon a resistive circuit gives rise to a voltage wave across the terminals of the circuit which is in phase with the current and therefore starts at zero; whereas a negative sine wave of current impressed upon a reactive circuit gives rise to a voltage wave which is 90 deg. out of phase with the current and therefore, neglecting high frequency transients, starts at its crest value.

This may also be seen from Equations (5), (5a), and (5b) in that the real part of the operator Z, which arises from the resistance of the circuit, gives rise to sine terms in the recovery voltage, whereas the imaginary part, which arises from the reactance of the circuit, gives rise to cosine terms in the recovery voltage.

Of two short-circuit currents having the same magnitude and phase angle and the same system voltage, but in one of which the current is limited by a resistance and reactance in series, and in the other by resistance and reactance in parallel, the parallel circuit is the easier to clear, for in the series circuit, the voltage rises at a high rate to a value $L \frac{d i}{d t}$, whereas in the parallel circuit the voltage rises from the start along a comparatively slow exponential curve.

b. Type of Short Circuit. The effective impedance of the network to the point of short circuit depends upon the constants of the network and also upon the type of short circuit that is interrupted.

Where the effects of stationary apparatus predominate over those of rotating machinery, for single-phase short circuits, either line-to-ground or line-to-line, the effective impedance of the network is the same as the impedance limiting the short-circuit current:

$\frac{Z_0 + Z_1 + Z_2}{3}$ for line-to-ground short circuits and $Z_1 + Z_2$ for line-to-line short circuits.

Thus the voltage after interruption will be the same, except for high frequency effects, as the voltage before establishment of the short circuit. This is confirmed by the oscillogram of Fig. 1, shown in Part I.

In the case of two-phase-to-ground short circuits on a grounded system and three-phase short circuits on a grounded or ungrounded system, however, one phase always clears ahead of the others, and the effective impedance which the network offers to the breaker in the phase clearing first may be different from that which limits the short-circuit current.

These impedances are calculated by the method of symmetrical phase-sequence components in Appendix A. For the case of the first phase to clear of a three-phase short circuit the impedance is given by

$$Z = \frac{3 Z_0 Z_1 Z_2}{Z_0 Z_2 + Z_1 Z_0 + Z_1 Z_2} \quad (6)$$

This solution is, of course, for a grounded neutral system and the effect of neutral impedance may be taken into account in the value assigned to Z_0.

It should be noted at this point that a difference in the recovery voltage characteristic of a three-phase grounded short circuit arises depending on whether

the neutral impedance is a resistor or a reactor. If it is a reactor, the voltage rises immediately to $i\,Z$, where Z has the value given by Equation (6). If the neutral impedance is a resistor, however, the circuit is that of Fig. 16, where x_a is the reactance to neutral offered to the phase clearing first and x_b and x_c together represent the reactance of the other two phases. The voltage rapidly attained is that calculated by considering the resistance as a short circuit:

$$e = i\,\frac{3\,X_0\,X_1\,X_2}{X_0\,X_2 + X_1\,X_0 + X_1\,X_2} \qquad \textbf{(6a)}$$

The peak value is that indicated by Equation (6). The curve joining the two is a logarithmic whose time constant is that given by the neutral resistor in series with the reactances x_b and x_c in parallel. This time constant is, in electrical radians

$$t_0 = \frac{X_1\,X_2 + X_2\,X_0 + X_0\,X_1}{2\,R\,(X_1 + X_2)} \qquad \textbf{(6b)}$$

where R is the neutral resistance.

The relative values $X_1 = X_2 = 2\,X_0 = \frac{R}{4}$ are such as to give a time constant somewhat longer than average, but by no means unusual. Substituting these values,

$$t_0 = 0.125 \text{ rad} = 7.2^\circ$$

In an ungrounded system, or in a grounded system when the ground is not involved in the short circuit, Z_0 is equal to infinity and equation (6) becomes

$$Z = \frac{3\,Z_1\,Z_2}{Z_1 + Z_2} \qquad \textbf{(7)}$$

and if $Z_1 = Z_2$, as is the case except in rotating machinery, equation (7) may be still further simplified:

$$Z = \frac{3}{2}\,Z_1 \qquad \textbf{(8)}$$

If the impedance of stationary apparatus predominates, the current in a three-phase short circuit is given by

$$I = \frac{E}{Z_1} \qquad \textbf{(9)}$$

where E is the normal system voltage to ground. Multiplying equations (8) and (9) we have

$$e = Z\,I = \frac{3\,E}{2} \qquad \textbf{(10)}$$

showing that the recovery voltage of the first phase to clear of a three-phase short circuit, where the impedance of stationary apparatus predominates, neglecting high frequency effects, is 1.5 times the Y-phase voltage existing before short circuit. This effect is shown in Fig. 2, Part I.

The impedance encountered by the first phase to clear of a two-phase-to-ground short circuit is also calculated in Appendix A. It is there shown to be,

$$Z = \frac{e}{i} = \frac{Z_0\,Z_1 + Z_0\,Z_2 + Z_1\,Z_2}{Z_1 + Z_2 + Z_0} \qquad \textbf{(11)}$$

Effect of Rotating Machinery

It is a familiar fact that in any fault at all close to a generator, the current is subject to considerable variation; decaying considerably from its initial value on machines subject to hand regulation; and starting to decay, but being brought finally to a value not greatly different from the initial value, in machines whose excitation is controlled by a quick response excitation system. The negative current at the time of interruption being equal and opposite to this short-circuit current, and the reactance encountered by this negative current being a constant, it follows that the recovery voltage is decreased in magnitude exactly in proportion with the a-c. component of current.

This same conclusion may be reached from another point of view, for the decay in short-circuit current is but a manifestation of decay in rotor flux, and a decay in rotor flux must also cause a proportionate decay in open-circuit voltage.

In Part I, the factor by which the recovery voltage is altered by this decay in flux is called the decrement factor and is designated by the symbol k_3.

In Appendixes B, C, and D, there is a rather involved mathematical discussion, in which the phenomena accompanying the flow and interruption of short-circuit current in rotating machinery are treated in some detail by the method first used by Messrs. Doherty and Nickle in their papers on synchronous machines. A brief discussion is offered at this point, however, in which an effort is made to base the argument on physical conceptions rather than on mathematics. The same results are obtained in the two cases.

In the case of a single-phase short circuit, either line-to-ground or line-to-line, the reactance determining the recovery voltage is the same as that initially limiting the short-circuit current. The recovery voltage, therefore, is equal to the decrement factor times the voltage before short circuit. There is very little distortion in the recovery voltage wave, even though the short-circuit current wave is considerably distorted.

These facts are well brought out in Fig. 17 which shows an oscillogram of a dead short circuit, single-phase, at 14,500-volts on a 100,000-kv-a. alternator without an amortisseur winding.

On this test,

The r. m. s. voltage before short circuit, as measured by a voltmeter is	14,500 volts
The decrement factor, as determined from the short-circuit current curve is	0.74
The calculated peak value of recovery voltage is, therefore, $14{,}500 \times \sqrt{2} \times 0.74$ or	15,200 volts
The value of crest voltage measured on the film is	15,900 volts

In a purely reactive three-phase short circuit, after the displacement has disappeared, when the current in any phase passes through zero, that phase is linking the quadrature axis of the alternator. The reactance which a negative current will encounter upon entering the machine at that instant, therefore, is the quadrature subtransient reactance of the machine.

Unless both short circuit and alternator are solidly grounded, the current of the first phase to clear passes through its own phase winding and also through those

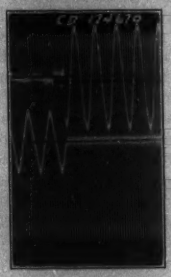

FIG. 17—OSCILLOGRAPHIC RECORD OF CURRENT AND VOLTAGE OF A SINGLE-PHASE LINE-TO-LINE SHORT CIRCUIT AT 14,500 VOLTS ON A 100,000-KV-A. SALIENT-POLE GENERATOR WITHOUT AN AMORTISSEUR WINDING WITH ZERO EXTERNAL REACTANCE

of the other two phases in parallel. The reactance is thus—

$$z = \frac{3}{2} x_q'' \tag{12}$$

and the voltage at the instant of clearing is

$$e_m = \frac{3}{2} i_m x_q'' \tag{13}$$

where i_m is the crest value of current flowing just before short circuit. The case of a grounded short circuit is solved in Appendix C. The solution is given by Equation (14:)

$$z = \frac{3 x_q'' z_0}{x_q'' + 2 z_0} \tag{14}$$

where z_0 is the zero-phase-sequence impedance at the point of short circuit. The remarks with regard to the comparative effect of resistance and reactance in the neutral also apply here.

The impedance representing the first phase to clear in a two-phase-to-ground short circuit is somewhat more difficult to visualize. The following equations are taken from the analysis in Appendix D and apply only for purely reactive circuits:

$$z = \frac{\begin{array}{l} 2\,(x_d''x_q'' + x_q''x_0 + x_0x_d'')\,(x_q''^2 + x_q''x_0 + x_0^2) \\ \quad + x_0(x_q'' - x_d'')\,(-x_q''^2 + 2\,x_q''x_0 + 2\,x_0^2) \end{array}}{x_d''\,(2\,x_q'' + x_0)^2 + x_0\,(2\,x_q'' + x_0)\,(x_q'' + 2\,x_0)} \tag{15}$$

$$e = E\,\frac{2\sqrt{3}\,(x_q''^2 + x_q''x_0 + x_0^2)^{3/2}}{x_d''(2\,x_q'' + x_0)^2 + x_0(2\,x_q'' + x_0)(x_q'' + 2\,x_0)} \tag{16}$$

It may be inferred from the fact that the instantaneous position of the generator winding with respect to the rotor is the principal factor governing the impedance encountered by the negative current that, with the change in the position of the winding with respect to the rotor which occurs throughout the cycle, the impedance of the winding will change so that a sine wave of negative current will give rise to a distorted voltage wave. That this is the case for other than single-phase short circuits is brought out quite clearly from the theoretical viewpoint in Appendix B and verified by the oscillograms of Figs. 4 and 18 for the three-phase case. In the case of single-phase short circuits, however, the distortion occurs in the current wave and the recovery voltage has the characteristic open circuit wave form of the machine, as shown in Fig. 17. In the case of a two-phase-to-ground short circuit on a grounded system, the wave forms of both the current and the recovery voltage of the first phase to clear are likely to be distorted.

On account of this distortion, initial values of voltage are considered instead of r. m. s. values.

Of all these cases, the two giving the highest value of voltage across the circuit breaker terminals are the first phase to open of a three-phase short circuit in which either short circuit or generator is ungrounded, as given by Equation (13), and the first phase to clear of a two-phase-to-ground short circuit with high neutral impedance. In the first case, the current i is limited only

FIG. 18—OSCILLOGRAPHIC RECORD OF THE RECOVERY VOLTAGE OF THE FIRST PHASE TO CLEAR OF A THREE-PHASE UNGROUNDED SHORT CIRCUIT AT 14,500 VOLTS ON A 100,000-KV-A. ALTERNATOR WITHOUT AN AMORTISSEUR WINDING AND WITH ZERO EXTERNAL REACTANCE

by x_d'' and decrement, so that for a three-phase ungrounded short circuit interrupted well up on the decrement curve,

$$i_m = \frac{E_m}{x_d''} \tag{17}$$

and

$$e_m = \frac{3}{2} E_m \frac{x_q''}{x_d''} \tag{18}$$

where E_m is the crest value of normal phase-to-neutral voltage.

In the general case, in which several machines feed the short circuit through various stationary reactances, x_q'' and x_d'' must be replaced by s_q'' and s_d'', which are the reactances of the system to the point of short circuit, using x_q'' for the reactance of all machines in determining s_q'', and x_d'' for the reactance of all machines in determining s_d''.

The ratio $\frac{s_q''}{s_d''}$ is, in general, less than the value of $\frac{x_q''}{x_d''}$ for the average machine of the system, so reactance external to the machines mitigates the effect of unequal reactances in the two axes. In Part I the ratio $\frac{s_q''}{s_d''}$ has been called k_q.

In the case of a two-phase-to-ground short circuit on a system whose neutral is grounded through a high impedance, the current is substantially that of a line-to-line short circuit, and the reactance encountered by the negative current at the time of clearing substantially that determining the short-circuit current; so that the recovery voltage is the full line-to-line voltage of the system (if the decrement factor is unity). Since at the time the first phase clears, the other pole of the breaker may continue to carry sufficient current for a fraction of a cycle to render it conducting, all of this voltage appears across a single pole of the breaker. This voltage may approach very closely

$$e_m = \sqrt{3}\, E_m \qquad (19)$$

In addition to the effect mentioned in connection with stationary apparatus, series resistance shifts the position of the winding with respect to the rotor axes at the time when its current ceases, and thus makes its reactance more nearly that corresponding to the direct axis.

In laminated rotor machines without amortisseur windings, the ratio $\frac{x_q''}{x_d''}$ is comparatively high, averaging about 2.0 or 2.5. This is because the field winding practically prevents, for the duration of a rapidly interrupted short circuit, any change in the value of rotor flux in the direct axis, thus limiting the value of x_d'' to slot and end turn reactance, whereas there is nothing except the air gap, and the metal rotor wedges of a round rotor machine, to limit the building up of flux in the quadrature axis.

Any conducting path which limits the amount of flux which can be built up, in a thousandth of a second or so, in the quadrature axis of a machine limits x_q'' in proportion. Thus an amortisseur winding reduces both x_d'' and x_q'', but the reduction in x_q'' is so much greater that the two become very nearly equal. The iron of a solid round rotor machine serves much the same purpose as the amortisseur winding of a salient-pole machine, and the ratio $\frac{x_q''}{x_d''}$ for solid rotor machines is only about 1.45.

The effect of an amortisseur winding is well brought out by the oscillograms of Figs. 4, 5, 18, and 19. All these oscillograms show the voltage across the first phase to clear of a three-phase grounded short circuit on an ungrounded machine, the generator used being the 100,000-kv-a. alternator used by the company with which the authors have been associated in its circuit breaker testing plant. The oscillograms of Figs. 4 and 18, however, were taken before, and the oscillograms of Figs. 5 and 19 after, the amortisseur winding was installed on that machine. Figs. 4 and 5 are taken with a comparatively slow film speed so that the entire short circuit could be recorded on a reasonably short film, while for Figs. 18 and 19, the film was run at a high speed so that the wave form could be observed. It will be noted that the overvoltage shown in Fig. 4 is much greater than that shown in Fig. 5 and that Fig. 18 shows a prominent third harmonic up until the time when the two later phases clear, which hardly appears at all in Fig. 19. Both Figs. 18 and 19 show a rather fuzzy curve up to the time when the two later phases clear, and in Fig. 5 this fuzziness shows up even on the slow film.

The difference in overvoltage between Fig. 4 and Fig. 5 checks with what is to be expected from the quadrature reactance factors for the two cases, as shown by the calculations below:

Quantities	Fig. 4	Fig. 5
Amortisseur winding	No	Yes
Decrement factor	0.63	0.50
Factor for three-phase short circuit on an ungrounded machine	1.50	1.50
Line-to-line voltage before short circuit (measured by voltmeter)	13,200	14,500
Peak value of recovery voltage	16,200	11,400
Quadrature reactance factor as calculated from above quantities:		
(a) $\dfrac{16{,}200}{\dfrac{13{,}200}{\sqrt{3}} \times \sqrt{2} \times 1.5 \times .63}$	1.58	..
(b) $\dfrac{11{,}400}{\dfrac{14{,}500}{\sqrt{3}} \times \sqrt{2} \times 1.5 \times 0.50}$	..	1.28
Quadrature reactance factor as determined from static tests—		
a. At zero field	1.73	1.38
b. At full field	..	1.28

As has been previously brought out, the distortion is to be expected with a high quadrature reactance factor.

The fuzziness or irregularity of the voltage wave is due to the inequality of the arc voltage on the two phases still carrying current, for the potential of the entire generator with respect to ground has a term equal

to one-half the difference of these two arc voltages. The departure of the actual voltage wave from its calculated value on account of this effect is limited to one-half the arc voltage of one of these other phases.

An induction motor is a laminated rotor machine with an amortisseur, whether it be a squirrel cage or polyphase wound rotor machine. Here the ratio $\frac{x_q''}{x_d''}$ is unity, due to the exact similarity of the two axes; for in an induction motor, the direct axis is distinguished from the quadrature axis only by the fact that for the moment it happens to be the axis of the rotor flux. The rotor flux of an induction motor is subject to about the same decrement as the d-c. component of a synchronous motor short circuit, so that short-circuit currents from induction motors are of no importance in a short circuit lasting more than four or five cycles, but a large induction motor connected to a short-circuited bus offers a low reactance for the negative current that flows when the short circuit is cleared, and thus makes the circuit easy to interrupt.

Fig. 19—Oscillographic Record of the Recovery Voltage of the First Phase to Clear of a Three-Phase Short Circuit at 14,500 Volts on Same Generator as Fig. 18, but After the Amortisseur Winding was Installed

Table III, Part I, gives the values of $\frac{x_q''}{x_d''}$ which may be expected for various types of machines.

Effect of Displacement

Displacement has the effect of shifting, with respect to the a-c. component, the time at which a short-circuit current wave passes through zero; hence it decreases the rate of change of current at that time and the initial instantaneous value of recovery voltage. In the case of rotating machines with unequal reactances in the direct and quadrature axes, displacement will have the additional effect of shifting the position of the winding at the time of current zero from directly in line with the quadrature axis of the rotor, to a position more nearly in line with the direct axis; thus it reduces the reactance of the circuit at the instant of clearing.

An oscillogram of a highly displaced short circuit is shown in Fig. 6, Part I.

High Frequency Phenomena

The high frequency phenomena accompanying the interruption of a short circuit are primarily attributable to the capacitances and leakage resistances existing at various points of the circuit—particularly, close to the circuit breaker. They may be calculated by setting up the circuit with the proper reactances and assigning the proper capacitance or resistance at each point, and solving.

A discussion of the types of apparatus important from the point of view of high frequency phenomena is given in Part I. A few details, however, which were omitted there, are conveniently mentioned at this point.

Transmission lines and cables are well and widely known to be distributed circuits capable of supporting traveling surges. In such a surge, a current of any given magnitude is always associated with a proportionate voltage, so that, from the point at which a surge is impressed, until reflections appear, the circuit appears to be a resistance. The ratio of voltage to current in such a surge is known as the surge impedance of the line. In overhead transmission lines the surge impedance is usually about 300 or 400 ohms; in cables it is much lower, varying from 20 to 50 ohms. The rate of propagation of waves on transmission lines is very close to the speed of light, about 1,000 ft. per microsecond. In cables the speed of propagation is only about 300 to 500 ft. per microsecond.

At an open end of a transmission line or cable a wave is reflected with reversed current and voltage of the same polarity. At a short-circuited end, the reflected voltage is reversed and the current has the same polarity as that of the approaching wave. Partial reflections occur at any discontinuities.

The minimum possible effective value of capacitance between the circuit breaker and transformer is the effective capacitance of the transformer winding plus the capacitance of one circuit-breaker bushing and one transformer bushing. This capacitance may be as low as 0.001 microfarad per transformer.

The inductance of the circuit is determined from the short-circuit kv-a.: If "kv." represents the transformer leg voltage before short-circuit in kilovolts, and "kv-a." the initial short-circuit kv-a. per phase for a fault right at the terminal of the transformer, at 60 cycles,

$$L = \frac{1000}{2\pi f}\frac{(\text{kv.})^2}{\text{kv-a.}}k_q = 2.65\frac{(\text{kv.})^2}{\text{kv-a.}}k_q \text{ henrys per phase}$$

$$\text{(at sixty cycles)} \qquad (20)$$

On the basis of this value of inductance and a capacitance of 0.001 μf. (10^{-9} farad) per transformer, the extreme frequency of the recovery voltage is given by

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{10^6}{2\pi\sqrt{0.26\frac{(\text{kv.})\,n\,k_q}{\text{kv-a.}}}}$$

$$= \frac{3200}{\text{kv.}}\sqrt{\frac{\text{kv-a.}}{n\,k_q}} \text{ cycles} \qquad (21)$$

and the maximum rate of recovery voltage

$$r = 20\, k_t \sqrt{k_d}\, k_v \text{ kv-a.}/n \text{ volts per microsecond} \quad (22)$$

in which

kv. = leg voltage of the transformer.
kv-a. = kv-a. per phase of the short circuit.

and

n = the number of transformers feeding the short circuit.

For a 200,000-kv-a. single-phase line-to-ground short circuit on one transformer at 38.1 kv. (66-kv. line-to-line) these values are f = 38,000 cycles and r = 10,500 volts per microsecond. It should be noted that these conditions are extreme and that under actual operating conditions the capacitance of bus structure and all transformers over one connected to the bus may be sufficient to cut both frequency and rate of recovery voltage to less than one-quarter of these values.

A rotating machine appears, from a theoretical standpoint, very similar to a transmission line, and it has been shown to react to lightning surges in much the same manner as a transmission line, even to the extent that oscillations due to reflection disappear when the correct value of resistance is connected to the machine neutral.

For the type of surge resulting from a circuit breaker interruption, however, the voltage at the terminals of the machine loses the straight lines and sharp corners that would be expected on a transmission line on account of the capacitance of buses, bushings, etc., connected at its terminals and on account of reflections at discontinuities within the winding. The oscillations in recovery voltage are, therefore, very nearly sinusoidal in shape, as would be the case in a lumped circuit. Their frequency, however, is equal to $\frac{1}{4\sqrt{LC}}$ as for a transmission line instead of $\frac{1}{2\pi\sqrt{LC}}$ as for a lumped circuit, L and C representing quadrature inductance per Y-phase and capacitance per Y-phase.

Approximate values of L, C, frequency, and rate of recovery voltage rise for the first phase to clear of a three-phase ungrounded short circuit are given in Equations (23), (24), (25), and (26), below, the values for the constants of these equations being given in Table IV. Fig. 9, Part I, shows a graph of Equation (26), giving the rate of recovery voltage rise for the first phase to clear of a three-phase ungrounded short circuit at the generator terminals.

It must be appreciated, of course, that such equations and curves as these require the assumption of specific values for many of the design constants of the machines, and that variations from assumed values in individual machines will cause variations from the values of L, C, f, and r given by these equations and curves. The one of these constants whose variation most affects these quantities appears to be the peripheral velocity. This has been assumed as 11,000 ft/min. in salient-pole machines with amortisseurs, 10,000 ft/min. in salient-pole machines without amortisseurs, and 22,000 ft/min. in turbo-alternators. The values actually used in water-wheel generators vary all the way from 5,000 ft/min. to 18,000 ft/min., the lower values occurring with low-head low-power installations, and the higher values with high-head high-power installations. Values of L and C are roughly inversely proportional to peripheral velocity, and values of f and r, directly proportional to peripheral velocity.

$$C = k_c \frac{M\,V\,A}{\sqrt{\text{kv.}}\,(1 + 0.08\text{ kv.})} \text{ microfarads per phase} \quad (23)$$

$$L = k_L \frac{(\text{kv.})^2}{M\,V\,A} \text{ henrys per } Y\text{-phase} \quad (24)$$

$$f = k_f \frac{\sqrt[4]{\text{kv.}}\,(1 + 0.08\text{ kv.})}{\text{kv.}} \text{ cycles per second} \quad (25)$$

$$r = k_r \sqrt[4]{\text{kv.}}\,(1 + 0.08\text{ kv.}) \text{ volts per microsecond} \quad (26)$$

TABLE IV
VALUES OF CONSTANTS

	k_c	k_L	k_f	k_r
Solid round rotors	0.0187	0.00048	83,500	679
Salient-pole generators without amortisseurs	0.0347	0.00172	32,300	420
Salient-pole generators with amortisseurs	0.0317	0.00061	57,100	334

If the neutral of a rotating machine is grounded through resistance, and if the short circuit is grounded, the neutral resistance is in parallel with the inductance of the last two phases to clear of a three-phase-to-ground short circuit. If the value of resistance is comparable with or less than the value of $\frac{1}{2}\sqrt{\frac{L}{C}}$ for these last phases it will damp out the oscillations of voltage across these phases rapidly enough to reduce appreciably the peak value of the second half-cycle, and to delay this peak several hundred microseconds.

Solution of Simple Circuits

Solutions are briefly presented here for five simple circuits as follows:

1. A three-phase-to-ground short circuit on a solidly grounded generator with three ohms external reactance in series per phase.
2. First phase to clear of a three-phase-to-ground short circuit on an ungrounded generator with three ohms external reactance in series per phase.
3. The first phase to clear of a three-phase ungrounded short circuit at the generator terminals.
4. Single-phase-to-ground short circuit approximately 10 miles out on a transmission line; circuit breaker directly connected to the high side of the transformer; transformer neutral solidly grounded; low-voltage side of transformer connected to an infinite bus.

5. Single-phase to ground short circuit on transmission line side of circuit breaker; a second transmission line approximately 10 miles long is connected between the circuit breaker and the transformer and is open circuited at the far end; transformer neutral is solidly grounded; low side of transformer is connected to an infinite bus.

1. The recovery voltage following a three-phase grounded short circuit on a solidly grounded generator with three ohms reactance in series per phase.

Generator voltage = 14,500 volts line to line, 8400 volts line to neutral.

Generator reactances: $x_q'' = 0.77$ ohm; $x_d'' = 0.60$ ohm; $x_0 = 0.05$ ohm.

The external reactance in this case is high enough so that $k_s = 1.0$.

Bus capacitance to ground = 0.008 μf per phase.

$$k_s = \frac{3 \times 3.05}{3.77 + 2 \times 3.05} = .93 \quad k_q = \frac{3.0 + 0.77}{3.0 + 0.60} = 1.05$$

The inductance of the reactor is $\frac{3.0}{377} = 0.0077$ henry

Hence, the value of voltage appearing immediately upon interruption, neglecting high frequency oscillations is

$$e_m = 8400 \times \sqrt{2} \times 1.05 \times 0.93 = 11{,}600 \text{ volts}$$

Before settling down, however, the voltage will oscillate between zero and twice that value, the frequency of oscillation being

$$f = \frac{1}{2\pi\sqrt{.0077 \times 0.008 \times 10^{-6}}}$$

$$= \frac{10^8}{2\pi\sqrt{.77 \times 0.8}} = 20{,}300 \text{ cycles}$$

The limiting conditions for the high frequency oscillations are as follows:

At time $t = 0$, $e = 0$ and $\frac{de}{dt} = 0$

When $t = \infty$, $e = 11{,}600$

These conditions are satisfied by the equation

$$e = 11{,}600\,(1 - \cos 2\pi \times 20{,}300\,t)$$

Thus far no attempt has been made to calculate the decrement associated with these oscillations, as this is not as a rule an important factor. In Fig. 10, Part I, this curve is plotted for comparison with the cathode-ray oscillogram obtained on this connection, the decrement exhibited by the oscillation of the cathode-ray oscillogram being assigned to the calculated curve.

Assuming a uniform rate of recovery of dielectric strength of the gap and no decrement, this rate must be that corresponding to the slope of the line $O\,A$ of Fig. 10. That is,

$r = 11{,}600 \times 4.37 \times 20{,}300 \times 10^{-6} = 1070$ volts per microsecond.

2. The recovery voltage of the first phase to clear of a three-phase-to-ground short circuit on an ungrounded generator with three ohms reactance in series per phase.

Generator voltage = 14,500 volts line to line = 8400 volts line to neutral.

External reactance per line = 3.0 ohms at 60 cycles, or 0.0077 henry.

Capacitance to ground between circuit breaker and reactor per phase = 8×10^{-9} farad.

Generator capacitance to ground = 0.8×10^{-6} farad

$$k_s = 1.00$$

$$k_g = 1.50$$

$$k_q = \frac{x_q'' + x_e}{x_d'' + x_e} = \frac{0.77 + 3.0}{0.60 + 3.0} = 1.05$$

Hence,

$$e = 1.5 \times 8400 \times \sqrt{2} \times 1.05 = 18{,}700 \text{ volts}$$

For the purpose of determining oscillating characteristics, the circuit may be regarded as that of Fig. 20A, which may be simplified to that of Fig. 20B, and again to that of 20C.

This circuit has two oscillating frequencies, as follows:

$$\frac{1}{2\pi}\sqrt{\frac{(3\,C_1 + C_2) + \sqrt{9\,C_1^2 - 2\,C_1 C_2 + C_2^2}}{2\,L_1 C_1 C_2}}$$

$$= 20{,}300 \text{ cycles}$$

$$\frac{1}{2\pi}\sqrt{\frac{(3\,C_1 + C_2) - \sqrt{9\,C_1^2 - 2\,C_1 C_2 + C_2^2}}{2\,L_1 C_1 C_2}}$$

$$= 2840 \text{ cycles}$$

The mathematics involved in an exact solution for the voltages associated with these frequencies is quite laborious, but a very close approximation to the correct result may be obtained by the following process of reasoning.

The high frequency oscillation is substantially that of inductance L_1 oscillating freely with capacitance C_1, capacitance C_2 acting as a short circuit, while the low frequency oscillation is that of inductance L_2 oscillating freely with capacitance C_2, C_1 being practically an open circuit.

In view of the fact that $L_1 = 2\,L_2$, two-thirds of the final voltage will appear across L_1 and one-third across L_2. Therefore, two-thirds of the final voltage is associated with the high frequency oscillation and one-third with the low frequency oscillation, and the equation of recovery voltage is

$$e = 12{,}500\,(1 - \cos 2\pi \times 20{,}300\,t) + 6300\,(1 - \cos 2\pi \times 2840\,t)$$

In Fig. 11 this curve is plotted alongside a cathode-ray oscillogram taken on the circuit for which the curve was calculated. As in Case 1, the decrement was not calculated for either frequency, but the high frequency decrement was taken from the cathode ray oscillogram and the low frequency oscillation is plotted without decrement.

3. The recovery voltage of the first phase to clear of a three-phase ungrounded short circuit at the generator terminals.

Generator voltage = 14,500 volts line to line, 8400 volts line to neutral.

$$x_d'' = 0.50 \text{ ohm}$$
$$x_q'' = 0.55 \text{ ohm}$$
$$L_q'' = 0.00146 \text{ henry per phase}$$

Generator capacitance = 0.26×10^{-6} farad per phase.

From Equation (18) the value of voltage appearing immediately, neglecting high frequency oscillations, is

$$e_m = 1.5 \times 8400 \times \sqrt{2} \times \frac{0.55}{0.50} = 19{,}500 \text{ volts}$$

Before settling down, however, the voltage will oscillate between zero and twice that value, the frequency of oscillation being

$$f = \frac{10^4}{4\sqrt{.26 \times .146}} = 12{,}800 \text{ cycles}$$

Assuming that all harmonics are obliterated by the discontinuities of the circuit, we have

$$e = 19{,}500\,(1 - \cos 2\pi \times 12{,}800\,t)$$

Assuming a uniform rate of recovery of dielectric strength of the gap, this rate must be

$$r = 19{,}500 \times 4.55 \times 12{,}800 \times 10^{-6} = 1135 \text{ volts per microsecond.}$$

4. The recovery voltage of a single-phase to ground short circuit 50,000 ft. (approximately 10 miles) out on a transmission line; circuit breaker directly connected to the high side of the transformer; transformer neutral solidly grounded; low-voltage side of transformer connected to an infinite bus.

Surge impedance of the transmission line = 400 ohms

Velocity of propagation on transmission line = 1,000 ft./microsecond.

Reactance of transmission line = 0.15 ohm/1,000 ft. or 7.5 ohms total.

System voltage = 66,000 volts line to line = 38,100 volts line to ground.

Transformer reactance = 10 per cent on 20,000 kv-a. at 38,100 volts = 7.25 ohms.

$$\text{Short-circuit current} = \frac{38{,}100}{14.75} = 2{,}580 \text{ amperes.}$$

Capacitance of bus structure between circuit breaker and transformer = 0.

Transformer and circuit breaker bushing capacitances = 0.1×10^{-9} farad each.

Effective transformer winding capacitance to ground = 1×10^{-9} farad.

Leakage inductance of transformer winding = 0.0192 henry.

In this case the voltage across the switch will have two components, for the voltage of the line side of the breaker will experience a change of one polarity on account of the negative current suddenly impressed on the transmission line, and the voltage of the transformer side will experience a change of the opposite polarity due to the negative current suddenly impressed on the transformer circuit.

The rate of change of current at the current zero is

$$\frac{di}{dt} = 2.58 \times 0.377 \times \sqrt{2} = 1.375 \text{ amperes per microsecond.}$$

The rate of change of voltage on the line side of the breaker is, therefore,

$$\frac{de_1}{dt} = 1.375 \times 400 = 550 \text{ volts per microsecond.}$$

This rate of change will continue until the wave has had time to travel to the end of the line and back, at which time it will reverse. At the time of reversal the voltage due to this effect will be $e_1 = 550 \times 100 = 55{,}000$ volts.

On the transformer side of the breaker the circuit consists of a capacitance of 1.2×10^{-9} farad in parallel with an inductance of 0.0192 henry. Impressing suddenly upon such a circuit a current starting from zero and increasing at the rate of 1.375×10^6 amperes per second gives a voltage oscillation

$$e_2 = 1.375 \times 10^6 \times 0.0192 \left(1 - \cos \frac{10^6 \times t}{\sqrt{19.2 \times 1.2}}\right)$$
$$= 26{,}400\,(1 - \cos 2\pi \times 33{,}000\,t)$$

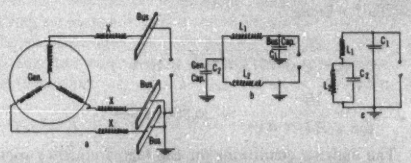

Fig. 20—Successive Simplifications of the Circuit for Analysis of the High-Frequency Oscillations of the Recovery Voltage of a Three-Phase-to-Ground Short Circuit on an Ungrounded Generator with External Reactance in Series

The equation for the voltage across the breaker is then

$$e = 550 \times 10^6\,t + 26{,}400\,(1 - \cos 2\pi \times 33{,}000\,t)$$

and the necessary rate of recovery of dielectric strength in the breaker is

$$r = 550 + 26{,}400 \times 4.37 \times 33{,}000 \times 10^{-6} = 4510 \text{ volts per microsecond.}$$

The calculated recovery voltage curve is plotted as curve No. 2 of Fig. 8 assuming a decrement $\epsilon^{-10{,}000t}$ for the transformer oscillation. It is interesting to note that this curve has two components of frequencies so different that the highest value of voltage is much lower than the sum of the two components. This may often hold true in the case of a short circuit out on a transmission line, but it will not always be the case.

5. Single-phase-to-ground short circuit at trans-

mission line side of circuit breaker; a second transmission line 50,000 ft. (approximately 10 miles) long is connected between the circuit breaker and the transformer and is open circuited at the far end; transformer neutral is solidly grounded; low side of transformer is connected to an infinite bus.

Constants of transformer and transmission line will be assumed the same as in case 4.

The bushing and transformer capacitances are now negligible, however.

$$\text{Short-circuit current} = \frac{38{,}100}{7.25} = 5{,}260 \text{ amperes.}$$

In this case there will be no voltage change at the line side of the breaker.

On the transformer side of the breaker the voltage may be calculated from circuit (3) of Fig. 8, assuming a current

$$i = 5260 \times 377 \times \sqrt{2}\, t = 2.80 \times 10^6\, t$$

The following equation holds for circuit (3) of Fig. 8 until reflections appear from the far end of the line.

$$\frac{di}{dt} = \frac{e}{L} + \frac{1}{Z}\frac{de}{dt}$$

where Z is the surge impedance of the transmission line, or, solving for e,

$$e = L\frac{di}{dt}\left(1 - \epsilon^{-\frac{Z}{L}t}\right)$$

and making the numerical substitutions

$$e = 0.0192 \times 2.80 \times 10^6 \left(1 - \epsilon^{\frac{-400}{.0192}t}\right)$$

$$= 53{,}700\,(1 - \epsilon^{-0.0208t})$$

(t expressed in microseconds)

This equation holds for the voltage at the transformer side of the circuit breaker until the reflected wave returns from the end of the transmission line and also for the front of the wave traveling down the line.

At 100 microseconds, however, the front of the reflected wave returns from the end of the line and there arises at the breaker terminal, over and above the voltage e_0, a voltage which may be calculated by the conventional transmission line theory.

This voltage is

$$e_1 = 2240\,(t - 100)\,\epsilon^{-0.0208\,(t-100)}$$

(t expressed in microseconds)

From $t = 100$ to $t = 200$ microseconds, $e = e_0 + e_1$. At 200 microseconds, however, the front of the next reflection returns from the end of the line and the voltage curve has another hump. The calculated recovery voltage curve for the first 300 microseconds is shown as curve 3 of Fig. 8.

A recovery rate of dielectric strength, $r = 53{,}700 \times 0.0208 = 1120$ volts per microsecond is rapid enough to prevent breakdown. This is only about one-fourth of what was required in case 4, in spite of the fact that the short-circuit current in case 5 is more than twice as great as in case 4.

Interruption of Line Charging Currents

In Part I there is described a process of breakdown of gap insulation, reversal of the potential difference between transformer terminal and transmission line, and interruption of the arc, leaving the line charged up to its new potential; the process repeating itself and increasing the transmission line potential to ground with each repetition.

This section inquires into the wave shape of current and voltage build-up in the transmission line and the rate of recovery voltage rise across the gap at the time the transient charging current supposedly reaches zero and is extinguished.

Fig. 21 applies to the interruption of the charging current of a transmission line when that transmission line is the only apparatus connected to the high side of the transformer. It is a calculated record of voltages and currents immediately after a breakdown of gap

FIG. 21—CURRENT AND VOLTAGE SURGES UPON THE SUDDEN CONNECTION OF A TRANSMISSION LINE TO THE HIGH SIDE OF A TRANSFORMER WHEN NO OTHER LINE IS CONNECTED THERE

insulation in the circuit breaker. The curves V_1, V_2, and V_3 of this figure represent the difference between the potential of the line at any instant and its potential just before the breakdown of the gap. The curves A_1, A_2, and A_3 represent the current flowing through the gap at any instant. Curves V_1 and A_1 apply to a line whose length is such that a wave can travel from one end of it to the other and back in a time equal to the time constant of a simple circuit consisting of resistance and inductance alone, whose resistance is equal to the surge impedance of the transmission line and whose inductance corresponds to the reactance which would limit single-phase line-to-ground short-circuit current fed by the transformer into a fault at its high side terminal. Curves V_2 and A_2 apply to a line twice as long as that to which V_1 and A_1 apply, and curves V_3 and A_3 to a line twice as long as that to which V_2 and A_2 apply. All six curves start together from the origin and separate only at the time when reflections return from the far end of the line.

For a transformer whose voltage and kv-a. ratings are 76-kv. to ground (132-kv. line-to-line) and 20,000-kv-a. per phase, operating solidly grounded and with a

single-phase total circuit reactance of 20 per cent at 60 cycles, curves V_1 and A_1 apply to a line 47.5 mi. long, V_2 and A_2 apply to a line 95 mi. long, and V_3 and A_3 to a line 190 mi. long. If the single-phase short-circuit reactance effective at the high side of the transformer is lower than assumed here due to lower voltage, higher kv-a. rating of transformer, more than one transformer in parallel, or a lower per cent circuit reactance, the lengths of line corresponding to the various curves would be proportionately reduced; thus if the voltage were 38-kv. line to ground or the single-phase circuit reactance 5 per cent on 20,000 kv-a. or 20 per cent on 80,000 kv-a., the lengths of line would be reduced to 12, 24, and 48 mi.

The equations for the curves are

1. For the voltage and current up to the time of the first reflection

$$e = E\left(1 - \epsilon^{-\frac{Z}{L}t}\right)$$

$$i = \frac{E}{Z}\left(1 - \epsilon^{-\frac{Z}{L}t}\right)$$

2. For the changes in voltage and current arising from the first reflection

$$e = 2\frac{Z}{L}t\,\epsilon^{-\frac{Z}{L}t}E$$

$$i = \frac{E}{Z}\left[2 - 2\,\epsilon^{-\frac{Z}{L}t} - 2\frac{Z}{L}t\,\epsilon^{-\frac{Z}{L}t}\right]$$

3. For the changes in voltage and current arising from the second reflection

$$e = E\left(2\frac{Z}{L}t\,\epsilon^{-\frac{Z}{L}t} - 2\left(\frac{Zt}{L}\right)^2\epsilon^{-\frac{Z}{L}t}\right)$$

$$i = \frac{E}{Z}\left[-2 + 2\,\epsilon^{-\frac{Z}{L}t} + 2\frac{Z}{L}t\,\epsilon^{-\frac{Z}{L}t} + 2\left(\frac{Z}{L}t\right)^2\epsilon^{-\frac{Z}{L}t}\right]$$

where

E = the potential difference across the circuit breaker just prior to breakdown of the gap.
Z = the surge impedance of the transmission line.
L = the inductance corresponding to the reactance limiting the line-to-ground short-circuit current fed by the transformer into a fault at its high side terminal, and
t = is time in seconds measured from the breakdown of the gap in case (1) and from the initial appearance of the reflections referred to in cases (2) and (3).

The recovery voltage characteristic at the end of the first loop of the high frequency transient current is similar to that of curve No. 2 of Fig. 8 except that the half amplitude of each component, instead of being about half of normal crest voltage is between 0.6 E and E, and E may be several times normal crest voltage, so that as far as rate of recovery voltage rise alone is concerned, the interruption may be much more difficult at this time than under any short-circuit condition. The conditions existing inside the breaker, however, are probably more favorable to interruption than in the short-circuit case, for in this case a moderate current has been flowing in the arc for a time less than one thousandth of a second (for a line less than 100 miles long) and the quantity of gas generated should be very small.

It is conceivable that the limit of the ability of the breaker to interrupt this transient current might limit the line overvoltage, for if this transient charging current is allowed to flow back and forth until it dies away, the transmission line will be brought, in approximately a steady-state condition, to the normal transformer voltage and the process of building up an overvoltage will have to start over again at the beginning. Now it

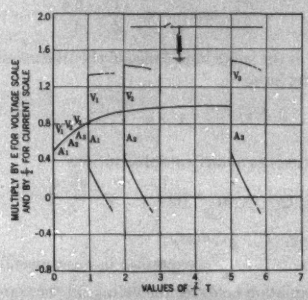

FIG. 22—CURRENT AND VOLTAGE SURGES OCCURRING UPON THE SUDDEN CONNECTION OF A TRANSMISSION LINE TO A POINT TO WHICH ARE CONNECTED THE HIGH SIDE TERMINAL OF A TRANSFORMER AND ONE OTHER LINE LONGER THAN THE ONE SUDDENLY CONNECTED

will probably not make much headway, due to the fact that a considerable gap will have been established between the circuit-breaker contacts.

Fig. 22 applies to the disconnection of a line energized from a bus to which one other line, longer than the one being deenergized, is connected.

The equations for the curves of Fig. 22 are

(1) for the voltage and current up to the time of the first reflection

$$e = E\left(1 - \frac{1}{2}\,\epsilon^{-\frac{Z}{2L}t}\right)$$

$$i = \frac{E}{Z}\left(1 - \frac{1}{2}\,\epsilon^{-\frac{Z}{2L}t}\right)$$

(2) for the changes in voltage and current due to the first reflection

$$e = E\left(\frac{1}{2}\,\epsilon^{-\frac{Z}{2L}t} + \frac{1}{2}\,\frac{Z}{2L}\,t\,\epsilon^{-\frac{Z}{2L}t}\right)$$

$$i = \frac{E}{Z}\left(2 - \frac{3}{2}\,\epsilon^{-\frac{Z}{2L}t} - \frac{1}{2}\,\frac{Z}{2L}\,t\,\epsilon^{-\frac{Z}{2L}t}\right)$$

With E equal to normal crest voltage, the rate of change of current through the switch at the time when the current passes through zero is approximately equal to the rate of change at a current zero of a dead short circuit at the high side terminal of the transformer.

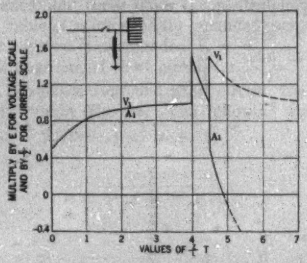

Fig. 23—Current and Voltage Surges Arising Upon the Sudden Connection of a Transmission Line to a Point to Which Are Already Connected the High Side Terminal of a Transformer and Another Line, Slightly Shorter Than the One Suddenly Connected, and Connected to an Infinite Bus at Its Far End

The impedance offered to high frequency transients passing through the switch is approximately a resistance equal to the sum of the surge impedances of the two transmission lines.

With E equal to normal crest voltage, therefore, the rate of rise of recovery voltage is roughly twice that given by the first part of curve 3, Fig. 8. E may, however, become several times normal crest voltage, so that the rate of rise of recovery voltage may reach 6 or 8 times that of curve 3.

From the point of view of the rate of rise of recovery voltage, therefore, interruption at the end of the first loop of this high frequency transient current is several times more difficult than the interruption of short-circuit current for the circuit of curve 3 Fig. 8, and probably about on a par with curve 1 of Fig. 8. The charging transient on this circuit is therefore considerably easier to interrupt than that of Fig. 21.

The remarks with regard to the condition of the gap which were offered in connection with the circuit of Fig. 21 also apply to the condition of the gap for this interruption.

Fig. 23 presents the voltage and current curves for a case differing from that of Fig. 22 in that the tap transmission line connected to the high-voltage bus is slightly shorter than the line being deenergized and is connected to an infinite bus at the far end.

The rate of change of current at the end of the first loop of the high frequency transient is nearly double that occurring in Fig. 22 and the rate of rise of recovery voltage is correspondingly greater.

Fig. 24 presents curves for the disconnection of a transmission line from an infinite bus, neglecting attenuation of the wave front. In this case it will be noted that the current passes through zero with an infinite rate of change. If the current is interrupted under the assumption of zero attenuation the voltage across the switch will rise immediately to a value E. Under this condition it is difficult to see how an interruption could occur. Actually, however, there will be a certain amount of attenuation which in long lines will probably be sufficient to make an interruption possible; in the case of short lines, two or three oscillations may occur before interruption. In this case interruption might occur with the transmission line charged to the same potential as before the breakdown of the gap, and in that case the process of voltage build-up would probably cease, for subsequent voltages across the gap would be only slightly greater than the value of E for this last breakdown.

Acknowledgment

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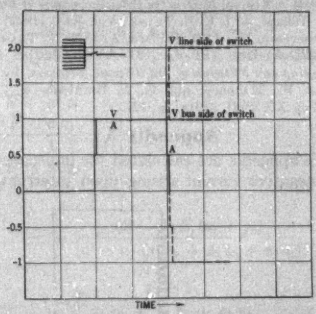

Fig. 24—Current and Voltage Surges Arising Upon the Sudden Connection of a Transmission Line to an Infinite Bus

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Appendix A

In this appendix are calculated the impedances met by the negative current arising upon interruption of

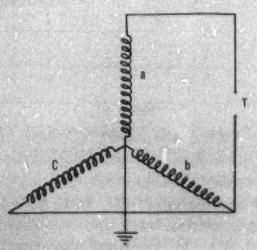


Fig. 25—Circuit for the First Phase to Clear of a Three-Phase Short Circuit

short circuits such that the short-circuit current is limited principally by stationary apparatus. Calculations are presented for three-phase short circuits and for two-phase-to-ground short circuits.

These impedances are determined by the method of symmetrical phase-sequence components as follows:

For a three-phase short circuit, the circuit is that shown in Fig. 25. The impedance of this circuit may be determined by assuming unit current flowing from one terminal to the other and calculating the resulting voltage. The equations applying are

$$e_b = a\, i_1 z_1 + a^2 i_2 z_2 + i_0 z_0 = 0 \qquad \text{(A-1)}$$

$$e_c = a^2 i_1 z_1 + a\, i_2 z_2 + i_0 z_0 = 0 \qquad \text{(A-2)}$$

$$i_a = i_1 + i_2 + i_0 = 1 \qquad \text{(A-3)}$$

Solving these three equations for i_1, i_2, and i_0,

$$i_1 = \frac{\begin{vmatrix} 0 & a^2 z_2 & z_0 \\ 0 & a\, z_2 & z_0 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} a\, z_1 & a^2 z_2 & z_0 \\ a^2 z_1 & a\, z_2 & z_0 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$= \frac{(a^2 - a)\, z_2 z_0}{(a^2 - a)\,(z_1 z_2 + z_2 z_0 + z_0 z_1)}$$

$$= \frac{z_2 z_0}{z_1 z_2 + z_2 z_0 + z_0 z_1} \qquad \text{(A-4)}$$

$$i_2 = \frac{\begin{vmatrix} a\, z_1 & 0 & z_0 \\ a^2 z_1 & 0 & z_0 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} a\, z_1 & a^2 z_2 & z_0 \\ a^2 z_1 & a\, z_2 & z_0 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$= \frac{(a^2 - a)\, z_1 z_0}{(a^2 - a)\,(z_1 z_2 + z_2 z_0 + z_0 z_1)}$$

$$= \frac{z_1 z_0}{z_1 z_2 + z_2 z_0 + z_0 z_1} \qquad \text{(A-5)}$$

$$i_0 = \frac{\begin{vmatrix} a\, z_1 & a^2 z_2 & 0 \\ a^2 z_1 & a\, z_2 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} a\, z_1 & a^2 z_2 & z_0 \\ a^2 z_1 & a\, z_2 & z_0 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$= \frac{(a^2 - a)\, z_1 z_2}{(a^2 - a)\,(z_1 z_2 + z_2 z_0 + z_0 z_1)}$$

$$= \frac{z_1 z_2}{z_1 z_2 + z_2 z_0 + z_0 z_1} \qquad \text{(A-6)}$$

But

$$z = e_T = e_a = i_1 z_1 + i_2 z_2 + i_0 z_0 = \frac{3\, z_1 z_2 z_0}{z_1 z_2 + z_2 z_0 + z_0 z_1} \qquad \text{(A-7)}$$

And, since $i = \dfrac{E}{z_1}$,

$$e = E\frac{z_2 z_0}{z_1 z_2 + z_2 z_0 + z_0 z_1} \qquad \text{(A-8)}$$

This equation may be used for solidly grounded systems, for systems grounded through a neutral impedance, and for ungrounded systems, for either grounded or ungrounded short circuits by adjusting the value of z_0 as follows:

For grounded short circuits

If the system is solidly grounded use the value of z_0 as calculated for the system at the point of short circuit.

At all points of the system where there is a neutral impedance, add three times the neutral impedance to the value of z_0 for a solidly grounded neutral.

If the system is ungrounded, $z_0 = \infty$

For all ungrounded short circuits, $z_0 = \infty$.

For a two-phase-to-ground short circuit, the circuit is that shown in Fig. 26. The following equations apply to this circuit:

FIG. 26—CIRCUIT FOR THE FIRST PHASE TO CLEAR OF A TWO-PHASE-TO-GROUND SHORT CIRCUIT

$$i_a = i_1 + i_2 + i_0 = 0 \qquad \text{(A-9)}$$

$$i_b = a\,i_1 + a^2 i_2 + i_0 = 1 \qquad \text{(A-10)}$$

$$e_c = a^2 i_1 z_1 + a\,i_2 z_2 + i_0 z_0 = 0 \qquad \text{(A-11)}$$

Solving for i_1, i_2, and i_0,

$$i_1 = \frac{\begin{vmatrix} 0 & 1 & 1 \\ 1 & a^2 & 1 \\ 0 & a z_2 & z_0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ a & a^2 & 1 \\ a & a z_2 & z_0 \end{vmatrix}}$$

$$= \frac{a z_2 - z_0}{(a^2 - a)(z_1 + z_2 + z_0)} \qquad \text{(A-12)}$$

$$i_2 = \frac{\begin{vmatrix} 1 & 0 & 1 \\ a & 1 & 1 \\ a^2 z_1 & 0 & z_0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ a & a^2 & 1 \\ a^2 z_1 & a z_2 & z_0 \end{vmatrix}}$$

$$= \frac{z_0 - a^2 z_1}{(a^2 - a)(z_1 + z_2 + z_0)} \qquad \text{(A-13)}$$

$$i_0 = \frac{\begin{vmatrix} 1 & 1 & 0 \\ a & a^2 & 1 \\ a^2 z_1 & a z_2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ a & a^2 & 1 \\ a^2 z_1 & a z_2 & z_0 \end{vmatrix}}$$

$$= \frac{a^2 z_1 - a z_2}{(a^2 - a)(z_1 + z_2 + z_0)} \qquad \text{(A-14)}$$

But

$$z = e_T = e_b = a\,i_1 z_1 + a^2 i_2 z_2 + i_0 z_0$$

$$= \frac{(a^2 - a)(z_1 z_2 + z_2 z_0 + z_0 z_1)}{(a^2 - a)(z_1 + z_2 + z_0)} = \frac{z_1 z_2 + z_2 z_0 + z_0 z_1}{z_1 + z_2 + z_0} \qquad \text{(A-15)}$$

The short-circuit current flowing previous to interruption in the circuit of Fig. 26 is determined by solution of the following equations:

$$e_b = a(E - i_1 z_1) - a^2 i_2 z_2 - i_0 z_0 = 0 \qquad \text{(A-16)}$$

$$e_c = a^2(E - i_1 z_1) - a\,i_2 z_2 - i_0 z_0 = 0 \qquad \text{(A-17)}$$

$$i_a = i_1 + i_2 + i_0 = 0 \qquad \text{(A-18)}$$

Equations (A-16) and (A-17) may be rewritten

$$a\,i_1 z_1 + a^2 i_2 z_2 + i_0 z_0 = a\,E \qquad \text{(A-19)}$$

and

$$a^2 i_1 z_1 + a\,i_2 z_2 + i_0 z_0 = a^2 E \qquad \text{(A-20)}$$

From Equations (A-18), (A-19), and (A-20)

$$i_1 = \frac{\begin{vmatrix} 0 & 1 & 1 \\ a E & a^2 z_2 & z_0 \\ a^2 E & a z_2 & z_0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ a z_1 & a^2 z_2 & z_0 \\ a^2 z_1 & a z_2 & z_0 \end{vmatrix}}$$

$$= \frac{(a^2 - a)E(z_0 + z_2)}{(a^2 - a)(z_1 z_2 + z_2 z_0 + z_0 z_1)}$$

$$= E\frac{z_0 + z_2}{z_1 z_2 + z_2 z_0 + z_0 z_1} \qquad \text{(A-21)}$$

$$i_2 = \frac{\begin{vmatrix} 1 & 0 & 1 \\ a z_1 & a E & z_0 \\ a^2 z_1 & a^2 E & z_0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ a z_1 & a^2 z_2 & z_0 \\ a^2 z_1 & a z_1 & z_0 \end{vmatrix}}$$

$$= \frac{(a - a^2)E z_0}{(a^2 - a)(z_1 z_2 + z_2 z_0 + z_0 z_1)}$$

$$= E\frac{-z_0}{z_1 z_2 + z_2 z_0 + z_0 z_1} \qquad \text{(A-22)}$$

$$i_0 = \frac{\begin{vmatrix} 1 & 1 & 0 \\ a\,z_1 & a^2\,z_2 & a\,E \\ a^2\,z_1 & a\,z_2 & a^2\,E \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ a\,z_1 & a^2\,z_2 & z_0 \\ a^2\,z_1 & a\,z_2 & z_0 \end{vmatrix}}$$

$$= \frac{(a - a^2)\,E\,z_2}{(a^2 - a)\,(z_1\,z_2 + z_2\,z_0 + z_0\,z_1)}$$

$$= E\,\frac{-\,z_2}{z_1\,z_2 + z_2\,z_0 + z_0\,z_1} \qquad \textbf{(A-23)}$$

$$i_b = a\,i_1 + a^2\,i_2 + i_0 = E\,\frac{(a - a^2)\,z_0 + (a - 1)\,z_2}{z_1\,z_2 + z_2\,z_0 + z_0\,z_1}$$

$$= (a - a^2)\,E\,\frac{z_0 - a^2\,z_2}{z_1\,z_2 + z_2\,z_0 + z_0\,z_1} \qquad \textbf{(A-24)}$$

The magnitude of the vector i_b is given by

$$|\,i_b\,| = \sqrt{3}\,E\,\frac{\sqrt{z_0^2 + z_0\,z_2 + z_2^2}}{z_1\,z_2 + z_2\,z_0 + z_0\,z_1} \qquad \textbf{(A-25)}$$

Thus multiplying Equations (**A-15**) and (**A-25**), the magnitude of the recovery voltage of a two-phase-to-ground short circuit in which the effect of the stationary apparatus predominates over the effect of rotating machinery is given by

$$e = \sqrt{3}\,E\,\frac{\sqrt{z_0^2 + z_0\,z_2 + z_2^2}}{z_1 + z_2 + z_0} \qquad \textbf{(A-26)}$$

Appendix B

The object of this appendix is the determination of the expression for the magnitude of recovery voltage, (neglecting the high frequency considerations), for a synchronous machine in which x_q'' differs from x_d'' and in a system in which the short circuit is opened before the direct-current component has vanished.

As in the remainder of the paper, the principle of superposition is used here. The voltage across the terminals of the switch is calculated as that which would be necessary to force through the circuit from one switch terminal to the other, the current i_a.

The distribution of the current in phases b and c is determined by considering that both i_b and i_c have one component equal to $-\frac{1}{2}\,i_a$, and a second component, which may be considered as a circulating current, which is equal and opposite in phases b and c. Thus the currents in phases b and c may be represented as follows,

$$i_b = i_{b1} + i_{b2} \qquad \textbf{(B-1)}$$

$$i_c = i_{c1} + i_{c2} \qquad \textbf{(B-2)}$$

where,

$$i_{b1} = i_{c1} = -\,\frac{1}{2}\,i_a$$

and

$$i_{b2} = -\,i_{c2}$$

The current which flows when a symmetrical three-phase ungrounded short circuit is placed on a three-phase machine, is given by Messrs. Doherty and Nickle in their paper *Synchronous Machines V.** The current in phase a which flows previous to interruption can therefore be expressed as,

$$i_a = \frac{e_0}{x_d}\cos(t - \alpha) + e_0\,\frac{x_d - x_d'}{x_d\,x_d'}\,\epsilon^{-\sigma_f t}\cos(t - \alpha)$$

$$+\,e_0\,\frac{x_d' - x_d''}{x_d'\,x_d''}\,\epsilon^{-\sigma_f t}\cos(t - \alpha)$$

$$-\,e_0\,\frac{x_d'' + x_q''}{2\,x_d''\,x_q''}\,\epsilon^{-\sigma_a t}\cos\alpha + e_0\,\frac{x_d'' - x_q''}{2\,x_d''\,x_q''}\,\epsilon^{-\sigma_a t}\cos(2\,t - \alpha) \qquad \textbf{(B-3)}$$

where

e_0 = open circuit terminal voltage before short circuit.

α = displacement between armature winding axis and axis of field pole at the instant of short circuit.

x_d = three-phase line-to-neutral synchronous reactance, direct axis.

x_d' = three-phase line-to-neutral transient reactance, direct axis.

x_d'' = three-phase line-to-neutral subtransient reactance, direct axis.

x_q = three-phase line-to-neutral synchronous reactance, quadrature axis.

x_q'' = three-phase line-to-neutral subtransient reactance, quadrature axis.

σ_a and σ_f are the decrement factors of the armature circuit and the field circuit with the armature short-circuited.

By making a change of reference axis, and by collecting the first and second terms, the current in phase a, under short-circuit conditions may be expressed as follows:

$$i_a = k_3\left[\frac{\cos\theta}{x_d''} - B\,\delta\cos\phi + C\,\delta\cos(2\,\theta + \phi)\right] \qquad \textbf{(B-4)}$$

where,

$$k_3 = \frac{i}{i''}$$

$$\delta = \frac{\epsilon^{-\sigma_a t}}{k_3}$$

*A. I. E. E. Trans., 1930, Vol. XLIX, R. E. Doherty and C. A. Nickle.

θ = time (numerically equal to radians), and is measured as the displacement in the direction of rotation of the direct axis of the field from the axis of magnetization of phase a.

ϕ = displacement of the axis of the field from the axis of phase a at the instant of short circuit.

$$B = \frac{x_d'' + x_q''}{2\, x_d''\, x_q''}$$

$$C = \frac{x_d'' - x_q''}{2\, x_d''\, x_q''}$$

In the following analysis, k_3 is omitted. Its effect may be considered by applying it to the final results.

To obtain the voltages generated by the interruption of the current represented by Equation (**B-4**), a current equal and opposite is assumed to flow in phase a, and the currents in phases b and c are considered split into two components as given by Equations (**B-1**) and (**B-2**). It is convenient then, to calculate the voltages in two parts, the first part being that arising from i_a, i_{b1}, and i_{c1}, and the second part being that arising from i_{b2} and i_{c2}. Accordingly, the voltages across phases at a, b, and c may be represented as:

$$e_a = e_{a1} + e_{a2} \qquad \textbf{(B-5)}$$

$$e_b = e_{b1} + e_{b2} \qquad \textbf{(B-6)}$$

$$e_c = e_{c1} + e_{c2} \qquad \textbf{(B-7)}$$

The flux linkages of phases a, b, and c due to the currents i_a, i_b, and i_c, may be calculated by the expressions given by Messrs. Park and Robertson in their paper *The Reactances of Synchronous Machines.** Since the neutral is ungrounded, the per unit linkages in phases a, b, and c may be represented as follows,

$$\psi_a = -\frac{x_d'' + x_q''}{3}\left[i_a - \frac{i_b + i_c}{2}\right] - \frac{x_d'' - x_q''}{3}\left[i_a \cos 2\,\theta + i_b \cos(2\,\theta - 120) + i_c \cos(2\,\theta + 120)\right] \qquad \textbf{(B-8)}$$

$$\psi_b = -\frac{x_d'' + x_q''}{3}\left[i_b - \frac{i_c + i_a}{2}\right] - \frac{x_d'' - x_q''}{3}\left[i_a \cos(2\,\theta - 120) + i_b \cos(2\,\theta + 120) + i_c \cos 2\,\theta\right] \qquad \textbf{(B-9)}$$

$$\psi_c = -\frac{x_d'' + x_q''}{3}\left[i_c - \frac{i_a + i_b}{2}\right] - \frac{x_d'' - x_q''}{3}\left[i_a \cos(2\,\theta + 120) + i_b \cos 2\,\theta + i_c \cos(2\,\theta - 120)\right] \qquad \textbf{(B-10)}$$

*R. H. Park and B. L. Robertson, A. I. E. E. Trans., 1928, Vol. XLVII, p. 514.

To facilitate calculation let

$$A = \frac{x_d'' + x_q''}{3}$$

and

$$D = \frac{x_d'' - x_q''}{3}$$

and

$$k_q = \frac{x_q''}{x_d''}$$

By letting i_a in Equation (**B-8**) be equal to the negative of the value given by Equation (**B-4**), and letting i_{b1} and i_{c1} equal minus one-half i_a, the expression for flux linkages in phase a due to the currents i_a, i_{b1} and i_{c1} may be written:

$$\psi_{a1} = \frac{(3 + k_q)}{4}\cos\theta + \frac{(1 - k_q^2)}{4\,k_q}\cos(2\,\theta + \phi) - \frac{\delta}{4}\,\frac{(1 - k_q^2)}{k_q}\cos\phi\cos 2\,\theta + \frac{(1 - k_q)}{4}\cos 3\,\theta + \frac{\delta\,(1 - k_q)^2}{8\,k_q}\cos(4\,\theta + \phi) + \frac{\delta\,(1 + k_q)^2}{4\,k_q}\cos\phi + \frac{\delta\,(1 - k_q)^2}{8\,k_q}\cos\phi \qquad \textbf{(B-11)}$$

The voltage due to these flux linkages is:

$$\frac{d\,\psi_{a1}}{d\,t} = e_{a1} = -\frac{(3 + k_q)}{4}\sin\theta - \frac{\delta}{2\,k_q}(1 - k_q^2)\sin\phi\cos 2\,\theta - \frac{3\,(1 - k_q)}{4}\sin 3\,\theta - \frac{\delta\,(1 - k_q)^2}{2\,k_q}\sin(4\,\theta + \phi) \qquad \textbf{(B-11a)}$$

Likewise, by using Equation (**B-9**), the flux linkages in phase b may be represented as:

$$\psi_{b1} = \frac{(-3 - k_q)}{8}\cos\theta + \frac{\delta}{8\,k_q}(1 - k_q^2)\sin\phi\sin 2\,\theta - \frac{(1 - k_q)}{8}\cos 3\,\theta - \frac{(1 - k_q)^2}{16\,k_q}\delta[\cos\phi - \sqrt{3}\sin\phi]\cos 4\,\theta + \frac{\sqrt{3}}{8}(1 - k_q)\sin\theta - \frac{\sqrt{3}}{8}\,\frac{(1 - k_q^2)}{k_q}\,\delta\cos\phi\sin 2\,\theta + \frac{\sqrt{3}}{8}(1 - k_q)\sin 3\,\theta + \frac{(1 - k_q)^2}{16\,k_q}\delta[\sin\phi + \sqrt{3}\cos\phi]\sin 4\,\theta + \frac{(1 + k_q)^2}{8\,k_q}\delta\cos\phi - \frac{(1 - k_q)^2}{16\,k_q}\delta\cos\phi - \frac{\sqrt{3}}{16}\,\frac{(1 - k_q)^2}{k_q}\,\delta\sin\phi \qquad \textbf{(B-12)}$$

The voltage generated in phase b due to these flux linkages is,

$$\frac{d\,\psi_{b1}}{d\,t} = e_{b1} = \frac{(3+k_q)}{8}\sin\theta + \frac{3\,(1-k_q)}{8}\sin 3\,\theta$$

$$+\frac{(1-k_q)^2\,\delta}{4\,k_q}\,[\cos\phi - \sqrt{3}\sin\phi]\sin 4\,\theta$$

$$+\frac{\sqrt{3}}{8}(1-k_q)\cos\theta + \frac{(1-k_q^2)\,\delta}{4\,k_q}[\sin\phi - \sqrt{3}\cos\phi]\cos 2\,\theta$$

$$+\frac{3\sqrt{3}}{8}(1-k_q)\cos 3\,\theta + \frac{(1-k_q)^2\,\delta}{4\,k_q}[\sin\phi + \sqrt{3}\cos\phi]\cos 4\,\theta \qquad \textbf{(B-12a)}$$

Also the flux linkages in phase c by the use of Equation (**B-10**), may be represented as,

$$\psi_{c1} = \frac{(-3-k_q)}{8}\cos\theta + \frac{(1-k_q^2)}{8\,k_q}\,\delta\sin\phi\sin 2\,\theta$$

$$-\frac{(1-k_q)}{8}\cos 3\,\theta - \frac{(1-k_q)^2}{8\,k_q}\delta[\cos\phi + \sqrt{3}\sin\phi]\cos 4\,\theta$$

$$-\frac{\sqrt{3}}{8}(1-k_q)\sin\theta + \frac{\sqrt{3}}{8\,k_q}(1-k_q^2)\,\delta\cos\phi\sin 2\,\theta$$

$$-\frac{\sqrt{3}}{8}(1-k_q)\sin 3\,\theta + \frac{(1-k_q)^2\,\delta}{16\,k_q}[\sin\phi - \sqrt{3}\cos\phi]\sin 4\,\theta$$

$$+\frac{(1+k_q)^2}{8\,k_q}\,\delta\cos\phi - \frac{(1-k_q)^2}{16\,k_q}\,\delta\cos\phi$$

$$+\frac{\sqrt{3}}{16\,k_q}(1-k_q)^2\,\delta\sin\phi \qquad \textbf{(B-13)}$$

The voltage in phase c due to these flux linkages is, therefore,

$$\frac{d\,\psi_{c1}}{d\,t} = e_{c1} = \frac{(3+k_q)}{8}\sin\theta + \frac{3}{8}(1-k_q)\sin 3\,\theta$$

$$+\frac{(1-k_q)^2\,\delta}{4\,k_q}[\cos\phi + \sqrt{3}\sin\phi]\sin 4\,\theta - \frac{3}{8}(1-k_q)\cos\theta$$

$$+\frac{(1-k_q^2)\,\delta}{4\,k_q}[\sin\phi + \sqrt{3}\cos\phi]\cos 2\,\theta - \frac{3\sqrt{3}}{8}(1-k_q)\cos 3\,\theta$$

$$+\frac{(1-k_q)^2\,\delta}{4\,k_q}[\sin\phi - \sqrt{3}\cos\phi]\cos 4\,\theta \qquad \textbf{(B-13a)}$$

According to Equations (**B-12**) and (**B-13**) there is a difference between the flux linkages in phase b and the flux linkages in phace c. The second components of current in these phases must be of such magnitude as to give rise to equal and opposite differences in flux linkages between phase b and phase c. The total voltage across phases b and c is obtained by differentiating the sum of the two components of flux linkages in each phase.

The currents i_{b2} and i_{c2} must be of such magnitude as to give rise to a flux difference between phases b and c which is equal and opposite to the flux difference $\psi_{b1} - \psi_{c1}$ which is obtained by subtracting Equation (**B-13**) from Equation (**B-12**).

$$\psi_{b1} - \psi_{c1} = \frac{\sqrt{3}}{4}(1-k_q)\sin\theta - \frac{\sqrt{3}\,\delta}{4\,k_q}(1-k_q^2)\cos\phi\sin 2\,\theta$$

$$+\frac{\sqrt{3}}{4}(1-k_q)\sin 3\,\theta + \frac{\sqrt{3}}{8\,k_q}(1-k_q)^2\,\delta\,[\sin(4\,\theta+\phi)]$$

$$-\frac{\sqrt{3}}{8}\,\frac{(1-k_q)^2}{k_q}\,\delta\sin\phi \qquad \textbf{(B-14)}$$

If the second components of flux in phases b and c due to the "circulating" components of current, are represented by ψ_{b2} and ψ_{c2}, the relation between the differences in fluxes between the phases b and c due to the two components in each phase is,

$$\psi_{b1} - \psi_{c1} = -\,(\psi_{b2} - \psi_{c2}) \qquad \textbf{(B-15)}$$

Referring again to Equations (**B-9**) and (**B-10**), the flux linkages in phases b and c due to the circulating current, may be determined by setting $i_{a2} = 0$, and $i_{b2} = -\,i_{c2}$. Thus the expressions for ψ_{b2} and ψ_{c2} are,

$$\psi_{b2} = -\,\frac{3}{2}\,A\,i_{b2} - D\,i_{b2}\,\{\cos(2\,\theta + 120^\circ) - \cos 2\,\theta\} \qquad \textbf{(B-16)}$$

$$\psi_{c2} = +\,\frac{3}{2}\,A\,i_{b2} - D\,i_{b2}\,\{\cos 2\,\theta - \cos(2\,\theta - 120^\circ)\} \qquad \textbf{(B-17)}$$

Likewise, by Equation (**B-8**), the flux linkages in phase a due to the second component of current in phases b and c may be represented as,

$$\psi_{a2} = -\,\sqrt{3}\,D\,i_{b2}\sin 2\,\theta \qquad \textbf{(B-18)}$$

The difference in flux linkages in phase b and c due to the second component of current is:

$$\psi_{b2} - \psi_{c2} = i_{b2}\,\{-\,3\,A + 3\,D\cos 2\,\theta\} \qquad \textbf{(B-19)}$$

By referring to Equation (**B-15**), it is obvious that the right-hand member of Equation (**B-14**) is equal to minus the right-hand member of Equation (**B-19**). This relation may therefore be written,

$$\frac{\sqrt{3}}{4}(1-k_q)\sin\theta - \frac{\sqrt{3}\,\delta}{4\,k_q}(1-k_q^2)\cos\phi\sin 2\,\theta$$

$$+\frac{\sqrt{3}}{4}(1-k_q)\sin 3\,\theta + \frac{\sqrt{3}}{8\,k_q}(1-k_q)^2\,\delta\sin(4\,\theta+\phi)$$

$$-\frac{\sqrt{3}}{8}\,\frac{(1-k_q)^2}{k_q}\,\delta\sin\phi = i_{b2}\,\{3\,A - 3\,D\cos 2\,\theta\} \qquad \textbf{(B-20)}$$

Equation (**B-20**) will be solved by letting i_{b2} be equal to a Fourier series and solving for the coefficients.

Thus

$$i_{b2} = \sum_{1}^{\infty} B \sin \theta + \sum_{0}^{\infty} C \cos \theta \qquad \text{(B-21)}$$

By substituting Equation (**B-21**) in Equation (**B-20**) the following sets of equations are obtained by equating coefficients of corresponding terms.

The constant term, C_0 is:

$$C_0 = -\frac{\sqrt{3}}{24\,A}\,\frac{(1-k_q)^2}{k_q}\,\delta \sin \phi + \frac{D\,C_2}{2\,A} \qquad \text{(B-22)}$$

The coefficients of the sine terms are:

$\sin \theta$:

$$\left(3\,A + \frac{3}{2}\,D\right) B_1 - \frac{3}{2}\,D\,B_3 = \frac{\sqrt{3}}{4}\,(1-k_q) \qquad \text{(B-23)}$$

$\sin 2\,\theta$:

$$3\,A\,B_2 - \frac{3}{2}\,D\,B_4 = -\frac{\sqrt{3}\,\delta}{4\,k_q}\,(1-k_q^2)\cos \phi \qquad \text{(B-24)}$$

$\sin 3\,\theta$:

$$-\frac{3}{2}\,D\,B_1 + 3\,A\,B_3 - \frac{3}{2}\,D\,B_5 = \frac{\sqrt{3}}{4}\,(1-k_q) \qquad \text{(B-25)}$$

$\sin 4\,\theta$:

$$-\frac{3}{2}\,D\,B_2 + 3\,A\,B_4 - \frac{3}{2}\,D\,B_6 = \frac{\sqrt{3}}{8\,k_q}\,(1-k_q)^2\,\delta \cos \phi \qquad \text{(B-26)}$$

$\sin n\,\theta\ (n > 4)$:

$$-\frac{3}{2}\,D\,B_{(n-2)} + 3\,A\,B_n - \frac{3}{2}\,D\,B_{(n+2)} = 0 \qquad \text{(B-27)}$$

The even coefficients B of the sine terms are zero, and the odd coefficients follow the relation,

$$m = \left.\frac{B_{(n+2)}}{B_n}\right]_{n=3}^{n=\infty} = \frac{(1-k_q^{1/2})^2}{(1-k_q)} \qquad \text{(B-28)}$$

Substituting the relation (**B-28**) in Equations (**B-23**) and (**B-25**), these equations may be solved simultaneously for B_1 and B_3, which are,

$$B_1 = \frac{3\sqrt{3}\,(1-k_q)\left[A - \frac{D}{2}\,(m-1)\right]}{36\,A^2 - 18\,A\,D\,m + 18\,A\,D - 9\,D^2\,m - 9\,D^2} \qquad \text{(B-29)}$$

$$B_3 = \frac{3\sqrt{3}\,(1-k_q)\,[A + D]}{36\,A^2 - 18\,A\,D\,m + 18\,A\,D - 9\,D^2\,m - 9\,D^2} \qquad \text{(B-30)}$$

By substituting Equation (**B-28**) in Equations (**B-24**) and (**B-26**), these equations may be solved simultaneously for B_2 and B_4, which are,

$$B_2 = -\frac{\sqrt{3}\,(1-k_q^2)\,\delta \cos \phi}{12\,A\,k_q} \qquad \text{(B-31)}$$

$$B_4 = 0 \qquad \text{(B-32)}$$

Likewise, by equating coefficients of the corresponding cosine terms, the following relations are obtained.

$\cos \theta$:

$$\left(3\,A - \frac{3}{2}\,D\right) C_1 - \frac{3}{2}\,D\,C_3 = 0 \qquad \text{(B-33)}$$

$\cos 2\,\theta$:

$$-3\,D\,C_0 + 3\,A\,C_2 - \frac{3}{2}\,D\,C_4 = 0 \qquad \text{(B-34)}$$

$\cos 3\,\theta$:

$$3\,A\,C_3 - \frac{3}{2}\,D\,C_5 = 0 \qquad \text{(B-35)}$$

$\cos 4\,\theta$:

$$-\frac{3}{2}\,D\,C_2 + 3\,A\,C_4 - \frac{3}{2}\,D\,C_6 = \frac{\sqrt{3}}{8}\,\frac{(1-k_q)^2}{k_q}\,\delta \sin \phi \qquad \text{(B-36)}$$

$\cos n\,\theta\ (n > 4)$:

$$-\frac{3}{2}\,D\,C_{(n-2)} + 3\,A\,C_n - \frac{3}{2}\,D\,C_{(n+2)} = 0 \qquad \text{(B-37)}$$

The odd coefficients of the cosine series are zero and the even coefficients are satisfied by the following relation,

$$m = \left.\frac{C_{(n+2)}}{C_n}\right]_{n=4}^{n=\infty} = \frac{(1-k_q^{1/2})^2}{(1-k_q)} \qquad \text{(B-38)}$$

Substituting Equation (**B-38**) in Equations (**B-34**) and (**B-36**), and using the value of C_0 in Equation (**B-22**), Equations (**B-34**) and (**B-36**) may be solved simultaneously for C_2 and C_4, as follows,

$$C_2 = -\frac{3\sqrt{3}\,D\,\delta \sin \phi\,(1-k_q)^2\,[A - D\,m]}{4\,k_q\,[36\,A^3 - 18\,A^2\,D\,m - 27\,A\,D^2 + 9\,D^3\,m]} \qquad \text{(B-39)}$$

and

$$C_4 = \frac{3\sqrt{3}\,\delta \sin \phi\,(1-k_q)^2\,[A^2 - D^2]}{2\,k_q\,[36\,A^3 - 18\,A^2\,D\,m - 27\,A\,D^2 + 9\,D^3\,m} \qquad \text{(B-40)}$$

The flux linkages in phase a due to the "circulating" or second component of current in phases b and c may be obtained by using Equation (**B-8**) and the additional conditions that,

$$i_{a2} = 0$$

and

$$i_{b2} = -\,i_{c2}$$

Thus the expression for flux linkages in phase a due to the second component of current in phases b and c is,

$$\psi_{a2} = -\sqrt{3}\,D\,[C_o + B_2 \sin 2\,\theta + \sum_{n_o=1}^{n_o=\infty} B_{n_o} \sin n_o\,\theta + \sum_{n_e=2}^{n_e=\infty} C_{n_e} \cos n_e\,\theta] \sin 2\,\theta \qquad \text{(B-41)}$$

or

$$\psi_{a2} = -\sqrt{3}\,D\,[C_o \sin 2\,\theta + B_2 \sin^2 2\,\theta + \sin 2\,\theta \sum_{n_o=1}^{n_o=\infty} B_{no} \sin n_o\,\theta + \sin 2\,\theta \sum_{n_e=2}^{n_e=\infty} C_{ne} \cos n_e\,\theta] \tag{B-42}$$

where n_o = odd term

and n_e = even term

But since,

$$\sin n\,\theta \sin 2\,\theta = \frac{\cos (n-2)\,\theta - \cos (n+2)\,\theta}{2}$$

and

$$\sin 2\,\theta \cos n\,\theta = \frac{\sin (n+2)\,\theta - \sin (n-2)\,\theta}{2},$$

Equation (B-42) may be written as follows, so that,

$$\psi_{a2} = -\sqrt{3}\,D\left[\frac{B_2}{2} + C_o \sin 2\,\theta - \frac{B_2}{2}\cos 4\,\theta + \sum_{n_o=-1}^{n_o=\infty} \frac{B_{(no+2)}\cos n_o\,\theta}{2} - \sum_{n_o=3}^{n_o=\infty} \frac{B_{(no-2)}\cos n_o\,\theta}{2} + \sum_{n_e=4}^{n_e=\infty} \frac{C_{(ne-2)}\sin n_e\,\theta}{2} - \sum_{n_e=0}^{n_e=\infty} \frac{C_{(ne+2)}\sin n_e\,\theta}{2}\right] \tag{B-43}$$

The voltage induced in phase a due to the flux linkages caused by the circulating current is,

$$\frac{d\,\psi_{a2}}{d\,t} = e_{a2} = -\sqrt{3}\,D\,[2\,C_0 \cos 2\,\theta + 2\,B_2 \cos 4\,\theta - \sum_{n_o=-1}^{n_o=\infty} \frac{n_o}{2} B_{(no+2)} \sin n_o\,\theta + \sum_{n_o=3}^{n_o=\infty} \frac{n_o}{2} B_{(no-2)} \sin n_o\,\theta + \sum_{n_e=4}^{n_e=\infty} \frac{n_e}{2} C_{(ne-2)} \cos n_e\,\theta - \sum_{n_e=0}^{n_e=\infty} \frac{n_e}{2} C_{(ne+2)} \cos n_e\,\theta\,] \tag{B-44}$$

Equation (B-9) may be used to obtain an expression for the flux linkages in phase b due to the circulating current in phases b and c.

$$\psi_{b2} = -\frac{3}{2}A\left[C_o + B_2 \sin 2\,\theta + \sum_{n_o=1}^{n_o=\infty} B_{no} \sin n_o\,\theta + \sum_{n_e=2}^{n_e=\infty} C_{ne} \cos n_e\,\theta\right] + \frac{3}{2}D\left[C_o + B_2 \sin 2\,\theta + \sum_{n_o=1}^{n_o=\infty} B_{no} \sin n_o\,\theta + \sum_{n_e=2}^{n_e=\infty} C_{ne} \cos n_e\,\theta\right]\cos 2\,\theta + \frac{\sqrt{3}}{2}D\left[C_0 + B_2 \sin 2\,\theta + \sum_{n_o=1}^{n_o=\infty} B_{no} \sin n_o\,\theta + \sum_{n_e=2}^{n_e=\infty} C_{ne} \cos n_e\,\theta\right]\sin 2\,\theta \tag{B-45}$$

By expanding Equation (B-45), the following expression is obtained,

$$\psi_{b2} = -\frac{3}{2}A\left[C_0 + B_2 \sin 2\,\theta + \sum_{n_o=1}^{n_o=\infty} B_{no} \sin n_o\,\theta + \sum_{n_e=2}^{n_e=\infty} C_{ne} \cos n_e\,\theta\right] + \frac{3}{2}D\left[C_0 \cos 2\,\theta + \frac{B_2 \sin 4\,\theta}{2} + \cos 2\,\theta \sum_{n_o=1}^{n_o=\infty} B_{no} \sin n_o\,\theta + \cos 2\,\theta \sum_{n_e=2}^{n_e=\infty} C_{ne} \cos n_e\,\theta\right] + \frac{\sqrt{3}}{2}D\left[C_0 \sin 2\,\theta + \frac{B_2}{2} - \frac{B_2 \cos 4\,\theta}{2} + \sin 2\,\theta \sum_{n_o=1}^{n_o=\infty} B_{no} \sin n_o\,\theta + \sin 2\,\theta \sum_{n_e=2}^{n_e=\infty} C_{ne} \cos n_e\,\theta\right] \tag{B-46}$$

Equation (B-46) may, by the substitution of trigonometric identities, be written as follows,

$$\psi_{b2} = -\frac{3}{2}A\left[C_0 + B_2 \sin 2\,\theta + \sum_{n_o=1}^{n_o=\infty} B_{no} \sin n_o\,\theta + \sum_{n_e=2}^{n_e=\infty} C_{ne} \cos n_e\,\theta\right] + \frac{3}{2}D\left[C_0 \cos 2\,\theta + \frac{B_2}{2}\sin 4\,\theta + \sum_{n_o=3}^{n_o=\infty} \frac{B_{(no-2)}}{2} \sin n_o\,\theta + \sum_{n_o=-1}^{n_o=\infty} \frac{B_{(no+2)}}{2} \sin n_o\,\theta + \sum_{n_e=0}^{n_e=\infty} \frac{C_{(ne+2)}}{2} \cos n_e\,\theta + \sum_{n_e=4}^{n_e=\infty} \frac{C_{(ne-2)}}{2} \cos n_e\,\theta\right] + \frac{\sqrt{3}}{2}D\left[C_0 \sin 2\,\theta + \frac{B_2}{2} - \frac{B_2}{2}\cos 4\,\theta + \sum_{n_o=-1}^{n_o=\infty} \frac{B_{(no+2)}}{2} \cos n_o\,\theta - \sum_{n_o=3}^{n_o=\infty} \frac{B_{(no-2)}}{2} \cos n_o\,\theta + \sum_{n_e=4}^{n_e=\infty} \frac{C_{(ne-2)}}{2} \sin n_e\,\theta - \sum_{n_e=0}^{n_e=\infty} \frac{C_{(ne+2)}}{2} \sin n_e\,\theta\right] \tag{B-47}$$

The total flux linkage in phase b may be represented by $(\psi_{b1} + \psi_{b2})$, or equal to the sum of the flux linkages

represented by Equations (**B-12**) and (**B-47**). Thus the total flux linkages in phase b are,

$$\psi_b = \frac{(-3-k_q)}{8}\cos\theta + \frac{(1-k_q^2)}{8\,k_q}\delta\sin\phi\sin 2\,\theta$$

$$-\frac{(1-k_q)}{8}\cos 3\,\theta - \frac{(1-k_q)^2}{16\,k_q}\delta\cos\phi\cos 4\,\theta$$

$$+\frac{(1-k_q)^2\,\delta}{16\,k_q}\sin\phi\sin 4\,\theta + \frac{\sqrt{3}}{2}D\,C_0\sin 2\,\theta$$

$$-\frac{\sqrt{3}}{4}D\,B_2\cos 4\,\theta + \frac{\sqrt{3}}{2}D\Big[\sum_{n_o=-1}^{n_o=\infty}\frac{B_{(n_o+2)}}{2}\cos n_o\,\theta$$

$$-\sum_{n_o=3}^{n_o=\infty}\frac{B_{(n_o-2)}}{2}\cos n_o\,\theta + \sum_{n_e=4}^{n_e=\infty}\frac{C_{(n_e-2)}}{2}\sin n_e\,\theta$$

$$-\sum_{n_e=0}^{n_e=\infty}\frac{C_{(n_e+2)}}{2}\sin n_e\,\theta\Big] + \frac{(1+k_q)^2\,\delta}{8\,k_q}\cos\phi \qquad \textbf{(B-48)}$$

since,

$$\Big[-\frac{3}{4}D\,C_{(n_e-2)} + \frac{3}{2}A\,C_{n_e} - \frac{3}{4}D\,C_{(n_e+2)}\Big]\cos 4\,\theta = \frac{\sqrt{3}}{16}\frac{(1-k_q)^2}{k_q}\sin\phi\cos 4\,\theta \qquad \textbf{(B-36)}$$

$$\Big[\frac{3}{2}A\,B_1 + \frac{3}{4}D\,B_1 - \frac{D}{4}B_3\Big]\sin\theta = \frac{\sqrt{3}}{8}(1-k_q)\sin\theta \qquad \textbf{(B-23)}$$

$$\frac{3}{2}A\,B_2\sin 2\,\theta = -\frac{\sqrt{3}}{8}\frac{(1-k_q^2)}{k_q}\delta\cos\phi\sin 2\,\theta \qquad \textbf{(B-24)}$$

$$\Big[-\frac{3}{4}D\,B_1 + \frac{3}{2}A\,B_3 - \frac{3}{4}D\,B_5\Big]\sin 3\,\theta = \frac{\sqrt{3}}{8}(1-k_q)\sin 3\,\theta \qquad \textbf{(B-25)}$$

$$\frac{3}{4}D\,B_2\sin 4\,\theta = -\Big[\frac{\sqrt{3}\,(1-k_q)^2}{16\,k_q}\delta\cos\phi\sin 4\,\theta\Big] \qquad \textbf{(B-26)}$$

$$\frac{3}{2}A\,C_0 = -\frac{\sqrt{3}}{16}\frac{(1-k_q)^2}{k_q}\delta\sin\phi + \frac{3\,D\,C_2}{4} \qquad \textbf{(B-22)}$$

$$\frac{3}{2}D\sum_{n_e=4}^{n_e=\infty}C_{(n_e-2)}\cos n_e\,\theta - \frac{3}{2}A\sum_{n_e=4}^{n_e=\infty}C_{n_e}\cos n_e\,\theta + \frac{3}{2}D\sum_{n_e=4}^{n_e=\infty}\frac{C_{(n_e+2)}}{2}\cos n_e\,\theta = 0 \qquad \textbf{(B-37)}$$

The total voltage induced in phase b is therefore,

$$\frac{d\,\psi_b}{dt} = e_b = \frac{(3+k_q)}{8}\sin\theta + \frac{(1-k_q^2)}{4\,k_q}\delta\sin\phi\cos 2\,\theta$$

$$+\frac{3(1-k_q)}{8}\sin 3\,\theta + \frac{(1-k_q)^2}{4\,k_q}\delta\sin(4\,\theta+\phi)$$

$$+\sqrt{3}\,D\,C_0\cos 2\,\theta + \sqrt{3}\,D\,B_2\sin 4\,\theta$$

$$+\frac{\sqrt{3}}{2}D\Big[-\sum_{n_o=-1}^{n_o=\infty}\frac{n_o}{2}B_{(n_o+2)}\sin n_o\,\theta$$

$$+\sum_{n_o=3}^{n_o=\infty}\frac{n_o}{2}B_{(n_o-2)}\sin n_o\,\theta + \sum_{n_e=4}^{n_e=\infty}\frac{n_e}{2}C_{(n_e-2)}\cos n_e\,\theta$$

$$-\sum_{n_e=0}^{n_e=\infty}\frac{n_e}{2}C_{(n_e+2)}\cos n_e\,\theta\Big] \qquad \textbf{(B-49)}$$

The voltage induced in phase c may be determined in a similar manner and will be found to be equal to that induced in phase b.

The total voltage induced in phase a will be,

$$\frac{d\,\psi_a}{dt} = \frac{d\,\psi_{a1}}{dt} + \frac{d\,\psi_{a2}}{dt} \qquad \textbf{(B-50)}$$

By adding Equation (**B-11a**) to Equation (**B-44**), the following expression for the total voltage in phase a is obtained.

$$\frac{d\,\psi_a}{dt} = -\frac{(3+k_q)}{4}\sin\theta - \frac{3\,(1-k_q)}{4}\sin 3\,\theta$$

$$-\frac{\delta\,(1-k_q)^2}{2\,k_q}\sin(4\,\theta+\phi)$$

$$-\Big[\frac{\delta\,(1-k_q^2)}{2\,k_q}\sin\phi + 2\sqrt{3}\,D\,C_0\Big]\cos 2\,\theta$$

$$-2\sqrt{3}\,D\,B_2\cos 4\,\theta - \sqrt{3}\,D\Big[\sum_{n_o=-1}^{n_o=\infty}-\frac{n_o}{2}B_{(n_o+2)}\sin n_o\,\theta$$

$$+\sum_{n_o=3}^{n_o=\infty}\frac{n_o}{2}B_{(n_o-2)}\sin n_o\,\theta + \sum_{n_e=4}^{n_e=\infty}\frac{n_e}{2}C_{(n_e-2)}\cos n_e\,\theta$$

$$-\sum_{n_e=0}^{n_e=\infty}\frac{n_e}{2}C_{(n_e+2)}\cos n_e\,\theta\Big] \qquad \textbf{(B-51)}$$

The total voltage across phases a and b is the difference between the voltages represented by Equations (**B-51**) and (**B-49**):

$$\frac{d\,\psi_{ab}}{dt} = -\frac{3}{8}(3+k_q)\sin\theta - \frac{9}{8}(1-k_q)\sin 3\,\theta$$

$$-\frac{3}{4}\frac{(1-k_q)^2}{k_q}\delta\sin(4\,\theta+\phi) - \Big[\frac{3}{4}\frac{(1-k_q^2)}{k_q}\delta\sin\phi$$

$$+3\sqrt{3}\,D\,C_0\Big]\cos 2\,\theta - 3\sqrt{3}\,D\,B_2\cos 4\,\theta$$

$$+ \frac{3}{2}\sqrt{3}D\Bigg[\sum_{n_e=-1}^{n_e=\infty}\frac{n_e}{2}B_{(n_e+2)}\sin n_e\theta - \sum_{n_e=3}^{n_e=\infty}\frac{n_e}{2}B_{(n_e-2)}\sin n_e\theta - \sum_{n_e=4}^{n_e=\infty}\frac{n_e}{2}C_{(n_e-2)}\cos n_e\theta + \sum_{n_e=0}^{n_e=\infty}\frac{n_e}{2}C_{(n_e+2)}\cos n_e\theta\Bigg] \quad \textbf{(B-52)}$$

To facilitate calculations of recovery voltage magnitudes for various values of k_q, it is convenient to convert all coefficients into functions of k_q. The terms $D\,C_0$, and the product of all coefficients by the term D may be expressed as functions of k_q. Thus the following substitutions may be made in Equation (**B-52**).

$$D\,C_0 = C_0' = -\frac{\sqrt{3}\,(1-k_q)^2\,\delta\sin\phi}{24\,k_q\,(1+k_q)} + \frac{(1-k_q)\,C_2'}{2\,(1+k_q)} \quad \textbf{(B-53)}$$

$$D\,B_1 = B_1' = \frac{(1+k_q) - \frac{1}{2}(1-k_q)(m-1)}{\sqrt{3}\left[4\left(\frac{1+k_q}{1-k_q}\right)^2 + 2\left(\frac{1+k_q}{1-k_q}\right)(1-m) - (1+m)\right]} \quad \textbf{(B-54)}$$

$$D\,B_2 = B_2' = -\frac{\sqrt{3}\,(1-k_q)^2\cos\phi}{12\,k_q} \quad \textbf{(B-55)}$$

$$D\,B_3 = B_3' = \frac{2}{\sqrt{3}\left[4\left(\frac{1+k_q}{1-k_q}\right)^2 + 2\left(\frac{1+k_q}{1-k_q}\right)(1-m) - (1+m)\right]} \quad \textbf{(B-56)}$$

$$D\,C_2 = C_2' = -\frac{\sqrt{3}\,\delta\sin\phi\,(1-k_q)^4\,[(1+k_q) - (1-k_q^{3/5})^2]}{12\,k_q\,[4\,(1+k_q)^3 - 2\,(1-k_q^{3/5})^2\,(1+k_q)^2 - 3\,(1-k_q)^2\,(1+k_q) + (1-k_q)^2\,(1-k_q^{3/5})^2]} \quad \textbf{(B-57)}$$

$$D\,C_4 = C_4' = -\frac{2\sqrt{3}\,\delta\sin\phi\,(1-k_q)}{3\left[(1+k_q) - 4\frac{(1+k_q)^3}{(1-k_q)^2} + 2\,(1+k_q) + \frac{2\,(1+k_q)^2\,(1-k_q^{3/5})^2}{(1-k_q)^2} - (1-k_q^{3/5})^2\right]} \quad \textbf{(B-58)}$$

By making the above substitutions the expression for the total voltage across phases a and b or a and c becomes

$$\frac{d\,\psi_{ab}}{dt} = \frac{d\,\psi_{ac}}{dt} = -\frac{3}{8}(3+k_q)\sin\theta - \frac{9}{8}(1-k_q)\sin 3\,\theta - \frac{3}{4}\frac{(1-k_q)^2}{k_q}\delta\sin(4\,\theta+\phi) - \Bigg[\frac{3}{4}\frac{(1-k_q^2)}{k_q}\delta\sin\phi + 3\sqrt{3}\,C_0'\Bigg]\cos 2\,\theta - 3\sqrt{3}\,B'_2\cos 4\,\theta + \frac{3}{2}\sqrt{3}\Bigg[\sum_{n_e=-1}^{n_e=\infty}\frac{n_e}{2}B'_{(n_e+2)}\sin n_e\theta - \sum_{n_e=3}^{n_e=\infty}\frac{n_e}{2}B'_{(n_e-2)}\sin n_e\theta - \sum_{n_e=4}^{n_e=\infty}\frac{n_e}{2}C'_{(n_e-2)}\cos n_e\theta + \sum_{n_e=0}^{n_e=\infty}\frac{n_e}{2}C'_{(n_e+2)}\cos n_e\theta\Bigg] \quad \textbf{(B-59)}$$

An inspection of Equation (**B-59**) reveals that the recovery voltage magnitude depends in part upon both the angle θ and the angle ϕ. Since we are interested in the values of these angles when the current in phase a is zero, the relation between θ and ϕ is obtained by equating the expression (**B-4**) to zero, and solving for the values of θ corresponding to various values of ϕ. For $k_q = 1.5$, 2.0, and 2.5, there are two values of θ for every value of ϕ considered.

Figs. 27 to 30 inclusive are families of curves of recovery voltage magnitudes plotted against the angle ϕ for various values of k_q and δ. Equation (**B-59**) was used in the solution for the recovery voltage magnitudes.

For values of k_q other than $k_q = 1.0$, and when δ does not equal zero the angle separating the two points of zero current is different from 180 deg. The slope of the current curve is different at the two zero points. Therefore, two different values of recovery voltage occur for each combination of ϕ, δ, and k_q.

When the armature transients have died away: *i. e.*, when $\delta = 0$, the two values of e are independent of ϕ and are both equal to $\frac{3}{2}k_q$, the value given in the body of the paper. For values of δ other than zero, one set of values of e is uniformly below $\frac{3}{2}k_q$. The other set of

values, however, is above $\frac{3}{2} k_q$ for values of ϕ between approximately 40 deg. and 90 deg.

Unless the breaker is timed to clear the circuit at a specific current zero, it may choose the zero associated with the lower value of recovery voltage. Thus the circuit may be more easily interrupted before the armature transients have died away than afterwards.

If the breaker is timed to interrupt at a specific current zero, however, its duty will be somewhat more severe for certain values of ϕ when the direct-current

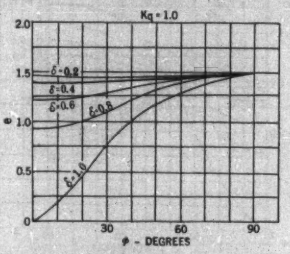


FIG. 27—VALUES OF RECOVERY VOLTAGE AS A FUNCTION OF ϕ AND δ FOR A THREE-PHASE SHORT CIRCUIT FOR WHICH $k_q = 1.0$

ϕ = Displacement angle between the phase axis and the direct axis of the rotor at the instant of short circuit

$\delta = \dfrac{\text{Per cent displacement at the time of interruption}}{\text{Per cent displacement of initial current wave}}$

component and the even harmonics are present than after they have vanished.

Appendix C

This appendix calculates the magnitude of the recovery voltage appearing across the circuit breaker terminals of the first phase to clear of a purely reactive three-phase grounded short circuit after the d-c. component and the second harmonic have decayed to zero for a synchronous machine.

The calculation follows:

The phase opening first is designated phase a, the others being, in normal sequence, b, and c. The circuit is, therefore, that of Fig. 25.

At the time when the current in phase a is passing through zero, that phase is in line with the quadrature axis of the machine.

The negative currents arising in the three phases upon interruption of current in phase a may be set equal to $i_a' t$, $i_b' t$, and $i_c' t$ respectively for a short time following the instant of interruption, t being measured in electrical radians from the instant of interruption.

Hence, flux linkages arising from the negative current are

$$\psi_q = \frac{2}{3} x_q'' [i_a' + (i_b' + i_c') \cos 120°] t$$

$$= \frac{2}{3} x_q'' t \left(i_a' - \frac{i_b' + i_c'}{2} \right) \qquad \text{(C-1)}$$

$$\psi_d = \frac{2}{3} x_d'' t [i_a' \cos 90° + i_b' \cos 30° + i_c' \cos 150°]$$

$$= \frac{1}{\sqrt{3}} x_d'' t (i_b' - i_c') \qquad \text{(C-2)}$$

$$\psi_0 = \frac{x_0 t}{3} (i_a' + i_b' + i_c') \qquad \text{(C-3)}$$

From these,

$$\psi_b = \psi_q \cos 120° + \psi_d \cos 30° + \psi_0$$

$$= -\frac{x_q'' t}{3} \left(i_a' - \frac{i_b' + i_c'}{2} \right) + \frac{x_d'' t}{2} (i_b' - i_c')$$

$$+ \frac{x_0 t}{3} (i_a' + i_b' + i_c') = 0 \qquad \text{(C-4)}$$

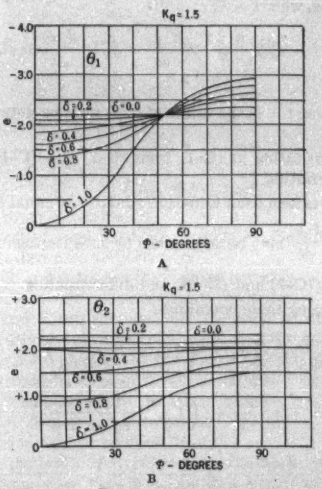


FIG. 28—VALUES OF RECOVERY VOLTAGE AS A FUNCTION OF ϕ AND δ FOR A THREE-PHASE SHORT CIRCUIT FOR WHICH $k_q = 1.5$

ϕ = Displacement angle between the phase axis and direct axis of the rotor at the instant of short circuit

$\delta = \dfrac{\text{Per cent displacement at the time of interruption}}{\text{Per cent displacement of initial current wave}}$

and,

$$\psi_c = \psi_q \cos 120° + \psi_d \cos 150° + \psi_0$$

$$= -\frac{x_q'' t}{3} \left(i_a' - \frac{i_b' + i_c'}{2} \right) - \frac{x_d'' t}{2} (i_b' - i_c')$$

$$+ \frac{x_0 t}{3} (i_a' + i_b' + i_c') = 0 \qquad \text{(C-5)}$$

ψ_b and ψ_c are set equal to zero because these two phases are short-circuited and hence can build up no flux linkages. The equality holds for all values of t, hence the t may be cancelled from the third members of Equations (C-4) and (C-5). In subsequent references to these equations, therefore, t will be considered cancelled.

Subtracting (C-5) from (C-4)

$$x_d''(i_b' - i_c') = 0 \qquad \text{(C-6)}$$

whence

$$i_b' = i_c' \qquad \text{(C-7)}$$

Substituting (C-7 in (C-4)

$$-\frac{x_q''}{3}(i_a' - i_b') + \frac{x_0}{3}(i_a' + 2\,i_b') = 0 \qquad \text{(C-8)}$$

$$i_a'(x_0 - x_q'') + i_b'(2\,x_0 + x_q'') = 0 \qquad \text{(C-9)}$$

$$i_b' = i_a'\frac{x_q'' - x_0}{x_q'' + 2\,x_0} \qquad \text{(C-10)}$$

Now

$$\psi_a = \psi_q \cos t - \psi_d \sin t + \psi_0 \qquad \text{(C-11)}$$

Whence, when $t = 0$,

$$e_a = \frac{d\,\psi_a}{dt} = \frac{d\,\psi_q}{dt} - \psi_d + \frac{d\,\psi_0}{dt} \qquad \text{(C-12)}$$

At time $t = 0$, the values of $\frac{d\,\psi_q}{dt}$ may be determined by substituting in (C-1) from (C-7) and (C-10) and differentiating,

ψ_d is seen from Equation (C-2) to be equal to zero, and $\frac{d\,\psi_0}{dt}$ may be determined by substituting in (C-3) from (C-7) and (C-10) and differentiating.

Performing these operations:

$$e_a = \frac{d\,\psi_a}{dt} = \frac{2}{3}x_q''\,i_a'\left(1 - \frac{x_q'' - x_0}{x_q'' + 2\,x_0}\right) + \frac{x_0}{3}\,i_a'\left(1 + 2\,\frac{x_q'' - x_0}{x_q'' + 2\,x_0}\right) \qquad \text{(C-13)}$$

$$= i_a'\left(\frac{2\,x_q''\,x_0}{x_q'' + 2\,x_0} + \frac{x_q''\,x_0}{x_q'' + 2\,x_0}\right) = \frac{3\,i_a'\,x_q''\,x_0}{x_q'' + 2\,x_0} \qquad \text{(C-14)}$$

But e_a is equal to e, the circuit breaker recovery voltage, and $i_a' = k_s\frac{E}{x_d''}$, where E is the voltage existing before short circuit at the point under consideration.

Hence

$$e = k_s\,E\,\frac{3\,x_q''\,x_0}{x_d''(x_q'' + 2\,x_0)} \qquad \text{(C-15)}$$

This equation may be written

$$e = k_s \cdot k_g \cdot k_q \cdot E, \qquad \text{(C-16)}$$

Where

$$k_g = \frac{3\,x_0}{s_q'' + 2\,x_0} \qquad \text{(C-17)}$$

and

$$k_q = \frac{s_q''}{s_d''} \qquad \text{(C-18)}$$

The substitution of s_q'' and s_d'' for x_q'' and x_d'' respectively in Equations (C-17) and (C-18) makes these equations suitable for use at any point on a transmission system or network.

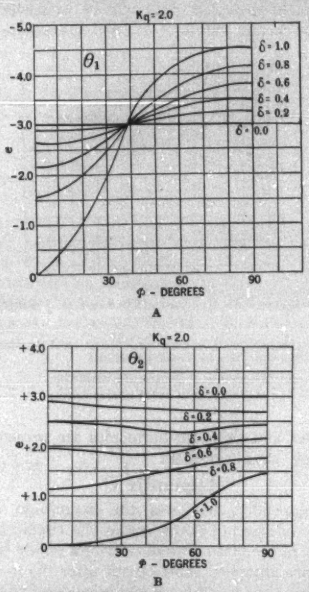


FIG. 29—VALUES OF RECOVERY VOLTAGE AS A FUNCTION OF ϕ AND δ FOR A THREE-PHASE SHORT CIRCUIT FOR WHICH $k_q = 2.0$

ϕ = Displacement angle between the phase axis and the direct axis of the rotor at the instant of short circuit

$\delta = \frac{\text{Per cent displacement at the time of interruption}}{\text{Per cent displacement of initial current wave}}$

The use of E as the voltage existing before short circuit neglects the effect of load current flowing previous to short circuit. The effect of load current is usually quite small, but it may be approximately taken into account by increasing the value of E in the ratio of the vector sum of load current and short-circuit current to the short-circuit current.

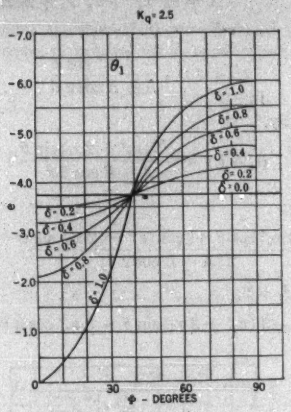
Appendix D

This appendix calculates the magnitude of the recovery voltage associated with the first phase to clear

of a two-phase-to-ground short circuit after the armature transients have died away.

The method of calculation is as follows:

1. The flux linkages for the two short circuited phases are set equal to zero, Park and Robertson's equations for these flux linkages being used. This gives two simultaneous equations which may be solved for the two currents.



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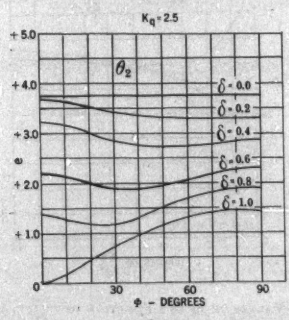

B

FIG. 30—VALUES OF RECOVERY VOLTAGE AS A FUNCTION OF ϕ AND δ FOR A THREE-PHASE SHORT CIRCUIT FOR WHICH $k_q = 2.5$

ϕ = Displacement angle between the phase axis and the direct axis of the rotor at the instant of short circuit

$\delta = \dfrac{\text{Per cent displacement at the time of interruption}}{\text{Per cent displacement of initial current wave}}$

2. From the current equations, the position of the rotor at the time when the current is zero is determined, and also the slope of the current wave at the zero.

3. A negative current is assumed to flow in one phase, equal and opposite to the short-circuit current there and the current induced in the other phase by this negative current is calculated by setting the flux linkages in this other phase equal to zero.

4. The flux linkages in the first phase due to the negative current in that phase and the induced current in the other phase are then calculated. The expression for flux linkages is then differentiated and values of angles are substituted for the time when the negative current is equal to zero. This gives the voltage occurring across the first phase to clear at the instant of interruption.

Park and Robertson give the following expressions for the flux linkages of phases b and c in terms of the instantaneous currents flowing in all circuits of the machine:

$$\psi_b = I_d \cos(\theta - 120°) - I_q \sin(\theta - 120°) - x_0 \frac{i_a + i_b + i_c}{3} - \frac{x_d + x_q}{3}\left(i_b - \frac{i_c + i_a}{2}\right) - \frac{x_d - x_q}{3}\left[i_a \cos(2\theta - 120°) + i_b \cos(2\theta + 120°) + i_c \cos 2\theta\right] \quad \textbf{(D-1)}$$

$$\psi_c = I_d \cos(\theta + 120°) - I_q \sin(\theta + 120°) - x_0 \frac{i_a + i_b + i_c}{3} - \frac{x_d + x_q}{3}\left(i_c - \frac{i_a + i_b}{2}\right) - \frac{x_d - x_q}{3}\left[i_a \cos(2\theta + 120°) + i_b \cos 2\theta + i_c \cos(2\theta - 120°)\right] \quad \textbf{(D-2)}$$

In these equations,

I_d and I_q are the field currents in the direct and quadrature axes respectively,

i_a, i_b, and i_c are the three phase currents,

x_0 is the zero phase-sequence reactance,

x_d and x_q are the machine reactances in direct and quadrature axes respectively, and θ is the angle from the axis of phase a to the direct axis at the rotor, being considered positive when the direct axis of the rotor is displaced from the axis of phase a in the direction of normal phase rotation.

Normal phase rotation is a, b, c.

In the work that follows, subtransient reactances will be used, so that it will be unnecessary to consider the current variations in the rotor.

For a two-phase-to-ground short circuit on phases b and c at normal excitation,

$$\psi_b = \psi_c = I_q = i_a = 0$$

and

$$I_d = 1.$$

Making these substitutions, Equations (D-1) and (D-2) become, respectively,

$$0 = \cos(\theta - 120°) - x_0 \frac{i_b + i_c}{3} - \frac{x_d'' + x_q''}{3}\left(i_b - \frac{i_c}{2}\right) - \frac{x_d'' - x_q''}{3}\left[i_b \cos(2\theta + 120°) + i_c \cos 2\theta\right] \quad \textbf{(D-3)}$$

$$0 = \cos(\theta + 120°) - x_0 \frac{i_b + i_c}{3} - \frac{x_d'' + x_q''}{3}\left(i_c - \frac{i_b}{2}\right)$$

$$- \frac{x_d'' - x_q''}{3}[i_b \cos 2\theta + i_c \cos(2\theta - 120°)] \quad \text{(D-4)}$$

Adding Equations (D-3) and (D-4):

$$0 = -\cos\theta - \frac{2x_0}{3}(i_b + i_c) - \frac{x_d'' + x_q''}{6}(i_b + i_c)$$

$$+ \frac{x_d'' - x_q''}{3}[i_b \cos(2\theta - 120°) + i_c \cos(2\theta + 120°)] \quad \text{(D-5)}$$

Subtracting Equation (D-4) from Equation (D-3):

$$0 = \sqrt{3}\sin\theta - \frac{x_d'' + x_q''}{2}(i_b - i_c)$$

$$- \frac{x_d'' - x_q''}{3}[\sqrt{3}\, i_b \cos(2\theta + 150°) - \sqrt{3}\, i_c \cos(2\theta - 150°)] \quad \text{(D-6)}$$

or, $$0 = -\cos\theta - \frac{2x_0}{3}(i_b + i_c) - \frac{x_d'' + x_q''}{6}(i_b + i_c)$$

$$- \frac{x_d'' - x_q''}{6}[(i_b + i_c)\cos 2\theta - (i_b - i_c)\sqrt{3}\sin 2\theta] \quad \text{(D-7)}$$

and $$0 = \sqrt{3}\sin\theta - \frac{x_d'' + x_q''}{2}(i_b - i_c)$$

$$+ \frac{x_d'' - x_q''}{2\sqrt{3}}(i_b + i_c)\sin 2\theta + \frac{x_d'' - x_q''}{2}(i_b - i_c)\cos 2\theta \quad \text{(D-8)}$$

Segregating terms involving $(i_b + i_c)$ from those involving $(i_b - i_c)$:

$$(i_b + i_c)\left[-\frac{2x_0}{3} - \frac{x_d'' + x_q''}{6} - \frac{x_d'' - x_q''}{6}\cos 2\theta\right]$$

$$+ (i_b - i_c)\frac{x_d'' - x_q''}{2\sqrt{3}}\sin 2\theta = \cos\theta \quad \text{(D-9)}$$

$$(i_b + i_c)\left[\frac{x_d'' - x_q''}{2\sqrt{3}}\sin 2\theta\right] + (i_b - i_c)\left[-\frac{x_d'' + x_q''}{2}\right.$$

$$\left. + \frac{x_d'' - x_q''}{2}\cos 2\theta\right] = -\sqrt{3}\sin\theta \quad \text{(D-10)}$$

Solving for $(i_b + i_c)$:

$$(i_b + i_c)\left\{\left[-\frac{2x_0}{3} - \frac{x_d'' + x_q''}{6} - \frac{x_d'' - x_q''}{6}\cos 2\theta\right]\right.$$

$$\left.\left[-\frac{x_d'' + x_q''}{2} + \frac{x_d'' - x_q''}{2}\cos 2\theta\right] - \frac{(x_d'' - x_q'')^2}{12}\sin^2 2\theta\right\}$$

$$= \cos\theta\left[-\frac{x_d'' + x_q''}{2} + \frac{x_d'' - x_q''}{2}\cos 2\theta\right] + \frac{x_d'' - x_q''}{2}\sin 2\theta \quad \text{(D-11)}$$

Simplifying:

$$(i_b + i_c)\frac{A - B\cos 2\theta}{3} = -x_q''\cos\theta \quad \text{(D-12)}$$

where $A = x_d'' x_q'' + x_q'' x_0 + x_0 x_d''$

and $B = x_0(x_d'' - x_q'')$

and $$(i_b + i_c) = \frac{-3x_q''\cos\theta}{A - B\cos 2\theta} \quad \text{(D-13)}$$

Similarly,

$$(i_b - i_c) = \frac{\sqrt{3}(x_q'' + 2x_0)\sin\theta}{A - B\cos 2\theta} \quad \text{(D-14)}$$

Adding,

$$i_b = \frac{-3x_q''\cos\theta + \sqrt{3}(x_q'' + 2x_0)\sin\theta}{2(A - B\cos 2\theta)} \quad \text{(D-15)}$$

When $i_b = 0$,

$$\sqrt{3}(x_q'' + 2x_0)\sin\theta = 3x_q''\cos 2\theta \quad \text{(D-16)}$$

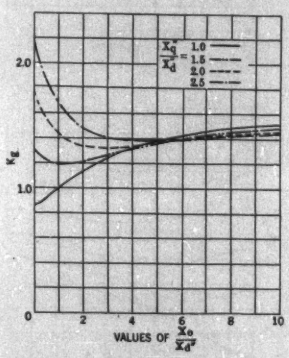

FIG. 31—VALUES OF k_g AS A FUNCTION OF $\frac{x_q''}{x_d''}$ AND $\frac{x_0}{x_d''}$ FOR A TWO-PHASE-TO-GROUND SHORT CIRCUIT

or $$\tan\theta = \frac{\sqrt{3}\, x_q''}{x_q'' + 2x_0} \quad \text{(D-17)}$$

Also,

$$\frac{di_b}{d\theta} = \frac{3x_q''(A - B\cos 2\theta)\sin\theta + \sqrt{3}(A - B\cos 2\theta)(x_q'' + 2x_0)\cos\theta + 6Bx_q''\sin 2\theta\cos\theta - 2\sqrt{3}(x_q'' + 2x_0)B\sin 2\theta\sin\theta}{2(A - B\cos 2\theta)^2} \quad \text{(D-18)}$$

Substituting the following, which are obtained from (D-17),

$$3\,x_q'' = 2\sqrt{3}\,\sqrt{x_q''^2 + x_q''\,x_0 + x_0^2}\,\sin\theta \qquad \text{(D-19)}$$

$$x_q'' + 2\,x_0 = 2\,\sqrt{x_q''^2 + x_q''\,x_0 + x_0^2}\,\cos\theta \qquad \text{(D-20)}$$

we have

$$\frac{d\,i_b}{d\,\theta} = \frac{\sqrt{3}\,\sqrt{x_q''^2 + x_q''\,x_0 + x_0^2}}{A - B\cos 2\,\theta} \qquad \text{(D-21)}$$

To determine the current, i_c' in c induced by the negative current, i_b' in b, we set ψ_c equal to zero.

$$0 = -\frac{x_0\,(i_b' + i_c')}{3} - \frac{x_d'' + x_q''}{3}\left(i_c' - \frac{i_b'}{2}\right)$$

$$-\frac{x_d'' - x_q''}{3}\,[i_b'\cos 2\,\theta + i_c'\cos(2\,\theta - 120°)] \qquad \text{(D-22)}$$

or

$$i_c'\left[-x_0 - x_d'' - x_q'' - \frac{x_d'' - x_q''}{2}(-\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)\right]$$

$$= i_b'\left[x_0 - \frac{x_d'' + x_q''}{2} + \frac{x_d'' - x_q''}{2}\cos 2\,\theta\right] \qquad \text{(D-23)}$$

or $i_c' =$

$$-i_b'\,\frac{x_0 - \dfrac{x_d'' + x_q''}{2} + \dfrac{x_d'' - x_q''}{2}\cos 2\,\theta}{x_0 + x_d'' + x_q'' + \dfrac{x_d'' - x_q''}{2}(-\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)} \qquad \text{(D-24)}$$

Substituting Equation (D-24) in Equation (D-1)

$$\psi_b = i_b'\left[-\frac{x_0}{3}\,\frac{\dfrac{3\,(x_d'' + x_q'')}{2} + \dfrac{(x_d'' - x_q'')}{2}(-3\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)}{x_0 + x_d'' + x_q'' + \dfrac{(x_d'' - x_q'')}{2}(-\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)}\right.$$

$$-\frac{x_d'' + x_q''}{6}\,\frac{3\,x_0 + \dfrac{3}{2}(x_d'' + x_q'') + \sqrt{3}\,(x_d'' - x_q'')\sin 2\,\theta}{x_0 + x_d'' + x_q'' + \dfrac{x_d'' - x_q''}{2}(-\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)}$$

$$+\frac{x_d'' - x_q''}{6}\,\frac{3\,x_0 + \dfrac{x_d'' - x_q''}{2}(3\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)}{x_0 + x_d'' + x_q'' + \dfrac{x_d'' - x_q''}{2}(-\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)}\cos 2\,\theta$$

$$\left.+\frac{x_d'' - x_q''}{2\sqrt{3}}\,\frac{x_0 + x_d'' + x_q'' + \dfrac{x_d'' - x_q''}{2}(-\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)}{x_0 + x_d'' + x_q'' + \dfrac{x_d'' - x_q''}{2}(-\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)}\sin 2\,\theta\right]$$

$$= -\,i_b'\,\frac{A - B\cos 2\,\theta}{x_0 + x_d'' + x_q'' + \dfrac{x_d'' - x_q''}{2}(-\cos 2\,\theta + \sqrt{3}\sin 2\,\theta)} \qquad \text{(D-25)}$$

But, from (D-17),

$$-\cos 2\,\theta + \sqrt{3}\sin 2\,\theta$$

$$= \frac{3\,x_q''^2 - x_q''^2 - 4\,x_q''\,x_0 - 4\,x_0^2 + 6\,x_q''^2 + 12\,x_q''\,x_0}{3\,x_q''^2 + x_q''^2 + 4\,x_q''\,x_0 + 4\,x_0^2}$$

$$= \frac{2\,x_q''^2 + 2\,x_q''\,x_0 - x_0^2}{x_q''^2 + x_q''\,x_0 + x_0^2} \qquad \text{(D-26)}$$

Substituting Equation (D-26) in Equation (D-25) and clearing of fractions in the denominator:

$$\psi_b = -i_b'\,\frac{2\,(A - B\cos 2\,\theta)\,(x_q''^2 + x_q''\,x_0 + x_0^2)}{x_d''\,(2\,x_q'' + x_0)^2 + x_0\,(2\,x_q'' + x_0)\,(x_q'' + 2\,x_0)} \qquad \text{(D-27)}$$

Differentiating, at the current zero,

$$k_g = \frac{d\,\psi_b}{d\,t}$$

$$= -\frac{d\,i_b'}{d\,t}\,\frac{2\,(A - B\cos 2\,\theta)\,(x_q''^2 + x_q''\,x_0 + x_0^2)}{x_d''\,(2\,x_q'' + x_0)^2 + x_0(2\,x_q'' + x_0)(x_q'' + 2\,x_0)}$$

(the second term of the differential is omitted, since $i_b' = 0$)

$$= \frac{d\,i_b}{d\,t}\,\frac{2\,(A - B\cos 2\,\theta)\,(x_q''^2 + x_q''\,x_0 + x_0^2)}{x_d''\,(2\,x_q'' + x_0)^2 + x_0\,(2\,x_q'' + x_0)\,(x_q'' + 2\,x_0)}$$

$$= \frac{2\sqrt{3}\,(x_q''^2 + x_q''\,x_0 + x_0^2)^{3/2}}{x_d''\,(2\,x_q'' + x_0)^2 + x_0\,(2\,x_q'' + x_0)\,(x_q'' + 2\,x_0)} \qquad \text{(D-28)}$$

Equation (D-28) may be applied to any point on a system by using the appropriate values of s_d'', s_q'', and x_0 in place of x_d'', x_q'', and x_0 respectively. The value of x_e can, of course, be adjusted to take into account the effect of neutral reactors.

Values of k_e are plotted in Fig. 29 for values of $\frac{x_q''}{x_d''}$ from 1.0 to 2.5 and for values of $\frac{x_0}{x_d''}$ from 0 to 10.

The recovery voltage of phase c, upon the assumption that that phase opens first, may be found by a process similar to that here used for phase b. It will be found to be identical with that of phase b.

Discussion

H. M. Wilcox: A similar investigation as that described has been under way for some time in the organization with which the writer is associated and it may be said that the conclusions drawn as a result of this investigation conform very closely to the conclusions reached in this paper. One or two points of variance are noted which may be worthy of discussion.

Reference is made in the paper to a series of current rupturing tests made on a standard testing circuit, in which the voltage to ground which could be interrupted by a single pair of contacts was approximately doubled by introduction of a shunt resistance of 1,000 ohms across the circuit breaker terminals. From the nature of the circuit used, it seems probable that this resistance may have been in the form of a water rheostat involving relatively large capacity to ground—large as compared with the capacity effect in the arc-drawing space of the circuit breaker. Tests in which the writer participated, using a shunt resistance of this nature, indicated that a value of 250 to 400 ohms would have secured the same result, and led to the conclusion that the introduction of additional capacity to ground was responsible for the improved performance to a far greater degree than the actual ohmic value of the resistance used. Other tests bearing out this conclusion indicated that the distribution of voltage across the two contact breaks of a single-pole oil circuit breaker could be varied over a wide range by introducing additional capacity effect on one or the other terminal, and it was found possible on these tests to throw so large a proportion of the impressed voltage across one of two contact breaks that the circuit breaker interrupted very nearly as much voltage on one break, the other being shunted with a conductor, as when both breaks were available for rupturing purposes. From statements made later in the paper, it appears that the authors themselves are fully aware of the significance of capacity to ground in the circuit and that the apparent difference of opinion is a matter only of misinterpretation of the text.

Again reference is made to a series of interrupting tests on an operating system at Wissota Station. The test circuit in this case partook largely of the nature of a laboratory circuit in that the test circuit breaker was located close to the generator, practically no transmission line being involved. The "overshooting" in voltage referred to undoubtedly accounts for the increase in duration of arcing noted on these tests as compared with a similar series of tests at Riverside station, involving 43 mi. of 110-kv. transmission line, but the authors leave the impression that these high frequency voltages were due to certain characteristics of the water-wheel generator at Wissota Station. In the lack of any parallel system to absorb oscillations, peculiarities of the generator would without doubt produce an effect on the voltage wave of the test circuit, but in the light of other data it seems logical to conclude that the capacity to ground, unquestionably present in the 43-mi. transmission line at Riverside and largely absent in the Wissota test circuit, is sufficient to account for the absence of "over-shooting" in the one case and its presence in the other. In the standard laboratory testing circuit with which the writer is familiar, fed by a 40,000-kv-a. motor-driven generator, records taken with a magnetic oscillograph show overshooting up to 125 per cent above normal frequency restored voltage. These voltage peaks rise abruptly at each current zero after the arc has reached an appreciable length and have been observed to continue for a number of cycles before final interruption. They appear to be of the nature of a high frequency voltage superimposed over the normal frequency wave and show oscillations of the order of 2,000 to 3,000 cycles, conforming very closely to the calculated natural period of a circuit of this nature. A series of short-circuit tests was made with a three-pole 15-kv. oil circuit breaker in the 250,000-kv-a. rupturing class, using this generator, and later a series of tests was made with the same circuit breaker on a large 12,000-volt cable system, over the same range of short-circuit currents. The restored voltage was substantially the same on both sets of tests but the maximum durations of arcing noted on the laboratory circuit were 40 per cent in excess of the maximum durations recorded on the field tests. In this case the variation in performance between the two series of tests was accounted for by the relatively large difference in capacity effect between the two circuits. Other statements in the paper lead to the belief that possibly the authors adhere to this conclusion also and that the apparent difference in opinion is again due to misinterpretation of the text.

From the standpoint of the circuit breaker designer, these fast-rising, overshooting voltages appearing within a few microseconds after a current zero, are of particular interest since they are of the voltages with which his circuit breaker must contend when interrupting short-circuit currents and in general are the only voltages the circuit breaker knows anything about. His design must be such that the rate of increase of dielectric strength in the arc path after a current zero will be greater than the rate of rise of impressed voltage if his circuit breaker is to function satisfactorily. It is not a matter of approaching an ideal; it is a matter of attaining that ideal or not having a circuit breaker. Various ways are open for him to secure this result. He may insert resistance across his breaker if he wishes—and sell it if he can—but, preferably, he will decide to incorporate in his design an interrupting medium which will be adequate to cope with any rate of rise of impressed voltage that may be encountered in a practical circuit. It may be said that the circuit interrupters discussed by the writer before the Institute in the last two years, were developed on a test circuit which showed voltage characteristics of the nature described in this paper and they have proven themselves adequate to interrupt such circuits satisfactorily.

D. C. Prince: The tests conducted by the Northern States Power Company called definite attention to the difference in the behavior of switches operating directly at generator terminals and connected to long lines.

An even more complete comparative test was afforded by tests made by the American Gas & Electric Co. at Philo, Ohio, between June 30th and July 6th, 1930. In these tests it was possible to connect varying lengths of transmission line to the circuit breaker under test and so determine the effect of different lengths of line. The two upper curves in Fig. 1 give the arc length in inches per break of a General Electric explosion-chamber breaker at two different values of current and over a considerable range of transmission line length. The curves are plotted in terms of calculated recovery time; that is, elapsed time from the current zero to the peak reestablishing voltage of the system. In approximate terms the recovery time is of the order of 10 microseconds per mi. of line attached. It is observed that nearly 50 per cent reduction in arc length occurs as the connected line is increased up to 150 mi., no further gain being observed after that point has been reached.

In considering the future course of development indicated by these studies and tests three courses are open:

1. Two types of oil circuit breaker should be provided; a smaller, less expensive breaker for line switching, and a larger more expensive breaker for generator switching.

2. Generating apparatus should be modified to give lower recovery voltage rates.

3. A switch should be developed which is not sensitive to changes in recovery rate within the limits likely to be encountered.

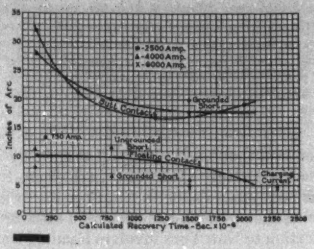

Fig. 1

Any further complexity in variety of oil circuit breakers is not likely to meet with favor. The second alternative means added expense which need not be incurred if the third alternative materializes. This third alternative seems on a fair way toward realization in the switch whose performance is recorded in the lower curve in Fig. 1.

W. F. Skeats: Mr. Wilcox in his discussion asks whether the 1,000-ohm resistor mentioned in connection with the tests described in the paper was a water box resistor and raises the question of the effect of the capacitance to ground of such a resistor. The resistor used was a combination 44 wire-wound, 90-ohm resistors arranged two in parallel and 22 in series, the entire assembly being hung in a vertical string near the breaker. It will be fairly obvious that the capacity to ground of such an assembly is very small even with respect to the capacitance of the remainder of the system.

In this connection Mr. Wilcox seems to be somewhat disturbed by the fact that some tests in which he participated indicated that a resistance of from 250 to 400 ohms would appear necessary in order to secure the results obtained from a resistance of 1,000 ohms in these tests. The comparative effect of any resistor depends probably to a very large extent upon the type of breaker upon which the tests are made and that while the data available on this point are far from complete, it is by no means the opinion of the authors that a resistance which will double the voltage at which one type of breaker will operate satisfactorily will have anything like the same result upon the performance of a breaker of any other type. Even for a given type of breaker, the comparative effect of a given ohmic value of the resistance is influenced by the voltage and current at which the breaker is operated, and also, to a very large extent, by the rate of rise obtaining before connection of the resistor. A relatively high value of resistance will be sufficient to produce a given percentage change on a very severe circuit, whereas a lower value of resistance must be used to produce the same percentage change on a milder circuit.

It would be expected that putting additional capacitance across one of the two breaks would affect the voltage distribution to such an extent that the voltage which a breaker would successfully interrupt with two breaks might be but very little greater than that which could be interrupted by a single break. It is very interesting, however, to learn that this effect has been tested and verified by Mr. Wilcox.

With reference to the tests on the Northern States Power Company system there were two factors contributing to the severity of the conditions at Wissota. One of these is the fact that the generators at Wissota are water-wheel machines, which are likely to have a much higher quadrature reactance factor than turbine-driven machines. This would increase the overshooting considerably, particularly as no other source of supply was connected to the bus and there was no external reactance. The other factor undoubtedly was, as Mr. Wilcox suggests, the large capacitance to ground present in the case of the Riverside tests but not in the Wissota tests. The capacitance to ground however does not influence the over-shoot, or magnitude of the recovery voltage peak, except to a very minor extent. What capacitance does do is to delay the rate of build-up of such voltage as does appear. The rate of voltage build-up appears to be fully as important as the magnitude of the voltage finally obtained.

Summarizing the content of the paper, the hitherto hidden phenomena of recovery voltage,—the puzzling harmonics which have sent the voltage vibrator of the oscillograph off the edge of the film and the mysterious forces which have caused the same breaker to require an arc length three times as great at one point of a system as at another point on the same system when voltage, current, and power factor were substantially the same,—are now subject to moderately precise calculation, and it can be predicted with some degree of accuracy what will be the performance at any point on any system, of any breaker whose characteristics are known.

Power Supply Facilities for Reading Suburban Electrification

BY C. L. DOUB*
Associate, A. I. E. E.

Synopsis.—*The Reading Company is now electrifying its Philadelphia suburban railroad service and facilities are planned so that the electrification may readily be extended to include through passenger and freight service between Philadelphia and New York City, Bethlehem, and Reading, Pennsylvania. Contract has been made with Philadelphia Electric Company for initial power requirements, and provisions are made for enlarging such supply to serve all of the above territory.*

The Railroad Company is constructing substation and transmission facilities to serve the suburban territory by a 12/24/36-kv., three-wire, single-phase, 25-cycle distribution system, with the view to serving the more remote area by superimposing 66-kv. transmission lines upon the initial system, at the time of future extension. The present and proposed system layout is described, together with the estimated power requirements and the manner in which the railroad system is to be served from the system of the power company.

THE present program of construction for electrification of Reading Company lines includes only suburban service in the vicinity of Philadelphia. This includes service to Lansdale, a distance of about 25 mi. along the Bethlehem Branch, to Langhorne, about 24 mi. along the New York Branch, to Hatboro, about 18 mi. along the New Hope Branch, to Chestnut Hill, 10.8 mi., and to Doylestown, 10 mi. from Lansdale and about 35 mi. from Reading Terminal.

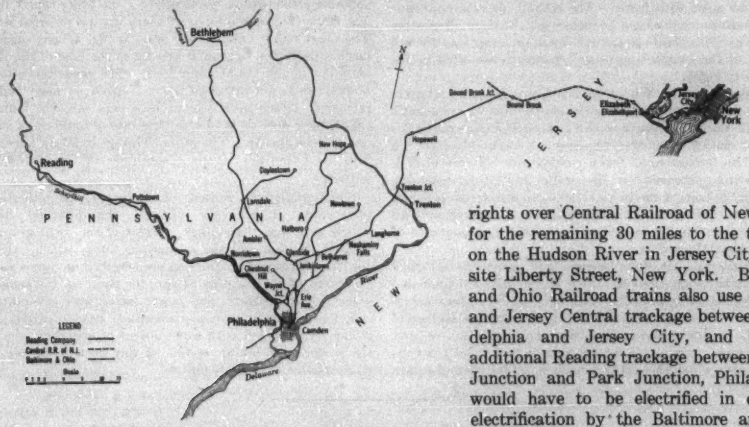

FIG. 1—PRINCIPAL READING COMPANY LINES IN TERRITORY FOR WHICH PLANS COVER POWER SUPPLY FROM PHILADELPHIA

SCOPE OF ELECTRIFICATION PLANS

All plans and designs, however, are made with the view to future extension for electric operation of through passenger and freight service. The Reading Company has announced that these plans will eventually include electrification to New York City, Bethlehem, Pa., and along the Schuylkill Valley to Reading, Pa. or beyond. Fig. 1 shows the physical arrangement of these branches. Electrification to New York involves Reading Company trackage only to Bound Brook, 60 mi. from Philadelphia, from which point this company has trackage rights over Central Railroad of New Jersey for the remaining 30 miles to the terminus on the Hudson River in Jersey City, opposite Liberty Street, New York. Baltimore and Ohio Railroad trains also use Reading and Jersey Central trackage between Philadelphia and Jersey City, and a small additional Reading trackage between Wayne Junction and Park Junction, Philadelphia, would have to be electrified in event of electrification by the Baltimore and Ohio to New York. The present suburban program includes approximately 60 route mi. and 130 mi. of track electrified, including yards, sidings, etc. An addition of the prospective through passenger electrification includes approximately 140 route miles and 380 track miles; and a further addition of freight electrification in the same area includes approximately 50 route miles and 490 track miles, making a total for all services of 250 route miles and about 1,000 track miles.

The Reading is well known as a heavy freight railroad, whose traffic density per mi. of track ranks near the top for all roads in the United States. It will be

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necessary to provide for electrification of a large number of heavy freight trains including a considerable proportion of coal trains averaging 4,200 trailing tons. The Reading passenger trains between Philadelphia and New York provide the highest type of passenger service and consistently run at speeds of 80 mi. an hr. or greater. The New York trains are of medium tonnage, carrying diners, chair cars, and club cars, but few sleeping cars on account of the relatively short distance. The Baltimore & Ohio trains using the same route include through equipment from the west and are of heavy weight. On the Bethlehem and Reading routes, the Reading provides through sleeping car service in conjunction with other railroads, and many of the passenger trains on these branches are of heavy weight.

All of the above, together with a heavy volume of Philadelphia suburban traffic and the addition of freight and passenger switching, etc., will include all forms of

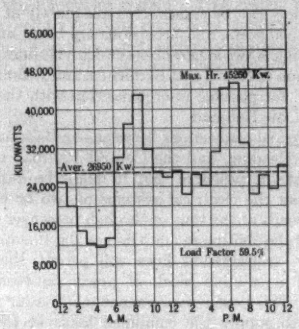

FIG. 2—ESTIMATED LOAD CURVE FOR ELECTRIFICATION BETWEEN PHILADELPHIA AND BOUND BROOK, BETHLEHEM, AND READING

Winter weekday traction load

electric railroad service, and the power system is being designed with all of these conditions in mind.

Calculations show that present loads will be about 9,000 kilowatts and 12,800 kv-a., average for maximum hr. Future loads on Reading trackage in the area which may be fed from Philadelphia will involve maximum-hr. loads of about 45,000 kilowatts and 65,000 kv-a. The load factor of the initial suburban service is typical of that of most large cities, and will be nearly 30 per cent. The addition of freight and through passenger trains, many of which are run throughout the night, raises the estimated load factor to about 60 per cent. The future load curve is shown by Fig. 2.

For the electrification of the Reading, the single-phase a-c. system was chosen, with nominally 11,000 volts on the overhead contact wires. The choice was based principally upon promoting system standardization in this territory and at the same time providing adequately for heavy freight and through passenger as well as suburban traffic. Actual design is for 12,000 volts upon the trolley.

Main Point of Power Supply. Due to the network of radiating branches from Philadelphia, it was important to choose a main point of power supply which would be convenient in serving the various lines as well as one which would be economical and readily accessible. It was early determined that in case of power being purchased, this should be at Wayne Junction, Philadelphia, which is the junction of several lines, and where the railroad had a considerable amount of property which would be available for the development of adequate system power facilities. This location is five miles from Reading Terminal, where the general offices of the company are located, together with division operating and dispatching headquarters. Negotiations with the Philadelphia Electric Company showed that this would be a convenient point of power supply for that company, being within about two miles of their Westmoreland substation, which has been developed as the Philadelphia center of the electric company's transmission and distribution system. Studies and estimates were also made on the basis of the railroad company building and operating a generating station.

The result of studies and negotiations was that the Philadelphia Electric Company would provide at Wayne Junction a 60- to 25-cycle frequency changing station to be owned and operated by that company and to supply single-phase 25-cycle power at 13,200 volts to the Reading Company at a single point. This voltage is a standard generator voltage, and must in any event be transformed by the railroad company. A contract was signed early in 1930 providing for an initial load of 9,200 kilowatts at a minimum (without penalty) of 65 per cent power factor, and 30 per cent load factor. The rates were established for initial service on the customary basis of primary or demand charge to apply to monthly maximum demand in kilowatts, and secondary or energy charge to apply to actual kilowatt hours consumed. The secondary charge is subject to correction based upon the actual cost of coal to the Philadelphia Electric Company and the actual cost of generation of hydroelectric power, properly weighed as to the proportion of steam and water power produced. The contract is for a term of 20 years, and includes provision that when the railroad company loads shall reach a predetermined value in kilowatts and load factor, which would result only from through passenger and freight electrification, charges shall be determined upon a "cost plus" basis.

The electric company determined upon a frequency changer unit of 15,000 kilowatts at 70 per cent power factor as the proper unit for installation, and two such sets will be installed in the initial station. These sets will be for outdoor installation, and will be the first of such installations in the country.

The layouts of the Philadelphia Electric and the Reading Company facilities are both made so that the Wayne Junction supply may be enlarged to a total of six such units with a capacity of 90,000 kilowatts and 128,000 kv-a. The frequency changer station will be fed by underground 13,200-volt, 60-cycle, three-phase cables from the Westmoreland Station, and duct lines with total capacity of six 18,000-kv-a. circuits are being constructed between Westmoreland and Wayne Junction, a distance of about 2 miles. Independent cables will be run for each present and future frequency changer unit, and will be fed from the duplicate 13,200-volt, 60-cycle buses at Westmoreland Station. This station is the Philadelphia terminus of the transmission lines from the recently completed Conowingo hydroelectric generating station of the Philadelphia Electric Company, and in addition to being served by lines from Conowingo (via Plymouth Meeting Substation), is also served by 66,000-volt underground transmission lines direct from Richmond and Schuylkill Stations and 13,200-volt underground lines from Delaware Station, all of which are steam generating stations of this Company in Philadelphia. The transmission lines from Conowingo, moreover, tie at Plymouth Meeting Substation into the 220-kv. ring which interconnects the power systems of the Pennsylvania Power and Light Company and the Public Service Corporation of New Jersey with that of the Philadelphia Electric Company, all of which have been described completely by Institute papers. It is thus apparent that the supply of power to the Wayne Junction Substation will be of the highest degree of reliability.

The Reading Company will construct its main substation adjacent to the frequency changer station, including transformers to step voltage up to the three-wire distribution system, and also step-up transformers for future transmission lines, together with a switching station for the trolley lines radiating from this station and headquarters for the power supervisor and load dispatchers. Cables will be run from each frequency changer set directly to generator breakers installed in the Reading Company Substation and then to the 13,200-volt 25-cycle bus provided by the Reading Company. This bus as temporarily installed will be made up of three sections, one transformer being fed from each, and the two frequency changer sets being connected to the two end sections. The bus is designed to be ultimately a six-section ring bus, each section to have one 15,000-kilowatt generator (21,400-kv-a.) and two 8,000-kv-a. step-up transformers. Adequate provisions are made for bus sectionalizing breakers together with current limiting reactors and reactor tie breakers. The bus will be installed initially as a single-phase bus, but provision is made to change this to a three-phase bus if it should be found that this is desirable for synchronizing purposes. Present calculations indicate that although motors will be paralleled through two miles of underground 13,200-volt cable, the machines will be sufficiently stable in operation that a three-phase tie will not be required on the generator sides to keep the machines in step.

Bus tie breakers will be owned and maintained by the railroad company, but will be controlled by the Philadelphia Electric Company operator. Generator breakers will be owned and operated by the electric company and transformer breakers will be owned and operated by the Reading Company. This plan is based upon the principle of the power company being responsible for maintaining an adequate supply of power on the bus, and the railroad company operator handling the power taken from the bus, considering the bus as an infallible source of supply. It will of course be necessary to arrange duplicate blocking of breakers when employees of either company work upon the bus or apparatus connected directly thereto.

Provisions for Transmission Lines. The Reading Company load calculations show that the ultimate power supply can be transmitted adequately at a voltage of 66,000 volts, and provisions are being made for the extension of two single-phase 66-kilovolt lines each in the direction of New York, Bethlehem and Reading, and southward for possible interconnection with the Baltimore and Ohio Railroad.

The physical connections between the Baltimore and Ohio, the Reading, and the Central Railroad of New Jersey make it probable that it might be desired to interconnect the electrical systems of two or more of these railroads at the time of through passenger electrification of the Baltimore and Ohio or the Reading, and the 66-kv. transmission lines will permit the interchange of a suitable block of power between these systems. This power requirement may be for normal purposes to supply the combined loads in the most reliable and economical manner, or for emergency supply when there is a deficiency of power from other normal connections. The layouts of the Philadelphia Electric frequency changer station and the railroad company main substation at Wayne Junction are both being made in such a way as to allow expansion for such purposes.

In addition to supplying power from Wayne Junction for the Reading Lines as far as Bound Brook, Bethlehem, and Reading, which cover a radius of approximately 60 miles from Philadelphia, it may be feasible to purchase power at other points near the extremities of these branches if this should prove advantageous. In this case, the 66-kv. transmission lines from Wayne Junction to such branches may or may not be run, as desired.

It will thus be seen that in the planning of the Reading Company power system for electrification, flexibility of facilities has been kept foremost so that there will be no limitations from the standpoint of power supply in the electrification of any or all of the Reading Company lines out of Philadelphia. Since the construction to Reading is not a part of the present program, there will remain further possible flexibility at the time of doing

such construction in providing arrangements for extensions beyond Reading into Central Pennsylvania, in the event it should prove desirable at that time to plan furnishing this power from Philadelphia. The distances are such, however, that it may be more economical to obtain additional power supply from some of the utilities in that territory or from a generating station located by the Reading Company itself in the mining region.

Three-Wire Distribution System. An early study of the economies and advantages of two-wire and three-wire (single-phase) transmission and distribution systems of various voltages led to the conclusion that the three-wire system having a voltage of 36 kv. between outside lines would have great flexibility for the Philadelphia area, would permit very simple and frequent substations, and would moreover be more economical in first cost and maintenance cost, even though expanded in the future by 66-kv. transmission lines paralleling the three-wire system. This system of distribution is used by the New York, New Haven and Hartford Railroad with 22 kv. between outside conductors and is used in a somewhat different arrangement by the Virginian Railway. Since the system is not altogether familiar to most engineers, the accompanying diagram (Fig. 3) will illustrate the typical connections. The trolley lines and rail connections are constructed in exactly the same manner as for the ordinary system. The operation of the third line, designated the "feeder," from the same single-phase transformer, provides a voltage between trolley and feeder of 36,000 volts which serves as a transmission line to outlying substations. There is thus a voltage of only 24 kv. between feeder and rail, and the insulation of the feeder conductors may be based upon this lower voltage.

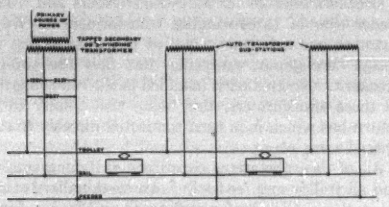

FIG. 3—TYPICAL THREE-WIRE SYSTEM CONNECTIONS

Some advantages of the three-wire distribution system are as follows:

(a) A transmission circuit is obtained by the addition of only one wire, and this wire is at a relatively low voltage to ground, as explained above.

(b) Substation transformers are simple and economical. The step-down transformers at outlying stations are merely auto-transformers, which are relatively small in size, and connections are very simple. It may be noted that a 12/24/36-kv. 2,000-kv-a. auto-transformer is physically and electrically the same as an ordinary 12/24-kv. 1,333-kv-a. two-winding transformer when one side of the primary is connected to the proper side of the secondary. The three-wire distribution system is in principle no different from the three-wire 110/220-volt distribution commonly used for either direct current or single-phase alternating current.

(c) Substation circuit breaker and switching equipment for transmission lines and transformers are reduced in number and voltage rating, thus permitting the use of frequent substations with reasonable economy. For most railroads, switching stations are in any event required at frequent intervals for trolley lines, and these may be extended in a simple manner to include auto-transformers and feeder switching equipment. The frequent installation of transformers promotes economy in distribution similar to that which prompted the frequent installation of synchronous converter substations for direct current interurban lines after the adoption of the automatic converter station. The frequent installation of transformers also reduces the amount of current flowing in the rail and earth return circuit, and thus reduces induction in nearby circuits.

(d) Auto transformers can be designed with a lower reactance than equivalent two-winding step-down transformers, and the transmission circuit consequently be made to furnish a larger proportion of the power than otherwise.

(e) Main supply transformers, such as indicated in Fig. 3 as the tapped secondary transformer, may be designed for various proportions of reactance between primary, trolley-rail, and trolley-feeder circuits, and distribution of current may be thus regulated within certain limits. This distribution of current may then be proportioned so as to give such resultant currents in the rail and earth return circuits as to reduce inductive troubles.

(f) The feeder being a single wire, and of relatively low voltage, a certain amount of flexibility may be obtained in its location with respect to the trolley and rail circuits, so as to tend to balance inductive effects. These effects are of course dependent upon the proportion of current flowing in the three conductors of the circuit, and must be calculated for the network under consideration.

The principal disadvantages of the three-wire distribution system are that it requires more painstaking calculation than the ordinary distribution system, and that interruption of all lines in the trolley circuit also constitutes an interruption in the transmission line for all stations beyond that point, excepting such power as may be fed through the feeder and rail conductors at lower voltage than the normal trolley-feeder transmission. The latter point must be given careful consideration in the design of substations, and in the protective measures employed.

Calculations and estimates show that for the Reading

lines in the Philadelphia area the three-wire distribution system is very suitable for transmission within a radius of approximately 25 miles but that it would not be economical, and voltage regulation would not be satisfactory, for the starting of heavy trains beyond this radius. The addition of transmission lines to points beyond will therefore adequately take care of the larger territory, and this territory may be served from one or more central points through similar three-wire distribution layouts fed from stations with step-down tapped-secondary transformers. The 66-kv. transmission line would thus become a very simple line between Wayne Junction and one outlying point on each branch, requiring the higher voltage circuit breakers at only the terminal points of the transmission lines. Manual or remote-control switching apparatus for sectionalizing and cross-tying the transmission lines at intermediate points may or may not be added, as desired. The flexibility of the layout will, however, permit any or all substations along the route to be fed directly from the 66-kv. transmission lines by installing tapped-secondary transformers instead of auto-transformers, and adding the required 66-kv. switching equipment. The most important present stations are so laid out that tapped-secondary transformers may replace the auto-transformers originally installed, by making very simple changes and additions to the stations. This would occur only during some expansion of the electrification, at which time there would be need of additional auto-transformer capacity at other substations.

Circuit Breaker Protection. It was decided at an early date to use the high-speed type of circuit breaker in all trolley circuits. Very satisfactory high-speed circuit breakers had been developed for 11,000-volt trolley lines by two American electrical manufacturers, and both types of breaker had been in heavy duty service for a considerable length of time. One available type was an oil circuit breaker, and the other was an air type breaker, but both have high ratings, and both perform very heavy rupturing duty for a number of duty cycles many times that generally specified for normal speed oil circuit breakers. Both breakers demonstrated speeds of automatic tripping in from one-half to one cycle of a 25-cycle wave, and while mechanisms were radically different, results obtained were very similar. Their use decreased frequency of inspection and maintenance, minimized burning of trolley lines and equipment due to short circuits, and greatly reduced inductive effects from the railway system, but the cost of either type of available breaker was very much higher than for the corresponding normal speed circuit breaker.

The General Electric type JRA-32 air-break high-speed circuit breaker was selected for this service with a rating of 1,500 amperes continuously at 25 cycles, 15,000 volts. It has an interrupting rating (single pole) of 50,000 amperes at 12,000 volts, and is specified to perform 50 "OCO" duty cycles consecutively at two minute intervals. Its principle of tripping on the short circuits is the so-called impulse tripping, depending upon the change of 25-cycle current from one-half cycle to the next. This scheme will be essentially the same as described in a paper by Mr. J. W. McNairy, A. I. E. E. JOURNAL, October 1928. "Back-up" protection of high-speed impedance type relays is provided to take care of slowly increasing overloads and trolley-feeder faults.

Circuit breakers selected for feeder circuits are Westinghouse type GO-2, 46-kv., 800 amperes continuous rating at 25 cycles, and 500,000-kv-a. interrupting capacity (three-phase basis). They are provided with the recently developed deion grid break as described in a paper by Messrs. Baker and Wilcox presented before the Institute. Although of the so-called normal speed type, it is expected that the deion grid type of break will give speeds of interruption materially faster than ordinary oil circuit breakers. A breaker of 46-kv. rating was chosen in order to provide a sufficiently high value of insulation to compare with line insulation used on these circuits, and also to provide ability to interrupt 36-kv. trolley-feeder faults.

Breakers selected for auto-transformers and trolley-feeder side of three-winding transformers at Wayne Junction are also type GO-2 of the same rating and design throughout, excepting that they are two-pole breakers. No breaker is installed in the rail connection to these transformers, this being tied solidly to the return bus which is in turn connected directly to rails, ground wires, etc.

All of the substations constitute switching stations, and all trolley and feeder lines are sectionalized at each substation. Circuits for each track and feeder circuits in duplicate are fed separately in each direction from every substation by single-pole circuit breakers. Each section of line is therefore of the through feed type. In case of any fault from feeder to ground or trolley to ground, the single-pole circuit breakers at each end of the faulty line will operate to clear such a fault. In the event of a fault between trolley and feeder not involving ground, which will constitute a 36-kv. short circuit, relaying is so arranged as to trip breakers on both circuits involved regardless of what combination of trolley and feeder is short-circuited. Since the trolley breaker is of the high-speed type, and the feeder breaker is of the normal speed type, there would be a tendency for the lower voltage trolley breaker to operate first and attempt to clear a 36-kv. trolley-feeder fault. This action is undesirable not only from the standpoint of breaker duty but in the event the trolley breaker should successfully open the fault circuit promptly, the feeder relays might not receive sufficient impulse to trip. The energized feeder would then be left crossed with the trolley, impressing 24,000-volts-to-ground upon any apparatus such as lightning arresters, car equipment, etc., connected to this trolley.

To obviate this contingency special relay circuits

have been set up in a comparatively simple manner to prevent the trolley breaker from tripping until the feeder breaker has tripped. This is accomplished by using a current balance scheme whereby currents in feeder circuits are opposed to currents in trolley circuits, thus eliminating the impulse trip function of the trolley breaker, if the trolley and feeder fault currents are the same. The fault will operate an impedance type relay installed in the trolley circuit for back-up protection, which will not complete the high-speed breaker tripping circuit, however, until a series relay connected in the common wire to all feeder trip circuits has been energized by the tripping of the feeder breaker. In the event of a fault between trolley and feeder involving ground, this will clear as separate trolley-rail and feeder-rail faults. It may be noted that trolley-rail faults will be by far the largest number, and trolley-feeder faults not involving ground are expected to be very infrequent. For fault purposes ground and rail are considered the same, for the ground side of insulators is the steel supporting structure, which is connected directly to the rail return circuit.

Each trolley is fed at both ends, and substations are located at the terminals of all electrified branches so as to make any stub end feeds unnecessary. The elimination of such stub feeds will aid materially in obviating induction in parallel lines. Since feeders are sectionalized at each substation they are naturally connected at both ends in a manner similar to the trolley circuits.

The reliability of the railroad power facilities is further protected by differential relay protection for all transformers and for all main bus sections. Since substations are unattended, remote electrical reset is provided for trolley bus tripping relays, so that an attempt may be made to restore service on all lines immediately.

Complete transfer buses are provided for all trolley and feeder lines, so that in the event of a circuit breaker or complete bus being out of service, all outgoing lines may be energized through other breakers.

Summary

The Reading Company's initial power supply is being obtained by purchase at one point from Philadelphia Electric Company. Power is metered at 13,200 volts, 25 cycles, single-phase, and is transformed by the railroad company to serve a 12000/24000/36000 three-wire single-phase, 25-cycle distribution system. Auto-transformer substations at various points step the 36-kv. voltage down to trolley voltage.

Future electrification will be served by expanding the present facilities without change, superimposing 66-kv. transmission lines upon the present system to serve the greater distances.

It is planned to receive power May 1, 1931, for testing of lines and trains, and to begin electric suburban operation in passenger service on July 1, 1931.

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Discussion

Sidney Withington: Mr. Doub's estimate of 30 per cent load factor is of interest and is quite in line with that normally experienced in suburban service. This applies not only to the power supply but to the distribution facilities and indeed to the tracks themselves. If the electrification is extended to include passenger through service the load factor will be increased and if handling of freight is added, it will again be increased. In normal railroad operation these track facilities are available for handling freight when passenger traffic is at a minimum and there is then a desirable filling in of the valleys of the load curve if electrification covers all branches of operation. The very high density of traffic of the Reading Railroad is mentioned. It may be said to be an axiom in electric traction that density is, other things being equal, a measure of the justification of electrification.

I should like to emphasize the importance of Mr. Doub's reference to the desirability of interchangeability among railroads of electric-motive power equipment. The multiplicity of "systems" of electrification toward which we in this country are gradually drifting, will ultimately cause a great deal of embarrassment and very careful thought must in the near future be given to standardization. The railroads have necessarily gone far towards standardizing of such items as track gage, air brake facilities, car couplers, etc., and they cannot afford to be hampered in operation by the growth of a diversity of systems of power distribution in electrification. It may safely be said, I think, that standardization of this important feature will do more to stimulate railroad electrification in this country than any other one thing.

The subject of purchase of power as compared to production by the railroad is of vital importance. Power companies are now well equipped physically to supply the needs of railroads. This was not the case 20 years ago. A railroad in installing an electrification project should expect to confine its immediate capital expenditures to its power distribution facilities which at best are expensive. The motive power can usually be purchased on an equipment trust basis and paid for over a term of years from savings. The local power supply company should be the organization to make the necessary expenditures for power generation facilities. There is no fundamental reason why the railroad should not purchase power from specialists just as it purchases let us say, its castings; few railroads operate their own foundries. The power company however obviously must present to the railroad a figure for price of power which will at least not be in excess of that for which the railroad can produce its own power, and must not burden the railroad with unnecessary guarantees and limitations which would unduly penalize the railroad in maintaining the flexibility of its supply. If the railroad purchases its power requirements, the power should, if possible, be delivered to the railroad distribution system in the

form (whether direct current or single-phase) in which it is to be used, that is, under normal conditions the power company should own and operate any substations facilities which may be necessary as part of the supply.

The reliability of railroad operation depends upon duplication of facilities. Railroads like many other public service organizations, must operate continuously 24 hours a day throughout the year, with no time out. In winter, blizzards and sleet, in summer, lightning and at any time of the year storms may be encountered; any unit of equipment may fail in service, and maintainers or material for repairs may be far away. Duplication of important items, that is the availability of an independent facility, is the only means of assuring continuity of operation. For this reason a power company's supply with its transmission network independent of railroad right-of-way may be ideal. A wreck or a fire on property adjacent to the right-of-way may cut the railroad in two and an independent supply is thus essential to continued operation. In the case of the Reading electrification this is not an essential consideration for its network of tracks provides opportunity for ring feeders of its own.

Mr. Doub refers to high-speed circuit breakers. Such protective facilities, designed to quickly clear any fault which may occur has done a great deal to advance the art of electric operation of railroads, whether on single-phase or d-c. distribution systems. From the very nature of the kind of operation, frequent grounds and short circuits are experienced in railroad power supply circuits and these on the grounded systems are often very severe. Quick clearing of the fault is essential to protect the equipment as well as to minimize transfer of induced power to neighboring communication facilities. The high capacity high-speed circuit breakers, whether air or oil, accomplish this very satisfactorily and simplify operation.

Substations of the Broad Street Subway of Philadelphia

BY H. M. VAN GELDER[1]
Member, A. I. E. E.

Synopsis.—*The Broad Street subway is the first step of a system of underground high-speed railways which is being designed and built by the City of Philadelphia. This paper describes the substations which furnish power for traction at 630 volts direct current and for the subway lighting and signals at 4,600 volts, 60 cycles.*

The first section of the Broad Street subway required three substations which were equipped with synchronous converters and completed in 1928. Later, when the subway was extended, a fourth substation was added, but by this time so much progress had been made in the design of substation apparatus, particularly in respect to increasing the current capacity of mercury arc rectifiers and perfecting iron-clad high-voltage switching equipment, that a radically new design of substation was decided upon embodying these types of apparatus. This substation was placed in operation this year.

This paper contains a description of the apparatus and method of operation provided in both types of substations and indicates the advantages of the equipment selected. Much thought was also given to special operating features for safety and reliability.

* * * * *

Introduction

THE construction of the Broad Street subway was started in 1924, the first section extending from City Hall to Olney Avenue, approximately six miles, with a half mile extension to the Fern Rock Yard. It was built for four tracks but only the two local tracks were installed. This section was placed in operation in 1928.

Early in the construction when clearances of the tunnel were determined it was decided to install a traction system using contact rail distribution at a potential of 600 volts direct current. Power was available from a public utility system at 13,800 volts, 60 cycles, three-phase, by underground transmission.

To supply traction power for this line and the yard three substations were constructed, spaced about two miles apart, at Louden, Cumberland, and Mt. Vernon Streets. They were built to contain sufficient equipment for the ultimate train operation on four tracks, but the initial installation was only for estimated 1930 traffic on the local tracks. The three substations are similar in design and equipment except for minor modifications to conform to the shape of the building sites.

Synchronous Converter Substations

The substation sites were selected just off Broad Street, generally in a fairly good residential district. As rotating apparatus was to be installed in these stations, considerable thought was given to the elimination of noise so as not to disturb the neighborhood. The buildings were constructed without windows and with walls having an inside brick veneer separated from the main wall by an air space. The main entrances were constructed with two sets of double doors having an air space between. The buildings were located on corners and an areaway or space was left on the sides adjacent to other buildings. This construction has proved very satisfactory, as almost no sound is transmitted outside of the station that can be heard across the street.

With the stations designed to be enclosed as described, it became necessary to give particular attention to the ventilation of the buildings, as all equipment was to be air cooled and no forced draft used. Liberal openings, fitted with louvers, were therefore provided in opposite sides of the basement. These openings

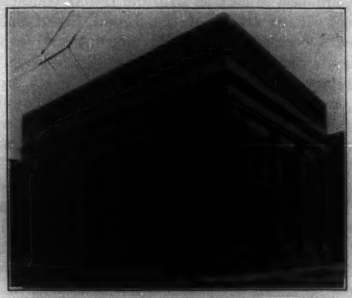

Fig. 1—Louden Substation

admit air from areaways in the side walk or between buildings. Large, specially designed, stationary ventilators were located in the roof over the converters and transformers and a division wall was constructed across the center of the basement separating the converter and transformer space. Openings with gratings were located in the main floor opposite the shaft ends of the converter, as it was known that the opening through the bed plate of the converter was not satisfactory for ventilation. It will be seen that with this arrangement of air inlets in the basement and ventilators in the roof, augmented by a stack effect of about 40 ft. between the basement and roof, a real system of natural ventilation

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was established. Architectural treatment for this type of building is well shown in Fig. 1, which is a corner view of the Louden Substation.

The requirements for the ventilation system were calculated, based upon the heat units to be dissipated at 150 per cent rating of the equipment with an assumed temperature gradient, taking into account the stack effect, but without adding the ventilating capacity that would be obtained due to any outside wind velocity. The inlet and outlet openings were sized to conform to these calculated requirements. The experience and results obtained by the 1925 and 1926 Committees on Power Generation and Conversion of the A. E. R. E. A. were made use of in planning the ventilation.

The capacity of the substations was determined from proposed schedules for initial and ultimate operation of trains on the two local tracks installed, and on two additional express tracks to be installed at a later date. Motor-generator sets, synchronous converters, and mercury arc rectifiers were considered in the studies for these substations. Rectifiers had not then been in service in large capacity units, and converters showed better economy and were lower in first cost than motor-generator sets, therefore converters were chosen. They were specified to be compound wound to give better regulation of the voltage in the subway as no feeder cables are used in parallel with the 150-lb. contact rail.

The assumed traffic of the Broad Street subway showed a decided peak during morning and evening rush hours, lasting in each instance about one hour, with momentary loads during acceleration of much greater value, so that the problem of supplying the power for the traction load resolved itself into a question of maximum ratings. It was found that the continuous rating of the converter was of little importance and that the two hour rating would probably not be reached for some time in the future, hence the peak loads during acceleration of four and six car trains determined the capacity of unit required. By installing a converter unit having sufficient capacity for the initial service it required three units for ultimate operation with four tracks. Provision was made in the substation for a fourth unit to be used as a spare. Therefore, two units were installed in each substation to take care of the initial load on two tracks and to provide a spare unit.

At the time these converters were installed they were the largest 60-cycle converters built for 630-volt traction loads. They are 360 rev. per min. and compound wound with slightly drooping characteristic. The actual ratings are 3,000 kw. at 630 volts continuously, when supplied with six-phase, 60-cycle, a-c. power at 100 per cent power factor; 4,500 kw. at 630 volts for two hours; and 8,000 kw. at 560 volts for three minutes.

Oil-insulated self-cooled transformers were installed in cells at the basement floor level. In order to guard against overheating for short periods of heavy load in excessively hot weather, when ventilation might be imperfect, a water coil was installed in the transformers to provide partial cooling equivalent to the radiation required for the transformer to carry 50 per cent of its nominal rating. The transformers have auxiliary expansion tanks to prevent oxidation of the oil.

Conventional types of brick masonry structure for the 13,800-volt bus and of concrete cells for the oil circuit breakers were installed on the main floor, or in a small balcony along the side of the building. Fig. 2 is a view of the main floor of Cumberland substation showing converters and transformer cells.

The control and instrument switchboard from which all circuits are operated is located on the main floor. The main positive and equalizer leads from the converters drop directly to the main converter switchboards located in the basement. From these switchboards the 630-volt main positive circuits run directly to the positive bus, thence through the high-speed circuit breakers to the feeder switchboard and out through the underground cables and conduits to the air circuit breakers and contact rail in the subway. The main

Fig. 2—Main Floor, Cumberland Substation

negative leads from the converter run directly to the negative bus. Thus it will be seen that no 630-volt power circuits are brought up to the main floor, and furthermore all of these circuits are run in the basement by means of copper bars, no cable being used until the underground cables are reached. Fig. 3 is a cross section of Louden substation and shows the location of apparatus and arrangement of ventilation.

The starting and control system for the converters, has some unique features which deserve a detailed description. It was designed for star-delta starting by the automatic switching of the high-voltage windings of the transformers; for assuring correct polarity; to avoid error in brush position; and to prevent closing a dead machine onto the bus. The sequence of operations when starting the converter is as follows: The field has to be excited from the control battery circuit, to insure correct polarity, before the main oil circuit breaker can be closed by the control switch. The closing of the oil circuit breaker completes the closing circuit of the star

oil switch, provided the brushes on the converter are raised. When the converter has attained synchronous speed a relay bridges a gap in the tripping circuit of the star oil switch, and this circuit is completed by another relay which closes at a definite value of the field current after the field switch has been thrown over to self-excitation. The star oil switch on opening completes the closing circuit of the delta oil switch, provided the oil circuit breaker continues closed. The oil circuit

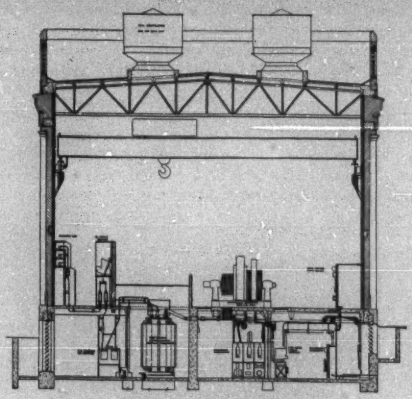

Fig. 3—Cross-Section, Louden Substation

breaker is trip free during the above operations so that full protection is assured. The d-c. switches and circuit breakers are electrically operated, but before the positive switch can be closed the delta oil switch must be closed, brushes lowered, equalizer switch closed, and positive circuit breaker closed.

The oil circuit breaker will open on over-current or low voltage, and, by interlock control, automatically opens either the star or delta oil switch and the d-c. circuit breaker. The d-c. breaker opens on over-current, reverse current, over-speed, or undervoltage, and on opening, opens the positive switch and the equalizer switch.

A ground relay opens both the oil circuit breaker and the d-c. breaker on a converter flash-over. Thus it will be seen that many of the operations are automatic, the operator performing only the initial movement of a sequence. The design also assures speed of operating and prevents an operator from making mistakes in putting a converter on the line. At the time of an unexpected failure of the power supply to a substation, the operator cleared all feeder circuits and auxiliaries, ordered power on again, started up the station, and established full d-c. feeder service in three minutes.

Two of the substations contain transformer and switching equipment for the supply of 4,600-volt, three-phase and single-phase power for lighting and signal feeders, respectively.

These three substations have been in operation for over two years and all equipment has functioned very satisfactorily. The converters have flashed over occasionally but have not averaged more than two or three per year per machine, and when they do occur the damage is slight and usually the machine does not have to be left out of service. The flash-overs generally occur at times when a heavy load is suddenly cut off, particularly where a limited short circuit occurs which does not open the high-speed breaker but clears itself or is cleared by a subway air circuit breaker. Apparently the flash-overs are due to large synchronizing currents caused by angular displacement of the armature after heavy momentary load.

One operator and a helper are on duty in each station at all times.

Mercury Arc Rectifier Substation

When the Broad Street subway was extended south from City Hall to South Street, a distance of about three-quarters of a mile, it was necessary to build a fourth substation to furnish traction power. The site of this substation, known as No. 7 and shown in Fig. 4, is just east of Broad Street and south of Pine Street, near the southern terminus of the subway extension. The site is 80 by 80 ft., and permitted the erection of a substation having access from both Juniper and Watts Streets.

Fig. 4—Substation No. 7

The design of substation No. 7 and its equipment was undertaken in 1929, by which time considerable experience had been obtained with the actual load and operating conditions existing in the three synchronous converter substations originally built for the Broad Street subway.

Some difficulty had been experienced with the synchronous converters flashing over under extreme load

variations, but this was not serious enough to warrant a change in the type of apparatus for this cause alone.

In the meantime much progress had been made in the design, and considerable experience gained in the operation of large capacity mercury arc rectifiers in this country. Two important companies were prepared to manufacture units for subway operation having maximum capacities of 12,000 to 15,000 amperes. Rectifiers also have many advantages over converters, such as much better efficiencies at light loads, the absence of large rotating parts with their attendant troubles and noise, the absence of commutation of heavy currents, better stability under short circuits, and the simplicity of bringing them on the line. Therefore it was decided to install rectifiers in this new substation.

Experience obtained on North Broad Street indicated that short peak loads would determine the capacity of the unit, and that these peaks usually lasted less than ten seconds or during the multiple motor operation on notching acceleration. Consequently the maximum rating of 300 per cent load was reduced from three minutes to one minute. A unit having a continuous rating of 4,000 amperes at 630 volts, 6,000 amperes at 612 volts for two hours, and 12,000 amperes at 550 volts for one minute was decided upon. As one of these units would carry the initial load, the first installation required two units, the second unit being installed as a spare. The building however was constructed to contain a total of four units which will provide for subway extensions adjacent to this substation.

Regulation of these rectifiers closely approximates that of the synchronous converters in the original substations, thus making it possible for the two types of substation to be operated in parallel even on the extreme fluctuations of load. There will be no necessity, and it is not intended, to install converters and rectifiers in the same substation for parallel operation.

Although cell structures built up of some form of masonry or steel have become fairly well standardized for high-voltage oil circuit breaker and bus equipment, live parts being more or less protected with barriers, doors, or covers, they have never proved entirely satisfactory from the standpoint of safety. Any accident or failure which occurs in such structure is sure to be serious from the standpoint of human life or interruption of service or both. There had been in use abroad and under development in this country for sometime a wholly enclosed and filled structure for bus and oil circuit breaker connections, known generally as the iron clad type of switchgear, which presented very attractive safety features. It was decided to use such equipment in the new substation.

The incoming 13,800-volt circuits, bus, oil circuit breakers, instrument transformers, disconnecting devices, and connections for this substation are all contained in an armor clad bus and switchgear structure which leaves no live part exposed. The structure is filled with hard compound, except the circuit breakers and potential transformers which are filled with oil, and the current transformers which are enclosed in a chamber filled with petrolatum. All cable leads to and from the armor clad structures are lead covered and wiped permanently to potheads which form a part of the structure. One feature of this structure is that all relays and connections are self-contained and mounted on the various switchgear units. Each armor clad structure is completely built and assembled in the factory, and no masonry structure is required in the station except for foundations. Consequently the field erection is comparatively simple. The main 13,800-volt, three-phase bus and circuit breakers are shown in Fig. 5.

The rectifiers are water cooled and the transformers are self-cooled, so that in the layout of the station building, which is located between two streets and with an areaway on each side, a central rectifier hall was provided and a separate transformer cell located in each

Fig. 5—Main 13,800-Volt Bus and Oil Circuit Breakers, Substation No. 7

of the four corners of the building. By this arrangement each transformer cell has an opening on two sides and a large roof ventilator at the top. One of these openings contains adjustable louvers and the other has a vertical lift door giving entrance from the street. This provides natural ventilation from two directions.

As there is practically no noise from the equipment of this substation no special precautions had to be observed regarding sound-proofing and no special ventilating problems were encountered except in the transformer cells as described.

The rectifiers are double six-phase, contain 12 anodes, and have automatic a-c. ignition and excitation, automatic water control, and an indicating vacuum regulator which cuts the rotary pump in and out at predetermined vacuum points, and also cuts the rectifier out of service if the vacuum falls below a predetermined value. Thermometers with electrical contacts also provide thermal protection for the rectifiers. The installation of rectifiers and their auxiliaries is shown in Fig. 6.

The water for cooling the rectifiers is taken from two city mains, two sources of supply being used to insure continuity. Due to the relatively light continuous loads and the prevailing low cost of water, it was found to be cheaper to waste the water than to provide cooling equipment for recirculating it.

Each rectifier has a static mercury vacuum pump and two rotary oil vacuum pumps, one of which is a spare. This spare pump was specified as an extra precaution

Fig. 6—Main Floor, Substation No. 7

against failure of the one piece of rotating apparatus included in the unit. It has been found that the rotary pump does not operate more than 25 per cent of the time.

The subway d-c. power load, a graphic chart of which is shown in Fig. 7, is of such a fluctuating character and has such a low sustained value that it was found of little use in forming or baking out the rectifier. In order to furnish a continuous load for this purpose a water rheostat which will absorb 1,200 amperes has been

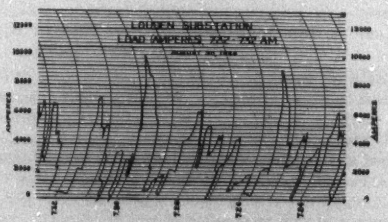

Fig. 7—Typical Substation Load of Broad Street Subway

installed in one of the areaways outside the substation. This rheostat may be operated in parallel with the traction load. If necessary the rectifier can be operated six-phase or three-phase at times of bake-out, thus increasing the current per anode.

An operating office was provided in the substation on the same floor as the rectifier hall. One side of this office consists of the main control switchboard from which all apparatus in the substation is operated and controlled. All other switchboards and all 630-volt d-c. circuits are located in the basement, following the same general design as for the converter substations, except that a single rectifier switchboard was used instead of separate converter panels under each machine as in the other substations.

The d-c. connections from the cathode of the rectifier pass directly through the floor to the rectifier switchboard. This board contains a remote-controlled positive switch and a special quick-acting circuit breaker for each rectifier, with auxiliary switches and interlocks, so that when the 13,800-volt oil circuit breaker of the rectifier unit is closed, all other functions of starting up are performed automatically; the operator has then only to close the positive d-c. circuit breaker control switch to bring the rectifier on the bus.

The four neutral connections of each main transformer are carried to the basement through wall ducts to its interphase transformer located immediately ad-

Fig. 8—Rectifier Switchboard and High-Speed Breakers, Substation No. 7

jacent to the negative bus. The neutral lead of this interphase transformer, which forms the negative connection of the rectifier unit, is connected to the negative bus.

The layout of the 630-volt d-c. buses, high-speed circuit breakers, feeder switchboard, and outgoing positive feeder cables is the same as in the other substations. Fig. 8 shows the rectifier switchboard and high-speed circuit breakers.

This substation also has the station auxiliary switchboard in the basement with the principal operating circuits controlled from the main control board. The control circuits of the substation are supplied from a 125-volt storage battery. The auxiliary a-c. circuits for the rectifier units have three sources of supply to insure continuity of service: an auxiliary station transformer supplied from the 13,800-volt main bus, and a second auxiliary transformer supplied from either

of two 4600-volt lighting feeders entering the substation from the subway and supplied from different substations of North Broad Street.

There has been no signal or telephone interference experienced due to the operation of this substation. Telephone circuits in this district are carried in underground cables.

This substation was placed in regular operation April 20, 1930, and up to the present time the equipment has functioned quite satisfactorily. During early operation a few back-fires occurred, doubtless due to insufficient baking-out of the rectifier before heavy overloads were carried. However, no damage was done, and now loads of 10,000 amperes are taken by one rectifier.

Discussion

Sidney Withington: One of the striking points touched upon by Mr. Van Gelder is his reference to the fact that the mercury arc rectifier supplanted the rotary converter in his more recent substation design. The development of the rectifier for traction purposes has recently been relatively rapid. This rapid development has been indicated especially in the electrification of the Cleveland Union Terminals and D. L. & W. Hoboken suburban electrification. These two projects both employing 3,000 volts direct current for distribution were inaugurated within a short time of one another. The Cleveland Terminal design is based upon motor-generators and the D. L. & W. upon mercury arc rectifiers. The few weeks intervening mark developments of vital consequence in the use of the static rectifier for 3,000-volt service in this country.

The growing use of the mercury arc rectifier for railroad work is making increasingly desirable a logical method of rating which will recognize the fundamental thermal and other load characteristics of the rectifier as compared with rotating apparatus. Mr. Van Gelder's load curve shows clearly the disadvantages of the present standard method of rating. It is to be hoped the work of standardizing the rating which has been undertaken by the Institute under the auspices of the American Standards Association will shortly be completed so that it may be adopted by manufacturers and railroads.

H. M. Van Gelder: Our units all have a 300 per cent rating for one or three minutes, given in detail in the paper. This is essential for the peak loads carried by these substations, and is the only rating which limits the use of the equipment, the continuous and 2-hr. rating not yet having been reached. With reference to a standard rating for this type of railway equipment it does not seem practicable to me to use the same standard for different classes of load. Where small train units are operated at frequent intervals, resulting in a fairly steady base load, a maximum rating of 200 per cent might be sufficient. On the other hand, with a 2-track high-speed line moving heavy train units at longer intervals, the only rating required would be a maximum rating of 300 per cent, or even more in some cases.

Utilization of Railroad Rights-of-Way
For Electric Power Transmission and Coordination with Railroad Electrification

W. W. WOODRUFF[1] and G. I. WRIGHT[2]
Associate, A. I. E. E. Member, A. I. E. E.

ON account of a combination of factors, Philadelphia has been obliged to take several steps in the matter of utilizing railroad rights-of-way for electric power transmission purposes and has probably made a further advance along these lines than any other city in this country.

While the same factors and their relative importance may not be operative in other localities, it is probable that in the immediate vicinity of any of the larger cities, there are, or will be in existence, a combination of circumstances which would make the utilization of railroad rights-of-way for electric power transmission purposes both expedient and essential.

It is felt that the results obtained in Philadelphia have been highly satisfactory and beneficial to both the railways and electrical industry and that wider knowledge as to the general features should be of interest to others.

In order to make the situation clear, it will be well to review the various factors involved and to briefly trace through several stages of the developments.

Philadelphia is located on a peninsula formed by the junctions of the Schuylkill and the Delaware Rivers. To the south of the Schuylkill River there is an industrial development which extends through the City of Chester and Marcus Hook to the State of Delaware. West of the Delaware River there is a section of congested urban development from one to six miles deep extending for about twenty miles along the water front. Back of this strip is an area of suburban and semi-rural development which forms a belt probably some twenty miles deep and extending from the vicinity of Trenton to the Mason-Dixon Line. (Fig. 1).

The main steam plants of the Philadelphia Electric Company that supply this region with electrical energy are necessarily located along the Delaware River. A glance at a map of the territory would indicate that plants located along the Schuylkill would be in a better strategical location for feeding the territory. Locations on this river, for large stations, are however impractical on account of the fact that the river flow is comparatively small. While the oldest station of the Philadelphia Electric Company is located on the Schuylkill and is still in operation, it has not been advisable to add to its generating capacity for a number of years and it is doubtful if it will ever be expedient to rebuild or increase its capacity. Consequently, a major transmission problem has been to find means of carrying power into Schuylkill Station, to establish other points of large capacity supply at strategic locations, and to tie these up with the larger generating stations in the territory and with outside sources of supply.

When we come to considering the transmission of large blocks of power into these urban centers, from outside sources, it is generally the case that the securing

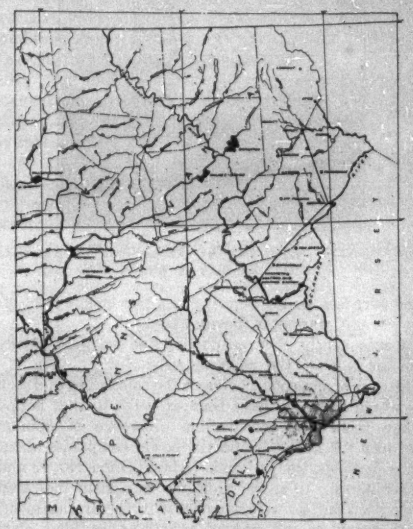

FIG. 1—MAP OF TERRITORY

of satisfactory right-of-way to the economic centers of distribution is almost an impossibility and that the various railroad lines form the only feasible routes through the highly developed districts. In practically all cases the railroads own these rights-of-way.

The first major problem of this kind in Philadelphia originated about 1915-16, when it became essential for the electric company to have a 66-kv. transmission line between Schuylkill Station in Philadelphia and the new Waterside Station in Chester in order to back up

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30-174

the older station, supply more power and take advantage of the more economical generation at the newer station.

The only feasible route between these two stations was along a freight line of the Reading Railroad and a short section of the Pennsylvania Railroad which ran between Philadelphia and Chester. Practically all the property on either side of this railroad route was either developed or had so much potential value for future manufacturing sites that its purchase for transmission

Fig. 2—66-Kv. Line Erected Along Side of Railroad Right-of-Way in 1915-16

line purposes, or the securing of a permanent right-of-way over it, was out of the question.

Consequently, the transmission line was built largely on the side of the railroad right-of-way. (Fig. 2). The type of agreement between the companies was such as would naturally have been expected in that period when there was considerable difference of opinion between the railroads and the power companies about wire crossings, Pennsylvania General Order No. 13 and other items. In short, while the agreement indicated that both parties were satisfied with the arrangement, both agreed to and granted, as little as possible. Naturally the agreements had a revocation clause, as was then customary.

Practically this solution has worked out satisfactorily but this is primarily because of a disposition on the part of the companies to consider the matter in a wholly cooperative spirit rather than on account of the nature of the agreement and its legal provisions.

A similar situation developed in 1924 in connection with a project of the electric company's for interconnection between Philadelphia and Trenton. In this case the district immediately adjacent to the Richmond Station, from which circuits must run, was so highly developed that the only expedient was the running of underground cable for a distance of about four miles and thence it was possible to construct a 66-kv. steel tower transmission line on the Oxford Branch of the Pennsylvania Railroad for a distance of about five miles.

The electric company was rather loath to invest several hundred thousand dollars on a right-of-way where it had no really permanent and irrevocable rights. At the same time, the railroad company was loath to grant any permanent rights on its property owned for transportation purposes. In this case the question was solved by the electric company purchasing in fee simple the additional property outside of the line of proposed tracks, which it had been necessary for the railroad company to secure to allow the necessary cutting and filling for the road bed construction. In addition, the electric company gave back to the railroad the right to cross its fee simple right-of-way wherever necessary to reach adjoining industrial developments and in return received certain rights to maintain wires overhanging the railroad right-of-way. (Fig. 3).

This agreement was much preferable to the revocable type previously mentioned, and was obviously a step

Fig. 3—66-Kv. Line Erected on Right-of-Way Purchased from Pennsylvania Railroad

forward in cooperation, but could hardly be called complete coordination.

When the science of electric transmission had progressed to a point where it became advisable to take advantage of the diversity existing between adjacent systems and to transmit power from outside sources into the urban area, the electric company was naturally again confronted with the same problem; that is, the necessity of either making very long and expensive runs of underground high-voltage cable or utilizing existing

railroad rights-of-way which formed the only open lanes into the congested urban sections.

The lines which have been mentioned above were of comparatively small capacity. A much more serious situation developed when it was necessary for the electric company to make plans for bringing the Conowingo power output into its metropolitan system. In this project transmission facilities had to be provided not only for Conowingo power itself but also power involved in the interconnection between the Philadelphia Electric Company, Public Service Electric and Gas Company of New Jersey, and the Pennsylvania Power & Light Company. Transmission capacity of about 600,000 kw. was required.

Obviously, it was necessary to deliver this amount of power at as high a voltage as practicable well into the highly developed sections of the city and equally obvious was it desirable to bring it in as far as possible on aerial construction. It was quite evident initially

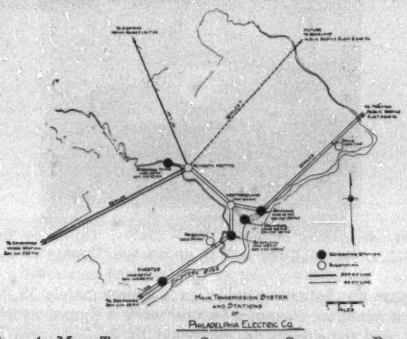

Fig. 4—Main Transmission System and Stations of Philadelphia Electric Company

that a considerable portion of this power must be distributed to the various generating stations in the city by means of 66-kv. underground cable, but on account of the investment, this length of cable must be kept at a minimum.

The only feasible route into the highly developed territory, reaching a point which was desirable from a distribution standpoint, lay along two lines of the Reading Railroad paralleling the Schuylkill River from a point slightly inside the city limits. The latter point could be reached by the conventional aerial construction on private right-of-way. The main features of the territory and the locations of the stations is shown by the map. (Fig. 4).

The situation, therefore, required the working out of an agreement between the railroad company and the electric company which would be mutually satisfactory. Obviously, a project which formed so vital a link for such a large and important undertaking must have a permanent status and could not depend for its continuity on any revocable clause or any other consideration which did not guarantee to the electric company permanent and perpetual rights. It is equally obvious that the railroad company was in the same situation and must preserve permanent rights for maintaining its facilities in operating condition, and also, preserve the ability to change and increase its facilities as conditions required.

Occupation of railroad right-of-way for electrical transmission can in no wise be considered as similar to the utilization of public highways. The railroad company, due to its prior development, owns outright and in fee simple the major portion of its right-of-way. Consequently, it cannot give up any of the rights invested in the ownership of property or in any way limit itself in the development of its facilities.

In the situation which developed in connection with the Reading Railroad, these factors were readily appreciated in the beginning, and it was obvious that the conditions required the consideration and the finding of a satisfactory solution for the following main problems.

(a) The designing of the structures so as to provide adequate strength to meet severe conditions of climatic loading, and the existing rules and regulations of any public service body which might apply.

(b) The designing and locating the structures so that they would not interfere with the operation of the railroad company's existing or proposed tracks or other facilities.

(c) The locating and designing of the structures so that they either serve as supports for or at least not interfere with, supports for the railroad company's future electrification circuits.

(d) In the event that alteration, augmentation or reconstruction of the railroad company's facilities became necessary, the providing for a joint study of the situation to determine the most economical means of making the improvement.

(e) The reconstructing or altering of the signal and communication system in such manner as to secure satisfactory operation under the inductive coordination conditions involved, considering future railroad electrification and train control requirements.

(f) The apportionment of costs which result directly or indirectly from the occupation of the railroad company's right-of-way by the electric company's transmission circuits.

(g) The determination of a rental charge for occupation of the right-of-way.

The agreement finally reached recognized all these factors satisfactorily.

The advantages of the arrangement to the electric company are mainly that it obtains a permanent right-of-way through a district where the securing of an independently owned right-of-way would be practically out of the question from either a financial or a public

policy point of view. However, it is also evident that the electric company assumes a considerable liability.

The benefits to the railroad company are:

(a) The control and coordination of the physical and inductive conflict which might arise were the electric company's lines to be located off the right-of-way but parallel to it.

(b) Obtaining supports for its electrification transmission and catenary circuits.

(c) The alteration of all signal and telegraph facilities so that these are unaffected by the operation of the electric company's facilities and will be unaffected by the railroad's electrical operation of its trains. In this particular case the railroad's signal system was changed from direct current to alternating current and the open wire signal and communication circuits were changed to cable and placed underground, so that the railroad will be relieved of the necessity of making these changes when it electrifies.

(d) An annual rental charge for the occupation of the right-of-way.

One very important feature should be remembered. If the electric company had been obliged to purchase a right-of-way, a considerable portion of it would in this case, either have paralleled or been in close proximity

Fig. 5—Design of 66-Kv. Four Circuit Tower on Reading Railroad Right-of-Way where Space is Available Outside of Tracks

to the railroad right-of-way. In this event, while the feature of physical interference would have been decreased, the inductive interference situation would have been almost the same as exists with the transmission circuits on the railroad right-of-way. On separate rights-of-way both parties would have equal rights and a very complicated situation might have existed as to the electric company's operation of facilities which affected the railroad company's facilities.

It is not intended here to give a detailed description of the structural features as these are more of interest to the structural than the electrical engineer, so only a few points of major interest will be mentioned. Figs. 5 to 8 illustrate various types of structures and some of the typical locations.

The general electrical features for the electric company's transmission circuits are very much the same as is now the practise for 66-kv. lines on the conventional steel tower type of construction. That is, the vertical

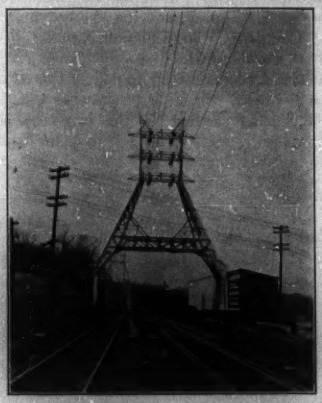

Fig. 6—Design of 66-Kv. Four Circuit Tower on Restricted Right-of-Way of Reading Company

separation between conductors is approximately 10 ft., the minimum clearance to steel work is figured at 2½ ft. and the circuits are protected by ground wires in order to avoid lightning disturbances as much as possible.

Most parties who see either the construction or photographs comment on the fact that every span is dead-ended at both ends. This is brought about by the fact that if the usual suspension insulator had been employed in order to obtain the necessary clearance between wires and structures and between circuits the length of the cross arm, the width of the structure, necessary to support the circuits would have been so great that the wires could not have been kept within the limits of the right-of-way.

In the original installation which was made for about 4½ miles on the Main Line and Richmond Branch of the Reading Railroad, provisions were made for 4 three-phase circuits, it being originally intended that two of these circuits would be reserved for the electric company and the two other positions reserved for the railroad company's transmission circuits for future electrification. With the further development of the railroad company's plans it proved feasible to change

the formation of the top of the structure so that on the recent Norristown Branch construction 3 three-phase transmission circuits were provided for the electric company and space for 2 two-wire circuits for the railroad company. On the newer construction it will also be noted that a bracket is provided for 36-kv. railroad circuits.

It was felt desirable by both parties to make the strength of the structure such that no question should ever arise as to their compliance with the Pennsylvania Public Service Commission's regulations covering the

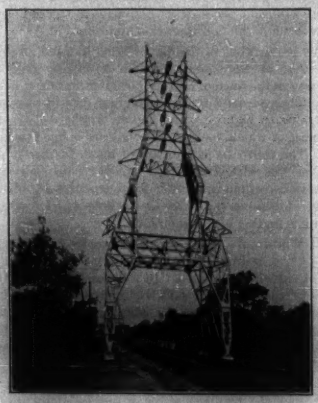

FIG. 7—LATEST TYPE OF TOWER OVER NORRISTOWN BRANCH RESTRICTED RIGHT-OF-WAY OF READING COMPANY

This provides for 3 three-phase 66-kv. power company circuits, two single-phase 66-kv. circuits for the railroad company, and two single-wire 36-kv. circuits for railroad three-wire system

strength of construction within the State of Pennsylvania. Consequently, the structures are designed to stand the full dead-end strain of the conductors on one side. While this requirement may seem very severe it actually developed during the course of detail designing that very little hardship was imposed. When sufficient strength is provided to stand the assumed design condition of all wires on one circuit being broken at the most disadvantageous location, that is, generally at the end of a cross arm, it was found that the structure was usually stronger than was required to meet Pennsylvania Public Service Commission's requirements.

In order to meet the requirements necessitated by future track construction, right-of-way, etc., several expedients were necessary in the design of structures. For the greater part of the installation on the Main Line and Richmond Branch, which is largely located on a cut on the side of a precipitate slope, there was sufficient room on the side of the cuts and fill to place the structures outside of the line of any present or proposed tracks, but as the more congested portion of the city was approached, the right-of-way conditions naturally became constricted and at some locations the conditions could only be met by a bridge type of construction which completely spanned a four track roadway. At other locations it was possible to meet the requirements by dividing the construction, placing the towers on opposite side of the right-of-way with one circuit going through the center of the structure and the other circuit overhanging the roadway.

In the construction on the Norristown Branch of the Reading Railroad the right-of-way conditions are very much restricted for about the first three miles out from the termination of the line. Throughout this section it was necessary to utilize a bridge type of construction spanning a proposed four-track roadway. A proportion of this roadway was involved in a grade crossing elimination project and several various interesting situations were involved.

In certain portions a retaining wall was to be built and the design of the foundations for the transmission structures was necessarily coordinated with the design

FIG. 8—DESIGN OF SPECIAL TWO-CIRCUIT TOWER ON SIDE OF READING TRACKS

Two additional circuits will be provided by similar construction on other side of tracks

of the retaining wall. Consequently, the tower footings formed portions of this wall. At another location a steel viaduct was to be constructed and the viaduct was so designed at the necessary location that the steel work for the transmission structures could be attached directly thereto.

At certain other locations where the actual railroad right-of-way was of insufficient width it was found preferable and cheaper to purchase additional right-of-way rather than to adopt the more costly bridge construction.

By the time this paper is presented one circuit of the new construction should be in operation, the initial construction having been in operation somewhat over two and one-half years with very satisfactory results. Throughout this two and one-half year period the two initial circuits have, at times, carried as high as 200,000 kw. In fact, they have continually carried, except for some 20,000 or 30,000 kw. which is diverted at Plymouth Meeting substation either to the Pennsylvania Power & Light Company's system or to Barbadoes Island plant at Norristown, practically the entire output of the Conowingo plant. During the present year interconnection is to be completed with the Public Service of New Jersey which will, to a certain extent, increase power flow both into and out of Philadelphia Electric Company's system at various times of the day.

The transportation of coal from the mines to our urban centers of population and of electrical energy from the sources of hydroelectric supply to these same centers affect all industry, and directly, or indirectly, a large proportion of the country's population.

The more intensive use of right-of-ways for the transportation of these commodities and the successful coordination of the problems involved is a tribute to the managements of the respective companies. Cooperation of this kind can not but result in substantial benefits to the public.

Discussion

Sidney Withington: This paper presents clearly the growing recognition of the fact that many problems which were formerly attacked individually are now recognized as joint problems among those whose duty is that of service to the public. Among other examples, are the problems of electrolysis, inductive coordination between power and communication circuits, and the joint use of wood poles among public utility companies. It is of interest to note that both parties to the arrangement under discussion are thoroughly satisfied. This speaks well for the spirit of the officers of both companies. Such cooperation reacts to the benefit of each company and through it to the public it serves.

Item "g" of the factors involved mentioned by the authors calls attention to rental charged for the occupancy of the railroad right-of-way. It is true that in other instances where such problems are met, the railroads may set an unduly high value on their rights-of-way as potential routes for other utility companies but it is also true that power companies sometimes express the thought that inasmuch as the railroad right-of-way exists and there seems to be room to spare, they should be allowed to locate their lines without obligation to pay any rental. Agreement upon a figure fair to both sides seems sometimes to be rather difficult.

In locating a high-tension line along and over railroad tracks it must be recognized by all concerned that there is introduced a hazard both to railroad operation and to the power facilities, which would not otherwise exist. If a derailment occurs which wrecks a tower and brings the high-tension circuits down upon the train, somebody must assume the responsibility. If the transmission line had not been within reach, the power supply would not have been interrupted, and conversely there might have been less property damage or personal injury to the railroad and its patrons if the transmission line had been absent.

It would be of interest to know what arrangements are made for patrolling and maintenance of the transmission line. Does the railroad allow the employees of the power company to use a car for patrolling purposes and for transporting labor and material or is there sufficient access from highways to the right-of-way at all points to make this consideration unnecessary?

The last paragraphs of this paper bring to mind an interesting picture—that of the transportation of energy along railroad right-of-way in the form of electric power rather than in the form of coal. The railroads are losing a great deal of passenger and freight traffic to highways and canals, to the construction and maintenance of which, incidentally they are obliged to contribute largely. The railroads cannot afford much further impairment in gross revenue without perhaps radical reconstruction of their rate structures, as many of their fixed expenses are independent of traffic. I believe railroads as a whole will look with some concern upon further loss of their load.

W. W. Woodruff: Mr. Withington asks information as to the use of the railroad right-of-way for patrolling and maintenance of transmission lines. In this particular case, the right-of-way is to a large extent sufficiently accessible from highways to make it unnecessary to use the railroad facilities for any ordinary maintenance or repairs. Also, the distance is so short that inspection or patrolling is, from a transportation standpoint, a very minor matter and can easily be done on foot.

A question is raised as to the reduction in railroad revenue resulting from the transportation of energy along the railroad right-of-way and the consequent reduction in coal tonnage. This is a question concerning which I choose to express only a very general personal opinion.

Everyone remembers the old cartoon showing two calves with two pails of milk; both calves trying to drink out of the same pail, with very poor success, while the other pail stands untouched. This applies rather pointedly to the thought expressed.

Translated into economic terms, the lesson is that the attempt by any organization, railroad or other, to block the advance of economic progress, in order to avoid the loss of a present advantage, is not in line with sound economic policy and doomed to eventual failure. Economic situations are always changing and are governed by factors entirely outside the control of any individual industry and any artificial measures introduced or old methods adhered to for the purpose of maintaining the *status quo* only postpones the evil day and makes the final readjustment more severe. A great many very disagreeable situations could have been avoided by recognizing this principle, and proper cooperation between various industries, before the time the situations became acute, could have resulted in benefits to all, rather than dissipating valuable assets.

Initiation of an Electrification Into Operation

BY H. C. GRIFFITH[1]

Non-member

Synopsis.—This paper describes the energizing and putting into service of the Pennsylvania electrification project. The normal and desirable procedure of placing an electrification into service is cited, and the departure from it brought about by the difficulties encountered in actual railway operation are explained.

The operating practise—functions of the power director, his assistants, substation operators, and foremen in connection with switching operations, grounding circuits, carding of circuits, and manner of obtaining releases—is described and illustrated. All of this is very essential to properly coordinate the work and safeguard the personnel of the construction forces, operating forces, and test men during the transition period of completion and placing into operation a railway electrification project.

* * * * *

THE recent completion and placing in service of the Pennsylvania Railroad's electrifications between Philadelphia and Trenton, and Philadelphia and Norristown, has emphasized the importance to the railroad of the transition stage which turns a construction job into an operating job, and which has been called initiation into operation.

The normal and desirable procedure on an electrification construction job would be to complete the construction work with the construction forces, release the construction men, and turn the project over to a test organization who would then energize the various circuits and apparatus and make all necessary potential tests, check control operations, protective relay operations, automatic switching sequences, etc.

Unfortunately, a major electrification project which when completed becomes a part of an electrification system already in operation can seldom follow this logical sequence. The management which has authorized the large expenditure naturally wants the operation to start at the very earliest possible date in order to obtain promptly the benefits for which the project was authorized. This necessitates the overlapping of the test period with the completion of the construction work, and results in having the construction forces working on portions of the circuits and apparatus while other portions are being energized and tested.

The situation is complicated further by the fact that the operating department requires a portion of newly electrified tracks turned over to them for a period before completion, for the operation of special electric trains for the training of crews. Also, if the project involves changing to a new transmission and distribution system for signal power, as was the case in the Trenton and Norristown electrifications, this supply of power must be available some time before completion of the construction work so that the signals and interlockings can be changed over prior to beginning of the operation. This results in having energized signal equipment, transmission, and trolley lines to contend with prior to the release of the construction forces.

To render this transition period as safe as possible for the construction force and others involved, when work of construction on and around apparatus at times energized to a high potential for testing and training operations was necessary, a general plan was prepared and adopted.

The section being electrified was divided into three or four divisions, using points where the circuits were sectionalized as limits. The section which it was desired to energize first was taken as No. 1, the second as No. 2, etc. The construction work program was then so arranged that in so far as possible No. 1 section would be pushed forward to completion—if necessary at the expense of the others; No. 2 would follow, and so on. This reduced to a minimum the amount of work the construction forces had left to do after the section in question had been energized, but did not entirely eliminate it, as test operation developed poor alinement of trolleys, hard spots in the catenary system, errors in substation wiring, and the need for readjustments in switches and circuit breakers.

A definite date and time was then set, after which all wires and apparatus in the first section should be considered energized to high potential at all times. The division superintendents were requested to issue a general order to be posted and signed for by the railroad employees, advising them of the exact time and the territory within which all overhead wires, substations and equipment must be considered energized.

A similar notice was forwarded to the superintendent of the construction forces, who was requested to have each man in his organization sign as having read and understood the notice.

The same thing was done for the field engineering and testing organization.

No voltage was applied to any of the equipment in each particular section until after the date and time specified in the notices applying to that section.

Prior to the date for energizing, various employees of the operating department were given a time allowance to go over in detail the new electrification project in order to become familiar with every phase of it. They were also given prints showing the arrangement and details of circuits. During this time allowance, they were expected to absorb sufficient information to enable them to pass a thorough examination to qualify in the particular class of work which they would be expected to perform when operation began; that is, the em-

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ployees who would operate as power directors were expected to learn thoroughly all phases of the project necessary in the transaction of business as power directors; the substation maintainers and inspectors to qualify along the line of their duties and the linemen along their special line. Each one was given a detailed examination by the supervisory force and was expected to pass this examination in a satisfactory manner.

A short time prior to the date and time specified in the

Fig. 1—Power Director's Headquarters at West Philadelphia Substation

General Notice issued, the power director was put on duty at the regular control headquarters to handle the energizing and testing of the new electrified section. A second and third trick power director were also

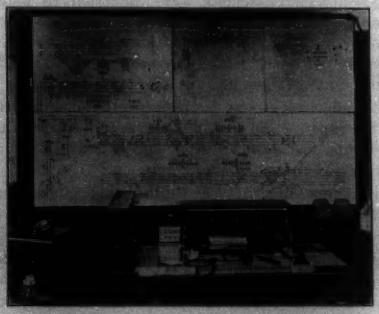

Fig. 2—Detail of Plug Board

Showing arrangements of trolleys and cross-overs with plugs at section-alining and supply points

assigned so that for the full 24 hours each day, each move of energizing various circuits and apparatus and testing of same was done under the control of a properly qualified man.

Qualified substation operators were placed in each substation of the section covered by the order, to take out clearances and ground all high-voltage circuits or apparatus on which construction work remained to be done. The protection of the construction force in each particular substation was made the definite duty of these men, and in a number of cases it was necessary to supply qualified substation men for each of the three tricks in the 24 hours of the day.

In a similar manner a qualified lineman was assigned

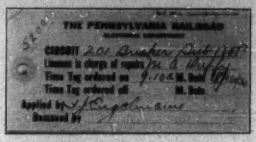

Fig. 3—Standard Clearance Tag to be Applied to Remote-Control Handle for Operating Switch

to each construction wire train, transmission gang, gang installing bridge protection, and each other gang whose duties involved its working over or around energized circuits. When necessary, in order to permit construction work to be performed after portions of the electrification had been energized, these qualified linemen would take out clearances and ground circuits for the performance of work by the construction gangs on or in close proximity to energized circuits.

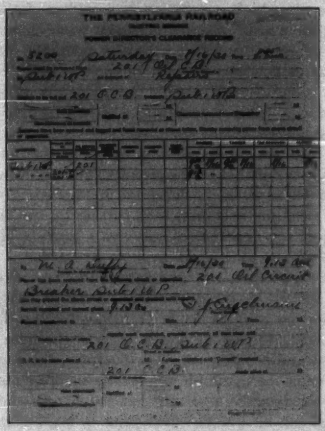

Fig. 4—Clearance Record Prepared by Power Director for Each Clearance Issued

The power director referred to above is located in the West Philadelphia Substation and has complete jurisdiction over the operation of the electrical circuits and apparatus on the section of the electrification to which he is assigned. This is illustrated in Fig. 1. In case of faults developing on any portion of the system, it is his duty to restore power at the earliest possible moment. In case a fault continues, he so sectionalizes

the system that the portion of the system out of service is reduced to an absolute minimum. He must deal with the train dispatcher in the handling of trains in the electrified zone so that restrictions in train operation and

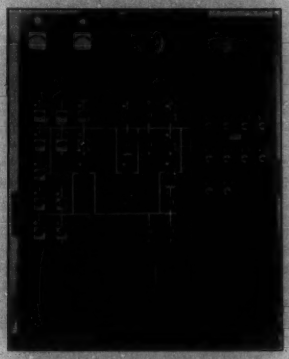

Fig. 5—Control Board

Showing remote-control handle blocked out and clearance tag applied to three control handles

the interruptions to train schedules are reduced to a minimum.

In case of a defective section of trolley developing, he must instruct the train dispatcher as to the condition

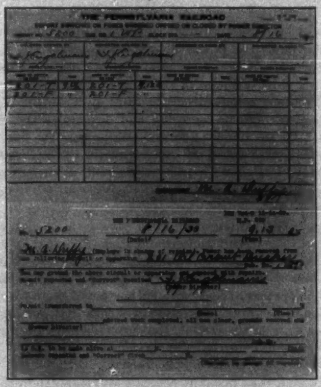

Fig. 6—Operation Report Form

Upper half—Standard switching blank filled out by operator

Lower half—Standard clearance blank filled out by qualified workman taking out clearance

existing in order to prevent the operation of trains into the defective section.

The responsibility for cutting out certain circuits or apparatus for normal maintenance is also a part of his duties. He selects the proper switches to be opened in order to remove power, and instructs the various operators usually located in the signal block towers, to open these switches and tag them prior to issuing the clearance to the qualified workman who performs the work. (Fig. 2.)

Fig. 7—Standard Grounding Stick

Showing clamp for conductor and clamp for ground connection

After the operator reports to the power director that proper switches have been opened and tagged with the name of the workman to perform the work, as ordered by the power director, the power director then issues to the qualified employee in charge of doing the work a

Fig. 8—Grounding Sticks Applied to High-Speed Trolley Circuit Breaker

formal clearance. (Figs. 3 and 4.) The switches thus tagged must never be closed until the removal of these tags at the instruction of the power director has been accomplished.

The power director is forbidden to order closed any switches, on which a clearance has been issued, prior to the formal release of the clearance by the qualified employee to whom it has been issued.

This rather elaborate formality has been found necessary to prevent the accidental closing of any switches energizing circuits on which men are working

and has proved to be an adequate safeguard. (Fig. 5.)

Where qualified substation operators and linemen are required to operate outlying disconnecting and sectionalizing switches not controlled from a remote control board, these operations must be performed only under definite instructions from the power director and a proper record kept on a switching report blank. The switching report blank and the clearance blank when completed must be forwarded at once to the operating headquarters for filing. The power director's record and the qualified employee's record of the clearance issued are checked and must be similar in all details. This checking is done periodically to prevent the development of carelessness on the part of any of the men involved in the handling of the clearances. (Fig. 6.)

After the qualified employee has received his clearance, before beginning work on energized equipment or circuits, he must apply the standard ground sticks which are supplied for that purpose. Upon completion of the work, these ground sticks must be removed before formally giving up the clearance under which he is working. (Fig. 7.)

The ground sticks are designed with a clamping hook on the end of a pole and an extremely heavy cable connecting to a clamp which clamps to a grounded connection or structure. Experience over a number of years has definitely proved that if these sticks are properly applied, the workmen will at all times be fully protected, not only from accidental energizing of circuits, but also in case of mistake in segregating the circuit or apparatus and the power not being removed. Ground sticks as illustrated in Fig. 8 have been applied to high-voltage circuits while energized without damage to the equipment or injury to the man involved.

The above general procedure has been reviewed in order to show the safety precautions found necessary in the handling of a major electrification project. It has been the experience that in spite of all precautions and safety features, accidents around high-tension circuits will happen and every effort must be made at all times to keep those directly involved in their operation thoroughly familiar with these hazards and compel strict adherence to all safety regulations. There is no period to which this applies more strongly than it does to the period of initiating electrical operation and closing out the final details of the construction work.

Discussion

Sidney Withington: Mr. Griffith calls attention to the haste on the part of railroad managements to place facilities in operation at the earliest possible moment as they approach completion. When it is considered that the capital charges against any given project begin as soon as expenditures start and that they are a direct function of the money expended, it is readily seen that these charges may reach to a very considerable sum per day when an installation of several millions of dollars is involved. Any time saved in placing the facilities in revenue service represents real money and is worthy of considerable effort.

Mention is made of the relations between the power director or load dispatcher and the train dispatcher. It is obviously very necessary for these two positions to function with the closest cooperation and when it can be accomplished without too much sacrifice it may be desirable to locate the two men in the same or immediately adjacent offices. The most reliable communication facilities are of course essential in any event, for defective communication inevitably means delay to train operation.

One of the outstanding characteristics of work on railroad power circuits having rail return is the absolute protection which each man has by the application of the ground stick described by Mr. Griffith. With this device the men working on such circuits are absolutely safeguarded in event of accidental energizing of the circuit. Each man can be made entirely and personally responsible for his own protection.

It would be of interest if Mr. Griffith were to mention the routine followed by the line crews after they have obtained a circuit upon which to work. Is the foreman provided with blank forms to fill out for the information of the linemen? On the New Haven Railroad the foreman gives to each lineman in his gang a carbon copy of a memorandum which states specifically what circuits have been deenergized and grounded. This does not relieve the individual lineman of responsibility for personally seeing that the ground stick is applied, but the act of the foreman writing out the information and the act of the individual lineman in receiving it, we feel, impresses the situation upon the minds of both and is of psychological value.

H. C. Griffith: The procedure followed by the line foreman in obtaining his circuit on which to perform work is as follows:

He telephones the power director, requesting the particular circuit desired, advises the general nature work to be performed and gives a close estimate of the length of time required. The power director, if conditions are suitable, orders open and tagged the proper switches to segregate and deenergize the circuit in question. He then issues a formal clearance to the line foreman who repeats same back to the power director. The foreman then advises each of his men the particular circuit or circuits on which he holds a clearance and points out the adjacent circuits that are not deenergized.

The grounded pantograph on the wire train is then raised to contact with the circuit and ground sticks installed on each side of the wire train. Each workman has definite instructions that he must personally insure himself that proper grounds have been installed before he proceeds with his portion of the work. At the completion of the work, the foreman checks that each individual lineman is clear of the circuit, that grounds have been removed and advises each man that the clearance is being given up, and the circuits must be considered energized.

The foreman then formally releases his clearance to the power director who then orders the necessary switches closed to energize the circuit and restore normal set up.

The Modern Single-Phase Motor for Railroad Electrification

BY F. H. PRITCHARD* and FELIX KONN*
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The past three years have seen a great increase of activity in the field of alternating current electrification of steam railroads and, consequently, much attention has been given to the design of the single-phase traction motors upon which the success of this type of electrification chiefly depends.

The object of this paper is to describe the up-to-date motors which have been developed to meet the requirements of modern American railroading.

IN order to meet the demands on traction motors incident to announced single-phase electrification programs, considerable developmental work has been done. Sample motors were built and extensively tested and a sample multiple-unit car was completely equipped and put into regular passenger service.

"Transformed" Voltage

The biggest handicap that besets single-phase commutator type motors lies in the presence of an alternating voltage between heel and toe of the brushes, which

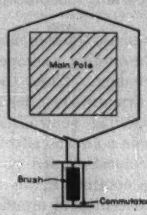

Fig. 1—Developed View of an Armature Coil Undergoing Commutation in Its Relation to the Face of the Main Pole

can be corrected at speed but which cannot be avoided at standstill.

This is illustrated in Fig. 1 which shows an armature coil undergoing commutation.

The torque field winding which is in series with the armature produces the torque flux. This flux instead of being constant, as in d-c. motors, is alternating and induces by transformer action a certain voltage in each armature turn. This "transformed" voltage has the maximum amplitude in the coils short-circuited by the brushes since the plane of these coils is at right angles to the direction of the flux.

With a sinusoidal flux wave of 25 cycle frequency, the r. m. s. voltage induced in these coils is:

Transformed voltage per turn = 1.11 ϕ maximum, where ϕ maximum is the maximum amplitude in megalines of the torque flux. If the brush short-circuits two adjacent turns, that is, if it touches three commutator bars, the voltage which is impressed between heel and toe of the brush is twice the voltage induced in each turn.

The transformed voltage is 90 deg. out of phase with the flux, that is, with the motor current.

Starting Commutation

We will first discuss standstill or starting conditions: Fig. 2 shows what happens to a typical brush between heel and toe of which an alternating voltage is impressed. A brush of the size used in the test normally carries a load current of 50 amperes and under overloads will carry up to twice that current.

If a heel-to-toe voltage is applied continuously, glowing will occur from about 2.5 volts up, and the glowing will be become very violent above the knee. Severe pitting and softening of the commutator bars will occur.

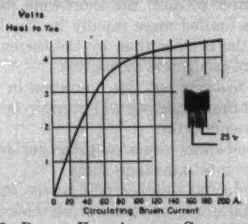

Fig. 2—Brush Volt-Ampere Characteristic

It is obvious that this condition, as far as its effect on the commutator bars is concerned, is most pronounced at standstill.

The curve shown in Fig. 2 was taken without any load current flowing through the brush; the presence of a load current increases the circulating current at any voltage.

Practical Limitations of the Transformed Voltage

Earlier designs of single-phase traction motors used heel-to-toe voltages of 10 volts or more and the present motors use values which are, roughly, half these.

It will be seen from the above that even these lower

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voltages, if maintained indefinitely, would damage the commutator. However, the operating experience abroad and the experience so far gained in this country with motors designed for these new constants indicates that, in service, the actual time during which these excessive circulating brush currents flow is well within safe limits.

Thus, it is seen that the design of a single-phase traction motor will largely hinge around the necessity of keeping the transformed voltage, from heel to toe of the brush, down to a permissible figure. In the motors under consideration, the brushes cannot short-circuit more than two coils and the transformed voltage per turn at standstill does not exceed 3.25 volts for the locomotive motors and 2.75 volts for the multiple-unit car motors.

Number of Brush Holders

These values of transformed voltage per turn at standstill determine the flux per pole under starting conditions. The torque of a motor is proportional to the product:

Total armature ampere-conductors times number of poles times the flux per pole.

The last term of this product is determined as we have just shown. Thus, we see that for an armature with a definite ampere-conductor loading, the starting torque goes up directly with the number of poles, or, in other words, with the number of brush holders.

Commutator Constants

As a matter of fact, the short-time loading of an armature is limited more rapidly by the load current density in the brushes than by the current density in the armature conductors themselves.

In order to bring out this limitation in the formula which gives the torque, we will express the total armature ampere-conductors as:

Total conductors times amperes per brush holder or, what amounts to the same thing:

Total conductors times current density in the brushes times commutator length times brush thickness.

If we substitute the above expression in the torque formula, we find that the latter is proportional to:

Total conductors times current density in the brushes times commutator length times brush thickness times poles times flux per pole.

It should be understood that all single-phase traction motors have armatures wound with one turn per coil; also, that the brush thickness is generally taken as twice the commutator bar pitch.

Hence, the brush thickness is generally proportional to the expression:

$$\frac{\text{Commutator diameter}}{\text{Total conductors}}$$

Substituting this expression in the last product, we find that the starting torque is proportional to:

Commutator length times commutator diameter times poles times current density in the brushes times flux per pole.

The last two terms are determined by the characteristics of the brushes; thus, we see that the starting torque which can be obtained from a single-phase traction motor is proportional to the area of commutator and to the number of brush holders.

For a given commutator diameter the new motors have from 50 to 100 per cent more brush holders than earlier motors and, as the disturbance at the commutator is proportional to the square or to some higher power of the flux per pole, the starting commutation is greatly improved.

Running Commutation

When the motor is running we must deal with two elements of commutation:

The a-c. or *quadrature* element of commutation, caused by the transformed voltage, is still present and we will show how this element can be effectively compensated; also, we are dealing with the *in-phase* or *rotational* element of commutation which is of the same nature as the sparking reactance voltage in a d-c. motor.

The modern single-phase traction motor has, in addition to the torque field winding, a compensating winding and an interpole winding. These two windings together neutralize the armature reaction very effectively; the compensating winding neutralizes the armature reaction under the main poles and effects a considerable improvement in commutation and power factor over that which would be obtained if the same number of ampere-turns were concentrated around the interpole. The interpole winding neutralizes that portion of the armature reaction which is in excess of the compensating winding ampere-turns and in addition provides a surplus of ampere-turns. This surplus produces a flux under the interpole which develops in the coils undergoing commutation a voltage to neutralize the reactance voltage induced by the reversal of the current, exactly as in a d-c. motor.

Rotational Element of Commutation

It should be strongly emphasized that the commutation of a single-phase motor can be no better on alternating current than it is on direct current. Consequently, in order to design a good a-c. commutator type motor, it is indispensable to put into it, first, all the elements which give good commutating characteristics to a multiple-wound d-c. motor, such as the following:

For the armature:

An odd integral number of slots per pair of poles,

An odd number of turns per slot,

A back-end winding pitch other than a multiple of the number of slots,

Sufficient fractional pitch to minimize the mutual inductance of coils undergoing commutation,

Adequate equalization.

For the stator:

A correct design of interpole having a face which properly covers the commutating zone backed up by a core dimensioned practically to obviate saturation,

Adequate neutral space between interpole and main pole tips.

For a better understanding of the rotational element of commutation in a single-phase motor, let us consider the variations of the current in any armature conductor, as illustrated in Fig. 3.

Assume that the peripheral speed of the commutator is 6,000 ft. per min., that is, 1,200 in. per sec., and that the brushes are ⅜ in. thick. The reversal of the current takes place in the time that it takes for a point on the commutator to pass a brush less the thickness of a mica, that is, in:

$$\frac{0.375 - 0.030}{1200} = \frac{1}{3500} \text{ second.}$$

With a 25-cycle power supply each alternation lasts 1/50 sec. In other words, the current reversal in commutation is 70 times faster than the reversal due to

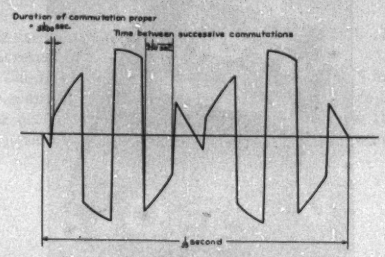

Fig. 3—Variations of the Current in One Armature Conductor

the alternating supply. If we look at a sine wave and pick two points $\frac{180 \text{ deg.}}{70} = 2\frac{1}{2}$ deg. apart, we realize that there is very little difference between their ordinates. In other words, during the time that a coil is short-circuited by a brush, the motor current varies so little that it might just as well be direct current. Fig. 3 also shows that, at normal speed in the example chosen, the frequency of commutation (250 reversals a second) is much higher than that of the supply. In other words, the fact that we are dealing with 25 cycles should not affect materially the phenomenon of current reversal which is 70 times faster and which occurs 10 times more frequently.

From the above, one could expect approximately the same excess ampere-turns in the compensating and interpole windings to commutate 25-cycle alternating current as to commutate direct current. This has been proved very conclusively by test.

Transformed Element of Commutation

To accomplish the compensation or neutralization of the transformed element of commutation, an external resistor shunting the interpole winding is used.

The current in the resistor leads the line current and the current in the interpole winding lags behind it.

Fig. 4 shows how the current in the interpole winding has two components; one, in phase with the line current, the magnitude of which is adjusted to take care of the rotational element of commutation, and one, 90 deg. out of phase with the line current. This latter

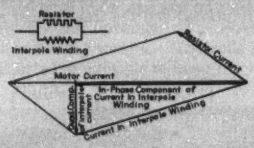

Fig. 4—Vector Diagram Illustrating the Phase Shift Obtained by the Use of the Shunting Resistor

component produces a flux which, by virtue of rotation, generates a voltage in the coils undergoing commutation, in this same phase relation, that is, 90 deg. out of phase with the line current. This opposes the transformed voltage both in phase, and, for definite speeds, in magnitude.

The simplicity of this scheme has given it a very wide application and practically all modern single-phase traction motors make use of it.

Theoretically, this scheme accomplishes a correct neutralization of the transformed element of commuta-

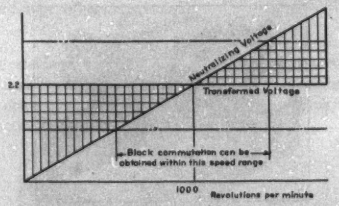

Fig. 5—Effect of Speed on the Neutralization of the Transformed Element of Commutation

Transformed volts per turn. The shaded areas denote under or over compensation

tion at all loads but only at one speed because the neutralizing voltage is proportional to the speed while the transformed voltage is not affected by the speed at a given load.

Fig. 5 illustrates this condition for a motor which is correctly compensated at 1,000 rev. per min. Below this speed there is a condition of under-compensation which is, of course, most pronounced at standstill. Above this speed there is over-compensation.

The fact should be kept in mind, however, that it is not necessary to have quite perfect neutralization in

order to obtain black commutation since it takes a certain voltage to produce a spark across the copper-brush contact. The two horizontal lines drawn above and below the line of ordinate 2.2 volts give an idea of the range of uncompensated voltages within which black commutation can be obtained. The lower the flux per pole, the wider is the corresponding speed range. Thus, improving the starting commutation also improves the running commutation.

Commutation Settings

It can be seen now how it is possible to segregate and correct the rotational and the transformed elements of

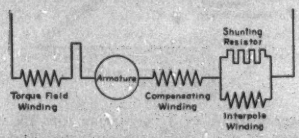

Fig. 6—Diagram of Connections of a Single-Phase Traction Motor

commutation in order to obtain black commutation over a wide range of speeds and loads. This consists in the adjustment of the number of turns of the interpole winding and of the value of the shunting resistance.

For motors required to operate over a very wide range of speeds and loads, two commutation settings are used. A change in the value of the shunting resistor and the introduction of a reactive shunt are effected by the control switches. Locomotive motors re-

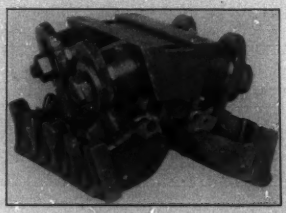

Fig. 7—Brush Holders and Supports with Micalex Insulators

quired to develop full rated horsepower up to the maximum operating speed come in this class.

The losses in the shunting resistor vary from one to two per cent of the motor input, the lower figure applying to the larger size and higher speed motors.

Diagram of Connections

The diagram of connections of the modern single-phase traction motor is a very simple one. All windings are in series; the shunting resistor is permanently connected across the interpole winding and there is no change in motor connections throughout the operating cycle.

Motor Voltage

Because of the limitation of the flux per pole, the single-phase traction motor is, inherently, a low-voltage machine. For example, the rated voltage of a locomotive motor is 225 volts and that of a multiple-unit car motor with a lower flux per pole is 170 volts.

If similar constants of design are used (flux per pole, commutator peripheral speed, commutator bar pitch) the voltage of machines of different sizes will be the same regardless of the number of poles and brush holders.

Power Factor and Efficiency

At their continuous rating, these motors have a power factor of approximately 95 per cent and an efficiency at the shaft of 85 to 90 per cent, the higher efficiency figures applying to the larger size and higher speed motors.

Field Shunting

Under severe starting conditions it may be advisable to weaken the main field at starting, preferably by

Fig. 8—View Showing the Accessibility of Brush Holders

means of a saturated inductive shunt, in order to keep the flux, and, hence, the transformed voltage, below a certain limit. In this case, the motor must draw a heavier current to develop the required torque but there is a net gain in commutation.

In some locomotive equipments being built the connections are changed from weak field to full field by means of a control relay which operates as a function of the locomotive speed. The relay is set to operate at the speed at which the commutation in the full field connection with its lower load current equals the commutation in the weak field connection with its lower transformed voltage.

Motor Construction

These features of design have been worked into a 1,250-hp. twin-armature locomotive motor and a 220-hp. axle-hung multiple-unit car motor, the construction of which is shown by the illustrations on these pages.

Brush Holders

In order to find room for the larger number of brush holders, provide proper clearances and allow for easy inspection and renewal of brushes, a new design of brush pressure finger has been worked out. At the same time, much greater uniformity of brush pressure as between new and worn brushes was incorporated in

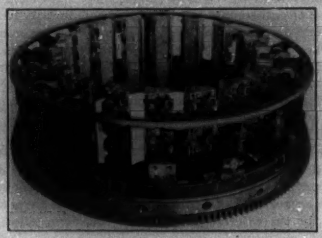

Fig. 9—Brush-holder Yoke for 1,250-Hp. Twin Armature, Locomotive Motor

the design and more accurate spacing of brush holders has resulted.

The brush holder and supports shown in Fig. 7 have been applied to the motors on the sample multiple-unit car previously referred to and Fig. 8 illustrates their accessibility. Their performance in actual service

Fig. 10—Rotor of 1,250-Hp. Twin-Armature Locomotive Motor

has been very gratifying; no trouble whatever has developed, the commutators run with a highly polished surface and the very small brush wear indicates at least two years of brush life.

The locomotive motor brush-holder yoke shown in Fig. 9 is arranged to revolve by means of gearing to facilitate inspection from one opening in the motor frame.

Armature

The armature winding is made up of folded cross-over bars which minimize eddy currents. The joints at the back end are silver soldered. There are three coils per slot and the armature slots are spiraled one-half slot pitch to minimize telephone interference and to smooth out the interpole flux. Fig. 10 shows a complete rotor.

Stator

Fig. 11 shows the 18 main poles and interpoles in each stator of the twin locomotive motor.

All stator windings are composed of rectangular conductors wound into coils of the requisite number of turns, insulated with mica tape and introduced into the slots through the slot openings. The slots are closed with bakelized canvas wedges. The coils of each circuit are connected in multiple to bus-rings by means of

Fig. 11—Frame of 1250-Hp. Twin-Armature Locomotive Motor with Stator Punchings Ready to Receive the Stator Windings

silver-soldered joints. After winding the stator is impregnated by the vacuum process with an insulating and water-proofing compound.

Bearings

The modern single-phase motors use grease-lubricated anti-friction bearings resulting in reduced bearing maintenance and maintaining uniform air-gaps which are of considerable aid in commutation.

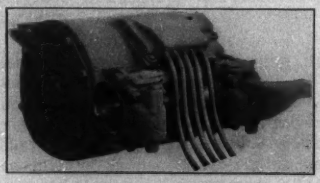

Fig. 12—215-Hp. Multiple-Unit Car Motor

Capacity

The horsepower capacities of these motors compare very favorably with those of d-c. traction motors of similar weight. For example, a 220-hp. multiple-unit car motor weighs 5,600 lb. and a 625-hp. freight locomotive motor weighs 14,000 lb. These weights do not include the gear, pinion, gear cover and axle linings.

Conclusion

It will be noted that in the attempt to improve the single-phase motor the greatest effort has been applied to the commutation at starting. Fortunately, this also aids the commutation at speed.

The other problems encountered in the design and construction of single-phase motors are similar to those met with in all other kinds of traction motors.

With this very material improvement in commutation, the single-phase traction motor can be counted upon to give good reliable service with low maintenance cost.

Discussion

C. E. Skinner: The paper forms an interesting presentation of the relations that exist in the single-phase series commutator motor and the successful results indicated are notable.

The fundamental principles have been rather well known. In an address in February 1908 Mr. B. G. Lamme stated: "The broad statement may be made that it is no more difficult to commutate an alternating current than an equal direct current."

Obviously, if the flux per pole be kept to a sufficiently low value, it is possible to obtain black commutation.

The recent advances in the art which permit a close approach to the desired value of low flux per pole are:

1. Increased peripheral speed of commutator.
2. Increased peripheral speed of armature.
3. Increased number of armature ampere turns per inch of armature diameter.
4. Use of anti-friction bearings.
5. Development of suitable auxiliary and control apparatus.

Increased peripheral speed of commutator has permitted a larger commutator diameter for any given rev. per min. as well as an increased number of bars per commutator. The increase in diameter of commutator together with improvements in detail of construction has permitted the placing of more brush holders around the commutator.

The increased number of bars per commutator, together with improvements in mechanical structure and in insulation, has permitted a greater number of conductors per armature.

The increased peripheral speed of armature obviously obtains a given horse power with less torque at the air gap.

The increase in armature ampere turns, both from increase in conductors and in amperes per sq. in., permits a decrease in total flux required for any given horse power.

The use of anti-friction bearings insures uniformity in air gap.

A uniform air gap lessens losses and duty on the cross equalization. This also insures more uniform current balance in the main and interpole field coils and hence improves the rating of the stator.

With the increase in number of poles of any given motor the external amperes input to the motor increases since the current per brush arm is maintained at the same value.

It is of interest to note that some twenty years ago Mr. R. E. Hellmund developed and tested, a low flux motor for multiple unit car service. This motor was not considered commercial at that time because the unit switches, reversers, etc., available and required for the external current involved, were objectionably large for car mounting.

During recent years the intensive development program warranted by important electrification prospects has secured the reduction of flux per pole to the permissible economical value obtainable with present developed limits of over-all design of both motors and control.

The paper of Mr. H. G. Jungk, printed on page 278, forms an interesting contribution to this whole subject.

The pioneer motor which has stimulated this development was constructed of full capacity about three years ago and was demonstrated to interested parties shortly thereafter and led to the construction of the high-speed locomotives which are now in operation.

The operating results have been most successful and some 20,000 miles service has already been reached.

A Cooperative Electrolysis Survey in Louisville, Kentucky

BY W. C. WHITE[1]

Non-member

Synopsis.—*A cooperative electrolysis survey in the city of Louisville, Kentucky, under the direction of an electrolysis committee is described. An analysis of a portion of the survey data and indicated mitigation measures are given as typical examples. The advantages of cooperative action in a general electrolysis survey are shown.*

Introductory

THE electrolysis problem has existed in the city of Louisville probably as long as in any other city of the United States since Louisville was the third city in which street railways were electrically operated. Up to 1926, the method of attack had been largely upon the basis of individual action, by each utility owning underground metallic structures, in an attempt to mitigate specific cases of electrolysis hazard. While these had resulted in generally satisfactory conditions of the underground cable plants, they had not been as successful with respect to water and gas pipes. The absence of complete electrolysis data covering the city of Louisville augmented the difficulty of reducing the hazard to the latter structures.

It was realized that the best engineering solution of the electrolysis problem as affecting all utilities could be obtained only upon the basis of very complete data covering the electrolysis condition of the affected structures and detailed information concerning the railway system, including those characteristics of the system affecting the magnitude and direction of stray return current. It was for these reasons that the Gas and Electric Company suggested a conference of all interested utilities to consider the desirability and practicability of cooperative action in the making of a comprehensive study of the Louisville electrolysis situation and determining suitable measures for improving conditions generally. Such a meeting was held early in 1926 and attended by representatives of all interested utilities including the City Water Department, the Railway Company and the Louisville Public Utilities Bureau.

The outcome of this conference was the formation of the Louisville Electrolysis Committee consisting of executives representing each interested utility with the chief engineer of the Louisville Public Utilities Bureau as chairman. This committee appointed a working or technical committee of engineers representing each utility and included a consulting electrolysis engineer retained by the Gas and Electric Company. The expense of the investigation was prorated among the interested utilities through the furnishing by each utility of its prorata share of engineering, labor, tools, meters, circuits, and materials as agreed upon by the general committee. This obviated the necessity for special accounting and inter-billing among the various utilities.

The technical committee prepared an outline of a proposed test procedure which was endorsed by the general committee and steps were taken to put it into operation. The first item in the outline of test procedure called for a general check up of the various underground structures by the individual utilities in order to correct so far as possible all obviously undesirable or abnormal electrolysis conditions. This check up included on the part of the railway company a comprehensive track bond test of their entire system and the repair of all bonds found defective; on the part of the cable owning companies investigations to make certain that all cables were properly cross bonded, that electrical drainage connections were in satisfactory operating condition and that there were no undesirable metallic contacts to other underground structures; on the part of the pipe owning utilities investigation of all known or suspected electrical drainage connections to determine which of these connections were in effective operating condition.

When the above work was completed, the technical committee started the field work of the general survey, which in general comprised a potential survey, a current survey, over-all track drops and gradients, and soil resistivity tests.

Potential Survey

A potential survey was made of the entire city, tests being made at more than 600 locations.

It is desired to emphasize the importance of a potential survey in the investigation of an electrolysis problem. The potential survey is the qualitative indication of the condition of the various sub-surface structures at the point and at the time tests are made. Therefore, for all data to be comparable, tests, in general, should be made under normal street railway load conditions, and the potential relations of the various structures at each test location measured simultaneously. The potential readings may be taken either with indicating or recording instruments. In the Louisville investigation, due to both the number of structures involved at each test location and to the large area to be surveyed, it was

1. Southern Bell Tel. & Tel. Company, Atlanta, Georgia.

Presented at the Southern District Meeting No. 4, of the A. I. E. E., Louisville, Kentucky, November 19-22, 1930.

found advantageous to use recording instruments. Twelve recording meters of the smoked chart type were mounted on a truck and suitable switching arrangements provided for connecting these by flexible insulated conductors between various structures and between structures and earth (See Fig. 1). To assure that connections to the various structures were of low resistance, provision was made for testing such connections with indicating meters and batteries. Six-hour clock movements were used. As the period of test at each location was approximately 20 minutes, this permitted a full day's test of each particular structure at approximately 10 locations to be recorded on a single chart.

Test locations were selected approximately 1,000 ft. apart in the outlying areas and approximately 500 ft. apart in the more congested areas, or in areas of positive-to-rail potentials in the case of piping systems

Fig. 1—Typical Truck Equipped for Use in Cooperative Electrolysis Survey

and positive-to-earth potentials in case of cable systems.

The criterion of electrolysis hazard to metallic subsurface structures is the potential thereof to immediately adjacent earth. The best practical approximation of this in the case of lead sheathed cables is the measurement of potentials to earth via the bottom of a manhole or an adjacent spare duct. In the case of pipe systems, due to the fact that they are in such intimate contact with the earth that these values are relatively small and due to the usual inaccessibility of adjacent earth, the best practical index is the potential to trolley rail and other nearby structures.

At each location, records were made of potential differences between all structures and rail, between all structures and water pipes, (the water system having been selected as a reference due to its being the most extensive system), and between all cable systems and earth. It is the purpose of cable-to-rail readings to determine which structure is probably the disturbing element *in a particular location* as well as to determine the points where drainage may be most advantageously applied; for instance a cable may be positive to earth yet negative to rails, which would indicate some other structure as the disturbing element. The use of water pipes as a reference system provided for tie readings which served as a check on the reliability of all readings taken.

Current Survey

A current survey of sub-surface structures supplements the potential survey in determining probable hazardous conditions, and furnishes valuable data for consideration in the application of remedial measures.

A current survey of cable systems is a relatively simple procedure and in a great many cases will indicate hazardous conditions where they would not otherwise be located. For example, the potential survey may indicate safe conditions at the points of measurement, while the current survey may show an abrupt change in the magnitude, or even a reversal, of the current in adjacent manholes such as to indicate hazardous conditions between the two manholes.

Current surveys of pipe systems are generally difficult and expensive to conduct unless suitable test stations have been provided for this purpose, since the readings require the exposure of a suitable length of pipe usually by excavation. For this reason current test stations should be installed on pipe systems, either when maintenance work is being done on existing lines or during the installation of new lines, their proper location having been previously determined by a study of the pipe systems in conjunction with the railway system.

Approximately 72 such test stations were installed on the water and gas systems at Louisville and 24-hr. smoked chart records obtained of the current. Also in the study of the problem numerous potential drops were taken between services for determining the direction of the current.

Over-all Track Potential Drops and Track Potential Gradients

The magnitude of stray currents, in general, depends upon the magnitude of over-all track potential drops and track potential gradients. High over-all track potential drops and track potential gradients result from long feeding distances with inadequate return circuit conductivity and heavy concentration of return current in the rails.

A large number of over-all track potential drops and track potential gradients was made on the Louisville Railway system. The over-all drops were 24-hr. records and were made from various points of the track system to the negative bus, or to the point of low potential in the case of insulated return feeder systems. The potential gradient measurements were also made with recording instruments and covered periods from 30 min. to one hr. under peak load conditions. These data are not only necessary in the study of a general electrolysis problem but are of value to the railway

engineers in the design and maintenance of their negative feeder system from the standpoint of railway economies. In the design of a feeder system, the peak load conditions are in general the determining factors.

The obtaining of over-all track potential drop data would generally be difficult other than in a cooperative investigation where the telephone company can furnish spare cable pairs to various locations to serve as voltmeter leads. These leads, of course, should be tested each time use is made of them to insure reliable results.

Soil Resistivity Tests

Soil resistivity tests were made of some 180 samples and while there was a wide range in the resistance of some samples, it was found that for the most part the soil averaged about 5,000 ohm cm. In general this would not be considered a high resistance soil and from the fairly consistent results obtained it would appear that the potential measurements gave directly a fair indication of relative stray current interchange between affected structures and that the resistance of the soil did not by itself account for the areas of high-or-low potential differences between structures.

Assembly of Test Data

At the time potential or current tests were made at selected locations, a card was prepared giving the number and location of the test point and cross referenced with the corresponding smoked chart records. On the reverse side of this card was shown a diagram of the test point and the location of structures to which connections were made. This was done in order that any test could be repeated with identical connections at any time. At the conclusion of each day's test the smoked chart records and cards were sent to the office where the data on the smoked chart records were averaged and recorded on the cards.

Those portions of the data capable of graphical presentation were plotted on suitable maps. These maps were skeleton maps of the street railway system. The water and gas pipe potentials were plotted to scale using the railway track system as a base line. The ordinates of positive potentials were shown to the north and east of tracks and negative potentials to the south and west These ordinates were then connected with straight lines and the positive areas colored red and the negative areas blue. The cable data were plotted in a like manner using, however, duct runs as base lines and plotting the potentials of cables to rails and earth. Over-all track potential drops and gradients were also shown on the maps but the actual numerical values were used.

This manner of presenting data is of advantage, especially in a cooperative study, as it furnishes a composite picture of conditions and assists in determining the disturbing element in any particular location.

Analysis of Data and Mitigative Measures

The technical committee arranged to meet once each week during the field survey, for the purpose of considering the data as they were obtained and to interchange information as to the operation or rearrangements of any plants which might affect the electrolysis situation.

When sufficient data were obtained in any section from which definite conclusions could be reached, mitigative measures were recommended in order that these might be applied as the survey progressed.

While it will not be possible to go into the details of this phase of the work some of the more important points considered in the analysis of data and in the application of remedial measures may be mentioned.

From various data obtained such as track drops and gradients, drainage currents, and potential readings it was found that a very considerable improvement in the electrolysis conditions could be made by beginning the operation of the Number 3 substation of the railway

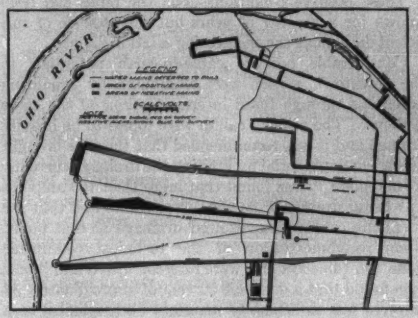

Fig. 2—Illustration of Graphical Presentation of Data

company, one hour earlier in the morning and continuing its operation until one hour later in the evening, and by operating the Number 2 substation for a 24 hr. period instead of part time. This recommendation was carried into effect.

From a study of the track drops and gradients it was felt that some of these were excessive. Additional substations, with the resultant shortening of feeding areas, were recommended as a means for reducing the high over-all track drops and these are now being considered.

Even where over-all track potentials are not of themselves excessive, unequal drops on long paralleling or approximately paralleling lines may produce hazardous electrolysis conditions. This is illustrated by the following case which also shows the necessity for complete reliable data in the solution of the problem.

In the Number 3 substation area there are three long

paralleling lines, *viz.:* the Broadway, Madison and Market Street lines, with no cross lines for interconnection. The potential survey showed that positive pipe-to-rail potentials existed throughout the length of the Madison Street line, which was the center line. The over-all track drops to the Number 3 substation negative bus were approximately 9, 3, and 8 volts respectively from Broadway, Madison and Market Streets (see Fig. 2). The Broadway and Market Street Lines were of older construction and heavily loaded while the Madison line was of new welded rails and lightly loaded. It was therefore evident that the positive potentials were due to shunting currents from the Broadway and Market lines to the lower potential line on Madison Street. The method which was recommended and carried out for correcting this condition provided for additional negative feeders to the Broadway and Market lines, insulating the negative bus in the Number 3 substation and installing resistance grids in the short feeder attached to rails near the junction of the Madison Street line.

These positive conditions upon the pipes had been known for a number of years but the absence of sufficient data made the engineering solution difficult.

Considerable positive cable-to-earth potentials in the downtown section were eliminated by equalizing potentials through fused interconnections of the cable systems. The railway cables were found to be heavily drained and it was recommended that these be brought nearer earth potential by limiting the drainage currents. At one point it was found that more than 100 amperes was being drained from the power cables and that the telephone cables also required drainage at that point. Upon recommendation of the technical committee these two cable systems were interconnected and it was then found that a drainage current of approximately 35 amperes was sufficient to maintain both systems at a satisfactory negative potential to earth.

While reduction of stray current is of primary importance in mitigating electrolysis hazard to underground structures, cable systems usually require, as a supplementary measure, the application of a certain amount of drainage for the reason that the corrosive action on lead is rapid and the sheath relatively thin. Fortunately a cable system lends itself very readily to drainage due to the continuity and good conductivity of the sheath. In general, cable drainage should be so designed as to keep the cable system slightly negative to earth.

It has generally been found that electrical drainage is only a partially effective measure in the protection of pipe systems. With pipe systems a more satisfactory supplementary measure, to the limitation of stray current in the earth, is the reduction of pipe system conductivity by the installation of insulating or high resistance joints. While there may be some isolated cases where drainage would be beneficial, pipe drainage should be resorted to only after careful study and the installation should be so made as to be readily accessible for test purposes.

Conclusion

In conclusion, it is worthy of note that in the conduct of the general survey, of subsequent special tests, and of all the work undertaken by the Technical Committee, the active interest and cooperation of the represented utilities were continuously in evidence. In a large measure the results attained and the work accomplished were due to this interest and cooperation. Realizing that it would be a serious loss to the cooperating utilities individually, and collectively if the benefits gained through this cooperation were allowed to lapse, the Technical Committee has continued in active existence and functions as a central clearing house for the investigation of all matters relating to electrolysis in the city of Louisville.

Electric Power in the Lumber Industry

BY A. H. ONSTAD*

Non-member

Synopsis.—In the preparation of this article an attempt has been made to show the development or growth of the use of electric power transmission in the lumber industry. More particularly in that branch of the industry that is engaged in falling trees, hauling them to the mill and sawing these trees into lumber units of the size used by builders and manufacturers of finished articles in which wood is the raw material.

An attempt has been made to show the general scope to which electricity is used, without including any technical descriptions of individual applications. It was felt that this can be best accomplished by a general description of its use throughout one of the most recently constructed and most modern manufacturing units in the Pacific Northwest.

* * * * *

THE advantages of the use of electric power transmission and electric drive in the lumber industry are many. Fuel economy, however, has not, until very recently, been counted with the advantages.

When considered, only from the angle of fuel economy, electric power had no advantage, except in a few isolated instances, over any other power as applied to the lumber industry until within the last decade. The raw material of the lumber industry, namely trees, is one of the earliest fuels used, and is still used as such in large quantities outside of the lumber industry.

The lumber industry has such an over abundance of this fuel that when cutting the trees into logs, preparatory to hauling them to the mill, the logger is very careful not to send to the mill logs that will not produce a high percentage of merchantable lumber. The sawdust produced in sawing boards is usually enough for all power needs. Any additional wood waste must be burned in refuse burners which are expensive to build and maintain and which usually scatter partly burned particles of wood over a great area with many attendant evils.

It would be hard to say when electricity was first used in the lumber industry. The writer recalls "shooting trouble" on an Edison belted bi-polar machine, which was connected to its load through a switchboard having the switches and such instruments as there were, mounted on a wooden panel, having an elaborate molded trim which would be a credit to any cabinet maker or furniture maker. When installed, this equipment was the last word in electrical equipment.

The advantages of the use of electricity for the distribution of power around a lumber manufacturing plant, aside from that of fuel economy, was early recognized and its use for this purpose began about twenty-five years ago. Prior to the adoption of electric power transmission a lumber plant had two or more complete power plants each with its complement of boilers, engines, pumps, etc., and a full operating force. One for the sawmill, one for the planing mill and sometimes one for the box factory or other side line or power user that could not be reached by belt or rope drive from the sawmill power plant.

The development of the low pressure and mixed pressure turbo generators stimulated the adoption of electric power distribution. By installing a mixed pressure turbo generator in the sawmill power house, arranging it to take the exhaust steam from the sawmill engine which was usually a Corliss engine and all the steam from the many steam cylinders in the mill, enough power could be generated to run the planing mill and other departments without drawing on the boilers for very much live steam. The cost of the installation of the generating equipment, plus the cost of the electric drives in the planing mill frequently exceeded the cost of a separate power plant for the planing mill, but the investment was justified by the reduction of the operating force and maintenance costs.

Many of the early applications of electric drive, or uses of electric power in the lumber industry that are still in use, would be hard to justify as being improvements of older and different methods. However, considering the equipment the designers and builders had to work with, the lumber industry rates as well as any other industry in this respect.

The multitudinous advantages of electricity for the distribution of power is now so fully recognized that it is accepted without question as the most economical method.

Changes being made in the method of lumber milling are steadily increasing the kilowatt hour consumption per unit of lumber, reaching as much as 121 kw-hr. per 1,000 ft. board measure in some mills that carry the manufacture to a fine degree. This change is also increasing the demand for process steam.

These changes, along with the possibility of selling all surplus power to public utility companies, or other power consumers, are placing a higher value on the mill refuse so that instead of surplus mill refuse being a liability it now has monetary value at many plants.

With such a contract for the sale of surplus power the mill company can run its generating equipment at nearly 100 per cent load factor, and if it has sufficient power plant equipment it can turn all its waste fuel (mill waste) into revenue.

This has made it imperative that efficient power

*Weyerhaeuser Timber Company, Tacoma, Wash.

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plant equipment be installed for the conversion of fuel into power. Also that the motor applications are made with a view to conservation of power as well as continuity of service.

An example of this is the recently completed plant of the Weyerhaeuser Timber Company at Longview, Washington.

The generating equipment at this plant consists of two 7,500-kw., 80 per cent power-factor, 2,300-volt, three-phase, 60-cycle, turbo generators. These turbines operate from steam at 300-lb. gage per sq. in. at the throttle; superheated to 580 deg. fahr. When operating at full speed load 150,000 lb. of steam per hr. at a pressure of 125 lb. gage, may be bled from each machine. This bled steam is used in the heating coils of the lumber drying kilns.

Two 500-kw., 250-volt, d-c. generators driven by synchronous motors supply current for 27 cranes used for handling lumber, as well as direct current for other d-c. motors and for the exciting current for the synchronous motors.

In use at the present time are 1,200 motors, not including motor-driven tools, such as portable drills and grinders, motor-driven electrical welding sets, electrical-driven clocks, and motor-driven office appliances. These 1,200 motors aggregate 23,849 hp.

The motors are divided as follows:

71,	2200-volt synchronous motors	total 10,645 hp.
30,	2300-volt induction motors	total 2,875 hp.
1021,	550-volt induction motors	total 8,575 hp.
78,	220-volt direct current motors	total 1,754 hp.

The load on this power plant with its 23,849 hp. averages 7,000 kilowatts. By the use of synchronous motors as here applied the power factor of the load is 96 per cent leading.

Electric power is now being used advantageously in every branch of the lumber industry. It is being used in the woods in a limited way for driving portable saws for falling trees and bucking them into log lengths. These saws consist of chains, with cutter teeth attached to one side of the links. These run over two sprockets, one mounted at each end of a light steel frame that serves to hold the sprockets in position and forms a guide for the saw chain. An especially designed motor of about five horsepower capacity, geared to the driver sprocket furnishes the power for driving the saw. Power for the operation of these saws is taken from nearby transmission lines, or from small gas-driven generator sets, which may be used for supplying power to several such saws working within a limited area.

Electric power is being used in a limited way for "yarding and loading" logs. This is the operation of dragging the log from where it was cut out of the tree and loading it aboard the cars for shipment to the mill.

The equipment in use varies for various sections or growth of timber. On the Pacific Slope of the Cascade Mountains, (the area where this equipment is used to the greater extent) the equipment resembles heavy duty hoisting equipment, both in mechanical construction and load characteristics.

The yarder is that part of the equipment that is used for dragging the log from the point where it has been cut out of the tree to the railroad siding for loading onto cars. Two full load line speeds are provided. A high speed being 1,800 ft. per min. and the low speed being 900 ft. per min. The higher speed is used on the smaller logs, and the lower speed used for the heavier logs. This variation of speed is obtained through reduction gears of different ratios connected to the drum by friction clutches. The usual equipment is a 300-hp. wound rotor type motor having a wide range of speed control through secondary resistance. These motors are designed for a pull out torque of approximately 300 per cent. The service is such that these motors are frequently loaded to the stalling point.

The loading end of this logging equipment usually consists of two, 75- or 100-hp. wound rotor type motors. The hoisting line of the loading equipment passes through sheaves properly supported over the load. The usual procedure in loading a log is to fasten one hoisting line at each end of one log; the log can then be manipulated through these double hoists so that the one end may be lowered onto the car ahead of the other, to conform to the requirements of binding the logs into the load in such a way that they will safely arrive at their destination at the mill pond.

For a large operation, on which load factor or days of operation per year is high, and in which life of operation is long enough to permit of reasonable obsolescence charges, this has proven profitable.

One serious disadvantage to the use of electrical equipment for woods operation, however, is the danger of loss of equipment by fire. The cost of electrical logging equipment is so great that the units are only built on order. The time required to replace a machine or repair one damaged by fire would be so great as to result in too severe a loss of production to warrant the adoption of electric power for logging operations except in a few special cases.

While the loss by fire is insurable, as far as property damage is concerned, it would hardly be practical to carry "use and occupancy" coverage on account of the cost of such coverage.

Electric power for the propulsion of the trains hauling the logs out of the woods is being used in one or two instances. Insufficient tonnage on the average logging railroad will prevent its general adoption for this purpose. The load center of a logging operation is anywhere from 15 to 30 miles from the center of the mill. The average number of trains over this main line section of a logging railway would probably be three trains each way per day. The branch lines radiating into the woods from this main line railway are seldom in service over a period of more than six months. In the case of the main line, electrification of the road would represent a large investment in the electrical

installation for comparatively few trains. For the branch lines this same electrification investment would be necessary and on account of the short period of service of these branch lines, electrification would not be economical.

There is no specific rule governing the application of motor drive to sawmill machinery. The power required for driving the various sections can be computed using the same laws that govern the application of electric drive to any other line of machinery.

Before attempting to compute the motor requirements for these applications, however, one making these computations should be thoroughly familiar with the possible loading of the various sections to be driven. In applying motor drive to live rolls and transfer chains the matter of obtaining sufficient starting torque is usually the prime consideration. The length of the roll trains and transfer tables is so short that if they are left in operation for a period of a minute or two they will completely unload all of the material that is on them.

Full voltage across the line starters is used for all motors, except wound rotor type and direct current motors.

Selecting the proper starting equipment requires considerable thought for with the frequent starting and stopping of the various sections the duties imposed on the starting equipment is even more severe than that imposed on the motors. Maintenance men spend more time keeping the starting equipment in good operating condition than they do the motors themselves.

There are many applications where thermo type overload relays will not function if the doors of the cabinets enclosing the starting panels are left closed. This is due to the rise in temperature within the cabinets caused by the arc created in opening the circuit of the motor. Some of the transfer tables in the mill are started on an average of four to five times per minute.

One of the advantages of motor drive is the ease with which the machinery can be started and stopped, which permits running the machinery only through the period that it is actually engaged in moving lumber. As this in many cases is only a small fraction of the total working time of the shifts, considerable wear and tear of the machinery is prevented by stopping its movement when not actually needed.

Among recent applications of starting equipment that has proven worth while is the automatic starting and stopping trains of live rolls and transfer chains to permit the movement of lumber over them. For instance a train of several sections of live rolls may be used for carrying timbers to the timber dock. As these timbers do not come with any degree of regularity the rolls need operate only a small portion of the time, with a trip pan arrangement placed between successive rolls throughout the length of the route. These trip pans are mechanically connected to the starting stations of the motors driving the sections, in such a manner that when the pan is depressed by the timber going over it, the motor is started and is kept in operation until all of the pans in that train of rolls are free to raise to the up position. With such an arrangement each section of rolls is started as the timber approaches the rolls and automatically stops when the timber has gone over the section.

Adapting individual motor drive to the machinery of a lumber plant presented many perplexing problems. With few exceptions, the speed of the final drive of sawmill and wood working machinery can be placed in two groups; a slow-speed group and a high-speed group.

Log hauls, transfer tables and conveyors belong to the slow-speed group, with final speeds, ranging from 1½ to 25 rev. per min. Live rolls drives ranging from 100 to 175 rev. per min. can also be placed in this class.

It was not until the so-called herringbone type gear was developed and it became practical to build speed reducing power transmissions of large reduction ratios that individual motor drive for these slow-speed groups became practical and economical. Now reduction ratios of 150 to 1 with high efficiency may be obtained. This permits the use of motors of 1,200 to 1,800 rev. per min. to be used for practically every slow-speed drive, resulting in better electrical load characteristics and greater economy of installation.

The standard ratios for gear reduction units in use by the Weyerhaeuser Timber Company are in multiples of ten; *i. e.*, units having ratios of 10 to 1, 20 to 1, 40 to 1, 60 to 1, 80 to 1, 90 to 1 and 120 to 1 seem to fit all our needs. A further refinement of speed adjustment being in the chain drive from the reduction unit to the driven machine. Steel roller chain of numbers 6, 7, 8 and 10, with double 8 and double 10, being used for extra heavy drives, are used for these final connections.

The cutting speed for the teeth of saws and the knife type cutters of wood working machines range from 8,000 to 14,000 ft. per min., which calls for speeds ranging from 300 to 7,200 rev. per min. These can be classed as high-speed drives.

The mechanical connection between these and motors of economical speed characteristics did not present the difficult problems involved in the slow-speed drives.

Recent developments in planing mill machinery such as matchers, molders, and saws, have opened a new field for the use of high-speed direct-connected motors. A large modern matcher has eight motors, totaling 270 hp., built into the machine. For these applications the machine manufacturers buy the rotor without a shaft, and the stator without a frame, and incorporate these in the construction of the machines.

It might be interesting to know that the modern

matcher (commonly called a planer) has the following motors, placed as follows:

Top cylinder	Two, 40-hp., 3600-rev. per min. motors
Bottom cylinder	One, 40-hp., 3600-rev. per min. motor
Outside side head	One, 30-hp., 3600-rev. per min. motor
Inside side head	One, 25-hp., 3600-rev. per min. motor
Top profiler	One, 30-hp., 3600-rev. per min. motor
Bottom profiler	One, 25-hp., 3600-rev. per min. motor
Feed work	One, 40-hp., 1200-rev. per min. motor

As the cutter head cylinders all turn in ball bearings and are very carefully balanced and heavy, they will continue to run for 10 or 15 minutes after the current has been cut off the motors. To make it possible to stop these quickly the control is equipped for plugging service so that when it is necessary to stop a head or cylinder for any purpose the motor is plugged, bringing it to a stop in a few seconds.

Modern molders are built for variable speeds of the spindle. This variable speed is obtained by varying the frequency of the circuit by the use of frequency changes. For normal work the cylinders are equipped with six or eight knife cutters and usually operate at 3,450 rev. per min. For the exposed surfaces of the molding that require a finer finish the particular heads being used for this exposed surface are speeded up by changing the frequency to 6,000 or 7,200 rev. per min. The control equipment is so arranged that the speed of any one or all of the five cutter heads or cylinders of the molder may be increased.

Two types of loads are selected for synchronous motor drive. First; widely fluctuating loads having a low-load factor, such as bandmills, edgers, slasher, trimmers, and hogs: Second; large loads of high-load factors such as fans, pumps, and air compressors.

Synchronous motors are not used on drives that require frequent starting.

The synchronous motors range in size from 50 to 400 hp. and in speed from 225 to 1800 rev. per min.

All synchronous motors are wound for 80 per cent power factor and have a pull-in torque of 100 per cent of full load torque. All synchronous motors are started with automatic, full voltage, across the line starters and with few exceptions are all excited from the 250-volt buses or feeders that supply power to the cranes and direct motors throughout the plant.

Synchronous motors located so far from the d-c. lines that the cost of a d-c. circuit to the motor for excitation would cost nearly as much as an exciter, are equipped with direct-connected exciters.

Synchronous motors have proven very satisfactory for widely fluctuating loads. The application to driving hogs being a striking example. Motors of 250 hp. at 600 and 720 rev. per min. and 400 hp. at 600 rev. per min. are used for this. These motors have the field control relays so adjusted that when the motor pulls out of step due to overload, the field excitation is cut out, and will automatically be restored when the motor again approaches synchronous speed. This may occur as many as 100 times during an 8-hr. shift.

The 2,300-volt induction motors range in size from 50 to 250 hp. These are mostly wound rotor type motors driving machinery requiring frequent starting, or machinery which will not permit of the severe strains imposed on it by the full voltage across the line starting. Some equipment requires limited speed control and for this purpose the wound rotor motor with drum type controllers is also used.

Induction motors of 50 hp., and smaller ratings are wound for 550 volts.

Except in sizes of 7½ hp. and smaller, the 550-volt induction motors are of the so-called double wound rotor type. These have the starting characteristics of the high-resistance rotor motor and the running characteristics of the squirrel-cage type motors.

One unusual motor application in this mill is that employed for driving the main bandsaw in the mill. For the regular run of fir logs a saw speed of 10,500 ft. per min. is the ideal speed. For a certain species fir log, one of exceptionally dense growth or one that has been dead for such a long period as to cause the wood to partially dry out, or for the logs that are frozen, such as occurs during the winter months, a saw speed of 8,000 ft. per min. is the most practical speed at which to run the saws. As these logs that require the slow-speed saws for sawing are mixed promiscuously with the softer texture logs, the custom has been to saw them all with the higher speed saw which has resulted in erratic sawing of the hard or frozen logs. At this Longview mill we arranged for practically instantaneous speed changes from 10,500 to a speed of 8,400 ft. per min. This was accomplished by putting two synchronous motors on one motor base, with the shafts rigidly coupled together, driving the same pulley. A 400-hp. 600-rev. per min. motor was installed as the main motor to use in starting the bandmill and for sawing the average run of logs. The other motor was a 250-hp. 450-rev. per min. motor. While the large motor is used for furnishing the power the small motor is disconnected from the line so that the only power required to drive it is that necessary to overcome the windage losses. When a hard log is sawn the sawyer, by pressing two push buttons, disconnects the large motor from the line and connects the small motor, which drops the speed of the saw to 8,400 ft. per min. The change from one speed to the other being made as frequent as the sawyer deems it necessary to make the changes.

Another unusual application of electric power to sawmill machinery is in driving the feed works of the larger edgers. The edger is the machine that rips the boards and timbers into the desired widths. The edger arbor on which the saws are mounted is direct-connected to a 400-hp. 80 per cent power-factor, 1200-rev. per min. synchronous motor. The feed rolls that feed the stock into the edger are driven by a 10-hp. series wound motor through reduction gears and chain drive. These are so proportioned that with no load on the motors the surface speed of the rolls will pull the lumber into

the edger as rapidly as good work can be done with the saws. As larger and thicker pieces are sent through the edger the load on the feed works motor increases and automatically decreases the feed so that the thicker stock is fed into the edger more slowly. The speed characteristic of the 10-hp. motor is such as to automatically provide the proper rate of feed into the saws for the varying thickness of material being sawn.

Another application of electric current is by the use of solenoids for remote operation of cylinder valves. By use of solenoids for this purpose the valves of cylinders can be operated from several distant points.

Among unusual applications of electric power to sawmill machinery is that used by the Weyerhaeuser Timber Company at their Mill "C" Everett, Washington. This mill is located approximately three-quarters of a mile away from the power plant. The mill is equipped with individual motor drive throughout but differs from the usual mill in the respect that no steam is used in the mill for any purpose. This mill is so far from the power plant that the cost of a well constructed steam line was such that it was decided to use compressed air in place of steam for operating all lifting cylinders, loaders, niggers, etc.

A two stage air compressor having a capacity of 1,500 cu. ft. of free air per min., driven by a 250-hp. synchronous motor furnishes all of this compressed air.

For driving the sawmill carriage a slow speed d-c. traction type elevator motor was used. This motor differs from the elevator motors in that it is not rated in horsepower. The specifications for the motor called for a torque of 12,500 lb. rather than the conventional horsepower rating. Power for this motor is taken from a motor-generator set consisting of a 75-hp., three-phase motor driving two d-c. generators; one for furnishing current to the armature of the motor and a smaller one for furnishing the current for exciting the fields of the generator and the motor.

The armature leads of the motor are directly connected to the armature terminals of the generator. Full field is left on the motor at all times. The speed of the motor is controlled by varying the voltage of the generator through field resistance placed in the field circuit of the generator. Reversal of the motor is effected by reversing the fields of the generator.

This motor is connected to the carriage through wire ropes, one end of which is securely fastened to the carriage; the other end being fastened to a 30-in. diameter sheave on the motor shaft.

The cycle of operation of this unit is two revolutions in one direction and then two revolutions in the opposite direction. Under normal operation this unit makes from seven to thirteen cycles per min.

Owing to the lack of ventilation, because the motor does not set up any windage through the coils by its own revolutions, it was necessary to provide artificial ventilation. This was accomplished by enclosing the motor in a galvanized iron housing and forcing cool air through this housing by the use of a motor-driven fan.

Accurate and quick speed control of a sawmill carriage is absolutely essential. The method used in the control of this drive, which is the Ward Leonard System, has proven highly satisfactory in every respect.

The newest use to which electricity has been placed in the sawmill industry is that of the Taglibue-Heppenstal moisture indicator. This consists of a case in which is mounted a battery, some amplifying tubes and transformers and an ammeter. Long flexible leads from this cabinet are connected to two sharp terminals in a small portable handle that the lumber tester carries with him when inspecting lumber.

By driving the steel terminal points that are on the handle into the wood, a current is set up in the circuit, the amount being governed by the moisture content of the wood. The value of this current is calibrated as percentage moisture content of the wood.

This method of testing for "dryness of lumber" has proven so dependable that it is now a standard for testing lumber that is bought on a specification calling for a definite moisture content.

Storage battery locomotives usually of seven ton capacity having 225-ampere hr. batteries are used for the transportation of lumber from one department to another, where these departments are so arranged that transportation cannot be readily accomplished by crane service. These locomotives run on a 30-in. gage track pulling four wheel industrial cars equipped with roller bearings. As the tracks are in good alinement and the grades are practically eliminated, one locomotive will haul a train of 15 cars or 60 tons on one charge of the battery and stay in service an 8-hr. shift without recharging the battery.

Small storage battery trucks and lift trucks are used in some departments for stacking certain items of lumber and for short distance movement of lumber.

With modern industrial lighting units it is possible to provide illumination so suited to the work that production can be carried on at night with as high a rate of efficiency as that obtained during daylight operation.

Progress in the Design of the Single-Phase Series Motor

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Associate, A. I. E. E.

Synopsis.—*The paper deals with the development of the design of the single-phase series motor and some of the main features in both electrical and mechanical proportions which determine the service capacity in the application of this type of motor to railway electrification.*

Intensive study and development begun in the early part of 1927 resulted in the production of a singe-phase series motor with more than enough capacity to meet the present day requirements of the American railroads. A brief description is given of this motor and of some of the novel features which made this recent progress possible. Since this motor was developed, built and tested, further possibilities of increasing rating or decreasing the size and weight for the same rating have been found and will result in further progress in the design of the single-phase series motor.

THE single-phase series motor for 25 cycles has been developed to the stage where it fulfills the present day railroad requirements, that is, high horsepowers per axle at high speeds. A high-speed passenger locomotive equipped with single-phase series motors with shunted interpole developed and built by the Westinghouse Electric & Manufacturing Co. has been operating since July, 1930. The continuous rating is

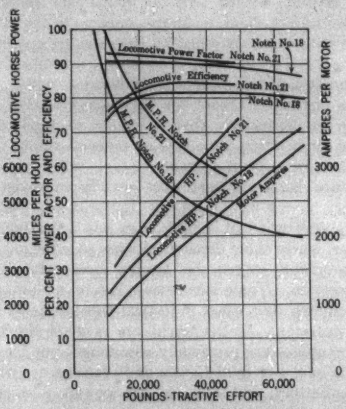

FIG. 1—PERFORMANCE CURVE OF PASSENGER LOCOMOTIVE EQUIPPED WITH FOUR NO. 422-A SINGLE-PHASE COMMUTATOR MOTORS

Includes input to all auxiliary apparatus and gearing

more than 1,000 hp. per axle at 56 mi. per hr. In service the locomotive has exceeded the performance for which it was originally designed and has clearly demonstrated that it is now practical to build a 25-cycle single-phase locomotive for as high as 75,000 lb. axle loading geared to operate at a maximum sustained speed of 90 mi. per hr. The characteristic curves are shown in Fig. 1.

This development was started more than three years ago in spite of the fact that at the time opinions were expressed that such a design was impossible. The outstanding features which made this possible were:

a. Roller bearings.

b. Commutators and armature windings designed to operate at higher peripheral speeds than heretofore considered good practise.

c. More compact ventilating scheme.

d. Improved quality of magnetic material.

e. Rolled steel housings and welded frames.

f. The disk spring commutator.

g. Improved designs of gears, pinions and mechanical drive.

Considerable credit is to be given to the European designers and manufacturers of single-phase railway apparatus in Germany, Austria, Switzerland, and Sweden, especially to the German engineers who were the first to apply generally roller bearings to locomotive motors in order to take advantage of the higher peripheral speeds on 16⅔-cycle single-phase series motors.

Once the fact was demonstrated that such high ratings were also possible on 25-cycle single-phase series motors, a number of further improvements has been found which permit the design of motors of even larger capacity in the same space and weight. The development of the single-phase commutator motor cannot be attributed to any one manufacturing firm or group of individuals. Its development is nearly as old and as broad as that of alternating current. The purpose of this paper is to describe briefly its history and how evolution led to the present day designs.

The single-phase commutator motor, though the fact is not generally recognized, followed closely the conception of the synchronous motor which was the first a-c. motor. The original designs were patterned after the d-c. series motor. The first appearance in literature of the mention of the possibility of such a motor was in 1884 by Alexander Siemens. This was followed by the experiments and attempts of Elihu

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Thomson in 1887 with the repulsion motor. Although not entirely successful the idea was followed up and in 1892, E. Arnold brought out a more successful repulsion motor which embodied the principles of the repulsion, the doubly fed and the series motor. This led to the Wagner and to the Deri motors which were developed for industrial application on 50 cycles or higher frequencies. The difficulties of commutation on the higher frequencies at this time, together with the advent of the induction motor, pushed the a-c. series motor into the background until after 1900.

The first real impetus to a practical application of the series motor for traction purposes came in 1902 when B. G. Lamme brought forth a single-phase series motor for railway application and emphasized the importance of a low frequency, having chosen 15 cycle for his motor. Independent developments in Europe at about the same time brought out a test locomotive for the Prussian State Railways in 1903 as a result of the endeavors of Winter, Eichberg, and Latour.

The first outstanding electrification was that of the New York, New Haven and Hartford Railroad in 1907 in which Lamme and Storer designed and built resistance lead series motors which are running successfully today. The principal objectives of Lamme for this development of the series a-c. motors, notwithstanding certain limitations in design, are given as follows:

a. To make use of the outstanding advantage of the series motor for traction purposes in conjunction with the alternating current high-voltage contact system which permits the maximum concentration of power in a given section with the minimum size of conductors and supporting structure and substation capacity, as well as high transmission efficiency.

b. To obtain a motor with minimum power requirements for acceleration.

c. To obtain a motor whose tractive effort could be maintained within limits over the entire speed range by varying the voltage of the motor, irrespective of motor combinations, by change in transformer taps, thus obtaining a maximum output.

Several outstanding features resulted from this New Haven Electrification:

a. Some of the passenger locomotives were operated at speeds of over 100 mi. per hr. but at a limited capacity because the axle loading was around 40,000 lb. and gearless motors were used.

b. The individual axle drive of the flexible quill type was developed for

1. Gearless motor (No. 130 motor).
2. Geared single motor (No. 403 motor).
3. Geared twin motor (No. 409 motor).

The gear twin motor with flexible quill drive has made it possible to design for considerably more horsepower per axle with smaller high-speed motors and narrow gear face.

c. Single-phase multiple unit cars.

Considerable independent development took place in Europe at this time. The work of Dr. Behn Eschenberg, R. Richter, and M. Latour produced the series motor with the compensating winding and series interpole shunted by resistance. The limitations of short-circuit currents in coils undergoing commutation were investigated and the combination of a brush, two bar pitches wide with a limit of 7 volts from one tip to the other was established, limiting the maximum flux per pole at start to

1.6 megalines at 50 cycles.
3.2 megalines at 25 cycles.
5.3 megalines at 15 cycles.

Dr. Behn Eschenberg in 1908 had investigated telephone disturbances caused by armature slots and recommended that the slots be skewed to minimize this effect. These general limits of electrical design characterize the designs of today.

The first experimental series motor with interpoles which the Westinghouse Electric & Manufacturing Co. built was in 1911. This motor had low flux per pole and interpoles shunted with resistance. Mr. Hellmund, in his tests with this motor, found most of the relations which characterize the designs of today but could not take full advantage of them at that time because both switches and reversers were then too big and heavy to handle the larger currents economically.

The doubly fed motor with repulsion start worked out better with the available control apparatus and this type of motor was applied to multiple unit cars. Considerable development of the doubly fed motor took place in which Arnold, Richter, Hellmund, Alexanderson, Punga, Latour and others did considerable to advance the art. The original and major number of motors used on multiple unit cars of the Pennsylvania Railroad at Paoli are of the doubly fed type, and the armatures of these motors were the first to be wound with the split-throw[2] or fractional-pitch windings to improve commutation.

The World War retarded progress a second time—more in Europe than in America of course—and it was not until 1919-20 that a multiple-unit car equipped with a single-phase series motor with resistance shunted interpoles was tried out in service on the Paoli line at Philadelphia. Weakening of the exciting field at start was tried on this equipment by connecting the fields of two motors in parallel at start and then changing to fields in series for the running connection. A second trial equipment in 1924 was tried in service with the field shunted by a reactor to reduce the field strength at start. This motor is still operating satisfactorily in daily service.

In 1923-24 a large locomotive motor of the single-phase series type with shunted interpole was designed

2. Patent No. 1298705, R. E. Hellmund, April 1, 1919.

and built for gear and side rod drive on the Pennsylvania Railroad.

As Europe recovered from the war considerable progress was again made in Germany, Austria, Switzerland and Sweden, where the single-phase systems were adopted as standard for electrification. The single-phase series motor with shunted interpole was generally applied with a definite trend toward individual axle drive. The doubly fed motors and resistance-lead motors were set aside. The necessity for smaller motors with greater capacity brought on an intensive development, both in drive and in mechanical construction. Attempts were also made to exceed the established limits in electrical design.

In Germany, Siemens-Schuckert concentrated on Geh. Reichel's objective of a single axle-hung motor for passenger locomotives and succeeded in developing 600 hp. per axle with 44,000 lb. axle loading for a maximum speed of 62 mi. per hr. The application of roller bearings and an increase in peripheral speed of commutators to 9,000 ft. per min. were the main features which made this possible.

AEG.[3] engineers directed their efforts toward producing the Kleinow locomotive with twin motor per axle drive and succeeded in producing a locomotive with 700 hp. per axle within the limits of 44,000 lb. axle loading and 60 mi. per hr. maximum speed.

In Austria, Siemens-Schuckert produced a passenger locomotive with vertical motors and, although a novel departure in drive construction, has given good results.

Brown Boveri Co. built locomotives with their single motor per axle drive and omitted the compensating winding in the pole face but used the shunted interpole in an effort to increase the number of poles on the motor. The outstanding result of their work was to develop a compact design of brush holder so as to place the maximum number around the commutator. This was reflected in the motor built for American application where 24 holders were placed around a 48-in. commutator and later 10 holders around a 17-in. commutator.

The European designs, not being hampered with the limitations of higher frequency, had commutation constants which were quite reasonable. The recent efforts of the Europeans described above may be summarized by saying that their attempts were directed toward obtaining more motor capacity for a given weight and displacement, higher efficiency and better power factor. This was made possible fundamentally by the application of roller bearings and improved armature and commutator construction for higher speeds together with the use of more poles, resulting in more capacity per pole and more poles for the same space and weight.

The development and successful application of the roller bearing has had considerable effect on designs. Long life, less lubricant and maintenance of a uniform air gap, together with a much higher journal speed, permit a considerable increase in armature rev. per min. An armature combination designed for a given flux per pole and ampere-turns when rotated at a higher speed will of course have a proportionally higher output. Compared with older designs a higher speed armature requires less flux per pole for the same total armature ampere-turns to produce the same output. This allows us to shorten the length of core, all of which results in higher efficiency and power factor for less weight of active material. The commutator design must be improved to stand higher peripheral speeds and likewise the armature core and windings.

In 1926-27 the problems in connection with the electrification of some of the larger American trunk lines became more active. Our twenty-five years experience in a-c. electrification and our knowledge of the 16⅔-cycle development in Europe gave us considerable data with which to proceed. The handicaps of 25 cycles over 16⅔ cycles is of fundamental importance in motor design. Starting conditions of commutation definitely limit the flux per pole. For the same voltage between commutator bars, the 25-cycle motor can have only ⅔ the flux per pole of a 16⅔-cycle motor. Hence for the same torque a 25-cycle motor must have either 3/2 times the armature ampere turns or 3/2 times as many poles as a 16⅔-cycle motor, or a partial increase in both armature ampere turns and poles to make up for ⅔ the flux. In addition to this, the axle loading in Europe on the 16⅔-cycle motors is not more than 44,000 lb. while here in America with 25 cycles, the axle loading is as much as 75,000 lb.

Hence the problem here is to design for $\frac{75,000}{44,000}$ or 170 per cent of the axle loading and starting tractive effort with ⅔ the flux per pole and either 3/2 the poles or 3/2 the armature ampere turns required in Europe. Besides this, the maximum locomotive speed in Europe is 62 mi. per hr. (although this may be increased to 75-80 mi. per hr.) while here in America we are designing for 90 to 100 mi. per hr. maximum. Hence with 170 per cent of the starting tractive effort per axle, 150 per cent of the maximum running speed with ⅔ the flux per pole and either 3/2 the poles or 3/2 the armature ampere turns, the proportions of our motors differ somewhat from those of the European motors.

Our engineers studying the problems of electrification economies, application, etc., put before the design engineers the problem of designing and building a motor suitable for freight service as an axle-hung motor and suitable for passenger service as a twin motor with flexible quill drive with motor parts interchangeable, which entails design difficulties. Railroad limitations at the time were 60,000-65,000 lb. axle loading, 45-50 mi. per hr. maximum speed for freight and 90-100 mi. per hr. maximum speed for passenger locomotives for

3. Allgemeine Elektricitaets Gesellschaft.

operation on 25 cycles. Designs were worked up and the first test motor was built for 25 per cent adhesion at 60,000 lb. axle loading geared for 90 mi. per hr. maximum speed, with a continuous rating of 1,000 hp. per axle at the rail at 55 mi. per hr. The motor was tested early in 1928 and the fact was clearly demonstrated that such a capacity was possible with 72 in. drivers. A change in ventilation was then made with the result that a motor rating of 1,220 hp. per axle at 56 mi. per hr. could be obtained by an increase in current without exceeding A. I. E. E. temperature limits.

Following these tests twin motors were built in mechanical parts designed to mount in a locomotive. Development was then concentrated on the commutator construction and the new disk spring type was developed in which the bars are held together by the force exerted by a disk spring pressing against the vee ring. Tests on this commutator show that the limit of 9,000 ft. per min. is a conservative one, in fact as soon as sufficient service data are obtained it may be possible to go to 10,000 ft. per min. or even higher with this disk spring construction. Increase in commutator speed for a given locomotive speed will produce more service capacity for a given motor weight and displacement.

The field of the motor is shunted at start and allows the possibility of designing for maximum flux at continuous rating independent of the value necessary at start. In fact the motor has the same flux at standstill as at continuous rating with the present locomotive control scheme.

A ventilation scheme was developed to eliminate fan members on the armature in order to increase the overall length of active parts. At the same time the arms of the spider and coil support provide an internal blower which assists in the circulating of the air through the armature passages.

To eliminate the difficulties of securing adequate cast steel frames of the size required, a new welded rolled steel frame was developed. Housings and commutator parts, also of rolled steel, were developed for the motor to take up less space because they are relatively lighter and stronger. The motor frame and housings were dimensioned and designed to take electrical parts of either 12, 14, 16, 18 or 20 poles. The original motor was designed for the minimum number of poles in order to get the maximum voltage and minimum current. This required a design wherein three commutator bars are spanned by the brush in order to get a brush of sufficient mechanical strength and smooth riding qualities on the commutator.

The motors for the existing locomotive were ready in the latter part of 1929 and the locomotive was placed in service the first week in July 1930. The actual weight on drivers is higher than the 60,000-lb. axle loading for which the motor was designed, hence the motor will be tested with more severe starting conditions than originally contemplated. The performance of the locomotive has exceeded expectations. The experience gained through the operation of this locomotive in service with the increased axle loading and the knowledge and experience gained in the past 3 years, make it possible to design for as high as 75,000 lb. axle loading at the same speeds.

In order to design these motors and take advantage of the best proportions, the major relationships of the fundamental limitations in design were comprehensively analyzed on the following basis. The required capacity of any motor applied to a locomotive or car is determined by starting tractive effort and the maximum sustained running speed at which the locomotive or car must operate. The gear ratio and wheel size must be so proportioned that the commutator speed will be within safe limits at the maximum sustained running speed of the locomotive. The maximum tractive effort which may be obtained with this gear ratio and wheel size is limited by the current carrying capacity of the brushes and commutator or armature windings. The continuous capacity is then the output at some voltage and speed with this gear ratio, wheel size and current carrying capacity at which the temperatures do not exceed certain safe values.

For multiple wound armatures:

Gross tractive effort = flux per pole × brush current density

$$\times \text{ poles} \times \frac{\text{peripheral speed commutator}}{1414 \times \text{mi. per hr.}}$$

$$\times \frac{\text{Width of brush}}{\text{Bar pitch}} \times \text{active length of commutator}$$

$$\times \cos . \alpha \times \text{gear efficiency}$$

where active length of commutator is the total length of brushes in a brush holder.

The gross tractive effort can be converted into net tractive effort at the rail by deducting rotational losses and gear losses which are of the order of 5 to 10 per cent. At standstill the cos . α may be of the order of 90 per cent but is practically unity above ¼ speed.

The limiting factors in the motor design are

1. The maximum peripheral speed of the commutator which determines both the diameter and length of the commutator.

2. The permissible current density in the brush which, for a given number of brush holders, limits the size of the brushes and the active length of the commutator but depends on the flux per pole.

3. The flux per pole which not only determines the magnetic sections of the punchings and the length of core but also the voltage between commutator bars at standstill and the circulating currents in a coil being commutated. This circulating current in turn limits the permissible current density in the brush.

4. The bar pitch of the commutator which limits the width of the brush for a given flux per pole and hence the current per brush holder.

5. The number of brush holders which can be

placed around the commutator which limits the number of poles.

The maximum peripheral speed of a commutator is a mechanical limitation. The dimensions of the individual bars and the method of holding them together determines the safe maximum speed at which the commutator may rotate without overstressing the parts. Disk spring commutators have been built to operate at 9,000 ft. per min.

The flux per pole at start, the permissible current density in the brush and the maximum number of bars spanned by a brush at any time are all very closely related and depend entirely on the characteristics of the carbon brush. The resistance of contact of any carbon decreases with temperature rise. On brushes with high contact drop, the circulating current in the coil short circuited by the brush varies about as the square of the voltage induced in the coil, reaching a critical condition between 3.5 and 3.75 volts above which the current increases very rapidly with a slight increase in voltage. Since this characteristic is caused by local heating it is evident that, with line current also carried by the brush, the voltage induced in the short-circuited coil must be held below this critical point. Tests have shown that some carbons will stand as high as 2.9 volts between bars and a load current density of 150 amperes per sq. in., where the brush is two bar pitches wide and is in contact with three bars most of the time.

If we substitute these values in the equation for tractive effort we have at 25 cycles:—

Gross tractive effort at start

$$= \frac{2.6 \times 150}{1414} \times \text{poles} \times \frac{9000}{\text{Max. mi. per hr.}} \times 2 \times$$

$$\text{active length of commutator} \times \cos . \alpha = \frac{4960 \times \text{poles}}{\text{Max. mi. per hr.}}$$

$$\times \text{ active length of commutator} \times \cos . \alpha.$$

Therefore with the tractive effort required for starting and the maximum mi. per hr. for sustained running speeds, the poles and active length of commutator may be readily determined. This brings us to the requirements of the car or locomotive.

On multiple unit cars, the weight on the driving axles will probably not exceed 40,000 lb. Since sand is seldom used the maximum adhesion will not be over 18 per cent giving a possible starting tractive effort of 7,200 lb. per axle. Geared at 70 to 75 mi. per hr. it would be possible to accelerate such a car at 1 mi. per hr. per sec. with only one axle driven by a 12-pole motor with 10 in. active commutator. With motors on two axles it would be possible to haul a trailer.

For a locomotive of 60,000 lb. axle loading and a starting adhesion of 25 per cent each axle would exert 15,000 lb. tractive effort at start. This would require a 12 pole motor with a 12 in. commutator to bring the locomotive to 45 mi. per hr. with one motor per axle. Two motors per axle would give a maximum speed of 90 mi. per hr.

Assuming the maximum weight per axle held at 75,000 lb., then at 30 per cent adhesion, each axle could exert 22,500 lb. tractive effort at start. This would require 18 poles and 12 in. of commutator length to bring this locomotive to 45 mi. per hr. maximum speed and with a twin motor of the same dimensions the locomotive could be operated at 90 mi. per hr. maximum speed.

The design of the locomotive then becomes one of grouping the required number of axles under a cab to give the most economical operating unit. The grouping of axles under one cab depends on the required size of driver and this is where the dimensions of core and windings play an important part. At the present time twin motors can be built to go on 72 in. wheels and develop 25 per cent adhesion at start geared for 90 mi. per hr. maximum running speed. With the developments now under way, however, this wheel size may be brought down to 68 in. or the maximum speed on 72-in. wheels may be raised if desired, because the experience of the last few years indicates that considerable further improvements can be made to permit the application of smaller motors for the same tractive efforts or of the same size of motors for greater tractive efforts.

Summary

1. The series single-phase commutator idea was conceived in 1884.

2. The developments on the high frequencies during this period diverted the efforts to the repulsion motor and the induction motor.

3. In 1902 B. G. Lamme gave the first real impetus to the application of the series single-phase motor to railroad application and recommended a low frequency.

4. In 1907 the New York, New Haven and Hartford Railroad electrification was the first outstanding application of the motor and these motors are operating successfully today on both alternating current and direct current.

5. Contemporary development in Europe led to standard system of alternating current only and brought out the shunted interpole motor.

6. In 1908 the electrical limitations were established of 2 bars per brush and 7 volts across the brush at start, also armature slots skewed to minimize telephone interference. The limitations hold today.

7. After the World War, a second big impetus was given to electrification by mechanical improvements.

   a. Increased commutator peripheral speed.
   b. Improved insulation.
   c. Roller bearings.
   d. More compact brush holders.

8. Further mechanical improvements followed in America.

   a. Disk spring commutator.

b. More compact ventilation design.
c. Rolled steel and welded frames and housings.
d. Better punchings.

9. A united effort by all firms to produce a unit axle drive to simplify the design of locomotive.

10. Finally a return to the series single-phase commutator motor with shunted interpole.

In conclusion, it should be noted that our new design of series commutator motor and control equipment, for 25-cycle railway electrification, has exceeded expectations as to horse power per axle. A 4-axle locomotive, with the new motor, has developed more service capacity than was expected with the 6-axle locomotive which we proposed four years ago and which was then considered to be a marked improvement over existing designs. Further, it should be noted that this new design, while representing an entirely new analysis and proportioning or arrangement of the various essential parts, to obtain better power factor, efficiency, commutation, and speed-torque characteristics, is nevertheless based on the sure foundation of nearly thirty years of constant practical experience in a-c. electrification, and over three years of intensive development work and testing of this new design.

Lightning Investigation at Alcoa, Tenn.

BY J. ELMER HOUSLEY[1]

Associate, A. I. E. E.

Synopsis.—*The cooperative lightning laboratories near Alcoa, Tenn., have been in operation since July, 1928. This paper outlines the purpose of the installation and describes the transmission line on which the studies are being conducted. A background is provided for a more complete appreciation of the technical data which appear from time to time. Conclusions drawn from an operating viewpoint are presented.*

* * * * *

EARLY in 1928 there was a crystallization of the ideas concerning the use of the cathode ray oscillograph for field studies of lightning. The immediate result of this was the desire to establish two lightning laboratories in the field.

The Aluminum Company of America, through its subsidiary the Knoxville Power Company, afforded a transmission line of high insulation value and recognizing the importance of lightning investigation, joined the Westinghouse Electric and Manufacturing Company in a cooperative field research endeavor.

At the inception of this work the determining factors governing the selection of East Tennessee were the great number of thunderstorms, and the existence there, of the most heavily insulated steel tower transmission line in operation in the southeast at the time.

The Line

The transmission line selected for the investigation consists of two circuits operated at 150,000 volts and extends from Alcoa, Tennessee to the Santeetlah power house, a distance of thirty miles.

The elevation of Alcoa is about 700 ft. and the line travels over a rolling country until Tower No. 72 is reached when the line suddenly climbs to an elevation of 2,600 ft. A rugged country is traversed with several long river crossings until the Cheoah power house, at tailrace elevation of 1,080 ft., is reached. A further mountainous section is traversed to Santeetlah power house at elevation 1,270 ft. Data covering the first circuit of this line have been given in an Institute paper.[2]

The towers are double circuit to tower No. 60 and single circuit with horizontal configuration of conductors to tower No. 125 where double-circuit construction is resumed to the terminus at tower No. 152 at Santeetlah power house. See Fig. 1.

The configuration of the cables is vertical on the double-circuit towers with the middle wire offset 4 ft. 6 in. toward the tower, and the vertical separation is 11 ft. 9 in. at the top and 12 ft. 8 in. at the bottom. The single-circuit lines have a flat horizontal configuration with a spacing of 17 ft. 6 in. The conductor is 500,000 cir. mils A. C. S. R., 40 per cent steel by weight. The diameter is 0.904 in. The insulator strings are composed of ten disks in suspension and twelve in each of the triple strings used in a single dead-end assembly. Strap iron horn-gaps are used on the line and ground side and a horn is placed on each side of the insulator string over the axis of the cable in suspension units. The horns are arranged above the insulator for horizontal strings. The gap spacing is 3 ft. 9 3/16 in. The span between towers on the line varies from 650 ft. to 2,400 ft. The single ground wire of 1/2-in. high strength steel is common to both types of tower. This wire is located at the top and center of all towers. The single-circuit towers are provided with holes for the attachment of two additional ground wires, one between and above each outside cable and the center cable.

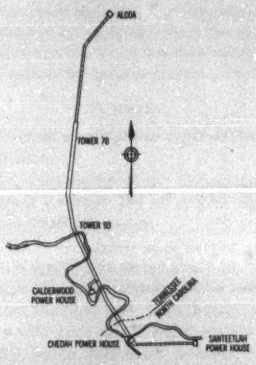

Fig. 1—Diagram of Transmission System of Knoxville Power Company and Tallassee Power Company

Operating Experience

The lightning season extends from March 15th to October 1st. The insulator flash-overs occur mainly in the mountainous section of the line from tower No. 81 to tower No. 152. The original line, twenty-five miles long, has been in operation since April, 1919 and on this line the interruptions have averaged about ten per year. Lightning, with one exception, has been the sole cause of interruptions chargeable to the high-tension line.

1. Knoxville Power Company, Alcoa, Tennessee.

2. *The 150,000-Volt Transmission Line of the Knoxville Power Co.*, by Theodore Varney, A. I. E. E. Journal, June, 1920, p. 563.

Presented at the Southern District Meeting No. 4, of the A. I. E. E., Louisville, Kentucky, November 19-22, 1930.

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The exception concerns an obsolete type of strain insulator which failed several years ago on a special long span. No insulators have been found damaged by flashovers. The second circuit was added two years ago and five miles of double-circuit line was added making the total length of the line thirty miles. The second circuit and the added length of line has not increased the number of interruptions in the past two years.

Selection of Sites

After deciding upon two stations for the field investigations the location of the stations was selected after a study of the usual path of the thunderstorms. Previous observation had indicated that many storms traveled along the ridge of the Chilhowee Mountain which extended east and west and the line crossed the ridge at right angle. Along the south base of the mountain flowed the Little Tennessee river for a short distance until it reached a water gap and turned north. Many storms traveled up the short valley from west to east, probably due to the prevailing winds which are southwest.

It seemed appropriate to locate one station at the base of the mountain at the north near tower No. 70 and the other at tower No. 93, along the river near the base of the mountain on the south. See Fig. 2. The locations were near roads which would permit the transportation of building material and the apparatus required.

Satisfactory results were obtained from the station at

Fig. 2—Lightning Laboratory at Tower No. 93

tower No. 93, but the location at tower No. 70 was too remote from the storms on the ridge of the mountain and this station will be discontinued.

Klydonograph Equipment

Three klydonograph installations were established in conjunction with each station. All klydonograph potentiometers were of the three-phase type, one was located adjacent to each station and one on either side of the stations one mile distant. This is illustrated in Fig. 3. The klydonographs are equipped with eight-day clocks and the film records are removed weekly. A klydonograph was placed in a parallel circuit between the top of the tower and the ground at tower No. 93,

Fig. 3—Three-Phase Potentiometer and Klydonograph Installation

to study the potential gradient existing on the tower during a storm.

Description of Station

The station building at tower No. 70 is a frame building and at tower No. 93 is a sectional steel building. A partition provides a dark room and a kitchen, while a tent with a wood floor, provides the living quarters. An electric refrigerator furnishes storage for food supplies and a supply of cold water for the development of films.

A shallow well equipped with an electric pump supplies water to the station at tower No. 93. The other station pumps water from a mountain stream.

The frame building is lined with a heavy wire screen, well grounded, in the laboratory section and in the steel building it was only necessary to ground the structure at several points.

Equipment

The equipment in each station was identical. A gasoline-driven 15-kv-a.,110-volt, three-phase, 60-cycle, a-c. generator supplied power for station use. The direct-connected 1-kw. exciter furnished 125-volt direct current for use in the concentration coils on the oscillograph.

The Norinder oscillograph with its auxiliary equipment, consisting of vacuum pump, switchboard, linear timing device, oscillator, and high-voltage rectifier has been used throughout the tests and many improvements have been made in the apparatus and increased reliability has been obtained.[3] See Fig. 4.

An osiso was used for recording the output of a

3. *A Cathode Ray Oscillograph with Norinder Relay*, by O. Ackerman, A. I. E. E. Journal, April, 1930, p. 285.

microphone located some distance from the station to detect the location of the stroke by recording sound from the thunder, but the induction picked up on the long leads during the storms interfered, so that good results were not obtained. A potentiometer connected to an antenna was used to study air gradient potentials existing in the river valley and on the top of the ridge of Chilhowee Mountain.

Staff

Each lightning laboratory was continuously attended during the lightning season by two engineers from the Westinghouse Company, who made their residence at the station. During the winter various parts of the equipment were returned to the factory for alterations suggested by the preceding season's work.

The oscillogram No. C-3095-B, Fig. 5, was made during a storm July 10, 1930. These data have not yet been analyzed. The storms during the 1930 period, up to

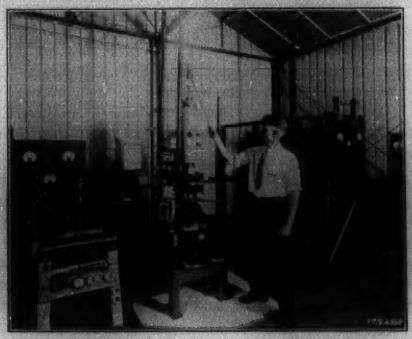

Fig. 4—Cathode Ray Oscillograph and Auxiliary Equipment in Laboratory at Tower No. 93

August 1, 1930, have been local thunderstorms and have covered only a very small area, and a small number of surges has been recorded at tower No. 93 and one at tower No. 70. Fig. 6 oscillogram shows a typical switching surge.

The small number of oscillograms obtained since the stations went into operation does not give a complete picture which would enable an analysis to be made of the waves appearing on the line with their exact point of origin and probable voltages which existed there. The data obtained and observations made during the past three years have been valuable in relation to the development of the cathode ray oscillograph and demonstrated the importance of this device in the field.

Conclusions

Since the location of storms must be very close to the station, chance plays a large part in the volume of data obtained. Accordingly it is hoped that by continuing the investigation over a period of time that a sufficient number of surges will be recorded so that a complete analysis may be made of the various types and determine the efforts of the natural strokes.

One of the first observations made indicated that the

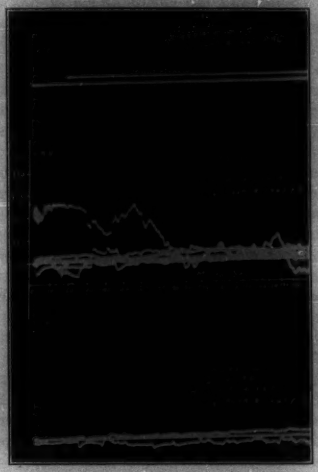

Fig. 5—Oscillogram of One Large Lightning Surge and Several Minor Surges

line trip-outs were caused by direct strokes and not by induced charges. Many strokes come very close to the line and near the oscillograph with no record obtained although a surge of two times normal would be quite legible.

Fig. 6—Switching Surge when Deenergizing Line

One conductor alone is involved during a stroke on the line and often it is the first wire adjacent to the approaching storm. The oscillograph is attached to only one conductor and a line tripout has resulted from

a stroke near the station but with a very small surge recorded on the oscillograph. A heavy surge would be recorded by the klydonograph on some other conductor. Conversely a heavy surge may be recorded on the oscillograph and the klydonograph on the same conductor while the other two conductors will have very light surges shown.

The klydonographs located one mile from the station have shown surges beyond the calibration range and the oscillograph would show a light surge indicating the attenuation which occurs.

Low resistance tower footings and a low resistance ground wire were indicated as desirable for decreasing flashovers. Due to the line being built over an almost solid rock country with a light covering of weathered rock, some artificial means of lowering tower footing resistance was indicated.

No damage to insulators from flashovers has been experienced in ten years operation and no burns on the cable were found during the last inspection made two years ago when each tower was climbed. Ground current relays have not been used but will be installed in the future. The generating capacity on the line varies from 130,000 to 150,000 kv-a., and at times the system operates in parallel with another large system through a 25,000-kv-a. auto-transformer bank.

Discussion

J. J. Toroki: The investigation work carried on in Tennessee has been exceedingly fruitful. The number of records obtained has not been astonishingly large but the nature of those which were obtained is such that it exposed serious discrepancies in the early theories and aided considerably in establishing a more consistent concept of the origin and manner in which surges appear on the line and their nature.

The early concepts of the formation of surges on lines were predicted on the inductive theory, that is that the bound charge on the transmission line was released suddenly when the cloud discharged. Accordingly, all the transmission line designs were based purely on this theory. The value of ground wires was intimated on their protective ratio. The insulation on the lines was determined by height and ground wire location and many other minor details were governed purely on this basis, yet these lines do not show a very marked improvement over those which had been built according to the dictates of construction economics only.

The first season's operating experience bore strong evidence against the induced voltage theory. Many storms raged about the station and over the line yet only one record was obtained in which the voltage rose above three times the operating value. The records of the operators showed clearly that of the hundreds of strokes appearing over or near the line only one or two resulted in a surge on the line which might be harmful in its character. The oscillograms of these surges showed that they were of long duration ranging from 30 to several hundred microseconds which again was contrary to the earlier concepts which dictated waves of short duration ranging from perhaps one or two microseconds to 15 or 20. Using these records as a basis and also those which were obtained later on oscillograph stations located in Chicago, New Jersey, and Arkansas, a series of calculations was made to determine the probable magnitude of induced surges. These calculations showed that if the clouds discharged in a period of time indicated by the oscillograms that the induced voltages would hardly exceed one or two hundred thousand volts which is a value quite harmless to high-voltage transmission lines. Then the direct stroke theory was ushered into prominence. In the past direct strokes were always considered as an irresistible force which man could not yet cope with. However, this realization, that the surges which we have been protecting against in the past were the results of lightning volts coming directly in contact with the line instead of originating by induction, gave the engineers renewed hope in their strive for the ideal—the lightning-proof line.

The direct stroke theory dictates that if the line is to be made immune from lightning it must have some protection against these direct strokes. The ground wires which have been used in the past still seem to be a good solution for the lightning problem, however, modifications must be made so that the new concepts can be satisfied, namely, that these ground wires should be so placed that the stroke itself must terminate on the ground wire rather than on the conductors. This meant that the ground wires must be placed well above the conductor and also in such a position that the ground wires protect not only one but all three conductors. Furthermore, the estimates and measurements of the currents in the lightning bolt dictated that the tower and footing impedances should be very low so that the tower itself will, when carrying these heavy currents, not rise up to a high potential above earth and thus cause flashover from the tower to the conductor. Analysis of a number of transmission lines indicated very strongly that this direct stroke theory will be much more useful than its predecessor. Lines which either by chance or design were located in a terrain where low tower footing resistance could readily be obtained had exceedingly good operating records, one especially which had an average tower footing resistance around two or three ohms has only had one outage in five years, whereas a line of similar design but going over mountainous country where the tower footing resistance ranged from a few ohms to several hundred has an average of five or six outages per year. Further analysis on the position of the ground wires also shows that where only one ground wire is situated in the middle and close to the conductor the performance is very poor. An exceedingly marked improvement is obtained in installations where two ground wires are so situated with respect to the conductors that they shield them physically.

Another question which has been less pressing in the engineers' mind is that of switching surges. Here again the experimental stations in Tennessee have been of considerable importance. They have made records of switching surges that occurred during the normal operation of the line. These records indicated that a normal switching surge will seldom exceed three times normal and only rarely rise to four times normal. Such switching surges are of great value in that they will indicate the minimum insulation a transmission line can have and still give good operating records. All these data and much more can be attributed directly or indirectly to the Tennessee investigation.

Operating Experience With Reactance Type Distance Relays

BY E. E. GEORGE[1]

Member, A. I. E. E.

Synopsis.—*In view of the fact that reactance type distance relays have been in service in this country less than a year, operating experience should be of general interest to the industry at this time. This paper presents the performance record during the 1930 lightning season of two new types of reactance relays. One of these types is a high-speed American built relay, using a stepped time-distance characteristic. The other relay, using a sloped time-distance characteristic is European built, but designed for a more conventional speed of operation.*

These relays have performed satisfactorily on long transmission lines with wide variation in short-circuit conditions. The correctness of the reactance principle has been demonstrated. Certain difficulties with these relays have arisen on interconnections operating near the stability limit. Both definite and tentative conclusions from this year's operating experience are included in the paper.

* * * * *

Dates of Installation

On February 9, 1930, the Tennessee Electric Power Company installed three reactance relays at their Great Falls Hydro Plant. These relays were of the normal speed distance type (LB-1) and were furnished by the American Brown Boveri Company, but were built in Baden, Switzerland, by Brown Boveri, Limited. This is believed to be the first installation of reactance relays in this country.

On February 19, 1930, the Tennessee Electric Power Company set up a temporary test installation of three General Electric high-speed reactance type (GAX) distance relays at their Ocoee No. 1 Hydro Plant. These same relays were permanently installed at West Nashville Substation on February 23, 1930. The Ocoee installation was the first in this country of American built reactance type distance relays.

Another installation of Brown Boveri type LB-2 relays was made at Ocoee No. 1 on February 25, 1930, and another installation of GAX distance relays was made at Centerville Substation on March 30, 1930.

Since the above time 21 more Brown Boveri type LB-1 relays and 21 more General Electric type GAX relays have been installed, making a total of 54 distance relays of the reactance type on this system. In addition to the above, six high-speed distance relays of the Westinghouse impedance type (HZ) are now being installed.

Figs. 1 and 2 are typical wiring diagrams for the two types of reactance relays and show how the test facilities preferred by this company have been applied. If it is desired to use the auxiliary equipment offered by one manufacturer for securing ground protection as well as phase protection, the test scheme becomes more complicated.

Reasons for Installation

The reasons for the rapid installation of these new types of relays may be of general interest. Herewith is given a tabulation of the variation in short-circuit kv-a. on the various high-tension buses of this system. This variation is due chiefly to changes in operating conditions with steam and hydro generation at widely different locations. The tabulation covers the total short-circuit kv-a. on the bus, and any one outgoing line has substantially less current.

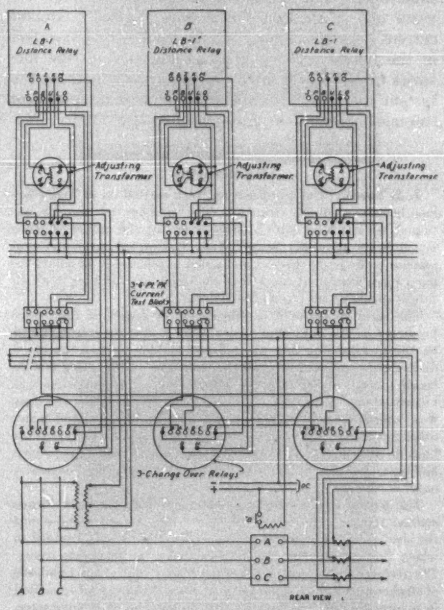

Fig. 1—Wiring Diagram for Type LB-1 Distance Relays

1. Supt. of Electrical Operation, Tennessee Electric Power Company, Chattanooga, Tennessee.

Presented at the Southern District Meeting No. 4, of the A. I. E. E., Louisville, Kentucky, November 19-22, 1930.

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Fig. 3 shows the location of circuit breakers on the 154-kv. and 110-kv. systems together with the type of phase protection now in service at each. Many of the

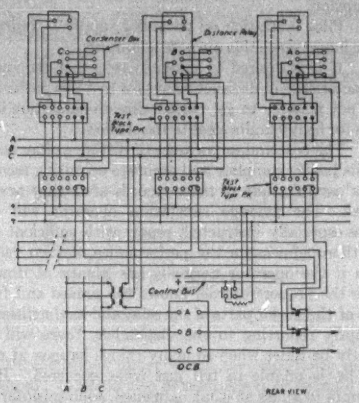

Fig. 2—Wiring Diagram for Type GAX Distance Relays

TABLE I
VARIATION IN SHORT-CIRCUIT KV-A. ON TENNESSEE ELECTRIC POWER SYSTEM

	Maximum	Minimum
Arlington	680,000	160,000
Centerville	360,000	170,000
Great Falls	480,000	210,000
Hales Bar	600,000	250,000
Ocoee No. 1	780,000	250,000
Ridgedale	560,000	220,000
West Nashville	390,000	180,000
Wilson Dam	800,000	150,000

line sections are of No. 2/0 copper or equivalent. This map together with the table of short-circuit kv-a. shows why the short circuit at the end of a long section may be even lower than the normal load of the line. Furthermore the opening of a section under fault frequently requires the adjacent sections to carry as much as twice normal load for a minute or more. Over-current relays (and relays having over-current pick-up) have proved inadequate and almost useless under these conditions.

The principle of the distance relay finds a ready application in handling this situation. Reactance type relays were preferred over the impedance type since it is thought that the former give a more accurate measurement of the distance to the fault by being independent of arc resistance. This was given considerable weight on account of the fact that the resistance of many lines on this system constitutes an appreciable proportion of their impedance. Reactance type relays have another advantage over impedance relays as now built when applied to lines having low minimum short circuit compared to maximum load. Since distance measurement is taken care of by the reactance meter, the impedance release or starting unit may be designed to have its release characteristic vary with the voltage. At 20 per cent of normal voltage, both types of reactance relays considered will release with current below 10 per cent of that required to release them at normal voltage.

All future installations on the 154-kv. and 110-kv. systems will probably be reactance type distance relays, although one short line will soon be equipped at both ends with an experimental installation of high-speed impedance type (HZ) distance relays. In selecting distance relays to date more attention has been paid to getting low pick-up under minimum generating conditions for long lines, than in securing high speed, although high speed will be more of a factor as high-speed breakers are installed generally over the system.

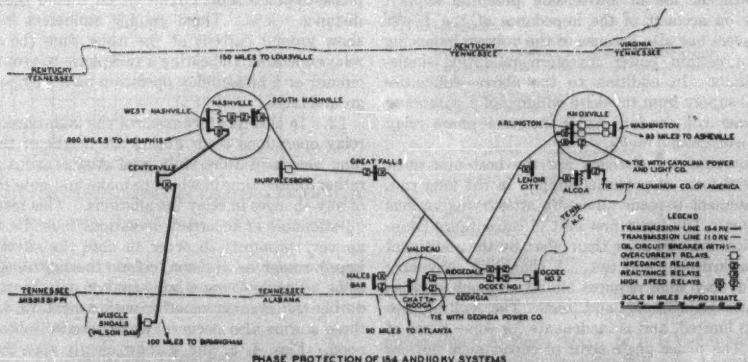

Fig. 3—Location of Distance Relays on Tennessee Electric Power System

Operating Experience

Both types of reactance relays have, in the opinion of the writer, demonstrated the following conclusions beyond question:

1. Reactance relays will protect each section of line selectively without cascading settings. Each section of line is thus protected independently of other sections.

2. The amount and location of generation has no appreciable effect on the time of clearing faults. Any secondary current of about 2 amperes or over will operate the relay in a predetermined time if the voltage is low enough to indicate a fault in the protected zone.

3. Loads much beyond the sustained thermal capacity of a transmission line can be carried without danger of tripping. With normal bushing current transformer ratios, the relays will carry from 500 to 1,000 amperes primary current, whereas the normal rating of the average high-tension transmission line is probably 250 to 350 amperes. Sustained currents of about ten times normal rating will fuse the average line. Sustained currents of possibly five times normal rating will cause failure of a line under normal tension. Frequently it is desired to carry twice normal load on a transmission line for a few seconds and there is no thermal limitation to doing this, but it is usually impossible with the ordinary overload relay. In case of a non-attended station, it may be necessary to back up the distance relays with some other type of relay following more closely the thermal characteristic of the transmission line. Ordinarily the operator or load dispatcher will notice promptly any unusual diversion of power and take steps to reduce the load.

4. Impedance releases or starting units on reactance relays should be set rather high—ten amperes or more, at normal voltage.

5. High-tension potential supply is necessary in practically all cases. If this is not provided there will be a certain percentage of incorrect operation due to trouble on the lower voltage system. Selectivity will suffer with the use of low-tension potential supply, not only on account of the impedance of the power transformers but also because of the voltage balancing action of certain power transformers during single-phase faults. In addition to the above difficulties potential supply from the delta winding of a grounding transformer will tend to cause incorrect phase relay operation on ground faults.

6. Potential transformers are the best and most economical form of potential supply, in the long run. This statement is made advisedly after trying various substitutes. The lower first cost of capacitance coupling devices may be more than offset by the additional cost of adjustment and test. A change in secondary loading at any time requires almost as much work as the initial adjustment and test. The volt-ampere output is limited, and is inadequate for some types of relays. The phase angle error of capacitance devices is considerable at very low voltage, where accuracy is most essential for reactance relay potential supply. Furthermore, when high-tension potential supply is installed for relays, there is a tendency to use it for synchronizing, voltage indication, frequency indication, indicating meters, watthour meters, etc. It is clearly impossible for capacitance devices to handle such burdens, whereas 500-volt ampere potential transformers will safely handle all the potential burdens in the station.

7. Pilot indication on potential supply is desirable. Neon lamps serve this purpose admirably.

8. If it is desired to take advantage of the special characteristics of distance relays for fast bus protection, it is not possible to get correct directional action in all cases for heavy faults close to the station.

9. The cost and importance of distance relay installations justify complete and accurate operating records, and a careful study of these records after they are obtained. The average substation operator will not, unless especially instructed, report with sufficient care and thoroughness on the performance of such relays. Even if the operator normally has plenty of time to make the proper records, knows the names and functions of the major parts of the relay and has facilities for promptly reporting to the dispatcher, there will frequently be times when so many things happen at once that he is unable to tell just what occurred. If he attempts to guess at what happened, such misinformation is much worse than no information at all. In spite of such handicaps there will be occasions when a good operator, well trained, can give very valuable information on relay operation.

10. Automatic oscillograph installations should be very valuable in analyzing distance relay operation, especially on occasions when the station operator's time is limited.

11. Quick trip ammeter installations as used to record neutral ground current, have been very helpful. The absence of a ground current record indicates a phase-to-phase short circuit which should operate the distance relays. These graphic ammeters frequently show ground current at the same time the distance relays operated, indicating a two-phase-to-ground short circuit, or a phase relay operation on a single-phase-to ground fault.

12. In this territory most of the high-tension phase relay operations occur during the height of the lightning season in June, July, and August, thus giving a rather limited time in which to make and note the effect of any changes in relay installations. This means that questionable or incorrect operations must be followed up very promptly in order to note the effect of any improvement or revision before the lightning season ends. The lightning storms are not only concentrated during the summer months, with a peak in July, but these storms also occur most frequently in the afternoon. Figs. 4, 5, and 6 show clearly that the major portion of each year's lightning occurs between the hours of 1:00 p. m. and 5:00 p. m. in June, July, and August. This extreme concentration of trouble greatly complicates relaying and dispatching problems due to independent troubles occurring close together, or even

simultaneously. A second line may relay out before the first line can be closed back in, thus leading to confusion in the operating records as well as to interruptions to customers provided with two or more sources of supply, even though none of the circuits go in trouble permanently.

13. One type of reactance relay is equipped with a distance indicator which has been very helpful in studying the conditions under which the impedance release operated. It would also be desirable to have more than one target on the step-by-step distance relays so that there would be a record of whether the relay tripped on instantaneous, intermediate or back-up time. It has been found impracticable to determine this by watching the relay, even on staged tests.

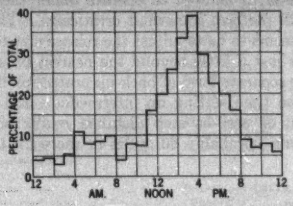

Fig. 4—Hourly Distribution of Interruptions due to Lightning

14. No entirely satisfactory method of low-voltage routine tests has been devised for distance relays. They have been checked by staged tests during installation, and occasionally thereafter in case of questionable operation.

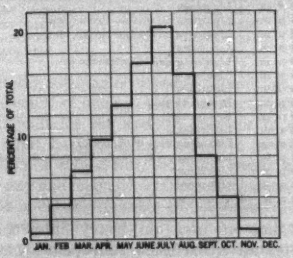

Fig. 5—Monthly Distribution of Lightning Storm Days

15. Both types of reactance relays have been found to operate on arc faults in the same time as on solid short circuits, even up to the longest arcs that could be maintained on the system.

Operating Problems

In addition to the foregoing conclusions regarding distance relay performance, there are certain questions which, in the opinion of the writer, have not been definitely answered as yet. These are outlined below, together with some of the evidence that may be helpful in reaching conclusions, especially when and if corroborated by further experience.

Tables II, III, and IV cover the operation of distance relays on this system from the dates of their installation up until the present time. From these detailed records the reader may analyze the available evidence in brief and form his own conclusions in the light of his own experience and his own particular operating conditions.

1. Apparently the operation of all types of distance relays is highly satisfactory on radial feeders. The only difficulties arise on trunk lines and on interconnections.

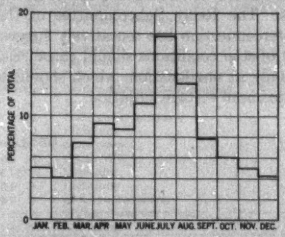

Fig. 6—Monthly Distribution of Relay Operations

2. The performance of distance relays under out-of-step conditions is still uncertain. This liability is not confined to these relays, as it is definitely known that the old impedance relay will operate quickly on any out-of-step condition if the current peaks exceed the pick-up value for a short time. Both laboratory and field tests show that one type of reactance relay will operate on out-of-step conditions if the frequency of surging is very low or very high. Another type seems

TABLE II
SUMMARY OF REACTANCE RELAY OPERATIONS
Through September 30, 1930

Correct operations	47
Questionable operations	7
Wrong operations	0
Inadequate potential supply	18
Total	72
Analysis:	
Correct operations	
Line relayed at one end only	3
Same line relayed simultaneously at other points	28
Line relayed while being supplied from one end only	5
Back-up protection	3
Low-tension bus failure	1
Bad synchronizing	5
Bus protection	2
Total	47
Questionable operations	
Out-of-step conditions	7
Total	7
Inadequate potential supply	
Cross phased	3
44-kv. trouble	5
Bushing potential devices	5
110-volt secondary trouble	5
Total	18

3. Even if out-of-step conditions should cause relay operation, many interconnections are subject to more or less prolonged surging somewhat below the stability to be reasonably immune to ordinary out-of-step conditions but severe surges may cause incorrect operation. The recently completed tie in East Tennessee interconnecting about 5,000,000 kw. in the north and 1,000,000 kw. in the south is particularly subject to bad surges. This problem is being actively studied at the present time. Unfortunately there is no precedent for saying that a line should or should not relay on out-of-step conditions. The Relay Subcommittee of the Southeastern interconnection has tentatively suggested that severe out-of-step conditions should cause relay operation, but only at one location, which should be at the point of maximum voltage variation.

TABLE III

DETAILS OF LB RELAY OPERATIONS

6 Installed in February 1930
3 Installed in May 1930
3 Installed in June 1930
6 Installed in July 1930

Station	Line	Date	Time	Cause
Ocoee No. 1	Nashville	3-20-30	11:20 a. m.	Open potential circuit
Great Falls	Cleveland	3-23-30	10:27 a. m.	Potential network
Great Falls	Cleveland	3-23-30	1:18 p. m.	Potential network
Great Falls	Cleveland	3-23-30	1:41 p. m.	Potential network
Great Falls	Cleveland	3-26-30	11:58 a. m.	Potential network
Ocoee No. 1	Nashville	6-21-30	3:07 p. m.	Lightning
Great Falls	Cleveland	7- 2-30	11:21 p. m.	Wilson Dam out of step
Great Falls	Cleveland	7-10-30	2:11 p. m.	Nashville out of step
Great Falls	Cleveland	7-10-30	2:12 p. m	Bad synchronizing
South Nashville	Great Falls	7-13-30	2:31 a. m.	Bus failure
Arlington	Maryville	7-13-30	6:35 a. m.	Lightning
Arlington	Cleveland	7-13-30	5:49 p. m.	Lightning
Arlington	Maryville	7-16-30	7:43 p. m.	Lightning in Georgia
Arlington	Cleveland	7-23-30	12:08 p. m.	Lightning
Arlington	Cleveland	7-23-30	1:04 p. m.	Lightning on double-circuit line
Arlington	Maryville	7-23-30	1:04 p. m.	Lightning on double-circuit line
Arlington	Maryville	7-23-30	1:14 p. m.	Lightning
South Nashville	Great Falls	7-23-30	5:08 p. m.	Lightning near Centerville
Great Falls	Cleveland	8- 9-30	2:04 p. m.	Lightning
Ocoee No. 1	Nashville	8- 9-30	2:04 p. m.	Lightning
Arlington	Maryville	8-10-30	3:20 p. m.	Bus protection
Arlington	Cleveland	8-10-30	3:20 p. m.	Bus protection
Arlington	Cleveland	8-14-30	7:26 a. m.	Lightning
South Nashville	Great Falls	8-14-30	7:57 a. m.	Lightning near Wilson Dam
South Nashville	Great Falls	8-14-30	6:32 p. m.	Lightning on W. Nash. line near S. Nash
Arlington	Cleveland	8-21-30	3:40 p. m.	Lightning
South Nashville	Great Falls	9- 8-30	8:38 a. m.	Lightning
Arlington	Cleveland	9- 8-30	10:44 a. m.	Lightning

limit and relays should hold in all such surges. This is one of the problems on the East Tennessee interconnection.

4. Wide variations in power factor during load swings seems to be one of the chief difficulties in applying reactance type relays to trunk lines and interconnections.

5. Impedance releases or starting units should have reset values fairly close to the release values, particularly if the starting unit is non-directional. Otherwise a fault may be cleared promptly at some other location, but the resulting surging or readjustment of power flow may be sufficient to hold the relay released and cause incorrect operation.

6. It would be advantageous for the starting units to be made more readily adjustable. At present they are designed and factory adjusted so as to give back-up protection to two or three average sections of line, while the distance measuring element is very flexible and can be adjusted for most any combination of sections. This arrangement is quite satisfactory as far as actual short circuits are concerned, but does not take care of the difficulties of load and power factor swings encountered on interconnections. Obviously a reactance meter, if allowed to operate under these conditions, will perform somewhat erratically. The logical remedy seems to be to adjust the release unit to operate on faults just a little beyond the nearest breaker, and use some separate form of back-up protection if desired.

7. The time setting on intermediate and back-up operation of step-by-step distance relays should be rather long—probably 60 cycles or more for the intermediate time—if used on heavy interconnections. This is a tentative conclusion from operating experience on the East Tennessee interconnection.

8. It may be possible to have the instantaneous element on a distance relay operate too quickly. Incorrect operation may occur due to breaker adjustment if the relay trips under two or three cycles.

9. There seem to be certain complications in using the bus protection feature of distance relays, and it is recommended that important stations be protected by some independent form of bus protection so that distance relays can be used for outgoing protection only.

10. It does not seem to be wise to try to speed up the initial time too much on certain types of relays. A certain definite time is required for proper directional discrimination.

TABLE IV
DETAILS OF G A X RELAY OPERATIONS
3 Relays installed in February 1930
3 Relays installed in March 1930
6 Relays installed in September 1930

Station	Line	Date	Time	Cause
West Nashville	Centerville	3-29-30	12:01 p. m.	Air break switch arced between phases
Centerville	West Nashville	4- 6-30	4:17 p. m	44 kv. bus switch opened by mistake
Centerville	West Nashville	6- 7-30	4:14 p. m.	Potential circuit opened by mistake
West Nashville	Centerville	6-19-30	10:02 a. m.	Insulator failure
Centerville	West Nashville	6-30-30	3:15 p. m.	Lightning near Great Falls
Centerville	West Nashville	6-30-30	3:17 p. m.	Bad synchronizing
Centerville	West Nashville	7- 7-30	6:45 p. m.	Lightning
West Nashville	Centerville	7- 7-30	6:45 p. m.	Lightning
Centerville	West Nashville	7-10-30	2:11 p. m.	Lightning near Great Falls
West Nashville	Centerville	7-10-30	2:22 p. m.	Lightning
Centerville	West Nashville	7-10-30	2:53 p. m.	Lightning on 44 kv.
West Nashville	Centerville	7-10-30	3:10 p. m.	Lightning
West Nashville	Centerville	7-10-30	4:10 p. m.	Lightning
Centerville	West Nashville	7-10-30	4:10 p. m.	Lightning
Centerville	West Nashville	7-11-30	12:12 p. m.	Lightning near Great Falls
Centerville	West Nashville	7-11-30	3:09 p. m.	Centerville transformer failed
Centerville	West Nashville	7-17-30	5:58 a. m.	Bad synchronizing
West Nashville	Centerville	7-18-30	1:34 p. m.	Bad synchronizing
West Nashville	Centerville	7-23-30	5:08 p. m.	Lightning
Centerville	West Nashville	7-23-30	5:08 p. m.	Lightning
West Nashville	Centerville	7-23-30	5:23 p. m.	Centerville by-passed
Centerville	West Nashville	7-24-30	1:22 p. m.	Lightning
West Nashville	Centerville	7-24-30	1:22 p. m.	Lightning
West Nashville	Centerville	7-24-30	1:26 p. m.	Bad synchronizing
Centerville	West Nashville	7-24-30	1:57 p. m.	Wilson Dam out of step
Centerville	West Nashville	7-25-30	3:24 p. m.	Lightning
West Nashville	Centerville	7-25-30	3:24 p. m.	Lightning
West Nashville	Centerville	7-25-30	3:48 p. m.	Lightning
Centerville	West Nashville	7-29-30	2:31 p. m.	Lightning on 44 kv.
West Nashville	Centerville	8- 9-30	2:04 p. m.	Lightning south of Centerville
Centerville	West Nashville	8-14-30	7:25 a. m.	Lightning on 44 kv.
West Nashville	Centerville	8-14-30	11:06 a. m.	Lightning south of Centerville
Centerville	West Nashville	8-14-30	2:29 p. m.	Loose connection
Centerville	West Nashville	8-14-30	6:32 p. m.	Loose connection
Centerville	West Nashville	8-14-30	8:32 p. m.	Loose connection
Centerville	West Nashville	8-14-30	9:17 p. m.	Lightning
West Nashville	Centerville	8-14-30	9:17 p. m	Lightning
West Nashville	Centerville	8-17-30	3:12 p. m.	Lightning
Centerville	West Nashville	8-17-30	3:12 p. m.	Lightning
West Nashville	Centerville	9- 8-30	6:11 a. m.	Lightning
West Nashville	Centerville	9- 8-30	8:38 a. m.	Lightning
West Nashville	S. Nashville	9- 8-30	8:38 a. m.	Lightning
Centerville	Wilson Dam	9-13-30	12:27 p. m.	Lightning
Centerville	Wilson Dam	9-24-30	6:19 a. m.	Insulator failure

11. Relay settings or schemes should not be modified on account of one report of incorrect operation. It is too difficult for the average operator to determine exactly what happened, and except in case of careful observation on staged test, too much weight should not be given any one reported erratic performance.

12. It is entirely feasible to use separate and independent ground protection with distance relays. It has always been the practise of this company to use current directional ground relays for ground protection, and no change was made in this practise when distance relays were installed for phase protection. However, both manufacturers have developed schemes of ground protection using distance relays and star potential supply.

13. Distance relays can be used successfully on the same system with other types of relays. However, if the ordinary types of relays operate slowly under minimum generating conditions the distance relays will frequently operate ahead of them.

Summary

Operating experience has shown the correctness of the major principles of design and application of reactance type distance relays on this system. Several difficulties have been encountered but these have been given less weight than the prime considerations of carrying high load safely during emergency and of clearing the majority of faults selectively during all operating conditions.

Discussion

J. B. MacNeill: Mr. George states:

"Potential transformers are the best and most economical form of potential supply in the long run."

This is admittedly so where very large secondary burdens are to be handled, or where extreme accuracy of phase angle is required. However, the fact remains that successful operation on many large systems, including relaying, indicates definitely the worth of bushing potential devices when used within their ratings. The low cost of these devices and the elimination of high-voltage potential transformers justify the bushing potential device where usable.

Looking to the future, it is reasonable to expect that auxiliary devices, such as relays, volt meters, synchronoscopes, etc., will all have reduced burden, thus broadening the application of low output sources of potential. Recent improvements in bushing potential devices have reduced phase-angle error considerably over wide voltage range. The error now obtainable at 10 per cent of normal operating voltage is not greater than 20 per cent of previous value.

Lighting Airway Beacons
Direct From High-Voltage Transmission Lines

BY F. W. CARTLAND[1]
Associate, A. I. E. E.

Synopsis.—Airway beacons requiring from 1½ to 2½ kw., may be lighted direct from low-voltage distribution lines, gas engine-driven generators or from high-voltage transmission lines. The latter requires a high potential transformer of very small capacity, although most standard high potential transformers are made only in ratings of 15 to 100 kv-a.

The "capacitap" is a potentiometer device similar to that used with condenser bushing terminals, the capacities being composed of suspension type capacitor units. The capacitap will give satisfactory performance for certain types of loads. The device is very well adapted to the supply of power to airway beacons having nearly constant load and approximately 100 per cent power factor where not more than 3 kw. is required. The construction and performance are discussed in detail.

Introduction

THE rapid progress of aviation during the last few years has emphasized the need of adequately lighted airways. The airports have been located near the cities where electric power for flood lights and searchlights was readily available. However, the supply of power to airway beacons in isolated places has been a more difficult problem. Beacons in such localities have been operated from low-voltage transmission lines or where these were not available, gas engine-driven generators have been used. The power required for such installations is approximately 2 kw., and low capacity transformers are available where the system voltage does not exceed 23,000 volts. When higher voltage lines are encountered the smallest standard transformers built vary from 15 to 100 kv-a., increasing with the system voltage. A 100-kv-a. transformer used to deliver but 2 kw. is not only expensive but also inefficient.

A paper presented by Messrs. Spracklen, Marshall, and Langguth at the Winter Convention in New York, February 13-17, 1928, described a potentiometer device for use with condenser type bushings, which supplied sufficient power for operating relays and synchronizing equipment. With the development of high-voltage outdoor capacitors of the suspension type, developed the possibility of adapting them to a potentiometer circuit which would supply greater amounts of power than could be obtained from a condenser bushing. While the capacitor could be thus used to supply very great quantities of low-voltage power, the capacitors as built at present are of such a size and construction that it is not considered desirable or economical to obtain more than 2 or 3 kw. from such a circuit.

Description of Capacitor Power Supply

The capacitor power supply or "capacitap," consists essentially of a number of suspension type capacitors connected in series with a portion paralleled by the load and a reactor in series as shown in Fig. 1. Since in most cases the load voltage is 110 volts, the use of a small transformer (11,000/110) as shown in Fig. 2 is more desirable considering the capacitor economy. Another modification of this circuit, as shown in Fig. 3, has an additional advantage in that the full load

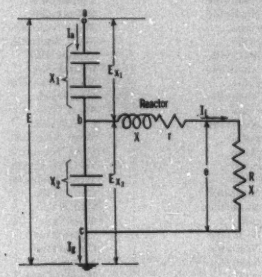

FIG. 1—THEORETICAL CIRCUIT—SINGLE LINE TO GROUND

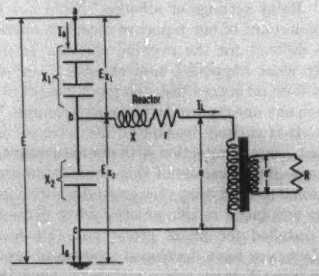

FIG. 2—SINGLE LINE TO GROUND USING SMALL TRANSFORMER

ground current is much less than that of the equipment shown in Fig. 2. (Considering circuits which produce the same voltage regulation.)

A capacitap as shown in Fig. 4 was built for studying the performance and practicability of this circuit. This equipment, built for supplying 2½-kw. unity

1. Capacitor Engineer, Westinghouse Elec. & Mfg. Co., E. Pittsburgh, Pa.

Presented at the Southern District Meeting No. 4, of the A. I. E. E., Louisville, Kentucky, November 19-22, 1930.

30-179

power factor at 110 volts from a 66-kv. line had load voltage curves as shown in Fig. 5. The curve of Fig. 6 shows the result of varying the size of reactor. The complete circuit as shown in Fig. 7, consisted of eight low-voltage capacitors, sixteen high-voltage capacitors shown in Fig. 8, a standard transformer, a reactor and two enclosed spark gaps. Both transformer and reactor were tapped to give the desired adjustments. Each of the low-voltage capacitors was 0.020 μf., 10,000 volt and each of the high-voltage capacitors was

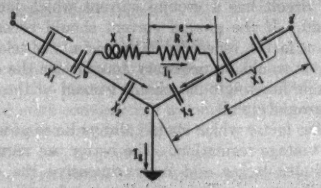

Fig. 3—Theoretical Circuit—Double Line to Ground

Fig. 4—Experimental Installation (2½ Kw. from 66 Kv.)

0.008 μf., 16,000 volt. The reactor was designed for 14,000 volts and had 176 henrys maximum inductance. The spark gaps were placed in parallel with capacitors $b\,c$ and $b'\,c$.

The individual capacitor unit is shown in Fig. 8. It is similar in design to those used for carrier current applications, but is a special rating. The construction is such that the capacitor will flash over the outside of the porcelain before it will flash through the inside. The characteristics are similar to those for line insulators. The wet flashover at 60 cycle is 70 kv. and the dry flashover is 116 kv. The weight of each capacitor is approximately 115 lb.

Installation

The capacitap may be installed in several ways such as suspending from an existing tower or from a tower or pole erected especially for carrying this load. The weight of the complete equipment shown in Fig. 4, is about 3,000 lb. The equipment may be connected

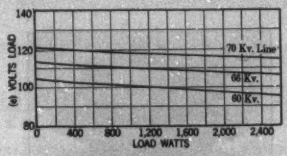

Fig. 5—Load Voltage Curves. (Experimental Installation)

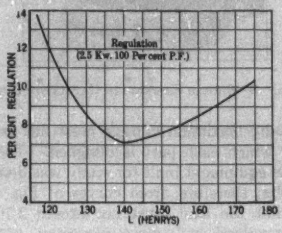

Fig. 6—Curve of Regulation vs. Inductance of Reactor

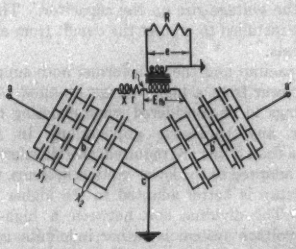

Fig. 7—Complete Circuit of Equipment Shown in Fig. 4

direct to the transmission line, however, the operating companies will probably use disconnecting switches. As this type of equipment can be mounted suspended above ground, it will not be necessary to build any enclosing fence as is required for equipment mounted on the ground.

Application

A study of the various possible circuits has definitely shown that the capacitap may be well adapted to certain classes of loads, while for others, the operation

would be greatly unsatisfactory. For example, either the supply of power to a beacon lamp load or the supply for charging a storage battery through a rectifier would be a satisfactory load; however, the operation of a large motor, or the closing of a circuit breaker would not be suitable. A current surge as encountered in starting a motor should not be delivered by the capacitor power supply as at present designed. Reference to Fig. 9 will show that as the load current increases, the voltage in various parts of the circuit rises, and in

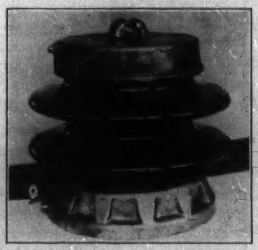

Fig. 8—Single Suspension Capacitor Unit

all places the maximum voltage must not exceed the rating for which each part is designed. The reactor used in the equipment built for study was designed for 14,000 volts; therefore, from curves of Fig. 9 it will be seen that the load must not exceed 2,700 watts. Also an overload will cause a current which may saturate the reactor. This condition would then tend to limit the voltage rise on the capacitor. The spark gaps were installed to protect the circuit from excessive voltage rises.

The capacitap and the transformer both supply low-voltage power from a high-voltage line and there are many things to be considered when choosing between them for any particular application. In general, where less than 3 kw. are required, the transformer will be better adapted to the lower voltage systems, whereas the capacitap is better adapted to the higher voltage systems. The dividing line between a high-voltage and low-voltage system is rather indefinite and will depend somewhat on the applications. For example, the supply of 2 kw. from a 44,000-volt line (25,400 volts to ground) will be best obtained from a transformer, while the supply of 100 watts at 110 volts for charging a storage battery (through a rectifier) may be economically obtained with the capacitap. For all ratings, the efficiency of the transformers and capacitaps is very high at full load, whereas a 50-kv-a. transformer (high voltage) used to supply 2 kw. will be much less efficient.

There are so many variable factors in the design of a capacitap, that all study has been limited to a grounded neutral system. Other arrangements will likely follow which will be adaptable to ungrounded or delta-connected systems.

Single-phase power may be obtained from the use of a line-to-ground connection, (Fig. 2) or a double line-to-ground connection, (Fig. 7). Often one may be considered as a substitute for the other, however, the inherent characteristics of each will greatly affect the choice. As will be shown later, the double line-to-ground circuit has a ground current which is constant and independent of load; however, the single line-to-ground circuit has a ground current which increases with load. If the ground current should become an important factor, especially in the vicinity of telephone circuits, it would be necessary to consider the use of the double line-to-ground circuit instead of the single line-to-ground circuit.

Another factor which should always be considered is that of voltage regulation. The better the regulation required, the larger and more expensive the equipment will become. The regulation is a function of the total kv-a. of capacitors and the ratio between the impedance of the line and ground branches of the circuit. The single line-to-ground circuit will be found to have slightly better regulation than the double line-to-ground circuit, using the same capacitors in either circuit.

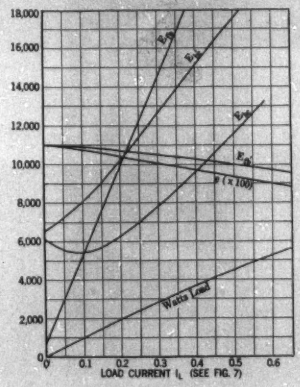

Fig. 9—Load and Voltage Characteristics

Summary

The capacitor power supply (capacitap) is an assembly of capacitors and associated apparatus which supplies power at a reduced voltage from high-voltage lines. The present practical limit is 3 to 5 kw. Tests have shown that such an equipment has characteristics which are suitable for only certain types of loads, such as airway beacons. The capacitors are very reliable and are designed with large factors of safety. The equipment may be suspended above ground making an

enclosing fence unnecessary. Two single-phase circuits are possible each having different characteristics with regard to ground current and voltage regulation. Only applications to grounded neutral systems have been considered. The capacitap is especially adapted to high-voltage systems.

The condenser bushing potential device as developed by Mr. J. F. Peters, supplied the basis for development of the capacitor power supply. The author wishes to acknowledge with thanks, the cooperation of Mr. Peters in this present development.

Appendix I

Since the regulation of any standard transformer is known, the characteristics of the circuits may be studied more conveniently by omitting the transformer and using the circuit of Fig. 1.

Let E = line-to-ground voltage.
x_1 = reactance of the upper capacitor group.
x_2 = reactance of the lower capacitor group.
x = reactance of the reactor.
r = resistance of the reactor.
R = load resistance.
X = load reactance.
e = load voltage.
e_0 = no load voltage.
y_2 = admittance of capacitor.
y_L = admittance of reactor-load circuit.

Referring to Fig. 1.

$$y_2 = \frac{1}{-j\,x_2}$$

$$y_L = \frac{1}{(r+R) + j\,(X+x)}$$

$$y_{bc} = \frac{x_2 + j\,[(r+R) + j\,(X+x)]}{x_2\,[(r+R) + j\,(X+x)]}$$

$$= \frac{(x_2 - X - x) + j\,(r+R)}{x_2\,[(r+R) + j\,(X+x)]}$$

$$Z_{bc} = \frac{x_2\,[(r+R) + j\,(X+x)]}{(x_2 - X - x) + j\,(r+R)}$$

$$Z_{ac} = \frac{x_2(r+R) + j\;x_2(X+x) - j\;x_1(x_2 - X - x) + x_1(r+R)}{(x_2 - x - X) + j\,(r+R)}$$

$$= \frac{(x_1+x_2)\,(r+R) + j\,[(x_1+x_2)\,(X+x) - x_1\,x_2]}{x_2 - (X+x) + j\,(r+R)}$$

$$I_a = \frac{\bar{E}\,[x_2 - (X+x) + j\,(r+R)]}{(x_1+x_2)\,(r+R) + j\,[(x_1+x_2)\,(X+x) - x_1\,x_2]}$$

$$I_L = \frac{I_a\,Z_{bc}}{(r+R) + j\,(x+X)}$$

$$I_L = \frac{\bar{E}\,x_2}{(x_1+x_2)\,(r+R) + j\,[(x_1+x_2)\,(X+x) - x_1\,x_2]}$$

$$= \frac{\bar{E}}{\dfrac{x_1+x_2}{x_2}\left[(r+R) + j\,(X+x) - j\,\dfrac{x_1\,x_2}{x_1+x_2}\right]}$$

$$e = \frac{\bar{E}\,(R + j\,X)}{\dfrac{x_1+x_2}{x_2}\left[(r+R) + j\,(X+x) - j\,\dfrac{x_1\,x_2}{x_1+x_2}\right]} \qquad (1)$$

If a non-inductive load is considered, $X = 0$

$$\bar{e} = \frac{\bar{E}}{\dfrac{x_1+x_2}{R\,x_2}\left[(r+R) + j\left(x - \dfrac{x_1\,x_2}{x_1+x_2}\right)\right]} \qquad (2)$$

If x is chosen to make the value of e a maximum, thus securing best regulation,

$$x = \frac{x_1\,x_2}{x_1+x_2}$$

$$\bar{e} = \frac{\bar{E}}{\dfrac{x_1+x_2}{x_2}\left(\dfrac{r}{R} + 1\right)} \qquad (3)$$

If a reactor could be made with $r = 0$, perfect regulation would be obtained,

$$\bar{e} = \frac{\bar{E}\,x_2}{x_1+x_2} \qquad (4)$$

The ground current or line current using equation (1) will be

$$I_g = I_a = \frac{\bar{E}\,[x_2 - (X+x) + j\,(r+R)]}{(x_1+x_2)\,(r+R) + j\,[(x_1+x_2)\,(X+x) - x_1\,x_2]}$$

Considering $X = 0$ and $x = \dfrac{x_1\,x_2}{x_1+x_2}$

$$I_g = \frac{\bar{E}\left[\left(x_2 - \dfrac{x_1\,x_2}{x_1+x_2} + j\,(r+R)\right)\right]}{(x_1+x_2)\,(r+R)}$$

$$I_g = \bar{E}\left[\frac{x_2^2}{(x_1+x_2)^2\,(r+R)} + j\,\frac{1}{x_1+x_2}\right] \qquad (6)$$

It is interesting to note that for $R = \infty$ (no load)

$$I_g = \frac{j\,\bar{E}}{x_1+x_2}$$

Appendix II

A similar analysis of the double line-to-ground circuit as shown in Fig. 3 follows:

Let

$\bar{E}_{ca} = E\,(-0.5 + j\,0.866)$
$\bar{E}_{ca'} = E$
$\bar{E}_{ca} = -j\,x_2\,(I_{ab} - I_L) - J\,x_1\,I_{ab}$

$$\bar{E}_{ea} = -j\,(x_2 + x_1)\,\bar{I}_{ab} + j\,x_2\,\bar{I}_L \tag{1}$$

$$\bar{E}_{ea'} = -j\,x_2\,(\bar{I}_{a'b'} + \bar{I}_L) - j\,x_1\,\bar{I}_{a'b'} \tag{2}$$

$$o = -j\,x_2\,(\bar{I}_{ab} - \bar{I}_L) - [(r + R) + j\,(x + X)]\,\bar{I}_L + j\,x_2\,(\bar{I}_{a'b'} + \bar{I}_L) \tag{3}$$

Solving equations (1), (2), and (3) simultaneously for $\bar{I}_L$

$$\bar{I}_L = \frac{E\,(1.5 - j\,0.866)}{-\left(\frac{x_1 + x_2}{x_2}\right)(r+R) + j\left[2\,x_1 - \left(\frac{x_1 + x_2}{x_2}\right)(x+X)\right]} \tag{4}$$

$$\bar{e} = \frac{E\,(1.5 - j\,0.866)\,(R + j\,X)}{-\left(\frac{x_1 + x_2}{x_2}\right)(r+R) + j\left[2\,x_1 - \left(\frac{x_1 + x_2}{x_2}\right)(x+X)\right]} \tag{5}$$

$$\bar{I}_{ab} = \frac{E + j\,x_2\,\bar{I}_L}{-j\,(x_1 + x_2)} \tag{6}$$

$$\bar{I}_{a'b'} = \frac{E\,(-0.5 + j\,0.866) - j\,x_2\,\bar{I}_L}{-j\,(x_1 + x_2)} \tag{7}$$

$$\bar{I}_e = \bar{I}_{ab} + \bar{I}_{a'b'} = \frac{E\,(0.5 + j\,0.866)}{-j\,(x_1 + x_2)} \tag{8}$$

Assuming a load having only resistance then $X = o$ and equation (5) becomes

$$\bar{e} = \frac{E\,(1.5 - j\,0.866)\,R}{-\left(\frac{x_1 + x_2}{x_2}\right)(r+R) + j\left[2\,x_1 - \left(\frac{x_1 + x_2}{x_2}\right)(x)\right]} \tag{9}$$

let $x = \dfrac{2\,x_1\,x_2}{x_1 + x_2}$ then $\dfrac{(x_1 + x_2)}{(x_2)}\,x = 2\,x_1$

$$\bar{e} = \frac{E\,(1.5 - j\,0.866)}{-\left(\frac{x_1 + x_2}{x_2\,R}\right)(r + R)} \tag{10}$$

Not only does this expression give conditions for best voltage regulation, but also the voltage $\bar{e}$ is in phase with $\bar{V}_{aa'}$.

Experiences with Grounded-Neutral, Y-Connected Potential Transformers on Ungrounded Systems

BY C. T. WELLER[1]

Member, A. I. E. E.

Synopsis.—The application of Y-connected transformers to three-phase, three-wire systems, in such a manner that the third and its multiple harmonics of the exciting currents would ordinarily be suppressed, is not in accordance with the best practise. However, such applications of grounded-neutral, Y-connected potential transformers, particularly to temporarily ungrounded systems, have been made for relaying and other purposes. The resulting complications, which are of a different nature than would be anticipated from the suppression of the third and its multiple harmonics, may not be generally appreciated, so this paper outlines experiences with two such applications. The peculiar and dangerous "saturation phenomena" obtained and the methods of preventing their occurrence are described and considerable data thereon included. The phenomena are discussed from theoretical standpoints in companion papers by A. Boyajian and O. P. McCarty and by C. W. La Pierre.

Introduction

THE application of Y-connected transformers to three-phase, three-wire systems, in such a manner that the third and its multiple harmonics of the exciting currents would ordinarily be suppressed, is generally regarded as objectionable. For power transformers, this difficulty has been overcome (1) by connecting the secondaries in delta or, if that is not permissible, (2) by adding tertiary windings connected in delta or (3) by adding a fourth conductor (metallic or ground) to connect the transformer and system neutrals. For potential transformers, only method (3) can usually be considered, since the closed-delta secondary or tertiary connection of methods (1) or (2) makes possible comparatively large circulating currents under certain system conditions. Potential transformer applications that do not utilize method (3), therefore, are not in accordance with the best practise for power transformers; however, applications of this nature have been made.

Grounded-neutral, Y-connected potential transformers with Y- or "broken-delta"-connected secondaries have been applied to three-phase, three-wire systems, the neutrals of which are temporarily or permanently ungrounded. The purposes of such applications include the protection of equipment against grounds, the economy resulting from the use of line-to-neutral[2] (L/N) instead of line-to-line[2] (L/L) potential transformers and the maintenance of the isolated-phase scheme for such installations.

It may not be generally appreciated that such applications frequently endanger connected equipment in addition to creating special relaying, synchronizing, and metering problems. This paper, therefore, outlines experiences with two applications under actual operating conditions which illustrate the inherent danger and afford a basis for the "General Recommendations" made. The paper is written from the operating standpoint, so the pertinent relaying and synchronizing problems are discussed briefly under "Significance of Results;" the metering problems, which involve questions of accuracy rather than of operation, are not discussed.

One of the systems operates at 12 kv., 60 cycles and

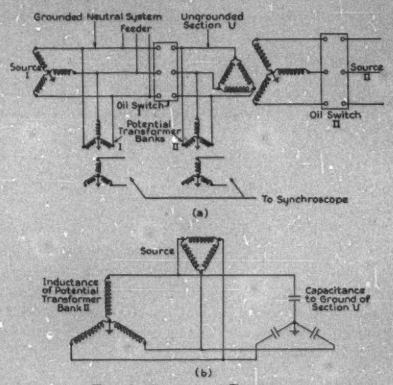

FIG. 1—SCHEMATIC DIAGRAMS

a. Typical system connections
b. Elements of circuit U

the other operates at 110 kv., 25 cycles. Each system is grounded at one point only, so sections separated temporarily from the system are, in general, ungrounded. The fact that the neutrals of the potential transformers connected to these sections are grounded does not change their designation as ungrounded sections or systems. The peculiar and dangerous phenomena to be described occurred when voltage was applied suddenly to ungrounded sections of the systems. This condition is indicated in Fig. 1a, when *Oil Switch I*

1. General Engineering Laboratory, General Electric Company, Schenectady, N. Y.

2. The abbreviations L/L for "line-to-line" and L/N for "line-to-neutral" are used throughout the paper.

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is open and voltage is applied to *Ungrounded Section U* from *Source II* by closing *Oil Switch II*.

The principal factors involved are the capacitance to ground of the immediate circuit, the inductance of the potential transformers and the point on the saturation curve at which the potential transformers normally operate; the first two factors are represented in Fig. 1b, but the third factor, although not represented, is most

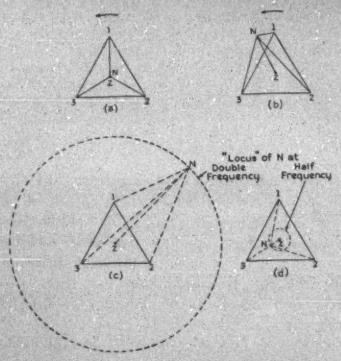


FIG. 2—REPRESENTATIONS OF VOLTAGE PHENOMENA BY MEANS OF VECTORS

a. Normal condition. Neutral N coincides with center of gravity Z of delta voltage triangle 1-2-3

b. Phenomenon A. Neutral N located near one vertex of delta voltage triangle 1-2-3

c. Phenomenon B. Neutral N oscillates at double frequency outside of delta-voltage triangle 1-2-3

d. Phenomenon C. Neutral N oscillates at half frequency inside of delta-voltage triangle 1-2-3

important, as the phenomena are fundamentally "saturation phenomena."[3]

The potential transformers were designed for normal operation at L/N voltage and for emergency (two-hour) operation at L/L voltage. Temporary operation in case of a ground on one line of the ungrounded section, which would apply L/L voltage to two of the potential transformers, was thus provided for; their ratings are given in Table I.

TABLE I
RATING OF POTENTIAL TRANSFORMERS

Cycles	Volt-amperes	Volts	
		Operating (L/N)	Circuit (L/L)
60	200	7,200:120	12,000
25	1000	63,500:63.5	110,000

SUMMARY OF RESULTS

The peculiar and dangerous saturation phenomena described briefly here were obtained frequently when voltage was applied to grounded-neutral, Y-connected potential transformers on temporarily ungrounded systems by closing an oil switch; at other times, the anticipated normal results, consisting of balanced L/N voltages equal to approximately 58 per cent of the applied L/L voltages, were obtained under the same system conditions. Connections are indicated in Figs. 1a and 3. The phenomena will be considered under three classifications, A, B and C (a fourth classification, D, covers a somewhat special case).

Phenomenon A consists of a displacement of the neutral (or ground) of the potential transformers from its normal position at the center of gravity of the delta-voltage triangle to a position near any vertex; the displaced neutral may be either inside, on the edge or outside of the triangle. Approximate L/L voltage then appears on two of the potential transformers, while a small voltage of very distorted wave shape appears on the third potential transformer. The normal voltage condition is indicated in Fig. 2a; one aspect of Phenomenon A is represented in Fig. 2b.

Phenomenon A occurred frequently under some conditions but not at all under other conditions on both the

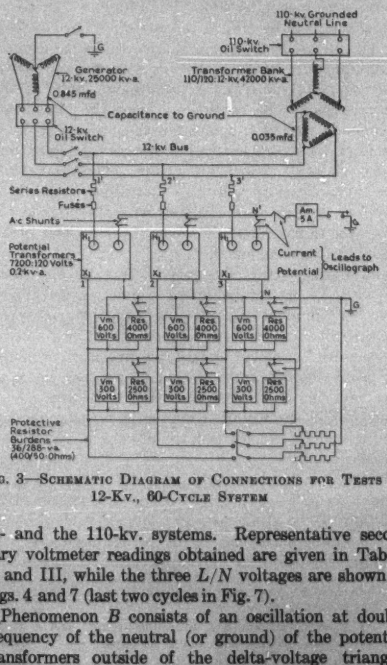


FIG. 3—SCHEMATIC DIAGRAM OF CONNECTIONS FOR TESTS ON 12-KV., 60-CYCLE SYSTEM

12- and the 110-kv. systems. Representative secondary voltmeter readings obtained are given in Tables II and III, while the three L/N voltages are shown in Figs. 4 and 7 (last two cycles in Fig. 7).

Phenomenon B consists of an oscillation at double frequency of the neutral (or ground) of the potential transformers outside of the delta-voltage triangle. Equal or balanced voltages of unsymmetrical wave shape and about 2.7 times normal magnitude appear

3. For discussions from theoretical standpoints, see following papers, *Physical Nature of Neutral Instability*, by A. Boyajian and O. P. McCarty, and *Theory of Abnormal Line-to-Neutral Transformer Voltages*, by C. W. LaPierre.

on the three potential transformers. These voltages are accompanied by balanced exciting currents of sharply peaked wave shape and about 10 times rated full-load current in magnitude; operation well up on the saturation curve is indicated thereby. Distinctly audible beats occur with a frequency of from about 30 to 170 per minute; excessive corona also occurs. Phenomenon B is represented in Fig. 2c.

TABLE II

12-KV., 60-CYCLE SYSTEM
7.2-KV. POTENTIAL TRANSFORMERS
COMPARISON OF PHENOMENA A, B, AND C WITH NORMAL SECONDARY VOLTMETER READINGS
(Approximate Primary Volts = Tabulated Volts × 60)

Line-to-line (L/L)				Line-to-neutral (L/N)			
Per cent*	1-2	2-3	3-1	1-N	2-N	3-N	Phenomenon
Capacitance to ground of 12-kv. circuit, 0.035 μf							
94	194.6	194	196.5	113	115	111	..
94	194.2	194	195.2	195	†	180	A
100	206	206.5	206.8	195	†	183	A
100	206.5	206	207.4	†	174	194	A
100	206.5	206.5	208	187	187	†	A
104	215.2	215	218	125	126	122	..
104	213.4	212.8	215.4	194	†	180	A
104	215.8	214	217.6	123	125	122	..
104‡	209	207.6	211.5	334.5	331.5	336	B
Capacitance to ground of 12-kv. circuit, 0.845 μf.							
100	207	208	209.2	120	119	122	..
100‡	197.5	199	200	147	148	149	C
113	233.5	235	234.8	134	133	138	..
113‡	224	225	227	155	155	155	C

*Approximate—based on potential transformer rating of 208 volts L/L ($= 1.73 \times 120$ volts L/N).
†Volts too low to read on 600-volt voltmeter.
‡Supply voltage practically unchanged from previous reading.

Phenomenon B started frequently under some conditions but held long enough to take instrument readings only at rated L/L voltage or higher on the 12-kv. system and even then only at certain times. Representative voltmeter readings are given in Table II; as compared with the normal condition, a drop of about 3 per cent in the indicated L/L voltages during B is noticeable; this is evidently caused by the excessive exciting currents. The three L/N voltages are shown in Fig. 5a for the 12-kv. system and in Fig. 7 (part of one cycle) for the 110-kv. system. Fig. 5b shows the voltages 2-N and 3-2 and the primary current in line 2′ on the 12-kv. system; the double-frequency current from the primary neutral N' to ground G was about 0.58 ampere.

TABLE III

110-KV., 25-CYCLE SYSTEM
63.5-KV. POTENTIAL TRANSFORMERS
COMPARISON OF PHENOMENON A WITH NORMAL SECONDARY VOLTMETER READINGS
(Approximate Primary Volts = Tabulated Volts × 1000)

Line-to-line (L/L)				Line-to-neutral (L/N)			
Per cent*	1-2	2-3	3-1	1-N	2-N	3-N	Phenomenon
100‡	110.3	111.8	109.5	64.1	64.2	64	..
110	122.7	123.8	121.4	70.4	71.3	70.5	..
110	119	121.8	119.2	138.2	22†	129.8	A
Duplicate bank of potential transformers added							
100	110.2	111.9	109.8	63.5	64	63	..
100	110	110.5	108.3	16†	113.8	122.5	A
110	122.2	123.2	122	70.3	71.2	70	..
110	121.5	121.7	120.8	13†	114.3	121	A

*Approximate—based on potential transformer rating of 110 volts L/L ($= 1.73 \times 63.5$ volts L/N).
†Estimated value.
‡Phenomenon A not obtainable.

FIG. 4—7.2-KV. POTENTIAL TRANSFORMERS—PHENOMENON A (SEE FIG. 3 FOR CONNECTIONS)

Curve A, voltage 2-N
Curve B, voltage 3-N
Curve C, voltage N-1

Phenomenon C consists of an oscillation at half frequency of the neutral (or ground) of the potential transformers inside of the delta-voltage triangle. Equal or balanced voltages of unsymmetrical wave shape and about 1.2 times normal magnitude appear on the three potential transformers. These voltages are accompanied by balanced exciting currents of sharply peaked wave shape and about 15 times rated full-load current in magnitude; operation at a point on the saturation curve well beyond that for Phenomenon B is indicated thereby. Distinctly audible beats occur with a frequency of about 240 per minute. Phenomenon C is represented in Fig. 2d.

a. Curve A, voltage 3-N
Curve B, voltage 2-N
Curve C, voltage 1-N

b. Curve A, current line 2′
Curve B, voltage 2-N
Curve C, voltage 3-2

FIG. 5—7.2-KV. POTENTIAL TRANSFORMERS—PHENOMENON B

Phenomenon C occurred frequently under one condition only on the 12-kv. system. Representative voltmeter readings are given in Table II; as compared

with the normal condition, a drop of about 4 per cent in the indicated L/L voltages during C is noticeable; this is evidently caused by the excessive exciting currents. Fig. 6 shows the voltages 3-N and 1-3 and the primary current in line 3′; the half-frequency current

Fig. 6—7.2-Kv. Potential Transformers–Phenomenon C

Curve A, voltage 3-N
Curve B, current line 3′
Curve C, voltage 1-3

from the primary neutral N' to ground G was about 0.88 ampere.

The occurrence of Phenomena A, B, and C, as shown by sustained instrument readings, was prevented in all cases by adding sufficient secondary burdens of resistance. The values (at 120 volts) of the protective resistor burdens required on the 12-kv. system are as follows: Phenomenon A, at all voltages tried, 72-volt-amperes; Phenomenon B, about 20 volt-amperes; Phenomenon C at rated L/L voltage, 72 volt-amperes—a burden of 72 volt-amperes, therefore, was provided for each transformer. Similarly, the burden values (at 63.5 volts) required on the 110-kv. system are: Phenomenon A (and B), at all voltages tried, 900 volt-amperes per transformer with one bank of potential transformers or 600 volt-amperes per transformer with two banks of potential transformers.

Results similar to those just described were obtained on the 12-kv. system, when other potential transformers of the same rating were substituted for the first group tried.

In further tests made on the 12-kv. system (capacitance to ground 0.035 μf.), Phenomena A and B also occurred when a bank of 13.2-kv. potential transformers was connected in multiple with the bank of 7.2-kv. transformers; however, neither A nor B occurred when the 7.2 kv. bank was disconnected.

Phenomenon B occurred when a bank of 6.9-kv., 1.5-kv-a. distribution transformers was connected in alone. A different phenomenon (D) occurred when the 5.96-kv. tap on this bank was used (this tap, of course, would not be used in practise on a 7.2-kv. circuit). Phenomenon D consists of an oscillation at triple frequency of the neutral (or ground) outside of the delta-voltage triangle; equal or balanced voltages of about 3.5 times normal appear on the three transformers; audible "buzzing" and excessive corona occur. Phenomenon D did not hold long enough at any time to obtain a complete set of instrument readings; evidently D is easier to prevent than B.

Significance of Results

Phenomena A, B, and C and the conditions under which they occur are not characteristic of any particular ungrounded system or type of transformer. Phenomena B and C endanger the potential transformers themselves; the former also endangers other connected equipment. All three phenomena tend to cause incorrect synchronizing and incorrect relay operation; the difficulties depend to some extent on the secondary connections used.

Y-Connected Secondaries. Ordinarily, if any one of the L/N voltages should be chosen for synchronizing purposes, it is evident that the indications frequently obtained would be considerably in error both in magnitude and in phase position, to say nothing of wave shape and frequency. This particular difficulty could be readily overcome by using any one of the L/L voltages. However, the correct effective secondary values would not be indicated by the voltmeters for Phenomena B or C, although this error would be small (see Table II); also, there might be some disturbance caused by throwing together two systems, one of which was being subjected to Phenomena A, B or C.

Broken-Delta-Connected Secondaries. Ordinarily, the high-impedance "ground relay" connected across the broken delta would operate frequently, although no accidental ground existed on the system.

These difficulties can be overcome in various ways such as by grounding the sections of the system, by using the L/L connection for all potential transformers, by providing protective resistor burdens for L/N rated potential transformers or by using potential transformers of considerably higher than L/N-voltage ratings for the L/N connection, and thereby lowering

Fig. 7—Six 63.5-Kv. Potential Transformers (Two Y-Banks)—Start of Phenomenon A (and B)

Curve A, voltage 3-N
Curve B, voltage 2-N
Curve C, voltage 1-N

the point on the saturation curve at which the transformers normally operate. The last two methods in particular are amplified under "General Recommendations."

General Recommendations

These recommendations are intended to serve as a guide in the application of grounded-neutral, Y-connected potential transformers to three-phase, three-wire systems or sections thereof, which are ungrounded temporarily or permanently.

Protective resistor burdens should be used in practically all cases if the occurrence of Phenomena *A*, *B*, and *C* is to be prevented under all possible operating conditions. The approximate relative values of such secondary burdens are given in Table IV; secondary connections are discussed in subsequent paragraphs.

TABLE IV

APPROXIMATE RELATIVE VALUES OF PROTECTIVE RESISTOR BURDENS REQUIRED TO PREVENT OCCURRENCE OF PHENOMENA *A*, *B*, AND *C*

System or section neutral	Y-connected potential transformers		
	Primary neutral	Normal voltage rating	Relative value of burdens
Ungrounded temporarily	Grounded	L/N	1.0
	Grounded	L/L	0.5
	Grounded	$2 \times L/N$	..
Ungrounded permanently	Grounded	L/N	1.5
	Grounded	L/L	0.75
	Grounded	$2 \times L/N$	0.25

It is always desirable to keep the volt-ampere ratings of the protective resistor burdens as small as possible and this involves a consideration of the characteristics of each proposed installation. The magnitude of the burdens provided for the potential transformers of L/N voltage design on the 12-kv. and 110-kv. systems was determined experimentally and a rough idea of the general requirements was thereby obtained; the magnitude of these burdens constitutes the standard of reference for the relative values in Table IV.

Potential transformers of L/L voltage design without protective burdens were entirely satisfactory on the 12-kv. system from the standpoint of non-occurrence of the phenomena described. Transformers of similar design without protective burdens have been recommended, therefore, for similar service and so far as is known, have also proved entirely satisfactory from that standpoint wherever installed. However, theoretical considerations and laboratory tests indicate that the phenomena may occur under favorable conditions if standard potential transformers are normally operated at approximately 58 per cent of rated voltage, as in the above cases, but not if operated at or below 50 per cent of rated voltage; these values depend, of course, on the actual flux densities involved. Protective burdens are recommended, therefore, for transformers of L/L voltage design; the ratio between the volt-ampere ratings of the burdens for transformers of L/L and L/N voltage design, respectively, is about 0.5 to 1; small burdens are recommended for transformers of twice L/N-voltage design.

A differentiation is made in Table IV between temporarily and permanently ungrounded systems. The ratio of the volt-ampere ratings of the protective burdens recommended for the two conditions is about 1 to 1.5. It is believed that a greater margin of safety is desirable for permanently ungrounded systems, since they are always subject to disturbances tending to displace the neutral from its normal position; installations of potential transformers on such systems have not been investigated.

In order to make this section more complete, two general cases involving Y-connected potential transformers, but somewhat outside the scope of the paper, are included. The first is their application with ungrounded or isolated neutral to any three-phase system; in this case, the L/N voltages are of little value for any purpose; further, if the secondaries are Y-connected to permit obtaining L/L voltages, the L/N voltage waves are distorted because of the suppression of the third and its multiple harmonics in the exciting currents, as is well known; such installations are not recommended. The second case occurs when the system and potential transformer neutrals are connected by means of a fourth conductor, as in method (3); this case is covered by standard practise for three-phase, four-wire systems.

Secondary Connections. The protective resistor burdens recommended for potential transformers with grounded primary neutrals must be connected L/N if the secondaries are Y-connected. The Y-connection is recommended when synchronizing is involved in order to permit utilizing the L/L voltage; special considerations apply when metering is involved.

An equivalent burden may be connected across the broken delta, if the secondaries are so connected, when metering is not involved. A similar result may be obtained with Y-connected secondaries by utilizing a small Y-broken-delta-connected auxiliary transformer bank to supply the equivalent burden; four conductors must be used to connect the potential and auxiliary transformer banks.

The closed-delta secondary connection should not be used when the primary neutral is grounded, since a ground on any line of the system would undoubtedly cause sufficient circulating current to destroy the transformers.

The secondary neutral (preferably), if Y-connected, or one line, if broken-delta-connected, should always be grounded in accordance with standard practise.

Methods of Test

12-Kv., 60-Cycle System

(1) Operation of Potential Transformers

(*a*) *Supplied from 42,000-Kv-a. Transformer Bank.* Connections are indicated in Fig. 3. Tests were made

with the three bus-disconnect switches open; the 12-kv. oil switch was open also. Voltage was applied to the 12-kv. bus and the bank (s) of potential transformers by means of the 110-kv. oil switch; this switch was closed (and opened) more than 600 times during the tests.

Voltage control of the 110-kv. circuit was not available so the usual variations in voltage occurred. Two bus voltages differing by about 9 per cent were obtained by means of taps on the high-voltage windings of the three-unit power transformer bank; the ratings used were 120 to 12 and 110 to 12 kv., respectively. The phase rotation of the 12-kv. circuit was 1′-2′-3′.

A 60-ohm series resistor and a fuse were connected in the primary circuit of each 7.2-kv. potential transformer. The neutral N' was isolated, when desired, by opening the switch between N' and ground G.

The current from N' to G was read on the 5-ampere ammeter (if of sufficient magnitude) and its wave shape was determined with the oscillograph (not shown) by taking the voltage drop across the adjacent a-c. shunt. Similarly, the wave shapes of the currents in primary lines 1′, 2′ and 3′ were determined from the voltage drops across the respective shunts next to N'; the drop leads are labelled "current."

One 30-kv. electrostatic voltmeter (not shown) was connected between lines 1′, 2′ or 3′ and ground to check roughly the secondary L/N voltmeters. Except for this voltmeter, all instruments used were of the portable type.

The secondary L/N voltages 1-N, 2-N, and 3-N were read on 600-volt voltmeters (lowest scale point, 100 volts); the L/L voltages 1-2, 2-3, and 3-1 were read on 300-volt voltmeters (lowest scale point, 40 volts). The wave shapes of these voltages were determined with the oscillograph; the leads are labelled "potential." In most cases, series resistances of 4,000 ohms each for the three L/N voltages and of 2,500 ohms each for the three L/L voltages were used; the small series switches shown were open when taking oscillograms of the respective voltages and closed at other times in order to keep the minimum burdens constant. Each minimum burden thus consisted of two parts, one of about 4 volt-amperes actual L/N and the other of about 29 volt-amperes L/L or equivalent L/N, both values being in terms of 120 volts.

The lowest resistance tap on the L/N oscillograph resistors that could be used even temporarily up to 350 volts was 1,000 ohms; this constituted a nominal burden of about 15 volt-amperes at 120 volts. Larger burdens were obtained with the protective resistors, which could be connected to give either 36, 72, 144 or 288 volt-amperes (400, 200, 100 or 50 ohms, respectively). These burdens were sometimes connected equivalent L/N by isolating their neutral; single burdens were sometimes connected L/L.

The general procedure in preventing the occurrence of the phenomena was to determine the amount of burden necessary on each transformer to reduce any particular existing phenomenon to normal. Starting with a small burden, successively larger burdens were connected until reduction to normal took place. The correctness of the burden value so determined was checked by applying the three equal burdens by means of the three-pole switch during several subsequent occurrences of the phenomenon. It was found that the phenomenon would not occur if the oil switch were closed with such burdens or somewhat smaller ones connected. The significant value was that necessary to prevent the occurrence of the most obstinate phenomenon. An approximation was sometimes obtained by connecting one burden L/N. Also, Phenomenon A could be obtained from normal by adding a single burden of 288 volt-amperes to any one of the three 7.2-kv. transformers, when otherwise supplying voltmeters (and oscillograph resistors) only.

Many sets of instrument readings and oscillograms were taken in order to cover adequately all phenomena encountered or produced, with particular emphasis on those associated with actual or possible operating conditions. For each set of readings, the instruments were read as quickly as possible, so as to obtain comparable values. Accurate instrument corrections were not available but the approximate corrections did not exceed two per cent, so no corrections were applied. For the oscillograms, 12-in. films were generally used at fairly high speed for stable conditions and 48-in. films at a somewhat slower speed for starting or transient conditions. Calibrations were taken for each set of oscillograms.

(*b*) *Supplied from 25,000-Kv-a. Generator.* Connections are indicated in Fig. 3. The tests were made with the generator ground, the three disconnect and the 110-kv. switches open. The potential transformers were connected between the disconnect and 12-kv. switches for this test only. Voltage was applied by means of the 12-kv. oil switch. Generator field control was available, so the test voltage range was somewhat wider than for (*a*). The generator voltage was set approximately by the field ammeter before closing the 12-kv. switch. Instrument readings and oscillograms were taken and protective burdens were tried as described under (*a*).

(2) Determination of Capacitance to Ground

System connections are indicated in Fig. 3. The capacitance to ground of the 12-kv. bus and connected equipment was determined by the volt-ampere method. Voltage was applied to the generator lines by means of a high-potential testing set excited from a separate source. A 13,200 to 110-volt potential transformer and a 150-volt voltmeter were used to determine the applied voltage. The charging current was read on a 5-ampere ammeter, which was connected in the grounded line. Readings were taken (*a*) with the generator and potential transformer ground and 110-kv. switches open and

the bus-disconnect and 12-kv. switches closed and (*b*) with the bus-disconnect switches open.

The capacitance of the transformers and bus to ground equals the difference between the results of tests (*a*) and (*b*).

In view of the fact that the capacitance value so obtained (0.035 μf.) was relatively small, a check test was made by using one of the 7.2-kv. potential transformers as a step-up transformer, the other two potential transformers being disconnected from ground. The applied voltage was determined by multiplying the secondary voltmeter reading by the nominal transformer ratio. The charging current was determined with the oscillograph by taking the drop across a shunt connected in the grounded line. Readings were taken with the bus-disconnect and 110-kv. switches open.

110-Kv., 25-Cycle System

OPERATION OF POTENTIAL TRANSFORMERS

The set-up was similar to that indicated in Fig. 1a with *Oil Switch I* open and voltage applied to *Potential Transformer Bank II* from *Source II* by closing *Oil Switch II*. Field control of the *Source II* generator (it and its transformer bank not shown) was available. Tests were made with one bank of 63.5-kv. potential transformers and also with two 63.5-kv. banks in multiple, one bank being connected Y-Y and the other bank Y-broken delta.

Two expedients were resorted to in order to obtain the desired number of operations without using the oil switch. First, a three-pole, air-break disconnect switch was inserted between the power and potential transformers, and voltage applied to the latter thereby. Arcing occurred every time just before the switch closed and in every case normal voltages were obtained, which was not the case when using the oil switch. Second, one leg of the Y-connected secondary was short-circuited through a fuse; this generally produced Phenomenon *A*, with the short-circuited phase "low," before the fuse melted.

One end of each potential transformer primary winding was solidly grounded so no current shunts or "neutral ammeter" could be used. Voltmeters (150-volt) were connected L/L and L/N on the secondaries as indicated in Fig. 3 for the Y-Y bank; the ratings of the voltmeters used for the broken-delta bank ranged from 10 to 300 volts. No attempt was made to keep the oscillograph resistors balanced because of the comparatively large transformer rating (1 kv-a). Voltmeter readings and oscillograms were taken and protective resistor burdens were tried as described under (1a); also, single protective resistor burdens were applied to the broken-delta bank by means of a single-pole switch. It was impracticable to determine the capacitance to ground of the immediate circuit because of the solid ground at the neutrals of the potential transformer banks.

a. Curve A, current N-G
Curve B, voltage 2-N
Curve C, voltage 3-2

b. Start
Curve A, voltage 1-N
Curve B, voltage 2-N
Curve C, voltage 3-N

FIG. 8—7.2-KV. POTENTIAL TRANSFORMERS—NORMAL CONDITION

Detailed Results

Results supplementing those under "Summary" are given in this section; most of the results were obtained on the 12-kv. system. A description of the "Methods of Test" has been given under that title. The designation used for the secondary L/L and L/N voltages and the primary currents refers to the numbers of the corresponding lines in Fig. 3; the phase rotation is 1-2-3. The term *rated voltage* always refers to the voltage rating of the potential transformers under test.

In practically every case, the important features are illustrated by oscillograms. Eighteen of the thirty oscillograms show the start of the several phenomena; seventeen show initial transient voltage oscillations, which occurred at the rate of approximately 3,500 cycles

per sec. on the 12-kv. system. As far as possible, oscillograms showing more than one phenomenon were chosen. Any L/L voltage, when shown, constitutes a timing wave, particularly for Phenomena B, C, and D.

Instrument readings taken in connection with the oscillograms are given in Tables V, VI, VII, and IX; electrostatic voltmeter readings taken directly on the 12-kv. circuit are not included in the first three tables as they checked the corresponding secondary voltmeter readings within expected limits.

12-Kv., 60-Cycle System

(I) Operation of Potential Transformers

(1) *Supplied from 42,000-Kv-a. Transformer Bank-Capacitance to Ground of 12-Kv. Circuit, 0.035 μf.*

(a) Three 7.2-Kv. Potential Transformers

Suppression of Third and Multiple Harmonics. Reference was made under "Introduction" to the suppression of the third and its multiple harmonics of the exciting currents, the effects of which are well known, in order to bring out the fact that the installations were not in accordance with the best practise in this respect; also, the phenomena might be attributed to this suppression. Actually, however, the capacitance to ground of the immediate circuit apparently absorbed practically all harmonics in the exciting currents, including the comparatively large harmonics accompanying the approximate saturation of the potential transformer cores.

This is well illustrated for the third harmonics in Fig. 8a, which shows voltages 2-N, 3-2 and current N'-G. The L/N and L/L voltage waves show practically no distortion; the small neutral current is apparently all third harmonic current. Thus the normal L/N voltage readings obtained were practically the same as if taken on a three-phase, four-wire system.

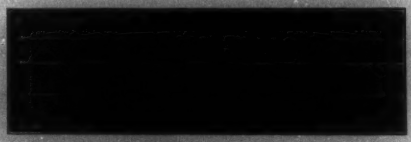
Out of 350 operations of the oil switch with one particular bank of potential transformers over a L/L voltage range of about 5 per cent, 187 (about 54 per cent of total) sets of normal voltage readings were obtained, although perhaps one-third started out as Phenomenon B (q. v.) and a small number as Phenomenon A (q. v.)

The start of the three L/N voltages is shown in Fig. 8b. The start (as Phenomenon A) of voltages 1-N, 2-1 and current 1′ is shown in Fig. 10b; oscillations in the current are not apparent; the normal condition is shown at the right.

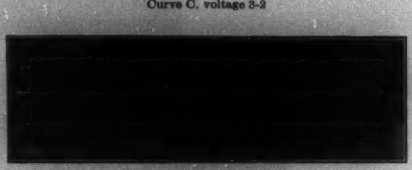
Voltmeter readings taken in connection with all of the figures described in this section (a) are given in Table V; when possible, ammeter readings of the primary neutral-to-ground current N'-G were taken and are included in Table V.

The effect of isolating the primary neutral and thereby actually suppressing the third and its multiple harmonics is clearly shown in Fig. 9a, in which the third harmonic in the two L/N voltage waves is very prominent. In this case, the neutral oscillates at triple frequency inside of the delta-voltage triangle. Fig. 9b shows the same voltages as Fig. 9a, but the L/N wave shapes have been somewhat improved by the addition of resistor burdens of 72 volt-amperes (200 ohms) per transformer.

Phenomenon A. Audible evidences of distress in the potential transformers during the occurrence of Phenomenon A were entirely lacking. It was not possible, therefore, to determine how many operations started out as A and finished otherwise. Out of 350 operations of the oil switch, however, 142 (about 40 per cent of total) sets of A voltage readings were obtained; of these, voltages 1-N, 2-N, and 3-N were too low to read on their respective voltmeters in the approximate ratio of 10 to 60 to 30. Differences in the characteristics of the transformers and (or) in the magnitudes of the three capacitances to ground are doubtless indicated thereby.



a. Curve A, voltage 3-N
Curve B, voltage 2-N
Curve C, voltage 3-2



b. 72-v-a.—resistor burdens
Curve A, voltage 3-N
Curve B, voltage 2-N
Curve C, voltage 3-2

Fig. 9—7.2-Kv. Potential Transformers—Third Harmonic Phenomenon

The three L/N voltages are shown in Fig. 4, voltage 2-N being low; the start of these voltages is shown in Fig. 10a. The start of voltages 1-N, 2-1 and current 1′ (crest value about 0.17 ampere when symmetrical) is shown in Fig. 10b; the relation between voltage 1-N and current 1′ is clearly shown near the center, one current peak coinciding approximately with each zero-voltage point; A quickly relapsed into normal. The start of voltages 3-2, 2-1 and current N'-G is shown in Fig. 10c. Current N'-G, which has a distorted wave shape (crest value about 0.42 ampere at start, 0.17 ampere subsequently), is absorbed by the circuit capacitance; the stability of A is indicated by the unvarying wave shapes at the right; the distortion of the L/L voltage waves is apparently negligible, as would be expected from the fact that the exciting currents are comparatively small. The three primary line currents

TABLE V

12-KV., 60-CYCLE SYSTEM. CAPACITANCE TO GROUND OF 12-KV. CIRCUIT, 0.035 μf. THREE 7.2-KV. POTENTIAL TRANSFORMERS. SECONDARY L/L AND L/N VOLTMETER READINGS TAKEN IN CONNECTION WITH OSCILLOGRAMS

(Approximate Primary Volts = Tabulated Volts × 60)

Refer to Fig	Line-to-line (L/L)				Line-to-neutral (L/N)					Sec. resis. bdn.	Pri. amp. N'-G
	Per cent**	1-2	2-3	3-1	1-N	2-N	3-N	Phenomenon	Comments		
8a	104	217	216*	218.7	125	125*	123	..	Normal	Vms.	†*
b	104	215.5	215	217.5	125*	126*	123*	..	Normal	Vms.	†
9a	104	216.6	215.4*	218.4	142	138*	142*	3rd	N' isolated	Vms.	..
b	104	215.2	215*	217.6	127	125*	126*	3rd	N' isolated	72 V-A.	..
4	101	210.5	210	212	195*	†*	180*	A		Vms.	†
10a	103	214	214	216	196*	†*	181*	A		Vms.	†
b*	102	212*	211	213.5	120*	125	120	A-	A didn't hold	Vms.	†
c	103	214.8*	214*	216	197	†	182	A		Vms.	†*
d	102	213.4	212	214.8	122*	122*	120*	A-	A didn't hold	72 V-A.	†
5a	101	210	209.5	212.7	331*	327.5*	333*	B		Vms.	0.57
b*	102	212	211*	213.2	340	337*	340	B		Vms.	0.58
11a	104	216	215	217.2	125*	120*	122*	B-	B (and D) didn't hold	Vms.	†
b	101	210.4	209.2	212.8*	338*	335	340*	B	1.5 cycles of D	Vms.	0.575
c*	102	212	211.5	214	197	†	182	A B	B didn't hold	Vms.	†
d*	101	210.2	209.4	212.8	336.5	332	336.5	B	Beats	Vms.	0.575*
e*	101	209.5	208.4	211.4	335	331.5	336	B		Vms.	0.575*
f*	100	208.5	207.6	211	120*	122	119	B-	307* volts N-"N"‡	288 V-A.	†

Item shown in Fig.; primary line current also shown when Fig. No. is marked thus (10b).

**Approximate—based on potential transformer rating of 208 volts L/L ($= 1.73 \times 120$ volts L/N); values for B are about 3 per cent too low—See note in Table II.

†Value too low to read on instrument used.

‡Voltage between transformer secondary neutral N (or G) and resistor neutral "N" during B.

are shown in Fig. 11c, A existing at the start and finish and B through the center; these currents combine to form the neutral current N'-G shown in Fig. 10c.

A was always reduced to normal by adding a protective resistor burden of 72 volt-amperes (at 120 volts) per transformer; the effect of these burdens on the three L/N voltages is shown in Fig. 10d. A (and possibly B) started, existed for a few cycles and then relapsed into normal. A short time delay should be provided, therefore, for a ground relay connected across the broken delta, when the secondaries are so connected, in order to prevent incorrect operation during the starting transients. In any case, the voltage across the broken delta equals approximately 3 times the voltage N-Z, which is represented for various conditions in Fig. 2.

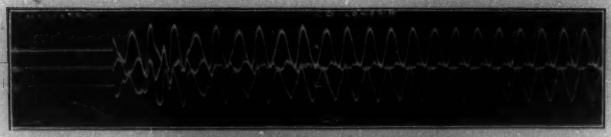
If sine-wave voltages are assumed for the readings in Tables II and V, the neutral N will be found to be located just outside the delta-voltage triangle at 94 per cent voltage as in Fig. 2b, just on the edge in most cases at 100 per cent voltage and just inside at 104 per cent voltage; in no case is the low voltage reversed in phase.

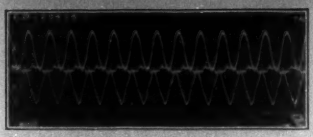
Phenomenon B. There was no mistaking the occurrence of Phenomenon B; in fact, the audible evidences of distress in the potential transformers, consisting of magnetic beats and excessive corona, first called attention to the situation, when preparing to synchronize under the conditions indicated in Fig. 1a. B is the most spectacular and at the same time the most sensitive to external conditions of any of the phenomena which held for an appreciable length of time: for example, at rated L/L voltage or higher, B failed to hold on a rainy night; also on a dry night when the bus-disconnect switches in Fig. 3 were closed, thus adding the short bus section up to the oil switch.

Out of 350 operations of the oil switch, 21 (about 6 per cent of total) sets of B voltage readings were obtained; however, probably three times as many started out as B and relapsed into normal, while perhaps two or three relapsed into A. It is interesting to note that heavy input-current surges to the 110-kv. power transformer bank were occasionally observed (measuring equipment not shown in Fig. 3) when the oil switch was closed; these surges apparently coincided with the start of B.

The three double-frequency L/N voltages are shown in Fig. 5a; the maximum crest value is about 615 volts (36.9 kv. primary, or 3.6 times normal crest). Voltages 2-N, 3-2 and current 2′ are shown in Fig. 5b. The relation between voltage 2-N and current 2′ is shown for a few cycles, the current peaks coinciding approximately with the zero-voltage points of alternate cycles; the maximum current crest value is about 1.3 amperes (33 times rated full-load crest). The relation between the two dents or teeth per half-cycle of voltage wave 3-2 and the peaks of current 2′ is clearly shown, although current 3′ evidently contributes in some places. The excessive exciting currents cause large unsymmetrical voltage drops in the transformer primary windings and thus distort and reduce the induced voltages, which combine in the secondary circuits to form the L/L voltages shown in the figures.

The start of the three L/N voltages is shown in Fig. 11a. The maximum crest value after stabilization is





Curve A, voltage 1-*N*
Curve B, voltage 2-*N*
Curve C, voltage 3-*N*

b. (Also "normal")
Curve A, voltage 1-*N*
Curve B, current line 1′
Curve C, voltage 2-1

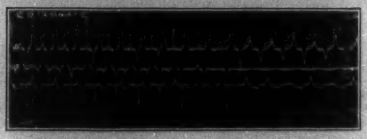
c. Curve A, voltage 3-2
Curve B, current *N′-G*
Curve C, voltage 2-1

d. 72-v-a. resistor burdens (also "normal")
Curve A, voltage 1-*N*
Curve B, voltage 2-*N*
Curve C, voltage 3-*N*

FIG. 10—7.2-KV. POTENTIAL TRANSFORMERS—START OF PHENOMENON A

a. Start (also D)
Curve A, voltage 1-N
Curve B, voltage 2-N
Curve C, voltage 3-N

b. Start (also D)
Curve A, voltage 1-N
Curve B, voltage 1-3
Curve C, voltage 3-N



c. Start (also A)
Curve A, current line 1′
Curve B, current line 2′
Curve C, current line 3′

Fig. 11—continued next page

[redacted]

[redacted]

d. Beats
Curve A, current line 1′
Curve B, current line N'-G
Curve C, current line 3′

[redacted]

[redacted]

e. Curve A, current line 3′
Curve B, current line N'-G
Curve 3, current line 1′

f. 288-v-a. isolated-neutral resistor burdens
Curve A, current line 1′
Curve B, voltage between transformer and burden neutrals
Curve C, voltage 1-N

FIG. 11—7.2-KV. POTENTIAL TRANSFORMERS—PHENOMENON B

TABLE VI

12-KV., 60-CYCLE SYSTEM. CAPACITANCE TO GROUND OF 12-KV. CIRCUIT, 0.035 μf. POTENTIAL TRANSFORMER COMBINATIONS AND DISTRIBUTION TRANSFORMERS. SECONDARY L/L AND L/N VOLTMETER READINGS TAKEN IN CONNECTION WITH OSCILLOGRAMS

Refer to Fig.	Line-to-line (L/L)				Line-to-neutral (L/N)				Comments	Transformer ratio	Pri amp. N'-G
	Per cent**	1-2	2-3	3-1	1-N	2-N	3-N	Phenomenon			
					Six 7.2-kv., 0.2-kv-a. potential transformers						
12*	100	209*	208.2	211	125*	120	115	*A B D*-	Normal not shown	60:1	†
					Three 7.2-kv. and three 13.2-kv. potential transformers						
13	103/56	106*	108	107	62.5*	62.5*	62	*A B D*-	Vms. changed	120:1	†
	103/56	108	108	108	91.5	93	†	*A*	For *A* in Fig. 13	120:1	†
					Three 6.9-kv., 1.5-kv-a. distribution transformers						
14a	107	212*	211	213.5	125*	120*	124	*A*-	Normal not shown	60:1	†
b*	104	207*	206	209.5	123*	114	122	*A B D*-	Normal not shown	60:1	†
	106	210	209.5	212.5	284	282	287	*B*	For *B* in Fig. 14b	60:1	0.5
					5.96-kv. tap on 6.9-kv. distribution transformers						
15	122	242.8*	241.5	244.5	140*	140*	142	*A B D*-	Normal not shown	51.8:1	†
	119	‡	‡	236	455	480	483	*D*-	For *D* in Fig. 15	51.8:1	‡
	120	238	238	240	233	†	238	*A*	For *A* in Fig. 15	51.8:1	†
					Two 7.2-kv. potential transformers connected L/N						
16*	102	212*			135*	125		*A B D*-		60:1	†
	102	213			173	175		*A*	For *A* in Fig. 16	60:1	†
	102	212.5			220	219		*B*	For *B* in Fig. 16	60:1	0.3

Item shown in Fig.; primary line current also shown when Fig. No. is marked thus (12).
**Approximate—based on 1.73 × L/N voltage rating—values for *B* (and probably for *D*) are too low—See note in Table II.
†Value too low to read on instrument used.
‡Reading not obtained.

about 615 volts, same as in Fig. 5a, but the maximum crest before stabilization is about 655 volts; the latter is attributed to the momentary existence of Phenomenon *D*. The slow variations in alternate successive positive (or negative) crest values are at the rate of approximately 90 per min. and are accompanied by

changes in wave form. Somewhat similar remarks apply to Fig. 11b, which shows voltages 1-N, 3-N, and 1-3. The maximum D crest is about 675 volts (40.5 kv. primary, or 4 times normal crest); variations are noticeable in the wave shape of the L/L voltage, which also serves as a timing wave.

The start of the three primary line currents is shown in Fig. 11c. This operation started and finished as A,

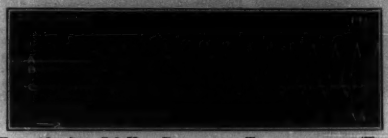


Fig. 12—Six 7.2-Kv. Potential Transformers (Two Y-Banks)—Start of Phenomena A, B, and D

Curve A, voltage 1-N
Curve B, voltage 2-1
Curve C, current line 1′

but B held long enough to show the relations between the three currents, which combine to form the neutral current N'-G shown in Figs. 11d and e. The peaks in the different lines are not in phase, except (approximately) when one peak is small; this means that the crest values of the neutral current N'-G do not greatly exceed those of the line currents.

Fig. 11d shows particularly the variations in the alternate successive positive (or negative) crest values

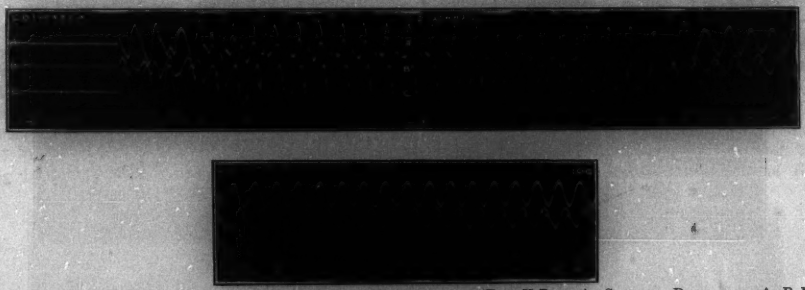


Fig. 13—Three 7.2-Kv. and Three 13.2-Kv. Potential Transformers (Two Y-Banks)—Start of Phenomena A, B, D and "Normal"

Curve A, voltage 1-N
Curve B, voltage 2-N
Curve C, voltage 2-1

of currents 1′, 3′, and N'-G, which occur at the rate of approximately 260 per min. for the double-frequency neutral current and 85 per min. for the line currents. The approximate maximum crest values are 1.75 amperes for current N'-G and 1.6 amperes for currents 1′ or 3′. The same currents are shown more spread out in Fig. 11e; the differences between the adjacent negative crest values of the neutral current are quite pronounced.

Fig. 11f shows voltage 1-N and current 1′ with an isolated-neutral burden equivalent to 288 volt-amperes per transformer. The voltage between the transformer neutral N and the burden neutral "N" is also shown; its wave shape is somewhat more symmetrical than that of voltage 1-N. The small current drawn by the oscillograph and volt-meter prevented B from holding; when these devices were not connected B held indefinitely.

B failed to hold with one group of transformers, because one transformer arced over. When the defective transformer was replaced, this difficulty disappeared. This transformer was later given a standard high-potential test at 25 kv. and showed no evidences of distress. The arc-overs might have been due to momentary cycles of D.

B was usually reduced to normal by changing the L/N oscillograph resistor settings to 1,000 ohms (14.4 volt-amperes) from 4,000 ohms (3.6 volt-amperes). The increase in actual L/N burden thus amounted to about 11 volt-amperes at 120 volts; much greater L/L or equivalent L/N burdens had little effect in this respect, as is evident from the description of Fig. 11f.

(b) Miscellaneous Transformer Combinations

Phenomena A, B, and D

This section includes results on five different transformer combinations. Voltmeter and ammeter readings taken in connection with the figures described are given in Table VI; supplementary sets of readings, corresponding to the various phenomena shown in the figures but not taken at the same time, are included when available.

Fig. 12 shows the start of voltages 1-N, 2-1 and current 1′ for two banks of 7.2-kv. potential transformers; the primaries but not the secondaries were connected in multiple. A trace of Phenomenon A and perhaps three separate cycles of D are shown at the start and

several cycles of B are shown subsequently. The maximum voltage and current crest values for D are somewhat higher than for B; the existence of a large triple-frequency current to ground during D is indicated by the three teeth per half cycle of the L/L voltage, whereas there are only two for the double-frequency current to ground during B. The approximate L/N voltage for B was 290 volts; this value was obtained during the brief intervals when B started and held temporarily in 17 out of 28 operations of the oil switch; the other 11 operations apparently started out as, and the 17 relapsed into, normal.

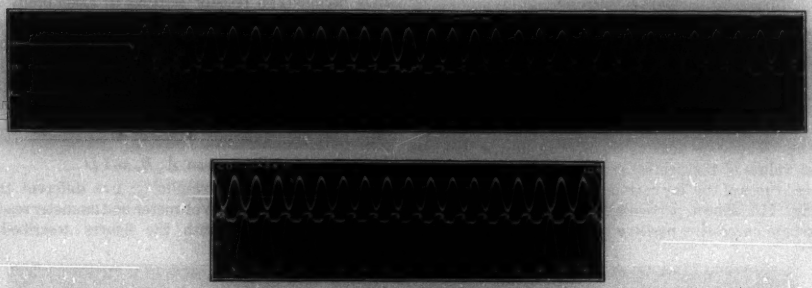
TABLE VII

12-KV., 60-CYCLE SYSTEM. CAPACITANCE TO GROUND OF 12-KV. CIRCUIT, 0.845 μf. THREE 7.2-KV. POTENTIAL TRANSFORMERS. SECONDARY L/L AND L/N VOLTMETER READINGS TAKEN IN CONNECTION WITH OSCILLOGRAMS

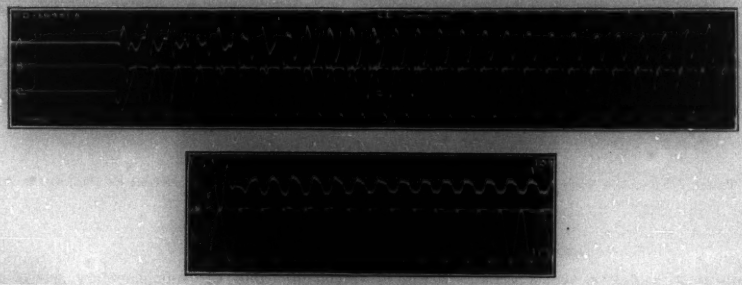
(Approximate Primary Volts = Tabulated Volts × 60)

Refer to Fig.	Line-to-line (L/L)				Line-to-neutral (L/N)				Comments	Pri. amp. N'-G
	Per cent†	1-2	2-3	3-1	1-N	2-N	3-N	Phenomenon		
6*	96	197.5	199	200*	147	148	149*	C		0.88
17a	95	196	197.4	199	148*	148*	149*	C		0.89
b*	96	198	199.2	201*	149	149	150*	C		0.88
c*	96	198	198.8	200.5	147	148	149	C	Beats	0.88
d	96	197.5	198.8*	200.5*	146	148	149	C		0.87*
	100	207	208	209.2	120	119	122	..	Normal	‡

Item shown in Fig.; primary line current also shown when Fig. No. is marked thus (6).
†Approximate—based on potential transformer rating of 208 volts L/L ($= 1.73 \times 120$ volts L/N); values for C are about 4 per cent too low.
‡Value too low to read on 5-ampere ammeter.



a. A
Curve A, voltage 1-N
Curve B, voltage 2-N
Curve C, voltage 2-1



b. A, B and D
Curve A, voltage 1-N
Curve B, current line 1′
Curve C, voltage 2-1

FIG. 14—6.9-KV., 1.5-KV-A. DISTRIBUTION TRANSFORMERS—START OF PHENOMENA

FIG. 15—5.96-KV. TAP ON 6.9-KV. DISTRIBUTION TRANSFORMERS—START OF PHENOMENA A, B AND D

Curve A, voltage 1-N
Curve B, voltage 2-N
Curve C, voltage 2-1

FIG. 16—TWO 7.2-KV. POTENTIAL TRANSFORMERS (CONNECTED L/N)—START OF PHENOMENA A, B, D AND "NORMAL"

Curve A, voltage 1-N
Curve B, voltage 2-1
Curve C, current line 1'

The start of voltages 1-N, 2-N and 2-1 is shown in Fig. 13 for one bank of 7.2-kv. and one bank of 13.2-kv. potential transformers in multiple; the voltage readings and oscillograms were taken on the 13.2-kv. transformer secondaries. Traces of A and D are shown at the start and several cycles of B and normal are shown subsequently. Practically no distortion appears in the L/L voltage, indicating that the phenomena were generated solely by the 7.2-kv. bank.

Fig. 14a shows the start of voltages 1-N, 2-N and 2-1 in A for one bank of 6.9-kv., 1.5 kv-a. distribution transformers; voltage 2-N is low. Fig. 14b shows the start of voltages 1-N, 2-1 and current 1'. Perhaps three separate cycles of D are shown; the crests of the accompanying currents are higher than those for B (see remarks for Fig. 12). A is shown also, with voltage 1-N low.

Fig. 15 shows the start of voltages 1-N, 2-N and 2-1 for the 5.96-kv. tap on the 6.9-kv. distribution transformers. Traces of A and several cycles of B are

a. Start
Curve A, voltage 1-N
Curve B, voltage 2-N
Curve C, voltage 3-N

Fig. 17—continued next page

b. Start
Curve A, voltage 1-3
Curve B, current line 3′
Curve C, voltage 3-*N*

c. Beats
Curve A, current line 1′
Current B, current line 2′
Curve C, current line 3′

d. Start
Curve A, voltage 1-3
Curve B, current *N′-G*
Current 3, voltage 3-2

FIG. 17—7.2-KV. POTENTIAL TRANSFORMERS—PHENOMENON C

shown. Two trains of several successive cycles each of D are shown; in each case, D terminated in an arc-over. The maximum crest value shown is about 650 volts (33.7 kv. primary, or 4 times normal crest). This is the only connection for which D held for more than about one cycle, thus making voltmeter readings possible; however, D did not hold long enough even in this case to obtain a complete set of readings. It is evident, therefore, that D is much less likely than B to sustain itself.

The start of voltages 1-N, 2-1 and current 1′ is shown in Fig. 16 for two 7.2-kv. potential transformers connected L/N; the third transformer was disconnected by removing its fuse. Secondary burdens 3-2 and 1-3 were disconnected to prevent partial excitation of the secondary from the other two transformers. Traces of A and possibly D and a few cycles of B and normal are shown. The maximum crest values for the B voltage and current are about 510 volts and 1.2 amperes, respectively. Evidently the phenomena are not suppressed by omitting one transformer.

TABLE VIII
CAPACITANCE-TO-GROUND TEST RESULTS

	12-kv. Circuit				
		Calculated			
Voltmeter Reading	Ammeter Reading	Volts	Impedance ohms	Capacitance μf.	Description (see Fig. 3)
100	3.975	12,000	3020	0.880	Complete circuit
100	3.815	12,000	3145	0.845	Generator only
				0.035	Bus and transformers only

(2) *Supplied from 25,000-Kv-a. Generator-Capacitance to Ground of 12-Kv. Circuit, 0.845 μf.*

THREE 7.2-KV. POTENTIAL TRANSFORMERS

Phenomenon C. The occurrence of Phenomenon C was evidenced audibly only by magnetic beats, since the L/N voltages were too low to cause corona. Voltmeter and ammeter readings taken in connection with the figures described are given in Table VII.

Fig. 6 shows voltages 3-N, 1-3 and current 3′; the relation between voltage 3-N and current 3′ is shown for a few cycles, the current peaks coinciding approximately with the zero-voltage points of alternate cycles; the maximum current crest value is about 1.75 amperes (45 times rated full-load crest). Fig. 17a shows the start of the three L/N voltages; slow variations in wave shape occur at the rate of about 53 per min. Fig. 17b shows the start of voltages 3-N, 1-3 and current 3′. The distortion of the L/L voltage wave caused by the excessive exciting currents is shown more clearly in Fig. 17b than in Fig. 6.

Fig. 17c shows the three line currents, which combine to form the neutral current N'-G shown in Fig. 17d. Slow variations in successive positive (or negative) crests occur at the rate of about 120 per min.; the maximum crest value is about 2.1 amperes (54 times rated full-load crest). Fig. 17d shows the start of voltages 1-3, 3-2 and current N'-G. Variations in the wave shape of the latter are noticeable; the maximum crest value at the start is about 2.9 amperes and after stabilization, 2.3 amperes.

At rated voltage, C was reduced to normal by adding a protective resistor burden of 72 volt-amperes (at 120 volts) per transformer. A larger burden was necessary at higher voltages, but it was not anticipated that such voltages would be encountered under normal operating conditions.

(II) DETERMINATION OF CAPACITANCE TO GROUND

The approximate test results outlined here were obtained by the volt-ampere method as described under "Methods of Test." Representative instrument readings taken are given in Table VIII, together with the calculated impedance and capacitance to ground of the 12-kv. circuit. Owing to the fact that the capacitance to ground of the 12-kv. bus and transformers was only a small fraction of either of the two calculated values from which it was determined, a check test was made by a more direct method. The result obtained was about 5 per cent greater than 0.035 μf.; part of this was due to the fact that no allowance was made for the voltage rise in the potential transformer. This close agreement between the two sets of results is very satisfactory under the circumstances.

FIG. 18—SIX 63.5-KV. POTENTIAL TRANSFORMERS (ONE Y-Y BANK; ONE Y-BROKEN-DELTA BANK)—PHENOMENON A

Curve A, voltage across broken delta
Curve B, voltage 3-N
Curve C, voltage 1-2

It was not possible to determine the capacitance to ground of each of the three phases under the different conditions. However, there is no reason to believe that it should differ appreciably from one-third of the proper total value obtained from test.

110-Kv., 25-Cycle System

OPERATION OF 63.5-KV. POTENTIAL TRANSFORMERS

Phenomenon A. The occurrence of Phenomenon A at rated L/L voltage or higher was evidenced audibly by increased corona, due to the comparatively high voltage (110 kv. or more) to ground on two of the lines. The voltage waves obtained for A (and B) and the normal condition were of the same general shapes as those on the 12-kv. system, so only two figures are

included, both of which were obtained when two banks of potential transformers (one Y-Y; one Y-broken delta) were connected in multiple; this represents the usual operating condition. Voltmeter readings for these figures are given in Table IX; for *A*, the neutral is displaced from its normal position by approximately one-third the reading of the voltmeter connected across the secondary of the Y-broken delta bank.

Out of 27 operations of the oil switch at rated voltage, 15 (about 56 per cent of total) sets of normal voltage readings and 12 (about 44 per cent of total) sets of *A* voltage readings were obtained; of the latter, voltages 1-*N*, 2-*N* and 3-*N* were too low to read on their respective voltmeters in the ratio of 50 to 33 to 17.

The start of the three *L/N* voltages is shown in Fig. 7; initial oscillations occur at the rate of approximately 4,200 cycles per sec. About one-half cycle of Phenomenon *B* and two cycles (after stabilization) of *A* are shown.

Fig. 18 shows voltages 3-*N* and 1-2 on the Y-Y bank and the broken-delta voltage on the bank so connected. This extreme example of *A*, in which the greatest *L/N* voltage is 50 per cent greater than the *L/L* voltage and in which the neutral has been displaced by the approximate amount of the *L/L* voltage (one-third of 258 volts) from its normal position, could be obtained only by reducing the voltage after once establishing *A*.

The occurrence of *A* (and *B*) was prevented for two banks in multiple at all voltages tried by adding protective resistor burdens of 600 volt-amperes (at 63.5 volts) per transformer. The single protective burden connected across the broken delta of one bank, found to be equivalent to the six 600-volt-ampere *L/N* burdens (about 6.7 ohms each) or, preferably, to the three 1,200 volt-ampere *L/N* burdens (about 3.4 ohms each) in terms of one bank, amounted to approximately 10 ohms.

It was not possible to obtain *A* at rated voltage with only one bank of potential transformers connected; however, *A* did occur frequently at 110 per cent of rated voltage. Voltmeter readings are given in Table III.

Protective resistor burdens of 900 volt-amperes per transformer were necessary to prevent the occurrence of *A* at 110 per cent of rated voltage. The single protective burden for the Y- broken-delta bank, found to be equivalent to the three 900 volt-ampere *L/N* burdens (about 4.5 ohms each) for the Y-Y bank, amounted to approximately 13.5 ohms.

It is evident that the total resistance of the necessary protective burden is independent of its connection; for example, substantially the same protective efficiency will be obtained with 15 ohms connected across a broken delta as with three 5-ohm burdens connected Y, for the corresponding secondary connections.

If sine-wave voltages are assumed for the readings in Tables III and IX, the neutral *N* will be found to be located outside of the delta-voltage triangle in all cases, as in Fig. 2b. For two banks in multiple, *N* is near the edge at 110 per cent voltage, considerably outside at 100 per cent and far outside the triangle at 76 per cent voltage. For one bank only, *N* is considerably outside the triangle at 110 per cent voltage; in this, as in the two previous cases, the low voltage can be said to be partly reversed in phase.

TABLE IX

110-KV., 25-CYCLE SYSTEM. SIX 63.5-KV. POTENTIAL TRANSFORMERS. CONNECTIONS: ONE BANK, Y-Y; OTHER BANK Y-BROKEN DELTA. SECONDARY VOLTMETER READINGS TAKEN IN CONNECTION WITH OSCILLOGRAMS

(Approximate Primary Volts = Tabulated Volts × 1000)

Refer to Fig.	Line-to-line (*L/L*)				Line-to-neutral (*L/N*)				Comments	Broken-delta vm.
	Per cent†	1-2	2-3	3-1	1-*N*	2-*N*	3-*N*	Phenomenon		
7	110	121	121.5	120.3	13*‡	114.5*	121*	*A*	Part cycle of *B*	196
18	76	84.5*	84	82.2	44.5	108.2	124.7*	*A*	Volts reduced after taking Fig. 7	258*
	110	122	123	121.8	69.9	71.1	70	...	Normal 110%	4.1
	78	86.1	86.7	85.4	49.8	50	49.5	...	Normal 76%	1.8

*Item shown in Fig.
†Approximate—based on potential transformer rating of 110 volts *L/L* (= 1.73 × 63.5 volts *L/N*).
‡Estimated value.

Discussion

For discussion of this paper see page 342.

Physical Nature of Neutral Instability

BY A. BOYAJIAN[1] and O. P. McCARTY[1]

Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*Experience has established the fact that the neutral of a three-phase system may become subject to certain strange phenomena of instability under apparently normal conditions. The disturbances are of two distinct classes: (1) persistent shift or inversion of the neutral, resulting in unequal leg voltages; and, (2) persistent oscillation of the neutral (with equal effective voltage in all three legs) at approximately one-half, double, or triple frequency. While in the ultimate analysis saturation is at the basis of the phenomena, a more definite explanation is given as follows:*

1. Neutral shift or inversion is a fundamental frequency phenomenon, and is due to the fact that the volt-ampere curve of the combination of an iron core reactor (transformer magnetizing current) in shunt with a suitable capacitor has one zone which is lagging and one which is leading. In a Y-Y bank of transformers, with suitable balanced line capacitances to neutral following a switching disturbance, one leg may act leading, the others lagging, and thus invert the neutral.

2. Oscillations of the neutral tend to take place at its natural frequency, but since, due to inevitable losses, not all oscillations can persist, in course of the starting transient the oscillation is resolved to the nearest lower frequency which is able to draw energy from the circuit by approximating harmonic relationship to it. The even harmonics are accounted for by the persistence of residual in the core, whether left from previous excitation or brought about by the direct current component of starting transient.

In single-phase circuits, the half-frequency oscillations of the neutral are exactly half frequency, but in three-phase circuits it may deviate from this somewhat, producing a continued phase-shift between the neutral potential and the impressed frequency, and resulting in beats.

It is commonly recognized that in a core subject to saturation energy can be put in at fundamental frequency and drawn out at a higher harmonic. This principle is here amplified and made reversible by postulating that energy may flow not only from fundamental to its harmonics but also from these harmonics to fundamental, if a suitable source of harmonic energy is connected to the circuit. This principle is then generalized whereby energy may flow not only from a higher harmonic to fundamental, but also from fundamental to an oscillation at subnormal or fractional frequency, the fundamental acting as a higher harmonic of the oscillation at subnormal frequency.

Laboratory tests are described (a) reproducing some of the disturbances observed in the field, and (b) supporting the theories outlined above.

Preventative measures and laboratory tests with them are also discussed.

* * * * *

Introduction

FROM time to time the attention of the Institute has been drawn to some peculiar troubles accompanying the operation of Y-Y connected transformers with their neutral grounded on an otherwise isolated system.[2] Such a scheme of operation is objectionable for other reasons besides the peculiar phenomena observed, and power transformers are not operated in this fashion nowadays, unless equipped with tertiary delta windings. The peculiar phenomena referred to above were, therefore, of only theoretical interest for many, until it was learned that such a scheme of operation is being used in a number of potential transformer installations, constituting a possible hazard to the system and connected apparatus. In view of the importance of the problem, and the obscurity of the phenomena, a combined theoretical and laboratory study was recently undertaken by the authors to elucidate the physical nature of the phenomena. The conclusions arrived at here are in some respects different from those of earlier investigators reported to the Institute, although not inconsistent with the works of Heegner[3] and Lampert[4] in Germany. The studies of these latter authors, however, have a bearing on only one group of the phenomena here discussed. Although the theories here advanced need not be considered complete in all detail, they are believed to interpret correctly the physics of these disturbances.

The phenomena under discussion are found only in circuits with capacitance and saturation, and therefore, it is proposed to build up the present discussion starting with an analysis of the characteristics of iron-core reactors in combination with capacitors.

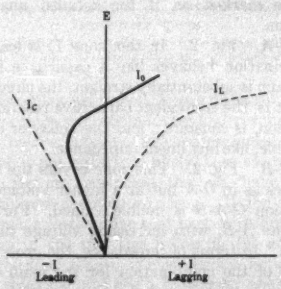


Fig. 1—Volt-Ampere Curve of an Iron-Core Reactor and a Capacitor in Parallel

I_L, curve for reactor; I_C, curve for capacitor; I_O, resultant of the two in parallel

Iron-Core Reactor Shunted by a Capacitor

In Fig. 1, the dotted curve I_L is the volt-ampere characteristic of the saturating inductance, such as the magnetizing current of a transformer, and dotted curve I_C is that of the shunt capacitance. The resultant

1. Both of General Electric Company, Pittsfield, Mass.
2. For references see Bibliography.

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characteristic curve of the combination is shown by the heavy curve I_o. It is realized that the exciting current of a transformer is much more complicated than that of a capacitor. While the latter may be considered as yielding a pure leading current, the former will always contain a lagging component, a power component and harmonics. As the lagging component is frequently 80-90 per cent of the total exciting current, it is important to know its behavior in the network, and Fig. 1 is drawn

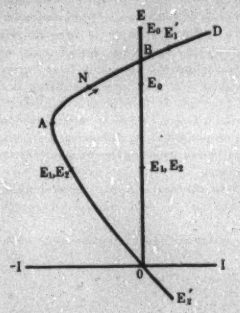


Fig. 2—Details of I_o from Fig. 1

for the fundamental frequency reactive components of the currents. The effect of losses and harmonics will be discussed later.

A magnified curve of I_o is given in Fig. 2, and three zones are marked on it for detailed analysis and comparison.

Zone O-A. Fig. 2. In this zone I_o is leading, and the combination behaves like a capacitive reactance. As the graph is substantially straight, the circuit parameter; that is, the equivalent capacitive reactance of the combination, is constant; and the behavior of the circuit is stable, like any linear impedance.

Zone A-B. Fig. 2. This zone covers the same current values as in *O-A* but at a higher voltage, so that the function *O-A-B* is double valued. Furthermore, in the zone *A-B*, with increasing voltage the current decreases. In terms of impedance, the gross apparent reactance of the combination for the main current is still leading or capacitive, but its incremental reactance; that is, the reactance for superposed smaller currents, is lagging or inductive.

$$X, \text{ considered as } \frac{E}{I} \text{ is negative, leading, capacitive; but,}$$

$$X, \text{ considered as } \frac{\Delta E}{\Delta I} \text{ is positive, lagging, inductive.}$$

Now such a characteristic is a source of instability *under suitable conditions*, somewhat analogous to the case of an arc of which the incremental resistance is opposite to its gross resistance and leads to instability under certain conditions. If, in a network, such a combination operating at the point N is in equilibrium, in the sense that the terminal conditions will be satisfied thereby, such an equilibrium may be unstable when disturbed, the point N moving either up into zone *B-D* or down into zone *O-A* to find a point of stable equilibrium, or it may fail to reach equilibrium and the point N may remain in a state of persistent oscillation. Corresponding to these two cases to be discussed later in detail, namely, where an abnormal condition of equilibrium is established, or, where the point N oscillates indefinitely, we have the two categories of troubles experienced in the Y-Y operation of potential transformers, namely, inversion or shifting of the neutral and the oscillation of the neutral.

Zone B-D. Fig. 2. In this zone, the current is lagging, and increases with the voltage (even though not in direct ratio); so that both the gross and the incremental reactance of the combination are inductive, and the behavior of the combination is substantially that of a plain saturating inductive reactance. An equilibrium point in this zone tends to be stable.

We may now proceed to build up the characteristics of such combinations in networks, with particular reference to the inversion of neutral and the oscillation of neutral.

Inversion of Neutral

(*a*) *Inversion in a Single-Phase Circuit.* Referring to Fig. 3A, let branches 1 and 2 be duplicates. A point

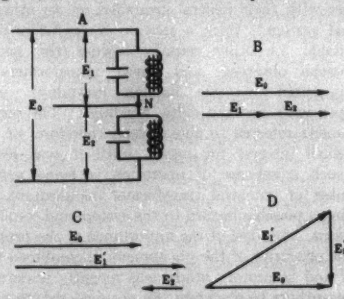


Fig. 3

A. Single-Phase Circuit with Unstable Neutral
B. Vector Diagram for Normal Condition of Neutral
C. Vector Diagram for Inverted Neutral
D. Vector Diagram for Neutral Inversion with a Prominent Quadrature Component

of equilibrium for N is obviously half way between the potentials of the terminals, as shown in Fig. 2, and also by the vector diagram of Fig. 3B:

$$E_1 = E_2 = E_0/2,$$

where E_0 is the line voltage, and E_1 and E_2 are the branch voltages. If the branch voltages E_1 and E_2 are

within the zone O-A (as shown in Fig. 2), the potential of N will be normal and stable, once so established. In Fig. 3B the line current will be leading, both branches acting capacitively. Assume, however, that when the line switch is closed, the starting transient unbalances the voltages and carries one of the branch voltages, say E_1, into zone A-D. A new and abnormal condition of equilibrium may now become established as follows:

Considering the characteristic curve, Fig. 2, it is seen that not only

$$E_1 + E_2 = E_0,$$

but also,

$$E_1' + E_2' = E_0.$$

In the latter case, E_2' is negative, E_1' is greater than the line voltage (see also the vector diagram of Fig. 3C), and the neutral potential is outside of the line voltage. Obviously, one leg or branch of the circuit is acting inductively, the other capacitively.

With a different line voltage, say E_0'', (Fig. 2), a still different type of abnormal equilibrium may take place with the leg or branch voltages assuming the values E_1'' and E_2'' respectively, in which case then, both branches are operating capacitively, one overexcited, the other underexcited.

A still different position of the neutral may correspond to one leg operating in the zone A-B, the other in the zone B-D, both legs overexcited. Such a case was observed in single-phase tests.

Effect of Losses and Harmonics. The effective resistance of each branch will obviously give a voltage drop in quadrature with the reactive drop, and since the ratio of resistance to reactance may not be the same in both branches, E_1' and E_2' need not be parallel to each other, but may be out of phase, as shown in Fig. 3D. The losses may thus modify the position of the neutral, but are not at all inconsistent with the existence of two different conditions of equilibrium, such as (E_1, E_2) and (E_1', E_2').

Harmonics cannot directly influence the position of the neutral of the fundamental frequency voltages, except indirectly by modifying the character of saturation. Oscillograms of inversion show that the harmonics are of relatively small value and exert no controlling influence on the phenomena.

So far as the effective values of the voltages are concerned, such as measured by r. m. s. voltmeters, harmonics always act as if they were at right angles to the fundamental (and to each other), and thus give an apparent shift to the neutral at right angles to the line voltage, as well illustrated by the laboratory tests.

(*b*) *Inversion in a Three-Phase Circuit.* If the neutral of a Y-Y or Y-broken-delta bank of transformers is grounded on a circuit which is otherwise isolated, the neutral of the transformers is joined to the neutral of the line capacitances to ground, and we have an equivalent circuit such as shown in Fig. 6A. Each leg of this network will have a volt-ampere characteristic similar to Fig. 2, capable of acting either as a capacitive reactance or as an inductive reactance.

Assuming the three legs as duplicates, the normal position of the neutral will be at the center of gravity of the line voltage triangle. However, other positions of equilibrium also exist, as for instance, two of the legs operating in the zone B-D with a lagging power factor, and the third in the zone O-A with a leading power factor, as explained for the single-phase circuit, in which case, one leg will be reversed, the other two will be greatly overexcited, and the neutral will fall outside of the triangle of line voltages. The alternative case, in which all the legs operated capacitively, one in the zone O-A, the other two in the zone A-B, as discussed in connection with single-phase neutral shift, the neutral falling just inside the line voltages, has also been observed.[8]

Since any one of the legs may become underexcited, depending on the instant of switching, there will be at least three abnormal and stable positions of the neutral.

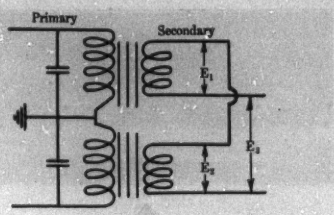


FIG. 4—CIRCUIT CONNECTION FOR SINGLE-PHASE NEUTRAL INSTABILITY TESTS

Voltage corresponding to normal neutral is neutralized and eliminated from E_3 which therefore exhibits only shifts and oscillations of the neutral

In the foregoing discussion, it was assumed that two legs may become overexcited, one underexcited. The other alternative, namely, two underexcited and one overexcited, is not ordinarily likely, because, no matter which two legs are considered, they will have to stand the line voltage common to them. If the tests are made at such a low voltage that even with line voltage the density in the two legs is low in the zone O-A; then, for the same reason, it may become impossible to carry the third leg into the high density zone B-D by any switching transient, or find an equilibrium point there.

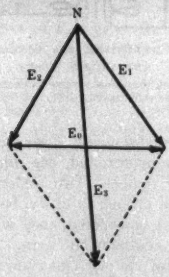
Losses and harmonics may shift somewhat the position of an abnormal neutral, but, unless intensified far beyond usual proportions, will not affect the persistence of an abnormal neutral potential.

Single-Phase Inversion Tests. If the theory of inversion outlined above is true, it should apply to a single-phase circuit as well as to three-phase. To verify this, two duplicate standard potential transformers, rated 13,200 volts to 110 volts, were connected in series to yield a neutral point, as shown in Fig. 4, and each trans-

former was shunted by a variable capacitance. The settings of the capacitances of the two branches were made alike in all cases, so that the potential of the neutral would normally be halfway between those of the terminals. When the circuit was connected to the generator, and the voltage gradually brought up by field control, the neutral was found to be normal in all cases. However, when the generator was first excited to the desired voltage, and the voltage suddenly thrown on the load, inversion of the neutral point was observed over a

A

B



C

FIG. 5

A. AND B. OSCILLOGRAMS OF NEUTRAL SHIFT TAKEN ON CIRCUIT OF FIG. 4

C. VECTOR PLOT OF A AND B

fairly wide range of capacitances, this range depending on the excitation of the transformers.

Referring to Fig. 4, the secondaries (low voltages) of the potential transformers will be seen connected in series opposition. The purpose of this was to segregate the abnormal voltage component of the neutral. Obviously, with balanced branch voltages the terminal voltage of the secondaries (as connected up) would be zero, but if the neutral should shift, invert or oscillate, the phenomenon will then appear in the resultant voltage of the two secondaries in its pure form, unmixed with normal voltages.

With line voltages, representing from 100 to 120 per cent excitation on the potential transformers, the neutral of this single-phase bank would frequently come in inverted, with capacitances from 0.025 to 0.050 μf. across each unit (on the high voltage side). A representative set of readings is as follows:

TABLE I

Line voltage representing 120 per cent excitation:

E_1	E_2	E_3	Oscillogram	Vector diagram
193%	178%	328%	Figs. 5A, 5B	Fig. 5C

(Note: The voltages are in per cent of the values corresponding to normal neutral).

The oscillograms in Figs. 5A and 5B show that all the voltages are of fundamental frequency and of reasonably good wave shape. Considering the vector diagram in Fig. 5C, the branch voltages are seen as of the same order of magnitude, and that the neutral has shifted almost in quadrature with the impressed voltage. The significance of this is that one of the branches is operating capacitively in the zone *A-B* (Fig. 2) fairly close to *B*, the other inductively in the zone *B-D*, but again fairly close to *B*. Only such a combination will give an inversion in which E_1 and E_2 will be so nearly equal. There were a good many cases of inversion in which the branch voltages were very unequal, but these were usually complicated by superposed oscillations. Inversions with unequal leg voltages, unmixed with oscillations, were more easily obtained in three-phase tests.

It is further to be observed in connection with the vector diagram (Fig. 5C) that the voltage E_3 is vectorially equal to twice the neutral shift, as theory would demand.

Three-Phase Inversion Tests. The circuit utilized is shown in Fig. 6A. It should be observed that when the neutral is normal, the voltage across the corner of the delta will be zero, but when the neutral moves from this position, it will show in the corner of the delta. Thus, the phenomena showing across the corner of the delta are entirely those of zero sequence components: whether they are third harmonic, or something else, is purely incidental. That is, any voltage of any frequency in the leg voltages will show across the corner of the delta if it is of zero phase sequence, and will cancel out if it is symmetrical polyphase. In more common terms, only those phenomena which disturb the neutral can appear across the corner of the delta, all others cancel out. Voltage oscillograms taken at this point, therefore, show clearly the nature of the disturbance of the neutral. The magnitude of this voltage will be three times the corresponding component in each leg.

Data on the typical cases of inversion follow:

TABLE II

1.	Vector diagram of readings, Fig. 6B		
	Oscillogram Figs. 6C, 6D		
	Capacitance 0.020 μf. per leg.		
	Impressed voltage	100	per cent of rating
	Observed, leg 1	96	" " of line voltage
	" 2	110	" " " " "
	" 3	21	" " " " "
	Cor. of delta	190	" " " " "
	" " "	300	" " " neutral shift
2.	Capacitance 0.010 μf. per leg.		
	Impressed voltage	75	per cent of rating
	Observed, leg 1	28	" " " line voltage
	" 2	114	" " " " "
	" 3	103	" " " " "
	Cor. of delta	207	" " " " "
	" " "	300	" " " neutral shift

(Note: Wave shape and vector diagram of case No. 2 were very similar to those of No. 1 and were therefore omitted).

Comments:

1. The oscillograms (Figs. 6C and D) show that the phenomena are of fundamental frequency and of reasonably good wave shape.

2. The data show that two legs are overexcited (acting inductively) and one underexcited (acting capacitively). The fact that the neutral shift is not exactly parallel to the normal leg voltage of the underexcited leg is largely due to the losses.

3. It was observed that any one of the three legs might come in reversed, on switching-in.

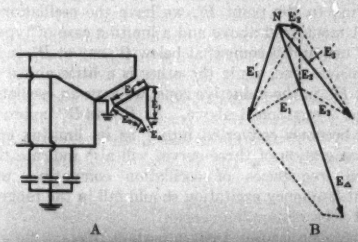
4. Inversion did not, of course, take place at every switching-in operation. As the upper or lower limit of the capacitance range was approached (0.005 and 0.020 μf. respectively), inversion occurred less and less frequently.

5. Inversion phenomena were not observed at excitations less than 55 per cent of normal. This figure, however, need not be taken very seriously as we may not have properly exhausted all possible combinations of density and capacitance conducive to inversion, but it is in accord with the theory outlined above that if the densities are kept so low that during switching-in transients the voltage will not go into the zone *A B* (Fig. 2), the neutral should be stable.

OSCILLATING NEUTRAL

When the neutral is disturbed in any way, the transition to a state of equilibrium must take place through an oscillation at the natural frequency of the network. In the group of phenomena classified as inversion of the neutral, this oscillation dies out, and the remaining phenomena are the fundamental frequency currents and voltages, and their usual harmonics. This is beautifully illustrated by the oscillogram in Fig. 7. In many other cases, the oscillation is found to persist.

The Equivalent Circuit of Oscillation. The single-phase network shown in Figs. 3A and 4 has two possible ways of natural oscillation. (1.) The capacitance and reactance of each leg or branch could oscillate against each other, and if the phase of oscillation of one branch or leg is 180 deg. from the oscillation of the other, the neutral will oscillate without showing any oscillation voltage or current in the supply lines. (2.) One leg or branch may show a net inductive reactance, the second, a net capacitive reactance, and thus the neutral may again oscillate without any oscillation voltage in the supply lines, but with an oscillation current in the



C

D

FIG. 6

A. CIRCUIT USED FOR THREE-PHASE NEUTRAL INSTABILITY TESTS. WITH NORMAL NEUTRAL E_Δ IS ZERO, AND THUS E_Δ EXHIBITS ONLY SHIFTS AND OSCILLATIONS OF THE NEUTRAL

B. VECTORIAL PLOT OF A NEUTRAL SHIFT OBSERVED ON THE CIRCUIT OF FIG. A

C. OSCILLOGRAMS OF NEUTRAL SHIFT TAKEN ON CIRCUIT OF FIG. A

supply lines. But as the ohmic value of the net effective reactance of each leg for the oscillation increases (by parallel resonance), the current will decrease for a given oscillation voltage. Thus, the difference between 1 and 2 being merely a matter of the magnitude of the oscillation current in the lines, the two types of phenomena are not radically different, but merge into each other, and when the oscillation current in the lines is small, the phenomenon may equally well be considered

one way or the other. These and other important relations may be illustrated as follows.

In Fig. 8, *O-A-B-C* is the volt-ampere curve for normal impressed frequency, while *O-A'-B'-C'* is that for half frequency, and the dashed curve for a multiple of normal frequency. The reader may easily check that the lower frequency curves must fall inside, higher frequency curves outside, of the normal frequency curve. For a given order of voltage, the normal frequency operation may be around A, the half-frequency oscillation around B'. If both units are oscillating corresponding to the point B', we have the oscillation of type 1 mentioned above and a limiting case of type 2. But if one unit is somewhat below B', say at B'', in the condensive zone, while the other is a little above B', say at B''', in the inductive zone, we have an oscillation of type 2 mentioned above. As B'' and B''' approach B', 2 becomes converted into 1 as its limiting case. A consideration of these curves will also indicate that natural frequencies of oscillation compatible with normal frequency excitation should fall in the range of observed oscillation frequencies.

Experience shows two important facts; manely, first, that these oscillations may be loaded with con-

Fig. 7—Oscillogram of Single-Phase Neutral Instability Exhibiting an Initial Oscillation and Gradual Transition into Inversion

siderable losses and still persist, indicating that they are able to draw a considerable amount of energy from the circuit; and second, that in the simpler cases, with comparatively little wave distortion, the oscillation frequency is seen to be in a zone corresponding to the nearest harmonic relationship; that is, either the second harmonic, or frequently, one-half harmonic, although not necessarily these values exactly. These two facts call for explanations as to how these oscillations can draw power from the impressed frequency, and how the 2 to 1 or 1 to 2 (that is, even harmonic) relationship can be maintained without direct current excitation.

Flow of Power From One Frequency to Another

To consider a well-known case, Fig. 9 shows a static frequency converter, converting from an impressed frequency to its triple frequency. The production of third harmonic voltage by saturation and by suppression of third harmonic exciting current is too well known to require any proof or explanation here. The loading characteristics of the third harmonic circuit depend on its no-load voltage and short-circuit current and may be represented reasonably well by the conventional circle diagram. By using higher and higher values of saturation, both the no-load voltage and the short-circuit current and hence the maximum output of the triple frequency side, are increased. For instance, at nominal 60-cycle densities around 170 kilolines per sq. in., third harmonic outputs over 100 kw. per thousand lb. of core can be obtained. Of course, the corresponding amount of power (plus the high losses) flows from

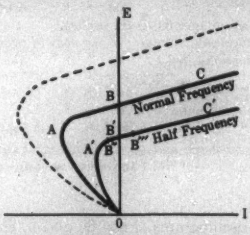


Fig. 8—Characteristic Modification of Fig. 2, for Various Frequencies

the primary mains into the primary windings at fundamental frequency.

Energy flow from one frequency to many other frequencies besides the third, is also possible. For instance, second harmonic. This, however, requires that the core be biased, a condition easily brought about by d-c. excitation.

Very large amounts of power at double frequency may be drawn from a static frequency multiplier at low nominal 60-cycle excitations, with the aid of a high d-c. excitation.

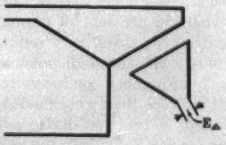


Fig. 9—Static Frequency Converter. E_Δ is a Reasonably Pure Triple Frequency Voltage

Reversibility of Power Flow

The flow of power in a frequency multiplier from a lower frequency to a higher frequency must be reversible.

Assume that I_3 is the current in a certain third harmonic load, drawing from the transformer a power P. If the current I_3 is reversed by the application of a suitable third harmonic voltage into the third harmonic circuit from an external source, the power P must also be reversed, delivering power to the transformer instead of drawing power from it. We are, therefore, justified in postulating that power may flow not only from a

fundamental to its harmonics (by virtue of saturation), but that it may also flow back from the harmonics to the fundamental, if the harmonics have access to a suitable external source of supply.

When the reversibility of power flow is recognized, the arbitrariness of calling the lower frequency fundamental becomes obvious. When power is flowing from 180 cycles to 60 cycles, we would be entirely justified in calling the 180-cycle circuit primary fundamental, and the 60-cycle circuit secondary one-third harmonic. But, aside from use of words, it should be obvious that if power may flow from 180 cycles to 60 cycles, it may also flow from 60 cycles to 20 cycles, or from a *fundamental to a subnormal harmonic*, under a suitable circuit condition.

One of the important features of those suitable circuit conditions is that the lower frequency should be able to take care of its magnetizing current, drawing it either from a generator or from a capacitor.

If the impressed frequency and the oscillation are related to each other as a fundamental and an exact harmonic, whether as higher or lower harmonic, the availability of energy to sustain both circuits may be granted in the light of the foregoing.

If the harmonic ratio is even, a source of bias is implied and must be accounted for; and, if the higher frequency is not an exact multiple of the lower, the possibility of sustained power flow from one frequency to the other must be accounted for.

Even Harmonics and Source of Bias

In the absence of sustained direct current, an available source of bias is residual. The direct current component of the switching transient might well start even harmonic oscillations, but as this direct current must soon die out, while the oscillations persist indefinitely, residual might sustain the bias.

We are accustomed to believe that an unexcited transformer core may carry a residual flux in it indefinitely, but that when an alternating flux is superposed on it, the residual must decay. It appears, however, that under favorable conditions the residual flux might persist; favorable conditions being those in which an even harmonic is capable of self-excitation by drawing its magnetizing current from an oscillating circuit and losses from a primary circuit. In view of the fact that the persistence of residuals under any condition, and decay under any other condition, are empirical facts, we need not speculate here on their theoretical explanations, but may well be satisfied with the observation of the mathematical fact that the superposition of two harmonic fluxes, having an even frequency ratio, makes the positive B_{max} different from the negative B_{max} in the core, as if the zero axis has been shifted. This fact may favor the persistence of a residual.

Non-Integral Ratios

Since the natural frequency of oscillation of the circuit is independent of the impressed frequency, the oscillation need not bear an integral ratio to the impressed frequency. In the majority of the cases produced in the laboratory, the frequency ratio was not an integer, and, there was a continuous phase shift between the impressed voltage and the oscillating voltage. A permanent residual can not, of course, support such an oscillation—the bias must also shift phase; that is, it must be an alternating bias at the heterodyne beat frequency. Saturation leads to the production of such alternating bias by modulation when fluxes at two different frequencies are combined in one core. The principle that saturation will cause modulation under such conditions, has already been commercially made use of by Mr. Alexanderson in his well known "magnetic amplifier." Oscillations of the neutral practically always show such low frequency components in the waves and in the accompanying noises and flickers.

If the natural oscillation at approximately even frequency ratio continually shifts phase with respect to

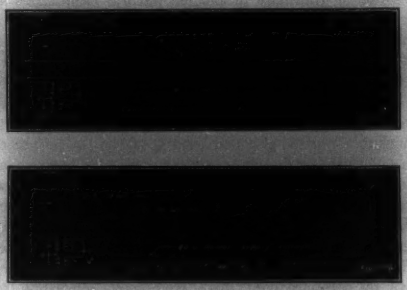


Fig. 10—Oscillograms of Oscillating Single-Phase Neutral

the impressed frequency, it cannot draw power from the impressed frequency uniformly, but must rise and fall. If the power flow were dependent exclusively on a fixed residual, the power flow to a shifting oscillation should not only pulsate but should also reverse; but, if dependent on a low frequency, alternating bias, then it need not reverse even though it must pulsate.

Harmonics of Natural Oscillations

It is to be expected that in the presence of saturation, not only the impressed frequency currents and voltages may become distorted and be given higher harmonics, but the oscillation of the network may also become distorted and be given higher harmonics. An oscillation at half impressed frequency may have a third harmonic of its own, and this will obviously appear as a one-and-a-half harmonic for the fundamental. Consequently, when a complex wave is being analyzed, confusion may arise by attempting to resolve everything into harmonics of the impressed frequency. Fortunately, many instances are found

in which the secondary harmonics of both the impressed frequency and the oscillating frequency are small, and by suitable arrangement the oscillation can be segregated and shown to be of surprisingly good wave shape, as discussed below.

Single-Phase Tests of Oscillating Neutral

Case 1. Fig. 7 illustrates perfectly how the transition from the initial unstable condition to the final

Fig. 11—Oscillogram of Oscillating Three-Phase Neutral

inverted condition of the neutral takes place through a series of gradually intensified natural oscillations at approximately half the impressed frequency. In this instance, the oscillations gave way to inversion, but in range of capacitance values above 0.025 μf. up to 0.050 μf., the oscillations once started usually persisted.

Case 2. Figs. 10A and 10B illustrate a case in which the oscillation persisted indefinitely. Note how remarkably pure is the wave shape of the oscillation of the neutral (single-phase neutral), and how the oscillation frequency is exactly one-half of the impressed frequency.* Examining the waves of the leg voltages (E_1 and E_2) nothing else of importance is found in them but a fundamental and a half-frequency harmonic. The flicker in the lamps connected across each phase was of comparatively high frequency, according to the oscillograms.

The indicated range of capacitance (0.025 to 0.050 μf.) was found to be adequate to furnish the necessary magnetizing current for the oscillation.

Examination of the oscillation on a long film does not reveal any definite pulsation in its amplitude. This is consistent with the integral ratio of the oscillation.

Tests with single-phase neutral oscillation were made primarily to verify the theory that the peculiar phenomena observed in three-phase circuits are not due to any virtue inherent in three-phase connection, but that they could occur in any circuit consisting of members with characteristics such as shown in Fig. 2. Having established this point both with reference to inversion and to oscillations, tests with single-phase circuits were not carried much further.

*The reason for the comparative simplicity of wave shapes observed in these tests was the fact that the circuit was made resonant for a subnormal frequency and would therefore not appreciably affect the higher harmonics of the magnetizing current. If the capacitance had been adjusted for oscillation at double normal frequency, it would have been near enough to third and fifth harmonic resonance to intensify them a great deal and, thereby lead to a very complicated wave shape, as had been observed by a number of the investigators. No attempt was made in this investigation to reproduce double or higher frequency oscillations, since the nature of the phenomena could be studied more conveniently at the lower frequencies.

Tests with Oscillations of Three-Phase Neutral

The circuit connection for neutral oscillation was the same as for inversion (Fig. 6A), the phenomena changing from one type to the other in going from one capacitance range to the other. The two ranges of capacitance overlap, and so do the phenomena.

a. Similarity to Single-Phase Tests. Fig. 11 gives the timing wave (supply voltage), leg voltage, and the neutral voltage for a typical case of oscillation. The neutral voltage is observed across the corner of the

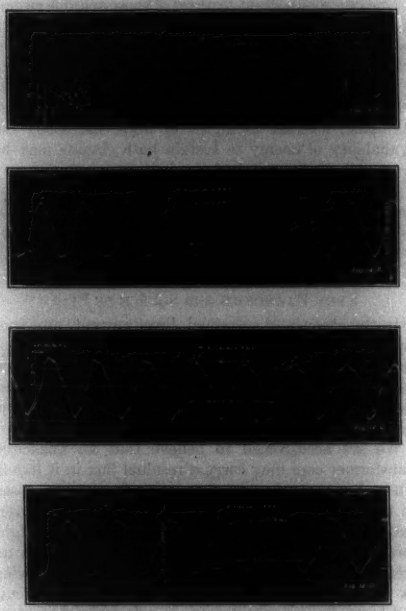


Fig. 12—Oscillograms of Three-Phase Neutral Showing One-Half Harmonic Oscillation with a Slow Phase Shift with Respect to Supply Voltage

delta. The characters of these waves will be seen to be substantially the same as in the single-phase tests: the three-phase connection has not materially modified the phenomena.

b. Continuous Phase-shift. While in the single-phase test, one leg oscillates against the second leg; in the three-phase case one leg oscillates against the other two in parallel. This has made enough change in the constants of the oscillating circuit so as to make

its frequency definitely different from half of impressed frequency, and consequently we observe (in the long film, Fig. 12) a slow continuous shift of phase of neutral oscillation with respect to the leg voltages and to the timing wave. Examining the leg voltage, we see that it contains nothing else of importance but the impressed frequency and the oscillating frequency. Furthermore, we see that due to the continuous phase shift

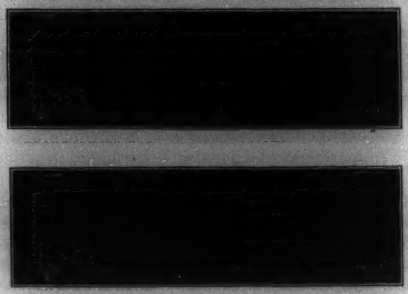


FIG. 13—OSCILLOGRAM OF THREE-PHASE NEUTRAL OSCILLATING AT HALF-FREQUENCY COMBINED WITH A SMALL NORMAL-FREQUENCY NEUTRAL SHIFT

Note the characteristic shape of the exciting current I_L, for this condition

between these two components, the wave shape of the resultant leg voltage continually changes, repeating at a low beat frequency. On Fig. 12, the oscillation frequency is seen to be 0.48 of the impressed frequency, producing one beat cycle every 50 cycles of the impressed frequency. Quarter periods of this beat cycle are shown in Fig. 12 by means of vertical lines. This period varied, of course, a great deal with varying circuit constants, as follows.

TABLE III

Excitation	Capacitance per leg	Beat frequency cycles per sec.	Capacitance per leg	Beat frequency cycles per sec.
120 per cent	0.045	0.53	0.025	0.33
100 " "	0.045	0.50	0.025	
90 " "	0.045	0.57	0.025	0.33
82 " "	0.045	0.67	0.025	0.40
73 " "	0.045	1.00	0.025	0.53

c. Peculiarities in the Transformer Exciting Current. Considering the current in each transformer primary (I_L in Fig. 13), the curious fact is observed that in certain zones of the film this current is negligible for a whole cycle of impressed frequency, and two appreciable peaks for the adjacent cycles of the impressed frequency. Other peculiar wave shapes will be seen in other zones. These are very easily explained by combining two fluxes of fundamental and half frequency together (in various phase relationships) and plotting the corresponding magnetizing current: the characteristic wave shapes in the different zones will all be reproduced.

d. The Capacitor Current. Considering the current in the capacitors (I_c Fig. 14), it is seen to contain three components: (1), a fundamental, (2), a half frequency, and (3), a small high frequency ripple. It will be further observed that the ratio of the half-frequency component to the fundamental is a great deal less in the current, than in the voltage wave, as it should be, theoretically, one-half.

e. Effect of Extraneous Capacitances. To make certain that extraneous capacitances to ground were not influencing the phenomena more prominently than those which were deliberately connected into the circuit, comparative tests were made with the neutral alternately grounded and isolated, and the conclusion was drawn that these concealed capacitances did not exceed 10 per cent of the main capacitances.

f. Variation in the Intensity of the Oscillation. Fig. 12 shows that the amplitude of the oscillation varied at the slow beat or modulation frequency. This is in accordance with theory, but one may wonder why the variation is not larger. The reason for it probably is that the power factor of the oscillation is very low: the hysteresis loss is largely furnished by the impressed (higher) frequency, while the eddy current loss is one quarter of that at impressed frequency.

FIG. 14—OSCILLOGRAM OF CAPACITOR CURRENT AND CAPACITOR (LEG) VOLTAGE UNDER CONDITIONS OF FIG. 13

g. Amplitude of the Oscillation. Some of the data are tabulated below:

TABLE IV
VOLTAGES IN THREE-PHASE TESTS

Test freq.	Cap. μf.	Fund. per leg volts	Osc. per leg volts per cent		Complex wave Measured volts	Complex wave Calc. volts
50	0.035	92	42	45.5 per cent	100	101
60	0.045	80	81.3	102 per cent	112	114
"	"	90	79	88 " "	118	120
"	"	100	75	75 " "	123	125
"	"	110	71	65 " "	126	131
"	"	120	62	52 " "	135	135
"	"	130	53	41 " "	140	140
"	"	140	43.5	31 " "	145	146
"	0.025	80	70	87.5 per cent	105	106
"	"	90	66.5	74 " "	111	111
"	"	100	61.5	61.5 " "	115	117
"	"	110	53.5	48.5 " "	122	122
"	"	120	43.5	36 " "	128	128

Some of the more interesting features of these data are:

1. Oscillation voltages of the same order of magni-

tude as the impressed frequency voltages are observed. Curiously enough the higher oscillation voltages have taken place at the lower excitations close to the border where the oscillation would stop altogether. With varying capacitances also, the higher oscillation voltages occurred close to the limiting value of capacitance at which the oscillation would stop altogether. In single-phase tests oscillations as high as 117 per cent of fundamental were observed. In either case, no effort was made to produce the maximum possible amplitude of oscillation.

2. The amplitude of the oscillation per leg was determined by taking one-third of the voltage across the corner of the delta. The fundamental was obtained from the line voltages. There is good agreement between the measured values of the complex voltage per leg and the corresponding values calculated with the aid of the two components.

h. Comparison of Wave Shapes of Three Legs. Fig. 15 shows the three leg-voltages. The differences in wave-shape are due to the fact that while the oscillation voltage is of the same magnitude and phase in all three legs (as a zero phase sequence phenomenon), the impressed frequency voltages are 120 deg. away from each other and therefore combined with the oscillation voltage at different phase angles. The wave shapes A, B, C, of this figure are reproduced by A', B', C' in Fig. 16, by combining with a given half frequency oscillation three fundamentals spaced 120 deg. away from each other.

Damping the Oscillations

Since the oscillation, unmixed by impressed frequency, appears only across the corner of the delta, the loading of the oscillation (without loading the impressed frequency) was done by connecting an adjustable rheostat at that point. This rheostat, however, was not the only load; the voltmeter and the ammeter were additional loads, besides the internal losses of the oscillating outfit. The data follow:

TABLE V

CAPACITANCE PER PHASE 0.035 μf.

Excitation on lines	Oscillatory watts in external load to stop oscillation	Equivalent watt rating of load per phase (See note below)
140 per cent	0	0
130 " "	2	..
118 " "	25	87
113 " "	35	79
109 " "	49	91
100 " "	68	96
91 " "	78	63
82 " "	78	15
72 " "	67	11
64 " "	47	8
55 " "	3	..
50 " "	0	0

(The column headed "Equivalent Load per Phase" gives the watts that would have been drawn per phase at impressed frequency if the damping resistances had been connected across each phase, instead of across the corner of the delta).

It is seen from the above data that less loading is necessary at the lower densities to damp out the oscillations. The reason for this is not that the oscillations are weaker, but rather that the magnetizing volt-amperes are very much smaller at the lower densities and, therefore, a smaller load brings up the power factor of the oscillating circuit to the necessary value. As a general conservative estimate, we would be inclined to suggest that the loading be at least equal to the magnetizing volt-amperes of the transformer. At densities 50-60 per cent of normal, this would constitute no hardship, whether the resistance is connected across the opened corner of the delta or line to neutral. The minimum resistance across the corner of the delta in these tests was about 375 ohms. In cases where potential transformers with line-to-line rating are connected line-to-neutral, operating at 57.7 per cent density, relay and other possible useful loads connected line-to-neutral (or across the corner of the delta) may almost accomplish this damping service.

Fig. 15—Oscillogram of Three-Phase Leg Voltages Under Condition of Oscillating Neutral

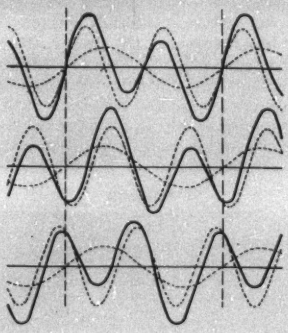


Fig. 16—Graphical Resolution Showing the Fundamental and Half-Harmonic Components of the Three-Leg Voltages of Fig. 15

Strange though it may sound, potential transformers with high quality low loss materials may give more trouble in this respect than those with inferior quality of materials.

Both theoretically and from foregoing test data, units operating at 50 per cent density appear safe from oscillations, but the margin of safety is small and we would doubt the wisdom of omitting all protection by merely going 10 to 15 per cent lower in density (as compared with 57.7 per cent excitation), because the

line voltages may sometime rise that much and wipe out the margin of safety.

Since the present investigation has concerned itself largely with the lower frequencies of oscillations, and since higher frequencies are also possible, question may arise as to the applicability of the present damping data to them. We believe such application would be on the safe side in view of the generally recognized fact that circuits are usually associated with higher losses and damping at higher frequencies.

In considering different methods of applying damping or balancing loads (such as across corner of delta, or line to neutral, or through an auxiliary low-voltage Y-delta potential transformer bank connected to the low side of the high-voltage potential transformer bank) the effect of such loads on metering accuracy is also to be considered, if metering is contemplated.

Conclusions

1. Operation of potential transformers in Y-Y with neutral grounded on an otherwise isolated system (whether permanently or occasionally isolated system, or section of a system) is a somewhat questionable practise unless protective measures are used. With many miles of lines connected to the potential transformers the line-to-neutral capacitances may be large enough to short-circuit all harmonic phenomena, especially in the higher voltage circuits, but when the connected portions of the lines are shortened in length by (temporary) isolation, various kinds of trouble may be anticipated. Systems which are using such connections, may be harboring a possible source of hazard to their equipment.

2. A well-known source of trouble is third harmonic voltage intensification, not discussed in the present paper because it is so well known.

3. It is found that with favorable capacitance values from line to ground, natural oscillations are possible around half normal frequency, as well as double normal frequency. The ratio need not be integral, in which case there is a continuous phase shift between the impressed frequency waves and the natural oscillation frequency waves, resulting in beats and pulsations, due to modulation by saturation.

4. Aside from oscillations, there is the possibility of the inversion of the neutral. The limiting values of resulting overvoltages in both of these cases, were not determined as they were unsafe for the equipment.

5. The explanation of the neutral shift or inversion is based on the fact that the combination of a saturable inductance and a constant capacitance has a multi-valued characteristic curve, with leading and lagging zones. If one leg operates in a leading zone, the other two in a lagging zone, the potential of the neutral is thrown outside of the triangle of line voltages. If all legs operate in leading but different zones of the characteristic curve, the neutral is again shifted but falls just inside the line voltage triangle. Losses and harmonics give a small quadrature shift to the neutral.

6. The persistence of oscillations is accounted for as follows: It is noted that the oscillations satisfy or approximate harmonic relationship to the impressed frequency. When the oscillation is at a higher than impressed frequency, the energy flow to it is similar to that from fundamental to higher harmonics as in static frequency multipliers. When the oscillation frequency is lower, such as one-half of the impressed frequency, the impressed frequency stands in the relationship of a higher harmonic of the oscillatory frequency, and the energy flow is reversed because the location of power and load are interchanged.

7. In potential transformer banks intended exclusively for relaying purposes, the secondary (or a tertiary) could be connected in delta (closed through a resistance just high enough to protect the bank from burnout in times of line-to-neutral short circuits), and then troubles due to either inversion or oscillation would be entirely eliminated. However, if the bank is to be used for metering as well as for relaying, currents circulating in the delta due to unbalanced line-to-neutral conditions in the system may adversely affect the metering accuracy of the outfit.

8. Loading the potential transformer from line to neutral also tends to stabilize the neutral and prevent inversion and oscillations; but, then of course, this will be a constant load at impressed frequency. It is believed that such a load should be at least of the same order of magnitude as the magnetizing volt-amperes of the transformers at normal operating voltage.

9. Operation of potential transformers approximately at half rated voltage is found to reduce greatly the chances of self-excited oscillations. The damping load, equal to the magnetizing volt-amperes at such low excitations, is very small and well worth while. Low density operation and protective load are also desirable from the standpoint of third harmonic phenomena.

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Discussion

For discussion of this paper see page 342.

Theory of Abnormal Line-to-Neutral Transformer Voltages

BY C. W. LAPIERRE*

Associate, A. I. E. E.

Synopsis.—*The abnormal voltages considered here are due to the occurrence of a large harmonic of fractional, even, or odd frequency in the line-to-neutral voltages of certain Y-connected transformer circuits. These phenomena were first described before the Institute in 1915, and other papers on the same subject have appeared since. The results of a detailed study of the instantaneous currents, voltages, and transformer flux densities in the type of circuit involved are presented in this paper. From this study the origin and characteristics of the abnormal voltages have been deduced.*

In general the voltages are found to be self-excited by successive alternate saturations in the transformer cores. The first saturation, which starts the phenomena, occurs within one cycle after the voltage is applied, due to the starting conditions of voltage and residual core density. When coupled with the system capacitance in the circuit of this discussion, each saturation establishes the condition for producing a succeeding and alternate saturation so that the process is self-continuing.

The frequency of the harmonic voltage generated by a succession of alternate saturations is dependent only upon the rate at which they occur. It is not surprising, therefore, that harmonics of unusual frequencies have been observed. The wide variations between the initial core conditions in the three transformers result in highly irregular saturations and correspondingly irregular voltages at the start. Such irregularities are of a transient nature. The occurrence of a sustained harmonic voltage is dependent upon the formation of proper multiple saturations which have been found to possess the necessary stabilizing properties. Such stabilizing multiple saturations only occur, for harmonic voltages having a frequency of either one-half, equal to, double, or triple the supply voltage frequency. Consequently, these are the only frequencies which can occur in the sustained condition.

The minor characteristics of the observed data, such as the wave shape, magnitude, and phase of the currents and voltages, beat frequencies, etc., have a definite place in the complete theory.

The present paper is one of a group of three presented at this time. The others, one by C. T. Weller and one by A. Boyajian and O. P. McCarty, cover different aspects of the same phenomena.

Introduction

THE abnormal voltages considered in this paper are due to the occurrence of a large harmonic of fractional, even, or odd frequency in the line-to-neutral voltages of certain Y-connected transformer circuits. These phenomena were first described before the Institute by L. N. Robinson[1] in 1915. Other papers on the same subject have appeared since, namely, a second one by Robinson[2] and those of King E. Gould[3] and Wilhelm Lampert.[4] These present results of a general nature from a wide variety of observations.

At this time three additional papers on the subject are being made available. C. T. Weller[5] is presenting specific data from an unusually complete series of field observations. A. Boyajian and O. P. McCarty[6] are presenting data from laboratory tests together with a descriptive interpretation of some phases of the phenomena. In the present paper a complete theory of the abnormal line-to-neutral voltages is developed which explains their fundamental origin and their detailed characteristics such as wave shape, magnitude and frequency.

The Abnormal Voltages

The circuit in which the abnormal voltages occur is represented by Fig. 1A. This circuit consists of a Y-connected bank of single-phase grounded-neutral transformers connected to an ungrounded source of supply. Three capacitors represent the capacitance of lines, bushings, etc., to ground. The three-phase line-to-line voltages are fixed by the source and may be assumed balanced and sinusoidal. The transformer or line-to-neutral voltages contain the large harmonics which have been observed. Thus the abnormal voltages are a peculiarity of only that portion of the circuit shown by Fig. 1A and do not involve the supply voltages which are independent and fixed. The transformer secondaries also contribute nothing toward

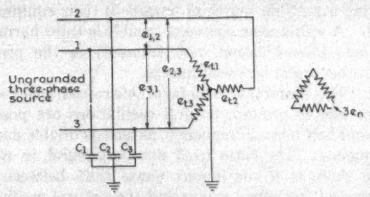

Fig. 1A—Representative Circuit

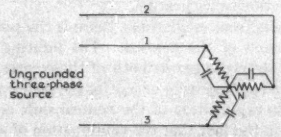

Fig. 1B—Simplified Representative Circuit

starting or maintaining the phenomena, as they have often been open-circuited while the abnormal voltages were being observed. Finally, the various observations have been made with a wide variety of Y-connected single-phase transformers, and in general no special importance can be attached to any particular type or rating.

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1. For references see Bibliography.

Presented at the Southern District Meeting No. 4, of the A. I. E. E., Louisville, Kentucky, November 19-22, 1930.

The large harmonics which occur are abnormal with respect to magnitude and frequency. The resulting transformer voltages may be as much as three or four times normal; consequently, the subject is of importance from the standpoint of insulation as well as operation. Some of the harmonic voltages occur at frequencies which are very unusual if not totally unex-

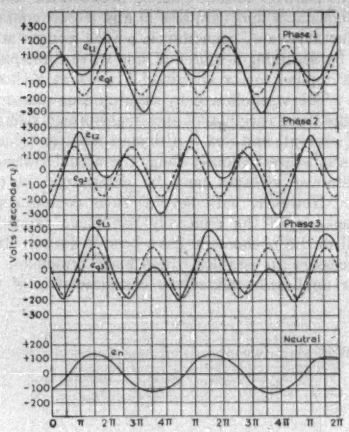

Fig. 2—Half Frequency Neutral Voltage

$e_l = e_n + e_g$

pected in ordinary alternating current circuits. In the transient state the voltages are usually very irregular and have no definite frequency. In the sustained condition, the frequency of the harmonic voltage has been found to be either one-half, equal to, double, or triple the supply voltage frequency. Two such frequencies do not occur simultaneously. While the harmonic voltages themselves are not purely sinusoidal, it is significant that other frequencies have not been observed to predominate in the sustained condition. In particular installations it was often found that the mere opening and closing of the supply switch would change the frequency of the predominant harmonic.

In Figs. 2, 3, 4, and 5 the above frequency conditions are illustrated by reproductions and derived curves from oscillograms obtained by C. T. Weller.[5] Other illustrations are given in his paper where harmonic voltages of one-half, equal to, double, and triple the supply voltage frequency have been designated phenomena *C*, *A*, *B*, and *D* respectively. A. Boyajian and O. P. McCarty[6] have segregated the phenomenon of equal to system frequency and designated it "neutral shift or inversion." All of the other frequencies which occur are termed by them "oscillation of the neutral." This classification arises from the fact that the voltage relations in the one phenomenon can be described by means of a conventional vector diagram, whereas the others cannot.

In addition to the characteristics discussed and illustrated above, a low frequency audible beat of from a fraction to one or two cycles per second is associated with the one-half and double frequency phenomena in three-phase circuits. Oscillograms with condensed time axis confirm these beats as variations in the currents and voltages within the transformers. As a result of the beats, the one-half and double frequency phenomena do not have exactly these relations to the supply frequency, but differ therefrom by a small amount.

Self Excitation of the Abnormal Voltages by Successive Alternate Saturations

For the purpose of describing the phenomena it has been desirable to classify the abnormal voltages according to whether they are transient or sustained and, in cases of the latter, according to frequency. Regardless of the various classifications, it has long been recognized or assumed that they have a common origin, namely, that all are the result of saturation* in the transformer cores. For the time being, therefore, it

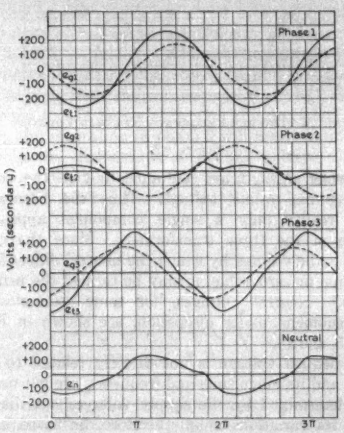

Fig. 3—Fundamental Frequency Neutral Voltage

$e_l = e_n + e_g$

is desirable to forget the various classifications and to investigate the circuit of Fig. 1A with a view toward determining its properties under conditions of transformer saturation, rather than for the purpose of explaining a particular set of voltmeter readings.

*By saturation is meant the operation of a transformer at flux densities on or above the knee of the magnetization curve. The term thereby includes quite a wide range of possible densities, each being associated with a current much larger than the normal excitation current of the transformer.

Of primary importance in Fig. 1A are the relations between the various voltages. The abnormal conditions which occur are the result of harmonics in the voltages across the individual transformers. These harmonics have zero phase sequence and do not appear in the line-to-line voltages which are fixed by the source. In other words the three harmonics are identical in phase and magnitude and must cancel when any two

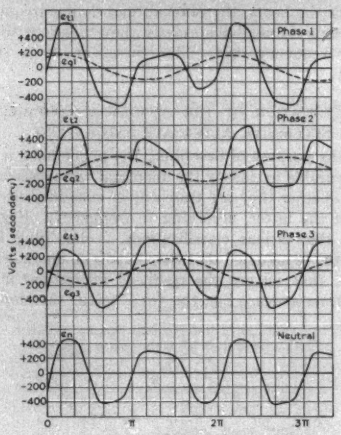

FIG. 4—DOUBLE FREQUENCY NEUTRAL VOLTAGE

$e_t = e_n + e_g$

transformer voltages are combined to give the corresponding line-to-line voltage. Thus the harmonics are in reality only a single component appearing in each of the three transformers. Since the component is the voltage by which the neutral differs from its normal or zero value, it may be termed the neutral voltage.* Neutral voltages, for some examples of the phenomena under discussion, are shown in Figs. 2, 3, 4, 5, and 13.

The neutral voltage is not directly related to the source, and its magnitude and frequency are not necessarily determined by the supply voltages. A discussion of its variations must, therefore, be based upon instantaneous values, since average or effective values have no physical significance in themselves but are only applicable to voltages of known wave shape. Then too, the transformers operate over such a wide range of flux densities that their instantaneous inductance values may lie any where within a range corresponding roughly to the ratio, 5,000 to 1. It is difficult to conceive of an average or effective value which could possibly represent such a variable inductance.

Consequently, the theory of this paper is based upon the results of a detailed study of instantaneous voltages, currents, and flux densities in the circuit of Fig. 1A. The study consisted of a series of computations making use of the known relations existing between the fundamental constants and variables in any circuit containing inductance and capacitance. Each computation started by assuming a source of sine wave three-phase voltages applied to the circuit of Fig. 1A at a chosen instant. This was followed by simultaneous consideration of the three variables of each transformer, namely, voltage, current, and flux. Although the step-by-step computations are tediously long, the actual transient and sustained voltages may be determined in this manner. However, with several such computations as a background the relatively few basic principles involved in the phenomena have been deduced. It is only necessary to establish these basic principles, for by their application all of the characteristics of the

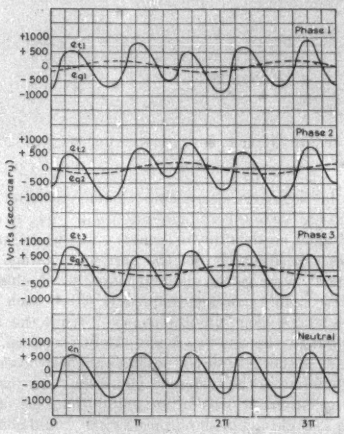

FIG. 5—TRIPLE-FREQUENCY NEUTRAL VOLTAGE

$e_t = e_g + e_n$

The voltages shown have not quite reached the steady state

phenomena may be adequately explained. In the paragraphs immediately following, the theory is outlined and its basic principles introduced. A more complete discussion of the various factors involved is contained in the later sections of the paper.

Although Fig. 1A represents the circuits in which the phenomena have been observed to occur, the circuit of Fig. 6A may be expected to produce the same transformer voltages and is identical in that respect to Fig. 1A under certain conditions. (See Appendix). In

*The neutral or harmonic voltage at any instant may be calculated by adding the instantaneous line-to-neutral voltages and dividing by three. The normal voltage components always add to zero leaving three times the neutral voltage. The addition may be physically accomplished by connecting the secondaries as shown in Fig. 1A and measuring three times the neutral voltage across the broken delta.

Fig. 6A the capacitor voltage is equivalent to the neutral voltage defined above, and may be visualized directly without the necessity of separating it from the transformer voltages as is the case in Fig. 1A. It is the variation in the voltages across this capacitor which introduces the harmonics in the transformer voltages. Furthermore, in Fig. 6A, the capacitor or neutral voltage is the only voltage that must be determined. All of the others are related to it and the

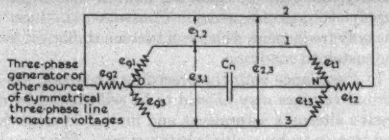

FIG. 6A—EQUIVALENT CIRCUIT

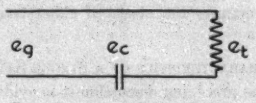

FIG. 6B—SINGLE-PHASE CIRCUIT

known supply voltages by the simple algebraic formulas,

$$\left.\begin{aligned} e_{t1} &= e_n + e_{g1} \\ e_{t2} &= e_n + e_{g2} \\ e_{t3} &= e_n + e_{g3} \end{aligned}\right\} \qquad (1)$$

The validity of these equations is evident when the voltages in each of the closed circuits are traced out.

Thus by adopting a particular type of supply, it is possible to reduce the problem to a consideration of the voltage across a single capacitor. While it is theoretically immaterial which figure is used in the following discussion, the processes to be described will have far greater practical significance if they are based upon Fig. 6A.

The Origin of the Abnormal Voltages. Saturation during the first cycle after connecting a transformer to a source of voltage is a common occurrence. Such saturations are the result of initial conditions of voltage and residual core densities. In transformers connected directly across a single-phase supply, the initial saturation results in the familiar starting currents represented by Fig. 7. This curve actually consists of a succession of saturations, all in the same direction. The magnitude of the peaks gradually diminish until normal densities are not exceeded. Each saturation is an individual event consisting of a rise of the current and flux along the hysteresis loop of Fig. 10 and their subsequent decay to normal values. The large current accompanying a single saturation is unidirectional; it cannot become a large reversed current until after the core density reaches saturation values in the opposite direction.

Referring to Fig. 6A (or Fig. 1A), assume the chance occurrence of a single saturation a short time after the supply switch is closed. The resulting unidirectional current must flow into the system capacitance since the unsaturated transformers will not pass an appreciable part of this current. The current flowing into the capacitance builds up a unidirectional capacitor voltage. This voltage continues to rise until the cycle of saturation is completed, at which time the transformer is restored to normal densities. With all of the transformers at normal densities, their impedances are too high to permit the capacitor to be discharged. Consequently, the capacitance voltage will be maintained practically constant for the time being. This action of a single saturation in the circuit is not unlike that of a valve. It injects a large current into the capacitance and then closes the entrance path to prevent its discharge. The magnitudes of the various changes resulting from a single saturation are dependent to a large extent upon the circuit constants, such as capacitance, normal transformer flux densities, size of core, number of turns, etc. It is not necessary to consider these factors at the outset, as the fundamental nature of the phenomena may be explained on the assumption that a sufficiently large capacitance is present and that saturations do occur.

The voltage changes resulting from the saturating process may be readily visualized with the aid of Fig. 8. The dashed line curves in this figure will be recognized as the three normal line-to-neutral transformer voltages. Assume the switch to be closed at t_0 and that due to the initial conditions the transformer of phase 2 saturates at time t_1. In accordance with the preceding discussion, the saturation of phase 2 will result in an abrupt rise in the capacitance or neutral voltage. For the purpose of illustration this rise may be assumed as vertical. After the single saturation is completed, the

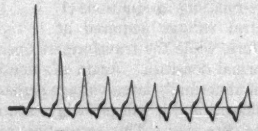

FIG. 7—TRANSFORMER STARTING CURRENT

high impedance of the transformers prevents the capacitance from discharging and the neutral voltage remains constant for the time being. However, the voltage which the capacitance has acquired adds to the voltage of phase 1 to result in an abnormally large voltage across its transformer. The large voltage, which in the figure is about twice normal, will saturate transformer No. 1 at about t_2. The second saturation is in the opposite direction to the first and consequently results in a reversal of the neutral voltage.

In case it is not clear that the second saturation occurring at t_2 would reverse the capacitor voltage, reference is made to the hysteresis loop of Fig. 10. Once a saturation starts, it will increase in intensity

until the transformer voltage reaches zero; for as long as there is some voltage, the flux continues to increase. After the flux reaches a maximum, at the instant the transformer voltage reaches zero, it must decline to its residual value. This partial collapse of the magnetic field induces a transformer voltage in the opposite direction. Consequently in Fig. 8, whenever a saturation is assumed to occur, the affected transformer voltage should be reversed by an appreciable amount

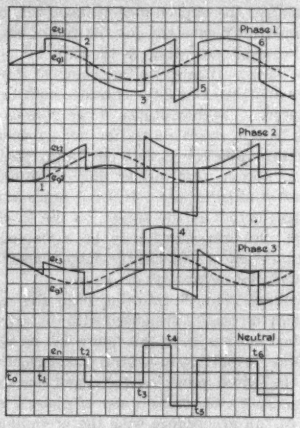

FIG. 8—HYPOTHETICAL VOLTAGES RESULTING FROM AN ARBITRARY SUCCESSION OF SATURATIONS

$e_t = e_n + e_g$

and the neutral and other voltages determined accordingly by the relations in equations (1).

The neutral voltage acquired at t_2 remains constant as before, while the transformers operate in the range of normal densities. Again the neutral voltage results in an abnormally large voltage across the transformer of phase 1 and it will probably saturate at t_3 in the opposite direction. This saturation reverses the neutral voltage a third time and the process may be continued for the fourth, fifth, and sixth saturations and so on indefinitely.

In essentials, Fig. 8 illustrates the origin of the abnormal voltages of this discussion, although the actual wave shapes obtained are not rectilinear. It is evident that the frequency of the voltages is dependent only upon the rate at which the saturations occur and it is not surprising that unusual frequencies have been observed. At the start when there are wide differences between the core conditions of the three transformers, the saturations occur in a more or less hit or miss fashion and the resulting neutral voltages are highly irregular. As the initial differences in core conditions begin to be smoothed out, the time of saturation becomes more and more dependent upon how the neutral voltages combine with the supply voltages. In order for the voltages to reach a sustained condition it is necessary that some sort of stabilizing process be present which will maintain the voltages within a stable zone. This requirement is satisfied by the formation of multiple saturations or the saturation of two or more transformers in parallel and in the same direction.* The proper stabilizing multiple saturations only occur for a limited number of neutral voltage frequencies, namely, one-half of, equal to, double, and triple the supply voltage frequency. Consequently these are the only frequencies which can become stable and reach the sustained condition.

In accordance with the preceding discussion the abnormal voltages may be said to be self-excited by successive alternate saturations and limited in magnitude and frequency by multiple saturations. The complete theory based upon these two principles explains in an adequate manner the observed characteristics of the phenomena.

THE CHARACTERISTICS OF A SINGLE SATURATION

From the preceding discussion it is evident that the phenomena originate as a result of a succession of individual saturations. It is desirable, therefore, to investigate in detail just what happens in Fig. 6A when a transformer saturates. The wave shape and magnitude of the resulting neutral voltages are of major importance.

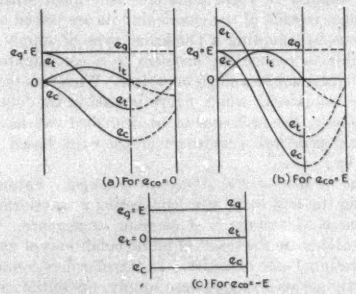

FIG. 9—STARTING VOLTAGES—CONSTANT INDUCTANCE AND CAPACITANCE IN SERIES

$e_t = e_g + e_c$

Wave Shape of the Neutral Voltage. Changes in the voltage across a capacitor in any circuit are dependent upon current and the time during which the current flows. The capacitances associated with the abnormal voltages in Fig. 6A are relatively large and require considerable currents to alter their charge in time inter-

*Direction of saturation as used here has a very definite meaning when referred to either Fig. 1A or Fig. 6A. Two or more saturations are in the same direction when their respective currents flow in the same direction at the neutral, that is, either out or in.

vals of the same order as the period of the supply voltage. At low densities the transformers possess high inductance, and the resulting small currents are insufficient to alter appreciably the charge upon the capacitor over considerable intervals of time. For that reason the normal operation of the transformers at low densities produces no appreciable effect upon the capacitor voltage. On the other hand, at high densities the inductance decreases to a small fraction of its normal value while the currents are often hundreds of times greater. Consequently, in deriving the effect of saturation in a single transformer the effect of

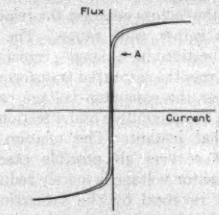

FIG. 10—HYSTERESIS LOOP

the two unsaturated transformers may be neglected. For such a case it is only necessary to consider a single phase portion of Fig. 6A such as is shown in Fig. 6B.

Fig. 6B is a relatively simple circuit and no particular difficulty is encountered in deriving the fundamental nature of the changes which occur even though the inductance may be variable. However, in order to connect the phenomena with more familiar circuits, assume first that the inductance is constant and apply a unidirectional voltage to the circuit. The resulting voltages and currents will oscillate as shown in Fig. 9a. The damping due to resistance may be neglected since the first half cycle or so is of sole interest here. In Fig. 9b curves are shown of voltages and currents for an initial capacitor voltage in one direction, and in Fig. 9c for an initial capacitor voltage in the other direction. In both of the latter figures, the initial capacitor voltage is equal in magnitude to the applied voltage.

The same surges of energy occur for a transformer and capacitance in series as for a constant inductance and capacitance in series. The wave forms of the resulting currents and voltages are different for the two cases due to the hysteresis loop relation between the transformer current and flux. However, just as constant inductances produce characteristic curves, namely, sine curves, due to the linear relation between current and flux, so also do iron core inductances produce characteristic curves as a result of the hysteresis loop relation between current and flux. Once established, the iron core inductance curves may be readily identified. In this manner voltage variations in an iron core inductive circuit can be described by reference to characteristic curves which are just as definite as the familiar hysteresis loop. Such curves are established in Figs. 11 and 12 which are the iron core inductance or transformer curves, from the circuit of Fig. 6B, corresponding to the constant inductance curves, Figs. 9a and b. Figs. 11 and 12 are based upon computations for transformers and capacitance approximating those used in some of the tests described by C. T. Weller.[5] The chief characteristics of these curves are explained on the basis of the hysteresis loop of Fig. 10. The curves were started at the point A on the loop in order to reach saturation more quickly. The small currents associated with the initial flux produce a negligible change in the capacitor voltage at first. However, the flux soon rises along the knee of the curve and the currents become abruptly larger resulting in an abrupt rise of the capacitor voltage. The flux continues to rise until the transformer voltage reaches zero. At this point there is no voltage tending to maintain the high currents in the transformer which are associated with the peak flux so that the magnetic field tends to collapse. The partial collapse of the magnetic field induces a reversed transformer voltage which may carry the capacitor voltage to a value far in excess of the applied voltage. Especially is this true when the proper initial capacitor voltage is present as in Fig. 12. The decay in the magnetic field continues until the current reaches zero. The density is now restored to a region of high permeability and consequently the transformer possesses a high inductance. The currents which then flow have very little effect upon the capacitor voltage so that it tends to be maintained at the value acquired during saturation.

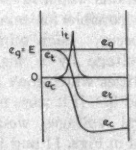

FIG. 11—SATURATION VOLTAGES FOR $e_{co} = 0$

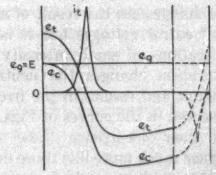

FIG. 12—SATURATION VOLTAGES FOR $e_{co} = E$

The negative transformer voltage in Fig. 12 resulting from the first saturation is maintained by the continued decay in flux through zero and down along the negative portion of the hysteresis loop. Eventually saturation in the negative direction will occur and the cycle of events is repeated as illustrated by the dashed line continuation of the curves. Although Figs. 11 and 12 have been derived with unidirectional applied voltage, their characteristic shape is also applicable to alternating applied voltage, since the transformer passes through the complete cycle of saturation and is restored to low density in a short time relative to variations in the applied voltage. In other words the alternating supply volt-

age would not vary appreciably while the phenomena occurs and may therefore be considered as unidirectional. The application of the characteristic curves to alternating supply voltages is shown in Fig. 14.

The abnormal voltages of this discussion are the result of a succession of alternate saturations each occurring in the manner just described. The remarkable similarity between the actual neutral voltage curve of Fig. 13 as obtained in the field and the computed curves of Figs. 11 and 12 is almost conclusive proof of the preceding statement. However, this close similarity is

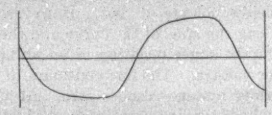

FIG. 13—FUNDAMENTAL FREQUENCY NEUTRAL VOLTAGE CURVE

not usually present although the neutral voltages practically always show the chief characteristics of the derived curves, namely, an abrupt reversal and rise of voltage followed by a slow droop toward zero. These characteristics are present in most of the neutral voltages of Figs. 2, 3, 4, and 5. The other variations present give insight into the processes whereby the voltages become regular and sustained as discussed in the following sections.

The Magnitude of the Neutral Voltage. Heretofore the discussion has proceeded upon the assumption that each saturation results in a reversal of the neutral voltage. Such is not always the case. It is true that the large voltage changes are the result of saturations which reverse the neutral voltages but it is also possible for minor saturations of small intensity to occur. Each minor saturation changes the neutral voltage by a slight amount and results in the irregularities which may be observed in the curves of Figs. 2 and 3. If it were not for their occurrence these neutral voltages would appear much more like those derived in Figs. 11 and 12.

The conditions which determine the intensity of a saturation and the magnitude of the neutral voltage resulting from it may be found from the instantaneous energy relations in the circuit. The various stages through which the voltages and currents pass during the saturation of a single transformer are investigated in detail in the Appendix. As a result of the deductions derived there, it has been possible to set up quantitative relations between the initial voltages and the final capacitor voltage. The final capacitor voltage has been found to depend upon its initial value just before saturation, the magnitude and direction of the supply voltage and the peak flux density which is reached. These factors are related as follows:

$$e_{cf}\,|\,e_{cf}\,| = -\,e_{go}\,|\,e_{go}\,| - P_w\,\{\,e_{co}\,|\,e_{co}\,| + e_{go}\,|\,e_{go}\,|\,\} \tag{28}$$

Where:

e_{cf} = the final capacitor voltage.

e_{go} = the supply voltage at the instant the transformer voltage crosses zero.

e_{co} = the initial capacitor voltage just before saturation.

P_w = the ratio of the energy released to the energy stored in the transformer core during saturation.

The vertical bars enclose absolute values of the voltages. It is necessary to write the products in this form instead of as squares of the algebraic symbols in order to retain the proper signs. Had the terms been squared all of the signs would have been positive and, therefore, not generally valid.

In general the factors entering the relation may vary throughout a rather wide range. The only definite result of saturation in a single transformer is that the voltage across the saturated transformer must reach zero. Whether the capacitor voltage reverses or not depends upon the magnitude and direction of the supply voltage at that instant. The relation expressed in Equation (28) covers all possible cases. However, when the capacitor voltage is merely reduced in magnitude and not reversed by the saturation of a single transformer, the bias voltage is still maintained so that one of the other transformers will saturate later in the same direction and accomplish the reversal. In some cases as many as four single saturations occur in the same direction before the capacitor voltage reaches its maximum reversed value. The general term *saturation* as used here includes the complete group of single transformer saturations occurring in one direction at a given time. Although the minor saturations are more or less incidental and do not directly result in major

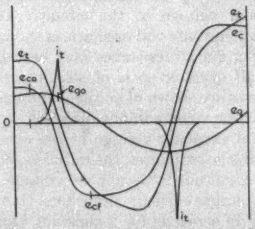

FIG. 14—TWO ALTERNATE SATURATIONS—A-C. SUPPLY

$e_t = e_g + e_c$

voltage fluctuations they function to stabilize the voltages in the sustained condition as explained below.

MULTIPLE SATURATIONS AND SUSTAINED VOLTAGES

Voltage Limitation and Stability. If the neutral voltages arising from a succession of alternate saturations are to become stable at a particular value, some process must be present to maintain and limit the voltages within the stable range. This process is found to exist as a result of multiple saturations. As ex-

plained before, a complete saturation resulting in a reversal of the neutral voltage may involve one or more transformers. Each single transformer saturation contributed a definite part of the total energy change. Multiple saturation refers to the condition where at least two such single transformer saturations occur in the same direction* to accomplish the neutral voltage reversal.

In general each succeeding major and alternate saturation tends to build up the alternating capacitor voltage just as indicated by Fig. 14 and by the transition from Fig. 11 to Fig. 12. As a matter of fact it has already been indicated that all sorts of saturations occur, resulting in voltage changes of from a slight reduction to a complete reversal. But the major saturations are the result of abnormally large transformer voltages which only occur when the neutral and supply voltages are in the same direction and add directly. This is the condition which produces the

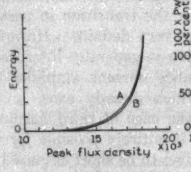

FIG. 15—MAGNETIC ENERGY OF OUR $C\ m^3$ TRANSFORMER STEEL
A. Stored or absorbed
B. Released

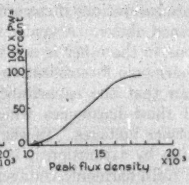

FIG. 16—RELEASED ENERGY IN PER CENT OF STORED ENERGY

largest reversed voltage and, therefore, the largest build up effect. As the voltages continue to rise, other transformers begin to reach saturation densities in the same direction. If the saturations fall into the proper phase with respect to the line-to-neutral supply voltages, a position will be reached in which a minor saturation occurs just before the main saturation. Such a condition is illustrated in Fig. 18 where the saturation of transformer 1 precedes the main saturation occurring in transformer 2. In accordance with the quantitative relations discussed in the preceding section and in the Appendix, the preliminary saturation of transformer 1 will reduce the initial voltage for the main saturation which in turn reduces the magnitude of the resulting reversed voltage. Thus when these preliminary saturations start to occur, the neutral voltages soon reach a limiting value where the preliminary saturation reduces the voltage by the amount of the build up effect. When each saturation occurs as a multiple saturation in this manner the condition is a stable one because a reduction of the neutral voltages, for any reason, makes the preliminary saturations disappear and the succeeding saturations will again tend to build up the voltages. On the other hand, if the voltages increase above the stable value the preliminary saturations will become more pronounced and cause their reduction.

This process is plainly evident in the neutral voltage curve of Fig. 3. Here a preliminary saturation reduces the voltage to a fairly small value, after which the main saturation occurs and results in the voltage reversal. In the case of Fig. 2 the reduction occurs in several steps as three or four saturations occur in

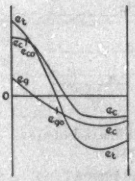

FIG. 17—SATURATION—REDUCTION OF REVERSED VOLTAGE

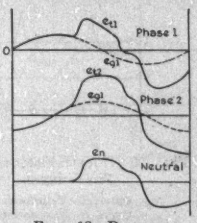

FIG. 18—DOUBLE SATURATION

multiple for each voltage reversal. All of the abnormal voltages observed possess these multiple saturations after they reach the stable state. Their effect is not always evident in the voltage curves since under certain circuit conditions the auxiliary stabilizing saturations occur too close to the main saturation and the effect of the first or stabilizing saturation merges into the effect of the second. This is the case in Fig. 13. The small auxiliary saturations are always evident in the current oscillograms, however, as shown in Figs. 19 and 20.

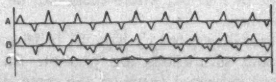

FIG. 19—FUNDAMENTAL FREQUENCY NEUTRAL VOLTAGE PHENOMENON (*C. D.* 120409 *B*)
Curve *A*—Transformer current phase 1
Curve *B*—Neutral current
Curve *C*—Transformer current phase 2

In general, therefore, the abnormal voltages tend to be cumulative but reach a limiting value when they give rise to the auxiliary stabilizing saturations just described.

Possible Sustained Neutral Voltage Frequencies. As a result of the stabilizing process the successive alternate saturations can only occur in the sustained condition at those frequencies which give rise to the proper stabilizing saturations. It should be emphasized, however, that the fundamental process involved in the generation of these voltages does not possess a definite

*Loc cit.

frequency characteristic. The saturations and resulting voltage reversals occur whenever the transformers reach saturation densities, and the neutral voltage frequency is dependent solely upon their rate. Consequently it is not to be expected that the possible sustained frequencies will necessarily be familiar multiples of the supply voltage frequency. In Figs. 21 to 26 the flux curves which result from the normal line-to-neutral voltage components are combined with the flux components resulting from various trial frequency

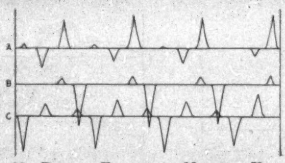

FIG. 20—DOUBLE FREQUENCY NEUTRAL VOLTAGE PHENOMENON (C. D. 120394)

Curve *A*—Transformer current phase 1
Curve *B*—Transformer current phase 2
Curve *C*—Transformer current phase 3 reversed with respect to *A* and *B*

neutral voltages in order to determine which possess the proper stabilizing conditions. The necessary conditions for a minor saturation to be effective in stabilizing the voltage are that it must precede the main saturation and that it will increase in importance as the neutral voltage increases, and decrease in importance as the neutral voltage decreases. An increase in neutral voltage causes an earlier rise in the flux curve so that these conditions are not difficult to recognize. Although sine curves have been used in these trials the flux curves resulting from the actual voltages are not materially different. The possible stabilizing saturations are numbered in the order of their occurrence except in Fig. 21, where there can be no doubt of their presence.

In each case illustrated by Figs. 21 to 26 some possible stabilizing saturations are present. In Fig. 23 both of those indicated occur on the same side of the cycle. Since the limitation must affect both halves of the wave such a condition is hardly stable. In Fig. 24 only two stabilizing saturations occur for the two cycles of neutral voltage. As one is positive and the other negative, stability is possible even though two peaks out of four are not affected. The conditions surrounding the actual formation of the double frequency voltages are rather critical. This fact fits in with the lack of complete stabilizing saturations. In Fig. 25 the stabilizing saturations continually diminish in magnitude, and the resulting voltages would be forced up to the next higher frequency. The six saturations per cycle illustrated by Fig. 26 have complete stabilizing saturations and should result in very stable voltages.

More than six saturations per cycle would require at least one transformer to saturate twice in the same direction for one half cycle. For uniform neutral voltages this is impossible. The second of the two required saturations would always occur too early and at a lower voltage than the first.

An inspection of Figs. 21 and 22 and the preceding discussion show that only 1, 2, 4, and 6 saturations per cycle possess the proper stabilizing saturations. Trials with other frequencies than those of Figs. 21 to 26, give strong indication that these include the only possible conditions although an infinite number of trials could be made. It is safe to conclude, therefore, that the frequency of the sustained neutral voltages obtained as the result of a stable succession of alternate saturations must be either one-half, equal to, double, or triple the supply frequency, since each complete saturation represents one reversal of the voltage.

In connection with the triple frequency voltages, it is necessary to distinguish here between what is ordinarily known as third harmonic intensification and triple frequency voltages which are excited by successive alternate saturations in accordance with the principles established above. Actually the transition in passing from one to the other is not very definite. However, third harmonic intensification is generally interpreted to mean that the capacitance current amplifies the normal third harmonics which already exist in the transformer voltages. In the case of third harmonic excitation by saturation it is entirely possible for the normal third harmonic voltages to be short circuited by the capacitance as has actually been observed.[8] With the proper conditions a closing of the switch in the same

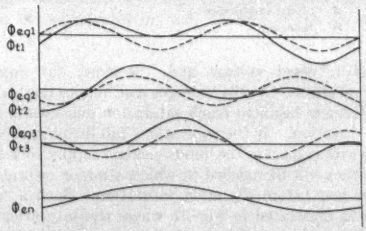

FIG. 21—ONE SATURATION PER CYCLE—MULTIPLE STABILIZING PEAKS

ϕ_t = total flux
ϕ_{eq} = supply voltage flux component
ϕ_{en} = neutral voltage flux component

circuit may give rise to abnormal triple frequency voltages which are self excited by saturation. In the case of third harmonic intensification the sustained voltages would occur as sustained phenomena after every switch closing, while in the case of triple frequency voltages from saturation they may occur only occasionally. As a matter of fact the terms used give sufficient distinction. Third harmonic intensification indicates an increase in third harmonic voltages which already exist, whereas the self excitation of third

harmonics by successive alternate saturations indicates that they need not necessarily be present in normal operation but may occur at times as a result of an abnormal condition, namely, transformer saturation.

Low Frequency Beats. With respect to the positive and negative flux curves the phenomena may be classified according to whether the neutral voltages result in symmetrical or unsymmetrical flux densities within the transformers. Figs. 21, 22, 24, and 26 permit the identification of each of the frequency conditions according to this classification. The one and four saturations per cycle phenomena obviously produce unsymmetrical densities. In three-phase circuits a certain degree of instability is an inherent accompaniment of the unsymmetrical densities and results in the low frequency variations in the flux densities and currents of the transformers which have been described.[1,4,5,6]

The instability occurs in the mean neutral voltage. For symmetrical densities a displacement of the mean neutral voltage increases the transformer losses in a way for which no energy is available and symmetry is

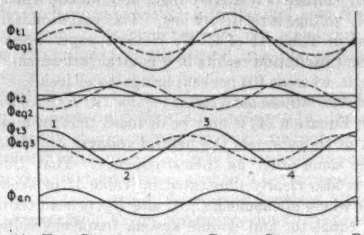

FIG. 22—TWO SATURATIONS PER CYCLE—STABILIZING PEAKS NUMBERED IN THE ORDER OF OCCURRENCE

restored just as the displaced single-phase transformer starting currents become symmetrical after a time. On the other hand the one-half and double frequency phenomena result directly in unsymmetrical densities so that the means exist inherently for their maintenance. The unsymmetrical densities, however, tend toward unequal positive and negative neutral currents since the shape of the magnetization curve precludes the possibility of the transformers operating at widely different densities and at the same time result in equal currents. The excess current occurring in a particular direction builds up a mean voltage on the capacitor of Fig. 6A. This voltage continues to rise until it builds up the flux density of the transformers to result in equal currents. The mean voltage then stops rising but the flux density resulting from it keeps on increasing. Unequal currents in the opposite direction result, which reduce and reverse the mean capacitor voltage until the process is repeated on the other side of zero. The continual surging between the mean capacitor voltage and the average density of all the transformers considered together gives rise to the beat phenomena. The beat phenomena are, therefore, inherent characteristics of the one-half and double frequency phenomena in the three-phase transformer circuit. The densities associated with both frequency conditions are sufficiently high to set up audible stresses within the transformers. The variation in the magnitude of the stresses produces the audible beats.

As a result of the beats the voltages due to the satura-

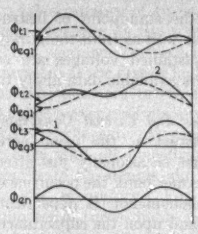

FIG. 23—THREE SATURATIONS PER CYCLE

Stabilizing peaks numbered in order of occurrence

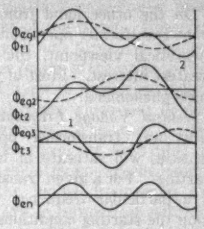

FIG. 24—FOUR SATURATIONS PER CYCLE

Stabilizing peaks numbered in order of occurrence

tions never occur at the exact rate of one-half or double the supply voltage frequency but differ therefrom by a small amount.

SOME APPLICATIONS OF THE THEORY TO THE OBSERVED VOLTAGES

Perhaps the most abundant proof of the foregoing explanation or theory of the abnormal voltages is afforded by a study of the irregular transients which

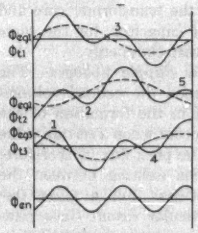

FIG. 25—FIVE SATURATIONS PER CYCLE

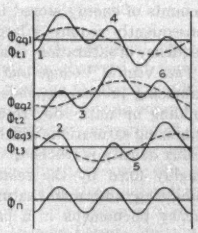

FIG. 26—SIX SATURATIONS PER CYCLE

have been recorded. These voltages are so irregular in magnitude and so indefinite in period that they must arise from such a chance process as the occurrence of successive saturations. The transients are important because they occur in every Y-connected transformer circuit. When delta-connected windings short circuit the neutral voltages they are of course suppressed in magnitude but the saturations still occur at the start

and the resulting circulating currents in the delta produce neutral voltages equal to their impedance drop. Also there must be quite a few Y-connected transformer circuits, of the type considered here, in which the system capacitances are not of the proper value to result in sustained voltages. Such circuits are subjected to abnormal voltages which go unnoticed due to their transient nature, but which may result in insulation failure.

On the other hand from the standpoint of instrument transformer operation as well as from the purely theoretical viewpoint, the sustained voltages are of greatest interest. Most of the available data apply to these phenomena.

Neutral Voltage Frequencies and Circuit Constants. Most any transformer operated at the usual core densities will saturate at least occasionally just after starting. For a given transformer bank the occurrence of a particular frequency phenomenon is dependent upon the starting conditions and upon the capacitance of the circuit. The higher values of capacitance are associated with the lower frequency neutral voltages and each frequency condition has a range of capacitance within which the phenomenon occurs. The ranges for the different frequencies overlap so that a given value of capacitance may result in either one of two frequency conditions, depending upon which of the two corresponding stabilizing conditions the voltages happen to fall into during the transient state.

For a particular system or value of capacitance the larger transformers are associated with the higher frequencies, assuming that they operate on the same voltage. A general relation between the transformer size and capacitance can be arrived at from the standpoint of energy. The larger transformers have greater amounts of energy stored in the transformer core and consequently require a larger capacitor, for the same frequency of saturation, to absorb the energy.

The Neutral Voltage and the Supply Voltages. The stabilizing process depends upon the neutral voltages building up until they result in the formation of the stabilizing saturations which occur when a certain value of flux density is reached. The peak flux is, therefore, roughly fixed by the relations existing between the stabilizing and main saturations. For a given frequency phenomena in a particular circuit these relations are approximately constant so that limitation occurs at a roughly constant peak flux. These relations can be expressed best in mathematical form. If ϕ_p represents the value of peak flux at which stabilization occurs and ϕ_e represents the normal peak flux the neutral of capacitor voltage may be thought of as due to the variation in the difference between these two flux values. Thus

$$\phi_c = \phi_p - \phi_e \qquad (2)$$

and the r. m. s. neutral voltage may be roughly expressed as

$$E_c = k f_c \phi_e \qquad (3)$$

For a given frequency, f_c, as the supply voltage increases, *i. e.*, if ϕ_e increases, ϕ_c and E_c must decrease. Thus over the range of supply voltage for which a particular phenomenon is sustained, the neutral voltage will tend to vary in an inverse relation to the supply voltage. This fact is excellently illustrated by the data in Table IV of the companion paper by A. Boyajian and O. P. McCarty[4] and also in some of the tabulations of C. T. Weller.[5]

For neutral voltages of the same as system frequency C. T. Weller has pointed out that the neutral lies sometimes within and sometimes outside of the delta voltage triangle. In the first case the neutral voltage is not quite sufficient to offset the normal line-to-neutral voltage and the neutral lies inside. In the second case the neutral voltage is greater than the normal line-to-neutral voltages and the neutral lies outside the delta voltage triangle. In accordance with Equations (1) and (2) the mere raising of the supply voltage will reduce the neutral voltage and bring the neutral inside the triangle. Thus the neutral lies inside when the supply voltage is relatively high, and outside when the supply voltage is relatively low. This variation is illustrated in Table II of C. T. Weller's paper where 94 per cent excitation results in a neutral just outside the triangle, whereas 104 per cent brings it well inside.

Relation between the Neutral Voltage and Its Frequency. From Equation (4) it may be deduced that for a given bank of transformers the neutral voltages will increase in the same order as their frequency. That this is true is also clearly illustrated in Table II of Weller's paper where phenomena C, A, and B represent the one-half, equal to, and double system frequency voltages respectively.

Equation (4) cannot always be applied so successfully to phenomena of different frequencies, due to the variation in the peak densities, or ϕ_p, at which the stabilizing saturations form. In general ϕ_p will be much lower for the equal to system frequency phenomenon than for either the one-half or double frequency phenomena.

Characteristics of Voltages Having System Frequency. In the two companion papers this phenomenon is termed neutral shift or phenomenon A. In both papers the voltages are represented by a conventional vector diagram.

The successive saturation theory of these voltages requires that the neutral voltage curve fall between the voltage curves of the transformers giving the main and stabilizing saturations. As a result the neutral voltage must always be about opposite to the third transformer voltage which lies half way between the two other voltages. This relation is illustrated in Fig. 22. The neutral, consequently, always falls in the general direction of one apex of the delta voltage triangle in a vector diagram.

As discussed elsewhere, variations in capacitance vary the phase of the neutral voltages so that the phase

relations are not always exactly the same. However, the preliminary saturation tends to be smaller and must occur in the transformer occupying the leading phase with respect to the main saturating transformer. Since the transformer in the leading phase thereby operates at a slightly lower density, its voltage should ordinarily be slightly lower than that of the main saturating transformer. In connection with the data on phenomenon *A* in Tables II and III (C. T. Weller[4]) it will be noted that the smaller of the two large voltages does occur in the transformer occupying the leading phase.

The Phase of the Stabilizing Saturations as Dependent Upon Capacitance. Within the range of capacitance which will sustain a given frequency a decrease in capacitance results in a more rapid rise of the neutral voltage. This causes the stabilizing saturations to occur earlier or in other words lengthens the time interval between the stabilizing and main saturations. It is to be concluded that the neutral voltages of Fig. 3 were obtained with relatively small capacitances whereas the neutral voltages of Fig. 13 were obtained with a relatively large capacitance. In the first case there is an appreciable interval between the stabilizing and main saturations; in the second, the two practically coincide.

Peak Flux Density and Capacitance. In accordance with the relations derived in the Appendix, the peak flux density during saturation tends to increase with increase in capacitance. It is to be expected in general, therefore, that as the capacitance is increased the neutral voltage would show some increase although not in proportion to the capacitance due to the relations involved in equations (3) and (4). This variation is illustrated by Table IV of the companion paper by Boyajian and McCarty.

Relation of Beat Frequency to Capacitance and Supply Voltage. The increase in flux density accompanying the rise in capacitance increases the asymmetries of the currents in cases of one-half and double frequency neutral voltages. The variations of the mean neutral voltage therefore occur at an increased rate since the excess currents are accumulated faster. This variation of the beat frequency with capacitance is illustrated by Table III of the companion paper by Boyajian and McCarty. Also illustrated in that table is the increase in beat frequency with decrease in supply voltage. The decrease in supply voltage causes larger neutral voltages, also giving rise to greater asymmetry in the currents and consequently more rapid beats.

Basis for Predicting the Occurrence of Any Given Phenomena. A complete discussion of this subject obviously includes a large number of detailed considerations. A basis for considering any particular set of transformers rests in the stabilizing process. This depends upon the flux density reaching a given value, namely, that at which the stabilizing saturations occur. Thus regardless of the size of core, number of turns, etc., for a given frequency of successive saturations the flux-density-time curves must have a fairly definite shape. With this as a basis the occurrence of a particular frequency neutral voltage can be predicted from the total capacitance and the transformer structure specifications.

Capacitance within the transformer must be included in determining the total capacitance. To maintain a given phenomenon with a particular size of core and a variable number of turns, the total capacitance must vary inversely as the square of the turns. However, the capacitance between layers and turns of a winding increases as the first or higher power of the turns so that as the turns are increased a point is reached where further increase offsets any decrease which may be possible in the external capacitance. This may be the reason why all investigators have not been able to duplicate the double and triple frequency phenomena in the laboratory. For the higher frequencies the number of turns should be low in comparison with the size of core. In general this would mean transformers with a fair load capacity instead of small potential transformers and the like.

Prevention of the Phenomena. The circuit considerations indicate that the phenomena may be prevented by decreasing the capacitance to well below that necessary to sustain the triple frequency phenomena or to increase it to well above the limit for sustaining the one-half frequency phenomena. The wide range of capacitance within which the phenomena are sustained makes this procedure of doubtful practical value in most cases.

The phenomena may be damped out with transformer line-to-neutral loading sufficient to dissipate, in one-half cycle of neutral voltage, the energy acquired by the capacitor from the supply voltage during the previous saturation.

Conclusions

It is to be concluded that the abnormal line-to-neutral voltages which have been observed at various times in the field and laboratory are the direct result of a succession of saturations in the transformer cores.

In the transient state the initial asymmetries of the core flux densities result in saturations of a highly irregular order and produce correspondingly irregular voltages having no definite frequency. The abnormal voltages are sometimes limited within a stable zone as a result of multiple saturations and thereby become sustained. The stabilizing process only occurs for neutral voltage frequencies of either one-half, equal to, double, or triple the supply voltage frequency; consequently, these are the only frequencies which can exist in the sustained condition.

The successive saturations giving rise to the abnormal voltages do not possess a definite frequency characteristic, and as a result frequencies occur which are not ordinarily expected in ordinary alternating current circuits.

The minor and incidental characteristics of the published data bear a definite relation to the process of

successive saturations and are adequately explained by this theory.

Appendix

Proof for the Equivalent Circuit. Mathematical analysis has a limited use in the treatment of iron-clad circuits. In this discussion, however, its application enables the substitution of a more simple circuit than that actually found in practise. All of the relations established in the following analysis involve instantaneous values only and thereby apply to all wave shapes.

The circuit of Fig. 1A comprises a Y-connected bank of transformers (with neutral grounded) connected to ungrounded three-phase delta voltages. The line-to-ground capacitance of lines, transformer bushings, switches, etc., are represented by the lumped capacitances C_1, C_2, and C_3. Since the ground only serves as the connection between the capacitors and the transformer neutral the connection may be made direct and each capacitance shunted across its transformer as in Fig. 1B. This circuit is quite complicated. A complete treatment would involve simultaneous consideration of six currents to say nothing of the transformer flux-current relations.

The three transformer voltages, however, may be resolved into three known components and one unknown, that is symmetrical components and components of zero phase sequence respectively. The three symmetrical components are so related that:

$$e_{a1} + e_{a2} + e_{a3} = 0. \tag{4}$$

For the zero phase sequence components the relation is:

$$e_{b1} = e_{b2} = e_{b3} = e_b \tag{5}$$

Each transformer voltage is the sum of its components, thus,

$$\left.\begin{aligned} e_{t1} &= e_{a1} + e_{b1} \\ e_{t2} &= e_{a2} + e_{b2} \\ e_{t3} &= e_{a3} + e_{b3} \end{aligned}\right\} \tag{6}$$

and from the definition of neutral voltages as given in an early part of the paper, namely:

$$\frac{e_{t1} + e_{t2} + e_{t3}}{3} = e_n \tag{7}$$

it is evident that

$$e_n = e_b$$

For normal operation of the transformers from balanced supply voltages the three transformer voltages add to zero, and, consequently, the neutral voltage is zero. For abnormal operation each transformer voltage consists of a symmetrical component plus the neutral voltage. The latter voltage component being common to each of the transformers.

In Fig. 1A the transformer voltages are functions of theirr espective currents. The three currents are so related that,

$$i_{t1} + i_{t2} + i_{t3} = i_n \tag{8}$$

The three capacitor currents are similarly related,

$$i_{c1} + i_{c2} + i_{c3} = - i_n \tag{9}$$

In terms of the transformer voltages the three capacitor currents are:

$$\begin{aligned} i_{c1} &= C_1 \frac{d\,e_1}{d\,t} = C_1 \left(\frac{d\,e_{a1}}{d\,t} + \frac{d\,e_n}{d\,t} \right) \\ i_{c2} &= C_2 \frac{d\,e_2}{d\,t} = C_2 \left(\frac{d\,e_{a2}}{d\,t} + \frac{d\,e_n}{d\,t} \right) \\ i_{c3} &= C_3 \frac{d\,e_3}{d\,t} = C_3 \left(\frac{d\,e_{a3}}{d\,t} + \frac{d\,e_n}{d\,t} \right) \end{aligned} \tag{10}$$

thus from equation (9):

$$- i_n = C_1 \frac{d\,e_{a1}}{d\,t} + C_2 \frac{d\,e_{a2}}{d\,t} + C_3 \frac{d\,e_{a3}}{d\,t} + (C_1 + C_2 + C_3) \frac{d\,e_n}{d\,t} \tag{11}$$

For the special case where

$$C_1 = C_2 = C_3 = C_0$$

the capacitance may be factored from the differential coefficients of the symmetrical components after which the latter will add to zero in accordance with the result of differentiating equation (4), so that,

$$i_n = - 3\,C_0 \frac{d\,e_n}{d\,t} \tag{12}$$

In other words the neutral current, for balanced line-to-neutral capacitance, is dependent upon the total capacitance and the neutral voltage. It is independent of the voltage-current relations within the transformers except indirectly as these alter the neutral voltage.

Any equivalent circuit will be satisfactory for balanced capacitance, provided the transformers retain the same connections and have the same neutral current as in the actual circuit. Only the neutral current is dependent upon conditions external to the transformers.

That these conditions are satisfied by Fig. 6A may be readily demonstrated. Here a definite supply voltage is assumed in the form of a three-phase Y-connected generator. The three symmetrical line-to-neutral generator voltages are related by:

$$e_{g1} + e_{g2} + e_{g3} = 0 \tag{13}$$

Each of the three single-phase closed circuits comprise voltages so related that,

$$\left.\begin{aligned} e_{g1} &= e_{t1} + e_c \\ e_{g2} &= e_{t2} + e_c \\ e_{g3} &= e_{t3} + e_c \end{aligned}\right\} \tag{14}$$

From equations (6) and (13):

$$\frac{e_{t1} + e_{t2} + e_{t3}}{3} = - e_c = e_n \tag{15}$$

From which the neutral current becomes,

$$i_n = - C_n \frac{d\,e_n}{d\,t} \tag{16}$$

Therefore, in so far as the transformers are concerned the circuit of Fig. 6A is electrically identical to that of Fig. 1A provided:

$$C_1 = C_2 = C_3 = C_o \tag{17}$$

and

$$C_n = 3\,C_o \tag{18}$$

It is significant that these deductions are independent of anything which may go on within the transformers. In fact the lumping of the capacitances may be applied equally well to any type of Y-connected similar or dissimilar impedances which may be substituted for the transformers.

The total effect of unequal capacitances may be illustrated by assuming that C_1 is the smallest of the three capacitances. Equation (11) may be written for this case in the form

$$-i_n = (C_2 - C_1)\frac{d\,e_{a2}}{d\,t} + (C_3 - C_1)\frac{d\,e_{a3}}{d\,t} + (C_1 + C_2 + C_3)\frac{d\,e_n}{d\,t} \tag{19}$$

Thus the capacitance may still be lumped in so far as the current resulting from the neutral voltage is concerned. In addition, however, there are components of current equal to the products of the excess capacitances of two legs and the differential coefficients of their respective symmetrical component voltages. In other words, as long as the total capacitance remains the same, making the individual capacitances unequal will only result in additional neutral currents of the same frequency as the supply voltages. No change in the neutral voltage frequency currents will occur. For cases where e_n is large compared to e_a or where the capacitance unbalance is small the effect of the unbalance is negligible.

Energy and Voltage Relations During Saturation. The energy stored within a capacitor at any instant, is a function of the voltage at that instant, regardless of the variations which may have occurred previously. By means of this fact it is possible to derive quantitative expressions for the surges of energy between the capacitor and transformer during saturation without taking into consideration the time required for the completion of the process.

Let the sinusoidal supply voltage be represented by e_g in Fig. 14 which illustrates saturation voltage curves for the circuit of Fig. 6B. The three voltages of the circuit must maintain the relation

$$e_t = e_g + e_c \tag{20}$$

As indicated in Fig. 14 saturation starts at e_{co}. Its immediate effect is the reduction of the transformer voltage, e_t, to zero. This occurs at the point marked e_{go} where the capacitor voltage is equal and opposite to the supply voltage in accordance with equation (20). During the time e_t is reaching zero, e_c drops from e_{co} to zero and then rises to $-e_{go}$. The first step results in a loss of energy by the capacitor of

$$w_o = 1/2\,C\,e_{co}^2 \tag{21}$$

The second step results in an acquisition of energy by the capacitor of

$$w_1 = 1/2\,C\,e_{go}^2 \tag{22}$$

Thus the total change in energy of the capacitance is

$$w_{o1} = 1/2\,C\,(e_{co}^2 + e_{go}^2) \tag{23}$$

This energy change in the capacitor results in a corresponding rise of magnetic energy stored within the transformer core. The energy acquired by the core is a function of maximum flux density as shown by curve A of Fig. 15 which represents the energy of one cubic cm. of typical transformer steel. Consequently by means of equation (23) and curve A of Fig. 15 the peak flux density reached during any saturation may be computed for a given core volume. In this manner quantitative relations are available from the time saturation starts until the transformer voltage reaches zero. After the transformer voltage reaches zero the magnetic field collapses regardless of further variation in the supply voltage. A portion of the magnetic field energy is thereby returned to the capacitor and results in a further change of its voltage. The actual energy returned is given by curve B of Fig. 15. However, it is of considerable advantage to express this returned energy as a fraction of that acquired. This factor, P_w, is shown in Fig. 16. From equation (23) the energy returned in the collapse of the magnetic field is

$$1/2\,P_w\,C\,(e_{co}^2 + e_{go}^2) \tag{24}$$

As above described the complete change in the capacitor voltage from its initial to its final value occurs in three steps. First, the capacitor voltage reaches zero; second, it reaches a value equal and opposite the supply voltage at the instant the transformer voltage reaches zero; and third, the capacitor acquires the energy of the magnetic field and reaches its final value. The final energy stored within the capacitor may be written

$$1/2\,C\,e_{cf}^2 = 1/2\,C\,e_{go}^2 + 1/2\,P_w\,C\,(e_{co}^2 + e_{go}^2) \tag{25}$$

and the final voltage becomes

$$e_{cf}^2 = e_{go}^2 + P_w\,(e_{co}^2 + e_{go}^2) \tag{26}$$

The exponents in the energy equations eliminate the effect of signs so that other possibilities may occur which would not conform with equation (26). For instance, it frequently happens that the transformer voltage does not reach zero until after the supply voltage is reversed. The curves for such a condition are shown in Fig. 17. For this case the energy relations result in a final voltage such that

$$e_{cf}^2 = -\,e_{go}^2 + P_w\,(e_{co}^2 - e_{go}^2) \tag{27}$$

If a general relation is to be established, which will be applicable to all cases, the terms in the equation cannot be squared for in doing so they lose their signs. The difficulty may be avoided by the use of products of absolute and algebraic values. Such a product has the same magnitude as the square of the algebraic value but retains its original sign. The absolute values

may be represented by enclosing them between the usual vertical bars. The general relation written in this manner is:

$$e_{cf} \mid e_{cf} \mid = - e_{go} \mid e_{go} \mid - P_w \{ e_{co} \mid e_{co} \mid + e_{go} \mid e_{go} \mid \} \tag{28}$$

In any case the significant result is that the final voltage of the capacitance is dependent upon the initial capacitor voltage at the start of saturation and upon the supply voltage at the instant the inductance voltage reaches zero. Where equation (**26**) applies, the initial capacitor voltage is often many times the supply voltage and therefore dominates in the expression for the final reversed voltage. However, due to the factor P_w the final reversed voltage cannot exceed the initial voltage without the energy contributed by the supply voltage. These equations, therefore, illustrate the specific manner by which the capacitor voltage may be maintained. In equation (**27**) if e_g is quite large the capacitor voltage may not be reversed at all but merely reduced in magnitude. This tendency, for such a case, is amplified by the smallness of P_w for, when the energy contributed to the core is not the result of the sum of e_c^2 and e_g^2 as in equation (**26**), the peak densities reached are usually low so that P_w is small in accordance with Fig. 16. Thus the capacitor voltage resulting from a saturation is largely dependent upon the point on the supply voltage cycle at which the saturation occurs. This characteristic is of major importance in establishing the conditions for a stable succession of alternate saturations.

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Discussion

EXPERIENCES WITH GROUNDED-NEUTRAL, Y-CONNECTED POTENTIAL TRANSFORMERS ON UNGROUNDED SYSTEMS

(WELLER)

PHYSICAL NATURE OF NEUTRAL INSTABILITY

(BOYAJIAN AND MCCARTY)

THEORY OF ABNORMAL LINE-TO-NEUTRAL TRANSFORMER VOLTAGES

(LAPIERRE)

J. Fallou: I believe that the two phenomena analyzed by Messrs. A. Boyajian and O. P. McCarty can find an immediate and common physical explanation if based on the following experimental data:

1. Fig. 1 is a circuit comprising an a-c. source of frequency f_o, an air-core inductance L, and a capacitance C. It is known that if the conditions of such a circuit are altered, there appears across the terminals of the inductance and the capacitance a voltage comprising; (a) a steady state term of frequency f_o; (b) a transient term of frequency f'.

$$f' = \frac{1}{2\pi\sqrt{CL}}$$

If it is desired to maintain at the terminals of the capacitance a voltage of frequency f', it is necessary and sufficient that the frequency f_o of the source, or one of its higher harmonics, be equal to the desired frequency f'. In particular, it will never be possible to obtain, in a permanent manner, a frequency lower than f_o.

Let the air-core winding be now replaced by one with a magnetic core, with a self-inductance L (as determined by the straight portion of its magnetization curve) the same as before. The closing of the switch produces, as before, a transient voltage across the terminals of the capacitor, the sum of a free and forced state. But it is possible that the intensity of the current produced by the free oscillation be far greater than that arising from the permanent term, in some cases sufficiently large to saturate the magnetic core. The voltage across the terminals of the capacitor then contains a fundamental wave f' accompanied by a series of harmonics. Generally, such a free oscillation dies out following a very complex law, and its wave shape also changes as the current decreases.

However, I have shown experimentally in 1926* that if the frequency of one of the harmonics of such a free oscillation is near that of the source, the entire free oscillation, including its fundamental component, may become sustained.

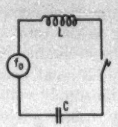

FIG. 1

Thus, a frequency lower than that of the applied e. m. f. may be found to persist across the terminals of the self-inductance or of the capacitance.

I have also found and described a static frequency reducer.† This apparatus operates on a principle totally different from that of frequency multipliers based on ferro-resonance used for a long time: it utilizes the maintenance of a free oscillation (which is necessary to start first) by one of its harmonics; while, in the frequency multipliers, on the contrary, a forced and distorted oscillation is produced, one of the harmonics being assumed as intensifying through an arrangement for resonance. Besides, frequency multipliers do not furnish even harmonics without the introduction of direct current into the circuit, while this arrangement furnishes directly an even multiple of the basic frequency.

2. Profiting by my experiences, Mr. E. Rouelle has been able to generalize the proposition which I have demonstrated; he has shown that in a circuit containing an iron-core inductance and a capacitance, fed by a sinusoidal e. m. f., it is possible to maintain various oscillations, the frequency of which can be not only even or odd sub-multiples of the source, as I have shown, but also even or odd multiples.

Thus, the following proposition is found definitely demonstrated:

In a circuit containing an iron-core winding and a capacitor, if a free non-sinusoidal oscillation of fundamental frequency f' is

*J. Fallou: *Revue Generale de l'Electricite* 1926—T. XIX, p. 987.
†American Patent No. 1,633,481.

started, such an oscillation can be maintained from a source of frequency f which is higher than, equal to, or lower than f', provided that f and f' have a simple ratio (even or odd, integral or fractional).

3. Finally, M. Rouelle has verified the following by test: Three iron-core windings $O\,A_1$, $O\,A_2$, and $O\,A_3$ (Fig. 2) have a common point O, and are connected respectively to three capacitors $A_1\,C$, and $A_2\,C$, and $A_3\,C$.

Excitation is applied between O and C from a source of frequency f; after which, on starting oscillations in one of the circuits, it is possible to maintain, across the terminals of $C\,A_1$, $C\,A_2$ and $C\,A_3$, three voltages at a frequency f' equal to one-third of f, each one of these voltages being spaced one-third of a cycle away from the adjacent circuit. M. Rouelle has thus shown the possibility of static transformation of a single-phase voltage of frequency f to a polyphase voltage of frequency f' sub-multiple of f.

It is clear that the last experiences of Rouelle demonstrates a phenomenon which is the reciprocal of that which happens in case of neutral oscillations.

As to the inversion of the neutral, it can be proven through the same mechanism, the sustained frequency f (oscillation of the neutral) is then almost equal to the sustaining frequency f'. The experiences of Messrs. Boyajian and McCarty have led the authors to sustain at the neutral point of the system oscillations at the frequency f equal to f' (inversion and oscillation at the frequency $f'/2$): by changing the test conditions, that is, the capacitances and the degree of the saturation of the iron, other frequencies could also have been put into evidence.

This explanation is otherwise very close to those given by Messrs. Boyajian and McCarty: it may not be without interest to state in passing, that these authors have been able to deduce

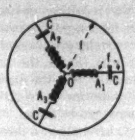

Fig. 2

a general law (maintenance of a harmonic or sub-harmonic), starting from a particular case (instability of neutral) while on our side, we have been led to traverse the same course in the reverse direction, M. Rouelle having explained the instability of the neutral starting from the law of maintenance which I demonstrated several years ago.

F. A. Hamilton, Jr.: The first of these papers outlines experiences with two applications of grounded-neutral potential transformers on portions of systems necessarily operated ungrounded at certain times. These two installations provide outstanding examples of the difficulties which may be encountered with such installations.

In addition to the two cases which are cited in the paper it may perhaps be interesting to call attention to several other similar cases.

A. A bank of grounded-neutral potential transformers was connected directly to the terminals of a bank of delta-connected power transformers. The potential transformer secondaries were connected in "broken delta", the circuit being closed through a potential relay. When the power and potential transformers were energized this relay operated and tripped the breaker. The difficulty in this case was obviated by the expedient of opening the relay trip circuit automatically just before closing the energizing breaker and closing the relay trip circuit automatically some time after the breaker was closed. The voltage which operated the relay when this automatic scheme was not used seemed to be a transient one which died out rather rapidly, making it possible to use the relay, which was provided for ground protection, at all times excepting when energizing the power transformer bank.

B. A case similar to that referred to under (A) was observed by the writer. In this case a delta-connected power transformer bank was arranged to be normally connected to a short section of bus by means of a breaker. A bank of grounded-neutral potential transformers was connected to the short section of bus. The potential transformer secondaries were connected in broken delta and the circuit closed through a potential relay. When the installation was first placed in operation, the closing of the breaker between the power transformer bank and the short section of bus invariably caused the operation of the relay in the secondary delta of the potential transformers. This difficulty was partially cured by the addition of a resistance burden to the secondaries of the potential transformers. As this method of curing the difficulty did not seem desirable to the operator, the potential relay was replaced by a lower resistance current relay and this cured the difficulty satisfactorily. As in the case discussed under (A), the unbalance voltage was a transient one which died out rather rapidly and which did not persist as in the cases described in the paper.

C. On another system employing distribution lines at generator voltage, the insulation on an oil circuit breaker broke down. Grounded neutral potential transformers with secondaries connected in broken delta, the circuit being closed through potential relays were used on this system. These should not have caused trouble as the generator neutral was normally grounded. Examination of the breaker which failed, and of others in the same station indicated the possible presence of overvoltages, and although no definite solution was ever arrived at, it is thought that at times the operator in the station may have left the generator ungrounded and that resulting abnormal voltages to ground may have been responsible for the disturbance.

D. On one other system tests were made to determine the effect of using grounded-neutral potential transformers on an ungrounded system. In this case the secondaries were also connected in broken delta and the circuit closed through potential relay. In this case, the neutral was shifted outside the delta of the system voltage and this unbalanced condition persisted. It was cured by adding a resistance burden to the secondaries of the potential transformers.

In the four cases mentioned above, one of the systems operated at 60 kv. one at 12 kv. and the others at about 13.8 kv.

The examples mentioned may serve to illustrate that the difficulty which may attend the use of grounded-neutral potential transformers on ungrounded systems or portions thereof, have been more frequent than might be believed.

John Auchincloss: While the most important, and probably the most frequent, use of Y-connected potential transformers occurs in connection with ground protective equipment operating on the zero phase sequence components of current and voltage there are nevertheless a few other aspects of the general problem of the Y-connection upon which the various phenomena disclosed by these papers may have a profound bearing. Quite apart from the hazard to the potential transformers themselves and all other equipment subjected to the stresses arising from the phenomena which they engender, there is the further question of the degree of confidence which may be placed in the indications of instruments or devices operating from the secondary L/N voltages of such potential transformers.

Two situations immediately come to mind in which the occurrence of the neutral phenomena described in the papers may give rise to very misleading or entirely false conclusions in attempting to interpret certain system conditions from the readings of instruments connected to Y-connected potential transformers. These involve the phase comparison of system voltages for purposes of synchronizing across grounded Y-delta power transformer banks, and the detection of accidental grounds

on isolated-neutral systems operating at voltages beyond the range of the ordinary electrostatic ground detector.

When it becomes necessary to synchronize across a Y-delta power transformer bank two parallel voltages, namely, the L/N voltage on the Y side and the L/L voltage on the delta side, are available for connection to the synchroscope, and if a Y-connected set of potential transformers on the Y-side is also available the prospect of using the L/N voltage on this side for purposes of phase comparison appears attractive by reason of its simplicity. The situation to which I have particular reference is that in which the paralleling breaker is located on the Y- or high-voltage side of the bank and a group of relaying potential transformers is present on the same side. Obviously the L/L voltages across the power bank are 30 deg. displaced at synchronism, requiring the use of phase-shifting auto-transformers, two-winding potential transformers or other special equipment to offset this displacement at the synchroscope. To avoid the added complication brought about by these auxiliary devices many engineers are prone to favor the L/N voltage on the Y-side with balanced Y burdens, for purposes of synchronizing. However, a little consideration will show that if the relaying potential transformers are to be used also for synchronizing they must be connected to the line side of the paralleling breaker which of course is also the side of the breaker away from the Y windings of the power bank carrying the ground. Consequently with the breaker open the potential transformers may or may not be connected to a temporarily ungrounded portion of the system, depending upon the switching arrangements and ground locations that may obtain for the time being on that part of the system to which the potential transformers are connected. Should these conditions be such that during the synchronizing period this part of the system is ungrounded, then obviously the potential transformers are operating under circumstances which these papers describe as ideal for the propagation of neutral phenomena, with its resultant distortion of the L/N voltages. Under conditions such as these the danger of misleading indications at the synchroscope is obvious, as is also the conclusion that although the practise of synchronizing on the L/N voltage may be condoned under certain favorable circumstances it should be rigorously shunned under conditions at all likely to give rise to vagaries in the position of the system neutral.

The other problem apparently involved with the phenomena described in the papers is that of detecting accidental grounds on high-voltage, normally ungrounded systems. Most methods so far devised for obtaining such indications have depended essentially upon the secondary Y voltages obtained from three potential transformers connected to the lines in Y formation, with the primary neutral point of the potential transformers solidly grounded. Since the system is ungrounded this situation also results in a set of conditions most favorable to the inception of the phenomena, with resulting erroneous indications on the voltmeters or other ground-detecting devices operating on the secondary voltages. Fortunately, however, and for several reasons, among them being the advantages accruing from the use of standard 110-volt indicating devices, the potential transformers so far used for this purpose have invariably been of a rating suitable for the L/L voltage. Although a number of these ground-detecting equipments have been in apparently successful operation for several years nevertheless since all of the papers indicate that instability may occur even down to 58 per cent of normal excitation it is not at all improbable that the accuracy so far obtained from them may be susceptible to improvement by the addition of suitable secondary loading.

W. Lampert: At frequencies, the ratio of which compared with the impressed frequency is *not* an integer, an alternating bias is necessary. But I do not think, that at even harmonics, the ratio of which *is* an integer of the fundamental frequency, a bias is at all necessary.

Take for instance the case, where there is no bias and no residual flux, and where in the leg voltage there is a fundamental and a second harmonic. Then a current of fundamental and double frequency will flow in the transformer winding. If the current of double frequency has such a phase that its positive maximum strengthens the positive maximum of the fundamental current and weakens the negative maximum of the fundamental current, the curve of the resulting magnetizing current has a short high positive portion and a longer lower negative portion. This current produces a flux curve, the integral value of which differs from zero; which means, that the alternating magnetizing current produces a direct flux besides an alternating flux. This direct flux is however of no importance, because it neither produces an e. m. f. nor requires a direct current or a direct voltage for its maintenance. The direct flux is only an accompanying phenomenon, and does not support the formation of the double frequency.

In the following we shall see, that it is even not always necessary, that a direct flux exist. In consequence of the saturation of the iron core, the direction of the direct flux is either positive or negative. This fact quite depends on the phase of the double-frequency current. In the example, where the positive maximum of the double-frequency current strengthens the positive maximum and weakens the negative maximum of the fundamental current, the direct flux is negative in spite of the high positive current peak. For the positive current peak produces on the average only a little higher flux value than the lower negative current values; but the negative current values exist much longer than the positive current values, so that the integral value of the flux curve, and with this the direct flux, become negative.

If however, the phase of the double-frequency current is shifted about 180 deg., so that the positive maximum of the fundamental current is weakened and its negative maximum is strengthened, then the resulting curve of the magnetizing current has only a short high negative peak and a longer lower positive portion. This current produces a flux curve, the integral value of which is positive, that is, the direct flux is positive.

Between these two states, where the direct flux is either negative or positive, such a phase for the double-frequency current, where the direct flux is zero, can be found. Then there is neither a direct flux, nor a direct current, nor a direct voltage. In other words, there is nothing which may resemble a bias.

W. W. Edson: Could this feature of neutral instability, or the half and double frequency oscillations, have any probable effect on the following installations?

1. Three 8050/115-volt star-delta connected potential transformers operating watthour and power-factor meters.
2. Same except operating directional relays.
3. Similar except using two transformers connected open star/open delta. In this case it was found that the three voltages to ground on the secondary leads are unbalanced, that is, the neutral is outside of the secondary delta.
4. Three 110-kv. condenser bushing network potential devices in effect equivalent to star-star connected potential transformers operating directional relays connected in delta. An auxiliary set of three potential transformers is connected to this secondary circuit and is arranged star-delta with a ground directional relay in the corner of this delta.

Walter L. Upson: The three companion papers dealing with abnormal line-to-neutral voltages of a Y-Y system present an admirable attack on a rather difficult subject. The contrasting theoretical studies are of considerable interest inasmuch as they represent such entirely different approaches to the problem that a reading of either paper would hardly suggest the other. Messrs. Boyajian and McCarty have built up their argument on the combined saturation curves of inductance and capacitance in parallel forming one leg of a three-phase system. Mr. LaPierre has used an equivalent circuit in which a capacitance is placed in series with the three inductive phases of his system, one terminal of the condenser being connected to the

neutral while the other is anchored to the assumed fixed neutral of the three-phase system supply. With this picture in mind, he considers what may take place under the provocation of saturated transformer cores. The effect of saturation in any one transformer is to make its inductive reactance negligible in comparison with the other unsaturated transformers. There results a heavy flow of current through this transformer into the condenser. Most of the voltage drop of this phase is across the condenser which is then charged up in such a direction that its voltage more or less opposes, for the moment, that of the other two phases. In the course of cyclic variation of voltage on the transformers, we soon reach a point at which one of the previously unsaturated transformers becomes subjected to the condenser voltage in addition to its phase voltage, with the result that the condenser discharges, saturating this transformer and then becoming charged in the opposite direction.

If there is the proper balance between capacitance, percentage of normal transformer voltage and perhaps other less important factors, this proceeding will be repeated and will become periodically recurring or stable. Of course something must start the phenomenon, and the author takes this to be the abnormal condition brought about by the starting transient with or without the help of residual magnetism in the transformer core. When the stable condition is reached all three transformers have presumably been brought to the same magnetic condition in which saturation passes successively from one to another, each time being accompanied by a reversal of the condenser potential. It is rather difficult to form a picture of this changing condition which will include the four possible periodic frequencies of the phenomena and enable one to see why any one of the frequencies should prevail under a given set of conditions, in spite of the author's painstaking effort to make it clear. However, if we start with a sine wave of flux which we will assume to be produced by a suitable magnetizing current, we can lay off a sine wave voltage, e_t, impressed on the transformer and leading the flux by 90 deg.

Lagging by 90 deg. will be the voltage drop across the condenser, and the sum of these two voltages will be the phase voltage of the system. Now suppose saturation to occur near the peak of the flux wave and that the peak is therefore flattened out. The voltage, e_t, falls quickly to nearly zero at the instant of saturation, and remains there until saturation ceases. At the same time, the condenser voltage rises being retarded by the falling system voltage, but continuing to rise while the system voltage builds up in the opposite direction and the transformer voltage hovers around zero. As soon as saturation ceases, the transformer voltage rises to something of a peak. If the transformer now saturates in the opposite direction, the procedure will repeat itself as outlined above. If it does not saturate, the condenser, being unable to discharge will retain its potential, permanently inverting the neutral unless it finds opportunity to combine with one of the other transformers to produce saturation in it and build up a charge in the opposite direction. Mr. LaPierre's theory is no doubt correct, and is greatly clarified in the last section of the appendix. However, even here we are left without directions which would enable us to predict the type of oscillations which might be expected. This is probably due to the necessity for restricting the extent of the paper. It would be interesting if a series of actual oscillograms could be shown based upon predetermined constants and conditions of operation.

A. Boyajian and O. P. McCarty: Mr. Fallon arrived at almost the same results and broad conclusions as ourselves but considerably earlier and through an entirely different route. We are glad to acknowledge his priority, and are pleased with the agreement in opinions.

Mr. Lampert has indicated a difference of opinion with reference to the necessity for bias. We all probably will agree that the integrals of the fundamental and second harmonic fluxes (in fact, of all the integral harmonic fluxes) taken over a cycle of the lowest frequency will add up to zero regardless of their phase relation, as is almost axiomatic. The same statement would also be true about currents. However, that does not exclude the possibility of a bias like the residual, requiring no external magnetomotive force in the form of a continuous unidirectional current in the windings, and therefore not appearing in any current integral. If an inexact even harmonic, which is continuously changing its phase with respect to the excitation frequency, needs an alternating bias, as Mr. Lampert seems to agree with us, then, it would seem to follow that the exact even harmonic of such a phase as to require neither a positive nor a negative bias would not be permanent, because it would not be able to draw power from the fundamental frequency to supply its losses.

Undoubtedly, in complex phenomena such as discussed in this group of papers, there are points on which opinions will differ, but it is satisfying to note that there is general agreement on the broader physics of the subject.

As indicated in our paper, we do not consider the subject as completely analyzed and closed. Since submitting our paper, we have attempted also a mathematical approach to the subject, and hope to be able to present it to the Institute in the near future.

C. W. LaPierre: The interest Professor Upson has taken in the successive saturation theory of these phenomena is greatly appreciated. His discussion brings out one or two points which might well be clarified further.

In interpreting these phenomena it is important to keep in mind several prominent characteristics of the observed data. In the first place a large portion of the oscillograms taken indicate that the phenomena are highly irregular in their nature. In many of the oscillograms shown by Mr. Weller the fundamental frequency is almost completely obscured. In most of the oscillograms showing the transient state there is no evidence of any definite frequency being present. Instead the voltages take the form of an irregular series of voltage swings.

Such transient states are highly important and should not be ignored for they in general produce the largest surges and insulation strains. Furthermore the sustained voltages are not lacking in irregularities of one sort or another. This highly irregular character of the phenomena is generally recognized.

On the other hand, and in the second place, the observed data for the sustained conditions do indicate certain regularities which require explanation. Such characteristics are amply shown in the tabulations of both the other papers of this group.

The phenomena, therefore, present a two fold aspect. Under one condition, largely transient, they are highly irregular and occur without definite magnitude or frequency. Under another condition they occur in a quite definite manner and possess easily identified characteristics. It is this dual nature of the phenomena which caused me to adopt the method of approach so nicely described by Professor Upson.

The results of this method are somewhat unexpected in that the highly irregular transients are very easily explained by the theory developed. The sustained conditions appear as special cases of the irregular transients and occur under proper circuit conditions when the transients more or less accidentally fall into a permissible sustained frequency. Due to the element of chance which enters it is not so easy to picture as a direct result of cause and effect the transition from the irregular to the definite frequency. However, such a procedure is not necessary. If the generation of the transient voltages is understood and it is further understood just what the necessary conditions are for a stable succession of saturations then one may readily see how the transition occurs. As to whether it takes one, ten, or fifty cycles to complete the transition is of no theoretical importance provided the mechanism itself is clear. In general the successive saturation theory is devoted to an explanation of just these fundamental processes which underlie the abnormal voltages.

As a result of its application it has been possible to devote a complete section of the paper to the explanation of detailed characteristics of the observed data, both for transient and sustained conditions.

In the latter part of Professor Upson's remarks a description is given of the voltage changes during the saturation process. It is necessary here to consider the relative magnitudes of the various circuit factors entering into the phenomena. Professor Upson's remarks apply largely to the condition where the circuit capacitance is relatively small. In that case the voltage does fall "quickly to nearly zero" and remains there until saturation is completed.

However, the transformer voltage cannot drop to zero without upsetting the voltage distribution between the individual capacitances making up the system total. If the system capacitance is relatively large the abrupt voltage change requires a large surge of current which must be supplied from the saturated transformer. In such a case of saturation, when the transformer voltage does reach zero the current surge is at its peak. With no voltage to maintain it the current tends to drop to zero. Its abrupt decay is accompanied by a partial collapse of the transformer flux which carries the transformer voltage far in the reversed direction. It is this process which generates the large voltages which are regularly observed in these phenomena. In this manner we are able to deduce from Figs. 2, 3, 4 and 5 of my paper that the system capacitances associated with these phenomena are relatively large, for, in those illustrations the voltages pass abruptly through zero to their reversed values at each saturation. Of course there may be, and undoubtedly are, minor saturations occurring in such an irregular process which do result in reversed voltages hovering around zero for a time, but these minor saturations are incidental to the main process. The magnitude of the reversal depends almost entirely upon how far the voltage must drop to reach zero after saturation sets in.

C. T. Weller: In regard to Mr. Edson's question, the connection mentioned in his first two items is referred to in the first paragraph of the paper and also under the section "Secondary Connections." The performance of two potential transformers connected L/N is indicated in Fig. 16, which is supplemented by Table VI and by a descriptive paragraph on page 315. Bushing potential devices consist primarily of two capacitances in series, the larger capacitance (and lower voltage) being shunted by a potential transformer; saturation in the L/N circuit as a whole, therefore, is not involved, so the phenomena should not occur.

Mr. LaPierre called attention to the erratic wave shapes and frequencies of the L/N voltages obtained during the starting transients leading to the temporary or sustained establishment of the phenomena. However, it would appear logical to assign a definite phenomenon letter or frequency to a particular half-cycle (or more) during the transient period, if such a phenomenon or frequency were also obtained in sustained form under similar conditions. Since this was true in most cases, I assigned phenomena letters to transient approximations of sustained phenomena for convenience. In the few cases where the phenomenon so designated was not obtained in sustained form, it is evident from the data that a modification of the circuit constants not then possible would have produced the phenomenon in sustained form.

Until recently, there has been no special interest in the Y-connection of potential transformers, although a growing realization of its advantages for ground-protection schemes has led to its more extensive adoption. It was therefore in order to point out some of the disadvantages of the applications with grounded neutral to ungrounded systems, particularly since they did not appear to be generally known. The phenomena described in the paper first presented themselves as potential transformer disturbances, which were quelled by the addition of suitable protective resistor burdens. It was not appreciated at the time that somewhat similar phenomena had been encountered previously (with power transformers), so comprehensive tests were made: the essence of the data obtained is presented in the paper.

Certain terms adopted in the paper might be considered for general use. The advantages of substituting "L/L" for line-to-line and "L/N" for line-to-neutral are self-evident. Also, "broken delta" implies a three-transformer connection with the secondary delta circuit either open or closed through a comparatively high-impedance device; a differentiation from "closed delta" and "open delta" (for two transformers) is thus obtained.

Figs. 2c and 2D and even 2B might be criticized because distorted wave shapes or harmonics cannot be represented properly by means of vectors. However, unusual phenomena justify unusual treatment and it is believed that the figures convey a fair idea of the effects of neutral displacement on the L/N voltages. A figure representing triple-frequency Phenomenon D was not included as this phenomenon was obtained only in transient form with potential transformers, although the transient L/N voltage crests were definitely higher than for double-frequency Phenonenon B. Even in the somewhat special case with distribution transformers, D was obtained in semi-sustained form only. Also, a figure representing the well-known third-harmonic phenomenon, due to suppressing the corresponding harmonics of the exciting currents, was not included. The apparent disregard of the effects of this well-known phenomenon afforded a convenient means of introducing the less-known phenomena.

The excellent reproductions of the oscillograms in the paper require no group comments when the sections are printed together. However, it was not possible to print three or four sections together, so descriptive references in the text to wave shapes shown "at the right" of the oscillograms or "subsequently" should sometimes be interpreted as "below." An inspection of the wave shapes of the "exciting" or line currents, which combine to form the neutral-to-ground current, and a consideration of the relative magnitudes involved will indicate why the phenomena were known from the first to be "saturation phenomena."

There are several points in the paper that require further emphasis. It should be noted that the phenomena are not characteristic of any particular ungrounded system or type of transformer. Phenomena B, C and D endanger the potential transformer themselves; B and D endanger other connected equipment also. All phenomena tend to cause incorrect synchronizing and incorrect relay operation.

The "General Recommendations" are intended as an approximate guide in the application of Y-connected potential transformers. It should be noted that it is possible to obtain the phenomena below 50 per cent of rated voltage, but that such an occurrence in practise is extremely improbable. The L/N voltage rating referred to in Table IV and elsewhere in the paper covers a somewhat special design of transformer, which can operate temporarily at L/L voltage in an emergency.

In conclusion, it is believed that a reasonably complete picture of the "saturation phenomena" is presented for the first time. While a review of the companion papers will indicate that the theory is somewhat complicated, there is no question as to what should be done to prevent the occurrence of the phenomena. In all cases, adequate secondary protective resistor burdens only are necessary; the magnitude of these burdens is related to the rating of the potential transformers and to the flux density at which they normally operate. The paper will have accomplished its purpose if it contributes to a better understanding of the operating difficulties involved and so prevent the inadvertent application of grounded-neutral, Y-connected potential transformers to temporarily or permanently ungrounded systems.

Power Transformer Noise
Its Characteristics and Reduction

BY ROBERT B. GEORGE*

Member, A. I. E. E.

Synopsis.—*Audibility characteristics of the ear and the units used for measuring noise are briefly discussed. A theoretical investigation was made to determine the different force components and other causes of transformer hum. Surface vibration measurements were made on transformer parts to determine the sources which cause noise in transformers and how these noise sources are affected by variations in operating conditions. Noise surveys were made to measure the range of intensities, the frequency characteristics and distribution of sound from transformers in service. The effects of substation acoustic conditions on noise distribution were investigated.*

CONSIDERABLE attention is being given to the reduction of unnecessary noises emitted by electrical apparatus. The scope of this paper is intended to cover the progress which has been made in the reduction of noise in power transformers. It represents only a part of an extensive investigation which is resulting in quieter electrical apparatus.

Practically all types of sound which cannot be classified as speech or musical tones may be classified as noise. Since transformer noise sounds like a hum having a definite pitch, it may also be classified as a musical tone. This tone may not be objectionable for short periods of time but when it is heard continuously for long periods of time it may be objectionable to some people.

The occurrence of sound involves the action of three elements,[1] the sound generator which is essentially a vibrating body, the transmitting medium and the sound detector. The vibrating body sends out simple harmonic pressure waves in all directions. These waves are transmitted by the air (or other medium) which surrounds the vibrating body. If the ear is placed in the sound field so that it may be acted upon by the incident pressure waves, the diaphram of the ear will be set in vibration by the alternating pressure. For a given frequency the apparent loudness is approximately directly proportional to the logarithm of the r. m. s. sound pressure. Fig. 1, which is the result of the work of Bell Telephone Engineers,[2] shows the maximum and minimum limits of audibility as a function of sound pressure and frequency. The minimum audibility curve is an average for many observers and corresponds to perfectly quiet conditions. If ordinary room noises were present, the lower audibility limits would be much higher than the values for this curve. Fig. 1 indicates that the ear is most sensitive to sounds having a frequency of approximately 3,000 cycles per second and the sensitivity decreases for sounds above or below this frequency.

In order to deal effectively with the noise problem, the sound analyzer developed by Mr. J. P. Foltz[3] was used in these investigations. The set[4] consists principally of a set of amplifiers and filter circuits which can be set to eliminate all but a certain frequency. The noise to be analyzed is picked up by a microphone and passed through the filter circuits. The setting of the filter circuit condenser indicates the frequency of the sound to be measured, and a meter indicates the intensity of the sound at that frequency in dynes per sq. cm. One dyne per sq. cm. equals approximately 0.00000035 lb. per sq. in. Dynes per sq. cm. are absolute units which are independent of the local noise level.

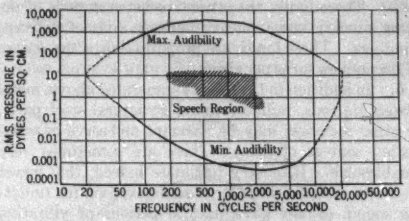

FIG. 1—MAXIMUM AND MINIMUM LIMITS OF AUDIBILITY

It is not desirable to make sound tests on comparatively quiet apparatus where the local noise level is high.

This analyzer is independent of the human ear for comparing sounds with reference standards, and it leaves nothing to personal judgment or the imagination of an individual listening to a telephone receiver.

A transformer has no rotating parts. An impulse from each half cycle results in a fundamental frequency of vibration of 120 cycles (double vibrations per second) from a 60-cycle transformer. These vibrations usually originate in the core and coil structure and are transmitted by the oil to the tank where they are in turn passed on to the air. When it is necessary to measure mechanical vibration, a calibrated phonograph pickup may be substituted for the microphone, and the sound

*Transformer Engineering Department, Westinghouse E. and M. Co., Sharon, Pa.

1. Churcher and King, *I. E. E. Journal*, Vol. 68, January, 1930.

2. "Useful Numerical Constants of Speech and Hearing," Fletcher, *Bell System Technical Journal*, Vol. IV, July, 1925.

Presented at the Southern District Meeting No. 4, of the A. I. E. E., Louisville, Kentucky, November 19-22, 1930.

3. *Study of Noises in Electrical Apparatus*, Spooner and Foltz, A. I. E. E., TRANS. July, 1929.

4. "Diagnosing a Case of Noise," J. P. Foltz, *Electric Journal*. March, 1930, p. 178.

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analyzer can be used to measure the frequency and amplitude of vibration. Since the active area, the frequency and the amplitude of vibration are related to the total noise emitted by the transformer at that frequency, the vibration of the tank surface can be measured on the test floor when the corresponding limits of vibration are known for satisfactory quiet operation in service.

A theoretical investigation was made to determine the different force components which might cause vibration, and to devise methods of dealing with each of these forces.

1. The force between the winding and the core is an attractive force, but when solid insulation is wedged in between the coils and core, this force is not serious.

2. There is an attractive force which acts at each joint of the core and tends to make the iron circuit continuous. This force is directly proportional to the cross section area and to the square of the magnetic density.

3. There are forces of repulsion between parallel laminations of the core. This is explained by the fact that like poles are induced in adjacent ends of parallel laminations.

4. There are electro-dynamic forces between the coils. These forces are attractive between coils of the same winding and repulsive between coils of different windings. These forces are proportional to the square of the ampere turns per group.

5. In addition to the forces enumerated above, metal plates, tie rods, radiator tubes, structural steel parts, or other members may be resonant and serve as unexpected sources of noise. These are sometimes very troublesome. It is very fortunate indeed, that power transformer tanks with large areas exposed for radiating sound, usually have natural periods of vibration below 60 cycles.

Some of the usual methods of obtaining quiet transformers consist of: (1) Using additional iron in the core in order to secure low magnetic density. This also means additional copper and a more expensive transformer. (2) Stiffening the bracing and supporting parts to reduce vibration and in some cases adding damping devices to change the natural period of vibration. (3) Adding cushions or padding between parts of the transformer.

Surface vibration measurements were made on the core and coil parts of several different transformers to determine the sources of noise due to excitation voltage only. The mid point of the high-voltage winding was grounded and the transformer was excited at 60 cycles, with 100 per cent voltage on the low-voltage winding during these tests. Vibration amplitude measurements were also made on the parts involved at no load and at different voltages between 80 and 120 per cent of rated voltage.

Vibration amplitude and frequency measurements were made on the tank surfaces of a number of representative sizes of completely assembled transformers at no load and at different voltage values. Then tests were made to determine the effect of line voltage regulation on the amplitude of vibration and the corresponding sound intensity. Fig. 2 shows the results of some of these tests on a large transformer at the elevation of the center of the core. The locations of the points are indicated on the plan view at the upper left hand corner of the figure. These curves are approximately straight lines within the operating range of the transformer. This indicates that the component due to voltage regulation can be considered to be directly proportional to the per cent change in line voltage for the region where these curves are straight lines. The ratio varies with different designs but it is usually of the order of 30 per cent change in sound intensity for 10 per cent change in primary voltage at no load.

Tests were made on the core and coil parts of a transformer to determine the characteristics of the parts of the transformer which serve as sources of noise due to load currents in the windings. These tests were made with one winding short-circuited and with sufficient voltage impressed on the other winding to obtain the desired values of load current in the windings. Vibration amplitudes and frequency measurements were made for each load condition. Similar tests were made on completely assembled power transformers and surface vibration measurements were made on the tank and other exterior surfaces. The curves of Fig. 3 show the effect of load current on the amplitudes of vibration of four representative points at different elevations on the side of a large transformer. It can be observed that the amplitude of vibration varies approximately as the square of the load current.

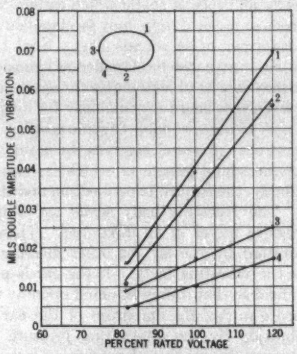

Fig. 2—Curves showing the Effect of Voltage Variation on Amplitude of Tank Surface Vibration

Tests at varying frequency were not made on large power transformers on account of the inconvenience of obtaining sufficient power at adjustable frequency.

Such tests are useful for determining resonant frequencies of various parts but they do not represent operating conditions on a constant frequency system.

Frequency analyses were made of points of maximum amplitude of vibration, with the transformer operating at rated frequency. Fig. 4 is a frequency analysis of

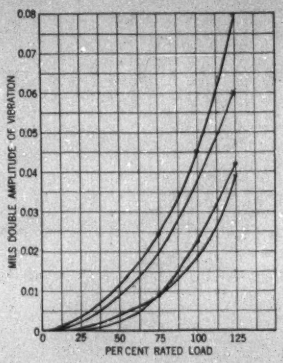

Fig. 3—Effect of Load Current on Amplitude of Tank Surface Vibration

vibration of the point of maximum amplitude of vibration of a 60-cycle transformer at 100 per cent voltage and no load. Fig. 5 is a frequency analysis of the same

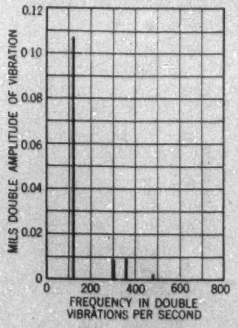

Fig. 4—Frequency Analysis of Vibration at 100 per Cent Voltage and No Load

point with the transformer at 150 per cent load current but without excitation.

It is interesting to note that different harmonic frequencies are present during the two conditions. This is evidently due to different parts of the transformer being affected by the load and the excitation components. All of these harmonic frequencies are exact multiples of the 60-cycle power frequency. The test range included frequencies up to 3,000 double vibrations per second but no frequencies above 600 double vibrations per second were noted.

The vibration amplitudes of this transformer at frequencies above the fundamental 120 cycles are so small that they can be neglected. There have been a few

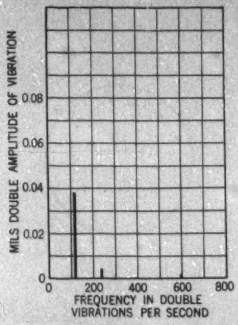

Fig. 5—Frequency Analysis at Location of Fig. 4 Except at 150 per Cent Load Current and Without Excitation

cases, however, where harmonic frequencies were appreciable and it was necessary to weigh them in proportion to the audibility curve of Fig. 1.

Noise surveys were also made to measure the range

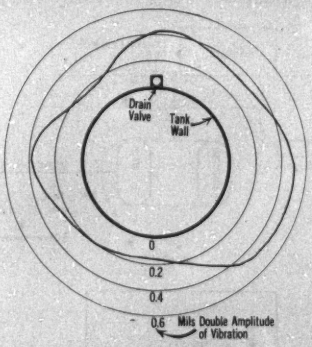

Fig. 6—Distribution of Vibration around the Tank of a Large Power Transformer

of intensities, the frequency characteristics and distribution of sound from power transformers in actual service.

Complete vibration measurements were made of the tank surfaces of each transformer where sound measurements were made, in order to obtain data on the relation of vibration of the tank surface to the sound given off by the transformer.

Fig. 6 shows the vibration of the tank wall of a 60-

cycle transformer at 97 per cent voltage and 41.6 per cent load. The curve is plotted in polar form with reference to the tank wall and at greatly enlarged scale in order to show the distribution of vibration.

Fig. 7 shows the distribution of the sound intensity of the 120-cycle fundamental note measured at a distance

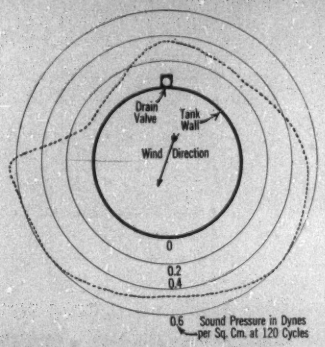

Fig. 7—Distribution of 120-Cycle Sound Intensity Measured at 10 Ft. from Tank Wall

of 10 ft. from the tank wall. There is a marked similarity between the distribution of vibration amplitude and the distribution of sound intensity. The substation structures at the upper left hand corner account

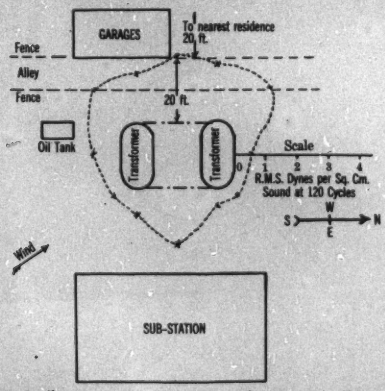

Fig. 8—Distribution of 120-Cycle Sound at 20 Ft. from Transformers

for the sound distribution in this region and the wind blowing from the direction indicated in the figure accounts for the increased sound intensity in the lower left hand region of the figure.

Fig. 8 shows the distribution of sound at 120 cycles measured at a distance of 20 ft. from the transformers of Fig. 9. Since there are two transformers in this station, lines are drawn to connect the two transformers and the resulting area is treated as the noise source. The curve is plotted similar to polar form with reference to the tank wall and the two connecting lines described above. At a distance of 20 ft., the average intensity is

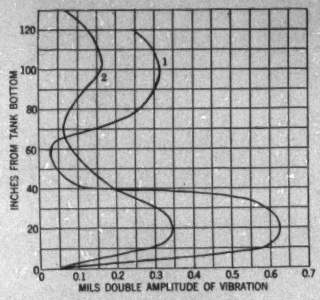

Fig. 9—Vibration Along Vertical Lines at the Sides of Two Large Transformers

1.142 dynes, the minimum intensity is 0.48 and the maximum intensity is 2.06 dynes per sq. cm., or a maximum to minimum ratio of 4.3 to 1. Reflections from the substation building and the neighboring garages are evident. The effect of wind is also evident in the upper right hand corner of the figure.

Curve 1 of Fig. 9 shows the distribution of the

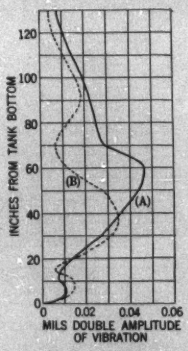

Fig. 10—Vibration Along Vertical Line of Tank.

(A) Complete Transformer
(B) With tie plates removed

vibration up the north side of the north transformer, and curve 2 shows the condition on the south side of the south transformer. Both transformers are large three-phase units and the load conditions are approximately 97 per cent voltage and 83.6 per cent load.

Curve *A* of Fig. 10 shows the distribution of vibration

up the nearest side of the transformer of Fig. 11. Since this is an unusually quiet transformer, the amplitudes are plotted to a larger scale. It is also of interest to note that the difference between curves *A* and *B* of Fig. 10 indicate a possible further reduction which can

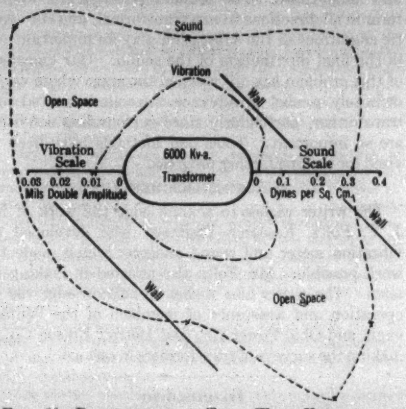

Fig. 11—Distribution of Tank Wall Vibration and Distribution of 120-Cycle Sound Pressure at 42 In. from Tank Wall

be accomplished by removing the steel plates which brace the core and coils to the tank wall. This practise is not generally recommended except in cases where the transformer will not be moved before the oil is lowered and the steel plates replaced.

Another interesting illustration of the effects of the acoustic conditions at the substation is given in Fig. 11.

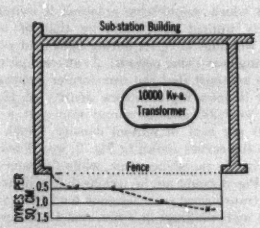

Fig. 12—Distribution of 120-Cycle Sound Pressure Across the Open Side of a Transformer Compartment

A smooth finish plastered wall was located diagonally as shown on the upper side of the figure and a hard surface wall of steel plates was located symmetrically on the opposite side of the transformer. The inner curve shows the distribution of vibration of the tank wall at 120 cycles and the outer curve shows the distribution of the intensity of sound at 120 cycles, and at a distance of 42 in. from the tank wall. Both curves are plotted similar to polar form with reference to the tank wall. The average 120-cycle intensity is 0.385 dynes per sq. cm. The maximum is 0.685 and the minimum is 0.31 or a maximum to minimum ratio of 2.2 to 1.

After the sound measurements for Fig. 11 were completed the transformer was deenergized, and the room 120-cycle noise levels at 42 in. from the two ends of the transformer, were found to be 0.152 and 0.162 dynes per sq. cm. respectively. This indicated that approximately one-third of the sound curve of Fig. 11 represented external noise which did not come from the transformer.

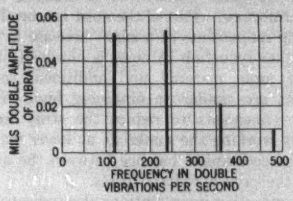

Fig. 13—Vibration Analysis at Mid Point of Transformer Side Wall

The distribution of sound at 120 cycles measured across the open side of a compartment 32 ft. wide and 22 ft. deep is shown in Fig. 12. The three walls were smooth surface brick with a glass window and glass door into

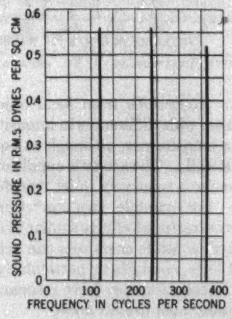

Fig. 14—Sound Analysis at 10 Ft. from Tank Wall at Location of Tests for Fig. 13

the substation building which formed the rear wall of the compartment. The 120-cycle sound intensities at the curb on the opposite side of the street, (approximately 50 ft.) were 0.0497 and 0.0416 dynes per sq. cm. opposite the left hand and right hand walls respectively.

Fig. 13 is a vibration analysis at the midpoint of the tank wall nearest to the open side of the compartment and Fig. 14 is a sound analysis at the center of the open side of the compartment.

Fig. 15 is a sound analysis made at a distance of 10 ft. from the drain valve of a transformer. When the distance was increased to 50 ft. from this transformer, the sound intensity was only 0.05 dynes per sq. cm. at the fundamental 120 cycle note and no higher frequencies could be detected.

On account of the very nature of a transformer, it will always have some hum, but this hum can be reduced for operation near hospitals or in residential districts. A quiet transformer is really a relatively quiet transformer. Part of the noise can be eliminated by adding refinements at a slight increase in cost and each further reduction in noise is obtained at the expense of more material and additional cost. The conditions are similar to obtaining high efficiencies, first by refinements to reduce the stray losses and finally by working the materials at lower densities to obtain the economical balance between the additional cost and the value of reduced losses.

These and other investigations also indicate that the sound intensities are not equal in all directions from the

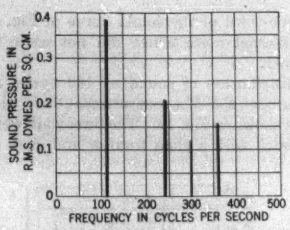

Fig. 15—Sound Analysis at 10 Ft. from a Transformer

transformer. Reflections from local objects near the substation cause wide differences in intensity at different points near a transformer, although at equal distances from the tank wall. The acoustic conditions at the substation can make a relatively quiet transformer appear to be noisy. For example, two converging walls or a corner can give directional effects similar to a megaphone. Some foundations, walls or objects can act as sounding boards. Oil lines, conduits or particularly sheet metal parts at the installation may have resonant frequencies which may cause them to emit noise.

If sufficient area is used to prevent crushing, cork pads are effective under transformer tanks in indoor vaults. Arbors, hedges, shrubbery and trees are very effective for absorbing sound from outdoor substations.

Conclusion

In addition to critical vibration frequencies of various metal parts, there are two distinct components which cause noise in power transformers. The load component varies as the square of the load current; the excitation component varies approximately 3 per cent for each per cent change in line voltage within the usual operating range, but this ratio and the proportions of load component to excitation component vary with different designs.

The surveys provide considerable advances in the knowledge of power transformer noise characteristics and magnitudes. The sound intensities are not uniform in all directions from a transformer, and the acoustic conditions at the substation play an important part in the final distribution of the sound. Our knowledge of this problem has not reached the stage where we can definitely predict in advance the sound emitted by a transformer, particularly since surrounding conditions are so important. Tools and methods have been devised for making further progress.

Acknowledgments

The writer wishes to acknowledge the work of Mr. J. P. Foltz, Research Engineer, who developed the vibration meter and sound analyzer which made this work possible. Mr. Foltz also assisted in making the tests. The writer also wishes to acknowledge the cooperation and assistance of members of the Pennsylvania and Ohio Power Co., and Detroit Edison Co., in making the surveys of transformers in service.

Discussion

F. W. Gay: The question of noise as related to electrical equipment is becoming of increasing importance and methods of reducing noise, or eliminating noise, will in the future receive correspondingly increased consideration. Mr. George does not bring out one very important point regarding the question of noise in electrical equipment; this point is the *time element*. During busy periods of the day and early evening, noise in electrical equipment is by no means as objectionable as during the quiet hours of the night. Fortunately the loads on electrical equipment during the quiet hours are relatively light and this fact at once suggests the advisability, if practicable, of lowering the densities at which materials are working during these light load periods.

A method which would seem practical if switching schemes were properly worked out, would be that of operating the transformer windings in series during light load periods, and in multiple during heavy load periods. As an example of such operation, if it be assumed that the transformer illustrated in Fig. 2 of the paper is operating at a core density of 11,000 gausses, a reduction to 7,500 gausses would represent a reduction to less than 70 per cent of normal density. Such a reduction, according to the curves shown in Fig. 2, would reduce the noise to very nearly the vanishing point, while the noises due to load as shown in Fig. 3 would not be serious during light load periods.

Now, if a transformer were designed for 7,500 gauss operation at light load with windings in series, and 15,000 gauss at heavy load with windings in parallel, such a piece of equipment would have an increased capacity of approximately 36 per cent and would only need to be operated in multiple for a few hours per day, or perhaps not at all for long stretches in the summer time when loads are generally light and noise is very objectionable.

A transformer designed for such series multiple operation could be operated to give a much higher average efficiency as well as very little noise during light load periods, and would have the added advantage of greatly increased overload capacity during periods of emergency.

Another method of practically eliminating noise would be to fully enclose the electrical equipment in noise-proof compartments and, in the case of transformers, resort to cooling by

radiators placed on the roof, using a volatile fluid to carry the heat from the transformer oil to the radiators on the roof. Such methods of carrying off heat have been used heretofore but have proven impractical by reason of the fact that the heat was dissipated from the radiators at only one temperature. It is suggested that several bodies of volatile fluid be used, each sealed separately from the others. One such body of volatile fluid may absorb heat at a high temperature from the oil at the top of the transformer tank and dissipate such heat from a radiator system on the roof at a corresponding high temperature while another body of volatile fluid may absorb heat from the oil at the bottom of the transformer tank at a low temperature and dissipate such heat from the radiator located on the roof at a corresponding low temperature.

A very great advantage can be obtained in the use of such a system of cooling, inasmuch as the radiator system dissipating heat at a high temperature need not receive air warmed by previous cooling operations as is necessarily so when the radiation devices are located around the transformer, and of necessity those radiation devices located at the top of the tank and adapted to radiate heat at a high temperature are bathed by hot air which has previously served to cool the lower portions of radiating surface which are radiating at a lower temperature.

This paper represents one of the first steps, which must be followed by practical design changes before the manufacturers can produce electrical equipment adapted to meet the probable exacting demands of the future.

Design changes should, if possible, be made in such a manner that not only will the noise be reduced, or eliminated, but other advantages will accrue which will at least in part compensate for the increased expense involved.

It would appear that there are many ways of reducing noise, some of which if properly worked out will produce equipment having better average efficiency, greater maximum capacity, smaller floor space dimensions, etc. These will go a long way to compensate for other disadvantages which must necessarily follow in departing from a design which has been so well worked out as the present oil-immersed self-cooled transformer.

W. W. Edson: As mentioned in the paper the noise from power transformers adjacent to a building is frequently quite troublesome, especially in a residential neighborhood. One solution which still appears to be satisfactory after several years' service places the transformers in a pit in front of the station. The noise is reflected upward and is not noticeable to the residents across the street.

R. B. George: The condition Mr. Gay calls the time element is really the combination of light load and a slightly higher voltage applied to the transformer terminals due to lower voltage drop in the line between the generating station and the transformers. The noise due to load current, illustrated by Fig. 3, is usually negligible for this condition. The effect of increased voltage, illustrated by Fig. 2, causes a slight increase in sound intensity from the transformer. When the traffic noise ceases in the vicinity, it may be possible to hear the transformer. Equipment for changing transformer taps under load can be used to control the voltage delivered to the transformers to obtain noise reduction by taking advantage of the characteristics illustrated in Fig. 2.

Both of the schemes described by Mr. Gay should be effective for reducing noise, but the writer favors simplicity in the apparatus and giving more attention to the acoustic conditions at the substation. Mr. Edson's scheme of placing the transformer in a pit should be effective. When this scheme is used, there should be no roof above the pit. There is, however, a possibility of the building walls at the sides above the pit serving as reflective surfaces which may reduce the effectiveness of a shallow pit. When a transformer is placed in a pit, it may be necessary to provide additional ventilation for cooling.

I wish to add that felt, cork, and other sound absorbing materials which are effective at frequencies above 500 cycles are not very effective for absorbing the low frequency 120-cycle note of a 60-cycle transformer.

One of the most effective schemes known to the writer consists of building four walls of either asbestos board, cork, pressed fiber board, or preferably more substantial material built similar to a chimney, extending to several feet above the top of the transformer, and serving to direct the sound upward.

Governor Performance During System Disturbances

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R. J. CAUGHEY,* Non-member
E. M. HUNTER,* Associate, A. I. E. E.
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Synopsis.—*This paper describes short-circuit tests which were made on a large power system under normal load conditions, in order to ascertain the operating characteristics of the system and the turbine governors during system disturbances. The introduction gives a brief description of the main features of the station and the connected system, a statement of the tests, and a list of the major conclusions. Part I covers the short-circuit tests on the system to determine the stability limits. Part II describes the load transfer and short-circuit tests to produce "governor pumping." Part III deals with the short circuits which were made to investigate the effect of a special valve mechanism on the stability limits. Part IV contains a discussion of miscellaneous tests, data, and general comments. In appendixes A and B, a detailed description of the system station and test equipment is given with estimates of the stability limits and short-circuit currents to be expected. Appendix C deals with the theory of the change in speed of a rotating body for a given energy differential.*

* * * * *

Introduction

In the past few years, the amount of analytical and test work on system disturbances and stability has increased tremendously, but very little of this theoretical or practical activity has dealt with the operation of steam turbine governors. Therefore when field tests appeared advisable to determine the cause of trouble evidenced by overspeeding and consequent tripping of the emergency overspeed governors on the two 50,000-kw. single-cylinder reheat type turbines at the Stanton Steam Plant during the lightning seasons of 1928 and 1929, this opportunity was used to make a thorough investigation of the operation of turbines during system disturbances.

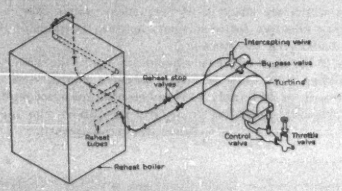

Fig. 1—Schematic Diagram of Piping and Valves of Stanton Turbines

This plant is located near Pittston, Pa., and is jointly owned by the Scranton Electric Company and the Pennsylvania Power and Light Company. The steam connections and valves of the turbine are shown schematically in Fig. 1. All governing is normally done by the main control valve at the first stage of the turbine. Each generator has its own transformer bank and 66-kv. bus. Power is normally supplied by generator No. 1 to the Scranton Electric Company, and by generator No. 2 to the Pennsylvania Power and Light Company. The 66-kv. buses are tied together by a bus-tie reactor. Due to this reactor, generator No. 1 tends to swing with the Scranton Electric system and generator No. 2 with the Pennsylvania Power and Light system.

Fig. 2 and Fig. 3 show a one line diagram of the two systems and of the stations respectively. Further details of the system, station, and test equipment will be found in Appendix A.

More than fifty tests were made during this investigation, which was started in July 1929 and continued over a period of several months.

The decision of an operating company to make tests involving major system disturbances is based on the reasoning that such disturbances are a recognizable part of normal operating experience and that these disturbances must be understood in order to insure safe and intelligent operation of any system. The best method for obtaining this information quickly is by actually carrying out a set of previously planned tests that will duplicate normal conditions.

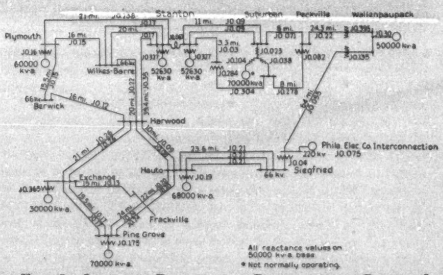

Fig. 2—One-line Diagram of Pennsylvania Power & Light and Scranton Electric Company's System. Only Equivalent Circuits Shown for Some Parts

General Summary of Results

The results obtained shed additional light on some phenomena which have occurred during system dis-

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turbances and lead to the following general conclusions:

1. It is possible for a generator to lose synchronism and pull back in again without giving any conclusive evidence in the control room or at the turbine that such a change has occurred. Graphic charts and indicating instruments are not fast enough to indicate out-of-synchronism conditions reliably. Thus, this condition occurs far oftener than has been generally appreciated.

2. Cases of sustained system oscillation, or surging, seldom occur unless some generator, or group of generators, loses synchronism. Any oscillation resulting from a disturbance that is not sufficiently severe to cause loss of synchronism quickly dies out, due to the inherent system damping.

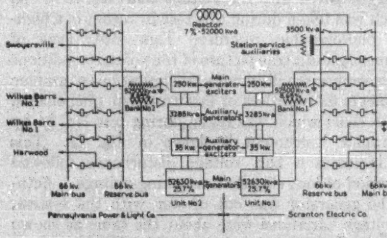


FIG. 3—ONE-LINE DIAGRAM OF STANTON STATION. REACTANCE VALUES EXPRESSED ON APPARATUS KV-A. BASE

3. Governors properly designed and adjusted will not contribute to any system oscillation as long as synchronism is maintained. When out of synchronism, however, the governor attempts to follow the speed change as the generator slips poles, and the result is a "pumping" phenomenon familiar to many station operators. This pumping action is a result rather than a cause of a sustained oscillation and is to be expected when the generator and the system are out of step at a certain slip frequency. It will cease if the generator pulls back into synchronism.

4. The speed of the turbine when out of synchronism is determined by the regulation of the governor and the induction generator characteristics of the generator. The average output during this period may be an appreciable portion of the output before loss of synchronism.

5. In these tests the short circuits resulted in considerable reduction in the kilowatt output of the generators with consequent overspeeding and loss of synchronism. Special valve equipment for quick reduction of steam input following a sudden drop in generator output will increase the stability limits. Under certain conditions it may be of considerable benefit in maintaining stability, but under other conditions the equivalent improvement in stability may be better obtained by other means.*

Part I

STABILITY LIMITS DURING SYSTEM SHORT CIRCUITS

Some preliminary tests were made by dropping and picking up load on one of the turbines to be sure that the governor and all other equipment was functioning correctly under normal conditions of system operation; these tests did not indicate anything erratic in the operation.

As short circuits were to be placed on one of the lines to the Scranton Electric Company from the 66-kv. bus supplied by generator No. 1, calculations were made to determine the magnitude of the fault currents and the stability limits of the system under short-circuit conditions. Fig. 4 shows the results of these preliminary calculations, which indicated that the system would be stable with a one-conductor-to-ground short circuit, but unstable with a two-conductor-to-ground short circuit, when the generator was carrying full load. These calculations also showed that if the load on the generator was reduced to 25,000 kw. and the short circuit was cleared in less than ¾ second, the system would be stable. The tests verified the calculations. For further discussion of the basis of these calculations, refer to Appendix B.

Test No. 11. The first short-circuit test consisted of a single-conductor-to-ground short circuit on the 66-kv. bus associated with generator No. 1. The auxiliaries for No. 1 turbine were supplied from the station transformer bank instead of from the house generator on this turbine, so that the full electrical output of the turbine would be effected by the short circuit. The voltage regulators on the main generators were left in service. Fig. 5 shows the resultant kilowatt swing in the output

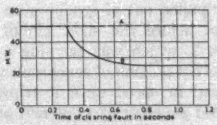


FIG. 4—POWER GENERATOR NO. 1 CAN CARRY AS A FUNCTION OF SWITCHING TIME

A. Rated megawatts of turbine
B. Two-conductor-to-ground fault outside generator transformer
Note: With a single-conductor-to-ground fault the stability limit is above rating of generator

*Such special valve equipment will vary in complexity with the size and type of the turbine and may be considered inadvisable for certain types. Its application to existing turbines may require a redesign of the valve controlling mechanism and the oil pressure system in order to get the necessary valve closing time of ⅛ to ¼ of the time at present required for normal governor operation. This equipment performs a function entirely different from that of the turbine speed governor and is not to be considered as a necessary adjunct to it.

of this generator. There was practically no movement of the governor beam during this short circuit and consequently no change in the steam input to the turbine. It should be noted that the inherent system damping caused the oscillation of the generator to die out very quickly. The short circuit caused a momentary reduction from 38,000 kw. to 25,000 kw. in the output of the turbine.

As predicted by the calculations and evidenced by

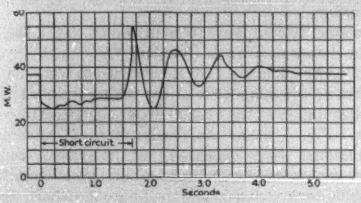


Fig. 5—Test 11. One-Conductor-to-Ground Fault. Both Generators Stable

this test, a much more severe short circuit would have to be imposed on the system in order to produce the instability of generator No. 1 which had been experienced during lightning conditions. All the following short-circuit tests were, therefore, two conductors to ground.

Test No. 12. Fig. 6 shows the record obtained from the next short-circuit test. The output of generator No. 1 was reduced from 40,000 kw. to 22,000 kw. and the generator pulled out of synchronism with the rest of the system before the short circuit cleared, but generator No. 2, which was carrying a load of only 31,000 kw., did not pull out of synchronism. Generator No. 1 increased in speed until it tripped the emergency overspeed governor at 1,960 rev. per min., shutting off all steam input to the turbine The turbine then drifted down to normal speed and pulled into step with the system 5½ seconds after the short circuit occurred.

There was very little reduction in steam input to the turbine until the emergency overspeed governor operated and closed both the throttle and intercepting valves. This was due to the stored energy in the reheat boilers and piping, and would not be true for an ordinary single-cylinder, non-reheat type of turbine. The results obtained from this test were exactly analagous to those previously experienced during lightning storm conditions. The indicating meters on the control board swung violently during the first one or two seconds of the disturbance, and again as the machine pulled into synchronism. During the intermediate time, the slip frequency was so high that the indicating instruments could not follow the changes. The entire disturbance was over in six seconds with nothing to indicate to the operator, except the tripped emergency overspeed governor, that the turbine had been out of step with the system.

A graphic voltage recorder with high-speed feature is normally used to record transient disturbances on this 66-kv. bus. The chart obtained from this instrument during this test is shown in Fig. 7A. Fig. 7B was obtained during a lightning storm when the emergency overspeed governor operated. The exact similarity of these charts indicates that the conditions existing during this test must have closely approximated those experienced during the storm disturbances. The voltage chart obtained from the graphic recorder should also be compared with the voltage chart shown in Fig. 8, which was taken during this test by means of a high-speed photographic recorder. This later type of instrument gives a true picture of the voltage conditions and the comparison illustrates the limitations of graphic instruments as recorders of fast transient phenomena. Another chart which was taken during test No. 34 by this graphic instrument is shown in Fig. 7E. This chart indicates a bad voltage disturbance followed by a period of low, slightly fluctuating voltage. Actually, the generator was out of step with the system, and voltage variations were about the same as shown in Fig. 8 obtained by the photographic recorder. Considerable caution must therefore be used in interpreting system disturbances from such graphic charts.

Test No. 13. Calculations had indicated that generator No. 1 would be stable with loads up to 25,000 kw., and a two-conductor-to-ground short circuit duration of less than ¾ second. A two-conductor-to-ground short circuit was thrown on the system with a 25,000 kw. load on generator No. 1, but due to the relay setting which kept the short circuit on for 1.7 seconds,

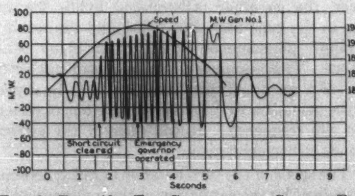
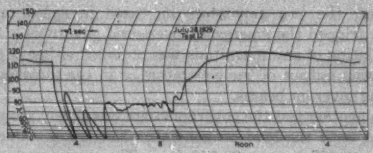


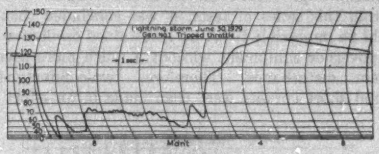
Fig. 6—Test 12. Two-Conductor-to-Ground Fault. Generator No. 1 Unstable, Generator No. 2 Stable

the generator again went out of synchronism but did not trip the emergency overspeed governor. Generator No. 2 being heavily loaded, 43,000 kw., lost synchronism with the rest of the system and tripped the emergency overspeed governor. Violent oscillation of the governor beam of generator No. 1 occurred before it resynchronized, but unfortunately, the oscillograph films had run through before this oscillation took place and no record of it was obtained.

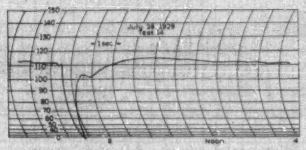
Test No. 14. The relay setting was changed to reduce the short circuit time to 0.65 seconds and the



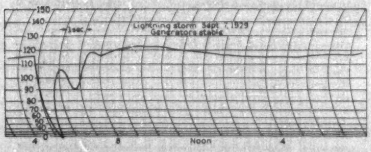
A. Voltage on Scranton Electric Co. 66-kv. bus at Scranton obtained by graphic recorder
Test 12. Generator No. 1 unstable
Generator No. 2 stable



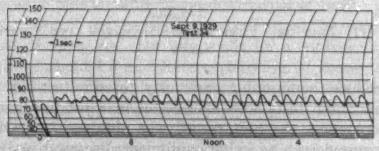
B. Lightning storm
Generator No. 1 unstable
Generator No. 2 stable



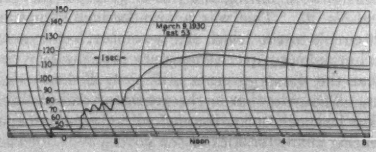
C. Test 14—Both generators stable



D. Lightning storm. Both generators stable



E. Test 34, Generator No. 1 unstable
Generator No. 2 stable during period of this chart



F Test 53, Generator No. 1 unstable
Generator No. 2 stable

FIG. 7

short-circuit test was repeated. Fig. 9 shows the resultant kilowatt swing of generator No. 1. The system was stable and there was very little movement

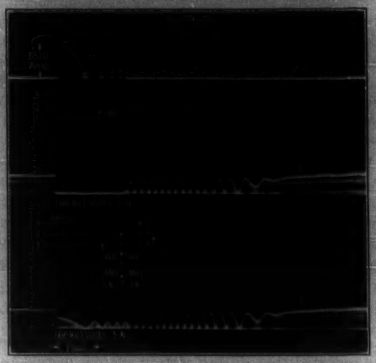


FIG. 8—TEST 12. GROUND CURRENT AND VOLTAGE ON SCRANTON ELECTRIC CO.

66-kv. bus at Scranton obtained by high-speed photographic recorder. Compare B and C with Fig. 7A

A—Fault ground current
B—Voltage phase 1-*N* 66-kv. bus
C—Voltage phase 3-*N* 66-kv. bus

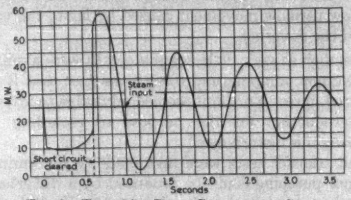


FIG. 9—TEST 14. BOTH GENERATORS STABLE

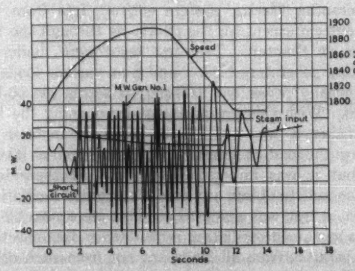


FIG. 10—TEST 23. BOTH GENERATORS UNSTABLE, OUT OF SYNCHRONISM WITH THE SYSTEM AND WITH EACH OTHER

of the governor beam, consequently, no change in the steam input to the turbine. Fig. 7C shows the graphic voltmeter chart made during this test and Fig.

7D a very similar chart obtained during a lightning storm. These last two tests illustrate the value of fast clearing of the short circuit.

Test No. 23. After obtaining longer films for the oscillographs, another test was made under the same load and short-circuit conditions as in Test No. 13. Fig. 10 for this test shows that both generators were out of synchronism not only with the system but also with each other. This is evidenced by the uneven peaks in the curve showing the kilowatt output of generator No. 1.

From this group of tests it is evident that generators lose synchronism for a brief time and then pull back into step again without giving evidence of this change, except by the magnitude of the disturbance; and also that system disturbances sustained after short circuits have been cleared are caused by loss of synchronism.

Part II

Governor Pumping

Load Transfer Tests. Several operating companies have reported that during certain system disturbances the governor beams of their turbines have oscillated, and that this oscillation has continued for 2 or 3 minutes, or until some means was taken to correct the condition, such as holding the governor beam, or disconnecting the turbine from the system. This phenomenon has generally been termed "governor pumping" and should not be confused with governor hunting which is a slow periodic movement caused by lag of the valve behind the governor.

Inasmuch as very little adequate data were available on governor pumping and due to the fact that it had appeared in test No. 13, it seemed advisable to attempt its reproduction for the purpose of recording and analyzing this disturbance.

Load transfer tests were first made to determine if governor pumping could be produced by a disturbance of a minor magnitude. In these tests the 66-kv. tie from Suburban to Wallenpaupack was opened leaving the Scranton Electric system entirely free from the other system except for the 66-kv. tie through the bus reactor at Stanton. The first tests were made by operating generators Nos. 1 and 2 at various loads so that initially power was transferred across the 66-kv. bus tie. With this arrangement, by opening the bus tie breaker it was possible to suddenly increase or decrease the load on generator No. 1 as much as 20,000 kw., but it was impossible with any of these load transfer tests to obtain any sort of sustained oscillation.

Next, a test was made to see if it were possible to start a sustained oscillation by suddenly disconnecting the generators from all loads but not from each other when they were carrying unequal portions of the load. Generator No. 1 was loaded to 30,500 kw. and No. 2 to 10,000 kw., and then the oil circuit breaker to the load was tripped. Generator No. 1 dropped to 5,000 kw. load as a generator and No. 2 took 5,000 kw. as a motor; the two generators increased in speed until they tripped the emergency overspeed governors. No oscillation of any magnitude was obtained either in the kilowatt output of the generators or in the movement of the governor beams.

In another test the governor beam was slowly pressed down until the load on generator No. 1 was reduced from 40,000 kw. to 22,000 kw. The governor beam was then released and the generator picked up its previous load of 40,000 kw. without any appreciable oscillation of governor beam or kilowatt output.

From this series of tests, it was concluded that it would be impossible to set up a sustained oscillation in the governor equipment without a much more severe system disturbance. Therefore, two-conductor-to-ground short circuits were then made in an effort to

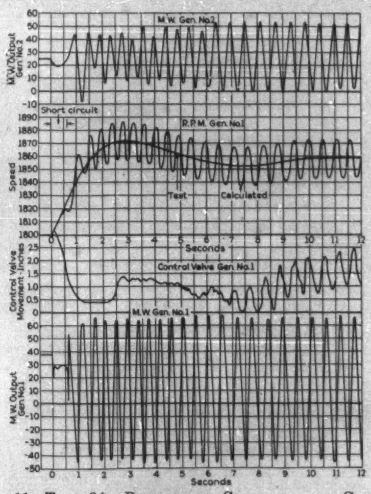


Fig. 11—Test 34. Pumping of Governor and Control Valve

reproduce the pumping previously noted in test No. 13. While these tests were successful in reproducing the phenomenon, the best record was obtained in testing the special turbine valve equipment discussed in Part III. The records from test No. 34 are therefore used as a basis for the following analysis of governor pumping.

Test No. 34. This was a two-conductor-to-ground short circuit test in which the generator lost synchronism but did not trip the emergency overspeed governor so that the speed governor still controlled the steam input. The reduction in the generator output due to the loss in synchronism coupled with the normal speed regulating characteristics of the governor resulted in a sustained prime mover speed above that of the system.

Fig. 11 shows the results of this test. During the

first 2.5 seconds the movement of the main control valve was controlled by an auxiliary pilot valve which was being used in a preliminary form in an attempt to keep the turbine in synchronism; this pilot valve, however, had no effect on the governor or control valve after this initial period. Seven seconds after the short circuit occurred the governor mechanism started to oscillate at a period coinciding exactly with the kilowatt oscillation of the generator. The speed curve shown is a

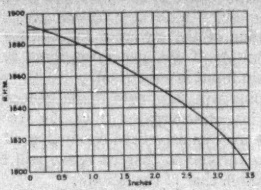


Fig. 12—Test 34. Control Valve Position Versus Turbine Speed

composite curve, composed of an average speed curve obtained from the oscillograph record, and a superimposed speed change calculated from the output of the generator and added to the average speed curve. From this composite curve it can be seen that if the governor follows the speed of the turbine it must alternately open and close the control valve an appreciable amount. The curves show that the governor was doing its best to follow these speed changes, although with about a quarter second time lag, and was thus varying the steam input in accordance with the speed, which is exactly what a speed governor is designed to do.

Fig. 12 shows the relation between speed and control valve position for this particular test. About 8 seconds after the occurrence of the short circuit, the speed of the turbine was approximately 1,860 rev. per min. with a periodic plus and minus speed variation of 15 rev. per min. This 30 rev. per min. speed change at the average speed of 1,860 rev. per min. calls for a control valve stroke of about 1.3 in. Reference to Fig. 11 shows that the stroke of the control valve at this time was about 1.3 in. and the average position was about correct for the average speed.

During the interval from the third to the seventh second, the periodic speed change caused by the generator slipping poles was so fast that it was impossible for the control valve to follow it. However, as soon as the average turbine speed came down to a point where the governor mechanism and control valve could follow the periodic changes in speed, the control valve started a pumping movement in which it assumed the proper position corresponding to the speed. This pumping of the governor beam was therefore a normal action and should be expected under conditions such as existed at the time of its occurrence.

Fig. 13 shows the speed regulation curve for the governor of this turbine at the load carried when this test was made. If the turbine was completely unloaded it would run about 5 per cent above the normal system speed. The figure also includes a curve showing the relation between speed of the turbine and the induction generator output of the generator for the voltage conditions which existed on this system during this test. The point where the induction generator curve crosses the speed regulation curve determines the speed of the turbine and the load which it will carry when it is out of synchronism. The average output of the generator during the period covered by the curves was about 13,000 kw. and the speed was about 3.3 per cent above normal.

Due to the steam storage in the reheat boilers on this particular type of turbine, the oscillation of the main control valve had very little pulsating effect on the steam energy input to the turbine. This was evidenced by practically constant steam input pressure obtained on a pressure recorder at the eighth stage where steam is admitted to the turbine from the reheat boilers.

During the period that the governor beam was oscillating in synchronism with the kilowatt oscillation of the generator, the speed of the generator was about 1.8 cycles above the speed of the system. This represents a slip frequency of 108 cycles per minute. The natural period of oscillation of the governor weights and springs, without the frictional resistance of the pilot valve and oil system, is about 200 cycles per minute. Due to this difference in natural period of oscillation, it seems reasonably sure that the governor mechanism itself contributed nothing to the oscillation of the governor beam and control valve. The speed change must

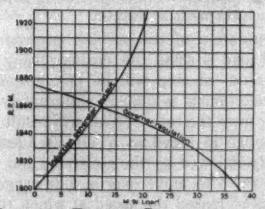


Fig. 13—Test 34. Turbine Regulation and Induction Generator Power

at least be considered the primary cause of the oscillation or pumping.

An effort was made to stop the pumping of the governor beam by steadying it by hand; the oscillation itself was damped by this means, but the turbine did not slow down and resynchronize due to the fact that no reduction was made in steam admission. The turbine was finally pulled into synchronism with the system when the main throttle valve was closed by hand, allowing the turbine to come down in speed. Considerable oscillation had been noticed in the meantime

on generator No. 2 and it was removed from the system by tripping the generator oil circuit breaker.

There are two ways of curing this governor pumping after it has once been started. The first method, which may be preferred in many instances, is to reduce the speed of the turbine so that it will synchronize with the rest of the system. The second method is to remove the generator from the system by tripping the generator oil circuit breaker. The operator in the switchboard gallery can apply the first method by operating the synchronizing motor on the governor spring, and the second method by tripping the generator oil circuit breaker. The turbine man can also apply the first method by tripping the throttle valve. This valve, however, should be reopened the instant the generator pulls into synchronism to avoid burning the turbine blades. As the turbine speed will decrease rapidly when the throttle valve is closed the generator will usually resynchronize by the time the turbine man is ready to reopen this valve.

Previous investigations of this peculiar governor pumping have shown that it was apparently possible to damp out such an oscillation during some tests by merely introducing frictional resistance on the governor beam, but during other tests this frictional resistance method was not successful. In all previous investigations it has only been possible to set up governor pumping by some very severe system disturbance, which in all probability meant that the generator was out of synchronism with the rest of the system. The results of these previous investigations also suggest that the primary cause of such governor pumping is loss of synchronism and consequent periodic speed change of the generator involved as it slips poles as found in the tests made at Stanton.

If the regulation of the turbine governor from no load to full load is small the slip frequency when the generator is out of step will be low, consequently a relatively long time is available to alternately store and take energy from the rotor. This will result in a large periodic change in speed at a low frequency which can be easily followed by the governor, and will cause the control valve to move over a wide range. If the regulation is increased the speed at which the turbine will run when out of synchronism will be higher, the slip-frequency between the generator and the rest of the system will be greater, and the speed change as the generator slips poles will be less. All of these factors tend to reduce the effect on the governor and make it more difficult for it to follow the periodic speed changes due to pole slipping. This may explain why it has apparently been possible to cure some cases of governor pumping by merely broadening the regulation range of the governor.

Reducing the field of the generator as has been done in some previous tests, would also tend to reduce the magnitude of the periodic speed change as the generator slips poles, and consequently reduce the effect on the governor.

From this series of tests the following facts seem to be evident:

1. The turbine governors operate correctly under all normal conditions of load transfer and system disturbances as long as the generators remain in step with the rest of the system.

2. Pumping of the governor mechanism occurs primarily because of a periodic change in speed of the turbine.

3. Pumping of the governor mechanism may continue indefinitely if the generator loses synchronism and the steam input is not cut off.

4. Pumping will terminate if the generator is synchronized with, or segregated from, the system. Synchronization may be expedited by cutting off the steam input to the turbine so that its speed will be reduced and again match that of the system.

Part III

Tests with Special Valve Equipment

In the previous short-circuit tests, all really severe system disturbances were accompanied by a loss of synchronism of one or both of the generators. When the generator lost synchronism under light load conditions and did not trip the emergency overspeed governor, considerable oscillation, or pumping, of the governor beam was noted To prevent both the severe system disturbances and the pumping of the governor beam, it was evidently necessary to keep the generator in synchronism with the rest of the system. One solution of the problem was, of course, to reduce the short-circuit time as shown by Tests Nos. 13 and 14. However, it was quite doubtful that sufficient decrease in short-circuit time could be secured and selective relaying also maintained with existing system connections and equipment. Therefore, the possibility of holding this turbine in synchronism by rapid reduction of input steam following a system disturbance seemed in this case to justify investigation. A special power surge relay and a valve mechanism were therefore designed to shut off the steam input to the turbine by closing the main control and intercepting valves temporarily when a short circuit occurred.

The power surge relay was a three-phase wattmeter element arranged with a normally open pair of contacts which would not close for a slow change in power output of the alternator, but for a sudden decrease in the output would close quickly and then remain closed for a time proportional to the magnitude and duration of the power change. This measure of the reduction in the kilowatt output of the turbine determined the time that the steam valves of the turbine remained closed, and thereby reduced the steam input in proportion to the reduction in the output. The valves were then allowed to reopen and the turbine to resume the same load that it was carrying before the disturbance.

In order to obtain a fast reduction in steam input to the turbine it was necessary to operate both the control valve and the intercepting valve on the reheat loop. The control valve with its oil operating cylinder and pilot valve was not changed as it was found that the closing time of this valve was short enough to give the desired results due to the fact that there was only about 25 per cent of the total turbine output under control of this valve during the first two seconds. A solenoid operated valve was added to the pilot valve to permit its control by the power surge relay. This arrangement also provided for the necessary free control of the pilot valve by the operating governor under normal load conditions.

The intercepting valve on the reheat loop, however, was found to operate too slowly. This necessitated a new valve cylinder with larger ports, and a stronger operating spring. A separate trip valve was in-

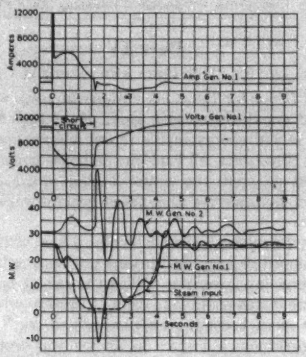


FIG. 14—TEST 51. SPECIAL VALVE EQUIPMENT. BOTH GENERATORS STABLE

stalled, large enough to insure dumping of the operating cylinder oil in the required time. This trip valve was also oil operated through a solenoid operated pilot valve. As originally designed the intercepting valve was given a long stroke to insure a small pressure drop through it, but while making these tests this stroke was reduced so that the steam flow was restricted as soon as the valve started to close. In this manner a closing time of about ¼ second was obtained for these valves.

Tests of the special valve equipment prior to these changes did not give the desired increase in stability due to the slow closing of the intercepting valve.

Test No. 51. Tests Nos. 14 and 23, the results of which are shown in Figs. 9 and 10, respectively, were two short circuits with similar load conditions on the turbine, but with a different duration of the short circuit. These tests were made with standard governor control equipment. In Test No. 23 both generators No. 1 and No. 2 lost synchronism with each other and with the rest of the system.

Fig. 14 (Test 51) shows the results of a two-conductor-to-ground short circuit on the 66-kv. bus with the kilowatt load on generator No. 1 the same as for Tests Nos. 14 and 23, and the short-circuit time the same as for Test No. 23. The load on generator No. 2 was, however, somewhat reduced from its previous load. The steam input curve on this figure shows how the high-speed valve mechanisms used in this test, but not in Tests Nos. 14 and 23, reduced the input so that the

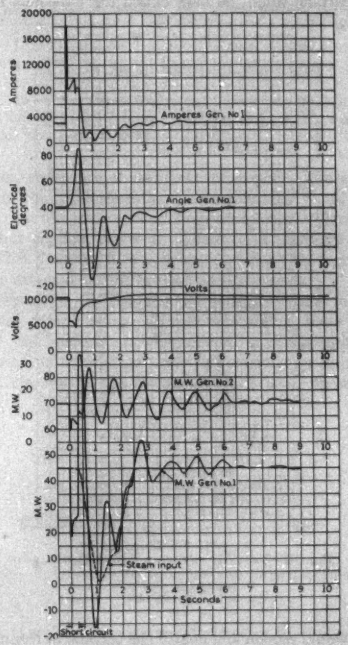


FIG. 15—TEST 52. SPECIAL VALVE EQUIPMENT. BOTH GENERATORS STABLE

generator not only did not lose synchronism with the rest of the system, but even drifted back in angle so fast that it was taking power as a motor when the short circuit cleared.

Test No. 52. Another short-circuit test was then made with full kilowatt output from generator No. 1, but with the relay set so that the oil circuit breaker opened the short circuit in 0.33 seconds. Fig. 15 shows the results of this test. Note that the generator did not pull out of synchronism.

Test No. 53. In order to see if the equipment would

hold the generator in synchronism irrespective of the duration of the short circuit, another test was made with full load on generator No. 1, but with the relay adjusted so that it cleared the short circuit in 1.78 seconds. The results of this test are shown in Fig. 16. The steam input for generator No. 1 is not available on account of faulty operation of some of the recording devices. Generator No. 1 pulled out of synchronism with the rest of the system, and after slipping 16 poles, pulled back into step again. A special asynchronous relay had been included in the test equipment so that if the generator did lose synchronism the steam valves of the turbine would be kept closed for 7 seconds before

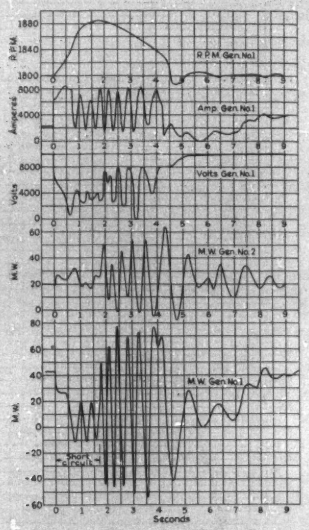


FIG. 16—TEST 53. SPECIAL VALVE EQUIPMENT

Generator No. 1 unstable
Generator No. 2 stable

they would reopen and enable the turbine to resume its full load.

Although generator No. 1 pulled out of synchronism with the system during this test, it is felt that by a few further refinements in the relay and valve mechanism, it would be possible to keep this generator in synchronism with the rest of the system even for short circuits as severe as in this test.

These tests evidence the possibilities of this method for improving system stability and indicate results which are comparable with those obtained by the use of the more commonly proposed means for attaining the same end.

It should be borne in mind that the application of this scheme to certain complex types of turbines can not always be economically justified owing to the large amount of supplementary control equipment required, and the unreliable operation which may ensue.

TABLE I
SUMMARY OF TESTS

Test No.	Initial load in mw. Gen. No. 1	Initial load in mw. Gen. No. 2	Duration of short circuit cycles	Valve control	Stability of system	Fig. No.	Remarks
11	38	30	100	Standard	Stable	5	1 C. G. Fault
12	40	31	100	Standard	Unstable	6, 7, 8	2 C. G. Fault
13	25	43	100	Standard	Unstable		2 C. G. Fault
14	25	43	39	Standard	Stable	9	2 C. G. Fault
20	35	43	..	Standard	..	20	Load Transfer Test
23	25	43	100	Standard	Unstable	10	2 C. G. Fault
34	38	25	40	Semi-Special	Unstable	7, 11	2 C. G. Fault
50	35	20	..	Special	..	20	Load Transfer Test
51	25	31	98	Special	Stable	14	2 C. G. Fault
52	44	20	19½	Special	Stable	15	2 C. G. Fault
53	45	20	106	Special	Unstable	16	2 C. G. Fault

Part IV

MISCELLANEOUS TEST DATA AND GENERAL COMMENTS

Load Dropping Tests. A number of tests was made to determine the increase in speed of the turbine caused by the stored energy in the reheat boilers and piping. During these tests, all load on the house generator was removed so that the entire turbine load could be dropped by opening the main generator oil circuit breaker.

With the reheat boilers in service, a load of 32,000 kw. was dropped causing the turbine to overspeed and trip the emergency governor. With the reheat boilers out of service it was possible to drop 31,500 kw. without the turbine speed becoming high enough to trip the emergency governor; but when 36,500 kw. was dropped, the emergency governor tripped at 1,955 rev. per min. The turbine speed increased to a maximum of 1,965 rev. per min. during this test.

With the reheat boilers in service, one test was made by connecting the generators so that load was dropped from both generators, but the generators themselves were left connected together. Generators Nos. 1 and 2 initially carried 30,500 kw., and 10,000 kw. respectively. Generator No. 1 continued to operate as a generator, driving generator No. 2 as a motor, and under these conditions, the machines overspeeded together and tripped both emergency governors.

The standard governor was used in all these tests and it was found that the pilot and main control valves were very fast in following the movement of the governor beam. The pilot valve moved within 2 cycles and the main control valve within 4 cycles after the movement of the governor beam. The governor itself responded in two or three cycles to any change.

Load Transfer Tests. Of the many load transfer tests made on the Stanton turbines, two were of outstanding interest in showing the effectiveness of the

special valve equipment. In these tests, system connections were such that the initial 35,000 kw. output of Generator No. 1 was divided, 11,000 kw. being sent through a long tie line by the way of Wallenpaupack to the Pennsylvania Power & Light Company, and the remaining 24,000 kw. being supplied direct to the Scranton Electric Company. The Scranton Electric breaker was then opened, forcing the entire 35,000-kw. output over the Wallenpaupack tie line. The kv-a. loading of this tie line was limited to 18,000 kv-a. by

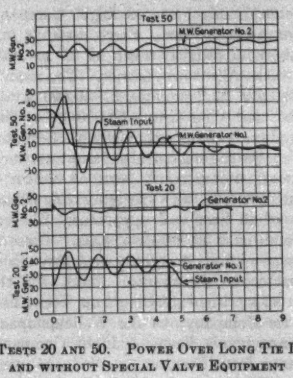


Fig. 17—Tests 20 and 50. Power Over Long Tie Line with and without Special Valve Equipment

relays at Wallenpaupack. For Test No. 20, Fig. 17, the turbine was equipped with a standard governor. After the breaker controlling the Scranton Electric load was opened, the generator load oscillated about an average output of 35,000 kw., until the breaker at

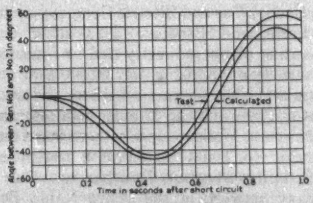


Fig. 18—Test 52. Comparison of Test and Calculated Relative Angle Between Generators Nos. 1 and 2

Wallenpaupack tripped on overload. In Test No. 50 (Fig. 17), the same switching arrangements were used, but the turbine was equipped with special valve mechanism. In this test, the valves started to close as soon as the Scranton Electric breaker opened and the output of the generator was sufficiently reduced so that the breaker at Wallenpaupack did not open.

Damping. The data on the Scranton tests also gave some indication of the change in damping of the generators with different values of resistance in the armature circuit.

Fig. 9 shows the damping of generator No. 1 with small external armature resistance. The amplitude of the oscillations of the generator decreases quite rapidly and after a few swings dies out entirely. Fig. 17 shows the oscillations of the same generator with considerable

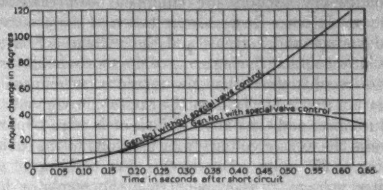


Fig. 19—Calculated Swing of Generator 1 During a Two-Conductor-to-Ground Fault with and without Special Valve Equipment

external resistance in the armature circuit. The amplitude of the oscillations decreases quite slowly and indicates that under these conditions the generator damping was but slightly positive. These tests therefore indicate that damping in a synchronous machine decreases as the external armature resistance increases.

Comparison of Tests and Calculation. Many mathematical computations of system stability have been made but seldom has there been an opportunity to make actual checks on these calculations by applying short circuits to the system while it is in operation.

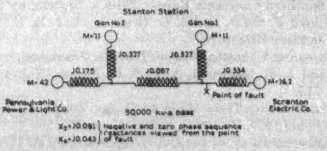


Fig. 20—Equivalent Reactances and Inertia Constants of the Systems

It is, therefore, interesting to compare the values obtained from the tests with values calculated for the same conditions. Such a comparison is given in Fig. 18 which shows the relative angle between the two Stanton generators during a two-conductor-to-ground fault. It will be noted from this figure that the calculated curve has the same relative shape as the test curve and follows it quite closely, especially during the first swing of the generators.

Calculations were also made to determine the increase in stability to be expected with the special valve control mechanism. The curves shown in Fig. 19 show the angular change of generator No. 1 due to a two-conductor-to-ground fault with and without the special valve control equipment. These curves indicated that

generator No. 1 could be made to remain in synchronism with the system with the special valve control equipment. This was substantiated by test.

Effect of Voltage Regulators. The main generators have direct-connected, self-excited, 250-volt exciters. The rate of exciter voltage build up at the voltage required for full load on the generator is 80 volts per second. The exciters are equipped with rheostatic type regulators. Records were taken of the generator field voltage and current, and the voltage across the regulator contacts during some of the short-circuit tests. These records show that the regulator attempted to maintain maximum field on the exciter throughout the disturbance, and that about 2.5 seconds were required to raise the exciter voltage from 175 volts to the exciter ceiling of 275 volts. Due to this slow build up of the exciter voltage, it is evident that the regulator had very little effect during the time the short circuits were on the system.

ACKNOWLEDGMENT

The authors wish to express their appreciation of the cooperation of Messrs. C. H. McKnight and J. E. Geue of the Stanton Operating Company in the performance of these tests and arrangement of system operation with connecting companies. They also wish to acknowledge the assistance of Mr. R. H. Park who initiated the tests with the high-speed valve mechanism, Mr. J. W. Dodge for assistance in the preparation of the test results and Mr. D. M. Jones for criticism of the paper.

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Appendix A

DESCRIPTION OF SYSTEM, STATION, AND TEST EQUIPMENT

Description of System. The Scranton Electric Company and the Pennsylvania Power & Light Company serve a considerable portion of eastern Pennsylvania. The Scranton Electric Company supplies power to the section in and around the city of Scranton. This is normally received from the Stanton Plant, and distributed over the 66-kv. and 22-kv. networks. The Suburban Plant is located in the city of Scranton, and has an installed capacity of approximately 70,000 kv-a. This plant is not normally in operation, but was used during these tests to insure continuity of load supply. The 22-kv. network is normally fed through a 30,000-kv-a. transformer bank located at Suburban.

The Pennsylvania Power & Light Company serves the section around Wilkes Barre, west as far as the Northumberland district, south as far as Harrisburg, and east to the Allentown district. The main generating plants are Wallenpaupack, 40,000 kw., Hauto 55,000 kw. and Pine Grove 55,000 kw. The transmission system is composed chiefly of 66-kv. lines.

Fig. 2 shows the main transmission network of the two systems, unimportant portions shown only as equivalent networks, with the generating capacity and reactance of all lines and apparatus. The reactance values are all on a 50,000-kv-a. base. These two systems have two interconnections, one, through a 66-kv. bus tie reactor at the Stanton Plant, and the other through a 66-kv. line extending from Peckville (Scranton Electric Company) to the Wallenpaupack Plant of the Pennsylvania Power & Light Company. The line distance from Stanton to Wallenpaupack is 44 miles; from Stanton to Siegfried, 37 miles; and the tie from Wallenpaupack to Siegfried is made by a 64-mile, 220-kv. line.

The Pennsylvania Power & Light Company has a low reactance tie to the Philadelphia Electric Company by means of a 220-kv. line from Siegfried to Plymouth Meeting. It also has 66-kv. ties with the Luzerne County Gas and Electric Company at Stanton, Wilkes Barre, and Berwick. This latter company has generating plants indirectly tied to Harwood with a total capacity of approximately 50,000 kw.

Because of the bus tie reactor between the systems at the Stanton Plant and the 32-mile, high reactance line from Scranton to Wallenpaupack, there was a tendency for generator No. 1 and the Scranton system to swing as a unit during transient disturbances. Due to the fairly heavy network of the Pennsylvania Power & Light Company, and the low reactance tie from Siegfried to Wallenpaupack and Plymouth Meeting, this part of the interconnected system also tends to swing as a unit under transient conditions.

Station Equipment. Fig. 3 is a one line diagram of the Stanton Station showing the main electrical connections. The high-voltage bus scheme is employed, that is, the generator and transformer are operated as a unit, no switching being done on the low-voltage side of the transformer bank. The station auxiliaries are normally fed from each house generator although it is possible to feed them from a transformer connected

to the 66-kv. bus on the Scranton Electric side of the station.

The two main generators are each rated 52,630 kv-a. 0.92 power factor, three-phase, 60 cycle, 11,000 volts. Generator No. 1 normally supplies power to the Scranton Electric Company, and No. 2 to the Pennsylvania Power & Light Company. Power is generated at 11,000 volts and transmitted at 66,000 volts. The two systems are separated at the plant by a 7 per cent bus tie reactor connecting the two 66-kv. buses.

Each turbine shaft also carries a 3,283-kv-a. 0.7 power factor, three-phase, 60-cycle, 2,300-volt house generator, and exciters for the main and house generators. The exciter for the main unit is a shunt wound machine rated 250 kw. at 250 volts; its voltage is controlled by a rheostatic type voltage regulator. The main step-up transformers are each rated 52,500 kv-a., and are solidly grounded on the 66-kv. side. The 66-kv. bus arrangement is practically identical on both the Pennsylvania Power & Light Company, and the Scranton Electric Company's side of the station. The switching scheme consists of a main and reserve bus having a circuit breaker selector scheme.

The 66-kv. reactor connecting the two buses formed a part of the ultimate station plan in which it was necessary to limit the breaker duty by use of reactors. It was also felt that in the present initial installation that the reactor would limit somewhat the magnitude of voltage troubles that might be reflected from one system into the other.

The relaying for the outgoing lines from this station is provided by balanced current or balanced power protection where parallel lines are involved, and straight over-current protection for single circuit lines.

The steam turbines are rated 50,000 kw., 1,800 rev. per min., 19 stages with initial steam pressure of 600 lb. per sq. in., and temperature of 725 degrees. The steam is reheated between the 7th and 8th stages to 725 degrees.

Steam extractions are provided for feed water heating from the 12th and 16th stages; additional extraction from the 6th stage is used for operating the air jet for the condenser when this pressure is over 100 lb. per sq. in., or about 20,000 kw. load on the turbine. The reheat system is shown diagrammatically on Fig. 1, and consists of a 20-in. diameter line from the 7th stage outlet to the reheaters and a 24 in. diameter line from the reheaters to the 8th stage admission. The reheater is by-passed by shutting the valves in the lines to and from the reheater, and opening the by-pass valve located at the turbine.

There is approximately 25 per cent of the total turbine power generated in the high-pressure section, and 75 per cent in the low-pressure section. At ful lload the reheater and connections contain approximately 270 lb. of steam, while the turbine flow is approximately 100 lb. per second. This gives, therefore, a large reservoir of energy for the low-pressure section of the turbine.

The turbines are designed for base load, with a single control valve connected through a relay system to the operating governor. There is also an intercepting valve in the reheat steam line at the inlet to the 8th stage of the turbine. This intercepting valve as well as the control valve and turbine throttle valve are under the control of the emergency governor which operates about 10 per cent above normal speed. The movement of the control valve, intercepting valve, and turbine throttle valve in response to the governors is accomplished by oil pressure. The control valve follows the operating governor movement by means of a pilot valve; return motion from the control valve recenters the pilot valve, so that the control valve takes a definite position for each position of the operating governor. The intercepting valve and throttle valve, having no return motion, are either open or shut. These valves are shut by relieving the oil pressure in the valve operating cylinders.

Test Equipment. In order to obtain simultaneous records of all the varying elements during a disturbance, it was necessary to use six oscillographs with a total of 22 vibrators. One element of each oscillograph was used as a timing wave. This wave was interrupted every second, so that it was possible to line up all the films after the test, and obtain the proper sequence of events. Preliminary tests with films five feet long did not give sufficient time to obtain a complete record of all the disturbances. Films 15 ft. long were obtained for the remainder of the tests. These were run at a speed of about one ft. per sec.

Two three-phase wattmeter elements were used to measure the kilowatt output of the two generators in the station. Current and voltage were measured during all tests involving short circuits; voltage from the 66-kv. buses to ground was obtained during some of the tests by means of two photographic, high-speed recorders.

The recording of the mechanical measurements on the oscillograph films raised several unique problems. Records of the pressure at several points in the hydraulic system of the turbine governor equipment, and of the steam pressure at the 8th stage of the turbine, were obtained by means of pressure recorders which had been developed for testing pressure in oil circuit breaker tanks during short-circuit tests. These recorders consist of a small solenoid coil in a magnetic frame with a flexible steel diaphram. The pressure applied to this diaphram varies the air gap in the magnetic circuit of the coil. A 500-cycle a-c. voltage is impressed on the coil, and the resultant current which changes with the pressure can be recorded by an oscillograph.

It was desired to record the movement of the governor beam very accurately. Contact resistance devices were tried during the first tests, but were unsatisfactory due to the force necessary to move the device and the space necessary between the contacts. Finally a special photoelectric cell position indicating device

was designed and used for the latter portion of the tests. This device consisted of a small disk with unequal holes around its periphery. The light was arranged to shine through these holes on a photoelectric cell; the change in the resistance of the photoelectric cell was used to operate an amplifying tube, the output current of which was recorded by the oscillograph. Each time a hole in the disk admitted light to the photoelectric cell, a deflection of definite magnitude was obtained on the oscillograph. This disk was fastened to the governor beam by means of a steel wire so that any movement of the governor beam would rotate the disk. By means of this device it was possible to measure the movement of the governor beam to 0.01 in. over its total travel of 4 in. The movement of the main control valve and intercepting valve was recorded satisfactorily by means of a contact resistance device.

To obtain the average speed of a turbine, a small synchronous converter was equipped with a large flywheel and run from the direct current end. The a-c. voltage of the converter was compared with one phase voltage of the house generator of the turbine by connecting the oscillograph so that it would read the vector difference of the two voltages. Before each test the converter was adjusted to the same speed as the turbine, so that the vector difference of the two voltages was about constant. The change in this vector difference during the test was used to obtain the speed of the turbine. The house generators on the two turbines are arranged on the shaft so that they have the same phase relation. The relative angular movement of the two turbines was recorded by taking the vector difference of like phases of the two house generators.

Appendix B

Calculation of Stability Limits and Short-Circuit Current

To study the electrical system stability, the system reactances were reduced to an equivalent four machine system; the two machines at Stanton were considered as generators tied together through the bus tie reactor, and equivalent motors represented the Pennsylvania Power & Light Company system and the Scranton Electric Company system. One- and two-conductor to ground faults were considered outside of the generator No. 1 high-tension bus as shown in Fig. 20. The positive, negative, and zero phase sequence reactances viewed from the point of fault together with the inertia constants for these machines are also shown in this figure.

Before making the tests at the Stanton Station, calculations were made to determine the stability limits of the system under short-circuit conditions and the short-circuit currents to be expected. For the preliminary calculations, the system was considered as a two machine problem; that is, the Stanton generator No. 1 was assumed to supply power to the rest of the system represented as an equivalent motor. The results of these calculations are shown in curve form in Fig. 4. With a one-conductor-to-ground fault, the stability limit was far above the rated kilowatt output of the generator, while the stability limit for a two-conductor-to-ground fault was approximately 25,000 kw. for a switching time of not more than three-quarters of a second.

Due to the low reactance tie between the generators in the Stanton Station, there was some doubt if reducing the system to a two machine problem could be justified. Close checks on these stability limits were made considering the Pennsylvania Power and Light Company and Scranton Electric Company systems as motors being supplied by the two Stanton generators as shown in Fig. 20, that is, swing curves were made for a four machine problem.

The real check on these calculations came when short circuits consisting of one and two conductors to ground were put on the system. As predicted by the calculations, the short-circuit tests showed that the system was stable with a one-conductor-to-ground fault but with a two-conductor-to-ground fault and a load of 40,000 kw. the system was decidedly unstable; but with a load of 25,000 kw. and short switching time, the system was stable. These tests are described in detail in the group from Test No. 11 to Test No. 14.

The short-circuit current calculated and checked by the oscillographs during a two-conductor-to-ground fault, was approximately 7,000 amperes in the ground with a corresponding 5,700 amperes in the faulty phases.

Appendix C

Change in Speed of a Rotating Body for a Given Energy Differential*

The formulas derived below were found to be quite useful in calculating a speed change for a given energy differential.

The fundamental equation for rotor motion is given by the equation

$$T = M \frac{d\,\omega}{d\,t} \tag{1}$$

where the per unit torque (T) is equal to some inertia constant (M) multiplied by the time rate of change of per-unit speed. The inertia constant (M) may be defined as the time required for the rotor to come to rest with unit decelerating torque applied at normal speed and maintained constant until zero speed is reached.

To evaluate (M), formula (1) is integrated from $\omega = 1$ to $\omega = 0$ with $T = -1$.

$$t = -M \int_1^0 d\,\omega = -M\,\omega \Big]_1^0 = M \tag{2}$$

Thus M is the time to come to rest with unit retarding torque applied.

*By Mr. I. H. Summers.

The energy differential is given by

$$\Delta E = \int_{t}^{t+\Delta t} T\,\omega\,dt = \int_{\omega}^{\omega+\Delta\omega} M\,\omega\,d\,\omega \qquad (3)$$

Where ΔE is the differential energy divided by the base power. This may be written

$$\frac{\Delta E}{M} = \int_{\omega}^{\omega+\Delta\omega} \omega\,d\,\omega = \frac{1}{2}(\Delta\,\omega)^2 + \omega\,(\Delta\,\omega) \qquad (4)$$

The quantity $\frac{\Delta E}{M}$ may be thought of as the differential energy divided by twice the rotor stored energy at normal speed.

The expression for the change in speed is

$$\Delta\,\omega = \sqrt{\omega^2 + 2\frac{\Delta E}{M}} - \omega \qquad (5)$$

Several modifications of the formula for $\Delta\,\omega$ have been found useful.

Starting from zero speed $\omega = 0$ and

$$\Delta\,\omega = \sqrt{2\frac{\Delta E}{M}} \qquad (5a)$$

Starting from unity speed

$$\Delta\,\omega = \sqrt{1 + 2\frac{\Delta E}{M}} - 1 \qquad (5b)$$

or starting from unity speed if $\frac{\Delta E}{M}$ is small there is approximately

$$\Delta\,\omega = \frac{\Delta E}{M} - \frac{1}{2}\left(\frac{\Delta E}{M}\right)^2 + \frac{1}{2}\left(\frac{\Delta E}{M}\right)^3 \qquad (5c)$$

To illustrate the use of formula (5c), a numerical example is given. The energy differential for one of the swings of the rotor during test 34 was 9,230 kw. sec. The constants for this machine are, $M = 11$, base power = 50,000 kw., normal speed 1,800 rev. per min. Then the speed change is

$$\Delta\,\omega = \left[\frac{9230}{\frac{50{,}000}{11}} - \frac{1}{2}\left(\frac{9230}{\frac{50{,}000}{11}}\right)^2 + \ldots\right] \times 1800$$

$$= 30 \text{ rev. per min.} \qquad (6)$$

Discussion

T. E. Purcell: These tests were made with about the same objective as those conducted under the writer's supervision several years ago and reported in a paper* presented at the winter convention of 1930. It is gratifying to learn that, in general, the results of these tests, although conducted under different conditions and in a different manner, are almost identical with results we obtained.

*Operating Characteristics of Turbine Governors, by T. E. Purcell and A. P. Hayward. A. I. E. E. Trans., April 1930, p. 715.

I am greatly surprised to learn from this paper that the turbines under test, overspeeded enough to trip the emergency governors when loads in excess of 60 per cent of the rated capacity were dropped and I am also surprised at the endeavor of the authors to point out that the correction of this most undesirable performance is a rather complex problem.

Trouble of this kind, that is machines tripping emergency governors due to loss of load, was encountered on the Duquesne Light Company's system, principally at the Colfax Power Station, for several years, until in 1925 the writer in cooperation with engineers of the Westinghouse Electric and Manufacturing Company agreed that this difficulty could be eliminated and diligently set about the task. The results of this work are outlined in a paper entitled "Load Dump Tests Made on Colfax Turbines" by the writer, which appeared in *Power* in 1929.

This feature of turbo generators, that is loss of load without tripping of emergency governors, is considered so important by us that it is one of the acceptance tests we conduct upon all new turbines installed. We have conducted tests upon a 60,000-kw. single-cylinder Westinghouse turbine, recently placed in service at our James H. Reed Power Station. The maximum load, about 70,000 kw., can be suddenly dropped by opening the generator oil circuit breaker without the emergency governor tripping.

E. E. George: Where it is desired to produce out-of-step conditions without too much shock to the system, this can be accomplished by lowering the excitation on the plant while building up on the kw. output. By juggling the excitation and governor opening it is fairly easy to control surging between the plant and the rest of the system. The effect of various out-of-step conditions on relays is very interesting.

One point particularly worthy of comment in this paper is the possibility of getting machines back in step without taking them off of the bus. This saves the time required to resynchronize the machine which may be excessive if there is trouble on the system.

For several years we have instructed our operators, in case of out-of-step conditions, to juggle the field rheostat and governor controls and try to get the equipment back in step without disconnecting and paralleling. In most cases it has been possible to accomplish this successfully.

Phillip Sporn: This paper forcefully brings out the fact that although governors have been in use in connection with the operation of steam turbines for many, many years, the real action of such governors, particularly their action under times other than normal, has not been well understood and certainly not understood clearly enough to be able to deduct therefrom the amount ultimately contributed by the governor itself toward the total disturbance. The tests which were carried out as described in this paper show that the governor's contribution to the total disturbance under certain conditions of system fault is no negligible portion.

As is also brought out in the paper, this out-of-step condition between generators in the same station or between generators and the system, is no uncommon occurrence. It undoubtedly happens many times but before anything really serious develops, the hunting dies down, the generators pull back into synchronism and the entire disturbance is summarized and frequently forgotten under the general heading of "surge on the system." Fig. 1 shows a high-speed chart taken at the Turner substation of the Appalachian Electric Power Company during a condition of a severe short on a low-voltage bank, approximately 50 miles away. It will be noted that this condition is very analogous to the condition of instability shown in Fig. 7E of the paper under Test 34. In this case the taking off by hand of a turbine unit at Cabin Creek, a generating station located approximately 25 miles from Turner, resulted in the system pulling into synchronism.

Fig. 2 shows another chart taken at the Turner substation during lightning disturbance on the Turner-Cabin Creek line. It will be seen in this case that the instability which developed at Cabin Creek on unit No. 6, due to a fault condition on the Turner-Cabin Creek line, was sufficient to pull the unit out of synchronism, with the result that it tripped off the line from overspeed. The situation is very similar to case B in Fig. 7.

Fig. 3 shows a record taken on a Hall recorder at the Glen Lyn generating station. The upper record shows ground current and the lower three records show line-to-line voltages on the 132-

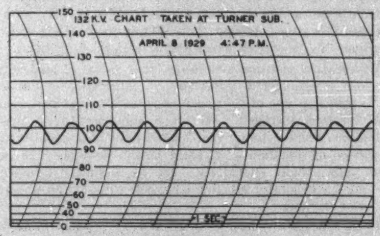

Fig. 1

kv. bus. During a lightning storm a double circuit fault occurred between Glen Lyn generating station and the Switchback substation to the west. Both circuits tripped and the station was practically unloaded resulting in violent swinging for about three seconds and subsequent less severe swinging for another three seconds until the generators finally pulled in step and the voltage became normal.

It is obvious that in all these conditions, while the governor may not have been a direct contributing cause for the violence of the fluctuations, it certainly did not contribute anything

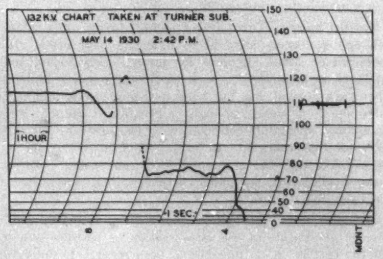

Fig. 2

definite early enough in the disturbance to materially reduce or minimize the disturbance. On the other hand, the governor offers a definite point in the system at which to apply restraining forces under disturbance conditions. The authors show in this paper a development which offers a definite remedial force toward the mitigation of some of these disturbances. There is no question but that the intelligent application of the watt principle in governing as brought about by the authors will be of material assistance in the operation of large power systems under disturbance conditions.

It may be interesting in connection with this to tell of some of the other remedial measures that have been taken at the Stanton Plant to overcome some of the difficulties that have been experienced with instability between two generators. The first step after the experience with instability was to reduce the load on the generating station under disturbance condition to about 60,000 kw., which is only 66 per cent of the normal system rating. This is a very uneconomical way of bringing about stability and could not be maintained for any extensive period. The problem was carefully analyzed from every angle and as a result the following measures were decided upon:

1. As a first step, high-speed relays in the form of plunger type ground relays were installed on all outgoing circuits and set so that they would operate on ground faults for about 80 per cent of the distance from the generating station to the nearest substation.

2. This station has two 66,000-volt buses, one a main and the other a reserve, each with its own circuit breakers. One set of these breakers, *i. e.*, the breakers on one entire bus, is being cut over for high-speed operation. It is estimated that this

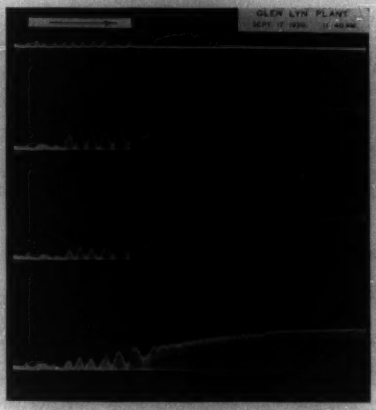

Fig. 3

combination of high-speed relays and circuit breakers will clear ground faults in a maximum of 10 cycles, for faults occurring on any outgoing line for about 80 per cent of the distance between the generating station and nearest substations.

3. The installation of watt governor mechanisms along the lines pointed out in the papers is being studied at the present time with a probability that the installation will be proceeded with as soon as some of the pertinent details are worked out. Subsequent to the installation of the high-speed relays but before the breakers were cut over to high speed, a very severe lightning storm occurred during which two faults occurred near the Stanton station, one on the Scranton system, and the other a little later, on the Pennsylvania Power and Light Company system. During this period the generators were carrying 80,000 kw. and no trouble of any kind was experienced at the plant. The generators rode through the storm in excellent shape although before these changes were made they had invariably developed instability and tripped off the line.

R. C. Buell: Mr. Purcell expresses surprise "that the tur-

bines under test overspeeded enough to trip the emergency governors when loads in excess of 60 per cent of rated capacity were dropped." It should be remembered that the turbines at Stanton are of the reheat type with the speed governor controlling only the high-pressure steam entering the turbine, no control is provided for the steam as it comes from the reheat boiler. At full load there is sufficient stored energy in this boiler to cause a speed rise in excess of 10 per cent, if full load is dropped. The intercepting valve located in the return line from this boiler and operated only from the emergency overspeed governor prevents excessive rise above 10 per cent.

Calculation showed that even with the reheat boiler out of service and the by-pass valve open and reheat valves shut, the entrained steam in the lines to the boiler contained sufficient steam to overspeed the turbine.

In the ordinary turbine without reheat the speed governor is fast enough to easily prevent tripping the emergency overspeed governor at 10 per cent above normal when full load is dropped.

We believe Mr. Purcell has interpreted our remarks in regard to the complexity of the problem of holding the generators in synchronism with the rest of the system as applying also to the load dropping test. The two problems are entirely different, for in one case the turbine speed must not increase more than 1 or 2 per cent, while in the other a 10 per cent rise is permissible.

The Ohio Falls Hydroelectric Station at Louisville, Kentucky

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and

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Synopsis.—*This paper is a description of the Ohio Falls Hydroelectric Station at Louisville, Ky. The development is a low-head, run-of-river plant and a combined navigation and power dam. The dam is a part of the U. S. Government Ohio River canalization project.*

Eight units, with a total installed capacity of 105,000 hp. operate on a maximum head of 37 ft. and deliver power to the systems of the Louisville Gas and Electric Company.

Several novel electrical features are embodied in the design of this plant. Both automatic and supervisory control equipment is employed and is housed in closed steel cabinets.

A miniature switchboard with complete control and supervision of each unit on a panel four in. wide is another new idea that is used in station control.

* * * * *

Introduction

When in 1923 the U. S. Government, in carrying out its plans for the complete canalization of the Ohio River, decided to construct a dam at Louisville of a height sufficient to give a maximum drop of 37 ft. between the upper and lower pools, the economic and engineering aspects made feasible the construction of a hydroelectric generating station.

Actual construction was begun in 1925 and the plant was turned over to the operating company in the early part of 1928.

Site

The Ohio River at Louisville descends about 28 ft. in a distance of two miles over a series of rapids. This is the only fall of any size in the entire length of the river, the general slope of which is very flat.

This natural fall has always been looked upon as a potential power source and was actually used on a relatively small scale by a flour mill built by the Tarascan Brothers early in the nineteenth century. It is interesting to note that these early pioneers in water power development chose a site only a few hundred feet away from that now used by the hydroelectric development.

Test borings made in the exposed limestone at the foot of the rapids indicated that foundation conditions there were sufficiently good to support the contemplated structure.

A power house site was chosen on the Kentucky side of the river within the limits of the city of Louisville. This latter fact was important because it insured a ready market for the power to be generated; more important still, steam generating capacity in sufficient amount to back up the intermittent power available from the run-of-river development was already installed within a few miles of the site.

In granting a license to the Louisville Gas and Electric Company, the Federal Power Commission instructed that company and the District Office of the U. S. Engineer Corporation to work together on the plans for the development. The engineers for the company prepared the plans for the power plant, the U. S. Engineers worked out the design of the dam, and the entire project was built by the construction forces of the Byllesby Engineering and Management Corporation.

Water Supply

The area drained by the Ohio River above Louisville is extensive, amounting to over 91,000 sq. mi. and extending well into New York, Pennsylvania, Virginia, and Maryland.

Despite this large area and the generous rainfall on the western slopes of the Appalachians, the natural flow of the river is unstable, varying from about 792,000 sec.-ft. at the time of maximum flood to 5,000 sec.-ft. or less during summer droughts. Due to a restricted river channel for a number of miles below the city, this variable flow causes a maximum variation in the river stage at Louisville of 65 ft. This results in a variable head at the power house that ranges from 37 ft. at times of low flow to zero at high flow.

The Dam

The dam is a concrete structure with steel gates and guides and timber wickets resting upon a rock foundation into which was cut a 4 by 3 ft. keyway. It extends upstream from the north end of the power house, paralleling the river banks approximately 7,000 ft., thence northward to the Indiana Shore. It is the longest and highest dam on the Ohio River, the entire length being 8,680 ft., including two 91-ft. beartraps, 3,600-ft. boule weir and 860-ft. chanoine pass.

The movable sections, operated by derrick boats by the U. S. Engineer Department, have a discharge capacity of 350,000 sec.-ft.

The beartraps are operated by hydraulic pressure and have a discharge capacity of 32,000 sec.-ft. During normal flow conditions, pool regulation is accomplished through maneuvering of boule and pass, leaving

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the beartraps to be operated during emergency cases and high water.

Fig. 1 shows a map of the river adjacent to Louisville with the dam and hydro station. Fig. 2 is an airplane view taken from below the hydro station also showing the dam and a portion of the city of Louisville in the background.

Power Station

The water-level variation had much to do with the design of the station. The entrance is located four feet above maximum high water. To save in superstructure, the generator-room floor is depressed 16 ft. below the entrance level. The water wheels are placed near low-water level.

Each unit passes a flow of about 4,000 cu. ft. per sec. so that the total water capacity of the plant is 32,000 cu. ft. per sec. In order to pass this large flow under a comparatively low head and maintain the plant efficiency, the water passages were made large, the racks for each unit being 40 ft. deep and the units being spaced on 58 ft. centers. Between units 2 and 3 and between 5 and 6 the spacing is increased to 64 ft. where expansion joints are carried through the structure.

Eight units, each having a capacity of 13,500 hp. at 37 ft. head are installed and provision is made for two additional units at the northern end of the station.

The generator room is 44 ft. wide and 508 ft. long. There are two floor levels below the generator floor. The lowest floor contains the tanks for governor oil and pumps supplying cooling water to the closed generator ventilating system. The intermediate floor holds the governor pressure tanks, oil pressure pumps, and pumps supplying cooling water for the transformers. At the southern or shore end of the station is located a sump with automatic pumps which takes care of all leakage from the stuffing boxes on the turbine shafts and at the expansion joints.

A standard gage railroad track leads into the shore end of the building where there is an unloading platform served by the house crane.

Fig. 3 is a view of the station taken from the upstream side showing a 25-ton gantry crane for serving the headworks. Headworks for two additional units are shown on the right.

Fig. 4 shows the interior of the generator room.

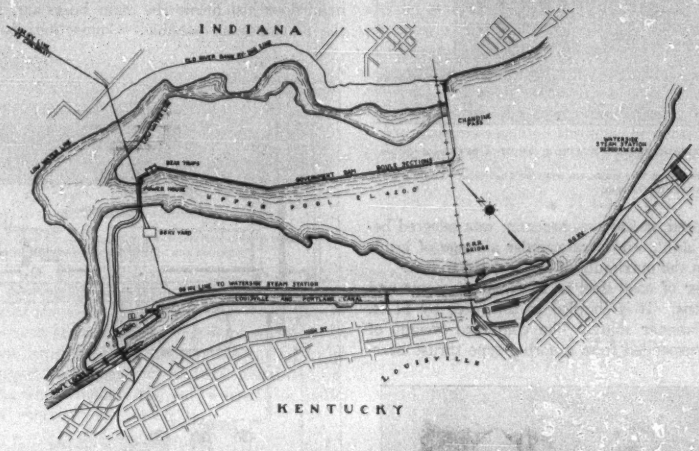

Fig. 1—Map of Dam and Hydroelectric Station Site

Water Wheels

With a maximum flow of 4,000 sec.-ft. through each unit, three intake openings with a total cross-sectional area of 1,400 sq. ft. are necessary in the concrete scroll case of each unit. These openings are equipped with

Fig. 2—Airplane View of Dam and Hydroelectric Station

Headworks for future two units are seen at the left of the station. Pennsylvania R. R. Bridge and sky-line of Louisville in background

trash racks and necessary guides for roller gates and stop logs to be used in case of emergency.

The hydraucone is used, which, due to decreasing the velocity of discharge, keeps the loss in energy in the tailwater down to a minimum.

A five-blade propeller type axial flow runner, designed to produce 13,500 hp. at 37 ft. net effective

Fig. 3—Hydroelectric Station from Upstream Side

Headworks for future two units are seen on the right. Gantry crane for serving headworks can be seen

head at a speed of 100 rev. per min. was selected because of its ability to operate under a range of heads varying from the maximum of 37 ft. down to 8 ft.

The runner of cast steel is 15 ft. in diameter and weighs 20 tons. It is coupled to the rotor by 61 ft. of 23 in. diameter shaft. The combined weight of 125 tons is suspended from a spring type thrust bear-

Fig. 4—Generator Room

8-12,550-kv-a. water-wheel-driven generators with surface air coolers. Oil circuit breakers, switchboard, and governor for one unit can be seen

ing. Four guide bearings, three of which are babbit and one lignum vitae are used because of the long shaft.

Electrically-driven fly-ball governors, automatically operated, provide positive control to servo motors and guide vane latch. The servo motors and latch are mounted directly on the cover plate, thus eliminating strains on the shift ring.

General Electrical Layout

The extreme length of the power house was determined solely by hydraulic requirements and made necessary an unusual arrangement of the generator electrical equipment to fit it into the available space.

The unit type of construction is used as regards the generator oil circuit breakers, auxiliaries, and switchboard panels.

The main buses are supported on a steel structure along the upstream wall of the station just below the windows and are totally enclosed in a sheet steel housing.

Generator oil circuit breakers are located on the main floor just below the main buses and as close to the machines as possible. Connections between the

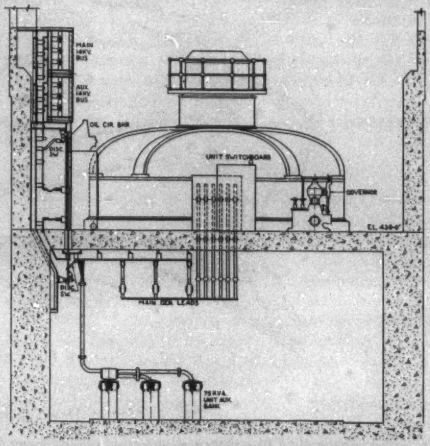

Fig. 5—Cross-Section of Station Showing Generator Lead Connections

Main and auxiliary buses are shown on upper left with generator gang operated disconnect switches and oil circuit breaker below. Switchboard enclosure is in center and generator governor to the right. The unit 75-kv-a. auxiliary transformer bank is on floor below with connections made solidly to the generator leads

generators and oil circuit breakers are of flat copper bar housed in sheet steel flues.

The generator switchboard panels are also located on the main floor near the generator and governor.

This grouping of the generator electrical equipment in close proximity to the individual units results in exceptionally short bus connections between the generators and oil circuit breakers and short control conduit and cable runs between the generators, switchboards, governors, and oil circuit breakers.

Fig. 5 is a section through the generator room showing a generator with associated oil circuit breaker, disconnect switches, switchboard housing, and governor.

The main and auxiliary buses are also shown with connections to the generator.

Fig. 6 is a view on the generator floor showing one unit with its associated oil circuit breakers for connection to the main and auxiliary buses, the unit switchboard and housing, and the governor.

Generators

Generators are of revolving field type, rated at 12,550 kv-a., 14,000 volts at 100 rev. per min. and equipped with direct-connected exciters mounted on

Fig. 6—Automatic Switching Equipment

Truck mounted oil circuit breakers for connecting unit to main or auxiliary bus are seen with the unit switchboard and governor

the thrust bearing housing with field current collector rings mounted above the exciter.

The generator stator frame which is of welded steel construction, supports the weight of the rotor, thrust bearing, upper guide bearing, and the remaining rotating parts which include the entire length of shaft, water-wheel runner and water thrust.

Armature Winding

From the armature winding 12 leads are brought out to a terminal board to permit differential protection and also for the purpose of changing the operating characteristics. For full load operation the windings are normally connected two-circuit Y but for light loads they may be connected one-circuit delta.

This change of connections is readily accomplished by means of gang operated disconnect switches. The improved efficiency of the delta connection as compared with the Y connection is computed as being 2.1 per cent at 10 per cent full load to 0.5 per cent at 75 per cent full load, 0.8 power factor. Above 75 per cent full load the Y connection is again used and normal efficiencies are obtained.

Ventilation

Generator ventilation in this station is similar to that used for many years on steam turbo-generators, heated air from the generator armature and field coils being forced by fans on the rotating member through water coolers located at four points on the periphery of the armature frame and at floor level. The air thus cooled is driven down through suitable openings in the concrete foundation to a space below the machine, formed by the circular foundation and a horizontal steel diaphram. This cooled air is then drawn by the generator fans up through the machine to be forced out again and through the coolers.

This is the first application of coolers to hydro-generators and has proved exceptionally successful and valuable in this instance because of the high air temperatures existing through a large part of the year and the smoke and dust in the outside atmosphere. The latter is due to railroads and factories surrounding the development.

Leakage of air is made up by the installation in the suction side of the system of Reed air filters which remove dust and dirt from entering make-up air.

The generator armature and field coils are found to be practically as clean as when installed and thus the efficiency of the units is kept at the original value and the life of the installation is preserved.

Fig. 7 shows a section through the generator and the cooling equipment with the direction of air flow indicated by means of arrows

Oil Circuit Breakers

The generator oil circuit breakers are truck type, motor operated, having an interrupting capacity of

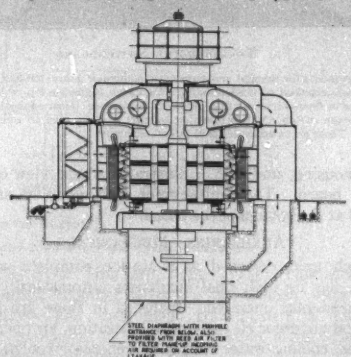

Fig. 7—Cross-Section of Generator and Air Cooling System

Arrows indicate the direction of air flow through armature and field coils. Filters for supplying make-up air are not shown

500,000 kv-a. standard duty cycle. One set of disconnect switches on the bus and one on the machine side of each oil circuit breaker are operated from a single hand operating mechanism located at the breaker.

Necessary current and potential transformers for metering and control are connected in the short bus runs between the machine and breakers.

Switchboards

Manual and automatic control of the generators, outgoing lines and station service banks is performed

by means of standard panels and control equipment located in close proximity to the equipment controlled. The switchboards with the associated relays, contactors and control switches are mounted in sheet steel housings for protection of the equipment.

Control and meter circuits are brought to the switchboards by means of Greenfield cable through multi-circuit train couplers so that the board can be entirely isolated and removed.

Fig. 8 shows the front of a generator switchboard

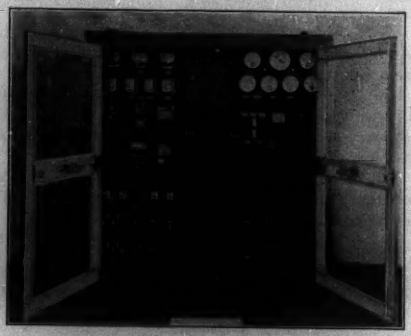

Fig. 8—Generator Switchboard

Instruments for manual and automatic control of generator unit shown on panels enclosed in steel housing. Two pilot lights on top of housing indicate to floor attendant that unit is to be started. Rear connections to pull-button switch which is operated to start unit are seen on right-hand door. These doors are closed and locked normally

and housing, and Fig. 9 illustrates a rear view of the same board. The multi-circuit train couplers are located at the bottom of the board.

Auxiliaries—Mechanical

Each unit, designed as a station complete within itself, has its individual auxiliaries automatically and independently operated. Included in these are bearing and governor oil systems, oil filtration, air operated brakes, cooling water supply to generator coolers and thrust bearings and water supply to lignum vitae bearings.

Realizing that auxiliary failures mean jeopardized service, certain additional features were included in order to insure more dependable operation. A complete station bearing oil system, automatically operated, supplies each generator with the one and one-half gallons per minute required for its operation. This oil can be distributed to the unit bearings and the overflow in the unit system returns to the station filter.

The governor oil pressure systems are equipped with pressure control relay gages, limiting normal operating pressure between 150 and 175 lb., designed to shut down a unit in the event the oil pressure drops to 130 lb. The tanks are connected to headers in groups of three, both on the pressure and storage side, in order to insure continuous operation in case of failure of any individual unit system.

A bank of three 4-in. centrifugal cooling water pumps supplies the house header and transformer banks. The house header which is connected to the city water supply, is also connected to all generator cooling units.

All waste water, leakage and seepage, drains to one common sump, from which automatic pumps discharge the water into the tailrace, pumping it over the high water level.

Auxiliaries—Electrical

The scheme of unit auxiliary equipment is also carried out as regards electrical equipment in that all of of the auxiliaries, with the exception of compressed air for the brakes and d-c. control voltage, are fed directly from each generating unit. The auxiliaries associated with the operation of the individual units are: one 30-hp. governor oil-pressure pump, one 7.5-hp. generator cooling-water pump and one ¾-hp. governor fly-ball motor. These auxiliaries are fed from the main generator by a step down transformer bank consisting of three 25-kv-a. single-phase 13,800- to 230-volt transformers. The 13,800-volt side of this transformer bank is connected directly to the generator leads, no fuses, oil circuit breakers or disconnect

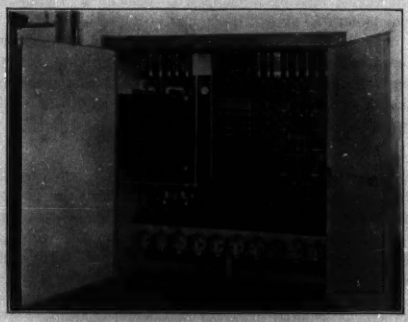

Fig. 9—Rear of Generator Switchboard

Multi-circuit train couplers through which meter and control circuits pass are at the bottom

switches being used. In starting a unit the governor fly ball and oil pressure pump motors are fed from a 230-volt station auxiliary bus. As the generator comes up to speed and 85 per cent of normal voltage is reached the governor fly-ball motor is automatically transferred to the unit feed. The governor oil-pressure motor is transferred from the station feed to the unit feed manually. The generator cooling-water pump motor is fed direct from the unit transformer bank at all times. The generator exciters are direct-connected to the main shaft.

The station auxiliary units used for house and emergency service for the generating units are as follows:

Two, 35-hp. battery charging motor-generator sets, two 15-hp. sump pumps, emergency 100 hp. sump and lignum vitae bearing water supply, three 25-hp. transformer and emergency thrust-bearing cooling-water pumps, two 15-hp. ventilating system motors, one 10-hp. air-compressor motor automatically controlled for use on brakes and governors, one 15-hp. air compressor motor for station purposes, one 3-hp. vacuum pump motor, two ¾-hp. and one 1-hp. oiling system motor, power house crane of 225 total hp. head works Gantry crane of 57.5 total hp.

The motors that are in duplicate are fed from two separate 230-volt buses. These two buses are supplied by a 600-kv-a. 13,800- to 230-volt transformer bank which can be energized from any one of three separate sources, namely the main or the auxiliary 14-kv. station buses, or an incoming 13-kv. feeder from the steam station. The selection of the feed to this 600-kv-a. bank is entirely automatic in that should a failure occur on the normal source, the source of next preference is automatically selected and connected to the transformer bank. In case it is necessary to take the 600-kv-a. bank out of service the station auxiliary motors essential to the operation of the plant can be fed from one of the 75-kv-a. transformer banks associated with each generator.

The station lights are normally fed from one 13,800- to 230-115-volt transformer energized from the same source as the 600-kv-a. power bank. In case of extreme emergency the most important lights are automatically transferred over to the station battery.

Fig. 5 shows the location of the unit 75-kv-a. transformer bank and connections to the generator leads.

Transformer Banks

Two transformer banks of 62,500 kv-a. and 56,250 kv-a. respectively, are located on a platform outside the building. The 62,500-kv-a. bank is two-winding, stepping from 14,000 to 66,000 volts and supplying a 75,000-kv-a. transmission line to Waterside Station.

The 56,250-kv-a. bank is three-winding, stepping from 14,000 to 66,000 and 132,000 volts. A 132-kv., 82-mile transmission line supplies power to Columbia Steam Station southwest of Cincinnati. The 66-kv. winding connects to the present line to Waterside.

The 132-kv. winding is supplied with a tap changing under load equipment which permits a 10 per cent variation of voltage in four 2½ per cent steps, up and down, to suit load conditions.

The low-tension connections of these banks are made to the main and auxiliary station buses through 3,000-ampere gang-operated disconnect switches on the bus and transformer sides of cell-mounted, motor-operated oil circuit breakers with an interrupting capacity of 800,000 kv-a. standard duty cycle. This breaker equipment together with the low-tension delta bus structures and instrument transformers is located in an extension of the power station along side the transformer platform.

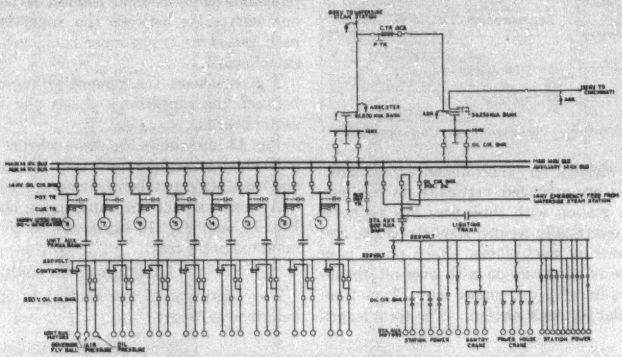

Fig. 10—One-Line Diagram of Station

14,000-, 66,000- and 132,000-volt buses are shown and also 230-volt unit and station auxiliary buses

The 132-kv buses extend over the roof of the station where connections are made to the Cincinnati line.

The 66-kv. connections from the two banks are carried overhead to the switch yard adjacent to the station where the tie for the Waterside line is completed.

A one-line diagram of the high and low tension connections is shown in Fig. 10.

Fig. 11 shows the 14,000-volt connections to the 56,250-kv-a. transformer bank.

14,000-Volt Buses and Connections

Main and auxiliary 14,000-volt buses extend the entire length of the station with the generator switching equipment located near the individual generators. This arrangement eliminates the long generator con-

nections that would be necessary if the typical scheme of a switch gallery at some remote point in the station, had been used. It also results in a corresponding reduction in the length of control and instrument conduit runs and brings all of the main equipment associated

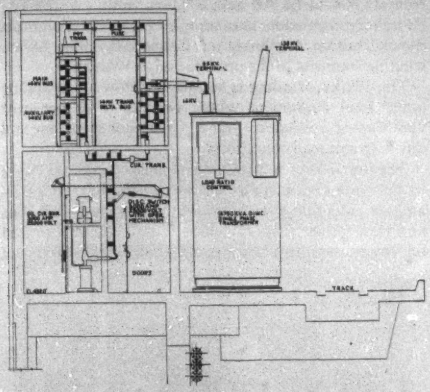

Fig. 11—14,000-Volt Transformer Connections

with the control within a very short distance of the unit.

The two buses are arranged vertically on the outside wall of the station with three-way bus supports mounted on an electrically welded steel structure, and the whole enclosed by steel plates. Each phase is of the box-bus type of construction with over-all dimensions of 4 by 4½ in. of ½ in. copper bar. Taps from the buses are of flat bar construction. The total length of the two buses is approximately 620 ft. with no portion of the bus or connections taped.

Supervisory Control

A miniature switchboard is used to provide centralized control of the operations of the station. This supervisory board is centrally located on the generator floor between units 4 and 5. The control equipment consists of a two-position control key for each circuit breaker or other device controlled, with three indicating lamps for each key. The red and green lamps indicate the closed or open position of the breaker respectively, and the white lamp indicates that the position of the controlled device does not correspond with the position of the control key.

In addition to the supervisory control keys, the miniature switchboard contains voltmeters, reactive volt-ampere meters, wattmeters, switches for raising and lowering load and switches for regulating voltage. Insulating potential transformers, 110/110 volts, are used in the secondary circuits of the generator and transformer bank instrument potential transformers for meters on the supervisory panels and with 10/1-0.5 ampere transformers in the secondary circuits of the current transformers.

Control and meter circuits are taken from the supervisory panels by means of low-voltage telephone wires to a terminal box below the generator room floor. From this terminal box to an auxiliary terminal box associated with the unit switchboards, lead-covered, 16-pair telephone cable in rigid conduit is used, there being one such cable between the supervisory panels and each generator, transformer bank, and station auxiliary switchboard.

Fig. 12 shows the general arrangement of circuits between the supervisory panels and the various unit switchboards.

Fig. 13 also shows the same scheme as applied to an individual generating unit, the two transformer banks, and the station auxiliary transformer bank.

A front view of the supervisory board is shown in Fig. 14, and Fig. 15 shows the associated relay panel mounted in the rear.

Generator Control

The operation of the control equipment is briefly described as follows:

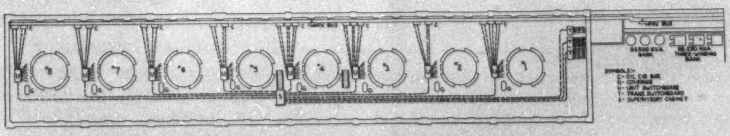

Fig. 12—Control Schematic

Circuits between supervisory panels and generator, station auxiliary, and transformer switchboards are shown, also control circuits from these switchboards to the equipment controlled

The operator at the supervisory panel selects the bus to which the machine is to be synchronized by turning the bus selector key. To give the starting indication he turns the "Start and Stop" control key to the "Start" position which notifies the floor attendant to stand by for starting the unit. He also selects the type of synchronizing to be used, whether automatic or self synchronizing. The floor attendant then checks to see that the gate limit stop is set for full-speed no-load value of head and closes a pull-button control switch on the generator cubicle door to give the actual starting indication to the unit. If all protective devices are in

their normal or reset positions, the governor solenoid picks up and opens the pilot valve which allows oil pressure to act on the gates to open them. At the same time the water-wheel brakes are released and the machine starts. The fly-ball motor of the governor is connected to the 60-cycle station auxiliary bus which prevents the machine from overspeeding and the governor synchronizing motor runs to the maximum speed position to allow rapid acceleration of the water wheel.

If automatic synchronizing has been selected, the field contactor is closed to apply field to the generator when the exciter voltage has built up to about 80 per cent and the fly-ball motor is now connected through potential transformers to the terminals of the generator.

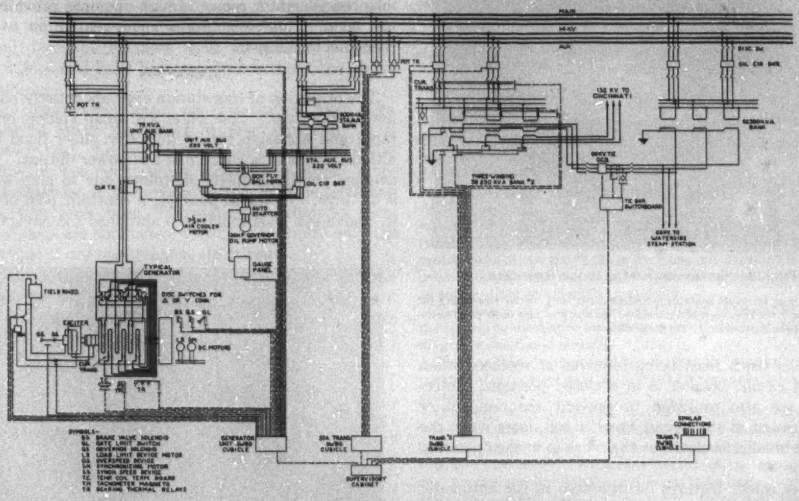

FIG. 13—THREE-LINE SCHEMATIC OF GENERATOR

Station auxiliary and transformer connections and control

When the machine reaches 98 per cent synchronous speed, the speed regulating relay automatically adjusts the speed until the generator is synchronized with the bus at which time the synchronizing relays operate to close the oil circuit breaker to connect the generator to the bus. The load is then adjusted by the operator at the supervisory panel.

If self synchronizing has been selected, the machine comes up to 98 per cent synchronous speed without field current and is closed to the bus selected as an induction motor. Seven cycles later the unit is pulled into step by the application of generator field current.

SHUT DOWN

The unit may be shut down at any time by turning the "Start and Stop" key to the "Stop" position. The water-wheel gates are closed, the generator is disconnected from the bus at the "running light" position, the field is deenergized, and the brakes are applied after the water-wheel has reached as low a speed as it will obtain without the application of the brakes.

PROTECTIVE DEVICES

The automatic equipment will shut down the unit in case of a-c. overvoltage, grounded windings, overheated bearings, loss of field, overspeed, or low oil pressure. If the shut down is caused by overvoltage or grounded windings, the generator is immediately disconnected from the bus and the field is killed. If the shutdown is due to any other cause the load is first reduced before the generator is disconnected.

TRANSFORMER EQUIPMENT CONTROL

The operator gives the closing indication at the supervisory panel to either breaker of either transformer bank by turning the control key to the "Close" position. The equipment is designed for either stub or multiple feed. The selective control relay and the synchronizing devices determine the type of feed and give the closing indication to the circuit breaker when conditions are correct. This circuit breaker is tripped in case of reverse over current.

The circuit breakers may be controlled manually by placing the transfer switch in the "Manual" position.

STATION SERVICE EQUIPMENT CONTROL

The station service bus is normally connected through a power transformer bank to the main bus.

If the voltage on this bus fails, the breaker is opened and the station service equipment is connected to the auxiliary bus, if energized, or to the emergency bus if this bus is energized and the auxiliary bus is not. When normal voltage returns to the main bus, the station service equipment is disconnected from the auxiliary or emergency bus and is connected to the main bus.

Safety Devices

The generator and station service oil circuit breakers are equipped with mechanical interlocks which prevent the truck from being removed or replaced when the oil circuit breaker is in a closed position. Interlocks are also provided to prevent the removal or replacement of the circuit breaker cell doors when the circuit breaker is closed or the truck is in the operating position.

Fig. 14—Supervisory Miniature Switchboard

Each four in. panel contains complete meter and control equipment for supervising the operation of a generator, station auxiliary bank or outgoing line. Space is provided for future additions

Other safety features incorporated in the automatic control of the generators include the following:

A generator cannot be synchronized with the bus or the breaker closed unless the truck is completely in its housing.

A machine will be shut down and locked out in case of overspeed, excessive a-c. voltage, low governor oil pressure, overheated bearing, excitation failure and short circuit or ground in the armature windings.

The generator cannot be reconnected to the bus without going through the normal sequence of starting in case the generator breaker is opened for any reason.

Heating and Ventilating

Both heating and ventilating are taken care of in combined units. The major layout consists of two complete systems, one at each end of the station designed to take air from the outside or from the generator room for recirculating. In either case air is drawn into the fan room through air filters and heating units. It is then forced through ducts to the lower floor from which point it rises to the generator room through stairways and the other openings and escapes through windows and louvers on the roof. None of this air passes through the generators as is customary in plants of normal design.

Two types of heating medium are used. In extremely cold weather and at times when the plant is shut down due to high water, steam is furnished by a 125-hp. boiler located in an annex of the power station. At other times, electrical heating coils supplied with off-peak power are used.

The generator field rheostats are located on the floor directly below the generators and advantage is taken of their heat losses, this heat being discharged directly into the generator room through openings provided in the floor. This also affords good ventilation to the rheostats in summer.

Operation

The operation of this station depends entirely on the flow of the river. Low water and small volume or the rapid and excessive rise of tailwater under flood conditions results in a curtailment of power output. The average yearly complete shutdown due to high water is 45 days. As far as possible, a constant pool of 420 ft. above sea level must be maintained for navigation purposes. This necessitates operation at highest point of efficiency during low water periods, at which time the plant operation is the governing factor of the pool stage. In case of emergency, for short periods, all available equipment can be put into service within two minutes. During periods when the flow exceeds the discharge capacity of the station, efficiencies are disregarded and pool stages are controlled by manipulation of dams and plant operation combined.

Fig. 15—Rear of Supervisory Miniature Switchboard

Economics of Operation

There has been a considerable saving in both time

and labor costs in operating the Ohio Falls Hydro Station; this saving resulting from the use of automatic control equipment.

Only one switchboard operator is required on a shift. To bring a unit in on the line he turns a control key, thereafter the automatic equipment functions to start auxiliary motors, release brakes, open water gates, apply excitation and synchronize the unit on the bus. It is estimated that these automatic features have effected a yearly savings, equivalent to two operators' salaries. There has also been a saving in the number of hydraulic operators needed at this station. Several factors account for this saving, simplicity and flexibility of operation, general arrangement of equipment, and the complete use of automatic protective equipment which in particular has proven to be dependable. Where we use three hydraulic men per shift for the actual operation of these eight generators, five men per shift would have been required without the above features.

The maintenance costs on this station have been somewhat lower than were at first expected. It is believed that the use of automatic equipment has resulted in less maintenance rather than more. Once the equipment is properly adjusted, routine tests and inspection have been the smallest maintenance item. These tests are conducted weekly and require only the part time of one maintenance electrician.

The miniature switchboard will permit further economies and may be located in a future adjacent steam station in which case one operator can control both stations.

Discussion

Chester Lichtenberg: The electrical features constituting the control of the Ohio Falls Hydroelectric Station represent a distinct advance in the art. They utilize well established principles. They combine these in a unique fashion, and permit not only a lower than usual installation cost, but also permit extensions to be made from time to time in the most economical fashion.

The design is unique in that each generating unit is complete in itself. The water-wheel, generator, exciter, governor, electrical control, and oil circuit breakers, are individual for each unit. There are only 4 points in common. They are as follows: (1) forebay, (2) high-voltage buses, (3) supply for auxiliaries (4) centralized control.

Each of these features, will, of course, be recognized as embodying well established principles. However, an outstanding characteristic of the Ohio Falls Station is the fact that it utilizes all of these simultaneously and to the utmost. The foresight and courage of the designers of this station thus enabled them to not only construct it at a cost well within the limits for conventional designs, but also permitted them to incorporate in it features which are now becoming recognized as essential for successful and economical operation.

Simplicity is characteristic of the design of this station. The elimination of almost all of the conduit and its wiring has been previously emphasized. One of the basic ideas, however, advanced by Mr. Stanley was to have an electrical control unit near the machine corresponding to the hydraulic control unit. This feature has been accomplished by grouping all of the electrical control features and placing them in a cubicle located adjacent to the governor. Consequently, each unit has its electrical control concentrated at one point and its hydraulic control concentrated at a closely adjacent point. As pointed out in Mr. Stanley's discussion this arrangement while very desirable from an installation standpoint did not seem to permit of economical operation. Consequently, a centralized supervisory control was incorporated. This permitted centralized supervision and control of all of the units through wires in telephone type cables. An immediate result of the decision to use centralized supervision of this character was to reduce to a minimum the number of connections between the centralized supervisory point and each of the electrical control cubicles. Instead of a large amount of conduit and wiring between each generating unit and a central switching point there is only one telephone cable between the electrical and governor control of each unit and the centralized supervising point.

Other important features to be emphasized in the unique design are that the main power circuits and the main control circuits have been shortened to a minimum.

An evaluation of these features indicates that while the electrical controlling equipment may have cost more than a conventional design, yet its installed cost is appreciably less than a conventional design due to the tremendous saving in conduit and control wiring. Besides, the elimination of the majority of this conduit and control wiring has appreciably reduced the inspection and maintenance expense. The net result appears to be a gratifying profit on the investment together with the possibility of very economical extension in the future.

R. M. Stanley: Making one of the largest automatic hydroelectric stations in the world automatic or semi-automatic does not entirely solve the operating problem. With the ordinary switchboard, control panels, instruments, etc. in a centralized control room at one end of the powerhouse or alongside the power house the superimposing of automatic control would constitute an added cost of considerable magnitude.

If, however, ways and means can be found to lessen the cost of the control equipment or its arrangement then automatic control may be justified economically.

This has been accomplished in our plant by using unit control panels and equipment near the units and transformer equipments controlled.

Obviously some centralized control location must exist since we cannot station operators in different parts of the plant; neither can we arrange for operators to travel from one point to another rapidly or efficiently. We substitute, therefore, the miniature switchboard and low-voltage multi-conductor lead-covered control cable for the large central control board and the usual 19/22 control cable and iron conduit.

Part of what we saved in the above manner we expended on automatic equipment.

There is, of course, no particular reason why a non-automatic station could not be equipped and controlled by a centralized miniature switchboard.

When we omitted the large centralized control board we saved the cost of a control room, skylight and foundation (costly on this installation because of the depth to bedrock). Altogether, therefore, the installation is not only convenient and safe but is justified on an economic basis.

The designing engineer must be visionary but practical, and one of his chief problems is to anticipate the future growth of the system, or, if some one else sets up the growth as a basis for a particular design problem, he must foresee clearly how to prepare for this growth so that at the proper time his design proves susceptible of proper expansion, and the future requirements are easily and economically taken care of. Quite often a number of years elapse before it becomes apparent that the design was not quite comprehensive enough and that major alterations are required. These changes in plans or layouts are expensive and

are economic losses which the industry has to bear, especially if rebuilding is necessary.

The miniature switchboard and unit control scheme readily meet this situation.

Suppose for example at the hydro plant a large switching center becomes a necessity. Assume it will be one-third 66 kv.; one-third 13.2 kv.; and one-third 132 kv. A designing engineer working under the old order of things would plan for certain panels—either bench or vertical panels, arranged perhaps in a semicircle or a hollow square with an operator's desk in the center. From the panels would be carried many control wires and conduits buried in walls and floors or run in trenches, on shelves or in large iron pipes or whatnot.

Back of the panels would perhaps be other panels forming another semicircle or hollow square on which would be mounted relays, watthour meters, etc.

The designing engineer must, of course, be prepared with his design for flexibility of panel groupings in case the ratio of number of panels is different from that assumed. Should there be more 132-kv. lines than were figured on or more 66-kv. lines, etc. his design must permit of complete flexibility or extra costs will result and costly changes will be required. Furthermore, the operators may not find the arrangement to their liking after a few years; and, should practises and standards change, the whole arrangement may become out of date, or the number and arrangement of control conduits may be wrong.

Further than this a great deal of money may be wasted in preparation for future equipment—control wire and conduits which may never be required or the requirements for which may be different.

Contrast the foregoing with the situation existing in our hydro plant. We have spent practically no money in advance for future requirements yet how easily such requirements can be taken care of.

If new 66-kv. or 132-kv. or 13.2-kv. or even 4-kv. feeders are required, switching equipment can be installed in the yard adjacent to the plant; unit switchboards—singly or in groups—can be placed near the equipment to be controlled. The miniature board can be extended 4 inches at a time perhaps and low-voltage, low-current multi-conductor cable installed between the miniature board and the unit switchboard. The original hydro plant layout need not be disturbed and a minimum of cost has resulted.

The ease and economy with which additions can be made was realized during the past spring and summer at which time we added a 132-kv. incoming transmission line, a 60,000-kv-a. three-winding 132/66/13.2-kv. transformer bank, a 66-kv. high-capacity oil circuit breaker. No changes in the hydro station have been required and very little money was expended in advance.

The transformers were located on a platform at one end of the plant adjacent to the 13.2 bus and oil circuit breaker rooms.

The 66-kv. oil circuit breaker is located about 600 feet from the plant.

The transformers have load ratio control on the 132-kv. side and there is no 132-kv. breaker.

The 66-kv. winding is connected to the system through the 66-kv. breaker.

The 13.2-kv. winding is connected to the station generating buses through two 13.2-kv. high capacity switches.

A unit type switchboard located at the end of the station controls the 13.2-kv. oil circuit breakers, and the load ratio transformer control on the roof of the 13.2-kv. bus and switchroom is a weather proof electrically heated steel cubicle containing additional panels necessary to the load ratio control equipment operation. Between these is a small number of relatively short control conduits and wiring. Between these unit switchboard cubicles and the miniature switchboard is the small low-voltage low-current multiple lead-covered control cable run in one conduit. Similarly at the 66-kv. oil circuit breaker is a unit type control switchboard in a similar weatherproof electrically heated steel cubicle supervised from the miniature switchboard through the low-voltage low-current multi-conductor lead-covered cable.

One of the benefits realized in this station is that these extensions and additions have not required dangerous work around operating equipment. The hydro station has not been cluttered with construction tools, materials and debris. The operation of the station has not been jeopardized by working on control wiring and conduits near operating equipment as is customary under the old order of things.

We find, therefore, that after three years of successful operation of the original layout and the successful installation and operation of the recent additions that the decided departure from conventional layouts has been entirely justified. We find that the miniature switchboard permits centralized control and supervision in a successful manner and that the application to this station solved the problem of automatic operation and centralized supervision of a considerable amount of apparatus in a very satisfactory manner with a minimum of space and cost and that the future can be taken care of in a cheaper and better manner and without requiring capital expenditures in advance of the actual requirements. It is possible that in the future there may be required a steam turbo generating station near the hydro station.

This station may contain two or more units. There may be two or more step-up transformers. These may step up from generator voltage to 66 kv. or 132 kv. or some other voltage. We may or may not distribute at generator voltage. The old nightmare of the designing engineer does not disturb us.

We can provide space outside the steam plant for any or all of such equipment in any combination which may be required as our system grows and as new apparatus and methods are developed. Unit type switchboard control units in steel weatherproof cubicles or other later types which may be developed can be mounted near the equipment to be controlled and an additional miniature switchboard or switchboards can be installed, connected together by the low-voltage low-current multi-conductor lead-covered cable in one or two iron conduits. And this is not all.

Should we decide that the control of the steam plant and the hydro plant should be in one place we can move the miniature switchboard and its cubicle from its present location to the steam plant. We will not abandon a large amount of control wire and conduit; we will merely move a relatively small amount of low-voltage low-current multi-conductor lead-covered cable. Thus our problem of looking into the future becomes a very simple one and we need not attempt to decide far in advance requirements for the future which at best can only be guessed at and speculated on, especially in situations similar to ours. Therefore, if engineers and executives will study this method of centralized control by miniature switchboards there will surely be found many applications where economic advantages may be realized and great relief from troublesome system planning may be had, and better all around results obtained when actual developments are made.

The Application of Hydrogen Cooling to Turbine Generators

BY M. D. ROSS*

Associate, A. I. E. E.

Synopsis.—*Hydrogen has many advantages over air when used as a cooling medium for rotating machines such as reduction of windage losses, increase in available output for a given amount of active materials and the absence of corona effects on the insulation in high-voltage machines.*

The paper describes a liquid sealing gland developed for use with turbine generators to prevent escape of gas along the shafts. Two 7,500-kv-a., 3,600-rev. per min. generators with hydrogen cooling have been built and tested. The second machine with the control apparatus is described in detail, and the operating performance over a period of 3½ months is given.

With the development of suitable sealing glands and control apparatus to the point where reliable operation can be expected over long periods, the use of hydrogen cooling for turbine generators appears to be desirable at the present time in ratings of 30,000 kw. and up.

Introduction

THE development of large turbine generators has recently progressed very rapidly, so that the turbine generator of today is considerably smaller, more reliable, and more efficient than the corresponding machine of five years ago. The improvement is largely due to such factors as better methods of ventilation, stronger rotor steels, lower loss armature laminations, and improved methods of insulating rotor windings. The present indications are that future improvement along the lines of present construction will be relatively slow. In recent designs for the largest machines at any given speed, there are indications that we are approaching the limits of possible ratings using the present materials and methods of construction, and accepted performance characteristics. Any marked change in performance or maximum size will probably be the result of a radical change in design.

A great deal of research has, therefore, recently been carried out in connection with the ventilation of rotating machinery by means of gases other than air. A survey of the available gases shows that hydrogen is the most suitable gas for this purpose, due to its low density (about 7 per cent that of air) and high heat transfer characteristics with forced convection. As the windage losses vary approximately in proportion to the gas density, they can be reduced to a negligible figure with hydrogen. This is an important feature in turbine generators where the windage losses are relatively high. Hydrogen can be obtained readily in large quantities and is a relatively low priced gas. Helium, which has somewhat poorer properties as a cooling medium and is non-explosive, is not available in sufficient quantities for this purpose. Its density is about twice that of hydrogen so that windage losses in helium would be somewhat higher.

Advantages of Hydrogen as a Cooling Medium

The advantages of hydrogen as a cooling medium have been covered by a number of writers.[1] Briefly, they are as follows:

1. Windage losses are reduced to about 10 per cent of their value in air with a gas mixture containing 97 per cent of hydrogen by volume. The efficiency of large turbine generators would be improved 0.6 to 1 per cent at full load, 1.2 to 2 per cent at half load, with hydrogen cooling, depending on the size and type of ventilation of the machine.

2. With hydrogen the temperature rise of the gas in passing through the machine is lower than with air, due to the lower losses to be absorbed. The surface temperature differences are considerably reduced with hydrogen and the thermal drops, through insulation where air pockets are present, are lower than with air. For a given amount of active materials the rating of the generator with hydrogen cooling would be at least 25 per cent greater than the rating available with air cooling.

3. Corona has little, if any, effect on insulation in an atmosphere of hydrogen. Insulation operating in hydrogen should, therefore, have a much longer life than in air. The windings would also remain free from dirt, a condition which is difficult to obtain in air cooled machines, even where recirculation of the cooling air is used, as there is always some leakage of air at the shafts and around the duct work.

4. Fires in the generator cannot occur, due to the absence of oxygen. No special fire protection apparatus would, therefore, be required with a hydrogen cooled generator.

5. Due to the better heat transfer and the lower total losses with hydrogen, the gas coolers would be smaller and would require less cooling water as compared with an air cooled machine.

Application of Hydrogen Cooling to Synchronous Condensers and Turbine Generators

A number of hydrogen cooled synchronous condensers is now in service and this type of machine is growing in popularity, as indicated by the number now on order. As the synchronous condenser does not

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1. For references see Bibliography.

have to be mechanically coupled with any other apparatus, it can be readily enclosed in a gas tight casing with provision for cooling the hydrogen gas and recirculating it. The problem so far as hydrogen cooled turbine generators are concerned is somewhat different as the generator shaft must be brought out through the gas tight enclosure, in order to couple it to the turbine. It is also desirable to bring the collector end shaft outside the enclosure, in order to permit work on the field collector ring brushes while the machine is in operation. This is an important feature in turbine generators which are sometimes run six months or more at a time without a shut-down.

Shaft Seals for Turbine Generators

One of the most important developments in connection with hydrogen cooling for turbine generators has been the design of sealing glands to prevent gas leakage around the shafts where they come out of the gas tight enclosure. Experimental work was started in 1925 to develop a seal. A survey of various methods

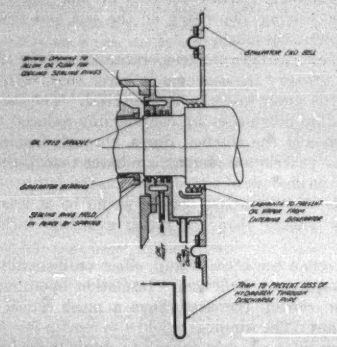

Fig. 1—Cross-Section of Sealing Gland for Turbine Generators

was made and a liquid seal employing oil as the sealing medium was adopted. The seal must be fitted to a shaft somewhat larger in diameter than the generator bearing and an oil seal was best adapted to the relatively high shaft speeds involved. Moreover, a suitable supply of oil under pressure could be obtained from the turbine lubrication system. The sealing gland is shown in Fig. 1. The main body of the sealing gland consists of a casting surrounding the shaft, split on the horizontal center line. A part of the inner bore of the casting is machined to have a clearance around the shaft slightly greater than the clearance in the generator bearing. Oil under pressure is fed into a groove in the center of the seal and flows both ways over the shaft. The flow of oil is throttled to a minimum by fitting three small brass rings in grooves in the seal casting, which are held lightly on the shaft by coil springs. These rings are free to move radially with the shaft. The oil film between the shaft and the seal casting is sufficient to stop the flow of gas at this point, provided the oil pressure is higher than the pressure of the gas in the machine. Oil leaving the hydrogen side of the seal is collected in a chamber in the seal casting and is returned to the main oil supply through a trap, which prevents loss of gas through the drain pipe. Due to the rings in the seal, only a small quantity of oil passes over the shaft and the amount of hydrogen carried out in the oil is very small. No special detraining tanks are required to separate the hydrogen from the oil before returning the oil to the main system.

The power loss in the seal is relatively small. To insure low ring temperatures, some oil is by-passed through a passage around the outside of the ring assembly for cooling purposes. This oil and that discharged along the shaft is returned to the system along with the oil from the generator bearings.

An experimental sealing gland was built in 1926 to determine the performance that could be expected in seals for large generators. The seal was designed to be suitable for a 7,500-kv-a., 3,600-rev. per min. generator. Tests showed that the loss of hydrogen in the oil was negligible, but that a certain amount of air in suspension in the oil entered the enclosure. This amounted to about 0.7 cu. ft. per day. Assuming two such seals in operation, about 70 cu. ft. of hydrogen of 99 per cent purity would be required to maintain the percentage of hydrogen in the machine at 97 per cent. The cost of this gas would be about 70 cents per day. It is not likely that the cost of gas for normal operation would be more than two dollars per day with the largest turbine generators. This figure does not include the cost of carbon dioxide and hydrogen for filling the generator after a shut down.

Turbine Generator Built and Tested in 1928

A 7,500-kv-a., 3,600-rev. per min. generator designed for hydrogen cooling and incorporating the sealing gland described above, was built and tested in 1928. While the rating of 7,500 kv-a. was considerably smaller than the sizes of machines in which hydrogen cooling was considered to be commercially feasible, the machine was large enough to allow construction such as would be used in large generators. Tests were made with the generator running as a synchronous motor, and temperature tests were made at zero power factor. With hydrogen cooling, the generator could be rated at 9,375 kv-a., 80 per cent power factor, with approximately the same temperature rises in stator and rotor as when carrying 7,500 kv-a., 80 per cent power factor, with air cooling. The frame structure was built along standard lines with certain modifications to make it gas tight. Tests on this machine indicated that frame structures for hydrogen cooling would have to be considerably different from the air cooled designs, in order to obtain the minimum gas leakage.

Description of Second Turbine Generator Built

A second 7,500-kv-a., 3,600-rev. per min. generator was built and put on test early in 1930, and is in operation at the present time. This machine is

Fig. 2—7,500-Kv-A., 3,600-Rev. per Min. Turbine Generator on Test Bedplate

The generator was operated as a synchronous motor during tests

shown in Fig. 2 and in section in Fig. 3. In line with standard practise on 3,600-rev. per min. generators, the generator rotor was designed to be coupled to the turbine through a flange coupling, thus eliminating one generator bearing. The generator frame was fabricated from steel plate, with a minimum length of bolted joints. The generator was mounted on a structural steel bedplate at East Pittsburgh for tests. When coupled to the turbine in normal service it would be mounted directly on the foundation side rails. The active parts of the generator are the same as the standard air cooled machine parts, and no particular effort was made to design the machine to obtain the maximum results with hydrogen cooling.

The finned tube gas coolers of Griscom-Russell Co. make are built into the generator frame above the armature core. Straight tube coolers were adopted in order to simplify the construction and to make the cooler sections more accessible for cleaning. Location of the coolers above the generator was considered to involve no operating hazards, as experience with coolers for a large number of machines has never indicated any troubles due to leakage or fractured tubes. One end of the cooler is bolted solidly to the frame and the other end is allowed to move relative to the frame, in order to take up expansion of the cooler tubes. The joint at the free end is made gas tight with a heavy rubber diaphram. The coolers are in two sections, one at each end of the frame, each section being divided into two parts, each of which can be taken out of service and cleaned without disturbing the other parts.

Double enclosing end-bells, split horizontally at the shaft, are provided to carry the gas from the coolers to the fans mounted on the rotor. The frame is designed for the multiple path radial system of ventilation with a single intake chamber at the middle.

The turbine end gland seal is shown in Fig. 4. Two baffle plates are provided in this seal to prevent churning of the oil by the heads of the coupling bolts. These plates are not required in the collector end seal. The seals are bolted rigidly to the ends of the bearing housings, and the sealing rings are located very close to the bearing, so that proper alinement can be readily main-

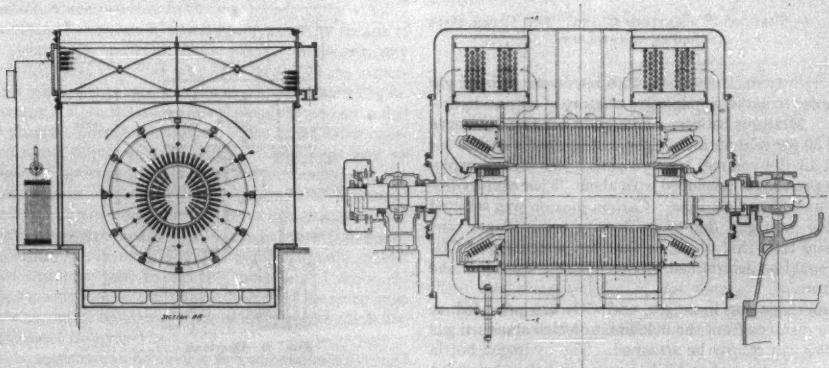

Fig. 3—Cross-Section of 7,500-Kv-A. Generator Shown in Fig. 2

tained. Flexible copper diaphrams are provided between the gland castings and the end-bells, in order to take care of any movement between the pedestal type bearing and the end-bells.

The armature leads are brought out through six gas tight condenser bushings located in a flange on the bottom of the generator. The field leads are carried through holes drilled in the shaft to a point outside the bearing where they are connected to the collector rings. Insulating stuffing nuts are provided to keep the gas from leaking around the field leads.

Description of Gas Control System

A special system for controlling the gas in the machine was developed. The aim in designing this control was to make it as simple and sturdy as possible, so that its operation would be readily understood by the average station attendant. Provision was made for repairing any part of the control mechanism without shutting down the generator.

In filling the generator with hydrogen or in removing

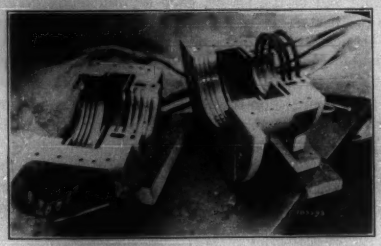

Fig. 4—Turbine End Sealing Gland. The Upper Half is Shown at the Left

the hydrogen, it is necessary to introduce an inert gas in order to avoid an explosive mixture of air and hydrogen. Mixtures between limits of 70 per cent hydrogen and 30 per cent air and 10 per cent hydrogen and 90 per cent air (by volume) are explosive. The highest explosion pressure is developed with about 35 per cent hydrogen and 65 per cent air. Carbon dioxide was selected as the inert gas, due to its density being 50 per cent greater than that of air, so that the percentage of CO_2 gas could be determined by checking the density of the mixture.

Manifolds for hydrogen and CO_2 are provided at some distance from the machine to which standard gas bottles can readily be attached. The hydrogen bottle pressure is reduced to 15 lb. per sq. in. before being fed to the generator. In case of failure of the reducing valve, a safety valve keeps the pressure in the line within safe limits.

The control equipment is mounted in a panel bolted to the side of the generator. This construction was adopted to simplify the piping between the control apparatus and the machine. The layout of the gas piping is shown in Fig. 5, and the electrical circuits in connection with the control in Fig. 6. Both hydrogen and CO_2 supply lines are led to a four-way gas valve on the right hand side of the control, as shown in Fig. 7. With

Fig. 5—Gas Control Cabinet

the handle in the "off" position, the control circuits are disconnected from the three-phase, 110-volt supply, and no gas can enter the generator. With the handle in the CO_2 position, CO_2 gas is admitted and a cam

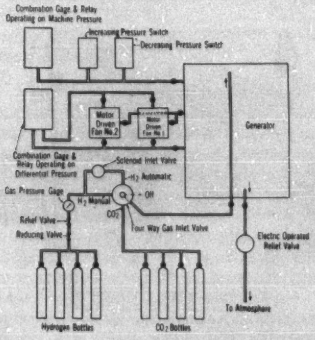

Fig. 6—Diagram of Gas Piping

operated switch on the valve shaft energizes the control circuit.

A small motor-driven fan running at constant speed draws a small quantity of gas from the generator and forces it through an orifice before returning it to the machine. The pressure difference between the two

sides of the orifice is proportional to the density of the gas passing through it. This differential pressure is registered on a gage mounted on the front of the panel. The gage is equipped with contacts controlling a signal which operates when the gas density exceeds a certain value or when the fan pressure drops to zero, indicating that the motor-driven fan has stopped. The motor-driven fans are provided in duplicate, with valves in gas lines, so that one fan can be taken out for repair, without affecting operation of the generator.

When sufficient CO_2 gas has been introduced, as indicated by an increase in the differential pressure gage reading, the control handle is moved to the position marked "Hydrogen Manual," where hydrogen gas can be introduced rapidly through a large valve opening. The differential pressure gage will then show a reduction in pressure as the percentage of hydrogen in the machine increases. When a satisfactory mixture is obtained, the main valve is moved to the automatic position where pressure in the machine is automatically maintained at about 1½ to 2 in. of water by means of an electrically operated needle valve. An electrically operated relief valve allows gas to escape to the atmosphere when the pressure in the machine reaches 2½ in. of water. Both the inlet and relief valves are controlled by pressure operated switches. The relief valve operates when filling with CO_2 and hydrogen as well as when the hydrogen is admitted automatically, but the inlet valve is electrically interlocked so that it cannot open except when the four-way valve is in the automatic position. In removing the hydrogen gas the above procedure is reversed, and the control handle moved from the automatic to the CO_2 position. Carbon dioxide is introduced until the differential pressure gage shows a low enough per centage of hydrogen to allow the machine to be opened up safely.

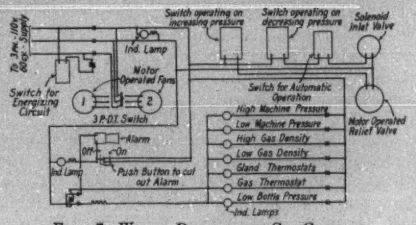

Fig. 7—Wiring Diagram for Gas Control

The machine can be filled with hydrogen or scavenged of hydrogen with the generator field rotating or at standstill. With the rotor running so as to insure nearly perfect diffusion, a volume of CO_2 gas equal to about one and one-half times the gas capacity of the machine is required to remove the air. About three volumes of hydrogen are needed to fill the generator with gas of 95 per cent purity. The gas volume of the 7,500-kv-a. generator is approximately 650 cu. ft. About two hour's time is required in filling the machine with hydrogen.

Relays are provided to sound a horn and light a signal light under the following conditions:

1. Machine pressure too high or too low.
2. Gas density too high, indicating contamination of the gas.
3. Gas density meter reading zero, indicating that the motor driven fan has stopped.
4. Low pressure in the hydrogen line feeding the machine.
5. Excessive gland seal temperature.
6. Excessive temperature of the gas in the machine in case of failure of the cooling water supply.

Operating Experience with 7,500-Kv-A. Generator

The generator has been operated over a period of 3½ months up to the time of writing this paper. Operation has been satisfactory, and no troubles were encountered with the control and sealing gland features. The glands, when inspected after three months running, showed no evidences of wear, with the tool marks still visible on the sealing rings. The glands required no special attention during this period and the valves admitting oil to them were left open at all times, so that the seals were in operation whenever the oil pump was running. When the generator was shut down with hydrogen in it, the oil pump was kept in operation, in order to maintain the seal at the shafts. Without oil pressure there was some leakage of gas through the sealing glands. In case of failure of the oil pressure, the infiltration of air would probably be very slow.

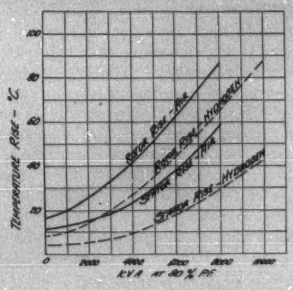

Fig. 8—Temperature Rises of 7,500-Kv-A. Generator with Air and Hydrogen Cooling

Performance of 7,500-Kv-A. Generator

The temperature rises of the stator and rotor of the second 7,500-kv-a. generator with air and hydrogen cooling are given in Fig. 8. With hydrogen cooling, the machine could be rated 9,375-kv-a., 80 per cent power factor. The windage loss with hydrogen was 7 kw. as compared with 73 kw. with air, which means an improvement in efficiency of 1.1 per cent at 6,000 kw.,

80 per cent power-factor load. The cooling water required for 9,375 kv-a., 80 per cent power factor with hydrogen cooling was approximately 40 gals. per min., as compared with 80 gals. per min. with air cooling and 7,500 kv-a., 80 per cent power-factor load.

Safety Precautions

The first question usually raised in connection with hydrogen cooling is: what will happen in case of an explosion? With the worst mixture of air and hydrogen, the explosion pressure will be 50 to 75 lb. per sq. in., depending upon the size and shape of the gas chambers in the generator. If the frame is provided with a number of diaphrams which will blow out at relatively low pressure, the pressures developed will be lower than the above values. The operating hazard for a hydrogen cooled generator with control system such as that described above is probably no greater than that involved in operating other types of control station equipment, and it is, therefore, questionable as to whether frame structures for hydrogen cooled generators should be made explosion proof. No attempt was made to make the two 7,500-kv-a. generators described above to withstand explosion pressures. However, if an explosion-proof frame is considered necessary, it can be built with some increase in cost over that of the lighter frame designed on the basis of gas tightness only.

Desirability of Adopting Hydrogen Cooling

With the development of sealing glands and control apparatus to the point where their operation can be relied upon in continuous service, there is no apparent reason why hydrogen cooling should not be adopted for large turbine generators. The cost of hydrogen gas should not exceed two dollars per day for the largest generators, which would be negligible as compared to the saving in generator losses. The labor cost for operating the generator would be no greater than for an air cooled generator, as the turbine floor attendant can take care of the gas control equipment. While the hydrogen cooled design would be somewhat harder to take apart than an air cooled design, the absence of dirt and corona effects in the machine would probably more than balance that point in estimating the cost of maintenance. With built-in gas coolers, as in the hydrogen cooled machine, considerable space under the generator would be available for removal of condenser tubes and for auxiliary apparatus which is not available with air coolers located below the generator.

The following example will indicate the savings in operating expenses with hydrogen cooling, and the possible capitalization of these savings.

Assuming the following factors:

1. A generator of 100,000 kw. rating at 1,800 rev. per min.
2. Reduction in windage losses with hydrogen cooling—600 kw.
3. Operating time per year—7,000 hr.
4. Value of power at the bus—0.4 cents per kw. hr.

The savings can be evaluated as follows:

Value of power at bus (4,200,000 kw. hr.)	$ 16,800
Less cost of gas at $2.00 per day and cost of hydrogen and carbon dioxide for 3 fillings after shut-down	1,100
Net saving per year with hydrogen cooling	$ 15,700
Capitalization of savings at 15 per cent	$105,000

Preliminary studies of large turbine generators indicate that hydrogen cooling would be desirable in ratings of about 30,000 kw. and up, where the additional cost of the gas-tight-enclosure and the control equipment would be relatively small in comparison with the cost of the generator, and the saving in losses would justify the slight additional complication due to this method of cooling.

Acknowledgment

The writer wishes to acknowledge the early work on hydrogen cooling carried out by Mr. C. J. Fechheimer, Mr. G. W. Penney, and Mr. C. M. Laffoon and others with whom the writer is associated. The sealing gland and method of measuring gas density were developed by Mr. Penney.

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1. "Fundamentals of Heating Calculations of Electric Machines," by R. Pohl, *Archiv Fur Elektro technik*, Vol. 12, 1923, p. 361.
2. *Hydrogen as a Cooling Medium for Electrical Machines*, by Edgar Knowlton, Chester W. Rice, and E. H. Freiburghouse, A. I. E. E. Trans., Vol. 44, 1925, pp. 922-934.
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5. *Outdoor Hydrogen-Ventilated Synchronous Condensers*, by Robert W. Wieseman, A. I. E. E. Trans., Vol. 48, 1929, pp. 1221-1226.

Discussion

S. L. Henderson: With our present knowledge of materials, the maximum possible rating of turbine generators in single machines is approximately 31,250 kv-a. at 3,600 rev. per min. and 200,000 kv-a. at 1,800 rev. per min., with air as the cooling medium. Cooling with hydrogen, therefore, offers the possibility of increasing this upper limit at least 25 per cent. One of the difficulties in the application of hydrogen cooling to turbine generators has been to obtain a proper seal between the shaft and the end-bell, a problem which could be eliminated in the hydrogen cooled synchronous condenser. The paper shows that the leakage can be controlled within commercial limits, and the density of the gas can be maintained automatically above an explosive mixture.

There is some question whether it is necessary to build these machines explosive proof, and this question is one that can only be answered by experience. The arrangement of coolers within the confines of the machine should be of interest as the machine is complete in itself and does not require the addition of ducts or cooler supports. Such a machine lends itself readily to the outdoor station for being gas proof, it must of necessity be weather proof.

The Newfoundland-Azores High-Speed Duplex Cable

BY J. W. MILNOR,* and G. A. RANDALL,†
Fellow, A. I. E. E. Non-member

Synopsis.—A description is given of the cable laid by the Western Union Telegraph Company in 1928 between Newfoundland and the Azores. The plan of this cable is unique in that it consists of a combination of non-loaded and loaded cable, the latter being tapered throughout certain portions. By means of this arrangement it has been possible to secure duplex or two-way operation at a speed of 1,400 letters per minute in each direction. The general methods used in obtaining the duplex balance are described and the fundamental theory of the cable is given.

* * * * *

IN September 1928 the final splice was made in a submarine telegraph cable of a new type connecting Bay Roberts, Newfoundland, and Horta, Azores. This cable adds an important link to the transatlantic cable system of the Western Union Telegraph Company. From Newfoundland it is extended to New York by land line connections, and at Horta connections are made with German and Italian cables.

This cable combines the advantage of high-speed operation characteristic of the new continuously loaded cable, with the facility of duplex or two-way operation inherent in the old non-loaded type of cable. Tests have shown that a signaling speed of 1,400 letters per minute cable code (42 cycles per second) simultaneously in both directions is now practicable, and the indications are that the ultimate duplex speed may be somewhat higher. If operated simplex, *i. e.*, in one direction only, the speed is somewhat over 2,500 letters per minute. The duplex speed is several times as high as any long cable has previously been duplexed, and provides the greatest message capacity of any existing transocean cable.

Prior to 1890, Heaviside had shown the important relation which exists between the inductance of a cable and its operating speed. However, it was not until recently that this knowledge could be applied to long telegraph cables, as a result of the development by the Bell Telephone Laboratories of a new magnetic alloy, permalloy, and of means for applying it to submarine cables.[1] The first cable of this type was laid by the Western Union Telegraph Company between New York and Azores Islands in 1924. The results obtained on that cable fully justified expectations, and since then a total of eight other long-loaded cables has been laid by various companies in the Atlantic and Pacific Oceans.

It has long been customary for the relatively slow speed non-loaded submarine cables to be operated simultaneously in each direction.[2] The electrical characteristics of loaded cables are such that they are much more difficult to duplex. For this reason all loaded cables laid between 1924 and 1927 were designed to be operated simplex only. Special devices were developed for quickly reversing the direction of transmission at regular intervals which may be as short as one minute. There are many situations, however, where duplex operation is decidedly advantageous, and it was inevitable that attempts should be made to develop a cable capable of high-speed duplex operation.

The desired result was obtained in the cable here described through the use of a combination of loaded and non-loaded cable, the latter being adjacent to the ends.

General Theory of Duplex Cables

In order to duplex a cable it is necessary to provide at each terminal station a so-called artificial line which closely simulates the impedance of the cable as measured from its end. The highest practicable accuracy of balancing is desirable, as the cable efficiency is thereby increased. The two impedances should closely match throughout a range of frequencies from about half the fundamental signaling frequency up to about 1.6 times that frequency; a somewhat less accurate balance is necessary at frequencies outside of that range.

A loaded cable is more difficult to balance than non-loaded cable for three reasons:

1. There are irregularities in the impedance as measured at different points along a loaded cable, which are much more pronounced than if non-loaded. Difficulties in manufacture are such that variations from the mean values of inductance are appreciable and unavoidable.

2. The effect on balance of such irregularities is more

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1. "Permalloy, An Alloy of Remarkable Magnetic Properties," by H. D. Arnold and G. W. Elmen, *Jour. Franklin Inst.*, V. 195, 1923, p. 621.

The Loaded Submarine Telegraph Cable, by O. E. Buckley, A. I. E. E. Trans., 1925, p. 882.

A bibliography on loaded cable practise appears in an article by Foster, Ledger, and Rosen, *Jour. I. E. E.*, London, April 1929, p. 475.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

2. Duplex operation was first introduced in 1871.

prominent in a loaded cable. The reason for this is that the velocity of wave propagation is slower and therefore the reflections from different parts of the cable tend to appear distinct, while in the non-loaded cable they are relatively merged. A more accurate balance is therefore necessary with loaded cable.

3. The impedance of a loaded cable is not constant. The inductance due to the loading material, as well as the effective resistance, vary with current as indicated in Fig. 2. The signaling current, of course, continually changes in magnitude, consequently the cable impedance undergoes frequent small variations.

Each of the above difficulties is reduced in importance by loading only the central portion of the cable, the end sections being non-loaded. For the non-loaded portion, a duplex balance is obtained having about the same order of accuracy as has been customary in the past in cables which are non-loaded throughout their length. In the part of the artificial line corresponding to the loaded portion, a duplex balance of a much lower degree of accuracy is sufficient, since it is distant from the terminals.[3]

The correctness of the above will be made evident in the Appendix, in which a more complete exposition is given of the influence of various factors. Some of these factors may be briefly summarized as follows:

The permissible attenuation from end to end of the cable as measured at the working frequencies, is determined by the degree of accuracy with which the artificial line as a whole can be adjusted to match the impedance of the cable. With higher accuracy a greater attenuation is permissible. The attenuation which is permitted from end to end of the *loaded* portion of the cable is determined by the accuracy with which the impedance of that portion of the cable can be matched in the corresponding portion of the artificial line.

In order that the most economical design of cable may be obtained, it is necessary that special consideration be given to methods for improving the stability of the impedance of the loaded portion of the cable, as affected by changes in the inductance of loading caused by varying current. The amount of change of inductance with current may be controlled to some extent through variations in the core design and in its treatment during manufacture. By the use of sea earths of proper length at the ends to hold the level of extraneous interference to as low a value as readily practicable, the use of a lower battery voltage at the opposite end of the cable is made possible. This results in lower current strength throughout the cable, with a corresponding gain in stability.

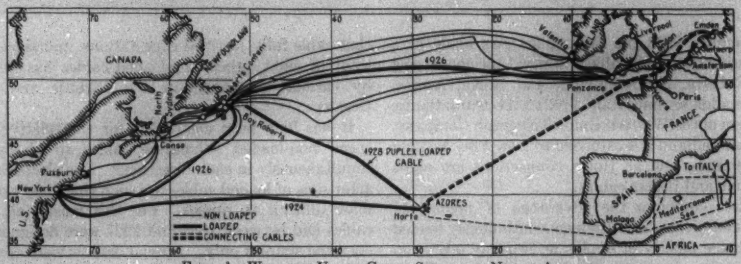

FIG. 1—WESTERN UNION CABLE SYSTEM IN NORTH ATLANTIC

DESCRIPTION OF 1928 CABLE

A theoretical study completed in January 1928, had indicated the commercial practicability of this type of duplex cable. At that time telegraph traffic on the simplex loaded cable previously laid between New York and Azores, was growing so rapidly that some increase in the facilities was urgently needed. This growth could be met only by providing a second connection at an early date, and it was considered desirable to take

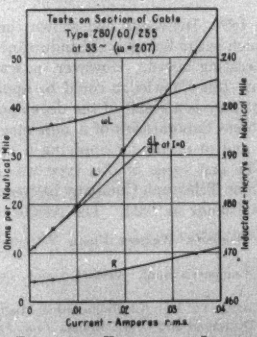

FIG. 2—EXAMPLE OF VARIATION OF LOADED CABLE RESISTANCE AND INDUCTANCE WITH CURRENT

3. It should not be inferred from these statements that it would be impossible to duplex an all-loaded cable. If by proper design the attenuation from end to end were held to a reasonably low value, and other factors are correct, a balance should be practicable. The combination of loaded and non-loaded cable was used in the present case in order to obtain the desired results at a minimum expense.

advantage of this new development in the cable about to be laid.

Because of the nature of the new technical problems involved in the design of the cable and its balancing equipment, and the lack of time in which to make proper experimental confirmation of some of them, it was considered advisable that the cable be so constructed as to permit simplex operation, if for any reason the duplex balancing should not prove satisfactory. However, it was found that in order to provide for simplex operation at the higher speed, it was unnecessary to make any important change from the design which was found to be best for duplex working.

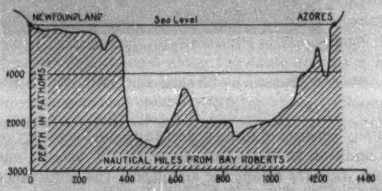

FIG. 3—PROFILE OF OCEAN BOTTOM ROUTE OF 1928 CABLE

The formal order for the manufacture was placed in March, 1928, although some details of the design were not completed until much later. The cable was manufactured by the Telegraph, Construction and Maintenance Company, Ltd. of London, and laid by their cable ship *"Dominia,"* the laying being completed on September 1st of that year.

The route followed by the cable is indicated in Fig. 1. A profile chart showing its depth is given in Fig. 3.

Plan of Cable. The total length of the cable is 1341.2 nautical miles. The non-loaded end portions are each 160 miles in length. The remainder of the cable is loaded, two different weights of conductor and loading being used. In Table I are given further details of the design, together with the results of electrical tests.

TABLE I
PHYSICAL AND ELECTRICAL CHARACTERISTICS OF EACH CORE TYPE

Factory values per nautical mile, at 75 deg. fahr.

	Ends	Intermediate	Center
Weight of copper—lb.	508	329	284
Weight of loading—lb.	non-loaded	51	60
Weight of gutta percha—lb.	303	275	258
Resistance d-c.—ohms	2.34	3.60	4.18
Capacitance—μf.	0.357	0.344	0.342
Dielectric resistance—megohms	746	919	1004
Section length—miles	2 x 160	2 x 70	881

Conductor. The conductor consists of a solid copper central wire closely surrounded by five or six thin copper tapes.

Loading. The loading throughout the central portions is Mumetal wire which was supplied by the Telegraph, Construction and Maintenance Company. This material is similar in general composition and characteristics to permalloy. The wire is wound directly on the copper conductor in a close continuous wrapping. The heaviest loading wire is about 0.008 in. in diameter. It is estimated that the total length of loading wire in this cable is about 50,000 miles.

An inductance range from 0.056 to 0.230 henrys per mile was obtained in different portions of the cable through controlled variations in the annealing of the loaded conductor. The average values of inductance in the various sections are indicated in Fig. 4. The permeability (corresponding to small magnetizing forces) ranges from 2,000 to 6,800. This maximum value materially exceeds any previously obtained in cable construction.

A special asphaltum flux was used to fill the interstices of the loaded conductor, for the purpose of equalizing pressure upon this strain-sensitive loading material.

Insulation and Armoring. The conductor, either loaded or non-loaded, was covered with a conservative thickness of a gutta percha mixture for purposes of insulation, in accordance with usual cable practise. This in turn was covered with wrappings of jute and a sheath of iron or steel wires to provide tensile strength and protection against injury. The portions near shore have much heavier sheathing than the main sections, and in addition have a wrapping of brass tape applied to the core as a precaution against injury by the teredo.

Sea Earths. The cable is single cored except near the ends. In order to reduce as far as possible the extraneous voltages picked up in shallow water, it is customary in cable practise to carry a second and sometimes a third core in the sections of cable near the ends. These additional cores are grounded at their outer ends to the cable sheath. At Bay Roberts the distance out to the end of the longer or receiving sea earth is 50.10 nautical miles. At Horta, Azores the corresponding distance is 2.39 nautical miles. The receiving sea earth connection at Bay Roberts differs from the other sea earth connection in the fact that a manganin resistance of 160 ohms is connected at sea in series with the sea earth core. This resistance assists in equalizing any

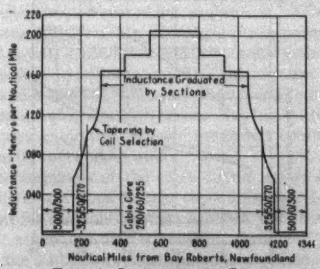

FIG. 4—INDUCTANCE OF CABLE

extraneous voltages picked up by cable and sea earth cores between that point and the receiving apparatus.

Cable Tests

Various tests were made during manufacture and upon the laid cable in order to ascertain its performance characteristics.

The results of the d-c. tests are as follows:

Total d-c. resistance of cable	4521	ohms
Total d-c. capacity of cable	463	μf.
Total dielectric resistance, mean of first 5 minutes	24.65	megohms

The attenuation of the cable was measured in each direction at various frequencies and with several sending voltages. The results of some of these tests are shown in Fig. 5. The curves show the attenuation of the fundamental frequency of a square-topped transmitted wave. The effect of changing the sending voltage was found to be small.

The time of propagation of signals over the cable is 0.235 seconds at 60 cycles. Based upon the measured values of time of propagation and attenuation at various frequencies, it has been estimated that during the process of laying, the strains placed on the cable caused an average loss in the inductance of about 13 per cent.

In Fig. 6 is shown a record of interference obtained at Newfoundland, using the 50-mile sea earth. At

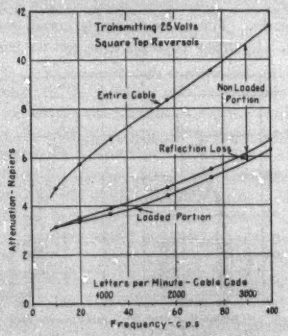

Fig. 5—Attenuation of Laid Cable

Azores where the cable reaches deep water within a few miles, the interference is much less.

Artificial Line Equipment

An essential part of this cable project has been the provision at each cable station of an adequate artificial line for completing the Wheatstone Bridge arrangement which is necessary for duplex operation. This line is a network which is an electrical image of the cable, designed to match it mile for mile as accurately as is necessary for the purpose. A certain amount of work has been done by others[4] with a view to providing a network for general use which would contain less electrical capacity than the actual cable, and would therefore be relatively inexpensive. The resultant network has always been complicated in theory and possibly difficult to adjust. In the present work, no attempt was made to use such devices.

The present equipment is somewhat more complicated than in the case of the usual artificial line for low-speed cables. For a non-loaded cable the network

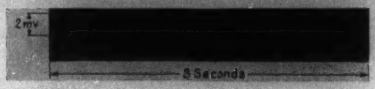

Fig. 6—Record of Interference in Cable, High Amplification

usually consists of units of resistance and capacity only. In addition to these two elements, the present equipment includes means for simulating the inductance of both non-loaded and loaded portions, eddy current losses in the loading, sea return losses, and dielectric losses. A compensating arrangement is introduced to prevent filter oscillations between the condenser and inductance units of the artificial line, in order to avoid the use of an excessive number of coils. The artificial line is adjustable in every portion, the size of the steps at each point of the line depending upon the accuracy of impedance matching required at that point.

As originally installed, the artificial lines were set up on the basis of computed values, the initial error in matching being of the order of one per cent. Uncertainties in the exact distribution of the cable constants themselves prevented a closer initial balance. The final balances were obtained by a systematic adjustment of the various elements composing the line.

As in the case of the cable itself, the change of inductance with current in the coils of the artificial line must be given consideration in their design. In theory it might be practicable to provide artificial line coils whose inductance would vary with current in the same manner that the inductance varies in the corresponding portion of cable, thus permitting the use of relatively larger inductance variation in designing the cable. Such procedure would greatly increase the complexity of the artificial line. It was therefore considered desirable to hold the current-inductance coefficients (defined in the Appendix) of both the cable and artificial line to relatively small amounts, the values for the artificial line being kept smaller than those in the cable.

Because of the use of inductance coils, some other unusual precautions were necessary in this line. In the usual artificial line, the resistance elements are of wire having low temperature coefficient. This wire did not

4. One of these methods is described by H. Salinger and H. Stahl, "The Simulation of Long Submarine Cables," *Electrische Nachrichten Technik*, August, 1926.

appear to be practicable in the inductance coils, and in order that the line should not be unusually susceptible to temperature changes, the copper resistance of the coils has been kept at a relatively low value. This necessitated larger coils than would otherwise have been used.

Special precautions have been taken to prevent pick-up of extraneous disturbances by the coils. This is accomplished by a method of mounting and wiring the coils which effectively transposes them with respect to external fields.

Each artificial line occupies a space of approximately 210 cu. ft. These lines are mounted in heat-insulated cabinets. To correct for variations in the balance due to temperature changes along the cable and to a less extent in the artificial line, five adjustable resistances have been provided in the operating room. Observations over a period of more than a year indicate that these resistances need not be altered more than once daily to maintain a satisfactory balance.

The method used in making the final field adjustments to the artificial lines is not new, being similar in

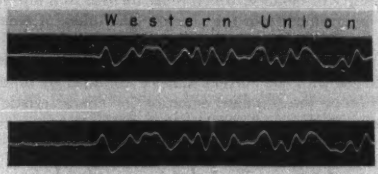

Fig. 7—Received Signals at Bay Roberts—Cable Code

Upper record—simplex, westward transmission
Lower record—duplex, eastward and westward transmission

principle to that used for many years with non-loaded cables, except that a high-speed recording device was used to observe the balance. During the greater part of the work a single reversal at a time was transmitted into the cable, while a record was obtained of the resulting current caused by lack of perfect balance. In the final refinements of balance different combinations of telegraphic signals were sent at various speeds.

A Western Electric Signal Shaping Amplifier[5] is used both during balancing and for commercial operation, the amplifier being standard except for a few minor alterations and additions.

In Fig. 7 are shown records of characters received, both simplex and duplex, over the cable. The speed here is 1,400 letters per minute cable code. A 12-volt battery was used for sending, and the strength of received signal with reversals at the above speed is about six millivolts. It has been estimated that the impedance of the artificial line matches that of the cable to an accuracy of roughly one part in 4,000, as determined when telegraph characters are transmitted. Such characters may be considered to be made up of many frequencies sent simultaneously. When determined with a sinusoidal voltage in the frequency range between 35 and 55 cycles, the estimated accuracy is better than one part in 7,000.

Relative Merits of Simplex and Duplex High-Speed Cables

In determining the kind of cable facilities best suited for handling telegraph traffic between two given areas, there are many factors which must be considered, and individual consideration for each project is therefore required.

The relative advantages and economics of simplex and duplex cables depend to a certain extent upon the length of the circuit and the desired message capacity. Consideration must be given to the relative amounts of urgent and deferred traffic; the difference if any in the load in the two directions; and the difference in time between the terminal areas. The nature of any existing cable facilities between the terminals must be taken into account in determining the method of increasing the facilities. It is also important to consider the manner in which the traffic would be handled in the event of a failure of one or more cables.

Where the load between two points is sufficient to justify only a single cable, and the difference in time between terminals is not excessive, the duplex type usually offers the more convenient method of handling it. An alternative arrangement is a simplex cable with automatic direction reversing equipment. The latter plan necessarily results in part of the traffic being somewhat delayed, a condition which in some cases is objectionable. In addition, much of the terminal equipment is idle half the time, and when working must be operated at relatively high speed. On the other hand, the duplex cable requires the somewhat complicated artificial line networks.

If the terminals of the cable circuit are so located that there is a considerable time difference between them, as for example between America and Japan, the simplex cable has the advantage, since the greater part of the traffic flows in one direction at a time.

Where the amount of traffic is enough to justify two or more cables, there are various possible solutions. The equivalent of a duplex circuit can, of course, be obtained through the use of two simplex cables worked in opposite directions. Ordinarily, however, the rate of traffic growth is not sufficient to justify laying two cables at the same time or within short periods. In some cases, a combination of simplex and duplex cables may be found to be the best ultimate solution when all factors are taken into account.

It appears probable that neither the simplex nor the duplex cable has as yet reached its limit of economical development, hence it is hardly practicable at this time to give a final answer to the above questions.

5. "Applications of Vacuum Tube Amplifiers to Submarine Telegraph Cables," by Austen M. Curtis, *Bell System Technical Jour.*, July, 1927.

ACKNOWLEDGMENTS

The fact that the application of non-loaded ends to a loaded cable would facilitate duplex working was discovered independently by Mr. J. J. Gilbert of the Bell Telephone Laboratories, and by the first named of the authors.

The cable under discussion and the duplexing equipment were developed, designed, and balanced by engineers of the Western Union Telegraph Company. Important contributions were made by Messrs. C. H. Cramer and W. D. Cannon. Most of the field adjustments of artificial lines were in charge of Mr. E. L. Newell. The route and the sheathing structure for the cable were worked out by Mr. V. H. Rowland.

Appendix

NOTES ON THEORY OF DUPLEX LOADED CABLE

Accuracy of Balance. The general theory of duplex balancing was published in a previous paper.[6] It was shown that the required degree of accuracy by which the impedance of the artificial line must match the impedance of a uniform cable, is proportional to the napierian base having as an exponent the attenuation. In the case of a composite cable of the type under discussion, the same theory is applicable providing that the proper value of attenuation is used.

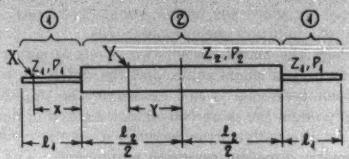

FIG. 8—NOMENCLATURE

Section 2 is long, *i. e.*, $\epsilon^{-2\alpha_2 l_2}$ is neglected
Z_1 and Z_2 are the characteristic impedances
P_1 and P_2 are the propagation constants per unit length
α_1 and α_2 are the attenuation constants per unit length

The full theory of a tapered loaded cable involves expressions which are somewhat unwieldy. The theory will be discussed here on the assumption that the loaded portion is uniform throughout, the cable being as indicated in Fig. 8. The manner of taking into account tapering of the inductance will be discussed later.

For determining the necessary accuracy of the duplex balance of the cable in Fig. 8, the following is the proper value of attenuation:

Total attenuation

$$= \log_\epsilon \left| \cosh P_1 l_1 + \frac{Z_1}{Z_2} \sinh P_1 l_1 \right|$$

$$+ \log_\epsilon \left| \cosh P_1 l_1 + \frac{Z_2}{Z_1} \sinh P_1 l_1 \right| + \alpha_2 l_2 \qquad (1)$$

This value is also found to be the natural logarithm of the ratio of sent to received voltage with the cable terminated in an impedance equal to the cable impedance as measured from that end. This expression, as well as those immediately following, have been developed by straightforward methods from the well known formulas for current and voltage of lines having distributed resistance, capacity, etc. In applying these expressions, they must in general be considered throughout a range of frequencies, preferably including all frequencies from zero to about twice the fundamental signaling frequency.

At points other than at the head of the line, the required accuracy of adjustment of the artificial line is less, since the effect of any irregularity is attenuated. It may readily be shown that at point X, Fig. 8, the required degree of accuracy is proportional to the napierian base with an exponent as given in equation (1), but with x written in place of l_1. At point Y, the required accuracy of adjustment is proportional to $\epsilon^{2\alpha_2 y}$.

Stability of Balance. The manner in which the inductance and resistance of loaded cable varies with current was indicated in Fig. 2. This tends to cause an instability of balance, which must be held to a low value as determined from the necessary accuracy of balance. An expression for the fractional rate at which the cable impedance as measured with alternating current changes with the current may be obtained by differentiation of the standard equation for characteristic impedance. For loaded cables the result is approximately:

$$\frac{dZ}{Z\,dI} = \frac{1}{2}\left(\frac{dL}{L\,dI} - \frac{j\,dR}{\omega L\,dI}\right) \qquad (2)$$

In general it is found that the effect on balance of the change of resistance is less important than the change of inductance.

It is convenient to introduce the expression, "current coefficient of inductance," as a measure of the fractional rate of change of inductance as measured with small values of alternating current; *i. e.*, the quantity $(dL/L\,dI)_0$. Its value depends upon the characteristics of the loading material and the diameter, in accordance with the following equation;

$$\left(\frac{dL}{L\,dI}\right)_0 = \frac{0.566}{d+t}\left(\frac{d\mu}{\mu\,dH_{max}}\right)_0 \qquad (3)$$

in which I is in amperes r. m. s.; d is the diameter of the copper conductor, and t is the thickness of loading material in centimeters. The quantity $(d\mu/\mu\,dH)_0$ is a property of the loading material. It might be expected to be constant for given material but in fact is found to vary to some extent with the absolute value of permeability, and to a lesser amount with the design and treatment of the core.

The necessary strength of received signal is determined from an estimate of the expected level of interference at each end of the cable, together with the expected amount of extraneous voltage due to the inaccuracy of balance. Knowing this value, the

6. *Submarine Cable Telegraphy*, by J. W. Milnor, A. I. E. E. TRANS., 1922, p. 30.

necessary signaling voltage at any point Y may be determined by the use of the following expression for attenuation:

Attenuation from point Y to opposite end of cable

$$= \log_e \left| \cosh P_1 l_1 + \frac{Z_2}{Z_1} \sinh P_1 l_1 \right| + \frac{\alpha_2 l_2}{2} + \alpha_2 y \qquad (4)$$

The value of signal current at the point is readily determined from the signaling voltage.

Tapered Loading. There are two advantages of tapering (decreasing) the inductance near the ends of the loaded position: (1) tapering reduces the loss of signaling energy caused by reflections from the non-loaded portion; (2) tapering permits the use of lower permeability in the loading material in this portion, which results in a smaller current-inductance coefficient.

On the other hand, the use of lower inductance has the disadvantage of causing the attenuation constant per mile to be increased; and to avoid having the attenuation become too large it was advantageous to make use of a heavier copper conductor within 70 miles of each end of the loaded portion of the 1928 cable. This has the additional advantage that the current-inductance coefficient is also lowered when the size of conductor is increased, as indicated in equation (3).

A solution of the equations of signal propagation in a circuit having linearly tapered inductance and resistance, and uniform capacitance and leakance is as follows:[7]

The parameters of the circuit are assumed to be:
$R_0 + K_R x$, the resistance per unit length
$L_0 + K_L x$, the inductance per unit length
C and G, the capacitance and leakance per unit length
K_R and K_L, the rates of tapering
x = the distance from the head of the taper
Subscript zero always refers to the point $x = 0$

The fundamental differential equations of current and voltage at any point are:

$$-\frac{dE}{dx} = [(R_0 + K_R x) + j\,\omega\,(L_0 + K_L x)]\,I \qquad (5)$$

$$-\frac{dI}{dx} = [G + j\,\omega\,C]\,E \qquad (6)$$

Differentiating and substituting,

$$\frac{d^2 I}{dx^2} = (G + j\,\omega\,C)\,[(R_0 + j\,\omega\,L_0) + (K_R + j\,\omega\,K_L)\,x]\,I \qquad (7)$$

7. This solution was worked out by Mr. John W. Arnold, and was published in the *Proc.* I. R. E., February 1931, p. 304.

Substituting

$$\theta = x\sqrt{(G + j\,\omega\,C)\,(R_0 + j\,\omega\,L_0)} \qquad (8)$$

and

$$\alpha = \frac{K_R + j\,\omega\,K_L}{\sqrt{(R_0 + j\,\omega\,L_0)^3\,(G + j\,\omega\,C)}}$$

this becomes

$$\frac{d^2 I}{d\theta^2} = (1 + \alpha\,\theta)\,I \qquad (9)$$

This equation may be solved by infinite series with the result

$$I = k_1 S_1 + k_2 S_2 \qquad (10)$$

where

$$S_1 = 1 + \frac{\theta^2}{2\,!} + \frac{\theta^3\,\alpha}{3\,!} + \frac{\theta^4}{4\,!} + \frac{4\,\theta^5\,\alpha}{5\,!} + \frac{\theta^6\,(1+4\alpha^2)}{6\,!} + \frac{9\,\theta^7\alpha}{7\,!} + \frac{\theta^8(1+28\,\alpha^2)}{8\,!} + \frac{\theta^9(16\alpha + 28\alpha^3)}{9\,!} + \frac{\theta^{10}(1 + 100\alpha^2)}{10\,!} + \ldots$$

$$S_2 = \theta + \frac{\theta^3}{3\,!} + \frac{2\,\theta^4\,\alpha}{4\,!} + \frac{\theta^5}{5\,!} + \frac{6\,\theta^6\,\alpha}{6\,!} + \frac{\theta^7(1 + 10\alpha^2)}{7\,!} + \frac{12\,\theta^8\,\alpha}{8\,!} + \frac{\theta^9(1 + 52\,\alpha^2)}{9\,!} + \frac{\theta^{10}(20\,\alpha+80\,\alpha^3)}{10\,!} + \ldots \qquad (11)$$

(NOTE: There are no simple expressions for the general terms of these series. Usually seven to nine terms are sufficient in practise.)

Combining (6) and (10), we have

$$E = -Z_0\left[k_1\frac{dS_1}{d\,\theta} + k_2\frac{dS_2}{d\,\theta}\right] \qquad (12)$$

in which

$$Z_0 = \sqrt{\frac{R_0 + j\,\omega\,L_0}{G + j\,\omega\,C}} \qquad (13)$$

The constants k_1 and k_2 may be obtained from a consideration of the boundary conditions at $x = 0$, with the result,

$$I = I_0 S_1 - \frac{E_0}{Z_0} S_2$$

$$E = E_0\frac{dS_2}{d\,\theta} - I_0 Z_0\frac{dS_1}{d\,\theta} \qquad (14)$$

If Z_a is the impedance terminating the tapered section, the voltage and current at this point are defined by the equation:

$$E_a = I_a Z_a \qquad (15)$$

Substituting these values for I and E in equation (14), we may eliminate I_0 and finally obtain:

$$\frac{E_0}{E_a} S_1 \frac{dS_2}{d\theta} - S_1 = \frac{Z_0}{Z_a} \frac{dS_1}{d\theta} + \frac{E_0}{E_a} S_2 \frac{dS_1}{d\theta} \tag{16}$$

A study of the functions S_1, S_2, $\frac{dS_1}{d\theta}$ and $\frac{dS_2}{d\theta}$ shows that

$$S_1 \frac{dS_2}{d\theta} = 1 + S_2 \frac{dS_1}{d\theta} \tag{17}$$

Therefore:

$$\frac{E_a}{E_0} = \frac{1}{S_1 + \frac{Z_0}{Z_a} \frac{dS_1}{d\theta}} \tag{18}$$

Similarly, it can be shown that

$$\frac{I_a}{I_0} = \frac{1}{\frac{Z_a}{Z_0} S_2 + \frac{dS_2}{d\theta}} \tag{19}$$

When the rate of taper is zero, α becomes zero and

$$S_1 = \frac{dS_2}{d\theta} = \cosh \theta$$

$$S_2 = \frac{dS_1}{d\theta} = \sinh \theta \tag{20}$$

that is, equations (14) reduce to the usual form of the equations of the current and voltage at any point in a uniform line.

Discussion

E. R. Shute: A study of the map given in Fig. 1 shows two loaded cables and seven non-loaded cables connecting Newfoundland with points east, while only one loaded cable and four non-loaded cables connect with the United States. The difference is made up by a system of land wires connecting the long cable sections with New York and other cities, thereby providing, by means of another route, additional assurance of uninterrupted service.

The Bay Roberts-Horta cable forms a diagonal of a quadrilateral. These triangular circuit layouts are greatly desired by the traffic forces because of the protection they afford in case of stoppage or failure of any one of the lengths.

The Commercial Cable Company has similar protection in its Canso-Horta cables.

The wisdom of these layouts was shown following the earthquake of November 18, 1929, when the loaded cable between New York and Horta was interrupted in deep sea.

There are many factors entering into the determination of whether a duplex or simplex cable shall be laid between two points. Generally speaking, where there is only one cable on the route, as in the case of the Bay Roberts-Horta cable, a duplex cable is indicated. A combination of two cables, one a high-speed simplex and the other a duplex cable, in most cases presents an ideal solution. Because of the time difference between terminals, the load generally is lopsided. With this combination you can turn the simplex cable in the direction of the larger file and thus have three times the capacity (assuming that the simplex cable works at twice the duplex cable speed) in one direction as in the other and have continuous communication in the opposite direction for fast services and replies.

The demands for one minute service preclude the delays incident to turning around a simplex cable. This demand is not a theoretical one, for without such service our brokers cannot take advantage of the difference in quotations on a stock on the exchanges on either side of the Atlantic.

Both the Western Union and the Commercial Cable Company support the New York-Horta-Emden route by the via London connections.

As mentioned before, where the permalloy parallels old type cables, as in the case of the New York-Bay Roberts-Penzance-London cable, shown in Fig. 1, the difference in time between the terminals tends to favor simplex operation. The New York-Bay Roberts-Penzance permalloy handles to a large extent only bulk traffic. It could be conveniently arranged to turn around every two minutes, but in practise it is not turned as often as every two hours. The New York-Bay Roberts-Horta cable in contrast is duplexed and handles traffic of all classes.

All of the loaded cables shown on the chart are operated by multiplex printers which are attended by experienced cable operators.

J. W. Milnor: Since this paper was prepared, further work has effected an improvement in the balancing with the artificial lines. The accuracy of balance is now about one part in 7,000 when telegraph characters are transmitted, and one part in 13,000 for any sine wave in the range between 35 and 60 cycles per second. The duplex speed has been increased also, and it is now practicable to transmit 1,600 letters per minute (cable code or multiplex characters) simultaneously in each direction, or a total of 3,200 letters per minute. By using multiplex equipment, five messages may be sent in each direction simultaneously over the cable.

Submarine Cable Telegraphy: Influence of Interference

BY J. W. MILNOR*

Fellow, A. I. E. E.

Synopsis.—*The design and operation of long submarine telegraph cables are affected to an important degree by extraneous electrical interference near the ends. The amount of interference in different cases is dependent upon a number of factors, some of which can only be determined by measurements made in the vicinity.*

The relation is first given between the amount of interference and the operating speed of the cable, assuming the interference to be sinusoidal. This is extended to include actual interference which is not sinusoidal. Since the computations with non-sinusoidal interference are too tedious for general field use, an experimental method is given which provides results which may be used directly in determinations of the effect upon cable operation.

Examples are given of the computation of the response of the receiving instrument under the influence of a transient extraneous voltage, the frequency characteristic of the receiving equipment being known. Conversely, examples are given of the computation of the frequency characteristic from the results of an experimental determination of the response of the receiving instrument to a transient impressed voltage.

* * * * *

IN the design of submarine telegraph cables most of the pertinent electrical properties are known or can be computed to reasonable accuracy. However, there is one factor, namely, extraneous interference near the ends, which is often highly uncertain yet which usually has a large influence both upon the optimum design and upon the limiting performance of the completed cable. In view of this, it is quite desirable that the effects of interference should be examined and that methods for measuring interference should be standardized.

In some cases, electrical interference may be caused by other cables, or by electrical railway or power systems in the vicinity. In general, however, the interference is chiefly of natural origin, akin to the atmospheric disturbances or static which affect radio systems.

Interference into cables from natural causes decreases with increasing depth of the cable.[1] In general it appears to be greatest at some moderate frequency, and decreases at higher and at lower frequencies. The interference depends to some extent upon the design of the sheathing of the cable, upon the electrical shielding if any, and upon the design of the loading, if a loaded cable. It appears generally to be greatest in low latitudes. Where justified, the interference may to a considerable extent be controlled by installing a sea earth, the proper length being dependent upon local and economical considerations.

In a previous paper,[2] the theory of telegraphy with particular reference to long cables was outlined, based upon a consideration of the manner in which sinusoidal alternating currents throughout a band of frequencies are transmitted through the system. With this knowledge at hand, the transient response of the receiving equipment to any form of applied voltage may be determined by performing one or two definite integrations as formulated in the paper referred to. For many purposes, however, it is unnecessary to compute the transient response, since the necessary transmission characteristics for sinusoidal currents may be determined once for all and subsequent computations may be restricted to the latter; in this respect this method of analysis affords important advantages for purposes of design. In the present paper the same method of analysis will be followed, and in order to avoid repetition, occasional references will be made to the previous paper.

The various curves in these papers are for the purpose of indicating the general principles as applied to a representative long cable, rather than giving results which may be accepted as final for use under all conditions. The characteristics of cable receiving equipment vary to some extent according to the design and speed of the cable with which the equipment is associated, and also because of differences of opinion of the cable operating attendants. The methods given for computing transients are of course not limited in usefulness to cable systems, but instead are applicable to various other classes of circuits.

Relations Between Amount of Interference and Cable Speed

A standard expression for the useful single-frequency

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1. "Extraneous Interference on Submarine Telegraph Cables," by J. J. Gilbert, *Bell System Technical Journal*, July 1926.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

2. *Submarine Cable Telegraphy*, by J. W. Milnor, A. I. E. E. Trans., February 1922, p. 20.

current received through a long non-loaded or loaded cable is the following:[3]

$$I_r = \frac{E_s}{Z_o + Z_s} \cdot \epsilon^{-Pl} \cdot \frac{2\,Z_o}{Z_o + Z_r} \qquad (1)$$

The corresponding deflection of the receiving instrument at any frequency is

$$D = \frac{E_s}{Z_o + Z_s} \cdot \epsilon^{-Pl} \cdot \frac{2\,Z_o M}{Z_o + Z_r} \qquad (2)$$

In studying the effects of interference, it is convenient to consider the disturbance to be introduced as a voltage E_d directly in series with the cable near the receiving end. For the present, the interference will be assumed to be sinusoidal; other transient forms of interference will be considered later.

The deflection of the receiving instrument due to interference is

$$D_d = E_d \cdot \frac{M}{Z_o + Z_r} \qquad (3)$$

The quantity $M/(Z_o + Z_r)$ is one form of frequency characteristic of the receiving equipment; it is a measure of the response both to signal voltages transmitted through the cable, and to interfering voltages. General information as to its character may be obtained by any one of three methods, as follows:

1. Sinusoidal voltages of various frequencies may be impressed in series between the cable and the receiving equipment, the deflection of the latter and (if desired) its phase relation being observed.

2. By analyzing the response of the receiving equipment to a pure impulse, as described in connection with Figs. 9 and 10.

3. By computation from the known electrical characteristics of the cable, the over-all arrival curve of the entire cable system having first been determined. Thus, plots of each term of equation (2) were given in the original paper[4] for a representative cable (3,000 ohms, 800 μf., worked cable code at a dot frequency of 10 cycles per second), assuming fairly efficient receiving equipment. From these, a curve of $M/(Z_o + Z_r)$ against frequency may readily be computed. Such curves, shown here in Fig. 1, give the deflection of the receiving instrument per peak volt of sinusoidal disturbance as a function of frequency.[5]

In this figure, the two curves marked "Beat transmission" show the effect of curbing of transmitted signals, assuming that the shaping elements of the receiving equipment have been so altered that the resultant received signal is the same as with "block" signals. Such a change results in greater susceptiveness to low frequency interference, and less to higher frequency interference.

For satisfactory operation, a limiting value of D_d is probably one-fifth of the size of the deflection when reversals at dot frequency are transmitted through the cable. A safer value of D_d might be somewhat less. For cables operated duplex, the effect of any lack of perfect balance must also be included in determining the performance of the cable.

The above information may be made applicable to cables other than the one described by noting that the value of $M/Z_o + Z_r)$ which measures the susceptibility to interference, is proportional to $\epsilon^{\alpha l}$. The point of maxima tends to move to the right for cables operated at higher attenuation.

Where the interference originates some distance out from the receiving end of the cable, its effect is reduced. If a voltage E_x is induced along the cable due to an external magnetic field at a distance x from the receiving end of a long cable, its effect on receiving equipment is equal to that of a voltage $E_x\,\epsilon^{-Px}$ at the receiving end. Similarly, a current I_x induced electrostatically through the insulation due to a potential variation of the sheath, is readily shown to be equivalent to a

3. The nomenclature is as follows:

E = sending battery voltage.
l = cable length.
f = $p/2\pi$ = frequency.
F = dot or fundamental signaling frequency.
α = attenuation constant per unit length of cable.

The following are vector quantities,—functions of the frequency:

E_s = sending voltages as modified by curbing, if any, of transmitted signals.
P = propagation constant per unit length of cable.
Z_o = characteristic impedance of cable.
Z_s = impedance to outgoing current of equipment at sending end.
Z_r = impedance looking from cable, of complete receiving equipment.
I_r = peak current at junction of cable and receiving equipment.
D = peak deflection of receiving instrument at any frequency (see D', below).
M = magnification of complete receiving equipment at any frequency = D/I_r.

The following is a function of time t:

D' = instantaneous value of deflection of receiving relay or recorder. The maximum deflection (neglecting a momentary overthrow) is taken as unity, the deflection at any instant being expressed as a fraction of the maximum.

It is assumed throughout that all characteristics in the system are linear, with effects proportional to causes within reasonable limits. If the receiving relay is of the type whose armature moves between fixed stops, its motion is of course non-linear and the theory is inapplicable to its motion; however with proper allowances the quantities D and D' might represent some other factor, such as the armature torque or relay current.

The quantity M takes into account all equipment in the receiving-amplifying-shaping network, and is of course a complicated function of the frequency. The method of analysis makes it unnecessary to set up the solution of the components of the network.

4. Milnor, loc. cit., Figs. 12 to 17 inclusive.

5. Instead of giving the modulus and the angular lead as in Fig. 1, the frequency characteristic may be specified by giving the real and imaginary components, which will be designated respectively by u and v.

voltage $I_x Z_o \epsilon^{-Px}$ at the receiving end. The above value of E_d may therefore be considered to be an equivalent interference as measured at the receiving end, equal to

$$E_d = \int_0^l \epsilon^{-Px} (E_x + I_x Z_o)\, d\,x + E_{SE} \qquad (4)$$

In this equation, E_x and I_x are the values per unit of length and E_{SE} is the potential variation, if any, of the

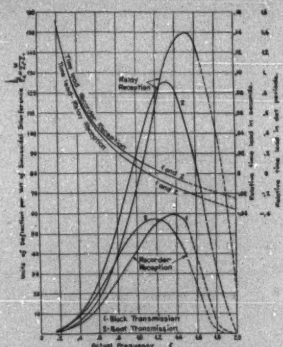

FIG. 1—FREQUENCY CHARACTERISTIC OF CABLE RECEIVING EQUIPMENT

earthing connection at the end of a short sea earth. If the sea earth is long the formula for E_d is somewhat modified.

ANALYSIS OF INTERFERENCE OTHER THAN SINUSOIDAL

In Fig. 2 is shown a typical record of interference on an actual cable, using a distortionless amplifier with high impedance input. This section will deal with the effect on signal reception of interference of this nature. The maximum peak in the record is 2.5 millivolts; however, it is found that its effect on reception is equivalent to that of a much smaller sinusoidal interference of the frequency at which the equipment is most sensitive, the effective value in different cases ranging from 5 to 60 per cent of the peak.

FIG. 2—RECORD OF INTERFERENCE ON CABLE. DISTORTIONLESS AMPLIFIER

In order to illustrate the manner in which interference affects operation, the curves in Fig. 3 have been computed showing the manner in which the receiving relay of the representative cable responds to various simple transient forms of interference. The method of computing is given below. An examination of these results indicates that an impulse of interference which is fairly short in time, causes a peak of response which is nearly proportional to the product of the voltage multiplied by the time in seconds. Where the interference is prolonged, the response is less than given by the above

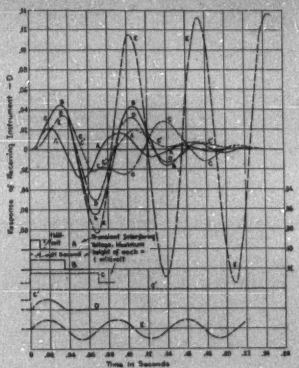

FIG. 3—RESPONSE OF RECEIVING INSTRUMENT TO INTERFERENCE IMPULSES

product. However, the response is greatest when two or more pulses occur in a manner to set up forced oscillation of the equipment.

Curves such as in Fig. 3 might be computed directly when the frequency characteristic as in Fig. 1 is known.[6]

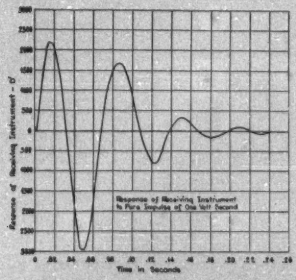

FIG. 4—IMPULSE CHARACTERISTIC

However, a more convenient method was first to compute an *impulse characteristic*, shown in Fig. 4, and then to build up the curves in Fig. 3 by means of the principle

6. Milnor, loc. cit., Table I and equations (10), (12) and (13) of Appendix. For a pure impulse of one volt second, $s = 0$ and $c = 1$. In the present case the integrations have been performed by breaking the curve representing u into several parts each of which was integrated analytically.

of superposition. The impulse characteristic as here used is defined as the transient response of the receiving instrument, when an instantaneous or pure impulse of voltage is impressed (across a small resistance) in series between the cable and the receiving equipment. The impulse characteristic is computed from the frequency characteristic by means of the equation[5,6]

$$D' = \frac{E_i T_i}{\pi} \int_0^\infty (u \cos p t - v \sin p t) \, d p \qquad (5)$$

$E_i T_i$ being the strength of the pure impulse in volt-

Fig. 5—Record of Cable Interference. Amplifier Tuned to 30 Cycles per Second; Decrement = 0.8

seconds. By taking advantage of the fact that D' is zero for negative values of t, this may be simplified into

$$D' = \frac{2 E_i T_i}{\pi} \int_0^\infty u \cos p t \, d p \qquad (6)$$

which is valid for positive values of t only.

It is, of course, entirely possible to determine the effect on signal reception by computation from a complete record such as shown in Fig. 2, by building up a curve of response of equipment by superposition using the impulse characteristic. The result would be similar in appearance to Fig. 5. However, such procedure

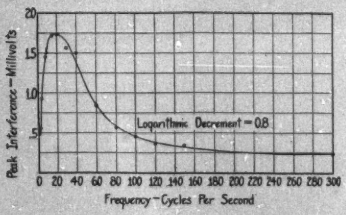

Fig. 6—Example of Variation of Measured Interference with Frequency of Tuned Circuit

would be rather tedious and impracticable in general studies of interference.

Field Measurements of Interference

The most accurate field method for determining the practical effect of interference would be to use receiving equipment adjusted to give properly shaped signals on the given cable. This method has the objection that the shaping may not be known in advance of laying, and comparative studies of interference in different locations or in relation to the cable design would be difficult. The importance of interference in a given case is measured by the maximum or peak response that occurs during a suitable period of time.

Since the frequency characteristic of efficient receiving equipment is somewhat similar in form to that of a simple electrically tuned circuit, the method suggests itself of making use of such tuning during the obtaining of the field records. Such a method offers advantages over the other methods, and appears to be sufficiently accurate for the purpose. The point of tuning must correspond to the frequency at which the cable equipment is most sensitive, and the broadness of tuning should approximate that of the operating equipment.

A convenient arrangement of apparatus for the purpose is a vacuum tube amplifier working into an oscillograph, the amplifier having a symmetrically tuned circuit between two of the stages, and being otherwise distortionless. The tuning may be provided by means of a low-loss inductor and condenser in parallel, the combination constituting the external plate load of the preceding tube. In order to control the broadness of

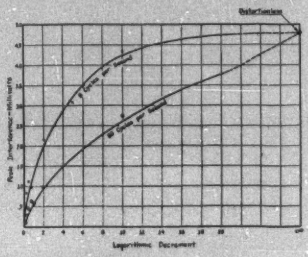

Fig. 7—Example of Variation of Measured Interference with Decrement of Tuned Circuit

tuning, an adjustable resistor is placed in parallel with the inductor and the condenser. The broadness of tuning of a symmetrically tuned circuit is conveniently specified by stating the "logarithmic decrement" as used in radio parlance. A low value represents relatively sharp tuning.

A sample record obtained with such equipment is shown in Fig. 5. In Figs. 6 and 7 is shown the manner in which the peak interference varied at one location with different frequencies of tuning, and with varying logarithmic decrements. It is evident from the latter figure that the response of cable receiving equipment to interference is reduced with sharper tuning of the equipment, (that is, by adjusting it to be relatively less sensitive to the higher frequencies which are unnecessary for transmission). For efficient cable equipment, the broadness of tuning appears to be approximately represented in different cases by decrements from 0.7 to 1.2; for less efficient equipment the value is higher.

The measured interference is in general increased if the length of the record is increased, since the highest

peak of interference on the record is recorded as the amount of interference under the given conditions. In Fig. 8 are shown three examples of the manner in which the peak interference varied with the length of record.

Under some conditions it is convenient to replace the oscillograph with an arrangement for automatically recording the peak voltages. There are different methods by which this may be accomplished, use being made of vacuum or gas filled tubes or of sensitive mechanical relays.

Computation of Frequency Characteristic from Impulse Response Curve

By means of a simple experiment, it is possible to determine the impulse characteristic of an actual cable circuit. The method is to insert between the cable and the receiving equipment a resistor which is small in value as compared with the characteristic impedance of the cable, and to discharge a condenser across the resistor. Under proper conditions such an impulse may for the purpose be regarded as an instantaneous or pure impulse, and its strength in volt-seconds is the

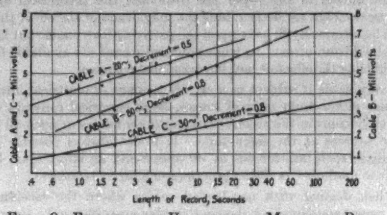

Fig. 8—Examples of Variation of Measured Peak Interference with Length of Record

product of the resistance, the capacitance and the voltage at which the condenser is charged. To avoid possible amplifier overloading, the deflection of the receiving instrument should be not greater than during regular operation. In the upper parts of Figs. 9 and 10 are shown examples of impulse characteristics obtained experimentally on two cables.

A casual examination of such an impulse characteristic yields valuable information as to the approximate frequency at which the equipment is tuned, and also gives an indication of the sharpness of tuning and the sensitiveness.

The voltage frequency characteristic of the cable receiving equipment under investigation may be computed directly from the experimentally determined impulse characteristic. This operation is the inverse of that in equations (5) and (6). While such computations may not be justified in the average routine investigation, they are quite instructive as a check of the theory. The results of two such computations are shown in the lower parts of Figs. 9 and 10. Certain data regarding these cables are as follows:

For Fig. 9, non-loaded cable, resistance 4,740 ohms, capacity 806 μf., operated at a dot frequency of 5 cycles per second. Beat signal transmission, 0.8 contact. Unshunted receiving condenser. Heurtley magnifier working into syphon recorder, but signals shaped for relay operation.

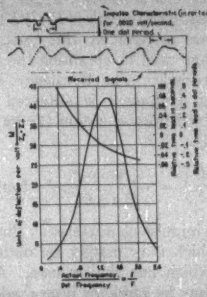

Fig. 9—Frequency Characteristic of Cable Receiving Equipment Determined from Impulse Characteristic

Fig. 10, duplex loaded cable with non-loaded end portions, operated at a dot frequency of 40 cycles per second. Block signal transmission. Western Electric vacuum tube amplifier working into oscillograph and relay.

These frequency characteristics are similar in general form to the characteristics for relay operation in Fig. 1, which were obtained by quite a different method. The differences between the curves are indicative of the

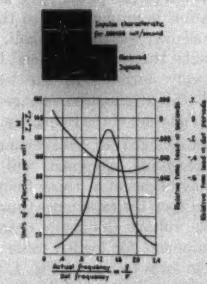

Fig. 10—Frequency Characteristic of Cable Receiving Equipment Determined from Impulse Characteristic

variations which may occur in practise between cables operated at different efficiencies and with somewhat unlike signal shaping. The heights of the curves vary because the cables are operated with different strengths of received voltage.

The method of determining the frequency characteristic from the impulse characteristic is to integrate

through the transient in accordance with the formulas,[7]

$$\left.\begin{aligned} s &= \int D' \sin p\,t\,d\,t \quad \text{between limits where } \cos p\,t = 0 \\ c &= \int D' \cos p\,t\,d\,t \quad \text{between limits where } \sin p\,t = 0 \\ b &= (D'_{-\infty} + D'_{+\infty})/2 \end{aligned}\right\} \qquad (7)$$

The frequency characteristic of the pure impulse which was applied to the circuit is $R_i C_i E_i \underline{/0^\circ}$, in which the quantities are respectively the resistance, capacitance, and voltage used in obtaining the experimental impulse characteristic. The frequency characteristic of the receiving equipment is obtained by dividing the values obtained from equation (7) by the product just given, that is,

$$\frac{M}{Z_o + Z_r} = \frac{D_d}{E_d} = \frac{\sqrt{s^2 + c^2}}{R_i C_i E_i} \underline{/-\tan^{-1} s/c} \qquad (8)$$

Instead of the impulse characteristic, there is another characteristic which may readily be obtained experimentally, and which has similar uses. A small continued voltage may be suddenly impressed across a low resistance in series between the cable and the receiving equipment. The resulting response of the receiving instrument is similar in form to curve C' in Fig. 3

Application of Theory to Loaded Telegraph Cables

The attenuation of a slow light-weight non-loaded cable increases nearly proportionately to the square root of the frequency. With heavy non-loaded cables, especially if operated at relatively high speed, the self inductance of the cable tends to reduce the attenuation, but this gain in transmission is partly offset by losses introduced by the sea return path. The usual result is that the attenuation is proportional to the frequency raised to a power slightly less than one-half.

In a loaded cable[8] there are additional losses due to hysteresis and eddy currents in the loading. The manner in which the attenuation changes is a much more complicated function of the frequency.

If a cable transmits a greater proportion of the higher frequencies, the receiving equipment will be required to provide less magnification at those frequencies, and vice versa. It has been the experience thus far that the appearance of the voltage frequency characteristic of the receiving equipment for loaded cable operation is generally similar in form to that in Fig. 1, with modifications in height to take account of the changed characteristics of the cable. The curve of time lead is more nearly flat, since currents at the different frequencies are retarded more nearly alike by the cable. The general method which is suggested for studying interference in the field is applicable to both non-loaded and loaded cables, provided that proper allowance is made for the sensitiveness and tuning of the equipment, in accordance with the design of the particular cable.

Acknowledgment is due to Mr. K. B. Eller for assistance especially in connection with the experimental studies.

7. Milnor, loc. cit., equation (8) of Appendix.

8. "High-Speed Ocean Cable Telegraphy," O. E. Buckley, *Bell System Technical Journal*, April 1928.

Discussion

A. W. Breyfogel: The subject matter of the paper is of paramount importance to cable engineers for it is the existence and magnitude of extraneous electrical interference that places a definite limit on the speed of a cable. This statement is not to be construed to mean that if all electrical interference could be eliminated completely, that the speed of any cable could be increased indefinitely. The electrical characteristics of a cable are a measure of its signaling speed, but this speed is never quite reached in practise because of the detrimental influence of interference.

By utilizing the methods outlined in the paper, the deflection of the receiving instrument per peak volt of sinusoidal or transient forms of interference as a function of frequency may be predicted. In like manner, the deflection of the receiving instrument per volt of received signal may be predicted and from the results of these calculations a fairly accurate predetermination of the ultimate speed is obtained.

The author of the paper has presented the theoretical and practical means for determining the magnitude of the effect that such interference will have upon the receiving instrument but has refrained from presenting what practical means are available for minimizing the effect of such electrical disturbances.

Every cable is subject to a certain amount of electrical interference either from natural causes, from electrical railways or power systems in the vicinity or from other cables or from all of these sources at the same time. It is not always possible to eliminate such interference at its source and an effort must be made to minimize its effect upon the receiving instrument.

When dealing with non-loaded cables where the receiving equipment consists of a hot-wire Heurtley magnifier and a Brown drum relay or a stretched-wire relay the effect of extraneous electrical interference can be lessened to some degree by adjusting the free period of the moving parts of these instruments to a frequency near the dot frequency or a little higher than the dot frequency of the signal. The instrument, in this case, would approach a maximum deflection for the dot frequency of the signal and a deflection somewhat less than the maximum for higher and lower frequencies. The instruments' response to interference frequencies above or below the dot frequency would thus be reduced to a minimum.

Where interference is present, it is always of advantage to tune the receiving equipment to the lowest practical frequency consistent with reception of satisfactorily shaped signals. Such tuning may be somewhat harmful to the reception of the signal itself but generally it permits reception at a higher speed than might otherwise be obtained.

Mr. Milnor has shown that for satisfactory operation of non-loaded cables where the three-element cable code is used, the limiting deflection produced in the receiving instrument by interference cannot be greater than about 20 per cent of the deflection produced by the dot frequency. It is interesting to note that in practise this percentage is found to be about the maximum that can be tolerated successfully. A deflection of this magnitude will not carry the pointer of the drum relay beyond the boundaries of no-man's-land and will, therefore, not produce failures. Should the interference become of greater magnitude, the pointer will cross the borders of no-man's-land and cause failures.

The receiving networks can also be used to a more limited

degree to minimize the effect of extraneous interference. The adjustments of the shaping networks associated with the Heurtley magnifier can be made more or less responsive to the signal, thus discriminating against interference.

When dealing with loaded cables where the prime receiving instrument is a vacuum tube shaping amplifier, the effect of interference can also be minimized to some extent by the adjustment of the shaping networks. In the amplifier, the tuning is done electrically and a certain amount of discrimination against extraneous interference is obtained.

During some recent tests on a loaded cable at speeds as high as 135 cycles, the effect of extraneous interference was successfully shaped out and excellent printing ranges were obtained with a resulting high degree of accuracy. If the interference could be completely eliminated, we believe that even higher speeds could be obtained with receiving apparatus that is now available.

Telegraph Transmission Testing Machine

BY F. B. BRAMHALL[1]

Associate, A. I. E. E.

Synopsis.—*In the study of telegraph transmission efficiencies and the analysis of signal distortions a suitable signal recording device is a first requisite. This paper describes a machine developed for the purpose. This machine furnishes a graphical and permanent record of the received signals and shows their relation to sent signals, in a form which is most convenient for comparison and analysis. The unusual arrangement of the recorder is such as to adapt it especially to the study of both characteristic and fortuitous distortions and, in general, to the comparison and absolute measurement of short time intervals. Time intervals of one ten-thousandth of a second in the opening and closing of electrical circuits are graphically recorded without resort to photographic means.*

Introduction

It is the object of this paper to describe an improved method of measuring the quality of transmission obtained from telegraph or other signaling circuits or apparatus. A description of the transmission testing machine developed for this purpose is followed by a discussion of the typical results obtained in various classes of work and an explanation of the method of their analysis.

Although the machine was conceived and used several years ago by Mr. J. W. Milnor, Research Engineer of the Western Union Telegraph Company, the principle of the method has not previously been published. While the fundamental principle remains unchanged the machine has naturally gone through a considerable evolution as its many adaptations have become apparent. In its present form it is used very extensively in laboratory and field investigations of telegraph signal transmission, and in the testing and comparison of relays and repeating instruments.

Quantitative Measurements of Signal Distortion

Any measurement of the quality of telegraph transmission is of necessity a measurement of the extent to which the received signaling pulses are distorted from the form in which they were originally transmitted. For the purpose of this paper, signal distortion will be defined as the deviation in length and time relationship of the received signal pulses from the absolute length and time relation of their transmission. This definition confines the measurement of distortion to the measurement of the time relationships which obtain between the signaling pulses as repeated by a relay or other repeating instrument.

The nature and effect of telegraph signal distortion as here defined was admirably explained in a paper called, *Measurement of Telegraph Transmission*, by Messrs. Nyquist, Shanck, and Cory, presented before the Winter Session of the Institute in 1927.

It should suffice here to point out only the distinction between the two general types of distortion designated as *fortuitous* and *characteristic*. Fortuitous distortion is that shortening or lengthening of the signal pulses which results from extraneous disturbing currents in the signaling circuit or from irregular and improper functioning of repeating or receiving instruments. From the very nature of its causes then this type of distortion may be expected to be quite irregular, fluctuating rapidly and perhaps very widely. Characteristic distortion may be defined as that shortening or lengthening of signal pulses which is governed by the sequence of the pulses themselves and by the electrical and magnetic characteristics of the signaling circuits and apparatus. Problems of the cause and elimination of characteristic signal distortion are most effectively attacked through an accurate knowledge of the current wave shapes which delay or hasten the time of operation of the repeating instrument. In the study of fortuitous distortions it is frequently desirable to supplement a knowledge of their effect on the signals with frequency analyses of the disturbing currents.

This paper will deal chiefly with the effects of signal distortion. The wave form distortions and irregularities will be dealt with only in so far as seems necessary in the explanation of the results obtained by the method of measurement described.

In all experimental studies of telegraph circuits and apparatus a method for measuring the fidelity with which the transmitted signals are reproduced in the receiving terminal is of prime importance. It is important to know not only the maximum signaling speed which can be obtained on a given circuit, but as well to know quantitatively what particular forms of signal distortion limit that speed. The most common method of determining the efficiency of transmission is to transmit into the circuit test signals of known characteristics while observing the integrity of their reception. The receiving element then, which constitutes the novel feature of the transmission testing machine, is the principal subject of this paper.

Methods of transmitting test signals which are substantially perfect are well known. In automatic signaling systems generally, the transmitting unit may usually be relied upon for substantially perfect transmission. The transmission testing machine here described, however, carries with it, in addition to the receiving element, a transmitting element which may be used for experimental testing work with both ends of the circuit available at a point.

Some of the common uses to which the machine is put are:

1. Engg. Department, Western Union Telegraph Co., 60 Hudson Street, New York, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

1. Absolute measurement of over-all transmission efficiency.
2. Detection and analysis of characteristic signal distortion.
3. Detection and analysis of signal distortion resulting from extraneous disturbances.
4. Study of repeating relay characteristics.

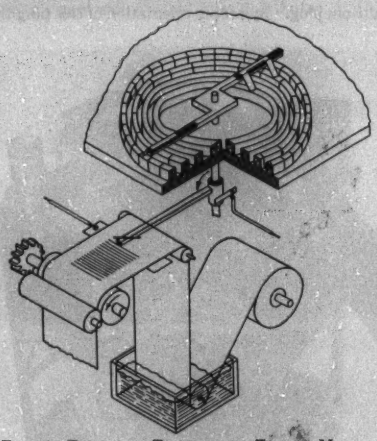

Fig. 1—Telegraph Transmission Testing Machine

Drawing of chemical recorder mechanism showing relation to transmitting commutator

5. Detection of hunting or irregularities in transmitting apparatus.
6. Study of duplex balance problems.

Transmission Testing Machine

Fundamental Principle. A conception of the principle of the machine can most readily be formed if it be considered first as arranged for testing circuits both ends of which are available at a point. Referring to Figs. 1 and 2, let the commutator transmit signals to the circuit under test. Consider the signals to operate a relay at the receiving end of the circuit. The contacts of the relay are made to control a potential applied between the metallic needle 13 and the platen 12. The arm which carries the metallic needle revolves on the shaft which carries the transmitting brushes. The metallic needle passes over a saturated paper tape 15 in light contact with its surface. The solution with which the paper tape is saturated and the material of the metallic marking needle are such that when the proper potential is applied to the needle a mark results from the electrochemical action. If in response to a transmitted signal pulse the receiving relay energizes the marking needle during the time of its transit across the paper, a mark is made continuously until the needle is again deenergized at the termination of that pulse. Thus, if the transmission over the circuit under test be perfect, the revolving needle will draw a line on the paper tape which is a true projection of the length of the transmitting segment.

The idea of transmission and reception being done on a common rotating member is, of course, no longer essential to a conception of the principle of this method of measuring. These two functions may obviously be performed by separate rotating members so long as proper synchronism of rotation is maintained. It is by this method that point to point testing is done.

Detailed Description. A detailed description of the recording mechanism is given with reference to Figs. 1 and 2. Fig. 1 shows the relation of the essential parts of the machine, Fig. 2 is a schematic wiring diagram.

Fig. 2 shows, developed, a transmitting commutator having a solid ring and a segmented ring. Each of the segments of the segmented ring is arranged for connection to either positive or negative transmitting potential, so that by the manipulation of switches any desired combinations of positive or negative pulses may be transmitted. The signaling currents pass from the solid ring through the circuit or apparatus under test to the coils of a receiving relay which in turn repeats the received signals to the recording device.

A suitable grade of paper tape is pulled by rollers over a contacting surface or platen after passing through a chemical bath. The marking needle attached to the rotating arm is carried on the shaft which drives the

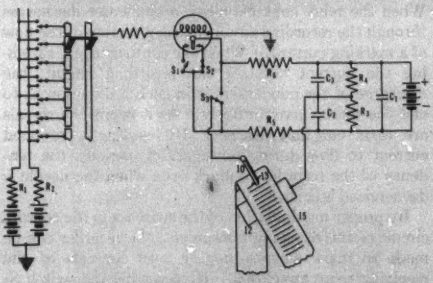

Fig. 2—Schematic Wiring of Test Circuit and Chemical Recorder

commutator brushes. When an electric current is passed through the marking circuit in such a direction that the marking needle is positive with respect to the platen a mark is recorded on the paper as a result of oxidation of the material of the needle.

The principle of the chemical recording feature is well known. Telegraph systems which made use of the chemical tape recorder for signal reception have been used to some extent in commercial telegraphy.

The chemical recorder of this machine uses an iron needle marking on a paper tape which is saturated with a solution of potassium ferricyanide and ammonium

chloride. The mark made with this solution is blue. Other solutions and needles of other materials produce different colored marks. The combination of the iron needle and potassium ferricyanide solution is perhaps the fastest acting of any that has been commonly used.

A common fault of chemical recording devices is the tendency of the metallic needle to mark lightly even when no potential is applied to it, and the tendency for the beginning and ending of the marks to be somewhat indistinct.

The marking circuit as arranged for this machine makes use of a capacity and resistance network to produce marks with a clear cut and sharp beginning and ending. With the switches of the marking circuit thrown as shown in Fig. 2, the needle marks when the marking contact of the receiving relay is closed. The resistances, R_3, R_4, R_5, and R_6, are so proportioned that when the circuit is open at the relay contacts current flows from positive battery through resistance R_4 then in part through the recorder and resistance R_5 to negative battery. The needle thus being negative with respect to the paper, no mark is made on the tape. When the circuit is closed at the relay contacts, however, current flows through resistance R_6, the relay contacts, through the recorder from needle to tape and through resistance R_3 to the negative pole. Under this condition a mark is made on the tape. The capacities C_1, C_2, and C_3 serve the purpose of accelerating the electrochemical action at the beginning of the mark and retarding the action at the ending of the mark. When the relay contacts close, a condenser discharges through the recording circuit in the direction of the flow of a marking current. When the contacts open, reversing the current through the recording circuit, the discharge of the condensers is in such a direction as to end the mark more abruptly. As a result the signals are clear cut and distinct. By causing a reversed current to flow during non-marking periods, the tendency of the recorder to mark even when the needle is deenergized is curbed.

By proper manipulation of the switches in the contact circuit of the receiving instrument, the recorder can be made to mark when either or both contacts of the receiving relay are closed. By reversing the switch S_3 and making suitable changes in the values of the resistances in the potentiometer, the recorder can be made to mark only when the relay contact circuit is open. The desirability of these arrangements will become evident as the methods of testing are explained.

The commutator and brush carriage of the machine are similar to those of the standard distributor used in multiplex telegraphy. They are, however, made somewhat larger to afford more perfect transmission. One segmented transmitting ring bears twice the number of segments of the other. One ring can, if desirable, be made movable like the receiving ring of the multiplex distributor. The inner ring has 20 segments. This ring is used for all testing work at normal telegraph signaling speeds. The outer ring is used for transmission at higher signaling speeds and for a variety of other purposes, such as maintaining synchronism with a distant transmitter and for range finding work. The two pairs of rings are each connected to standard multiconductor receptacles of the type used on multiplex apparatus and connection is made between the battery control switches and the commutator by means of the multi-pin plug. By proper orientation of this plug then,

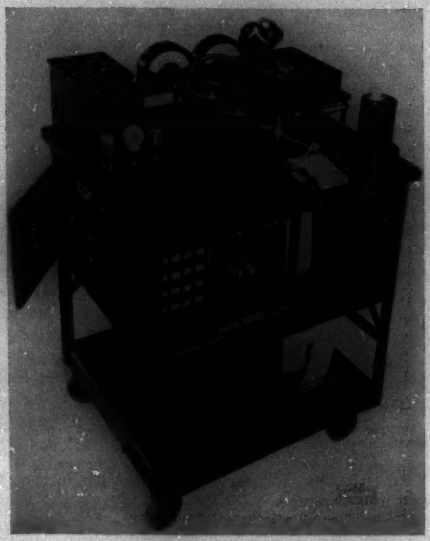

Fig. 3—Transmission Testing Machine

One of the newer types, showing recorder and control apparatus

it is possible to observe the reception of any chosen element of the transmitted test signals. In loop testing work, where the test signals are transmitted from the machine, the characteristic lag of the circuit or apparatus may be accommodated by orientation of this plug.

Numerous motor-drive arrangements have been used in transmission testing machines. An early machine was driven by a small shunt-wound d-c. motor. A train of gears and a friction disk afforded a wide range of speed adjustment in this machine. Such a drive proved satisfactory when the machine was used for relay testing and other work on which the time lag of the circuits under test was not appreciable.

The application of the machine to the over-all measurement of circuits of appreciable lag and to straightaway testing necessitated the development of a more constant drive. The drive used in the present machine to obtain uniform rotational speed is a synchronous

motor drive. The driving unit is shown in Fig. 4. In this unit the rotating members of two driving motors are rigidly connected to the common shaft. One of these, a standard shunt-wound low-speed d-c. motor may be seen at the bottom of the unit. The synchronous motor is a standard phonic wheel commonly used in multiplex distributors. Above this motor will be seen the large mercury filled flywheel which is also commonly used with the multiplex distributor.

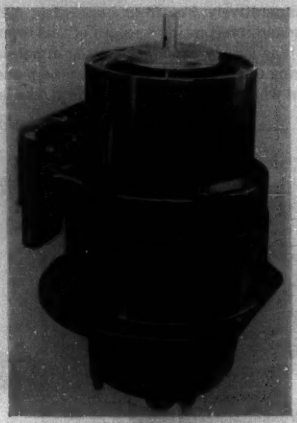

Fig. 4—Drive Unit, Assembled

Showing "phonic wheel" motor with flywheel above and direct-coupled d-c. starting motor beneath

A low resistance potentiometer is used to control the potential applied to the armature of the d-c. motor. The d-c. motor alone may be used when great constancy of driving speed is not required. It is also used as a starting motor for the phonic wheel drive or as an auxiliary drive to the latter.

The phonic wheel motor derives its power through the contacts of the tuning fork commonly used in connection with this motor in the multiplex automatic equipment. Synchronism of the phonic wheel motor with the driving fork is indicated both by a pilot lamp and by a meter in the phonic wheel circuit.

The driving fork is adjusted to vibrate at the desired frequency by means of removable weights on its tines. Small changes in its frequency are made by means of sliding weights. These may be moved without stopping the fork. A push-button switch is arranged to control a damping network so connected to the driving winding of the fork as to change its frequency of vibration slightly. By this means a predetermined change may be made in its frequency without disturbing the adjustment of the weights. When the machine is running synchronously with a transmitter located at a distant point, this control is very useful.

The circuits and controls of the transmitting commutator are not unusual. The telegraph transmitting potentials are brought through suitable protective resistances to a battery reversing switch. A rotary signal setting switch provides a convenient means of setting up on the commutator a number of predetermined combinations of testing signals. Ten different combinations of test signal pulses which have been found useful are shown by Fig. 6. These combinations are those used in the making of specimen records shown in this paper.

Local Testing Work

Polar Relay Testing. In order that the operation of the machine may be more readily understood the method of obtaining the test records in a very simple local test is outlined in some detail. Possibly the work in which the measurements are most simply made is that done in the study of polar relay operation. For the purpose of this description then, the case of a test of a polar relay in a local circuit will be considered.

In work of this class it is generally most convenient to transmit the test signals from the commutator of the machine. In the case of the polar relay test, bi-polar

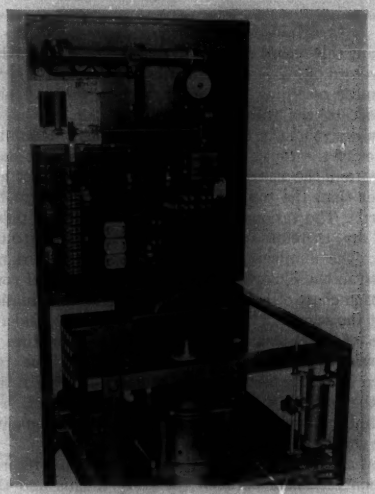

Fig. 5—Standard Semi-Portable Machine with Top Removed

transmitting potentials are used. Suitable protective resistances in the transmitting potential leads are a part of the transmitting circuit arrangements of the machine. The solid ring on the transmitting commutator is connected to the windings of the relay through a suitable current controlling resistance. Or if the test is one intended to show the operation of the relay in a particu-

lar network circuit, this network is used. The machine is driven at the desired operating speed for the test. Whether the test signals be alternate marking and spacing pulses or whatever combinations of signaling pulses may have been chosen, the operation of the relay armature against the contact stops is recorded on the chemical tape.

A typical record made in this class of work is that shown by Fig. 7. The signal combinations used in

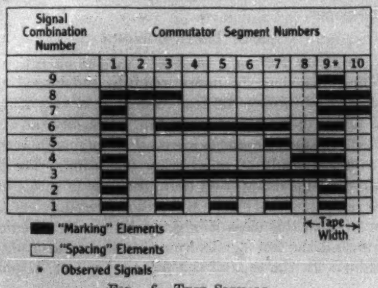

Fig. 6—Test Signals

Combination of test signal pulses used in the making of specimen records

making this record were those shown by Fig. 6. When the record of Fig. 7 was made the marking needle of the chemical recording mechanism was so oriented with respect to the transmitting brushes that the signal pulse designated in Fig. 6 as No. 9 was recorded in the center of the tape. Since the tape moves as it does, away from the axis of the recording needle, the bottom, rather than the top, is actually the beginning of the record. The various combinations of test signal pulses used are, therefore, numbered from the bottom to the top. The numbers which appear on the record not shown in the illustration, are those of the signal combination chart of Fig. 6. If on this signal combination chart the segments blocked out in white are considered to be of such polarity as to cause the armature of the relay to move to its marking contact the test record is readily seen as a graph of the chart. In this test the machine was allowed to record the reception of a number of signals under each different signaling pulse combination.

Just as the lower half of the record of Fig. 7 records the contact made between the relay tongue and the marking contact stop, so the upper half records the contact of the tongue with the spacing stop. The upper half of the record is made by reversing the polarity of the transmitting potentials and connecting the spacing contact of the instrument, instead of the marking, to the recording circuit.

At the lower end of each half of the record it will be noted that the recorder made a nearly continuous mark across the tape. This portion of the record was made by arranging the recorder circuit to mark when the relay tongue was in contact with either contact stop. The only breaks in the mark recorded under this condition are those which represent the time during which the tongue was in transit from one stop to the other.

Had there been any appreciable difference in the magnitude of the potentials of the two transmitting batteries or had the receiving relay been biased, the marks recorded on the lower half of the record would have been of a different length than those on the upper half. This particular record shows the repeated signals to be comparatively free from bias. The method of measuring and interpreting a record of this type is very simple. It is largely explained by the dimension lines and lettering of the figure. The distances p and p' are those from which the basis of comparison, the length of

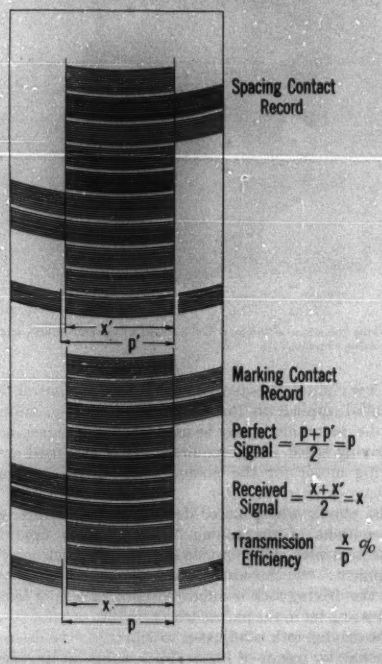

Fig. 7—Specimen Record of Good Polar Relay Operation

the perfect signal, is taken. These distances are measured on the record of the operation of the relay on a-c. pulses. The distance p is proportional to the time elapsed between the departure of the receiving relay tongue from the spacing contact stop and its departure, at the next succeeding signal pulse of opposite polarity, from the marking contact stop. The distance p', measured on the other half of the record, is proportional

to the elapsed time between the end of a repeated marking signal and the end of the succeeding repeated spacing signal. The length of the perfect signal is then taken as the average of the measured distances p and p'. As already mentioned the length of the perfect signal may be determined from the length of the transmitting

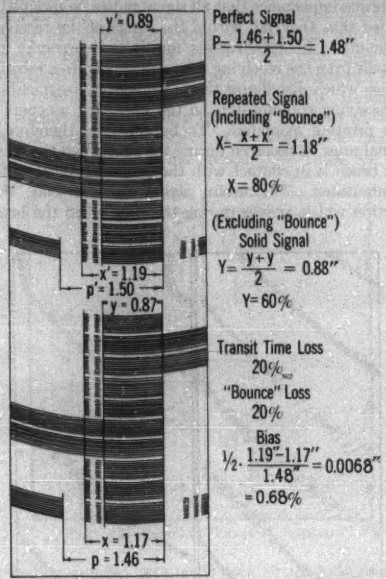

FIG. 8—SPECIMEN RECORD OF IMPROPER POLAR RELAY OPERATION

segment and the radius of the path of the recording needle. However, since this radius may be altered slightly by adjustment of the needle a more accurate standard may be obtained from the measurements described. The repeated solid signal is taken as the average of the two measurements x and x'. The lines on which the x and x' measurements are made are drawn parallel to the edge of the tape and pass through the beginning and ending of those marks which record the signal latest to arrive and that earliest to be terminated. The average of the distances x and x' then, expressed in percentage of the length of the perfect signal, is taken as the transmission efficiency. In this particular case, the only appreciable difference between the repeated signal and the perfect signal is that which represents the transit time of the relay tongue.

Quite obviously, if the recorder is sufficiently quick in its response, a mark will be made on the chemical tape even though the time duration of the electrical contact of the relay tongue with its stop be exceedingly short. If the characteristics of the relay be such that its tongue rebounds from the stop one or more times, the record, to be perfectly useful for this class of work, should permit a complete analysis of the repeated signal. Fig. 8 shows a record taken from a relay which repeated the signals less faithfully than did that used for the first record. In this case, the bouncing action of the relay tongue against its contact stop is very pronounced. The measurement of this record has been made in such a way as to determine not only the length of the solid portion of the signal but also the time elapsed between the initial contact and the final contact of the tongue after which it comes finally to rest firmly on its stop. For the sake of clearness the actual measurements made in analyzing this record are given. In this case the difference between the perfect signal measurement P and the total repeated signal X gives the transit time loss as before. A third measurement Y gives the length of the solid unbroken portion of the repeated

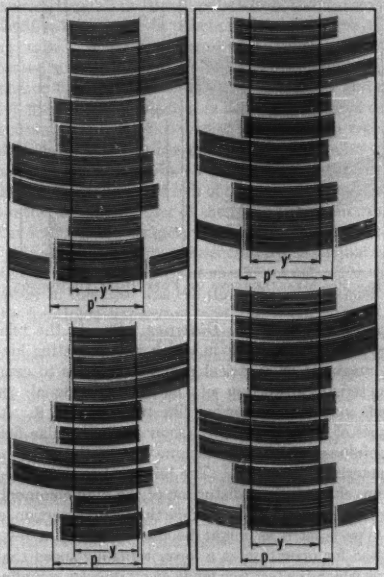

FIG. 9—TYPICAL CHARACTERISTIC DISTORTION

signal. The difference between this and the X measurement is commonly called the "bounce" loss. It will be noted that the signals repeated from the marking contact are somewhat shorter than those repeated from the spacing contact. This bias is readily expressed in per cent of perfect signal.

The time required for the relay tongue to move from

one contact stop to the other and the time it consumes in bouncing are referred to as *loss*. The term *loss* is used in a limited sense. In the general case the transmission efficiency of circuits or repeaters is not impaired to anything like the extent that this *loss* would indicate. It is not the purpose of this paper to discuss the efficiency of operation of repeating instruments, but this point is brought out to avoid any misinterpretations.

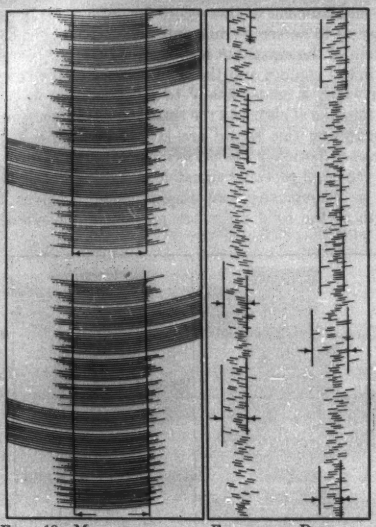

Fig. 10—Measurements of Fortuitous Distortion

Typical Characteristic Distortion. Should the relay under test or the circuit in which it is operating have such magnetic or electrical characteristics as to cause characteristic distortion of the repeated signal, the records may look somewhat like those of Fig. 9. An analysis of these records will show the particular field of usefulness which the machine has in the study of characteristic distortion. The left hand record of Fig. 9 shows characteristic distortion of the kind commonly designated as negative or underthrow. Distortion of this type is such as might be introduced by a relay designed for operation at a lower signaling speed than that at which the test was made or by sluggish signaling circuits or networks.

Distortion of the type shown by the right hand record of Fig. 9 is frequently the result of a highly damped current oscillation in a relay operating circuit. This distortion from the nature of its cause is called positive or overthrow distortion.

It is perhaps unnecessary to discuss the importance of an accurate knowledge of characteristic distortion in telegraph signaling circuits. However, the following consideration will be helpful in drawing an analogy between the transmission testing machine measurements and the familiar range finding operation commonly used to determine the quality of transmission over circuits operated by automatic means. In automatic signaling systems in which the transmitting and receiving apparatus run in synchronism it is, of course, of prime importance that all signal pulses be received so timed that they fall within the range of the receiving segments. In synchronous multiplex telegraphy the signals from the receiving relay are applied to a rotating brush which distributes them, through the segments of a receiving commutator, to the operating magnets of the printing machines. To be effective, therefore, a signal must be received during the time when the rotating brush is in contact with the proper segment of the commutator. The useful signal then is only that portion which arrives during the time when the brush

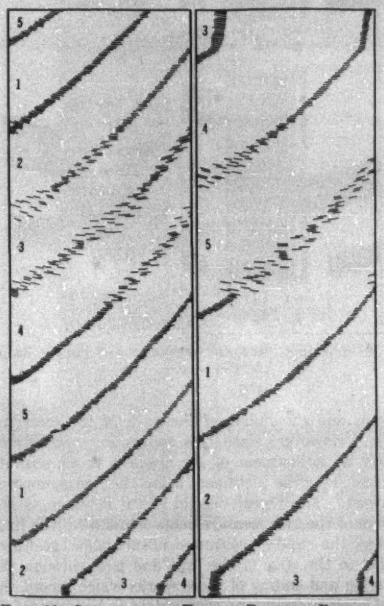

Fig. 11—Straightaway Testing Drifting Receiving Machine

is passing over the segment for which the particular signal pulse is destined. A general practise in handling automatic printing systems is to determine the "printing range," the angle through which the receiving segments may be oriented before failures occur in the printing machine. This range is a reliable measure of the transmission efficiency of the circuit and is entirely

analogous to the operation of measuring the received signal from the transmission testing machine record. Making proper allowances for apparatus failures, relations are readily obtainable between "percent perfect signal" received and "printing range."

Fortuitous Distortion Measurement. It is, of course, not characteristic distortion alone that limits transmission efficiency and printing range. In land line telegraphy fortuitous distortions are generally far more serious. The effect of these distortions may be quantitatively determined from the transmission machine records. The left hand record of Fig. 10 shows the signals received over a telegraph line into which a periodic disturbance current of a considerable magnitude was induced. This record, like those of the

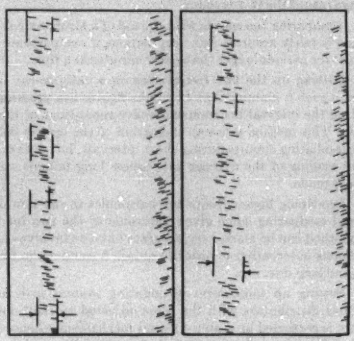

Fig. 12—Typical "Traffic" Signals

preceding samples, was made on a circuit both ends of which were available at the test point. In general appearance the record is not unlike those already explained. The unevenness of the beginning and ending of the received signals is the result of extraneous currents in the line. The periodic nature of the disturbance is evident in the beats it produces in the beginning and ending of the signals.

The right hand record of Fig. 10 presents a different appearance from any yet discussed. In this case the marks record the open circuit time of the relay contact circuit when both the spacing and marking contacts are tied together. In other words, the marks record the transit time of the relay armature. Fortuitous distortion is as readily measured from a *negative* record of this type as from a *positive*. For the isolation of fortuitous distortion the test signals are generally continuous a-c. pulses. This record may be considered as a *negative* in yet another sense. The measurements show not the percentage of signal repeated but the percentage made useless by the disturbing current.

The disturbing currents which cause fortuitous distortion are frequently, as in this case, of a very erratic nature, fluctuating widely from moment to moment.

Straightaway Testing

Obviously, any of the specimen records shown are as readily obtained when the signals are transmitted from a synchronous transmitting machine as when transmitted locally. Fig. 11 shows two records obtained in this manner. Both of these records were made with the receiving machine and the synchronous transmitter running at the same nominal rotational speed. At the bottom of each record the recorded signals show the two machines to be in very close synchronism. After the speeds had been thus approximately matched the receiver was caused to drift slowly with respect to the transmitter. The left hand record illustrates the signals from a standard four-channel multiplex distributor having 20 transmitting segments. The right hand record shows the signals from a three-channel, 15-segment, distributor.

The numbers on these records are arbitrary numbers assigned to the segments of the transmitting distributor. By drifting the receiving element with respect to the transmitting, all the signal pulses from the transmitting distributor are made to pass in review across the recording tape. When the transmitter and receiver are located at two distant points and the transmitter is sending a particular combination of positive and negative pulses, one of which it is desired to observe on the recording tape, it is convenient to drift the receiving machine in this manner until the desired pulse of the combination is located. The distorted appearance of the signals in the upper half of these tapes is the result of a rather severe hunting of the transmitting distributor. The hunting in this case was introduced intentionally.

Fig. 12 shows two specimen records of normal traffic signals on a multiplex automatic circuit. The transmitter in this case was the three-channel distributor of the multiplex apparatus, and the maximum dot frequency of the signals was approximately 45 cycles per second. The signal pulse combinations of traffic signals, as may well be imagined, cover every conceivable combination of positive and negative pulses. In the record of Fig. 11, with test signals which are continuous alternating current pulses, a mark is made by the chemical recorder for every pulse reversal. There is, therefore, a mark on the tape showing the beginning and ending of the particular pulse which is being observed for every revolution of the transmitting distributor. In the record of traffic signal pulses, however, many revolutions of the transmitter may be made with no marks recorded on the tape. From such a record it is not known whether the armature of the receiving relay is at rest on the marking contact or on the spacing contact. In the record of Fig. 12 the total losses of the circuit under test is recorded. The displacement of a particular mark on this record may be the result of any one of the various factors which introduce signal distortion, either characteristic or fortuitous, or more likely may be the result of a cumulative effect of all the distortions.

In tests of transmission made for the purpose of analyzing signal losses on a circuit it is frequently necessary to obtain a number of measurements. One may determine the characteristic distortion, another the fortuitous, and yet others may be required to analyze the latter and determine the magnitude of the components resulting from the various disturbing influences. A check test of the total losses is usually made by recording traffic signals. Such signals, because of the great variety of pulse combinations of which they are made, result in a maximum of characteristic distortion loss and are also subject to all the disturbances which produce fortuitous distortion.

If periodic disturbances, such as the currents induced by power distribution systems, are present in the circuit the losses resulting from these may not ordinarily be so readily separated from the normal telegraph "crossfire" loss. The transmission testing machine does permit, however, a separation of these two losses by a method which is rather unique. Measurements by this method depend for their success upon the condition that the frequency of the disturbing potential be fairly constant. If the frequency is sufficiently constant the testing machine may be driven at such speed that the signal displacement which results from this part of the disturbance is a constant amount. The effect of the power disturbance distortion as evidenced on the tape record is then a permanent shortening or lengthening of the observed pulse. It has the appearance of a uniform bias of the signals. That part of the distortion which results from the normal telegraph crossfire may then be measured by the method used in the measurement of the record of Fig. 12, without regard to the absolute length of the signal.

In general, measurements of over-all circuit losses are as readily made on a circuit which contains a number of repeaters as on a single repeater section. Obviously, however, the analysis of the losses may not be carried to the same length as it may in tests of a single section.

In this paper no mention has been made of the application of the machine to transmission measurement work on either ocean cable circuits, employing a three-element code, or on single Morse circuits. In work on circuits of the latter class, it is usually convenient to interpose between the receiving instrument and recording mechanism a polar relay which has two live contact stops from which the records may be made. It is difficult when recording from a single Morse relay of the conventional type to determine if the received signals are entirely free from bias. Frequently, in this class of work a special recording instrument is substituted. The terminal apparatus used for the three-element cable code presents no special problem. The records obtained from three-element signals are as conveniently interpreted as are those of the examples presented.

These are but a few of the more common measurements to which the machine is readily adapted. The field of work which the machine may be made to cover in telegraph transmission measurement, as may readily be appreciated, is very broad.

Discussion

H. H. Haglund: In his paper Mr. Bramhall has hewn very close to the line and described the machine only for the actual testing of transmission. As a matter of fact the machine has many other uses, one of which is described below.

For comparing the speed of oscillating systems of comparatively low frequency such as electrically driven tuning forks, the machine is particularly well adapted. In the past, five methods have been commonly employed for this purpose:

1. Comparing the rates of clock mechanisms driven by the two oscillators. This method is accurate only for comparing the average rate of the two systems.

2. Comparing the speeds with the aid of a stroboscope. This method is fairly accurate but very tedious if comparisons are to be made for periods longer than a few minutes at a time.

3. Setting up the two frequencies on a cathode-ray oscillograph in such a manner that Lissajous figures are produced and counting the interval between successive repetitions of the same figure. This method gives an indication of the erratic behavior of the oscillating circuits during short intervals, but does not give a good reading of the average rate unless long tedious observations are made.

4. Producing higher frequency harmonics in the two oscillators and comparing some given harmonic of the two for beats. This method can be made very accurate but also involves tedious continuous observation in order to obtain a comparison for both short and long intervals.

5. Setting up some form of counting system such as two multiplex distributors with the rates adjusted so that one distributor is corrected by a step-by-step mechanism whenever the speed of one gains or loses with respect to the other and counting the number of such steps that occur in a given interval of time.

The transmission testing machine may be used to make the comparison much more conveniently and usually with greater accuracy in the following manner:

Drive the motor on the testing machine from a relay controlled by the output of the oscillator which for the purpose of the test is considered as the standard. The motor makes one revolution for every ten cycles.

Run the second oscillator at the same speed or at a speed differing from the standard by multiples of one-half cycle. Operate the recording mechanism from a relay controlled by the second oscillator so that the needle of the machine marks during the relay travel time.

If the speeds are matched exactly, the short lines or traces on the paper will line up in two perfectly straight columns parallel to the edges of the paper. If the rate of one of the forks now changes, the lines of the traces will run off at an angle which can be readily measured.

The product of the tangent of this angle and the distance the paper feeds out per minute divided by the distance between the traces on the paper gives the change in rate in half cycles per minute. In practise the change of rate can often be obtained by observation. For example: If the traces begin to drift off at such an angle to the edges of the paper that in five minutes the right hand trace has moved over to the position previously occupied by the left hand trace, the change in rate is obviously one-half cycle in five minutes or one-tenth cycle per minute.

Since the machine will operate for several hours with practically no attention, a long run can be obtained during which a continuous record is produced. This involves no tedious count-

ing and permits the variations to be carefully measured after the test has been run. Average rates, rates for successive intervals, maximum and minimum rates can all be obtained from the same record. Accuracies to one part in a million can easily be obtained except for short intervals.

R. B. Shanck: The device is a valuable addition to the tools which are available for the measurement of telegraph signal distortion. I believe that it is particularly well suited to the investigation of distortion which, with a regularly-recurring signal, oscillates through a definite cycle so as to form a regular pattern, as is the case with a-c. interference, for instance. The result of changes which affect distortion of this type is clearly evident immediately.

It is gratifying to note the tendency, in connection with the discussion and investigation of telegraph distortion, to employ the same approach and terminology throughout the world. Thus the separation of distortion into components, called bias and characteristic and fortuitous distortion, as now widely done in this country, has also been taken up abroad. Recently distortion was measured and reported in this way in connection with a voice-frequency telegraph investigation by a subcommittee of the International Consulting Committee on Telegraphy.

A Printing Telegraph Concentrator

BY WILLIAM B. BLANTON*

Non-member

Synopsis.—The central telegraph office of a city is the focal point for the collection and delivery of commercial telegrams within that city and the surrounding territory. This central office is connected telegraphically to the public branch offices within this area as well as to the offices of those patrons who have sufficient telegraph business to warrant the telegraph company incurring the expense of providing such a connection.

Printers employing the start-stop method of synchronism have been found exceedingly adaptable for use in circuits providing such telegraph connections, and the telegraph companies are using these printers in increasing numbers for this purpose.

The termination in a central office of a large number of these circuits, the majority of which is worked intermittently for the greater part of the time, requires the provision of some means of concentration in order that the traffic may be handled efficiently and economically.

This paper describes a multipled turret printing telegraph concentrator designed and used by one of the telegraph companies to meet this need.

* * * * *

Introduction

During recent years great improvements have been made in printing telegraph systems, particularly those using the start-stop method of synchronism.[1]

Some of the designations which are employed by the manufacturers and various users for telegraph printers adapted to start-stop systems are start-stop printer, simplex printer, teletype, telephone typewriter, typing telegraph, and teleprinter.

The telegraph companies are using in increasing numbers printers of this type on private tie lines which connect patrons' offices with a telegraph office, on public circuits which connect the public branch offices within a city to the central telegraph office, and in many cases on lightly loaded trunk circuits extending between telegraph offices in different cities.

For services of this kind, one of the telegraph companies has installed, since 1925, upwards of 20,000 of these printers. About 5,000 of this number were installed in patrons' offices. In fact, the greater portion of printer installations now being made are necessitated by the increasing number of printer tie lines which connect patrons' offices with a telegraph office.

The purpose of this paper is to describe a concentrator which was developed for use in central telegraph offices. Essentially, this concentrator provides for terminating a large number of start-stop printing telegraph circuits before a smaller number of operators so that any one of the circuits can be worked by any one of the operators.

Throughout the paper the terms "printer" and "printing telegraph circuit" should be understood to refer to equipment and line operated on the start-stop principle.

The Advantages of Concentration

The termination in a central telegraph office of a large number of printing telegraph circuits, the majority of which is worked intermittently for the greater part of the time, presents the problem of providing suitable facilities for efficiently and economically handling the traffic on these circuits.

Earlier methods included the termination of the heavily loaded circuits on operating tables individual to each circuit, and of the lightly loaded circuits in small concentrators each having four operating positions and a capacity of eight lines. While these arrangements are still satisfactory for small offices where few circuits are involved, they are unsuitable for handling the numerous circuits which are now terminated in the larger offices. For the latter conditions it has been found desirable to provide concentrators accommodating a large number of circuits and operating positions in order to obtain the advantages discussed in the following paragraphs.

The termination of a large group of intermittently worked circuits in a single concentrator permits the most efficient utilization of printer equipment and floor space due to the fact that any one of the concentrator positions can work any one of the circuits. Therefore, the number of operating positions provided for a given group of circuits need not exceed those required to handle the maximum number of these circuits which it is anticipated will be busy simultaneously during the peak load. The smaller floor area required to accommodate circuits when concentrated has been of a very distinct value in many offices, in that it has postponed for the time being the necessity of obtaining additional floor space to care for the increasing number of printer circuits.

The ability to work any one of many circuits from a concentrator operating position tends to utilize fully the time of each operator for useful purposes. Of course, this may be accomplished to some extent even with the termination of circuits on individual operating positions and in small eight-wire concentrators by shifting the operators from one operating position to another to meet the load changes. However, the impracticability of doing this to realize the maximum operating economies and at the same time not degrade the speed of service will become

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1. T. E. Herbert, "Telegraphy," p. 632, Sir Isaac Pitman & Sons, Ltd.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

more apparent as the number of circuits is increased.

Besides the increased productiveness of each operator, several other operating economies accrue from the concentration of many printer circuits in one unit. The use of automatically operated supervisory arrangements, which give a continuous and accurate check on the number of operators required to handle the traffic load, permit a quick adjustment of the operating staff to meet the changing traffic load. The centralization of the operating force reduces the supervisory staff to a minimum. The bringing together of the operating force into a small group during the light load periods, particularly at night, permits further savings through the shutting down of lights, belt conveyors and other telegraph equipment in the unused portion of the office.

These economies are not obtained through any sacrifice in the speed of service. On the contrary, the speed of service obtained on each circuit in a properly staffed concentrator is consistent with that obtained on a circuit terminated on an individual operating table staffed continuously with an operator. Furthermore, better supervision of the movement of traffic is achieved due to the centralization of the operating force and the centralization of the various operating functions of each circuit.

The more intermittent the traffic load is on a circuit, the greater are the advantages in concentrating. Obviously, there would be no gain in providing concentration facilities for a heavily loaded circuit which requires the individual attention of an operator all of the time. However, on most heavily loaded circuits there occur periods, particularly at night, when the continuous services of an operator are not required. During such times it is desirable to work such circuits in a concentrator; and in order that advantage may be quickly taken of these periods, it is the present practise to terminate all start-stop printing telegraph circuits entering the larger offices in concentrators. This also has the advantage of uniformity of circuit arrangements in an office.

The concentration of heavily loaded circuits, the busy periods of which will probably occur simultaneously, does not effect any appreciable saving in operating positions. In fact, the addition of such a circuit to a concentrator practically requires the addition of an operating position. Thus, the value of a concentrator in which heavily loaded circuits will be terminated is greatly enhanced if the burden of concentration expense carried by each operating position is small.

Methods of Concentration

Before the equipment described in this paper was decided upon, three methods of concentration were considered; machine switching, manually attended switching board, and lines multipled before each operator. The following brief descriptions of these methods of concentration are not intended to apply to any particular existing concentrators but merely explain in very general terms the fundamental features of each method.

A machine switching concentrator would comprise essentially equipment which, in response to a calling signal from an outer office, would automatically connect the line of that office to the printer apparatus at an idle concentrator operating position, and also equipment which would enable a concentrator operator to initiate a call to any one of the outer offices. This latter equipment might consist of switches which would respond to electrical impulses, caused by a concentrator operator dialing the call numbers or letters of any one of the outer offices, so as to connect this operator's printer apparatus to the line of the desired office; or this equipment might be similar to that of a multipled turret concentrator, *i. e.*, the lines would be multipled before each sending operator thus enabling an operator to connect her printer apparatus to any desired line in order to initiate a call.

A manually attended switching board concentrator might comprise an arrangement somewhat similar to a manual switching telephone private branch exchange. Thus, the lines would be terminated on jacks located in the switchboard. The wires to the printer apparatus at the concentrator operating positions might be similarly terminated in jacks in this switchboard, or be terminated in cord circuits appurtenant to the switchboard. In either case, the switching operator or operators stationed at this switchboard would note the incoming calls, indicated by lamps or other means associated with the line jacks, and would connect these calling lines to printer apparatus at idle operating positions by inserting plugs, attached to cord circuits, into the proper jacks.

Outgoing calls might be handled in the same manner, the switching operator connecting the printer apparatus at the calling operating position to the desired line. Each cord circuit which the switching operator uses to connect a line to an operating position would be provided with a supervisory signal arrangement so as to permit a concentrator operator to give the switching operator a "disconnect signal" at the completion of the exchange of traffic with an outer office. Multiples of this switchboard would be necessary if the number of switching operators exceeded the number that can be stationed at one switchboard.

A concentrator in which the lines are multipled before each operator would also comprise means, generally lamps, which indicate incoming calls, and means at each operating position which enable an operator to connect her printer apparatus to any desired line. Thus, in answering or initiating calls, the concentrator operator would make the actual connection of her printer apparatus to the desired line.

A study of the economics and operating advantages of these three methods and their adaptability to the present methods of handling telegraph traffic led to the

choice of the arrangement whereby the lines are multipled before each operator, or, in other words, a multipled turret concentrator.

Description of the Concentrator

The methods which have been generally followed in the design of multipled turret arrangements are to provide for each line a jack, calling lamp, and sometimes a busy lamp in each of the turrets, which are nothing more or less than small switchboards connected in multiple. Generally, one turret suffices for several operators, the number of turrets required being dependent on the maximum number of operators required at any one time to handle the traffic on the lines terminated in the turrets. Each operator is provided with one or more cord circuits, each terminating in a plug by means of which she connects an operating set to any desired line by inserting the plug in the proper line jack.

It can be readily appreciated that the cost of the turrets and the cabling required to connect the jacks and lamps in multiple comprises no small part of the total cost of a multipled turret arrangement. With the printer concentrator, which in some cases requires 70 operating positions for a unit terminating 100 lines, it becomes imperative that the cost per operating position of the turrets and cabling be held to a minimum. Two methods of accomplishing this suggest themselves: first, let each turret accommodate a large number of operating positions; second, simplify and reduce the cost of each turret and the cabling between the turrets.

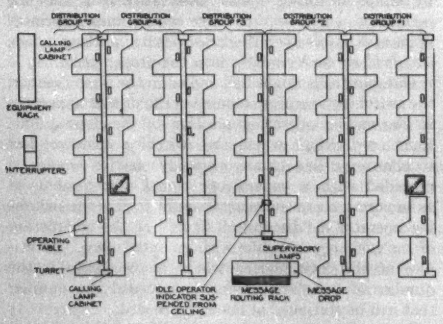

Fig. 1—Typical Concentrator Floor Plan

It is impracticable to follow the former because the distance between operators and the bulky equipment installed on the operating tables necessitate that the turrets be individual to each operating position if they are to be easily accessible. In order to simplify the turrets and reduce the number of conductors which will have to be multipled to each turret, several departures are made from earlier practises; no busy lamps are provided, the calling lamps are removed from the turrets and placed in lamp cabinets so located that a number of operators can observe one cabinet, and the concentrator line circuit is designed on a single conductor basis; in other words, each line appears in the turrets as one conductor without any associated local or private wires. Therefore, the cabling which is multipled from turret to turret consists of only one conductor for each line circuit which may be terminated in the concentrator, and the cabling required for the calling lamps is simplified since the number of calling lamp cabinets

Fig. 2—General View of a Concentrator

A Calling lamp cabinet
B Turret
C Idle operator indicator
D Message routing rack
E Message belt conveyor
F Message drop

is considerably less than the number of operating positions.

Layout of Concentrator. Fig. 1 shows in schematic form the relative location of the various items in a typical concentrator installation. Fig. 2 is an illustration of an actual concentrator installation having 50 operating positions and a capacity of 100 lines. The layout of this installation is identical with that of Fig. 1. Reference to these two figures when reading the following description will aid in correlating the various items.

Operating Table. The physical size of the printer and accessory operating equipment required at a central office operating position, even when it is not equipped as a concentrator operating position, requires considerable operating table area. The top of the standard table is 49½ in. (126 cm.) long, 21 in. (53 cm.) wide and on the right hand end of the table an extension projects forward. One of these tables equipped as a 100-line concentrator operating position is illustrated in Fig. 3.

The front edge of the printer is alined with the left hand edge of the extension which forms a 120-degree angle with the front edge of the table, while the gum-

ming desk is alined with the front edge of the table. The printer is a "tape printer" as contrasted with a "page" printer. Each message, whether being transmitted or received, is printed on one side of a paper tape, the other side of which is coated with dry gum or glue. This tape issues from the left end of the printer. When transmitting, the operator allows this tape to go through a hole in the table top into a metal receptacle, whereas during the reception of messages, the operator by means of a thimble tape cutter and hand tape

Fig. 3—Concentrator Operating Position

H Belt conveyor
J Time stamp
K Gumming desk
L Busy line lamp
M Idle operator lamp
N Answer-next lamp
O Turret
P Jack-plug
R Calling lamp cabinet
S Live message file
T Printer
U Number stamp

moistener pastes the tape on a telegram blank which is held on the gumming desk by means of a spring clip.

The other equipment on this table which is required at any standard central office operating position is as follows: the two-compartment, "V" shaped, live message retainer into which clerks called routing aides place messages awaiting transmission, full rate messages being placed in the bottom compartment and reduced rate messages in the top compartment; the number stamp used to number consecutively all messages transmitted from this operating position; the two compartment box located to the left of the printer into which the outgoing messages are placed after they have been transmitted; the three-compartment box directly behind the gumming desk which is used as a storage for message blanks; the time stamp which is used to stamp the time on all received messages; and the "goose neck" arrangement into which the operator inserts any message requiring the attention of the supervisor.

Along the rear edge of the table may be seen the belt conveyor on which all received messages are placed.

Turrets. To avoid using for the concentrator operating positions a table larger than the standard operating table, it is necessary that the turrets be of a small and compact design. Fig. 3 shows a turret, having a capacity of 100 lines, located near the rear of the table, midway between the sending and receiving portions of the operating table. It is not intended that the capacity of the concentrator be limited to 100 lines; in fact, installations have already been made where two of these turrets for 200-line concentrators have been mounted on the standard operating table. Moreover, it is quite practicable to install turrets for more than 200 lines on the same size table.

The use of a one-conductor multiple permitted a

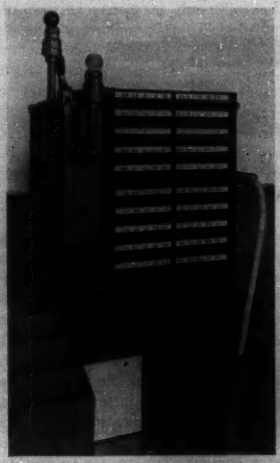

Fig. 4—Signal Assembly and 100-Line Turret

departure from standard telephone and telegraph practise in the design of the turret. Instead of jacks, the line conductors are terminated on solid plugs or studs molded in a bakelite panel.

Fig. 4 gives a close-up face view of a turret equipped with two panels, each panel having a capacity of 50 lines. The line designations are printed on white cardboard and inserted in cellulose acetate tubing. By loosening either of the thumb nuts in the upper corners of each panel, the associated metal angle may be swung outward thus permitting the removal or replacement of the designation strips.

That portion of the face area of the panels which is used for the studs is approximately the same as it would be if telephone type jacks were used. However, this design of panel presents possibilities of greater face area reduction if that should become advisable in the future. For example, the horizontal spacing of the studs was governed by the desire to provide ample designation space for two or three letter designations.

The immediate gains obtained with this panel as compared with an equivalent arrangement of jacks and designation strips are: reduced cost of apparatus and installation, saving of space by use of a shallower turret, better insulation between adjacent circuits, and more robust construction. In connection with the last item, it will be noted that all springs and moving parts are included in the device, described later, which the operator uses to connect her printer to a line stud.

Cord Circuit. Associated with each operating position is an operator's cord circuit which includes the receiving and transmitting portions of the printer, the three lamps and lever type key contained in the signal assembly mounted on the left hand side of the turret, certain relays, jacks, resistance units, a vacuum tube, and other apparatus and equipment. This circuit connects to a three-conductor cord, the other end of which terminates in a "jack-plug," so termed because it has the features of both a jack and a plug as generally thought of in the communication art. It is by connecting this jack-plug to a line stud in the turret that the operator connects her printer to any desired line. When not in use, this jack-plug rests in a plug seat bushing fastened in the end of a short length of pipe located directly in front of the turret.

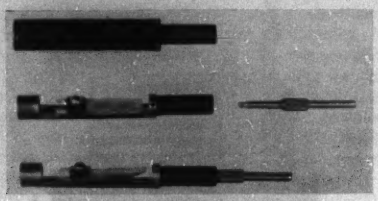

Fig. 5—Several Views of a Jack-Plug

A metal cabinet is provided at each position for housing some of the equipment included in the operator's cord circuit and also other electrical equipment such as connecting blocks, fuse blocks, power receptacles, etc. The metal cabinet has a top plate somewhat larger than the cabinet proper. This enables the cabinet to be installed on the table by lowering it into a rectangular opening until the top plate rests on the top surface of the table. Access is gained to the equipment inside of the cabinet by means of doors which open underneath the table top.

Several views of the jack-plug are shown in Fig. 5. The top view shows a completely assembled jack-plug. The middle view is of the same arrangement except that the large bakelite sleeve is removed and also, a stud, of the type that is molded in the panels, is shown. The bottom view illustrates the operation of the jack-plug when it is plugged on a stud. The body of the jack-plug makes contact with the stud, and this stud operates a lever that extends up into the small barrel of the jack-plug so as to close the two springs.

Calling Lamps. The calling lamps serve to register an incoming call when an outer office initiates a call to the central office. As has been previously mentioned, lamp cabinets, each containing one lamp for each line terminated in the concentrator, are distributed among the operating positions so that each cabinet is visible to and used by a number of operators. It is the general practise to locate these cabinets so that no operator is more than four and one-half table lengths distant from a cabinet. Since the tables are arranged in double-sided rows, one cabinet may therefore serve as many as ten operators.

A view of one of these cabinets supported by a structure that is fastened to the belt conveyor guide angles may be obtained from Fig. 3.

There are no designations on the calling lamps, whereas, as has been noted, the line studs in the turrets are designated with the office letters. However, the calling lamps are in the same relative arrangement as the line studs with which they are associated. An operator, to answer an incoming call, notes the position of the lighted lamp and then attaches her jack-plug to the line stud having the same relative position. To aid the operators in performing this function, the 100 lamps and studs are arranged in the cabinets and turrets, respectively, in groups of 25 each. Thus the operator notes at a glance in which group the lighted lamp is located and its relative position in this group. Actual practise has demonstrated that the operators become exceedingly skillful in selecting the proper line in the turret by this method.

Busy Line Indication. The absence of busy lamps appurtenant to the line studs makes it essential that each operator's cord circuit be equipped with means that will give a "busy indication" when an operator plugs in on a busy circuit and, at the same time, not interfere with or mutilate signals which might at that moment be passing over the line circuit.

To obtain this feature with the one-conductor multiple, the concentrator circuit is designed so that positive battery[2] is applied to a line when it is idle and negative battery is applied to a line when a concentrator

2. In this paper, the word "battery" is used to denote any source of electrical energy which supplies current to telegraph circuits. The "batteries" referred to have one pole permanently grounded and are spoken of as "positive" or "negative" depending upon the polarity of the ungrounded pole.

operator has her printer connected to it. Then by the use of a unidirectional current device, the operator's cord circuit is arranged so that when an operator plugs in on an idle line, the printer motor starts and the printer with its associated negative battery is automatically connected to the line. On the other hand, if the operator plugs in on a busy line, the unidirectional current device will function so as to keep the printer motor from starting and to prevent the printer with its associated negative battery from being connected to the line. Furthermore, the characteristics of the unidirectional current device are such that no interference is caused to the line signals which might then be passing over the line circuit.

When an operator plugs in on a busy line, the busy line lamp, having a red colored lamp cap and located in the lower portion of the front of the signal assembly (see Fig. 4), will light steadily until the operator removes her jack-plug from this busy line or until the line ceases to be busy. If the operator fails to see this visual signal and attempts to send on her printer, the printer will not function due to the fact that the printer motor is not running.

Answer-Next System. The answer-next system serves to expedite the answering of incoming calls in that it avoids delay that might result from the inattentiveness of the operators and confusion that might arise from several operators endeavoring to answer the same call. By virtue of the answer-next arrangement, the responsibility for answering an incoming call is placed upon the lowest numbered idle operator.

A white lamp, termed the answer-next lamp, is provided on the front of the signal assembly at each operating position as shown in Fig. 4. This lamp is always lighted on the lowest numbered idle operating position thus indicating to this operator that she is to answer the next incoming call. For example, if the first 10 operators are busy, the answer-next lamp on position 11 will be lighted. When the operator at this position answers a call, the lamp will be extinguished at position 11 and will light on position 12, provided of course that the operator at number 12 position is idle and that all lower numbered operators are still busy. If, on the other hand, a lower numbered operator than 11 becomes idle before operator 11 answers a call, then the answer-next lamp on position 11 will be extinguished and the lamp on the lower numbered idle position will light.

The answer-next lamp on the lowest numbered idle operating position glows steadily as long as there are no incoming calls, thus warning the operator that she should be in readiness to answer the next call. If there is a long interval of time before a call appears, it may be expected that the attention of the operator will wander and that she will not keep her eyes continuously on a calling lamp cabinet. Therefore, to attract the operator's immediate attention when a call does appear, the answer-next lamp begins to flash the instant a call is registered in the lamp cabinets.

By means of the lever type key located in the top of the signal assembly (Fig. 4) any of the operating positions which are not staffed, either because of light loads or of repairs being made to the table equipment, may be cut out of the answer-next system, or in other words, the positions may be "busied."

Operating Practises. Since many of the features of the concentrator about to be described are necessitated by the present practises followed in handling telegraph traffic in central telegraph offices, it is thought best to briefly outline these practises.

All messages received over printer circuits are placed on continuously moving belts which run along the rear edge of the operating tables and which carry the messages to the end of the table row. Other belts, located at the end of the table row, comprise a part of a mechanical conveyor system that carries all received messages (including those received by other facilities such as multiplex, telephone, pneumatic tube, and Morse) to the main routing center of the office. Here are located the requisite number of route clerks who ascertain the destination of these messages and dispatch them by means of other conveyors to "drops," points at which messages are discharged from the conveyor, located near the proper line wires. Where concentrators are not used, each outgoing message is removed from these drops by a clerk, called a routing aide, and carried to the operating position at which is terminated the line wire of the office to which the message is destined.

The handling of the received messages presents no problem to concentration. On the other hand, the present practises followed in handling the outgoing messages introduces many of the problems in the design and operation of large concentrators.

There are two requirements to be met in handling this outgoing traffic. First, each message must be expedited to its destination. To insure a satisfactory speed of service, there is specified a time limit within which each message must be retransmitted after it is received in the office. Second, full rate traffic must be given precedence over reduced rate traffic, such as day letters and night letters. In order to meet these requirements with a concentrator, messages for an office, the line of which is idle are taken to an idle operator for immediate transmission, while messages for an office the line of which is busy are taken to the operator then working that circuit.

At first thought it may seem that a more desirable method of designing and operating a large concentrator would be to divide the operating positions into sending and receiving positions and that all outgoing messages should be taken to sending operators who would transmit them to the proper offices at the first moment the lines to these offices became idle. However, it is

apparent in considering the second requirement mentioned in the preceding paragraph, that this procedure would be open to the objection that a circuit might be busy at a receiving position then handling day letters, while at the same moment one or several full rate messages might be awaiting transmission at a sending position.

Sub-Routing Center. A sub-routing center, centrally located at one end of the rows of operating tables, is provided for each concentrator. All the messages received at the main routing center of the office which are destined for the circuits terminating in a particular concentrator, are dispatched by a mechanical conveyor to a drop located at the sub-routing center of this concentrator. This drop is located at one end of a message routing rack which has one compartment for each circuit terminated in the concentrator. The front view of a message routing rack provided with 100 compartments and with the message drop on the

Fig. 6—Front View of Message Routing Rack

right hand end is shown in Fig. 6. Each of these compartments bears on both the front and rear of the rack the designation of the office with which it is associated.

One or more routing clerks stationed at the rear of the rack remove the messages from the drop and sort them into the proper compartments according to the destination of the messages. These messages awaiting transmission may be thought of as falling into two classes, those for offices which are being worked at that moment by a concentrator operator and those for offices the line wires of which are idle.

Group Position Finder Facilities. To facilitate the distribution of messages destined for transmission on lines which are busy, the operating tables of a concentrator are divided into distribution groups. For example, the operating tables in the typical layout shown in Fig. 1 are arranged in five distribution groups. The number of tables included in a distribution group is not definite but depends on the layout and total number of tables in each particular concentration unit, the one requisite being that all the tables in any one distribution group shall face on the same aisle. In practise, one single-sided table row is designated as a distribution group if the table row is long, or two single-sided table rows facing on the same aisle, if the table rows are short.

On the front of the message routing rack a vertical row of lamps, one lamp for each distribution group in the concentrator, is provided underneath each compartment and its associated designation card. The rack illustrated in Fig. 6 has five lamps per compartment, one for each of the five distribution groups shown in Fig. 1.

When none of the lamps under a particular compartment is lighted, this indicates that the associated line is idle. The distribution of messages destined for transmission on idle lines is covered in the section below.

If one of the lamps under a particular compartment is lighted, this indicates that the associated line is being worked by a concentrator operator. The routing aide when removing a message from this compartment notes which one of the lamps is lighted. This serves to guide her to the proper distribution group. Then she views the turret on each busy operating position in this group and ascertains by the location of the jack-plug which operator is working the line in question.

An operator indicator system was devised which would denote under each compartment the exact location of the operator working the associated line. This arrangement was more intricate and costly than the group position finder arrangement. An analysis of the two showed that the individual position finder possessed very few advantages over the group position finder, and that these advantages were not sufficient to justify the increased cost of installation and maintenance.

Idle Operator Indicator. Messages for offices, the line wires of which are idle, are given to idle operators for immediate transmission. To facilitate this, an idle operator indicator, Fig. 7, is located within view of the routing aides when they are standing in front of the message routing rack.

The idle operator indicator contains one lamp for each operating position in the concentrator. The lighted lamps denote staffed operating positions where the operators are not working a line wire, whereas, the unlighted lamps denote unstaffed positions or positions where the operator is busy. The lamps are not numbered or otherwise designated. Instead, the layout of the lamps corresponds to the layout of the operating tables in the particular concentrator and the routing aides are required to note the relative location of the lighted lamps when they desire to know where the idle operators are located.

As a further aid to locating the idle operators, each

operating position is equipped with an idle operator lamp mounted on the top and towards the rear of the signal assembly (see Fig. 4) and colored green to distinguish it from the white answer-next lamp. This lamp is lighted on each operating position where the operator is idle.

The idle operator indicator indicates to the routing aides the location of all of the idle operators. After the routing aides have left the message routing rack, the idle operator lamps on the positions guide them

Fig. 7—Idle Operator Indicator and Supervisory Lamps

to the idle operators and also immediately indicate to the aides any change in status that may have taken place since they left the message routing rack.

Messages for idle lines are distributed to the highest numbered idle operators, the reverse order in which incoming calls are answered.

Supervisory Lamps. Three supervisory lamps are located at some point about the concentrator, generally on the supporting structure of one of the calling lamp cabinets as shown in Figs. 6 and 7, so that they are visible to the supervisor while he is performing his duties about the concentrator. These lamps serve to warn the supervisor immediately of conditions which require attention. The first lamp lights when there is one or more incoming calls. The second lamp lights when all the operators assigned to the concentrator are busy.

With the second lamp unlighted and the first lamp lighted for an appreciable interval of time, the supervisor is advised that the operator, on whose position the answer-next lamp is lighted, is not promptly answering the incoming call. Such a condition might exist through no fault of the operator if her answer-next lamp fails to light due to a burnt out filament. However, the supervisor is instantly made aware of that condition by the third supervisory lamp which lights at any time that an answer-next lamp fails because of an open filament.

When the second lamp lights denoting all operators are busy, the supervisor is warned that there is no "readiness to serve." A constant check must be kept on this feature in order that there will always be an operator available to answer an incoming call.

The idle operator indicator is utilized by the supervisor in determining the size of operating staff required throughout the day.

Equipment Rack. Fig. 8 shows a rack equipped with relays, resistances, fuses, and other apparatus required for a concentrator terminating 100-line circuits. Various alarm arrangements and testing facilities are also included on this rack.

Signal and Motor Control at Distant Office. When an operator at the concentrator plugs in on a line circuit, this automatically causes a visual signal to function and the printer motor to start at the distant office. The starting of the motor permits the concentrator operator immediately to supplement the visual signal with an audible signal by repeatedly depressing the proper key on her printer, thereby actuating the bell signal in the distant printer. The outer office operator acknowledges the call and proceeds to exchange traffic, having extinguished the visual signal by operating a conveniently located switch.

The outer office calls the concentrator by depressing

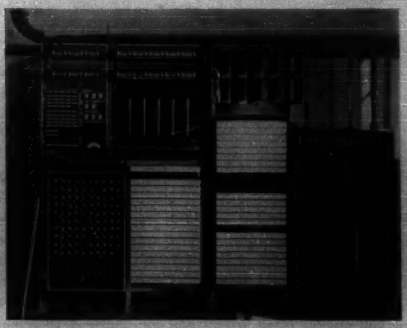

Fig. 8—Equipment Rack for a 100-Line Concentrator

any one of the printer keys, first starting the printer motor if it is not already in operation.

Circuit Operation. An explanation of several of the salient points of the concentrator circuit, shown in Fig. 9, may be of interest.

Positive battery, designated "signaling battery" in Fig. 9, of a potential of 110 volts, is applied to each idle line, resulting in a continuous current flow over the circuit. An outer office operator to initiate a call to the concentrator, throws her switch to the operating posi-

tion, thereby starting her printer motor. Then by depressing any key of her printer the flow of line current is momentarily interrupted which causes the signal relay at the concentrator to light all of the calling lamps associated with this line. A concentrator operator to answer this call attaches her jack-plug to the proper line stud which applies the positive signaling battery to the plate of the vacuum tube in her cord circuit. A small plate current results which causes certain relays in her cord circuit to function so as to start her printer motor and connect her printer and its associated negative battery, designated "operating battery" in Fig. 9, to the line. The current flow which then results on the line circuit energizes a line relay that in turn energizes a cut-off relay which disconnects the signal relay and the signaling battery from the line.

It is essential that the cut-off relay remain continuously in the energized position during the entire time that the concentrator operator has her printer connected to the line. Since this must be accomplished on the same conductor over which are passing the line signals, consisting of intervals of current and no current, it is necessary to provide a slow-to-release arrangement for the cut-off relay which will not allow it to release except after a predetermined interval of no current on the line circuit.

The start-stop printers shown in the illustrations of this paper employ a seven unit code transmitted at approximately 22 cycles per second, *i. e.*, each unit of the code takes 1/44 second. The first unit is always the start signal of no current and the seventh unit, the stop signal of current. The intermediate five units are in accordance with the Baudot code and consist of various combinations of current and no current depending on the character being transmitted. Since the seventh unit is always a current impulse regardless of what character is being transmitted, it is evident that the longest interval of no current that can occur when only one printer is transmitting is 6/44 seconds. If the second printer on a circuit is operated to "break" the first printer, longer intervals of no current may occur. However, experience has shown that even in such cases, at least one impulse of current is generally transmitted over the line every second.

The slow-to-release arrangement employed to maintain the cut-off relay continuously operated while an operator has her printer connected to a line consists of two locking circuits through the cut-off relay, either one or both of which when closed will maintain the cut-off relay in the operated position. A motor-driven interrupter is also included which momentarily opens at regular intervals and in a definite sequence these two locking circuits. The arrangement so functions that if an impulse of line current passes through the line relay at least once every second, these locking circuits will be restored after being opened by the interrupter in such a manner that one or the other will be continuously energizing the cut-off relay. When the concentrator operator disconnects, all battery is removed from the line and the cut-off relay will be deenergized shortly afterwards since there is no line current to restore the locking circuits after they are opened by the interrupter.

When the cut-off relay is deenergized, the signal relay with its signaling battery are connected to the line and are in readiness to receive another incoming call. It will be noted that the line current from the signaling battery does not pass through the line relay, thus it is evident that the cut-off relay remains in the deenergized condition during the idle period of the line circuit.

The signal and motor control device at the outer office is a polarized relay which lights the signal lamp and starts the printer motor when negative battery is

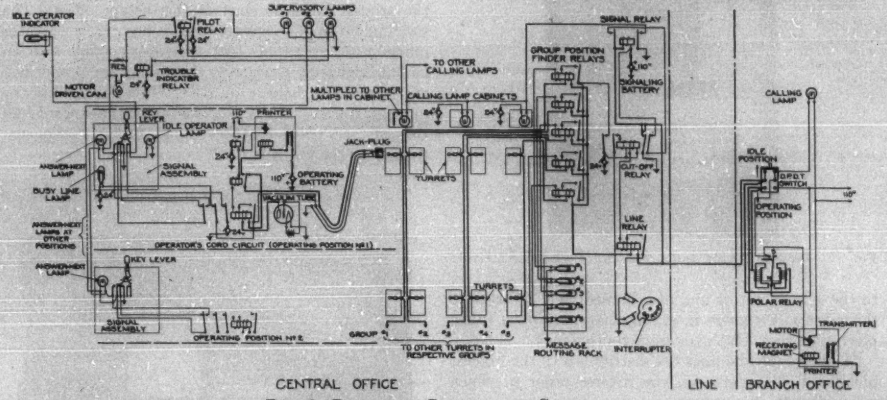

Fig. 9—Diagram of Concentrator Circuit

applied to the line and extinguishes the lamp and stops the motor when positive battery is applied to the line.

The vacuum tube is the unidirectional current device employed to prevent a second operator from connecting her printer and operating battery to a line circuit which is busy at another concentrator operating position. If this second operator attaches her jack-plug to the stud of a busy line, the plate of the vacuum tube in her cord circuit encounters the negative operating battery (or no battery when spacing signals are being transmitted from the concentrator position) on this busy line and consequently no plate current will result, thus maintaining the relays in her cord circuit in the deenergized position. The impedance of the plate of the vacuum tube to ground is so high that no interference is caused to the line signals which might be passing over the line circuit.

Application of Concentrator

Numerous installations of the multiple turret concentrator have been made. Most of these concentrators were planned and installed on the basis of terminating 100 lines, while in a few cases concentrators accommodating 200 lines were installed. In some of the smaller offices having only a few circuits, concentrators were installed and equipped for less than 100 circuits, the installations being made, however, so that the capacity of the units can readily be increased to 100 or 200 hundred lines without unduly disturbing the initial installations.

Although this type of concentrator is adaptable to a larger number of lines, 100- and 200-line concentrators were selected as the standard sizes of concentrators because of the relative ease of finding the operator working any particular line, as well as the fact that these two sizes of units, particularly the former, meet the requirements of a large number of offices. The former size unit is essentially used for terminating heavily loaded circuits whereas the latter size unit is used for terminating comparatively lightly loaded circuits. In offices having one hundred or less circuits, no distinction is made between heavily and lightly loaded circuits as they are all terminated in one concentrator. At the present time most of the offices fall in this class which explains the greater usage of 100-line concentrators. In the larger offices, more than one concentrator is required. For example, ten 100-line and three 200-line concentrators have been installed in the New York City central office.

The traffic load on the circuits terminating in a concentrator determines the number of operating positions which is provided. The number of positions provided for the 100-line concentrators has varied from 20 to 70 positions and for the 200-line concentrators, from 30 to 40 positions.

The present practise is to terminate all start-stop printer circuits entering a central office in concentrators. Short city lines are connected directly to a concentrator multiple. Longer circuits, particularly trunk circuits, in which it is not practicable to operate the printers directly in the line circuit, are terminated in single line or duplex-half repeaters. The local sides of these repeaters connect to the concentrator multiples.

Duplex circuits which permit simultaneous transmission in both directions on one line wire are terminated in terminal duplex sets. From the local sides of these sets, a sending and receiving leg are each connected to a concentrator multiple. These two legs appear on adjacent studs in the turrets, the sending leg terminating on the left hand stud. In Fig. 4 will be seen adjacent studs having identical designations, for example the first two studs are designated *A B* and *A B*, respectively. In this particular case, this represents a duplex circuit.

During that part of the day when the exchange of business is heavy enough to keep both channels of a duplex circuit busy, the concentrator supervisor temporarily assigns two adjacent operating positions to this circuit. When the traffic falls to the point where it can be handled with one channel, certain switches on the terminal duplex set are operated so as to change the set for half duplex working, that is, transmission can be in only one direction at a time. In this condition, a concentrator operator can both send and receive on the left hand stud (the sending leg) in the same manner as she does on any single-channel circuit. When a duplex circuit is arranged for half duplex working, it is allowed to "float" in the concentrator, that is, it is not definitely assigned to any particular operating position.

Sometimes an outer office has such a heavy message load as to require the equivalent of a duplexed circuit but the economics of the situation dictate that a second single-channel circuit be supplied this office rather than install duplexing equipment with its attendant maintenance charges. Therefore, two adjacent studs having identical designations may also represent two separate circuits to an outer office; in fact, more than two circuits are sometimes required to public branch offices having a very heavy message load. Generally, however, such cases are taken care of with pneumatic tubes if the distance is not too great.

At night, the traffic decreases to such an extent that it is not sufficient to keep one channel continuously busy on some of the trunk circuits which are multiplexed during the day. It is then advantageous to convert such circuits to start-stop printer operation during these dull periods and to terminate the circuits in concentrators, utilizing concentrator multiples which connect to offices that are closed at night. This permits centralizing the night staff which results in operating economies due to employing the operators to better advantage.

It is the general practise to segregate the various types of circuits terminating in a concentrator. Thus the upper two rows in Fig. 4 contain public circuits

and the lower eight rows, private tie lines; the circuits in each case being arranged in alphabetical order. Where more than one concentrator is installed, the private tie lines and public circuits are terminated in separate units. Segregating the private tie lines permits assigning operators for working these circuits who are best qualified and especially trained for dealing directly with the public.

Discussion

C. M. Brentlinger: The concentrator which Mr. Blanton has described was developed because the traffic department required a means of giving very fast service without incurring prohibitive operating expense. By fast service we mean the answering of incoming calls on private tie line or public branch office circuits within a very few seconds—as quick or quicker than a telephone is ordinarily answered. The unit as developed has enabled us to give this fast service at approximately the same operating expense as with the older type of equipment.

There are two general reasons why we do not put all circuits in a given office in one large unit, instead of splitting them into several 100- or 200-wire units, one reason having to do with equipment and the other with operation. The equipment on each operating table occupies so much space that it would be difficult to place a turret of the standard telephone exchange type before each operator; also the cabling and other expense involved in running several hundred wires in multiple to a large number of tables would be much higher than it is when the units are smaller and the lines limited to 100 or 200.

Another item where additional expense might be incurred in using one large instead of two smaller units, is in the provision of equipment to care for growth. Assume a 50-position 100-wire unit, which has become inadequate due to an increase in the number of circuits. We could add a few operating tables to this unit, put an additional turret on each of the existing operating tables, and thus accommodate the additional circuits by enlarging the unit; or, we could install a second unit of 50-wire capacity with say 10 tables. The latter would be the cheaper procedure on account of the excessive wiring expense involved in adding a turret to each of the 50 original operating tables, and extending the multipled circuits to them.

Aside from the equipment and installation expense, a large unit would be unwieldy to operate. For example, the handling of outgoing messages would be more difficult in a large unit. With our present layout, the circuit for which a given message is destined is either idle or is being worked in one of say 70 tables, and by means of the group-position finder is rather easily located. Multiply the number of tables and circuits by 3, 4, 5, or even 10, and the problem is not so easy, the group-position finding equipment would be very expensive, and the time required to carry the message from the message routing rack to the operating table would be increased. The use of one large unit in an office would require the distribution of all simplex traffic from one center, instead of the present convenient arrangement with a distributing point for each 100 or 200 circuits.

There is also an advantage in the smaller units with respect to incoming calls, for if the unit were very large, it would be impossible to establish the priority of calls and answer them in order. Obviously answering calls in sequence during peaks is absolutely necessary if the maximum answering time is to be kept down to a very few seconds. Of course equipment could be provided to establish the sequence of incoming calls, but this would be expensive.

There is no particular advantage to a large unit. A 70-position unit with 100 heavily loaded circuits requires a maximum of 4 supervisors and 70 operators. This is a sufficiently large force to permit efficient assignment, and no gain would result from having the whole simplex staff in one large group.

Another phase of the application of concentration units is the resulting saving of operating tables and floor space, as compared to separate tables or eight-wire telegraph type concentrators. In the new central office at 60 Hudson Street a striking comparison can be drawn between the old and new types of equipment. The new office requires only 608 operating tables for terminating 1,600 circuits. If old type equipment had been used nearly 1,000 operating tables would have been required for the same number of lines. These additional tables would have required 9,000 sq. ft. more floor space. The savings in operating tables and floor space can probably be better appreciated if we consider that they are the equivalent of the whole simplex departments in the Dallas, Jacksonville, Indianapolis, Milwaukee and Houston offices.

H. Mason: Apart from the mechanical and electrical details of this development, many interesting points in its application to telegraph service could be discussed. Fifty-nine installations have already been made in twenty-seven offices embracing perhaps 6,700 lines and 2,200 operating positions, some 25 per cent being allowed for growth. As yet the equipment is so young and the possible variations in practise are so numerous that we have not settled down to what we may safely consider the best operating routine for this equipment. As intimated in the paper, there may well be a difference in the proper technique for answering a customer's tie line and for answering a branch office wire where the distant operator is an employee of our own. It is not unusual to find operators who are highly successful on the branch wires but whose method with the customer leaves something to be desired. On the other hand, other operators get along excellently on the private tie lines but are less efficient when operating on the busier branch wires.

Evidently it is necessary for us to give an exceedingly prompt answer on customers' tie lines. On branch wires, when economy so dictates, we can impose a slower answer. In practise we average about a three-second answer on the former and a six-second answer on the latter. If, then, we establish two standards of speed of answer with lighter staffing on the slower group we must, on that account as well, segregate our tie lines and our branch wires into separate units.

The distribution of business outgoing from a concentrator presents a problem. This is a manual function with no automatic guide except an indication if the desired line is already being operated. Shall we endeavor to place outgoing business with the lowest numbered idle operator or shall we take it to the higher numbered positions where an idle operator may more likely be found? If but one operator is idle shall we give her an outgoing message, thus leaving no one available to quickly answer an incoming call? As yet our practises vary considerably. On a concentrator of tie lines it is sometimes required that two, or in extreme cases, three operators be left idle before outgoing business is distributed. On branch wires this is rarely done.

Another problem remaining to be solved is in connection with branch office operation where the branch is big enough to warrant two wires during the busy hours. When the branch opens up the second wire by calling on it the answering operator will probably be quite remote from the one working the first wire although the pins for the two wires, as stated in Mr. Blanton's paper, are adjacent to each other in the multiple. With this condition it might well be that on the second wire deferred business was being transmitted while on the first full rate business was awaiting its turn. At present nothing but close attention by an ambulatory supervisor can take care of this situation.

When twenty or thirty operators are working on one concentrator one or another operator is becoming free and idle with relative frequency. If, then, the "answer next" lamp appears before a

high-numbered operator and an incoming call is received and said operator stalls along for but a few seconds before answering it may well be that the answer next light will jump to a lower-numbered position. We are considering changing the wiring so that when an answer next lamp once lights it will stay lighted until that operator does something.

In the New York office an interesting comparison may be made. Besides a considerable number of these concentrators there is also installed the modern equipment for the recording of telegrams telephoned in by patrons. Adopted from telephone company developments an automatic position finder is used so that an incoming telephone call is placed upon an idle operator. In this equipment the search for an idle operator does not always begin at the bottom but, in a manner of speaking, begins where it left off seeking on the last call. For this reason all assigned operators are likely to have been equally busy by the end of the day. This differs from the simplex concentrator scheme where evidently the low numbered operators are much more frequently occupied than those of higher numbers. We had thought that this would help in operator assignment by finding, when over-assigned, that the last operator would sit continuously idle hence clearly indicating the desirability of releasing her. This does not seem to work out in practise, probably in part due to our holding of several operators idle for the readiness to serve feature and also due to our method of distributing outgoing business. We are at present unable to say which method of automatic operator selection is preferable. They are both working very well.

That feature of the sub-distribution center which indicates if a desired line is already being worked so that the outgoing message can be taken to the operator thereon is of considerable benefit in concentrators handling branch office lines. Depending, of course, upon the load density of the wires, perhaps 85 per cent of the business carried from the distributing racks to the positions may have been subject to this feature. In concentrators full of tie lines, however, wires are in use such a small fraction of the day that the probability of an incoming message finding the line already being worked is quite small and in such case the distribution feature, while of course not a detriment, presents little or no advantage.

In actual practise these equipments have resulted in some extremely desirable savings in floor space. They have postponed major moves that the growth of simplex operation otherwise would have necessitated. They have not as yet resulted in payroll economies, partly due to their newness and our unfamiliarity with just the best procedures; also because they were installed during a period of declining loads and it is concededly difficult to cut operating costs in linear ratio to reduced files; but chiefly to the fact that we have elected to materially improve the speed of answer coincident with their installation. I have in mind a certain city, more or less typical, where the cost per handling is approximately the same now as it was before but where, prior to the installation of the concentrator, the average speed of service from branch to main was 4.1 minutes and six months later, after the installation, it was 2.7 minutes.

Most of us believe that along with its other advantages, labor savings as well will result when loads increase and when operating practises through trial and experience have crystallized into the most desirable forms.

Modern Practises in Private Wire Telegraph Service

BY R. E. PIERCE*

Member, A. I. E. E.

Synopsis.—*Private wire telegraph service forms an important part of the communications service of the United States. This pape outlines briefly the practises which are followed by the Bell System in furnishing a million and a half miles of such service. Some of the practises discussed are: Central office arrangements, line transmission systems, and station apparatus. The paper closes with a description of a few typical applications for press associations, brokers, police and airways.*

TELEGRAPHY has the distinction of being the earliest application of electricity to the service of mankind and it still ranks as an important factor in our commercial, educational, and social activities. This paper is devoted to a statement of the present practises of the telephone companies comprising the Bell System in the furnishing of more than a million and a half miles of private wire telegraph service to press associations, brokers, financial houses, public service companies, and other organizations and individuals.

The term "private wire telegraph service" as used in this paper covers that type of service by which a complete telegraph communication system is furnished to the patron for his exclusive use. This type of service is sometimes also referred to as special contract telegraph service. The service is complete in that all the facilities including the station apparatus at the patron's premises are furnished and maintained. The circuit may be arranged for either the manual or machine method of operation to suit the requirements of the organization using the service. Throughout the paper the machine method will be referred to as the "printer method" since this is the term most universally used. Other terms which are used by various organizations to describe the same method are "typing telegraph," "telephone typewriter," "teletype," and "teleprinter."

It might be well to consider briefly some of the unique features which have influenced the telegraph engineering in connection with this private wire service and which have brought about the development of the particular methods and means as described in this paper. The commercial telegraph message business is essentially a point-to-point service and the telegraph facilities are designed and operated for the most part so as to furnish direct communication channels between two points. The private wire telegraph service, on the other hand, usually requires circuits with sending and receiving equipment at more than two points and often at many points and because of this fact the engineering problems involved are somewhat different than in the commercial telegraph field.

Another unique feature of the private wire telegraph service is that it is furnished very largely over plant which is used jointly for telephone and telegraph purposes. For many years the use of the telephone plant for telegraph was accomplished by the superposition of two direct current telegraph channels on a telephone pair below the voice range. There came a time, however, when this method did not provide a sufficient number of telegraph facilities to handle the rapidly growing private wire service over the major routes. It was at this time that the high-frequency carrier telegraph system[1] was developed to provide telegraph trunk channels above the voice range on open-wire lines. By this method it has been possible to obtain 10 two-way telegraph circuits on a pair of wires in the frequency range from 3,000 to 10,000 cycles.

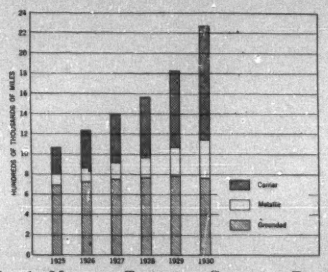

FIG. 1—MILES OF TELEGRAPH CIRCUIT IN PLANT

When it was decided to put the toll telephone plant in congested areas into cables, it was necessary to develop telegraph systems which would also be satisfactory in these long toll cables. To meet this need the metallic system and the voice-frequency system[2] were developed. The latter system provides for 12 two-way telegraph circuits on a standard four-wire telephone circuit but since it operates in the voice range it precludes the use of the circuit for telephone at the same time.

The chart shown in Fig. 1 illustrates mileage of the various types of telegraph facilities in the plant for several years past. The mileage given covers those

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

1. For references see Bibliography.

circuits in private wire service, the spare circuits maintained for protection of the plant in service and the circuits used by the telephone companies for their official business. The interesting point in connection with this chart is that a very large percentage of the growth has been accomplished with the carrier telegraph system. It will be noted that there are now more than a million miles of carrier telegraph trunk circuits in the plant. The development of the private wire telegraph service on the main routes is illustrated by the fact that on September 30, 1930 New York and Chicago were connected by more than 300 private wire telegraph circuits over the telephone wire plant. Of these 300 circuits, 120 were direct New York-Chicago voice-frequency carrier channels without intermediate drops and the growth of this direct business is such that another system of twelve channels is being added between these points about every six months. If these 300 telegraph circuits were each operated over a separate wire on a pole line it can be seen that more than seven complete 40-wire pole lines would be required to handle this service.

Central Office Telegraph Arrangements

In the private wire telegraph services where all of the stations are within a small area, wires are assigned to the

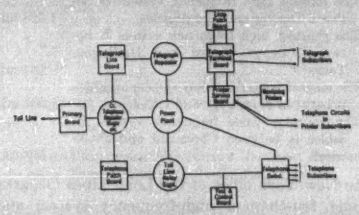

Fig. 2—Central Office Equipment for Telephone and Telegraph

telegraph service from the telephone plant without any superposition. Since these networks are relatively simple from an engineering standpoint the longer services are chiefly considered here.

The joint use of the outside plant for telephone and telegraph purposes requires that provision be made in the central offices, where the toll lines terminate, for testing and repeating equipment for the telegraph circuits as well as for the telephone circuits. The items of equipment which are generally involved in such a termination are shown in Fig. 2.

In addition to the items of equipment directly associated with the telegraph line circuit, there is, of course, a number of auxiliary facilities required in giving the private wire service. There is a complete network of test wires connecting all of the central offices where telegraph equipment is installed for use by the service maintenance people in routine tests and in clearing special trouble. There is also an intra-testroom communication system required in large offices so that men in different parts of the office may intercommunicate.

The printer service position, which is indicated in Fig. 2, is a comparatively recent development and provides a centralized location for terminating the printer subscribers' loops, the monitoring printers and the special telephone circuits for communication between the subscribers and the service people.

If the private wire subscriber having printer service wishes to communicate with the attendants who are in charge of the maintenance of his service, he has merely to call on the telephone provided for the purpose and usually mounted adjacent to the printer installation. Calling over this instrument connects him directly with the attendant at the printer service position in the nearest telegraph test room. If some difficulty on the circuit is reported, the telegraph attendant can immediately communicate with distant repeater attendants over test wires provided for the purpose, and can make measurements over the private wire network in question with the monitoring printers, transmission measuring apparatus and other testing means at his disposal. If the trouble cannot be located and remedied at once, spare facilities are patched into the network and service is resumed.

New Arrangements of Central Office Equipment. With the rapidly increasing number of telegraph circuits entering the repeater offices, it has been found desirable to completely redesign the central office equipment from the standpoint of mounting arrangements. At the present time, practically all of the central office telegraph apparatus being installed for use on the systems which have been mentioned above, is designed to mount on relay rack bays which are completely assembled and wired at the factory. A typical layout of such apparatus is indicated in Fig. 3 which shows a rack upon which are mounted four channel terminals of a voice frequency telegraph system together with a common monitoring set. This monitoring set can be plugged into any one of the four channel terminals or into the channel terminals on adjacent bays for supervision, testing or line-up purposes. This feature of a common monitoring set for several telegraph repeaters is a recent development which is now in quite general use on all of the telegraph systems being installed for giving the private wire telegraph service.

Methods and Means for Supervising, Testing, and Lining Up. In providing private wire telegraph service, it is necessary that the plant forces be equipped with efficient testing and measuring devices for determining the condition of the telegraph facilities at all times and for making the necessary adjustments and changes in cases of trouble. The extensive introduction of printer methods of operation has required the application of many new monitoring and testing facilities. Monitoring printers are provided in all of the principal telegraph offices with special switchboard arrangements for con-

necting these printers to the private wire telegraph facilities. In the Chicago office of the long lines department there are more than 60 of these monitoring printers which are used exclusively in the maintenance and supervision of the private wire telegraph service.

It will not be possible to go into detail concerning all of the interesting practises which are being followed in giving this private wire service, but a few of the more important items will be mentioned, first in connection with the line transmission systems and second in connection with the methods and means of sending and receiving.

Line Transmission Systems

The telegraph circuits required for a private wire service may be divided into three general classes, which

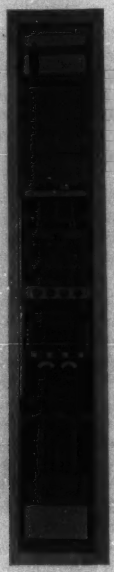

Fig. 3—Voice-Frequency Telegraph System

Four channel terminals with common monitoring set

are illustrated in Fig. 4. First, the trunks or backbone circuits between telegraph repeater offices; second, side "legs" which connect the main telegraph offices with outlying offices, and third, subscribers' loops which run from the telephone offices to the station equipment on the subscribers' premises. One requirement which must be kept in mind in considering the facilities used in private wire networks is that all of the different systems used must be so engineered as to allow interconnection between them. It may be necessary to build up any one network out of a number of different systems. The references mentioned previously give descriptions of carrier and metallic systems and the methods of arranging the circuits of the terminal apparatus to allow this universal interconnection. These arrangements are typical of those used in all of the telegraph terminal apparatus.

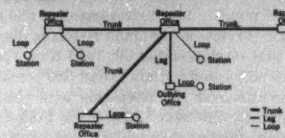

Fig. 4—Typical Layout of a Private Wire Telegraph Circuit

Carrier Telegraph Systems. Reference has been made to the rapid growth of carrier as shown in Fig. 1 and in this connection the following approximate data as of September 30, 1930, may be of interest.

Total mileage of carrier telegraph channels	962,000 mi.
Channel mileage of carrier on open wire lines	522,200 mi.
Channel mileage of voice frequency carrier in cables	439,800 mi.
The longest high frequency system is between Denver and Los Angeles	1,528 mi.
The shortest high frequency system is between Corpus Christi and Harlingen, Texas	134 mi.
The longest voice frequency system in cables is between New York and Chicago	1,001 mi.
The shortest voice frequency system in cables is between Cleveland and Pittsburgh	143 mi.

The New York office of the Long Lines Department has one ten-channel high-frequency system and 50 twelve-channel voice-frequency systems. These systems reach the following points.

One high-frequency system to Buffalo.

Voice-frequency systems as follows:

Albany	2	Cleveland	3	Rochester	1
Boston	6	Detroit	2	Syracuse	1
Buffalo	5	Indianapolis	1	Toledo	1
Charlotte	1	Pittsburgh	8	Toronto	1
Chicago	10	Portland, Me	1	Washington	5
Cincinnati	1	Richmond	1		

In addition to the carrier development mentioned above, there have been certain cases where it has been desirable to operate the voice frequency carrier telegraph systems on channels of carrier telephone systems.[3] The carrier telephone system is designed to operate on a pair of open wires and provides three two-way telephone circuits in the frequency range from 5,000 to 30,000 cycles. Fig. 5 illustrates the different items of equipment involved in the operation of the voice frequency telegraph systems on these channels. The hybrid coils which are usually used for terminating two one-

way carrier telephone channels in single two-way telephone circuits have been removed and standard twelve-channel voice-frequency telegraph terminals have been connected.

One feature which has led to the use of carrier telephone channels for telegraph purposes is the possibility of operating voice-frequency telegraph for part of the distance over cables and part of the distance over open wires without telegraph equipment of any kind, intermediate between the two-voice frequency telegraph terminals. A good example of such an application was recently made by the Pacific Telephone and Telegraph Company. A toll telephone cable which was under construction between San Francisco and Los Angeles was completed from San Francisco to San Jose. The ultimate plan called for the operation of a large number of telegraph circuits direct from San Francisco to Los Angeles through this cable by means of the voice-

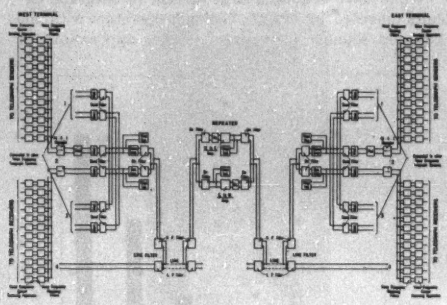

Fig. 5—Voice-Frequency Telegraph Systems Superposed on a Type C Carrier Telephone System

frequency telegraph system. It was very desirable, however, to obtain additional telegraph facilities over this route in advance of the completion of the cable. The plan which was adopted to obtain these telegraph facilities was to install three standard voice-frequency telegraph system terminals in San Francisco and Los Angeles. These three systems were then put in service on the three telephone circuits of a standard carrier telephone system on open wire between Los Angeles and San Jose. Thirty-six telegraph circuits were then obtained from a single pair of wires in addition to the 1½ telephone circuits of the ordinary type. At San Jose these three carrier telephone channels were connected to three standard four-wire telephone cable circuits which terminated in San Francisco in the three-voice frequency telegraph terminals. This method of operation was continued until the cable was completed between Los Angeles and San Francisco when the same voice-frequency systems were put into service entirely on cable conductors. It is expected that this general method of superposing telegraph in the frequency range above the voice will have wide application in the future.

Two-Path Direct Current Systems. It has been the common practise in providing telegraph operation in both directions over a direct current system to use the well-known balance method. In many cases, it is found that a satisfactory balance can be maintained so that the speed at which full duplex operation can be carried on is only slightly less than the of speed transmission in one direction at a time. When the telegraph circuit is operated over line facilities which are liable to change in their electrical characteristics, it is found that frequent adjustment of the balancing networks is required if the circuit is to be operated full duplex at anything near its maximum efficiency, and in some cases considerable attention is required to maintain satisfactory transmission in one direction at a time. This instability requires attention from skilled attendants and at times there are delays or interruptions on a circuit. To overcome these difficulties, certain use is now being made of what is termed the "two-path method of operation." With this system, a separate telegraph channel is used for operation in each direction, these two channels being connected together and to the subscribers' loops at the terminals by appropriate apparatus. This, of course, is the application of a principle which is employed in long telephone circuits and in all of the carrier telegraph systems. With this arrangement, there is no balancing required, the full duplex speed is the same as the one-way speed and no skilled attendants are required on the circuit. Fig. 6 illustrates a terminal set arranged to operate on this principle. This unit is designed with two complete terminal sets on a relay rack mounting with complete power plant and is intended for installation in small outlying telephone offices where no skilled telegraph attendant is available. The only maintenance or lining-up required with this apparatus is to regulate the line current and adjust the relays, which can be done by the regular telephone central office personnel.

Regenerative Repeaters. When a private wire telegraph service involves a very complicated network it sometimes becomes desirable to divide the circuit from a transmission standpoint so that the signal distortion occurring in one part of the circuit will not be cumulative throughout the entire circuit. This result is accomplished in the case of printer service by introducing regenerative repeaters at regular repeater points where the circuit is to be divided. These repeaters receive signals from either direction on a distributor mechanism and then transmit them from another distributor. In this way the repeater will transmit perfect signals into one part of the network provided the signals it receives from the other part of the network are not distorted beyond the point where they could be recorded perfectly on a receiving printer.

Start-Stop Printer System. The multiplex and the start-stop[4] printer systems in common use differ in one

important feature. In the multiplex system the signals are transmitted to the line and from the line to the receiving apparatus by distributing mechanisms which are operating continuously in approximate synchronism. In the start-stop system, the signals are transmitted to the line and from the line to the receiving apparatus by mechanisms which start from a given position, transmit one character and stop. In this latter arrangement, the maintenance of synchronism between the sending

Fig. 6—Two-Path, D-C. Telegraph System
Two terminal sets with power plants

and receiving devices is necessary during the transmission of only one letter or character. The multiplex system is essentially a point-to-point system whereas the start-stop system is adapted to circuits involving a large number of stations, any one of which may send or receive. The advantage of the continuously synchronized system of transmission is the possibility of using a shorter code. It is only necessary to transmit the intelligence-bearing signals while in the start-stop system a signal is sent to start and another to stop the distant distributor mechanisms.

The fundamental advantage of start-stop over continuously synchronous systems is the fact that in the start-stop system the lag in the transmission of a signal from one point to another point in a circuit is automatically taken into account. With the continuously synchronized system and only two stations on a given circuit, an initial adjustment can be made to properly orient the receiving device with respect to the sending device which will hold as long as the lag remains fixed. With more than two points on a circuit any one of which may send to all the others or with transmission systems which are apt to be changed in a way that affects the lag, the adjustment to compensate for lag becomes difficult if not impracticable. This feature would considerably restrict the field of use for continuous synchronism in private wire telegraph service where such a large portion of the circuits require service between more than two points and where it is essential to maintain a flexible plant in order to maintain continuous service. This explanation will indicate why the multiplex system which is designed as a continuously synchronized system is used in the commercial telegraph message service while the start-stop system is used in the private wire telegraph service.

Station Apparatus

While there is quite a large mileage of telegraph circuits operated by expert telegraph operators with the key and sounder method, the newer applications of

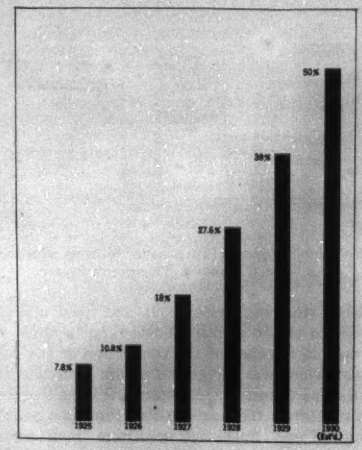

Fig. 7—Trend Toward Machine Operation in Private Wire Telegraph Service Bell System

private wire telegraph service are, in most cases, being made with machine or printing telegraph methods of operation. The data in Fig. 7 indicate the change in the percentage of the private wire service mileage of the Bell System which is operated by the manual and the printer methods.

In the private wire service which is furnished with printers, the machines for sending and receiving are set to operate at a speed which best suits the requirements of the subscriber. At the present time two standard speeds are provided. These are known as 40 speed and

60 speed which represent traffic handling capacities of 40 words per min. and 60 words per min., respectively. At the present time approximately 20 per cent of the printer private wire service is furnished at 40 speed and 80 per cent at 60 speed.

Recent Developments in Printers. In general two types of printers are used in telegraph service. One of these prints on a narrow tape while the other prints in page form. The commercial telegraph companies are almost universally using the tape printer. In private wire telegraph service, the brokers have adopted the tape printer, while the press and the commercial customers other than the brokers, in general, use the page printers. The comparative advantages of the tape and the page printers are quite generally understood, and need not be discussed in this paper.

The page printer which is shown in Fig. 8 has recently been developed to meet the exacting requirements of

Fig. 8—Page Printer

the press and commercial private wire subscribers. This machine is equipped with a stationary paper platen and a moving type basket. This feature allows the copy to be easily read while the machine is in operation and also reduces the possibility of trouble with the paper feeding mechanism.

Modern business methods call for the use of many printed forms in the routine correspondence of commercial concerns and to meet this requirement arrangements have been designed for handling organized printed forms in this new printing telegraph machine. Absolute registration is assured between different copies when carbons are used and between all the copies on different machines on the same circuit by feeding the paper with pins on the platen roller which engage perforated holes in the printed forms. Fig. 9 illustrates samples of forms which are used by different commercial houses in this type of machine.

This new page printer is also equipped with arrangements for starting and stopping the printer motors from a distance without any additional wire facilities. When this feature is used all printer motors on a circuit are normally idle. The motors on all machines start when a spacing signal is received over the line and stop when a particular combination of impulses is received from the sending keyboard.

Another practise which is expected to eliminate one of the most difficult problems in maintaining reliable printer service is the recent application of synchronous motors for driving the printers. It is of course understood that all machines on a circuit must operate at approximately the same speed so that the distributor

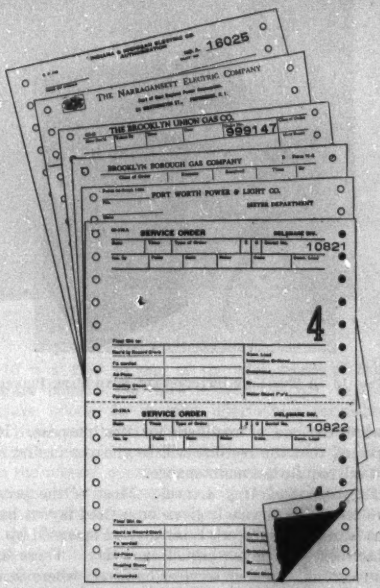

Fig. 9—Samples of Forms for Use with Printer Equipped with Positive Paper Feeding Attachment

mechanisms will remain in synchronism through one letter cycle. In the past where a-c. motors were used, they were usually of the ordinary series type with a contact governor. While these motors run at very constant speeds, the governor requires careful adjustment and occasional maintenance. Since the power systems of the country are becoming very closely regulated as to frequency, it has been found practicable to use synchronous motors of the self-starting type as the source of power in the printers. With this arrangement the speed regulation is in general better than with the carefully adjusted governor on a series motor and there is no adjusting or cleaning required to maintain accurate

speed. It might also be mentioned that, in certain cases where the printers are operated in close proximity to high-gain radio receivers, the prevention of interference in the radio set from the printer operation has given an added incentive to the application of the synchronous motor.

In a great many printer installations, it is necessary to provide a local source of direct current power. In the past, (where only alternating current power was available) this has usually been accomplished by the use of a motor-generator but the present tendency is

Fig. 10—A Ten-Line Switchboard with Tape Printer

toward the use of rectifiers for this purpose. It is expected that the rectifier will be cheaper in first cost and will require less maintenance.

Telegraph Switching Systems. Most of the private wire telegraph service is given on a fixed layout basis, that is, a given network is set up and operated by the subscriber for a given period each day. There have been certain types of service, however, where it was desirable to be able to interconnect stations on a network at the will of the subscriber. A typical application of this kind is found in the service for the Pennsylvania State Police. In this case, switchboards are provided which allow certain stations to be interconnected at the will of the operators at the switchboards and when a state broadcast is desired the operator at general headquarters may "seize" control of all the stations in the state so that more than one hundred police stations are connected to one sending machine and can be notified simultaneously.

Fig. 10 shows a typical private wire switchboard. This switchboard provides for ten lines which are connected to the operator's printer or together through a repeater by the operation of keys. Lights appear on the board to indicate a line is calling and the operator may monitor on a connection or disconnect the monitoring printer as desired.

For services where it is desirable to select particular stations and it is impracticable to provide a switchboard with individual circuits to each station, a selective calling arrangement has been developed. With this plan all stations are connected to the same circuit and selecting signals are transmitted from one station to another over the circuit used for passing the telegraph signals. The apparatus used to accomplish this consists of telephone dials, step-by-step switches and relays. Any point on the circuit may be called by dialing any other point on the circuit. A station may also dial by a single three-digit code all of the other stations or a group of stations. Busy signals appear at all stations when the circuit is in use. An audible or visible alarm is provided which will call the attention of the attendant to an incoming message. The motors of all printers are stopped when the station is idle but are automatically started when the station is called and automatically stopped when the calling station releases the circuit.

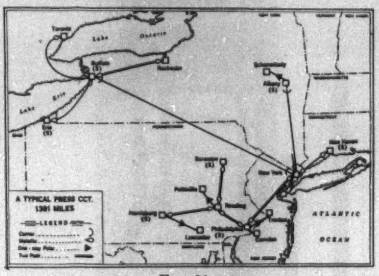

Fig. 11

Secrecy is obtained in this system because it is not possible for a station which has not been called to "listen in" when the circuit is in use by other stations.

Typical Application of Private Wire Telegraph Service

Press. The press associations are engaged in the collection and the distribution of news. This service must be accomplished quickly, accurately and without interruption, and the private wire telegraph service is the means used for accomplishing this result. A typical telegraph network—one of the many circuits of one of the leading press associations in the United States—is illustrated in Fig. 11. It will be noted that this network contains approximately 1,400 miles of telegraph circuits and connects sixteen telegraph stations, five of which are arranged for both sending and receiving while eleven are for receiving only. The various types of telegraph systems involved in this particular service are indicated by the designations on the chart.

In a service of this kind most of the sending would be

done from New York; it will be noted, however, that there are other points indicated on the network as sending stations. The circuit must be arranged with a "break" feature so that when desired any one of these other sending stations may take control of the entire network and transmit to all stations. To accomplish this the operator at the station transmitting the message operates the break key on his machine which transmits a break signal over the circuit locking up the machine which has been sending thereby indicating to the operator at that point that another station wishes to send.

The printers involved in the press work are, in most cases, of the page type using a continuous roll of paper. The sending machines are usually equipped for what is known as automatic transmission. The sending operator manipulates a keyboard which controls a perforator. This perforator prepares the message to be transmitted in the form of a perforated tape which is fed into a transmitter connected to the line. This transmitter is operated at a speed for which all the printers on the line are set. The perforator may be operated at any speed desired, and if the speed of the line is exceeded, as is usually the case, the message is stored up in the perforated tape and is transmitted to the line at a regular rate. With this method of transmission there is a continual flow of traffic over the line and any interruption in the operation of the keyboard does not interrupt the signals going to the line. This arrangement obtains the maximum efficiency from the line.

In certain cases a "bulletin" or "flash" of special

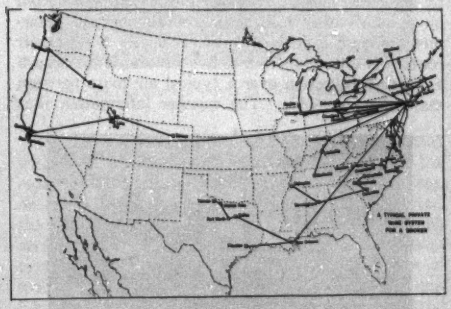

Fig. 12

importance must be transmitted. Arrangements are provided so that by throwing a key the transmission of signals from the perforated tape is stopped and the keyboard is connected directly to the line so that as each key is depressed the proper signals are transmitted. In this way the bulletin or flash may be sent directly to the line from the keyboard without any delay, and when it is complete the regular flow of traffic may be resumed by restoring the key previously mentioned. In order to attract the attention of the operators at the newspaper offices to such a flash or bulletin a bell is provided on the printers, which is caused to ring by the transmission of a particular combination of impulses from the sending keyboard.

Brokers' Service. A large percentage of the business involved in the trading on the stock and commodity exchanges is handled over the private wire systems of the various brokerage concerns. Fig. 12 shows a typical broker's service which comprises several separate circuits. This service is operated by the printer method.

Fig. 13—Wire Room of a Broker

Fig. 13 is a view in one of the wire rooms of this brokerage house. The wire network is used in the brokerage houses for passing information regarding orders which are placed before the market opens and for distributing certain market letters or market gossip. When the market opens the wires are kept busy with the passing of orders, confirmations and questions and answers concerning specific securities. It will be noted from the illustration that the tape printer is used and the cover of the machine is cut away in such a fashion as to allow the tape to be removed from the machine immediately after the last character of an order is printed. The orders are gummed to "buy" or "sell" order blanks and are then handled in the usual way. After the market closes in the afternoon the wires, in cleaning up the day's business, continue to handle a large volume of traffic. The records are checked and verified and all matters cleared before the opening of business on the following day. It is reported that during four days of the record month of October, 1929, one of the brokers using this service handled the remarkable total of 49,860 orders, confirmations and general messages over the firm's six private wire telegraph circuits.

Police Systems. The various police organizations throughout the country are finding the private wire telegraph service a reliable means of quickly disseminating police information. Fig. 14 illustrates the extent to

which this service is being used in the cities and states along the Atlantic Seaboard. These police systems are essentially broadcasting networks for transmitting police information simultaneously to a large number of receiving stations from one or more central points. The information usually originates in a telephone call or a personal visit to a police station from which the information can be quickly transmitted to the central

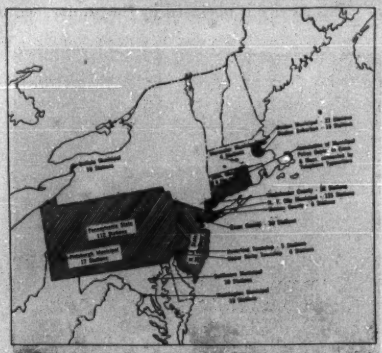

Fig. 14—Police Telephone Typewriter Systems Eastern States

headquarters and from there sent out on the broadcasting network. The receiving machines make a record of the message regardless of whether or not the station is attended. In certain cases the receiving machines are equipped with an audible alarm so that a police officer at a distance may be advised that a message is being received. In certain cases it is desirable to pro-

Fig. 15—A Police Switchboard and Sending Printer

vide arrangements whereby the headquarters from which the broadcast is transmitted will know whether the message is received at each point to which it was transmitted. In these cases an acknowledgment signal is arranged so that after each message is transmitted the ringing of a bell on the receiving printer indicates that the transmitting station requests an acknowledgment which can be sent back by the pushing of a button on the receiving machine. This acknowledgment lights a lamp on the switchboard at headquarters indicating that the message has been received.

Other police networks are set up with all stations

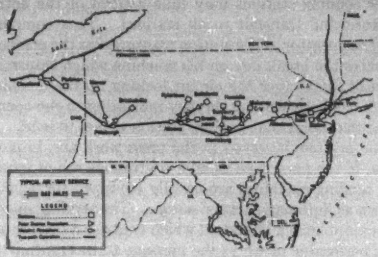

Fig. 16

connected to the same circuit so that any one may send and all of the others receive. Such a circuit connects seventeen of the principal municipalities in Connecticut. In this way police information originating in any of these towns can be broadcast simultaneously to all of the other towns on the network.

Fig. 15 shows one of the switchboards and sending machine in a state police system.

Weather Reporting Systems for Airways. The Department of Commerce has contracted for quite a large network of private wire telegraph circuits for use in collecting and distributing weather information for use by aeroplane pilots. Weather information is collected on an hourly basis in sequence from the principal intermediate landing fields and critical points along an air route. It takes from four to five minutes to make such a weather collection over a typical air route between major landing fields. The weather information thus

Fig. 17—An Air-Way Communication Office

obtained is furnished to the air pilots before they leave the field or to pilots in the air by means of the radio telephone equipment of the Department of Commerce. Along the air route the printers are usually installed

adjacent to the field in what is known as a watch-house. In addition to the service mentioned in certain cases local private wire circuits are provided for the purpose of distributing weather information between the Weather Bureau, radio stations, airports, and air transport companies' offices.

In addition to the collection of weather information the private wire circuit is used in some cases for the reporting of the departure, arrival, and passing of planes. Fig. 16 shows a typical airway network and Fig. 17 gives a view in one of the Department of Commerce airway communication offices. It will be noted that the printers are of the tape type. The stations along the air route are usually connected in series so that each station receives the information sent from all other stations. In certain cases the bell signal on the printer is used to call the attention of a particular station with which it is desired to communicate.

Fig. 18—An Industrial Concern Communications Office

Industrial Service. Factories are generally located with regard to the availability of raw materials and the ease of transportation. Executive and sales offices, on the other hand, are usually near the distributor, the exporter or the consumer. Private wire telegraph service is used extensively to provide direct communication channels between the widely separated units of modern industrial organizations. Fig. 18 shows a number of printers in the Chicago offices of an industrial company which are used for communication with several plants near Chicago and to units as far away as Indianapolis and Cincinnati. With the aid of this service the far-flung units of this organization are kept in constant touch with each other, thereby speeding up production and improving the service rendered to their customers.

In looking forward into the future, there seems to be every indication that private wire telegraph service will continue to grow and expand, and with the requirements continually changing it is to be expected that many new and interesting developments will be applied to this service.

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Discussion

J. O. Carr: It is apparent from the paper that the goal in this development is flexibility of the system and simplicity of the station apparatus.

One realizes the accomplishments better by comparing them with the earlier Morse system which was notably simple. A wire, a few batteries, a key, and relay and sounder at each station constituted the entire plant.

Mr. Pierce shows quite clearly the progress that has been made. In the Bell system various kinds of transmission channels have been developed which have far greater operating margins than the older types and suitable repeater apparatus has been devised so that all types of transmission channels may be interconnected quickly and interchangeably to produce any sort of a private wire system, completely flexible either with Morse or printer operation.

As Mr. Pierce has pointed out, when the use of printing telegraph passed beyond the stage of communication between two stations new problems arose. He has spoken of the matter of the method of synchronism. There were other problems which required deep study to solve. In a private wire system it may be necessary to permit transmission from a dozen stations and it is necessary that when the transmission shifts from one station to another that the signals be received correctly by all stations without the need for any change of adjustments. To meet this requirement it was necessary to greatly refine the transmitting apparatus to secure a high degree of uniformity in the character of the signals from each transmitter. The receiving mechanism also had to be refined in order that it would continue to print correctly through as great a range of variation in the received signals as possible.

That considerable progress has been made in improving the operating margins of both the transmission channels and apparatus is indicated by the statement that 80 per cent of the private wire service is at the 60-word per minute speed.

It is interesting to note from this paper, the use of the two-path d-c. method of operation. It was formerly considered necessary to utilize the telegraph channel to the fullest possible extent. Apparently it is proving economical to be somewhat wasteful of transmission channels if by so doing the continuity of the service can be increased and the need for expert attendance reduced.

After all the growth of the service is dependent upon the degree of satisfaction the subscriber secures from its use. The data on the growth of the service given in the paper indicate that it is giving satisfaction to the subscriber.

H. H. Nance: The discussion in Mr. Pierce's paper of the operation of voice-frequency carrier telegraph systems on channels of carrier telephone systems is of special interest. The Long Lines Department of the American Telephone and Telegraph Company has been operating a system between Chicago and San Francisco in this manner since September, 1929, and

since that time other similar systems to the Pacific Coast have been established between Chicago and Seattle and between Denver and San Francisco. The Chicago-Seattle and Chicago-San Francisco systems are each over 2,300 miles in length and are the longest carrier telegraph systems we have in operation.

This method of operation is particularly valuable in times of line trouble because of the flexibility in rerouting the telegraph systems that is possible as a result of the extensive application of carrier telephone systems. In a recent storm in the middle west two of the Long Lines main transcontinental routes were severely affected. In order to maintain complete telegraph service to the Pacific Coast two Chicago-Los Angeles and two Chicago-San Francisco voice-frequency telegraph systems were temporarily set up using cable circuits between Chicago and St. Louis, carrier telephone systems St. Louis to San Bernardino, Cal., which systems are routed over the Long Lines southern transcontinental line through Texas, New Mexico, and Arizona and was not affected by the storm, and cable circuits from San Bernardino to Los Angeles and San Francisco. By this procedure complete service was maintained and in addition telegraph circuits were furnished to other wire using companies also affected by the storm.

Symposium on Coordination of Power and Telephone Plant*

Part I—Trends in Telephone and Power Practise as Affecting Coordination

BY W. H. HARRISON[1] and A. E. SILVER[2]

Member, A. I. E. E. Fellow, A. I. E. E.

Synopsis.—*The general trends in telephone and electric power systems are outlined and the reactions of certain of these trends on coordination are described.*

In the telephone system, brief mention is made of the rapid growth of the dial system of operation, improvements in subscriber-station apparatus, rapid extension of new types of facilities for toll circuits and the growth of connections to foreign countries. Improvements in telephone service increase the importance of securing adequate coordination. The advantages of the use of cable facilities for toll circuits, of repeaters, and of carrier current systems as regards coordination of long distance and interurban telephone circuits are discussed. The benefits accruing from improved subscriber-station apparatus, central office equipment, abandonment of iron wire for the short tributary toll circuits and new methods of making sleeves at joints in open wire lines are outlined.

In the power system, brief mention is made of increasing use of larger generating units, and growing use of automatic devices to replace manual operation. Improvements in power service generally react favorably on coordination. The general trends toward higher voltages for transmission and distribution and the improved standards of construction accompanying these trends are described. The important matter of system stability and the practises as regards grounding of transmission circuit neutrals, lightning control and current limiting devices, and the reactions of these matters on coordination are outlined. Reference is also made to grounding of distribution system neutrals, service taps on transmission lines, general practises as regards transformer connections and improvements in wave shape in so far as these matters react on coordination.

In conclusion, it is pointed out that, while there have been influences working both favorably and unfavorably toward coordination, the preponderant trend is definitely toward an improvement. The benefits which have accrued from the activities of the Joint General Committee and the important function of the Joint Subcommittee on Development and Research are also mentioned.

General Trends

THE important benefits resulting from the cooperative handling of questions arising from the proximity of the physical plants of the telephone system and the electric power systems of the United States are emphasized when consideration is given to the extent and the rapid growth of these two industries. This growth is illustrated by Fig. 1 which shows that during the past decade, while the population of the country has increased 16 per cent annual telephone messages have increased 96 per cent and annual kilowatt hour usage of power 107 per cent. Another indication of the growth of these utilities is given by Fig. 2 which shows that during the past decade customers telephone stations have increased 88 per cent and customers of central stations 127 per cent. The leaders of both utilities confidently expect that, apart from temporary setbacks associated with recessions in general business, the recent rapid growth of these utilities will continue throughout the next decade.

Such a rapid growth of the two utilities both of which must supply the same customers with services essential to their comfort and prosperity, necessarily brings with it a large number of cases of physical proximity between the plants of the two utilities where, due to the widely different characteristics of the circuits involved, difficulties may arise. The necessity for active study of the coordination of the different systems and for the current handling of large numbers of individual situations will continue for a long time to come.

Associated with this rapid growth there has been another trend in these two utilities which has an important effect on coordination work. This trend is the steady improvement in the quality of service afforded to their customers.

In the telephone system the improvement in the standards of service, if considered by itself, tends to increase the noticeability and the reaction on service of inductive effects from outside sources. Such changes as the improvement in the characteristics of transmitted speech, including the extension of the band of frequencies efficiently transmitted, and the avoidance of cases in which interfering noises are produced from sources within the telephone plant, tend to increase the effect of moderate amounts of noise current induced in the telephone circuits from outside sources. Similarly increases in the extent of the service and in the speed of completing calls have led to increased reliance on prompt telephone communication which tends to increase the importance of avoiding interruptions. Five years ago the average interval of time between the placing of a long-distance toll call by a subscriber and the commencing of the conversation was 7½ minutes. At the present time it is a little less than 2½ minutes. Telephone users have now come to rely on the almost immediate establishment of telephone connections and are correspondingly more critical of interruptions or delays.

*Joint work of the National Electric Light Association and Bell Telephone System.

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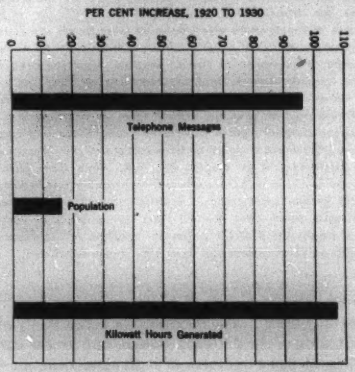
The improvement of service has been associated with a particularly rapid growth of very long haul telephone business and a consequent increase in the average length of telephone circuits used for interurban and long distance work. This is illustrated by Fig. 3 which shows the growth in the last few years and the expected growth for the next few years of typical circuit groups of different lengths. In the period 1925 to 1929 while telephone toll business as a whole increased 59 per cent New York-Chicago business increased 170 per cent and the



1930 Population..... 123,900,000
Telephone messages.....21,600,000,000
Kw. hours generated.....90 000,000,000

FIG. 1—PER CENT INCREASE, 1920 TO 1930, IN POPULATION AND IN TELEPHONE AND POWER USAGE

Note: Values for 1930 are estimates based on best available data. Telephone data refer to Bell System

combined Chicago and New York business to Los Angeles and San Francisco 380 per cent. From the standpoint of coordination with other electric circuits the very long telephone circuit offers a more difficult problem than the circuit of moderate length because of the cumulative effect of exposures in different sections.

In the power industry one of the most important items in the improvement of service has been the steady decrease in the number of service interruptions. This has been brought about mainly by better standards of construction, including more systematic mechanical and electrical arrangement of circuits and apparatus, and increased numbers of circuits and sources of supply. The interconnection of power systems has figured largely in the last mentioned factor contributing to service reliability, by making available greater numbers of sources and by multiplying the routes over which power can be received at specific locations. While the increasing numbers of interconnecting and other types of lines bring new conditions for the coordination of power and telephone plants, improved construction and increased security of circuits and apparatus have a definitely beneficial effect upon matters of coordination by reducing the number of abnormal conditions of operation.

Other items in the improvement of the service given by the power industry are better voltage regulation and a great increase in the number of types of power consuming appliances and apparatus made available for the customer. Accompanying better voltage regulation are certain factors which definitely aid coordination, among these being better balance of currents in the separate phases of the circuits and more effective arrangements minimizing the tendency for currents to flow in the earth. The effect of increased numbers of types of utilization apparatus on coordination is problematical, though probably not of sufficient magnitude to be of practical importance.

Other trends which have a bearing on the improvement of power service are discussed in the section of this paper devoted to the power system.

While in some respects the general trends indicated above, namely, the extent and rapid growth of the two utilities, and the improvement of service standards, have by themselves tended to increase the importance and the difficulties of coordination work, these adverse tendencies have been offset by beneficial effects of improvements in plant design and construction and by the cooperative endeavor which has been carried on by the two utilities during recent years. It is a tribute to the effectiveness of this cooperative work that the degree of satisfactory coordination between the two systems is

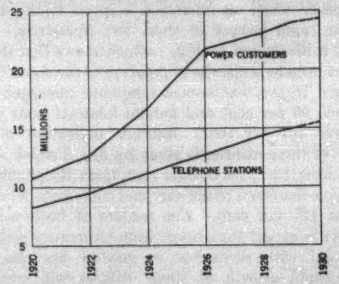


FIG. 2—TELEPHONE STATION AND POWER CUSTOMER GROWTH

Note: Values for 1930 are estimates based on best available data. Telephone data refer to Bell System

steadily improving. Fig. 4 shows that during the past 10 years the mileage of telephone toll circuits has increased 250 per cent and the mileage of power transmission lines over 100 per cent. The effect of such growth on the number of situations of proximity is illustrated by the fact that during the past three years the exposures of interest from a noise standpoint have increased from the equivalent of about 10 miles to about 14½ miles per 100 miles of open-wire telephone toll lead; while on the other hand the exposures not as yet adequately coordinated have in the same period de-

creased from the equivalent of 2.6 miles to 1.5 miles per 100.

While the trends of practise in the design, construction and maintenance of the plants have necessarily been largely controlled by the fundamental requirements of service and economy in developing the two systems, and while the trends naturally have not all been in the same direction as regards their effect on the coordination problem, still the general trend of plant practise at the present time is in the direction to facili-

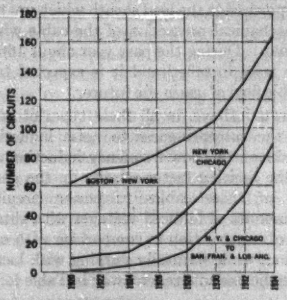


Fig. 3—Long Haul Telephone Circuit Growth Typical Circuit Groups

tate the coordination of the plants of the two utilities. In the following pages brief statements are made descriptive of the more important of these trends in the respective systems.

Trends in Telephone System

The telephone plant is at the present time rapidly changing in its physical character through the application of important developments and changes in engineering and construction practise.

Probably the most fundamental and far reaching of these changes is the progress of conversion from manual to dial system operation. When present plans are completed this will result in the operation of approximately 80 per cent of the telephones of the Bell System on a dial basis, and a large part of the existing manual central office equipment will have been removed from service. With the application of the dial system there is a trend toward a greater concentration of central office equipment in one building, so that in the future as many as 100,000 telephones may be switched by the various central office units in a single building. While these trends are of the greatest and most fundamental importance from the standpoint of the development of the telephone business they do not affect the coordination problem in any material way and therefore need not be further discussed here.

An important trend in telephone practise has been the provision of apparatus designed for higher standards of service and greater convenience for use at the customers' station. This includes the hand set, new types of private branch exchanges and of auxiliary telephone station apparatus, and improvements of transmission characteristics. These changes in some respects affect the coordination problem and these effects are indicated below.

Another important fundamental change in the telephone plant and one of great importance from the coordination standpoint is the rapid extension of new types of facilities for toll circuits, that is, long distance and interurban circuits whose use involves what is called a toll charge. These changes and their effects on the coordination problem are discussed in this paper.

One of the most spectacular trends of development of the Bell System at the present time is the increase in the number of connections to foreign countries. Earlier connections to Canada and Cuba were supplemented in 1927 by service to Mexico and by transoceanic radio links providing service from New York to London through which connection is made to the principal European countries and in 1930 a similar radio link from New York to Buenos Aires through which connection is made to Montevideo, Uruguay, and Santiago, Chile. During the next few years it is expected that

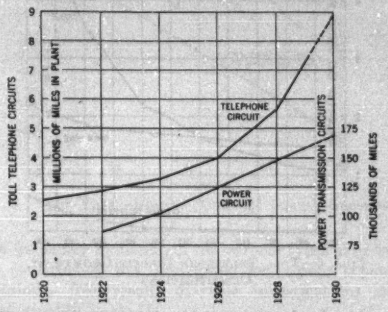


Fig. 4—Toll Telephone and Power Transmission Circuit Growth

Note: Values for 1930 are estimates based on best available data. Telephone data refer to Bell System

these foreign connections will increase to include generally all important points in South America, Australia, Japan, Honolulu and all other points which may offer an appreciable demand for service.

These intercontinental circuits are not of such character and location as to be directly affected by the physical proximity of power circuits, but their efficiency is affected by the noise currents on connected circuits in the same way as other very long circuits are affected and this is discussed briefly below.

Toll Cable. The change in methods of designing and constructing toll circuits which is of greatest importance from the standpoint of general development of telephone

plant is the great increase in use of cables for those circuits, including both the very long distance circuits and the shorter interurban circuits. This increase is shown by Fig. 5. A single cable may provide for from 250 to 500 telephone circuits and several hundred telegraph circuits, that is, as many circuits as would be provided by five to ten heavily loaded pole lines of aerial wire construction. This concentration of circuits in a single cable, a number of which can be placed on a single route, is in itself of great assistance in coordination problems by greatly reducing the number of routes for which coordination arrangement must be made. Furthermore the presence of the lead sheath together with the twisting of the cable conductors, the high degree of balance with respect to ground, and the mutual shielding effect of the many circuits in one cable practically prevent noise currents from being induced directly into

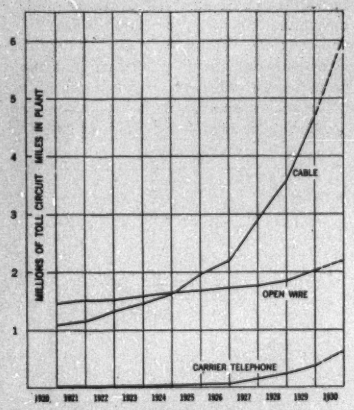


FIG. 5—TOLL TELEPHONE CIRCUIT GROWTH BY CLASSIFICATIONS

the cable circuits from outside electrical sources. The shielding effect of the lead sheath when suitably grounded also provides substantial reductions in the voltages of fundamental frequency which may be induced along the cable conductors at times of trouble on neighboring power systems.

A telephone toll cable with its associated equipment costs about the same per mile as a twin circuit power transmission line of the 110-kv. class. This high cost has led to a large use of private right-of-way for new extensions of these cables, particularly for aerial cable construction. This, of course, has an added advantage from the coordination standpoint in tending to keep these important telephone routes off the highway, which are so much used for the distribution systems of both utilities. In the more rapidly growing cable routes underground conduit construction is employed and these in most cases are located along the highways. In these cases, however, the close proximity of several cables in the same conduit run offers a considerable amount of mutual shielding effect which reduces the susceptiveness of circuits in these cables to values approaching that obtainable by a single tape armored cable.

This tape armored cable, which recently has been placed in use in this country, is designed for burying directly in the ground, and has an increased degree of magnetic shielding. This is provided by two wrappings of steel tape outside the lead sheath which are necessary for the mechanical protection of the cable when ducts are not used. During the past year about 160 miles of this cable were installed and it is expected to have a considerable field of use in the future.

As indicated above, in all these types of cable construction the susceptiveness to noise induction is so greatly reduced that low frequency induction generally becomes the limiting factor relative to the permissible proximity of these cables to power circuits. The relative amounts of induced voltages with these different types of construction in comparison with open wire construction, while naturally varying with local conditions, are indicated in a general way in Table I.

TABLE I

Type of construction	Approximate relative volts on telephone circuits per ampere of inducing current at 60 cycles
Open wire	1.0
Single cable, aerial or underground—sheath well grounded	0.5
Buried tape armored cable—well grounded	0.2

Note: All values for cables assumed full size, *i. e.*, 2¼-in. diameter.

The above figures are based on favorable conditions for obtaining low resistance ground connections on the cable sheaths. Such ground connections are necessary to provide the full shielding benefits, since the shielding is brought about by induced currents on the cable sheath flowing along the sheath and through ground. These sheath currents, because of the close coupling between the sheath and pairs, induce voltages into the pairs tending to neutralize the voltages induced into the pairs directly from the power system. The use of the tape armor, which is a magnetic material, increases the coupling between the sheath and pairs. The grounding conditions necessary for satisfactory shielding effects can usually be obtained, but situations sometimes arise in the case of aerial construction where it is difficult or impossible to obtain them.

While as noted above, the cable circuits are effectively protected from noise induction, the efficiency obtainable over the long circuits is limited in part by the noise currents occurring in the open-wire lines which may be switched to the long cable circuits. This is because the efficiency of the long cable circuits depends upon voice-

operated switching devices which must not be operated by the noise currents. This is also true of the intercontinental circuits mentioned above. The extension of the circuits controlled by voice-operated devices tends therefore to increase the importance of good coordination of the entire plant.

Telephone Repeaters. Another important trend of practise is the extended use of telephone repeaters. The purpose of these devices is to amplify the voice currents and thus make possible higher efficiency and greater extension of long distance telephone circuits. Their use is essential to the great development of toll cable. Moreover, they are used widely on open-wire circuits. Without repeaters it was necessary on the long open-wire circuits to permit the power level of voice currents to sink to relatively low values. An extreme example of this is given by the New York-Denver circuit which, before repeaters were available for use on this circuit, had an over-all equivalent, using the highest grade of telephone construction which had been developed up to that time, of about 31 *db.*[3] With the application of repeaters to this circuit the level of voice currents could be kept relatively high throughout the circuit. This is illustrated in Fig. 6 giving level diagrams for the circuit as originally set up and later when provided with repeaters.

The use of repeaters contributes to reducing the susceptiveness of the telephone plant and thus aids coordination. On such a circuit as the original New York-Denver circuit just mentioned, a relatively small amount of noise current greatly impaired transmission because of the weak incoming voice currents. Although the repeaters naturally amplify the noise currents as well as the voice currents, the fact that the voice level is kept high throughout, results in great benefit which in this case, assuming similar exposure conditions in the various repeater sections, gives an improvement in the ratio of voice currents to noise currents of slightly over five.

Repeaters probably also have some effect in reducing certain of the effects of low frequency induction by the fact that they sectionalize cable lines at about 50-mile intervals and open-wire lines at intervals of 200 miles or less, and limit the power which can be transmitted from section to section. There is some evidence that this tends to limit acoustic shocks.

Carrier Telephone Systems. A third important trend in telephone practise is the extension in the use of carrier telephone systems for long circuits and the associated changes in aerial wire construction practises. The growth in use of this type of circuit is indicated in Fig. 5. The carrier systems are much less influenced by noise induction from power circuits because they occupy a range of frequencies (5,000 to 30,000 cycles) in which the harmonic power voltages or currents ordinarily are extremely small. Furthermore, in order to obtain economies inherent in the use of large numbers of carrier systems on the same telephone pole line it has been necessary to design systems of transpositions of much increased effectiveness and even to change the configuration of the wires in order to greatly reduce the inductive effects between the telephone circuits. These changes also result in reduced susceptiveness to outside inductive influences. The type of construction now recommended for new aerial wire lines in cases where the extensive use of carrier is anticipated is shown in Fig. 7. The two wires of each pair, except pole pairs, are spaced 8 in. apart compared with the previous standard of 12 in. Often transpositions are made as frequently as every second pole and are of an improved type giving better balance between circuits; also on the circuits on which carrier telephone is used the phantoms are abandoned. The relative susceptiveness to

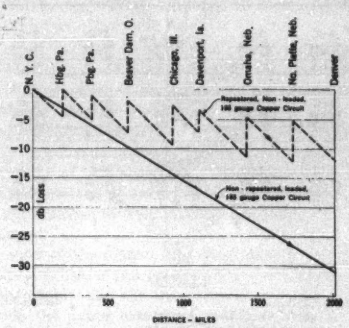


FIG. 6—NEW YORK—DENVER CIRCUIT LEVEL DIAGRAMS

noise frequency induction of the various types of aerial wire construction has been tested for various typical conditions. The results of these investigations are summarized in Table II.

TABLE II

Facility	Type of transposition*	Approximate relative susceptiveness†
12 in. phantom	Voice (brackets)	1.00
12 in. side	Voice (brackets)	0.50
8 in. pair	Carrier (break irons)	0.25 or less

*Voice circuits are not so frequently transposed as carrier circuits. Bracket type transpositions require two spans to complete the transposing whereas the break iron type completes the transposing on a single crossarm.

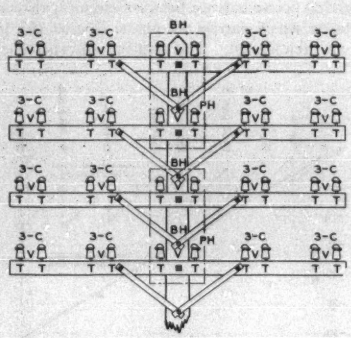
†Susceptiveness is used in the sense defined by the Joint General Committee, namely, "Those characteristics of a signal circuit with its associated apparatus which determine, so far as such characteristics can determine, the extent to which it is capable of being adversely affected in giving service, by a given inductive field."

Subscribers' Station Apparatus. To a large extent the trend of development in subscribers' station apparatus is toward new arrangements which provide greater convenience and more closely meet the needs

3. This means that the ratio of output power to input power of this circuit is 0.0008.

of the users and which have no material effect upon the coordination problem. An important group of developments, however, centers about the improvement of the electrical performance of the station apparatus by removing impairments caused by the earlier types of apparatus. These changes, by improving the quality of speech as reproduced by the telephone system, tend to make more noticeable the impairments caused by the effects of currents induced from external sources.

The tendency toward an increase in the range of voice frequencies efficiently reproduced by the telephone system tends to increase the range of frequencies of induced currents which may cause noise interference as discussed in the introductory section. An extreme



LEGEND

SYMBOL	FACILITY	TOTAL CIRCUITS	
V	VOICE FREQUENCY-PHYSICAL	20	
PH	VOICE FREQUENCY-PHANTOM	2	
3-C	CARRIER TELEPHONE	48	
T	D-C TELEGRAPH	40	
BH	CARRIER TELEGRAPH (10 CHANNELS)	40	
	TOTAL TELEPHONE		70
	TOTAL TELEGRAPH		80

FIG. 7—POLE LINE CONFIGURATION

Non-phantomed construction—8-inch spacing between wires of non-pole pairs

illustration of this is the circuits designed to transmit programs for radio broadcasting stations. The transmission characteristics of these circuits have been improved by including both higher and lower frequencies and in their most modern form these circuits efficiently transmit currents of frequencies in the range between 35 cycles and 8,000 cycles and are therefore capable of being affected by inductive noises over this wide range.

The room noise conditions at the subscribers' premises have an effect on telephone transmission. This noise besides acting directly on the ears of the telephone user is converted by the transmitter into electrical currents, a part of which actuates the receiver, thus producing noise. The present trend in telephone practise is very strongly toward a reduction of these effects. This will tend to bring into increasing prominence noise caused by induction in the telephone circuits which now in many cases is partially overshadowed by the reproduction of the noises in the room.

As partly offsetting this tendency steps have been taken to improve the degree of balance to ground of new station apparatus, particularly in the case of party lines. The new station apparatus with the improved transmission characteristics discussed above will be designed for reduced effect of noise currents entering from the line. Also, in extending the selective signaling features to rural areas, higher impedance ringers and a newly developed high impedance relay are being used in order to limit susceptiveness to noise from exposures between the rural open-wire extensions and rural power distribution circuits. Where central office equipment is being modified to permit of increased range of direct current signaling, or for some other reason, the reduction of susceptiveness is always a consideration. All of the newer repeating coils used for supplying talking battery to subscribers in common battery areas, which comprise the bulk of the local plant, possess a much higher degree of balance than the coils which were standard a few years ago.

Other Items. So far the changes which are associated directly with the major trends of development in the telephone plant have been described. The broad outlines of these developments depend on all of the factors affecting telephone service as well as coordination with power circuits. There are other features not directly associated with these main trends which, while introduced into the telephone plant largely because of the advantages to be gained in reducing susceptiveness to electrical influences, have also afforded other benefits. A few of the more interesting examples of these changes are given below.

Referring to the toll plant, there may be mentioned the recently adopted general practise of soldering aerial wire sleeve connections in order to insure a permanently high degree of series balance. Heretofore reliance had been placed on the contact between the wires and the twisted sleeve. The practise of soldering will be supplemented in the near future by a cold-rolled sleeve method, and it is confidently expected that these practises will result in material noise improvements. They will also probably reduce the maintenance required on open-wire toll circuits, particularly where exposures are involved.

Another item is the abandonment of the use of iron wire and substitution of copper for short tributary toll circuits. Coordination of the iron wire circuits is relatively difficult because of the development of resistance unbalances at the wire joints. The transmission efficiency is also improved by the reduced resistance afforded by the copper but this effect is generally of secondary importance in the short tributary circuits.

In toll offices improvements have been made in the

balance of coils and condensers used for superposing telegraph on the telephone circuits. The use of repeating coils, commonly used for side-circuits, has been extended to phantom toll circuits. These coils act as insulating transformers to prevent noise voltages from the outside conductors being impressed upon the intricate cabling and equipment of the office.

Referring to the local plant, there are several noteworthy examples of modifications made principally for the purpose of reducing susceptiveness. Investigation such as that of the coordination between power and telephone distribution plants conducted at Minneapolis by the Joint General Committee, stimulated the development of means for reducing the susceptiveness of the telephone distribution plant. Present practises call for the interconnection of aerial and underground cable sheaths and the grounding of the aerial sheath in order that the benefits of the shielding action of the sheath currents as previously described, may be realized for noise induction. In cases where electrolysis conditions do not permit direct grounding, condensers of the electrolytic type are employed to prevent the flow of direct currents.

The telephone circuits have long been equipped with over-voltage protectors for the purpose of protecting apparatus and cables against lightning waves and against power frequency transients from the lower voltage distribution circuits, also with fuses for opening the lines in cases in which heavy currents flow. The trend in development of these devices has been principally toward more uniform operation and lower maintenance costs. With the rapid increase of voltage and capacity of power circuits generally, experimental studies have been undertaken of further means for maintaining the safety of persons working on or listening on the telephone circuits. At the present time, development work is being done on various devices for this purpose, some of which are fundamentally different in design and operation from those previously used. It is hoped that these devices, which are discussed in one of the following papers, will afford increased protection against overvoltages and improve coordination conditions.

Trends in Power System

In the field of power generation marked attention has been paid, from the start, to methods of improving the efficiency of the generating process and reducing the investment per kilowatt of generating capacity. This has led to the development of larger and larger generating units. A single shaft unit of 160,000 kw. capacity and a triple-element unit of 208,000 kw. capacity are in operation. The latter consists of one high-pressure and two low-pressure turbines with their respective generators. Single shaft units of 200,000 kw. capacity are under construction and it seems probable that the trend in the future will be toward even larger units of both types. This trend toward larger units instead of the equivalent in small units has resulted in improved wave shape but otherwise does not directly affect coordination except in so far as it may reflect the general trend toward larger concentrations of power with the accompanying tendency to increased magnitude of system abnormals.

Another definite trend in the power industry, but one which is not of importance from the standpoint of coordination, is the increasing use of automatic devices to replace manual operation. Complete automatic operation is being practised to some extent in hydroelectric generating stations and is widely practised in substations of various types. The trend is definitely toward wider use of automatic devices and new types and applications of such devices are being constantly developed.

In view of the remarkable development and rapidly multiplying uses of thermionic tubes and related devices in other fields, and the theoretically potential applications in the power art, the question will doubtless be asked as to the trend of their application in the power field. However, other than application for current rectification, such as in railway work, it cannot be said that progress has advanced to the point of establishing a trend.

Those trends in power system development which are more directly concerned with matters of coordination are discussed in the following.

System Voltages. Referring to Table III, it is of

TABLE III

TOTAL CIRCUIT MILES OF TRANSMISSION LINES. BY VOLTAGES, YEARS 1926-1929 INCLUSIVE

Voltages	1926	1927	1928	1929	Per cent of total 1929	Average annual increase per cent 1926-1929
220,000	1,054	1,257	1,442	1,442	0.9	11.0
132,000	3,125	3,343	4,010	4,448	2.8	12.5
110,000	7,875	8,661	9,114	10,159	6.4	8.9
66,000	12,157	15,212	18,716	21,236	13.3	20.4
60,000	8,801	9,257	8,076	8,174	5.1	−2.4
44,000	7,517	8,492	8,732	8,761	5.5	5.2
33,000	23,831	24,706	27,451	28,523	17.9	6.2
22,000	10,130	10,429	11,545	12,583	7.9	7.5
13,200	19,496*	18,441*	19,551	21,340	13.4	3.1*
11,000	8,072	9,145	10,007	10,860	6.8	10.4
All other over 11,000	28,223	28,535	29,843	31,916	20.0	4.2
Total	130,281	137,478	148,487	159,442	100.0	7 0

*This apparent discrepancy is believed to be due to reclassification of these lines as between transmission and distribution facilities

interest to note that the rate of increase of transmission line mileage, as a whole, is lagging behind the rate of growth of both installed generator capacity and electricity production. Furthermore, mileages of the higher transmission voltages, 220 kv., 132 kv., 110 kv. and particularly 66 kv., are growing at a faster rate than the group average. These comparisons reflect the increasing utilization of the higher voltages with the greater circuit capacities they provide. As power industry growth requires the handling of larger blocks of power and as greater distances between sources and markets are encountered, the development and use of circuits and apparatus to transmit at voltages higher than the 220 kv. initiated in 1923 must be expected as an economic necessity.

In the distribution field also, coincident with the development of rural service, there has been a movement to higher voltages in primary circuits, and indications point to the continuance of this trend in the future. Due to the distances involved, voltages from 6,600 to 13,200 (and even higher) have been used in rural work. In urban areas the high load densities encountered in some districts require the handling of large blocks of power in the primary circuits, and the lower primary voltages have often been replaced by higher voltages for such conditions. In addition to the greater capacities provided by the higher voltages, possibilities of system simplification by combining rural and urban systems and eliminating voltage transformations are of considerable economic importance.

While at first glance the pronounced trend to higher transmission and primary distribution voltages may appear to enhance the difficulties of coordinating communication and power lines, certain factors enter to offset this. As transmission voltages increase, line construction as a whole becomes more massive, greater clearances and wider rights-of-way become necessary and construction costs per mile rapidly rise. These greater space requirements weigh against the use of highway locations and, together with the higher construction costs, which make the shortest possible lengths desirable from an economic viewpoint, frequently influence the selection of direct cross-country private rights-of-way providing generally greater separation from communication circuits in the same territory.

The use of the higher voltage circuits, each transmitting many thousands of kilowatts, of itself tends to increase the problems of coordination. However, the greater separations obtained by the use of private rights-of way for these main transmission circuits in most cases eliminate the need for coordinative measures to control normal induction (manifested as noise in the telephone circuits) and, in case noise presents a specific problem, the greater separations simplify and render less extensive those specific coordinative measures which may be required. Induction due to power system abnormals too is mitigated or rendered easier of control.

In the case of distribution lines, the adoption of increasingly higher voltages is accompanied by more systematic grades of construction and greater clearances from communication circuits. The result, of course, is that fewer abnormal conditions of operation occur and the number of related disturbances in the communication circuits is correspondingly reduced. The possibility of contact between power and communication circuits is also reduced. This trend toward better grades of construction applies also to transmission lines and as noted previously to other parts of the power system.

System Stability. During recent years considerable attention has been paid to the development of methods for improving system electrical stability. One of the most important of these methods is the use of higher speed switching,—at present, faults can be cleared in 15 cycles, or less, of a 60-cycle wave. So far, high-speed switching has been applied mainly to transmission circuits. However, as development proceeds and cost of equipment required is reduced, the field of application of high-speed switching may naturally be extended to distribution systems. The result in the case of either transmission or distribution will be of course, to reduce the duration of transients. Akin to high-speed switching, the use of high-speed excitation of rotating equipment has been developed. This may tend to increase the maximum fault current values somewhat which would make coordination more difficult. However, the reduction in the severity of instability surges, in so far as such surges involve faults-to-ground, affords definite benefits from the coordination standpoint. It requires further study and observations to determine what, if any, inherent limitations or advantages it may possess with respect to coordination work.

The way has been paved for the development of high-speed switching by steady improvement in relaying practise. Selective operation of protective relays in power systems, during the early stages of relay development, was largely dependent upon an additive sequence of time intervals which might aggregate a considerable period in the case of the more remote units in the sequence. The development of relaying practise has included various methods of securing selectivity independently of time. This has accomplished large increases in the over-all speed of operation, at the same time improving selectivity. Coincident with these improvements there has also been a substantial gain through greater precision in design and workmanship and improved application of relays and related devices. These trends definitely aid coordination by reducing duration of transients, eliminating faulty relay operation, and steadily reducing the radius of influence of system abnormals.

With the growth in power systems and major interconnections, the use of bus or feeder current limiting reactors or other means of limiting the concentration of fault current flow is being given increasing application. Such practise acts to restrict the magnitude of inductive transients. In distribution systems the growing use of

feeder reactors has a similar effect in matters of coordination.

For well-known reasons, among which are the avoidance of transient over-voltages resulting from arcing grounds and the economies made possible in apparatus insulation, it is predominant practise in America to ground the neutrals of transmission systems at important transforming centers, sometimes through resistors or reactors but usually solidly. In view of the prevalence of the latter method, a large proportion of higher voltage transformers now in service have been constructed with insulation between the neutral ends of the grounded windings and the core and tank, designed to support only the neutral potentials produced by fault currents regulating through the unavoidable impedance of grounding connections. The economies resulting from this method of construction become greater as rated operating voltages rise. The use of solidly grounded neutrals tends to make coordination more difficult in view of the possibilities for increased flow of earth currents.

On some large power networks with relatively great possible concentrations of short-circuit power and solidly grounded neutrals tendencies towards instability of operation have appeared. In some instances also oil circuit breaker characteristics, particularly as regards the older breakers in service, have become a source of concern. For these reasons, in these situations, increasing study and consideration are being given to the use of current limiting devices in the neutral where the characteristics of the apparatus and limitations of relaying will permit of such operation.

In some European countries, particularly in Germany, where grounding for the purpose of power system voltage stabilization is excluded by governmental regulation, dependence is extensively placed on the Petersen coil as a substitute. This device may be regarded as a special type of neutral impedance. The Petersen coil has been applied to but limited extent in this country although its possibilities for moderate voltage systems, especially for situations warranting only single circuit supply, are receiving consideration.

In this country, the increasing use of neutral impedance, as well as the use of other types of current limiting devices is an aid to coordination since it reduces the magnitude of abnormal induction.

Lightning Control. The major problem of the transmission art at the present time is the control of lightning in its effects on service. In those sections of the country in which lightning is prevalent, this natural hazard accounts for a large proportion of transmission circuit faults, approaching 100 per cent in the case of the heavier, higher class trunk transmission lines. The seriousness of this problem and the researches which some of the larger power utilities and apparatus manufacturers are conducting for its solution are being fully reported from time to time before the Institute and need not be discussed here. It is sufficient to say there is encouragement that methods for the solution of this problem, as it affects high-voltage trunk circuits, will be known in the not too distant future. Where adequate methods are found and applied the results of course, will be a decrease in the number of system disturbances which induce transients in communication circuits.

Present measures in power system practise, especially at the higher voltages, directed toward the control of service interruptions caused by lightning include: improved application of overhead ground wires, improved grounding connections at the supporting structures, the improved use of wood for lightning insulation, and the use in shunt with line insulators of fused gaps or other valve devices to "spill" the surge without dynamic current follow up. There is also under consideration the application on grounded neutral systems of single-phase switching. All of these measures, with the exception of the last, are helpful from the coordination viewpoint since their effect is to avoid or reduce system faults or at least to decrease the magnitude of earth fault currents and hence of the accompanying voltages induced in nearby communication circuits.

Single-phase switching involves the use of individually controlled and operated single-phase circuit breakers. Upon the occurrence of a single-phase fault-to-ground, the breakers on the faulty phase only would open, leaving the other two-phase conductors in circuit to maintain connection momentarily between source and load. In a short interval the breakers controlling the faulty phase would be reclosed automatically.

Single-phase switching has not progressed beyond a preliminary consideration of its possibilities. If applied in situations of proximity, the residual voltages and load currents while one phase of a three-phase grounded neutral system is momentarily open circuited may constitute a problem in coordination.

Underground Construction. The use of underground construction in distribution systems is seldom economical but is increasing in high-load density districts and in some residential areas primarily due to requirements for civic improvements and the relieving of surface congestion. The reduced influence on communication circuits of such underground circuits as compared to overhead construction, is too well known to need repeating here. Coincident with the more recent developments in underground distribution certain special situations have brought about the development of underground cable suitable for use in high-voltage transmission circuits, inclusive of 132 kv. Underground installations involving these transmission voltages are highly special, comparatively few in number and small in extent. However, they have a definitely favorable effect upon coordination problems within the territories surrounding them.

Aerial cable construction for both distribution and transmission circuits has been used to a limited extent and has a definitely beneficial effect upon coordination

matters. Whether this type of construction will be extended in the future is not evident.

Grounding of Distribution System Neutrals. One of the difficult tasks encountered in distribution systems is that of obtaining adequate grounding of primary and secondary circuits. Because of this difficulty the establishment of neutral networks grounded at many points has become a practise. In most cases in the past two separate neutral networks have been provided, one for the primary and one for the secondary system. However, in several localities these two separate neutrals have been combined into a common-neutral arrangement providing in this way an increased multiplicity of ground connections to both the primary and secondary neutral conductors. Further extension of the use of this system is probable. This arrangement introduces features of interest from the coordination standpoint, because of the increased opportunities for the flow of currents through the ground. Experience and investigations so far, however, indicate that with adequate attention to coordination this arrangement is comparable in its effect on neighboring communication circuits, to other types of distribution systems.

Service Taps on Transmission Lines. In some rural situations, it has been found economically impracticable to initiate distribution lines due to distances involved. However, in many such situations immediate electric service is urgently required and in some of these cases, transmission lines may be located relatively close to the point where service is desired. In such cases the only alternative to a long distribution line is to tap the high-tension transmission line when this can be done by some less expensive method. Such methods have been developed and applied to a limited extent. More study and field experience are needed to determine the effects of these installations on inductive coordination should they become extensively employed.

Transformer Connections. In distribution practise, the trend toward higher primary voltages has been accompanied by the use of the Y connection of the primaries of transformers as a step in the transition from one voltage class to another. Thus 2300-volt delta systems have become 2,300/4,000-volt Y connected systems, 6,600-volt delta systems have become 11,000-volt Y systems, and the 7,620/13,200-volt Y connection is being used. The use of the Y connection of the primary of distribution transformer banks is sometimes necessarily accompanied by a similar connection of the secondary. Such YY connections are usually in urban situations. Also, these banks usually represent only a small portion of the total transformer capacity on the circuits.

On large transformer banks and in the higher voltages delta-Y connections have long been the prevailing practise. However, where the YY connection is used for purposes of grounding, especial attention has been given to controlling the effects of this connection in situations of coordination, and for the absorption of triple harmonic currents it is common practise to use delta-connected tertiary windings in such installations. This subject is discussed more fully in another paper in this symposium.

Wave Shape. The connection of primary circuits directly to generating station buses results in service and economic advantages by eliminating transformations thereby improving voltage regulation and aiding system simplification. This practise, however, tends to make coordination more difficult as those harmonics which may be present in the generated voltage can flow directly out over these circuits. However, the important bearing of the wave shape of generators and apparatus of various kinds on the coordination problem has long been realized and is receiving increasing attention. Even before the formation of the Joint General Committee the general problem of apparatus wave shape was being studied both as to the amounts of various harmonic components which were present in apparatus wave shape and as to the relative effect of these components when appearing in communication circuits by induction from power circuits. As a result of this study an instrument was developed for measuring "telephone interference factor" of a voltage wave. With this instrument as an aid a better understanding of the bearing of wave shape has been gained by the apparatus manufacturers and there has resulted a gradual improvement in the wave shape of new apparatus.

It is recognized that there is a median line beyond which general improvement in the inherent wave shape of apparatus would not justify the attendant increased difficulties of design and increased manufacturing costs, —to avoid the alternative of applying in specific cases, available and less expensive methods of externally correcting wave shape. Work is now in progress cooperatively between the manufacturers and users looking toward the establishing of a measure of wave shape in apparatus design which will strike an economic balance between benefits and burdens.

The increasing use of rectifiers for conversion from alternating to direct current has an influence on inductive coordination. Considerable study has been devoted to this matter as a result of which methods for control of the distortion of the d-c. voltage wave caused by the rectifiers have been applied in several instances and a solution of this part of the problem appears to be in hand. More study and experience are needed as regards the specific conditions under which the wave shape distortion of the alternating current supply would require consideration.

With the progress being accomplished in the design and application of apparatus and the better understanding of the influence of circuit and transformer connections on inductive relations, problems concerned with wave shape can be expected to steadily decrease. The status of the cooperative study of this subject is described in this symposium.

Conclusion

A brief outline has been given here of the general trends in plant development and operating practise in telephone and power systems with special regard to those trends which affect the problem of coordination. While naturally there have been influences working favorably and others working unfavorably toward the problem it is clear that the preponderant effect of the development now being applied in the two industries is reducing the proportion of new situations in which specific coordinative measures are necessary. While to a considerable extent, as indicated in the body of the paper, this is due to the natural trends of plant design associated with new developments within each of the industries, it is also true that the extent of the progress made is due in no small measure to the careful study of all phases of the problem being conducted by the Joint Committee of the National Electric Light Association and the Bell Telephone System.

Under the guidance of this Committee and soon after its formation, the types of situations of physical proximity were classified and certain broad principles of cooperation were recommended. Soon thereafter more complete principles and detailed practises were formulated. These principles and practises were printed and widely distributed to companies and individuals directly interested in the problem of coordination.

The principles and practises thus set up were largely qualitative and the need for an organized program of research to establish quantitative data and to develop improved physical facilities for coordination was early recognized. Accordingly, the Joint Sub-Committee on Development and Research was organized, and assigned the work of determining both experimentally and by field experience quantitative data covering the various aspects of coordination problems, and the development of detailed methods of effecting physical coordination. Under this Sub-committee a very large volume of research work has been undertaken. Results of some of this work have been published and a considerable amount is now in progress. The three papers to follow in the symposium discuss much more fully three of the most important aspects of coordination work at the present time and tell of the work being done in these fields by the Joint Subcommittee on Development and Research and by the other branches of the Joint General Committee's organization.

In reviewing this subject one is impressed by the number of ways in which the coordination problem touches both the telephone and power fields, and with the very large amount of cooperative work which has already been done. This work, as has been indicated, has resulted in great progress in the satisfactory handling of coordination matters of all types. This matter concerns two industries both of which are in a period of rapid development and change, both as regards their size and as regards the physical arrangements which constitute their plants. Many new developments in each plant require consideration from the standpoint of coordination. It is evident, therefore, that if the ground already gained is to be held and further progress made the channels of cooperation between the two industries must be kept in operation both for the consideration of new problems arising with new developments in the industries, as well as for the further perfection of the cooperative methods of handling specific problems. These papers in other words do not constitute in any sense a final report. They are intended to show the present status of two very active and rapidly changing arts and to indicate the highly satisfactory results which have followed from a number of years of sincere cooperative effort between the telephone and power industries.

Part II—Status of Joint Development and Research on Noise Frequency Induction

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Foreword

THE Joint Subcommittee on Development and Research is the agency through which the National Electric Light Association and the Bell Telephone System carry out technical work on problems of physical relations which vitally affected their respective growth and operating practises. In the present paper and companion papers the status of this joint development and research work is described.

The present paper, Part II of the Symposium, is concerned with problems of induction in telephone circuits under normal operating conditions of power systems which result in noise. Part III of the Symposium treats of induction at the power-system fundamental frequency, principally that occurring at the time of grounds, short circuits or other abnormal conditions of power systems. Part IV of the Symposium treats of the physical relations and of the special noise-frequency and low-frequency problems brought about by the close proximity of the two types of service when occupying the same poles.

The Joint Subcommittee on Development and Research has subdivided its work among eleven project committees and assigned to each the actual carrying on of specific research work. Certain of the project committees are engaged on the problems described in this paper, while the remainder are concerned with the development and research problems of the companion papers, Parts III and IV of the Symposium. The names of these project committees, together with a statement of the phase of the problem considered by each, is given in Volume I of "Engineering Reports of the Joint Subcommittee on Development and Research."[1]

Naturally the first steps taken by the Joint Subcommittee were the review and appraisal of existing information and the exchange of data between the two interests represented. This paper includes a statement of the problem, with some review of the factors involved, the results accomplished by the subcommittee and the work projected in connection with each factor.

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1. For references see Bibliography.

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Classification of Factors Contributing to Induction

There are certain characteristics of a power circuit with its associated apparatus that determine the character and intensity of the electric or magnetic field which is set up in the surrounding medium. These characteristics are termed "influence factors."[2]

Likewise, there are certain characteristics of a communication circuit with its associated apparatus which determine its responsiveness to external electric or magnetic fields. These characteristics are termed its "susceptiveness factors."[2]

There is a third group of factors which refer to the interrelation of neighboring power and communication lines by electric or magnetic induction or both. These are termed "coupling factors."[2]

Inductive interference is thus the manifestation in the telephone circuit of a combination of influence, susceptiveness and coupling; and inductive coordination consists in the control of factors in all three of these classes to the degree required for satisfactory operation of both services.

Methods of Control

Physical Separation. The first method which comes to mind for the control of inductive effects is that of physical separation obtained by placing the power and telephone lines on separate routes. A separation between lines of a few hundred feet practically eliminates the noise-frequency problem whereas the low-frequency problem may exist with much greater separations. Since the same customers desire both communication and power services, the two kinds of distribution lines are necessarily often located on the same streets and highways. Power transmission lines and toll telephone lines do not, in general, have to be placed on particular routes and, therefore, separation can often be employed where such lines are involved. Cooperative advance planning on the part of the utilities in laying out their plants makes it possible to employ separation where it is readily feasible and economical.

Frequency Separation. Another method of fundamental importance is the use of frequency separation. By this method, circuits to be coordinated are arranged so as to be responsive to different frequencies or bands of frequencies, and comparatively unresponsive to the frequency or band of frequencies employed for the other circuits. It is thus possible to make many different uses of electricity involving transmission in the same

medium. This solution is familiar to us in the coordination of radio services.

Fig. 1 shows a diagram of the various uses of the frequency spectrum for electrical transmission and the manner in which power and communication services are coordinated by means of frequency selectivity.

The first commercial electrical energy available was in the form of direct current. Shortly thereafter, alternating current was used for the transmission of power. The nominal frequencies of the current used for this service in the earlier days range from 16⅔ cycles to 133 cycles. In American practise the frequencies used for power purposes have practically settled down to either 25 or 60 cycles. There is one extensive 50-cycle system and a few odd frequency systems. These latter of 30, 33, and 40 cycles, and

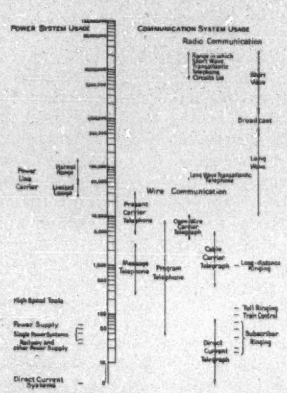

Fig. 1—Frequencies Used for Electrical Transmission

perhaps others, are being rapidly eliminated, due to the importance of interconnecting them with 60-cycle systems. At the present time, there is some tendency for the use of higher frequencies in special machine shop applications. This use, at present, is principally at 180 cycles and need not concern us here as its extent is usually confined within a factory building.

In message telephone transmission, the prime consideration is the transmission of intelligible speech. While the range of response of the human ear is from about 16 cycles to 15,000 cycles per second, human speech occupies a narrower range and a still narrower band is adequate for intelligibility. The present voice-frequency telephone circuits, especially the longer ones, operate within a frequency band of about 250 to 2,750 cycles per second. The frequency selectivity at the edges of the band is not sharp, however, so that extraneous currents at frequencies outside of this band may also give rise to noise. This is particularly true at the lower end with some of the local exchange circuits. High quality telephone circuits for program transmission cover a wider range. This may be on certain circuits as much as from about 35 to 8,000 cycles per second, thus overlapping the fundamental frequencies used for power transmission.

Control of Power Levels. Coordination by frequency separation becomes inadequate when the power levels of the various classes of services differ greatly as with power and telephone services. Thus, although incidental powers at harmonics of the power circuit fundamental frequency are negligible in comparison to the power at the fundamental frequency, they are large compared to the power employed in the telephone circuits and fall directly within the frequency range of the telephone circuits.

While the powers involved in telephone transmission are small as compared to those on power lines, they are in turn large as compared to the acoustical power received from the talker or delivered to the listener. The ordinary telephone transmitter is an amplifier, delivering to the line several hundred times the voice power which actuates the diaphram. On the other hand, the receiver requires an electrical power a hundred or more times that which it delivers as sound to the listener's ear.

It is obvious that the relative levels of harmonic frequency power in the power circuits and voice-frequency power in the telephone circuits are of major importance in inductive coordination. These considerations have had large influence in the power field in the control of wave shape of rotating machinery and transformers, and in the telephone field in fixing limitations on such factors as wire sizes, spacings of repeaters and instrument efficiencies.

Balance. Among the most important methods of coordinating power and communication circuits is the control of their respective balances to ground and to each other. A power circuit with absolutely balanced voltages and currents impressed, and with the various conductors arranged in such a way that they would not establish external electric or magnetic fields, would not have any effect on any type of neighboring communication line.

Likewise, a telephone circuit in which there were no unbalances and in which the conductors were arranged in such a way that in the presence of an electric or magnetic field they would not have any voltages induced between them would not become noisy from any neighboring power circuits. Such an ideal state is impossible, but much has been accomplished by care in the design of the lines and equipment and by the transpositions of the conductors.

Shielding. It is possible to materially reduce electric fields by interposing between disturbing and disturbed conductors grounded conductor surfaces known as

shields. Magnetic fields can likewise be reduced by interposing conducting paths which circulate current to set up counter magnetic fields. The power and telephone cables in use are probably the simplest examples of shielding. A cable sheath is almost 100 per cent effective as a shield for electric induction, either on a power cable or on a telephone cable. The sheath is less effective as a shield for magnetic induction, because of its finite conductivity. It does not seem feasible at this time to obtain anywhere near perfect magnetic shielding.

Factors Contributing to Noise Frequency Induction

Influence Factors

There are two characteristics of a power system which are of primary importance in determining its inductive influence upon neighboring telephone systems, *i. e.*, its wave shape and its balance. The wave shape is determined by characteristics of apparatus associated with the system. The balance is determined by the degree of symmetry of the supply voltages, load impedances, and of the series impedances and shunt admittances of the lines. While it is not practicable to design rotating machinery or other apparatus using magnetic cores entirely free from harmonics, or to realize ideally balanced three-phase systems, it is practicable to control both these factors within limits which, in conjunction with a similar degree of control on the coupling and in the susceptiveness of the communication circuits, permit satisfactory operation of both services without unduly burdening either.

The work on influence factors which has been conducted by the Joint Subcommittee on Development and Research has, therefore, been directed for the most part toward the study of the wave-shape characteristics of power systems and apparatus and methods for their improvement and the investigation of factors affecting the balance of the power systems and methods for their control.

Wave Shape. In initiating its work on influence factors, the Joint Subcommittee found little information available as to wave shape which might be expected on operating power systems equipped with various types of apparatus. In order to obtain a broad picture of wave-shape conditions as they exist in the field, the Subcommittee conducted an extensive survey of wave-shape conditions on 34 operating power systems in the eastern half of the country. The program was arranged to obtain information as to the average and range of magnitudes of harmonics present in various types of transmission and distribution systems under normal operating conditions, to observe the relation between the wave shape of generating machinery under open-circuit conditions and under load, to study the effects of various transformer connections on wave shape, and to observe the effects on wave shape of various types and magnitudes of load.

The measurements made included analyses of the phase-to-neutral and phase-to-phase voltages and phase currents on a large number of generators, transmission lines and distribution feeders. Wherever practicable, data were obtained as to the balance of the operating systems by measurements of residual voltages and residual currents. Measurements were also made of the telephone interference factors[3] of the voltages and currents. Where telephone circuit exposures suitable for test purposes existed, noise measurements were made on the communication lines to aid in determining the relation between power system wave shape and balance and telephone circuit noise. The actual measurements were for the most part conducted by the operating companies with the cooperation, during the first part of the testing program, of representatives of the Joint Subcommittee.

The mass of data accumulated during this survey is being summarized in several technical reports which it is anticipated will yield much valuable information pertaining to the wave-shape problem. An important practical application of these data will be in connection with the prediction of wave-shape conditions on new lines which are to be involved in exposures with communication systems and on which noise estimates are desired.

In general, the survey data indicate that the magnitudes of the harmonics present in voltage and current diminish with increasing frequency, with the exception that a pronounced dip occurs in the region from 800 to 1,500 cycles. This is, no doubt, a result of the efforts of the machine designers to closely control the harmonics in this important region. Frequencies above 2,000 cycles become extremely small except where these may be introduced on the power circuits by superposed carrier communication or signaling services. In general, the frequencies used for such services have been in the range from 50 to 200 kc., which is above the range employed for carrier communication on telephone lines.

In the general survey of wave shape, no efforts were made to select feeders involved in cases of inductive interference. Aside from the survey work, however, representatives of the Subcommittee have participated in a number of investigations of such cases in which power system wave shape was an important factor. Much valuable data as to wave-shape conditions under which coordination difficulties are experienced have been obtained from these studies, while in obtaining these data the subcommittee representatives have been of service to the local companies in the solution of the particular problems.

A limited amount of theoretical work has been carried on having to do with the effects of load on the harmonics observed in the open-circuit voltage of rotating machines. This work which was based on Blondel's two-reaction theory was supplemented by laboratory tests on several small machines. It was found that the reactions which take place within the rotating machines, particularly when two or more are operating in parallel, are so complicated as to practically preclude accurate computations of the effects. However, the

data obtained from this investigation have been valuable in connection with later studies.

Balance. A balanced power circuit is one in which the voltages between the various phase conductors and ground are equal in magnitude and sum up vectorially to zero and in which the phase currents are also equal in magnitude and sum up vectorially to zero. In a three-phase system where the currents or voltages are not equal but do sum up to zero, the currents or voltages can be resolved into two balanced three-phase systems, one of positive phase sequence and one of negative phase sequence. In cases where the currents or voltages do not sum up to zero they contain a single-phase component which is usually termed residual or zero-phase sequence component. Any three-phase system can be resolved into its balanced and residual components and each treated separately. The coupling for the residual components is usually much larger than for the balanced components and is therefore frequently of major importance in coordination problems. Differences in the magnitudes or departures from phase symmetry of the three impressed phase-to-neutral voltages, load or line unbalances, give rise to residual currents or voltages.

Experience has indicated that the outstanding factor in the unbalance of power systems is the existence of triple-harmonic voltages and currents which may arise either in rotating machinery or in transformers which are connected in star with grounded neutral. Since the triple-harmonic voltages in the three phase-to-neutral legs are in phase, they act in a path consisting of the phase conductors and an external return as, for instance, a metallic neutral or ground.

A large measure of control may be exercised on the magnitudes of the triple-harmonic residual voltages and currents by the use of certain transformer connections and by not operating the transformers at high flux densities.

The magnitudes of triple-harmonic residual currents in grounded-neutral systems may be minimized by the use of star-delta connected transformers, in which case nearly all the required triple-harmonic current circulates in the delta. The opposite extreme occurs with star-star connections in which case the full triple-harmonic magnetizing current flows in the two systems which the transformer interconnects, the relative magnitudes in each depending on their relative impedances. Where a star-star bank is connected at one terminal of a line, with a star-delta at the other, the neutrals at each end being grounded, practically the entire third harmonic required by the star-star bank may be expected to circulate in the line connecting the two.

An effective method of control for cases in which star-star connections are required due to phase relations is the provision of a third set of windings or tertiaries in the transformers, the impedance of the tertiaries with respect to the other windings being sufficiently low to furnish an adequate path for the triple-harmonic magnetizing current. An alternate method of control, which also provides like phasing on the two sides of the bank, is the use of zig-zag connected transformers.

In four-wire multi-grounded neutral distribution systems, it has been found helpful in controlling the residual triple-harmonic currents from the single-phase load transformers to provide star-delta connected banks at various points in the network with neutrals connected to the system neutral. In some cases, these have been three-phase load banks, in others, special banks installed as a method of control.

The subcommittee is continuing its work on wave shape and balance through a laboratory study of transformer harmonics and transformer connections. These tests are being made on small model transformers, typical of the designs which are used for large sizes on transmission systems. It is planned to develop the theory applicable to harmonics from transformers on three-phase systems from the work on these laboratory models. It is planned to supplement the work by tests on large transformers in the manufacturer's shops and in the field.

A number of severe noise situations has been created during the past few years when star-connected generators, operating with grounded neutral,[4,5] have been connected directly or through star-star transformer banks to transmission or distribution systems. The interference in these cases resulted from triple-harmonic residual components impressed on the system by the particular generator operating with the grounded neutral. The magnitudes of these currents depend on the triple-harmonic components in the generator phase-to-neutral voltage and the impedance to ground of the system. The methods of control which have been successfully applied in these cases include the following:

1. Isolating the generator neutral and supplying the system ground through a suitably designed transformer bank.
2. Grounding the neutral of only those generators designed to be free from triple harmonics in their phase-to-neutral voltage.
3. The use of selective devices such as reactors or anti-resonant circuits commonly called "wave traps" in the generator neutral for suppressing the disturbing triple-harmonic components.

Non-triple harmonic residual voltages and currents may exist from differences of phase-to-neutral load impedances and from differences in the capacitances to ground of the three-phase wires.

In multi-grounded neutral four-wire systems differences in the single-phase loads connected between the individual phases and the neutral may be important sources of residual current. A considerable measure of control may be exercised by restricting the size of single-phase areas and balancing the load on the different phases.

Capacitance unbalance to ground may be due to single-phase branches on three-phase distribution systems. Usually, the more important effect is on the single-phase branch where the residual voltage is practically equal to the phase-to-neutral voltage. The unbalancing effect on the three-phase system may be minimized by equalizing the lengths of the branches connected to the several phases. The residual voltage on the single-phase branch can, where necessary, be eliminated by the use of isolating transformers or by converting to a three-phase branch.

Capacitance unbalance may also be due to dissymmetry in the arrangement of the wires of the circuit to each other and to ground. These unbalances are lowest in triangular configurations of the wires and largest when all the wires are in the same vertical or horizontal plane. With multi-circuit lines, a considerable measure of control may be obtained by suitable phase interconnection of the circuits. Transpositions are also effective in controlling these unbalances.

Coupling Factors. The coupling between power and communication circuits is, of course, determined by the degree of their proximity, but it may be greatly modified by the balance of the two classes of circuits to each other and with respect to ground. While the most direct and certain method for reducing coupling is to avoid proximity, means are available for minimizing the coupling where necessary.

The work on coupling of the Joint Subcommittee on Development and Research, in the voice and carrier-frequency range, has been directed toward two objectives: (1) development of improved methods for predetermining the coupling to be expected in new cases of exposure, and (2) development of improved methods for reducing coupling for given degrees of proximity.

Several years ago the California Joint Committee on Inductive Interference[6] completed an extensive series of computations on coefficients of induction which was expressed in the form of curves for various physical relationships of power and telephone lines. These coefficients indicate the voltages induced in short, isolated, untransposed telephone circuits by unit voltage and current on similarly untransposed power circuits. They do not include the small separations involved with jointly used poles.

These curves and others based on them have been used for many years in determining relative coupling, when comparing different exposures, different routes involving various degrees of exposures, different configurations of power and telephone circuits and for other comparisons where all factors were substantially equal in the situations being compared, except those involved in determining the coefficient of induction. For these purposes they have been very useful. Methods have not, however, been available whereby these coefficients could be used for computing noise where transposed circuits were involved and where many telephone wires were on the line, which exert an important shielding effect on each other.

The Joint Subcommittee on Development and Research has been conducting experimental studies both for highway and wider separations, and those occurring with jointly used poles, so that the effects of transpositions and of mutual shielding of the many wires involved might be properly taken into account in determining the noise currents in the metallic circuits.

In determining the coupling between power and telephone circuits, it is desirable to differentiate between the effects of the balanced and residual components of the voltages or currents of the power circuit, between the effects of voltages and those of currents, and on the telephone line between induced voltage which acts directly in the metallic circuit, termed "metallic-circuit induction," and that which acts in the circuit composed of the wires with ground return, termed "longitudinal-circuit induction."

Since the residual components act in a circuit having ground as one side with the wires in parallel for the other, while the balanced components are confined to the wires of the system, the coupling for the residual components is much greater than for the balanced components. The coupling for the balanced components may be reduced by the use of power circuit transpositions, while such transpositions have no effect on coupling for the residual components.

The distance between the power and telephone wires is usually large as compared to the spacing of the wires of the telephone circuit, so that the longitudinal induced voltages are large as compared to the metallic-circuit voltages. The effect of the telephone transpositions being merely to equalize the relations of the two sides of the telephone circuit to the power circuit, such transpositions do not change the magnitude of the longitudinal voltages, but do reduce the metallic circuit voltages.

The relative magnitudes of inducing voltages and currents differ widely among various power circuits, and may vary greatly with time on any given circuit. They will also differ considerably at a given time and on a given circuit among the various frequencies involved. For this reason it is necessary to consider separately the coupling arising through the electric and magnetic fields.

Voltages induced in metallic circuits for the separations between lines usually encountered are practically proportional to the spacing of the wires of the telephone circuit. Voltages induced in 8-inch spaced pairs are thus approximately two-thirds of those induced in 12-inch pairs, while those induced in phantoms on 12-inch spaced side circuits are twice those induced in the sides. The longitudinal voltages are, however, practically independent of the wire spacing so that the contributions which these voltages make to noise in the metallic circuit are unchanged except as the change in spacing may affect the balance to ground.

Spacing of the wires on the power circuit and their configuration also have an important effect on the coupling for the balanced voltages and currents, the coupling, in general, increasing as the spacing increases. Coupling for the residual components is, however, affected only to a minor degree by the spacing and configuration. Much information bearing on these matters is included in the material on coefficients of induction published by the California Commission referred to above.

Measurements of coupling have been made by the Subcommittee in a number of situations. These have included cases of (1) exposure of overhead transmission lines and open-wire toll telephone circuits at highway separations, (2) overhead distribution lines and subscribers' telephone cables in joint use and at street separations and (3) overhead distribution lines and subscribers' open-wire circuits in joint use. Information was obtained on coupling both for voltages and currents and for the balanced and residual components. The results of the work on overhead distribution lines and subscribers' telephone cables have already been published.[7] The other data are to be published as soon as they are prepared in suitable form.

The work on overhead distribution lines and subscribers' circuits is relatively complete, covering a wide range of conditions typical of those encountered in the field. Various arrangements of primary and secondary conductors covering single-phase and three-phase, three-wire and three-phase, four-wire systems were investigated. The shielding effect of the telephone cable was determined and, with the open-wire subscribers' telephone circuits, the shielding effect of the various telephone wires on each other.

For telephone cable circuits when the sheath is grounded at either one or both ends, the inductive effect of the power circuit voltages on the wires enclosed is negligible as compared to that of the power circuit currents. Furthermore, because of the close association of the wires of a pair in the cable and the frequent twist, the metallic-circuit induced voltages are neligibly small as compared to the longitudinal voltages so that, in general, only the magnetic longitudinal coupling factors are of importance in these situations.

The work further indicates that, for most practical problems involving overhead distribution lines of the multi-grounded type and subscribers' cable circuits, a knowledge of the coupling for the residual or unbalanced currents is sufficient, the effect of the balanced currents being relatively unimportant. However, in cases where the line currents are particularly heavy or contain exceptionally large harmonic components, the balanced currents become important.

In the range of frequencies used for telephone transmission the ratio of open-circuit voltage induced on a telephone line through electric induction to inducing voltage on the power circuit is substantially independent of frequency. When the exposed section of line is connected to the remaining section of the telephone line or to terminal apparatus, a current is set up which is approximately proportional to the frequency of the induced voltage. The circuit will perform as if there were a small condenser connected between the power circuit and telephone circuit and the induced current experienced will be proportional to the frequency and the magnitude of the inducing voltage on the power circuit.

The coupling between power and telephone circuits for currents is in the nature of a mutual inductance, so that the voltage induced in the telephone circuit is proportional to the magnitude of the inducing current in the power circuit and its frequency.

This statement applies strictly only to induction from the balanced current components. Induction from residual current in the power circuit is complicated by the effect of the finite conductivity of the earth. With increase in frequency the earth currents tend to be closer to the surface and the coupling with the telephone circuit tends to increase less rapidly than would follow from proportionality with frequency. The departure from linearity is not large in the frequency range from 250-2750 for highway separations and for joint use.

Transpositions afford one of the most powerful means available for controlling coupling of power and open-wire telephone circuits in given situations of proximity. Transpositions operate by neutralizing, in one section, inductive effects which arise in a closely adjacent section. It is evident that, in order for transpositions to be fully effective, conditions must be substantially alike among the various sections to be neutralized as regards relations of the power and telephone circuits to each other, to ground, and among the various circuits on each line. This latter condition more often applies to the telephone lines, as they usually comprise many circuits.

These conditions require that balanced and coordinated systems of transpositions be provided between each point of discontinuity in the exposure. By "discontinuity" is meant any point at which an important change takes place in the physical or electrical conditions of the circuits, such as loads, branch circuits, series impedances, etc.; any change in configuration, in the separation of the two classes of circuit or in their position relative to ground or to some other circuits which may be associated with either power or telephone circuits closely enough to appreciably modify the induction.

In addition to meeting these conditions, the telephone transpositions must also satisfy the requirements for minimizing crosstalk among the various telephone circuits. This, in general, requires telephone transposition arrangements of considerable complexity. For this purpose standard transposition arrangements are available,[8] adapted for different lengths depending upon the distances between the successive discontinuities.

In most cases unavoidable irregularities occur in the spacing of poles, in distances between power and telephone circuits, in presence of shielding objects, such as trees, and in height of poles, which it is not possible to treat as discontinuities and take into account in the transposition design. In cases where these irregularities are large, the effectiveness of the transposition arrangements is greatly impaired. The extent to which the effectiveness of such arrangements is impaired due to these non-uniform conditions is a problem not easily susceptible to mathematical analysis and reliable information is not now available. The subcommittee is planning to investigate this problem experimentally by tests on a number of situations involving operating circuits.

Susceptiveness Factors. The degree to which telephone transmission is adversely affected by noise-frequency induction depends not only upon the magnitudes of the induced voltages as determined by influence and coupling factors, but also upon the susceptiveness factors of the telephone system. These include the manner in which the induced voltages and currents are propagated to the circuit terminals together with the reactions of the circuit unbalances, thus relating the current in the terminal apparatus to the induced voltages, the sensitivity of the receiving apparatus and the operating power level of the telephone circuits.

Propagation Effects and Balance. Important differences exist with respect to propagation effects and balance between open-wire and cable circuits and between toll and exchange systems.

As pointed out in the discussion of coupling, only the magnetically induced longitudinal voltages and currents affect telephone cable circuits. Because of the absence of electric induction and direct metallic-circuit induction and because of the important shielding effects exerted by the cable sheath and the various telephone circuits on each other, telephone cable circuits are much less susceptive than open-wire circuits.

In open-wire telephone systems consideration must be given both to electric and magnetic induction and to voltages directly induced in the metallic circuit as well as to those induced in the longitudinal circuit. In a line composed of a number of circuits, the currents set up in any one circuit depend, not only upon the voltage induced in that circuit and its impedance, but also upon the currents and voltages which are set up in the rest of the telephone circuits on the line. It is not possible, therefore, to calculate the induced currents merely from a knowledge of the magnitudes of the currents and voltages on the power circuits and the coupling between the power circuits and isolated pairs of wires on the telephone line, considered independently.

These mutual effects among the various telephone circuits exist both within and without the exposed sections. Thus, the propagation of the induced voltages and currents along any one circuit is influenced both by the electrical conditions of this circuit and also by the conditions of all other wires on the line. Additional complexities arise in the propagation of the induced voltages and currents, because of non-uniformity in impedances to ground at terminals, points where circuits join or leave the line, and where lengths of cable may be used at terminals or at intermediate points. The impedances to longitudinal induced voltages and currents vary over a wide range depending on the number of wires on the line, the relative position of the exposure and the circuit terminals and the occurrence of sections of cable. Due to reflection effects from these irregularities, peaks of current and voltage may exist along the circuits which are large as compared to the corresponding magnitudes at the exposure terminals. If circuit unbalances happen to exist at these maximum points, metallic-circuit voltages and currents thereby introduced are increased.

While the distribution of longitudinal voltages and currents among the various wires upon the telephone line depends upon the nature of the inducing field in which it is placed, the experiments of the committee have shown that a satisfactory degree of approximation for studying propagation effects can be obtained by energizing all wires on the line simultaneously at the same potential from a common source. An extensive experimental study has been made in this way by the committee in which the magnitudes of the longitudinal voltages and currents at various points along the line have been measured as well as metallic-circuit currents set up through the unbalances at the sending and receiving ends of the line.

By making measurements of this sort on a considerable number of lines of different types of construction and different transposition arrangements, it is hoped to obtain statistical data whereby the metallic-circuit voltages and currents at the circuit terminals may be determined from the magnitudes of longitudinal voltages and currents as measured at exposure terminals.

Unbalances in toll circuits are the result of commercial variation from the balanced condition, since the circuits are designed to be symmetrical. These unbalances may consist of resistances in joints, capacitance or inductance unbalances due to irregularities in transposition spacing or to omitted or unspecified transpositions, or differences in the impedances of apparatus connected in series with the wires or between them and ground. These unbalances are fortuitous both as regards their magnitudes and location along the toll circuits. Some increase in importance with frequency and others decrease. These, combined with the irregularities in the propagation of the longitudinal voltages and currents, cause the resulting metallic-circuit currents in individual circuits to vary in an erratic fashion with frequency. The general trend is one of proportionality, independent of frequency within the important range, between the longitudinal currents and voltages at the exposure and current in the metallic circuit at the terminals. Taking into consideration

the effects on coupling, the currents at the terminals increase approximately in direct proportion to the frequency of the inducing voltage or current on the power circuits.

Because of the lower susceptiveness of cable circuits together with the high degree of balance of the terminal apparatus and because of the more general use of private rights-of-way, cases of noise-frequency induction into toll cable circuits have been comparatively infrequent. For this reason the attention of the subcommittee as far as toll systems are concerned, has been directed toward open-wire circuits.

In exchange circuits certain inherent unbalances exist due to the arrangements employed for supervisory signaling, for selective ringing, and for coin-box service. The supervisory system utilizes a low impedance relay connected in series with one side of the central-office interconnecting circuit. The selective ringing scheme involves connecting the ringer windings from one side of the line to ground at the station set. For the coin-box service, a coin-collect relay winding is connected between one side of the station set and ground. These unbalances have been investigated in detail by the committee and the results have been published[7] as described later.

The unbalance of party lines due to the ringer ground is usually much more important than that of the central-office interconnecting circuit due to the supervisory relay. Both are, in general, more important than the cable unbalances. Coordination difficulties between telephone exchange systems and power distribution systems thus usually involve the party line circuits before the individual line circuits are affected.

The controlling unbalance in the exchange plant when in cable being in the nature of an inductance between one side of the line and ground, its importance decreases with increasing frequency of the induced longitudinal voltage. This effect largely counter-balances the increase in coupling with frequency. Thus, in most situations involving joint use of poles by distribution circuits and exchange cable telephone circuits, induced currents of the third and fifth harmonics of the power circuit fundamental frequency assume chief importance.

Exceptions are cases where outstanding harmonics in the range between 800 and 1,500 cycles are present on the power circuits. In these cases, particularly where the exposures are long, the central-office apparatus unbalances may be more important than those of the party-line station apparatus.

The method which has been found most generally applicable for reducing the susceptiveness of exchange cable circuits is the grounding of the cable sheaths. This reduces through shielding the magnitudes of the longitudinal voltages and currents. Special station sets having lower susceptiveness have been used in specific cases where their use appeared to be the best method.

Power Level and Sensitivity. The magnitudes of the induced currents in the telephone system having been determined by the influence factors, the coupling, and the susceptiveness of the telephone circuits, the degree to which they impair telephone service depends upon their intensity as compared to the intensity of the telephone currents.

Consideration has been given by the subcommittee to the possibility of increases in power levels (a) on local exchange circuits and (b) on toll circuits. Little promise has been found in the proposal to raise voice power levels in the local exchange plant as a means of reducing the effects of noise. As previously pointed out, present telephone transmitters materially amplify the power received from the voice so that the electrical power on the telephone line is some hundreds of times greater than the acoustic power applied. In development work on telephone transmitters, telephone engineers are proceeding on the basis that more is to be gained by improving the frequency response of the transmitter than can be gained by mere increase of power. This line of development has, of course, the effect of raising power levels at frequencies where they have been relatively low.

Two proposals for application to the toll telephone plant were studied. One would involve changing the repeaters now in use at terminals and at intermediate points to a more powerful type and equipping all toll circuits with terminal repeaters of this same type. This would permit raising the power levels without altering the relative levels of the various telephone circuits and thus would not change the crosstalk. Another would involve such changes only on certain long toll circuits, leaving the remainder of the circuits at their present levels. As the result of a trial installation, it was found that to realize any appreciable change in level on these circuits, very extensive changes would be required to avoid crosstalk from the higher level circuits to the remaining ones which were not changed.

The levels employed in carrier telephone circuits, while somewhat lower than those used on voice-frequency open-wire telephone circuits at the receiving end, are higher at the sending ends than the corresponding voice-frequency levels. Since the power-system harmonics in the carrier-frequency range normally are small as compared to those in the voice-frequency range, carrier-frequency open-wire systems experience considerably less noise from power systems.

Effects of Noise. The actual voice power level on telephone circuits varies over a wide range, depending upon the particular user, his distance from the telephone central office, and by the transmission loss in the connection between the two subscribers. Impairment caused by a given amount of line noise on the circuit may also vary over a considerable range, depending upon the voice power level and the noise in the room where the telephone is being used. The method in use by the Bell System for an engineering basis in considering the effects of noise on telephone conversation is to

substitute for the noise increases in the transmission loss of the circuit. Thus, the circuit with its actual loss and noise is represented by a circuit of lower noise and increased transmission loss. These added losses are known as Noise Transmission Impairments and are abbreviated N. T. I. The N. T. I.'s were determined from articulation tests and judgment tests made on noisy and quiet circuits, and were set up on the basis of typical talker volumes, transmission equivalents, and amounts of room noise at the station terminals. Additional transmission loss was added to the quiet circuits so that noisy and quiet circuits gave equal articulation or were judged by the observers to be equivalent in their transmission performance. Thus, the N. T. I.'s are used to indicate an additional transmission loss or impairment which is occasioned by the presence of the noise.

The articulation and other tests on which these N. T. I. ratings were based are now being supplemented by tests conducted under the direction of the subcommittee. Measurements are being made of the effects of various magnitudes and sorts of line noise in the presence of typical amounts of room noise, as determined from a room noise survey made by the subcommittee, and for representative toll connections and talker volumes. The line noises being employed are those found typical from an extensive survey made by the subcommittee on open-wire toll circuits throughout the country. These tests will afford a basis for comparing various methods of measuring line noise, including ear comparison methods now in general use and new visual meter methods now under development. Thus, this work should lead to a mutually acceptable method for measuring noise and a basis upon which agreement may be reached as to the impairment in telephone transmission caused by noise.

Published Results. As various phases of the technical work being done are completed, they are published in the form of Engineering Reports which are released by the Engineering Subcommittee of National Electric Light Association and Bell Telephone System. Eight reports, of which five refer to matters concerning noise-frequency induction, have already been issued. Other reports dealing with this subject have been recently approved by the Engineering Subcommittee and will soon be issued. Certain other technical results which have come from the Subcommittee's work have been presented by various individuals connected with the work in papers before the A. I. E. E. Still other results have been published in brief articles in the N. E. L. A. *Bulletin.*

One of the problems, of which the technical work of the committee has been completed and published, is that of inductive coordination of primary distribution systems and exchange telephone circuits in cable. The results of this work are given in detail in a report[7] entitled "Minneapolis Joint Use Investigation." This report includes information on influence factors applying to various types of power distribution systems, including three-phase, three-wire and three-phase, four-wire systems with various arrangements of neutral grounding, data on coupling between various typical arrangements of these systems and telephone cable circuits, and information on susceptiveness characteristics of telephone systems, including unbalances of lines and apparatus. To facilitate the use of this information in the day-by-day coordination problems handled by the operating companies, a summarizing report[9] entitled "Short-Cut Methods for Calculating Noise in Local Telephone Subscribers' Circuits in Cable Due to Exposures to Power Distribution Circuits" has been prepared. This report presents empirical formulas for estimating noise-frequency induction and includes a brief discussion of the technical factors involved and the approximations underlying the formulas. Means are described for reducing influence by the control of triple-harmonic exciting currents and load unbalances of power distribution circuits, and for reducing susceptiveness by grounding telephone cable sheaths and by controlling the unbalances of the telephone station equipment. The information should be useful to engineers of the operating companies in the cooperative planning to avoid induction troubles.

While a large part of the experimental work connected with the problem of joint use of local open-wire subscribers' circuits and power distribution circuits has been completed, the detailed technical reports have not yet been completed for publication. However, a summarizing[10] report which it is believed will largely fill the needs of the engineers of operating power and telephone companies has been completed. This is entitled "Short-Cut Methods for Calculating Noise in Open-Wire Subscribers' Circuits Due to Joint Use Exposures to Power Distribution Circuits."

Three reports have been issued dealing with the problem of coordination of open-wire toll circuits and overhead transmission and distribution lines. The first[11] discusses the "Termination of Isolated Exposure Sections to Obtain Normal Metallic-Circuit Currents," which affords a means of taking into account the shielding effects present when the line is in normal operating condition. The second report[12] describes "A Method of Measuring the Balance of Open-Wire Telephone Circuits with Respect to Longitudinal-Circuit Induction," which should be useful to the field in the making of special tests and in supplying statistical data of value for estimating noise effects on open-wire line circuits. The third report,[13] dealing with "Methods of Measuring Noise on Open-Wire Toll Circuits," is a detailed presentation of the various types of tests for studying noise problems on toll lines, and includes a discussion of the method of analyzing the test data.

Another report[6] deals with "The Effects on Inductive Coordination of Generators Feeding Directly on the Line and Operating with Grounded Neutrals." This report includes a detailed discussion of the factors

involved and describes methods which have been developed for control of the triple-harmonic residual currents and voltages which occur with this method of operation.

The results of the work done by the subcommittee on a survey of room noise in telephone locations was described in a recent paper.[14] While this was an incidental phase of the general study on effects of noise on telephone transmission, it was felt to be of timely value, particularly in respect to the methods of measurement employed. Using the results of the data obtained in surveys of wave shape on operating power systems and analyses of noise current on telephone circuits, a paper[15] was prepared on the frequency response characteristics of telephone transmitters and receivers. This paper indicated that there appeared to be no advantage, in reducing effects of noise, in shifting the resonance points of telephone transmitters and receivers from their present region, as the frequency distribution of the noise currents was such as to give a minimum in this resonance region.

At the time that the joint work was started the need arose for considerable special apparatus to make the measurements which were required. Some of the important pieces of apparatus for the work in the voice-frequency range were sensitive single-frequency voltmeters and ammeters. These needs were taken care of by the development of sensitive analyzers whereby single-frequency voltages or currents could be selected from complex wave shapes on either power or telephone circuits. One form of this apparatus has been described in a paper before the Institute[16] and another in a serial report[17] of the National Electric Light Association.

In connection with the survey of room noise, a room noise meter was developed. This was described in the paper[14] previously referred to which presented the results of this survey.

Further Work of the Subcommittee

When the subcommittee started its work there was before it an accumulation of technical problems which had arisen as the arts developed without such close cooperation as now exists. The statements given above regarding various phases of the subcommittee's work on noise-frequency induction indicate the substantial progress which has been made in the solution of these accumulated problems. They convey also a general picture of the work which the subcommittee has immediately before it.

It must not be thought, however, that when these accumulated problems have been solved the work of the subcommittee will be completed and its efforts discontinued. This cooperative work must always bear a relation to the total development efforts of both the power and communication fields. As has already been pointed out, this work is concerned with two electrical arts which have been particularly noteworthy for their success in constantly developing their technical methods and expanding their services. These developments will surely continue and constant consideration of the physical problems of coordination is needed to insure that such developments act to steadily improve rather than to make more difficult the coordination of power and communication circuits.

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Part III—Status of Joint Development and Research on Low-Frequency Induction

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INDUCTION at power system fundamental frequency, commonly called "low-frequency" induction, has different characteristics and produces quite different effects from induction at the noise frequencies discussed in the paper by Messrs. Blackwell and Wills. Since very little has been published on low-frequency induction, it seems desirable, in order to make clear what the Joint Subcommittee on Development and Research is doing on this subject, to explain the problem in some detail.

The disturbances in communication circuits due to low-frequency induction are in general discrete occurrences, coincident with accidental grounds or other faults on neighboring power lines, rather than being continuous and due to normal power line operation.

Three-phase power circuits, when operating normally, are so nearly balanced with respect to earth at their fundamental frequency, and telephone circuits of the ordinary type are relatively so insensitive at frequencies of 60 or 25 cycles, that induction at these low frequencies under normal power line conditions is rarely a practical problem. But when abnormal conditions, particularly faults to ground, occur on power lines, large unbalanced voltages and currents at fundamental frequency exist temporarily and at such times there may be induced in neighboring telephone circuits voltages which are hundreds of times as great as under normal operating conditions. The induced voltages under abnormal conditions may reach values sufficient to cause hazard to telephone employees or interruption to service. Although such abnormal conditions occur infrequently and usually last for only the very short period required to interrupt or clear the power circuit, the effects which may be produced are so serious that protection against this type of induction is an outstanding problem in the coordination of power and telephone systems. A large part of the subcommittee's work has for its object the development of means for controlling and minimizing such induced voltages and their effects.

While low-frequency induction is not usually severe except under abnormal conditions, power circuits which operate at any time on an unbalanced basis, or which are closely coupled to grounded wires capable of carrying large currents may, even under normal operating conditions, create a problem of low-frequency induction in paralleling telephone circuits in addition to setting up high-frequency disturbances as explained in the Blackwell-Wills paper. This is particularly true where the exposed telephone circuits are used for special services such as the transmission of radio broadcasting programs. Grounded types of telegraph and other signal circuits also are sensitive to low-frequency induction.

Classification of Factors Responsible for Inductive Effects

The same three classes of factors which combine to underlie the noise-frequency problem appear also in the low-frequency problem. As they appear in the latter, these are:

1. "Influence factors" in the power system, which are concerned with the magnitude, duration, and frequency of occurrence, of unbalanced voltages and currents.

2. "Susceptiveness factors" in the communication system, which are concerned with the nature and seriousness of the effects produced by the induced voltages.

3. "Coupling factors" which determine the magnitude of the voltages induced in the communication system, per unit unbalanced voltage or current of the power system.

In the low-frequency induction problem, the coupling factors are largely dependent upon the characteristics of the earth and the relations of power and telephone systems to the earth. If the earth were an insulator instead of a conductor there could, of course, be no such thing as fault current in the earth and the coupling between power and communication circuits would be much less. It would not then be hazardous for a lineman when in contact with earth to touch a charged wire. Or if the lines were not in proximity to the earth, there would be no chance for a lineman working on the wires to get in contact with earth. Neither in power systems nor in telephone systems is it actually necessary that the earth be used as part of an operating circuit but, as the earth is a conductor, and power and telephone lines and apparatus are located on its surface, it is essential in both systems that the earth be taken into account in circuit problems and that paths to earth for protective purposes be established at certain points.

It must not be assumed, however, that the earth is a perfect conductor. For the most part the materials of which the earth's crust is composed are of relatively

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low conductivity. From numerous measurements in various places the average conductivity over considerable volumes of earth has been found to range from 10^{-11} to 10^{-15} abmho per cm. cube. The resistance of an earth path is therefore not zero but may be many ohms or even in some cases many hundreds of ohms. Most of this resistance is in the immediate vicinity of the electrodes and can be reduced by increasing the surface area of contact between electrode and ground.

In discussing the low-frequency induction problem, it is convenient to consider the factors controlling:

(1) The magnitude of induced voltages, (2) the frequency of occurrence of induced voltages, (3) the duration of induced voltages, and (4) the effects produced by induced voltages.

Factors Controlling the Magnitude of Induced Voltages

The magnitude of voltages induced in telephone systems in specific cases depends chiefly on the magnitude of residual currents and voltages resulting from power circuit faults to ground and on the exposure conditions.

Residual Currents and Voltages. A balanced power circuit is one in which the voltages from the various phase conductors to ground are equal and sum up vectorially to zero and in which the phase currents also are equal and sum up to zero. Under this condition all the currents in the circuit are *balanced* currents and all the voltages are *balanced* voltages. If, however, one phase develops a fault to ground, this relation becomes disturbed, the voltages to ground of the phases become unequal, and their vector sum, which is the *residual* voltage (3 times the so-called uniphase or zero-phase sequence voltage) of the power circuit, is no longer zero. The currents in the three phases likewise become unequal and when added vectorially their sum, which is the *residual* current (uniphase or zero-phase sequence current) of the power circuit, is no longer zero. In most low-frequency induction problems residual current is far more important than residual voltage.

Residual voltages and currents are equivalent to single-phase voltages and currents applied to a circuit consisting of the three line conductors in parallel as one side, and the earth as the other side. Their large inductive effects are due to the great dimension of the loop formed by this earth return circuit, much of the return current being effectively so deep in the earth that its neutralizing action is small. In the case of the balanced components, the inductive effect due to the voltage or current of one conductor is largely neutralized by the voltages or currents of the other two conductors.

The chief characteristics which determine the magnitude of the residual voltages and currents are (1) the power circuit voltage, (2) the impedances of the neutral ground connections, (3) the line and apparatus impedances, (4) the fault and earth impedances, (5) the sources of power supply, (6) the character of ground wires if used, and (7) the circuit configuration including ground wires.

When a fault occurs between a phase conductor and earth on a power system having neutral ground connections, these neutral connections, together with the fault, line conductors and earth, form a closed circuit for the residual current. Unless the neutral impedance is very high, *e. g.*, approaching that of an isolated system, the shunting effect of the capacitance to ground of the line conductors may for most purposes be neglected and practically the same value of residual current exists at all points along the line between the fault and the neutral connection to ground. For simplicity, a system with a single line and single neutral ground connection may be assumed. With this picture in mind, it is clear that the value of the neutral impedance may be an important factor in determining the magnitude of the residual current. If the fault occurs near the point where the neutral is grounded, the line and apparatus impedances being low, a small impedance in the neutral may control the current. On the other hand, for faults occurring at points remote from where the neutral is grounded, the impedance in the neutral connection may have to be relatively large to materially reduce the residual current.

As one limit there is the solidly grounded neutral, *i. e.*, no impedance is inserted and as good a ground as practicable obtained. This obviously permits maximum residual current when ground faults occur. Unless the grounding impedance is very high the residual current, and not the residual voltage, is the controlling factor in grounded-neutral systems.

As the other limit there is the isolated neutral, *i. e.*, the impedance from neutral to earth is infinite. In this case no residual current passes through the neutral. At the ends of the line the residual current is zero, gradually increasing to a maximum at the point of fault. The circuit for residuals is through the capacitance of the line to ground, the magnitude of this capacitance controlling the magnitude of the residual current, which is much less than with grounded neutral systems except in cases of double faults when it may be very large. With a single fault the residual voltage may be a more important factor in respect to induction than residual current.

The impedance of the fault itself depends upon a number of things, including the type of line construction and the earth conditions. The subcommittee has under way investigations to gather data on the range of fault impedances under different conditions. To determine the maximum residual current, the fault impedance may be taken as zero. In many instances, this approximation gives sufficiently close results, particularly if the fault is remote from the grounded neutral so that line, neutral and apparatus impedances are controlling.

In case of conductors falling upon the ground, local earth conditions largely determine the fault impedance. On a steel tower line an insulator breakdown results in a relatively short arcing path to grounded metal, whereas, in wood pole construction, the pole itself introduces considerable impedance unless nullified by guys or other metal.

The foregoing discussion of residual current has been confined practically to the situation brought about by single faults to ground. Double faults at separate locations sometimes occur and these are equivalent to a phase-to-phase short circuit through the earth, giving a large residual current in the intervening section of line. If the two faults in such a case are on opposite sides of an exposure, very severe induction may result. Experience shows that double faults at separate locations constitute only a few per cent of the total faults occurring on grounded-neutral power systems but are a much larger percentage of the total faults on systems normally isolated from ground.

The presence of ground wires on a line may have considerable influence on fault impedance. Being connected to ground at frequent intervals, such wires decrease the impedance to ground where a breakdown occurs between a phase conductor and a ground wire or any metal in contact with a ground wire. A ground wire tends to increase the total residual current but on the other hand its controlling function, from the induction standpoint, is that of a shielding conductor tending to decrease the induced voltage.

Circuit configuration does not have a large influence on unbalances due to abnormal conditions, but it has an important effect upon any unbalance of a power circuit under normal operating conditions. To be balanced, the phases of the power circuit must be symmetrical with respect to each other and to earth. To the extent that the capacitances and inductances of the several phase conductors differ, residual voltages and currents will result. Transpositions afford a means for compensating for these circuit unbalances.

In cases for which protective measures are being considered, it is important to be able to estimate the magnitude of the residual current when faults occur at different points on the power system. Apart from inductive effects, this is a question of importance to power companies, since forecast of currents under different fault conditions is essential in the design and setting of protective relays. Much work has therefore been done by different investigators on methods of predetermining these currents. Helpful mathematical methods have been developed, though sometimes the results obtained by their use are open to question due to lack of accurate values of some of the important impedances. Proper allowance for fault impedance and the effect of ground wires is sometimes difficult to determine and in cases of complicated networks approximations usually have to be made. To facilitate the numerical computations, calculating boards of varying degrees of elaborateness have been developed. The subcommittee is investigating this matter and by experimental work is checking the results of estimates and acquiring further knowledge of the range of the variable factors. Through this work, it is hoped to increase the convenience and accuracy of these important computations.

Exposure Conditions. The relationship between power and telephone lines with respect to the exposure conditions is defined by the "coupling coefficient" or "coefficient of induction," a factor which, when multiplied by the value of current (or voltage) in the power line, gives the resulting voltage set up in the telephone line. A power line and a neighboring telephone line have several different coupling coefficients corresponding to different conditions, such as, whether the induced voltages are due to power current or power voltage, to balanced or residual components, and whether they are voltages induced along the conductors (or to ground) or are induced directly in the metallic circuit. Low-frequency induction is predominantly magnetic in character and the coupling which is most significant is that between the power conductors and the telephone conductors, both considered with earth return. The induced voltages are due principally to "longitudinal circuit induction."

A number of dimensional factors affects the magnitude of this coupling, such as the length of the exposure, the separation between lines, and the locations of ground connections on the two systems. Local conditions as to earth conductivity and the arrangement of geological strata for some distance below the earth's surface, constitute other important factors. An accurate mathematical evaluation of the coupling between earth return circuits is difficult. Formulas have been developed under simplifying assumptions as to symmetry and homogeneity, which aid in explaining and interpreting experimental results and in predicting approximate values of coupling in cases where experimental measurements are not available.

Assuming uniformity of exposure conditions the coupling varies directly with length of parallelism, except for end effects or interactions between ground connections of the two lines. Increase in separation of the lines diminishes the coupling but the exact relationship depends upon the distribution of current in the earth which in turn depends upon the frequency and the earth conductivity. In many cases different strata of different conductivities are involved in the path of the earth current, which adds to the difficulty of correlating experimental and theoretical results. The effect of earth conductivity on coupling is accentuated as the lines are more widely separated. At roadway separation, large differences in earth conductivity affect the coupling only moderately; but at separations of one-half mile to one mile, coupling values may differ by 20 to 1 or more, due to the range

in value of earth conductivity. Irregularities in exposure conditions such as changes in direction of one or both lines, crossovers, and angular exposures of varying separation, are complications which frequently occur in practise.

The voltages set up in neighboring communication circuits by power currents are due usually to inductive coupling but in some cases are due partly or wholly to resistive coupling. It is seldom necessary in practical studies to try to segregate these two components of voltage, since their effects in the telephone system are not a function of the phase relationships of these components to the power line current which produces them. It is not unusual to speak of inductive coupling as including both inductive and resistive coupling.

Any grounded circuit in proximity to power and telephone lines within an exposure brings about a certain amount of shielding through the reaction of the currents induced in this conductor upon the primary magnetic field set up by the residual current in the power circuit. In this respect a shield wire acts like a short-circuited turn on a transformer. The effectiveness of the shielding depends upon the conductance of the shield wire, the manner and effectiveness of its grounding, and its position with respect to the power and telephone wires. Such a wire affords maximum shielding when closely coupled to either the power wires or the telephone wires, when its conductance is high, and when its ground connections are of low resistance.

The variation of coupling with separation and with earth conditions is of great practical importance in the coordinated location of lines. Most of the subcommittee's study of coupling, therefore, involves field investigations of the variation of coupling with separation under different earth conditions, and is furthermore directed toward devising convenient and accurate methods of predetermining coupling in practical cases. Also, by studying and correlating experimental data derived under different conditions and from widely separated parts of the country, the subcommittee hopes to arrive at a better empirical basis for estimating coupling. Some of the work on this subject has already been presented.[1]

Factors Controlling the Frequency of Occurrence of Induced Voltages

The frequency of occurrence of induced voltages in paralleling communication lines, while chiefly dependent on the frequency of occurrence of faults on the power line, is also somewhat affected by the location of the exposure with respect to the location of neutral grounding points. For example, if there is only one neutral ground, faults occurring between it and the exposure will produce relatively little induced voltage.

The frequency with which faults occur is usually traceable to features of electrical and mechanical design, the character and amount of insulation, and the location of the lines. Specifically, the factors which appear to be responsible for the majority of faults on power lines are: poor configuration, inadequate spacing and clearances, inferior insulation, lightning, fog, smoke and dirt, birds and animals, proximity of lines to external objects apt to interfere with operation mechanically or electrically, and certain mechanical features of design affecting the strength of construction, such as ineffective anchors, guys, or conductor and ground wire supports, particularly at angles and dead-ends, and insufficient bearing areas of subsurface structures.

Factors Controlling the Duration of Induced Voltages

The length of time faults are permitted to remain on a power system is controlled by the kind of protective relaying employed and by the type and condition of the circuit breakers and other terminal equipment. The type of relay system, the degree of sectionalization obtained, the adequacy of the circuit breaker as to speed and rupturing capacity, and the maintenance of the equipment are the most important factors.

Generally, the type of fault has little effect on the duration if there is sufficient current to operate the relays. Conditions have been noted, however, where the fault is of such high impedance that the current is not adequate for the operation of the relays. Such high impedance faults usually occur on wood pole lines and may result in burning of pins, crossarms and poles. They may also occur on steel tower lines as the result of branches of trees getting in contact with conductors.

Effects Produced by Induced Voltages

Low-frequency and transient voltages induced on telephone circuits may produce a variety of effects depending upon their magnitude and duration. These effects include service interruption, false signals, telegraph signal distortion, damage to plant, electric shock, and acoustic shock.

Telephone circuits are very low energy circuits, the voltage for talking purposes rarely exceeding one or two volts, with maximum current measured in milliamperes. For signaling purposes a maximum of 165 volts peak, is used with currents limited to about 0.10 ampere. For telegraph service the voltages are limited to 135 volts between wire and ground, while the current is limited to less than 0.10 ampere. By contrast, the voltages due to induction, in some cases of exposure, may be a thousand volts or more.

Service Interruption. When the telephone protectors are operated by induced voltage the behavior of the protector discharge gaps depends upon the magnitude of the voltage and current and the length of time the discharge lasts. In cases where the discharge is not promptly extinguished or where the current is very

1. For references see Bibliography.

high, the discharge gaps may become permanently grounded. This causes interruption to service until the affected protectors can be replaced, the time necessary for such replacement depending, of course, upon the protector locations.

False Signals. False switchboard signals are likely to be coincident with protector operation They produce a bad service reaction due to operators answering false calling signals and cutting off connections because of false disconnect indications.

Distortion of Telegraph Signals. The induced voltages appear in just the same paths over the wires as the operating voltages of grounded telegraph. The effect of such induced voltages depends on their magnitude, character, and duration. Voltages much lower than those sufficient to operate the protectors may cause detrimental effects ranging from a slowing down of speed to complete failure. Where the duration is short, the effect may be limited to distortion of signals, or, if the voltages are high enough, to momentary interruptions.

Damage to Central Office or Other Telephone Plant. The dielectric strength of the telephone plant is adequate for the voltages used in communication service, with appropriate factors of safety, but higher voltages may sometimes, notwithstanding the protective devices, cause dielectric failure, thus damaging the plant, particularly cables and wiring or apparatus in telephone offices.

Electric Shock. Telephone linemen in the course of their work upon wires at relatively close spacing, cannot avoid getting in contact with the wires and if the wires were subject to sufficient induced voltage, the men would be liable to receive electric shocks. On severely exposed lines such voltages are liable to occur at any time, suddenly and without warning. Electric shock might either injure a linemen directly or startle him and cause him to lose his hold and fall from the pole. Voltage to ground due to induction appears not only within the exposed section of line but considerably beyond. A similar, and in some respects worse, condition may exist with respect to employees working on cable circuits which are either exposed or directly connected to exposed circuits. In cables the wires on which the foreign voltage appears are very close to the grounded metal sheath and usually also to other wires at approximately earth potential, as well as to the earth itself. This problem has become more difficult with the rapid growth of the telephone and electric power systems and is engaging the subcommittee's serious attention.

Acoustic Shock. Acoustic shocks are liable to occur with the breakdown of telephone protector discharge gaps, which temporarily unbalances the circuit and causes a sudden and abnormally large current in the receivers. This current gives rise to sudden and severe flexures of the receiver diaphram, which produce loud sharp noises in the ear of a person using the receiver. Telephone operators, due to the nature of their work, are particularly liable to acoustic shocks, the effects of which range from minor reactions to severe general disturbances of the nervous system which may be painful and of long duration. In addition, if danger of severe shocks exists, the operating force may become fearful and the impaired morale seriously affect the service.

Types of Protective Measures

The foregoing effects of induction from paralleling power lines may be reduced by: (1) measures in the power system to limit the influence, (2) measures in the communication system to limit the susceptiveness and (3) coordinated location of lines or other means to reduce the coupling. As a solution in a specific situation, one measure may be sufficient or two or more measures may be required, depending on the conditions. The solution should afford the necessary protection without hampering the development or operation of either system. Where there are two or more alternative solutions, the one which is best from the engineering standpoint, including both the technical and economic aspects, should of course be applied.

Cooperative planning in advance of construction is especially important in situations involving low-frequency induction, because of the wide ranges in magnitude both of coupling factors and of residual currents. By advance notifications of construction it is possible to bring up for analysis the low-frequency effects which the proposed construction would bring about and, if necessary, to agree upon changes in the plans to prevent or reduce these effects.

As to the physical dimensions and relations of power and telephone lines which constitute an exposure there are no blanket rules for guidance; each case requires specific consideration. Due to differences in geological conditions and other variable factors, a given length of parallelism at a given separation might give satisfactory results in one location, whereas an exactly similar physical relationship of lines in another location might result in the communication system being rendered inoperative at times of power system fault. This fact emphasizes the necessity of advance planning and cooperative study of situations as they arise. Such cooperation may easily lead to a satisfactory solution of situations which at first seem very difficult. On the other hand situations which at first appear devoid of any possibilities of trouble may on careful study be found to require protective measures.

Protective Measures for Power Systems. It will be evident from the foregoing discussion that protective measures to reduce the inductive influence of power systems should be directed to limiting the magnitudes of unbalanced currents and voltages, particularly under abnormal conditions, and to reducing the duration and frequency of occurrence of abnormal conditions. Of such protective measures some are concerned with fun-

damental questions of line and system design and must be incorporated in the construction plans, while other measures are of such a character that they may either be incorporated in the original construction or added later if found necessary as a result of subsequent experience or developments in either the power or telephone system.

Fault-Resistive Design and Construction. As mentioned in the paper by Messrs. Harrison and Silver the methods employed in reducing the frequency of occurrence of faults are primarily involved in the design and construction of the power line, *i. e.*, adequate insulation, clearances, and spacings, and so arranging the component parts of the structure that the line will in effect be fault-resistive. Increasing demands for better service by the public combine with considerations of inductive coordination to justify greater attention to fault-resistive line construction.

Faults may result from improper guying of poles, *i. e.*, guys so located that the spacing between guys and conductors is inadequate, or the path from insulator to crossarm brace and thence to the guy is insufficient to withstand the voltages imposed. The conductor spacing may be inadequate or the configuration of the circuits may be such that the sudden unloading of conductors coated with sleet will result in their whipping together, or, if a ground wire is used, it may be so located that the unloading of sleet will cause the conductors to whip into the ground wire, or the design of the line, either steel tower or wood pole, may be such that inadequate strength is provided for the mechanical loads incurred.

Attention is being given to the location of lines as a material factor in limiting the number of outages resulting from external sources, such as lightning, broken trees, blasting, and automobiles. For example, lines built in valleys are less subject to failures due to lightning and wind storms than lines built over hills.

There is little need to call attention to the grade of insulation employed on power lines as recent lightning studies and papers have emphasized the importance of rationalization of insulation throughout the plant. By this method it is hoped that preferential points of failure would be established, thus permitting prompt restoration of service without damage to expensive equipment since most of the faults would be confined to the line.

The amount of insulation to be employed on lines is affected by topographical and climatic conditions. Lines in areas relatively free from lightning or shielded from lightning disturbances may, of course, employ less insulation without increasing the number of faults. On the other hand, lines built in areas where lightning is prevalent may justify not only higher insulation but also, on steel tower lines, the use of ground wires as an additional protection. Areas where salt fog, smoke, or chemical fumes are prevalent require special treatment as to the form of insulation used.

Laboratory tests and limited field experience indicate that a proper utilization of the inherent insulating properties of wood in structures may result in considerable improvement in line operation. The subcommittee is investigating the service performance of wood pole lines of differing designs with a view to determining how much may be accomplished in reducing the number and severity of faults by suitable arrangements of metal braces, fittings, guys, etc., to avoid so far as possible shunting out the insulation of the wood.

To the experienced designer the protective measures to be employed on lines subject to frequent faults are obvious, namely, the rearrangement and reconstruction of the tower or pole top to obtain greater spacing between conductors or greater clearance between conductors and other metal parts. In some cases spacing and clearances would be materially improved by utilizing a triangular configuration so that the conductors are not likely to come in contact with each other or the ground wire when sleet or other conditions cause whipping or dancing of the conductors. In other cases, merely a relocation of the point of attachment of guys would improve conditions without materially decreasing the strength of the structure.

Fault-Current Limiting Measures. Resistors, or reactors, in the neutral ground connection of a power system provide a means of directly limiting the magnitude of the residual currents, except in cases of double faults. In cases where the residual currents can be so far reduced as not to set up induced voltages of high values in the communication system without reacting unfavorably on power system operation, this method alone may afford a satisfactory solution. In such cases it has the further advantage of reducing the stresses to the power system due to the fault current. Where it is impracticable to clear up a situation by residual current limitation alone, this method may be effectively used in combination with other protective measures.

The reduction in residual current which will be brought about by adding a given amount of impedance in a neutral ground connection can be estimated with reasonable precision. It is not so much this question therefore, that requires study by the subcommittee as it is the question of the limitations and costs of this protective measure, and its reaction upon the power system. Included in this work is a study of the relative advantages of inductance as compared with resistance for accomplishing such current limitation. The subcommittee has under observation a number of installations of current-limiting devices and is engaged in experimental and theoretical studies and in field observations by means of recording instruments to determine the possibilities of this type of protection.

In non-grounded power systems a single fault on a phase conductor results in the charging current of the system flowing to earth through the fault. The other phases, rising to full line voltage above the grounded phase, create a system unbalance which may manifest

itself by induction in paralleling communication lines. In such cases the problem is one of electric induction except for the magnetic induction set up by the charging current.

When double faults occur on either grounded or non-grounded systems, severe magnetic induction is liable to result and under these conditions it is difficult to limit the residual current.

Shielding. Ground wires on a power line, while tending to increase the total residual current, serve the purpose of shielding by reducing the strength of the external electric and magnetic fields set up by the residual voltages and currents. The net effect of ground wires from the low-frequency standpoint is to reduce the voltages induced in paralleling communication circuits under abnormal power circuit conditions. The effectiveness of such shielding depends on the impedance of the shielding conductor and its ground connections. Under favorable conditions the induced voltage at 60 cycles in paralleling communication circuits may be reduced about 40 per cent by this method. Such ground wires, if used on wood pole lines, have a disadvantage in that they impair to some extent the insulating property of the poles.

High-Speed Circuit Breakers and Relays. Very sensitive high-speed relay systems have been developed which, together with high-speed types of circuit breakers reduce the time duration of a power line fault to approximately 1/10 second, as compared with one-half second to three seconds required by the older forms of relays and circuit breakers, thus tending to minimize the effects of induction. On the other hand inadequate relaying, or the omission of automatic circuit breakers, may extend the duration of faults to a point where the hazards to power apparatus are serious. High-speed breakers and relays are expensive and it is difficult to justify them solely as a remedial measure for induction, particularly as the speeds of operation now available for relays and breakers on power systems, have not reached values which make them a complete solution of coordination problems. However, with the increasing size and interconnection of power systems, high-speed relays and circuit breakers are playing an increasingly important part in promoting power system stability.

Periodic testing of relays and circuit breakers accompanied by complete overhauling at regular intervals, will do much to reduce the duration of faults and to prevent improper functioning of the equipment.

The subcommittee is following the developments in high-speed breakers and relays with much interest. If such devices should come into general use for all classes of service it is expected that they would materially improve the whole inductive situation.

Improvement in Balance. As mentioned above, low-frequency induction between power and communication lines is sometimes experienced under normal operating conditions. On grounded telegraph and signal lines the trouble usually manifests itself by a chattering of telegraph instruments or by false signals. Improvement in balance of the power line by transpositions will in some cases correct the difficulty.

Protective Measures for Communication Systems. In general, measures applicable to the communication system to prevent or reduce the effects of induced voltages take the form of arrangements or devices for removing or counteracting the voltages to ground or the currents in the telephone circuits which might be produced by the induced voltages.

Bell System Standard Protectors. It is Bell System standard practise to equip all telephone circuits which are exposed to the liability of foreign voltages, with electrical protective devices. These devices are made in various forms and combinations for different plant and exposure conditions. The protector used at central offices and at subscribers' stations includes a discharge gap which operates at approximately 350 volts and a fuse which opens the circuit at about 10 amperes. Such devices are intended to offer a measure of protection against lightning discharges and against the voltages and currents resulting from accidental contacts with foreign wires or from low-frequency induction.

In order to protect telephone linemen or others working on open-wire lines against electric shock from induced voltages, it is necessary that the voltages between line wires, and between each line wire and ground, be kept low. The use of protectors at central offices does not so protect the linemen as the impedance drop on the line wires permits high voltages between wires and ground at other points, such as the terminals of the exposed section.

It appeared however, that protectors of the Bell standard type might be used on open-wire lines at locations immediately adjacent to exposures to limit induced voltages to ground. A number of installations of this kind has been made but observations over a period of time show that they introduce serious troubles as the protectors, being subjected to heavy discharges, often become permanently grounded thus interrupting service. It also sometimes happens, as all the line wires are not always equally exposed, that some of the protectors operate and others do not, resulting in objectionable voltages between line wires.

Relay Protectors. In view of the inadequacy of existing forms of protectors for such use, the subcommittee is experimenting with a "relay protector." This device includes Bell standard protectors in combination with a relay which operates to short-circuit them upon the occurrence of a discharge, thus relieving the protectors of the duty of carrying the large discharge current and greatly reducing their tendency to become permanently grounded. In more recent types all the relays at a protector point are electrically interlocked, so that when any relay operates all line wires are grounded within a few cycles.

Several trial installations of relay protectors have

been made and are under observation. To guard against voltages to ground within the exposure these protectors have to be placed within, as well as at the ends of, the exposed section of line. Where the longitudinal induced voltage is large, protectors are required at a number of points within the exposed section.

The effective application of such protectors requires grounds of the order of one or two ohms and an important feature of the investigation is to devise methods of constructing and maintaining such grounds at remote points along the line.

The subcommittee is investigating in the field and in the laboratory the effectiveness, cost, reaction on service, and other practical questions relating to the installation and maintenance of this method of protection.

Acoustic Shock Reducers. Since acoustic shock due to induced voltages involves dissymmetrical discharges across the two sides of the protector, efforts have been made to devise a protector which would break down and discharge symmetrically, *i. e.*, provide two reliable low-impedance paths for heavy discharges, which would at all times have very closely the same arcing impedance. Thus far the subcommittee has not been successful in developing a practicable protector of this kind.

For the purpose of equalizing the voltages on the protector during the discharge period, an accessory device termed a "discharge balance coil" is under investigation. It consists of two equal windings on a common core, each in series with the discharge gap of one side of the line, and so arranged that the fluxes set up by the circuits in the two windings are in opposition. The "booster" action of this coil tends to equalize the discharge currents. This reduces acoustic shock from induced voltages, provided all protectors are so equipped and the line itself has no large unbalances. When however, voltage is impressed on one wire only of a telephone circuit, as by accidental contact, these coils have a detrimental effect on the action of the protector in reducing voltage to ground, as they introduce impedance in the protector discharge path.

Development work is also being conducted on other types of acoustic shock reducing measures which do not attempt to prevent unbalanced current but merely to shunt it out of the telephone receiving circuit. Obviously a device acting on this principle to be successful must be practically instantaneous in operation. One of the most promising of such devices consists of a high ratio step-up transformer with its primary connected directly across the receiver to be protected. The secondary is connected to a low-voltage discharge gap. Any abnormal voltage across the primary operates the discharge gap and the transformer becomes a low-impedance shunt. A number of field trials of these reducers applied to operator's receivers has been made. While not affording the full degree of protection desired they have been found to reduce substantially the severity of acoustic shocks and it is believed that they will be of considerable benefit in cases where some form of protection against acoustic shocks to operators is urgently required.

Another device based on the shunting principle consists of opposingly poled copper oxide rectifiers connected across the receiver. These have the property of greatly diminishing impedance with increasing voltage. The problem is to obtain a sufficiently sharp change in impedance with voltage, while avoiding a normal impedance so low as to cause serious transmission losses. As an aid to this end, biasing batteries are under investigation.

The committee has also investigated the saturating characteristics of a vacuum tube for acoustic shock reduction. The properties of a vacuum tube are such that the output current cannot be increased substantially beyond a definite value regardless of the input voltage. This feature can be made use of to limit shocks by a design which will pass currents substantially without distortion up to approximately the highest value of signal current used, thus cutting down the shock voltages which exceed the normal signals. While quite effective, this method involves apparatus which is more bulky and expensive than the transformer and spark-gap type reducer. Telephone repeaters accomplish this result to some extent and are being investigated by the subcommittee, to determine the quantitative reduction of acoustic shock by this means under practical conditions.

In cases where toll or trunk lines are exposed, an acoustic shock reducing device which could be placed at the ends of the lines would have the advantage of protecting subscribers as well as operators. Development work to obviate certain difficulties in using such a device is under way.

An effort is being made to develop a telephone receiver which will saturate between the values of current required for effective speech transmission and values of current which produce acoustic shock. This requires a sharp bend in the saturation curve of the iron employed in the receiver magnetic circuit. Until the development of permalloy, this feature was not approachable, but experimental permalloy receivers have now been developed, and, while it has not yet been possible to achieve the end sought without serious sacrifice in transmission, work along this line is continuing.

Improved Insulation. A slight reduction of susceptiveness to interference by low-frequency induction could be secured by providing increased dielectric strength to ground in communication circuits and their associated apparatus. Another method would be to insulate or isolate all conducting parts of the communication system so as to prevent contact by employees or others with wires or apparatus which may carry a dangerous voltage. Neither of these appears practicable at this time.

Drainage. Drainage is a method for controlling the

parts of the circuit in which the induced voltages appear and causing these voltages to be consumed in those parts where they are least harmful. This is accomplished by connecting the telephone conductors to ground, preferably through balanced impedance coils, at certain points throughout the exposure. Assuming low resistance grounds at the drainage points, the resulting voltage to ground at such a point after drainage is established is limited to a value corresponding to the voltage drop over the impedance of the coil and ground connection. If this impedance is small compared to the other impedances in the drainage section, the voltage to ground at the drainage point is a small part of the total voltage induced in that section.

Under present conditions, the application of drainage is limited to special situations where interference with circuit testing and maintenance is of relatively minor importance and where superposed d-c. telegraph and carrier telephone are not used.

Neutralizing Transformers. The neutralizing transformer is a device for introducing into an exposed communication wire a voltage in opposition to the voltage induced by the disturbing circuit, thereby to a certain extent neutralizing the latter. The neutralization is effected by means of transformer action, the primary coils of the neutralizing transformer being connected to conductors which are grounded at the terminals of the exposure (or section of exposure), so that the voltage induced in these conductors will send currents through the transformer primaries. These primary currents induce in the secondaries of the transformers voltages substantially in opposite phase to the voltages induced in the telephone wires by the power circuit. The secondaries being connected in series with the exposed communication wires, the neutralizing action is obtained.

On account of introducing crosstalk and adversely affecting telephone transmission and carrier, application of neutralizing transformers has been confined chiefly to telegraph circuits. No applications of these devices to power line exposures have been made. They are, however, being studied by the subcommittee to see whether the objections mentioned above can be overcome and to determine their possible field of application.

Shielding. Shielding on a telephone line may be effected by special grounded conductors, by working conductors, or by cable sheaths. Miscellaneous structures such as pipe lines or rails in the immediate vicinity of an exposure also introduce more or less shielding. The employment on a telephone line of a high conductance shield wire, well grounded at the ends of the exposure and at intermediate points, may reduce the induced voltage by as much as 40 per cent at a frequency of 60 cycles. As bearing on the prevention of electric shock from induced voltages on telephone lines, shielding has a disadvantage in that it may, depending somewhat on the method of construction, add to the chance of a lineman making contact with grounded metal.

Use of Cable. A metallic sheath enclosing the conductors of a cable is a type of shielding. The lead sheath of a 2⅝ in. diameter aerial telephone cable, if effectively grounded at the ends, as when directly connected to an underground cable sheath, reduces the voltages induced in the conductors within the cable by about 50 per cent at 60 cycles. The additional shielding brought about by the surrounding earth when such a cable is placed underground is negligible at low frequencies, although underground construction has an advantage in affording a low-resistance ground for the sheath. The large number of conductors in a cable affords mutual shielding which varies from a negligible to a considerable amount depending upon many factors, important among which is the extent of the cable beyond the ends of the exposure. If two or more cables are close to one another through an exposure, each benefits by the shielding action of the others, so that the shielding increases with the number of cables.

If the lead sheath of the cable is surrounded by magnetic material as by armoring or placing cable in iron pipe, the shielding may be largely increased. With the form of iron tape armored cable referred to in the Harrison-Silver paper, which is now in trial use, shielding at 60 cycles is about 80 per cent, assuming effective grounding. Armoring a cable increases its cost substantially but has an advantage apart from shielding in that the cable being protected by the armor against mechanical injury may be buried directly in the earth without conduit. The armor is protected by impregnated wrappings but its life has yet to be determined. The shielding afforded by this type of cable has been studied experimentally under practical field conditions. Other installations and studies have been made abroad. It is probable that there may be a field of use for this type of cable in situations for which it is best adapted.

Coordinated Location of Lines. Since the magnitude of induced voltages for given power line conditions depends upon the inductive coupling of the two classes of lines, which in turn is dependent upon their relative location, particularly their separation and length of parallelism, it is possible by advance cooperative planning of new power and telephone line locations to minimize and in some cases to forestall inductive effects in the telephone system. If the cost of remedial measures which inductive exposures would render necessary can be avoided, additional expense in locating lines to avoid such exposures may be justified and where a complete solution is obtained in this way both parties secure greater freedom in the construction and operation of their lines. However, with the rapid expansion of both services, the possibilities of complete solution by separation of lines alone are becoming more and more rare, particularly for lines along highways.

Coordination of Grounding Practises. The occurrence of a fault on a power system usually results in raising the ground potential at the points of grounding as well as at the point of fault, but if steps are taken to coordinate the grounding of the power system and the telephone

system serving the power company, particularly at transformer and generating stations, the effects in the telephone system of the earth potential gradient caused by a power fault may be minimized. For example, if in a switching station the same ground should be used for the power system neutral and for the telephone system, a power fault might cause the switching station ground to rise many volts above the distant telephone exchange ground, and result in operating the telephone protectors and possibly interrupting service. If, however, independent grounds sufficiently separated are used at the switching station, or an insulating transformer is placed in the telephone circuit, the power neutral ground may rise in potential without unduly affecting the telephone system.

Comprehensive consideration of the low-frequency coordination problem involves a study of the reactions between the grounding practises employed by power companies and those employed in telephone and telegraph systems. There is considerable diversity in practise with respect to methods of grounding. Some power transmission lines and primary distribution lines are not provided with any designed grounds, although most such lines have grounded neutrals and a few lines are grounded in such a way that operating current flows through the earth. In built-up communities there are underground pipes, cables, and other structures along which current in the earth will flow to a greater or less extent. These structures have varying degrees of conductivity and some of them have, either by design or by accident, high-resistance joints. Consequently the paths of earth currents are exceedingly complex. The conditions as to earth currents and earth potentials necessary to be known in order to work out any coordinated scheme of grounding would usually have to be determined by tests.

The different kinds of grounds to be considered include those on: power transmission circuit neutrals, lightning arresters, power distribution primary neutrals, power distribution secondaries, railway systems, building conduits, telephone protectors, batteries, ringers, telegraph circuits, lightning rods, electrolysis protection systems, various types of signal circuits such as fire and police alarm systems, and so on. The grounding practises for all these different systems should be carefully studied and coordinated in order to prevent so far as possible harmful reactions among them. Such a study of course goes considerably beyond the scope of this subcommittee.

Comparison of Different Protective Measures. The ideal protective measure would be one which furnished adequate protection and had no unfavorable reaction from an economic or service standpoint on the system to which it is applied. However, the work thus far has not disclosed any measure which fully meets this ideal.

The relative advantage of different measures resolves itself into a question of the best technical results which can be obtained at the least over-all cost. The solution of problems consists of finding measures which afford the highest degree of protection which is practicable and reasonable under the circumstances. In the investigation of a specific case it may be found that certain protective measures can be combined with other work in such manner that the cost is not wholly chargeable to coordination for the reason that other results of value are secured. For example, shielding may be obtained at small cost if improvement of performance of a transmission line justifies the installation of ground wires; or, the benefits of shorter duration of induced voltage by the use of high-speed circuit breakers and high-speed relays may be secured in connection with a program for improving the stability of power systems.

No other measure affords such complete protection against all effects of induction as adequate separation. However, measures applied to power systems such as fault current limitation which strike directly at the source of low-frequency induction are of a basic character and permit a closer association of the two classes of lines, a very important consideration in congested areas. Measures which affect only the frequency of occurrence of faults, or their duration, while very helpful, are not as effective from a protection standpoint as measures which limit the magnitude of residual currents and voltages.

As to measures which would allow telephone circuits to operate through a strong inductive field, the use of lead-sheathed cable surrounded by magnetic material seems to offer the physical possibility of affording the most effective protection. Precautions would be required, however, to prevent the shielding structure itself from rising to a dangerous potential with respect to earth. On open-wire lines where the occurrence of high induced voltages cannot be prevented, some form of protector for limiting the magnitude of voltage to ground seems to be a logical line of development.

Devices such as acoustic shock reducers, which protect only against a single effect of induced voltages, do not afford a solution of most specific situations, but have to be used in combination with other protective measures. In many situations, no single protective measure is adequate and if the exposure is severe several may be required.

In considering the effects which a new exposure may produce, all the relevant factors are capable of advance determination except frequency of occurrence of induced voltages, which has to be estimated on the basis of experience or judgment and a statistical analysis of line failures.

Selection of measures to be employed in specific cases should be made with the above considerations in mind to the end that the best engineering solution may be obtained irrespective of whether the protective measures are applied to the telephone system, to the power system, or to both.

Reaction of Physical Exposures and Lightning on Low-

Frequency Induction Problem. As telephone circuits which are exposed to induced voltages may also be exposed to possible contact with power circuits and to lightning, any comprehensive scheme of protection must take into consideration the high currents resulting from contact and the high voltage due to lightning. In this connection there are some points of difference in the reactions on the protection scheme of induction, contact, and lightning.

Contacts between power and telephone wires may occur at crossings or conflicts or they may occur on joint pole construction as described in the paper by Messrs. Huber and Martin. In any event such contacts can occur only where the two lines are in close proximity, whereas in cases of inductive exposure, a fault outside as well as inside the exposure may produce disturbances in the telephone circuits. Moreover, in cases of contact, wire or structure failures are generally involved while faults may cause induction which do not involve falling wires. Contacts impose on the telephone line the full voltage to ground of the power conductor at that point, whereas induced voltage is usually only a fraction of the power circuit voltage. This does not mean that the imposed voltages due to contact are always higher than those due to induction, because the majority of exposures to contact do not involve the higher voltage circuits while the opposite is true regarding inductive exposures. In cases of contact only part of the wires of the telephone line are usually involved whereas in the majority of induction cases substantially the same voltage is induced on all the wires. The voltages imposed on a telephone line by contact as well as those by induction may extend over the full length of the conductors involved.

In addition to the effects of contact between wires of the two systems, there is a distinct class of hazard to linemen of both utilities introduced by situations of insufficient clearance due to improper construction or inadequate maintenance on the part of one or both utilities.

Voltages on telephone lines by lightning produce effects somewhat similar to the effects produced by power lines but lightning voltages differ from the other voltages in that their duration is much shorter. Lightning makes necessary protector discharge gaps of very high speed of operation in order to prevent serious overvoltages on the telephone system, whereas contacts with power circuits make necessary a protector of high current-carrying capacity.

Committee's Program of Work

The program of work on low-frequency induction undertaken by the Joint Subcommittee on Development and Research through its project committees is laid out to develop the essential facts bearing on the problem of telephone protection in a broad sense, including causes, effects, and remedial measures. The program covers not only the technical but also the economic aspects of the problem. The problems of lightning and physical contact under conditions of conflict or joint use are also included, as the measures finally adopted must protect against voltages from these sources as well as voltages induced by power systems.

Extensive field trials of all promising protective measures, are under way in order to determine their practicability under operating conditions. As the work progresses, it is expected to issue from time to time reports covering the applicability, efficacy, limitations, and conditions of use, of various measures. This should result in a better understanding of the problem and more effective and economical solutions of specific situations as they arise.

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Part IV—Status of Cooperative Work on Joint Use of Poles

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Associate, A. I. E. E. Member, A. I. E. E.

Synopsis.—*Because of the necessity of reaching the same customers, electric supply and telephone lines commonly use the same streets and highways. In urban communities, the joint use of poles for these two services has been very widely adopted and practises for joint use construction have been established from experience gained in past years. In rural communities, joint use is not always practicable or economical. Joint use involves many engineering and economic problems which have received the careful consideration of the Joint General Committee of the National Electric Light Association and Bell Telephone System.*

This paper describes some of the problems which have been encountered in joint use, and briefly outlines the work which is being conducted by the Joint General Committee in connection therewith.

It is concluded that in specific cases proposed for joint use all factors should be studied cooperatively by the companies concerned and that everything practicable should be done to facilitate joint use construction and extend its usefulness.

* * * * *

TELEPHONE and electric light and power services are supplied in the same areas and to customers who are to a large extent common to both utilities. It is therefore necessary that both types of service be carried along the same streets and highways.

Experience has shown that safer and more satisfactory conditions can often be secured if the power and telephone circuits are carried on the same poles. This is due in part to the fact that clearances and climbing space can be more readily maintained where both classes of circuit are carried on the same poles rather than on separate poles on the same side of the street. Where separate lines are placed on opposite sides of the streets and alleys, it is difficult to secure and maintain proper clearances for service wires to buildings where these cross the line of the other utility.

Joint use of poles by the power and telephone companies results in the use of fewer poles on streets and highways and better appearance of aerial lines. It is, therefore, more desirable from the public point of view. It conserves pole timber and in many cases is more economical to both classes of utility than separate lines.

Because of the above mentioned advantages, joint use of poles by power and telephone companies has been widely adopted. No complete data are available as to the extent of such joint use at the present time, but it is estimated that there are at least five million poles jointly used by the power and telephone companies in the United States.

Both of these classes of utility have been growing rapidly in the past twenty-five years and the development, design, and construction of the physical plant of each has kept pace with the growth in territory and number of customers served.

While earlier types of distribution plant were such that the possibility of contacts between wires of the two utilities and other hazards could be satisfactorily met by proper construction methods, protective devices, etc., later developments have increased the use of types of power distribution circuits regarding which questions frequently arise as to how service can be properly maintained and extended on jointly used poles.

These questions have received and are receiving careful consideration by the Joint General Committee of the National Electric Light Association and Bell Telephone System. This committee has recommended certain principles and practises for the joint use of wood poles which are intended for use as a basis on which electric supply companies and communication companies should work out their mutual problems and has undertaken important research work in connection with these matters through its Joint Subcommittee on Development and Research.

The principles and practises mentioned were presented in a report of the Joint General Committee under date of February 15, 1926, and while it is beyond the scope of this paper to consider these principles in detail, the following recommendations are of interest in that they indicate the way in which this matter is generally being approached:

Each party should:

(a) Be the judge of the quality and requirements of its own service, including the character and design of its own facilities, both now and in the future.

(b) Determine the character of its own circuits and structures to be placed or continued in joint use, and determine the character of the circuits and structures of others with which it will enter into or continue in joint use.

(c) Cooperate with the other party so that in carrying out the foregoing duties, proper consideration will be given to the mutual problems which may arise and so that the parties can jointly determine the best engineering solution in situations where the facilities of both are involved.

It will be observed that while each party retains full responsibility for facing and meeting its own problems, it is recommended that both parties cooperate in working out mutual problems involving the joint use of poles and in finding the best over-all engineering solution in

1. Middle West Utilities Co., Chicago, Ill.

2. Engr. on Foreign Wire Relations, American Tel. & Tel. Co., New York, N. Y.

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each situation. These are among the most important of the principles which have been recommended and are the basis upon which practically all cooperative work is being carried forward.

It is the purpose of the following paragraphs to describe what has been done and what is being done by the Joint Subcommittee on Development and Research in connection with the engineering and economic problems which have been encountered in joint use work.

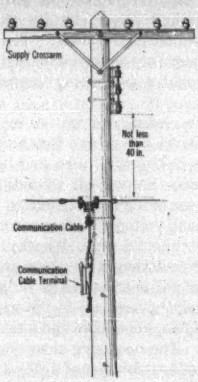

Fig. 1—Typical Jointly Used Pole

Construction Practises

Joint use construction practises have undergone almost continual change and improvement from the time joint use was first adopted and continued development is to be expected in the future. However, many of the fundamental requirements for securing satisfactory conditions on jointly used poles were recognized at an early date and form the basis for present day practise.

In the matter of relative levels it has been recognized that power wires should as far as practicable be carried in the upper position. In general, they are larger and stronger than the telephone wires. This is inherently so because of the current carrying capacity required. Placing power wires in the upper position on jointly used poles avoids the necessity of telephone linemen climbing through power circuits, the exact nature and characteristics of which they are not always familiar with.

Clearances must be provided which give sufficient space below the power wires so that power linemen will not have to come in contact with telephone wires while they are working on power wires. This neutral space must also provide sufficient clearances above the telephone equipment so that telephone linemen may work on the telephone plant without danger of coming in contact with power equipment. Clear climbing space must also be provided so that linemen may climb poles without having to be extremely careful to avoid falls or contacts with circuits from which they may receive physical injuries.

Fig. 1 shows one method for securing satisfactory conditions on a jointly used pole carrying circuits which both the power and telephone groups have recognized as being suitable for joint use.

In the matter of mechanical strength, joint use follows the practise in the construction of separate lines. That is, strength of construction should be provided such as to stand, with reasonable factors of safety, storm conditions which experience indicates are likely to occur from time to time in any particular area.

With regard to the matter of insulation and electrical strength, practises as to the size and type of power insulators have followed developments in the general field of power construction. Wires to street lights and underground connections to aerial plant require vertically run wires on jointly used poles. The location, insulation and mechanical protection of these have received special consideration to eliminate hazards to workmen.

Sufficient clearances between vertically run circuits of one type and the equipment of another utility on jointly used poles have also been found to be very important from the standpoint of avoiding interruption to power and telephone services.

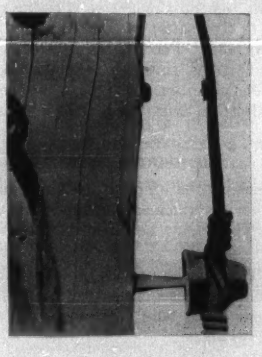

Fig. 2—Frayed Insulation Showing Breakdown of Insulation Between Power and Telephone Plant*

In the course of electrical storms, lightning may induce high voltages on either supply or communication

*In order to obtain a satisfactory photograph of the points of arc the vertical drop has been straightened out so that the clearance shown is much greater than existed at the time of the arc.

wires. If the separation between the supply and communication facilities is not adequate at any point, these induced voltages may break down the insulation and arc between the two as illustrated in Fig. 2. Damaged plant may, of course, result from lightning alone. However, when lightning has established an arc between the power and communication circuits the normal voltage of the supply circuit may maintain the arc. This results in the transfer of power into the telephone plant at voltages which may be well above that for which it is insulated and may cause trouble on both the power and telephone system. This sort of abnormal belongs to the general class that includes insulator flashovers, short circuits on cables, tree grounds and similar power system occurrences that always carry the probability of damage to the power system or service.

While vertically run attachments with improper clearances have played a large part in causing such occurrences, any situation where insufficient clearance between power and telephone facilities is provided may result in similar trouble.

Emphasis has, therefore, been placed in present day standards on the necessity of maintaining proper clearances as well as strength of construction to prevent this kind of abnormal. Experience has shown that where these clearances are adhered to this type of abnormal is kept to a reasonable minimum.

The Joint General Committee is giving careful consideration to the matter of construction standards on jointly used poles. Pending the development of complete specifications covering recommended practises under various conditions, they have recommended the National Electrical Safety Code to be used as a guide to practise.

Protective Devices

Both telephone and supply circuits are equipped with protective devices which are fundamentally the same in principle. They may be divided into two general classes:

1. Those which provide protection from abnormal voltages consisting of protector blocks in the telephone plant and lightning arresters in the supply system.
2. Those which provide protection from abnormal currents consisting of heat coils and fuses in the telephone plant and fuses and circuit breakers in the supply systems.

These protective devices are a secondary defense against abnormal conditions which it is impracticable to avoid either by design or through adherence to construction standards.

Even when all practicable precautions with regard to clearances, strength of construction and insulation have been taken, accidental breaks occur in both power and telephone wires. In some cases there are direct contacts between such wires. Higher than normal potentials are also introduced into the telephone and power circuits by lightning and other causes.

It is because of the limitations of protective devices and other protective measures that joint use with certain types of circuits has been in question. Considerable differences of opinion exist between engineers as to the degree of hazard involved in joint use between telephone plant and power circuits of various types, voltages, and connected power. The problem has increased in importance as the use of higher distribution voltages and greater generating capacity have been employed.

This matter is under investigation by the Joint Subcommittee on Development and Research. Studies are now in progress in one rural area and in one suburban area to determine the over-all advantages and disadvantages of the use of higher distribution voltages and of joint use with these voltages under present conditions.

The first experimental work done by the Joint Subcommittee in connection with these problems was a detailed study of the characteristics of various types of fuses. This study covered all of the well-known commercial types of telephone fuses and a number of experimental models. The operating characteristics of these fuses were obtained at voltages of 2,300, 4,000, 7,500 and 13,200. The current range was from 16 to 1,000 amperes. These tests were carried on in a laboratory where 20,000 kva. of generating capacity was available. The tests showed the dependability that could be placed upon the various fuses for interrupting voltages of the range from 2,300 to 13,200 volts. They showed under what conditions the fuses could be depended upon and the ranges where the available type of fuses could not be depended upon for safe operation.

A number of the experimental models showed considerable promise of improvement over existing models, and this work will be carried further to determine what improvements can be made in the operating characteristics of fuses.

The next phase of the problem taken up included a study of the operating characteristics of various types of overvoltage protectors suitable for use on communication circuits. The experimental work covered breakdown with direct current, 60-cycle alternating current and a complete study with a cathode-ray oscillograph of the behavior under steep wave fronts for carbon block protectors, neon and vacuum tubes.

These tests showed that the carbon block protector has a breakdown point with all types of applied wave fronts which is sufficiently fast and low to protect the insulation that is now used in the communication plant, as shown by similar tests on condenser and cable paper. The shortcomings of these blocks lie in their tendency to permanently ground the circuit when carrying current for any appreciable length of time.

The tests with steep wave fronts were carried to a rate of rise of 36,500 volts per microsecond, and it was determined by tests of propagation of steep wave-front voltages through telephone cable that it was practically impossible to subject the plant to voltages with any faster rate of rise than those used in the protector tests.

The problem of adequate protection of the telephone plant in joint use, obviously, cannot be solved by the development of the telephone protective devices alone. The protective devices in the power system are equally important.

One of the important functions of the power system protective devices is that of clearing power system faults in a reasonable time interval. Obviously, telephone protective equipment cannot be expected to prevent damage to telephone plant in case of contact between the wire circuits of the two utilities when power system protective devices fail to operate and the physical contact of the circuits is maintained over an indefinite period of time.

One problem in the development and research work is the fixing of the part that the protective devices on each system must play in abnormal conditions. It is necessary that the over-all protective equipment be adequate and that the burden of overcoming weaknesses in the protective equipment of one system be not thrown on the protective equipment of the other. There are inherent limitations in both classes of protective equipment that must be defined.

Therefore, the next step in this investigation is a determination of the over-all characteristics of power circuit and telephone circuit protection under typical conditions of contact between the two plants.

While protective devices are an important element in connection with joint use involving certain types of power circuits employing the higher distribution voltages, there are also other important considerations. The general insulation of the telephone plant must also be considered, especially in connection with drop loops attached to and entering subscribers' premises. These matters are also being studied by the Joint Subcommittee.

All of these problems, as is the case of others being studied by the Joint General Committee, are being approached on the basis of determining the best over-all engineering solution such that both systems can provide their services in the most convenient and economical manner.

Inductive Coordination

In the early history of joint use, noise induction problems involving street lighting circuits appeared. Other interesting problems were encountered such, for example, as the accidental grounding of one corner of an isolated delta power system with its resulting unbalanced voltage inductive effects on open-wire telephone circuits, which type of telephone construction then predominated.

As these problems arose they received careful study and with the development and extended use of telephone cables and the use of improved operating methods in power and telephone distribution generally, inductive coordination of power and telephone distribution systems in the urban communities became less troublesome and did not for a time receive any large amount of consideration.

However, during recent years the introduction and extended use of various types of multi-grounded distribution systems described in the paper by Messrs. Harrison and Silver and the existence of certain types of signaling on local telephone circuits, have contributed toward making important the consideration of noise inductive effects in connection with joint use. This matter is discussed more fully in the paper by Messrs. Harrison and Silver.

The technical factors involved in inductive coordination problems under joint use conditions are complicated. The details regarding these factors and the results of the extensive studies of these matters by the Joint Subcommittee on Development and Research are described in the paper by Messrs. Wills and Blackwell.

The various operating problems which have arisen almost since the birth of the power and telephone industries and the investigations conducted by the Joint Subcommittee on Development and Research indicate the importance of giving careful consideration to the inductive coordination features of joint use and of including this factor in studies of the relative advantages and disadvantages of joint use as compared with separate lines. This factor should, of course, be considered from both its technical and economic aspects.

Much can be accomplished in the inductive coordination of the two distributing systems by cooperative advance planning. In urban areas where the telephone circuits are largely in cable, there is about a two to one ratio in the inductive effects between a joint line and separate lines across the street. In rural areas where the telephone circuits are largely open wire, the ratio of the inductive effects on joint lines as compared with separate lines across the highway, is much greater, other things being equal.

In urban areas the power and telephone companies can through cooperative planning frequently arrange to establish important power feeders and telephone circuits on separate streets and thereby avoid large inductive effects and permit more extensive joint use of branch lines. A careful review of the equipment used on the power and telephone circuits and the introduction of operating practises designed to limit the inductive susceptiveness of the telephone circuits and the inductive influence of power circuits, form an important part of advance planning and cooperation.

As described in the paper by Messrs. Wills and Blackwell, these latter factors include such items as limitation of the odd triple frequency series arising in

Y-connected generators feeding directly on the line and in single-phase service transformers. Suitable limitations of the unbalances existing among the loads connected between the three-phase conductors and the neutral, limit the ground return components.

Grounding of aerial telephone cable sheaths to provide for increased shielding and the use of central office and station equipment providing a higher degree of balance with respect to ground are helpful.

The matter of joint use may involve both rural and urban communities. It is more generally associated with the latter because of the severe limitations in physical space available for utility use. In the case of rural lines where the telephone circuits are largely in open wire and the exposures between particular circuits are likely to be long, joint use is not always practicable. In these cases locations for separate lines are usually available.

Furthermore, joint use in rural areas is not always economical from a purely construction standpoint due to the fact that relatively longer spans can often be used on the power lines and both utilites are able in many instances to use shorter and lighter poles than would be practicable in joint use. Joint use with telephone toll circuits or power transmission lines has not, in general, been found desirable. Types of construction vary so widely and service requirements and inductive effects are such that it becomes uneconomical to carry out such construction.

Conclusions

Joint use of poles by power and telephone companies has many advantages, both from the standpoint of the public and from the standpoint of the wire using companies. This is especially true in built-up communities.

Important problems brought about by developments in practises, particularly in the use of high-voltage distribution, remain to be solved.

Careful adherence to generally accepted practises with regard to clearances, strength of construction, insulation and inductive coordination is necessary in order that the advantages of joint use can be secured.

In considering specific cases proposed for joint use, it is advisable that all of these factors be studied cooperatively by the companies concerned, to the end that good service, safety and economy by both classes of utility may be promoted.

It is important that everything practicable should be done to facilitate joint use construction and extend its usefulness. The Joint General Committee of the National Electric Light Association and Bell Telephone System is continuing its efforts in this connection.

Discussion

J. J. Smith: The paper by Messrs. Harrison and Silver is very interesting in showing how in spite of the approximate two-to-one growth in both the telephone and power systems in the last decade which necessarily brings with it a large number of cases of physical proximity, the difficulties of coordination have not correspondingly increased in this interval. Sometime ago I had occasion to review some estimates that were made in 1920 as to what the situation in 1930 would be. Looking backward it is now easy to see that these estimates were altogether too pessimistic and for this I believe the joint work of the N. E. L. A. and Bell System can take some of the credit.

Considerable discussion is devoted to conditions on the power system which do not seem to be representative of the more usual type of practise on power systems. I have in mind more particularly their references to the use of transformers with graded insulation and Y-Y transformers. The consensus of opinion of engineers with whom I have discussed the matter seems to be that if 220-kv. transformers alone are considered, a large proportion of the transformers are constructed as described in the paper. If, however, we consider systems 66 kv. and above the majority of the transformers have sufficient insulation on the neutral end of the winding to permit considerable impedance to be inserted between line and ground. With reference to the section on transformer connections I feel that by far the more common practise is the use of the delta-Y connection and that the Y-Y connection referred to is the exception rather than the rule.

From Messrs. Wills and Blackwell's paper it would appear that with reasonable balance on both power and telephone systems the problem of noise is not normally a serious one. The difficulties with regard to noise lie mainly in special situations involving unbalance on one or the other of the systems. Within the next decade it seems, at the present time, that we shall witness the introduction of vacuum tube devices on a fairly large scale. As Messrs. Harrison and Silver have pointed out, their uses up to the present time have not been sufficient to establish a definite trend. Experience so far would tend to show that they will present new types of problems, but there is reason to believe that with a continuation of the cooperative work described in this symposium these will be solved satisfactorily.

Attention may be called to the fact that some of the work being done by these committees is of interest in other fields. For instance, work on methods of measurement of noise is being carried on by other committees and societies in connection with machinery, street, and other types of noise. There is sufficient correlation among the various bodies working on this subject to ensure that they are all working along somewhat similar lines and also to avoid undue duplication of effort. The relation between the effects of lightning on transmission lines in causing outages and the problem of low-frequency induction is another example.

The subject of coupling between ground return circuits has always been a very difficult one and a large amount of theoretical work has been done on it, all directed towards estimating induced voltages by calculation. In some cases these calculations checked reasonably well with test and in others there was no agreement. As is pointed out by Messrs. Conwell and Warren for separations of one-half mile to one mile the coupling coefficients may vary as much as 20 to 1 due to the range of earth conductivity. I believe that at the present time in order to make a reasonably accurate estimate it is necessary to conduct tests in a given location. I should like very much to hear what hopes the sub-committee have of establishing an empirical basis such as they refer to in their paper for estimating coupling.

In the foreign literature on the subject suggestions have been made at various times as to the amount of energy required to produce acoustic shock. I wonder if the committee's work has shown any correlation or a complete lack of correlation between these quantities.

If one unfamiliar with the subject were to attempt to estimate the interest in the coordination of the power and telephone plant by reference to papers presented in the last ten years before the Institute or published in electrical journals, he would be led to conclude that little attention was being given to the subject, since the amount published has been comparatively small. This

conclusion, however, would be entirely incorrect as is apparent to anyone familiar with the large expenditure of time and money on the program being carried out by the Joint Committee of the N. E. L. A. and Bell System. It is to be hoped that this symposium is only the start and that further papers giving more detailed accounts of the studies will be presented before the Institute in due course.

M. D. Hooven: There are apparent several outstanding points of interest to power company engineers who in the field must cope with these problems of coordination in their infinite form and variety. As Chairman of the Foreign Systems Coordination Committee, National Electric Light Association, I may, to some extent at least, present the general viewpoint of these engineers.

Most important is the educational value of the material presented. Nowhere is there available in such concise and understandable form any review of the whole problem as here presented, such a review filling an often expressed need. With particular reference to the problem of low-frequency induction, it has seemed evident during the past few years that this problem is becoming of increasing importance; not all of its ramifications are yet understood. The Conwell-Warren paper supplies a needed source-book of information. It is the intent of the Foreign Systems Coordination Committee to bring all the material of this symposium to the attention of every engineer working on coordination problems.

The last three papers of the symposium each stressed the importance of cooperative advance planning and the Harrison-Silver paper showed how interwoven were the advances of the art in each of the two systems. The Development and Research work will continue to bring forward facts and findings, but much of the good of these results will be lost if the two systems are not planned cooperatively. The Foreign Systems Coordination Committee, in disseminating the results of this work, is fostering the adoption of routines of cooperative advance planning in all the geographic divisions of the Association.

L. T. Robinson: When the experimental work of the Joint Development and Research Subcommittee was started in 1923 there was very little information available which would allow an attempt to be made to proceed in a logical manner from cause to effect in dealing with the problem of coordination. The California Railroad Commission Report of 1919 deals almost wholly with conditions on the power circuit and with coupling between the power and telephone circuits. There is practically nothing in it to allow a determination of the effects on the telephone system of voltage induced in it by external means. It was necessary therefore to develop apparatus for use in such work and this resulted in the various forms of wave analyzers, etc. referred to in these papers.

As a result of the more accurate measuring devices it was possible to accumulate data from different sources which might be correlated among themselves. The principal sources of such data were:

1. Tests made on both power and telephone apparatus under carefully controlled conditions to determine their characteristics when considered alone.

2. Tests made in the field, such as those given in Engineering Report No. 6 on the Minneapolis tests where the relation between the systems were studied.

3. Tests of a statistical nature such as the survey of wave shape on 34 systems and the noise measurements on paralleling telephone lines.

4. Investigation of field cases which arose from time to time in which the Committee cooperated in obtaining data.

As a result of this work I believe it is now safe to say that for a given situation, if the various influence factors are known, a reasonably close estimate of the conditions to be expected can be made. In order to obtain the influence factors for a given location certain data may have to be obtained by test. For instance, due to the variation of harmonics in different power systems it would be desirable to make a direct determination in any given case. Variations in ground conductivity may make individual tests necessary in a given location, also the conditions of parallelism between the power and communication system must be obtained from observation, maps or records.

For these reasons it is evident that it would be extremely expensive to study in detail every contemplated extension of both power and communication systems *a priori*. The more logical way to proceed is to set up reasonable standards for both power apparatus and telephone apparatus which will be such that, if they are met, satisfactory conditions will obtain in the average situation. Special situations will of course arise needing more detailed study but these can be handled individually. The setting up of such standards should be based upon the most economical scheme of supplying power and communication to the public. Advantages have come from general acceptance of such an idea without the fixing of limits for either kind of plant equipment or apparatus. These may, perhaps, advantageously follow in due course. Using the means and methods which have been developed during the work of the Development and Research Committee there has been, as pointed out in the papers, marked progress made.

Reference to the paper by Messrs. Harrison and Silver shows that the number of telephone and power customers has about doubled within this 10-year period. Both telephone service and power service has been improved. It cannot of course be claimed that this improved situation, in spite of the increased congestion in many areas, is due entirely to the work of the Committee but this work has played an important part in the results.

It now appears that we are faced with a similar problem in the relation of the use of radio to other electrical industries. There is some tendency at the present time to expect that the whole burden of reducing troubles with radio reception should be borne by other industries such as by adding corrective equipment or redesigning the apparatus. However, we again find the three factors—"influence factor," "susceptiveness factor," and "coupling factor"—have the same importance and that an economic solution may involve not only changes in the electrical distribution system but also in the radio broadcast and receiver system as well.

Steps looking toward a cooperative study of this situation are now being taken and it is felt that the results will be as successful as the work described in connection with the coordination of power and telephone plants. The degree of success will depend in my opinion upon the extent to which general principles so successfully applied to the problems discussed in these papers are followed.

H. S. Phelps: Frequently one of the most difficult tasks confronting those responsible for the administration of coordination studies is that of acquainting their executives with the fundamentals surrounding the problem. The paper by Messrs. Conwell and Warren should be most helpful to one called upon to outline the essential features of low-frequency induction.

Speaking of several of the factors affecting the problem of low-frequency induction, I am glad to see that attention has been called to the importance of the frequency of occurrence of the induced voltages as well as their duration.

In studying specific problems these factors must be given serious consideration. It is entirely unreasonable to presume that the same coordination measures should be expected or desired in situations involving the most sturdy types of construction, built for maximum service reliability, that might be justified where less reliable transmission systems are involved.

In determining the importance of the effects produced by induced voltages due consideration must be given to a number of factors. Among these are included:

Character of communication service normally rendered over

the lines involved in an exposure situation and the degree to which it might be affected adversely by abnormal potentials.

The likelihood of damage to physical plant as the result of abnormal potentials.

The physical hazard that might be imposed on employees at times of abnormal potentials.

Outlining the chief characteristics that determine the magnitude of residual currents and voltages should be particularly helpful. Making specific reference to such items as fault and earth return impedance and also the character and arrangement of ground wires immediately makes it evident that they are important factors.

Previous to the work that this symposium is reporting, information concerning fault and earth return impedances was extremely meager. As a result, there has been much controversy during many coordination studies resulting from attempts to name a mutually satisfactory value of fault impedance for substitution in the classic equations for determining "fault current." There is little wonder that such has been the case when values have been recommended in good faith that varied in value all the way from zero ohms to fifty ohms.

In pointing out some of the factors that have a material influence upon the inductive coupling between transmission circuits and communication circuits attention has been called to the important part played by earth conductivity. Also, there is a rapidly growing recognition of the widely varying earth conductivity conditions. That computed and observed values of inductive coupling differ by no more than a ratio of two to one is truly a matter of surprise when it is realized that earth conductivity conditions vary by as much as fifty to one within a distance of a few miles in some localities. On the other hand the earth conductivity conditions may remain practically constant throughout the area served by an extended power transmission system.

As the result of all these things marked changes have taken place in the procedure for calculating inductive coupling. Whereas it was formerly the practise to substitute more or less arbitrary values for earth conductivity generally based on personal opinion for the coefficients in the equations for calculating coupling, recent developments include a preliminary field study to determine earth conductivity values that apply in particular situations. The new method makes possible much more reliable data, from which conclusions may be drawn concerning the probability of experiencing trouble as the result of projected exposures.

Now that the benefits of ground wires are being given more serious consideration, as the result of the careful studies of the lightning protection problem, the benefits that may be derived from them in the coordination problem should also be taken advantage of wherever possible. While ground wires do increase the magnitude of fault currents, still at the same time they provide, in general, a decided net benefit through their shielding effects. The benefits to be derived through the use of ground wires warrants additional study to determine more fully the influence on their effectiveness of such factors as conductivity, resistance of their earth connections, location with respect to the power conductors and their influence on the coordination of systems in situations where currents of large magnitude are being transmitted.

Very serious consideration should be given the recommendation that advantage be taken of cooperative advance planning. Such studies invariably result in better appreciation by all of the parties at interest of the problems with which the individual parties are confronted. Also, such cooperative efforts develop ways and means for solving the problem in a manner that is mutually satisfactory.

Livingston P. Ferris: Among the factors "blamed" for the inductive effects of a power system on a communication line, we find the earth itself, particularly in the case of low-frequency induction. Messrs. Conwell and Warren first blame it for being a conductor at all, as they indicate that the problem would vanish if the earth were an insulator. Furthermore, they point out that inductive effects may differ by 20 to 1 due to differences in earth conductivity between different locations, other conditions being the same. Of course, we have to take the earth as we find it, and it would be as well to consider how it may affect our practical problem.

Assume, in a region of high and homogeneous conductivity, that we have power and telephone lines parallel to one another at a separation of 100 ft., and for just enough length so that with a ground on the power line the voltage induced in the telephone line reaches a value, beyond which we agree that some remedial measures should be taken to control it. Adhering to this same value of induced voltage and length of exposure, what then is the effect of diminishing earth conductivity on the permissible closeness of these lines, without resorting to any remedial measures? This is shown graphically by the curve on Fig. 1, using scale A. If we start off at a wider separation, say 1,000 ft., giving, of course, a lesser induced voltage per unit of length, and correspondingly, a longer exposure, the effect of diminishing earth conductivity is shown by the same curve, using scale B. The range of conductivities on this chart corresponds to conductivities which have been met in practise. Thus, to compensate for differences of earth conductivity within the practical range,

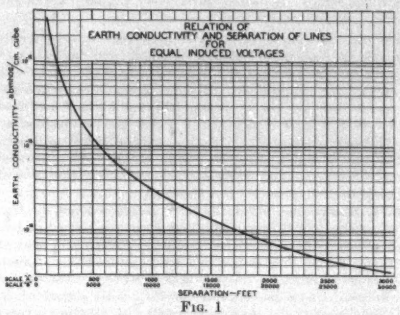

Fig. 1

our lines may have to be separated 3,000 ft., instead of 100, surely of great practical moment. If it were possible to superimpose on this chart a cumulative plot of the number of telephone lines within different increasing distances from power lines throughout the country, this would be even more forcibly evident.

In many practical cases, therefore, the answer to the question—"To be, or not to be—an exposure?" lies in the bosom of the earth, which we are powerless to change. But we can provide more information concerning earth conductivity, its variation with conditions and localities, and convenient means for predetermining its effective value in specific cases. With an appreciation of the need and importance of this information for practical purposes, I am sure that both power and telephone engineers will further lend their aid and support to the committee in its researches on this problem.

H. M. Trueblood: In the presentations and discussions of the papers, several allusions have been made to the wide range of subjects which the Joint Subcommittee on Development and Research has had to cover in its studies. The division of the work is such that some of the project committees are engaged upon questions relating very largely to telephone systems, while others are giving their attention to questions which concern power systems almost exclusively. Each project committee is made up of members representing both industries, so that the natural and desirable result has been that the power and tele-

phone engineers who carry on the detailed technical investigations are getting well acquainted with each other's problems.

A good example of this is to be found in the work of the project committee assigned to study the possibilities of neutral impedance as a measure of coordination. It is true that one or two points affecting the value of this device in reducing induction require some further study. For example, there is the question of the relative importance of induction from unbalanced voltage and charging current, both of which tend to increase as neutral impedance is increased, and of induction from short-circuit current through the neutral, which decreases with increasing neutral impedance.

There is also the matter of the relative advantages of different types of neutral impedance from the standpoint of inductive coordination in the voice-frequency range. But, as the paper by Messrs. Conwell and Warren points out, the direct effects of neutral impedance in reducing induced voltages are not primarily the objects of our study, since to a large extent they can be calculated with all the precision necessary in practical work. On the contrary, the principal questions involved in the use of neutral impedance are those arising from its reactions on power system operation, such as the possibilities of overvoltages, of relaying difficulties, of improvements in system stability, and of reductions in stresses on equipment or in damage due to short-circuit currents.

While it is obvious that some of these reactions upon the operation of the power system will, in turn, react upon the inductive coordination problem, their larger aspect is in their relation to the functioning of the power system as such, and it is here that the subcommittee's work in connection with neutral impedance is now mainly concentrated.

These questions, of course, are of great interest to the power industry, apart from their bearing on inductive coordination, and the subcommittee has received much valuable assistance from power operating and manufacturing companies and from other committees working on similar questions. The continued support and cooperation of the industry is necessary in this work, since to a large extent practical results can be arrived at only through observations and tests on operating systems.

D. H. Gage: In the telegraph field of communication we are particularly concerned with the problem of low-frequency induction. We find in practise that our telegraph and printer circuits, where exposed to paralleling power lines, are not only interrupted during abnormal conditions on the power circuits, but are subject to interference from what might be termed normal induction.

These results are sometimes very puzzling and it seems that there is a field for investigation where unexpected interference results from an exposure of a communication line to an apparently well-balanced power circuit with very little current in the neutral.

Then again, there may be a number of comparatively short exposures at different places on the same line, none of which alone would cause serious interference, but the results of the different exposures may combine in such a way as to materially affect the operation of our circuits.

We find in the papers that double faults occur with greater frequency on systems isolated from the ground than do double faults on systems with grounded neutrals. In case of a double fault on a non-grounded system do the faults occur simultaneously or does one follow the other and is the second one a result of the first, if so, why?

It is generally conceded that the joint use of poles by power and communication companies has advantages.

In the consideration of this problem the committee has apparently given a great deal of attention to the engineering and economic problems involved, but there is one phase of this joint use problem that is apparently not considered an engineering feature. Perhaps it has no place in an engineering study, but it is an important part of the problem and there should be no one better qualified to pass on it in some respects than the engineer. It is the problem of joint line agreements.

There is usually little difficulty in reaching an agreement on the questions of construction specifications, clearances, or division of costs. The character of circuits on jointly used poles is one for discussion. In the present state of the art it is generally conceded that joint use with certain types of circuits is undesirable, but the art is changing all the time and who can say what the future will develop? Furthermore, while the characteristics of the power circuits may be acceptable to the communication companies for joint use today, future developments may require the power companies to change the character of its circuits, what then shall be the procedure?

Again, take the question of liability, what is a fair provision in case of injury to person or damage to property resulting from joint use of poles? These are questions that should be covered in a joint use agreement and while they may be outside the scope of an engineering study, is it not reasonable that the engineers should give these questions careful consideration.

A. E. Silver: Mr. J. J. Smith, in discussing the sections dealing with Y-Y connections of transformers and graded insulation of transformers and apparatus, has expressed a preference for the delta-Y connection, and questioned the extent of the use of graded insulation in equipment now in service.

It was not intended by the authors to bring into question the theoretical preference of the delta-Y connection as compared to the Y-Y connection. However, in making the selection in any situation, due weight must be given to the nature of the apparatus and other existing power system features with which new equipment must coordinate. As indicated in the paper, the trend in the larger units, particularly those associated with transmission lines, has been toward the use of the delta-Y connection, or its equivalent, the provision of the tertiary winding. Nevertheless, in the distribution field, this trend is less marked and there is extensive and satisfactory application of the Y-Y connection, as for instance the widely used 2,300/4,000-volt system, in which single-phase lighting units distributed between the different phases of the primary constitutes the equivalent of the Y-Y connection.

As to the extent of the presence of graded insulation in various types of existing apparatus, I am unable to give a definite figure. However, graded insulation is present in many systems and in those cases, constitutes a limitation on the use of the neutral impedance.

Notwithstanding steady growth in capacity and in spread and in intimate intermeshing of the plants of the communication and power industries, the better understanding of the problem on all sides that is coming from the cooperative treatment in general and the Development and Research work in particular, will continue the trends firmly in the direction of more simple and effective coordination.

H. L. Wills and O. B. Blackwell: The third paragraph of Mr. J. J. Smith's discussion which refers to the Wills-Blackwell paper, is interesting in two ways. The first part of it might lead to the impression, while undoubtedly not intended by its author, that little remained to be done on the subject of noise induction and that the remaining difficulty was principally a matter of balance. While the unbalance difficulties stand out particularly we should not lose sight of the other factors and their relation to unbalance. For example, if the wave-shape distortion increases, the balance must, in general, be correspondingly improved to obtain an equivalent degree of coordination. In power distribution systems, particularly, this may introduce a considerable handicap on account of difficulties with single-phase extensions and branches. Furthermore, wave shape needs careful consideration in connection with new developments in the art. The reaction of large mercury arc rectifiers on the wave shape of power circuits from which they are supplied is a case in point. In

certain situations the effects of these loads have created difficulties on telephone lines exposed to single-phase branches as supplied from the same power system.

Mr. Smith mentions in the latter part of this same paragraph another interesting development in the possible introduction of vacuum tube devices in power systems on a fairly large scale. We can, however, all agree with him in taking a hopeful view of the situation based on the continuation of the close cooperative work in solving the new problems as they arise.

H. S. Warren: From a technical standpoint the most interesting feature of the low-frequency induction problem is undoubtedly the great distance to which the inductive influence of power transmission lines extends. It might be supposed that if a telephone line is separated from a paralleling power line by 1,000 ft., or in any event, by 2,000 ft., it would not be disturbed by the power line. However, both experience and careful experimental tests have shown that such a supposition is not correct and that substantial induced voltages are experienced in some cases where the separation between lines is a mile or more. It also is of great technical interest that the coefficients of induction between the parallel power and communication lines have widely different magnitudes in different places depending on the geological structure of the earth at the different locations. One of the most important problems in low-frequency induction therefore is the development of simple means for determining coefficients of induction.

In reply to Mr. J. J. Smith's question as to whether the Committee is able to hold out hope of being able to predict coefficients of coupling in new situations, I may say that where the lines are parallel at roadway separations, the coefficient of coupling may now be estimated, by assuming an average earth conductivity, with a precision not greatly inferior to the precision with which we may estimate short-circuit currents. However, the dependability of our estimates of coupling falls off rapidly with increasing separation of lines, and for separations of several hundred feet or more, there are at the present time no means of making dependable predictions without some kind of actual exploration or electrical survey of the local conditions. Of course average values can always be assumed but in view of the wide variations of these coefficients I think it probable that some method of determining them locally will always be advisable in important cases. Of course, as time goes on, tests made in connection with specific cases, or for other reasons, are adding to our knowledge of the characteristics of the earth in different localities with respect to coefficients of induction, and this tends gradually to reduce the amount of specific testing work required in connection with new situations.

The Committee's problem is largely one of devising methods of making such determinations, which will be simple to carry out and the results of which will be easy to apply. The Committee is also interested in gathering data on the variation of effective earth conductivity with geographical location and geological conditions, in order that the correlation, if any, of these factors may be established. Such a known correlation would, of course, be of practical value in estimating coefficients of coupling. The above may serve also to answer the question asked by Mr. L. T. Robinson as to the possibility of setting up standards so as to avoid the necessity of studying each specific situation.

In reply to Mr. Gage's question, double faults on power transmission lines with isolated neutrals occur in sequential order, a ground on one phase raising the voltage to ground of the other phases, so that the insulation fails at some point where it is weaker than elsewhere. For practical purposes, however, particularly from the inductive coordination standpoint, the two faults in most cases may be regarded as simultaneous. The last sentence in the paragraph to which this refers would stand as written. The Committee expects to make tests with a view to determining more exactly the characteristics and properties of arcing grounds and other faults.

It appears to be a fact that the seriousness of voltages imposed on communication wires by induction is less generally recognized than in the case of voltages impressed on communication wires by actual contact with high-voltage wires. If a power wire breaks or otherwise makes contact with a telephone wire and thus imposes on it hundreds of volts, the seriousness of the condition is apparent to everyone, but that voltages of the same order of magnitude and capable of producing equally injurious effects, may be set up in communication circuits by induction, is not always recognized. This may be due in part to the fact that the introduction of induced voltage into a communication circuit is not accompanied by any visible effects such as the arcing and burning which often occur at points of contact. It is hoped that our paper will aid in bringing about a better understanding of the low-frequency induction problem and also that it may indicate to operating companies, at least in a general way, methods of protection against the effects of low-frequency induction.

Field Tests on Thyrite Lightning Arresters
Using Artificial Lightning of 1,500,000 Volts

BY K. B. McEACHRON* and E. J. WADE*
Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The impulse characteristics of thyrite station type arresters of various ratings have been studied when connected to a 45-mile transmission line. The surges were supplied by a 1,500,000-volt impulse generator.

The volt-time curves for arresters of various ratings were obtained under the conditions corresponding to the arrester on each incoming circuit, on the bus with two incoming circuits and on the bus with one incoming and one outgoing circuit. The oscillograms were found to check the calculations closely.

With the arrester 90 ft. from a nearby tower the effect of tying arrester and tower grounds together was studied, and also the effect on flashover at nearby towers.

The effect of flashover at a tower 1,660 ft. away on the operation of the arrester was determined.

Oscillograms are given comparing the protective characteristic of a single-stack arrester with that of a two-stack arrester of the same rating. There is no discernable difference between the two characteristics.

It has been proposed that line insulation for a distance of one-half mile from the station should be limited. Oscillograms are given for two arrester ratings, showing that, for the waves used, no protection is offered by the arrester at a distance of 2,180 ft. and a large part is lost at 500 ft. It is concluded that the insulation should be limited at the arrester if the arrester is to protect it safely.

The effect of the length of arrester connections is given, showing that on the 69-kv. arrester a loop 30 ft. in length reduces the potential 35 kv. The conclusion is that a long arrester circuit allows reflection of original wave at the transformer, while a long transformer arrester circuit allows the reflection of the arrester potential. Both the arrester circuit and the arrester transformer circuit should be kept as short as possible, but the former is more important than the latter.

* * * * *

THE study of the protection of lines and apparatus against the effects of lightning naturally divides into two parts. One deals with natural lightning itself, while the other is concerned with the use of artificial surges made to simulate natural lightning. The purpose of the study of natural lightning is to disclose the form of wave which may appear on the line, whether lightning strikes the line or some other point nearby. To study the effects of natural lightning on the operation of apparatus is very unsatisfactory and frequently will give rise to erroneous results. It is necessary to be able to apply the same impulse over and over again to determine how apparatus behaves and the effect of lines or apparatus on the waves themselves. The studies with natural lightning will determine the character and form of waves which may appear in service, while the investigation with artificial lightning will disclose the performance characteristics of lines and apparatus.

Testing Equipment

This investigation is a continuation of the one made last year[1,2] in cooperation with the Consumers Power Company on their S-19 line in western Michigan, extending from Grand Rapids to Croton Dam, a distance of about 45 miles. In all of the tests to be described, which deal only with problems relating to lightning arresters, the impulse generator was located at the Croton end of the line.

The impulse generator consisted of 60 banks of oil capacitors charged in parallel to 25 kv. and discharged in series, according to the Marx circuit. Each bank had a capacitance of 0.50 μf., giving a total capacitance of 0.00833 μf. for the entire generator. The open circuit potential of the impulse generator was 1,500,000 volts. In order that the generator might be used in wet weather a canopy was constructed over the generator, as seen in Fig. 1.

A portable cathode ray oscillograph designed by Mr. E. J. Wade was found to be of great value when taking oscillograms at different points along the line.

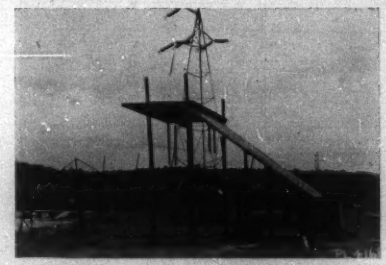

Fig. 1—1,500,000-Volt Impulse Generator Connected to the S-19 Line of the Consumers Power Company

This oscillograph is of the cold cathode type and is provided with many features making for simple and reliable operation in the field. Automatic vacuum control, a fool-proof film changing mechanism, elimination of a large number of joints in the oscillograph, and its small size, are some of the noteworthy improvements incorporated in this equipment. The oscillograph is mounted in a special body Ford truck intended to supply not only transportation for the oscillograph,

*Both of the General Electric Co., Pittsfield, Mass.

1, 2. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

but also power for its operation. Because of lack of time, the oscillograph went into service before its power supply equipment was complete so that it was necessary to use a portable gasoline driven generator for the oscillograph power supply.

Tests were made using the station type Thyrite arrester under a variety of circuit conditions. In all the tests described here, the arresters were located at Tower 14, a distance of 1.46 miles from the lightning generator at Croton Dam. This distance was great enough so that the arrester operation was practically complete before reflections from the impulse generator could return to the arrester.

Negative impulses were used in all of the tests, because the negative wave attenuates less rapidly than the positive, and favorable results required the highest test voltage possible at the arrester.

Thyrite Arrester Performance

When applying arresters, an exact knowledge of the arrester performance for any assumed condition is

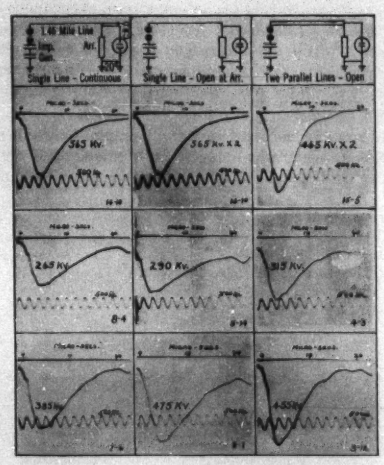

Fig. 2—Arrester Protective Characteristics as Measured at Tower 14

Oscillograms—Top row—With no arrester connected
Middle row—69-kv. arrester
Bottom row—115-kv. arrester

desirable. Calculations[3,4] have been made for the thyrite arrester which show methods for determining the arrester performance under assumed conditions. The volt-ampere curve for the thyrite arrester is a straight line on log-log paper and is conveniently represented by the equation $E = CI^{(1-a)} = C$ where E and I are the arrester voltage and current respectively, while a and C are constants of the thyrite material. The exponent a is a fixed value of 0.72, while C is the resistance of the arrester with one ampere flowing and obviously varies directly with the number of thyrite units in series.

To check the calculations which have been published for the arrester, various numbers of arrester units were studied so that all arrester ratings from 15 to 220 kv. have been tested under conditions representing closely those of service. Three conditions were studied for each arrester rating, one corresponding to the arrester

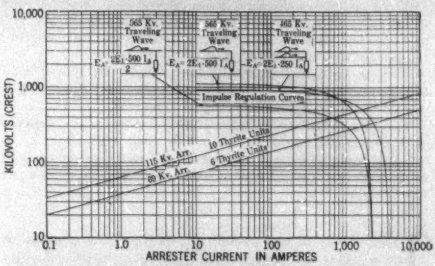

Fig. 3—Calculated Arrester Performance for Applied Impulses. Corresponding Cathode-Ray Oscillograms shown in Fig. 2

at the end of a single circuit line, another with the arrester on the bus at the end of a double circuit line, and the third with the arrester at an intermediate point in a line. These three conditions are illustrated in the circuit diagrams of Fig. 2.

The potential without the arrester at Tower 14 is shown in the upper oscillograms in Fig. 2. The middle oscillograms (8-4, 5-14, 4-3) show the potential measured across the terminals of 6 thyrite arrester units and the lower oscillograms show the potential across 10 units. These units form one leg of a 69-kv. and 115-kv. arrester, respectively, for grounded neutral systems.

The calculated values for the two arresters may be obtained from Fig. 3. The impulse regulation curves are plotted for the three conditions indicated, using the usual surge impedance value of 500 ohms, and the incoming wave voltages corresponding to those used in Fig. 2. The intersections between the arrester characteristics and the impulse regulation curves give the maximum arrester voltages for the various conditions.

The comparison between the test and calculated values are given in the table and the agreement is quite good. If, however, the surge impedance of the line is taken as 530 which is the calculated single line value, the agreement is even more favorable as the effect of increased surge impedance is to reduce the voltage across the arrester. It is also probable that the arresters have a slightly better exponent than 0.72 which also has the effect of giving test results which are more satisfactory than those shown by the calculation.

THYRITE ARRESTER PERFORMANCE
COMPARISON OF CALCULATED AND TEST VALUES USING SURGE IMPEDANCE OF 500 OHMS

Circuit condition	(1) Single line continuous		(2) Single line open		(3) Parallel lines open	
	Calculated	Test	Calculated	Test	Calculated	Test
69-kv. arrester	280	265	305	290	340	315
115-kv. arrester	400	385	475	475	505	455

It is to be noted that the arrester gap breakdown is not readily apparent, although a slight oscillation appearing on the original negatives indicates that the breakdown occurred at from 30 to 35 kv. crest per 11.5 kv. arrester unit. Before these oscillograms were taken in the field it was expected that when gap breakdown occurred a sudden change in potential would take place which on account of its steepness might occasion turn-to-turn stresses in nearby inductive apparatus. The oscillograms from the lowest to the highest rating show the arrester volt-time characteristic to be free from such a sudden change of potential. Laboratory tests do show such a sudden change in potential at gap breakdown, but this effect may be due to the fact that

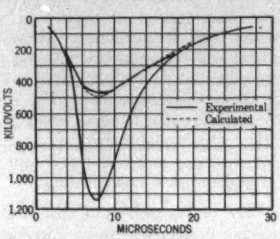

FIG. 4—COMPARISON OF CALCULATED AND MEASURED VOLT-TIME CURVES OF 115-KV. THYRITE ARRESTER

true traveling wave conditions are not reproduced in the present laboratory equipment. Other tests[5] with two arrester units in series made on the Pittsfield 66-kv. line of the Turners Falls Power Company agree in giving an arrester breakdown of about 35 kv. per unit with a wave reaching arrester breakdown in one microsecond, and in not showing a sudden drop in potential after gap breakdown.

Using methods of calculation previously referred to[4] the wave shape of voltage across the arrester for any of the three given conditions may be calculated and found to agree with the test results. In Fig. 4 the original wave without the arrester is reproduced together with the voltage across the 115-kv. arrester when at the end of the transmission line. These are the top and bottom oscillograms in the middle row of Fig. 2. The calculated volt-time characteristic of the arrester is also shown.

EFFECT OF GROUND RESISTANCE

The oscillogams referred to dealt with the potential across the arrester. In practise it is frequently necessary to know the effect of ground resistance in the arrester circuit.

The tower footing resistance at Tower 14 as measured with a megger was 35 ohms. A 12-unit thyrite arrester was located about 90 ft. away with a driven pipe ground about 15 ft. from the arrester. This ground tested 50 ohms. The tower footing and the arrester ground were connected together with a cable approximately 100 ft. in length. This test set-up is shown in Fig. 5, together with the oscillograms obtained. The S-19 line was open at Tower 14, giving an open circuit potential of 1,130 kv., as shown in Fig. 2.

With the oscillograph connected to a ground remote

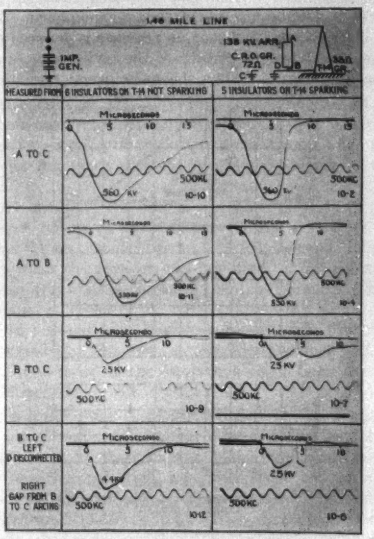

FIG. 5—OSCILLOGRAMS OF ARRESTER PERFORMANCE AS AFFECTED BY ARRESTER GROUNDS

Also showing protection of insulators on Tower 14. Tests made using 138-kv. thyrite arrester

from the other grounds, the voltage across the arrester plus the drop in the ground resistance was taken and is shown in the first oscillogram at the left of Fig. 5. On the tower at the time this test was made were six-suspension insulators spaced 4¾ in. which did not flash over until the string was later reduced to five insulators. The upper oscillogram on the right (10-2) shows the wave being chopped on the tail by the insulator flash-over.

The second group of oscillograms on the left and right (10-11 and 10-4) show the arrester potential with six and five insulators on the tower. The difference of 30 kv. between the crest potential of the first and second

set of oscillograms is in the *IR* drop due to the ground resistance. The third set of oscillograms (10-9 and 10-7) taken between the arrester ground and the separate oscillograph ground shows a crest potential of 25 kv. which corresponds well to the difference between the first two oscillograms having crest values of 560 and 530 kv. respectively. Oscillogram 10-9 also determines the point on the arrester voltage wave where the arrester began to take current. This point may be seen more clearly by referring to Fig. 6 showing the plot of the second and third oscillograms of Fig. 5 in which the insulators on the tower did not arc over. The breakdown of the arrester gap does not appear on the oscillogram, the rise being quite smooth except for a slight oscillation during the time of breakdown. This oscillation is so small that its presence is deduced only from the decreased density of the oscillogram on the front of the wave. Since the crest current through the

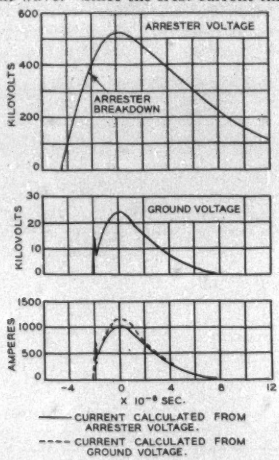

FIG. 6—CURVE SHOWING ARRESTER BREAKDOWN AS DETERMINED FROM OSCILLOGRAM OF GROUND RESISTANCE VOLTAGE

Arrester current and ground current curves

arrester will occur at the time of crest potential across the arrester, the ground voltage and the arrester voltage curves of Fig. 5 are plotted so that the zero of the time scale occurs at the voltage crests. At a time of 2.2 microseconds before the crest of the arrester potential is reached the gap breakdown occurs at a potential of approximately 375 kv.

Using the standard volt-ampere curves for the 138-kv. arrester for grounded neutral service a curve of current is plotted, as shown by the full lines in the lower portion of Fig. 6. The dotted lines show the current determined from the ground resistance and the ground voltage curve. The agreement is seen to be very satisfactory. It is well to note in connection with Fig. 6 that the arrester current reaches 1,150 amperes which constitutes quite a satisfactory test on an arrester rated 138 kv.

Referring again to Fig. 5, an oscillogram (10-12) was taken with the arrester ground disconnected, with the result that 44 kv. was measured across the tower footing resistance. Noticeable oscillations appear in the front of the ground voltage waves apparently resulting from reflection through the length of the ground wire. With both the tower ground and the arrester ground connected in parallel and with the conductor insulation reduced to 5 units, oscillogram 10-7 shows two voltage peaks of about equal magnitude. The first occurs when the arrester current flows to ground and the second when the five insulators flash over on the tail of the wave. In oscillogram 10-6 the sparkover of a small gap connected between the oscillograph ground and the tower ground is shown. This oscillogram is interesting since it shows time lag displayed in completing the sparkover of the gap.

The oscillograms of Fig. 5 indicate that when making arrester calculations one is justified in adding the ground resistance to the arrester resistance to determine the total protective characteristic of the arrester. The straight line characteristic of thyrite is very well adapted to this form of calculation. However, if the arrester ground and the apparatus grounds are bused together the potential on the apparatus will be independent of the ground resistance. The connections between the grounds should be as short as possible. The protection of an insulator string on a nearby tower has been demonstrated and flashover shown when the insulation is reduced below a certain limit.

EFFECT OF FLASHOVER OF THE LINE

There has been considerable discussion concerning the possible effect on the arrester protective characteristic of a flashover of an insulator string located at some distance from the apparatus. To simulate this condition the insulation on Tower 11 was reduced to 5 disks and the voltage at Tower 14 measured with the oscillograph.

In order to get a reasonable length of chopped wave, the front of the surge was slowed down by inserting an inductance coil between the surge generator and the line. The wave was chopped at Tower 11 where the tower footing resistance was 35 ohms.

The upper left oscillogram of Fig. 7 (11-14) shows this chopped wave as measured at Tower 14, where the line was open. The reflections back and forth between the point grounded by the spark over and the open end of the line are clearly evident.

The sparkover at Tower 11 evidently occurs after the reflection from Tower 14 arrives back at that point, since the wave at Tower 14 is 8 microseconds long before sparkover occurs, while the distance between Towers 11 and 14 is 1,663 ft. The subsequent reflections apparently have a period of 8.5 microseconds, which, on the assumption that the length of the line equals one-quarter wavelength gives a propagation velocity of

approximately 800 ft. per microsecond. Thus the beginning of the original impulse which had a crest value of about 480 kv. and wave front of 10 microseconds reached the open end of the line at Tower 14 and began to reflect, the voltage at the open end always being doubled. The reflected voltage wave traveled back toward the impulse generator adding its potential to that of the original incoming wave so that the potential at Tower 11 was about 550 kv. when the insulator flashed over. For two microseconds after the flashover occurred on Tower 11 the voltage at Tower 14 continued to rise on reflection, because of the time taken for the wave of reduction, due to the flashover on Tower

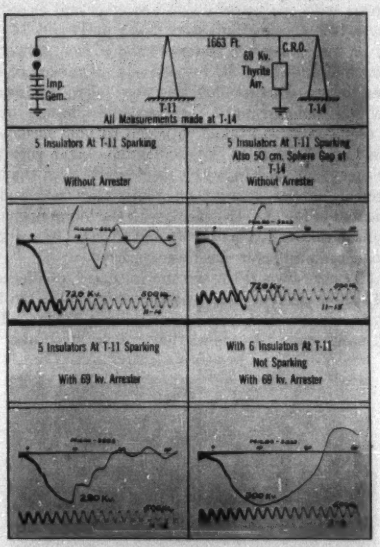

FIG. 7—OSCILLOGRAMS SHOWING THE EFFECT OF ARRESTER ON THE TAIL OF A WAVE CHOPPED BY THE FLASHOVER OF AN INSULATOR 1,663 FT. FROM TOWER 14

11, to travel back to Tower 14. Calculation of the reflection at Tower 14 indicates a potential of about 800 kv., although but 720 kv. was measured. The difference is probably due to corona loss. Thus a certain portion of the energy of the original wave was trapped between the flashover at Tower 11 and the open circuit at Tower 14. Since, however, the arc on Tower 11 has resistance which is to be added to the tower footing resistance, the voltage at Tower 11 was not zero, so that a certain small portion of the energy trapped will be absorbed in this resistance and some of it transmitted to the rest of the line. Some may also be received from the impulse generator end of the line because other oscillations are to be found trapped between Tower 11 and the impulse generator. Because of these and other losses of energy the oscillation decreases rapidly in amplitude.

When a 6-unit 69-kv. arrester was connected between line and ground the oscillations as measured at Tower 14 were quite different. The lower left oscillogram (11-12) shows a maximum potential of 290 kv. while the original traveling wave had a crest of about 480 kv. This means that instead of a positive* reflection taking place at Tower 14, a negative reflection occurred such that the crest of the reflected wave was equal to the difference between 290 and 480 kv. or 190 kv. Up to the time when the arrester began to take current from the surge, the reflection was positive, but as the arrester resistance decreased with greater current flowing through it, a point was reached where the arrester resistance was equal to the line surge impedance and no reflection occurred. A little later at higher voltage the arrester resistance became less than the surge impedance and the reflection changed from a positive reflection to a negative reflection, passing through zero at the instant when the arrester resistance equaled the surge impedance of the line.

As a result of the decrease in potential due to the arrester reflection, the flashover of the insulators on Tower 11 was delayed about 2½ microseconds longer than when the arrester was not used. The insulator flashed over on the tail of the resultant potential which appeared at Tower 11.

This test indicates that in cases where a flashover takes place on the line the arrester resistance will be such that most of the dangerous oscillations due to double reflections will not take place.

The lower right oscillogram (11-8) of Fig. 7 shows the potential allowed by the arrester when the insulators on Tower 11 did not flash over. This oscillogram shows the negative reflection returning from the impulse generator after about 22 microseconds. The upper oscillogram (11-15) is interesting in that a 50-cm. sphere gap arced on the third reflection, while the insulator on Tower 11 sparked over on the first reflection. From standard curves the voltage at the time of sparkover is less than the 60-cycle spark potential of the gap. The early stages of arcover occurred at higher voltage although the arcover was not completed until the third half cycle.

EFFECT OF TWIN ARRESTER STACKS

It has been customary to divide the total length of high-voltage arresters into two, or sometimes three parallel stacks, electrically connected in series. This practise has been criticized occasionally because of the number of turns or bends in the arrester circuit. It has also been maintained that the total length of arrester circuit was the criterion rather than the number of bends. In Fig. 8, two oscillograms are shown which were taken

*A positive or a negative reflection at a point means a reflection with the same or opposite polarity respectively, as that of the incoming wave which is being reflected.

with the idea of determining, if possible, the effect of subdivision of an arrester into two stacks. For this purpose a 12-unit 138-kv. thyrite arrester was used, as shown in Fig. 9. This figure also shows the location at Tower 14 where all of the arrester tests were made.

The oscillograms show that the arrester potential is not measurably increased by dividing the arrester into two stacks. The additional length of circuit involved in such subdivision is too small to give a measurable

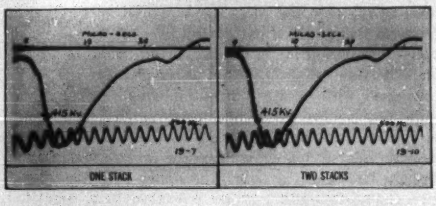

Fig. 8—Oscillograms showing the Effect of a Two-Stack Arrester Compared with a Single-Stack Arrester

indication on the oscillogram. This additional length was about 12 ft. being equal to the height of an arrester stack plus the connection between the two stacks. In the case of a three-stack arrester there would be less increase in length of circuit through the arrester from the line-to-ground terminal, compared to a single-stack arrester having the same number of units.

Although this increase in length did not make an appreciable increase in potential, with a 138-kv. arrester, yet with a low-voltage arrester such an increase in circuit length gives rise to an appreciable increase because of the lower potential allowed by the arrester. Thus a 10-kv. increase could not be detected on a 220-kv. arrester characteristic, but would be of importance in the case of a 3-kv. arrester.

Protection Afforded Transmission Line by Station Arrester

When attempting to coordinate apparatus insulation in stations with adjacent transmission insulation the use of a standard level of insulation extending out from the station for one-half mile has been suggested. From the standpoint of the lightning arrester protection the use of a single point where reference level insulation is maintained is superior. This level can well be established by the use of a horn or other gap close to the arrester and to the transformer to be protected. The gap may be set according to the values suggested by Montsinger and Dann,[6] which have a high enough spark potential so that the arrester will prevent the gap from sparking for anything but a severe direct stroke at the arrester. The use of such a gap gives the designer of both the arrester and transformer a definite condition to follow which may be stated in terms of volts and time.

If, however, the line insulation is kept at a coordinated level for a distance of one-half mile from the station, beyond which the insulation is increased materially, an arrester at the station will be unable to offer protection to insulators one-half mile away. As the arrester is approached the protection becomes better, and calculations are readily made for any given situation which will show the degree of protection offered at any location.

In order that the calculations might be checked tests were made on the S-19 line with the arrester at Tower 14, the impulse generator being connected to the line to give a steep voltage rise. Two arresters were investigated; one a 6-unit 69-kv. arrester and the other a 12-unit 138-kv. arrester. Tests were made using each arrester with the transmission line open and closed at Tower 14. Oscillograms were taken at Towers 14, 13, and 10, the distance of Towers 13 and 10 from Tower 14 being 500 and 2,180 ft. respectively. On each of the three towers when oscillograms were taken the insulation was reduced until the insulators flashed over. An oscillogram was taken with and without flashover.

In Fig. 10 are given the oscillograms taken with the line open at Tower 14. The oscillograms with the line closed are similar, except the potentials are decreased as indicated in Fig. 2. In general with the line continuous the insulation on Towers 14, 13, and 10 could be reduced by one insulator before flashover occurred.

At Tower 14 where the arrester is located, oscillogram 10-11 shows that 6 disks were protected by a 138-kv. thyrite arrester, the crest potential being 530 kv. The next oscillogram 10-4 shows that 5 disks arced over on

Fig. 9—Thyrite Arrester Set Up in Two Stacks

Tower 14 in the background. Oscillograph truck at extreme right

the tail of the wave. Moving on to Tower 13, oscillograms 16-11 and 16-7 show that the arrester at Tower 14 protects 8 disks but 7 disks arcover on the tail of the wave having a crest potential of 660. At Tower 10 the conditions are the same as at Tower 13. These results show that for this condition the arrester protection does not extend back from the arrester for a distance of more than 500 ft. If the incoming potential had been higher the effect of the arrester would have been felt at a

greater distance from the arrester. When comparing insulator disks flashed over, it should be remembered that the disks used were spaced 4¾ in. so that seven of these units are approximately equal to six units spaced 5¾ in.

The tests made with the 69-kv. arrester show that the zone of protection extends somewhat further than was the case with the arrester discussed in the last paragraph. At Tower 14 the arrester protected four disks but three disks arced over on the tail as seen in oscillograms 10-15 and 11-2. The crest potential measured at the tower was 315 kv. which was 25 kv. higher than the potential measured across the arrester terminals as shown in Fig. 2. At Tower 13, oscillograms 16-3 and 16-6 show a crest potential of 480 and 450 kv. with five disks protected and four disks arcing on the front. At Tower 10 reference to oscillograms 18-3 and 18-2 will show that the effect of the arrester is negligible although on the first oscillogram seven disks did not arc over, while on the second seven disks did arc over. It is clear, comparing the 138-kv. arrester results with those for the 69-kv. arrester measured at Tower 10, that the arrester is having no effect on the insulator flashover. Of course the results would be somewhat different with waves having longer tails and with waves having steeper fronts.

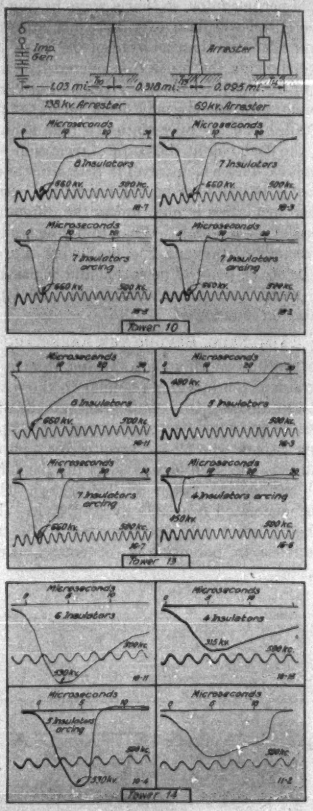

FIG. 10—OSCILLOGRAMS SHOWING THE DEGREE OF PROTECTION OFFERED TO INSULATORS ON INCOMING LINE WITH ARRESTER IN THE STATION

Tests with 138-kv. and 69-kv. arresters

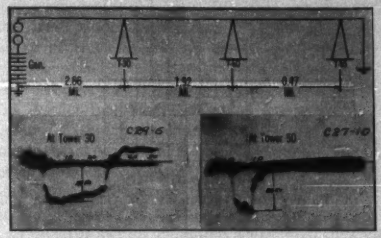

FIG. 11—OSCILLOGRAMS TAKEN AT TWO DISTANCES FROM A GROUNDED POINT SHOWING THE PROGRESS OF THE NEGATIVE REFLECTION BACK ALONG THE LINE

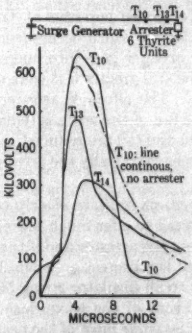

FIG. 12—PLOT OF THE OSCILLOGRAMS OF FIG. 10 SHOWING THE PROGRESSIVE CHANGE IN THE FORM OF THE WAVE AS IT TRAVELS ALONG THE LINE

If the arrester had reduced the potential at Tower 14 to zero, the zone of protection would not have been materially extended. The oscillograms in Fig. 11 taken in connection with another investigation show this condition plainly. With a short circuit to ground at Tower 55 the potential at Tower 50 was 86 kv. and 103 kv. at Tower 30. The oscillogram at Tower 50 shows the complete front of the wave, the tail being

cut by the negative reflection at Tower 55. An insulator flashover at Tower 50 on the tail might have been prevented by the ground at Tower 55, but a flashover on the front would not have been so prevented.

In Fig. 12 the oscillograms for the 69-kv. arrester shown in Fig. 10 have all been plotted to the same scale.

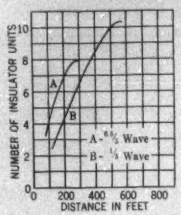

FIG. 13—PROTECTION OFFERED BY AN ARRESTER AT POINTS BACK ALONG THE INCOMING LINE

Traveling waves assumed to be 0.5/5 microsecond and 1/5 microsecond with a crest of 1,500 kv. Number of insulators used is based on front of the wave flashover. (See references 7 and 8 for impulse ratio curves). Calculations based on insulators spaced 5¾ in.

The broken line shows the voltage at Tower 14 with the line continuous. With the line open at Tower 14 all of the voltage ordinates should be doubled to indicate the open circuit potential at Tower 14. It will be noticed that the crest potential at Tower 10 is somewhat higher than the potential with line continuous measured at Tower 14.

The difference in crest potential at Tower 10 is due to the positive reflection which occurs from the end of the line before the arrester goes into operation. This difference would be still greater if it were not for the effect of attenuation. The effect of arrester may be plainly seen in that the tail of the wave is reduced. At Tower 13 both the crest and tail are reduced, while at Tower 14 the curve becomes practically the arrester characteristic itself.

These oscillograms indicate clearly that the arrester cannot be expected to offer much protection more than 500 ft. away from the arrester and if the wave front had been steeper still less protection would have been given by the arrester to an insulator string 500 ft. back from the arrester. Front of the wave flashover occurs frequently in practise while most of the flashovers shown in Fig. 10 were on the tail of the wave. This means that still steeper waves at higher crest potentials would have flashed over in less time and the arrester would need to be closer to give protection. As an illustration of the effect of wave front, two curves have been plotted in Fig. 13 showing the distance from an arrester that protection is offered if the insulators are considered as arcing on the front of the wave. The arrester is assumed to hold the potential constant at 350 kv. while the incoming traveling waves are assumed to have straight line fronts reaching a crest potential of 1,500 kv. in 0.5 and one microseconds. The tail is assumed to decrease to 750 kv. in five microseconds. The curves show that for the 0.5 microsecond front no protection is offered at a distance of more than 250 ft. against front of the wave flashover. With the one microsecond front this distance is increased being equal to approximately one-half the length of the wave front expressed in feet.

Flashover on the tail of the wave requires a lower potential per insulator, so that a wave having the same crest potential will flash over more insulators on tail of the wave flashover than on front of the wave flashover. A curve showing the protection offered by an arrester on tail of the wave flashover is given by Fortescue.[9]

In practise flashovers may occur on either the front or the tail, but if the over voltage is high as in a direct stroke, the flashover at the point of inception will be on the front. Waves which travel in from a distance will be more likely to flash over on the tail at the point where decreased insulation is met, unless the difference in insulation levels is great. When applying transmission line arresters to prevent line flashovers the arresters should be at the point of inception and to prevent front of the wave flashovers, need to be close together, preferably on each tower.

POTENTIAL BEYOND THE ARRESTER

With the line continuous to Grand Rapids a set of oscillograms given in Fig. 14 similar to those shown in Fig. 10 were taken. One oscillogram (20-16) was taken at Tower 29 and shows the appearance of the wave after traveling beyond the 69-kv. arrester for a distance of 1.38 miles. The oscillogram (8-4) on the left is the original oscillogram taken at Tower 14. The crest potential decreased from 265 kv. to 240 kv. the wave shape being practically the same as at the arrester. The decrease in potential is, of course, caused by attenuation.

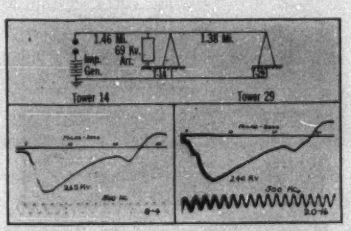

FIG. 14—OSCILLOGRAMS SHOWING EFFECT OF DISTANCE ON TRANSMITTED WAVE

These oscillograms demonstrate clearly one of the fundamentals of wave reflection and transmission. The transmitted wave beyond the arrester is the wave measured across the arrester itself. The reflected wave is the difference between the original applied wave and the voltage across the arrester. The reflected wave returns

toward the point from which it came, while the transmitted wave passes on never to return, unless a reflection point is reached before the losses have reduced the wave to zero as it travels along the line.

Arrester Location and Connections

When discussing the application of lightning arresters it has long been pointed out that the connection to the arrester and to ground should be kept as short as possible and that the arrester should be located close to the transformer. Oscillograms given in Fig. 15 were taken to show the effect of arrester connections.

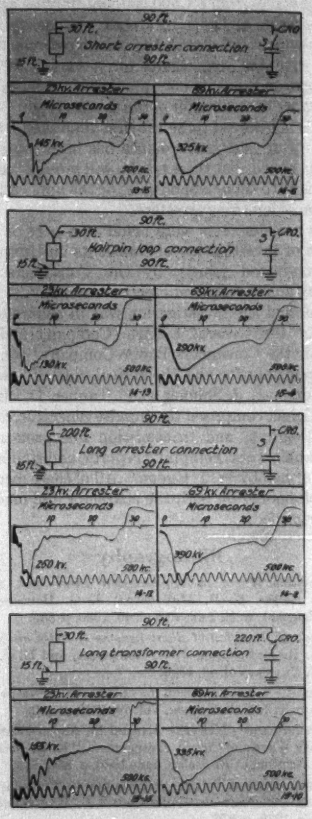

Fig. 15—Oscillograms showing the Effect of Length of Arrester Connections and Location with Respect to the Transformer

The test was made with a two-unit thyrite arrester rated 23 kv. and with a 6-unit arrester rated 69 kv. The arrester was located at Tower 14 as in the other tests. The distances involved are shown in the figure. The distance from the arrester to the transmission line was 30 ft. while the distance from the arrester to the arrester ground was 15 ft. The measured arrester ground resistance was 40 ohms while the tower footing ground resistance was 35 ohms. The two grounds were connected together by a conductor 90 ft. in length. The line was open at Tower 14 and a conductor run down to a capacitor which represented the electrostatic capacity of the transformer to be protected. Tests were made with the oscillograph connected between the top of the tower and the ground bus, so that the oscillograms do not include the effect of the tower footing resistance, except as it changes the total current flow through the arrester which will, of course, change the arrester potential slightly.

Since the oscillograms taken with the switch S (Fig. 15) closed are all of the same relative characteristics as with the switch open, only those with the switch open will be reproduced. There is a tendency for the crest voltage to be slightly higher with the capacitor connected, this being due to the manner in which the reflections between the arrester and the capacitance add. With different values of capacitance and different distances between the arrester and the capacitor the voltage might be more or less than that without the capacitor.

The first pair of oscillograms, 13-15 and 14-6 were taken with the connections shown and give a crest potential of 145 kv. at the oscillograph with the 23-kv. arrester, and 325 kv. with the 69-kv. arrester. Using the hairpin loop the voltages are reduced to 130 kv. and 290 kv. respectively, as seen in oscillograms 14-13 and 15-4 in the second group. The reason, of course, for this reduction in potential is that the line is brought down to the arrester so that the time involved in getting from the transmission line through the arrester and to the ground is made as short as possible. In this connection the arrester is acting as a resistance between line and ground, which in itself does not delay the impulse in reaching the ground. Only distance delays the impulse unless time is involved in changing the arrester from an open circuit to a resistance which is not the case for the arrester used in this test. In order that the impulse on the line be reduced it is necessary that a negative reflection return from the ground through the arrester so that the time delay involved is the time taken for the wave to travel from the line through the arrester to ground and back up to the line. In the meantime the surge on the line has not been waiting, but has traveled on toward the transformer. If the length of the arrester connection line to ground was made equal to half the length of the wave front of an incoming wave, the potential at the transformer would become double the crest value of the incoming wave by reflection and the arrester wave of reduction would reach the transformer only in time to cut the tail of the wave.

To illustrate the effect of long arrester circuit to ground the third group of oscillograms was taken in which 200 ft. of conductor was inserted between the arrester and the transmission line. Its effect is clearly seen on oscillograms 14-12 and 14-8. From the best case with the hairpin loop and 23-kv. arrester, the transformer voltage has increased from 130 to 250 kv. and with the 69-kv. arrester from 290 to 390 kv. With steeper waves or with longer arrester circuit these differences would be increased up to the time that the original wave crest was reached.

The condition of a long arrester circuit is similar to those existing in connection with Fig. 10, in which it was shown that the arrester could not give any protection at Tower 10 when the arrester was at Tower 14, 2,180 ft. away.

The fourth group of oscillograms shows the effect of adding 220 ft. to the distance between the arrester and the transformer. These oscillograms may be compared with the corresponding ones of the first group. Since the arrester circuit is the same, it will be seen that only 10 kv. was added to the transformer voltages by such an increase in circuit length. With either a steeper wave or with more distance a greater increase in potential would have been noted. It is of interest to note that the potential previously measured directly at the arrester terminals was always 122 kv. with the 23-kv. arrester and 290 kv. with the 69-kv. arrester. Arrester volt-ampere curves show that since the open circuit potential without the arresters was 1,130 kv. the current through the 23-kv. arrester was over 2,000 amperes, while that for the 69-kv. arrester was about 1,500 amperes.

The oscillograms indicate that the length of the arrester connection is more important than the length of the connection between the arrester and the transformer. This is true because with long arrester connections the transformer reflects the incoming wave, while with short arrester connections and long transformer connections the transformer reflects only the arrester potential. If it is necessary to choose one or the other, naturally the second condition is to be preferred. To be assured of the best results from the arrester it should be connected close to the transformer, the arrester circuit being as short as possible between line and ground.

The effect of the length of circuit is not dependent on arrester rating. It depends on the steepness of surge. However, the question as to whether or not a certain length of lead is seriously objectionable does depend on arrester rating since an increase in potential which is negligible on a high-voltage arrester may well mean insufficient protection with a low-voltage arrester.

When considering the length of the arrester circuit the entire length from the line to ground must be considered. If the arrester is connected to an extensive steel structure which is thoroughly grounded, the arrester circuit distance can be taken from the line to the steel structure. In cases where both the arrester and the transformer are elevated but are bonded together, as may be the case in a small industrial substation, the voltage in the transformer insulation will be the arrester voltage if the leads are short. The potential on the line will be higher than this if the ground connection is long but the transformer will be protected.

Conclusion

Tests have been made under conditions where true traveling waves from a 1,500,000-volt impulse generator were impressed on lightning arresters of various ratings. It was demonstrated that the calculated results agree closely with the test results for a variety of circuit conditions, so that one may predict with assurance the performance of an arrester of the thyrite type.

As this is believed to be the first time such a demonstration of arrester performance has been made in the field, it is hoped that these results will prove of considerable benefit to those responsible for the application and operation of lightning arresters.

These arrester tests were made as a part of a large program of investigation of transients on transmission lines, conducted jointly by the Consumers Power Company and the General Electric Company.

In the field the investigation was in charge of E. J. Wade for the General Electric Company and J. R. Eaton for the Consumers Power Company. O. Brune was responsible for the theoretical side of the work, while Messrs. E. J. Shimek, C. C. Tanner, D. Blackwell and D. Geiger comprised the impulse generator, cathode-ray oscillograph, and transmission line groups. The photographic work, compiling reports, and similar work was performed by W. J. Rudge. All of these men are to be complimented on both the character and amount of work performed.

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Discussion

R. H. Park: Messrs. McEachron, Wade, and others have shown that it is now possible to calculate the performance of lightning arresters as far as concerns the effect of voltages due to lightning. In determining the effect of normal frequency voltages, it is usually only necessary to know the maximum voltage that an arrester can safely withstand continuously. I understand that this figure is given by the ungrounded arrester rating. For grounded circuits, these data would appear to be practically all that is required to permit intelligent application of arresters.

On the other hand, it would appear, on the basis of our present information, that in the case of ungrounded circuits the arrester is practically certain to be subjected to overvoltages resulting from arcing grounds. I believe that, according to present understanding, the voltage during an arcing ground periodically builds up for a few cycles and then discharges through the arrester, the discharge energy being roughly proportional to the length of line affected. If this is the case, it would follow that any arrester should be capable of discharging the voltages occurring during arcing grounds provided that the length of line involved is short enough, while the maximum safe length of line would depend upon the type of arrester used. These considerations suggest the desirability of obtaining quantitative information in regard to the performance of arresters when subjected to voltages due to arcing grounds.

Series Resistance Method of Increasing Transient Stability Limit

Series Resistance Inserted into Generator Circuit Loads Machines During Fault and Retards Subsequent Overswing

BY R. C. BERGVALL[1]

Associate, A. I. E. E.

Synopsis.—This paper considers the effect of resistance inserted in series with generators during and after a short circuit to increase the transient stability limit. The series resistance loads the machines during a fault and greatly reduces the normal tendency to overspeed. After the fault has been cleared the series resistance exerts a powerful influence in retarding excessive over-swing by taking advantage of the low generator stored energy relative to that of the load center. It is shown that by using series resistance a typical hydro system can maintain synchronism through a three-phase fault with speeds of circuit breaker and relay operation now available. A three-phase resistor normally short-circuited by a circuit breaker and connected into the neutral of each generator provides the most desirable arrangement. The cost will generally be much lower than the combination of other measures commonly taken such as high inertia generators, low reactance of generators and transformers, damper windings, transformer neutral impedance, special simultaneous relay schemes or other remedial measures such as lightning proof lines, and the expenditure can be deferred until it is justified by operating experience.

* * * * *

THE demand for service continuity from large power stations that transmit their output over long transmission lines has resulted in the development of high-speed circuit breakers and relays. For a short circuit within 10 per cent of the end of a transmission line the circuit breaker nearest the fault can be opened in approximately 8 cycles and the circuit breaker at the other end of the line in 16 cycles, the delay being necessary to obtain proper relay selectivity with the other parallel line. Faults within the 80 per cent zone at the middle of the line can be cleared simultaneously from both ends in 8 cycles. Various methods of obtaining simultaneous circuit breaker tripping for the severe type of faults within the 10 per cent end zones have been considered, but have not been extensively adopted on account of cost and the possibility of dropping the entire station due to faulty operation.

Hydroelectric units have a relatively low stored energy per kilowatt and in general will not stay in synchronism for three-phase faults within the 8-cycle simultaneous clearing zone, or for double line to ground faults within the 8-16-cycle sequential clearing zone when transmitting rated kilowatt output. The electrical output is greatly decreased during short circuits on account of the low power factor of the fault current circuit while the prime mover input remains substantially constant for the time intervals under consideration. The difference between the mechanical input and the electrical output must be expended in accelerating the generator rotor, and depending upon the stored energy, the severity of the fault, and the time of circuit breaker operation, a velocity away from synchronism is imparted to the rotor that must be removed by the synchronizing forces present after the fault is cleared, without permitting sufficient over-swing to result in loss of synchronism.

This paper considers the effect of automatically inserting a relatively high value of series resistance into the generator circuit immediately upon the occurrence of a short circuit, and of removing the resistance a predetermined time after the fault has been cleared. The series resistance can be inserted into the fault current path in about six cycles and will increase the generator output

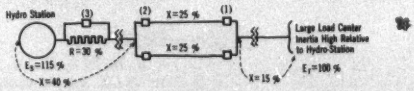

FIG. 1—HYDROELECTRIC TRANSMISSION SYSTEM USED

Example: 60 cycles. Per cent impedances based on generator-kw. rating, represent typical proportions. The 30 per cent series resistor is normally short-circuited

to around full load for the remaining two to ten cycles of fault duration, thereby preventing the rotor from attaining an excessive velocity away from synchronism during the fault. In addition the series resistance is beneficial after a fault has been cleared because it heavily loads the hydro-generators at the expense of delivering less power to the load center. The hydro-generators, which have a relatively low stored energy compared to the load center, are retarded at a much greater rate than the load center so that the series resistance tends to draw them together. The added losses will cause a slight decrease in system frequency, but this is of no consequence as the governors can restore the frequency as long as synchronism is maintained.

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

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The action of the series resistance during and after a fault will be considered in detail with reference to the hydroelectric transmission system shown in Fig. 1. The impedance proportions are typical and the stored energy of the hydro station is low relative to the load center. A 30 per cent resistance short-circuited by circuit breaker (3) is shown in series with the generators. Actually the preferred location is in the neutral of each generator in accordance with Fig. 2. The per cent resistance on the generator rating will then be the same regardless of the number of machines in service.

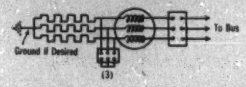

Fig. 2—Preferred Location of Series Resistor in the Neutral-Phase Leads of Each Generator

The conditions produced on the system by a three-phase fault near the load end, with and without the series resistance are compared in Figs. 3, 4, and 5 for 8-16-cycle sequential circuit breaker clearing. The initial angular displacement for 100 per cent load and both lines in service is 31 deg. When the short circuit occurs the generator electrical output is decreased from 100 per cent to 12 per cent on account of the low power voltage is now partially restored and some power is delivered to the load center. This accounts for the maximum power of curve *b* being 80 per cent even though the fault is not entirely removed. The difference between the ordinates of curve *b* and the 100 per cent line represents the percentage of full-load torque accelerating the rotor and at 16 cycles the angular displacement is 176 degrees and synchronism has been lost.

Fig. 4 shows the same fault and the same line circuit breaker sequence with 30 per cent resistance inserted in series with the generators six cycles after the fault occurred by the opening of circuit breaker (3). In the six- to eight-cycle period the fault current produces losses in the resistor that increase the generator output from 12 per cent to 110 per cent even though no power can be delivered to the load center through the three-phase fault. The opening of circuit breaker (1) at eight cycles transfers operation to curve *b*. The losses in the series resistance plus the power delivered to the load center increases the maximum power to 152 per cent as compared to 80 per cent without the resistor. At 16 cycles circuit breaker 2 opens and the generator output will be in accordance with curve *c*, Fig. 4, which has a crest of 190 per cent as compared to 150 per cent for the corresponding curve *c*, Fig. 3, without series resistance. The shaded area below the 100 per cent line representing

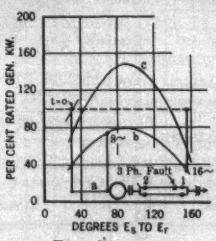

Fig. 3

Fig. 3—Generator Power-Angle Diagram. No Series Resistance

Curve	Time	Fault	Bkr. 1	Bkr. 2
a	0-8 ~	On	Closed	Closed
b	8-16 ~	On	Open	Closed
c	16-∞	Off	Open	Open

After fault is cleared there is no restoring power as curve *c* is below 100 per cent line

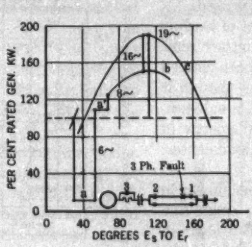

Fig. 4

Fig. 4—Generator Power-Angle Diagram. 30 Per Cent Series Resistance Inserted in 6 Cycles

Curve	Time	Fault	Bkr. 1	Bkr. 2	Bkr. 3
a	0-6 ~	On	Closed	Closed	Closed
a	6-8 ~	On	Closed	Closed	Open
b	8-16 ~	On	Open	Closed	Open
c	16-* ~	Off	Open	Open	Open

*Breaker 3 will be reclosed in from 30 to 200 cycles depending on the system. Operation then will be on curve *c* in Fig. 3.

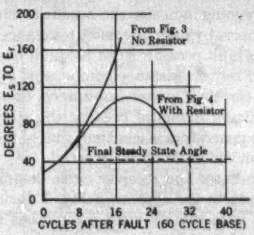

Fig. 5

Fig. 5—Angular Displacement vs. Time After Occurrence of Fault

Without series resistance the hydro generators are 180 deg. out of phase in 18 cycles. With series resistance synchronism will be maintained by a wide margin

Figs. 3-4-5—Three-Phase Fault Near Receiving End of Hydroelectric System Shown in Fig. 1 With and Without Series Resistance. Circuit Breaker 1 Opened in 8 Cycles, Circuit Breaker 2 Opened in 16 Cycles

factor in the fault current circuit. The prime mover continues to apply 100 per cent torque and the difference, or 88 per cent torque, accelerates the rotor.

Fig. 3 shows conditions without the use of series resistance. The angular displacement at eight cycles has increased to 69 degrees and the opening of circuit breaker (1) transfers operation to curve *b*. The system prime mover input is a measure of the energy expended in accelerating the rotor, and the equal shaded area above the line indicates that this energy has been absorbed and the crest of the angular swing away from synchronism has been reached. It is evident that there is ample area available under curve *b* and synchronism would have been maintained even though circuit

breaker 2 had not opened for a considerably longer period of time. The proper time for short-circuiting the series resistance will be discussed later. When this is done operation will be transferred to curve *c*, Fig. 3, which represents normal conditions with one line out of service.

The angular displacement *vs.* time with and without the use of series resistance is shown by Fig. 5. These

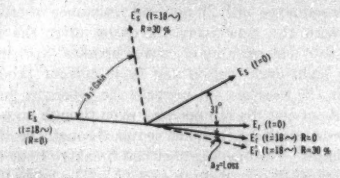

FIG. 6—VECTOR POSITIONS OF E_s AND E_r 16 CYCLES AFTER FAULT

Angle between E_s and E_r decreased by series resistance. Low inertia generators retarded at greater rate than high inertia load center slows down

curves were calculated by point-by-point methods using the power angle diagrams of Figs. 3 and 4 and typical hydro generator stored energy. It is evident that synchronism will be lost without the use of series resistance and will be maintained by a wide margin if it is used.

The foregoing indicates the beneficial effect of loading the generators by inserting series resistance in the circuit during the fault period. After the fault has been cleared the added series resistance component, even though it increases the total circuit impedance, exerts a substantial stabilizing influence because the stored energy of the hydro generators is less than that of the load center. This action will be analyzed in detail.

The generator and receiver circle diagrams shown in Fig. 7 apply to the system of Fig. 1 during single-line operation immediately after a fault has been cleared. With no series resistor the maximum power from the generator and receiver circles is practically the same. The addition of 30 per cent series resistance as shown by the dotted curves increases the maximum generator output from 150 to 190 per cent, but decreases the maximum received power from 140 to 92 per cent due to the added transmission loss. The power angle diagrams for these conditions calculated from Fig. 7 are shown by Figs. 8 and 9.

Fig. 8 shows the generator output plotted against angular displacement with and without series resistance. The area above the 100 per cent line represents the energy available for retarding the rotor after fault has been cleared. The 30 per cent series resistance greatly increases this area and also permits a maximum angular swing of 188 deg. instead of 138 deg.

Fig. 9 shows the corresponding receiver power angle diagrams indicating that the received power is considerably reduced by the series resistance loss.

The influence of the series resistance on the angular positions of the generator and receiver voltages with typical stored energy proportions between the hydro generators and the synchronous machines at the load center is illustrated by Fig. 6. The line circuit breakers were assumed to operate in 8 and 16 cycles respectively. The initial angle with both lines in service was 31 deg. Without series resistance the energy represented by the shaded area under the 100 per cent line in Fig. 3, acting on the relatively small hydro generator stored energy moves E_s to E_s' in 16 cycles. Using the series resistance the shaded area above the 100 per cent line in Fig. 4 beings to retard the hydro generators after six cycles so that E_s only moves to E_s'' in 16 cycles. The series resistance, therefore, decreases the angular displacement of the generators by the large angle a_1.

Considering the load center the series resistance reduces the delivered power as may be ascertained from Fig. 8. Due to the relatively high load center inertia this reduction in delivered power only moves E_R apart from E_s by the small additional angle a_2. At the end of 16 cycles the total angular displacement has been decreased $(a_1 - a_2)$ deg. due to the insertion of 30 per cent series resistance, and the velocity away from syn-

FIG 7.

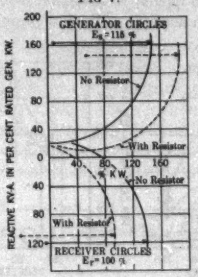

FIG 8.

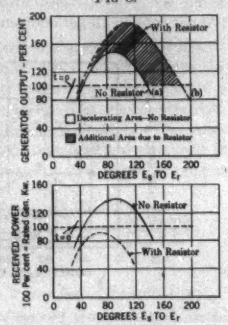

FIG. 9

FIG. 7—CIRCLE DIAGRAMS WITH ONE TRANSMISSION LINE SERVICE

FIG. 8—GENERATOR POWER-ANGLE DIAGRAM

FIG. 9—RECEIVER POWER-ANGLE DIAGRAM

EFFECT OF 30 PER CENT SERIES RESISTANCE ON CHARACTERISTICS OF HYDROELECTRIC TRANSMISSION SYSTEM IN FIG. 1 DURING SINGLE-LINE OPERATION AFTER FAULT HAS BEEN CLEARED

chronism has also been practically removed as may be ascertained from Figs. 4 and 5.

A test set-up was made in the shop using proportions similar to the hydroelectric transmission system shown in Fig. 1 for the purpose of checking our methods of calculation. Fig. 10 is an illustration of the shop system. The hydro station was represented by a 100-kva. synchronous motor-generator set with the d-c. end

acting as a prime mover. Sufficient external resistance was used in series with the motor armature to give it approximately constant torque characteristics over a sufficient range of speed. The detailed test data cannot be presented in this paper on account of space limitations. The tests checked calculations closely, and showed that for the same fault and time of circuit breaker operation, up to greater than 50 per cent more

Fig. 10—Shop Test Using 100-Kva. Generator

power could be delivered and still maintain synchronism by the use of series resistance. The tests showed that quick response excitation functions very effectively in conjunction with the series resistance. The series resistance would hold the system in synchronism until the internal machine voltages could be actually built up to produce an exceedingly rigid system by the time the series resistance was removed.

For illustrative purposes the preceding discussion dealt with a simple system having infinite capacity at the load center, except for the discussion regarding Fig. 6. The transients produced by a two-phase-to-ground short circuit on a system where the synchronous equipment at the load end is smaller in proportion to the hydro station than is the case on the usual system are shown in Fig. 11. The angular oscillations were calculated by point-by-point methods in conjunction with the a-c. calculating board. The dotted line shows the rapidity with which the hydro generators go out of synchronism with reference to the synchronous machinery at the load end if no series resistor is inserted. The full lines show the angular oscillation with 30 per cent series resistance on the generator rating inserted in the circuit during the 6-25 cycle period following the fault. It is evident that synchronism will be maintained, as the angular deviation has been restored to about normal in 60 cycles and will stabilize itself after a few oscillations. The downward slope of the curves indicates that the system frequency has decreased slightly due to the losses while the series resistance was in circuit, but the slight decrease in frequency is of no consequence as long as synchronism is maintained.

Three-phase faults, which are the most severe type, were used for the analysis of faults on the system of Fig. 1. Single-phase and double line-to-ground faults do not lower the generator output to such an extent as three-phase faults and these faults in the 8-16-cycle tripping zones could be handled even if the series resistance were not inserted until the fault was closed. The increased area under the generator power-angle curve as shown by Fig. 8 would be sufficient to prevent over-swing, and inserting the resistance during the fault period, while not as effective as for three-phase faults, has beneficial effect and serves to increase the margin of stability. It is evident that if a system can be designed to withstand the severe three-phase faults, it will as a consequence withstand other faults by a wide margin.

The magnitude of the series resistance can be varied over a wide range with practically the same results. The resistance should be inserted as soon as possible after a fault occurs. The length of time that it is left in the circuit will depend on the individual system under consideration.

A number of methods for determining the time of removing the resistor will be considered:

1. Remove the resistor at the crest of the first swing, or slightly afterwards, but before the rotor has gathered enough return velocity due to large area under curve *c*, Fig. 4 to go out on the next swing along the normal operation curve in Fig. 3.

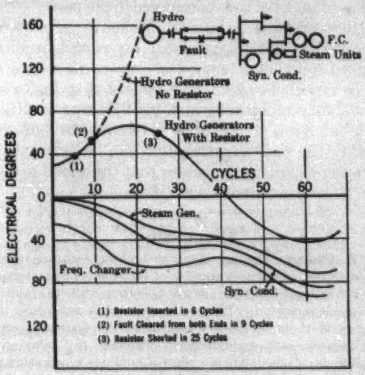

Fig. 11—Transients on Typical System Showing Effect of Inserting Series Resistance

From point-by-point calculations made in conjunction with a-c. calculating board

2. Leave the resistor in the circuit until the severe oscillations are damped out, which occurs in two or three seconds. The resistor can then be short-circuited without producing any appreciable disturbance as there is only four degrees angular difference with and without the series resistance at 100 per cent load as shown by Fig. 8. Quick response excitation will in this time have

increased the internal voltages so that the crest of the the curve *a*, Fig. 8, will approach curve *b* forming a rigid system for a short time after the resistance is short-circuited.

3. Short-circuit out the series resistance in steps and gradually "notch" from curve *b*, of Fig. 8 curve *a*.

4. There is usually a number of machines in a station and the neutral circuit breakers short-circuiting the resistors can be closed successively, producing the same effect as cutting the resistance out in steps.

The series resistor used in the neutral of each generator as shown by Fig. 2 will have approximately the same physical size as an ordinary grounding resistor. The circuit breaker short-circuiting the resistor need not have a high rupturing capacity and the duty is light since it is shunted by resistance.

Conclusion

The foregoing indicates that synchronism can readily be maintained on a hydro system of normal proportions for any type of a fault with the speeds of circuit breaker and relay operation now available, by inserting series resistance in the circuit for a short time. The cost of the series resistor and its circuit breaker will generally be less than the combination of other measures such as high rotor inertia, low generator and transformer reactance, damper windings, impedance in the transformer neutrals and special simultaneous operation relay schemes that have been considered in the past.

After confidence has been established in the series resistance method of maintaining stability, consideration may be given to building lower cost transmission lines that will be expected to flash over, instead of trying to solve the stability problem by building lightning proof lines.

Unlike most of the other measures for improving stability the resistor installation can be deferred until operating experience indicates that the added expenditure to maintain service is justified.

Discussion

S. B. Crary: The use of series braking resistors increases the generator output at a sacrifice of the receiver or motor input and of the simplicity of the cycle of operation while the fault is on and after it is cleared. This method then does not belong in the category of those methods which make the system inherently more reliable, such as high-speed switching, fast relaying and lightning proof lines; but in a class of methods which attempt to increase the reliability at a sacrifice of other characteristics of the system.

The effect of series braking resistors has been analyzed over a whole range of switching times rather than at a particular set of switching times as was done by Mr. Bergvall. By doing this a more complete picture of the problem is available.

Fig. 1 illustrates graphically the results of calculations made to determine the relative gain that is obtainable by the use of a series braking resistor. Calculations were based on a typical 60-cycle system using the reactances shown, which were the same as used by Mr. Bergvall. A 30 per cent series resistor, which is about the optimum value, was used and the same as used by Mr. Bergvall for this particular system. Conditions as assumed are ideal for the series resistor, in that an infinite bus receiver was assumed, which is a system so large compared with the generator that its frequency or voltage does not change during any fault conditions, and is not affected by the decreased amount of power that it receives from the generator while the series resistance is inserted in the circuit. The theoretical gain that can be realized for a sequence of switching of 6, 8, 16 cycles (6 cycles being the time required for breaker 3 to open, 8 cycles for breaker 1 to open, and 16 cycles for breaker 2 to open) is in the neighborhood of 50 per cent. This total gain, however, is not all necessary as the rated output of this typical system is unity.

The practical gain to be realized from the use of the series braking resistor decreases with faster switching time. It has already become practical to consider actual switching times under favorable conditions of the order of 6 cycles or less as is shown in the paper by Messrs. Spurek and Strang. At the point of rated output for the particular system being considered, 6 cycles switching is sufficiently fast to allow the system to ride through a three-phase fault at full load. The curves given on this figure are for a fault at the receiving end of the transmission line, which is just as severe as a fault at the generating end of the line as breaker 2 is not opened until the end of 12 to 16 cycles for the case of a fault at the receiving end as against opening in 6 to 8 cycles for a fault at the generating end.

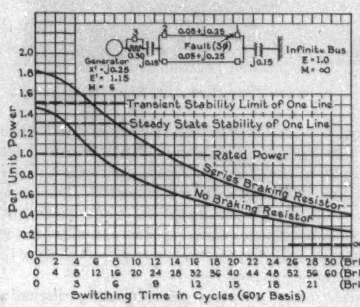

Fig. 1—Power That Can Be Carried Through the First Swing with a Three-Phase Fault with and without Series Braking Resistors

Calculations show that these faults are of equal severity. Faults outside of the 10 per cent zone at either end will be cleared at both ends in 6 to 8 cycles so that the curves given above are for the most severe faults that could be obtained on the transmission lines. For three-phase faults in the center section of either line the theoretical gain to be realized from the series resistance over the case when no braking resistors are used is reduced to 34 per cent for 8-cycle switching of the line breakers and 6-cycle switching of the resistor breaker. This reduction in the theoretical gain for a fault in the mid-section is accompanied by a reduction in the practical gain also, for 7 to 7½ cycle switching of the line breakers at either end is sufficiently fast to allow the system to carry full rated output through the first swing without the use of braking resistors for this particular location of the fault.

With the recent improvement in switching times the possibility of three-phase faults occurring on a system has been reduced. Even in the past three-phase faults have been fairly infrequent. For faults other than three-phase, the power limit is greater. As Mr. Bergvall has stated "if a system can be designed to withstand the severe three-phase faults it will as a consequence withstand other faults by a wide margin." High-speed switching then for this particular type of system will normally give a margin of power limit above the rated output and, therefore, eliminates

the need of braking resistors and the accessory apparatus that must be used with them.

If the system under consideration has slower speeds of switching, it is problematical whether it would be good engineering to equip the generating station with series braking resistors, the necessary additional circuit breakers, and the required control equipment to protect against the occurrence of three-phase faults, when the manufacturers are already in a position to supply breakers and relays with an over-all operating time of approximately 8 cycles or better.

All of the foregoing has been based on a system having a comparatively large receiving end inertia as compared with that of the sending end.

Consideration, however, should also be given to a system which depends considerably on the hydro-station for its power when the hydro-station is operating at or near full load. Such a system would suffer from the loss of the hydro-station sufficiently so that a large portion of the load would have to be dropped, while with the system of the previous case the loss of the generating station would not necessarily be accompanied by a loss of load.

With a system having a comparatively large hydro-station as compared with the load center, conditions are different from that

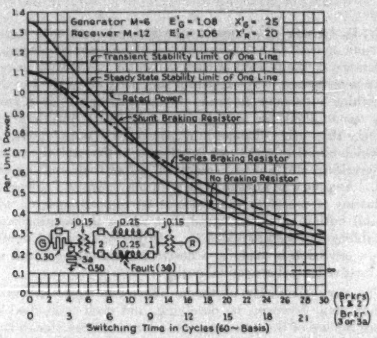

Fig. 2—Power That Can Be Carried Through the First Swing with a Three-Phase Fault with and without Braking Resistors

of the case when the load center has a relatively large inertia. A system of this type may have a receiving end inertia which is only twice as large as the generating station. Upon the occurrence of a fault on the lines connecting the hydro-station to such a load, the motor may swing to a considerable angle away from the generator before the fault is cleared, so that it may not be possible for synchronism to be maintained when the restoring forces become effective, *i. e.*, upon the clearing of the fault. The stability problem of such a system is much more acute as may be seen by referring to Fig. 2. This is the type of system which needs a means of improving its stability limit. The stability characteristics of such a system have been studied.

Mr. Bergvall in his paper selected as his second case a system having a finite receiving end inertia. As the constants of the system were not given, a system was set up having the constants as shown on Fig. 2. This figure gives the switching time curves for a system having an equivalent receiving end inertia of twice the sending end inertia. The load is taken as 160,000 kw. The hydro capacity is taken as 100,000 kw. and the steam capacity as 60,000 kw.

Mr. Bergvall in his calculations used a two-phase to ground fault, at the center of the line, however, since a three-phase fault was considered in the previous case it seems logical to examine the case of three-phase faults on this system as well, so the results will be more closely comparative. Fig. 2 is a switching time curve for this system with a three-phase fault in the center of the line. On such a system the series braking resistor gives a much smaller gain (10 per cent at 8 cycles) in the stability limit for the switching sequence assumed (the braking resistor breaker opens in ¾ of the time it takes the line breakers to open) the gain in power limit is reduced due to the smaller relative inertia of the receiver. This lower receiving end inertia is affected more by a decreased power input, which is brought about by the insertion of the series resistance, than it would be if it were very large. A decrease in power input causes the motor to slow down and therefore the generator must slow down at a faster rate in order that the angle between them will be decreased and synchronism maintained. This results in a greater dip in frequency and receiving end voltage than would normally be obtained. The standards of system requirements are already fast traveling beyond the point at which it is only necessary to consider means of holding the system together, but it also becomes necessary to keep the voltage and frequency at as near a constant value as possible.

There are other methods by which the stability limit may be increased for this particular type of system that parallel the series resistance method somewhat in principle. Fig. 2 has included on it a curve for a shunt braking resistor, this braking resistor to be thrown across all three phases to ground after the occurrence of a fault. The particular value of resistor used in calculating the above curve is 50 per cent. That is, with 50 per cent of normal voltage across the resistor, the loss in the resistor will be normal or 100 per cent kw. The speed at which this resistor was connected to the circuit was taken as 6 cycles for 8-cycle switching of the line breaker, the same time as Mr. Bergvall selected for inserting the series resistance after the occurrence of the fault. A breaker can be built for this duty such that the necessary relay time and the speed of closing is 6 cycles, or less. The shunt braking resistor curve shows a greater power limit than the series for switching times less than 11 cycles when used in this particular system.

The use of either series or shunt braking resistors, if used at all, should be strictly limited to the period during which the station is operating at or near full load. The value of resistance selected to result in any appreciable gain in power limit at full load must, of necessity, be of such a value as to load the generators down considerably. The loss in the resistor during a large portion of the period while it is connected in the circuit is the rated kw. output of the station. If the generator or the generators of the station are lightly loaded, switching on the braking resistors with the occurrence of a fault would increase the magnitude of the oscillation rather than decrease it. This objection, however, could be solved by the use of a wattmeter control scheme.

In general, therefore, the following conclusions may be made as to the benefit to be derived from the use of the shunt and series braking resistors.

Approximately 75 per cent of the generating capacity of the country is steam whose stability problems can in most cases be solved by other means. Out of the remaining 25 per cent of hydro capacity those stations having stability problems may be grouped into two general classes; systems with relatively large receiving end inertias, and systems with relatively small receiving end inertias.

a. For systems having a relatively large receiving end inertia, high-speed switching will, in general, be sufficient to carry the system through the most severe faults.

b. For systems having a relatively small receiving end inertia, series resistance method shows very little gain in power limit with the shunt resistor method showing a slightly greater advantage for the switching times now available.

J. W. Anderson: As pointed out by Mr. Bergvall, hydro-electric generators because of their relatively low inertia have a

very marked tendency to overspeed upon loss of load immediately after a short circuit occurs. Some actual figures of acceleration will serve to emphasize this effect. Tests made some time ago at Conowingo to determine governor action upon sudden loss of load showed that generator acceleration during the first half-second after dropping full load averaged over 900 elec. deg. per sec., or, in other words, the generator advanced more than a full cycle in the first half-second after dropping full load. During this period the governor had closed the gates only a negligible amount. A similar test dropping half load indicated an average acceleration of about 500 deg. per sec. for the first half-second after loss of load. However, for these tests the load was dropped by opening an oil circuit breaker which entirely disconnected the generator from the system. Actually under the usual short-circuit condition of one or two phases faulted to ground, some synchronizing torque will exist between the hydroelectric generator and the steam system which will tend to retard acceleration of the former. An oscillogram of an actual two-phase-to-ground fault on the Conowingo system which occurred at a time when four generators were on the line carrying approximately full load indicated that the waterwheel generators during the 30 cycles on the Philadelphia system immediately following the fault accelerated at an average rate of about 580 deg. per sec., or in other words, Conowingo was 180 deg. out of phase with Philadelphia in a little less than 20 cycles. These figures, of course, reflect the slight tendency for the steam system to slow down referred to by Mr. Bergvall.

The use of resistance to load hydroelectric generators on the occurrence of a short circuit is apparently an effective method of preventing overspeeding of waterwheels to the point where instability results. The idea of the installation of resistors to prevent excessive overspeeding has, of course, been under consideration for some years and at least one installation of a resistor in the neutral of high-voltage transformers has been made. It was formerly thought by some engineers that the resistance of an arc might be high enough to exert an appreciable effect in this direction and some stability calculations have been made using various assumed values of arc resistance. However, both accidental faults and staged short-circuit tests have indicated that arc and ground return resistance are not sufficiently high to produce the braking effect required to preserve stability.

The installation of resistors in the individual generator circuits offers some advantages over the installation of resistors in the neutral of high-voltage transformers, since with the individual resistors the amount of resistance in use varies automatically with the number of generators in service, and maximum advantage can be taken of graded transformer insulation since overvoltages accompanying the installation of transformer neutral resistors are eliminated, as are the effects of these voltages on lightning arresters and transmission lines. Another advantage of individual resistors in generator neutrals is their effect in reducing the terminal voltage of the generators, thus forcing the voltage regulating system to produce a higher internal machine voltage.

Resistor installations of the type proposed by Mr. Bergvall may constitute the only means of preserving stability or they may supplement or be supplemented by other methods such as the use of high-speed oil circuit breakers, high-speed excitation systems, damper windings, and apparatus of special design. In installations where the resistors would permit the use of generators of normal design, transformers of normal reactance, and normal speed oil circuit breakers and relays, it is probable that the resistors will be justified economically by a wide margin.

The choice of a suitable resistor for a given installation may require careful consideration. Although the amount of resistance is automatically varied with the number of generators in service, the total short-circuit current supplied to any fault will not vary in direct proportion to the number of generators in operation and the amount of short-circuit current supplied by any generator will vary with the number of generators in service for a given fault location. Further, to secure best results, the value of resistance should change for each type and location of fault. Under certain system conditions it may be desirable to operate hydroelectric units at reduced loads with the result that generators in service will operate at partial gate openings and a resistor designed for sufficient retardation of full gate opening may cause too great a retardation at partial gate openings. It is probable, however, that some average value of resistance can be found which will satisfactorily meet all conditions. Certainly in the interests of simplicity and good operation it would be undesirable to alter the resistor set-up for different operating conditions.

As Mr. Bergvall points out, the use of resistors may reduce the necessity for solving the problem of lightning-proof lines. We do not, however, subscribe to his suggestion that consideration be given to building lower cost transmission lines that will be expected to flash over. Although it is true that a saving of a few per cent such as might be obtained by building a line of reduced insulation might be greater in a long, high-voltage line than the cost of the resistor installation, it would appear that in any high-voltage, high-power installation where stability is a problem that required high grade of service would not permit deliberately inviting flashover even though there was no actual interruption to service.

G. D. Floyd: The idea of inserting resistance into a system at time of fault is not new, but the advantages of placing this resistance in the neutral of each generator in a station have not been investigated. The apparent simplicity of the device, which is nothing but a dynamic brake can be easily appreciated, and for this reason alone the arrangement is likely to find favor more quickly than other changes in generator or system layout, most of which added in greater or less degree to the capital cost of an undertaking, and had to be defended before they could be specified. Anyone who has had to do with the design of a power plant or layout of a system will welcome an improvement on which capital outlay can be deferred, but where the contingency of instability has been provided for. Objections are seldom raised where provision is made in a design for something which does not involve immediate capital outlay.

The very simplicity of the arrangement suggested by the author may, however, be deceiving, and each individual case where the use of the series resistor is contemplated should be examined carefully, before assuming that its use will prevent instability. A case in point is a system having hydraulic generators, a transmission line, and a large proportion of synchronous load, such as pulp grinders. It would appear from Mr. Bergvall's analysis that conditions at such a load may be made worse actually, by use of series resistors at the generating end.

In the calculations and tests covered by the paper, zero resistance at point of fault may be inferred. This is justified for three-phase faults, but in the case of single and double line-to-ground faults where there may be high resistance at fault location, the added resistance in the generator leads may cause a drop in speed of the unit sufficient to cause loss of synchronism. Cases are known where there was sufficient resistance at point of fault to cause drop in speed of the generators, where no other resistance was present except that of the line itself. The same argument would apply for those systems where the high-voltage neutral is grounded through impedance made up of appreciable resistance, unless special provision were made to allow the resistor in the generator neutral to be introduced only in the case of three-phase faults.

H. S. Phelps: In concluding his paper Mr. Bergvall states that following the establishment of confidence in the series resistance method of maintaining stability, consideration may be given to building lower cost transmission lines that will be expected to flash over instead of trying to solve the stability problem by building lightning proof lines.

That conclusion must, of course, be studied in conjunction with such papers as those presented by Harrison and Silver and Conwell and Warren presented as a part of the Symposium on Coordination of Power and Telephone Plant, see page—.

In the former paper the authors indicate various ways by which system reliability has been improved and likelihood of line flashover has been decreased.

In the latter paper the authors consider in some detail the influence on the coordination problem of such a proposal as that offered by Mr. Bergvall. It is discussed in their paper in the section entitled "Factors Controlling the Frequency of Occurrence of Induced Voltages."

It is, therefore, evident that before reaching a decision which would favor increasing the probable number of line flashovers there are several related problems that must be considered in order to assure that a satisfactory solution has been reached in a particular situation.

R. H. Park: Mr. Bergvall has presented a valuable analysis of the benefits to the stability of a hydroelectric transmission system which result when a resistor is momentarily inserted in series with the generator leads on the occurrence of a short circuit on the high-voltage lines. An alternative procedure would be to momentarily connect a shunt resistor across the generator terminals on the occurrence of a line fault. I should like to ask Mr. Bergvall what he considers to be the relative advantages and disadvantages of these two arrangements.

R. C. Bergvall: Mr. Crary's discussion is based on two figures showing the transient power limit plotted against switching time. Unfortunately, he varied the switching time of the series resistor whereas in any practical installation this time would remain constant at 6 cycles or less. This assumption places the series resistance method in an unfavorable light and, therefore, general conclusions cannot be drawn based on these figures.

I recognize that there are limitations to the use of series resistance in maintaining stability and each individual system must be considered by itself in applying the series resistance method. Fig. 11 in the paper represents such an analysis made in conjunction with the a-c. calculating board and shows the effectiveness of the series resistance in maintaining stability with a 2-phase-to-ground fault.

As indicated by Mr. Anderson and Mr. Floyd, the principal advantage of the series braking resistance method is that the expenditure can be deferred until operating experience indicates that service improvement will justify it. The use of damper windings, special reactance in generators and transformers, etc., is an expenditure that must be made at the time of original construction when the necessity for such expenditure may not be evident.

In analyzing an individual system it would be well to give consideration to the shunt resistance proposed by Messrs. Park and Crary as there may be a field for both the shunt and series resistance method.

Oil Circuit Breaker Tests—Philo 1930

Aims, Setup and Results from a System and Operating Viewpoint

BY PHILIP SPORN* and HARRY P. ST. CLAIR*
Fellow, A. I. E. E. Member, A. I. E. E.

Aims and Objects of Tests

BEFORE undertaking the Philo 1930 132-kv. oil circuit breaker tests, which involved the placing of short circuits directly on the 132-kv. bus of the largest generating plant on the interconnected system of the American Gas and Electric Company group, careful consideration was given to the benefits to be obtained. Among these considerations were the following:

First, the proposed tests offered a means of determining positively whether a high capacity 132-kv. breaker would be able to withstand its full rated duty. This in itself was considered of great value. In addition, past experience justifies the conclusion that the carrying out of such tests almost invariably results in the discovery of one or more weak points in the breaker assembly; the remedies which are devised to overcome such weaknesses often constitute important advances in the art of circuit breaker design. An instance of this kind was provided by the tests on the 132-kv. breakers made at Canton, Ohio,[1] in 1925 and 1926, in the course of which tests weakness showed up in the explosion chamber assembly. The result was not only an immediate redesign of the explosion chamber assembly, but the beginning of the development which later, as the result of the Canton tests as well as subsequent tests on other systems, produced the non-metallic explosion chamber.

Second, the tests promised to materially contribute to the development of high speed in the explosion chamber type of breaker, a considerable number of which is in service on the Ohio Power Company's System and on the systems of other subsidiaries of the American Gas and Electric Company. At the time these breakers were installed speed as an item in the performance of a breaker was, even from an academic standpoint, of only minor interest. The closing speed was, of course, considered, but only from the standpoint of the breaker's ability to successfully synchronize large generators. At the present time, however, the high-speed breaker is recognized as a very important development which promises considerable relief from the damaging effects of system disturbances, such as flashovers, insulator failures, etc., and by means of it system stability in many cases may be greatly improved. In addition, the high-speed breaker is expected to prove of great value from the standpoint of service to customers by shortening the time of voltage surges and dips. This is particularly important to many customers using motors which either inherently or by arrangement of control apparatus have a tendency to drop off the line on voltage dips.

Again, the tests offered an opportunity to further demonstrate the feasibility of the multi-duty cycle to replace the present standard duty cycle for circuit breaker performance. As power systems grow more extensive and standards of service are raised, it is becoming more and more evident that the present standard duty cycle of two breaker openings separated by a two minute interval is hopelessly inadequate. It is well known that during storms, breakers are frequently called upon to operate many times without any intervening inspection, and that the interval of two minutes is entirely too long, and cannot be tolerated in most well-operated systems. In the Canton tests previously referred to, several groups of as many as six to eight shots with considerably less than two minute intervals were made on different breakers, the successful outcome of which indicated that a more severe duty cycle could be adopted. One of the aims of the Philo tests was, therefore, to carry out further tests of a similar nature to provide further evidence of the feasibility of changing the present duty cycle.

It was felt, too, that the carrying out of heavy duty short-circuit tests directly on the high-tension bus of a large generating plant would give valuable information not only on the performance of a breaker located at such a point, but also on the performance and operation of the plant itself under short-circuit conditions. Carrying out the tests at this location also made it possible to obtain experimental confirmation of a recently enunciated theory regarding recovery voltage phenomena in oil circuit breaker operation.[2] In order to obtain this experimental confirmation it was necessary to carry out short-circuit tests with various numbers and lengths of 132-kv. lines connected to the test bus, a condition which was met by the Philo setup.

System Setup and Arrangements for Philo Oil Circuit Breaker Tests

The system setup for the Philo short-circuit tests is shown in Fig. 1. It will be observed that practically the entire system of the American Gas and Electric Company group, together with some of the important interconnections was tied together in a practically normal setup.[3] No special provisions were made to take care of any portions of the system load from sources not connected with the system supplying the tests; normal interconnections with West Penn. Power Company

*Both of American Gas and Elec. Co., 30 Church Street, New York City.

1. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

on the east, with Chicago on the west, and with North Carolina on the south, were maintained. With this setup a short circuit of over 1,500,000 kva. was obtained at the Philo 132-kv. bus. The major portion of this total, or approximately 1,000,000 kva., was supplied by the Philo plant itself, while the remainder was supplied by the various generating plants distributed at more or less distant points throughout the system. These remote generating plants in some cases maintained only their usual light load (Sunday) running capacity, but in other cases, such as at Windsor, Cabin Creek, Kenova and Glen Lyn, one or more additional generators were put on the line to increase the available capacity for the tests. The tests made at Canton[1] in 1925 and 1926 involved only about one-half of the short-circuit kv-a. obtained in the Philo tests and this was supplied from several generating plants all located at some distance and connected by transmission lines. Therefore, in severity of duty on any one generating

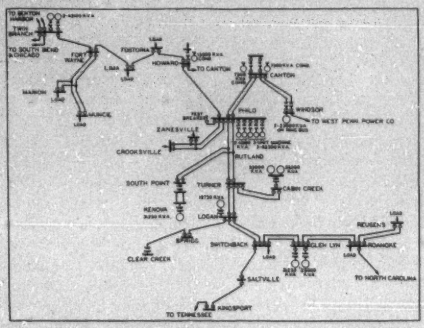

FIG. 1—DIAGRAM OF THE AMERICAN GAS AND ELECTRIC COMPANY 132-KV. INTERCONNECTED SYSTEM AS SET UP FOR THE PHILO OIL CIRCUIT BREAKER TESTS

plant and on the breaker, the Philo tests represented a step of a considerable proportion.

Figs. 2 and 3 show the location of the test breaker with respect to the plant and switch yard respectively. The test breaker was mounted on a concrete mat foundation located on a cinder fill and was connected by means of short overhead 250,000 cm. copper conductors to the end of the 132-kv. reserve bus. By a special arrangement of the main, reserve and transfer buses, and by using both the transfer breaker and one line breaker, it was possible to place two breakers in series to serve as backup breakers during closing-opening tests and as closing and backup breakers respectively during opening tests.

In Fig. 3 immediately to the right of the test breaker, in the background, may be seen the temporary building which was erected to house the magnetic and cathode-ray oscillographs, together with dark room and other necessary equipment.

RESULTS OF TESTS

A companion paper[4] gives detailed quantitative data on the results of the Philo tests with particular emphasis on an analysis of the effects of varying rates of rise of recovery voltage on the breaker performance. Briefly, however, the tests comprised a total of 81 shots divided into three separate groups or programs carried out on

FIG. 2—VIEW OF TEST OIL CIRCUIT BREAKER AND PHILO PLANT

three separate days. In order that the tests might be carried out at full rating, a 60-in. tank breaker having a rating of 1,500,000 kva. was used.

The first group, consisting of 28 shots on the breaker equipped with standard butt type explosion-chamber contacts, included 7 shots with Philo generators alone

FIG. 3—VIEW OF TEST OIL CIRCUIT BREAKER AND PHILO 132-KV. SWITCHYARD

supplying the short circuit and with no transmission lines connected to the short-circuited bus. These shots, of which 4 were made with full capacity of Philo plant, undoubtedly imposed a severe duty on the breaker as evidenced by length and duration of arc. The greatest distress was shown by the breaker on two of the shots in which approximately two-thirds of the Philo capacity

was used. While the breaker succeeded in rupturing the arc on both of these shots before the opening of the backup breaker, there was evident an unusual amount of smoke and some oil discharge from breaker vents, and the oscillographic records showed arcing continued to the end of the stroke. Subsequent tests under the same recovery voltage conditions but with maximum Philo plant capacity showed no repetition of this distress, the arc being cleared considerably before the end of the stroke. In addition to a number of shots taken with

A

B

C

Fig. 4—View of Contacts of Standard Butt Type Explosion-Chamber Oil Circuit Breaker After Test of June 29

part or all of Philo capacity and with from one to three open transmission lines connected to the short-circuited bus, five shots were taken with the maximum available system capacity, resulting in a duty on the breaker either at or above its full rating.

Inspection at the conclusion of the 28 shots revealed one of the explosion chambers completely shattered, with evidence from the manner of burning on the contact rod that the breaker had been operated a number of times in this condition. While it was thought that the breaking of the explosion chamber might have occurred on the 12th or 13th shot, on which the maximum distress was apparent, no other evidence as to the particular shot on which the breakage occurred was available.

Fig. 4, *A*, *B*, and *C*, shows all of the contacts of this breaker as they appeared at the conclusion of the tests. It will be noted that except for the broken explosion chamber on phase 1, all of the contacts were in excellent condition considering the number of shots that had been taken on the breaker.

The second group, consisting of 40 shots on the same breaker equipped with the new "oil-blast explosion chamber"[5] included a total of 16 shots with the maximum rate of recovery voltage rise obtained as above with no lines connected to the bus. Seven of these shots were made with the full capacity of the Philo plant behind the short circuit giving a duty on the breaker varying from a little below 1,000,000 kva. to 1,285,000 kva. interrupted. Six shots were made with the maximum available system capacity, all of them at a duty of more that the full breaker rating of 1,500,000 kva. with a maximum of 1,735,000 kva. on the last shot.

The performance of the breaker was satisfactory throughout and all of scheduled shots were taken notwithstanding the coincidence that on the last test a contact rod was lost through breakage of a crosshead. This breakage was due to abnormal stress resulting from the use of a rod which was too short to be properly threaded into the crosshead. Oscillograms showed that the breaker carried its full three-phase short-circuit current on this last test, although, of course, one phase would not have closed had any further tests been made. This and other breakage occurring on the contacts of the same phase on this last shot are described in another paper[6] and are illustrated in Fig. 5, shown herewith. It will be noted that the contacts on the other two phases appeared in good condition following the total of 40 shots.

As a final group of tests it had been planned to take around 20 shots on the same breaker equipped with the older type segmental contacts and explosion chambers. Due, however, to the occurrence of distress resulting in a broken bushing on the breaker (Crooksville No. 1) used as a closing breaker for opening shots on the test breaker, the program had to be cut short with only 13 shots. After taking 6 shots at intervals of from ¾ minute to 1½ minutes with light duty both in generating capacity and rates of recovery voltage rise, a total of 6 shots was made at intervals of from 1 to 3 minutes with the maximum available system capacity. On all of these 6 full system shots, which were handled by the breaker without apparent distress, the duty was approximately 1,400,000 kva., or slightly under the rated capacity of 1,500,000 kva. As stated above, the tests were cut short after the completion of the 13th shot. This shot, intended to be the first of a series, was taken

with the full capacity of the Philo plant and with medium rate of rise of recovery voltage as obtained with one short line connected to the short-circuited bus.

A

B

C

Fig. 5—View of Contacts of Oil-Blast Explosion-Chamber Breaker After Tests of July 4

As in the tests on the standard butt type contacts, inspection after the tests disclosed one of the explosion chambers shattered. Some idea of the force with which this explosion chamber blew up was evidenced by holes in both thicknesses of the tank linings made by pieces of the explosion chamber. Fig. 6A shows the shattered explosion chamber on phase 1, together with the remaining contacts on all three phases of this breaker following

A

B

C

Fig. 6—View of Contacts of Segmental-Contact Explosion-Chamber Breaker After Tests of July 6

the tests. It will be seen that the remaining contacts came through in fairly good condition.

The two explosion-chamber failures encountered on these tests were apparently the result of a general weakness on the part of the chamber structure and were not due to other faults of design. It is believed that subse-

quent changes considerably increasing the strength of the chambers, will be sufficient to prevent the recurrence of such failures.

In addition to the three groups of interrupting tests just described, several charging current interrupting tests were made with the circuit breaker equipped

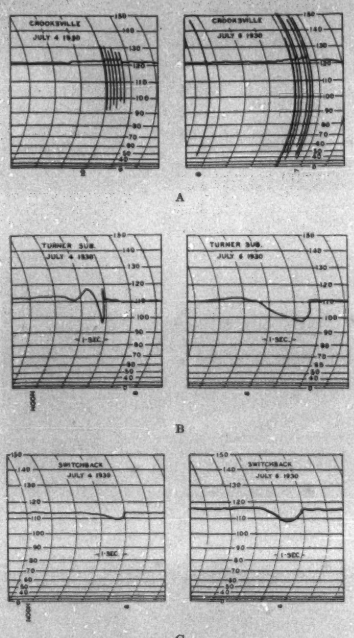

FIG. 7—VOLTAGE CHARTS TAKEN AT CROOKSVILLE, TURNER, AND SWITCHBACK SUBSTATIONS, SHOWING COMPARATIVE VOLTAGE DIPS CAUSED BY HIGH-SPEED AND ORDINARY CIRCUIT BREAKERS TESTED AT PHILO, JULY 4 AND JULY 6 RESPECTIVELY

with the oil-blast explosion chambers. These tests were made immediately following the group of 40 shots and consisted of the interruption of charging current on individual 132-kv. lines, varying in length from the short line from Philo to Crooksville to a line of 128 miles in length from Philo to Windsor. These tests were carried out without incident, the charging current involved being interrupted with comparatively short duration and length of arcs.

EFFECT ON SYSTEM OPERATION DURING TESTS

On the whole the test programs were carried out smoothly and with very little delay or deviation from the schedules laid out in advance. Also disturbances on the system due to the tests were not very severe and caused very little trouble or complaint except at nearby points such as the city of Zanesville. At this place voltage dips were naturally fairly heavy and as a result of some of the heavier or more prolonged short circuits, a number of complaints of voltage surges were received.

During the tests, recording voltmeters, some of them equipped with a high-speed tripping device, were in service at all of the important substations on the 132-kv. system. Typical charts from these voltmeters are shown herewith in Figs. 7 and 8. Fig. 7A shows comparative records obtained on a quick-trip recording voltmeter at Turner Substation on July 4 with the high-speed breaker, and on July 6 with the segmental contact breaker. While inertia and over travel of the voltmeter

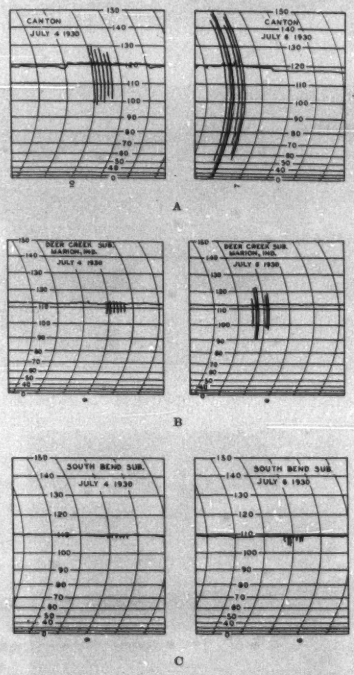

FIG. 8—VOLTAGE CHARTS TAKEN AT CANTON, MARION, AND SOUTH BEND SUBSTATIONS, SHOWING COMPARATIVE VOLTAGE DIPS CAUSED BY HIGH-SPEED AND ORDINARY CIRCUIT BREAKERS TESTED AT PHILO, JULY 4 AND JULY 6 RESPECTIVELY

pen on this type of instrument are responsible for some of the variations in these charts, nevertheless a very clear comparison is obtained between the severity of disturbances caused by the high-speed and ordinary circuit breaker. A similar comparison, both as to

severity and duration of the voltage dip is evident in the charts obtained at Switchback reproduced in Fig. 7B.

A very striking comparison of the severity of voltage dips caused by the tests at a point very close to the Philo plant is afforded by the chart shown in Fig. 7C taken at Crooksville. While it is obvious that the exact extent of the voltage dip is not obtainable from the moving pen instrument since inertia plays a large part in the record obtained, at the same time the comparative length of the lines drawn on the charts during the tests on the high-speed and low-speed breaker respectively, may be taken as a rough indication of the severity of such disturbances on rotating equipment connected to the system, since this equipment is also subject to inertia effects. In other words, a voltage dip, whose duration is too short to allow the recording voltmeter pen to follow it all the way down, may also be too short to cause a synchronous machine to fall out of step.

Figs. 8A, B, and C show voltage charts taken at Canton, Marion, and South Bend, respectively. The Canton charts show almost the same degree of disturbance as Crooksville, whereas the voltage surges at Marion and South Bend are successively less severe. In each case, however, the comparison between the effects of the high-speed and the low-speed breakers is very clearly indicated.

Benefits Obtained

In general the objects for which the tests were undertaken were quite fully realized; the tests have been and will be of real benefit from the standpoint of the power system in many ways. Some of these are the following:

For one thing, the tests have without question increased confidence in the higher ratings of modern breakers. A few years ago 132-kv. breakers of 1,500,000 kva. rupturing capacity were considered adequate for any location on the system, and, as a matter of fact, higher ratings were not available. Unforeseen developments, however, such as the installation at the Philo plant of three-unit turbines of much higher capacity than the single-unit machines originally planned have made it necessary, to employ breakers of considerably more than 1,500,000 kva. rating. At the same time later developments in the art of designing and building breakers have given the manufacturers sufficient confidence to offer breakers of 2,500,000 kva. rating. Not only has a number of breakers of this rating with 72-in. tanks been purchased, but it has been possible, with certain changes to re-rate existing 72-in. tank breakers at 2,500,000 kva. While these ratings for the 72-in. tank breakers had been accepted, it was felt that such ratings involved rather liberal extrapolation of existing knowledge into unexplored regions, and quite naturally the acceptance was accompanied by a certain amount of skepticism. In view of the recent Philo tests, however, in which a duty of 1,735,000 kva. was successfully handled by the 60-in. tank breaker, there is little question but that the 2,500,000 kva. rating can be obtained on the 72-in. tank breaker. This positive assurance of obtaining the desired circuit breaker ratings is, of course, very valuable in that it is now possible to engineer the system along the most desirable lines without being hampered by fears of circuit breaker limitations.

Again, the definite realization of the 8-cycle breaker, as demonstrated by the tests, marks a forward step of real practical value to the power system. In the past the types of relay systems used have employed such a large element of time in most cases that the saving in breaker time was of no great importance. At the present time, however, more and more instances are being encountered where it is desirable and necessary in meeting continually improving standards of service to devise means of reducing relay time to an absolute minimum. In such cases the time consumed in breaker operation becomes a very large factor.

On the system of the American Gas and Electric Company, at least two instances of the type just mentioned have occurred and are under consideration at the present time. One of these, a fairly typical situation, occurs in the Appalachian Electric Power Company territory on the portion of the 132-kv. system east of Glen Lyn, where the important loads to the east and south, including Roanoke and Lynchburg, are supplied from the long 132-kv. double-circuit lines from the Glen Lyn station. A very long single-circuit 132-kv. line, passing by and supplying Fieldale and Martinsville, extends all the way to the North Carolina state line where it ties in with the Carolina Power and Light Company. Faults on this single-circuit line or on the double-circuit line from Roanoke to Glen Lyn result in severe voltage dips at Roanoke and Lynchburg, due to the absence of any appreciable amount of generating capacity at either of these points. These voltage dips have been the cause of serious disturbances to service on a number of power users employing synchronous apparatus which either drops out inherently or is designed to be dropped off the line in case of loss of voltage. While some progress has been made in the way of remedying this difficulty by suitably revamping the users' control apparatus, at the same time many of the cases cannot be taken care of in this way and the necessity of doing everything possible to reduce the duration of voltage dips remains very urgent.

Progress has also been made in the application of recently developed relays for shortening the time required for clearing sections of single-circuit lines, but there has been little incentive to adopt new and faster relays for the protection of balanced double-circuit lines because the operating time of existing relays for this service is already small compared with the breaker operating time, so that comparatively little reduction in over-all time could be obtained. By employing the high-speed circuit breaker, however, and making use of faster relays which will thereby be justified, it will be possible to cut down by at least half and probably more,

the time required to clear one circuit of balanced double-circuit lines.

The high-speed circuit breaker offers great possibilities in reducing the percentage of two-line outages on double-circuit balanced lines. As is well known, the percentage of outages on double-circuit lines in which both circuits are involved is small compared with those in which a single circuit only is involved. However, recent experience seems to indicate that the percentage of two-circuit outages is increasing. This may be due to power system growth without accompanying improvements in the way of reducing relay and breaker operating time, with the result that the greater concentration of power in the arc following the flashover increases the tendency for the flashover to be communicated to the second circuit. It has been suggested, therefore, that if one circuit on a balanced double-circuit line could be cleared very quickly by means of fast relays and a high-speed breaker, the number of outages involving both circuits could be substantially reduced.

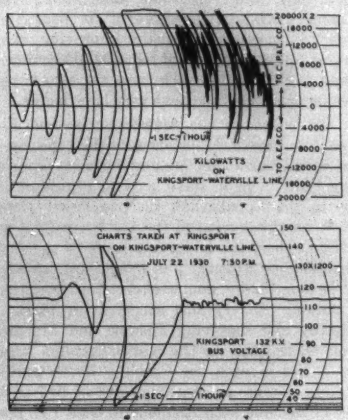

Fig. 9—Voltage and Kilowatt Charts Taken at Kingsport, Tennessee, During the Breaking Apart of the Interconnection with Carolina Power and Light Company Due to Instability

While the principles governing the phenomena of static stability limits in transmission lines have been well known for some time, comparatively few actual cases of static instability have been reported. It may be of interest, therefore, to describe a case which has occurred on the Appalachian Electric Power Company's System at Kingsport, Tenn., where an interconnection is made with Carolina Power and Light Company. This interconnection may be characterized as a comparatively weak tie between two large systems. While it would be highly desirable to maintain the interconnection during emergencies on either system, at the same time certain disturbances may occur on one of the systems resulting in a burden on the interconnection in excess of the static stability limit of the lines involved. Such a disturbance occurred on the Carolina system, with the result that the two systems pulled entirely apart and voltage at Kingsport dropped to practically zero. A graphic record of this stability breakdown is reproduced herewith in Fig. 9 in the form of power and voltage charts taken at Kingsport by means of quick-trip high-speed recording instruments on the interconnecting line to Waterville. It will be noted that the power goes considerably beyond the maximum scale reading of 40,000 kw. and that the voltage drops to zero in approximately two seconds.

In order to more definitely establish the value of kilowatts at which these two systems pulled apart and to determine the effect of using instantaneous relays to trip the line instead of the induction type relays previously used, the conditions on the systems in Tennessee

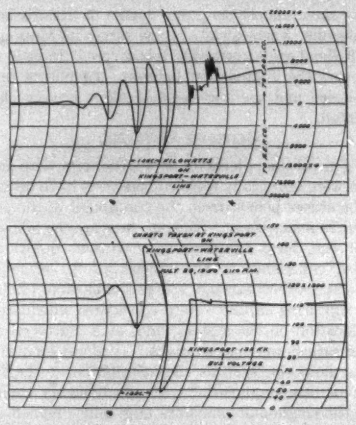

Fig. 10—Voltage and Kilowatt Charts Taken at Kingsport During the Breaking Apart of the Carolina Power and Light Company Interconnection After the Installation of Instantaneous Relays

causing the excessive loading of these interconnections were deliberately reproduced and a record was obtained as shown in Fig. 10. It will be noted that the total time from the beginning of the voltage dip to its recovery after separation of the two systems was reduced to less than one second as compared with two seconds in the previous case. Also the kilowatt charts, this time with a maximum scale of 80,000 instead of 40,000 kw., showed a power swing to approximately full scale or 80,000 kw. which checks fairly well with approximate calculations of the stability limit for these interconnections.

Since such a case of trouble necessitates the separation

of the two systems in any event, it would obviously be of distinct advantage to bring about the separation before the complete loss of voltage occurs due to instability. In order to accomplish this some relay scheme based preferably upon power flow, or perhaps on current only, could be employed to open up the line when power or current reaches values at which pulling apart from instability will be likely to result. To be effective, however, the scheme will have to include a high-speed circuit breaker in order to quickly open the line once the load reaches the predetermined critical value.

The carrying out of the Philo tests has undoubtedly resulted in a distinct benefit in the training and experience obtained by the operating personnel of the Philo plant in handling the equipment under short-circuit conditions, particularly the new 165,000-kw. three-unit turbine generator. This experience will be useful in giving a greater degree of confidence in the ability of the plant and equipment to withstand heavy short circuits, particularly since subsequent inspection of all the Philo generators showed no damage to any machine.

The Philo tests, particularly when viewed in the light of many other groups of tests, such as those carried out at Canton and on other systems in the past few years, will it is hoped result in a concerted effort to bring about the adoption of a new duty cycle for oil circuit breaker performance. Among operating people it is hardly questioned that the multi-duty cycle is necessary since breakers are frequently required to stand repeated shots during lightning storms. The carrying out of the Philo tests in which the breaker operated through a long series of shots without intermediate inspection or change of oil indicates that a multi-duty cycle is practicable.

The authors wish to acknowledge the cooperation of The Ohio Power Company and the interconnected system without which the full value of the tests could not have been realized, and the valuable assistance and cooperation of the late Mr. Joseph A. Bergin, Superintendent of the Philo Plant.

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Discussion

For discussion of this paper see page 521.

The Oil-Blast Circuit Breaker

BY D. C. PRINCE* and W. F. SKEATS*

Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*The principle of operation of oil circuit breakers has been conceived to the present time as deionization of the arc stream during the time required for the recovery voltage to appear across the circuit breaker terminals. An alternative principle is here developed in which the arc products are swept away by the scavenging action of an oil blast. As a logical development from this conception a new type of oil circuit breaker is described which owes its efficiency to careful provision for a scavenging blast of oil which is driven by the gas pressure from one break and which displaces the arc in the space between the electrodes of another break.*

THE oil circuit breaker has been used for more than thirty-five years as a circuit interrupter for alternating current circuits, and still is the most practical, economical, and effective device for that purpose so far developed. In recent years much study has been given to the determination of the fundamental characteristics of the arc in oil as encountered in oil circuit breakers. One of the first important facts brought to light by these studies was that an unconfined arc in oil is extremely erratic in length, duration, and amount of gas formed. To illustrate this point, the arc lengths and arc duration of a typical plain-break breaker for a number of tests are shown in Fig. 1. It is apparent, from a study of this figure, that the arc length and arc duration are subject to capricious variation so that any prediction of their behavior must be a matter of probability and any effective design must be based on maximum values. Even the maximum arc lengths are different in different sizes of breakers at the same currents and voltages, and in the same breaker are different with various opening speeds. These maximum values for various conditions and voltages have been determined from actual tests and are used as the design basis in plain-break breakers.

It is not difficult to see why this variation exists. Photographs of the operation of plain-break oil circuit breakers have been made which show the formation of gas bubbles surrounding the arc, (see Fig. 2). These gas bubbles are rather poor insulators and only as portions of the arc stream are accidentally interrupted by oil spray is their dielectric strength increased to the point where the arc will go out. Even so, the plain-break oil circuit breaker has given an excellent account of itself for many years and only where increased rupturing capacity or higher speed is required has it been displaced by more complicated structures.

As early as 1906 explosion-chamber breakers were developed, as in Fig. 3, in which the arc stream was confined and drawn through a hole into which fresh oil and gas from the explosion chamber discharged. This greatly increased the rate at which the arc gases were replaced by dielectric and raised the dielectric strength of the arc path, multiplying the rupturing capacity of a given circuit breaker structure by several fold.

Recently an excellent analysis of oil circuit breaker phenomena has been presented[1,2] in which the qualitative factors governing deionization rates in arcs have been explained. The principle of confining the arc and maintaining a supply of oil in its immediate vicinity has been confirmed, although the means suggested differ considerably in operation from the method employed in the explosion chamber. The importance of low dissipation of energy in the arc in keeping down the mechanical stresses accompanying an interruption has been set forth, and a new structure has been developed for the purpose of accelerating the deionization of a mixed gas-and-oil arc path, so as to raise its dielectric strength during the short interval while the restoration of voltage across the breaker contacts is taking place. The deionization processes suggested, however, operate

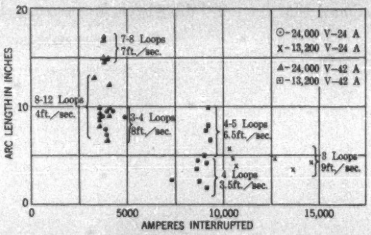

FIG. 1—ARC LENGTH VS. CURRENT FOR PLAIN-BREAK OIL CIRCUIT BREAKERS OF DIFFERENT PROPORTIONS

vigorously throughout the current cycle, requiring, for an arc of given length, high voltage and energy dissipation and causing, therefore, a rapid evolution of gas. This is unavoidable where effective deionization at the current zero must be secured in a fairly homogeneous arc path. But the arc path at the time of rupture need not be homogeneous and it is the purpose of this paper to describe a process operating along entirely different lines.

The arc path at the time of rupture for existing circuit breakers has an effective dielectric strength of from

*Both of the General Electric Co., Schenectady, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

1. *Extinction of a Long A-C. Arc*, by J. Slepian, A. I. E. E. TRANS., Vol. 49, 1930, p. 421.

2. *The Use of Oil in Arc Rupture*, by B. P. Baker and H. M. Wilcox, A. I. E. E. TRANS., Vol. 49, 1930, p. 431.

about 2,500 volts per in. in plain-break circuit breakers up to 10,000 volts for the deion grid breaker and as high as 15,000 volts for standard explosion-chamber circuit breakers. Clean oil has a dielectric strength of over 200,000 volts per in., with a strength under impulse over 600,000 volts per in. The introduction of any considerable amount of solid oil into the gap at the current zero, therefore, holds possibilities of great improvement over other methods of interruption.

In order that we may know accurately the time available for introducing dielectric following the current zero, an analysis has been made of the rate at which voltage may be expected to appear across breaker

Fig. 2—Typical Shape of Gas Bubble in Plain-Break Oil Circuit Breakers

terminals. These results have been presented in another paper.[3] From this analysis it appears that high rates of applied voltage need be looked for only across breakers connected directly to generator or transformer terminals or to current limiting reactors. Under the worst conditions line voltage may be impressed across a 15-kv. breaker in 10 microseconds. The rate of voltage rise is then

$$\frac{\sqrt{2} \times 15{,}000}{10 \times 10^{-6}} = \sqrt{2} \times 1.5 \times 10^{9} \text{ volts per sec.}$$

For the extremely short time involved, the oil in an oil circuit breaker may be said to have the dielectric strength it presents to impulse voltages, or about $\sqrt{2} \times 6 \times 10^{8}$ volts per in. Therefore, if fresh dielectric is to be introduced at a rate equal to the rate of voltage rise, it must be introduced at the rate

$$\frac{\sqrt{2} \times 1.5 \times 10^{9}}{\sqrt{2} \times 6 \times 10^{8}} = 2.5 \times 10^{3} \text{ in. per sec. or 200 ft.}$$

per sec. This velocity corresponds to a pressure of 255 lb. per sq. in., which is not an unusual explosion-chamber pressure.

A high-voltage transformer will have a voltage recovery rate corresponding to $\sqrt{2} \times 5 \times 10^{9}$ volts per sec. while the rate for a long distance 220-kv. transmission line will be about $\sqrt{2} \times 0.8 \times 10^{9}$ volts per sec. even though the full voltage corresponds to 220 kv. line to line. It is quite possible to design a circuit breaker so that oil will sweep away the arc at the rate required by the foregoing calculations.

The effectiveness of the explosion-chamber breaker

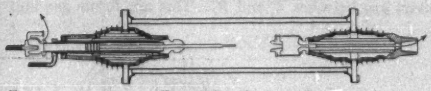

Fig. 3—Section of Early Oil Circuit Breaker Showing Explosion Chamber

may be explained in this way. Fig. 4 shows a standard high-speed explosion-chamber circuit breaker in four positions. In the first diagram the breaker is shown in the closed position. In the second, the contacts have separated and a small arc has been drawn which has decomposed enough oil to place the remaining oil in the explosion chamber under considerable pressure. In the third diagram, the moving contacts have reached the explosion-chamber throat and oil is being crowded in upon the arc in the throat. In the fourth diagram the oil has divided the gas bubble into two parts, interposing a solid wall between them, and thus effectively opened the circuit.

As the gas bubble expands, driving the oil through the throat of the explosion chamber, the pressure varies from hundreds of pounds in the large part of the volume of the explosion chamber to a very low value actually in the throat. That is, the potential energy of pressure has been converted into kinetic energy of velocity. At the same time, since the pressure is low in the throat, the oil can very easily rush into this space so as to cut the arc in two.

That this is actually what happens is proved very well by Fig. 5, in which, for a large number of tests at widely different voltages, the arc is uniformly extin-

3. *Circuit Breaker Recovery Voltages: Magnitudes and Rates of Rise*, by R. H. Park and W. F. Skeats, A. I. E. E. Trans., March 1931, p. 204.

guished while the contact tip is passing through the explosion-chamber throat.

In order to make this action even more sure and positive the form of explosion chamber shown in Fig. 6 has been developed. It contains an upper butt contact

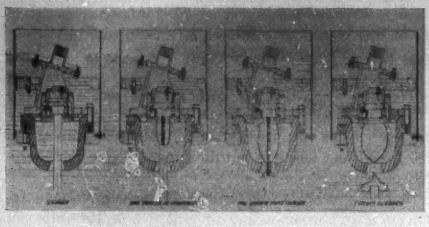

Fig. 4—Diagram Showing Operation of Explosion Chambers

A, a floating contact B, and a hollow contact C. In the closed position current is carried by these three members firmly pressed together. When the breaker is tripped, the floating and hollow contacts, B and C, start down, driven by the opening springs, thus drawing a short arc between A and B. This arc forms gas and

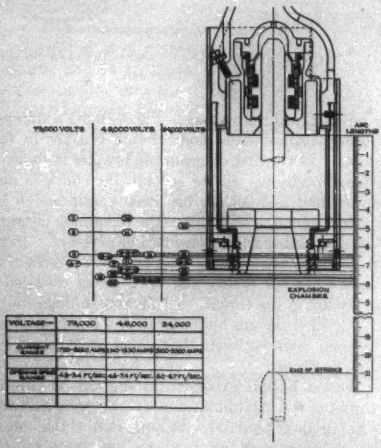

Fig. 5—Sketch Showing Metallic Explosion Chamber and Various Arc Lengths for Different Voltages. Circled Numbers are Serial Test Numbers

places the oil in the explosion chamber under considerable pressure for which there is very little relief until the floating contact reaches its stop. The secondary break then occurs between contacts B and C and a second arc is drawn between these contacts. But as the oil glides through the passageway in the contact rod, this arc is carried before it and exists alongside it without mixing, and hence with a minimum of loss throughout the current cycle. Then, as soon as the current zero is reached, the arc products are displaced by a smooth flowing wall of solid oil, the dielectric strength of which has not been impaired by carbonization or entrained ionized gas.

Several beneficial effects are obtained by the use of this modification. In the first place, the necessary arc length at generator voltages is reduced from something slightly in excess of the inside length of the explosion chamber to a little over an inch per break. This shorter

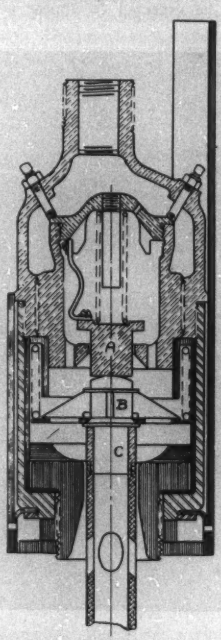

Fig. 6—Special Explosion Chamber

arc length can be more rapidly established with a given contact speed, which is a decided advantage where it is particularly important to cut down the duration of short circuit. Furthermore, both on account of the shortness of the arc and its brief duration, the energy which it dissipates is much reduced. The importance of reducing the energy from the point of view of (1) keeping down the mechanical stresses and oil deterioration resulting from a given interruption, (2) increasing the interrupting capacity of a structure of given size, and (3) minimizing the demonstration accompanying an interruption, has been brought out by other investigators as well as the authors. In addition, the shorter arc duration results

in less burning and longer life of the contact parts and of the oil. Also, both arcs are quite effectively kept away from insulating material so that burning of insulating parts is reduced. These two latter effects operate to cut down inspection and replacement requirements. Finally, the oil is more effectively directed into the second gap, so that the current range over which the chamber operates effectively is increased.

FIG. 7—OSCILLOGRAM SHOWING INTERRUPTION OF 24,800 AMPERES AT 13,200 VOLTS, 24 CYCLES, BY CIRCUIT BREAKER WITH OIL-BLAST EXPLOSION CHAMBERS

Curve *A* Voltage across contacts of breaker
Curve *B* Line current
Curve *C* Pressure in tank

Over 250 tests have been made at currents from 900 to 25,600 amperes at 13,200 volts and up to 56,400 amperes at lower voltages. Above 5,000 amperes the arc is uniformly extinguished at the first current zero occurring after the secondary contacts have been separated by as much as an inch, and in many cases the arc has been extinguished with a contact separation of but one-quarter of an inch. Fig. 7 is a typical oscillogram showing the operation of this circuit breaker.

The need for high speed of interruption is fully as urgent at transmission voltages as at generator voltages. Accordingly, a somewhat larger explosion chamber was modified to correspond to Fig. 6. This arrangement was found to be effective up to 44,000 volts, as is demonstrated by the data in Table I. By comparison of the data shown in this table with those given by Fig. 5, the improvement is apparent. Whereas with the standard explosion chamber, arc lengths varied from four to eight

TABLE I
TESTS OF OIL-BLAST CIRCUIT BREAKER FHKO-139-54-B-110 KV., 600 AMPERES
Feb. 6, 7 & 18, 1930

Volts	Amperes	Arc duration: Cycles	Arc duration: In. per break	Volts per in.
22 kv.	1570	0.9	0.9	13.2 kv.
22 "	1600	0.9	0.8	13.75 "
22 "	1740	2.2	2.6	4.23 "
22 "	1880	0.8	0.9	12.2 "
22 "	1820	1.5	*	
22 "	1730	1.2	1.3	8.46 "
22 "	1530	1.2	1.3	8.46 "
44 "	3870	0.5	0.75	29.7 "
44 "	3930	0.5	0.5	44.0 "
44 "	3240	1.5	1.7	13.0 "
44 "	5000	0.5	0.5	44. "
44 "	3620	0.5	0.5	44. "

*No record obtained.

TABLE II
TESTS OF OIL-BLAST CIRCUIT BREAKER FHKO-139-54-B-110 KV., 600 AMPERES
March 18 & 19, 1930

Volts	Amperes	Arc duration: Cycles	Arc duration: In. per break	Arc energy Kw-sec.	Kva. x 1000	Watt-sec. per Kva.	Volts per In. in arc
66 kv.	2940	2.25	5.75	.23	194	0.118	5.75 kv.
66 "	2590	1.85	7.50	.32	171	0.187	4.55 "
66 "	2920	2.25	5.25	.28	193	0.145	6.3 "
66 "	2830	.3	7.50	.35	187	0.187	4.4 "
66 "	2020	1.75	6.00	.25	193	0.130	5.5 "
88 "	1890	3.75	10.5	.58	166	0.350	4.2 "
88 "	2020	2.75	6.25	.15.2	177	0.086	7.0 "
88 "	1890	2.62	5.25	.14	166	0.084	8.4 "
88 "	2210	2.25	7.50	.36	194	0.185	5.9 "
88 "	1950	3.62	11.00	.50	171	0.292	4.0 "
110 "	1740	3.25	8.75	.29	191	0.152	6.3 "
110 "	1560	2.75	6.00	.19	172	0.110	9.2 "
110 "	1650	3.75	9.00	.36	181	0.199	6.1 "
110 "	1540	3.25	8.00	.21	169	0.124	6.9 "
110 "	1690	2.25	5.25	.11	186	0.059	10.5 "
132 "	1350	2.25	4.50	.19	178	0.107	14.7 "
132 "	1570	.3	6.50	.18	207	0.087	10.2 "
132 "	1480	4.25	10.75	.41	195	0.210	6.1 "
132 "	1250	3.5	7.25	.18	165	0.109	9.1 "
132 "	1340	.5	12.75	.56	177	0.316	5.2 "
132 "	1220	.4	17.80	.22	161	0.137	3.7 "
132 "	790	4.5	18.70	.25	104	0.240	3.5 "
132 "	1075	.6	21.90	.65	142	0.458	3.0 "

Contact burning negligible. No load speed 7.8 ft./sec.
Oil deterioration negligible.
No measurable pressure above oil.

inches, those in Table I vary from one-half to two and one-half inches.

At 110 kv., however, it was found more desirable to substitute molded explosion chambers for the steel ones used at the lower voltages. The detailed arrangement of these chambers is shown in Fig. 9. Fig. 10 shows an oscillogram typical of the behavior of this combination. The arc duration is about three cycles, and both arc drop and arc energy are very low. The results of a considerable number of tests are shown in Table II.

In comparing these data with those shown in Fig. 5 for tests on a standard explosion chamber at 73 kv., it should be noted that all tests recorded in Table II were made with one terminal of the breaker grounded, whereas the tests of Fig. 5 were made with the transformer neutral grounded. In addition, the recovery rate of the plant used for the tests of Fig. 5 was somewhat lower than the rate of the plant used in the more recent tests.

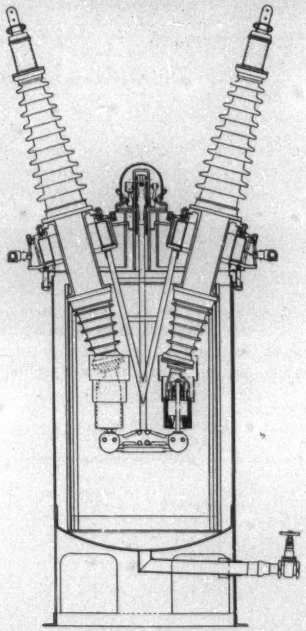

Fig. 8—Type of 110-Kv. Oil Circuit Breaker Used in Tests with Oil-Blast Explosion Chambers

On July 4 the explosion-chamber arrangement just described was installed in a three-phase oil circuit breaker at the Philo plant of the American Gas and Electric Company. The breaker rating was as follows: FHKO-139-60B, 132,000 volts, 600 amperes. This oil circuit breaker interrupted a large number of short circuits. These tests are being reported in another paper.[4] The results of these tests have been calculated and plotted in terms of calculated recovery rates for the systems attached, as shown in Figs. 11 and 12. In Fig. 11 is shown a comparison of the arc lengths for the butt-contact explosion chamber and the oil-blast explosion chamber. The arc length with the oil-blast is much less than with the butt contacts. Fig. 12 shows the actual over-all time for these two circuit breakers figured from the application of current to the trip coil.

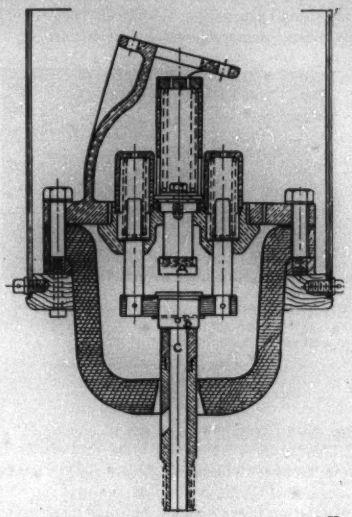

Fig. 9—Oil-Blast Explosion Chamber for 110-Kv. Oil Circuit Breaker

Fig. 10—Oscillogram Showing Interruption of 1,560 Amperes at 110,000 Volts, 60 Cycles, by Breaker with Oil-Blast Explosion Chambers

Curve *A* Voltage across contacts of breaker
Curve *B* Line current
Curve *C* Breaker travel between impulses equals ½ in.
Curve *D* Trip coil current

4. *Circuit Breaker Field Tests on Standard and Oil-Blast Explosion-Chamber Oil Circuit Breakers*, by R. M. Spurck and H. E. Strang, p. 513.

The gain in the case of the oil-blast contacts is more impressive when it is remembered that the mechanical time required to separate the contacts is the same in both cases. Of the single-phase laboratory tests, those at 88 kv. are probably the most comparable with those at Philo, which were made with the same explosion chambers, (Fig. 11). The maximum arc lengths were about the same, although the recovery time of the test transformers is only 60 per cent of the shortest time in

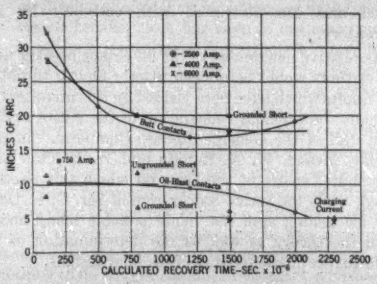

FIG. 11—TESTS AT PHILO, OHIO, ON THE FHKO-139-60B, 132,000-VOLT, 600-AMPERE, OIL CIRCUIT BREAKER, SHOWING THE MAXIMUM ARC LENGTH VS. RECOVERY TIME

the Philo tests. An especially reassuring feature of these tests is that the factory testing plant appears to stress the circuit breaker as severely as a large central station connection providing two and one-half times the short-circuit energy.

During the tests it has been found that the circuit has, in many cases been successfully opened with chamber pressures less than those calculated above as theoretically necessary. This is explained by a more

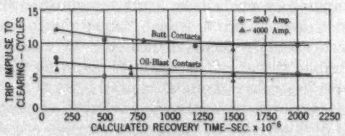

FIG. 12—TESTS AT PHILO, OHIO, ON THE FHKO-139-60B, 132,000-VOLT, 600-AMPERE, OIL CIRCUIT BREAKER SHOWING THE DURATION OF SHORT CIRCUIT VS. RECOVERY TIME

complete analysis of the oil flow during the arcing period.

Fig. 13 shows a series of drawings representing the locations of arcs, gas, and contacts during an operation of the breaker. Fig. 13A shows the contacts in the closed position. In Fig. 13B the upper contacts have parted, establishing the pressure-generating arc. The lower contacts have parted in Fig. 13C and the current is still considerable, giving a fat arc, which is, however, beginning to be carried down the contact tube, which has been freed of oil, and gas is commencing to escape from the vent holes. As the arc dwindles with the cyclic decay in current, the oil moves radially inward along an area many times larger than the bore of the contact tube. At the current zero, Fig. 13D, the velocity of gas moving downward into the tube becomes many times the velocity of the inward-flowing oil. In effect the rate at which solid oil dielectric replaces

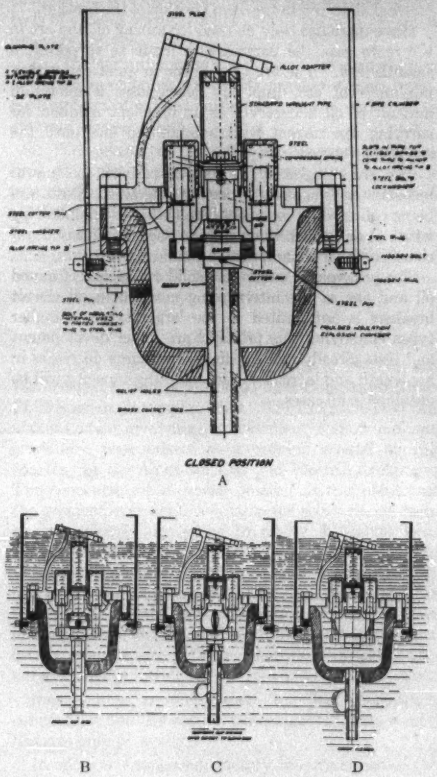

FIG. 13—DIAGRAM SHOWING LOCATIONS OF ARCS AND GAS IN OIL-BLAST EXPLOSION CHAMBER

ionized gas is the velocity with which the gas can go through the bore of the contact rod. The gas may pass through the contact tube at the velocity of sound, 2,300 ft. per sec. in oil circuit breaker gas at ordinary temperatures, while at arc temperatures it probably exceeds 10,000 ft. per sec., which accounts for the extraordinary rate at which the space once occupied by the arc recovers its dielectric strength.

In old types of plain-break oil circuit breakers, the

fact that the voltage divided by arc length gave values of approximately 2,500 volts per in., or approximately one per cent of the strength of the oil, has frequently caused comment. With a breaker of the type described above, the voltage across the working gap reaches, in actual test, values of 44,000 volts per in., which is quite comparable with the ultimate strength of the oil and considerably higher than should be allowed across the internal dielectric in the open position of the breaker.

Since there has been no need for mixing oil and arc in the main gap, the parts are so made as to permit a smooth flow. The arc then burns in quiet gas with a minimum of arc drop. It decomposes a minimum amount of oil and serves as an excellent medium for carrying the current with a minimum loss until the arrival of a current zero.

The arc energies in the tests have been too low to measure satisfactorily from oscillograms. Data are being collected by means of a ballistic galvanometer by which it is hoped that more refined comparisons of circuit-breaker behavior can be made in the future.

The principle of employing solid oil instead of mixed oil and gas as the interrupting medium in oil circuit breakers is not limited to the larger types. Smaller types employing this principle are under development and tests already made indicate that large decreases in arc length and increases in interrupting capacity will be obtained in all cases.

In developing the oil-blast principle it has not been necessary either to prove or disprove any of the numerous theories which have been advanced to explain the operation of plain-break oil circuit breakers. The plain break is simply set aside with all its vagaries and mysteries. The arc with its varying gas composition, varying pressure, uncertain temperature, varying conductivity, unknown state of ionization, etc., is merely swept away and its place taken by solid oil of known and high dielectric strength, something that is understood and can be tested with certainty by any laboratory.

As relative newcomers in the oil circuit breaker field, the authors would have been utterly unable to obtain the results which have been yielded by this investigation without the splendid cooperation of the entire switchgear staff of the General Electric Company. In particular, Messrs. W. K. Rankin and W. E. Paul have contributed many of the ideas on which this paper is based, while the active cooperation of Mr. G. F. Davis and his testing staff has made possible many checks upon the theory as well as the determination of the performance characteristics of the final designs.

Discussion

For discussion of this paper see page 521.

Circuit Breaker Field Tests

On Standard and Oil-Blast Explosion-Chamber Oil Circuit Breakers

BY R. M. SPURCK* and H. E. STRANG*
Associate, A. I. E. E. Associate, A. I. E. E.

INTRODUCTION

THE principle of interrupting short circuits inside an oil-filled explosion chamber immersed in the oil in a circuit breaker tank has been in use many years and its efficacy recognized. Nevertheless, many tests made in the High Capacity Laboratory and on power systems indicated that there was in breaker performance, some factor not thoroughly understood. It has been known that the apparent severity of duty on an oil circuit breaker during interruption is not entirely a function of line voltage and short-circuit current. It has also been recognized that at the instant of interruption, the recovery voltage reaches a value frequently in excess of normal line voltage, and it was considered probable that the recovery voltage characteristic had something to do with variations in breaker performance. An analysis of data collected from field and factory tests made during the last ten years, showed that variations in breaker performance were not caused solely by differences in the magnitude of recovery voltage. Calculations were therefore made on the probable *rate* of such voltage rise under the various prevailing test conditions and it was found that this rate, together with the magnitude, furnished a satisfactory explanation for much of what had previously been considered erratic behavior of test breakers. A very complete discussion[1] of the effect of circuit arrangements on the magnitude and rate of rise of recovery voltage has been presented to the Institute, with examples of the effect on oil circuit breaker performance which results from such variations.

A companion paper[2] describes the development of a modification of the conventional explosion-chamber assembly on which the tests in the factory indicated a reduction in arc duration for all rates of rise of recovery voltage and a definite improvement in performance.

It is the object of this paper to describe field circuit interrupting tests on an oil circuit breaker planned for the purpose of determining:

1. The actual rate of rise of recovery voltage across the terminals during the interrupting period under varied system conditions.
2. The performance of a new oil-blast explosion-chamber type of oil circuit breaker during interrupting tests of various current values and varied rates of rise of recovery voltage.
3. The performance of a standard butt contact type of explosion-chamber oil circuit breaker during interrupting tests of various current values and varied rates of rise of recovery voltage.
4. The performance of a standard segmental contact type of explosion-chamber oil circuit breaker during interrupting tests of various current values and varied rates of rise of recovery voltage.
5. The contact burning, deterioration, and other changes in the condition of the breaker after repeated interrupting tests in order to form a basis for recommended operating duty.

SYSTEM

The tests were made in the summer of 1930 on the 132,000-volt system of the Ohio Power Company at their Philo Station, Zanesville, Ohio. The extensive facilities of that system, involving an investment of many millions of dollars, were offered as a proving ground by the courtesy of the American Gas and Electric Company. The concentration of power possible at this point was the greatest ever made available for short-circuit tests at any voltage, amounting to nearly 1,750,000 kva. Thus it was possible to test for the first time a triple pole 132,000-volt oil circuit breaker at short-circuit currents not only approximating its rating but far in excess of the rating. Power concentration sufficient to test breakers at the duty possible at Philo is not to be found in any manufacturer's testing plant and probably also not available in the higher voltages on any other system.

Inasmuch as another paper[3] describes the system connections and test set-up, this paper will discuss such features only generally.

In order to supplement factory investigations of the effect of recovery voltage characteristics, arrangements were made to control the rate of voltage rise during these tests. A convenient means of manipulating this characteristic was presented by transmission lines which could be connected to or disconnected from the high-voltage bus to which the test breaker was connected. Under this condition, the recovery voltage characteristic is the sum of several outgoing and reflected voltage waves. The initial rate of rise is determined by the

*Both of the General Electric Co., Philadelphia, Pa.

1. *Circuit Breaker Recovery Voltages—Magnitudes and Rates of Rise*, by R. H. Park and W. F. Skeats, A. I. E. E. JOURNAL, December, 1930, p. 1017.

2. D. C. Prince and W. F. Skeats, p. 506.

3. P. Sporn and H. P. St. Clair, p. 498.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

short-circuit current and the number of lines connected to the bus, and the time required to reach a maximum value also involves the length of the lines. The severity of the recovery voltage characteristic is thus reduced as the number and length of lines connected to the bus are increased.

In order to best determine the effect of this difference in rate of rise, the two possible extreme conditions and one intermediate condition were chosen, as follows:

1. Calculated rate of rise of recovery voltage 270 volts per microsecond.

Three lines connected as follows:

One Rutland line 64.7 mi. long.
One Canton line 73.0 mi. long.
One Howard line 80.6 mi. long.

These lines were left open at the far end except on

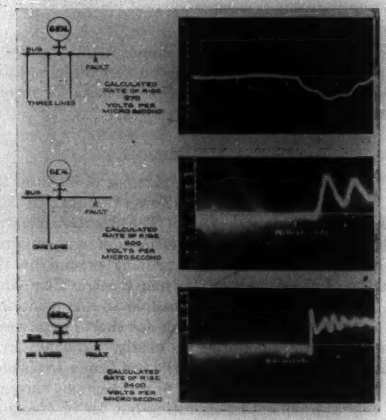

Fig. 1—Rate of Recovery Voltage Rise Under Varying System Connections

full system capacity tests during which they delivered power to the short circuit.

2. Calculated rate of rise of recovery voltage 600 volts per microsecond.

One Crooksville line 21.8 miles long connected.

3. Calculated rate of rise of recovery voltage 2,400 volts per microsecond.

No lines connected.

Representative cathode-ray oscillograms taken under these three conditions are shown in Fig. 1.

Breaker Tested

The breaker chosen for these tests was a triple-pole, 132,000-volt type FHKO-139-60B, having an interrupting rating of 1,500,000 kva. and operated by a standard SD-19 trip-free solenoid.

In one series of tests, this breaker was equipped with the oil-blast explosion chambers described in Mr. D. C. Prince's paper referred to above. Such a chamber in the form used for the tests is shown schematically in Fig. 2 and as designed for regular service in the photo-

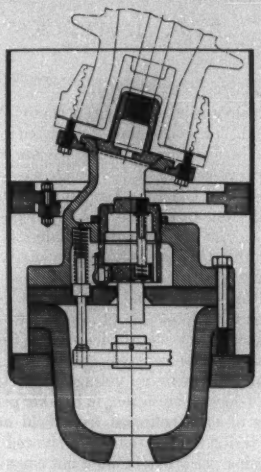

Fig. 2—Oil-Blast Contacts for First Series of Tests

graph Fig. 3. In these chambers, the upper contact is similar to the conventional butt type with compression springs to provide proper contact pressure.

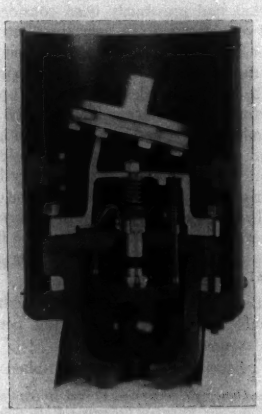

Fig. 3—Typical Oil-Blast Contact Explosion-Chamber Assembly

Between this upper contact and the moving contact rod, is supported a movable horizontal insulating baffle, having mounted in its center a conducting cylindrical

section arranged in such a way that when the breaker is in the closed position, this section provides the connection between the moving contact rod and the upper contact. This intermediate baffle, with its conducting center cylinder, is supported and its movement restricted by four studs attached to the cover of the explosion chamber. Compression springs between the movable baffle and the upper explosion chamber support tend to force the baffle downward. The main movable contact is a hollow tube, 1¾ in. outside diameter and 1¼ in. inside diameter, fitted with removable arcing tips open on the ends. The action during opening is such that as

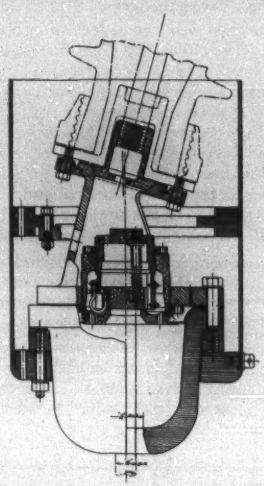

Fig. 4—Butt Type Contacts for Second Series of Tests

the crosshead moves downward, the contact between the upper fixed member and the upper face of the conducting cylinder separates, and an arc is formed between them. This arc decomposes the oil surrounding it and forms gas which creates a pressure in the explosion chamber. By this time, the main contact has moved downward such a distance that the intermediate baffle has reached the limit of its permissible stroke, and the moving contact separates from the lower face of the conducting cylinder in the baffle. This action uncovers the opening in the end of the hollow moving contact and permits the pressure already generated by the opening of the first gap, to blast a continuous stream of clean oil through this hollow contact. This continuous stream of high dielectric oil, obviously, must pass across the arc formed between the contact rod and the lower face of the conducting cylinder of the moving baffle and in so doing, introduces insulation into that arc stream at an extremely high rate.

The same chamber of molded compound which has been standard for the last two years for high-voltage breakers is used. In common with previous designs, there are no pockets which can trap carbon or foreign material, and such parts as are subjected to burning are easily renewable; also, a vent is provided in the upper part of the contact to allow the escape of gas which collects in the chamber during interruption and to permit it to refill through the throat opening with fresh oil.

Standard Explosion-Chamber Butt Contacts

The standard explosion-chamber butt contacts used in the second series of tests were similar to the contacts in the oil-blast explosion chamber, except that the moving baffle was omitted and contact in the closed position made directly between the hollow moving member and an upper fixed contact. This chamber is shown schematically in Fig. 4. These contacts were

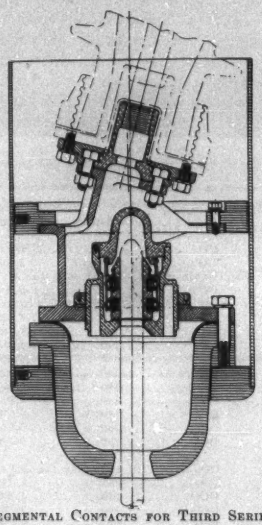

Fig. 5—Segmental Contacts for Third Series of Tests

mounted in the same breaker used on the first series of tests on the oil-blast explosion chamber.

Standard Explosion-Chamber Segmental Contacts

In the third series of tests, the breaker used for the previous series of tests was equipped with standard explosion chambers with segmental contacts. Such a chamber is shown in Fig. 5.

Test Procedure

A very definite program was laid out for each series of tests covering a wide range of short-circuit current on both CO and OCO duty cycles.

During all the tests, the breaker was tripped by means

of bushing current transformers which operated instantaneous solenoid relays, the contacts of which completed the d-c. trip coil circuit. No attempt was made to trip the test breaker by means of an auxiliary switch on a closing or back-up breaker, so these results show actual breaker operating time. Closing-opening tests, of course, required no other breaker for switching under short-circuit conditions. On opening tests only, however, a breaker in regular service at the Philo Station was used to throw on the short circuit.

Tests on Oil-Blast Explosion Chamber

A series of 40 tests was made on the breaker equipped with the oil-blast explosion chambers shown in Fig. 2. The results of this series of tests, arranged in groups according to the rate of recovery voltage rise, are summarized in Table I. This entire series of tests was run off in about six hours, including the necessary time required for switching to change the system conditions. The maximum value of short-circuit current which was recorded was 1,735,000 kva. at 140,000 volts and many of the tests were at values of short-circuit current in excess of the breaker's published rating. The last six tests listed, which were at interrupting values in excess of the rating of the breaker, were made as rapidly as the oscillograph film holders could be loaded, or approximately two minutes between tests. This completed the originally contemplated program. No inspection of contacts was made during the entire series of tests.

Oil samples taken and tested by the regular maintenance crew of the Ohio Power Company averaged 30 kv. before the tests. The average of four samples from different parts of each tank following the tests was 28 kv. and a comparison showed almost no discoloration of the oil. The average tank pressure was 8 lb. per sq. in. with a minimum of zero and a maximum of 11 lb. There was not a drop of oil thrown and only a slight amount of gas during this series.

After the tests were completed, an inspection of the breaker disclosed that one pole was failing to make contact because of breakage in the end of the crosshead which allowed the contact rod to drop to the bottom of the tank. This breakage was near the tapped hole in the end of the crosshead into which the contact rod is screwed and was caused by the use of too short a contact rod, which did not permit fastening properly into the threaded socket provided in the crosshead. This caused an unusual strain to be put on the crosshead socket and broke it off. A subsequent check of the oscillographic

TABLE I
OIL-BLAST CONTACTS

Rate of recovery voltage rise	Duty	Line volts kv.	R m. s. amperes initial in arc	Kva. interrupted	Short-circuit duration cycles	Breaker operating time cycles	Arc duration cycles	Arc length in.
270 Volts per microsecond	CO	140	2,550	617,000	5.5	5.2	3.0	4.0
	CO	140	3,450	835,000	5.5	5.2	3.0	4.2
	OCO	140	No current records					
	OCO	140	3,000	725,000	6.0	5.7	3.0	4.5
	CO	140	3,550	860,000	6.0	5.7	2.8	6.0
	CO	140	4,900	1,188,000	6.0	5.2	3.2	5.0
	CO	140	4,450	1,080,000	6.0	5.5	2.5	5.1
	CO	140	4,300	1,042,000	5.0	4.5	1.6	2.7
	OCO	140	4,250	1,030,000	5.0	4.5	2.0	3.0
	CO	140	6,950	1,690,000	6.0	5.2	2.4	5.3
	CO	140	6,350	1,540,000	5.0	4.2	2.0	2.5
	OCO	140	6,800	1,650,000	6.0	5.5	3.0	5.0
	OCO	140	6,550	1,590,000	5.5	4.5	2.4	3.5
	OCO	140	6,550	1,590,000	6.0	5.7	2.9	5.2
	OCO	140	7,150	1,735,000	6.0	5.5	3.0	6.4
600 Volts per microsecond	CO	140	3,250	787,000	7.0	5.7	4.5	8.0
	CO	140	2,700	650,000	7.5	7.0	4.7	9.1
	OCO	140	3,250	788,000	7.0	6.7	3.9	8.0
	OCO	140	3,000	726,000	7.0	6.5	4.0	8.0
	CO	140	3,650	885,000	7.0	6.2	3.7	6.5
	CO	140	3,450	835,000	7.0	6.2	3.5	8.5
	CO	140	4,700	1,140,000	6.0	5.5	2.7	5.3
	CO	140	4,050	983,000	6.0	5.7	2.7	5.5
	OCO	140	3,900	946,000	6.5	5.5	3.6	7.1
2,400 Volts per microsecond	CO	140	2,800	680,000	7.0	6.2	4.2	8.5
	CO	140	3,300	800,000	6.5	6.2	4.1	8.2
	OCO	140	3,550	860,000	6.5	6.2	3.5	6.0
	OCO	140	3,100	750,000	7.0	6.5	4.0	8.1
	CO	140	2,100	510,000	8.0	7.2	4.7	10.5
	CO	140	2,700	655,000	8.5	7.7	5.5	11.5
	CO	140	3,100	752,000	7.0	6.2	4.2	8.5
	CO	140	730	177,000	9.0	8.0	5.7	9.3
	OCO	140	800	194,000	8.5	7.2	5.0	8.0
	CO	140	5,300	1,285,000	6.5	6.2	4.0	8.1
	CO	140	4,100	995,000	6.5	5.7	2.7	6.0
	CO	140	4,100	995,000	8.0	7.0	4.6	14.5
	CO	140	3,800	922,000	6.0	5.5	3.2	8.0
	OCO	140	3,650	885,000	6.5	6.0	3.5	6.7
	CO	140	4,500	1,090,000	8.0	7.7	4.4	13.3
	CO	140	4,400	1,068,000	6.5	6.2	3.2	7.0

records indicated that this breakage occurred during the last test and that the breaker was, therefore, in this damaged condition only during the four charging current tests.

On the same pole, all the fibre posts supporting the movable baffle in both explosion chambers were found broken. It is probable that this occurred during the last scheduled test, at which time the contact rod

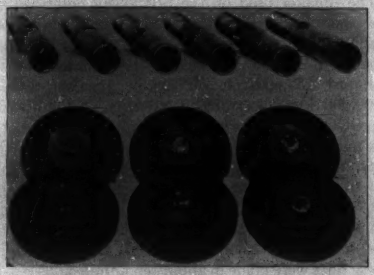

Fig. 6—Oil-Blast Contacts Following 40 Tests

support was broken. Materials of increased strength and section have been substituted in the final design to overcome this difficulty.

Fig. 6 shows the contacts after all the tests were completed. Actual measurement indicates that a total of less than ¼ in. of contact metal had been burned from the arcing surfaces. In order to determine their suitability for further operation, they were returned to the factory and subjected to a heat run of 1,200 amperes at 60 cycles showing less than a 30 deg. cent. rise.

Fig. 7 shows a pair of typical oscillograms obtained during one of the tests on the modified contacts. These oscillograms record the current in three phases, pressure

Fig. 7—Typical Oscillograms from First Series of Tests

in three tanks, one line-to-line voltage, two arc voltages, travel record, trip coil current, and cathode-ray oscillograph trip record.

Tests on Butt Type Contact

A series of 28 tests was made on the standard or butt

TABLE II
BUTT CONTACTS

Rate of recovery voltage rise	Duty	Line volts kv.	R. m. s. amperes initial in arc	Kva. interrupted	Short-circuit duration cycles	Breaker operating time cycles	Arc duration cycles	Arc length in.
270 Volts per microsecond	CO	136	3,250	765,000	9.5	9.2	7.5	18.5
	CO	136	3,200	755,000	9.0	8.5	7.0	17.2
	CO	136	2,600	612,000	9.0	8.5	6.5	16.2
	OCO	136	2,700	635,000	9.5	9.2	5.7	16.2
	OCO	136	2,700	635,000	10.0	9.5	6.0	14.7
	CO	136	3,250	765,000	9.5	9.2	7.2	18.2
	CO	136	2,700	635,000	9.5	9.0	7.2	16.7
	OCO	136	2,550	600,000	10.0	9.5	7.4	14.4
	CO	140	4,500	1,090,000	8.5	8.0	5.5	16.7
	OCO	140	4,400	1,065,000	10.0	9.7	7.5	17.2
	CO	140	4,700	1,138,000	9.0	8.7	5.8	16.7
	OCO	140	4,550	1,100,000	10.0	8.2	5.7	16.6
	CO	140	6,550	1,585,000	8.0	7.5	5.0	16.3
	CO	140	6,000	1,450,000	8.0	7.2	5.0	15.5
	CO	140	6,750	1,640,000	7.5	7.0	4.5	11.5
	OCO	140	6,200	1,500,000	9.0	8.5	7.1	14.7
	OCO	140	6,650	1,610,000	9.0	8.2	5.5	12.0
600 Volts per microsecond	CO	136	2,900	683,000	9.5	9.2	6.0	15.0
	OCO	136	2,750	648,000	11.0	10.5	7.5	17.2
	CO	140	4,000	268,000	9.0	8.5	5.7	17.2
	OCO	140	4,350	1,052,000	11.0	10.2	7.2	18.7
2,400 Volts per microsecond	CO	136	2,500	590,000	12.0	11.2	8.2	24 0
	CO	136	2,500	590,000	39.0	38.0	36.0	32.0
	OCO	136	2,600	613,000	34.0	33.0	31.0	32.0
	CO	140	4,300	1,040,000	12.5	11.0	9.5	28.0
	CO	140	3,900	943,000	10.5	9.7	7.2	20.4
	OCO	140	4,000	967,000	11.5	9.5	7.7	18.2
	OCO	140	3,750	907,000	11.0	10.2	7.5	18.7

type of contact as shown in Fig. 4. A summary of the results arranged in groups according to the rate of recovery voltage rise is given in Table II. All of the tests in this series were made in about eight hours, including the time used in the switching required to modify the system conditions. Many of the tests were made in groups having intervals between tests of approximately two minutes, which was the approximate time required to reload the oscillographs. No inspection of the contacts was made during the entire series of tests. The contacts which were removed from the breaker following these tests are shown in Fig. 8, and although somewhat burned, were in such shape that they could have been kept in operation for many more tests. The average pressure developed within the breaker tanks was 10 lb. per sq. in. with a minimum of zero and a maximum of 46 lb. per sq. in. Prior to the tests, the oil measured 30 kv. average and after the tests, the average value was 15 kv.

On Nos. 12 and 13 of this series of tests, at a voltage of 140,000 volts and at an interrupting duty of approximately 600,000 kva. with the system connections such as to give the maximum rate of recovery voltage rise of

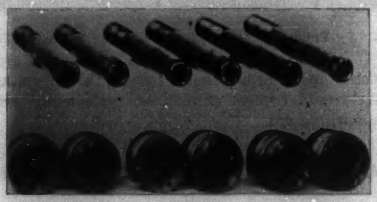

Fig. 8—Butt Type Contacts Following 26 Tests

2,400 volts per microsecond, the breaker arced to the end of the stroke. However, it cleared the circuit without causing an operation of the back-up breaker. Even under such severe conditions only a slight amount of oil was thrown out of the vent pipes on one tank during one test and from two tanks in the other test. In performance as to oil throwing, this compares with no oil throw on any of the other tests in this series up to interrupting duties of 1,640,000 kva.

On test 22, after the breaker had interrupted 19 short circuits ranging in value from 750,000 to 1,000,000 kva. and three short circuits equal to or in excess of its interrupting rating, one of the bushings developed a crack in the cement joining the lower clamping ring to the bottom porcelain and permitted oil to leak out. All other bushings withstood the entire series of tests on both types of contacts. This breakage did not prevent completing the series of 28 tests as originally planned.

The inspection of the breaker after the series of tests was completed showed that one of the explosion chambers had broken but on just which test this occurred is not known. It is of interest to note that improved manufacturing methods have resulted in materially increasing the strength of chambers of this type so that even though the operation of the breaker was not seriously impaired, chamber breakages even under severe conditions of interrupting duty are a remote possibility.

Tests on Standard Explosion-Chamber Segmental Contacts

The breaker used on the previously described tests was then equipped with standard molded explosion chambers and segmental contacts. Such a chamber is shown on Fig. 5. As such, it represented the then standard 132,000-volt oil circuit breaker rated at 1,500,000 kva. interrupting capacity on the standard two OCO two-minute interval cycle. A total of 12 tests was made at 140,000 volts. On the first group of these tests, approximately 140,000 kva. was interrupted. The time required for making those five tests was about sixteen minutes. After the group of tests, the system connections were changed and another group of six tests was made and about 1,400,000 kva. interrupted. These tests were made in about ten minutes.

On the last test of this group, which was an OCO test at 1,360,000 kva., the back-up breaker cleared the circuit. Shortly after that, however, and without inspection, the test breaker was again given a CO test. On this test, current appeared only in one phase because of difficulty with the back-up breaker. The current interrupted on the one pole was equivalent to 940,000 kva. An examination of the breaker after the test showed one explosion chamber broken. This had probably occurred on the next to the last test. On none of the tests except the next to the last one was any distress shown or any oil thrown, but on that one, however, some oil was thrown from one pole. Other than that, the only evidence that the breaker had opened was a slight puff of smoke from the gas vents. The oil before the tests were started had a breakdown of 30 kv. and after the tests averaged 23 kv., measurements in both cases made in a standard test box. The contacts were only moderately burned. A summary of these tests is given in Table III.

Breaking Line Charging Current

The ability of any type of breaker or contact to quickly rupture line charging currents has been a question of considerable interest, although not one of prime importance; therefore, following the series of 40 tests on the oil-blast explosion chamber and before any inspection had been made, four shots were taken interrupting the charging current of two different lines from the Philo station.

The average arcing time during these tests was 2.7 cycles with a maximum of four cycles and an average arc length of 3.2 in.

Discussion of Results

Table IV gives a summary of the two principal series of tests and indicates clearly the effect of the difference

TABLE III
SEGMENTAL CONTACTS

Rate of recovery voltage rise	Duty	Line volts kv.	R. m. s. amperes initial in arc	Kva. interrupted	Short-circuit duration cycles	Breaker operating time cycles	Arc duration cycles	Arc length in.
270 Volts per microsecond	CO	140	5,800	1,400,000	12.5	14.2	5.7	14.8
	CO	140	5,800	1,400,000	12.5	12	5.0	14.3
	CO	140	5,800	1,400,000	11.5	11	4.5	13.2
	OCO	140	5,650	1,370,000	17.5	15	5.8	14.2
	OCO	140	5,600	1,360,000	17.5	15	5.5	13
	OCO	140	5,600	1,360,000	Cleared by backup breaker			
	CO	140	3,900	940,500	15	13.7	7.7	13.2
2,400 Volts per microsecond	CO	140	No current record					
	CO	140	580	140,000	14	12.5	6.2	11.5
	CO	140	600	145,000	14	12.5	7.2	13
	OCO	140	600	145,000	16.5	15	7.0	12.2
	OCO	140	580	140,000	18	15.7	7.2	12.8
	OCO	140	580	140,000	17	14	6.7	11.3

in rate of rise of recovery voltage on arcing time and arc length. An analysis of these data shows that for comparable values of rate of recovery voltage rise, the average values of arc duration in the oil-blast explosion-chamber breaker are about half the duration obtained with the butt type contact; also, that the average arc length with the modified contact is approximately 40 per cent of the arc lengths with the butt contacts under comparable rates of rise of recovery voltage.

The average speed of the contacts during the opening stroke under load on the oil-blast explosion-chamber breaker was 9.5 ft. per sec., whereas that of the butt type contact chamber breaker was 13 ft. per sec. The conclusion drawn is that although high speed of cross-head travel has never been essential to the proper functioning of explosion-chamber breakers, it is even less important in breakers provided with the oil-blast explosion chambers.

TABLE IV
SUMMARY OF TEST DATA

Lines connected to bus	Calculated volts per microsecond	No. of tests	Arc duration cycles av.	Arc length in. av.
Butt Contacts				
3	270	17	6.2	15.5
1	600	4	7.1	17.0
0	2400	7	15.0	24.5
Total 28 tests	Average		8.6	18.0
Oil-Blast Contacts				
3	270	15	2.6	4.5
1	600	9	3.7	7.1
0	2,400	16	4.1	8.8
Total 40 tests	Average		3.5	6.8

Fig. 9 shows a composite diagram of the butt type and oil-blast explosion-chamber breakers with the values of the range of arc lengths as measured during all the interrupting tests at all rates of rise of recovery voltages compared with the breaker stroke.

Fig. 10 shows graphically the relation which existed between arc duration and current interrupted, for both types of contacts during all interrupting tests and for all rates of rise of recovery voltage. Although the difference in rate of voltage rise had an effect on arc duration as previously shown yet the performance of the oil-blast explosion-chamber contacts is remarkably consistent over a wide range of current and recovery voltage conditions.

General Comments

Owing to previous inability to obtain short circuits of this order of magnitude for test purposes, there has been on the part of purchasers of oil circuit breakers, a certain amount of skepticism regarding the higher ratings. It is therefore interesting that these tests have demonstrated the ability of the standard breaker, without modification of contacts, to meet its rating.

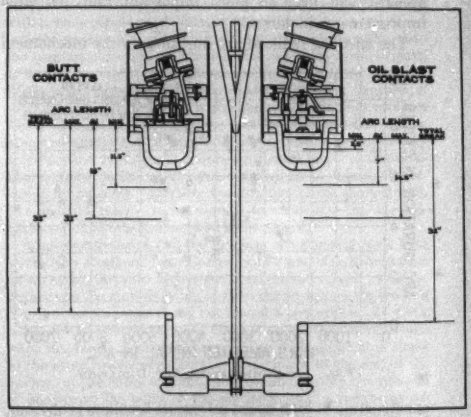

Fig. 9—Comparative Arc Length Diagram

The higher rate of rise of recovery voltage on these tests (2,400 volts per microsecond) is approximately the same as obtained with the high current laboratory test

equipment at Schenectady, which is in the order of 3,000 volts per microsecond. Inasmuch as this seems to be about as high a rate as is met in practise, it would appear that breakers which successfully meet their rating in the laboratory are adequate to meet the most severe field conditions and have a very wide margin for normal conditions, where the average rate of rise is probably in the order of 300 to 600 volts per microsecond.

Although the circuit breaker designers' problem has always been to consider the worst possible operating conditions, a better knowledge of what these conditions are is now made available through the newly developed theory of recovery voltage characteristics, and is substantiated by these tests, typical oscillograms of which are shown. Thus, a much more accurate idea may be obtained of the rate at which insulation must be inserted between the electrodes of an interrupting device, and designs are now available in which the required rate of insulation insertion is realized, so that the effects of such wide differences in the rate of recovery voltage rise are minimized.

Fig. 9 indicates the increased factor of safety obtained by applying the oil-blast explosion chamber to breakers of present standard design even under the worst operating conditions. Considering the results so far realized, it may not be too optimistic to expect that this development will lead to more important and desirable improvements in design.

The effect of reduced arc duration on the mechanical

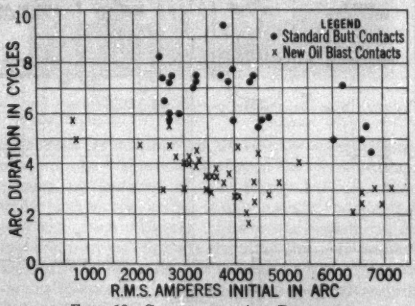

Fig. 10—Comparative Arc Duration

construction of high-speed oil circuit breakers is obvious, as it is not necessary to design moving parts for tremendous speeds, hence not necessary to provide extra heavy opening springs to obtain such speeds. Conventional types of operating mechanisms may be used, and when provided with high-speed trips to reduce as far as possible the time lost between energizing the trip coil and the separation of contacts, the full benefit of this very material reduction in arcing time is obtained.

The ability of the oil-blast explosion-chamber contacts to carry currents up to 1,200 amperes without danger of overheating, even after performing severe interrupting duty, has been demonstrated.

The oil-blast explosion chamber has the same over-all dimensions as the older style and can be applied to existing breakers by substituting an assembly similar to that shown in Fig. 9 for the old style explosion chamber but with the proper type of crosshead having larger contact rods. The oil-blast explosion chambers may also be installed on many older style breakers now in operation with the plain break or finger contacts, without involving more extensive changes than are often

Fig. 11—Segmental Contacts Following Tests

economically justified by the increased interrupting capacity and shorter operating time.

Even in cases where system stability is not a pressing problem, there is an increasing demand for faster operating breakers in order to reduce the duration of shock to connected apparatus. This need has been emphasized particularly by users applying electricity to processes where voltage dips of appreciable duration may affect their finished product. Such shorter duration of faults, and particularly the shorter arcing time, materially reduce the required maintenance on the breakers themselves, and for that reason this development, in addition to being most valuable as a solution to high-speed switching problems, should find a very general application.

Conclusions

A high-voltage high capacity breaker has now been demonstrated to have the ability to interrupt 40 short circuits up to 115 per cent of its rating in a short period of time without inspection or attention and still be in condition to operate. This would indicate that the general impression of an oil circuit breaker as an unreliable device, and one requiring undue maintenance, may in some respects be subject to revision. Such a development appears to meet the requirements set up only a few years ago by some operating engineers for a breaker that could operate 50 times under short circuits without requiring attention.

A standard breaker was given a number of interrupting tests at short-circuit current approximating its rating and gave a very satisfactory performance. It seems pertinent to emphasize again that this is the first demonstration ever made of the interrupting performance of a high-voltage breaker at short-circuit values approximating 1,500,000 kva.

Also, much data were obtained on burning of contacts and oil deterioration under repeated tests at high interrupting duties. These data will be very useful as a basis for possible changes from the standard operating duty cycle.

There is an increasing appreciation of the beneficial effect on operation of short duration of fault, and from that standpoint the minimum possible breaker operating time is to be desired. Nevertheless, the majority of the requirements for fast switching demanded by present day standards of service are fairly well met by a breaker with an operating time of not over eight cycles.

Furthermore, it is not improbable that additional refinements in mechanical construction in order to cut down elapsed time until contact separation, and possibly even more efficient means of arc extinction, will in the future permit the design of breakers having still shorter operating time.

The authors desire to express their appreciation to the organization of the American Gas & Electric Company and the Ohio Power Company for their courage, planning, and cooperation, without which these tests could not have been so successfully carried out. Mr. Skeats' assistance, particularly in comparing test results with his previous work on the theory of recovery voltage, is gratefully acknowledged.

Discussion

OIL CIRCUIT BREAKER TESTS—PHILO 1930

(Sporn and St. Clair)

THE OIL-BLAST CIRCUIT BREAKER

(Prince and Skeats)

CIRCUIT BREAKER FIELD TESTS

(Spurck and Strang)

R. H. Park: Recent developments in the design of oil circuit breakers have made possible very much more rapid switching than could be obtained formerly. With such quick switching the ratio of the direct current to the symmetrical component of short-circuit current is apt to be much greater than formerly, and as a consequence the calculated circuit breaker duty may be affected to a larger extent by the presence of d-c. component.

In the past it has been customary to assume that, as far as involves interrupting capacity, the effect of a displaced current was the same as the effect of a symmetrical current of the same mean square value. Since the d-c. component was generally a small fraction of the total short-circuit current, it actually did not make much difference what assumption was made in regard to this effect. However, with increased switching speeds, it appears to be desirable to review the situation in order to determine whether or not the original assumption was a reasonable one.

Undoubtedly, the only final test of any rule for determining the importance of the d-c. component is one which is deduced directly from actual tests in which the magnitude of the d-c. component is varied. Granting this, it should still be of interest to determine what conclusions may be arrived at on the basis of our present conception of the factors involved. As I understand it, the two principal factors are recovery voltage and arc energy. From the standpoint of recovery voltage, it can be shown that the presence of a d-c. component tends to lower circuit breaker duty. Suppose, however, that we disregard this tendency. It would then follow that the equivalence of a symmetrical and displaced wave is determined by the equality of their arc energies. But the volt-ampere characteristic of an arc at any given separation of contacts may be roughly approximated by the equation $e = k\,i^n$, where n is negative instead of $+1$ as would be the case with a resistor. The power loss in the arc is then $e\,i = k\,i^{n+1}$ and the average power loss per cycle is measured by the root mean $n + 1$ power of the current instead of by the root name square as in the special case then $n = +1$. For instance if $n = -\frac{1}{2}$, the average energy loss per cycle is measured by the root mean square root of the current.

The following table gives the root mean $n + 1$ power of a displaced wave as a function of arch exponent n and amount of displacement.

Per cent displacement	$n = +1$	$n = 0$	$n = -\frac{1}{2}$
0	100	100	100
20	104	103	102
40	115	108	105
60	131	118	112
80	151	134	120
100	173	157	141

These data suggest that the established practise of evaluating breaker duty in terms of the r. m. s. value of the current in the arc may result in an overemphasis of the effect of d-c. component. I should like to ask the authors of the three papers on circuit breakers whether they have any data bearing on this conclusion.

R. Treat: For a long period, the oil circuit breaker was regarded in many quarters as about the crudest and most unsatisfactory piece of equipment with which the electrical industry had to contend.

It is refreshing, therefore, to find from the papers presented and from those which have been presented to the Institute in the last year or two that the present trend in design of circuit breakers is to replace brute strength with some degree of subtlety, based on better knowledge of the phenomena involved.

Frequently in pursuing the paths of progress we find that an advance for the purpose of achieving a definite objective brings with it some corollary advantages which were more or less, if not entirely unexpected and in some cases are more valuable than the original objective. This condition seems to exist, in part, at least with respect to the development of high-speed breakers. While it has been generally appreciated that short duration of short circuits was desirable, it was the system stability problem which brought the real urge for fast breakers and fast relays. Many calculations on a large number of systems indicated very definitely the considerable increase in power limits of transmission systems during short circuits which could usually be attained by reducing materially the duration of the short circuit. The demand became so insistent as to necessitate the developments which have been described in these papers.

A few months ago I had the privilege of witnessing some tests on the New England Power Company's Fifteen Mile Falls system on which are installed high-speed breakers and relays. In some of the tests the total duration of short circuit was on the order of 6 cycles and on many it did not exceed 8 or 9 cycles. It was a revelation to observe the small effect of even severe short circuits when the duration was kept low. From the standpoint of shock to the synchronous apparatus, of disturbance to the rest of the system, of the effect on paralleling communication circuits and of damage at the point of the arc itself, the improvement occasioned by the short duration of the fault was astonishing. It was my own impression and I think that impression was shared by the engineers of the Power Company that high-speed breakers and relays were entirely justified from these standpoints, regardless of any effect they might have on system stability.

The other advantage of short duration of fault lies in the breaker itself. A careful examination of the facts brought out in these papers indicates that the achievement of speed has been accompanied by reduction in severity of short circuit on the breaker structure. It is not too much to hope that some time in the future advantage of this reduction may be had in giving us more breaker rating for the money. A present breaker of given voltage and interrupting capacity might conceivably have its size, that is, height reduced on account of reduced arc

length and, therefore, stroke, and at the same time have its interrupting capacity raised possibly 50 or even 100 per cent. Further, the tests reported arouse the hope that there will soon be available breakers in which the period between necessary maintenance inspections can be significantly increased.

I do not by any means have the feeling that the designers of circuit breakers have reached the end and that there is no more progress to be made. Quite the contrary. I do feel, however, that the progress that has been reported to the Institute in the ast two or three years is most gratifying and a happy augury of what may be expected in the next few years.

J. Slepian: The great progress in the field of circuit interruption which has come in recent years was due first and foremost, apart from the driving force of necessity—that well known parent of invention—to the clarification and precision of ideas relating to the extinction of a-c. arcs under oil. If further progress is to be made, these ideas must be still further clarified and made still more precise, even to the point of complete expression in mathematical form. The very excellent paper by Messrs. Park and Skeats* presented last fall is an example of the advance toward greater precision, and is to be highly commended. Relapses to the old loose ways of glossing over the happenings in those vital few microseconds after a current zero in which the interruption of the circuit takes place should not be tolerated, however, and I feel compelled, in the interest of progress in this art, to criticize adversely very many statements made in the paper by Messrs. Prince and Skeats. Let me leave the detailed citation of the various incorrect and loose statements to the end of my discussion, however, and proceed first to the discussion of the theory of operation of the explosion pot oil circuit breaker as given by the authors of this paper.

In Fig. 4 of the paper are given diagrams of an explosion pot breaker contact in four positions; the first closed, the next two with different lengths of arc, and the last, with a barrier of oil across which, a little while before, had been the arcing path. Everything has been shown in this figure except what is important from the standpoint of theory, namely, how the circuit is interrupted by the interaction of the arc and the oil. As the authors state earlier in the paper, the space between the contacts which is highly conducting up until a current zero must change over into a good insulator in less than ten microseconds. It is what is happening in these ten microseconds which is important in the circuit interruption, and about which the authors profess to give information. But the first three diagrams of Fig. 4 refer to times thousands of microseconds before the current zero, and the last diagram refers to a time thousands of microseconds after the current zero. There is no doubt that an arc in a continuous gas pocket or bubble, as shown in the second and third diagrams, exists prior to the current zero, and that some time after the circuit interruption, oil will displace gas from the bubble and will form a barrier between the open contacts, somewhat as shown in the fourth diagram, but what is wanted is information as to the circuit interruption period itself. Where are diagrams showing conditions one microsecond after current zero, two microseconds after current zero, and so forth up to ten microseconds after current zero? These are the times in which the space is recovering dielectric strength, and which should be portrayed to explain the authors' theories.

If the authors had attempted to draw such diagrams, the fallacy of the theory which they present would have been evident. The authors account for the growing dielectric strength of the arc space after current zero by the formation of a barrier of oil whose growing thickness keeps pace with the growing voltage impressed upon the electrodes by the external circuit. Using the very large figure of 600,000 volts per in. for the dielectric strength of the oil next to the arc, the authors calculate that this barrier of oil must grow in thickness at the rate of 2,500 in. per sec. to keep pace with the 1.5×10^9 volts per second rate of rise of the voltage impressed upon the electrodes tending to reignite the arc.

The authors believe this speed of formation of an oil barrier is attained through the use of the explosion pot. But consider the first microsecond of the extinction period. An obstructing film of oil must form across the arc space with a thickness equal to $2{,}500 \times 10^6$ or 0.0025 inches, capable of withstanding $1.5 \times 10^9 \times 10^6$ or 1,500 volts. This film must be completely continuous. Any hole or break in it will cause reignition of the arc. In the next microsecond, the film will have grown to 0.005 in. and will be withstanding 3,000 volts, but the film must not wave in its continuity. The slightest hole in it, and the arc will reignite. Even more, the film must not waver in its thickness. If it is anywhere less than the required 0.005 inches, the arc will reignite.

What extraordinary hydromechanics will be required which will project such wonderfully continuous and uniform films of oil across the gas space precisely at a current zero within a microsecond.

The incorrectness of the theory given here is also revealed at once by the experimental fact that the arc length required to interrupt a circuit was found by the authors to vary with circuit voltage. But if the theory given by the authors was correct, the interruption of the circuit, whether of high or low voltage, would be accomplished by the formation of a barrier of oil growing in thickness at the rate of 2,500 in. per sec. and withstanding 600 volts per thousandth inch. For a 15,000-volt circuit, the oil barrier would need to reach a thickness of only 0.025 inches, but it must reach this thickness certainly or the arc will reignite. Increasing the separation of the electrodes will not help (according to the authors' theory) if the required thickness of oil barrier is not obtained, for the authors are not depending upon deionization of the arc space which other writers have stressed. Therefore, in a breaker which is operating according to the authors' theory in a 15,000-volt circuit, the 0.025-in. thick oil barrier must be formed within 10 microseconds. But if this oil barrier is growing in thickness at this rate for the first ten microseconds, it may surely be expected to continue to grow at this rate for some time after, since the pressures which are responsible for the formation of this oil barrier (according to the authors' theory) are continuing to act. It is not unreasonable to expect the barrier to continue to grow for 100 microseconds, and in this time the oil barrier will have attained a thickness of 0.25 in. and will be capable of withstanding 150,000 volts. Hence, since the rate of voltage rise is about the same for high-voltage circuits as for low, any practical breaker which operates according to the authors' theory, and which is capable of interrupting 15,000 volts, should also be capable of interrupting 150,000 volts. It should be 150,000 volts or nothing! But this is in glaring contradiction with the experimental facts.

I cannot leave Fig. 4 of the paper without pointing out the serious misconception of the nature of the arc in the oil circuit breaker which is revealed in diagrams two and three of Fig. 4. The arc is shown in a gas pocket or bubble, but the profound influence of the rapid vaporization and decomposition of the oil-wall boundaries of the bubble are not illustrated. The arc in the gas pocket under oil is in a very different state from an arc of like current in a gas volume of like size and pressure but in a chamber whose walls are of non-gas-forming material such as glass or other refractory. This is revealed by observation of the arc with the camera, as well as by the fact that the alternating-current arc in the gas pocket bounded by oil can interrupt greater volts per inch than the arc in the gas pocket bounded by glass. The arc in the glass vessel has a definitely columnar structure similar to that shown by the authors in diagrams two and three of their Fig. 4. There is an intensely luminous cylindrical core of fairly definite section joining the two electrodes, and surrounded by regions of lesser luminosity. But in the gas pocket in the oil, the camera shows that the columnar structure of the arc nearly

*A. I. E. E. Trans., March 1931, p. 204.

disappears. The whole gas bubble is filled with a rather uniform luminosity. The gas projected from the decomposing oil walls breaks up the structure of the arc continually. The highly ionized conducting paths are swirled around in all parts of the gas bubble. This is the action which is responsible for the interrupting capacity of the oil circuit breaker. On this account, in discussing the oil circuit breaker in my paper, *Extinction of a Long A-C. Arc*,* I do not show a columnar structure of the arc in the illustrating diagram.

What then, is the theory of the explosion pot circuit breaker? What are the gains which it gives over the plain break? In the above mentioned paper, after having pointed out, as I believe, for the first time, the important part played by the rapid evolution of gas from the oil walls of the gas bubble, I showed that the action of the gas could be intensified by keeping the oil walls of the bubble close to the arc at all times. In this way, the rate of evolution of gas from the oil wall is greatly increased, and the distance through which this gas must move to mix with the arc is greatly reduced. The normal growth of the gas bubble must be combatted, as this growth will move the oil walls farther from the bulk of the arc. A method which I showed in my paper for combatting this growth of the bubble was by using oil soaked baffles which stayed close to all parts of the arc even though the liquid oil moved away. A still better method was revealed by Messrs. Wilcox and Baker† in their paper on the deion grid. Still another method which is more obvious, but which was not discussed in my paper, is to use obstructions or baffles which directly prevent the motion of the oil away from the arc, and physically prevent the expansion of the oil bubble. This method has the disadvantage of requiring the development of high pressure, but it is the method which is used in the explosion pot.

In the explosion pot, by the confining of the oil, the gas bubble is prevented from attaining the large size it would have, if the oil were free to move. Hence, within the explosion pot, there is a more intense rate of gas evolution, and greater volts per inch can be interrupted than with the plain break, in those cases where the interruption takes place with the arc entirely within the pot. This action is still more pronounced for that section of the arc which is in the throat of the chamber. Here, the throat walls with the adhering oil press still more closely upon the arc and force the gas bubble section to be small. Furthermore, upon emergence of the electrode, there is at this place an intense blast of gas as the gas stored in the chamber vents itself through the throat as the moving electrode emerges. These are reasons why the likelihood of extinction is greatest at the moment when the electrode emerges from the throat.

There is still a fourth method of keeping the gas bubble around the arc small, which was not mentioned in my paper but which was described in a patent application filed at about the time of its publication, and which has a very direct relation to the oil blast explosion pot described in the present paper. This is to use a hollow electrode, which will permit venting of the gas from the bubble around the arc without requiring the bubble walls to recede from the arc. Thus the bubble around the arc itself is greatly reduced in size.

The question then naturally presents itself as to how much of the advantage of the oil blast explosion pot arises from the venting of the gas bubble around the second arc through the hollow electrode *C*, and how much from the arc between the fixed contact *A* and the intermediary contact piece *B*. May it not be that if the intermediary piece *B* were removed, still better results might have been obtained, due to the earlier venting of the gas within the explosion pot through the hollow electrode, giving a smaller bubble around the arc with a reduced pressure within the pot? It is true that in the tests described by Messrs. Spurck and Strang on the standard explosion pot breaker a hollow moving contact was used, but apparently there were no holes in the sides of these contacts which would permit the gas to escape through the hollow electrode. I would like to ask the authors whether they tried omitting the piece *B* from their present structure with no other essential change, so that venting of gas through the hollow electrode would still be permitted, and what results they obtained. This question is all the more pertinent because in the paper by Messrs. Spurck and Strang, Fig. 9 shows that in many of the operations of the oil-blast explosion pot breaker, the circuit interruption occurred with the moving electrode far outside the pot.

Besides being in contradiction with the explanation given by Messrs. Prince and Skeats in connection with their Fig. 6, it makes it seem extremely improbable that the little short arc between *A* and *B* could have the profound influence upon the long arc between *B* and *C* which the authors claim, particularly when so much of this long arc lies outside of the explosion pot. A diagram explaining the authors' theory in relation to the 14.5-in. arc length shown in Fig. 9 of the Spurck and Strang paper would be very helpful.

Now for the loose and inaccurate statements which I find throughout this paper and which are of the type which I believe are inimical to progress in the art of circuit interruption. On page 506, column one, second paragraph, "These gas bubbles are rather poor insulators, etc." Gases at atmospheric and higher pressures are normally very good insulators. Air at normal pressure, and for ordinary distances, has a dielectric strength of about 50,000 volts per inch. It is the capacity for the rapid recovery of this high dielectric strength which gases enjoy under proper conditions which makes arcs in gases so useful for circuit interruption purposes. It is through the recognition of the great insulation value of gases and the discovery of those conditions which accelerate the recovery of this insulating state that progress is to be obtained.

Continuing with the same sentence, "and only as portions of the arc stream are accidentally interrupted by oil spray is their dielectric strength increased to the point where the arc will go out." How can a spray which consists of discrete drops be able in any way to "interrupt" a gas path? If the drops are expected to coalesce into a continuous barrier, what an extraordinary "accident" that within a few microseconds including a current zero, they should coalesce into a film of the proper continuity and thickness as to interrupt the gas path, and withstand full circuit voltage!

In the next paragraph, "This greatly increased the rate at which the arc gases were replaced by dielectric and raised the dielectric strength of the arc path, etc." Where do the "replaced arc gases" go in the few microseconds in which ability to withstand voltage is obtained? If they themselves do not recover the ability to withstand voltage by rapid deionization, must not their continuity be broken by the "introduced dielectric," and must not the authors show that this "introduced dielectric" really does break the continuity of the gas path?

Beginning at the bottom of page 506, "The arc path at the time of rupture for existing circuit breakers has an effective dielectric strength of from about 2,500 volts per inch in plain-break circuit breakers up to 10,000 volts per inch for the deion grid breaker, etc." This is a very inaccurate statement of the dielectric properties of the arc space near current zero. The dielectric strength is a continually increasing one in time from a very low value to strengths of the order of 50,000 volts per inch. In the deion grid breaker dielectric strength is recovered more rapidly than in the plain break breaker, but it is likely that the ultimate or final dielectric strength is not much greater than in the plain break breaker.

In the same paragraph, "The introduction of any considerable amount of solid oil into the gap at the current zero, therefore, holds possibilities of great improvement over other methods of interruption." It is difficult to see how the introduction of "solid" oil into the gap will increase dielectric strength in itself

*A. I. E. E. Trans., April 1930, p. 421.

†*Use of Oil in Arc Rupture*, A. I. E. E. Trans., April 1930, p. 431.

unless the solid oil actually breaks the continuity of the gas path. This should be mentioned and some reference to the difficulty of causing this introduced solid oil to interrupt the gas path in the short time available, should be mentioned.

Again in the second column of page 507, "Therefore, if fresh dielectric is to be introduced at a rate, etc.," here again the provision that from the very beginning and throughout the period of "introducing the fresh dielectric" the fresh dielectric must always be breaking the continuity of the gas path is not mentioned, even though this requirement seems impossible of practical attainment.

In the next paragraph, "oil will sweep away the arc." Where to, in the few microseconds available?

At the bottom of page 507 it is stated that the experimental fact that the arc in the explosion pot breaker is most often extinguished at the time that the moving contact emerges from the throat of the chamber proves that the authors' theory as to the interrupting action of solid oil is correct. This is not at all the case. All that can be concluded is that probably the arc space of that portion of the arc which is at the throat recovers dielectric strength at current zero much more rapidly than that of other portions of the arc.

At the bottom of the first column, page 508, "But as the oil glides through the passageway in the contact rod, this arc is carried before it and exists alongside it without mixing, etc." Without inquiring into the justification on hydrodynamical grounds of the authors' assurance that the flow of oil at so high speeds as they postulate will be as smooth as they describe, we might ask as to whether this oil which is receiving heat from the arc alongside of it at the rate of thousands of kilowatts per square inch, may not be decomposing into gas, and whether this gas might not be mixing into the arc space and causing some disturbance of the arc.

Continuing, "Then, as soon as the current zero is reached, the arc products are displaced by a smooth flowing wall of solid oil, etc." By what magic is the flowing oil caused to leave the gas space so severely alone up to the current zero, and then immediately after current zero to change its course suddenly and "displace the arc products?" Where are these "arc products displaced" to in the first microsecond, the second, etc.? They must be "displaced" even after the first microsecond, as 1,500 volts are recovered by the circuit even in that short time. A few diagrams would be very helpful.

In Table I the volts interrupted per inch are stated to be the extraordinary value of 44 kv. This, however, is probably obtained by dividing the voltage of the circuit by the length of the second arc which is drawn and wholly neglecting the length of the first arc. But are the authors so sure that the first arc was contributing nothing at all to the recovered dielectric strength of the switch? It would seem that in all fairness they should allow at least 2,500 volts per inch for the "effective dielectric strength" of this first arc, since earlier in the paper they credit the arc in the plain-break breaker with this figure. It might even be expected that the advocates of an explosion pot breaker would allow an even larger figure for "effective dielectric strength" of an arc which is entirely within an explosion pot. In comparing the advantages of the "fairly homogeneous arc path" (p. 506, second column) with the non-homogeneous, will it not be fairer for the authors to make comparison on the basis of the total arc length making up the break?

In the first paragraph of page 512, the authors again state that in actual test 44,000 volts per inch of arc were interrupted. Again may we ask whether the several inches of first arcs drawn have not been omitted in calculating this figure?

In concluding, the authors say in their next to the last paragraph, the arc "is merely swept away, and its place taken by solid oil." May I conclude with whither, and when?

H. M. Wilcox: Messrs. Prince and Skeats are to be congratulated upon their contribution to the theory of arc phenomena in oil, in the form of the oil-blast principle. It is to be regretted, however, that in presenting to the engineering profession a new theory of arc rupture, distinct from existing theories, they have not seen fit to set forth more fully the fundamental principles underlying the theory which they have evolved. A careful study of the data they present leaves some points not sufficiently clear to warrant unconditional acceptance of their theory.

Referring to Fig. 6 of their paper, it is stated that a short arc is drawn between contacts *A* and *B* which forms gas and places the oil in the explosion chamber under considerable pressure. This action is evidently the result of volatilization of oil in contact with the arc since there is no other gas-generating agency present. Immediately following this the second arc is drawn between contacts *B* and *C*, and it is stated that as the oil glides through the passageway in the hollow contact rod this arc is carried before it and exists alongside it without mixing, remaining at the current zero a smooth flowing wall of solid oil the dielectric strength of which has not been impaired by carbonization or entrained ionized gas. The reason for this marked difference in performance in the contact between oil and arc in the two cases is not apparent from the subject matter of the paper, or in the light of other existing theories of arc rupture in oil.

Again, referring to Fig. 13, the authors leave the impression that the contact tube is discharging gas at high velocity during the current cycle and that this gas is still flowing at the current zero. On the other hand Mr. Spurck, in a companion paper, evidently discussing the same structure, speaks of pressure generated by the opening of the first gap as blasting a continuous stream of clean, high-dielectric oil through the hollow contact, this continuous stream passing across the arc formed between the contact rod and the lower face of the conducting cylinder, thus introducing insulation into the arc stream at an extremely high rate. The authors should clarify these apparently conflicting statements before definite conclusions can be drawn as to the theory they advance.

A comparison of the structure in Fig. 6 with the earlier explosion chamber shown in Fig. 5, leads to an evaluation of the differences between the two structures. Judging from the performance data given for the new chamber, the interruption occurs while the lower terminal of the arc is in the throat of the chamber which is precisely what is claimed for the older chamber. The throat effect has been modified by the substituton of a hollow electrode for the former throat bushing, thus increasing its effective length but tending to decrease the velocity of the oil or gas passing through it, although this may be offset by the increase in pressure due to the reduction in volume of the oil space within the chamber. An intermediate baffle has been added with its attending complications. The oil reservoir space within the chamber has been shortened and, since the arc is extinguished in the throat in both cases, the amount of shortening may be taken as being roughly a criterion of the reduction in arc length on circuit interruption.

In this connection the question arises as to just what is considered as arc length in the test data reported in these papers, particularly in view of the high value of volts per inch of arc interrupted as claimed by the authors. Messrs. Prince and Skeats show an oscillogram (Fig. 10) in which contact separation takes place after five impulses of the breaker travel wave, or a lift rod travel of 2½ inches, the arc duration being considered as starting at this point in the travel. Mr. Spurck in Fig. 9 of his paper, shows clearly arc lengths measured from the point at which the second break takes place and, since the various tabulated data show repeated instances of arc lengths of one inch or less, the conclusion must be drawn that the arc in the first gap has been disregarded in preparing the data given.

There is grave doubt as to whether the authors are justified in ignoring arc lengths of such magnitude in determining the performance of a circuit breaker. From the data given it appears

that the first arc drawn is of the order of 2½ inches long per break, giving an arc length of five inches for the two breaks. This arc length represents energy dissipated and is subject in all ways to the ills cited by the authors against arcs drawn under previous existing arc-rupturing methods and should, therefore, be included in determining the volts per inch of effective dielectric strength of the arc path.

Assuming the length of the first arc drawn to be 2½ inches per break or five inches in the two breaks, a comparative study of the two conditions may be made for the 140-kv. tests shown in tabulation I of Mr. Spurck's paper. For the 15 tests made with a rate of recovery voltage rise of 2.70 volts per microsecond, which is a comparatively easy circuit to interrupt, the average effective dielectric strength of the arc path in terms of normal-frequency recovery voltage, as calculated from the tabulated data, was 9,000 volts per inch, assuming that recovery voltage is 100 per cent of initial voltage. Adding five inches of arc length for the first arcs drawn, this value drops to 5,800 volts per inch. For the nine tests made with a 600-volt per microsecond rate of rise of recovery voltage, the average effective dielectric strength of the arc path as calculated from the tabulated data was 5,500 volts per inch, while this value drops to 4,050 volts per inch when the length of the initial arc is included. For the sixteen tests made with a 2,400-volt per microsecond rate of rise, a very "fast" circuit, the average value calculated from the data is 4,500 volts per inch, but this value becomes 3,550 volts per inch when the length of the first arc is included. It will be noted in reviewing these revised values that in changing from the 270 volts to the 600 volts microsecond rate of rise of recovery voltage, the average decrease in performance as measured by volts per inch interrupted, was 1,750 while on changing from the 600 to the 2,400 volts per microsecond rate of rise, the decrease was only 500 volts per inch. It may be assumed then that the interrupting performance at the 600-volt rate was representative of the breaker performance over a relatively wide range of voltage recovery conditions and that the value of about 4,050 volts per inch represents its interrupting ability under average operating conditions when the total length of arc drawn is taken into consideration.

A comparison has been made in these papers between the oil-blast circuit breaker and oil circuit breakers equipped with deion grids. No tests have been made on an operating system at 140 kv. with the latter device but field tests made at 66 kv., at 110 kv. and at 220 kv. indicate that its performance in volts per inch interrupted is substantially constant over this range of voltages so that a comparison may be drawn between the two methods of circuit interruption from these data. Published results of tests with deion grids on the 220-kv. system of the Philadelphia Electric Company at Plymouth Meeting Station, November, 1929 show 12 single-phase, line-to-ground tests, at current values from 1,000 to 3,000 amperes, with an average arcing time of 2.8 cycles, giving an average effective dielectric strength of 7,500 volts per inch of total length of arc in the two breaks.

Published results of tests made with a breaker taken from service and equipped with deion grids, on the 110-kv. system of the Alabama Power Company at Birmingham, Alabama, November, 1929 show 9 three-phase tests and 2 line-to-ground tests, at current values from 2,000 to 4,000 amperes, with an average of two cycles of arcing for all tests, giving an interrupting value of 8,000 volts per inch of total arc length in the two breaks. Two additional single-phase, line-to-ground tests were made at 3,500 amperes, with an average arcing time of 2½ cycles, giving an interrupting value of 12,000 volts per inch. Tests made on the 66-kv. system of the Union Gas & Electric Company at Cincinnati, Ohio in June 1930, hitherto unpublished, show 9 three-phase tests at currents from 1,200 to 6,500 amperes, with an average of 2½ cycles of arcing, giving an interrupting value of 7,000 volts per inch of total arc length.

Magnetic oscillographic equipment only was used on these tests and definite quantitative values for rate of rise of recovery voltages cannot be determined from the records. They were all large systems, however, with many ramifications, and were among the leaders in their respective voltage classes so that the tests may be accepted as representing average conditions to be found in operating service. It will be noted that the average performance of the deion grid circuit breakers on all of these tests ranged from 7,000 to 8,000 volts per inch with a grand average of 7,500 volts per inch of total arc length as compared with a performance of 4,050 volts per inch on the 140-kv. tests with the oil-blast circuit breaker under average circuit conditions, when the full length of arc drawn is taken into account, and a performance of 5,800 volts per inch under the best circuit conditons tested.

Philip Sporn: The paper by Messrs. Prince and Skeats brings out forcefully again, as was brought out some time ago by Dr. Slepian and his associates, how by proper study and research an apparently undesirable characteristic of engineering material can be overcome and turned to advantage. In this case the natural tendency of transil oil to gasify under an arc has been made the means of forcing a supply of fresh oil into the arc path, thus reducing both the time and the energy of the arcing period. The evolution of gas is thus in effect made the means of restoring or regenerating the dielectric strength of the oil in the arc path. In theory it is stated, this is quite different from the deionizing process taking place in the slot of the deion grid. It may, however, be that one general theory could be made to explain both principles. The fact that the arcing periods have been brought down to approximately the same time by the two methods would tend to lend plausibility to this idea.

J. B. MacNeill: Messrs. Prince and Skeats have given an interesting description of the oil-blast circuit breaker. Both the manufacturers and the operators involved are to be congratulated on the tests made on this breaker and the amount of power procured on short circuit. The idea of using tubular electrodes through which hot gases may escape has been given considerable attention in the past, and the following quotation from a patent specification written in 1926 shows the general idea as previously worked on.

> "We are aware that it has been proposed heretofore to employ tubular electrodes through which hot gases emanating from the arc may escape; to enclose these electrodes in a gas-tight chamber; and to initiate compression of gas in such chamber prior to separation of the electrodes to break the circuit of which they form a part."

Some of the ideas advanced seem at variance with well-established facts concerning arc behavior. For instance, pictures given show the arc as forming between the contacts (see Fig. 4 of the Prince and Skeats paper) concentrically all around the periphery of the arcing tip and in such a way that oil or gas passing down the hollow electrode will pass through the arc stream. This does not happen, but on the other hand, the arc will form a crater at some point on the arc tip which will move about but will never permit the arc to envelop the tip. What actually will occur is an arc holding to one spot on the tip momentarily with gas and oil rushing past it partly into the hollow electrode and partly out of the explosion pot throat through the regular contact clearance. The effect is entirely similar to the old-fashioned explosion pot action, except that the relation of arc gas and oil to the crater on the moving contact is more intimate.

The claim for very high voltages ruptured per inch of arc length given in this paper is not borne out by the results of actual tests given in the Spurck and Strang paper. The claims seem to be based on the inadmissible assumption that the arcs between contacts A and B (Fig. 6 of Prince and Skeats paper) can be disregarded. At any rate, one finds no data to substantiate claims for 44,000 volts per inch of arc and it appears that a reasonable average is more nearly 4,000 volts per inch.

Messrs. Prince and Skeats wave aside the older forms of breakers, "The plain break is simply set aside with all its vagaries

and mysteries." But still the new device they propose as shown in Fig. 9 of the Spurck and Strang paper gives arc lengths on the same voltage and system location varying all the way from 2.5 in. to 14.5 in. The device, therefore, does not give the consistent operation under all conditions which is claimed for it in the papers.

Incidentally, the Prince and Skeats paper predicates arc rupture on action within the explosion pot, or at the worst when the arc tip is leaving the pot. Reference to Fig. 6 of the Spurck and Strang paper, however, shows that the average arc length at rupture gives the arc tip well outside the pot and in the worst case of 14.5 in. arc length between contacts *B* and *C* the greater part of the arc is outside the pot. What becomes of the theory of arc interruption given in such cases?

It is pointed out by Messrs. Sporn and St. Clair that an explosion pot was ruptured probably on the 12th or 13th shot, but that the test was continued through a series of 28 shots. The remarkable point is that on the shots succeeding the chamber destruction the breaker continued to give performance on the same rate of recovery voltage rather better than on the earlier shots when the explosion pot was intact. Even on the 600-volt per microsecond circuit on succeeding shots of the same series the operation with the broken pot is comparable to that on the early shots made with the less severe circuit. The longer arc time on the 2,400 volt per microsecond is naturally attributed to the relatively severe circuit.

On page 512, Prince and Skeats say, "the arc energies in the tests have been too low to measure satisfactorily from oscillograms." This is certainly not because of low arc energy. For illustration see Fig. 7 of Mr. Spurck's paper, showing the operation of the *G* oil-blast breaker, in which the arc voltage is easily 8,000 volts on the cycle preceding rupture. Surely arc voltages of this nature indicate large arc energy and are well within the accuracy limits of ordinary instruments.

One is inclined to question the desirability of the extra breaks introduced within the explosion pots and shown as *A-B* on page 508 of the Prince-Skeats paper. Two more contact surfaces exceedingly inaccessible for inspection and maintenance are introduced and the number of replacement parts is increased. Furthermore, all these contacts are of the "butt" type with no relative motion between them while in contact and, therefore, subject to welding when closed on heavy currents. We must assume that the authors have found the complication necessary even though the discussion in the papers does not make the necessity of it quite clear.

In conclusion, I wish to congratulate the authors on having made improvements in explosion pot breakers; the fact remains, however, that the results obtained still permit arcing outside the pots and, therefore, in some cases do not meet the claims that have been made for years even on old constructions.

J. N. Mahoney: The several papers on this general subject, indicate that the general trend of development of devices for improving the process of a-c. circuit interruption, follows the methods investigated, tried out, or adopted in connection with high power and moderate voltage d-c. circuit interruption. This is a natural trend as the problem of interrupting a d-c. circuit of equivalent power and voltage is more difficult than alternating current because the d-c. power does not cyclically pass through zero as in the individual legs of any a-c. circuit.

The third sentence first paragraph of the paper by Messrs. Prince and Skeats, states that "One of the first important facts brought to light by these studies was that an unconfined arc in oil is extremely erratic in length, duration and amount of gas formed." It has been the writer's experience that an unconfined or undirected arc, in either air or oil of either direct or alternating current of any considerable power, is likely to be extremely erratic and out of definite control both as to the problems of insulation and interruption.

The whole problem is to efficiently confine and direct the arc while limiting its volume and effects. All of the schemes described in the several papers and discussed herewith have these general objects in view.

The use of alternating current and oil immersion reduced the dimensions of switching apparatus in a ratio of at least ten to one, compared with the use of direct current in air, at the higher transmission voltages.

Many of the schemes suggested or tried out for d-c. circuit interruption in the last 40 years, and used mostly in special or experimental applications because of the relatively moderate voltage of present d-c. circuits, are now being brought forward in various combinations for the higher powered and higher voltage a-c. circuit interruption.

One of the oldest arc directing means is the magnetic blow-out, inherent or applied. This was supplemented by the so-called "arc chute," for confining the arc, and later by "arc horns," for directing the arc, and arc splitting plates, both conducting and non-conducting, for the purpose of reducing the arc gas volume and effects.

One of the design defects of an otherwise very good and much used line of d-c. railway power plant breakers, was in having the blow-out coils in shunt to the main contacts and no provision for clearing an arc starting on the main contacts, because of the blow-out coil circuit impedance.

This caused the industry to adopt the so-called "carbon circuit breaker" for heavy current moderate voltage service largely because of the superior arrangement of the main, auxiliary, and arcing contacts which caused the arc to travel upward away from all contacts by means of the inherent magnetic and expulsion blow-out action.

Modern air type breaker designs combine the structural arrangements of the carbon type with powerful series type magnetic blow-out and liberal arc chutes containing energy reducing and absorbing means to quickly quench the arc. Some of the lower voltage forms include a large number of arcing circuit gaps to utilize what we used to call the "inherent arc counter electromotive force" of the several series gaps as indicated by arc stream exploration.

About 20 years ago in certain 5,000-volt d-c. railway experimental work we tried out the water-break type of breaker with good results. Later in circuit interrupter development for auxiliary circuits on 3,000-volt railway equipment, a multiple metal-screen enclosure for directing and condensing the arc gases, similar in principle to that used in the Davy miners lamp, was found to be effective in confining an arc in air. This device was practically the same as that shown in Fig. 5 of Mr. Torok's paper—*Experimental Lightning Protector for Insulators.*

A successful application of the air-blast principle of circuit interruption has been used on the pressure governor switches of electric railway air compressors for the past twenty years. It was found necessary to have an appreciably persistent blast of air to prevent arc re-establishment on inductive circuits.

An old application of the carbon dioxide blast circuit interrupter utilized a small metal container of the compressed gas as an arcing contact. These containers were automatically replaced for a limited number of circuit interruptions.

With reference to the paper by Messrs. Spurck and Strang, my experience indicates that both the rate and magnitude of the recovery voltage rise as well as the magnitude of the current determine circuit interrupter duty in oil, air, or other atmosphere on either alternating or direct current.

One of the most severe field test setups of my experience was clearing a short circuit made at the load end of several underground cables in parallel with a breaker cut in between the bus and the battery of cables. Ordinarily a breaker is only required to handle a short circuit through one of these cables but in this setup the entire output of the bus was temporarily carried by this test breaker. The current magnitude and rate of voltage rise

were both high, the rate approaching that of a simple d-c. circuit containing resistance only.

W. F. Skeats: The first three illustrations are for the conditions of Fig. 1, in Messrs. Spurck and Strang's paper. The first one, Fig. 1, is the recovery voltage characteristic for the last phase to clear of a three-phase grounded short circuit at the full capacity of the station and with three lines, varying from 62 or 63 to about 80 miles in length, connected to the bus. The timing of the phenomena is correct, but the calculated amplitude is a little greater than that shown by the cathode-ray oscillogram, a part of the discrepancy being accounted for by the decrement of the short-circuit current up to the time of interruption.

Fig. 2 shows a comparison for the same conditions except that only about 65 per cent of the station generating capacity was used, and only one line, twenty-two miles long, was connected to the bus. The calculation was not carried any further, because the line had a tap which made the figuring very complicated, due to reflections. The check, so far as it goes, is good, however.

Fig. 3 is for the same generating capacity as in Fig. 2, but with no transmission line connected to the bus. This curve is a composite of two oscillations at somewhat different frequencies. The calculated frequencies and amplitudes are approximately the same as those of the test curve, although a slight difference in the ratio of the two frequencies and probably a difference in the rate of decay of the two oscillations in the cathode-ray oscillogram causes the curves to have a slightly different appearance.

Fig. 4 is for the case of the second phase to clear, the conditions otherwise being the same as for Fig. 3.

Oscillations appear at two different frequencies because of the mutual capacitance of the buses in these cases of the various phases. Only a single frequency appears in the case of the first phase to clear of a grounded short circuit or for any phase of an ungrounded short circuit.

Some surprise may be felt at the fact that the arc lengths on grounded short circuits were greater than the arc lengths on ungrounded short circuits, in spite of the fact that the magnitude of the recovery voltage peak is more than 50 per cent greater on the first phase to clear in the ungrounded case than for any phase in the grounded case. Three factors combined to cause this situation:

1. In the case of ungrounded short circuits, the recovery voltage is split, two-thirds being positive with respect to ground, and one-third negative, or vice versa, whereas in the case of grounded short circuits, on all phases, the entire recovery voltage has the same polarity with respect to ground.

2. The time required to reach the peak was somewhat less in the case of grounded than in the case of the ungrounded short circuits.

3. Each of the three poles of the breaker is attempting to clear its phase in the case of the first phase to clear of an ungrounded short circuit, whereas in the case of the last phase of a grounded short circuit, only one pole of the breaker is operating. While this does not make it any easier for the individual poles to clear in the ungrounded case, purely on the basis of probability, other things being equal, one of the three poles will usually clear before the one pole of the last phase to clear of the grounded case.

One of the principal sources of the demand for high-speed breakers has been the requirement for keeping the generating station in synchronism with its load. It is therefore of interest to system operators to know whether the longer or shorter arc durations will obtain when the question of stability is involved. Under such conditions, there will always be at least one line connected to the bus, so that recovery voltage conditions will tend to be lenient, and the shorter arc lengths may usually be expected.

Reviewing the test as a whole, there are several features which

Fig. 2—Recovery Voltage Characteristic for Last Phase to Clear of a Grounded Short Circuit with One Line Connected. 730,000-Kva. Short Circuit

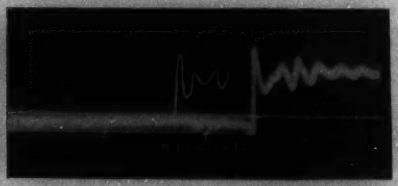

Fig. 3—Recovery Voltage Characteristic for Last Phase to Clear of a Grounded Short Circuit with No Lines Connected. 680,000-Kva. Short Circuit

Fig. 4—Recovery Voltage Characteristic for Second Phase to Clear of a Grounded Short Circuit with No Lines Connected. 860,000-Kva. Short Circuit

Fig. 1—Recovery Voltage Characteristic for Last Phase to Clear of a Grounded Short Circuit with Three Transmission Lines Connected. 1,030,000-Kva. Short Circuit

distinguish them from previous field tests on circuit breakers. These features are:

1. Field interrupting capacity tests have been made on a high capacity breaker designed for normal operation with the recovery voltage characteristic of the system under control.

2. Cathode-ray oscillograms have been recorded for at least one phase under each recovery voltage condition.

3. These oscillograms check calculated recovery voltage characteristics.

4. The breaker has been found to respond to recovery voltage manipulation substantially in the manner expected.

In view of these facts it is not too much to expect that the lack of confidence which has existed as to the significance of field tests at one location with regard to performance of the breaker at any other point on the same or another system may cease to worry system operators.

G. T. Tretjak: The development of the oil-blast circuit breaker marks a new advancing step in the technical solution of the problem of arc extinction in oil circuit breakers.

The test data given by D. C. Prince and W. F. Skeats and the tests described by K. M. Spurck and H. E. Strang yield an excellent proof of a considerable improvement in the oil circuit breaker by introducing the principle brought out in the papers.

However, the test data undeniably show (see Fig. 9 of the Spurck and Strang paper) that the arc extinction proceeds slightly otherwise than described by Messrs. D. C. Prince and W. F. Skeats, *viz.*, the arc in most cases is extinguished outside the explosion chamber, *i. e.*, at the time when the end of the hollow tubular contact has already left the limits of the chamber.

Taking into consideration that the most advantageous conditions for the arc extinction are created when the end of the tubular contact is in the throat of the chamber and the current passes its zero value I conceive that the height of the throat of the explosion chamber might be chosen so that at the given speeds in breaking the oil circuit breaker, the time of the passing of the hollow tubular contact end through the throat of the explosion chamber should not be less than half a cycle. Thus the passing of the current through zero would inevitably meet with the movement of the tubular contact end through the throat of the explosion chamber, and the length of the arc would not extend beyond the limits of the explosion chamber. It is very possible, that a 2- to 3-in. height of the throat would answer the purpose.

Philip Sporn: One of the principal motives behind our participation in the Philo tests was the hope that the tests would give us sufficient data to demonstrate the feasibility of the multi-duty cycle. The present OCO, 2 minutes, OCO breaker duty cycle of the Institute is admittedly archaic and does not meet actual operating conditions or practise. Where it takes a good part of a day to empty the oil of a breaker to take a good look at it and then refill it with oil, it is a physical impossibility to inspect a breaker after two openings, even if the expense were not objectionable. The result is, therefore, that engineers buy a breaker guaranteed on a 2-shot duty cycle and then let it handle from 10 to 50 shots before inspecting it. It would seem time to bring Institute standards and service practise into closer conformity.

This was difficult at one time. The test experience with the deion grid breaker and with the oil-blast explosion breaker has removed most of these difficulties however. A standard duty cycle (on high-voltage outdoor breakers at any rate) calling for from 20 to 30 shots would seem in order and would be welcomed by every operating man.

I submit this for the careful consideration of the Institute members and of the Protective Devices Committee.

D. C. Prince: Regardless of the conclusion which anyone may reach concerning the relative merits of the various theories presented, the fact must not be lost sight of that the papers presented have described a new form of circuit breaker which has given an excellent account of itself, both in factory and in field tests.

Mr. R. H. Park has raised the question of whether the influence of displaced waves is being properly considered in the determination of circuit breaker duty. At the present time, the actual variation in pressures obtained is rather greater than the range of factors presented by Mr. Park so that it would be difficult to make a differentiation of the sort that he has indicated. We are proceeding to refine our test methods with the hope that we will be able to separate out the variables influencing pressure. If these endeavors are successful, a refinement may result in which such factors as mentioned by Mr. Park may be identified.

Dr. Joseph Slepian has presented a large number of both general and specific criticisms of our papers. The general criticisms may be summed up by saying that our theory of the interruption of an arc under oil materially differs from his own and therefore tends to detract from the simple picture which would be obtained were only one theory in the field. We believe that we have quantitative proofs substantiating the accuracy of our theory and therefore submit that if this quantitative theory be substituted for the qualitative theories heretofore advanced, the resultant picture will be more clear and precise than the one which he has presented.

I will endeavor to take up in order the specific criticisms which have been raised. If any are overlooked or slighted it will be through inadvertence as it is believed that adequate and straightforward answers can be made to all the points raised.

Fig. 4 has been criticised on the ground that the various views shown are at time intervals too great before and after the current zero. This was not our intention in drawing this figure. The third diagram should be interpreted to apply to time about 100 microseconds before the current zero and the last to a time say between 10 and 50 microseconds after the current zero. We are endeavoring to secure a photographic record of these events which will be published in the near future. Such records will, of course, settle this part of this question.

Regarding the requirement that the film of oil thrown across the arc space in the first few microseconds must be uniform, it will be apparent with a moment's thought that it need not be at all uniform but must simply increase at the weakest point at a rate to provide dielectric strength sufficient to withstand the recovering voltage. The form of recovering voltage is such that the rate of rise is not large in the first few microseconds so that the initial formation of film is not required to be at as great a rate as would be indicated by the average rate of voltage rise. The oil blast velocities required to provide this growing film of oil are not such as to require "extraordinary hydromechanics" but are easily obtained with the pressure known to exist in these explosion chambers by the application of the well-known laws of hydraulics. For instance, a 132,000-volt system might have the abnormal recovery rate of 1,000 volts per microsecond requiring an oil velocity of 150 ft. per second corresponding to a pressure of 183 lb. A further explanation of this is given later in this discussion.

In the variation of arc length with circuit voltages cited against the validity of the theory which we have presented, there are several very good reasons why a greater arc length is obtained in a high-voltage circuit interruption than in a low-voltage interruption. Four of these are:

1. The oil film will not necessarily increase to ten times its original thickness in ten times the time.

2. At higher voltages, a thicker oil film is required in which the voltage distribution may be sufficiently uneven to overstress considerably a part of the dielectric although the average stress is below the breakdown point.

3. When a voltage wave having a steep front is impressed on an oil film, the puncture occurs at a value considerably higher than when a 60-cycle wave is used. As the time required to reach the crest voltage increases, the strength of the oil per

unit thickness falls progressively from the impulse value to the 60-cycle value. Even with the same rate of rise of voltage, the crest of a 150,000-volt wave will not be reached until ten times later than the crest of a 15,000-volt wave. During this lapse of time, the effective strength of the oil will have fallen much farther from the impulse value.

4. Recovery rates may run several times as high on high-voltage as on low-voltage circuits as illustrated by our own 1,200-volt per second maximum at 14,500 volts and 2,800 volt per microsecond at 110,000 volts.

With the lengthening of the arc, the effectiveness of the oil blast increases progressively. The pressure in the explosion chamber increases with time thus giving an increase in the oil velocity as the arc is lengthened. A given velocity of oil flow will produce correspondingly greater increase in the rate at which the oil film thickness grows. This will be apparent if we consider first an arc occupying a conical space 1 in. long and 1 in. in diameter. The oil must flow ½ in. from the circumference to reach the center and so establish 1 in. of dielectric between the contacts. If the arc is drawn to a length of 3 in., oil flowing ½ in. from the surface of the cone to the axis will establish an oil path 3 in. between contacts. Thus the oil-blast action is progressively more effective as the length increases and this increase of effectiveness is maintained even after the moving contact has passed through the explosion-chamber throat.

In using the cone figure, it must not be assumed that the gases surrounding the arc must assume a regular cone shape. There will be a good deal of solid oil inside the cone boundaries prior to the current zero but this does not detract from the usefulness of the figure in explaining what occurs.

Regarding the question as to whether the arc is columnar in structure, there are plenty of illustrations showing the arc both ways. Even when the arc drawn under oil appears diffused, the line of demarcation between the gas bubble and the oil refracts the light emitted by the arc considerably and may magnify the arc appearance. In addition to this, the carbon resulting from decomposition to the surface of the oil makes the bubble to some extent opaque and may prevent the actual arc structure from showing clearly in the illustration. There is nothing about the theory which would stand or fall depending on whether the arc is columnar or diffuse in structure.

All of the arguments which have been presented on the value of blasting gas through the arc stream apply with even greater force when oil is considered as the dielectric blasted across the arc stream. It is a matter of common knowledge that oil has a dielectric strength many times that of the gases formed by an arc under oil even when the latter are cooled. There appears to be no real evidence showing that the action of various forms of explosion chambers is due to gas effects while there is considerable evidence that it can be explained entirely by oil-blast effects.

The use of a hollow moving electrode is not new, having been introduced in a patent[1] granted in 1908. The authors have naturally not overlooked the possibility that such hollow contact rods might be nearly as effective as the preferred oil-blast arrangement. Tests were conducted in which the floating contacts (*b*) in the oil-blast structure were omitted while other conditions remained the same. The operation was markedly inferior. It has been General Electric practise for a number of years to use hollow contacts on explosion-chamber circuit breakers of certain ratings. Improvement is sometimes obtained in this way although it is not of the same order of magnitude as that obtained by the oil-blast explosion chamber finally developed.

A number of "loose and inaccurate statements" are then listed by Dr. Slepian. Regarding the rate of recovery of dielectric strength by gas, little has been published. The data given by Dr. Slepian in his paper, "Theory of the Deion Circuit Breaker,"[2] is referred back to Fig. 6 of his paper, "Extinction of an A-C. Arc."[3] The data of Fig. 6 was obtained in tests in which a very short arc between plate electrodes was studied under alternating-current conditions. It is well known that this configuration with the plates reasonably cool presents a pronounced rectifying effect. The ability of such a gap to withstand voltage in the reverse direction immediately after a current zero is undoubtedly due to this rectification effect and is not a measure of rate of recovery of dielectric strength by the gas. This accounts for the fact that in Fig. 1 of the latter paper, the curve of recovery does not commence at zero but instead at approximately 260 volts per gap. The slope of the curve thereafter may be a partial measure of the gas deionization. However, attention is directed to the fact that in 10 microseconds it has recovered less than 100 volts out of about 3,500 required for a completely deionized gap of this size. Such a rate of increase is too small to be of interest.

The use of the words "oil spray" seems to have confused Dr. Slepian. Anyone who has watched the play of an ordinary garden hose will recall that the water first emerges in a sheet which is only later broken up into discrete drops.

The question is raised as to where the gases go when displaced by a film of oil. Obviously they have to go only far enough to permit the interposition of a thin film of oil. After the circuit is once interrupted, the contacts move much farther apart and a great deal more oil is interposed so that there is no danger of the slowly moving oil bubbles causing any serious weakening of the dielectric strength.

Regarding the difficulty of introducing "solid" oil so as to actually break the continuity of the gas path, we have found that the methods which we have employed overcome this difficulty very acceptably. At the current zero, the cessation of gas generation allows the oil walls to close in and the pressure from the gap insures that the oil will effectively cut the continuity of the gas path between the lower contacts.

In his next to the last paragraph, Dr. Slepian reiterates his wonder that an oil film can break up the continuity of the gas path in such a short time. We can only submit that the evidence indicates that, remarkable as this may seem to Dr. Slepian, it nevertheless occurs.

In answer to the question, "Where to, in the few microseconds available?". The arc products need be swept only far enough away to permit interposition of a very thin oil film.

From the fact that the arc is most often extinguished in the throat of an explosion chamber, we see no reason to conclude that gas is the active medium rather than oil. We merely have Dr. Slepian's unsupported statement that it is gas and not oil, to which we take exception.

There may be some disturbance between the oil and the arc while current is high but as the current approaches zero, the rate of formation of gas also decreases rapidly so that just prior to the time current zero occurs, there is no very pronounced gas formation to prevent the interposition of oil in solid form.

It is not magic that the oil is unable to invade the gas space during the time of considerable current. Such oil as does invade the arc is gasified immediately and after the current zero, the oil coming in finds no energy to gasify it and therefore remains oil. This oil displaces gaseous arc products at first in only a thin sheet breaking the continuity of the gas path but this oil barrier quickly grows as the gaseous products escape by way of the hollow contact.

The first arc drawn in each explosion chamber in the 44-kv. tests referred to in our paper was approximately ¼ in. long and was ignored in arriving at the figure of 44,000 volts per inch of arc. In giving this figure, we had in mind a value which represented the effectiveness of the actual break. If the preliminary arc is included, the average values of volts per inch

1. Patent No. 887,723, J. D. Hilliard, May 12, 1908.

2. A. I. E. E. Trans., April 1929, p. 845.

3. A. I. E. E. Trans., Oct. 1928, p. 1407.

of break reduce from 44,000 volts to 29,000 volts. From a purely statistical point of view, however, this preliminary arc should have been included even though the authors are sure that the preliminary arc contributes practically nothing to the recovered dielectric strength of the breaker.

Dr. Slepian's concluding question of whether and when are the arc products swept away is merely a repetition of similar questions which have gone before.

Mr. Wilcox has made some very natural mistakes in interpreting the data presented. In the tests at Philo the pressure forming gap was only ¾ in. The actual travel before this arc is drawn is ½ in. making a total of 1¼ in. which he apparently read as 2½ in. In the second place, arc lengths drawn from tests at 600 volts per microsecond rate of rise are not based on "average operating conditions." Such a rate of rise corresponds to but 21.6 per miles of line attached to the bus which is much less than will be present under average operating conditions and probably a great deal less than connected to breakers during tests which have been reported during the last year. For the full system tests which more nearly represent average operating conditions, the arc lengths were materially shorter.

In discussing the paper, Mr. MacNeill has been confused by the lack of differentiation in Fig. 13 between the actual arc stream and the gas evolved by it. The arc stream is shown as a narrow column passed from one point on the intermediate electrode to a point on the rim of the lower contact, just as Mr. MacNeill believes it should be. The gas bubble should undoubtedly be somewhat less regular than it has been shown and would probably be indented more on the side opposite to that being occupied by the actual arc stream. Such condition would obviously make easier rather than more difficult interpolation of solid oil barrier between the electrodes.

Fig. 10 in the Spurck and Strang paper referred to by Mr. MacNeill does not distinguish between the arc lengths obtained with the different rates of voltage recovery. In the full system tests the arc length varies from 2½ to 6.4 in. In the tests with maximum rate of voltage rise, the variation was from 6 to 14½ in. The performance at any given rate of recovery is therefore quite consistent and the comparison of 2½ in. and 14½ in. obviously unfair. The oil-blast action does not break down as the moving contact leaves the explosion chamber throat but rather increases in its effect.

In deducing from Table II in the Spurck and Strang paper that the explosion-chamber breaker was more effective with one chamber broken, Mr. MacNeill is mislead by the arrangement of the test data in the tables. The arrangement used was selected in order to group tests in the same rate of recovery voltage rise.

On the subject of extra breaks, it will be noted that all arc of the butt type, are heavy and will withstand a large number of operations without serious burning. This is borne out by the number and severity of the tests performed without interruption at Philo.

In closing, the point which I would like to leave with you is that this paper is not presenting just one more oil circuit breaker theory. It is not merely a qualitative theory which has inspired improvements in circuit breaker structure although it certainly has done that. It is a quantitative theory for the oil circuit breaker which is susceptible of definite proof. The theory can be expressed by a simple mathematical relation. The rate at which voltage appears across the terminals of an oil circuit breaker after a current zero is subject to calculation.[4]

Let it be represented by $\frac{v}{t}$ volts per second. The dielectric strength of oil can be measured independently to be of the order of 30 kv. per 1/10 in. for a one minute high-potential test and perhaps 60 kv. per 1/10 in. for a momentary impulse. Let it be represented by $\frac{v}{1}$ *i. e.* volts per inch. Dividing the first by the second of these fractions we secure the relation

$$\frac{\frac{v}{t}}{\frac{v}{1}} = \frac{1}{t}$$

The fraction $\frac{1}{t}$ represents inches per second and is the velocity at which oil must be introduced into the space between the contacts in order to present a dielectric strength equal to the rate of increase of voltage across the gap. If the voltage rises at a higher rate than provided by this fraction, the gap will be broken down. If at a lower rate, the current will be successfully interrupted.

We have a quantitative proof of this proposition and are preparing to submit it at the earliest possible moment. (This proof has been published since the Institute meeting "Oil-Blast Breaker Theory Proved Experimentally," by D. C. Prince and E. J. Poitras, published in the February 28, 1931 issue of the *Electrical World*.

R. M. Spurck and H. E. Strang: The major part of the very interesting discussion that resulted from our paper divides itself logically into three subjects, *i. e.*, (1) volts per inch of break distance referred to by Messrs. MacNeill, Slepian, and Wilcox; (2) the interrupting characteristics of the plain and oil-blast explosion chamber referred to by Messrs. MacNeill, Slepian, Wilcox, Mahoney and Tretjak; (3) the effect of the new developments on the interrupting ratings and duty cycles of commercial oil circuit breakers mentioned by Messrs. Sporn and Treat.

Although volts per inch of arc has been used to compare the interrupting qualities of circuit breakers, it is not believed that too much emphasis should be placed on that figure alone as a criterion of breaker performance. In any breaker structure, the volts per inch of arc obtained is affected by many variables, among the most important of which are the magnitude and rate of recovery voltage rise and the speed of the moving contacts. As the rate of recovery voltage rise or the magnitude of recovery voltage increase, the volts per inch of arc decreases. Also, when other conditions remain the same, increasing the speed of the moving contacts will decrease the volts per inch of arc even though the actual arc duration may be shorter. Therefore, in order to obtain even general comparisons of the volts per inch of arc in the performance of a breaker, it is considered necessary to state the magnitude of rate of recovery voltage rise and also the speed of the moving contacts or its equivalent inches of arc and arc duration. Inasmuch as values of volts per inch of arc have been brought out in the discussion, it seems well to list herein the following tabulation showing the values obtained during the Philo tests on the oil-blast explosion chamber.

Recovery voltage rate volts per microsecond	Volts per inch of arc		
	Maximum	Average	Minimum
270	12,400	7,800	5,700
600	6,700	5,100	4,100
2,400	6,000	4,500	2,700

These values include the length of the auxiliary gap, which was ¾ in. and not 2½ in., even though the actual voltage drop across it is relatively small. Also, in order to permit discussion of the values submitted by Mr. Wilcox, the values given above are calculated on the assumption that the value of the recovery voltage is normal voltage. It is, however, forcibly emphasized

4. *Circuit Breaker Recovery Voltages*, by R. H. Park and W. F. Skeats, A. I. E. E. Trans., March 1931, p. 204.

that such an assumption is erroneous as will be seen from an examination of the cathode-ray oscillograms shown in Fig. 1 of our paper. A study of the oscillograms in that figure shows that the peak recovery voltage varied from about 170,000 volts at a recovery voltage rise rate of 270 volts per microsecond to 400,000 volts at a recovery voltage rise rate of 2,400 volts per microsecond. The speed of the moving contacts, as may readily be obtained from the tabulation in the paper, varied from 6 ft. to 18 ft. per second, depending on the severity of the duty on the breaker as indicated in general by the current interrupted and the rate of recovery voltage rise. In this connection, it is interesting to note that the two longest arc lengths obtained in the entire series of tests, namely 14.5 in. and 13.3 in., occurred at values of kva. interrupted of approximately 1,000,000 kva., at a recovery rate of 2,400 volts per microsecond and with a recovery voltage peak of about 400 kv. The speed of the moving contacts in these two tests was 18 and 15 ft. per second respectively, and the arc duration only 4.4 and 4.6 cycles respectively. In these two tests, the speed of the moving contacts was also greater than in any of the other tests.

The values of volts per inch of arc given by Mr. Wilcox do not give the speed of the moving contacts or the rate of rise or magnitude of the recovery voltage which makes it impossible to draw any conclusions from a comparison of his values with those obtained during the Philo tests. It may be assumed, however, that inasmuch as they were all system tests, the rate of recovery voltage rise would probably not exceed 150 or 200 volts per microsecond.

Referring to the comparison of the operation of the plain explosion chamber with that of the oil-blast explosion chamber, it is reasonable to expect that in the plain explosion chamber, the most favorable location of the moving contacts when the arc is broken is at or near the throat opening because it is not until the throat opening is approached that the blast of oil produced by the chamber becomes effective in contacting with the arc. There is also no reason to assume that the chamber is not an effective interrupting element even though the moving contact has moved out of the chamber throat. It is believed that the correctness of that statement will be obvious when it is realized that after the moving contact has left the throat opening without interrupting the circuit, the arc is playing through the throat opening to the contact inside the chamber. The arc inside the chamber is decomposing oil and forming gas which is forcing oil and gas out through the throat of the chamber. Therefore, when the current passes through zero, oil from the chamber displaces the gaseous arc stream in the throat opening and so introduces good insulation into the arc stream.

With the oil-blast explosion chamber, the hollow contact gives the effect of pushing the throat opening upward so that the oil blast produced by the auxiliary gap may become effective in contacting with the arc stream as soon as the second gap parts. It is our conception of the operation of this chamber that the oil and some gas flows from the chamber through the hollow contact rod to the lower part of the floating contact. As current zero approaches, however, the oil surrounding the arc gas bubble closes in, as explained by Mr. Prince, until a solid wall of oil is interposed between the moving and floating contacts.

Mr. Tretjak proposes moving the throat opening of the standard chamber upward several inches. Numerous schemes of that kind, including baffles, have been tried. Some of them have given improved performance but none have given the performance obtained with the oil-blast chamber which, as has been brought out above, gives something of the equivalent of a moving throat and does not require the end of the moving contact to come to the throat before the oil-blast is effective.

Commenting further on Dr. Slepian's comment on the improvement that might be obtained by omitting the primary gap in the oil-blast chamber, attention is directed to the fact that the butt contacts, the tests of which were reported in our paper, had hollow contact rods open on both ends. They did not give the results obtained with the oil-blast contacts.

Mr. MacNeill comments on the performance of the breaker with butt contacts after one explosion chamber on one pole had ruptured. The arrangement of the test data in the tables does not follow the actual sequence of testing due to our desire to group the tests in accordance with recovery voltage rise and progressive values of current interrupted. Also, it is not clear, as Messrs. Sporn and St. Clair stated, on exactly what test the chamber broke. It is significant, however, that even with one explosion chamber out of commission, the breaker performed well even under severe conditions of recovery voltage rise and high currents interrupted.

Mr. MacNeill questions the consistency of operation of the oil-blast explosion chamber as indicated by the values of arc lengths given. In this connection, attention is directed to the fact that the tests reported in reality cover three separate and distinct test conditions between which, as we have brought out, the severity of interrupting duty varies widely. For that reason, it is not considered proper to compare a minimum value taken from the series of least severity with a maximum value taken from the series of greatest severity. However, if values of arc duration—which in the final analysis are the values most pertinent in discussing rapid operation of breakers—are compared in the various groups, we have the following:

Recovery voltage rate volts per microsecond	Arc duration cycles		
	Average	Maximum	Minimum
270	2.6	3.0	1.6
600	3.7	4.7	2.7
2,400	4.1	5.0	2.7

We submit these as being consistent not only in their own group but relatively so over the range of conditions listed.

Messrs. Treat and Sporn have brought forward the suggestion that inasmuch as the interrupting performance of breakers had been greatly improved not only in the time of operation but also on the duty imposed on the breaker itself during interruption and the number of times that it would operate under high duty, the operation cycle now standardized might be liberalized. Such modifications are now under consideration.

Traveling Waves on Transmission Systems

BY L. V. BEWLEY*
Associate, A. I. E. E.

Synopsis.—*The purpose of this paper is two-fold: First; to present the theory of traveling waves on multi-conductor systems, and second, to compile a brief compendium on the general subject of traveling waves on transmission systems. While the application of the multi-conductor theory is more laborious than that of the single-wire theory, yet it does not involve much greater complication, and it becomes necessary to go to the more general theory when mutual effects are important, as in the study of ground wires, or when discontinuities exist in paralleled circuits carrying traveling waves. The origin, shape, and general characteristics of traveling waves are discussed. The equivalent circuits of terminal equipment and the corresponding reflections and refractions from such junctions are given for a large number of cases. The methods of computing a multiplicity of successive reflections by means of lattices are described. The effect of line losses in equalizing the subsidence of traveling waves and on their attenuation and distortion is also discussed.*

* * * * *

Origin of Traveling Waves

SURGES on transmission lines owe their origin to four different causes; induced from lightning discharges, direct lightning stroke, arcing grounds, and switching.

The maximum potential at the point of origin, and the crest of induced traveling waves are respectively:

$$V = \alpha G h$$
$$V' = \alpha' G h$$

where h is the height of the line conductor, G the field gradient, and α and α' factors depending on the law of cloud discharge and the distribution of bound charge.[1] For an instantaneous cloud discharge (an impossible assumption) $\alpha = 1.0$ and $\alpha' = 0.5$. (Fig. 1) As the time of cloud discharge increases both α and α' decrease, and rapidly approach equality; so that for a cloud discharge slower than 10 microseconds they are practically equal. The equation for α' based on the assumption of an exponential law of cloud discharge is:

$$\alpha' = \frac{1}{2}\left(1 - \epsilon^{-\frac{3x}{t}}\right)$$

where x is the length of the bound charge in thousands of feet, and t is the time of cloud discharge in microseconds. The length of the wave is:

$$L = x + t$$

and the front of the wave is x or t depending upon which is the smaller. It is evident that induced strokes become harmless for long periods of cloud discharge, and that high potential induced strokes are possible only with very short waves. For the accepted maximum value of $G = 100$ kv/ft., an average line height of $h = 60$ ft., a 3,000-ft. rectangular bound charge, and 10 microseconds for the cloud discharge, the maximum induced traveling wave that can occur on the line is:

$$V' = \alpha' G h = 0.30 \times 100 \times 60 = 1{,}800 \text{ kv.}$$

and this wave has a total length of only 13 microseconds. Had the time of cloud discharge been 30 microseconds then the induced voltage would have been only

$$V' = 0.13 \times 100 \times 60 = 780 \text{ kv.}$$

The induced voltages decrease in importance as the line insulation is increased, and the proportion of outages due to direct strokes increases. The induced voltages can be roughly halved by the use of one or more ground wires.

The most feasible way of correlating the direct versus

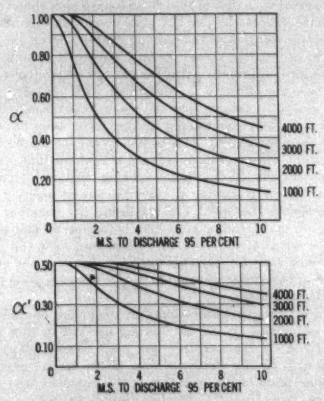

FIG. 1—REDUCTION FACTORS FOR INDUCED LIGHTNING POTENTIALS

induced stroke arguments is through the simple theoretical relationships between the traveling waves originating from these two causes. It was found in a previous investigation[1] that the law of cloud discharge has a greater effect on the shape of the traveling wave than the distribution of bound charge, and therefore that the assumption of a rectangular bound charge may be made without involving any marked departure from the true shape of the traveling wave. Then the determination of the wave shape reduces to the problem

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1. For references see Bibliography.

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of finding the law of cloud discharge. If the wave measured by the cathode-ray oscillograph station was due to a direct stroke, then the shape of the current wave in the lightning bolt is known. But if Q is the charge on the cloud, this current is

$$i = \frac{\partial Q}{\partial t} = Q_o \frac{\partial}{\partial t} F(t)$$

where Q_o is the total charge at the beginning of cloud discharge, and $F(t)$ is the law of cloud discharge. Therefore

$$F(t) = \frac{1}{Q_o} \int_0^t i \, dt$$

The actual numerical values of Q_o and i are unimportant. All that is necessary is to arbitrarily choose

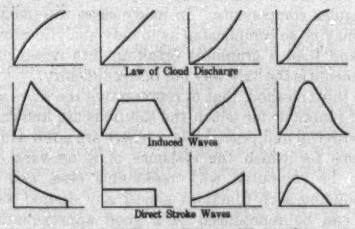

FIG. 2—INDUCED AND DIRECT STROKE WAVES CORRESPONDING TO DIFFERENT LAWS OF CLOUD DISCHARGE

Q_o of such value that $F(t)$ reaches a final value of unity at the completion of cloud discharge. And the cloud discharge is complete when i has ceased. Suppose that the direct stroke wave has the typical shape, Fig. 3D,

$$i = I(\epsilon^{-at} - \epsilon^{-bt})$$

Then the law of cloud discharge is

$$F(t) = \frac{I}{Q_o} \int_0^t (\epsilon^{-at} - \epsilon^{-bt}) \, dt$$

$$= \frac{I}{Q_o} \left\{ \frac{1 - \epsilon^{-at}}{a} - \frac{1 - \epsilon^{-bt}}{b} \right\}$$

In Fig. 2 are shown a few simple direct stroke waves, and the corresponding laws of cloud discharge and induced waves. Thus if a cathode-ray oscillogram is definitely known to be that of either a direct or induced stroke, then the character of the other may be derived. Of course the length and shape of the bound charge distribution will influence the shape of the induced traveling wave, but not nearly to the same extent as the law of cloud discharge. The above expressions follow from the premise that the electrostatic field of the cloud collapses *uniformly*. Possibly there is considerable departure from this assumption. Nevertheless, the above equations do correlate the two types of lightning waves as near as our present knowledge of the mechanism of cloud discharge will permit.

The third cause of abnormal voltage oscillations on a transmission line is arcing grounds.[7] Theoretically, voltages as high as 7½ times normal line-to-neutral voltage are possible. But many assumptions underlie the present theories of arcing grounds. When the results arrived at by theory are judiciously shaded to compensate for the assumptions introduced to simplify the analysis, it is doubtful if an arcing ground can actually cause a voltage in excess of 5 times normal line-to-neutral voltage on an isolated neutral system. Arcing ground surges are oscillatory in nature, and consist of a normal frequency oscillation which gradually builds up to several times normal, and a superimposed high frequency oscillation, whose frequency depends chiefly upon the length of line between the station and the arcing ground.

Switching surges exhibit the same class of characteristics as arcing grounds, although they are more irregular in shape. Their proper consideration is outside the scope of this paper.

SHAPES AND SPECIFICATIONS OF TRAVELING WAVES

The principal shapes of most natural waves are included in Fig. 3, that is, may be represented by the difference of two exponentials, but of course actual waves are usually serrated by minor irregularities.

As far as mathematical simplicity is concerned the

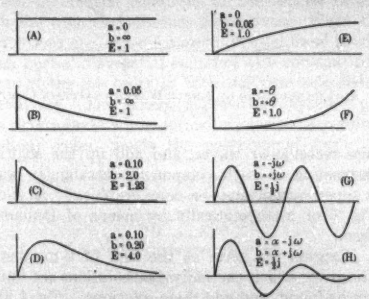

FIG. 3—EMPIRICAL WAVE SHAPES GIVEN BY $e = E(\epsilon^{-at} - \epsilon^{-bt})$.

most simple wave to calculate the effects of, is the infinite rectangular, shown in Fig. 3A. Also, as a rule, such a wave is the most dangerous to terminal equipment, and therefore calculations based on it are apt to err on the side of safety. Still other reasons that have favored its use in analysis are that it is by far the easiest to study pictorially, and that until recently the actual shapes of lightning surges were not known. However, during the past few years a great many cathode-ray oscillograms of natural lightning waves have been obtained under many different conditions, so that fairly definite information as to their general shape and characteristics is available. It is therefore essential that calculations be made with these characteristic

lightning waves, in order that the influence of the fronts, tails, and lengths of the waves may be evaluated.

Three different methods for calculating waves of arbitrary shape are given in the following:

1. Express in operational notation, combine with the function representing the reflection or refraction operator, and solve the resulting operational equations.

2. Consider the wave as made up of a series of

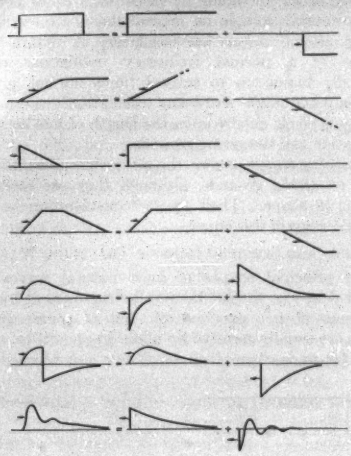

FIG. 4—COMPOUNDING OF SIMPLE WAVES TO OBTAIN COMPLEX WAVES

infinite rectangular waves, and add up the solutions corresponding to each component rectangular wave. This superposition may be done graphically as shown in Fig. 5 or mathematically by means of Duhamel's theorem.

3. Express the wave as the sum of a number of functions for which the individual solutions are known or can be found, and add these solutions. Fig. 4 illustrates a few simple examples.

Which of these methods to use is entirely a matter of convenience in any specific case, each method having certain advantages and limitations. The difficulty usually encountered with the first method is that it complicates the operational equations so that a solution either cannot be obtained at all, or else only by the most laborious and complex process. As a rule, in dealing with the behavior of traveling waves at a transition point, it will be found that the reflection (or other) operator acting on the simple unit function (an infinite rectangular wave) is just about as complicated a proposition as can be handled with engineering expedience, and that any further complications in the operational equations are prohibitive of a solution. The application of Duhamel's theorem suffers from substantially the same defects, except that the trouble with it is in making a difficult integration. It is

$$f(t) = E(o) \cdot \phi(t) + \int_o^t \phi(\tau) \frac{\partial}{\partial t} E(t-\tau)\, d\tau$$

where

$\phi(t)$ = solution corresponding to unit function

$E(t)$ = applied wave of arbitrary shape

$f(t)$ = solution corresponding to $E(t)$

The graphical representation of a wave of arbitrary shape as a set of rectangular components, is of course only an approximation, but in a great many cases of engineering importance it is quite sufficient, and has the advantage of simplicity. In any event it can always be made as accurate as required, merely by subdividing the wave into a sufficiently large number of small rectangular components. In many cases the incident wave may be so complicated as to defy analytic expression, and then a graphical break-up into rectangular components is the only way out of the difficulty.

The third method, that of representing the wave as a sum of functions for which the solutions are known, is very powerful and practicable. There are a few simple functions for which the response of a network can usually be computed with reasonable ease, and by compounding such functions almost any desired wave shape can be reproduced to a good approximation. These elementary waves are:

a. Infinite rectangular
b. Simple exponential
c. Uniformly rising front
d. Damped sinusoid
e. Difference of two exponentials

As a matter of fact, by a suitable choice of the parameters in the difference of two exponentials, all

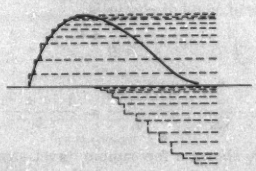

FIG. 5—APPROXIMATION BY RECTANGULAR COMPONENTS

of these elementary waves may be considered as special cases of e, as illustrated in Fig. 3. It so happens that the solution corresponding to

$$e_1 = E_1 \epsilon^{-at}$$

is easily effected by means of Heaviside's shifting theorem.

$$f(p)\, \epsilon^{-at} = \epsilon^{-at} f(p-a)$$

and in many cases with no greater mathematical difficulty than attains with the unit function alone.

The references should be consulted for more details concerning the specification of traveling waves.[1,2,4]

Attenuation of Traveling Waves

It is usually justifiable to calculate traveling waves on the assumption of no losses, and then to compensate for the attenuation by an exponential decrement factor, arrived at experimentally.

Corona is the chief factor in causing attenuation and distortion. It levels off the top and elongates the wave, but its effect varies with the weather and other conditions, so that there is no such thing as a definite attenuation on a given line.

The effect of the line losses on traveling waves are three-fold: (1), the waves of voltage and current are attenuated, (2), the shapes are distorted with time, (3), the current and voltage waves depart from exact similarity, so that they are no longer connected by the simple linear proportionality factors called the surge impedances.

Fig. 8 shows the effect of attenuation in the charging of an open-ended line from an infinite voltage source. Without losses, the cycle of oscillations repeats indefinitely, but when line losses are present the oscillations gradually diminish until the line eventually reaches a steady state condition. However, a distortionless line can never become fully charged to the terminal potential, throughout its length, for the distortionless feature requires the presence of both leakage conductance and series resistance. The flow of current in the leakage conductance results in a progressive voltage drop along the line. Therefore, the ultimate level charging of a line requires that there be no leakage currents. Referring to Fig. 8, which has been drawn for a linear rather than exponential attenuation, the voltage at various instances of time for an attenuation $(1-\alpha)$ per trip is

Sending end	Units of time	Receiving end
1	0	0
1	1	α
$1+\alpha^2$	2	2α
1	3	$2\alpha-\alpha^3$
$1-\alpha^4$	4	$2\alpha-2\alpha^3$
1	5	$2\alpha-2\alpha^3+\alpha^5$
$1+\alpha^6$	6	$2\alpha-2\alpha^3+2\alpha^5$
1	7	$2\alpha-2\alpha^3+2\alpha^5-\alpha^7$
$1-\alpha^8$	8	$2\alpha-2\alpha^3+2\alpha^5-2\alpha^7$
etc.		etc.

The voltage at the receiving end after 4 n units of time is

$$e = 2(\alpha - \alpha^3 + \alpha^5 - \alpha^7 + \ldots\ldots)$$
$$= 2(\alpha - \alpha^3)(1 + \alpha^4 + \alpha^8 + \ldots\ldots)$$
$$= 2(\alpha)(1-\alpha^2)\sum_{0}^{n}\alpha^{4r} = 2\alpha\,\frac{1-\alpha^{4(n+1)}}{1+\alpha^2}$$

After an infinite number of oscillations

$$e = \frac{2\alpha}{1+\alpha^2}$$

Thus if the attenuation is $(1-\alpha) = 0.5$ as in Fig. 8, and the line is distortionless, the open end of the line finally stabilizes at a voltage of

$$e = \frac{2 \times 0.5}{1 + 0.25} = 0.8$$

General Properties of Free Traveling Waves

In Appendix I the general differential equations for potentials and currents of a multi-conductor transmission line, (Fig. 6), are formulated. Each conductor, (Fig. 7), is assumed to have its own resistance to ground

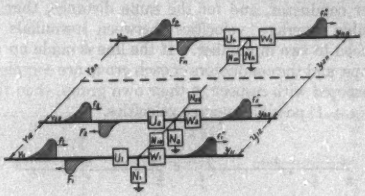

Fig. 6—General Multi-Conductor System

and to each of the other conductors, self and mutual inductance between all wires, and leakage conductance from each wire to earth and to all the other wires. For an n-wire system there results a set of n simultaneous partial differential equations of the second order in both the time and distance derivatives. When these n equations are solved for any potential there results a linear partial differential equation with constant coefficients of the $2n$ order in both the time and distance derivatives. This general differential equation is given as a determinate whose formal expansion, according to algebraic rules, yields the polynominal form. The

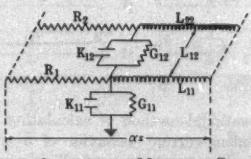

Fig. 7—Circuit Constants of Mutually Coupled Circuits

differential equation and its coefficients is identical for the potential on any wire of the n-wire system; but the boundary conditions, and therefore the integration constants of the solution, may be different for each wire. The general linear partial differential equation with constant coefficients has a formal solution, but it has not been included in this paper (see "Treatise on Differential Equations" by Forsyth). But the solutions are given corresponding to the following special cases:

The No-Loss Line
The Completely Transposed Line
Solution for Alternating Currents

If a line is free from losses the differential equations of the system are satisfied by pure wave functions. These waves travel without attenuation or distortion until a transition point on the circuit is reached, where they reflect and refract in accordance with the general laws given in Appendix II. In an n-wire no-loss system there are n possible velocities of wave propagation, but in the case of overhead conductors these n velocities all become equal to the velocity of light.

If the line is completely transposed, so that every conductor occupies the same relative position as every other conductor, and for the same distance, then the possible average velocities between terminals are reduced to two in number. If the line is made up of g groups, and the conductors of each group are completely transposed with respect to their own group, then there are $(g + 1)$ possible average velocities.

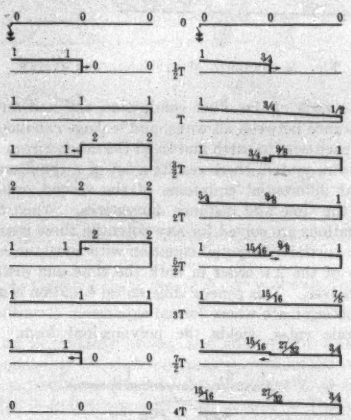

FIG. 8—EFFECT OF ATTENUATION ON CHARGING A LINE

The conventional method of calculating the steady state alternating-current behavior of a transmission line by using "constants to neutral" is a very special case, but is rigorous for a single-circuit, three-phase, completely transposed line. To obtain these simplified equations directly from the general alternating current solution is a matter of considerable manipulation. The reduction is carried out in detail in Appendix I.

In Appendix I it is shown that in an n-wire circuit the energy of free traveling waves is equally divided between the electrostatic and the electromagnetic fields. However, when these waves reach a transition point, or when waves traveling in opposite directions pass through one another, the energy balance is upset. Whether the energy resides mostly in one field or the other during a transition period depends entirely upon the nature of the transition.

Pure traveling waves of voltage and current on an n-wire system, are related to each other by a set of simultaneous linear equations having constant coefficients called the self and mutual surge impedances of the line. These equations[3] are

$$\begin{aligned}
\pm e_1 &= z_{11}\, i_1 + z_{21}\, i_2 + \ldots\ldots\ldots\ldots + z_{n1}\, i_n \\
\pm e_2 &= z_{12}\, i_1 + z_{22}\, i_2 + \ldots\ldots\ldots\ldots + z_{n2}\, i_n \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\pm e_n &= z_{1n}\, i_1 + z_{2n}\, i_2 + \ldots\ldots\ldots\ldots + z_{nn}\, i_n
\end{aligned}$$

where the (+) sign is used for waves traveling in the forward direction, and the (−) sign for waves moving in the reverse direction. Also

$$z_{rr} = 60 \log \left(\frac{2h}{\rho} \right) = \text{self surge impedance of conductor } r$$

$$z_{rs} = 60 \log \left(\frac{a}{b} \right) = \text{mutual surge impedance between } r \text{ and } s$$

h = height of r above ground plane
a = distance between r and image of s
b = distance between r and s
ρ = radius of conductor

These equations, in conjunction with the principle of superposition and the conditions of voltage and current continuity at a junction or transition point, define the behavior of pure traveling waves on transmission systems. The principle of superposition is involved in calculating the effect of more than one independent wave at the same point simultaneously, such as occur for successive reflections, meeting of circuits, etc.

Instead of writing the voltages in terms of the currents and the surge impedances, the arrangement may be reversed and the currents written in terms of the voltages and surge admittances as follows:

$$\begin{aligned}
\pm i_1 &= y_{11}\, e_1 + y_{21}\, e_2 + \ldots\ldots\ldots + y_{n1}\, e_n \\
\pm i_2 &= y_{12}\, e_1 + y_{22}\, e_2 + \ldots\ldots\ldots + y_{n2}\, e_n \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\pm i_n &= y_{1n}\, e_1 + y_{2n}\, e_2 + \ldots\ldots\ldots + y_{nn}\, e_n
\end{aligned}$$

If

$$D = \begin{vmatrix} z_{11}\, z_{21} \ldots\ldots z_{n1} \\ z_{12}\, z_{22} \ldots\ldots z_{n2} \\ \ldots\ldots\ldots\ldots \\ z_{1n}\, z_{2n} \ldots\ldots z_{nn} \end{vmatrix}$$

and

$$D' = \begin{vmatrix} y_{11}\, y_{21} \ldots\ldots y_{n1} \\ y_{12}\, y_{22} \ldots\ldots y_{n2} \\ \ldots\ldots\ldots\ldots \\ y_{1n}\, y_{2n} \ldots\ldots y_{nn} \end{vmatrix}$$

then the y's and z's are related to each other as

$$y_{rs} = \frac{(\text{minor of } D \text{ for which the cofactor is } z_{rs})}{D}$$

$$z_{rs} = \frac{(\text{minor of } D' \text{ for which the cofactor is } y_{rs})}{D'}$$

For instance, in the case of two conductors

$$z_{12} = \begin{vmatrix} & y_{21} \\ y_{11} & y_{21} \\ y_{12} & y_{22} \end{vmatrix} = \frac{-y_{21}}{y_{11}\,y_{22} - y_{12}\,y_{21}}$$

$$z_{11} = \begin{vmatrix} & y_{22} \\ y_{11} & y_{21} \\ y_{12} & y_{22} \end{vmatrix} = \frac{y_{22}}{y_{11}\,y_{22} - y_{12}\,y_{21}}$$

Traveling Waves on a Single-Conductor Circuit

In the majority of problems dealing with traveling waves, it is sufficient to make the calculations on the basis of a single-wire circuit. The return part of the circuit may be either a second conductor or the ground.

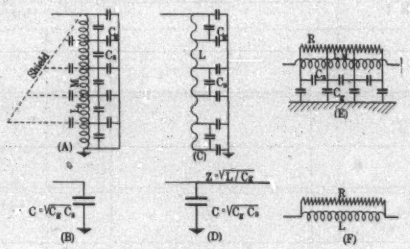

Fig. 9—Equivalent Circuits of Terminal Apparatus to Lightning Surges

(A) Transformer, ideal
(B) Transformer, approx.
(C) Rotating machine, ideal
(D) Rotating machine, approx.
(E) Reactor with shunt resistor, ideal
(F) Reactor with shunt resistor, approx.

When the single-circuit theory is applicable, the potential and current waves are proportional to each other by the surge impedance of the circuit;

$e\,(x - v\,t) = Z\,i\,(x - v\,t)$ for forward moving waves

$e\,(x + v\,t) = -\,Z\,i\,(x + v\,t)$ for backward moving waves

where

$Z = \sqrt{L/C}$ = surge impedance in ohms
$Y = 1/Z$ = surge admittance in mhos
L = inductance in henrys
C = capacitance in farads

It is to be noticed that for waves traveling in the positive or forward direction that e and i have the same sign; but that for waves traveling in the reverse or negative direction they have opposite signs. However, it is immaterial which direction along the line be chosen as positive or forward; provided that the positive sense be strictly adhered to throughout the calculations.

The reflection and refraction operators, by equations (18) and (19) of Appendix II are respectively

$$\frac{Z_o - Z}{Z_o + Z} \text{ and } \left(\frac{1}{1 + N\,(W + z)}\right)\left(\frac{2z}{Z_o + Z}\right)$$

where Z_o is the total impedance, expressed in operational form, at the transition point as viewed from the approaching incident wave, z is the surge impedance of the line, and N and W are impedances having the same meaning as in Fig. 6. When these operators are applied to any incident wave e, expressed as a function of time t counted from the instant that e arrives at the transition point, they derive the reflected and transmitted waves e' and e'' respectively.

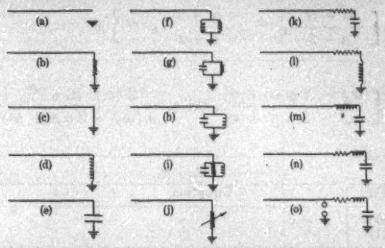

Fig. 10—Terminal Conditions on Single-Conductor Circuits

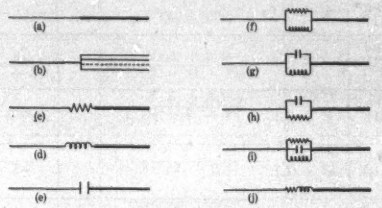

Fig. 11—Junctions Between Single-Conductor Circuits

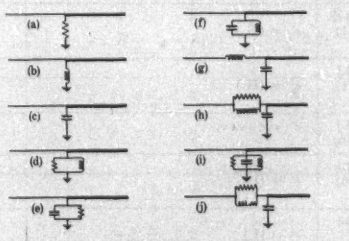

Fig. 12—Junctions Between Single-Conductor Circuits

The solutions and graphs of most of the thirty-five

circuits shown in Figs. 10, 11, and 12 have been given in previous papers but the equations will be repeated here for ready reference. By a fortunate coincidence, nearly all of these circuits are included by two general equations, through an adjustment of the coefficients thereof. Therefore, all that is necessary here is to give these equations, and a table with the proper coefficients for each case. The equations are

$$A\left[\frac{a+\alpha}{a-\beta}\,\epsilon^{-at} - \frac{\alpha+\beta}{a-\beta}\,\epsilon^{-\beta t}\right]E \qquad (1)$$

$$A\left[\frac{\omega_o^2 - 2\,a\,\alpha + a^2}{\omega_o^2 - 2\,a\,\beta + a^2}\,\epsilon^{-at} + \frac{2\,(\alpha-\beta)\,\epsilon^{-\beta t}}{\omega\,(\omega_o^2 - 2\,a\,\beta + a^2)}\right.$$

$$\left.\{(\omega_o^2 - a\,\beta)\sin\omega\,t + a\,\omega\cos\omega\,t\}\right]E \qquad (2)$$

$$\omega_o^2 = 1/L\,C \text{ and } \omega^2 = (\omega_o^2 - \beta^2),\ e = E\,\epsilon^{-at}$$

In the following tables the reflected wave e' and the transmitted wave e'' are given. In all cases the total voltage at the transition point is

$$e_o = e + e'$$

The reflected and transmitted current waves are respectively

$$i' = -\frac{e'}{z_1} \text{ and } i'' = \frac{e''}{z_2}$$

and the total current at the transition point is

$$i_o = i + i'$$

TABLE I

Fig.	Equation	α	β	A
10-a (1)*	$e' = e$			
10-b (1)	$e' = \frac{R-Z}{R+Z}\,e$			
10-c (1)	$e' = -e$			
10-d (1)	$e' = (\text{I.})$	$\frac{Z}{L}$	$\frac{Z}{L}$	1
10-e (1)	$e' = (\text{I.})$	$\frac{1}{C\,Z}$	$\frac{1}{C\,Z}$	-1
10-f (1)	$e' = (\text{I.})$	$\frac{Z\,R}{L\,(R-Z)}$	$\frac{Z\,R}{L\,(R+Z)}$	$\frac{R-Z}{R+Z}$
10-g (1)	$e' = (\text{I.})$	$\frac{R-Z}{Z\,R\,C}$	$\frac{R+Z}{Z\,R\,C}$	-1
10-h (1)	$e' = (\text{II.})$	$-\frac{1}{2\,C\,Z}$	$\frac{1}{2\,C\,Z}$	-1
10-i (1)	$e' = (\text{II.})$	$\frac{Z-R}{2\,R\,C\,Z}$	$\frac{Z+R}{2\,R\,C\,Z}$	-1
10-j (10)	$e' = e - z\,i_o$			
10-k (6)	$e' = (\text{I.})$	$\frac{1}{C\,(Z-R)}$	$\frac{1}{C\,(Z+R)}$	$\frac{R-Z}{R+Z}$
10-l	$e' = (\text{I.})$	$\frac{Z-R}{L}$	$\frac{Z+R}{L}$	1
10-m (5)	$e' = (\text{II.})$	$\frac{-Z}{2\,L}$	$\frac{Z}{2\,L}$	1
10-n (5) (6)	$e' = (\text{II.})$	$\frac{R-Z}{2\,L}$	$\frac{R+Z}{2\,L}$	1
10-o (5)	Same as 10-n before gap sparkover Same as 10-e after sparkover			

*See Bibliography references.

TABLE II

Fig.	Equation	α	β	A
11-a (1)	$e' = \frac{Z_2 - Z_1}{Z_2 + Z_1} e$ $e'' = \frac{2 Z_2}{Z_2 + Z_1} e$			
11-b (1)	$e' = \frac{1 - Z_1 Y_0}{1 + Z_1 Y_0} e$ $e'' = \frac{2}{1 + Z_1 Y_0} e$	Y_0 = total admittance of all outgoing lines in parallel		
11-c (1)	$e' = \frac{Z_2 - Z_1 + R}{Z_2 + Z_1 + R} e$ $e'' = \frac{2 Z_2}{Z_2 + Z_1 + R} e$			
11-d (1)	e' = (I.) $e'' = \frac{\alpha - \beta}{a - \beta} (\epsilon^{-\alpha t} - \epsilon^{-\beta t})$	$\frac{Z_1 - Z_2}{L}$ "	$\frac{Z_1 + Z_2}{L}$ "	1
11-e	e' = (I.) e'' = (I.)	$\frac{1}{C(Z_1 - Z_2)}$ 0	$\frac{1}{C(Z_1 + Z_2)}$ $\frac{1}{C(Z_1 + Z_2)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$ $\frac{2 Z_2}{Z_2 + Z_1}$
11-f	e' = (I.) e'' = (I.)	$\frac{R(Z_1 - Z_2)}{L(R - Z_1 + Z_2)}$ $-\frac{R Z_2}{L(R + Z_2)}$	$\frac{R(Z_1 + Z_2)}{L(R + Z_1 + Z_2)}$ $\frac{R(Z_1 + Z_2)}{L(R + Z_1 + Z_2)}$	$\frac{R + Z_2 - Z_1}{R + Z_2 + Z_1}$ $\frac{2(Z_2 + R)}{R + Z_2 + Z_1}$
11-g	e' = (II.) e'' = (II.)	$\frac{1}{2C(Z_2 - Z_1)}$ 0	$\frac{1}{2C(Z_2 - Z_1)}$ $\frac{1}{2C(Z_2 + Z_1)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$ $\frac{2 Z_2}{Z_2 + Z_1}$
11-h	e' = (I.) e'' = (I.)	$\frac{R - Z_1 + Z_2}{RC(Z_1 - Z_2)}$ $-\frac{1}{RC}$	$\frac{R + Z_1 + Z_2}{RC(Z_1 + Z_2)}$ $\frac{R + Z_1 + Z_2}{RC(Z_1 + Z_2)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$ $\frac{2 Z_2}{Z_2 + Z_1}$
11-i (1)	e' = (II.) e'' = (II.)	$\frac{Z_2 - Z_1 + R}{2RC(Z_2 - Z_1)}$ $\frac{1}{2RC}$	$\frac{Z_2 + Z_1 + R}{2RC(Z_2 + Z_1)}$ $\frac{Z_2 + Z_1 + R}{2RC(Z_2 + Z_1)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$ $\frac{2 Z_2}{Z_2 + Z_1}$
11-j	e' = (I.) $e'' = \frac{-2 Z_2}{L(a - \beta)} (\epsilon^{-\alpha t} - \epsilon^{-\beta t})$	$\frac{Z_1 - Z_2 - R}{L}$	$\frac{Z_1 + Z_2 + R}{L}$ $\frac{Z_1 + Z_2 + R}{L}$	1

Traveling Waves on Multi-Conductor Circuits

Equations (9) and (10) of Appendix II are the two general equations which must be satisfied by the incident, reflected, and transmitted waves on all wires at the transition points of the general system represented in Fig. 6. Therefore the solution of these simultaneous equations yields the reflected and transmitted waves in terms of the incident waves and the circuit constants of the lines and the transition points. These equations are considerably simplified when the networks N_{12}, N_{23}, etc., connecting the different lines at the junctions are equal to zero. In that case they become

$$(1 + N_r U_r)\,[Y_{r1}(e_1 - e_1') + \ldots\ldots + Y_{rn}(e_n - e_n')] - N_r(e_r + e_r') = (y_{r1} e_1'' + \ldots\ldots + y_{rn} e_n'')$$

$$(e_r + e_r') - U_r [Y_{r1} (e_1 - e_1') + \ldots\ldots + Y_{rn} (e_n - e_n')]$$
$$= e_r'' + W_r (y_{r1} e_1'' + \ldots\ldots + Y_{rn} e_n'')$$

where n is the total number of conductors of the system, and r is any particular conductor. Each of the above equations must be written for each r so as to provide the necessary $2\,n$ simultaneous equations from which the $2\,n$ unknowns $(e_1' \ldots\ldots e_n', e_1'' \ldots\ldots e_n'')$ can be found. A few examples will be given to illustrate the use of these equations. In the interests of brevity, the illustrations are for a two-wire and ground circuit.

Fig. 13a. One of two lines suddenly terminates.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$
$$y_{11} = 1/z_{11},\ y_{22} = y_{12} = 0$$

Substituting these values in the general equation there is

$$\left.\begin{aligned} Y_{11} (e_1 - e_1') + Y_{12} (e_2 - e_2') &= y_{11} e_1'' \\ Y_{21} (e_1 - e_1') + Y_{22} (e_2 - e_2') &= 0 \\ (e_1 + e_1') &= e_1'' \\ (e_2 + e_2') &= e_2'' \end{aligned}\right\}$$

The solution of these simultaneous equations gives

$$e_1' = \frac{z_{11} - Z_{11}}{z_{11} + Z_{11}}\, e_1$$

$$e_2' = e_2 - \frac{2\, Z_{12}}{z_{11} + Z_{11}}\, e_1$$

$$e_1'' = \frac{2\, z_{11}}{z_{11} + Z_{11}}\, e_1$$

where

$$Z_{11} = \frac{Y_{22}}{Y_{11}\, Y_{22} - Y_{12}^2}$$

TABLE III

Fig.	Equation	α	β	A
12-a	$e' = \dfrac{Z_2 R - Z_1 R - Z_1 Z_2}{Z_2 R + Z_1 R + Z_1 Z_2}$ $e'' = \dfrac{2\, Z_2 R}{Z_2 R + Z_1 R + Z_1 Z_2}$			
12-b (1)	$e' = \text{(I.)}$ $e'' = \dfrac{\alpha + \beta}{\alpha - \beta}\, \dfrac{E}{\alpha}\, (\alpha\, \epsilon^{-\alpha t} - \beta\, \epsilon^{-\beta t})$	$\dfrac{Z_1 Z_2}{L\,(Z_2 - Z_1)}$	$\dfrac{Z_1 Z_2}{L\,(Z_2 + Z_1)}$	$\dfrac{Z_2 - Z_1}{Z_2 + Z_1}$
12-c (1)	$e' = \text{(I.)}$ $e'' = E\, \dfrac{\alpha + \beta}{\alpha - \beta}\, (\epsilon^{-\alpha t} - \epsilon^{-\beta t})$	$\dfrac{Z_2 - Z_1}{Z_1 Z_2 C}$ "	$\dfrac{Z_2 + Z_1}{Z_1 Z_2 C}$ "	-1
12-d	$e' = \text{(I.)}$ $e'' = e + e'$	$\dfrac{R\, Z_1 Z_2}{L\,(R\, Z_2 - R\, Z_1 - Z_1 Z_2)}$	$\dfrac{R\, Z_1 Z_2}{L\,(R\, Z_2 + R\, Z_1 + Z_1 Z_2)}$	β/α
12-e	$e' = \text{(I.)}$ $e'' = -E\, \dfrac{\alpha + \beta}{\alpha - \beta}\, (\epsilon^{-\alpha t} - \epsilon^{-\beta t})$	$\dfrac{R\, Z_2 - R\, Z_1 - Z_1 Z_2}{Z_1 Z_2 R\, C}$	$\dfrac{R\, Z_2 + R\, Z_1 + Z_1 Z_2}{Z_1 Z_2 R\, C}$	-1
12-f	$e' = \text{(II.)}$ $e'' = e + e'$	$\dfrac{Z_1 - Z_2}{2\, Z_1 Z_2 C}$	$\dfrac{Z_1 + Z_2}{2\, Z_1 Z_2 C}$	-1
12-i (1)	$e' = \text{(II.)}$ $e'' = e + e'$	$\dfrac{Z_1 R - Z_2 R + Z_1 Z_2}{2\, Z_1 Z_2 R\, C}$	$\dfrac{Z_1 R + Z_2 R + Z_1 Z_2}{2\, Z_1 Z_2 R\, C}$	-1
12-g (2)	$e' = \text{(II.)}$ $e'' = \text{(II.)}$	See reference	See reference	See reference
12-h (2)	$e' = \text{(II.)}$ $e'' = \text{(II.)}$	See reference	See reference	See reference
12-j (2)	$e' = \text{(II.)}$ $e'' = \text{(II.)}$	See reference	See reference	See reference

$$Z_{12} = \frac{-Y_{12}}{Y_{11}Y_{22} - Y_{12}^2}$$

If e_2 was induced on line No. 2 by e_1 on line No. 1 then

$$e_2 = \frac{Z_{12}}{Z_{11}} e_1$$

Or conversely, if e_1 was induced by e_2, then

$$e_1 = \frac{Z_{12}}{Z_{22}} e_2$$

If, as would likely be the case, the line to the right is simply a continuation of No. 1 wire, then

$$z_{11} = Z_{11}$$

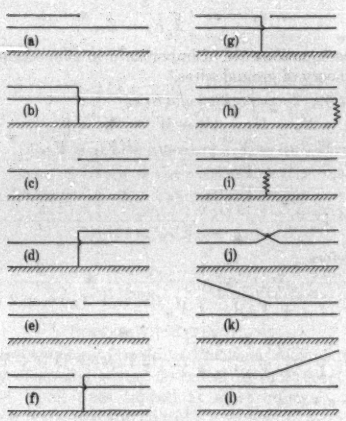

FIG. 13—TRANSITION POINTS OF A DOUBLE CIRCUIT

and the equations become

$$e_1' = 0$$

$$e_2' = e_2 - \frac{Z_{12}}{Z_{11}} e_1$$

$$e_1'' = e_1$$

that is, there is no reflection on line No. 1, and the full wave is transmitted. In this case, had e_2 been induced by e_1 there would be no reflection on No. 2 conductor either.

Fig. 13b. One of two lines is terminated and grounded.

$$N_1 = U_1 = U_2 = W_1 = W_2 = 0, \quad N_2 = \infty$$

$$y_{11} = 1/z_{11}, \quad y_{22} = y_{12} = 0$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' \\ (e_2 + e_2') &= 0 \\ (e_1 + e_1') &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Solving these simultaneous equations there is

$$e_1' = \frac{Y_{11} - y_{11}}{Y_{11} + y_{11}} e_1 + \frac{2Y_{12}}{Y_{11} + y_{11}} e_2$$

$$e_2' = -e_2$$

$$e_1'' = \frac{2Y_{11} e_1}{Y_{11} + y_{11}} + \frac{2Y_{12}}{Y_{11} + y_{11}} e_2$$

Fig. 13c. Isolated conductor introduced.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$Y_{11} = 1/Z_{11}, \quad Y_{12} = Y_{22} = 0$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') &= y_{11} e_1'' + y_{12} e_2'' \\ 0 &= y_{21} e_1'' + y_{22} e_2'' \\ (e_1 + e_1') &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{z_{11} - Z_{11}}{z_{11} + Z_{11}} e_1$$

$$e_1'' = \frac{2z_{11}}{z_{11} + Z_{11}} e_1$$

$$e_2'' = \frac{z_{12}}{z_{11}} e_1'' = \frac{2z_{12}}{z_{11} + Z_{11}} e_1$$

Thus if No. 1 is a through conductor, so that $z_{11} = Z_{11}$, there is no reflection.

Fig. 13d. Grounded conductor introduced.

$$N_1 = U_1 = U_2 = W_1 = W_2 = 0, \quad N_2 = \infty$$

$$Y_{11} = 1/Z_{11}, \quad Y_{12} = Y_{22} = 0$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') &= y_{11} e_1'' + y_{12} e_2'' \\ e_2 + e_2' &= 0 \\ (e_1 + e_1') &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{Y_{11} - y_{11}}{Y_{11} + y_{11}} e_1$$

$$e_1'' = \frac{2Y_{11}}{Y_{11} + y_{11}} e_1$$

$$e_2'' = 0$$

Fig. 13e. Break in one conductor.

$$N_1 = N_2 = U_1 = U_2 = W_1 = 0, \quad W_2 = \infty$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ (e_1 + e_1') &= e_1'' \\ 0 &= y_{21} e_1'' + y_{22} e_2'' \end{aligned}\right\}$$

Therefore, taking

$$Y_{11} = y_{11}, Y_{22} = y_{22}, \text{ and } Y_{12} = Y_{21} = y_{12} = y_{21}$$

$$e_1' = 0$$

$$e_2' = e_2 - \frac{z_{12}}{z_{11}} e_1$$

$$e_1'' = e_1$$

$$e_2'' = \frac{z_{12}}{z_{11}} e_1$$

Fig. 13f. Broken line—far section grounded.

$$U_1 = W_1 = W_2 = N_1 = 0, \quad U_2 = N_2 = \infty$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= 0 \\ (e_1 + e_1') &= e_1'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= 0 \end{aligned}\right\}$$

Therefore, since $e_2'' = 0$

$$e_1' = \frac{1 - Z_{11} y_{11}}{1 + Z_{11} y_{11}} e_1$$

$$e_2' = e_2 + \frac{2 Z_{12} y_{11}}{1 + Z_{11} y_{11}} e_1$$

$$e_1'' = \frac{2}{1 + Z_{11} y_{11}} e_1$$

$$e_2'' = 0$$

Fig. 13g. Broken line—near section grounded.

$$U_1 = U_2 = W_1 = N_1 = 0, \quad W_2 = N_2 = \infty$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ e_2 + e_2' &= 0 \\ e_1 + e_1' &= e_1'' \\ 0 &= y_{21} e_1'' + y_{22} e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{z_{11} Y_{11} - 1}{z_{11} Y_{11} + 1} e_1 + \frac{2 Y_{12} z_{11}}{z_{11} Y_{11} + 1} e_2$$

$$e_2' = - e_2$$

$$e_1'' = \frac{2 z_{11}}{z_{11} Y_{11} + 1} (Y_{11} e_1 + Y_{12} e_2)$$

$$e_2'' = \frac{z_{12}}{z_{11}} e_1'' = \frac{2 z_{12}}{z_{11} Y_{11} + 1} (Y_{11} e_1 + Y_{12} e_2)$$

Fig. 13h. One line grounded through a resistor at end of line.

$$U_1 = U_2 = N_2 = 0, \quad N_1 = 1/R, \quad y_{11} = y_{22} = y_{22} = 0$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') - (e_1 + e_1')/R &= 0 \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= 0 \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{R - Z_{11}}{R + Z_{11}} e_1$$

$$e_2' = e_2 - \frac{2 Z_{12}}{R + Z_{11}} e_1$$

If $R = Z_{11}$, then $e_1' = 0$ and there is no reflected wave on No. 1 wire. However, a wave

$$e_2' = e_2 - \frac{Z_{12}}{Z_{11}} e_1$$

is reflected on No. 2 wire.

Fig. 13i. Resistance ground on one wire.

$$U_1 = U_2 = W_1 = W_2 = N_2 = 0, \quad N_1 = 1/R$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' + (e_1 + e'_1)/R \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ e_1 + e_1' &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{- Z_{11}}{2 R + Z_{11}} e_1$$

$$e_2' = \frac{- Z_{12}}{2 R + Z_{11}} e_1$$

$$e_1'' = \frac{2 R}{2 R + Z_{11}} e_1$$

$$e_2'' = e_2 - \frac{Z_{12}}{2 R + Z_{11}} e_1$$

These equations are of importance in connection with the theory of ground wires.[3]

Fig. 13j. Transposition of a line.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$y_{11} = Y_{22}, y_{22} = Y_{11}, y_{12} = y_{21} = Y_{12} = Y_{21}$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ e_1 + e_1' &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{(Y_{11} - Y_{22})}{(Y_{11} + Y_{22})^2 - 4 Y_{12}^2} [(Y_{11} + Y_{22}) e_1 + 2 Y_{12} e_2]$$

$$e_2' = \frac{- (Y_{11} - Y_{22})}{(Y_{11} + Y_{22})^2 - 4 Y_{12}^2} [2 Y_{12} e_1 + (Y_{11} + Y_{22}) e_2]$$

$$e_1'' = e_1' + e_1$$

$$e_2'' = e_2' + e_2$$

If the two conductors are in the same horizontal plane so that $Y_{11} = Y_{22}$, then there are no reflections.

If the two incident waves are alike, that is $e_1 = e_2 = e$ then

$$e_1' = - e_2' = \frac{(Y_{11} - Y_{22}) e}{Y_{11} + Y_{22} - 2 Y_{12}}$$

Fig. 13k. Line entering a section parallel to another line.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$Y_{12} = 0, \quad Z_{11} = z_{11}, \quad Z_{22} = z_{22}$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ (e_1 + e_1') &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{[(Y_{11} - y_{11})(Y_{22} + y_{22}) + y_{12}^2] e_1 + 2 y_{12} Y_{22} e_2}{(Y_{11} + y_{11})(Y_{22} + y_{22}) - y_{12}^2}$$

$$e_2' = \frac{[(Y_{11} + y_{11})(Y_{22} - y_{22}) + y_{12}^2] e_2 - 2 y_{12} Y_{11} e_1}{(Y_{11} + y_{11})(Y_{22} + y_{22}) - y_{12}^2}$$

$$e_1'' = \frac{2\,Y_{11}\,(Y_{22} + y_{22})\,e_1 - 2\,y_{12}\,Y_{22}\,e_2}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - y_{12}^2}$$

$$e_2'' = \frac{2\,Y_{22}\,(Y_{11} + y_{11})\,e_2 - 2\,y_{12}\,Y_{11}\,e_1}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - y_{12}^2}$$

In a case of this kind it is highly improbable that both e_1 and e_2 would exist simultaneously, so that the equation could be simplified to that extent.

Fig. 13l. Line leaving a section parallel to another line.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$y_{12} = 0, \quad Z_{11} = z_{11}, \quad Z_{22} = z_{22}$$

$$\left.\begin{aligned} Y_{11}\,(e_1 - e_1') + Y_{12}\,(e_2 - e_2') &= y_{11}\,e_1'' \\ Y_{21}\,(e_1 - e_1') + Y_{22}\,(e_2 - e_2') &= y_{22}\,e_2'' \\ e_1 + e_1' &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{[(Y_{11} - y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2]\,e_1 + 2\,Y_{12}\,y_{22}\,e_2}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

$$e_2' = \frac{[(Y_{11} + y_{11})\,(Y_{22} - y_{22}) - Y_{12}^2]\,e_2 + 2\,Y_{12}\,y_{11}\,e_1}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

$$e_1'' = \frac{2\,[Y_{11}\,(Y_{22} + y_{22}) - Y_{12}^2]\,e_1 + 2\,Y_{12}\,y_{22}\,e_2}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

$$e_2'' = \frac{2\,[Y_{22}\,(Y_{11} + y_{11}) - Y_{12}^2]\,e_2 + 2\,Y_{12}\,y_{11}\,e_1}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

Successive Reflections

There are many important problems, such as in the theory of ground wires,[3] the effect of short lengths of cable,[11] trunk lines tapped at short intervals, etc.,[4] where it is necessary to consider a number of successive reflections of traveling waves. Sometimes it is exceedingly difficult to keep track of the multiplicity of these reflections. A lattice has therefore been devised which shows at a glance the position and direction of motion of every incident, reflected and refracted wave on the system at every instant of time. In addition, this lattice provides the means for calculating the shape of all reflected and transmitted waves and gives a complete history of their past experience. Even the effects of attenuation and wave distortion can be entered on the lattice, if the defining functions are known.

The principle of the reflection lattice is illustrated in Fig. 14. Three junctions, Nos. 1, 2, and 3 placed at uneven intervals along the line are shown. These junctions may consist of any combinations of impedances in series with the line or shunted to ground. The circuits between junctions may be either overhead lines or cables; having in general, different surge impedances, velocities of wave propagation, and attenuation factors. To construct the lattice, lay off the junctions to scale at intervals equal to the times of passage of the wave on each section between junctions. Now choose a suitable vertical time scale, shown in Fig. 14 at the left of the lattice, and draw in the diagonals. At the top of the lattice, at any convenient place centered on the junctions, place indicators with the reflection and refraction operators marked on them. In the notation of this paper these indicators are shown as little double-headed arrows marked as follows:

a = reflection operator for waves approaching from the left.

a' = reflection operator for waves approaching from the right.

b = refraction operator for waves approaching from the left.

b' = refraction operator for waves approaching from the right.

β, α = attenuation factors for section between junctions

It is understood, of course, that these operators are

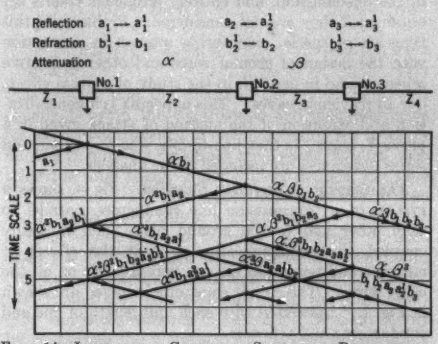

Fig. 14—Lattice for Computing Successive Reflections

operational expressions involving the impedance functions of the junctions, and no restrictions are placed on their generality. Now, starting at the origin of the initial incident wave at the top of the lattice, obtain the reflected and refracted wave at each junction by applying the operators of that junction to the incident wave arriving there, and proceed until the lattice has been completed. It will be observed that:

1. All waves travel downhill.
2. The position of any wave at any time is determined from the vertical time scale at the left of the lattice.
3. The total potential at any point at any instant of time is the superposition of all the waves which have arrived at that point up until that instant of time, displaced in position from each other by intervals equal to the instants of their time of arrival.
4. The previous history of any wave is easily traced,

that is, where it came from and just what other waves went into its composition.

5. Attenuation is included.

6. If it is desired to carry the computations to a point where it is not practical to place the various operators directly on the lattice itself, then the arms may be numbered and the operational expressions tabulated in a suitable table. No difficulty is encountered in this practise, and sometimes it is possible to devise purely tabular methods which can be filled in automatically.

Appendix I

General Equations of a Multi-Conductor System

The conventional treatment of transmission line transients is based on consideration of a single wire and its return, and ignores the presence of other conductors. However, there are many cases in the study of traveling waves where the effect of the other conductors cannot be neglected. Sometimes their influence is so vital as to completely change the characteristics of the phenomenon, and entirely erroneous results are obtained if they are not considered. Problems of this type are of special engineering interest in connection with the design of ground wires and other protective schemes, and in general, in the study of mutual effects due to traveling waves. This appendix is a generalization and extention of the method of attack given in a previous paper,[3] in that line-to-line, line-to-ground, and series resistances are included in the analysis.

The General Differential Equation. Fig. 6 shows a system of n transmission line conductors, parallel to each other and to the ground plane, and mutually coupled electromagnetically, so that the effects of currents and potentials on any wire are felt on all the other wires. The circuit constants involved are shown in Fig. 7. Associated with each unit length of line and conductors r and s there is

L_{rr} = self inductance coefficient of conductor r
L_{rs} = mutual inductance coefficient between r and s
K_{rr} = self capacitance coefficient of conductor r
K_{rs} = mutual capacitance coefficient between r and s
R_r = series resistance of conductor r
g_{rr} = leakage conductance to ground of conductor r
g_{rs} = leakage conductance between r and s

It will also be convenient to introduce the notation

$$\begin{aligned}
G_{rr} &= g_{r1} + g_{r2} + g_{r3} + \ldots\ldots + g_{rn} \\
G_{rs} &= G_{sr} = -g_{rs} = -g_{sr} \\
Z_{rr} &= (R_r + p\,L_{rr}) \\
Z_{rs} &= p\,L_{rs} \\
Y_{rr} &= (G_{rr} + p\,K_{rr}) \\
Y_{rs} &= (G_{rs} + p\,K_{rs}) \\
p &= \partial/\partial t
\end{aligned}$$

The charges per unit length on the conductors in terms of Maxwell's electrostatic coefficients and the potentials $e_1, e_2, \ldots, e_n$ are

$$\left.\begin{aligned}
Q_1 &= K_{11}\,e_1 + K_{12}\,e_2 + \ldots\ldots + K_{1n}\,e_n \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
Q_n &= K_{n1}\,e_1 + K_{n2}\,e_2 + \ldots\ldots + K_{nn}\,e_n
\end{aligned}\right\} \quad (1)$$

The magnetic fluxes linking the conductors per unit length in terms of the inductance coefficients and the currents $i_1, i_2, \ldots, i_n$ are

$$\left.\begin{aligned}
\phi_1 &= L_{11}\,i_1 + L_{12}\,i_2 + \ldots\ldots + L_{1n}\,i_n \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
\phi_n &= L_{n1}\,i_1 + L_{n2}\,i_2 + \ldots\ldots + L_{nn}\,i_n
\end{aligned}\right\} \quad (2)$$

The leakage currents flowing to ground and to the other conductors are

$$\left.\begin{aligned}
i_1' &= g_{11}\,e_1 + g_{12}\,(e_1 - e_2) + \ldots + g_{1n}\,(e_1 - e_n) \\
&\quad = G_{11}\,e_1 + G_{12}\,e_2 + \ldots + G_{1n}\,e_n \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
i_n' &= g_{n1}\,(e_n - e_1) + g_{n2}\,(e_n - e_2) + \ldots + g_{nn}\,e_n \\
&\quad = G_{n1}\,e_1 + G_{n2}\,e_2 + \ldots + G_{nn}\,e_n
\end{aligned}\right\} \quad (3)$$

The differential equations of the first conductor are

$$-\frac{\partial e_1}{\partial x} = \frac{\partial \phi_1}{\partial t} + R_1\,i_1 = Z_{11}\,i_1 + Z_{12}\,i_2 + \ldots\ldots + Z_{1n}\,i_n \quad (4)$$

$$-\frac{\partial i_1}{\partial x} = \frac{\partial Q_1}{\partial t} + i_1' = Y_{11}\,e_1 + Y_{12}\,e_2 + \ldots\ldots + Y_{1n}\,e_n \quad (5)$$

Differentiating equation (4) with respect to x and substituting the equations of type (5) there is

$$\begin{aligned}
\frac{\partial^2 e_1}{\partial x^2} &= Z_{11}\,(Y_{11}\,e_1 + Y_{12}\,e_2 + \ldots + Y_{1n}\,e_n) \\
&\quad + Z_{12}\,(Y_{21}\,e_1 + Y_{22}\,e_2 + \ldots + Y_{2n}\,e_n) \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
&\quad + Z_{1n}\,(Y_{n1}\,e_1 + Y_{n2}\,e_2 + \ldots + Y_{nn}\,e_n) \\
&= (Z_{11}\,Y_{11} + Z_{12}\,Y_{21} + \ldots + Z_{1n}\,Y_{n1})\,e_1 \\
&\quad + (Z_{11}\,Y_{12} + Z_{12}\,Y_{22} + \ldots + Z_{1n}\,Y_{n2})\,e_2 \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
&\quad + (Z_{11}\,Y_{n1} + Z_{12}\,Y_{2n} + \ldots + Z_{1n}\,Y_{nn})\,e_n \\
&= J_{11}\,e_1 + J_{12}\,e_2 + \ldots + J_{1n}\,e_n
\end{aligned} \quad (6)$$

where

$$\begin{aligned}
J_{rs} &= Z_{r1}\,Y_{1s} + Z_{r2}\,Y_{2s} + Z_{r3}\,Y_{3s} + \ldots\ldots + Z_{rn}\,Y_{ns} \\
&= Z_{1r}\,Y_{1s} + Z_{2r}\,Y_{2s} + Z_{3r}\,Y_{3s} + \ldots\ldots + Z_{nr}\,Y_{ns}
\end{aligned} \quad (7)$$

Let

$$A_{rr} = \left(J_{rr} - \frac{\partial^2}{\partial x^2}\right) \quad (8)$$

Then the complete set of differential equations for the n conductors may be written in the symbolic form

$$\left.\begin{aligned}
0 &= A_{11}\,e_1 + J_{12}\,e_2 + \ldots\ldots + J_{1n}\,e_n \\
0 &= J_{21}\,e_1 + A_{22}\,e_2 + \ldots\ldots + J_{2n}\,e_n \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
0 &= J_{n1}\,e_1 + J_{n2}\,e_2 + \ldots\ldots + A_{nn}\,e_n
\end{aligned}\right\} \quad (9)$$

where the J's are operators in the time derivative $p = d/dt$, and the A's are operators in both the time

and space derivatives. Solving these n simultaneous equations for any e, there results a determinate of which the numerator is zero on account of having a column of zeros. In order, therefore, that a solution other than zero can exist, it is necessary that the denominator also be zero (on the assumption that the indeterminate so formed will evaluate to a finite value). Therefore, there must be

$$\begin{vmatrix} A_{11} & J_{12} & \cdots & J_{1n} \\ J_{21} & A_{22} & \cdots & J_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ J_{n1} & J_{n2} & \cdots & A_{nn} \end{vmatrix} e = 0 \qquad (10)$$

Now, dropping subscripts

$$Z\,Y = (R + p\,L)(G + p\,K) = L\,K\,p^2 + (L\,G + R\,K)p + R\,G \qquad (11)$$

so that ultimately, the expansion of (10) will lead to a polynominal of degree n in $(\partial^2/\partial x^2)$ and degree n in $p^2 = \partial^2/\partial t^2$. The solution of this partial differential equation is the most general solution for a system of parallel conductors.

There are three conditions under which equation (10) may be considerably simplified in obtaining a solution. These are I, *The No-Loss Line*, II, *The Completely Transposed Line*, and III, *The Alternating Current Solution*.

Case I. The No-Loss Line. The solution for this case has been given in a recent paper,[3] and will not be repeated here. However, the following discussion of the energy relationships was not included in the previous paper.

Consider a system of n potential and current waves $(e_1 \ldots e_n,\ i_1 \ldots i_n)$. From electrostatics (see "Electricity and Magnetism" by J. H. Jeans, page 94) and equations (1) of Appendix I of this paper, the total energy residing in the electrostatic field is

$$W_e = \frac{1}{2} \int (Q_1 e_1 + Q_2 e_2 + \ldots + Q_n e_n)\, d x$$

$$= \frac{1}{2} \int (K_{11} e_1^2 + K_{12} e_2 e_1 + \ldots + K_{1n} e_n e_1)\, d x$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$+ \frac{1}{2} \int (K_{n1} e_1 e_n + K_{n2} e_2 e_n + \ldots + K_{nn} e_n^2)\, d x \qquad (12)$$

where the integration is to extend over the lengths of the waves.

The electromagnetic energy (Jeans page 443) by equations (2) of Appendix I is

$$W_m = \frac{1}{2} \int (\phi_1 i_1 + \phi_2 i_2 + \ldots + \phi_n i_n)\, d x$$

$$= \frac{1}{2} \int (L_{11} i_1^2 + L_{12} i_2 i_1 + \ldots + L_{1n} i_n i_1)\, d x$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$+ \frac{1}{2} \int (L_{n1} i_1 i_n + L_{n2} i_2 i_n + \ldots + L_{nn} i_n^2)\, d x \qquad (13)$$

The total energy of the waves is (considering only the waves moving in one direction, and calling the current waves i)

$$W = \int (e_1 i_1 + e_2 i_2 + \ldots + e_n i_n)\, dt$$
$$= \int (z_{11} i_1^2 + z_{12} i_2 i_1 + \ldots + z_{1n} i_n i_1)\, dt$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+ \int (z_{n1} i_1 i_n + z_{n2} i_2 i_n + \ldots + z_{nn} i_n^2)\, dt$$
$$= \int (y_{11} e_1^2 + y_{12} e_2 e_1 + \ldots + y_{1n} e_n e_1)\, dt \qquad (14)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$+ \int (y_{n1} e_1 e_n + y_{n2} e_2 e_n + \ldots + y_{nn} e_n^2)\, dt$$

But for free traveling waves on overhead lines,[3]

$$z = c\,L \qquad (15)$$

and

$$d x = v\, d t = c\, dt \qquad (16)$$

Therefore, equation (14) may be written

$$W = \int (L_{11} i_1^2 + L_{12} i_2 i_1 + \ldots + L_{1n} i_n i_1)\, d x$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$+ \int (L_{n1} i_1 i_n + L_{n2} i_2 i_1 + \ldots + L_{nn} i_n^2)\, d x$$
$$= 2\, W_i \qquad (17)$$

thus proving that the energy of the system is divided equally between the electrostatic and magnetic fields for free waves traveling in one direction. While waves moving in opposite directions are passing through each other the total energy is not equally divided, but may be distributed in any proportion between the two fields. This is also true at a transition point, where the incident waves give rise to reflected waves. In such cases the energies must be computed from equations (12) and (13) and added to find the total. Equation (14) applies only to waves moving in the same direction, and while it serves to determine the total energy by computing the energies in each system of oppositely moving waves and adding them, it does not hold for resultant potentials and currents.

Case II. The Completely Transposed Line. If all n conductors are completely transposed with respect to each other and to the ground, and if the conductors have the same resistance, then in effect

$$L_{rr} = L, \qquad K_{rr} = K, \qquad G_{rr} = G$$
$$L_{rs} = L', \qquad K_{rs} = K', \qquad G_{rs} = G'$$
$$Z_{rr}\, Y_{rr} = (R + p\,L)\,(G + p\,K) = Z\,Y$$
$$Z_{rr}\, Y_{rs} = (R + p\,L)\,(G' + p\,K') = Z\,Y'$$
$$Z_{rs}\, Y_{rr} = p\,L'\,(G + p\,K) = Z'\,Y$$
$$Z_{rs}\, Y_{rs} = p\,L'\,(G' + p\,K') = Z'\,Y'$$
$$J_{rs} = Z_{1r}\, Y_{1s} + Z_{2r}\, Y_{2s} + \ldots + Z_{rr}\, Y_{rs}$$
$$+ \ldots + Z_{sr}\, Y_{ss} + \ldots + Z_{nr}\, Y_{ns}$$
$$= Z\,Y' + Z'\,Y + (n-2)\,Z'\,Y' = J \qquad (18)$$
$$J_{rr} = Z_{1r}\, Y_{1r} + Z_{2r}\, Y_{2r} + \ldots + Z_{rr}\, Y_{rr}$$
$$+ \ldots + Z_{nr}\, Y_{nr}$$
$$= Z\,Y + (n-1)\,Z'\,Y' \qquad (19)$$

$$A_{rr} = \left(J_{rr} - \frac{\partial^2}{\partial x^2}\right) = \left[Z\,Y + (n-1)\,Z'\,Y' - \frac{\partial^2}{\partial x^2}\right] = A \tag{20}$$

$$(A - J) = \left[\,(Z - Z')\,(Y - Y') - \frac{\partial^2}{\partial x^2}\,\right] \tag{21}$$

Hereby the determinate of equation (10) becomes, upon dividing each column through by J and calling $A/J = a$, subtracting adjacent columns from each other, dividing out $(a - 1)$, adding all rows to the last, and finally expanding the remaining determinate in terms of the minors of which the lower right hand elements are the cofactors

$$\begin{vmatrix} A\,J & \ldots\ldots\ldots\ldots & J \\ J\,A & \ldots\ldots\ldots\ldots & J \\ \cdot & \ldots\ldots\ldots\ldots & \cdot \\ J\,J & \ldots\ldots\ldots\ldots & A \end{vmatrix} e = (A-J)^{n-1}\,(A-J+n\,J) \tag{22}$$

This is the operational equation of the completely transposed line in terms of the operators A and J. Upon substituting the values of A and J from equations (18) and (20) it takes the form

$$\left(v_1^2\,p^2 + w_1\,p + u_1 - \frac{\partial^2}{\partial x^2}\right)^{n-1}\left(v_2^2\,p^2 + w_2\,p + u_2 - \frac{\partial^2}{\partial x^2}\right) e = 0 \tag{23}$$

If the line is free of losses, as well as completely transposed, then (22) becomes

$$\left(v_1^2\,p^2 - \frac{\partial^2}{\partial x^2}\right)^{n-1}\left(v_2^2\,p^2 - \frac{\partial^2}{\partial x^2}\right) e = 0 \tag{24}$$

This is satisfied by the equation of wave motion

$$e = f\,(x + v\,t) \tag{25}$$

which substituted in (43) gives

$$v = \pm\,v_1 \text{ and } v = \pm\,v_2 \tag{26}$$

thus showing that there are only two possible velocities of wave propagation on the completely transposed no-loss line. In the case of overhead conductors in air both of these velocities approach equality with the velocity of light, in agreement with the findings of the previous section.

Case III. Solution for Alternating Currents. Suppose that the line is operating under steady-state alternating-current conditions, so that the potentials on the n conductors at coordinate x are given by

$$\left.\begin{aligned} e_1 &= E_1 \sin\,(\omega\,t + \theta_1) = \text{imaginary part of } E_1\,\epsilon^{j(\omega t + \theta_1)} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ e_n &= E_n \sin\,(\omega\,t + \theta_n) = \text{imaginary part of } E_n\,\epsilon^{j(\omega t + \theta_n)} \end{aligned}\right\} \tag{27}$$

where the amplitudes $(E_1 \ldots\ldots E_n)$ are functions of x. Substituting (27) in (6) there is

$$\frac{\partial^2 \dot{E}_1}{\partial x^2} = \dot{J}_{11}\,\dot{E}_1 + J_{12}\,\dot{E}_2 + \ldots\ldots + \dot{J}_{1n}\,\dot{E} \tag{28}$$

where

$$\left.\begin{aligned} \dot{E}_1 &= E_1\,\epsilon^{j\theta_1} \\ &\ldots\ldots\ldots\ldots \\ \dot{E}_n &= E_n\,\epsilon^{j\theta_n} \end{aligned}\right\} \tag{29}$$

$$\left.\begin{aligned} \dot{J}_{11} &= J_{11} \text{ when } p = j\,\omega \\ &\ldots\ldots\ldots\ldots \\ \dot{J}_{1n} &= J_{1n} \text{ when } p = j\,\omega \end{aligned}\right\} \tag{30}$$

The common factor $\epsilon^{j\omega t}$ has been canceled out on both sides of equation (28).

Now according to equation (10), the general differential equation is the same for every conductor of the n wire system, and therefore each $\dot{E}$ of (28) must follow the same function of x, but the integration constants are, in general, different for each $\dot{E}$. Since the differential equations are ordinary linear differential equations with constant coefficients of order $2\,n$ and homogeneous in $d^2/d\,x^2$ it follows that the solutions are of the form

$$\left.\begin{aligned} \dot{E}_1 &= \sum_1^n\,(\dot{C}_{1r}\,\epsilon^{\lambda_r x} + \dot{C}_{1r}'\,\epsilon^{-\lambda_r x}) \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{E}_n &= \sum_1^n\,(\dot{C}_{nr}\,\epsilon^{\lambda_r x} + \dot{C}_{nr}'\,\epsilon^{-\lambda_r x}) \end{aligned}\right\} \tag{31}$$

where the $\dot{C}$'s are complex integration constants and the λ's are the roots of equation (10). There are $2\,n^2$ of these integration constants, of which all except $2\,n$ are redundant. To prove this, substitute equations (31) into the original equations of type (28), obtaining n equations of the type

$$\begin{aligned} \sum_1^n \lambda_r^2\,(\dot{C}_{1r}\,\epsilon^{\lambda_r x} + \dot{C}_{1r}'\,\epsilon^{-\lambda_r x}) \\ = \sum_1^n \dot{J}_{11}\,(\dot{C}_{1r}\,\epsilon^{\lambda_r x} + \dot{C}_{1r}'\,\epsilon^{-\lambda_r x}) \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ + \sum_1^n \dot{J}_{1n}\,(\dot{C}_{nr}\,\epsilon^{\lambda_r x} + \dot{C}_{1r}'\,\epsilon^{-\lambda_r x}) \end{aligned} \tag{32}$$

Collecting terms

$$\begin{aligned} \sum_1^n \{[(\lambda_r^2 - \dot{J}_{11})\,\dot{C}_{1r} - \dot{J}_{12}\,\dot{C}_{2r} - \ldots\ldots - \dot{J}_{1n}\,\dot{C}_{nr}]\,\epsilon^{\lambda_r x} \\ + [(\lambda_r^2 - \dot{J}_{11})\,\dot{C}_{1r}' - \dot{J}_{12}\,\dot{C}_{2r}' - \ldots\ldots - \dot{J}\,\dot{C}_{nr}']\,\epsilon^{-\lambda_r x}] = 0 \end{aligned} \tag{33}$$

It is now necessary to digress long enough to prove (what is probably self-evident to many) that each of the coefficients in (33) is individually equal to zero. In the general case let

$$A\,f_1\,(x) + B\,f_2\,(x) + \ldots\ldots + N\,f_n\,(x) = 0 \tag{34}$$

where the functions $f\,(x)$ are all different. Assuming that each of these admits of expansion as a power series in x, by Maclaurin's theorem, there is

$$\begin{aligned} A\,(a_1 + b_1\,x + c_1\,x^2 + \ldots\ldots) + \ldots\ldots \\ + N\,(a_n + b_n\,x + c_n\,x^2 + \ldots\ldots) = 0 \end{aligned} \tag{35}$$

Collecting terms

$$(a_1 A + a_2 B + \ldots + a_n N) + (b_1 A + b_2 B + \ldots + b_n N)\, x + \ldots = 0 \tag{36}$$

But by the method of indeterminate coefficients, each term of this power series in x must individually equal zero, so that there are the simultaneous equations

$$\left.\begin{aligned} a_1 A + a_2 B + \ldots + a_n N &= 0 \\ b_1 A + b_2 B + \ldots + b_n N &= 0 \\ c_1 A + c_2 B + \ldots + c_n N &= 0 \end{aligned}\right\} \tag{37}$$

etc.

Since the coefficients $(a, b, c \ldots)$ are entirely arbitrary, the solution of these equations leads to a determinate of which the denominator is finite and the numerator is zero (by virtue of a column of zeros), and therefore

$$A = B = \ldots = N = 0 \tag{38}$$

Thus in any equation of type (34) the individual coefficients are separately equal to zero.

Returning now to the equations of type (33), and considering all n of these equations, there is

$$\left.\begin{aligned} (\dot{J}_{11} - \lambda_r^2)\, \dot{C}_{1r} + \dot{J}_{12}\, \dot{C}_{2r} + \ldots + \dot{J}_{1n}\, \dot{C}_{nr} &= 0 \\ \dot{J}_{21}\, \dot{C}_{1r} + (\dot{J}_{22} - \lambda_r^2)\, \dot{C}_{2r} + \ldots + \dot{J}_{2n}\, \dot{C}_{nr} &= 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{J}_{n1}\, \dot{C}_{1r} + \dot{J}_{n2}\, \dot{C}_{2r} + \ldots + (\dot{J}_{nn} - \lambda_r^2)\, \dot{C}_{nr} &= 0 \end{aligned}\right\} \tag{39}$$

and exactly the same relationships hold between the $\dot{C}'$ coefficients. Now in order that (39) may be satisfied by values of the $\dot{C}$'s other than zero, the denominator of the determinate must be equal to zero, that is

$$\begin{vmatrix} (\dot{J}_{11} - \lambda_r^2) & \dot{J}_{12} & \ldots & \dot{J}_{1n} \\ \dot{J}_{21} & (\dot{J}_{22} - \lambda_r^2) & \ldots & \dot{J}_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \dot{J}_{n1} & \dot{J}_{n2} & \ldots & (\dot{J}_{nn} - \lambda_r^2) \end{vmatrix} = 0 \tag{40}$$

Therefore, if (40) holds, there are $(n-1)$ independent relationships between the $\dot{C}$'s in equation (39), so that any $(n-1)$ of them may be eliminated. But since there are n values of r, there will remain n integration constants that must be determined from the terminal conditions. Likewise, there will remain n arbitrary integration constants among the $\dot{C}'$ coefficients.

Thus, the n-wire transmission system has associated with it n propagation constants (λ_r) and $2\,n$ integration constants $\dot{C}_r$ and $\dot{C}_r'$.

If the line is completely transposed, then by equation (22) there are only two independent roots to the differential equations, and therefore only four integration constants.

If the line is a completely transposed three-wire line, there is, by equations (22), (18), and (20)

$$\left[(Z - Z')(Y - Y') - \frac{\partial^2}{\partial x^2} \right]^2$$

$$\left[(Z + 2\,Z')(Y + 2\,Y') - \frac{\partial^2}{\partial x^2} \right] \dot{E} = 0 \tag{41}$$

Therefore the propagation constants are

$$\left.\begin{aligned} \lambda_1^2 &= (Z - Z')(Y - Y') \\ \lambda_2^2 &= (Z + 2\,Z')(Y + 2\,Y') \end{aligned}\right\} \tag{42}$$

But (λ_2^2) does not satisfy (40), so that there are no $r = 2$ constants. However, (λ_1^2) does satisfy (40) and therefore the solution for a completely transposed three-wire line is

$$\left.\begin{aligned} \dot{E}_1 &= \dot{C}_{11}\, \epsilon^{\lambda_1 x} + \dot{C}_{11}'\, \epsilon^{-\lambda_1 x} \\ \dot{E}_2 &= \dot{C}_{21}\, \epsilon^{\lambda_1 x} + \dot{C}_{21}'\, \epsilon^{-\lambda_1 x} \\ \dot{E}_3 &= \dot{C}_{31}\, \epsilon^{\lambda_1 x} + \dot{C}_{31}'\, \epsilon^{-\lambda_1 x} \end{aligned}\right\} \tag{43}$$

If for the complex number (λ_1) there be substituted

$$\lambda_1 = \alpha + j\,\beta \tag{44}$$

then equations (44) may be expressed in any of the following familiar forms

$$\begin{aligned} \dot{E} &= A\, \epsilon^{\lambda x} + B\, \epsilon^{-\lambda x} \\ &= A\, \epsilon^{\alpha x} (\cos \beta\, x + j \sin \beta\, x) + B\, \epsilon^{-\alpha x} (\cos \beta\, x - j \sin \beta\, x) \\ &= (A + B) \cosh \lambda\, x + (A - B) \sinh \lambda\, x \\ &= (A + B)(\cosh \alpha\, x \,.\, \cos \beta\, x + j \sinh \alpha\, x \,.\, \sin \beta\, x) \\ &\quad + (A - B)(\sinh \alpha\, x \,.\, \cos \beta\, x + j \cosh \alpha\, x \,.\, \sin \beta\, x) \end{aligned} \tag{45}$$

This is the so-called vector solution. The actual potential as function of x and t is

$$\begin{aligned} e &= \text{imaginary part of } \dot{E}\, \epsilon^{j\omega t} \\ &= \text{imaginary part of } (A\, \epsilon^{\lambda x} + B\, \epsilon^{-\lambda x})\, \epsilon^{j\omega t} \end{aligned} \tag{46}$$

It is worth noticing from (42) that λ_1 is in terms of the so-called "constants to neutral" used in practical transmission line calculations. For if

h = geometric mean height above ground
s = geometric mean spacing between conductors
r = radius of conductors

then

$$Z = R + j\left(\frac{1}{2} + 2 \log \frac{2\,h}{r} \right) 10^{-9}$$

$$Z' = \quad + j\left(2 \log \frac{2\,h}{s} \right) 10^{-9}$$

$$(Z - Z') = R + j\left(\frac{1}{2} + 2 \log \frac{s}{r} \right) 10^{-9} \text{ ohms}$$

$$(Y - Y') = (G - G') + \frac{j}{\left(18 \log \frac{s}{r} \right) 10^{11}} \text{ mhos}$$

Appendix II

Behavior of Waves at a Transition Point

In a previous paper[3] equations were given for the reflected and transmitted waves on an n-wire transmission system similar to that shown in Fig. 6, but lacking the mutual connecting admittances, N_{12}, N_{13}, etc. In this appendix these admittances are included in the analysis, and the more general equations obtained. The application of these general equations is illustrated

in the paper by a number of practical examples which have come up from time to time in the investigation of artificial lightning surges. Even the complicated circuit of Fig. 6 will not serve for every conceivable case, but it hardly seems profitable to generalize any further. If a particular transition point cannot be made a special case of Fig. 6, it will probably be as easy to solve it directly as to reduce it from anything more general. In any event, the procedure followed in setting up the equations and solving them is the same regardless of how complex the transition points may be.

Referring to Fig. 6 let

$Y_{11}, Y_{22}, \ldots, Y_{nn}$ = self surge admittances of lines on the left

Y_{12}, Y_{13}, etc. = mutual surge admittances of lines on the left

$y_{11}, y_{22}, \ldots, y_{nn}$ = self surge admittances of lines on the right

y_{12}, y_{13}, etc. = mutual surge admittances of lines on the right

$U_1, U_2, \ldots, U_n$ = series impedance network on the left

$W_1, W_2, \ldots, W_n$ = series impedance network on the right

$N_1, N_2, \ldots, N_n$ = admittances to ground

N_{12}, N_{23}, etc. = admittances from junction to junction

e, i = potential and current incident waves

e', i' = potential and current reflected waves

e'', i'' = potential and current transmitted waves

When the incident waves arrive at the transition points, they give rise to reflected and transmitted waves which satisfy the general equations of the transmission line, and are in accord with Kirchhoff's laws and the conditions of current and voltage continuity at the junctions.

The total potential at the junction of any incoming line r is the sum of the incident and reflected waves on that line

$$(e_r + e_r') \tag{1}$$

and the total current is[3]

$$(i_r + i_r') = Y_{r1}(e_1 - e_1') + \ldots + Y_{rn}(e_n - e_n') \tag{2}$$

The potential across the admittance N_r is

$$E_r = (e_r + e_r') - U_r(i_r + i_r') \tag{3}$$

and the current through N_r therefore is

$$I_r = N_r E_r \tag{4}$$

The current transmitted to the outgoing line is

$$i_r'' = y_{r1} e_1'' + y_{r2} e_2'' + \ldots + y_{rn} e_n'' \tag{5}$$

and the current transferred to the other junction is

$$I_r' = N_{1r}(E_r - E_1) + N_{2r}(E_r - E_2) + \ldots + N_{nr}(E_r - E_n) \tag{6}$$

The condition of current continuity is

$$i_r + i_r' = i_r'' + I_r + I_r' \tag{7}$$

The potential wave transmitted to the outgoing line is

$$e_r'' = E_r - W_r i_r'' \tag{8}$$

Substituting (2), (3), (4), (5) and (6) in (7) and rearranging, there is

$$\begin{aligned}[Y_{r1} + Y_{r1} U_r (N_r + N_{1r} + \ldots + N_{nr}) - Y_{11} N_{1r} U_1 \\ - \ldots - Y_{n1} N_{nr} U_n](e_1 - e_1') \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ +[Y_{rn} + Y_{rn} U_r (N_r + N_{1r} + \ldots + N_{nr}) - Y_{1n} N_{1r} U_1 \\ - \ldots - Y_{nn} N_{nr} U_n](e_n - e_n') \\ + N_{1r}(e_1 + e_1') + \ldots + N_{nr}(e_n + e_n') \\ - (N_r + N_{1r} + \ldots + N_{nr})(e_r + e_r') \\ = (y_{r1} e_1'' + \ldots + y_{rn} e_n'')\end{aligned} \tag{9}$$

Substituting (2), (3), (4), (5) and (6) in (8) and rearranging, there is

$$\begin{aligned}(e_r + e_r') - U_r[Y_{r1}(e_1 - e_1') + \ldots + Y_{rn}(e_n - e_n')] \\ = e_r'' + W_r(y_{r1} e_1'' + \ldots + y_{rn} e_n'')\end{aligned} \tag{10}$$

For an n-wire system n equations of type (9) and n equations of type (10) can be written, and these $2n$ simultaneous equations suffice for the determination of the $2n$ unknowns ($e_1' \ldots e_n'$, $e_1'' \ldots e_n''$). The other quantities may then be found from equations (1) to (8). These equations are therefore sufficient to completely formulate the behavior of the incident, reflected, and transmitted waves at a general transition point. Some simplifications and examples are given below.

Mutual Connecting Networks Removed. Suppose that N_{12}, N_{23}, etc., are all zero. Then equations (9) and (10) reduce to

$$\begin{aligned}(1 + N_r U_r)[Y_{r1}(e_1 - e_1') + \ldots + Y_{rn}(e_n - e_n')] - N_r(e_r + e_r') \\ = (y_{r1} e_1'' + \ldots + y_{rn} e_n'')\end{aligned} \tag{11}$$

$$\begin{aligned}(e_r + e_r') - U_r[Y_{r1}(e_1 - e_1') + \ldots + Y_{rn}(e_n - e_n')] \\ = e_r'' + W_r(y_{r1} e_1'' + \ldots + y_{rn} e_n'')\end{aligned} \tag{12}$$

These are the general equations derived in a previous paper.[3]

Single Wire Line. In this case only e_1, e_1' and e_1'' exist, and equations (11) and (12) become

$$(1 + N_1 U_1) Y_{11}(e_1 - e_1') - N_1(e_1 + e_1') = y_{11} e_1'' \tag{13}$$

$$- U_1 Y_{11}(e_1 - e_1') + (e + e_1') = (1 + W_1 y_{11}) e_1'' \tag{14}$$

Solving these two simultaneous equations for the reflected and transmitted waves, substituting $Z_{11} = 1/Y_{11}$ and $z_{11} = 1/y_{11}$, and dropping subscripts, there is

$$e' = \frac{(z + W)(1 + N U) + U - Z - Z N (z + W)}{(z + W)(1 + N U) + U + Z + Z N (z + W)} e \tag{15}$$

$$e'' = \frac{2 z}{(z + W)(1 + N U) + U + Z + Z N (z + W)} e \tag{16}$$

The conventional traveling wave theory is based on a single-wire line and is expressed by the above equations. In terms of the total impedance at the transition point,

$$Z_o = U + \frac{1}{\left(N + \frac{1}{W+z}\right)} = \frac{U + (1 + N\,U)\,(W + z)}{1 + N\,(W + z)} \tag{17}$$

the above equations take the more familiar form

$$e' = \frac{Z_o - Z}{Z_o + Z}\,e \tag{18}$$

$$e'' = \frac{1}{1 + N\,(W + z)}\;\frac{2\,z}{Z_o + Z}\,e \tag{19}$$

Equations and curves of a great many combinations have been worked out as special cases of these expressions. Those shown in Figs. 10, 11, and 12 are in the readily accessible literature. It will be noticed that many of the combinations illustrated are special cases of the more complicated circuits, merely by substituting limiting values (zero or infinite) for the constants R, L, C, and Z as required. Moreover, each of these cases is of practical importance in the study of high-voltage surges.

Energy Relationships at the Junctions. The energy of a free traveling wave is given by equations (12) to (17) of Appendix I. During the time that the incident waves are at the junction a redistribution of energy is taking place. The division of energy during this transition period furnishes a valid check on the reflection, refraction, and transfer operators, and is of interest on its own account. At any time t, counting from the instant when the system of incident waves $(e_1 \ldots e_n)$ arrive at the junction, there is

$$\int_t^\infty (e_1\,i_1 + \ldots + e_n\,i_n)\,dt$$

$$= \text{energy remaining in the incident waves} \tag{21}$$

$$-\int_0^t (e_1'\,i_1' + \ldots + e_n'\,i_n')\,dt$$

$$= \text{energy in the reflected waves} \tag{22}$$

$$\int_0^t (e_1''\,u_1'' + \ldots + e_n''\,i_n'')\,dt$$

$$= \text{energy in the transmitted waves} \tag{23}$$

$$\int_0^t (E_1\,I_1 + \ldots + E_n\,I_n)\,dt$$

$$= \text{energy absorbed by the networks } (N_1 \ldots N_n) \tag{24}$$

$$\int_0^t [(e_1 + e_1' - E_1)\,(i_1' + i_1) + \ldots + (e_n + e_n' - E_n)(i_n' + i_n)]\,dt$$

$$= \text{energy absorbed by the networks } (U_1 \ldots U_n) \tag{25}$$

$$\int_0^t [(E_1 - e_1'')\,i_1'' + \ldots + (E_n - e_n'')\,i_n]\,dt$$

$$= \text{energy absorbed by the networks } (W_1 \ldots W_n) \tag{26}$$

$$\int_0^t \sum_1^n \sum_s (E_r - E_s)\,.\,N_{rs}\,(E_r - E_s)\,dt$$

$$= \text{energy absorbed in connecting networks } N_{rs} \tag{27}$$

The $\sum_s$ summation in (27) ranges from $s = 1$ to $s = n$, excepting $s = r$.

Equating the sum of these energies to the energy in the original incident waves, by the conservation of energy,

$$\begin{aligned}
\int_0^\infty \sum_1^n e_r\,i_r\,dt = {} & \int_t^\infty \sum_1^n e_r\,i_r\,dt - \int_0^t \sum_1^n e_r'\,i_r'\,dt \\
& + \int_0^t \sum_1^n e_r''\,i_r''\,dt + \int_0^t \sum_1^n E_r\,I_r\,dt \\
& + \int_0^t \sum_1^n (e_r + e_r' - E_r)\,(i_r + i_r')\,dt \\
& + \int_0^t \sum_1^n (E_r - e_r'')\,i_r''\,dt \\
& + \int_0^t \sum_1^n \sum_s (E_r - E_s)\,.\,N_{rs}\,(E_r - E_s)\,dt
\end{aligned} \tag{28}$$

Combining the first term on the right with the term on the left, according to the rule

$$\int_0^\infty - \int_t^\infty = \int_0^t \tag{29}$$

and discarding the integrals, there is

$$\begin{aligned}
\sum_{r=1}^n [e_r\,i_r + e_r'\,i_r' - e_r''\,i_r'' - E_r\,I_r - (e_r + e_r' - E_r)(i_r + i_r') \\
- (E_r - e_r'')\,i_r'' - \sum_s (E_r - E_s)\,.\,N_{rs}\,(E_r - E_s)] = 0
\end{aligned} \tag{30}$$

The currents and voltages at a transition as determined by the reflection, refraction, and transfer operators, must satisfy equation (30).

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Discussion

For discussion of this paper see page 557.

Attenuation and Successive Reflections of Traveling Waves

BY JAMES C. DOWELL*

Associate, A. I. E. E.

Synopsis.—This paper describes the method followed in separating out the individual waves from the oscillographic record of the voltage at a transition point due to the superposition of several successive reflections. The separation is necessary in determining the attenuation of the waves on the transmission line. In the past it has not been unusual to find the reduction in the crest value of each successive reflection attributed to attenuation, whereas the major cause of such reduction was reflection at the terminal equipment. The paper discusses a class of lattices which offer a simple and convenient means for keeping track of successive reflections, so that the position, direction, and previous history of each separate wave may be ascertained at any instant of time almost at a glance. The reflection and refraction operators are defined and applied to specific examples. A few oscillograms of successive reflections are analyzed and a number of attenuation curves has been plotted.

* * * * *

Introduction

In the summer of 1929 a million-volt impulse generator and a cathode-ray oscillograph laboratory were installed on the double circuit 132-kv. Philo-Canton line of the Ohio Power Co. for the purpose of studying natural and artificial lightning surges. Details regarding this installation have been published before,[1] and will not be repeated here.

Conditions at this location were not favorable to the accurate determination of attenuation for artificial surges owing to the short periods of time that either circuit could be taken out of service for experimental purposes. It was, therefore, necessary to depend upon measurements of successive reflections at the oscillograph station. This method involves a change of wave shape at each reflection, and therefore introduces inaccuracies and complications in the segregation of the attenuation constants. The object of this paper is to describe how this problem was solved, since both the method and the results are believed to be of interest.

Tests were made with three different waves of less than one microsecond front and exponential tails decreasing to half value in four, seven, and twenty microseconds respectively. Waves of this shape were used in order to facilitate calculation. The crest voltages of the applied waves were obtained by sphere gap measurements, and the voltage of successive reflections by comparative deflections on the oscillograms. All impulses applied to the line were of negative polarity, but of course changed sign completely when reflected from a grounded end, and changed sign partially when reflected by the capacitance of the impulse generator.

All the waves on which attenuation data are included in this paper were applied on a single line, either grounded or open circuited at the far ends. After reflection from the end of the line the wave returned and charged the capacitance of the impulse generator. A capacitor acts like a short circuit at the first instant, but finally, if the wave is sufficiently long, the capacitor becomes fully charged, and thereafter acts like an open circuit. In other words, the reflected wave at a capacitor is entirely different in magnitude and shape than the incident wave which caused it. Moreover, the total voltage measured at the capacitor due to a single incident wave is the sum of the incident and reflected waves. If in addition, one or more successive reflections from the end of the line have reached the impulse generator before it has completely discharged, then the total voltage measured at its terminal is the sum of all incident and reflected waves which are present simultaneously at its terminals. This may happen if the initial wave is of the order of twice or more the length of the transmission line. In such cases it is impossible to obtain the attenuation constants until the individual waves have been segregated out from the total potential measurements as given by the oscillograms. This means that the oscillogram record must be separated into its component waves and the amplitudes of these waves compared.

This problem has been worked in the following stages:

1. Construction of a reflection diagram or lattice.
2. Derivation of the reflection operators.
3. Calculation of the traveling wave components.
4. Calculation of the total voltage at the generator.
5. Determination of the attenuation constants by comparison with the oscillograms.

Reflection Lattices

In analyzing the problem of successive reflections of traveling waves use has been made of the reflection lattices as described in the paper by L. V. Bewley, on page 532.

Consider Fig. 1 representing an impulse generator connected to an open end transmission line. As the capacitance of the impulse generator discharges, a wave e_1 travels out on the line, and when it reaches

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1. For references see Bibliography.

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the open end reflects without change of shape or sign. The reflection returns over the line to the impulse generator, and causes another reflection. But this second reflection is quite different in shape from the first reflection, depending upon the constants of the impulse generator circuit. The behavior of traveling waves for a considerable variety of terminal conditions has been worked out,[4] and general methods developed for their calculation. Theoretically, it is always possible to express the terminal equipment as generalized impedances in terms of Heaviside's operators; and it is also theoretically possible to calculate the reflection from any such generalized impedance for incident waves of arbitrary shape.[5] Suppose that the operator, which when applied to the wave arriving at the impulse generator derives the reflected wave, be designated by α. Then the zig-zag geometric scheme (lattice) in Fig. 1 shows how each wave arriving from the end of

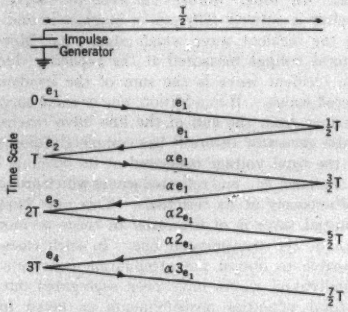

Fig. 1—Successive Reflections from Impulse Generator

the line experiences a reflection at the impulse generator, and that each successive reflection is α times the preceding or incident wave which caused it. Of course α is not a simple constant multiplier, but a function involving the constants of the circuit and the time derivative, $p = d/dt$. According to this scheme all waves travel "down hill" and it is therefore convenient to place a time scale along the side of the lattice so that vertical distances between points represent the differences in time that it takes a traveling wave to reach those points. Thus the lattice presents a clear time-distance picture of the phenomenon of successive reflections and shows at a glance just what is happening at all points along the circuit at all instants of time. For example, at the end of 1¼ units of time (counting one unit of time equal to the time that it takes a wave to travel twice the length of the line) a wave αe_1 reflected from the impulse generator has just reached the midpoint of the line. Prior to its arrival, two waves of equal size e_1 traveling in opposite directions had passed this point. The total resultant voltage as function of time at any point is the sum of all waves which have arrived up to the given instant, properly displaced, of course, by the intervals of their relative position on the line.

Fig. 2 shows a slightly more complicated situation in which the impulse generator is a distance $x_1/2$ from one end of a line and a distance $x_2/2$ from the other end. In this case a wave e arriving at the impulse generator gives rise to a reflected wave αe and a refracted or

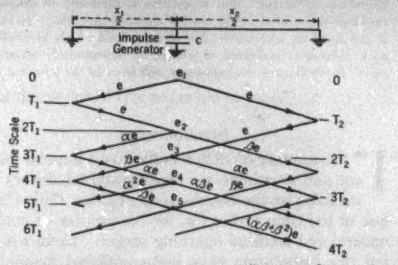

Fig. 2—Successive Reflections from Impulse Generator at Intermediate Point on Line

transmitted wave βe where α is called the *reflection operator* and β the *refraction operator*. An interesting feature of this condition is that at definite intervals (as at e_5 on the lattice) waves arrive simultaneously from the right and left and add up by superposition. Verification of this is shown on the oscillogram in Fig. 3. It will be noticed that the fifth wave has practically doubled up, due to the superposition of two waves

Fig. 3—Oscillogram Corresponding to Fig. 2

arriving simultaneously from the right and the left branches of the line. It is clear from the lattice that each of these time waves has traveled a distance equal to twice the length of the line, that is $x_1 + x_2$.

Reflection and Refraction Operators

A reflection operator is a general differential expression associated with a transition point of a transmission line circuit, such that when applied to the function of time representing an incident wave striking that transition point, yields a differential equation whose

solution gives the reflected wave. Similarly, the refraction operator is defined as that operator which gives the refracted or transmitted wave which passes the transition point and enters another line.[5] In this paper the simple condition of one outgoing line, and no series

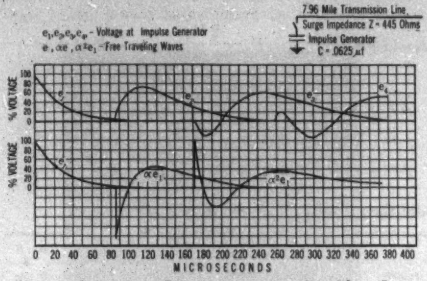

FIG. 4—SUCCESSIVE REFLECTIONS ON A 7.96-MILE LINE

Calculated free waves and total voltages

impedance networks, is under consideration. The following notation is used in subsequent derivations.

Z = surge impedance of the line.
$Z_o(p)$ = total impedance at the transition point.
e = $Z\,i$ = incident voltage wave.
i = incident current wave.
e' = $-Z\,i'$ = reflected voltage wave.
i' = reflected current wave.
e'' = $Z\,i''$ = total voltage = transmitted voltage wave.
i'' = transmitted current wave.

From traveling wave theory and the condition that there cannot be a discontinuity of potential at the transition point, there is

$$e'' = e + e' = (i + i')\,Z_o(p) = (e - e')\,\frac{Z_o(p)}{Z}$$

Solving for e' and e'' respectively.

$$e' = \frac{Z_o(p) - Z}{Z_o(p) + Z}\,e = \textit{reflected wave.}$$

$$e'' = \frac{2\,Z_o(p)}{Z_o(p) + Z}\,e = \textit{transmitted wave} = \textit{total voltage.}$$

When the impulse generator is simply a capacitor C, connected to a single transmission line,

$$Z_o(p) = 1/p\,C$$

$$\alpha = \frac{1/p\,C - Z}{1/p\,C + Z} = \frac{a - p}{a + p} \quad \text{where } a = \frac{1}{C\,Z}$$

and the total voltage is obtained from the refraction operator

$$\beta = \frac{2/p\,C}{1/p\,C + Z} = \frac{2\,a}{a + p}$$

The first wave due to the initial discharge of the impulse generator is

$$e_1 = E\,\frac{p}{p + a} \qquad 1 = E\,\epsilon^{-at}$$

Where 1 is Heaviside's unit function

The subsequent voltages at the impulse generator due to successive reflections then are

$$e_2 = \beta\,e_1 = E\,\frac{2\,a\,p}{(p + a)^2}\,1 = 2\,E\,\text{at}\,\epsilon^{-at}$$

$$e_3 = \alpha\,\beta\,e_1 = E\,\frac{2\,a\,p\,(a - p)}{(p + a)^3}\,1 = 2\,E\,\text{at}\,(a\,t - 1)\,\epsilon^{-at}$$

$$e_4 = \alpha^2\beta\,e_1 = E\,\frac{2\,a\,p\,(a - p)^2}{(p + a)^4}\,1$$

$$= \frac{2}{3}\,E\,a\,t\,(2\,a^2\,t^2 - 6\,a\,t + 3)\,\epsilon^{-at}$$

The traveling waves are

$$\alpha\,e_1 = E\,\frac{p\,(a - p)}{(a + p)^2}\,1 = E\,(2\,a\,t - 1)\,\epsilon^{-at}$$

$$\alpha^2\,e_1 = E\,\frac{p\,(a - p)^2}{(a + p)^3}\,1 = E\,(2\,a^2\,t^2 - 4\,a\,t + 1)\,\epsilon^{-at}$$

$$\alpha^3\,e_1 = E\,\frac{p\,(a - p)^3}{(a + p)^4}\,1 = E\left(\frac{4}{3}\,a^3\,t^3 - 6\,a^2\,t^2 + 6\,a\,t - 1\right)\epsilon^{-at}$$

These voltages have been plotted in Fig. 4 for the case of a 0.0625 μf. capacitor as impulse generator,

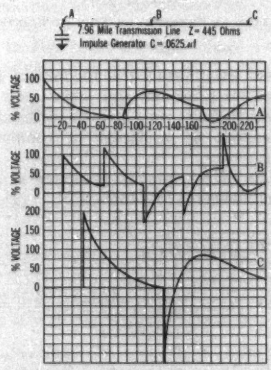

FIG. 5—VOLTAGE AS FUNCTION OF TIME AT POINTS A, B AND C

Line open at C

and a 7.96 mi. open end transmission line of $Z = 445$ ohms surge impedance. Some of these voltages have more than one maximum. The greatest of each of these is (excepting the 100 per cent peaks of $\alpha\,e_1$ and $\alpha^2\,e_1$).

e_2/E	e_3/E	e_4/E	$\alpha\, e_1/E$	$\alpha^2\, e_1/E$
0.735	0.618	0.546	0.446	−0.414

In Figs. 5 and 6, corresponding to an open and grounded line respectively, the voltage at three points along the line has been plotted as function of time; and in Fig. 7 the distribution of potentials along the line at different instants of time are shown. It is evident from these curves that successive reflections exhibit the following characteristics:

a. Reduced maximum crests
b. Increased lengths
c. Oscillatory components

Due to the elongations experienced by the reflections there are three or more pairs of incident and reflected

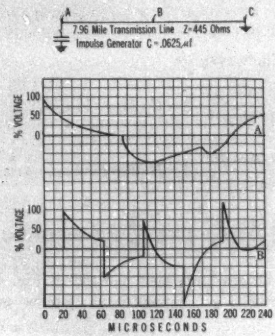

FIG. 6—VOLTAGE AS FUNCTION OF TIME AT POINTS A AND B

Line grounded at C

waves present simultaneously at the impulse generator after the second reflection; and while some of these components contribute very little to the total voltage, yet they cannot be neglected in computing the attenuation.

In the case of the impulse generator at an intermediate point along the line, there is

$$Z_o(p) = \frac{Z}{1 + Z\,C\,p}$$

$$\alpha = \frac{-p}{p+b} \quad \text{where } b = 2/C\,Z$$

$$\beta = \frac{b}{p+b}$$

Then according to the lattice of Fig. 2

$$e_1 = e = E \frac{p}{p+b} \; 1 = E\,\epsilon^{-bt}$$

$$e_2 = \beta\, e = E \frac{b\,p}{(p+b)^2} \; 1 = E\,b\,t\,\epsilon^{-bt}$$

$$e_3 = \beta\, e = E \frac{b\,p}{(p+b)^2} \; 1 = E\,b\,t\,\epsilon^{-bt}$$

$$e_4 = \alpha\,\beta\, e = E \frac{-b\,p^2}{(p+b)^3} \; 1 = \frac{E}{2}(b^2 t^2 - 2\,b\,t)\,\epsilon^{-bt}$$

$$e_5 = (\beta + \alpha\,\beta + \beta^2)\, e = E \frac{2\,b^2\,p}{(p+b)^3} \; 1 = E\,b^2\,t^2\,\epsilon^{-bt}$$

$$\alpha\, e = -E \frac{p^2}{(p+b)^2} \; 1 = -E\,(1 - b\,t)\,\epsilon^{-bt}$$

$$\beta\, e = e_2 = E\,b\,t\,\epsilon^{-bt}$$

and the greatest maximum of each of these is:

e_1/E	e_2/E	e_3/E	e_4/E	e_5/E	e/E	$\alpha\, e/E$	$\beta\, e/E$
1.000	0.368	0.368	0.230	0.549	1.000	0.134	0.368

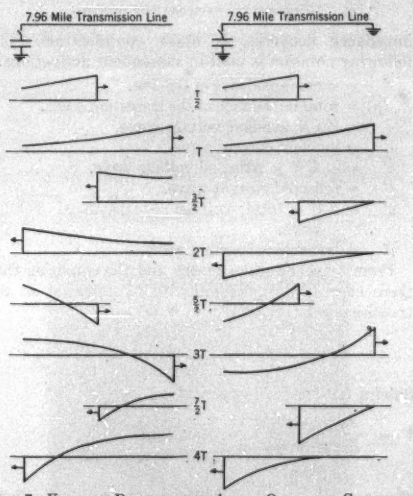

FIG. 7—VOLTAGE DISTRIBUTION ALONG OPEN AND GROUNDED LINES

ATTENUATION

As the waves travel along the line they suffer a decrease in amplitude and a distortion of shape due to the line losses, these being principally corona, and to a lesser extent skin effect. It has been fairly well established through numerous cathode-ray oscillograph studies of artificial lightning waves that they may attenuate considerably without an appreciable change in shape, other than some slowing down of the front. On the basis of this experimental evidence it is permissible to treat such traveling waves as though the

transmission line were a "distortionless line," in which case the attenuation is accounted for by a simple exponential decrement factor. This attenuation constant is most accurately determined by a comparison of the crests of the wave at different intervals along the circuit. These crests should be measured by a sphere gap or

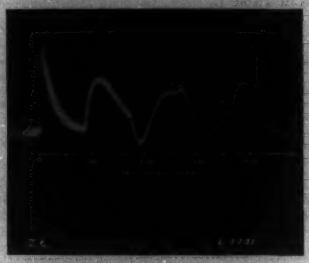

FIG. 8—OSCILLOGRAM OF SUCCESSIVE REFLECTIONS ON A 7.96-MILE TRANSMISSION LINE OPEN AT THE FAR END

cathode-ray oscillograph, although rough figures may be obtained with the surge recorder or klydonograph. However, if time or operating conditions do not permit measurements to be taken for the same wave at various distances along the line, then recourse may be had to oscillograms showing a number of successive reflec-

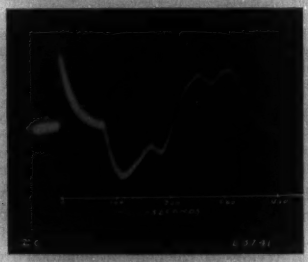

FIG. 9—OSCILLOGRAM OF SUCCESSIVE REFLECTIONS ON A 7.96-MILE TRANSMISSION LINE GROUNDED AT THE FAR END

tions. Figs. 8 and 9 show the oscillograms of several successive reflections on a 7.96-mi. transmission line, both open-circuited and grounded at its far end.

Figs. 10 and 11 show the records transcribed from the oscillograms in Figs. 8 and 9 respectively, and the calculated voltage components at the impulse generator as given in Fig. 4. From a comparison of the oscillograms and the calculations the attenuation constants α_1, α_2, α_3, α_4, etc., are deduced. These factors are different for each wave, for although the distance traveled between reflections is the same, yet each wave has a different shape and crest due to the influence of the impulse generator impedance on the character of the reflection.

It will be noticed that after the third reflection there is considerable deviation between the measured and calculated values. This apparent distortion appeared to be due to limitations of the cathode ray oscillograph circuit, rather than to line loss distortion.

The attenuation curves for a number of typical cases are shown in Figs. 12 and 13. The $(- -)$ curves correspond to the line open at its far end, while the

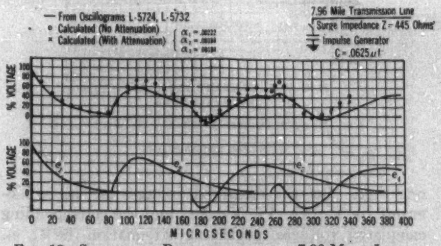

FIG. 10—SUCCESSIVE REFLECTIONS ON A 7.96-MILE LINE BETWEEN A CAPACITANCE AND AN OPEN END

$(+ -)$ or $(- +)$ curves correspond to the line grounded at its far end. Since a voltage wave reflects with reversed sign from a grounded point, a wave traveling over such a line is alternately positive and negative, as indicated by the $(+ -)$ or $(- +)$ sign. On the short 7.96-mi. line, the grounded line condition showed the least attenuation, as was to be expected, because the wave used was longer than the line, and consequently, as shown in Fig. 7, the average voltage over the line was less and the losses smaller. Moreover, the voltage

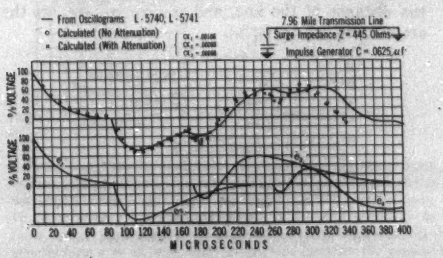

FIG. 11—SUCCESSIVE REFLECTIONS ON A 7.96-MILE LINE BETWEEN A CAPACITANCE AND A GROUNDED END

at the end of the line is at ground potential for the grounded line, but double the wave crest for the open line. As the corona loss varies approximately as the square of the excess corona voltage, this doubling up of voltage at the open end of a short length of line may have considerable effect on the attenuation. Of course, if the line is long compared with the length of the waves,

the importance of an open or grounded end is not so material.

As an example of how the attenuation constant is computed, take one of the cases given in Fig. 13 where the line is so long in comparison with the length of the wave that there is never more than a single incident wave and its reflection at the impulse generator at any

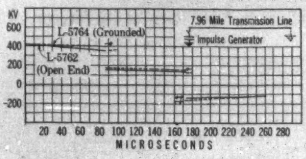

FIG. 12—ATTENUATION ON A 7.96-MILE LINE

Grounded and open ends

one time. Let the measured voltages on the oscillograms be V_1, V_2, V_3, V_4, etc., and the corresponding calculated voltages, neglecting attenuation, be e_1, e_2, e_3, e_4, etc. Then on the assumption of simple exponential attenuation there is

$$\frac{V_2}{V_1} = \frac{e_2}{e_1}\,\epsilon^{-\alpha_1\tau} \text{ or } \alpha_1 = \frac{1}{\tau}\log\left(\frac{e_2}{e_1}\,\frac{V_1}{V_2}\right)$$

$$\frac{V_3}{V_2} = \frac{e_3}{e_2}\,\epsilon^{-\alpha_2\tau} \text{ or } \alpha_2 = \frac{1}{\tau}\log\left(\frac{e_3}{e_2}\,\frac{V_2}{V_3}\right)$$

$$\frac{V_4}{V_3} = \frac{e_4}{e_3}\,\epsilon^{-\alpha_3\tau} \text{ or } \alpha_3 = \frac{1}{\tau}\log\left(\frac{e_4}{e_3}\,\frac{V_3}{V_4}\right)$$

etc. etc.

where τ is the time that it takes the wave to travel twice the distance of the line, and α_1, α_2, α_3, etc. are the attenuation constants.

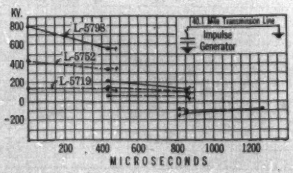

FIG. 13—ATTENUATION ON A 40.1-MILE LINE

Far end grounded

These attenuation constants are in general different, since each successive reflected wave is different in shape and magnitude.

If the line is so short that there is more than one pair of incident and reflected waves at the impulse generator at the same time, these waves may be added graphically as shown in Figs. 4, 5, and 6, and the comparison based on scaled values.

The attenuation curves shown in Figs. 12 and 13 are in reality curves of

$$V_{[k+1]}\left(\frac{e_k}{e_{k+1}}\right) = V_k\,\epsilon^{-\alpha_k\tau}$$

Some of the results obtained on attenuation are tabulated in Table I.

TABLE I

Kv. voltage range Initial	Final	Attenuation constant	Polarity		Total distance traveled in miles	Capacity of impulse generator (μ f.)
788	460	0.00127	−	+	80.2	0.0125
204	135	0.00097	+	−	"	"
125	85	0.00091	+	−	"	"
427	346	0.00050	−	+	"	0.0208
154	106	0.00088	+	−	"	"
98	81	0.00045	+	−	"	"
143	135	0.00014	−	+	"	0.0625
60	56	0.00016	+	−	"	"
52	(?)	(?)	+	−	"	"
427	405	0.00065	−	+	15.92	0.0208
180	165	0.00093	+	−	"	"
154	88	0.00644	+	−	"	"
427	364	0.00184	−	−	"	"
161	141	0.00155	−	−	"	"
131	126	0.00049	+	+	"	"
136	124	0.00106	−	+	"	0.0625
55	60	(?)	+	−	"	"
55	(?)	(?)	+	−	"	"
122	101	0.00222	−	−	"	"
45	39	0.00172	−	−	"	"
36	(?)	(?)	+	+	"	"
775	345	0.00226	−	+	66.4	0.0125
360	188	0.00154	−	+	"	0.0208
427	260	0.00580	−	−	15.92	"
427	187	0.00186	−	+	82.3	"

CONCLUSIONS

The data secured on attenuation by this investigation cannot be regarded as accurate and definite. But the study was of value in demonstrating the limitations of trying to calculate attenuation from the measurements of successive reflections—limitations in both the mathematical work and the oscillograph circuit. Moreover, the calculation demonstrates once more that the behavior of artificial lightning surges on a transmission line may be computed with a reasonable degree of engineering accuracy by treating the line as distortionless.

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2. *Critique of Ground Wire Theory*, by L. V. Bewley, A. I. E. E. TRANS., October 1930.

3. Discussion by L. V. Bewley on the paper *Effect of Short Lengths of Cable on Traveling Waves*, by McEachron, Hemstreet, and Seelye, A. I. E. E. TRANS., October 1930.

4. *Traveling Waves Due to Lightning*, by L. V. Bewley, A. I. E. E. Trans., July 1929, p. 1050.

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6. "Reflection of Transmission Line Surges at a Terminal Impedance," by O. Brune, *G. E. Review*, May 1929.

7. "Calculation of Voltage Stresses Due to Traveling Waves with Special Reference to Choke Coils," by E. W. Boehne, *G. E. Review*, Dec. 1929.

8. *Traveling Waves on Transmission Lines with Artificial Lightning Surges*, by McEachron, Hemstreet, and Rudge, A. I. E. E. Trans., July 1930, p. 885.

Discussion

TRAVELING WAVES ON TRANSMISSION SYSTEMS

(L. V. Bewley)

ATTENUATION AND SUCCESSIVE REFLECTIONS OF TRAVELING WAVES

(James C. Dowell)

C. L. Fortescue: With regard to some of the remarks that Mr. Bewley makes concerning waves due to lightning, I am inclined to agree with him that the characteristic wave of the direct stroke of lightning has an extremely steep front and an exponential tail, but caution is necessary in drawing inferences as to the general shape of the potential impressed by the lightning stroke from cathode-ray oscillograms because the surge that is propagated over the wire is entirely different from the surge impressed by the stroke since it is modified by reflections at tower points by flashover of the insulator string itself, by attenuation due to corona, and by several other modifying influences. For example, when a stroke hits a tower or ground wire there is no surge induced on the line wire as a result of the negative surge which flows over the ground wires. There is the usual induced positive wave caused by the release of the charge from the cloud which has been generally termed "the induced wave." This wave occurs whether the cloud is discharged by hitting the tower or ground wire or whether it is discharged by hitting earth at some neighboring point. On the line wire for a limited distance on either side of the stroke a negative potential will be induced, the value of which is reduced at each tower until finally it becomes negligible.

If the ground wire protection system fails as a result of an excessive difference of potential between ground wires and line wires then, and only then, will a negative surge be transmitted over the line but this surge will be chopped by the flashover of the insulator and will also be modified by the reflections at the towers adjacent to the point of stroke and therefore will be not at all representative of the actual impressed potential due to the stroke itself.

I was very much interested to find that Mr. Bewley uses the method of successive reflections in making computations. For lightning calculations to determine proper clearances and so forth, this method is simple and direct and is sufficiently accurate. The results can be worked out graphically with simple ways in a very short time.

H. L. Rorden: A purely mathematical treatment of a subject which might become extremely involved is often looked upon with skepticism. Experimental verification of such calculations is therefore valuable in that it strengthens our confidence in theoretical results. Moreover, where differences between the mathematical and experimental results are found, experimental data are particularly valuable in locating the cause of such deviation, and frequently suggests methods of obtaining better agreement.

In an extensive group of tests of traveling waves made with the cathode-ray oscillograph, the results obtained with line conditions as are given by Fig. 10D of Mr. Bewley's paper are illustrative of the general correlation found. A transmission line slightly over two miles in length was used, (11½ microseconds) and the waves applied to terminal impedances at the far end of the line were recorded with the cathode-ray oscillograph. In Fig. 1 the results obtained with the line as illustrated in Mr. Bewley's Fig. 10D are given, where the incident wave is assumed to be of the form $e = E\epsilon^{-at}$ for the purpose of calculation. No extraneous constants were introduced into the circuit to slow up the front of waves sent from the lightning generator. The oscillogram of the incident wave e illustrates how losses have caused it to deviate from the assumed form. The record of the total voltage at the terminal impedance e_o is plotted together with its calculated wave, in which it is seen that the maximum voltage obtained is but 57½ per cent of the theoretical value. The reflected wave e' is plotted at the lower left.

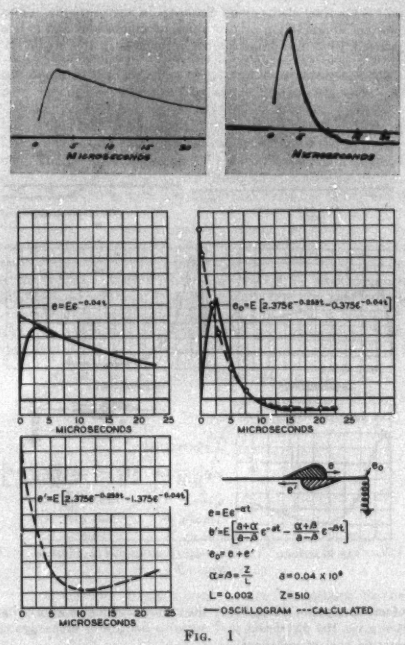

Fig. 1

From these results it appears obvious that the wave front cannot be neglected without introducing a serious error, since the wave used here has almost reached its crest in two microseconds. In order to obtain comparable results it becomes necessary to more exactly approximate the incident wave by employing the general form $e = E(\epsilon^{-at} - \epsilon^{-bt})$. In this instance the parameters a and b are determined from the oscillogram of the incident wave. By the method derived by Mr. Bewley in an earlier publication* the equations given in his present paper are very easily extended to apply to the case having the latter form of incident wave, and such as is illustrated in his Fig. 3C and D. Fig. 2 is a comparison of the same oscillograms shown in Fig. 1 with the waves calculated by more nearly approximating the incident wave. It is seen that the incident wave may be very

Shunt Resistors for Reactors, by Kierstead, Rorden and Bewley, A. I. E. E. Trans., July 1930, p. 1161.

exactly reproduced by the equation employing the difference of two exponentials. By Mr. Bewley's method, this could be extended to three exponentials if still closer approximation were found necessary. The agreement of the mathematical with the experimental results is obvious from a consideration of the plotted wave of the voltage at the terminal impedance e_0 where the circles represent calculated points and the curve is drawn from the oscillogram. At the lower left again is given the reflected wave as was obtained by both the mathematical and experimental methods.

Ernst Weber: I would like to call attention to the first equation in the paper which gives the length of the traveling wave in the form

$$L = x + t$$

where x is the length of the bound charge in thousands of feet and t is the time of the cloud discharge in microseconds. This is a strange combination of two different dimensions and it is not apparent what the dimension of L itself is in this case. It is

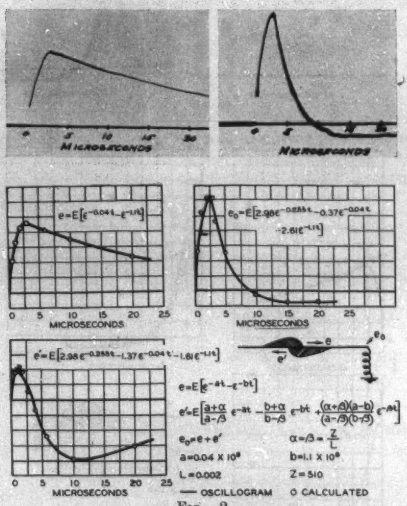

Fig. 2

unfortunately an example of the non-scientific writing which often gives the physicists just cause to belittle or disparage the scientific work of engineers.

I would like to call attention to a reference regarding the specification of traveling waves as indicated on the second page of the paper. The general expression of the given eight shapes of traveling waves by means of exponential functions with varying exponents is not new. Mr. W. Deutsch has given an ingenious variation of Heaviside's operational method, by expressing the applied voltages of different wave shapes in terms of exponential functions with a "determining exponent." In his paper (*Archiv. f. Electrotechnik*, 1918, volume 6) he uses the method of the complex function theory and obtains very elegant results, even in the case of transmission line problems.

F. W. Peek, Jr.: Mr. Bewley has presented a very important and valuable paper. In the early work on traveling waves it was generally necessary for the sake of simplicity to use a rectangular wave shape. Mr. Bewley has shown how to get results without much difficulty for any wave shape. He has presented a very important tool in the lattice. By means of this lattice any number of reflections may be taken care of readily. Mr. Dowell has shown in his paper just how this can be done and, by the examples he has given on measured traveling waves, shows the accuracy of the results.

O. Ackermann: The paper of Mr. Dowell is very interesting and remarkable as it shows how operational calculus methods, brought to increased attention and clarified by several papers of Mr. Bewley, can be applied to actual cases and result in a very desirable simplification in the calculating process.

The check with actual oscillograms (as demonstrated in Figs. 10 and 11) is so conspicuous that one is inclined to welcome the conclusions from these tests as reliable information with regard to attenuation characteristics of transmission lines. However, the results of this paper, as pertaining to attenuation, deviate from anything experienced before, in such alarming manner that it is imperative to look for the reasons of this wide discrepancy. The strange results are best illustrated by Figs. 12 and 13 which indicate that surges only decrease 10 to 20 per cent in a travel of 85 microseconds which corresponds to about 16 miles. All other tests of the General Electric Company and those conducted by the Westinghouse Company, as well as measurements made in Europe, show a very much higher rate of attenuation, indicating, in the average, that, over a 16-mile stretch, surges attenuate to about one-half their original value.

We believe that the error in Mr. Dowell's paper lies in an assumption as this: "The measured voltages at the surge generator check very well with those calculated; therefore, the free waves causing these voltages must have been as assumed in the calculation."

However, there is the fact that incident waves of quite different shape but conveying about equal charges may produce almost the same voltage at the surge generator because of the integrating action of the surge generator capacity.

We would like to ask Mr. Dowell whether a wave half as high but twice as long as the one taken as "reflected" wave would not produce oscillograms quite similar to the ones shown in the paper. We see the possibility of very great percentage variations in the attenuation decrement with very much smaller modifications in the shape of the reflected wave.

The attenuation data given in the paper postulate a resistance of line plus ground return of three to five ohms per mile and a leakage resistance of around 50,000 ohms per mile if a distortionless line is assumed as suggested in the paper.

Our own tests and those conducted by many others, restricted to direct measurements of attenuation, bring out values of the resistance of line plus ground return in the order of 40 ohms per mile, the largest part of which is contributed by the ground return, and from 500 to 2,000 ohms per mile leakage resistance, if the latter is pictured in series with some sort of a valve to present the characteristics of corona losses.

E. W. Boehne: There is quite a general agreement among various observers that the principle cause of attenuation is corona. Mr. Dowell is of the opinion that this attenuation is directly related to corona loss and suggests that since corona loss varies approximately as the square of the excess corona voltage, that the doubling up of voltage at an open end may have considerable effect on the attenuation.

Undoubtedly some part of the rapid attenuation in the presence of corona is due to this corona loss, however, it is the purpose here to point out in the light of the information gathered from field observations one additional way in which corona affects the attenuation of voltage surges.

Consider a simple transmission system on which a large lightning surge has originated, the crest value of the surge being well above the critical corona voltage as shown in Fig. 3.

That portion of the conductor subjected to voltages above the critical corona voltage, will due to formation of corona, experience a change in line constants with respect to the wave level at that point. Neglecting leakage and resistance, the principal changes will be of the capacitance and inductance of the conduc-

tor. For a simple demonstration of the distortion of such a wave we will assume that only the capacitance of the conductor is increased for each differential wave level in the presence of corona and that the inductance remains unaltered.

The fundamental relation which will be used is:

$$1/2\ L\ I^2 = 1/2\ C\ E^2 \tag{1}$$

Defining E/I as the surge impedance, we have

$$\frac{E}{I} = Z = \sqrt{L/C} \tag{2}$$

Similarly

$$v = \frac{1}{\sqrt{L\,C}} = \frac{1}{Z\,C} \tag{3}$$

Now allowing only the capacitance in the above relations to increase for that portion of the wave above the critical corona voltage and allowing this capacitance to be greater for each higher wave level in the corona region, let us predict what will happen to the wave as it travels to the right.

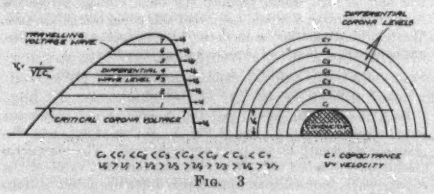

Fig. 3

1. The surge impedance of the conductor to the various wave levels in the corona region will be lowered. This follows from equation (2). Moreover the surge impedance will be minimum to the highest wave level and will hence follow the peak of the wave.

2. That portion of the wave above the critical corona voltage will travel at a slower speed than that portion below the critical corona voltage and the highest voltage level will travel at the slowest speed.

This is all we can predict physically, but let us see what apparent changes the above brings about. (See Fig. 4.)

a. We would expect that portion of the wave below the critical corona voltage to slip ahead of the rest of the wave at a constant rate depending upon the difference in velocity of the waves free from corona and in the presence of corona. Moreover we would expect this phenomenon to take place suddenly at the critical corona voltage. The actual oscillograph story of this physical fact has been observed by field studies made by Mr. McEachron in Michigan.

b. Each differential wave level above the critical corona voltage will recapitulate this slippage or shearing effect in that each differential voltage level will appear to slip ahead of the voltage level above it giving the effect of distorting the front of the wave in the corona range. This distortion will be to slope off the front of the wave above the critical corona voltage in much the same manner as does a lumped capacitor.

c. Because of the peaked nature of the usual lightning waves the top of the wave will be continually "toppling" over to the rear because of its slower velocity. This action will result in a rapid "apparent" attenuation of the wave and in any stage of the process the peak of the wave becomes flatter.

d. Because of this slippage to the rear there will be a building up of the tail of the wave. This fact has been actually observed and could not take place if the whole wave were considered to travel at one speed and attenuate due to losses.

e. The wave after traveling some distance will eventually resolve itself into a much longer wave traveling below the corona voltage.

f. These effects will be modified to some degree by what might well be termed "residual corona" or the corona which exists on the line immediately after the wave has passed.

This simple analysis of allowing only the capacitance in the presence of the various corona levels to change explains one possibility of the following observed facts.

1. The records of natural lightning show that most of the low-voltage surges are usually long surges while the high-voltage surges are comparatively short in length.

2. The fact that positive corona is much more severe than negative corona it is in agreement with the fact that the attenuation of positive surges is greater than that of negative surges, this being in agreement with the above.

3. Similarly the fact that the distortion of positive waves is greater than that of negative waves is readily explained.

4. It has been shown that the attenuation is a function of the wave shape in that long flat waves attenuate slowly and peaked waves attenuate rapidly. The above explanation shows why this is so.

5. The surge impedance in the presence of corona has been observed to be lower and is in agreement with this theory.

6. The slipping ahead of that part of the wave below the critical corona voltage has been measured with an oscillograph.

7. The building up of the tail of the wave has been observed oscillographically.

8. Positive voltage surges of *natural lightning* have been recorded which show a second increase in voltage. This second rise in voltage is more than likely that part of the wave held back by the corona. With a little study we will be able to measure this displacement and predict not only how far the wave has traveled but its proximate magnitude and shape at the point of origin and this in turn should give some clue as to how it originated.

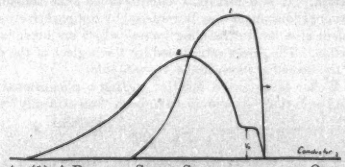

Fig. 4—(1) A Positive Surge Shortly after it Originated on an Overhead Line

(2) The Same Surge after Traveling Several Miles and Distorted in a Manner Described Above

Such a study would be impossible without the study of artificial surges on actual transmission lines as Mr. McEachron has carried out in Michigan during the past few years.

The general conclusion is that a large component of the observed phenomena of traveling surges on overhead lines, including attenuation, distortion, etc., can be attributed to a change in line constants in the presence of corona, the latter merely causing various sections of the wave to travel at speeds corresponding to the constants at that point and time.

In this concept it is true that these changes will *accompany* a certain amount of corona loss but that these changes are not caused by this loss.

L. V. Bewley: Mr. Fortescue states that the surge propagated over the line wire is entirely different from the surge impressed by the stroke, since it is modified by reflections at the tower, flashover of insulators, attenuation due to corona, and several other modifying influences. In a previous paper (Bib-

liography No. 3) I discussed the effect of successive reflections at some length, and gave lattices, equations, and curves to illustrate their influence in the case of both direct and induced surges. Mr. Fortescue's conclusions are in agreement with that study.

Mr. Rorden has just completed an experimental verification of the cases given in Tables I, II, and III of the paper, and in every case has found excellent detailed agreement between the calculations and the cathode-ray oscillograms, of which his Fig. 2 is an example. The results of his study are to be published

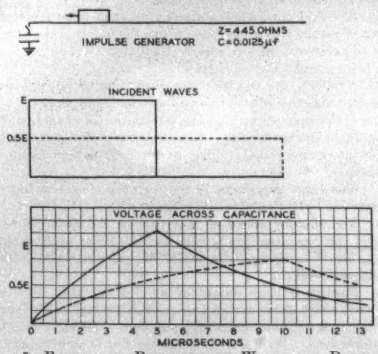

FIG. 5—EFFECT OF RECTANGULAR WAVES OF DIFFERENT SHAPE

in the near future; and will then provide the most complete and conclusive evidence available as to the validity of the methods of calculation. It is a fortunate circumstance that transmission line terminal equipment may be replaced by comparatively simple equivalent circuits (to traveling waves) which are amendable to calculation. The errors introduced by the neglect of the distortion of the traveling waves are on the safe side.

Mr. Weber takes me to task for writing a dimensionally unbalanced equation. It should have been "scientifically" written as

$$L = x + v\,t$$

where v is the velocity of propagation. However, if x is in thousands of feet and t is in microseconds, then

$$v = 1 \text{ (\textit{thousands of feet}) per (\textit{microsecond})}$$

and numerically

$$L = x + t$$

where L is in *thousands of feet* if measured on a *space* axis, or in *microseconds* if measured on a *time* axis.

Mr. Weber calls attention to an article by W. Deutsch (1918) as having been the first to represent different wave shapes by exponential functions. The idea of expressing any function (with certain restrictions relative to a finite number of discontinuities, etc.) over a finite range, as a series of exponentials is quite old. It is the basis of the *Euler-Maclaurin formula* (1730-1740). As a matter of fact, the Heaviside expansion theorem itself yields the solution in terms of a series of exponentials. The specification of an impulse (my Fig. 3D) as the difference of two exponentials appears in Steinmetz's books. Letting the parameters take on different limiting values in order to describe a few wave shapes is such an obvious thing to do that I did not consider it worth while to search the literature in order to anticipate any claim of priority.

I would like to remark that the exponential occupies a position of importance relative to operational calculus which is comparable to that of the sinusoid in Fourier analysis. The reason for this, of course, is because the exponential is the natural solution to the general differential equation with constant coefficients. It is therefore a great help to be able to express the applied forces as exponentials.

James C. Dowell: In explaining the low rate of attenuation, Mr. Ackermann stated that an error might be introduced by assuming the wrong shape of free traveling waves in the calculations. These free waves were calculated in each case, using the constants of the impulse generator and the transmission line and were not assumed in calculating the voltages across the impulse generator.

In regard to the question as to whether a wave twice as long and half as high would produce the same voltage across the impulse generator, it would seem that this could not be the case. The following equations were derived for two rectangular waves, one being half as high and twice as long as the other:

$$e = 2\,E\,(1 - \epsilon^{-t/\tau c}) \text{ for } t \leqq L = \text{length of wave}$$

$$e = 2\,E\,\epsilon^{-t/\tau c}\,(\epsilon^{L/\tau c} - 1) \quad \text{for } t \geqq L$$

Both the incident waves and the voltage waves across the impulse generator are plotted in Fig. 5. Waves of five- and ten-microsecond length respectively are compared. The ten-microsecond wave reached only 70 per cent of the crest value of the shorter wave, indicating an appreciable difference in the voltage across the impulse generator with this modification in wave shape. While the above is only a special case, the length of the waves and the constants used are comparable with those used in the tests and it would seem to be a fairly representative case. The possibility of getting the same voltage across the impulse generator with widely different wave shapes seems to be slight.

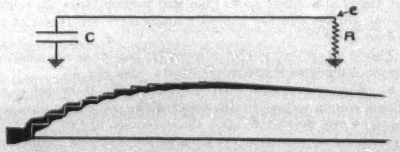

FIG. 6—SUCCESSIVE REFLECTIONS BETWEEN A CAPACITANCE AND A RESISTANCE

Since the above tests were made many oscillograms have been taken of traveling waves reflecting from various combinations of terminal impedance. Extensive calculations have been made for each condition and they check the oscillograms very closely in each case. An oscillogram of the voltage across a resistance placed at the end of a 600-ft. line with a long impulse wave applied is shown in Fig. 6. Note the agreement between the calculations and the oscillogram. It all goes to show that the behavior of traveling waves on transmission lines can be represented fairly simply by mathematical equations.

Impulse Tests on Substations

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Associate, A. I. E. E. Associate, A. I. E. E. Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*This paper presents a description and discussion of a series of impulse tests on two substations of the Public Service Electric and Gas Company's system.*

The 220-kv. substation of the Roseland switching station was surged through eight miles of overhead line. A study of the effects of various amounts of connected bus and station apparatus was made. The effect of different arrester locations on the protection afforded to the station insulation was determined.

The Elizabethport 26-kv. substation was surged through five miles of overhead line and two miles of cable. The effects of various arrester locations, both on the line and in the substation were determined. The attenuation in and surge impedance of the cable were measured.

* * * * *

DURING the last few years a great deal of experimental work has been accomplished in studying the nature of disturbances due to lightning, and in the determination of transmission line characteristics, in order to forecast the behavior of transients of this nature on the transmission system. The experimental work to date on terminal equipment has been largely confined to the study of individual units.[1,2] In the summer of 1930, the opportunity to study surge behavior on typical substation layouts presented itself, and the tests described in this paper are the result of cathode ray oscillograph work in this field.

Protection to terminal apparatus is even more important to the operating engineer than protection to line equipment, since higher installation and replacement costs are involved, and the hazard to service is in most cases greater. An effort was made to check graphically by means of these tests some of the theories generally accepted in regard to the action of a traveling wave striking typical substation structures, in the expectation that the results might serve to clarify the general protection problems. It was realized that actual layouts vary greatly, and that test results must depend largely upon the type of station under investigation and upon the chosen impulse wave. For these reasons, the results secured are largely qualitative in nature.

Two types of substation structures were employed. The first series of tests was made on the 220-kv. bus and transformers at the Roseland Switching Station of the Public Service Electric and Gas Company, a typical substation of the high-voltage class; and the second, on the structures of the Elizabethport 26.4-kv. substation, which is of the distribution type. The results are divided under these two locations.

Tests at 220-Kv. Switching Station

The Roseland Switching Station is the terminal point of two 220-kv. interconnections. The station feeds power to a 132-kv. transmission ring which connects the other switching stations of the system. At the time the tests were made, the 132-kv. equipment was in service. The 220-kv. structures, while complete, had not yet been placed in operation, pending completion of the transmission lines. A schematic diagram of the bus layout is shown in Fig. 1. For the purpose of these experiments, only one line and one bank of transformers were available for connection to the station bus structure and oil circuit breakers.

Fig. 2 illustrates the type of station construction. The transmission line is terminated at an A frame. The conductor distance from this point to the line disconnecting switches is 290 ft., and that from the bus disconnecting switches to the transformer 330 ft. Main and transfer buses are aluminum tubing 270 ft. in length. The transformer bank consists of three

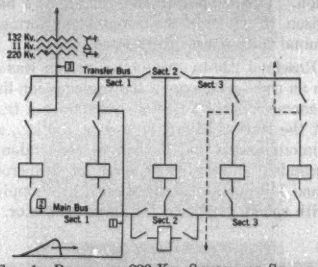

Fig. 1—Roseland 220-Kv. Switching Station

single-phase 30,000-kva. oil immersed self-cooled units. These were connected wye-wye with solidly grounded neutral on both 220-kv. and 132-kv. windings.

The high-voltage surge generator and the cathode ray oscillograph used in the tests have been described in previous papers. The equipment is essentially the same as that used at Stillwater, New Jersey in 1929.[3-8]

The voltage impressed upon connected apparatus in a station depends upon the magnitude of the incoming traveling wave; the surge impedance of the transmission line; and the complicated impedance network presented by the station bus, the apparatus itself, and other lines common to the terminal. This voltage is expressed by the equation:

$$E_i = \frac{2\,Z_i\,(p)\,E_1}{Z_i\,(p) + Z_1} \qquad (1)$$

*Public Service Electric and Gas Company, Newark, N. J.
†Westinghouse Electric and Mfg. Company, East Pittsburgh, Pa.

1, 2. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

where E_t = terminal voltage
E_1 = traveling wave voltage
Z_t = line surge impedance
$Z_t(p)$ = generalized terminal impedance

Maximum voltage is measured at the terminal when the line is disconnected from all apparatus, bus, and other lines; $Z_t(p)$ then being simply that of the measuring system employed to record the transient. If this be a potentiometer of sufficiently high resistance, the error involved is in most cases negligible. Where

Fig. 2—Roseland 220-Kv. Switching Station

necessary, the formula may be used to determine the correction. Using the open-end voltage as a basis for comparison, the effect of the bus and transformer upon the terminal voltage was investigated.

The Roseland 220-kv. switching station was surged through an eight-mile loop of the transmission line, the surge generator being adjusted to produce a traveling wave at the station having a crest of 169 kv., a front of 5.5 microseconds and a time to half value of 19 microseconds. A very short surge of 4-microsecond length and 140-kv. crest was obtained by chopping this wave with sphere gaps near the surge generator.

TABLE I

Station conditions	Per cent reduction of crest kv. 19 micro-second wave	4 micro-second wave
Section 1 transfer bus	4.5	
Transformer connected	11.2	32.0
Section 1 main bus line and transformer breakers	5.6	
Transformer connected	14.0	45.0
Complete main and transfer buses all breakers	24.0	
Transformer connected	35.0	65.0

Volt-time oscillograms were taken at the line end (1) of Fig. 1 with the transformer disconnected and at the transformer bushing (3) for each of the following station conditions:

(a) Line connected to section 1 transfer bus.

(b) Line connected to section 1 main bus, line and transformer circuit breakers.

(c) Line connected to complete main and transfer bus, and all circuit breakers.

Figs. 3 and 4 show oscillograms of the terminal voltage for the station conditions indicated. In general, the transient is lengthened and its crest lowered by the station network. Table I gives the reduction in crest voltage in per cent of the open end voltage. It will be noted that this effect is considerably greater for the short wave than for the long wave. Conversely, the effect would be less for longer waves.

Measurements made at positions (1) and (3) showed no difference in crest voltage for the same station conditions.

Arrester Location

The correct position for lightning arresters in large substations has been a matter of much discussion. If only one arrester be connected to a single bus leading directly to a transformer, the voltage at the arrester

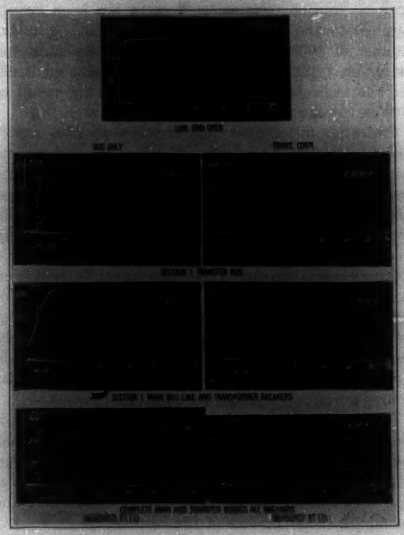

Fig. 3—Oscillograms Showing the Effect of Terminal Equipment on the Station Voltage

location will be a minimum; while that at any other point will depend upon the arrester voltage, the rate of rise of the traveling wave, and the distance from that point to the arrester. Assuming full reflection, the transformer voltage is expressed by the following relation when $\frac{d e_1}{d t}$ can be taken as uniform up to the arrester breakdown voltage, and $e_t \leqq 2 e_a$.

$$e_t = e_a + \frac{2 d e_1}{d t} \frac{D}{1000} \qquad (2)$$

where e_t = voltage at the transformer in kv.
e_a = arrester voltage in kv.
e_1 = traveling wave on the bus in kv.
D = distance from arrester to transformer in ft.

$\frac{d e_1}{d t}$ = kilovolts per microsecond

Fig. 5 shows oscillograms of the transformer voltage for three different locations of the arrester. The arrester used in these tests was a 25-kv. L V porous block unit,[1] located at 0, 595, and 920 ft. from the transformer, positions (3), (2), and (1) Fig. 1.

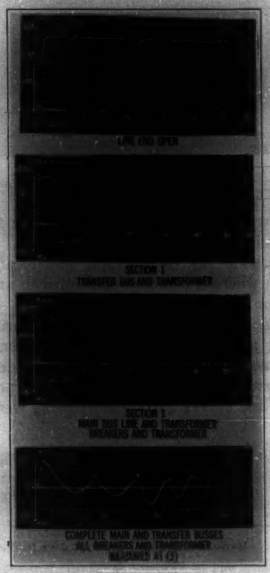

FIG. 4—OSCILLOGRAMS SHOWING EFFECT OF TERMINAL EQUIPMENT ON SHORT WAVE

The curve of transformer voltage plotted against the distance in feet from the transformer to the arrester is shown in Fig. 6. With $\frac{d e_1}{d t}$ and e_a taken from the oscillograms, the calculated values check the oscillograph measurements.

If D is increased to the value

$$D' = \frac{1000\, e_a}{2\, \frac{d e_1}{d t}} \tag{3}$$

$$e_t = e_a + \frac{2\, d e_1}{d t} \cdot \frac{1}{1000} \cdot \frac{1000\, e_a}{2\, \frac{d e_1}{d t}}$$

$$= 2\, e_a$$

that is, the transformer voltage will reach twice arrester voltage. This is the maximum for e_t regardless of the position of the arrester, and the relation fails when $D > D'$.

From Fig. 6, the rise in transformer voltage over arrester voltage for a distance of 350 ft. is 25 kv. This is based upon the front of the test traveling wave which rose at the rate of 43 kv. per microsecond. If the front of the lightning transient be taken as 150 kv. per microsecond, and the 25 kv. rise arbitrarily considered allowable, the arrester must be placed at only

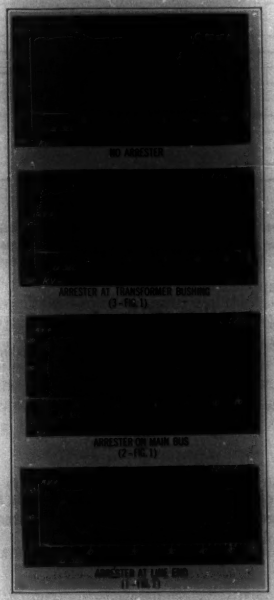

FIG. 5—OSCILLOGRAMS SHOWING EFFECT OF ARRESTER LOCATION ON VOLTAGE AT TRANSFORMER TERMINAL

$$D = \frac{25 \times 1000}{2 \times 150} = 83 \text{ ft.}$$

from the transformer.

As e_a decreases corresponding to the particular application, the per cent increase of transformer voltage over arrester voltage rises for any given arrester posi-

tion. As an illustration we may take the following cases:

220-kv. application

$$D = 600 \text{ ft.}$$

$$\frac{d\,e_1}{d\,t} = 150 \text{ kv. per microsecond}$$

$$e_a = 710 \text{ kv.}$$

$$\text{Kv. rise} = \frac{2 \times 600}{1000}\ 150$$

$$= 180 \text{ kv.}$$

$$= 25.4 \text{ per cent rise over } e_a$$

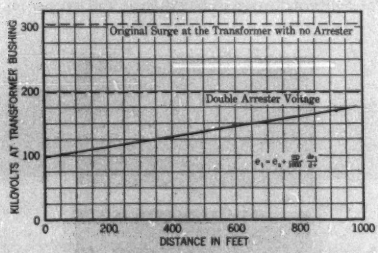

Fig. 6—Curve of Switching Station Transformer Voltage Against Distance of Arrester from Transformer

25-kv. application

$$D = 600 \text{ ft.}$$

$$\frac{d\,e_1}{d\,t} = 150 \text{ kv. per microsecond}$$

$$e_a = 100 \text{ kv.}$$

but

$$D' = \frac{1000 \times 100}{2 \times 150}$$

$$= 333 \text{ ft.}$$

Here the distance is evidently greater than D'; hence the transformer terminal voltage will reach twice arrester voltage.

Tests at 26.4-Kv. Substation

The Elizabethport Substation is designed to receive energy at 26,400 volts, for distribution at 4,150 volts. The type of construction is illustrated in Fig. 7, and a schematic diagram of the 26.4-kv. bus layout in Fig. 8. The completed substation will provide for eight 26-kv. feeders entering in cable. Two of these were available for test purposes. Two banks of transformers were used, the first consisting of two 4,000-kva., three-phase oil cooled units, the second a single 4,000-kva., three-phase unit. The 26-kv. substation bus is 150 ft. in length.

The line used in the impulse tests extends a distance of 31,370 ft. in aerial conductor on wood pole construction from the power station to a point 9,600 ft. from the substation, where it enters underground cable leading to the substation bus. The cable is of the belted type, 350,000 cm. with $\frac{10 \times 5}{32}$ paper insulation. The surge generator was located at the far terminal of the open wire in order to secure sufficient distance to establish a

Fig. 7—Elizabethport 26.4-Kv. Substation

traveling wave effect at the cable pothead; that is, the point of junction of open wire and cable.

The cathode ray oscillograph was located at the substation where it could be connected to any point in the outdoor structures.

The inaccessibility of the cable pothead location prevented a measurement of the wave entering the cable by means of the oscillograph. In order to determine the wave front at this point the cable pothead was grounded and sphere gap measurements were made at

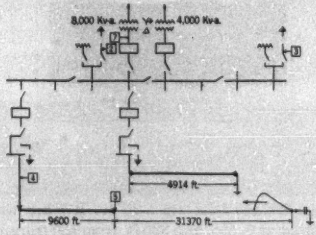

Fig. 8—Elizabethport 26.4-Kv. Substation and Lines

points several hundred feet apart and increasingly distant from the pothead. This procedure was continued until two equivalent readings indicated that the crest value of the wave had been reached. By this method the breakdown voltage of the gap plotted against twice the distance from the gap to the pothead, expressed in microseconds or time of wave travel, gives a point on the approximate curve of wave front.

Fig. 9 shows the front of the incident wave on the open wire plotted from the sphere-gap measurements. This wave has a crest value of 136 kv. and a time to crest of 2.7 microseconds. The tail of the wave could

not be determined except by comparison with the wave after arrival at the station end, where it was recorded by the oscillograph. Fig. 10, oscillogram C-6250-D, shows the wave measured at the transformer terminal.

It was next desired to determine the attenuation of the traveling wave in the 9,600-ft. length of cable. The incoming test feeder was connected through the station bus to the second cable. Thus measured, the crest voltage of the traveling wave in the substation was 16.4 kv. The three conductors of one cable were connected in series and a measurement made of the attenuation in twice its length. This method showed a reduction in crest of 3.4 kv. for 19,200 ft.

To find the surge impedance of the cable, the feeder was connected to ground through a resistance of known value. An oscillogram of the voltage across the resistor was taken. From this measurement and that of the traveling wave the surge impedance of the cable was calculated to be 30.2 ohms.

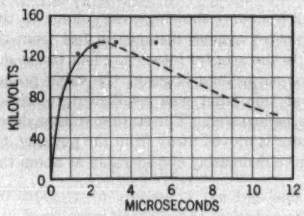

Fig. 9—Traveling Wave on the Line, Plotted from Sphere Gap Measurements (Curve)

Knowing this value, the surge impedance of the open wire under test conditions was calculated to be 447 ohms.

Terminal Conditions

The effect of connecting bus and transformers to the cable is to reduce the crest voltage of the incident wave below the open end value. This reduction is a function of both the station network impedance and the surge impedance of the entering conductor, as noted in the discussion of the Roseland tests. By reference to (1) it may be seen that for a given terminal impedance, the ratio of the terminal voltage, E_t, to the open end voltage, $2\,E_1$, will increase as the conductor impedance, Z_1, decreases in magnitude. A less pronounced modification at Elizabethport would therefore be expected due to the lower impedance of the cable as compared with that of the line at Roseland.

Fig. 10, oscillogram C-6250-D, shows the wave measured at the transformer bushing. By comparison with the measurement made at an open end, the crest voltage is found to have been reduced from 32 kv. to 30.5 kv. The difference of 1.5 kv. represents a modification of about 5 per cent, as compared to the 35 per cent reduction at the Roseland 220-kv. switching station with the entire bus and transformers connected.

Measurements made at the transformer bushing and at the cable terminal showed no noticeable difference in wave shape between these two points.

Under actual operating conditions several 26 kv. cables will be connected to the station bus. The effect of these added impedance paths in parallel will be to

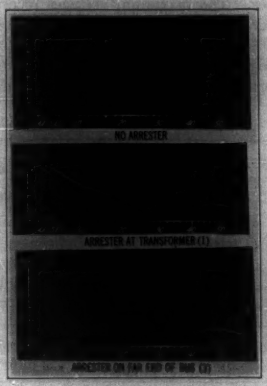

Fig. 10—Oscillograms Showing the Effect of Arrester Location on the Voltage at the Transformer, Elizabethport Substation

lower the equivalent total impedance of the station, thus reducing the impressed voltage on the station equipment. Fig. 11, oscillogram C-6251-B, shows that the second cable connected to the bus reduces the crest value of the wave at the station to 16.4 kv., from its original crest of 30.5 kv. This second cable was grounded at the far end.

Station Protection

In the study of protection of station equipment, lightning arresters were placed at several different

Fig. 11—Oscillogram Showing the Effect of Connecting a Second Cable to the Elizabethport Substation Bus

locations within the station and the voltage at the transformer bushing measured for each location. Due to limitations on voltage imposed by the insulation of the open wire line, the maximum test voltage obtainable at the station was about 30 kv. For this reason the arresters used were standard 6-kv. station type, which

had a breakdown voltage sufficiently under the incident crest voltage to secure comparative results.

Referring to Fig. 8, the arrester locations chosen were, as marked on the diagram; (1) the transformer bushing, (2) the main bus 90 ft. distant from the transformer, (3) the main bus 150 ft. distant from the transformer, and (4) the cable pothead 160 ft. distant from the transformer. The maximum voltages recorded at the transformer with the arresters located at positions (1), (2), and (3), were 21, 23.5, and 25 kv. respectively. The oscillogram obtained at position (4) showed a rise in transformer voltage too great for the distance involved, and cannot be explained except by the possibility of an erratic breakdown of the arrester due to the slight margin of incident voltage over the arrester breakdown value. The results for this position are therefore omitted.

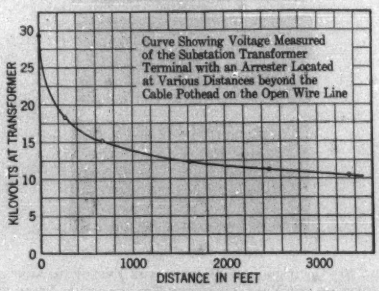

Fig. 12

Fig. 10 shows oscillograms of the voltage with no arrester and of the resulting voltage with the arrester located at position (1) and at position (3), which are the positions nearest to, and farthest from the transformer, respectively.

The difference in crest of four kv. is, then, the maximum to be expected with a wave of front similar to that of the test wave, having a rate of rise of 10 kv. per microsecond. Under extreme conditions the rate of rise to be expected on a lightning transient would be in the order of 150 kv. per microsecond. The greatest conductor distance probable in this type of substation is 200 ft. Referring to (2) an approximation of maximum station voltage variation due to arrester location may be obtained. For the conditions described this value becomes 60 kv. As shown by Fig. 10, oscillogram C-6251-F, such rise in voltage over the arrester breakdown value takes a shape similar to the first ripple on the wave crest.

The foregoing test results were obtained with only one cable tied to the bus. A second cable was available, but when this was connected, the voltage on the station was reduced below the point of arrester breakdown. Under normal operating conditions, the several cables feeding the bus will form a junction of parallel impedances; equivalent to a lumped impedance of considerably lower value than that of an incoming surge path. For this reason, a surge due to a lightning disturbance outside the station would, upon arriving at the station bus, be considerably reduced in magnitude because of the lower impedance at this point.

Station Protection by Line Arresters

Another problem under investigation at Elizabethport was the determination of an effective location of an arrester at or beyond the junction of cable and open wire. The locations chosen for test were at (5), Fig. 8, the cable pothead, and at distances of 267, 668, 1,610, 2,470, and 3,340 ft. from this point.

For these tests the arresters used were the 6-kv. line type. Fortunately ground conditions along the open wire line were good; arrester ground resistances varying in a narrow range of 0.5 to 1.5 ohms.

Fig. 12 is a curve showing crest voltage at the transformer plotted against distance in feet from the cable pothead to the arrester location. Due to the decreased conductor surge impedance at the cable pothead, the voltage at this point was insufficient to cause breakdown of the arrester. The station voltage decreases as the arrester is moved away from the pothead, the slope of the curve decreasing considerably at about the 1,000-ft. point.

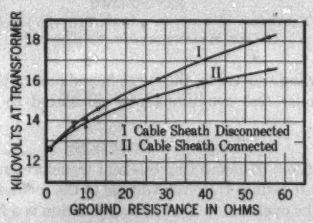

Fig. 13—Comparative Station Voltages with Varying Ground Resistances, with and without a Connection from the Cable Sheath to the Arrester

Arrester placed 1,610 ft. from pothead

Maximum voltage at the station occurred when the arrester was located at position (5) Fig. 8, the cable pothead, since the voltage at this point was less than the arrester breakdown voltage. In this case the maximum voltage at the pothead may be expressed by (1), where Z_1 (p) is replaced by the surge impedance of the cable. Considering the station bus as an open end, and neglecting attenuation in the cable, the maximum station voltage is twice the voltage at the line pothead. Had the crest voltage of the incident wave been high enough to cause the arrester to break down, the maximum station voltage would approach twice the arrester terminal voltage.

Minimum station voltage occurs when the arrester is moved a distance from the pothead, expressed in time of wave travel, equivalent to one-half the duration of the incident wave from zero to the point of arrester cutoff on the tail. The station voltage then approaches double that appearing in the cable as a result of a traveling wave on the line equal to the arrester voltage. Substituting the test values, a minimum station voltage of 5.3 kv. would be expected with the arrester placed about 11,000 ft. from the pothead.

Effect of Ground Resistance on Line Arrester Operation

The grounds used for the arresters in these tests were of exceptionally low resistance. Since such conditions are not always obtainable in practise, measurements were made to determine the effect of increasing ground resistance on arrester performance, and to compare these results with those obtained when the arrester was also connected to the cable sheath. Curve I of Fig. 13 shows the crest voltages measured at the substation transformer with the arrester on the line 1,610 ft. from the cable pothead, for resistances in the ground lead up to 55 ohms. Curve II, shows the voltage under the same conditions with an additional ground connection consisting of a wire laid on the ground from the arrester to the cable sheath.

Summary

1. The capacity to ground of the bus and circuit breakers in large substations tends to reduce the terminal voltage. In general, the effect of connecting the transformer to the bus system is further to slope off the front and reduce the crest. The total effect is more pronounced in the case of very short waves, but appears appreciable for those of moderate length.

2. When lightning arresters are placed on the bus, the voltage at the transformer will be greater than that at the arrester by an amount depending upon the steepness of the lightning transient, the arrester voltage, and the circuit distance from the arrester to the transformer. Oscillograms show results in agreement with calculations. The seriousness of such overvoltages decreases with increasing arrester voltage rating for any given arrester location.

3. The data indicate that the attenuation in cable of surges of the voltages used in these tests is of relatively small magnitude.

4. In small substations fed by cables the capacity of the bus and connected apparatus causes a relatively small decrease in the terminal voltage. This effect is slight because of the lesser terminal capacity and the low surge impedance of the entering feeders.

5. In small substations, no appreciable difference in surge wave shape is found at various points within the station.

6. A large measure of protection is afforded substations by a number of connected high-voltage cables.

7. In the case of a lightning arrester placed on a line joining a long cable leading to a substation, the station protection is increased as the arrester is moved away from the junction along the open wire. This is true only for a surge originating beyond the arrester. The greatest increase in protection occurred in the first thousand feet with the wave shape used. The exact location must be selected after weighing the possibility of a stroke between the pothead and arrester.

8. High ground resistance is found to decrease materially the protective value of line arresters. In the case of very high resistance grounds, arrester performance can be improved by running a connection from the high resistance ground to the cable sheath. Economic considerations will decide whether such a measure should be adopted.

Acknowledgment

The authors wish to express their appreciation to Mr. J. J. Torok for his suggestions in planning these tests, to Mr. H. K. Sels for his help with the paper, and to Messrs. J. M. Reel and W. A. Merriam for their assistance during the tests and in the analysis of the data.

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Discussion

For discussion of this paper see page 583.

Lightning Investigation on a Wood Pole Transmission Line

BY R. R. PITTMAN* and J. J. TOROK†

Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The purpose of this paper is to report the results of an investigation made on a wood pole line. The entire line was originally very highly insulated. The splintering of the structures by lightning indicated that excessively high voltages were appearing quite frequently. Because of the high insulation a cathode ray oscillograph station was installed on the line. A total of 38 significant surges was recorded, two of these being a five million and a four and one-half million-volt surge. Analysis shows that these high surges were the results of lightning bolts striking the lines causing faults between conductors but not to ground. The data also indicate that harmful effects are caused only by direct strokes. These and current measuring records indicate that currents varying from very low values to 100,000 amperes may readily be obtained.

Two methods of protection are dealt with, one utilizes a fuse, the other an expulsion lightning discharge device. The new device operated eight times, all of these operations being successful.

Introduction

THAT wood possesses a certain insulating quality to impulse voltage has been recognized by a number of investigators,[1,2] and a considerable amount of laboratory work has been done to determine quantitatively this value.

The use of this insulating property of the wood to reduce transmission line outages resulting from lightning flashovers has been attempted. A number of transmission lines so designed has failed under actual operating conditions to give the results desired. That the amount of insulation inherently present in a wood structure is insufficient to prevent lightning flashovers has been repeatedly demonstrated.

As a result, investigation of the character of voltages causing the flashover of the entire length of wood supporting structures was considered of great importance and interest, entailing the measurement of voltages considerably above the flashover value of the usual type of transmission line. Previous investigations,[3,4] of a similar nature have been made on steel tower lines, where the magnitude of the voltages to be measured has been limited to the flashover value of the insulator strings.

Description of Line

A 110-kv. line, 28 miles in length, extending from Camden to Magnolia in southern Arkansas, was selected for the investigation. No guyed structures were installed except at angles in the line. The customary storm guying of tangent structures at frequent intervals along the line was entirely omitted.

The oscillograph station was located at about the midpoint of the line as shown in Fig. 1 to avoid the recording of reflected voltage waves. For a distance of four and one-half miles on either side of the station no guying of any kind was installed, the angles in this section of the line being supported by bracing the structure with wood poles. Other angle structures beyond this section were insulated from ground by means of an 18-ft. length of wood in each guy wire.

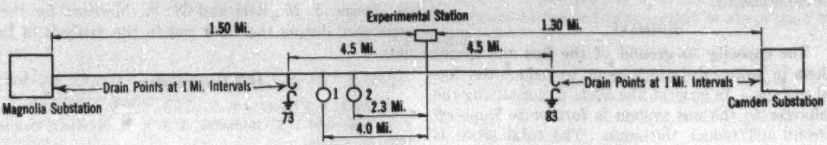

Fig. 1—Diagrammatic Arrangement of Line and Oscillograph Station

1. Structure No. 167 2. Structure No. 153

So-called drain points, equipped with fused horn gaps from each phase to ground, (Figs. 2 and 3) were installed at one mile intervals along the line, except in the nine-mile section in which the oscillograph station was located. All guyed structures were made drain points and the fused horn gaps on the poles (Fig. 2) performed the additional function of protecting the wood guy insulators from damage. The performance of these drain points assisted somewhat in coordinating the flashovers and circuit breaker operations with the oscillograph records. The line conforms to the following brief specifications:

Conductor	4/0 A. C. S. R.
Poles	50 ft. creosoted southern yellow pine
Arms	6 in. by 8 in. by 29 ft. creosoted southern yellow pine
Height of conductor at structure	37 ft. (approx.)
Average height of conductor	32 ft. (approx.)
Conductor spacing	14 ft.
Configuration	Flat horizontal
Insulators	6—10-in. suspension disks
Average span	600 ft.

*Arkansas Power & Light Co., Pine Bluff, Ark.

†Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

1, 2. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

Line History

As originally constructed, the 18-ft. length of wood in the guys necessary on angle structures was shunted by a 13 ft. 6 in. gap in each pole ground wire. This length of wood pole was damaged very severely by the lightning currents in a number of cases with little apparent decrease in line outages as compared with similar lines of conventional wood pole design. The horn gap arrangement was installed on these angle structures primarily to prevent damage to wood poles and the fuses added in an attempt to interrupt dynamic follow up current.

Such an arrangement prevented further damage to the guyed angle structures, but flashovers of the intermediate tangent structures were still rather frequent. These flashovers caused considerable damage to the structures in some cases by removing sections of the

Fig. 2—Angle Structure Arranged with Fused Horn Gap

treated sap wood from the full length of the pole, and at the same time caused interruptions to service due to circuit breaker operation.

In an effort to improve the operating performance of the line, as well as make possible better coordination of line flashovers with the oscillograph and circuit breaker operations, the number of these fused structures was increased by equipping certain tangent structures. As operating records indicated that the amount of damage to tangent structures had been considerably greater than on similar lines with uninsulated guys at one mile intervals along the line, it was hoped that the additional fused structures would prevent or reduce the damage to adjacent poles.

The substation protection at each end of the line consisted of a 20-in. gap between rings on the suspension insulators, with the top ring grounded to the station ground network through a 73-kv. five-ampere fuse. Such an assembly differs from the fused structure used on the line in that the line structures are provided with a five-foot air gap in series with the fuse and ground wire.

Oscillograph Station

The oscillograph station is in general very similar to those described in other papers,[3,5] and is shown in Fig. 4.

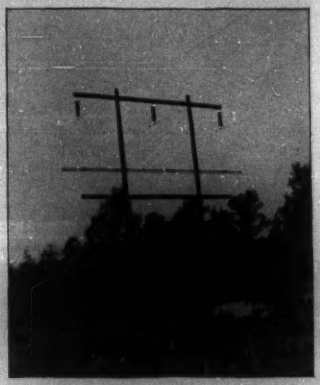

Fig. 3—Multiple-Fuse Horn Gap Arrangement

The oscillograph equipment is of the latest type comprising a modified Norinder oscillograph, pumping equipment, timing system, oscillator, high-voltage supply and capacity potentiometer. The potentiometer is of special interest as it is designed to withstand potentials in excess of seven million volts. The high-voltage

Fig. 4—View of Oscillograph Station

electrode is built up of fine copper screening stretched over a 20-ft. square pipe frame. It is believed that with an electrode of such size the corona formation on the edges will produce a relatively small increase in area, thus minimizing corona distortion. This upper plate is supported by four 60-ft. poles and four strings of insula-

tors each comprising seven 10-in. suspension units. More recently this potentiometer has been revamped into a resistance potentiometer so that the advantages of a delaying cable can be realized.[6] The resistors and cable are coupled to the high potential electrode by a special porous block arrester which by its valve action

Fig. 5A—Oscillogram of Lightning Surge

permits only transients exceeding 125-kv. in crest to appear across the resistors.

Inasmuch as the potentiometer is connected to only one conductor it was deemed necessary to bond the cross arm hardware on the line structure adjacent to the station, making it possible for the high potentials appearing on either of the other two conductors to flash over the insulators and so appear on the potentiometer. To prevent an outage due to this bonding and for the purpose of records the connections were made through fuses.

Results Obtained at the Oscillograph Station

During the season a total of 38 oscillograms of voltages imposed on the line by natural lightning was obtained, of these only six were of special interest, the others being relatively low in magnitude. The highest voltage recorded was 5,000 kv. as shown on Fig. 5A and Fig. 5B. It is of interest to note that this voltage was transmitted along the line for a distance of four miles, this being the length of line between the oscillograph station and the point on the line subjected to the lightning stroke. This was a negative surge reaching its crest in something less than two microseconds, the maximum rate of rise shown by the record is in the order of 4,000 kv. per microsecond. Since this potential is well above the corona voltage of the line, it is to be expected from the results reported by Conwell and Fortescue that the rate of rise at the point of origin was much greater, perhaps as much as 10,000 kv. per microsecond. The surge originated near structure No. 167 which is four miles from the oscillograph station and one-half mile from structure No. 173, the nearest drain point. This location is designated in Fig. 1 as (1). The entire length of the cross arm was marked and both poles slightly splintered, indicating that all three conductors flashed over to ground, causing a circuit breaker operation at the same time that the oscillograph was tripped. The three fuses at structure No. 173 were blown. A flashover also occurred from the middle conductor to a telephone line 22 ft. below. The telephone line crossed the transmission line at right angles at a point 150 ft. from structure No. 167. The fuses on the structure adjacent to the oscillograph station were not blown and this, together with the splintered arm on structure No. 167, indicates that the potential of all three conductors was approximately the same.

The current determinations in the above surge must be limited to the crest of the wave measured by the oscillograph as flashover to ground occurred at several different points, and it is to be understood that the results will indicate only a fraction of the total current. Experimental results and calculations[9,10] indicate the approximate surge impedance of three conductors in parallel as 200 ohms. Therefore, the current of the surge as it passed the experimental station was $\frac{5,000,000}{200}$ or 25,000 amperes. Since this current is only one branch of the traveling wave set up on the line the total current, neglecting attenuation, would be 50,000 amperes. If attenuation is considered in a manner suggested by O. Ackermann the total current flowing on the conductors in both directions prior to flashover would range from 100,000 to 160,000 amperes.

Another interesting oscillogram is shown in Fig. 6A and a replot in Fig. 6B, also a negative surge reaching 4,500 kv. in about one microsecond. The voltage was transmitted from a point designated in Fig. 1 as (2) which is about 2.3 miles from the station. An entire structure was flashed over, and other conditions in general were the same as those under which the record shown in Fig. 5A was obtained.

Fig. 5B—Replot of Fig. 5A

That there are certain voltages requiring considerable time for flashover is shown by the oscillogram in Fig. 7. Attention is also called to the positive reflection of about 1,000 kv. The oscillogram in Fig. 8 was about a 20-microsecond front, and a crest value of 1,000 kv.

There was no interruption to service on the line when the oscillogram in Fig. 9 was obtained, although the

voltage at the oscillograph station was about 2,000 kv. The voltage responsible for this oscillogram probably flashed over the line at some point without establishing a 60-cycle arc. The surge shown in Fig. 10, though of exceedingly long duration, was not accompanied with

Fig. 6A—Oscillogram of Lightning Surge

circuit breaker operation. The oscillogram shown in Fig. 11 cannot be definitely explained or tied in with blown fuses or damaged structures as flashover did not occur. No damaged structures could be found which could be correlated with these latter five records.

All of the higher voltage surges were negative in

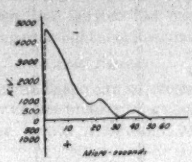

Fig. 6B—Replot of Fig. 6A

polarity. Such positive surges as were obtained were small in magnitude, and probably without effect on the operation of the line.

Direct Strokes

A number of lightning strokes to ground near the transmission line was observed by the laboratory

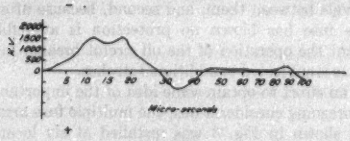

Fig. 7—Replot of Lightning Surge Oscillogram

operators during the season. In no case did such strokes affect the operation of the line, or induce sufficient voltage on the conductors to cause the operation of the oscillograph. A direct stroke 700 ft. from the oscillograph potentiometer flashed over several poles on a 33-kv. line. The station was in operation at the time, but the induced voltage was insufficient to trip the oscillograph circuit. At this time the oscillograph was adjusted to initiate operation at values slightly above normal crest line voltage.

Failure to obtain records on the oscillograph from these strokes near the line leads to the conclusion that

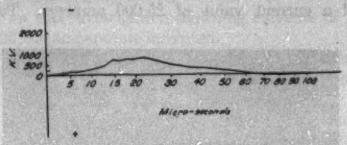

Fig. 8—Replot of Lightning Surge Oscillogram

any induced voltages from nearby lightning strokes are small in magnitude and of a harmless character to this type of transmission line.

Lightning Current Measurements

During the latter part of the lightning season kly-

Fig. 9—Oscillogram of Lightning Surge

donographs were placed on the wood poles to determine the current during flashover. About 10 ft. of the wood poles are shunted by a copper conductor in series with a zircon or carbon resistor having a resistance of about

Fig. 10—Oscillogram of Lightning Surge

0.15 ohms. The klydonograph is connected across the terminals of the resistor. This arrangement is shown in Fig. 12. The klydonograph which takes the structural form of a telephone receiver is mounted on a wooden box containing the resistor. By this arrange-

ment the current can be determined from the size of the klydonograph figure and the resistance of the zircon rod. Only one record was obtained and that at a drain point, when only one fuse was blown, indicating that the surge was very light. The klydonograph figure indicated a current value of 25,000 amperes. To this

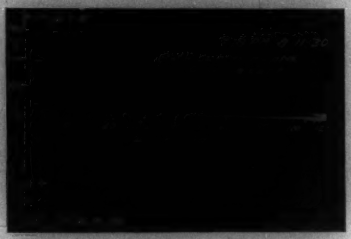

Fig. 11—Oscillogram of Lightning Surge

should be added the current flowing in each direction over the line conductor to obtain the total.

Operating Results

It was anticipated that the installation of the fused horn gaps in the ground circuit might act under certain conditions to discharge the current due to the lightning voltage and then quickly cut off the 60-cycle arc current,

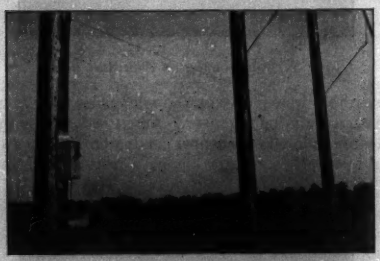

Fig. 12—Klydonograph Arrangement for Surge Current Measurements

thus avoiding the operation of the oil circuit breaker with the attendant service interruption.

The following tabulation shows the results obtained with 20 of these so-called drain points in the 28 miles of line.

Date	Fuses blown	Flashover on line not causing switch operation	Flashover on line causing switch operation
Aug. 4	2	2	0
Aug. 30	6	1	1
Sept. 3	2	1	0
Sept. 15	2	1	0
Sept. 26	1	1	0
Total	13	6	1

The flashover on the line incurring switch operation was caused by a stroke to the line near structure No. 167. The voltage on the line due to this stroke was measured at the oscillograph station as a 5,000-kv. surge. Circuit breaker operation was caused by a three-phase short circuit at structure No. 167. The same lightning stroke blew all three fuses at drain point No. 173. These results indicate that the protective effect of a drain point does not extend to one-half mile even with the high insulation used on this line.

The following tabulation shows where blown fuses were located with reference to structure numbers and conductors. It should be remembered that the average span on this line is 600 ft. The numbering begins at the Camden substation, and extends south. Structures 1 and 273 have no air gap in the ground circuit.

Struction No.	West Conductor	Middle Conductor	East Conductor
1	0	0	0
7	0	0	0
13	0	0	0
27	0	0	0
35	0	0	0
46	0	0	0
55	0	0	0
64	0	0	0
76	0	1	0
87	0	0	0
173	1	1	1
186	0	0	0
197	1	1	0
210	0	0	0
219	1	0	0
231	0	0	0
242	0	0	0
253	0	0	1
270	0	0	0
273	2	1	2
Total	5	4	4

The fuses used for this service were of the expulsion type, and rated by the manufacturer at five amperes, 73,000 volts. The maximum short-circuit kva. along the line investigated is about 175,000.

A single fuse in the ground circuit of a structure seemed inadequate, first, because many flashes of lightning are made up of a number of successive discharges which follow each other with very short time intervals between them, and second, because after the single fuse has blown no protection is available to prevent the operation of the oil circuit breaker in case the line is subjected to additional strokes.

In an effort to obtain some idea of the importance of the foregoing considerations, the multiple fuse arrangement shown in Fig. 3 was installed at six locations. This arrangement was intended to provide three fuses so disposed under each line wire that they would successively blow whenever the character of the lightning voltage was such as to cause the gap to flash over. Only four fuses were blown on this type of structure during the season, and these fuses were in each case the middle fuse of the group.

The results obtained from the fuses installed at drain points were quite encouraging, however the continuous

patrolling and replacements became a large factor in the economic considerations. Accordingly, an expulsion type of lightning arrester was later devised independently by Pittman consisting of a fiber tube 38 in. long, into each end of which was inserted a wire electrode reducing the effective arcing length to 30 in., thus insuring that the breakdown would take place within the tube. The device was inserted without additional gaps in the hardware grounding lead. Thus the path of the surge would be over the line insulator to the cross arm hardware and from thence by copper wire to the expulsion arrester which would break down inside and conduct the surge to ground. The subsequent 60-cycle arc initiated by the surge is then quenched out by the expulsion effect of the fiber tube.

The operating record of this experimental expulsion form of arrester is quite encouraging. During the period of time they have been in service on another similar 110-kv. line, eight successful operations were recorded.

During the season a number of flashovers occurred but the total physical damage to the line has consisted in one broken insulator unit, three slightly splintered cross arms, and two slightly splintered poles. No conductor has fallen, and service has been immediately restored following each lightning flashover.

Conclusions

1. The frequent flashovers of crossarms and poles accompanied by circuit breaker operations indicate that insulation alone is not a solution to the lightning problem.

2. Voltages of high magnitudes may be transmitted for several miles along highly insulated transmission lines to terminal apparatus.

3. The experimental results of this investigation show that voltages of harmful character to high-voltage lines appear only when direct strokes or their branches terminate on the conductors.

4. The fronts of surges at their points of origin may have a rate of rise of 4,000 kv. or more per microsecond.

5. Tripouts on ungrounded wood pole construction of the type described in this paper are usually the result of phase-to-phase flashovers.

6. In this investigation lightning currents of more than 25,000 amperes are indicated.

7. Transients appearing on transmission lines can be limited to values within the strengths of terminal equipment by the use of suitable protective devices.

8. Limited field investigations indicate that an arrester of the expulsion form will successfully function to interrupt dynamic current follow up.

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Discussion

For discussion of this paper see page 583.

1929 Lightning Experience on the 132-Kv. Transmission Lines of the American Gas and Electric Company

BY PHILIP SPORN*

Fellow, A. I. E. E.

Synopsis.—*A record of the 1929 lightning performance of a 1,200-mile 132-kv. transmission net works given, and compared with the record of the system during the previous three years. The system is almost entirely of steel tower construction with conductors in vertical configuration; and on most lines one ground wire is employed.*

It is shown that some 75 per cent of high-voltage transmission line outages on this system are due to lightning. Lightning performance of lines being dependent on the severity of lightning conditions, a record of these conditions should be kept from year to year if records are to be compared.

Data are presented on relative yearly lightning intensity, line outages, damage found to insulators and hardware, performance of double circuit lines, lines with and without ground wire, and with two ground wires, and also with and without grading shields. The apparent part tower footing ground resistance plays in line outages due to lightning is also given.

The conclusions are drawn that

1. Two circuit lines are far more reliable, from a lightning point of view, than single circuit lines.

2. One ground wire offers considerable protection against lightning, and two ground wires considerably more.

3. Grading shields properly applied do not reduce line outages, but decrease the damage to insulators, hardware, and conductors.

4. Both direct and induced lightning strokes have to be considered on these lines.

5. Flashovers do not always concentrate on high tower footing resistance towers.

6. If a ground wire is to offer a high degree of protection against lightning flashover at the tower, the tower footing resistance must be relatively low—in the order of five ohms or less.

* * * * *

Introduction

THIS paper, dealing with the 1929 lightning experience on the 132-kv. transmission lines on the system of the American Gas and Electric Company, is a continuation of a record, begun in 1926, which has been presented as a series of papers† before the Institute during the past five years.

The lightning problem, in so far as it affects the operation of electric light and power systems logically, divides itself into three parts:

(*a*) *Theoretical and Laboratory Investigations.* This includes, obviously, mathematical studies and analysis of various lightning phenomena under various system conditions: laboratory studies with artificial lightning on lightning phenomena, and on the performance of various insulating structures and of various portions of the electric light and power system under lightning conditions.

(*b*) *Field Research with Natural and Artificial Lightning.* The development to a high state of reliability of the klydonograph, or surge recorder, of the Dufour oscillograph, of the direct hit recorder, of flashover indicators, of field intensity recorders and the practical development of portable impulse generators has placed in the hands of the engineer and the investigator a large number of tools which makes possible extensive and important researches on actual apparatus that were not dreamed of a few years ago. This field of investigation is obviously a very fertile one in the search for a complete solution to the lightning problem.

(*c*) *Operating Records.* These comprise the recording of actual operating experience with various devices and remedies for the mitigation or elimination of lightning damage that may have been used, and the analysis of actual system performance under lightning conditions. While this would at first appear to be the least scientific and easiest avenue of approach to the solution of the problem, it is a fact, nevertheless, that it is actually the most difficult operation. Obviously, too, and this cannot be too strongly emphasized, it is the field experience, properly analyzed and interpreted that gives the final answer as to the tenability of the many

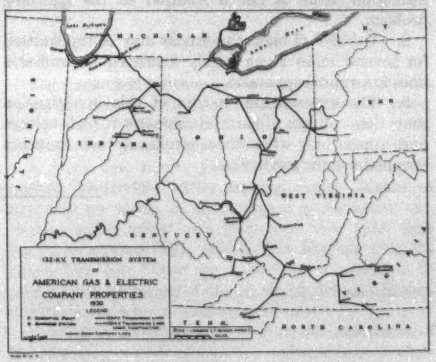

FIG. 1

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†See Bibliography.

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TABLE I

Column No.	1	2	3	4	5	6	7	8	9	10	11	12
		Crooksville So. Point	Fort Wayne Marion Muncie	Glen Lyn Roanoke	Glen Lyn Switchback	Lima Fort Wayne	Lima Fostoria	Logan Sprigg	Marion Kokomo	Philo Canton	Philo Crooksville	Philo So. Point Turner
Length of line (miles)		102.3	80.6	65.0	30.0	63.2	45.6	21.0	28.9	73.0	15.4	118.7[1]
Line placed in service (date)	No. 1	2/26/28	9/23/28	6/13/26	8/13/27	9/18/25	9/3/25	6/28/26	11/4/28	9/28/24	9/28/24	9/18/28
	No. 2		10/14/28	6/13/26	8/13/27				11/4/28	9/28/24		4/22/26
Number of circuits	1929		2	2	2	1	1	1	2	2	1	1
Number of ground wires	1929		1	1	1	1	1	0	1	1	1	1
Grading shields	Line No. 1		R & R[2]	R & R	R & H[3]	R & H	None	None	R & R	R & H	None	R & H
Grading shields	Line No. 2		R & R	R & R	R & H				R & R	R & H		
Grading shields installed (date)	Line No. 1		9/23/28	5/16/28	8/13/27	5/29/26			11/4/28	5/13/26		9/18/28
Grading shields installed (date	Line No. 2		10/14/28	4/30/28	8/13/27				11/4/28	5/13/26		
1926 Lightning Performance										1925-26		
Number of circuits				2		4	1	1		2–2	1	1
One circuit only tripped out				16			9	3		–13	3	11
Both circuits tripped out				4						–2		
Total outages				20			9	3		88–15	3	11
Circuit outages/100 miles of line/year				48[5]			19.8	31[5]		120–20.5	19.5	10.6
Circuit outages/100 miles of circuit/year				24[5]			19.8	31[5]		60–10.3	19.5	10.6
1927 Lightning Performance												
Number of circuits				2	2	4	1	1		2	1	1
One circuit only tripped out				21[6]			4	7		3[6]	1	1
Both circuits tripped out				3						1		
Total outages				24	0		4	7		4	1	7
Circuit outages/100 miles of line/year				37	0		8.8	33.3		5.5	6.5	5.9
Circuit outages/100 miles of circuit/year				18.5	0		8.8	33.3		2.8	6.5	5.9
Damage to insulators (no towers affected)				17	1		7[8]	0[7]		0	0	2
Damage to conds. and hardware (no towers affected)				14	3		5	0[7]		3	0	1
Total cases of damage				20	3		9	0[7]		3	0	2
Cases of damage per line outage				0.83	0		2.25[8]	0		0.75	0	0.29
1928 Lightning Performance												
Number of circuits		1	2	2	2	4	1	1	2	2	1	1
One circuit only tripped out		4		5	1		1	9		10	2	12
Both circuits tripped out		0		2	1					2		
Total outages		4		7	2		1	9		12	2	12
Circuit outages/100 miles of line/year		3.9		10.8	6.7		2.2	42.9		16.4	13	7.1[9]
Circuit outages/100 miles of circuit/year		3.9		5.4	3.4		2.2	42.9		8.2	13	7.1[9]
Damage to insulators (no towers affected)		0		2	1		2	5		10	2	10
Damage to conductors		0		2	2		1	0		6	1	6
Arcing rings burned (no towers affected)		3		5	3					10		0
Total cases of damage		3		6	4		2	5		11	2	11
Cases of damage/line outage		0.75		0.86	2		2.0	0.55		0.92	1.0	0.92
1929 Lightning Performance												
Number of circuits			2	2	2	1	1	1	2	2	1	2[12]
One circuit only tripped out	Operated as part of Philo-Turner Line.		5	19	6	0	4	16	1	9	1	24
Both circuits tripped out			1	11	3				0	2		3
Total outages			6	30	9	0	4	16	1	11	1	27
Circuit outages/100 miles of line/year			40.3	46	33.3	0	8.9	76	3.5	15.0	6.5	16.3
Circuit outages/100 miles of circuit/year			20.2	23	16.7	0	8.9	76	1.7	7.5	6.5	8.4
Damage to insulators (no towers affected)			1	9	4	2	5	5		9	1	22
Damage to conductors			1	16(18)[13]	4(18)	0	0	6(18)		4	1	8(28)
Arcing rings burned (no towers affected)			1	27	13	0				12		17
Total cases of damage			1	36	14	2	5	7	Records	12	2	29
Cases of damage per line outage			0.12	1.2	1.55		1.25	0.44	incomplete	1.09	2	1.04

[1]Philo to Rutland only.
[2]Ring and Ring.
[3]Ring and Horn.
[4]Operated as Lima-Twin Branch.
[5]Corrected for 1 calendar year.
[6]One circuit out of service at time of I. F. O.
[7]Superficial inspection—towers not climbed.
[8]Probably partly accumulation of 2 previous years.
[9]Rutland So. Point No. 2 (50.3 mi.) also on this circuit.
[10]Chargeable to 3 years' operation.
[11]Includes Howard Fostoria Line.
[12]Philo-So. Point-Turner (168 mi.) taken as 2 circ. line 161 mi. long.
[13]Indicates no. of total cases with severed strands.
[14]Installed Sept. 24, 1928.

TABLE I—*Continued*

Column No.	1	13	14	15	16	17	18	19	20	21	22	23	24
		Roanoke Danville	Roanoke Reusens	Saltville Kingsport	So. Bend Mich. City	Switchback Logan	Switchback Saltville	Turner Cabin Creek	Turner Logan	Twin Branch Fort Wayne	Twin Branch So. Bend	Windsor Canton	Philo Howard Fostoria
Length of line (miles)		65.0	43.0	56.0	40.0	50.0	46.0	23.6	40.2	65.4	4.9	55.0	125.5
Line placed in service (date)	No. 1	9/16/26	5/5/26	8/25/27	6/24/26	11/27/27	10/16/27	7/29/28	1/30/26	9/8/25	4/5/25	9/ /17	8/2/29
	No. 2		9/ 9/27			11/27/27		7/31/28	1/30/26		4/5/25	9/ /17	
Number of circuits	1929	1	2	1	1	2	1	2	2	1	2	2	1
Number of ground wires	1929	1	1	1	1	1	1	1	1[14]	1	1	2	1
Grading shields	Line No. 1	R & R	R & H	R & H	None	R & H	R & H	R & R	R & H	R & H	R & H	None	R & R
Grading shields	Line No. 2		R & H			R & H		R & R	R & H		R & H	None	
Grading shields installed (date)	Line No. 1	9/16/26	5/22/26	8/25/27		11/27/27	10/16/27	7/29/28	7/20/26	6/7/26	6/8/26		8/2/29
Grading shields installed (date)	Line No. 2		9/ 9/27			11/27/27		7/31/28	7/20/26		6/8/26		
1926 Lightning Performance													
Number of circuits		1	1		1				2	4	2	2	
One circuit only tripped out		0	13						16		0	9	
Both circuits tripped out									2		0	2	
Total outages		0	13						18		0	11	
Circuit outages/100 miles of line/year		0(A)	36.6[5]						44.8		0	20	
Circuit outages/100 miles of circuit/year		0(A)	36.6[5]						22.4		0	10	
1927 Lightning Performance													
Number of circuits		1	1	1	1	2	1		2	4	2	2	
One circuit only tripped out		16	16		4				20		1	2[6]	
Both circuits tripped out									4		1	0	
Total outages		16	16		4				24		2	2	
Circuit outages/100 miles of line/year		24.6	37.2	0	10				57.2		41	3.6	
Circuit outages/100 miles of circuit/year		24.6	37.2	0	10				28.6		20.5	1.8	
Damage to insulators (no towers affected)		14	2	0	1				2		1	0	
Damage to conds. and hardware (no towers affected)		5	7	0	0				4		0	0	
Total cases of damage		14	7	0	1				5		1	0	
Cases of damage per line outage		0.87	0.44		0.25				0.21		0.50	0	
1928 Lightning Performance													
Number of circuits		1	2	1	1	2	1	2	2	4	2	2	
One circuit only tripped out		13	6	5	7	16	5		12		0	2	
Both circuits tripped out			4			2			1		0	0	
Total outages		13	10	5	7	18	5		13		0	2	
Circuit outages/100 miles of line/year		20	23.3	8.9	17.5	36	10.9		32.3		0	3.6	
Circuit outages/100 miles of circuit/year		20	11.7	8.9	17.5	18	10.9		16.2		0	1.8	
Damage to insulators (no towers affected)		18	7	1	4	5	1		23[10]		0	0	
Damage to conductors		5	1	1	1	2			22[10]		0	1	
Arcing rings burned (no towers affected)			25	6		12	3		25[10]		0		
Total cases of damage		18	28	6	4	14	3		38[10]		0	1	
Cases of damage/line outage		1.38	2.8	1.2	0.57	0.78	0.6		1.07		0	0.50	
1929 Lightning Performance													
Number of circuits		1	2	1	1	2	1	2	2	1	2	2	1
One circuit only tripped out		30	11	13	9	11	16	4	11	7	1	2	3
Both circuits tripped out			5			9		3	4	0	2	0	
Total outages		32	16	13	9	20	16	7	15	7	3	2	3
Circuit outages/100 miles of line/year		49.3	37.2	23.2	22.5	40	34.8	29.6	37.4	10.7	61.2	3.6	8[11]
Circuit outages/100 miles of circuit/year		49.3	18.6	23.2	22.5	20	34.8	14.8	18.7	10.7	30.6	1.8	8[11]
Damage to insulators (no towers affected)		5	4	1	3	10	5	6	3	5	2	0	1
Damage to conductors		15	5(18)	5	3	6	4(18)	3(18)	3(38)	4	2	0	2
Arcing rings burned (no towers affected)		32	16	17		17	16	12	16	0	0	0	3
Total cases of damage		34	18	17	3	20	17	12	17	8	3	0	3
Cases of damage per line outage		1.06	1.12	1.31	0.33	1.0	1.06	1.71	1.13	1.15	1.0	0	1.00
	Operated as part of Philo-Turner Line.												

[1]Philo to Rutland only.
[2]Ring and Ring.
[3]Ring and Horn.
[4]Operated as Lima-Twin Branch.
[5]Corrected for 1 calendar year.
[6]One circuit out of service at time of I. F. O.
[7]Superficial inspection—towers not climbed.
[8]Probably partly accumulation of 2 previous years.
[9]Rutland So. Point No. 2 (50.3 mi.) also on this circuit.
[10]Chargeable to 3 years' operation.
[11]Includes Howard Fostoria Line.
[12]Philo-So. Point-Turner (168 mi.) taken as 2 circ. line 161 mi. long.
[13]Indicates No. of total cases with severed strands.
[14]Installed Sept. 24, 1928.

theories developed in connection with lightning and as to the practicability of any methods or structures that may be developed for the elimination of the harmful effects of lightning.

The above has been offered for what is hoped is the final *raison d'etre* for this particular type of approach to the lightning problem, since its permanent value has been questioned at times although, it is true, not by those intimately associated or acquainted with the problem.

General

The transmission network, shown in Fig. 1, comprised in 1929—1,198 miles of line; the total circuit miles of line in service during the same period was 1,825 miles. The system, it will be noted, has been enlarged by the addition of the Fort-Wayne-Marion-Muncie line, of the

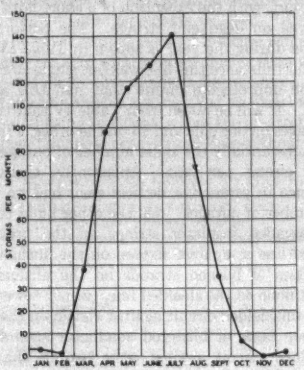

Fig. 2—1929 Lightning Storms Recorded at 24 Stations on 132-Kv. System

Total number of storms—652

Marion-Kokomo line, of the Turner-Cabin line, and of the Philo-Howard-Fostoria line. No changes of a fundamental nature outside of those described in the last paper were made during the year 1929, except the addition of one ground wire in the fall of 1928 on the Turner-Logan line.

While complete details with regard to the tower structures, configuration of conductors, etc., have been previously given, it might be well to summarize here a few salient points. All lines coming under the range of observation, as described in this paper, are of double-circuit steel-tower construction (except the Logan-Sprigg wood pole line) with the conductors in a vertical configuration, all lines with a single exception using an A. C. S. R. conductor and a single ground wire of the same material. The average insulation employed consists at suspension points of 10 disk units, their length varying from 4¾ to 5⅛ in., and employing two additional units on dead ends. The majority of all lines was, during 1929, equipped with some form of arcing protection consisting either of a ring and horn combination, the horn in that case being located at the ground end, or of a ring and ring combination.

1929 Performance

Table I gives a further résumé of the principal characteristics of the various circuits together with the 1926, 1927, 1928, and 1929 lightning performance. No attempt will be made to completely dissect the data given there, since it is believed that these data are given in a concise and readily enough comparable form to make comparison and analysis a simple matter.

The record of storms started for the first time in 1928 was maintained through 1929. Since no real knowledge of the lightning performance of a line can be obtained without a knowledge of the number of times the line was exposed to lightning, an attempt was further made this year to obtain even more thorough records of lightning storms in the vicinity of the lines than was obtained in 1928 when this feature of observation was inaugurated. The system of recording storms which was set up required the operators at all switching points on the 132-kv. system to obtain and send in records of all lightning-storms observed at their respective stations. A curve integrating the storms at all stations for each month is shown in Fig. 2. The records show a total of 652 storms reported over the 132-kv. system during the entire year. It must be realized, of course, that this represents a lower figure than the actual number to which the system was subjected, since it is possible for some storms to pass over the system at points where no observations were recorded, and further that it is not absolutely certain that all lightning storms were observed and reported at operating points on the 132-kv. system.

TABLE II—LIGHTING OUTAGES 1929

	Line Outages			
	Total	Sleet	Ltng.	% Ltng.
Fort Wayne-Marion Muncie	26	19	6	23.1
Glen Lyn-Roanoke	32		30	93.6
Glen Lyn-Switchback	12		9	75.0
Lima-Fort Wayne	0		0	
Lima-Fostoria	7		4	57.2
Logan-Sprigg	20		16	80.0
Marion-Kokomo	3		1	33.3
Philo-Canton	27		11	40.7
Philo-Crooksville	8	1	1	12.5
Philo-Turner	40		27	67.5
Roanoke-Danville	36		32	89.0
Roanoke-Reusens	16		16	100.0
Saltville-Kingsport	13		13	100.0
South Bend-Michigan City	13		9	69.2
Switchback-Logan	22		20	90.8
Switchback-Saltville	17		16	94.0
Turner-Cabin Creek	9		7	77.8
Turner-Logan	18		15	83.3
Twin Branch-Fort Wayne	13	2	7	53.8
Twin Branch-South Bend	6	3	3	50.0
Windsor-Canton	4		2	50.0
Philo-Howard	18	11*	3	16.6
Totals	360	36	248	69.

*Dancing conductors.

Per cent lightning outages 69%

Per cent lightning outages (omitting sleet and dancing conductors) 76.6%

The lightning outages over the entire system as shown in Table I, amount to 248. The total outages on the system from all causes was 360 and of this number, outages directly attributable to sleet and dancing conductors were 36. Lightning therefore accounted for 69 per cent of the total outages, or if sleet and dancing conductor troubles are eliminated, (the general experience being that troubles of this nature occur at rather infrequent and long spaced intervals) then lightning accounted for 76.6 per cent of the total outages. This checks data previously given to the effect that on a well-constructed and operated system, lightning today accounts for about 75 per cent of the total outages.

The 1929 lightning outages for the entire system considerably exceeded those recorded in 1928, the outages being 133 in 1928 and 248 in 1929. This increase is partly due to the added miles of transmission line since 1928, but chiefly to the more severe lightning conditions which existed in 1929. This statement is borne out, first from actual observations and comments from the operating people in the field; and second from the record of storms reported at six operating points on the system for 1928 and 1929 which were as follows:

TABLE A—LIGHTNING STORMS

	1928	1929	1929 % of 1928
Canton	21	38	180
Kingsport	23	24	104
Newcomerstown	23	34	148
Philo	31	40	129
South Bend	21	33	157
Turner	42	53	126
Per cent Storms	100	138	141

During 1929 the southern section of the system reported unusually severe lightning conditions which will account to a certain extent for the rather poor performance of such lines as the Glen Lyn-Switchback, Roanoke-Danville, and Switchback-Saltville lines.

Comparative line outages due to lightning during 1928 and 1929 are given below and indicate in every case the more severe lightning storm conditions over the system as a whole.

TABLE B—LINE OUTAGES PER 100 MI. OF LINE PER YEAR

	1929	1928	1929 % of 1928
All lines	21.6	13.5	160
Two circuit lines	23.3	17.7	136
One circuit line	17.8	10.0	178
With grading shields	20.8	13.8	151
Without grading shields	13.3*	8.1	165
Average			158

Line outages per 100 mi. of line per year on 4 new lines in operation in 1929 . . . . . 7.3.

*Exclusive of Windsor-Canton and Logan-Sprigg lines.

The average ratio of lightning outages in 1929 to 1928 for the entire system is 1.58; and the lightning storm record for five stations where records were kept for both years showed a ratio of 1.38 for the two years which is in the same general order as the outages. This shows conclusively that in analyzing the relative lightning performance of lines from year to year lightning conditions to which the lines are subjected must also be considered, or else erroneous and misleading conclusions may be drawn This is again clearly demonstrated by an analysis of outages on lines with and without grading shields discussed below.

Detail Discussion of Results

Effectiveness of Grading Shield Protection. During 1929 the Philo-South Point-Turner line operated for the entire lightning year with one circuit completely equipped with grading shields (ring and horn arcing combinations) while the other circuit had no protection of any sort, with the exception of a short horn that was employed on dead end strings. The performance of this line during the year is shown in Table IV. For a very short distance this line was but single circuit and it was the unprotected circuit that had this additional length. It will be noted from Table IV that there were 13 outages on one side and 14 on the other, a figure that gives substantially the same number of outages per 100 mi. of circuit per year. Apparently, therefore, the difference in protective devices used had no effect on the number of outages. On the other hand there were five cascaded insulator strings on the protected or shielded line as against sixteen on the unshielded line, four cases of burned strands on the unshielded line against none on the shielded line, twenty towers flashed on the unshielded circuit against eleven on the shielded and four cases of blistered conductor strands on the unshielded circuit against only two in the shielded circuit. The above was determined from careful inspection consisting of a complete climbing of each tower on the line.

Performance of Double Circuit Lines. In Table III there is given an analysis of the outages on the two circuit lines. It will be seen that out of a total of 146

TABLE III—OUTAGES ON TWO-CIRCUIT LINES

		Outages		
	Miles	One line	Two line	% Two line
Fort Wayne-Marion-Muncie	80.6	5	1	16.7
Glen Lyn-Roanoke	65.0	19	11	36.7
Glen Lyn-Switchback	30.0	6	3	33.3
Philo-Canton	73.0	9	2	18.2
Philo-Turner	161.	24	3	11.1
Roanoke-Reusens	43.0	11	5	31.2
Switchback-Logan	50.0	11	9	45.0
Turner-Cabin Creek	23.6	4	3	42.8
Turner-Logan	40.2	11	4	26.7
Twin Branch-South Bend	4.9	1	2	66.7
Windsor-Canton	55.0	2	0	0.0
	626.3	103	43	
Per cent outages		70.5%	29.5%	

outages 103 or 70½ per cent involved but one circuit. This shows a higher percentage where both lines were affected than was the case in 1928, where the figure was 19 per cent and considerably higher than the 1927 figure of 16 per cent. It is possible that the growth of the system and the concentration of additional capacity, and therefore short-circuit current on the system without a concurrent development along the lines of faster switch operation, has brought about a situation where a greater percentage of what were originally single-line flashovers are developed by arc communication into two circuit flashovers, but of course, no data are available on that score. Some system changes that are under discussion with a view of speeding up switch operation may throw further light on this question in the future.

From this record it is clear that the line outages are not materially different with grading shields from what they are when no grading shields are used. A study of Table B, however, would tend to indicate some 60 per cent greater outages with grading shields than without. The explanation for this apparent discrepancy lies in the fact that grading shields are installed on lines located in the worst lightning territory, where the greatest protection against damage is needed; and lines without grading shield protection are in the district where lightning conditions are less severe, and where less outages would be expected.

TABLE IV—COMPARISON OF DAMAGE TO LINE WITH AND WITHOUT RING AND HORN
PHILO-SOUTH POINT-TURNER LINE
(From Tower Climbing Inspection)

Tower location with	With ring and horn	Without ring and horn
Cascaded insulators	5	16*
Strands burned in two in wire	0	4‡
Number of flashovers	11	20
Blistered strands	2	4†
Lightning outages	13	14
Length of line (miles)	152.6	168
Outages/100 mi. of cir./year	8.5	8.3

*5 cases of badly stripped insulators.
†In addition to cases with strands burned in two.
‡Replaced about 50 ft. of conductor each side of tower, in one case.

Conductor Damage. The damage to the conductors was observed from tower climbing inspections and is given in detail in Table I. A total of 90 cases is recorded and in only 11 cases was the burning severe enough to sever one or more strands in the conductor. Analyzed on the basis of 100 mi. of circuit per year, there are 0.6 cases of severed strands against a total number of cases of five burned or blistered conductors. In no case was the damage to conductors severe enough to prevent any line being placed back into service, and practically all damage to conductors was observed from a tower climbing inspection on the semi-yearly patrol. In no case was burning of the conductor found within the limits of the arcing protection shield at the live side of the insulator string.

Grading Shield Damage. The damage observed on grading shields due to flashover was in all cases quite negligible, being confined to slight burning at the tips of the horns and similar burning on the ends of the rings. In no case has it been necessary this year, or in past years, to replace or repair a grading shield assembly due to damage from flashover. The burning on the grading shields is clear cut evidence that the shield is in most cases carrying and holding the 60-cycle arc instead of permitting it to concentrate on the conductor and hardware.

Location of Damage by Physical Inspection. In an attempt to determine the location where lightning first affected the line, that is on top, middle or bottom conductor, the results of the physical inspection have been tabulated in Table V, where the location of burning or other evidence of flashover have been recorded. From this table it appears that cases of trouble involving top or top and middle conductors comprise 44.2 per cent of the total cases. Those involving the middle conductor or bottom and middle conductor comprise 42.8 per cent.

TABLE V—LOCATION OF TROUBLE BY PHYSICAL INSPECTION

Line	T-M-B	Top	Mid.	Bot.	T&M	T&B	M&B
Fort Wayne-Marion-Muncie				1			
Glen Lyn-Roanoke	3	8	5	8		8	2
Glen Lyn-Switchback	2	5	3	3			1
Lima-Fort Wayne			1				
Lima-Fostoria		2		1			
Marion-Kokomo							
Philo-Canton	1	9		1			1
Philo-Crooksville		2					
Philo-Turner	1	6		1			
Roanoke-Danville	3	7	5	6	6	1	6
Roanoke-Reusens	2	4	2	7			3
Saltville-Kingsport		9	4	3			1
South Bend-Michigan City	1					1	1
Switchback-Logan		11	3	3	1	1	1
Switchback-Saltville		6	4	3		1	3
Turner-Cabin Creek		4	2	2	1	1	1
Turner-Logan	1	10	1			1	
Twin Branch-Fort Wayne		3	2	1	1	1	
Twin Branch-South Bend		1	1			1	
Windsor-Canton							
Philo-Howard		1					2
Howard-Fostoria							
Totals	13	90	33	41	9	16	22
Total locations 224							
Per cent of totals	5.8	40.2	14.7	18.3	4.0	7.2	9.8

Cases involving top or top & mid. only ..... 99 = 44.2%
Cases involving bot. mid. or top & mid. only ... 96 = 42.8%
Involving one phase only ..... 164 = 73.2%
Involving more than one phase ..... 60 = 26.8%

Assuming that those cases that involved the top conductor only or the top and middle conductor exclusively were all caused by direct strokes (although it is possible that some of these cases may have been caused by induced voltages), and assuming further that those cases that did not involve the top conductor at all were all caused by induced voltages, it would appear that, on this system at any rate, direct and induced lightning strokes are of practically equal importance in so far as they contribute to the system outages.

Another interesting conclusion drawn from this table

is that 73.2 per cent of the cases of trouble involved only one phase of the circuit and only 26.8 per cent involved more than one phase. It is also shown that in only 5.8 per cent of the cases were top, middle, and bottom conductors affected (that is, a complete three-phase short circuit developed). Conclusions drawn from this type of data are not infallible, since lightning flashover may occur on the line without a power follow-up arc; and also if such an arc does exist, it is not certain that the evidence of the 60-cycle follow-up will be left, in all cases, to be observed by physical inspection. Another method, however, has been used to check these results. This method is the determination of the phase wire affected, by the record of relay targets at the time the circuits tripped out.

Location of Damage by Relay Targets. A great many of the relays on the 132-kv. circuits are equipped with a target which indicates when the relay has operated. A

TABLE VI—LOCATION OF TROUBLE BY RELAY TARGET INDICATION

Line	Conductors affected						
	Top	Mid.	Bot.	T&M	T&B	B&M	T-B-M
Ft. Wayne-Marion							
Muncie	3						
Glen Lyn-Roanoke	5	6	3	5	1	3	1
Glen Lyn-Switchback	2	2					
Philo-Canton	1		1	1	1		
Philo-Turner	6	1	4	3	1	1	
Roanoke-Danville	2	2	5	5		2	2
Roanoke-Reusens	1	1	3		1		1
Saltville-Kingsport	3		1	2			
Switchback-Logan	4			1	1		1
Switchback-Saltville		1			1	1	
Turner-Logan	4	3	3			3	
Philo-Fostoria		1					
Totals	31	17	20	17	6	10	5
Per cent of total	29.2	16.0	18.9	16.	5.7	9.5	4.7
Total indications							106
Per cent involving top or top and mid. only					45.2		
Per cent involving bottom and mid. but not top					44.3		
Per cent involving top, mid. and bottom					4.7		
Per cent involving one phase only					64.1		
Per cent involving more than one phase					35.9		

record of these target indications is kept at each station and an analysis has been made to indicate which phases of a circuit have been in trouble. A summary of these data is given in Table VI. The analysis of this table shows that in 45 per cent of the cases the top or top and middle conductors have been affected and in 44.3 per cent of the cases the bottom or bottom and middle conductors have been involved. This checks very closely with the figures obtained by the method of physical inspection where the corresponding figures were 44.2 and 42.8 per cent. The table further shows that one phase was involved in only 64.1 per cent of the cases against 73.2 per cent as shown by physical inspection. It also shows 4.7 per cent of the cases involved all three conductors of a circuit against 5.2 per cent given in Table V.

Reasoning again on the assumption that all cases involving the top conductor were of direct stroke origin, and that cases not involving the top conductor were of induced stroke origin, the evidence seems to be very positive that on this system at any rate induced strokes as well as direct strokes must be considered in supplying protection to the system, and that the two are of about equal importance.

Tower Footing Resistances. On a great many of these lines the tower footing has been measured, in some cases with the ground wire in place, and in some cases with the ground wire electrically disconnected from the

TABLE VII—TOWER FOOTING RESISTANCE WHERE FLASHOVER OCCURRED ON PHILO-CANTON LINE 1929

Case No.	Tower No.	Grd. resist. ohms*	F. O. 1928	Resistance of preceding and succeeding towers	
1	30B	1.8	No	5.1	2.0
2	76A	2.2	No	10.3	2.0
3	89A	18.2	No	3.5	8.0
4	96A	21.2	No	8.2	27.0
5	126B	5.1	No	3.0	31.0
6	151C	2.4	Yes	10.3	5.0
7	152C	5.0	No	2.4	6.4
8	155B	2.8	No	5.8	10.1
9	191B	4.7	No	1.3	33.0
10	192B	33.0	No	4.7	1.5
11	225A	18.4	No	22.7	10.8
12	291A	2.3	No	7.8	1.9

Total number of towers in the line 299
Total number of towers with flashover 12
Average resist. per tower for whole line (ohms) 7.7
Average resist. per tower for those with flashover (ohms) 9.9

Ratio of resist. $\frac{\text{Flashed over towers}}{\text{Average of line}}$ 1.29

Per cent of cases where flashover occurred at tower having lowest resistance of three adjacent towers 25%

*Tower resistance measured with ground wire disconnected.

tower. Since it has been extensively pointed out of late that the tower footing resistance plays an important part in the lightning protection of a line, it is considered worth while to present operating data to throw some light on this aspect of lightning protection as found in service. Tables VII and VIII give the tower footing resistances on the Philo-Canton and Turner-Logan lines at those towers where flashovers were observed on inspection. There is also given the tower footing resistance of the tower on each side of the one where the flashover was observed. On both of these lines tower footing resistances were measured with the ground wire disconnected from the tower so that the ohmic values are true tower footing resistances and do not include any effect of the ground wire and adjacent towers.

There are two things to be noted in these tables: first, that the towers where flashovers occurred have a higher average footing resistance than the average for the entire line, the ratios being 1.29 on the Philo-Canton line and 1.67 on the Turner-Logan line, that is, the towers where flashovers occurred have a footing resistance averaging some 30 to 70 per cent higher than the average for the line. The other point which the operating data show is that in 25 per cent of the cases on the Philo-Canton line and in 47 per cent of the cases on the Turner-Logan line the tower where the flashover occurred has a lower resistance than either of the towers on

TABLE VIII—TOWER FOOTING RESISTANCE WHERE FLASHOVERS OCCURRED ON TURNER-LOGAN LINE 1929

Case No.	Tower No.	Gr. resist. ohms*	F. O. 1928	Resist. of preceding	and succeeding towers
1	18B	2.50	No	2.25	2.25
2	24C	2.50	No	1.25	1.50
3	25C	1.50	No	2.50	5.50
4	28C	3.50	No	5.50	1.25
5	40C	1.50	No	1.50	3.25
6	44C	8.50	No	6.25	7.50
7	45C	7.50	No	8.50	15.25
8	63C	6.00	No	60.0	6.25
9	72E	250.	Yes	11.0	250.
10	85B	250.	No	50.0	25.0
11	120E	28.0	No	20.0	8.75
12	122C	3.0	No	8.75	12.5
13	128C	6.5	No	7.5	18.5
14	140C	14.5	No	6.5	3.5
15	144E	2.75	No	3.0	13.0
16	149E	16.0	No	25.0	25.0
17	151E	21.0	No	25.0	22.5

Total number of towers in the line 153
Total number of towers with flashover 17
Average resist. per tower for whole line (ohms) 22.0
Average resist. per tower for those with flashover 36.8

Ratio of resist. $\frac{\text{Flashover towers}}{\text{Average of line}}$ 1.67

Per cent of cases where flashover occurred at tower having lowest resistance of three adjacent towers 47%

*Tower resistances measured with ground wire disconnected from towers

each side of it. On the theory that lightning flashovers will occur where the tower footing resistances are high, the condition just referred to is rather difficult to explain.

Tables IX and X give similar data on the Roanoke-Reusens and Glen Lyn-Roanoke lines. On both of these lines the tower footing resistances were measured with the ground wire attached to the towers. Here again the results show, first, that on the Roanoke-Reusens line the towers where flashover occurred have a

TABLE IX—TOWER FOOTING RESISTANCE WHERE FLASHOVERS OCCURRED ROANOKE-REUSENS LINE 1929

Case No.	Tower No.	Gr. resist. ohms*	F. O. 1928	Resist. of preceding	and succeeding towers
1	46B	20	No	40.0	20.0
2	67A	40	No	40.0	40.0
3	70B	58	Yes	25.0	14.0
4	79A	20	No	40.0	24.0
5	80B	24	No	20.0	25.0
6	89A	35	Yes	22.5	40.0
7	95B	20	No	40.0	20.0
8	96A	20	No	20.0	24.0
9	102A	50	No	4.0	22.0
10	103C	22	No	50.0	25.0
11	115A	12	No	50.0	19.0
12	116A	19	No	12.0	45.0
13	119A	11	No	90.0	21.0
14	124A	10	No	21.0	12.0
15	126A	12	No	12.0	6.0
16	132A	30	No	30.0	16.0
17	136A	20	No	30.0	14.0
18	140B	40	No	60.0	20.0

Total number of towers in the line 200
Total number of towers with flashover 18
Average resistance per tower for whole line ..... 32.6 ohms
Average resistance per tower for those with flashover 25.8

Ratio of resistance $\frac{\text{Flashed over towers}}{\text{Average of line}}$ 0.78

Per cent of cases where flashover occurred at tower having lowest resistance of three adjacent towers 27.8%

*Tower resistances measured with ground wire connected to towers.

footing resistance of 78 per cent of the average of the line and on the Glen Lyn-Roanoke line 1.84 times the average tower footing resistance for the line.

On the Roanoke-Reusens Line 27.8 per cent of the flashed-over towers had a lower tower footing resistance than the tower on either side; and on the Glen Lyn-Roanoke line in 13.9 per cent of the cases the same condition holds true.

The data in Tables VII, VIII, IX and X have been analyzed in a different way in Fig. 3. There it is shown

TABLE X—TOWER FOOTING RESISTANCE WHERE FLASHOVER OCCURRED ON GLEN LYN-ROANOKE LINE 1929

Case No.	Tower No.	Gr. res. ohms*	F. O. 1928	Resist. of preceding	and succeeding towers
1	12C	15.	No	8.25	250
2	13C	4.5	No	250	3.75
3	14C	3.75	No	4.5	2.25
4	17B	4	No	2.5	250
5	21C	250	No	250	250
6	22A	250	No	250	250
7	24C	250	No	250	250
8	26A	60	No	250	250
9	28C	250	No	250	250
10	31C	5.75	No	4.5	9.5
11	35C	250	No	13.0	7.5
12	40B	250	No	5.0	4.0
13	49B	17	No	7.75	10.0
14	53C	250	No	6.5	5.5
15	102B	8.5	No	1.5	6.5
16	135B	6.0	No	3.0	7.5
17	124A	3.5	No	2.5	9.0
18	123A	2.5	No	2.0	3.5
19	81C	2.0	No	1.75	4.25
20	80C	1.75	No	4.0	2.0
21	74A	8.5	No	4.0	4.3
22	72E	8	No	80	4.0
23	71E	80	No	250	8.0
24	69C	9	No	250	250
25	65B	250	No	250	250
26	64C	250	No	250	250
27	63B	250	No	250	250
28	51C	4.8	No	250	21.0
29	46B	3	No	17	3.0
30	22B	10	No	25	12.3
31	21B	25	No	25	10
32	20B	25	No	250	25
33	18A	250	Yes	13	250
34	14A	250	No	250	15
35	9A	10	No	17	13
36	6A	250	No	250	12.0

Average resist. of towers where measurements could be made ..... 7.9 ohms
Average resist. of towers measured taking 250 ohms for all towers higher than this ..... 47.1 ohms
Average of tower resistances at above locations 86.5

Ratio of resistance $\frac{\text{Flashed over towers}}{\text{Average of line}}$ 1.84

Per cent of cases where flashover occurred at tower having lowest resistance of three adjacent towers 13.9

*Tower footing resistances measured with ground wire connected to towers.

that in 50 per cent of the cases where towers were flashed over the tower footing resistance was 12 ohms or less. In about 65 per cent of the cases the tower footing resistance was 20 ohms or less and in 80 per cent of the cases was below 50 ohms. On the other hand, in 17½ per cent of the cases the tower footing resistance was about 250 ohms, which was the upper limit of the ground ohmer used to obtain the measurements.

It would appear from the above that if a criterion for immunity from flashover is to be established on the basis of tower footing resistance, values considerably lower than have heretofore been considered necessary will have to be obtained. For example, a value of 20 ohms, which has been considered by some as a sufficiently low value, would appear to offer very little protection, the experience on the four lines given in Fig. 3 being that 65 per cent of the flashovers occurred at points where the resistance was 20 ohms or less. The order of limiting resistance will depend of course upon the relative immunity desired, but a resistance of 5 ohms would not appear an unreasonably low one to seek, and it may be that even lower values will, under certain conditions, be found desirable.

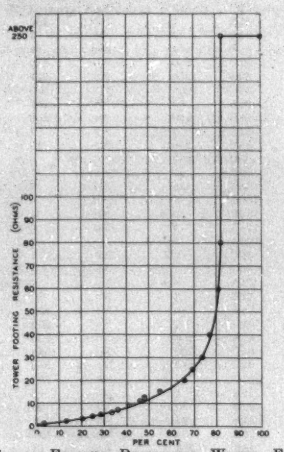

Fig. 3—Tower Footing Resistance Where Flashover Occurred—1929

1. Philo-Canton
2. Turner-Logan
3. Glen Lyn-Roanoke
4. Roanoke-Reusens

Ground Wire Protection. The beneficial effects of the ground wire are demonstrated from the 1929 records in several ways. First, on the Logan-Sprigg line which, while being a wood pole line, nevertheless has steel crossarms grounded at every pole, the outages were 76 per 100 mi. of line per year. This is 1.8 times as many as recorded last year and checks fairly closely with the more severe lightning conditions experienced in 1929. On the other hand, the outages on this line are 3½ times the average on the system as a whole, the rest of the system being in all cases equipped with at least one ground wire.

In the case of the Windsor-Canton line where two ground wires are employed the outages were only 3.6 per 100 mi. of line per year which is exactly the same figure found in 1928 and 1927. It is believed, however, that the Windsor-Canton line is in a district where lightning conditions are not nearly as severe as in the Logan-Sprigg district; but the discrepancies in outages are far too great to attribute to this difference of location only.

A very direct comparison of the benefits of the ground wire is found on the Turner-Logan line where one ground wire was in operation during 1929 for the first time. In 1928 the outages per 100 mi. of line were 32.3 and in 1929 the outages were 37.4. Correcting the 1928 outages on this line by increasing them 40 per cent (see Table A) due to the more severe lightning conditions encountered in 1929, and further increasing them 31 per cent to allow for the time (some seven weeks) the line was out of service for installing the ground wire, there would have been 59.3 outages if this line had not been equipped with a ground wire. On the above basis the installation of one ground wire resulted, therefore, in a decrease in outages of at least 37 per cent. This is most assuredly a minimum figure since the general experience has been that during a year when there is a great frequency of lightning storms, there is experienced a proportionately greater number of heavy storms than one would be led to expect from the numerical ratio of all storms.

Conclusions

The lightning experience obtained during the year 1929, thoroughly analyzed and considered in the light of previous experience on the same lines, leads to the following conclusions:

1. *Reliability of Two-Circuit Lines.* Two-circuit lines when installed in vertical configuration on the same towers enhance the reliability of the system in a greater proportion than the mere doubling of the circuits. Experience definitely indicates that the likelihood of both circuits tripping out is comparatively small, only 16 to 29½ per cent of the cases of the trip-outs involving both circuits. There appears a likelihood that this percentage can be reduced by higher speed switching.

2. *Effectiveness of Ground Wires.* The only line equipped with two ground wires, the Windsor-Canton line, had 3.6 outages per 100 mi. of line per year. The only line not equipped with a ground wire, the Logan-Sprigg line, had an outage record of 76. The Turner-Logan line operating with one ground wire in 1929 had 37.4 outages per 100 mi. of line per year, and in 1928, when no ground wires had been installed, had an actual outage of 32.3; this figure corrected on the basis of the more severe lightning conditions in 1929 becomes 59.3 outages per hundred miles of line if the no ground wire condition had obtained in 1929.

This shows clearly the benefits of one ground wire, and indicates definitely that two ground wires are more beneficial than one. It is of course necessary to keep

in mind that the Windsor-Canton line is built with a short span and therefore at a low height above ground, and is moreover located in a territory where lightning conditions are not as severe as on other parts of the system.

3. Effectiveness of Grading Shields. Grading shields have very little effect on the lightning outages of these 132-kv. lines, neither increasing nor decreasing the number of outages. On the other hand they definitely limit the damage to insulators and conductors; further the amount of cascading is reduced. The cascaded strings over the entire system (and most of the system is equipped with grading shields) were 23 per cent of the total number of towers flashed over. Those lines that had grading shields installed recorded only 21 per cent; but the lines that had no shields had a record of 70 per cent. This is a very striking indication of the benefit rendered by grading shields.

4. Lightning Storm Data. No reliable interpretation of performance of lines under lightning is possible, or is of real value, without an accompanying record of lightning storm conditions during the particular time discussed. Conclusions gravely misleading will otherwise be made.

5. Line Conductors Involved in Flashovers. Records of physical inspection and relay target indications show that in tripouts due to lightning, all three phases are involved, in from 4.7 to 5.8 per cent of the cases. In 44.2 to 45.2 per cent the top and middle conductors only are involved. In 42.8 to 44.3 per cent the bottom and middle conductors are involved. It is also shown that in from 73.2 to 64.2 per cent only one phase is involved. It is interesting to note that the number of cases where either the middle or bottom conductors only were involved is almost equal to the number of cases in which the top conductors only were involved.

This analysis indicates very strongly that both direct and induced strokes are of vital importance and play a great part in the performance of this 132-kv. network.

6. Effect of Tower Footing Resistance on Lightning Flashover. Detailed data on the tower footing resistances on four 132-kv. lines do not show any tendency for lightning flashovers to concentrate on towers of high-footing resistance in preference to those having low-footing resistance. In fact, whether lightning strikes a tower with low- or high-footing resistance seems to be, from these data, a matter of chance. Flashed-over towers were observed where the footing resistance varied from 1.5 to over 250 ohms.

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Discussion

IMPULSE TESTS ON SUBSTATIONS

(SOUTHGATE, BROOKES, WHITEHEAD AND ROMAN)

LIGHTNING INVESTIGATION ON A WOOD POLE TRANSMISSION LINE

(PITTMAN AND TOROK)

1929 LIGHTNING EXPERIENCES

(PHILIP SPORN)

R. N. Southgate: Mr. Sporn points out that cases of trouble involving top or top and middle conductors are about the same percentage of total flashovers as those involving middle or middle and bottom conductors. He assumes that those cases which did not involve the top conductor were caused by induced strokes. There seems to be a possibility here that with the single-ground wire and vertical configuration, streamers of the lightning stroke might come in contact with the middle or bottom conductor without involving the top conductor or ground wire.

In cases of direct stroke where the ground wire or tower is not involved, the possibility of insulator flashover would not depend on the tower footing resistance as flashover would occur due to over-voltage on the conductor rather than due to tower top voltage rise. This might account in part for the fact that a large number of flashovers occurred on towers of low footing resistance.

Results of 1930 lightning investigation with the cathode-ray oscillograph on 12 miles of unenergized 220-kv. line terminating at the Roseland Switching Station lend added weight to the opinion, drawn from previous experience, that induced surges on a line of this nature are of negligible magnitude.

This line has the same structural characteristics as that used in the 1929 investigation and described by Messrs. Conwell and Fortescue, A. I. E. E. TRANS., Vol. 49, with the exception that the two ground wires were in place. During the entire season, four positive surges were recorded, all below 100 kilovolts in crest and less than 10 microseconds in length. Two of these surges were definitely correlated with records of ground current in the tower structures, while two were of unknown and possibly induced origin. Tower footing resistances on the section under investigation were low; the average being 3.9 ohms and the maximum 20 ohms.

Several strokes were observed to strike very close to the tower line near the station without causing any measurable surges on the oscillograph. A considerable length of the 220-kv. line parallels on the same right-of-way, a 132-kv. transmission line which was in service. One heavy stroke was observed to strike the ground in almost direct line with the parallel 132- and 220-kv. lines, and was calculated by velocity of sound to have occurred at about 2,000 ft. from the station. A photograph was obtained of the stroke (see Fig. 1) and from this record the lo-

cation of the stroke was placed within 350 ft. of the 220-kv. line to which the oscillograph was connected, and within 275 ft. of the live 132-kv. line. No surge was recorded on the oscillograph which was in operation and adjusted to record surges of low magnitude, nor did any trip-out occur on the 132-kv. line.

This fact and other evidence would indicate the immunity of these and similar lines to strokes of this nature and lend weight to the theory that direct strokes are responsible for the major number of flashovers.

J. J. Torok: One of the great difficulties of the lightning investigation programs has been the lack of organization on the part of the utility engineers in obtaining operating data of the right kind so that a complete analysis of a line or type of line can be made. This paper presents data of the type that is needed for such analyses.

In his evaluation of the data and conclusions he makes the following statement: "Assuming that these cases that involved the top conductor only or the top and middle conductors exclusively were all caused by direct strokes (although it is possible

Fig. 1

that some of these cases may have been caused by induced voltages) and assuming further that these cases that did not involve the top conductor at all were all caused by induced voltages, it would appear that, on this system at any rate, direct and induced lightning strokes are of practically equal importance in so far as they contribute to the system outages."

The data presented by Mr. Sporn can be interpreted on another basis, purely that of the direct stroke. The difference between the following interpretations and that of Mr. Sporn's is that Mr. Sporn assumes that when ground wires are used that the lightning stroke in all or the majority of cases terminates on the ground wire or top conductor, and that the other flashovers are caused by induced voltage. On that assumption he is entirely justified in drawing the conclusions which he has, namely that flashovers have taken place under the conditions where the direct-stroke theory as it now stands will not prove adequate especially where very low tower footing resistances have been measured and a large number of flashovers has occurred. However, the analysis that has been made on several types of towers on which ground wire protection of various forms are used has shown that the presence of the ground wire does not necessarily indicate that the ground wire only was struck. In a large number of cases where only one ground wire is used the top conductor or middle conductor were struck directly. This evidence has been determined from the nature of the markings left on the porcelain insulator surfaces. If the data submitted by Mr. Sporn are analyzed on this premise entirely different conclusions may be obtained.

Our analysis will be confined to Tables V, VII, VIII, IX, and X. Table VII tabulates the tower footing resistance of the Philo-Canton line. These resistances are fairly low except for four cases. In Table V it is shown that on the Philo-Canton line the majority of the outages took place on the top conductor with only one or two on the bottom and middle conductor. This indicates that the majority of the outages were caused by direct strokes to the top conductor while strokes to the tower, the ground wire, or top conductor did not raise the tower potential to a value sufficient to cause flashover on the conductors. Table VIII which is a tabulation of the flashovers on the Turner-Logan line shows that the majority of the tower footing resistances are fairly low. Only five or six towers may be said to have tower footing resistances above safe values. Table V shows that this line had 10 flashovers on the top conductor and one on the top, middle, and bottom, one on middle and one on top and bottom. In this case the predominance of the flashover of the top conductor is evident, showing that direct strokes terminated on the top conductor. Table IX tabulates the flashovers occurring on the Roanoke-Reusens line. The tower footing resistance on this line is fairly high ranging between 11 and 58. These values are probably too high to prevent flashover when struck directly. Because of the lower coupling between the ground wires and the bottom conductor it is expected that the voltage across the insulators on the bottom conductor will be higher than on the others. This is borne out by the values taken from Table V showing that the bottom conductor flashed over seven times, the top four, the middle two, and all three, that is top, middle, and bottom, two, and middle and bottom three.

Table X lists the flashovers occurring on the Glen Lyn-Roanoke line. The tower footing resistance on this line is fairly evenly divided between high and low resistance values. The locations of flashover with respect to position of the conductor, as would be expected from analysis, would be that the flashovers occurring on the top and bottom conductors would be very nearly the same and in several instances flashover would take place on all three conductors. This is borne out by data given in Table V. There were eight flashovers on the top conductor, five on the middle and eight on the bottom conductor.

It would be interesting to analyze the above tables in more detail in the manner shown above. However, this would require the tying in of the flashovers on each tower with the tower footing resistance.

The data in this paper very markedly show the importance of adequate ground wire protection. For example the Turner-Logan Line had ten flashovers on the top conductor. It would be safe to predict that if two ground wires were to be used, and so situated that they will adequately shield the conductors from direct strokes, the number of flashovers would be reduced from thirteen to one or two. It is quite evident that one ground wire does not effectively shield the conductors. Furthermore, the additional ground wire increases the coupling between the ground wires and the conductors to such an extent that the effectiveness of the line insulation will be raised from 10 to 30 per cent.

F. W. Peek, Jr.: The oscillograms in Figs. 5 and 6 of the paper by Messrs. Torok and Pittman are especially interesting. These figures record voltages of the order of 5,000,000. The fronts are exceedingly steep and the effective duration short. In fact the effect of these waves on the sparkover of insulators, bushings and gaps would be quite similar to the ½/5 wave so long used in our laboratory to obtain steep wave front effects.

The highest measured line voltages as shown by these oscillograms have not yet exceeded the 5,000,000 volts first produced in the laboratory about two years ago and the effects correspond to the steep laboratory waves. Full size wood poles were also flashed over and split in the laboratory as they have been in the field.

Mr. Sporn's paper is a very valuable one and could well be used as a model for recording and presenting data on operating experiences. I am going to try to arrive at the conclusions from a different angle than Mr. Sporn in that part of the sparkovers on his 132-kv. system are due to direct hits and part to induced voltages. Referring to Table VII, in 8 out of 12 cases the tower footing resistance varied from 1.8 to 5 ohms. In the remaining 4 cases it covered a range of resistance from 18 to 33 ohms. If all sparkovers were caused by direct strokes, only 4 would be expected since the tower footing resistance on the remaining 8 is too low to make a direct-stroke sparkover likely except under the most extreme conditions. However, for these particular data the ground resistance is low enough so that the induced voltages would be practically the same for all towers. Thus there would be an equal chance of an induced voltage outage for any of these towers regardless of the footing resistance. The possible conclusions, therefore, are that all outages are due to induced voltages; or that 8 or more outages are due to induced voltages with the remaining 4 or less due to direct strokes. This is based on the assumption of all the direct strokes striking the tower. If, however, half of the direct strokes hit the center of the line it might be concluded that 50 per cent were due to induced voltages and 50 per cent to direct strokes. An analysis of some of the other tables will give the same conclusions.

L. V. Bewley: The paper by Southgate, Brooks, Whitehead and Roman gives a number of oscillograms showing the effect of bus structure, breakers and transformers on traveling waves. But beyond calling attention to the fact that the crest is lowered and the transient is lengthened, as additional station equipment is added to the transition network, they do not draw any conclusions as to the characteristic effect of station equipment on traveling waves. In other words, they do not attempt to determine $Z_t(p)$ of their equation (1) from the test results which they obtained. In the past it has been customary to regard buses, breakers and transformers as simple capacitances, as far as traveling waves are concerned, and to compute the behavior of traveling waves on that basis. It is therefore gratifying to notice from the oscillograms that the character of the transient is essentially that of a reflection from a lumped capacitance. I should like to ask the authors if they made any attempt to check the experimental work by calculations based on measured values of capacitances, or if they have determined by trial what values of capacitance check the oscillograms? It seems to me that such information would greatly increase the value of their paper; because after all, we are not so much interested in a specific group of tests as we are in the general conclusions that can be drawn from them.

The authors have also discussed the effect of the arrester location, and in equation (2), have given a useful formula for estimating purposes, based on complete reflection at the terminal equipment, and a uniformly rising front on the incident wave. If, however, the terminal equipment involves a large capacitance, or outgoing lines and cables causing a reduction of surge impedance, then the reflection is only partial, and the arrester will protect the equipment at greater distances than that given by the formula. In that event a lattice can be constructed and the successive reflections computed. When the arrester is connected to the system by an independent lead, (one of the cases described in the paper by McEachron and Wade), then a lattice is well nigh indispensable as a means for calculating the protection afforded by the arrester.

Messrs. Pittman and Torok in their paper speak of the "approximate surge impedance of three conductors in parallel." It may, therefore, be appropriate to include in this discussion the definitions and derivations of the self and mutual surge impedances of the equivalent conductor which replaces any number of conductors in parallel.

Let there be n conductors carrying equipotential waves e_0 and currents $(i_1, i_2 \ldots\ldots i_n)$. Then the total current is

$$i_0 = i_1 + i_2 + \ldots\ldots + i_n \tag{1}$$

and the self surge impedance of all wires in parallel is defined as

$$Z_{00} = \frac{e_0}{i_0} = \frac{e_0}{i_1 + i_2 + \ldots\ldots + i_n} \tag{2}$$

But the fundamental equations for traveling waves are

$$\left.\begin{array}{l} e_1 = Z_{11}\, i_1 + Z_{12}\, i_2 + \ldots\ldots + Z_{1n}\, i_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ e_n = Z_{n1}\, i_1 + Z_{n2}\, i_2 + \ldots\ldots + Z_{nn}\, i_n \end{array}\right\} \tag{3}$$

If $e_1 = e_2 = \ldots\ldots = e_n = e_0$, then solving (3) for the currents, there is

$$i_1 = \frac{A_1}{D} e_0,\ i_2 = \frac{A_2}{D} e_0, \ldots\ldots, i_n = \frac{A_n}{D} e_0 \tag{4}$$

where

$$D = \begin{vmatrix} Z_{11}\, Z_{12} \ldots\ldots Z_{1n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ Z_{n1}\, Z_{n2} \ldots\ldots Z_{nn} \end{vmatrix} \tag{5}$$

and

$$A = \begin{array}{c} \begin{array}{ccccc} (1) \ldots\ldots & (r) & \ldots\ldots & (n) \end{array} \\ \begin{vmatrix} Z_{11} \ldots\ldots 1 \ldots\ldots Z_{1n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ Z_{n1} \ldots\ldots 1 \ldots\ldots Z_{nn} \end{vmatrix} \end{array} \tag{6}$$

that is, A_r is the same as determinate D with the rth column replaced by a column of *ones*. Then

$$i_0 = i_1 + i_2 + \ldots\ldots + i_n = \frac{A_1 + A_2 + \ldots\ldots + A_n}{D} e_0 \tag{7}$$

and therefore

$$Z_{00} = \frac{e_0}{i_0} = \frac{D}{A_1 + A_2 + \ldots\ldots + A_n} \tag{8}$$

The mutual surge impedance between this group of wires and an independent wire k (not of the group) is defined as

$$Z_{0k} = \frac{e_k}{i_0} \tag{9}$$

where e_k is the potential induced in k by i_0. By the general equations of traveling waves this potential is

$$\begin{aligned} e_k &= Z_{1k}\, i_1 + Z_{2k}\, i_2 + \ldots\ldots + Z_{nk}\, i_n \\ &= \left(\frac{Z_{1k}\, A_1 + Z_{2k}\, A_2 + \ldots\ldots + Z_{nk}\, A_n}{D}\right) e_0 \end{aligned} \tag{10}$$

and therefore the mutual surge impedance between the group and the independent wire k is

$$\begin{aligned} Z_{0k} &= \left(\frac{Z_{1k}\, A_1 + Z_{2k}\, A_2 + \ldots\ldots + Z_{nk}\, A_n}{D}\right) \frac{e_0}{i_0} \\ &= \left(\frac{Z_{1k}\, A_1 + Z_{2k}\, A_2 + \ldots\ldots + Z_{nk}\, A_n}{A_1 + A_2 + \ldots\ldots + A_n}\right) \end{aligned} \tag{11}$$

As an example of the application of these equations consider the line described by Pittman and Torok specified on page 568. There is

$$Z_{11} = Z_{22} = Z_{33} = 60 \log_e \left(\frac{2 \times 12 \times 32}{.23}\right) = 487$$

$$Z_{12} = Z_{23} = 60 \log_e \left(\frac{65.6}{14}\right) = 93$$

$$Z_{31} = 60 \log_e \left(\frac{69.9}{28}\right) = 55$$

Of course, $Z_{21} = Z_{12}$ and $Z_{32} = Z_{23}$ and $Z_{13} = Z_{31}$.

$$D = \begin{vmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{13} \end{vmatrix} = \begin{vmatrix} 487 & 93 & 55 \\ 93 & 487 & 93 \\ 55 & 93 & 487 \end{vmatrix} = 115 \times 10^6$$

$$A_1 = \begin{vmatrix} 1 & Z_{12} & Z_{13} \\ 1 & Z_{22} & Z_{23} \\ 1 & Z_{32} & Z_{13} \end{vmatrix} = \begin{vmatrix} 1 & 93 & 55 \\ 1 & 487 & 93 \\ 1 & 93 & 487 \end{vmatrix} = 17.01 \times 10^6$$

$$A_2 = \begin{vmatrix} Z_{11} & 1 & Z_{13} \\ Z_{21} & 1 & Z_{23} \\ Z_{31} & 1 & Z_{13} \end{vmatrix} = \begin{vmatrix} 487 & 1 & 55 \\ 93 & 1 & 93 \\ 55 & 1 & 487 \end{vmatrix} = 15.26 \times 10^6$$

$$A_3 = \begin{vmatrix} Z_{11} & Z_{12} & 1 \\ Z_{21} & Z_{22} & 1 \\ Z_{31} & Z_{32} & 1 \end{vmatrix} = \begin{vmatrix} 487 & 93 & 1 \\ 93 & 487 & 1 \\ 55 & 93 & 1 \end{vmatrix} = 17.01 \times 10^6$$

The equivalent surge impedance of all three wires in parallel therefore is

$$Z_{00} = \frac{D}{A_1 + A_2 + A_3} = \frac{106.5 \times 10^6}{(17.01 + 15.27 + 17.01)\,10^6} = 216$$

as compared with the 200 ohms given by Pittman and Torok.

There are three points in Mr. Sporn's paper which I wish to discuss—his inference that a flashover of the bottom conductor (of a vertical arrangement) indicates an induced stroke, his data on the effects of tower footing resistance and his conclusions about the extra benefits of two ground wires. The voltage which can appear on a line conductor as an induced surge is

$$e = \alpha\, G\, h\, (P.\ R.)$$

where

α = reduction factor (less than unity) depending upon law of cloud discharge and distribution of bound charge,

G = field gradient at the earth due to the cloud,

h = average height of the line wire,

$(P.\ R.)$ = protective ratio of the ground wires.

Now α and G are the same for all wires, and therefore the voltages appearing on different line wires depend upon the product $h\ (P.\ R.)$, and while the protective ratio is the highest for the lowest conductor, h is the smallest for that conductor, so that the product is usually actually less for the bottom conductor than for the top conductor. On the other hand, the voltage across the insulator due to a direct stroke at the tower is

$$e_{1r} = \left(1 - \frac{Z_{1r}}{Z_{11}}\right) e_1 = \frac{Z_{11} - Z_{1r}}{Z_{11}}\, e_1$$

where

e_1 = voltage on ground wire,

Z_{1r} = mutual surge impedance between the ground wire and the line wire r,

Z_{11} = self surge impedance of the ground wire.

But Z_{1r} decreases with the separation between the line and ground wires, so that the maximum voltage across the insulation occurs for the *lowest* conductor. Curves illustrating the effect of increasing the separation are given in Fig. 13 of my paper *Critique of Ground Wire Theory*, A. I. E. E. Trans., March 1931, p. 1. Thus it is seen that a flashover on the bottom conductor may be due to a direct stroke on the ground wire rather than an induced surge. With regard to the value of low-tower footing resistances, the situation can be summed up as follows:

1. A lightning stroke will strike the nearest tower quite independently of the tower footing resistance.

2. If any tower of a continuous transmission line has an insulator flashover, it will be the tower that is struck, quite regardless of how its resistance compares with that of adjacent towers.

3. The lower the tower footing resistance the greater the reduction of voltage on the tower, ground wires, line conductors and across the insulators, and the more limited the danger zone.

These points were discussed in the above paper. The first point was illustrated in Fig. 7, the second and third points in Figs. 11 and 12. If the tower footing resistances of all the towers are not brought down to the same level, at least that of the more exposed towers should be reduced. Whether two ground wires are better than one depends on the location and arrangement of the ground wire, and the relative importance of induced and direct strokes. If a 132-kv. line with *any* kind of ground wire protection is immune from induced surges, then a single properly placed direct-hit wire with sufficiently low-tower footing resistances is as effective as any number of ground wires. But if the ground wires are placed at minimum elevations (as is dictated on the induced surge theory) then they will not be high enough to give the necessary shielding effect, nor will they have enough separation to prevent side flashes in case of a direct hit at mid-span.

It seems to me that the principal value of Mr. Dowell's paper is not so much the numerical data on attenuation which he secured, as in demonstrating how a difficult problem in traveling waves can be accurately calculated by a comparatively simple and straight-forward method. While it has been repeatedly shown that waves of relatively low potential may be treated as distortionless and excellent engineering agreement with tests obtained; yet for waves considerably above the corona limit the distortion cannot be ignored. There is a great need for a reasonably accurate and simple method for computing the distortion experienced by a traveling wave as it moves along the line.

H. L. Melvin: The results of the Arkansas Power & Light Company and Westinghouse Electric and Manufacturing Company cooperative lightning investigation has yielded two important contributions; first, the measured values of lightning voltage magnitudes have been extended and second, the practical possibility is indicated of draining lightning current from transmission conductors without permitting dynamic current follow-up for a sufficiently long interval of time to interfere with service or requiring relay and oil circuit breaker action to clear the fault.

At the present time two basic design principles present themselves for the improvement of line performance; first, diversion of strokes from the line conductors, requiring more complete knowledge concerning the characteristics and magnitudes of lightning currents and voltages, the impedance characteristics of the paths utilized for discharging the lightning as well as the nature and magnitudes of the voltages appearing across the line insulators when the lightning current is being discharged; second, the drainage scheme, the success of which will largely depend upon the development and proper application of the arc interrupting devices.

The investigation has also demonstrated that no practicable amount of insulation is effective for holding lightning voltages of the high magnitudes at least, though it may be that by utilizing the impulse insulation strength of the wood to a considerable degree, some flashovers are avoided that might have occurred with the conductors supported on normal amounts of porcelain insulation. The surges in Figs. 10 and 11 illustrate such possibilities. These oscillograms are extremely interesting and unusual as compared to the high-voltage surges causing flashover, which have been obtained from this and other lightning investigations. Many of the oscillograms of low voltages which have been held by line insulation are of long durations, however, and in my opinion the two shown in the paper may be more nearly representative of actual lightning than any which have been obtained to date. It is reasonable to expect that the transmission line would hold the one million-volt surge but the two million seven hundred thousand-volt surge would probably break to earth if it occurred anywhere on the line except in the nine-mile test section.

An explanation which seems reasonable is that a stroke of relatively low-current magnitude or a side flash from a main stroke occurred in the test section rather close to the oscillograph

station. A slightly increasing current value in the lightning stroke during the first 125 microseconds of the discharge would account for the rising voltage characteristic. Attenuation from 3,000 to 1,500 kv. or less in the four miles to the lower insulated section and to 600-kv. thirteen miles away at the substations also seems entirely possible. It must, of course, be remembered that the actual magnitudes of these high voltages are more or less approximate due to the potentiometer coupling calibration, also that they are measurements of voltage with respect to time at a point on the line referred to the earth connection at the oscillograph station. Regarding time it seems very reasonable to me that durations of lightning current flow of this order and even greater are normal rather than unusual.

C. E. Ambelang and F. E. Andrews: It would be of considerable interest if Messrs. Pittman and Torok would give an estimate of the value of lightning voltages at the points where flashover occurred. For example, in the case where the 5,000-kv. oscillograph record was taken from the lightning flashover four miles from the oscillograph station it would be interesting to know what value of lightning voltage could be estimated at structure No. 167 where the surge originated.

We assume that flashovers referred to are in general lightning flashovers, as distinguished from power arc flashovers, except that the latter is mentioned in the case of the flashover at structure No. 167. We have found in our work on 33-kv. wood pole lines that the condition under which 60-cycle power arc between conductors, or from conductor to ground, will follow a lightning sparkover determines to what extent the service on the line is affected by lightning flashovers.

We have established quite definitely by operating experience that 9 ft. of wood between conductors provides sufficient insulation to prevent line interruption due to the power follow-up, and that wood insulation in the path from conductor to ground over the guys is also effective in this same way. We have had a large number of lightning flashovers to ground on our 40-ft. poles of all wood construction without a 60-cycle power follow-up. We had approximately 90 miles of this type of construction in service during 1930.

The authors conclusion No. 1, stating that pure insulation is not a solution to the lightning problem, we interpret as referring to insulation with respect to lightning and our experience as indicated above, ties in with this conclusion.

Reference is made in conclusion No. 2, to voltages of high magnitudes being transmitted for several miles along highly insulated lines. Our observations indicate that this may not always be the case. We have observed a number of instances in which structures of high insulation value, that is, our all wood pole construction, have flashed over to ground due to lightning while adjacent structures of very low insulation have not flashed over. In one instance the highly insulated pole was only 150 ft. from the start of the pole line of lower insulated value. In another case a 40-ft. pole, 500 ft. from a structure with grounded guys, was splintered by lightning which apparently followed this path in preference to the nearby low insulation path. In no case has the travel been noted over a quarter of a mile.

In conclusion No. 5, the authors state that trip-outs on ungrounded wood pole construction are the result of line-to-line flashovers. This checks with our experience and was covered in the paper *Transmission Research and Design with the Field as a Laboratory*, by F. E. Andrews and C. L. Stroup, A. I. E. E. Trans., July 1930, p. 959. This experience has been borne out by information obtained during 1930. We have found that the steel crossarm brace used with the conventional type of 33-kv. construction short circuits a portion of the wood crossarm and must be eliminated in order to prevent power arcs from occurring on the 33-kv. construction.

We recognize that the problem with respect to preventing line outages is somewhat different at the lower voltage with which we have had experience, than with the 110-kv. lines on which the authors have conducted their investigation. However, the underlying principles are the same and our experience substantiates that which they cite, and corresponds to their conclusions with respect to line operation.

C. L. Fortescue: As the paper by Messrs. Pittman and Torok states, the line was freed from any drain points for the length of nine miles so as to provide the necessary conditions for obtaining the highest records due to lightning. Two such records were obtained during the last lightning season.

Fig. 11 has not been adequately explained in the text of the paper and I think it should be made clear that it cannot possibly be a cathode-ray oscillograph record of what happened on the line itself as on either side of the lightning experimental station four and one-half miles distant, the line was provided with fused drain points having a flashover voltage of 800 to 1,000 kv. for long surges.

While it is quite possible in the particular section in which the lightning station is located to have potentials as high as that recorded in Fig. 11, the duration of such a surge could not last more than 40 microseconds before being reduced to a value of about 1,000 kv. or less. The fact that, as stated in the paper, there was no record of flashover or blown fuses, this oscillogram must be considered as a false record probably due to something influencing the cathode-ray oscillograph circuit directly and not through the line.

The cathode-ray oscillograph is a very sensitive instrument and is susceptible to extraneous influences. Although precautions are taken to reduce such possibilities to a minimum, nevertheless they should be kept in mind in interpreting the results obtained.

It should be pointed out that with a line of this type of construction provided with drain points of the type described in the paper at one mile intervals, the duration of any surge of the magnitude indicated in Fig. 11 would not exceed 10 microseconds, before being reduced to a relatively small value.

H. C. Don Carlos: It would seem that the experience the Hydro-Electric Power Commission of Ontario has had with certain of its transmission lines would have some bearing on the points mentioned in these papers.

In 1920 a wood pole 110-kv. transmission line approximately 70 miles long and lying along the north west shore of Lake Superior, was placed in service operating at 60 cycles. The construction used embodied wooden crossarms and braces as shown in Fig. 2, the average span being about 350 ft. It will be noticed that the line is equipped with a grounded sky wire. This sky wire was grounded at approximately every fifth pole. This grounding wire was 9/32-in. steel, similar to the sky wire and after being wrapped around the bottom of the pole several times was carried up the pole and stapled to it, to within about 17 ft. of the top. At this point it was pulled off and clamped to the sky wire about three feet from the poles. It will be seen that since the standard pole was 45 ft. high, the average conductor height from the ground was about 29 ft. This line was located in the center of an 80-ft. right-of-way cut through the bush and while certain sections were in rock it is believed that about 50 per cent was through muskeg. It will be evident that this circuit is well shielded by its grounded sky wire and the nearby trees, etc. However, the district in which this transmission line route lies, is called the Thunder Bay District, which title is probably justified by the average of about 30 lightning storms per year (47 in 1925 and 20 in 1927) since the line was placed in service, with practically all of these occurring within five months, with July and August being, of course, the months when storms were most prevalent.

However, during the ten years service of this circuit, operating at approximately 120 kv. there is no record of an outage on it due to lightning. There have been several outages during this time on this circuit but in each case, these have been due either to interference with the circuit, or to other circuits which have been

solidly connected to it. During the first three and one-half years when we were solely dependent on this circuit for the supply of power to the city of Port Arthur, we were fortunate inasmuch as there were no outages on this line from any cause.

This right-of-way was widened about 60 ft. and a circuit mounted on steel towers was placed in service late in the summer of 1924. No ground wire was placed on these towers. Two outages on this new circuit this year, seven in 1925 and five in 1926 indicated the necessity for some additional protection to this circuit and a sky wire was placed in service on this tower circuit in 1927. Since we have had one outage in 1927 on this circuit due to a lightning flashover, one in 1928, two in 1929 and two in 1930, placing this ground wire in service would indicate some benefit from the sky wire.

A second circuit was erected on the steel towers in January 1930 and three outages during lightning storms in 1930 occurred on this circuit. None of these outages on this circuit occurred simultaneously with outages on the other circuit on these towers.

One or more outages per year is also experienced on the circuits of a 9-mile steel tower line extending on from the Port Arthur station which is fed from the above mentioned circuits.

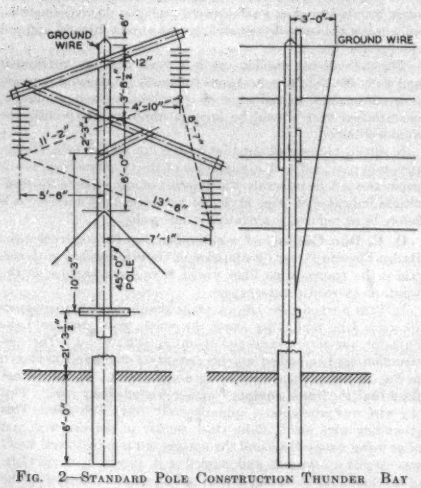

Fig. 2—Standard Pole Construction Thunder Bay System Line

Our system superintendent suggests that the majority of the flashovers in this district occur in some high ground which is supposed to contain low-grade iron ore.

Contrasting very sharply, however, with the excellent performance of this wood pole circuit (from the Nipigon River to Port Arthur) is the experience we have had with a 66-mile wood pole circuit in Southern Ontario (from St. Thomas to Sarnia) operating at 25 cycles. This circuit was placed in service in January 1926. About three-quarters of this circuit used a standard two-pole H frame construction with horizontal configuration of the conductors and 500 ft. spacing between structures, while the remainder used single poles with triangular spacing similar to the Thunder Bay System line.

No ground wire was erected on these poles when the line was placed in service but 16 outages in 1926 due to lightning indicated the urgency for some additional protection on this circuit. A sky wire was erected on this circuit in Feb. 1927 with ground wires being carried down every fifth pole. However, seven outages due to lightning were experienced in 1927, two in 1928, eleven in 1929 which and eleven in 1930, which indicate that the problem of protecting this circuit is not yet solved.

During 1929 the resistance of the earth connections at the various poles on this circuit was measured and it was found that many of these were quite high and some effort was made during 1930 to reduce these resistances by driving additional pipe grounds. A more careful survey of this circuit also indicated somewhat inadequate clearances from power conductor to guy wires. In some cases insulators, bridged by one inch gaps have been used in these guy wires. The poor performance of this circuit, which runs practically due west from St. Thomas, is hard to understand in view of the fact that a steel tower line approximately 103 miles long carrying two circuits south west from St. Thomas at an angle of about 40 degrees from this wood pole line is practically immune from lightning trouble having only one or two outages per season. Further study will be made of this circuit with a view of applying additional protection to reduce the effect of lightning on it.

In this connection a brief mention might be made of the performance of the 230-mile 220-kv. transmission circuit from the Gatineau River in Quebec to Toronto in Ontario. This circuit went into service on October 1st, 1928. Up to the present time this circuit has had a total of nine outages, of which four were due to external interference and five were due to lightning flashovers. It is believed that one of these lightning flashovers went directly to ground from the middle of the span. The second circuit on this right-of-way was placed in final service on July 28, 1930 and has not yet had an outage. The only outage on the original circuit since August 23, 1929 occurred on September 17, 1930 and did not result in any interruption to service since the second circuit was not affected.

E. R. Whitehead: I quite agree with Mr. Bewley that a knowledge of the functional impedance used in the Roseland tests is highly desirable. It was not possible to give complete mathematical treatment of these various conditions in the limited time available for preparation of the paper.

I may add, however, that methods of calculation outlined by Mr. Bewley have been used in computing complex waves resulting from capacitance reflections from the towers themselves. In large substations bus structures may be of such a length that considerable error would be introduced into calculations by considering the station capacitance as lumped at the end of the transmission line.

It is also pointed out that the equation for estimating the effect of the lightning arrester location is approximate, and cannot be used where only partial reflection takes place at the equipment or point under consideration. The results obtained show that the equation can be used where a transformer constitutes the terminal load. It was felt that such an expression would furnish a simple guide to arrester location where conditions were comparable with those of the tests, and that any errors arising from its use must be on the safe side.

J. J. Torok: Mr. Peek in discussing the 5,000,000-volt surge simulated it to the ½-5 microsecond wave. In the nature of wave shape it is true the 5,000,000-volt surge is similar to the ½-5 microsecond wave but in making this comparison one must remember that this is a chopped wave. It was chopped by flashover of the wood pole and also by a flashover from one of the conductors to a telephone line running under the line. This oscillogram does not give sufficient information to enable us to reconstruct the original wave, that is the wave that would have traveled over the conductor had not the pole flashed over. For this reason we cannot make comparisons with any particular laboratory wave.

Mr. Bewley goes to considerable length to calculate the surge impedance of three conductors. His results check our calculations, however, we assumed the lower figure of 200 ohms

simply because the mathematical calculation took no account whatsoever of corona. It is well known that corona increases the effective diameter of the conductor and consequently reduces its surge impedance. Through lack of definite data on corona we assume a somewhat lower value of 200 ohms which we believe would be more nearly correct than calculating the value assuming only the diameter of the conductor.

Fig. 11 has been referred to and is under considerable discussion. It is to be remarked here that the oscillogram shown in Fig. 11 is somewhat questionable. There are two sources in which error might creep into this set up. Both of these sources are due to the fact that a capacity potentiometer is used. A lightning stroke terminating nearby might cause an induced potential on the intermediate electrostatic plate. Since this plate is connected directly to one of the deflecting plates on the oscillograph it requires only fairly low potentials to cause a relatively large deflection in the cathode-ray oscillograph. Another possibility is that the stroke terminating nearby caused considerable difference in potential between two points in the earth, that is between the ground electrode of the potentiometer and the ground of the cathode-ray oscillograph in the oscillograph building. This difference also may be sufficient to cause a deflection. To overcome this the potentiometer has been changed to a resistance type. It is hoped that with this new type of potentiometer all external influences will be completely eliminated.

Mr. Ambelang and Mr. Andrews have brought up a very interesting point. They have pointed out that by the use of wood, power follow arcs will not be established. It is this property of wood construction that makes it so desirable as a form of line support.

Wood in forming a path for lightning streamers acts as a restricting agent and prevents the formation of power follow arcs, however, it is necessary to have sufficient length of wood in a circuit to make it completely effective. Wood, in itself, is not sufficient to prevent a lightning flashover, however, the deionizing properties of wood will serve to suppress the arc and prevent an outage which after all is the essential factor in transmission line construction.

Philip Sporn: We have gathered the data presented in this paper, endeavoring to prove or disapprove in our own mind a few of the various theories floating around; that is, whether our troubles due to lightning are the result of direct strokes or induced surges. According to the data which we have accumulated, both are to be treated alike in regard to protection, etc. In the near future we hope that some definite conclusions will be reached in allocating the direct stroke to high-potential lines and induced surges to low-potential circuits. Our data which we have collected gives a true picture of what has actually occurred and has not been distorted or influenced in attempting to give any particular theory preference. We can only analyze the data which we obtained and have merely pointed out what it seems to indicate to us. If others can draw different conclusions, in some cases, from these data, it merely shows that records are not sufficiently in detail to definitely prove some of the points which are now in question.

Electron Tubes in Industry

BY W. R. KING[1]
Non-member

General Introduction

ELECTRON tubes have been in everyday use in radio and other communication fields for a number of years. It is only recently, however, that the field of applications has been extended to include industrial usages, and while the number of industrial installations employing electron tubes is not enormous, great progress has been made in applying these new electrical tools to the problems of the industries. It is not the intention to cover the entire field of industrial tube applications in this paper, but merely to present the fundamentals of operation and the practical aspect of the application of some of the more important of these new and interesting devices.

The applications and prospective applications of electron tube systems may be broadly divided into two general groups, (1) those in which the electron tube devices perform operations for which no reasonably suitable apparatus of a conventional nature is available, and (2) those for which conventional apparatus already exists. Obviously the dividing line between what we may call "competitive" and "non-competitive" applications cannot be clearly defined. Photoelectric devices for example, have no strict competition from existing apparatus, although they frequently fall in the "borderline" class. Electron tube equipment for supplying adjustable voltage to d-c. motors is, on the other hand, competitive with conventional apparatus and must be so considered from a standpoint of performances and economy.

In weighing the advantages and disadvantages of electron tube systems consideration must be given to the absence of major moving parts. The maintenance cost for an equipment is undoubtedly decreased if it contains no bearings to be oiled and no wearing parts to be frequently adjusted and occasionally replaced by skilled labor.

Another advantage that is generally important is that electron tube systems can respond to controlling functions of a very low energy level. In adapting electrical control to automatic machinery of various types the ability to secure control from small parts or from slight movements is often the difference between the failure and success of the machine.

The fragility of electron tubes has been a much discussed disadvantage in their application. It is obvious that the assemblage of glass and small metal parts must be handled and mounted with some care. For most applications, however, the fragility of the tubes is a psychological disadvantage rather than an actual one. The tubes are really more sturdy than they appear and it is felt that few potential applications of electron tubes will be found impractical because of the seemingly fragile nature of the tubes.

The question of tube life and tube replacement is largely one of economics. Tubes operated at rated values of current and voltage may be expected to have a certain average life. If they are operated somewhat under their rating this life expectation figure will be materially increased. By way of illustration, typical estimates of tube life at rated load run from 2,000 to 10,000 hours. When used well below rated values, the tube life may be increased to as much as double its expected life at rated load. It is a matter of economics, then, to determine whether to use a certain tube at its rated load, or to use a larger and more expensive tube operating well under its rated load and thus secure a greatly increased life.

Theory and Characteristics of Photoelectric Tube

Fundamentally, the photoelectric tube is a two-element light sensitive device which depends for its operation on the emission of electrons from a light sensitive surface. There are other light sensitive devices such as the selenium tube and the electrolytic cells, but this paper is confined to a consideration of light sensitive devices in which electrons are liberated from a sensitized cathode and flow to a positively charged anode.

Sodium, potassium, rubidium, caesium, and other alkali metals have been used to form the sensitive cathode surface. Caesium is commonly used at present because its color sensitivity in the red end of the spectrum is higher than that of other alkalis.

There are at present two general types of photoelectric tubes, the gas filled tubes and the vacuum type tubes. In both types light liberates electrons from the sensitive cathode. If a source of potential is applied to the tube to make the cathode negative and the anode positive, the negatively charged electrons will be attracted to the anode. In the vacuum tubes this flow of electrons from cathode to anode constitutes the entire current passed by the tube. In the gas filled tubes however, the maximum current which can flow is much greater than that represented by the number of electrons liberated from the cathode. The reason for this is that the electrons, in traveling from the cathode to the anode, collide with the atoms of the inert gas introduced in the tube and liberate from the atoms additional electrons which are also attracted to the anode. In going to the anode these electrons collide with other atoms releasing

1. Industrial Engg. Dept., General Electric Co., Schenectady, N. Y.

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additional electrons, etc. This phenomenon is called gas amplification and its use makes possible the construction of photoelectric tubes capable of passing approximately ten times the current per unit light flux that can be passed by the vacuum photoelectric tubes.

The number of electrons emitted from the sensitive cathode (and therefore the current which the tube passes) is dependent upon the amount of light on the cathode. For the vacuum photoelectric tube the relation between light flux and anode current (see Fig. 1) is linear to a high degree of accuracy. For the gas filled tubes this relation has a slight curvature (Fig. 1) but for most purposes may be considered substantially linear. This proportionality of current to light flux is the most important characteristic of the photoelectric tube and it is upon this characteristic that most of its applications are based.

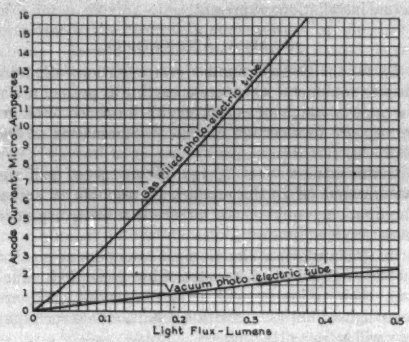

FIG. 1—LIGHT FLUX—ANODE CURRENT CHARACTERISTIC FOR GAS-FILLED AND VACUUM TYPE PHOTOELECTRIC TUBES

AMPLIFICATION OF PHOTOELECTRIC CURRENTS

To be readily useful, the minute photoelectric current (of the order of a few microamperes) must be amplified sufficiently to operate relays, ordinary recording and indicating instruments, or other devices. The most commonly used amplification systems employ a three-element thermionic amplifier tube similar to the type which has become familiar as a radio amplifier. This type of tube will be referred to as the pliotron.[2] The circuit is so arranged that a change in the photoelectric tube current changes the grid voltage of the pliotron. This results in a proportionate change of the pliotron plate current which is of the order of milliamperes and may be used to operate relays, etc. This type of circuit arrangement may be employed on alternating current or direct current. Because of the rectifying action of both the pliotron and the photoelectric tube the a-c. circuit is active only when the respective anodes are positive. The resultant output is a half-wave pulsating direct current. The effective value of this current depends chiefly on the illumination on the photoelectric tube but is affected to an extent by variations in the supply voltage.

A relay capable of operating on the few milliamperes output of the pliotron, however, cannot control power circuits directly, nor even the operating coil circuits of medium size contactors or solenoids. It is necessary, therefore, to employ an additional small contactor before currents of the order of amperes can be controlled. In some applications this is perfectly satisfactory but frequently applications are encountered in which the speed of response required is too great for this concatenation of relays. In such cases it has sometimes been found advantageous to employ a thyratron[3] in place of the relays. The operation of the thyratron in this capacity will be discussed more thoroughly later. For the present we may consider it as an electronic device which has an anode current of the order of amperes which may be controlled by certain functions of its grid voltage. In the system involving photoelectric tube, pliotron, and thyratron, the thyratron grid is controlled by the output of the pliotron in such a manner that when the illumination on the photoelectric tube reaches a predetermined value, the thyratron suddenly conducts and will carry currents of several amperes—sufficient to operate directly power size contactor coils, solenoids, etc. When the light decreases below a certain value, the thyratron stops conducting. In securing the amplification from microamperes to amperes by this system, the time lag is only a small fraction of that required by a series of relays to secure the same degree of amplification.

COLOR ANALYSIS AND COLOR MATCHING

A number of photoelectric equipments has been built for color analysis and color matching. The recording color analyzer and one type of color comparator is described here.

As its name implies the recording color analyzer is a photoelectric device which automatically draws for permanent record the color curve of a sample. The device is so constructed that the light reflected from the surface of a solid material or the light transmitted by a transparent substance may be the basis of the final curve drawn. By means of a flicker disk, the photoelectric tubes receive alternately monochromatic light reflected first from a standard then from the sample. A difference between these results in a pulsating photoelectric current which is amplified sufficiently to operate a motor and move a shutter in the standard light path. A recording pen indicates on a chart the position of the shutter at which the light from the standard equals that from the sample. As the chart is rotated

2. Trade name for electrostatically controlled high-vacuum discharge device.

3. Trade name for electrostatically controlled arc discharge device.

the monochromatic band of light is shifted through the visible spectrum. The curve obtained indicates at each wavelength the balance point at which the light from the white standard equals that from the sample; the ordinates of the curve become the reflection coefficients of the sample in terms of standard white.

The use of this device opens a new field to those working with color in view of the fact that the curve obtained from a certain sample is absolute identification for its color. It is not inconceivable that in the future numbers will be applied to the many curves and the arbitrary names such as royal blue, cardinal red, canary yellow, etc. will be discarded for purposes of specification.

The color comparator is a much simpler device than the color analyzer. Its most important use is in matching colors. The device consists of a simple optical system and an amplifier. A milliammeter indicates the difference between light reflected from two specimens placed successively in the sample stand. A tri-colored filter makes it possible to obtain comparisons in three parts of the spectrum in addition to white light. The accuracy seems to be as good as that of the eye for all samples and is better than that of the eye in the spectral regions where the eye is insensitive.

Recording or Indicating Light Conditions

Applications requiring the continuous recording or indicating of amounts of visible energy on the photoelectric tube may be divided into three general classes:

1. Light from a constant source passing through a substance of varying translucency or transparency to the photoelectric tube.
2. Light from a constant source reflected into the photoelectric tube from a surface of varying reflecting power.
3. Visible radiation from a varying source applied directly to the photoelectric tube.

In making such applications some inaccuracies are encountered due to voltage variations, changes in characteristics of photo-tubes, amplifying tubes, and light sources. Consideration must be given in each case to determine whether or not the accuracy secured will be acceptable, or if it is apparently not acceptable, whether or not the complication necessary to reduce the inaccuracies to acceptable values is warranted.

Photoelectric equipment of this nature has been used to make continuous records of smoke densities. In an application of this type a beam of light is transmitted through the duct carrying the smoke and onto the photoelectric tube. The variation in smoke density permits more or less light to fall on the photoelectric tube. When properly calibrated, then, the photoelectric current is a measure of the smoke density. A recording milliammeter measures the amplified photoelectric current and the resultant chart shows graphically the smoke conditions at all times.

Photoelectric tube and amplifying devices have been used for recording temperatures of hot bodies. At present, however, it is practically impossible for an electrical manufacturer to build a device which will be applicable to a number of different applications of this type. Each different application, then, will probably require some development, first to determine its feasibility, and second to determine the best operating scheme.

Other applications have been proposed in which a continuous record would be made of light transmitted through liquids or reflected from a moving web of paper or material. Variations in the photoelectric current would ensue from changes in color, shade, density or some other characteristic of the material affecting its light transmitting or reflecting properties. While it is felt that no insurmountable difficulties would be encountered in making such applications, it is known that the difficulties in working out a specific case would be considerable and the writer does not know of any such actual installations at present.

Relaying Applications

By far the greatest number of applications of photoelectric equipment which are operating at present are of the type in which an electric circuit is opened or closed in response to a certain change of illumination on the photoelectric tube.

In this manner street lights are being turned on in the evening and off in the morning in response to actual illumination requirements rather than according to a predetermined time schedule.

By similar devices electric signs are operated so that they are lighted when, because of low general illumination, their brilliance will attract attention, and are never lighted until illumination conditions are such that they will attract attention.

Room and factory lights can also be controlled in response to illumination requirements, with the result that sufficient illumination is maintained at a minimum cost for lighting energy.

Perhaps the simplest type of application of photoelectric equipment is that in which a beam of light falling on the photoelectric tube is intercepted by a practically opaque object and some electric circuit is closed or opened accordingly. By this means, for example, objects passing through the light beam may be accurately counted. The amplified photoelectric current operates a small relay which controls a magnetic counter. Or, if higher speed is required, the relay may be replaced by a thyratron which will operate the counter directly. Photoelectric counters are being used commercially in counting automobiles, people, steel bars and billets on rolling mill tables, drops of oil in an automatic oiling machine, etc., and are performing in such capacities in a thoroughly practical manner. Such a counting system offers distinct advantages in counting heavy objects which would soon destroy a mechanical counter, or small light objects which could not exert sufficient force to operate a mechanical

counter. Often it is difficult or physically impossible to place a mechanical counter in such a position that it will be operated by the passing of objects. In using the photoelectric equipment, the devices may be placed at a considerable distance and the light beam alone projected across the path of the moving object. The possibilities of photoelectric counting equipment have only been scratched and it is believed that the next few years will find many industrial plants using photoelectric devices to secure an accurate count of their products.

This brings us to a consideration of photoelectric devices for limit switch operations. Their use in this capacity has made possible the automatic operation of machines in a manner which has not heretofore been practicable. Mechanical devices are not suited in some cases because of mechanical wear and tear, or because sufficient force is not available to operate them, or because of space limitations. It requires no additional force for a moving object to intercept a light beam; furthermore, the beam is in no way worn by the interception. In the manufacture of small cotton bags, photoelectric relays are being employed to initiate operations on the bags when they are in the proper position. Photoelectric relays are being installed in ore bins to indicate when the ore has reached the proper level. Several large pipe facing machines have been equipped and are in operation with photoelectric devices to stop the traversing motion and advance the cutting tool when the pipes reach the proper position. In a number of instances photoelectric relays have replaced mechanical limit switches on rolling mill tables.

Photoelectric devices can be made sensitive to smaller changes in illumination than those characterized by the applications involving the interception of a light beam by an opaque object. For example, photoelectric equipment is being used to indicate the proper operation of a filter in a sugar refining process. A beam of light is passed through the solution and if the solution runs cloudy because of improper filter operation the decreased translucency causes the photoelectric device to operate an alarm. With proper illumination black spots of sufficient size on paper or other material may be used to operate a photoelectric relay either by changing the amount of light transmitted through the paper to the photo-tube or by reflecting less light into it. Sorting devices can be made to operate from changes of reflected or transmitted light.

The application of photoelectric devices to the simpler types of problems has been facilitated by the standardization of photoelectric equipments. Several electrical manufacturers have introduced complete photoelectric equipments including photoelectric tube, some type of amplifying arrangement, and a small relay. A device of this type which also includes a contactor is shown in Fig. 2. The ability to procure such devices makes it no longer necessary for the plant engineer or production engineer to begin at the bottom and learn all about photoelectric tubes and amplifying systems when he sees the possibility of employing photoelectric devices to advantage.

Pliotron: General Theory

In considering amplifying devices for the minute photoelectric currents the pliotron has been mentioned. This tube has other applications in the industrial field and no general discussion of electron tubes would be complete without at least a brief resume of its general theory and characteristics. The pliotron is a three-element electronic device depending for its operation on the thermionic emission of electrons from a filament heated by an electric current. With a positive potential on the anode (or plate), the negatively charged electrons are attracted to it. If a simple circuit of pliotron, battery, and meter is arranged so that the positive battery terminal is connected through the meter to the anode, and the negative terminal to the filament, which is heated from another battery, the

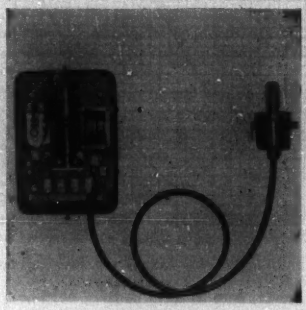

Fig. 2—Photoelectric Relay

stream of electrons from the filament to the plate will complete the circuit and the meter will indicate a flow of current. If the battery is reversed the anode will be negative and no current will flow. It is apparent, therefore, that the pliotron is a rectifying device. If the battery is replaced by an a-c. source the current which is passed will be half-wave pulsating direct current.

The grid is placed between the filament and the anode and controls the flow of electrons from the filament to the anode. If the grid is made negative it repels electrons itself and reduces the flow from filament to anode. It is possible to stop the electron flow completely by making the grid sufficiently negative. The relation that exists between grid voltage and plate current for different values of plate voltage is shown in Fig. 3. This characteristic is utilized in almost every application of the pliotron and it is this characteristic that has made the pliotron and similar vacuum tubes almost invaluable as oscillators, amplifiers of weak signals, etc.

The Pliotron as an Amplifying Device

When the grid is maintained negative its current will be very small. Therefore the grid potential may be controlled with very little power. For this reason the tube may be used as an amplifier. Consider the amplification of photoelectric currents for example. The currents are of the order of a few microamperes and the effective impedances may be many megohms. Although

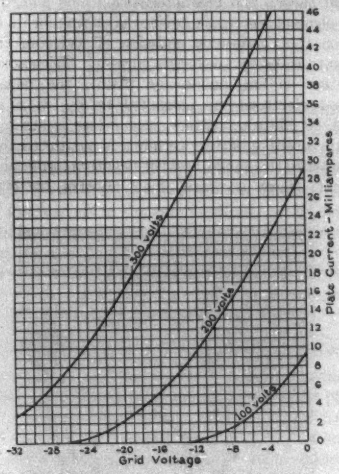

Fig. 3—Grid Voltage—Anode Current Characteristic for a Typical Pliotron

the energy level is very low, a slight change in photoelectric current causes a change of several volts in the high impedance circuit. This voltage change, applied to the pliotron grid, is sufficient to change the anode current by several milliamperes, enough for example, to operate a small relay.

This characteristic has been employed for several years in automatic train control. A small voltage is picked up from the tracks and amplified to operate signal devices in the engineer's cab.

The pliotron may be used as a very sensitive relay to respond to the opening and closing of delicate contacts or to contact between an electrode and a conducting liquid (such as water).

The Pliotron as an Oscillating Device

By a proper circuit arrangement in which the grid is inductively coupled to the plate circuit oscillations may be generated. Under certain conditions if the coupling between the plate coil and the grid coil is changed, as by placing an iron vane between the coils, the oscillations will be stopped. This characteristic of the oscillating circuit has been employed in the automatic leveling of elevators. An iron vane in the elevator hatchway passes between the plate and grid coil in an oscillating circuit on the car. When the iron vane is in the proper position the oscillations cease and a relay is closed, operating the control equipment which brings the elevator to a stop.

General Theory and Characteristics of the Thyratron

Practically speaking, the thyratron is a mercury-arc or mercury-vapor rectifier incorporating a control electrode or grid. This section is confined to a consideration of the hot cathode mercury vapor thyratron. The ionization of the mercury vapor results in an approximately constant internal voltage drop of about 15 volts.

The method of grid control is entirely different from that of the pliotron. For a given plate voltage there is a particular grid voltage at which ionization will just occur, thus allowing the tube to pass current. If the grid potential is below this critical potential, *i. e.*, more negative, no discharge will occur and no current will pass. As soon as the grid potential is increased above the critical voltage, ionization occurs and the tube passes current, providing the anode is positive with respect to the cathode. Once the anode current is

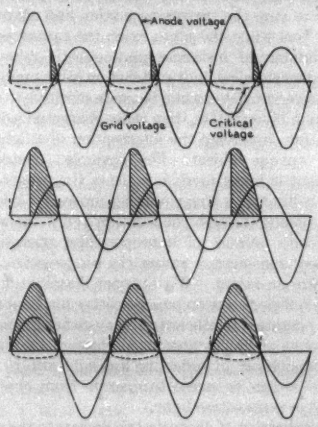

Fig. 4—Control of Thyratron by Shifting Phase of Grid Voltage

started, the grid has no appreciable effect on it. The grid cannot limit or stop the flow of current, but can regain control to keep it from starting again if the flow ceases long enough for the mercury vapor to deionize.

If an a-c. voltage is applied to the plate, the grid has an opportunity of regaining control once each cycle and can delay the start of the arc for as long a period during the cycle as the grid is sufficiently negative. Therefore,

if the grid as well as the plate is supplied with an alternating current, the phase relation between the grid and plate voltage determines the point in the wave at which current begins to pass in each cycle, hence the average amount of current passing through the tube (Fig. 4).

We may consider then, that there are two methods of controlling the grid of a thyratron supplied with alternating current; the voltage magnitude method and the phase shift method. The phase shift method makes it possible to vary the average current passed by the tube anywhere between zero and maximum while the voltage magnitude method can be readily adapted only to control between full on and full off.

Thyratrons are rated on an average and instantaneous current basis. For example, one size of thyratron has a maximum average plate current of 2.5 amperes averaged over a period not to exceed 15 seconds and a maximum instantaneous plate current of 10 amperes. This means that the tube will safely pass 10 amperes but that the duration of this current flow must not exceed a value which would make the average for 15 seconds greater than 2.5 amperes. Thus if a tube is operating as a half-wave rectifier on a 60-cycle or 25-cycle system it could have an average current of 2.5 amperes by passing 5 amperes during each positive half cycle, *i. e.*, half the time. Similarly two such thyratrons in a full wave rectifier system would have a maximum average d-c. output of five amperes as each tube could pass five amperes half the time, *i. e.*, one tube on one half cycle, the other on the next. One larger tube has a maximum average current rating of 12.5 amperes. Two such tubes in a full wave rectifier circuit could provide 25 amperes direct current. It is extremely probable that thyratrons of the future will be capable of passing current of the order of a hundred amperes or more.

The life of a thyratron will depend on the service in which it is placed. Typical life estimates run from 2,000 to 10,000 hours. If a very long life is required in a particular application it may be secured by using a tube whose current rating is higher than that actually required for the service. The question then is an economic consideration of first cost plus replacement costs of the small tube against first cost of the larger tube, allowing due consideration to the value of the greater continuity of service secured with the larger tube.

The Thyratron as a Relaying Device

The operation of the thyratron as a relaying device differs greatly from the operation of other relaying devices. It may be used as a valve to start d-c. current in a circuit but cannot readily stop the current flow as long as the anode remains positive. It can be used, though, to start and stop a flow of current if alternating current is applied. The current which passes however is not alternating current, but is half-wave pulsating direct current if one thyratron is used. The other major difference between a thyratron relay and an ordinary relay is that the thyratron has an approximately constant internal voltage drop of 15 volts.

The field of the thyratron as a relaying device is chiefly in controlling solenoids, contactor coils, magnetic brakes and clutches, etc. Such devices may be wound to operate from the half-wave d-c. voltage resulting from the application of commercial a-c. voltages to the thyratron. Such a system involves no contacts and furthermore very high-speed operation can be secured. The thyratron grid circuit may be controlled by a change in either voltage or current directly; or by a change in reactance, capacitance, or resistance resulting in a change in voltage or a shift in phase of the grid voltage; or by the opening or closing of delicate contacts.

For example, the amplified current of a photoelectric tube may be used to control the thyratron. To accomplish this, a resistor is connected in the plate circuit of the pliotron and the voltage drop across the resistor is used to control the grid voltage of the thyratron and thus determine whether it shall conduct or not conduct. In this manner a change in illumination on a photoelectric tube may be employed to operate a solenoid or similar device without the aid of any contact making or moving part device whatever.

Similarly the change in reactance in a coil caused by moving an iron core in it may be used to shift the phase of the thyratron grid voltage and control the operation of a magnetic clutch or brake.

One relaying device has been developed in which thyratrons are employed to short circuit the high-voltage secondary of a series transformer whose primary is connected in series with a resistance welding transformer. The change in the impedance of the primary of the series transformer as the thyratrons are alternately permitted by their grids to pass current and not pass current, makes it possible to control at high rates of speed the intermittent flow of current to the welding machine without any mechanical wear and tear or burning of contact tips as experienced in mechanical interrupters and contactors.

The Thyratron as a Controlled Rectifier

Although the thyratron operates as a "controlled rectifier" in most applications this term will be used herein to refer to the thyratron system in which alternating current is rectified to provide a pulsating direct current, the voltage of which may be varied smoothly and without "steps" by properly controlling the thyratron grids.

The simplest system of this type is the single-phase, full-wave, grid-controlled rectifier, which will be discussed in detail. Fig. 5 shows a schematic wiring diagram of such an arrangement. Considered as a plain rectifier, when the anode of tube No. 1 is positive it will pass current from anode to cathode, through the load, and back to the neutral point of the transformer. This current will continue to flow as long as the No. 1 anode remains positive. At the

end of that half cycle the anode of tube No. 2 becomes positive and tube No. 2 conducts, the current passing from anode to cathode, through the load, and back to the neutral point of the transformer. Each tube conducts on alternate half-cycles, but permits current to pass through the load in the same direction in each half cycle. The voltage across the load is shown in Fig. 6.

Now consider the effect of grid control upon such an arrangement with a resistance load. When the

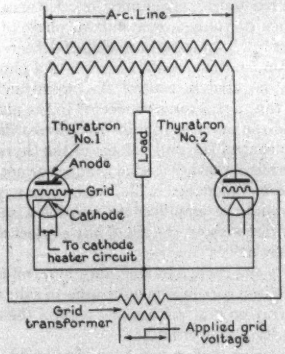

FIG. 5—FULL-WAVE GRID-CONTROLLED RECTIFIER

grid voltage is in phase with the anode voltage (*i. e.*, each grid in phase with its respective anode) the operation will be exactly the same as though the grids were not present because the grid of each tube will permit conduction as soon as the anode of that tube becomes positive. Furthermore, if the phase of the grid voltage is advanced to less than 180 deg. lead the operation will not be changed as the grid of each tube will be positive when its anode becomes positive, conduction will be permitted, and will continue for the remainder of that half-cycle despite the fact that the grid becomes negative meanwhile. If however, the

FIG. 6—FULL-WAVE RECTIFIER OUTPUT VOLTAGE WAVE FORM FOR RESISTANCE LOAD

grid phase is retarded from the "in phase" position, *i. e.*, made to lag the anode voltage, the grids will not be positive when their respective anodes become positive and conduction in each half-cycle will not commence until the grids have become less negative than the critical grid voltage. As the phase of the grid voltage is made to lag the anode voltage more and more the point at which conduction begins comes later and later in each half cycle, conduction occurs for shorter and shorter periods and consequently the average voltage across the load is decreased. Finally, when the grids lag their anodes by 180 deg., no conduction occurs because the grids are never positive when their respective anodes are positive. Fig. 7a shows the relation between load voltage and grid-phase angle lag. The inductance in the supply transformer accounts for the slight deviation of the practical curve from the theoretical.

In the case of an inductance load the operation is somewhat different because the energy storage in the inductance tends to make the flow of plate current hold over into the negative half cycle. A thorough discussion of the phenomena involved in this type of operation would be somewhat complex and probably is beyond the scope of this paper. Fig. 7b however, shows the theo-

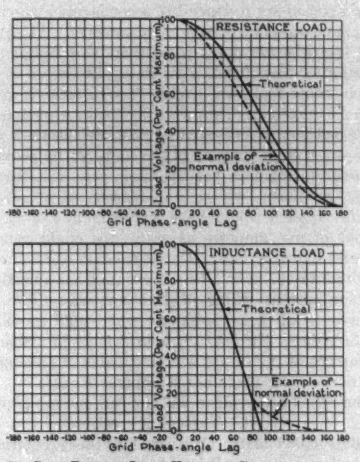

FIG. 7—GRID PHASE—LOAD VOLTAGE CHARACTERISTIC FOR A FULL-WAVE GRID-CONTROLLED RECTIFIER SYSTEM

With (a) resistance and (b) inductance load

retical relation between load voltage and grid-phase angle lag for a theoretical pure inductance, and also an example of typical deviation caused by the resistance present in a practical inductance.

To make the system serve a useful purpose it is necessary to devise means for shifting the phase of the grid voltage of the thyratrons in response to electrical functions or mechanical movements. One very simple method of securing such a phase shift is to set up a reactance-resistance bridge as shown in Fig. 8a. A change in the value of either the reactance or the resistance will cause the grid voltage phase to shift as shown in the vector diagram in Fig. 8b. If we make the reactance the coil of an ordinary a-c. solenoid, its reactance value may be varied by moving the plunger in and out. Thus by moving the plunger it is possible

to shift the grid voltage phase and change the output voltage of the thyratron system. A mechanical movement of an inch and a half can control the voltage from zero to maximum.

The a-c. coil of a small saturable core reactor may be connected in a circuit arrangement in such a manner that variation of the d-c. current through the d-c. coil of the reactor will shift the phase of the grid voltage. In this manner variations in a d-c. current of a few milliamperes may control the output voltage of the thyratron from zero to maximum.

A very small induction voltage regulator may be used in a circuit combination in which rotation of the regulator rotor causes the grid voltage to shift in phase and, consequently the thyratron output voltage to vary.

It is apparent, then, that there is a number of ways in which the phase shift of the grid voltage may be accomplished in response to changes in voltage or current, or in accordance with rotary or linear mechanical

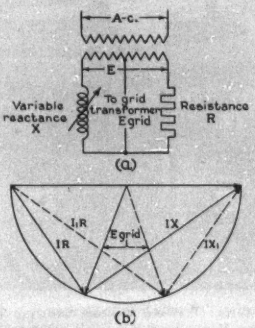

Fig. 8—Typical Circuit for Shifting Phase of Grid Voltage and Vector Diagram for this Circuit

motions. Undoubtedly many new schemes will be devised to fit particular applications as the art progresses.

The first important commercial installation of thyratron control was made in the Chicago Civic Opera House. In this installation full-wave grid-controlled rectifiers are employed to provide adjustable voltage direct current for saturable core reactors controlling the lighting effects for the stage and auditorium. The grids are controlled by means of small rotary regulator units. As the regulator unit is rotated the phase of the grid voltage is shifted, the thyratron output voltage changes, varying the degree of saturation in the saturable core reactors. The reactance of the a-c. coil of the reactor changes accordingly, permitting more or less current to flow through the lighting circuit, thus controlling the intensity of the lights. Thyratron control is also employed in this installation to control the speed of a d-c. motor by which it is possible, through a system of Selsyn[4] devices, to rotate all of the individual regulators for the various lighting circuits simultaneously, and thus control all of the lighting circuits from the rotation of one regulator.

Similar systems have been applied to other lighting control systems. The grid controlling regulators are so small that they may be continuously rotated by small synchronous motors similar to the kind used in electric clocks. This makes it possible to secure cyclic dimming effects without the use of motor operated rheostats.

The thyratron system may be used to supply small d-c. motor armature power, thus providing motor-speed control by armature voltage variation, somewhat after the manner of a non-reversing, non-overhauling Ward-Leonard system; or it may be used to supply motor field excitation to the conventional adjustable speed d-c. motors. Of course, a much greater speed range is secured by using armature voltage variation. The thyratron equipment can be controlled to deliver to the armature any voltage from zero to maximum and the motor speed will vary accordingly. Furthermore, the thyratron equipment may be controlled to vary the voltage delivered to the armature smoothly, *i. e.*, in infinitesimal steps. In applying the pulsating d-c. voltage to the motor armature, the ripple must be limited to secure good commutation. This is accomplished by connecting a suitable smoothing reactor in the d-c. circuit. As in other applications, the energy required to control the thyratron grids is very small.

The speed range which may be secured by using the thyratron equipment to supply field excitation is limited by the motor itself. The application of this system generally requires that both a-c. and d-c. power be available, since the thyratron equipment operates from alternating current and direct current must be supplied for the motor armature. The thyratron equipment in this case merely provides a d-c. source of excitation with unusual control features for conventional adjustable speed motors. The occasion that would warrant the installation of a motor-generator or rectifier to supply armature power, in addition to the thyratron equipment to supply controlled excitation, would be unusual although cases justifying such an arrangement may arise.

With the thyratrons at present available for industrial use it is possible to supply armature power for motors up to 5 hp., 230 volts, and field excitation up to 5 kw., 230 volts. Fig. 9 shows a thyratron panel for supplying armature power to a one-hp., 230-volt motor. Each of the thyratrons on this panel has an average current rating of 2.5 amperes so that the total average output may be up to 5 amperes, which is sufficient for a one-hp., 230-volt motor. Fig. 10 shows a 2.5 ampere tube with a 12.5 ampere tube. A panel similar to that shown in Fig. 9 but employing two of the larger tubes is

4. Trade name for self-synchronous motion transmitting apparatus.

capable of supplying up to 25 amperes direct current which is sufficient for a 5-hp., 230-volt motor. Power for motors larger than 5-hp. would have to be taken from a three-phase distribution and a three-phase grid-controlled rectifier circuit would have to be used.

An installation of thyratron motor control is being

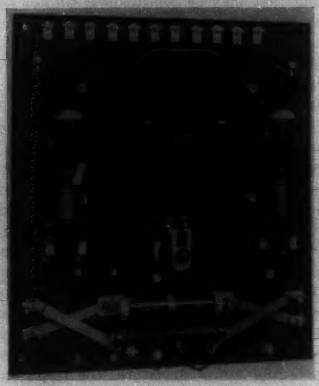

Fig. 9—Thyratron Panel for Supplying Variable D-C. Voltage to a One-Hp., 230-Volt Motor

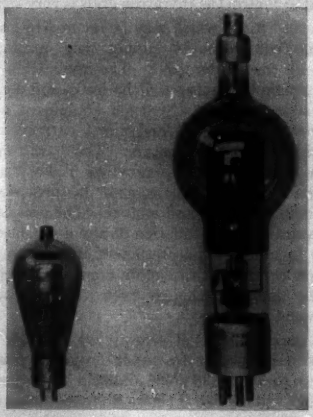

Fig. 10—The Relative Size of a 2.5-Ampere Thyratron and a 12.5-Ampere Thyratron

employed to maintain constant tension on a wire drawing winder. To successfully wind fine wire after its final drawing it has been found necessary to maintain a practically constant tension in the wire as the reel diameter increases and as the speed of the drawing machine varies. The thyratron equipment has been found to meet these requirements admirably. In this installation (Fig. 11) a solenoid armature is attached to a floating pulley and moves up or down as the tension of the wire under the pulley increases or decreases. The winder motor speed is accordingly decreased or increased by the thyratron control which supplies its armature power. As this operation is continuous and the response to a change in tension is almost instantaneous, a practically constant tension results.

Another installation of thyratron motor control is being made for controlling the speed of a motor driving a reeling machine for rubberized belting. Here again the control is to function in response to change of tension in the belting as the reel diameter increases.

Many additional possibilities are open to consideration from an engineering and economic viewpoint. The thyratron should be given due consideration wherever

Fig. 11—Wire Drawing Winder Showing D-C. Motor Supplied with Variable Voltage from Thyratron Panel and Tension Regulating Device Controlling Small Reactor in Grid Circuit of Thyratrons

Thyratron panel is mounted in enclosing case to left of machine

relatively small powers of adjustable voltage direct current are required and should be given special consideration when automatic control of such voltage is desired in response to mechanical movements or electrical functions.

Discussion

J. V. Briesky: Mr. King's paper gives a very interesting review of a number of industrial tube applications which have been made in various industries during the past few years. The importance of this general subject is attested to by the large number of articles that has been published on various phases of it, particularly during the last two years.

Electrical manufacturers have come to recognize the importance of electron tubes in industry and are devoting much time and thought to the development of industrial tubes and tube apparatus. While of course much of this tube development work is an outgrowth of the earlier work on radio, it should be realized that new lines of tubes are being developed for industrial work

because of the fact that different characteristics and sizes are required and also because of the fact that for this work long life, constancy, and sturdiness of tubes are of prime consideration.

The author deals in his paper with three general classes of tubes, *i. e.*,

1. Photoelectric tubes,

2. Grid-controlled vacuum tubes (referred to as pliotrons by the author),

3. Certain types of grid-controlled mercury vapor tubes (referred to as thyratrons in the paper).

Photoelectric tubes are fairly well known by now and many applications have already been made. Grid-controlled vacuum tubes are of course familiar to all of us because of their radio applications. Many new industrial uses are also being developed for these tubes. While their current capacity is quite small at this time, it is safe to predict that vacuum tubes handling larger currents will soon be available. This will bring about more widespread use of these tubes for a variety of applications.

The third type of tube described in Mr. King's paper, the grid-controlled mercury vapor tube employing a hot cathode, belongs to the family of grid-controlled, gas-filled tubes. This family may be subdivided into three groups:

1. Cold cathode tubes,

2. Hot cathode tubes,

3. Tubes employing a pool of mercury for a cathode.

The cold cathode tubes are the most sensitive, as far as grid control is concerned, but their current output is limited to milliamperes. The grid glow tube developed by D. D. Knowles of the Westinghouse Research Laboratories is a good example.

Hot cathode tubes may be subdivided into two types, those employing mercury vapor (this class is described in Mr. King's paper), and those using neon and similar gases. Each type has its advantages. For instance, the mercury vapor type has a somewhat lower voltage drop, while the neon type has a much smaller variation in characteristics with temperature. The hot cathode tubes are available in several sizes, from 0.1 ampere up to several amperes of continuous current carrying capacity.

Wherever large capacity tubes are required, the third group of tubes, the mercury pool type, should be considered, which can handle currents of several hundred amperes, and on which much research work is being expended at this time.

As pointed out by the author, quite a few applications of these grid-controlled gas-filled tubes have been made, but it is believed that during the next few years a tremendous increase in this activity will take place, particularly when the larger tubes become standardized and are generally available. Problems such as motor-speed control, speed matching, conversion of low to high frequency currents, will no doubt be solved by these tubes in many cases.

It is gratifying to note the increasing amount of interest taken by all industries in "industrial tubes," which interest will no doubt hasten the arrival of the day when applications of tubes will be as commonplace as those of the electric motor.

C. H. Sharp: The paper does not mention an important application of photoelectric cells and electron tubes which has been made in the electrical industry itself, namely, the photometry of incandescent lamps. At the present time the older method of photometry which depended upon the use of the eye has been to a very large extent superseded in the manufacture and testing of incandescent lamps by photoelectric photometry. The earlier photoelectric cells were not well adapted to this application because of their lack of red sensitivity. The caesium cell does not suffer from this disadvantage but has a spectral sensitivity distribution which is such that with a moderate amount of correction by color filters it is applicable to photometric purposes when the color differences between the lamps to be measured lie within the ordinary range of commercial lamps. An amplifier with any one of a number of hook-ups is added to the photoelectric cell so that only sturdy, easily-managed instruments are required. Measurements are made either by a deflection method or by a null method. The speed and precision of photometric measurements have been greatly enhanced by this application.

In one laboratory, at least, the photoelectric cell has been applied to the determination of the color temperature of lamps whereby a color match of a lamp against a standard is readily made to within five degrees. The procedure by which this is done has been described in the *Journal of the Optical Society of America*, volume 30, page 62.

H. S. Phelps: The proposal to use a photoelectric switch for control of street lighting circuits is of particular interest since:

1. Such an application should result in reduced capital investment and,

2. Municipal lighting would be scheduled in accordance with necessity for illumination in lieu of a schedule defined by the hours of the day.

Among the points that must be borne in mind in studying the possibilities of this application are:

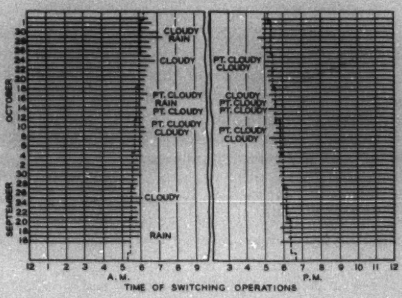

FIG. 1—PHOTOELECTRIC CELL STREET LIGHT CONTROL UNIT

The broken lines indicate the Philadelphia street lighting schedule. Unit installed on east gable of radio shack on roof of 2301 Market Street with window exposed north.

a. Reliability.

b. Cost of Maintenance.

An operating test of such a device for a period of several months has disclosed:

1. Very close approximation to the municipal street lighting schedule in effect where the tests were made.

2. Extension, both night and morning, of the scheduled lighting period on cloudy or stormy days.

3. Rare operation as the result of storms that occur during the day except those in progress near sunrise and sunset.

It still remains to determine:

If the kw-hr. consumption allowed by this device during an average year differs from the consumption under fixed schedules sufficiently to require revision of present flat-rate contracts.

Is the deviation of individual years from the average year sufficient to indicate that contracts for municipal lighting service controlled by photoelectric switches should be on a kw-hr. consumption basis rather than on a flat rate per lamp year.

Fig. 1 shows the relation observed between the Philadelphia street lighting schedule and that maintained by a photoelectric switch for the period indicated.

Synchronous Motor with Phase-Connected Damper Winding for High-Torque Loads

BY M. A. HYDE, Jr.[1]
Associate, A. I. E. E.

Synopsis.—Considerable progress has been made in recent years in the application of low-speed synchronous motors to industrial loads requiring both high starting torque and high pull-in torque with moderate starting current demand. The problems encountered are exemplified in the drive of grinding machinery in the cement and mining industries, to which several different adaptations of synchronous motors are discussed. A form of synchronous motor with phase-connected damper winding has been devised for this type of service, developing high-torque / kv-a. ratios in a simple mechanical construction.

Introduction

Of the many directions in which the synchronous motor has within recent years extended its field of usefulness, one of considerable importance is that particular field of application involving low-speed ratings,[2] where the conditions of service impose low-starting current with a combination of high-starting and pull-in torque requirements.

While the permissible magnitude of starting current varies greatly in different installations, in general low-starting current is a desirable qualification and in many cases one of prime importance. By way of comparison take as a reasonable criterion of low current starting performance the usual engine type or direct-connected compressor motor, which is ordinarily started on full voltage drawing approximately 275 per cent full load current.[3] In this particular application due to the feasibility of starting the machine unloaded, the exerted starting and pull-in torques are low, ordinarily about 50 and 40 per cent respectively, and the pull-out torque about 150 per cent.

Most industrial drives do not present such a fortunate combination of requirements. There are many applications requiring either starting torque or pull-in torque to be emphasized and this is readily accomplished with motors of moderate inrush through the development by designers of improved methods for calculating starting performance and the employment of a variety of damper winding materials.

A more difficult problem is presented by the class of drives requiring development of full load pull-in torque and starting torques substantially in excess of this value, with moderate current inrush. As an example of this type of application take the ball mills used in the mining industry for reduction of ore, and the tube and compartment mills used in the cement industry for the grinding of raw materials and cement clinker.

These machines consist essentially of a cylindrical steel shell mounted horizontally, either on rollers supporting tires on the mill shell, or on hollow trunnions on which the shell is rotated and through which it is usual to pass the material under reduction. While passing through the interior of the rotating shell, the material is subjected to the grinding action of a charge of steel balls or slugs. The grinding charge is restrained in the mill by partitions containing slots or other openings to allow passage of the ground material. Rotation of the mill is accomplished through suitable gearing to the shell. The pinion shaft for tube and compartment mills operates usually at 150 to 180 r. p. m. with a single reduction to the shell which turns at 19 to 25 r. p. m. depending upon the diameter. Motor sizes range commonly from 200 to 1,000 horsepower. Ball mills of sizes requiring 200 horsepower or over usually employ single reduction gearing with pinion shaft speeds from 200 to 360 r. p. m. although sometimes as low as 150 r. p. m. A low-speed motor on the mill pinion shaft provides an excellent drive not only for mechanical simplicity but because the operating speed is constant and the magnitude of the load makes high power factor a desirable characteristic.

The load on such a mill for a given shell diameter and speed, is determined almost entirely by the weight of grinding charge, the weight of material under reduction being small in comparison. The grinding charge remains in the mill at all times, it being impracticable to start with the shell empty. It is apparent that after the mill has been accelerated to a speed approaching synchronous speed, the effective pull-in torque of the driving motor must be approximately the full-load running torque. The required starting torque is quite variable, depending upon the type and condition of the mill bearings and the condition of the mill charge, which latter must be lifted through a portion of a revolution before "spilling" occurs. While the required starting torque is often as low as 125 per cent of the normal running torque, it is usually desirable to provide for starting torques in the order of 200 per cent, to take care of the worst conditions.

1. Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

2. The term low-speed ratings refers to the usual classification covering speeds of 450 r. p. m. and below.

3. Unless otherwise stated herein, starting currents expressed in percentages are based on the current corresponding to operation at full mechanical load at unity power factor, regardless of the power-factor rating of the motor.

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31-43

Since these units, particularly in a cement plant, may be few in number and yet constitute an important proportion of the plant load, it is usually desirable that the current inrush at starting be moderate.

Another consideration—a mechanical one—results from the large masses involved in these mills. For example, a certain cement compartment mill driven by a 600-hp. motor, has a total weight of moving parts, including grinding charge and material under reduction, of approximately 120 tons. The inertia of such masses makes it desirable to start the mill with the initial exertion of a moderate torque, only sufficient to start the mill under favorable conditions. This takes up any play in the drive with less impact and imposes less strain on gearing and shell than would result from the sudden application, each start, of the maximum starting torque of which the motor is capable as determined by ability to start the mill under the most severe conditions.

Review of Previous Practise on Applications Requiring High-Starting and Pull-in Torque

(*A*) *Motors of Standard Electrical and Mechanical Construction.* These have been used to some extent where the the high starting kva. is not objectionable. However the motor is physically oversized as compared with a motor of compressor service characteristics, which affects adversely its current inrush. Such a motor to operate at say 180 r. p. m., delivering 190 per cent starting torque and 110 per cent pull-in torque, will have a frame size corresponding to a compressor motor of almost twice its rating and an inrush of some 650 per cent, or starting torque / kva. ratio of about 0.30.

In 1927 Mr. H. V. Putman in his paper[4] examined the use of the double squirrel-cage type of damper winding in salient-pole machines, pointing out the greater suitability of this type of construction to applications requiring relatively high pull-in torques with low starting kva., the starting torque not materially exceeding the pull-in torque. The same paper brings out the combination of the torque of the bars of an ordinary damper winding with the torque of the field to produce high accelerating torques, the effect of the field being particularly important in low-speed machines. The double-cage form of damper winding is less effective in low-speed motors than in high-speed machines because of the limited space available in the low-speed pole head for submerging the high reactance element.

(*B*) *Mechanical Modifications of Standard Motors.* In the past the combination of starting characteristics required by tube mills and similar drives has been met largely by employing motors of ordinary low-torque electrical characteristics with various forms of clutches—magnetic, mechanical or the equivalent thereof—embodied sometimes in special mechanical construction, thus permitting the motor to be started and synchronized without load. The problems of starting and pull-in torque are thereby avoided entirely, it being necessary simply to make available adequate pull-out torque for starting and accelerating the load. Low-speed motors of this type are readily built with pull-out torques of 200 per cent and starting currents comparable to those of compressor motors. As an example of the extent to which this design may be carried, in a certain installation a 450-hp., 180-r. p. m., 80 per cent leading power-factor motor of this type delivers a maximum starting torque, or pull-out torque, of 216 per cent and starts on full voltage, drawing only 192 per cent of its full load running current. On the unity power-factor basis this percentage current would be 240 per cent.

This form of drive has the possibility of very smooth acceleration of the load without initial shock to the drive, when the loading device is properly constructed and controlled.

Although electrically advantageous, constructions of this type are likely to have mechanical drawbacks, the ideal arrangement being the simple two-bearing coupled type developing the necessary starting and pull-in characteristics electrically and without the need of any intermediate device for accelerating the load.

(*C*) *Motors of Ordinary Construction Applied with Special Control.* (1) Application of Over-voltage During Starting Interval: Temporary overvoltage may be applied to the motor stator throughout, or during any part of the starting interval. The induction motor torques will be increased as the square of the primary voltage, but the input kva. will also increase in the same ratio, so that there is no gain in the torque per kva. ratio. Due to the intermittent nature of overvoltage application, this scheme allows the motor to be built in a physical size more nearly normal for the running load than is the case with the straight full-voltage starting motor built for equally high torques. Depending upon the elaborateness of the voltage control, this system may or may not have the merit of smoothly handling the load during starting.

(2) Starting with Capacitor Across Motor Terminals: This improves the power factor and resultant input current of the combination. To reduce the cost of the capacitor it may be applied on an intermittent basis to operate at overvoltage, being disconnected at the end of the starting period, the capacitor kva. varying as the square of the voltage. This scheme improves the starting torque/kva. ratio by reducing the starting kva. rather than increasing the torque. It does not reduce the physical size of the motor and does not provide for an initial reduction in the starting torque for smooth handling of the load.

(3) Use of a Variable Resistor in Field Circuit: It is well known that the shape of the speed-torque curve is greatly affected by the amount of resistance in the field. The lower resistance values tend to flatten

4. *Starting Performance of Synchronous Motors*, A. I. E. E. Trans., January 1927, p. 39.

the speed-torque curve near synchronism giving low slip, but at the same time tend to break the curve at a lower torque, and produce cusps. To take most advantage of the torque exerted by the field, several machines have been built in which the field is closed on a high resistance initially, in the order of 100 times the field winding resistance, which in combination with a high-resistance, low-reactance damper winding, produces a high-resistance machine, resulting in high starting torque per kva. As the motor accelerates the field resistance is reduced in steps by the control to a value of three or four times that of the field itself producing a low-resistance field circuit and thereby improving the torque near synchronism.

The use of a high resistance across the field at starting increases the extent to which the voltage induced in the field winding appears across the field terminals. This voltage, being produced by the transformer action of the field winding as a single-phase secondary to the polyphase primary winding, is proportional to the number of turns in series in the field winding. Hence the fields of these machines were designed with several sections in parallel thus reducing the total turns in series and the induced voltage, but likewise reducing the d-c. excitation voltage. A 600-hp., at 150-r. p. m., 48-pole, 60-cycle motor of this construction with 120 per cent pull-in torque and 205 per cent pull-out torque developed 240 per cent starting torque with 495 per cent starting kva., a starting torque/kva. ratio of 0.48. The voltage across the field at starting is about 3,500 volts, the motor exciting from a 50-volt supply.

Very high-starting torque as compared to pull-in torque was found to be typical of this construction. In fact the starting torque was so high that on tube mill applications a primary resistor was employed in the control equipment whereby the initial starting torque could be considerably reduced. A 300-hp., 450-r. p. m. machine of this type belted to a large jaw crusher in a quarry delivered a starting torque of 275 per cent with a somewhat better torque/kva. ratio, a pull-in torque of 100 per cent at 5 per cent slip and a pull-out torque of about 290 per cent, characteristics very desirable in an application of this sort.

The Synchronous Motor with Phase-Connected Damper

Experience gained in the application of motors of the type just described, emphasized the desirability of a synchronous motor with the starting characteristics of a slip-ring induction motor. The commercial solution of this problem has long been the desire of synchronous motor engineers. In undertaking this problem it was decided to adhere to the salient-pole construction rather than to use the synchronous-induction form of machine, in view of the natural advantages held by the salient-pole type in physical size, efficiency, and excitation characteristics.

Within the past two years several installations have been made employing such a low-speed motor, developing a pull-in torque in excess of full load torque, with adjustable starting torques up to and exceeding 200 per cent, and with remarkably high-torque/kva. ratios. This motor is of salient-pole construction with the very desirable operating features of that type. Starting characteristics resembling those of the slip-ring induction motor are attained through the use of a special form of damper winding, in which the bars are of low resistance and, instead of being joined to end rings to form the usual cage construction, are phase-connected and brought out through slip rings to an external starting resistor which is varied by suitable control. The idea forming the basis of this development was included in Mr. Putman's paper mentioned above.

To develop the induction motor torques to the greatest extent the construction throughout is directed toward low reactance. Thus the stator employs a larger number of slots with more coils and fewer turns per coil than commonly used in ordinary low-torque

Fig. 1—A 1,000-Hp., 40-Pole Motor with Phase-Connected Damper Winding. Collector Housing Open

machines. The field is open-circuited at start thereby eliminating an undesirable low power-factor component in the primary current ordinarily reflected from the circulation of current in the highly reactive field circuit when closed upon a resistor of the proportions commonly used. The disadvantage generally resulting from open field starting is the high transformer voltage on the field winding at standstill as discussed above. On this account the first motors of this type were designed with fields for low-voltage excitation.

Fig. 1 shows one of two 1,000-hp., 180-r. p. m., 100 per cent power factor, 2,200-volt, three-phase, 60-cycle motors of this form, built in 1928 for cement compartment mill drive. The excitation of these units is at 45 volts, 8 kw. each. The induced voltage across

field terminals at start is approximately 3,300 volts. The following data on starting performance apply:

Pull-in torque at 5 per cent slip	125 per cent
Maximum starting torque	210 per cent
Locked starting kva	430 per cent
Ratio starting torque/kva	0.49
Pull-out torque	180 per cent

Since the transformer voltage in the field is proportional to the number of field turns in series, a field sectionalizing device suggests itself as a means of reducing this voltage. This parallels the practise of using a

Fig. 2—A 300-Hp., 164-R. P. M., Motor of the Phase-Connected Damper Type, with Control as Installed in a Cement Plant

field break-up switch on the synchronous converter, but with the very important mechanical difference that the synchronous motor field is conventionally on the rotor, where sectionalizing is less convenient. However, experiments were made with a 300-hp., 164-r. p. m. machine wound for 125-volt excitation and equipped with a centrifugally operated sectionalizing switch

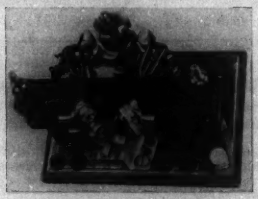

Fig. 3—Centrifugal Sectionalizing Switch-Cover Removed

mounted in the rotor, and these experiments were so successful that after extended shop tests on the durability of such a switch and the practicality of its use, an installation of such a motor was made in 1929 to drive a tube mill in a cement plant, where it has since been in regular operation. Fig. 2 is a view of this installation.

The following test values on this 300-hp. machine are quite similar to the characteristics of the preceding 1,000-hp. motor, as was to be expected since the only radical change introduced was the adoption of the field sectionalizing switch.

Pull-in torque, 5 per cent slip	115 per cent
Maximum starting torque	210 per cent
Locked starting kva	460 per cent
Ratio starting torque/kva	0.46
Pull-out torque	234 per cent

The voltage induced on each section of the field of this machine at start is about 3,000 volts. This voltage diminishes as the rotor accelerates, being proportional to the rotor slip. The response of the sectionalizing switch to speed is very sharp and consistent, since the centrifugal force on the actuating member varies as the square of the speed. Closure of the switch solely in response to speed, or slip, means closure only upon the existence of a predetermined low percentage of induced field voltage.

Fig. 4—Rotor of a 300-Hp., 164-R. P. M. Motor with Phase-Connected Damper Winding

Fig. 3 shows a switch designed for this service. The unit is enclosed in a tight welded steel housing so proportioned as to mount within a rotor as shown in Fig. 4 at suitable distance from the shaft center to develop the necessary actuating force at the desired rotor speed. As mounted on the rotor the switch is directly above the motor shaft when in the position shown in Fig. 3. At the closing speed the weight swings outward centrifugally against the restraining spring, closing the switch contacts. The mechanism is so arranged that the closing action, once initiated, proceeds very rapidly, and also that upon stopping the motor the switch maintains its closed position until a relatively low speed is reached, permitting the field to be connected to a discharge resistor and opened in the usual way by the field switch on the control panel. The sectionalizing switch is adjusted during the motor test, after which all moving parts are secured and, with the exception of the spring, made incapable of further adjustment.

The unusual appearance of the rotor shown in Fig. 4

results from a later improvement in the starting performance through the adoption of laminated rim construction to minimize the eddy currents in this member during starting.

Another feature apparent in Fig. 4 is the adoption of a comparatively broad pole face permitting the damper winding to occupy a greater pole embrace and to be more favorably distributed from the standpoint of phase balance.

This unit, rated 300 hp., 164 r. p. m., 100 per cent power factor, 2,200 volts, three phase, 60 cycles, developed the following starting performance:

Pull-in torque at 5 per cent slip	122 per cent
Starting torque with 15 per cent damper resistance	230 per cent
Corresponding locked kva	396 per cent
Ratio starting torque kva	0.58
Starting torque with 30 per cent damper resistance	169 per cent
Corresponding locked kva	280 per cent
Ratio starting torque kva	0.60

Fig. 5—350-Hp., 400-R. P. M. Installation on Preliminary Grinder in a Cement Mill

The transformer field voltage at start tested about 3300 volts per section when split into three parts by the sectionalizing switch.

Fig. 5 shows a 350-hp., 400-r. p. m., 80 per cent power-factor motor of this construction driving a centrifugal type rock grinder. This installation operates on an excitation supply of 250 volts direct current and in this case the calculated transformer voltage on the field was such as to be undesirably high even when split into three sections, so two switches were mounted diametrically opposite on the rotor and the break-up process carried to five sections, the voltage per section at standstill being then about 4,000 volts.

This machine being of higher speed would naturally tend to exhibit a somewhat higher starting torque kva. ratio, and to further improve the starting performance it was provided with starting grids of non-magnetic alloy to minimize the reactance in the damper winding circuit

The following starting performance was obtained:

Pull-in torque at 5 per cent slip	118 per cent
Starting torque with 25 per cent damper resistance	190 per cent
Starting kva. (based on 80 per cent power factor)	215 per cent
Ratio starting torque/kva., 80 per cent power factor	0.88
100 per cent power factor	0.71
Starting torque with 40 per cent damper resistance	152 per cent
Starting kva. (based on 80 per cent power factor)	166 per cent
Ratio starting torque/kva., 80 per cent power factor	0.91
100 per cent power factor	0.73
Pull-out torque	210 per cent

Control

The results attained in this type of motor are to an unusual extent bound up in the control equipment.

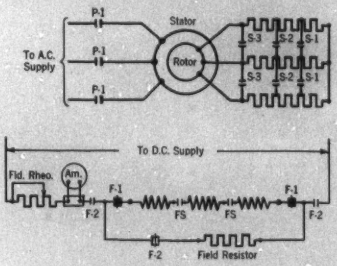

Fig. 6—(a) Typical Elementary Control Diagram

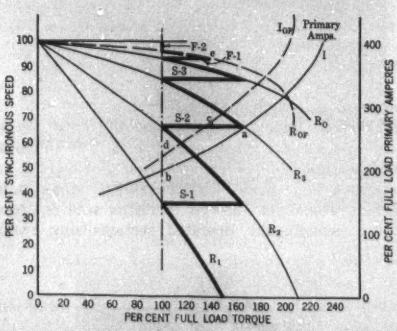

Fig. 6—(b) Typical Speed-Torque and Current-Torque Curves

Curve R_O represents speed-torque with damper winding short-circuited, field open

Curve R_{OF} represents speed-torque with damper winding short-circuited, field closed on resistor. Current-torque curve I corresponds to speed-torque curves R_1, R_2, R_3, and R_O. Current-torque curve I_{OF} corresponds to speed-torque curve R_{OF}

This employs the primary switch, field switch, and resistor, and necessary auxiliary and protective relays found in the usual full-voltage synchronous motor starter; and in addition two or more accelerating contactors and starting resistor in the damper winding circuit, as well as a spring-actuated auxiliary field

switch. The definite time sequence scheme of starting has been used thus far, in which the switching operations have been controlled by a motor-driven master drum. Fig. 2 shows a typical arrangement of control and also indicates the physical proportions of the starting resistor which in this case is mounted above the control panel.

The operation of the equipment can be readily understood by reference to the elementary control diagram shown in Fig. 6A, and the corresponding typical speed torque and speed-current curves shown in Fig. 6B. In starting, the first operation is to open the auxiliary field switch F-1, which, being a spring-closed contactor must have its coil energized to open. Next the primary switch P-1 is closed, thus starting the motor with such maximum resistance in the damper winding circuit as

Fig. 7—Auxiliary Field Switch Specially Designed High-Voltage Service

secures the desired initial torque, for example, 150 per cent. Assume that this torque is sufficient to break the static friction of the load and that the load torque then diminishes to 100 per cent. The motor accelerates along the speed-torque characteristic R_1 until a balancing speed is reached at the load torque. The motor continues to operate at this speed until the end of a predetermined interval when the first secondary contactor S-1 closed, transferring operation to the second speed-torque characteristic R_2. Now if the initial motor torque had not been sufficient to start the load, the motor would have remained at rest until closure of S-1, when a higher standstill torque (in this case 210 per cent) would have been exerted, and the starting current instead of being about 250 per cent as for 150 per cent torque, would then be about 350 per cent. If this torque is insufficient to start the load an overload relay can be arranged to remove the motor from the line. Assume acceleration along R_2 to a balancing torque of 100 per cent; then after the termination of a second predetermined interval S-2 closes and the motor accelerates along R_3 to the balancing speed, after which S-3 closes, short-circuiting all the external damper circuit resistance. The motor is now operating on a comparatively flat speed-torque curve R_o due to the low resistance of the copper damper winding. At the low slip now existing, the transformer voltage on the field has reduced to a small percentage of its initial value, and the centrifugal sectionalizing switch $F\,S$ has closed its contacts. The field will now exert an appreciable torque if closed on a suitable resistor, and this is done by deenergizing the coil of F-1 whose spring-closed contacts then complete the circuit through the field resistor, and the motor now operates on a still higher speed-torque curve R_{oF}. As the induction motor torques vary between the extreme values indicated by heavy lines on curves R_1, R_2, R_3, and R_o, the primary current varies between points a and b on curve I. Upon transfer from speed-torque curve R_o to R_{oF}, the primary current transfers to curve I_{oF} point c and recedes to point d when the motor reaches the highest induction motor speed at 100 per cent torque. The motor is now near synchronism, and when the field switch F-2 is finally closed the motor synchronizes and operates thereafter as an ordinary synchronous motor. In line with usual control practise, the field resistor is removed by opening of the back contact shown on F-2, at the time the main contacts on this switch are closed. The auxiliary switch F-1 is made spring closed so as to be deenergized during running operation and thus normally in position to carry both the exciting and discharge circuits for the field, allowing the field excitation switch F-2 to complete these circuits in the customary manner. At the same time the auxiliary field switch performs the functions of separating the high voltage on the motor field terminals from the remainder of the circuit at start, and of closing the field on its resistor later in the starting sequence, when the field torque is a substantial factor in the high pull-in torque delivered by the machine.

Fig. 7 shows a switch constructed for this service. The lower stationary studs are connected to the motor-field terminals. An interlock on this switch carries the initial closing circuit of the primary switch, which arrangement makes it impossible to start the motor without first opening this switch. The switch may be mounted on a separate small panel at the rear of the main control panel, thus removing the field voltage at starting from the main control panel. The installation view in Fig. 2 shows the two starting resistance contactors at the bottom of the panel and the operating mechanism for the oil-immersed 2,200-volt primary switch, the remaining switches and relays being enclosed under a dust tight steel cover as a protection against the dust conditions incident to cement mill service.

The construction described above yields pull-in torques somewhat in excess of full-load torque and

starting torques up to approximately twice full-load torque with starting torque/kva. ratios materially exceeding values heretofore published for low-speed motors without loading devices. The extent of improvement may be appreciated by comparison of the value 0.60 for the 300-hp., 164-r. p. m. motor described above, with the value 0.30 considered good for the usual standard machine of similar rating and torques. Or, in comparison with a compressor motor delivering 50 per cent starting torque and 40 per cent pull-in torque with 275 per cent kva., here is a motor delivering over three times these starting and pull-in torques with approximately the same kva. These high torques are developed in a manner similar to the starting characteristics of the slip-ring induction motor and are transmitted to the load through the simplest of mechanical parts. The salient-pole construction is used, with its advantageous operating characteristics including high efficiency, low excitation requirement, and commercial d-c. excitation voltage.

Discussion

For discussion of this paper see page 615.

Synchronous Motors

Design and Application to Meet Special Requirements

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Synopsis.—*Many present day applications of synchronous motors call for modifications of normal motor characteristics, as regards starting torques, thermal capacity of the starting winding, synchronizing ability, maximum synchronous torque, and mechanical design. The salient-pole motor with squirrel-cage amortisseur winding has advantages over other types which have led to the general adoption of this type as standard. Without abandoning this form, the starting and the synchronous characteristics can be adjusted to meet a variety of requirements by changing the proportions of the design and by using special forms of control equipment. Other forms, such as the synchronous induction motor, the phase-wound damper motor, and the supersynchronous motor, are sometimes justified for meeting strict starting current limitations or for accelerating extremely heavy inertia loads.*

In the mechanical design of machines for definite space requirements, fabricated construction allows changes in construction to be made at small expense, and often permits motors to be built integral with the driven machine.

INTRODUCTION

SYNCHRONOUS motors have been applied, in recent years, to drives for which only the induction motor was formerly considered suitable. This practise has resulted from the need for motors operating at unity or leading power factor and for motors having high efficiency at low speeds. Great improvements in starting characteristics and in mechanical design have made synchronous motors available for these applications. New methods of control and mechanical construction have also extended their use.

This paper describes the modern normal design and shows how characteristics may be varied through electrical design and control and how mechanical arrangements may be varied.

Normal Design Features

Before discussing the normal characteristics of conventional synchronous motor designing, it is well to consider why the salient-pole machine has been accepted as the conventional type, rather than one of the other possible designs.

To permit taking full advantage of separate excitation with the large number of field ampere turns required, it is desirable that the field copper space be a maximum. Therefore, very deep field slots are required, and for maximum space factor, only one slot per pole should be used. Hence, the salient-pole field winding is greatly preferable to the phase-wound field for a synchronous motor. For very high-speed machines, mechanical stresses prevent the use of salient poles, but otherwise they are almost universally employed.

Due to its greater excitation capacity, the salient-pole synchronous motor may employ a relatively large air gap, which is mechanically desirable, without impairment of electrical characteristics.

In order to reduce the magnetizing kva. induction motors, it is necessary to use small air gaps of the order of 0.25 per cent of the pole pitch. Mechanical limitations prevent the use of air gaps smaller than about 0.1 per cent of the diameter, so that induction motors with more than 8 poles are definitely handicapped. The normal design of the salient-pole synchronous motor permits a natural air gap of the order of 1½ to 2 per cent of the pole pitch and mechanical limitations are not reached until the motor has 60 poles or more. For this reason, the lower the speed, the more favorable are the synchronous motor characteristics in comparison with those of the induction motor.

In addition to its desirability from the mechanical standpoint the large air gap makes possible the use of larger stator slots with consequent improvement of space factor and efficiency. This comes about because the slot pitch is limited to a definite relation with the air gap to avoid high-tooth frequency iron losses. Furthermore, small high-frequency flux pulsations cannot cross a large air gap, hence thicker laminations can be used in the pole pieces.

While the rotating member may be either the field or the armature, the rotating field is the normal design with centrifugal stresses and simplicity dictating that it be inside the armature.

This normal design provides space in the pole face for an amortisseur winding with ample heat capacity to enable the motor to start most industrial loads. Actual starting torque and starting current characteristics vary with motor ratings as described later.

The conclusion is that the normal type of synchronous motor which is best for steady load operation has the following design characteristics:

1. Separate d-c. excitation. 2. Internal revolving field. 3. Laminated salient-field poles. 4. Air gap equal to about 1½ to 2 per cent of the pole pitch. 5. Amortisseur winding section of sufficient heat capacity to enable the motor to safely start the load. 6. Open armature slots, somewhat fewer in number than those of an equivalent induction motor.

Without abandoning this basic form, the electrical design may be proportioned to obtain the characteris-

1. Both of the General Electric Company, Schenectady, N. Y.

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tics required for many exacting applications; and where this is possible, it is preferable to use the normal type synchronous motor rather than to develop other types with a limited range of usefulness.

Normal Starting Characteristics

Starting and accelerating characteristics vary with speed and horsepower. A 100-hp., 60-pole motor will have high reactance, and consequently, low starting

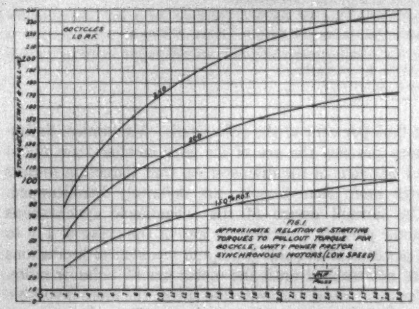

FIG. 1—APPROXIMATE RELATION OF STARTING TORQUES TO PULL-OUT TORQUE FOR 60-CYCLE, UNITY POWER-FACTOR SYNCHRONOUS MOTORS. (LOW SPEED)

and pull-in torque while a 1,000-hp., 20-pole motor will have low reactance and high starting and pull-in torques. The empirical curves of Fig. 1 approximate the relationship between torques at synchronous and subsynchronous speeds for 60 cycles, 450 r. p. m. and lower. Table I shows typical torque requirements, often the major consideration, for various types of drives.

TABLE I

Application	Torque requirements		
	Start Per cent	Pull-in Per cent	Pull-out Per cent
1. Reciprocating compressors (unloaded while starting)			
Air and ammonia (normal)	40	40	140
Low-pressure & vac. pumps	40–80	40–80	140–175
CO^2 compressors	40–90	40–60	150
2. Hydraulic pumps			
Reciprocating (starting unloaded)	40	40	140
Centrifugal	30–40	60–110	150
3. Fans-centrifugal compressors	15–40	50–120	150–175
4. Paper industries			
Pulp grinders	50	50	150
Jordans	50–60	50–60	150
Beaters	125	100	175
5. Rubber industries			
Mixers cracking mills	100–125	100	200–250
Plasticators	100–125	100	150
6. Rock Products			
Ball or Tube Mills	140–150	100–120	150–175
Hammer Mills	150–175	100–120	200–250
7. Coal pulverizers	150–175	100–120	200–250
8. Mining industry			
Ball or tube mills	175–200	100–125	175–200
Gyratory crushers (secondary cone type)	100–150	100–125	175–200
(usually started light, with lower torque)			
9. Steel			
Continuous rolling mills	50–100	100	250–350

(Subject to wide variations)

MODIFICATIONS OF NORMAL STARTING CHARACTERISTICS

It is evident that for many applications involving low-speed motors, it is necessary to find means of increasing the starting and accelerating torque: while for high-speed motors it is frequently desirable to reduce both the torque and the starting inrush. For the normal low-speed machine, the starting inrush is 250 to 350 per cent of the kva. rating at approximately 30 per cent power factor lagging; while for high-speed machines the inrush varies from 500 to 800 per cent of normal rating at 30 to 50 per cent power factor.

Reduction of Starting Current and Torque

Heretofore where reduction of starting current or torque has been required, autotransformers have been most widely used, while series reactors and resistors are sometimes used because of simplicity.

Increasing the reactance in the primary windings at starting reduces the inrush and torque without modifying the synchronous characteristics and without an external voltage reducing device. Fig. 2 shows the external connections for a motor having a double-circuit stator winding. One branch of each phase is energized at start, the other branch being connected in parallel as substantially full speed is attained.

The two windings are preferably arranged in alternate slots, although other coil groupings may be used. At starting, the reactance of the motor, with only half the coils energized in alternate slots, is much greater than with all the coils energized. The reduced current input creates less flux to act on the rotor and consequently less torque is developed. Fig. 3 compares normal and single-circuit, full voltage starting of a

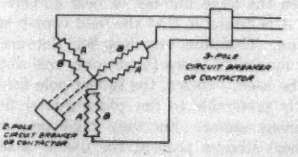

FIG. 2—PRIMARY CONNECTIONS FOR DOUBLE WINDING STARTING

large high-speed salient-pole motor. As in the case with reactor starting, the torque characteristics approach normal during acceleration. A torque characteristic of this type is particularly suitable for blower and centrifugal pump drives. The second inrush when energizing the second branch is relatively small. As shown in Fig. 3 the starting torque is about 50 per cent and the starting kva. is about 70 per cent of the normal full voltage values but practically the full torque is available at pull-in.

High-torque loads such as ball mills, tube mills, and pulverizers can be started without shock by the double winding motor. When the first circuit is energized, the motor develops sufficient torque to take up all gear and coupling slack. Energizing the second circuit starts and accelerates the load without shock to the mechanical parts. Voltage disturbances, caused by the starting of synchronous motors from lines fed by small generators having voltage regulators, are minimized by using double winding starting. The drop caused by the first and larger increment is corrected by the regulator before the coming of the lesser actual starting and accelerating increment of load. The total voltage disturbance is much less than would be the case with single-winding full-voltage starting.

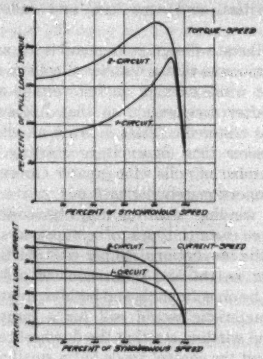

Fig. 3—Characteristics of 800-Hp. Unity Power-Factor, 1,200-R. P. M. Synchronous Motor with Normal Full-Voltage Starting and with Single-Circuit Starting

The use of double windings is somewhat restricted in small high-voltage motors because of the desirability of avoiding the use of very small conductors, necessitated by coils of many turns.

Increase of Starting Torques

The need for starting and pull-in torques greater than those of a normal motor occurs most frequently in applying low-speed motors, although the requirement of abnormally high pull-in torque is sometimes encountered also in applying high-speed motors. Considering the low-speed group, the following methods are available for increasing the subsynchronous torques:

1. Oversize high-torque motor, possibly with double squirrel-cage rotor windings.
2. Normal motor started on over voltage.
3. Delta-Y starting.
4. Normal motor started on tapped winding.
5. Oversize motor with phase-connected starting winding, and with salient or distributed pole-field windings.
6. The supersynchronous motor.

1. Oversize Motors. The oversize high-torque motor is similar to the normal motor but built to larger dimensions for a given rating to make room for a greater total magnetic flux. The wide pole pieces accommodate more bars without restricting the magnetic section. The bars may be arranged on a double squirrel cage using high- and low-reactance elements to improve the torque per ampere and to fit the speed-torque characteristic to the load requirement. The speed-torque curve shown in Fig. 4 is from a large low-speed rubber mill motor and it indicates the uniform torque obtainable from start to synchronism.

This type of motor is widely used on rubber mills, ball and tube mills, crushers, and on certain types of compressors. Simplicity of construction and in control, high efficiency, ample thermal capacity in the amortisseur winding and the design control of torque characteristics are the factors favoring this type of motor. Ample room makes it possible to reduce armature and field copper losses so that the efficiency exceeds that of normally designed motors. Iron losses tend to offset this gain only at light loads. Double-winding increment starting is available to meet objections to the sudden application of high torque, and to excessive voltage dips when high-torque motors have to start from a limited power supply.

Reliable intermittent rated capacitors of comparatively low cost are available for supplying part of the starting magnetizing current when connected in parallel with the motor; their use extends the high-torque motor field by modifying the starting current. While unity power factor starting is hardly feasible, the capacitor is justified by reduction in starting current and improvement in starting power factor particularly when the one capacitor can be used in starting several motors. By cutting out the capacitor in steps as the motor accelerates, the kva. demand may be regulated so as to prevent objectionable voltage variations.

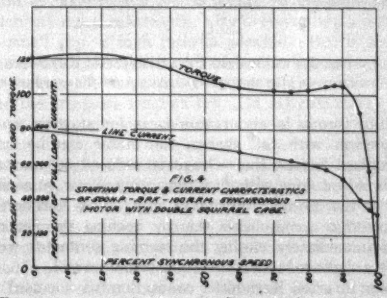

Fig. 4—Starting Torque and Current Characteristics of 500-Hp., 9-Power Factor, 100-Rev. per Min. Synchronous Motor with Double Squirrel Cage

2. Overvoltage Starting. Overvoltage starting can

well be applied to a normal motor to obtain high torque in very slow speed ratings in which high-torque design is at a disadvantage because of the excessive sizes, weights and costs involved.

Undoubtedly there will be some initial objection to the application of overvoltage to motor windings as there was against full voltage starting of normal motors. However, to the manufacturer overvoltage starting of normal low-speed motors does not present any difficulties. With 150-170 per cent starting voltage, a low-speed, low-torque motor is not subjected to any higher per cent of current or torque than is a high-speed motor started on full voltage. The short stiff end turns of a low-speed motor's stator coils require little or no bracing to withstand starting stresses.

Overvoltage starting may be accomplished by stepping up the line voltage through an auto-transformer or by applying line voltage to only a part of the primary winding. The favored arrangement of an inverted auto-transformer with taps is shown in Fig. 5. Starting torque varies as the square of the applied voltage plus a slight increase which comes from the fact that saturation decreases the starting reactance. If high

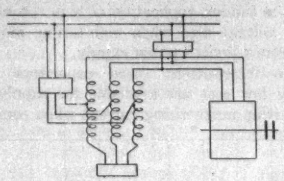

Fig. 5—Primary Connections for Overvoltage Starting of Synchronous Motor with Inverted Auto-Transformer

pull-in torque is also required, as for starting up a pulverizer with full charge, the motor can be synchronized before the voltage is reduced to normal. This is best done without interrupting power, so as not to let the motor drop out of step. The Korndorfer connection accomplishes this by opening the neutral and immediately closing the running contactor with only a slight reduction of voltage below normal.

The inverted Korndorfer connection may be used to provide increment starting leading up to overvoltage, thus avoiding mechanical shock to machines in which there may be slack between motor and load and for which high starting torque is required. At start the Y point of the auto-transformer is left open and the motor terminal voltage is below normal because of the reactive drop in the auto-transformer which is carrying the impedance current of the motor. Excess voltage may now be applied without interrupting power by closing the neutral point of the auto-transformer.

3. Delta-Y Starting. Delta-Y starting has been successfully used on motors required to start a heavy load with moderate inertia characteristic but, since the circuit must be interrupted in changing to normal connections after synchronizing, it is doubtful if large high-voltage circuit breakers are fast enough to keep the motor in step. Moreover, no adjustment of starting characteristics is possible except in the motor design.

In a factory test made to simulate load conditions the changeover was made successfully in 4¾ cycles or 0.08 seconds by standard low-voltage a-c. contactors.

4. Tapped Winding Starting. Where it is found that a relatively small increase in starting or pull-in torque is required the service man may cut out a few coils of each phase to decrease reactance and increase input. This procedure lowers synchronous reactance, raises densities, and core loss, and calls for more excitation.

In a deliberate design it is possible to arrange the end connections so that a number of coils in each phase are cut out while starting on full voltage and picked up again after acceleration so that full synchronous reactance is restored. Tests indicate an effect similar to but weaker than overvoltage starting because a reduced number of coils with greater current does not set up a proportionately higher flux.

Tapped winding starting may be considered for instance for certain types of compressors requiring more than the conventional 40 per cent starting torque. The greater weight and dimensions of a high-torque motor may be objectionable and besides, its steep torque angle characteristic would call for a large flywheel. With tapped winding starting the flywheel requirement is normal and the control needs but a small addition.

5. Phase Wound Starting Windings. All of the high-torque arrangements so far discussed have approximately the "torque efficiency" (ratio of starting torque to starting inrush) of the normal motor, although somewhat better ratio is possible in the high torque, double squirrel-cage motor, and further improvement may be obtained by using starting capacitors.

The wound rotor induction motor with external rheostats in the secondary circuit has excellent starting torque and current characteristics, so many attempts have been made to combine these with the advantages of synchronous operation. By using such a motor with slightly larger air gap and fewer and larger rotor slots than normal for an induction machine, the synchronous induction motor, widely used abroad, is obtained. The slip rings are connected to rheostats in starting and to a direct circuit source at speed. The three fundamental difficulties with this type of motor are high cost, low efficiency, and high ratio of starting to rotor excitation voltage. The limited space for field copper in the rotor slots requires an oversize motor and a more expensive field winding than for a salient-pole motor of the same output and temperature rise. The same cause results in a higher excitation loss and a lower efficiency. The third difficulty arises from the fact that the $I\ R$ drop required to circulate full current in the field winding with a reasonable power loss is generally less than 2 per cent of the voltage induced in the field winding by

normal flux at full frequency. The rotor starting voltage at full load torque is therefore some 50 times the excitation voltage, unless connection changes are made. These difficulties tend to limit the application of the synchronous induction motor to a few cases of high-speed motors, where the disadvantage of increased excitation loss is minimized.

Another proposal tried out both here and abroad has been to use a normal salient-pole synchronous motor with a phase winding in the pole faces. This has interesting possibilities, as it permits normal efficiency to be obtained, but it introduces a more complicated mechanical construction, and it does not permit as great a torque efficiency as the normal wound rotor induction motor construction. The phase winding must be carefully insulated and mechanically braced for high-speed motors, and this hampers accessibility of the field windings. To secure good starting characteristics and adequate heat capacity in the phase winding, it is desirable to use special pole construction, at added expense. To permit full flux and torque to exist at starting, the field winding must be open circuited, with resulting very high induced voltage or else special low-voltage excitation must be used. To avoid using very low-voltage excitation, it is customary to sectionalize the field winding while starting.

In general, it appears that the use of such special types of motors gives greater over-all cost for a given performance than when standard salient-pole motors with special control are used, though this conclusion is somewhat dependent on the manufacturer's facilities.

6. The Supersynchronous Motor. Controllable low-starting current combined with high torques is found in the low-speed supersynchronous motor. The external primary element is supported by bearings and is surrounded by a brake band. In starting, the primary is brought up to speed in the reverse rotation and synchronized while the field with its connected load stands still. Gradual application of the brake by hand or by pilot motor slows down the external element and starts and accelerates the field and load. The sum of the speeds of the two elements is synchronous speed and all the pull-out torque of the motor is available during starting and accelerating, and rated power factor prevails. When the primary comes to rest, say after 12 to 20 seconds, the unit operates as a normal synchronous motor. This motor is widely used in cement mills, on line shafts, mine fans, etc., not only for its low-starting current, but for the smooth control of the acceleration which is possible through regulation of the brake pressure. The design is not applicable to high-speed motors and unless its unique characteristics are important for a specific application, it is generally preferable to use a normal type motor with the proper control features.

Leading vs. Unity Power Factor for High-Torque Motors.

In normal low-torque designs, it is found that an 80 per cent leading power-factor motor develops more starting and pull-in torque than the unity power-factor motor of the same rating. Hence the question is often asked whether one should use a leading power-factor motor where high starting torques are the main objective.

The normal-torque, leading power-factor motor is larger than the unity power-factor machine because the excess magnetizing capacity must be capable of some adjustment without affecting the ability of the motor to carry its load. That is, the normal leading power-factor motor must have a higher pull-out torque than the unity power-factor motor, and will consequently be built in larger dimensions, with a more efficient amortisseur winding and higher starting torques.

In a leading power-factor motor designed oversize to obtain high starting torques, the advantage of the larger amortisseur cannot be fully realized; the magnetizing current of the motor itself must be kept low so that the excess field capacity may be used to supply magnetizing current to the line. Thus the magnetic densities must be lower than in a unity power-factor motor, and the leading power-factor motor will develop less starting torque than the unity power-factor motor, in which higher densities are used.

On the other hand, if high pull-out torque is the governing requirement, the leading power-factor motor may be built in smaller dimensions than the unity power-factor motor. Briefly, pull-out torque is the ratio of magnetizing field strength to demagnetiznig armature reaction strength. By over-exciting a motor of small gap, a high ratio is possible. But if unity power-factor is required, this high ratio is possible only by making the magnetizing current of the motor high and the armature reaction low. In extreme cases this results in a unity power-factor motor of larger size and larger air gap than are required for a leading power-factor motor. The extremely large gap penalizes torque efficiency, so that for applications requiring high pull-out as well as starting torque, the leading power-factor motor may be best adapted. However, it must be remembered that adjustment of power factor is sacrificed in order to maintain high pull-out torque.

Table II shows approximately how pull-out torque affects motor size at various power factors, and is based on 60-cycle motors at speeds up to 450 r. p. m.

TABLE II

Pull-out torque	Equivalent horsepower		
Per cent	1.0 power factor	0.9 power factor	0.8 power factor
150	1.0		
175	1.18		
200	1.37	1.37	
225	1.56	1.54	1.56
250	1.80	1.69	1.69
275	2.02	1.85	1.80
300	2.25	2.05	1.91
325	2.53	2.22	2.02
350	2.79	2.40	2.16

Accelerating and Synchronizing Heavy Inertias

The problem of synchronizing a salient-pole motor having heavy inertia load is discussed in other papers.[2]

In some applications of medium and high-speed synchronous motors, the pull-in torque requirement may be the limiting feature of design. Some of the most difficult cases arise in direct-connected drive of mine fans, oxide fans, and sintering fans. The air load on the fan may be as high as 100 per cent, since such fans are not always provided with shut-off gates. Typical values of inertia are such that the motor must reach 97.5-98 per cent speed before it can be synchronized. To develop 100 per cent torque at 98 per cent speed, the starting winding must be of very low resistance, with the maximum torque point occurring at approximately 95 per cent speed. Very few motors of standard design will meet such conditions. It is therefore, of interest to consider what can be done by special designs, both in the acceleration and the synchronizing of heavy inertias.

With the supersynchronous motor, the problem of pulling into step is entirely avoided, because the motor is in synchronism before the load is started. As regards heat storage and heat dissipation, the brake must be considered rather than the squirrel cage. Since the brake is outside the motor frame, it has a large radiating surface and considerable thermal capacity. It has been found practicable to apply this motor to heavy mine fans, line shafts (with large inertia in the various connected pulleys), and similar loads. However, this motor does not solve the problem on high-speed drive as it is not at present available for high speeds.

Both the synchronous induction motor and the phase wound damper motor are capable of accelerating a large inertia, since most of the loss in the secondary circuit is dissipated in the resistor. Normal type motors, however, can be built with amortisseur windings to accelerate heavy loads. Much of the heat generated in the bars during a gradual acceleration passes directly into the pole pieces, and the permissible temperature of the bars is not limited by contact with any insulating materials. A survey of a large number of designs of standard, medium, and high-speed motors indicates that for motors of 514-1,200 r. p. m., the inertia which can safely be accelerated is approximately as follows:

$$W R^2 = 25 \times H P \times \frac{(1000)^2}{(N)} \times \frac{1}{P F}$$

where

$H P$ = the rated horsepower of the motor
N = synchronous r. p. m.
$P F$ = rated power factor of motor

In any group of commercial designs in standard frames, some ratings will have more favorable characteristics than others, and in specific cases it is often possible to enlarge the amortisseur bars and increase the accelerating capacity well beyond the limits of this formula.

In low-speed motors the thermal capacity of the amortisseur seldom requires any modification of the normal design. Because of the relatively large diameter required for a low-speed motor, the amortisseur will normally contain enough material to give large thermal storage. While the $W R^2$ of many low-speed drives is very large, the stored energy at the low speeds is usually moderate, so that the size of the motor is generally fixed by other requirements.

At the other extreme, two-pole synchronous motors are found to have excellent accelerating capacity, in spite of the small motor diameter. These motors, built with round rotors and embedded field coils, are used principally for driving centrifugal compressors, many of which involve very high stored energy. By making the amortisseur in the form of very deep bars placed at the sides of rotor slots, a larger surface is provided for transferring heat to the rotor iron. In effect, the whole outer portion of the rotor is used to absorb the heat generated during acceleration.

In general, for synchronizing with a large inertia, either a phase winding or a squirrel cage can be designed with very low resistance to develop high torque at very low slip. It seems to be characteristic of most flywheel loads that the load at starting is not high, hence the moderate starting torque obtained with a low resistance squirrel cage of normal reactance is usually adequate for starting loads of this type.

Synchronous Characteristics

The use of synchronous motors on steady loads does not present any unusual problems involving the synchronous characteristics. But where irregular loads and high momentary peaks occur, it is necessary to modify the normal design to insure successful operation.

In any application which involves wide swings of load, the first question in selecting a motor involves the power supply. If the capacity is relatively large, and power swings corresponding in magnitude to the load swings are not objectionable, the first point is established in favor of the synchronous motor. If the supply is limited, and the load swings occur at very short intervals, the wound rotor induction motor with a flywheel and a slip regulator is still in demand. A number of large synchronous motor-generator sets has been installed in steel mills, to supply power to d-c. motors on reversing drives. Over a wide range of load, the losses of a high-torque synchronous motor are lower than those of an induction motor with slip regulating equipment, and the great concentration of power often permits taking advantage of this saving. But even in a steel mill the load swings of a reversing blooming mill motor are usually so great that they must be leveled out by a flywheel motor-generator set.

If it is found that the load swings can be assimilated

2. Edgerton, *Jour. I. E. E.*, Sept. 1930.
Edgerton & Fourmarier, see page 769.

by the power system, the characteristics of the synchronous motor itself must be considered. For any practical case involving peak loads of the order of one-half second duration or more, three alternatives are possible in applying a synchronous motor:

1. Provide a motor with sufficiently high sustained pull-out torque to carry the load without losing synchronism.

2. Remove excitation and allow the motor to run on the amortisseur winding, resynchronizing after load has been reduced.

3. Automatically apply excess excitation to motor field during heavy overloads.

The first method, which amounts to building an oversize motor, is particularly applicable where the load swings occur frequently. Fig. 6 is taken from a wattmeter chart of a synchronous motor driving a band saw. Since load peaks up to 250 per cent may occur every few seconds, the synchronous motor must inherently be able to develop this torque with rated excitation current, hence motors for this service are designed for 250-300 per cent pull-out torque.

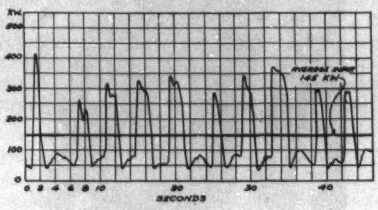

Fig. 6—Characteristic Input Curve of a Synchronous Motor Driving a Band Saw

Although unity power-factor motors can be designed to develop 300 per cent pull-out torque or more by resorting to a very large air gap and low armature reaction, it has been pointed out that such motors will in general be larger than leading power-factor motors which develop the same pull-out torque. Since the synchronous motor is often chosen primarily for power-factor improvement, motors of 80 per cent or even 70 per cent leading power factor are commonly selected.

Even with this torque available, motors are occasionally thrown out of step by extreme overloads. Then it must be determined whether the flywheel effect of the machines, together with the sustaining torque of the amortisseur winding, can carry through the peak load without too great a reduction in speed. It is found that medium and high-speed loads having severe fluctuations frequently include a very large WR^2. Loads of this character are found in lumber mills. They include band saws, fuel hogs and chippers, all traditionally driven by induction motors. These machines are built with large flywheel effect to take advantage of the slip of an induction motor in reducing the electrical input during momentary overloads. For synchronous motor drive, the speed reduction can be estimated closely if data are available regarding the inertia, the amortisseur torque, and the probable duration of the extreme load peaks. If the reduction is not large, the duty of accelerating back to synchronous speed will not endanger the amortisseur winding, since the heating of this element, for a given torque output, varies with the slip.

However, care must be taken in applying a synchronous motor where it must operate as an induction motor during extreme overloads. After a motor has been forced out of synchronism d-c. excitation must be temporarily removed, since the motor would otherwise generate a braking torque in opposition to the accelerating torque of the amortisseur winding. The load must be limited to a value which will enable the motor to resynchronize. In wood hog drive, this is done by automatically stopping the feed conveyor while the motor is out of synchronism. In sawing logs with band saws, where a heavy knot may overload the motor to the pull-out point, the loss of speed changes the whine of the saw to a lower pitch. The operator, accustomed to gaging the load by this sound with induction motor drive, can slow down the log carriage to let the motor resynchronize.

An element of judgment enters in estimating how often the motor will be pulled out of step. If the intervals are apt to be long, and the peaks short, the advantages of synchronous operation may be largely realized; but if the load is such as to cause pull-out frequently, the average power factor may be no better than that of a wound rotor induction motor, and the violent changes in power factor when synchronizing and when dropping out may seriously affect voltage regulation.

For meeting such conditions, the synchronous induction motor would be well suited, except for the inherently low pull-out torque of this type of motor. If designed for good starting characteristics, unity power-factor designs will have only 120-140 per cent pull-out torque. By designing for 0.8 leading power factor, 175-200 per cent pull-out torque can be obtained. If the overexcitation is further increased to obtain still higher synchronous torque, the loss in efficiency may be serious. Comparison of a synchronous induction motor with a salient-pole motor, both rated 125 hp. 0.8 power factor, 900 r. p. m., shows a differential of 2 per cent in efficiency at full load in favor of the salient-pole machine. In this case, the wound rotor machine is designed for a synchronous pull-out torque of 175 per cent at 0.8 power factor and can exert 300 per cent torque as an induction motor, without d-c. excitation. In designing this machine, the intention was to produce a motor capable of carrying moderate overloads in synchronism and to be used with a large flywheel to sustain the speed during high momentary loads. The salient-pole motor, also rated at 0.8 power factor, has

250 per cent pull-out torque, and can exert 200 per cent torque at 85 per cent speed without d-c. excitation. Obviously, the wound rotor machine can only be justified by very severe accelerating duty or starting current requirements.

Quick Response Excitation. The subject of high-speed excitation cannot be discussed within the limits of

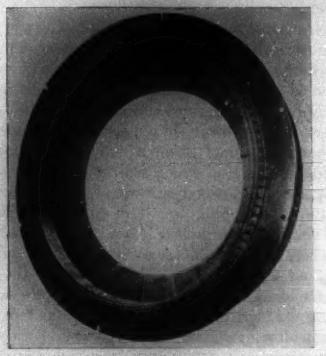

Fig. 7—Fabricated Frame of 500-Hp. 72-R. P. M. Synchronous Motor for Mounting in the Base of a Coal Pulverizer

this paper, but a few of the possibilities may be mentioned. If a synchronous motor is subjected to occasional peak loads much greater than the average load, and the field excitation can be quickly increased

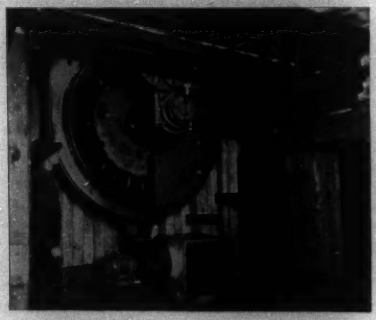

Fig. 8—Inverted Synchronous Motor Mounted on Lower Shaft of a Band Saw

to meet these overloads, a material reduction in the size and cost of the motor is sometimes possible. In addition, the motor may be designed for best efficiency at the average load condition, and the motor power factor will be held more nearly constant during the load changes. The problem is the same as that involved in overexcitation of generators to maintain synchronism during short circuits, except that most industrial motor installations cannot justify the refinement and expense of the equipment used with large generators.

When a heavy load is suddenly applied, a synchronous motor can momentarily exert a torque far greater than its sustained maximum torque. But unless excitation is quickly increased, the duration of this high transient torque will be only a fraction of a second, varying with the inductance of the field winding. Fortunately, this time interval is usually long enough so that the excitation of the motor may be controlled from the exciter shunt field, although in the larger sizes, modification of the exciter may be necessary to increase its rate of voltage rise. With small exciters, which inherently give

Fig. 9—Construction of Amortisseur for Heavy Starting Duty

quicker response, standard designs may suffice. The indication may be given by a quick acting current or power relay in the primary circuit of the motor.

For small motors such as 100 horsepower, it will usually be more economical to build a motor having inherently this higher torque, than to resort to overexcitation, with a larger exciter and additional control. But for a motor of say, 2,000 horsepower, the additional cost of field forcing equipment is relatively small. The gain in operating efficiency and the saving in first cost will often justify this equipment for large size motors on fluctuating loads.

Another method of overexcitation which is simpler but which involves some electrical loss has been used on a number of large synchronous motors on continuous rolling mills. With a normal excitation requirement of 125 volts the exciter is operated at 175-200 volts, the

difference being dissipated in a fixed resistor in series with the motor field. A contactor is used to short circuit the resistor when the motor load exceeds a predetermined value. In spite of the additional loss in the motor field circuit, a study of several cases has shown that the over-all efficiency is higher than that of an 80 per cent power-factor motor for the same maximum torque conditions, and the overexcited motor has lower first cost.

Interesting possibilities are found in the use of electron tubes for overexciting a synchronous motor, although this development has not yet, to the authors' knowledge, been applied commercially. Circuits can be arranged, using the thyratron tube, whereby the excitation voltage will be automatically regulated as a function of the primary current in the motor windings, the response being practically instantaneous. Factory tests on a standard motor with controlled thyratron excitation have shown a great improvement in stability on sudden reductions of voltage, as well as increased synchronizing ability with a heavy load of high inertia.

In applying any field forcing scheme, the motor field must be of liberal design so that the intermittent high excitation will not overheat the field coils. With adequate margin provided, however, the field winding can still be considerably smaller than that of an equivalent high-torque motor which carries high-excitation current continuously.

Mechanical Design

To adapt the synchronous motor to special applications without considerable development expense, there is required a greater flexibility in mechanical construction than is needed for normal applications for which lines of standard parts are available. The use of fabricated construction has materially assisted in promoting the "built-in" type of drive and has allowed the machinery manufacturers greater freedom in the design of the driven equipment. Fig. 7 shows the frame of a 500-hp. 100-pole 72-r. p. m. vertical motor for direct connection to a coal pulverizer. In this machine, it was necessary to keep the diameter to a minimum and space limitations did not permit an oversize high-torque motor. A normal-torque motor using overvoltage to obtain high starting and pull-in torques made it possible to keep within the required dimensions. A supporting ring built as part of the motor frame, serves as a foundation for the pulverizer. The result is a compact direct-drive unit in which all gears have been eliminated. The large air gap of the synchronous motor promotes mechanical reliability in this equipment in which considerable vibration may exist.

The flexibility of the welded steel construction often permits motorizing old compressors without rebuilding foundations, the motor frame being extended to meet specified dimensions. The forming of special stator frames to accommodate unusual mountings for other applications is illustrated in the case of the inverted motor, Fig. 8. For small, medium speed drives, overhung synchronous motors can sometimes be used thus eliminating the extended base, extra bearings and coupling. The rotors of these machines are often fabricated to permit short overhang.

Although the rapid progress of fabrication in recent years has received considerable attention, the art of casting in iron, steel, and alloys has also reached a high state of development. Each type of construction has certain advantages and disadvantages and the decision as to which type to use is always subject to full analysis from the standpoint of manufacturing facilities, economical production, and mechanical reliability.

The use of synchronous motors with loads requiring high-starting torques has made the amortisseur winding one of the most important parts in the machine. Fig. 9 illustrates the trend toward very sturdy construction to withstand heavy duty loads.

The progress in the improvement of insulations and of the magnetic and mechanical properties of iron and steel does not come within the scope of this paper other than to mention that these improvements have assisted in the extension of the synchronous motor to new fields of application.

Discussion

SYNCHRONOUS MOTOR WITH PHASE-CONNECTED DAMPER WINDING FOR HIGH-TORQUE LOADS

(Hyde)

SYNCHRONOUS MOTORS

(McLenegan and Ferriss)

Fraser Jeffrey: Unless an oversized machine is used, it is questionable whether enough thermal capacity can be built into a salient-pole synchronous motor with squirrel-cage amortisseur winding to take care of the acceleration and pulling into step of an extremely heavy inertia load. This type of load is frequently to be found in cement plant tube mills where the starting conditions are often unusually severe especially after an extended period of shut down in cold weather. Under this condition of starting, the heat losses that have to be dissipated in the squirrel-cage amortisseur winding approximately equal the kinetic energy of the rotating load at full speed. Therefore it is desirable to be able to get rid of this heat loss external to the motor, as is done with a slip-ring induction motor, rather than to try and dissipate a large loss such as would occur in the rotor of a squirrel-cage induction motor under such conditions.

While the authors stress among other things, the high cost of the so-called synchronous induction motor and its use being restricted mostly to high-speed drives, nevertheless and contrary to this opinion, such machines are being successfully applied to low-speed high inertia drives at costs competitive to other modified types of salient-pole machines. While it is true that the synchronous induction motor has a somewhat lower efficiency than the salient-pole type synchronous motor, still for the acceleration of unusually heavy inertia loads, it has certain other features that make it desirable, if not necessary, for this kind of service.

The stator of the synchronous induction motor is like that of any ordinary induction motor, but the rotor is usually made with deep open type slots, the winding of which resembles that of a simple d-c. armature. This is fully distributed and the three phases are connected 120 elec. deg. apart, so that in starting, smooth and even acceleration equivalent to a regular slip-ring type induction motor, is obtained. By referring to Fig. 1, it will be noted that the three-phase leads are brought out to three slip rings between which are interposed two other slip rings to which the source of excitation is connected. Thus the maximum phase

voltage in starting is separated by two ring spaces. The exciter armature, which is always in the circuit during both starting and running conditions, forms the neutral point of the rotor winding and thus has no excessive voltage across its terminals during the starting period. Starting is extremely simple and is usually accomplished by pressing a push button which closes the primary oil switch and then automatically starts cutting out sections of secondary resistance. The period of acceleration is governed by adjustable timing relays, and, as the motor has not over 1 per cent slip at full load as an induction motor, it easily pulls into step and operates as a synchronous machine, with very little line disturbance.

The additional complication of sectionalizing the fields to reduce initial starting voltages has not been found necessary because the highest open-circuit phase potential on say a 1,500 hp., 163½ r. p. m. synchronous induction motor has not exceeded 3,200 volts. This in turn required a direct current exciting potential of 65 volts. Neither of these conditions has been of any detriment to the successful operation of this type of motor. Such a machine is capable of exerting full load starting torque with 143 per cent line current (full load current being based on 100 per cent power factor) with 69.5 per cent power factor. The usual accelerating periods vary, depending largely on the local conditions, being anywhere from 30 to 60 seconds but rarely more than one minute under the most severe starting conditions.

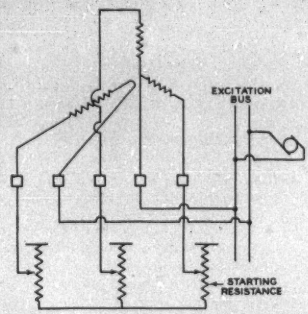

Fig. 1

Once set, the time of acceleration usually remains unchanged for any given installation, but varies somewhat for different installations.

There is a large number of low-speed synchronous induction motors in operation in this country today. They vary in size from 150 hp. to 1,500 hp., and are connected to high inertia loads. The fact that they are simple in mechanical and electrical construction as well as simple in operation and that they smoothly accelerate and pull into step these heavy loads and do it with minimum currents at high power factor, gives them an advantage over the other types of modified synchronous machines. Some of these motors have been in service for approximately five years, others somewhat less, and their operation over an extended period of time, has been uniformly successful.

F. K. Brainard: The authors are to be congratulated on their keen analysis and clear presentation of the problems connected with synchronous motor application. However, the subject is not complete without a brief discussion of the economic phases of the problem since one of the ultimate [illegible]ts of any engineering project large or small is the total ec[illegible]omic cost of the project under consideration compared with the cost of possible alternatives.

As stated by the authors the motive behind the change from induction to synchronous motors has been partly power-factor improvement and partly efficiency, both, of which are primarily economic questions. That is, the losses in many synchronous motors are less than in the corresponding induction motors and the synchronous motors either draw no reactive magnetizing kva. from the system or furnish reactive kva. to be used by other apparatus on the system while the induction motors always draw reactive kva.

Normal unity power-factor 60-cycle synchronous motors with suitable belted exciters and manual control cost less and have higher efficiency than squirrel-cage induction motors with similar control at speeds of about 514 r. p. m. or less. Hence for these speeds and for applications for which they are suitable synchronous motors should be used. For speeds from about 600 r. p. m. to 3,600 r. p. m. however the induction motor is cheaper and the efficiencies nearly equal. Hence for this range in speed the synchronous motor must usually justify itself on the basis of the lack of reactive kva. drawn by the induction motor. This comparison is usually favorable to the synchronous motor up to about 1,200 r. p. m. although this depends largely upon the value of reactive kva. in the particular installation, but above that the induction motor generally has the advantage, *i. e.*, the additional cost of eliminating the relatively small amount of reactive kva. drawn by the high-speed induction motor is usually unjustifiable.

Similarly in the comparison between unity power-factor and 80 per cent power-factor synchronous motors the additional cost of the 80 per cent power-factor motor must be justified by the reactive kva. which it feeds into the system, after making due allowance for its lower efficiency. This is largely a question of speed as before, since the additional cost as well as the additional loss per reactive kva. increases rapidly as the speed decreases. The point at which the 80 per cent power-factor motor becomes uneconomical however depends to a very large extent upon the power-factor clause in the power company's contract in case the power is purchased. Years ago the power companies generally made no charge whatever for reactive kva. but now they frequently make a very considerable charge for it, so that this point must be carefully considered in any installation where purchased power is used.

Similar comparisons could be made between the various types of motors mentioned by the author but in any case the proposed motor must be justified economically if it is to survive.

D. W. McLenegan and A. G. Ferriss: As mentioned in the paper, the authors feel that there is a limited field for the synchronous induction motor, principally in cases where very severe accelerating duty is combined with the requirement of very low starting current. However, in the authors' experience the additional cost, complication, lower efficiency, and special exciter required by the synchronous induction motor are justified only in a relatively small percentage of the high-torque applications. Tube mills in cement plants, cited by Mr. Jeffrey, do not present severe accelerating duty to a normal high-torque motor. In many cases it has been found that the inertia of the tube mill was equivalent to less than the inertia of the motor rotor, when corrected for the difference in speed. Experience gained in the application of salient-pole motors to a wide variety of high inertia loads indicates that the majority of such applications can be successfully handled by salient-pole motors at lower cost, and with better efficiency than with synchronous induction motors.

With salient-pole motors having squirrel-cage starting windings, open field starting is seldom resorted to except where the very maximum torque efficiency is required. Hence the problem of excessive starting voltages in the fields of these motors seldom arises. Where open field starting is necessary, the induced voltage can be kept within reasonable limits by sectionalizing the field winding, or by using lower excitation voltage without sacrificing the advantages of the salient-pole design. It is interesting to note that the figure given by Mr. Jeffrey for rotor starting voltage of a synchronous-induction motor is almost exactly 50 times the excitation voltage, which is the ratio given in this paper.

With reference to Mr. Brainard's discussion, it is recognized

that in the medium speed range the synchronous motor is seldom a competitor of the squirrel-cage induction motor except in cases where the slightly higher efficiency or the need for power-factor improvement justify the greater first cost of the synchronous motor. It appears, however, that at the higher voltages the differential in efficiency between synchronous motors and induction motors of standard design increases, since the synchronous motor is normally built with larger slots, and hence can better accommodate high-voltage insulation without causing excessive copper losses.

As pointed out in the paper, the choice between unity-power-factor and 80 per cent power-factor synchronous motors must be made by balancing the need for power-factor improvement against the increased cost and lower efficiency of the overexcited motor, since the torque requirements of most applications can be met with either design.

Lackawanna Suburban Electrification[1]

BY EDWARD L. MORELAND[2]
Fellow, A. I. E. E.

Synopsis.—*The initial electrification on the Delaware, Lackawanna & Western Railroad includes approximately 68 route miles and 158 track miles of line out of Hoboken, New Jersey, and covers the more important parts of the suburban service and a small amount of freight transfer switching.*

The 3,000-volt d-c. system is used. Power is purchased in the form of three-phase, 60-cycle, high-tension, alternating current delivered by the power companies to five substations owned and operated by the railroad located along the right-of-way. Four of the substations are manually operated and one is fully automatic. Mercury-arc rectifiers are used exclusively for converting the alternating current to 3,000-volt direct current. The rectifiers have excited grids and are compounded to give substantially flat voltage up to 150 per cent load. All d-c. distribution is through the catenary system, auxiliary feeders not being necessary. All d-c. circuits are protected by high-speed magnetic blow-out circuit breakers.

The catenary system is of the tangent-chord double-contact wire type supported on steel structures in multi-track sections and on creosoted wood poles on the single track branch.

Multiple-unit trains are used exclusively in the passenger service, made up of two-car units consisting of a motor car and a trailer car, semi-permanently coupled together. Motors are insulated for 3,000 volts, are operated in series-parallel under electro-pneumatic control with automatic acceleration, and are self ventilated.

The freight transfer service is handled by two locomotives of the "three power" type, arranged for operation from the 3,000-volt d-c. contact system or on internal power provided by a diesel engine generator set supplemented by a storage battery.

General Conditions

THE electrification program now in hand by the Delaware, Lackawanna & Western Railroad covers approximately 68 route miles and 158 track miles of line in the suburban district west of Hoboken, and includes the main passenger line from Hoboken to Dover (via Newark, the Oranges, Summit and Morristown) and the branch lines to Montclair and Gladstone. The steam service replaced by electrified trains aggregated approximately 23,000 revenue car-miles per day. The map shown in Fig. 1 indicates the lines now being electrified as well as other features, such as location of substations and tie stations, which will be referred to later.

The present electrification covers only local passenger service on the portion of the system now being electrified and electric transfer of freight trains between the freight classification yard at Secaucus and the terminal pier head yard in Jersey City, adjoining the Hoboken Terminal. Local passenger trains on the Boonton Line (via Paterson and Passaic) and all passenger trains to and from points beyond Dover will continue to be operated by steam power. Classified freight trains which do not need to be set out at the Secaucus yard for classification will also be taken to and from the Jersey City yard under steam power.

The principal reason for undertaking the electrification was a desire to give better service in the suburban territory by providing faster, cleaner, and more frequent trains possible only with electrification. One of the important factors in this connection was the discomfort to passengers caused by the comparatively long tunnels. These consist of two parallel tunnels just west of Hoboken, with two tracks each, approximately 4,300 ft. long, in which smoke conditions were bad under steam operation, particularly during rush hours. Under the present plan of operation one of the tunnels will be used exclusively by electric trains and all steam trains will be put through the other tunnel. During rush hours the one tunnel cannot handle all of the electric trains and it will be necessary to put some of them through the steam tunnel; the smoke conditions even in this tunnel will, however, be very much improved as compared with conditions under all steam operation.

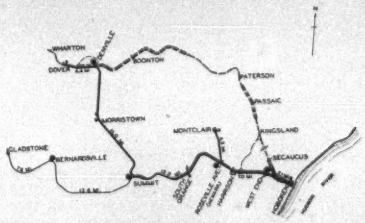

Fig. 1—Map of Lackawanna Suburban Lines East of Dover

Showing lines electrified and locations of substations and tie stations

Although the initial electrification is essentially confined to suburban passenger service, the choice of system used and all designs were worked out with the idea that the electrification must be suitable for both suburban service and for main line operation of all through passenger and freight service, which will presumably be electrified at some later time.

The 3,000-volt d-c. system is used, with power delivered to the trains by means of an overhead contact system fed from five substations owned and operated by the railroad located along the right-of-way. In the passenger service multiple unit cars are used exclusively.

1. This paper was written in October, 1930, at which time approximately one-half of the electrification was in operation, the program being scheduled for completion about the end of the year.

2. Jackson & Moreland, Consulting Engineers, Boston, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

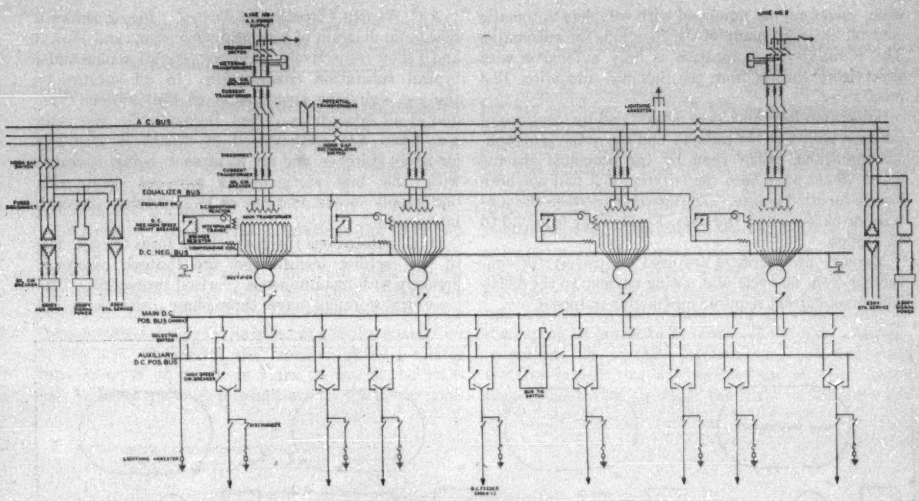

Fig. 2—Simplified Wiring Diagram of Typical Substation

In the freight transfer service "three-power" locomotives are employed.

Power Supply

Power for the electrification is purchased from the three power companies serving the territories in which the substations are located, and is delivered by the power companies to the five railroad substations in the form of three-phase, 60-cycle, alternating current, at the voltages available on the power companies' systems in the vicinity. There is no transmission of alternating current power by the railroad along the right-of-way for traction purposes.

The locations of the substations are shown on the map (Fig. 1). Power is supplied at the West End and Roseville substations by the Public Service Electric and Gas Company at 13,200 volts and 26,400 volts respectively, at the Summit substation by the Jersey Central Power and Light Company at 66,000 volts, and at the Bernardsville and Denville substations by the New Jersey Power and Light Company at 33,000 volts. The West End substation is supplied through three underground cables, any two of which are capable of transmitting the full requirements. Roseville substation is supplied by two underground cable circuits and each of the other three substations by two overhead circuits, in all cases one circuit being of sufficient capacity to supply the full requirements of the substation. In all cases, except Bernardsville, it is expected that all circuits will be kept in service during normal operation. At Bernardsville power will be taken normally from one circuit, with the second circuit open, but with provision for automatic throw-over to the second circuit in case of failure of voltage on the first circuit.

There are three separate power contracts with the three power companies. The contracts are all of the monthly-demand-charge plus energy-charge form, practically universal in railroad power contracts, with the demand determined as the average of the three highest clock-hour demands during the month, each month on its own basis (with certain minimum restrictions) and provision for elimination of unusual demands caused by unusual conditions and for compensation by the power companies for increased demands on the adjoining companies caused by failure of power supply. The fact that there are three separate contracts, each with its demand separately determined rather than one single contract for the entire power supply with a single demand, unquestionably adds materially to the total cost of power since it deprives the railroad of the benefit of diversity between the demands of the substations on different power systems. The three contracts were, however, necessary because the power companies would not offer a single combined proposition, and the cost of an alternating current transmission system along the right-of-way to distribute alternating current power to the substations, if all of the requirements could have been purchased from a single power company, would have far more than offset the added cost due to loss of diversity between substations.

Substations

Four of the substations, namely, West End, Roseville, Summit and Denville are at junction points and an operator will be kept in attendance at all times. They

were, therefore, not equipped with complete automatic control, although many of the functions are automatic. The Bernardsville substation is fully automatic with supervisory control from the Summit substation 13.4 miles away.

Substation locations were determined by locations of junction points on the railroad and by other geographical conditions rather than by the economic spacing which would have been the criterion if it had not been for the function points. The spacings between the substations as shown on the map, Fig. 1, vary from 7.0 to 15.0 miles.

Mercury arc rectifiers are used exclusively for converting from 60-cycle alternating current to the 3,000-volt direct current required for traction purposes.

A-C. Supply Circuits and Control. Fig. 2 shows a schematic diagram of a typical substation, and Figs. 3 and 4 show respectively, a typical substation plan and a typical substation cross-section. In all substations the a-c. switching structure is of the outdoor type, and at all substations, except Bernardsville, the main a-c. buses are sectionalized by means of horn-gap air-break switches, and are at present being operated with the bus sectionalizing switches open with one supply circuit and one or two rectifiers on each bus section.

In all substations the connections from the a-c. bus to the rectifier transformers are through oil-circuit breakers with instantaneous overload protection. The main transformers have three-phase delta-connected

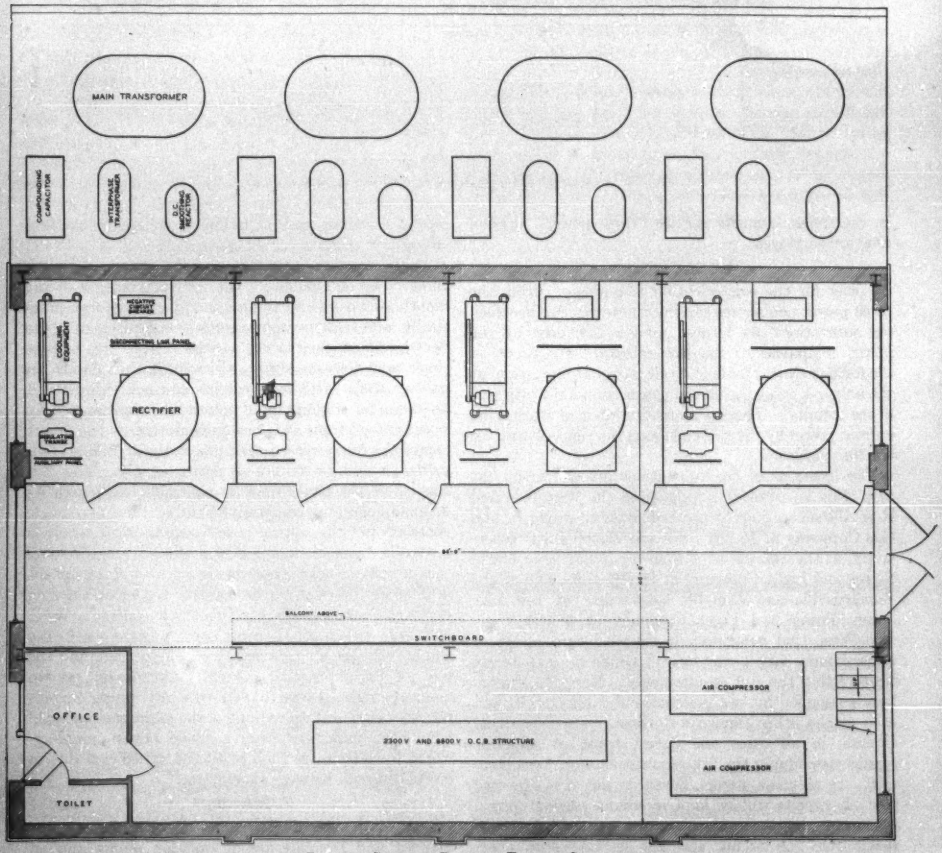

Fig. 3—General Plan of Typical Substation

primaries, with ratio adjusters externally operated from the ground, and twelve-phase zig-zig secondaries, and are provided with thermal protection against overheating.

At West End and Roseville substations the incoming a-c. power feeders are connected to the bus through oil circuit breakers and disconnecting switches. Each of these substations is equipped with four twelve-phase transformers each supplying power to a twelve-phase, 3,000-kw. rectifier tank.

At Summit and Denville substations the incoming a-c. power feeders are connected to the bus through horn-gap air-break switches. In these cases the oil circuit breakers in the power companies' substations supplying the feeder circuits are relied upon to clear the associated circuit in the event of a fault on a section of the bus. Provisions are, however, made for later installation of oil circuit breakers instead of the horn-gap air-break switches if conditions on the power companies' systems should change so that the present method of operation would no longer be satisfactory. These two substations are each equipped with two twelve-phase transformers each supplying power to two six-phase, 1,500-kw. rectifier tanks. Normally, both rectifier tanks of each two-tank unit will be operated together, but in event of failure of one tank of a pair the faulty tank can be disconnected by opening disconnecting switches provided in the leads to the anodes, and the other tank operated alone.

At Bernardsville substation the incoming a-c. power feeders are also connected to the bus through horn-gap air-break switches. As already pointed out, this station will normally be supplied from one circuit, the second circuit being used only in case of failure of power on the first circuit. At this substation there are two twelve-phase, 1,500-kva. transformers, each supplying power to a twelve-phase rectifier tank.

Fig. 5 shows a photograph of the outdoor structure at the Roseville substation, which is more or less typical of the kind of construction used throughout, except that at Summit, Denville, and Bernardsville the "strung bus" is used instead of the rigid type of bus employed at West End and Roseville. Lightning protection to the a-c. bus is provided by oxide film type lightning arresters.

D-C. Bus Arrangement and Circuit Control. On the d-c. side of each substation a main and auxiliary bus are provided, and each bus is sectionalized by a disconnecting switch, but is normally operated with the sectionalizing switch closed. The d-c. feeders connecting the substation bus to the catenary system are controlled by high-speed circuit breakers of the magnetic blow-out type, with holding coils energized from a storage battery. The feeders are staggered on the bus, as far as practicable; for example, if there are two feeders connecting to the catenary system west of a substation one feeder is connected to each section of the d-c. bus so that a single d-c. bus failure could not entirely cut off the supply of power to a section of the line. The arrangement of circuit breakers and disconnecting switches is such that any rectifier and any d-c. feeder may be connected to either the main or auxiliary bus with its associated high-speed circuit breaker in series. Any feeder circuit breaker can be by-passed and disconnected from the line for inspection or maintenance by connecting the feeder direct to the auxiliary bus, the feeder protection then being furnished by a high-speed bus tie breaker. This arrangement also permits the feeders supplying the contact system on both sides of any air gap to be tied together through disconnecting switches and the auxiliary bus so that live line maintenance or adjustment of air gaps can be carried out without danger of high voltage across the gap due to the opening of a circuit breaker. The d-c. bus, and the disconnecting switches on the d-c. feeder circuits and the rectifier positive connections, are mounted in an alberene stone cell structure, which in all substations except Bernardsville is in a balcony. At the Bernardsville substation the cell structure is on the main floor. The high-speed magnetic blow-out circuit breakers on the d-c. feeder circuits and on the rectifier positive circuits are mounted on top of the cell structure.

A lightning arrester of the electrolytic aluminum cell type with balancing resistor and shunted spark gap is provided on each d-c. feeder and on the d-c. main bus.

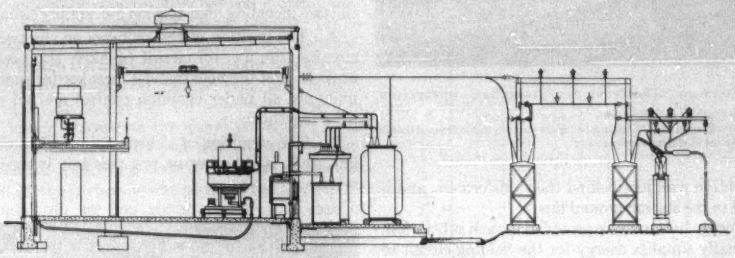

Fig. 4—Cross-Section of Typical Substation

Fig. 6 shows a photograph of the switchboard in the Roseville substation, and indicates the uses of the various panels. Each rectifier unit has a separate panel, whereas three feeders (with space for a fourth) are controlled from each feeder panel.

A negative bus is provided at each substation to which the rail returns and rectifier negative leads are connected. The negative bus is mounted outdoors on

Fig. 5—Outdoor Structure at Roseville Substation

Substation building in background
Derrick at left used to lift equipment from cars on the depressed tracks
Photograph taken during construction

the substation wall just behind the transformers, and is grounded to the station ground bus.

Two storage batteries are provided in each substation. One normally supplies energy for the holding circuit of the high-speed d-c. circuit breakers and the other normally supplies power for operating the high-speed d-c. circuit breakers, the a-c. oil circuit breakers, the pilot lights and the emergency station lighting. The circuits are, however, so arranged that any battery circuit can be switched to either battery. Two motor-generator battery charging sets are also provided. Normally one set is used to charge each of the batteries but either set can be used to charge both batteries.

Bernardsville Automatic Substation. As previously mentioned, four of the five substations are manually operated. The fifth, Bernardsville, is fully automatic with supervisory control from the Summit substation. The power supply at this substation will normally be taken over a single circuit, but provision is made for automatic throw-over to a second supply circuit in the event of failure of power on the first circuit. Once having changed over to the second circuit, it does not go back automatically to the first circuit on restoration of power on that circuit unless there is a failure of power on the second circuit.

The automatic features provide that either of the two rectifier units may be made the preferred unit and that the second unit will automatically come into service when the load on the first unit exceeds a pre-determined amount, and will automatically drop out after a pre-determined time delay when the total substation load drops below a pre-determined amount. The preferred rectifier may be kept on the line continuously or the control may be adjusted to bring it on only when there is a load demand on the substation as indicated by a drop in d-c. line voltage, and to take it off the line, after a definite time delay, when the load has decreased below a pre-determined amount as indicated by current flow. In the event of a fault on either rectifier or any of its auxiliary circuits that rectifier is automatically locked off the line. If the fault is on the preferred rectifier the second rectifier automatically takes its place.

Supervisory Control of Automatic Substation. The supervisory control is of the synchronous selector type, using four wires between the Bernardsville substation and the supervisory point at the Summit substation. Lights on the supervisory panel at the Summit substation automatically indicate:

1. The position (open or closed) of the switches on the two incoming high-tension a-c. supply circuits.

2. The position of the switches on the two rectifier circuits (the a-c. oil circuit breaker, the positive d-c. breaker, and the negative d-c. breaker for each rectifier unit, are all under common control and all open and close together).

3. The condition of either rectifier when not on the line, that is, whether or not the unit is available for service.

Fig. 6—Switchboard at Roseville Substation

4. Lock-out of a unit due to some fault which requires manual release of the lock-out.

5. The position of the switches on the two signal supply circuits fed from the substation.

6. The position of the switch connecting the signal power bus to the transformer from which the signal power is normally supplied.

7. Whether or not the switch between the signal power bus and the signal circuit connecting between the Bernardsville and Summit substations, if in the open position, is set up for automatically closing in event of loss of voltage on the Bernardsville signal power bus.

8. Lock-out of the automatic reclosing breaker on the d-c. feeder to the stub-end catenary line to Gladstone or on the stub-end a-c. signal circuit to Gladstone, or lock-out of the oil circuit breaker connecting the signal power bus to the signal power transformer.

9. An alarm indication if either of the charging units for the two storage batteries shuts down or current ceases to flow to the battery; and

10. A warning indication if the voltage on the signal power bus is low.

The supervisory control permits the operator at the Summit substation; (1) to take control of both incoming a-c. supply lines together, and having taken control, to operate them individually, or restore both to automatic control at will; (2) to take control and operate, or restore to automatic control either rectifier individually; (3) to open or close either d-c. feeder breaker; (4) to open or close any one of the three signal power switches, and if the switch between the signal power bus and the signal power circuit between Bernardsville and Summit is open to set up this switch for automatic closing in event of loss of voltage on the signal power bus, or to take off the automatic set up at will; (5) to determine the amount of load being carried by either rectifier as indicated by the current in one phase of the a-c. supply circuit; and (6) to trip simultaneously, in case of emergency, all circuits which can feed power into the substation, both alternating and direct current.

Signal and Auxiliary Power Supply. In addition to the 3,000-volt direct current supplied from the substations for traction purposes, each of the substations also supplies 2,300-volt, single-phase, 60-cycle power for signal operation. The West End, Roseville and Denville substations also supply 6,600-volt, three-phase, 60-cycle power for railroad auxiliary power circuits for station lighting and miscellaneous power requirements along the right-of-way.

Rectifiers. Rectifiers were selected rather than motor-generators for the conversion of the 60-cycle a-c. power to 3,000-volt d-c. power because of the relatively low first cost of rectifier equipment, the high efficiency of such equipment, its ability to carry heavy overloads without injury, and the economy of substation design resulting from the small space occupied by the rectifiers and the absence of heavy foundations required for motor-generators. Fig. 7 shows the comparative efficiencies of 3,000-volt mercury-arc rectifier sets and motor-generator sets.

The substation buildings housing four 3,000-kw. rectifier tanks are only 64 feet long by 40 feet wide by an average height of 27 feet (all inside dimensions), which is small compared with the space which would be required for the same capacity in motor-generator sets. The Lackawanna rectifiers were bought under a specification requiring guarantee of successful operation in service under loads of 150 per cent of current rating (50 per cent overload) for two hours and 300 per cent of current rating (200 per cent overload) for five minutes. During the acceptance tests short-time overloads up to 600 per cent of current rating (500 per cent overload) were also applied and successfully carried for periods of 10 seconds.

The rectifier tanks are equipped with the usual arc-striking anodes, holding (or exciting) anodes, vacuum pumps, and gages. The tanks are water-cooled, using a closed cooling system with forced circulation under thermostat control. The circulating water for each rectifier tank is re-cooled in a cooler unit associated

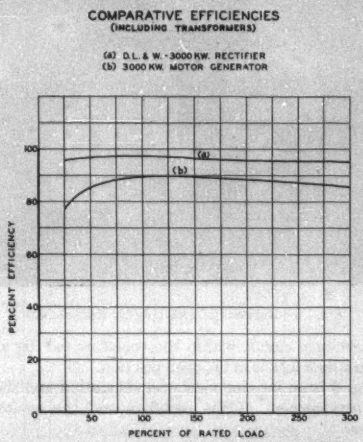

Fig. 7—Comparative Efficiencies, 3,000-Volt Mercury Arc Rectifiers and Motor-Generator Sets (Both Including Transformers)

with that tank somewhat similar to an enlarged automobile radiator, using a circulating pump and motor-driven fan. Heating elements under thermostat control are also provided to prevent various parts of the rectifiers becoming too cool when shut down or lightly loaded.

Direct current smoothing reactors to reduce the ripple in the d-c. voltage, and resonant shunts to drain off high-frequency harmonics which would cause interference on communication circuits, are also provided. The resonant shunts are tuned for 360, 720, 1,080 and 1,440 cycles. Surge suppressors made up of capacitors with limiting resistors are provided to protect the transformer secondary windings and the interphase transformer windings against the high-voltage surges induced when the residual current flow following an arc-back or internal short circuit is interrupted by the arc going out.

The shell of the rectifier tank and some of the associated equipment is at substantially cathode potential (3,000 volts above ground) when the rectifier is in operation. This equipment is therefore insulated from ground and carefully protected from accidental contact while in operation. This is accomplished by enclosing each rectifier tank, with its associated cooling equipment, auxiliary switch panel, auxiliary circuit insulating transformers, resonant shunts, surge suppressors and the negative circuit breaker with its associated load limiting resistor in a grille enclosure, so arranged with interlocks on the locks of the enclosure doors that no

Fig. 8—Rectifiers and Grille Enclosures

high-voltage circuit within the enclosure can be alive when a door lock is in the open position.

Fig. 8 is an interior view of a substation and shows the arrangement of the rectifiers and the enclosing grilles.

In addition to the overload protection provided by the oil-circuit breaker on the a-c. side of the transformer, each rectifier unit is protected by a high-speed circuit breaker of the magnetic blow-out type on both the positive side (cathode connection) and on the negative side (connection to mid-point of the interphase transformer). The positive breaker is set to open on reverse current only and is given a low setting with zero rate-of-rise adjustment. The positive d-c. circuit breaker is interlocked with the a-c. oil circuit breaker in such a way that the opening of the a-c. oil circuit breaker of any rectifier unit trips the d-c. positive breaker associated with that unit. The negative d-c. circuit breaker, when closed, short circuits a series load limiting resistor of three ohms resistance, which is cut into circuit when the negative breaker opens. This breaker is set to open only on very heavy overloads or short circuits. An auxiliary timing relay connected across part of the negative load limiting resistor is so arranged that if the current through the resistor drops to a pre-determined amount within a pre-determined time the negative breaker recloses. If the current does not drop within the pre-determined time the relay functions to trip the a-c. oil circuit breaker, which in turn trips the positive d-c. circuit breaker. Closing the a-c. oil circuit breaker automatically closes the negative d-c. circuit breaker.

Rectifier Grid Excitation. The outstanding features of the Lackawanna rectifiers, aside from the voltage and capacity of the units, are the anode grid excitation and the automatic compounding. Fig. 9 shows a cross-section of the rectifier tank. One of the main anodes, which are made of graphite, is shown at the left. In addition to the usual shields and baffles provided to shield the anode from the mercury thrown up from the cathode mercury pool by the arc stream, each anode is also surrounded by an inner shield suspended from a porcelain insulator. This shield supports a mesh grid, made of special high-melting-point metal, mounted just below the face of the anode. The inner shield and grid, which are insulated both from the anode and the tank case, are connected by a separate bushing through the anode seal and through a resistor to an excitation circuit, the opposite pole of which is connected to the cathode. Voltage applied to the grid ionizes the

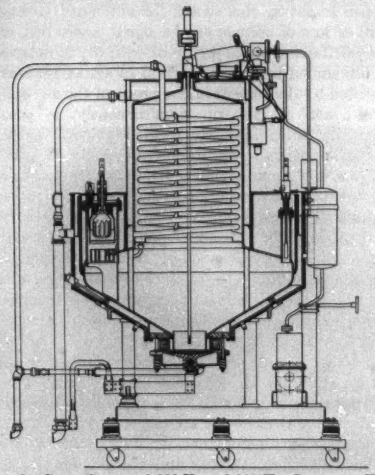

Fig. 9—Cross-Section 3,000-Volt, 3,000-Kw. Rectifier Tank

mercury vapor near the face of the anode and thus assists in establishing the power arc. This permits much closer shielding of the anodes than would be possible without the grid excitation, and consequently reduces the tendency for the rectifier to "arc back" or short-circuit internally from anode to anode and also materially reduces the amount of power necessary in the holding anode circuit.

Tests have been made both with d-c. grid excitation with a grid voltage of 96 volts (above cathode potential) and with twelve-phase a-c. excitation with the grid voltage leading the anode voltage by 12 to 15 degrees. When the rectifier units are run singly they operate satisfactorily with either d-c. or a-c. grid excitation. Difficulty has, however, been experienced in getting units to parallel satisfactorily with a-c. excitation, and consequently the units are now (October, 1930) being run with d-c. excitation supplied by tungar trickle charger battery sets pending further tests with a-c. excitation. The rectifiers parallel satisfactorily and

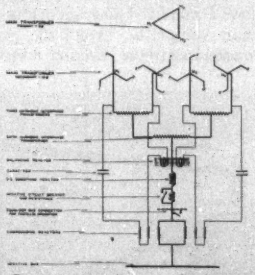

FIG. 10—COMPOUNDING CIRCUIT, SIMPLIFIED DIAGRAM

divide load perfectly with d-c. excitation, but a-c. excitation would be simpler if equally effective. The shielding of the Lackawanna rectifier is so complete that it will not establish the power arc and pick up load until the grids are excited, or if the grid excitation is cut off while a rectifier is carrying load the rectifier immediately drops its load.

Rectifier Compounding. The rectifier compounding circuit is shown diagrammatically in Fig. 10. The main power circuits are indicated by the heavy lines and the auxiliary compounding circuits by the lighter lines. As shown on the diagram the twelve-phase secondary of the main transformer is made up of four three-phase secondaries with their neutrals connected together through interphase transformer windings. From the standpoint of the external circuit the four three-phase windings make up a true twelve-phase secondary, each phase displaced 30 degrees from the next. Due to the action of the interphase transformers, however, when the rectifier is running non-compounded, each of the three-phase windings functions as a separate three-phase unit, and the voltage on the d-c. side of the rectifier is the voltage corresponding to a three-phase rectifier.

The shifting of the arc from anode to anode in the three-phase units results in triple frequency or third harmonic voltages, which are impressed on the third harmonic interphase transformers. If neutral N^1 were connected directly to neutral N^2 and neutral N^3 were connected directly to neutral N^4 instead of through the interphase transformer windings, the rectifier would function as two six-phase units and the d-c. voltage would be the voltage corresponding to a six-phase rectifier, which is approximately 15 per cent higher than the voltage of a three-phase rectifier with the same a-c. secondary coil voltage.

The compounding is accomplished by shunting the third harmonic interphase transformers (which connect neutral N^1 with N^2 and N^3 with N^4) by means of two compounding circuits each containing a capacitor and a winding of a compounding reactor. The d-c. windings of the compounding reactors are connected in series with the main power circuit, and consequently the saturation of the cores of the compounding reactors is varied with the output of the rectifier. Obviously, as the saturation of the core varies the reactance of the compounding circuit and its admittance to the third harmonic voltage across the third-harmonic interphase transformer winding also vary. The compounding circuits may, therefore, be tuned so that the capacity and reactance balance at the core saturation corresponding to the d-c. load at which the maximum compounding effect is desired. At that point the reactance of the compounding circuit to the third harmonic

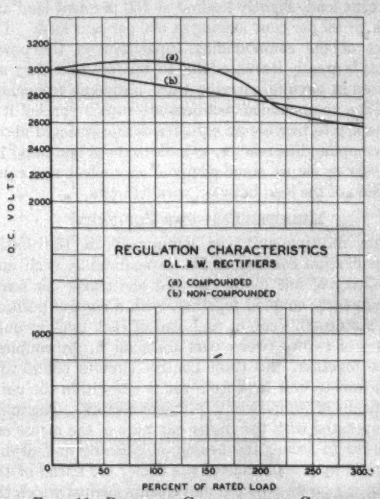

FIG. 11—RECTIFIER CHARACTERISTIC CURVES

voltage is substantially zero and the third harmonic interphase winding is therefore substantially short-circuited so far as the third harmonic current is concerned. The two three-phase windings then function as a six-phase winding, and the rectifier d-c. voltage corresponds to the voltage of a six-phase rectifier. In other words, at very light loads the d-c. voltage corresponds substantially to the voltage of a three-phase

rectifier and as load comes on, due to the shunting of the third harmonic current from the interphase transformer winding through the compounding circuit, the condition approaches more and more nearly that of the six-phase rectifier, and the voltage is varied accordingly. The two compounding circuits are linked together by means of the balancing reactor to insure balancing the load between the two groups of anodes. The compounding circuit can be adjusted to hold the d-c. voltage substantially constant up to as high as 225 per cent of rated load with relatively little increase in voltage at intermediate loads. The compounding of the Lackawanna rectifier is adjusted to give approximately 3,000 volts up to substantially 150 per cent load, using a total of 416 kva. of capacitors in the compounding circuits of each 3,000-kw. rectifier unit. The characteristic curve of one of the Lackawanna rectifiers, as determined by test, when running compounded, is shown in Fig. 11. For comparison the characteristic when running non-compounded is also shown.

The power factor of the compounded rectifier unit, including the transformer, is notably high, showing approximately 96½ per cent lagging at 25 per cent load and increasing with load to 99 per cent lagging at 50 per cent load, slightly leading at 150 per cent load and back to 99 per cent lagging at 300 per cent load. The effect of the compounding capacitors on the power factor is greatly increased due to the fact that they are located in circuits carrying third harmonic frequency.

When compounded rectifiers are run in parallel it is necessary to provide an equalizer bus connected above the compounding reactor, as indicated in Figs. 2 and 10, in order to assure equal compounding effect and equal division of the load between parallel units.

Multiple-Unit Car Equipment

The multiple-unit car equipment in the initial electrification consists of 141 two-car units, each unit consisting of one motor car and one trailer car semi-permanently coupled together, with a control position and motorman's cab at each end of each two-car unit. Trains of two to twelve cars are made up by coupling units together, the train control circuits connecting from unit to unit (and from car to car within the unit) by means of jumpers with plug connectors. The units are operated with the trailer car west of the motor car in order to secure the benefit of cleaning and drying the rails by the passage of the trailer car ahead of the motor car on west-bound movements against which the grades are heaviest.

All motor cars are new steel cars, 70 ft., 3¼ in. long over bumpers, seating 84 passengers, are equipped with vestibules with hinged doors and trap doors, and without toilets. All motor cars except five are equipped with roller-bearing trucks. The trucks on five-motor cars were equipped with standard journal bearings for comparative purposes. The motor cars, completely equipped, weigh approximately 147,000 lb. without passenger load.

All trailer cars are remodeled steel cars formerly used in steam service. They have been equipped with vestibules, seat 78 or 82 passengers, depending upon the type (except combination baggage and passenger cars and club cars, which seat less), are 70 ft., ¾ in. to 70 ft. 6 in. over bumpers, and weigh fully equipped from 105,000 to 110,000 lb., exclusive of passenger load. Each trailer car is provided with a toilet. All trailer cars, except five, have standard journal bearings on the trucks. The trucks of five have been equipped with roller bearings for comparative purposes.

Fig. 12 shows a view of an eight-car train, made up of four of the two-car units.

Fig. 13 shows a simplified diagram of the car wiring. All of the main equipment, except the pantographs, main fuse, master controller, heaters and heater bus

Fig. 12—Multiple-Unit Train with Four Two-Car Units
Showing typical overhead construction on curved track

jumper (connecting the trail car with the motor car), is mounted underneath the motor car.

Motor Equipment. Each motor car is equipped with four motors of 235 horsepower (one-hour rating) each, the capacity of the motors being fixed by the requirements of local service with frequent accelerations. On express runs the motors would be good for an additional trailer car on each unit, but all units were made alike in order to permit maximum flexibility in use. Advantage is being taken of this fact to permit hauling an extra express car (not electrified) on certain trains during non-rush hours. Fig. 14 shows the motor characteristic curves. The motors are operated with series-parallel connections, starting with four motors in series and cutting over to two parallel groups of two in series as the train comes up to speed. All motors are, however, insulated for the full 3,000 volts.

The motors are self-ventilated, using a series pull-push system. The cooling air is taken in through louvers at the top of the car, passes through settling chambers over the vestibules, then through ducts built into the sides of the car and under the car, and through flexible "pants-leg" ducts to the motors. The

air enters the motor at the commutator end through an end housing connection admitting the air to the armature inside the commutator bell. It is then sucked through the armature by the fan on the pinion end and forced back through the field windings to a discharge port at the commutator end of the housing. The air taken in at the roof of the car is relatively clean and the motor housing being under pressure there is no tendency to draw dirt into the motor. Results indicate that the design is effective in keeping the motor free from dirt or snow without the necessity for filtering the air.

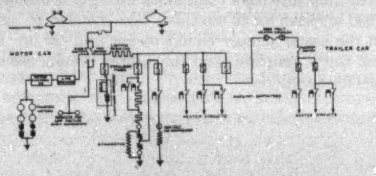

Fig. 13—Car Wiring, Simplified Diagram

There are four brush studs per motor, each brush stud holding two brushes. The armature shaft and axle bearings are of the constant oil level type. The gear ratio is 59 to 22 and the gears are equipped with special gear silencers. The gear cases are mostly malleable iron but some motors are equipped with pressed steel cases.

Control and Auxiliary Equipment. Each motor car is equipped with two pantographs, only one of which will normally be used at a time. The pantographs have four self-alining roller bearings, are spring raised and air lowered, and have a working range from 15 ft., 6 in. to 25 ft., 3 in. above top of rail. The standard pantograph has two contact shoes each equipped with two wearing strips with a grease groove between. For comparative purposes, however, fifty pantographs have been provided with single contact shoes, each with two wearing strips and a grease groove. These pantographs are idential with the standard pantographs in all other particulars and the parts are interchangeable. The normal pressures against the contact wire are 28 lb. for the standard pantograph (with two contact shoes) and 18 lb. for the single shoe pantograph. The maximum variation between upward and downward pressure throughout the working range of the pantograph is approximately 4 lb.

The control system operates at 32 volts, supplied from a storage battery carried by the motor car. A master controller is located in each end of each two-car unit. The main controller (which is under electro-pneumatic control by the master controller), the contactors, and auxiliary equipment are mounted beneath the motor car. The control equipment provides automatic acceleration adjusted for an acceleration of 1.5 mi. per hr. per sec. The operator can, however, take control at any point and hold up the automatic acceleration. With motors in the series parallel connection, and 50 per cent of the field shunted, the balancing speed of a six-car train is approximately 65 mi. per hr.

A dynamotor (with two windings in series), mounted under each motor car, provides 1,500-volt power for operation of a 1,500-volt air compressor and also drives, by direct shaft connection, a 4-kw. battery charging generator for charging the control battery, which also supplies the car lighting for the motor car and the associated trailer car. The dynamotor is self-ventilating, taking its ventilating air directly from beneath the car through a centrifugal air separator.

The air compressor has a displacement of 36 cu. ft. per min., and is adequate to supply the requirements of a four-car train with an ample margin of reserve in the event of failure of one compressor of a two-unit train.

In the event of failure of one unit in a train the control of that unit can be cut out and the faulty unit dragged as a trailer by the other units in the train. In the event of failure of one motor, or pair of motors, on a single-unit train that pair of motors can be cut out by a motor cut-out switch provided for the purpose

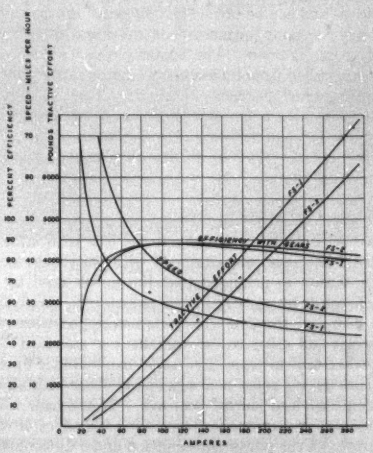

Fig. 14—Characteristic Curves for Multiple-Unit Traction Motor

underneath the car, and the unit operated on the other pair of motors.

Push-button control switches mounted in the operator's cabs provide for controlling the pantographs, the control circuit, the heater circuit, the main line breaker, the dynamotor, the number and gage lights and the head lights.

Heaters. Heating on all cars is supplied by totally enclosed resistance type heaters operated in series on 3,000-volt circuits. On the standard passenger cars the

heaters are mounted under the seats with 40 heating elements in series. The heating elements, which are themselves totally enclosed and insulated, are supported on porcelain block insulators mounted in steel cases, so designed as to guard fully against the possibility of coming in contact with the heating element. The heater cases and the conduits leading to them are thoroughly grounded. There are two separate heater circuits in each car with two heating elements in each heater case, one on each heater circuit. The two heater circuits are separately fused but are both under control of the same thermostat. Either circuit may, however, be cut out by the trainman, leaving the other circuit under thermostat control. In the baggage compartments of combination passenger and baggage cars and in the toilet rooms wall type heaters are used, and in the club cars the heaters are mounted along the sides of the cars just above the floor line in the position formerly occupied by the steam pipes. The total heater capacity in each passenger car is approximately 28 kilowatts. In addition, heaters are also provided in the motorman's compartments at the two ends of the unit. These are under the control of the operator (without thermostat control) and can be turned on only when the door of the cabinet enclosing the master controller and other control equipment is open. The heater circuit in the trailer car is supplied from its associated motor car by a heater bus jumper on the roof of the cars. There is no 3,000-volt connection between separate units, each unit being entirely independent of the other units in the train, except for the control circuits connected through by jumpers underneath the cars and for the air lines which are also connected through.

Metering Equipment. Ten of the motor cars (five of those with roller bearings and the five with standard journal bearings) are equipped with watthour meters with shunts so arranged that the power used by the whole car or by any circuit can be metered. With these ten cars and the five trailer cars equipped with roller bearings (all other trailer cars having standard bearings) one ten-car train (five units) can be made up with all cars having roller bearings and another with all cars having standard journal bearings. Thus means will be available for accurately determining the effect of roller bearings on the power consumption under all conditions of service.

Car Circuit Protection. The general scheme of the protection to the various circuits on the cars is shown by the simplified car wiring diagram, Fig. 13. The motor circuit is protected by a main line breaker consisting of four magnetic blow-out contactors in series and an overload relay. All auxiliary circuits are fed as a group through a limiting resistor of 1.5 ohms (which would limit a short-circuit current to a maximum of 2,000 amperes). Each auxiliary circuit is protected individually by an expulsion fuse, and the dynamo circuit has an additional permanent resistance in series of 13.5 ohms.

There is also a main fuse, of a new combination expulsion—compression type, mounted on the roof of the car through which all car circuits are supplied. In the event of a heavy short circuit on the car or a motor flashover not cleared by the main line breaker, the high-speed circuit breakers at the substations and tie stations through which power is supplied to the overhead contact system, will normally operate faster than the main fuse on the roof of the car. If, however, the high-speed circuit breakers should fail to function properly, the fuse will clear the fault. Only one case of this kind has as yet occurred.

The main fuse has a continuous rating of 300 amperes, or 650 amperes for 30 seconds. The dynamotor circuit and the heater jumper circuit to the trail car are each fused for 20 amperes, and the heater circuits are each fused for 8 amperes.

Fig. 15—Three Power Locomotive

Lightning protection is provided on each motor car by an electrolytic aluminum cell lightning arrester with balancing resistor and shunted spark gap.

Locomotives

Two locomotives of the three-power type shown in Fig. 15 have been provided for handling the freight transfer service between the Secaucus and the Jersey City yards, a distance of approximately four miles. When on electrified tracks, these locomotives operate from the 3,000-volt contact system, and when on non-electrified tracks on internal power provided by a 300 hp. diesel engine direct connected to a 200-kw., 750-volt generator supplemented by a storage battery of 360 cells with a rating of 242 kw-hr. (340 ampere hours). Each locomotive weighs 248,000 lb. with all the weight on four driving axles, is equipped with four motors of 400 horsepower each (one hour rating) and has a tractive effort of 60,000 lb. Each locomotive is capable of hauling a train of 2,500 tons, trailing load, at a balancing speed of approximately 28 mi. per hr. when operating on external power from the 3,000-volt contact system, or at approxi-

mately 12½ mi. per hr. when operating on internal power only. When running light on internal power the locomotive has a balancing speed of approximately 30 mi. per hr.

Distribution System

With two minor exceptions the 3,000-volt power for operation of the trains is distributed through the contact system only, without the use of auxiliary positive feeders. The two exceptions are at points where there is a change from three electrified main tracks to two electrified tracks, and the third circuit is carried through the two-track zones in order to make the three circuits independent throughout.

Type of Catenary. The catenary system is of the tangent chord type. On main line tracks it consists of a composite main messenger, an auxiliary messenger and two contact wires. The main messenger is made up of a core of seven strands of high strength bronze surrounded by thirteen strands of hard-drawn copper, the total having a breaking strength of 31,000 lb. and an equivalent conductivity of 350,000 cm. hard-drawn copper. The auxiliary messenger is 2/0 hard-drawn copper and the contact wires are two 4/0, grooved bronze wires of 55 per cent conductivity. A relatively small amount of special 80 per cent conductivity bronze contact wire has also been installed for comparative purposes. The total equivalent conductivity of the standard main line catenary system is 720,000 cm. hard-drawn copper per track. On yard tracks and cross-overs a lighter system is used with a 7/16-in. bronze messenger and a single 4/0, 55 per cent conductivity bronze contact wire.

Catenary Supports. Steel catenary supporting structures are used throughout except on the branch line from Summit to Gladstone, which is a single track line and on which wood poles with steel brackets are used except at special locations where steel structures are required. The standard steel supporting structure has H-columns and either an H-beam or a lattice truss cross member, the H-beam being used up to spans of 53 ft. and lattice truss for longer spans. Dead end structures have A-frame columns. Copper-bearing steel was used in all small members. The H-columns on standard structures are set with the web at right angles to the track and are bolted to the foundations with two bolts. On tangent track the standard spacing between catenary supports is 300 ft.; on curved track, or where there are overhead bridges or other obstructions, the spacing is reduced, or pull-off poles are located between supporting structures. In general the signals and catenary system are supported on the same structures. Signal bridges installed during recent years had been designed to take future catenary load, and the older signal bridges were replaced by new structures. In a relatively few cases it was impracticable to locate the catenary structure at the desired signal location, and existing signal bridges (raised if necessary) were left between catenary supporting structures.

Fig. 12, already referred to, shows typical catenary construction on three-track curve section. Fig. 16 shows typical construction on three-track tangent section and also illustrates the method of mounting signals on catenary supporting structures.

Tie Stations and Catenary Sectionalizing. Except on the single track branch line from Summit to Gladstone, tie stations (or circuit breaker sectionalizing stations) are located approximately midway between substations and also at or near the ends of stub end lines. The locations of the tie stations and the distances between substations are shown on the map, Fig. 1, already referred to. At the tie stations the catenary system is sectionalized and all circuits are brought to a common bus through high-speed circuit breakers, the holding coils of which are energized from the line. With this arrangement advantage is taken of multiple feed with

Fig. 16—Typical Contact System on Three-Track Tangent Section

Method of mounting signals on catenary supporting structure shown in foreground

reduced copper losses between substations and tie stations without sacrificing selective protection of the line. In the event of a line fault the faulty section is isolated by the substation and tie station breakers connected with that particular section of the line without disturbing the rest of the line. All tie stations are largely automatic in function but are also under remote or supervisory control from the nearest railroad signal tower. Lights on the control panel in the signal tower indicate the position of each feeder circuit breaker and a voltmeter shows the voltage of the control battery. The towerman, under direction of the load dispatcher, can operate any switch in the tie station or restore it to automatic control, at will. The high-speed circuit breakers connected with circuits which are also fed at the other end from a substation are arranged so that after opening under a line short circuit or overload they reclose automatically as soon as voltage is restored on the circuit (*i. e.*, when the circuit has been energized

from the substation end by the operator at the substation, reclosing the feeder breaker). On stub end lines, which receive power at one end only, the breakers, after opening under short circuit or overload, automatically reclose three times at definite intervals and then lock out if the fault has not been cleared. The remote control permits the towerman to close any breaker which has remained open as a result of automatic operation.

As previously pointed out, the holding coils of the high-speed circuit breakers on the feeder circuits and on rectifier circuits at the substations are energized from a storage battery, whereas the holding coils of the feeder breakers at tie stations are energized from the line. This arrangement aids in getting selectivity in the event of line fault since the holding power of the breakers at the tie station is reduced as the line voltage falls under short circuit.

The trip points of the high-speed magnetic blow-out circuit breakers are adjustable as to current and also as to sensitivity to the rate-of-rise of the current. New features of these breakers permit increased flexibility in the rate-of-rise adjustment. Means are also provided for calibrating the breakers with low-voltage battery current (maximum, 20 amperes at 12 volts).

With the breakers properly adjusted with respect to each other a short circuit on any part of the catenary system will normally open only the breaker or breakers directly connected with the faulty section of line. The time from application of the short circuit until the current starts to decrease following the opening of the breaker is as short as 0.012 seconds under maximum short-circuit conditions. Longer times may be required for lesser short circuits.

In addition to the sectionalizing provided at tie stations and substations, other intermediate line sectionalizing switches are provided on the catenary system to permit cutting out smaller sections of line in event of fault or for inspection and maintenance work. These sectionalizing switches are of the horn-gap air-break high contact pressure type and are mounted on the catenary supporting structures, and equipped with operating mechanisms permitting operation from the ground. Those in most important locations are equipped with motor-operated mechanisms under control from the nearest signal tower or other point where an attendant is always available who can operate the switches under instructions from the power dispatcher.

Ground Wires. In order to provide lightning protection both to the catenary system and to auxiliary power circuits which are carried by the catenary structures, two 1/0 hard-drawn copper ground wires are carried on bayonets on the catenary structure except on the Gladstone branch, which is referred to later. These ground wires are connected directly to the steel of the catenary structures and are also connected to the mid-point of approximately every alternate impedance bond. They also serve to give a low resistance return path in the event of a faulty insulator on the catenary system and thereby insure proper breaker operation. In addition they serve, to a minor degree, as auxiliary negative return circuits in parallel with the rails.

On the single track line from Summit to Gladstone a supplementary negative return cable is provided consisting of a 4/0 hard-drawn copper cable carried on the catenary supporting brackets close to the poles, and connected to the brackets. This negative return is bonded to the mid-point of impedance bonds at suitable intervals. Its most important function is to guard against the possibility of track workers being exposed to high voltages across open track circuits in event of broken rails or removal of rails during track repairs. It also serves to insure proper circuit breaker operation and prevent burning of the wood poles in event of a faulty catenary insulator, provides considerable lightning protection to the catenary system, and decreases the resistance of the return circuit by supplementing the rail return.

Track Bonding

The standard track bonding consists of a single 4/0, 8½-in., gas-welded, stranded-copper, steel-terminal, U-bond at each joint on each rail, except where only one rail is bonded through interlocking plants, at which locations two of the same size and type of bonds are installed on each joint of the one rail. Impedance bonds are provided at all signal locations, the distance between the impedance bonds averaging about 2,500 ft. Two sizes of impedance bonds are used, 500 amperes and 1,000 amperes, depending upon the location. Cross bonds are provided between mid-points of impedance bonds at suitable intervals ranging from 5,000 to 8,000 ft.

Shops

Periodic light inspection of the cars and locomotives is taken care of in a light inspection shop in the Hoboken yard. This shop is 480 ft. long and has three inspection pits, each sufficient to accommodate a six-car train, and two shorter truck tracks. It is also provided with galleries for facilitating inspection and adjustment of pantographs and with a crane at one end of the shop of sufficient capacity to lift one end of a car at a time to permit change of trucks. This shop is equipped for making light repairs only. The 3,000-volt contact system is not carried into the shop. The cars are moved in and out of the shop by 250-volt power supplied to the cars (from motor-generator sets) by means of jumper cables plugged into receptacles provided for the purpose on the motor cars, as indicated on the elementary car wiring diagram, Fig. 13.

Heavy repairs, general overhaul and painting will be carried out at the railroad's car shops at Kingsland approximately seven miles from Hoboken on the Boonton line. Two sections of the Kingsland shop have been modified to make them thoroughly suitable for this work and facilities for dipping and baking armatures and other motor repairs have been provided. As the Boon-

ton line is not now being electrified beyond Secaucus the cars will be hauled to the shop by steam switching locomotives or by the three-power electric locomotives running on internal power.

Signal System

In connection with the installation of the electrification program it was necessary for the railroad to make a complete change in its signal system. The signals and track circuits were formerly battery-operated and the signals throughout most of the territory electrified were of the old semaphore type. All signals in the electrified zone have been changed to color light signals with a-c. track circuits. They are now operated at 60 cycles, alternating current, but are so arranged that the frequency can be changed to 100 cycles if this should later be necessary in connection with automatic train control. Power for operation of the signals is supplied from signal power circuits carried on the catenary supporting structures. In most cases these circuits are in duplicate with provision for automatic throw-over from one circuit to the other in the event of failure of power on the first circuit.

Protection Against Electrolysis, Inductive Interference and Contact with Other Circuits

Extensive surveys and tests were made both before and after electric operation of trains was begun to determine the points at which there might be danger of electrolytic damage to underground cable sheaths and other metallic structures, and drainage cables have been installed where they were found desirable.

Where necessary to provide adequate physical clearance, communication circuits located along the right-of-way, consisting of the railroad's communication lines and some other telephone and telegraph lines, have been moved or put into cables. Inductive interference from the d-c. power current flowing through the catenary system has been practically eliminated by the use of twelve-phase rectifier transformers, d-c. smoothing reactors and resonant shunts, installed at the substations. Some slight difficulty on railroad communication circuits with long parallel exposures was experienced at first but was substantially eliminated by more careful adjustment of the resonant shunts.

The mercury arc rectifiers also cause a distortion of the wave form on the a-c. supply circuits, and where the a-c. power is supplied by overhead transmission lines inductive interference in neighboring telephone circuits may be experienced unless steps are taken to guard against it. This effect can, however, be largely eliminated by proper coordination and transposing of the power circuits and the neighboring telephone circuits. This has been taken care of by the cooperative effort of the power companies and the telephone companies.

Communication circuits and low-voltage power circuits crossing the right-of-way, except those on overhead highway bridges, have been removed.

No radio interference has been reported.

Methods of Handling Construction Work

The electrical equipment on new motor cars was installed by the car builders with the exception of the motors which were installed by the railroad forces after delivery of the cars. The electrical equipment on the trailer cars, consisting of control and heater equipment, was installed by the car builders adding the vestibules to these cars. The substation and tie station buildings were built under contract by building contractors, except for two tie stations where existing buildings were used, and all equipment in substations and tie stations was installed by electrical contractors. All work along the right-of-way, including catenary foundations, catenary supporting structures, erection of the catenary system, and auxiliary power and signal circuits, relocation and modification of signals and track bonding was carried out by railroad forces.

Discussion

H. M. Trueblood: This electrification has been of considerable interest from the inductive coordination standpoint, not only to the operating telephone and telegraph companies, but also to those of us who are studying this question from a more general point of view. For one thing, this is the first electrification in the United States depending solely on rectifiers as the source of propulsion power. For another, the contact-wire voltage is 3,000 volts as against 1,500 volts for the Illinois Central terminal electrification in Chicago, and 600 volts for the New York Central and Pennsylvania terminal electrifications in New York. This matter of trolley voltage is of interest because, other conditions being the same, it is a fair approximation to say that the voltage induced in exposed communication circuits, when trolley short circuits occur, is proportional to the trolley voltage. Again, in the territory traversed by the Lackawanna Electrification, there is a considerable concentration of important communication facilities. In a rather unusual degree, telephone central offices are located in close proximity to the railroad right-of-way and there was some concern at first because of the possibility of ground potentials sufficient to cause disturbances during periods of train acceleration, when fairly heavy currents are drawn for several seconds.

The telephone company has therefore followed the work of getting this electrification into operation with a great deal of interest. Through excellent cooperative arrangements, the voltages produced under short-circuit conditions, and also ground potentials under conditions of heavy current flow, have been quite fully studied in a series of field tests. While a few measurements are still to be made, the results of these tests indicate that little or no difficulty is to be expected in Bell System plant either from induction or from ground potentials. Measures mentioned by Mr. Moreland have been effective in the avoidance of noise from the trolley-circuit currents and voltages. Aside from such arrangements to avoid electrolysis as are mentioned in the paper, we therefore do not expect that specific measures of coordination will be needed, so far as concerns effects in Bell System plant arising from the d-c. propulsion system.

A number of tests on the effects of wave-shape distortion on the a-c. side of the rectifiers has been carried out, with the cooperation of the power companies supplying energy to the electrification. Here measures to which Mr. Moreland has referred will, we believe, take care of the situation so far as exposures to the rectifier feeders themselves are concerned. The problem, however, is not confined to such exposures, since the harmonics originating in the rectifiers tend to spread in the connected a-c.

system, and in fact, appear on the 2,300-volt buses used for general distribution to certain localities. The solution to this part of the problem is now being worked out. In certain places, it appears that specific measures of some sort will have to be taken to avoid difficulty.

W. B. Potter: The 3,000-volt multiple unit equipment principally because of its application to suburban service is of particular interest. The scheme of ventilation for the traction motors seems better than anything that has been done heretofore. Putting the air intake at the top of the car away from the dirt and brakeshoe dust, together with using the empty space at the end of the car roof for a settling chamber, is obviously a good way to supply the motors with clean air. It is surprising that this plan has not been more generally used.

The use of 3,000-volt rectifiers to the exclusion of any rotary conversion apparatus is probably the most novel feature of this electrification. Reference has been made to arcovers. Something that is akin to commutator flashing but different in that it is not destructive to the rectifier. The momentary even though infrequent interruption of power is, however, an annoyance. It is gratifying to know that progress is being made toward the elimination of this arcing phenomena.

C. H. Lydall: I have been very interested in Mr. Moreland's paper as I spent some years in South Africa on the Maine Line Electrification of the Natal Railway.

One hundred and seventy route miles, mostly single line, were electrified, primarily to increase the volume of traffic over the hilly portion of Central Natal. The railroad in the center of the section electrified climbs up to a height of 5,000 ft. above sea level and then drops down to 1,000 ft. above sea level at the southern end. This will give some idea of the gradients which the electric locomotives had to negotiate. Locomotives working on 3,000-volt d-c. were used throughout on the electrification and three 1,000-hp. units were used on all fully loaded freight trains.

Very little trouble was experienced in the construction and starting to work of the scheme with one exception, namely, trouble from lightning. In parts of South Africa lightning is, I suppose, as severe as in any part of the world. It was not realized how severe this was until we got to the preliminary running stage of the electrification. During that period it was found that in bad storms lightning struck the trolley lines and arced over, generally at the point where the pull-off arms were attached to the masts. The insulators at this point were smashed, the pull-off arms hung down, suspended from the trolley wire and the next locomotive that came along caused a short circuit. Maintenance gangs had to get to the point of breakdown and repair the line before traffic could be resumed. In South Africa, where the roads are few and very bad, the greatest time was taken in the gangs getting to the breakdown.

As soon as it was realized how severe the lightning was in Natal and the serious effect it would have on the running of the service, high-speed circuit breakers were installed on the feeders to the track. The breakers operated so satisfactorily that interruptions caused by lightning ceased entirely. When the lightning hit the trolley wire and the insulators flashed over the breakers opened so quickly that the flashover was extinguished before the insulators were damaged. Locomotive drivers were given two minutes in which to switch off their controllers, the line breakers were closed again and service continued in normal operation.

Transverse Fissure Detector Car

BY HARCOURT C. DRAKE[1]
Associate, A. I. E. E.

Introduction

TRANSVERSE fissures in rails are fatigue failures of internal origin, whose rate of growth depends largely upon the density of traffic. The typical air-tight fissure, Fig. 1, has a characteristically bright, silvery appearance produced by the peening action of the fissure faces when the rails are subjected to the impact of heavy wheel loads.

This type of failure was brought to the attention of railway engineers about 19 years ago when a passenger train was derailed by a rail which broke into 17 pieces, disclosing several transverse fissures. In the years which followed, rail failures due to transverse fissures were reported in ever increasing numbers. The presence of large transverse fissures in relatively new rail under the best conditions of track maintenance, together with the utter impossibility of seeing or locating such fissures, placed this type of failure in the class with a cyclone, a lightning stroke, or other "act of God."

Fig. 1—A Typical Internal Transverse Fissure

In the early part of 1927, internal fissures were successfully located in rails at the laboratory by the electric method and after several months of experimental work a fissure detector car (Fig. 2) was constructed for the American Railway Engineering Association. The car passed its acceptance test and was placed in testing service on the railroads of the country in November, 1928.

Theory of Operation

The fissure detector depends for its operation upon the fact that electric current flowing through a rail is compelled to pass around any fracture, inclusion, or separation in the metal. When a rail is sound the axis of current is substantially straight, but if a fissure is encountered, the current axis at that point is displaced. An inductive pick-up device is mounted directly over the rail head, where it cuts the magnetic lines of force surrounding the rail. In the case of a sound rail carrying current, the magnetic flux surrounding the rail is uniform and no e. m. f. is generated as the inductive pick-up passes along, but when the axis of current is deflected, as at a fissure, the flux density around the rail varies, resulting in a generated potential at the terminals of the pick-up. This output from the pick-up is amplified and led through relays connected to recorder pens, thus giving a record of the received impulse (see Fig. 3).

Factors Governing Design

In applying the electrical method of fissure detection to inspection of rails in track, the following factors materially affect the final design:

1. Many spots are encountered where locomotive drivers have slipped in starting and gouged out the rail.

Fig. 2—The First Fissure Detector Car, as Built for the American Railway Association

2. Worn or flat spots occur frequently, especially opposite the rail joints.
3. In certain stretches of track the rail surface has acquired a pronounced ripple, termed corrugated or washboard rail.
4. The rail is frequently covered with a hard insulating film, the resistance of which may run up to several thousand ohms.
5. The gage of the track varies between certain limits.
6. The rail surface may be level or canted.
7. The size of the rails varies from 70 to 130 lb. per yd.

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

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Description of the Detector Car

The single unit type of detector car (Fig. 4) is 45 ft. long, self-propelled, equipped with extra heavy frame and axles, and capable of a speed of 45 mi. per hr. when running to and from work. Just back of the driver's compartment are located the cook's gallery and comfortable living quarters for the crew. Back of the living quarters is the compartment containing the power plant of the detector equipment. This power plant consists of a specially designed double commutator, 6,000-ampere

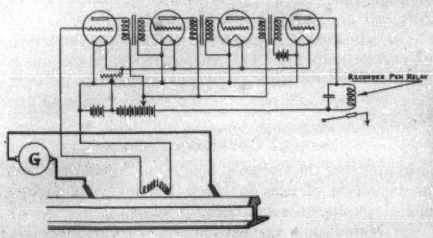

Fig. 3—Schematic Diagram of the Fissure Detector Circuit

2-volt d-c. generator, direct connected to a 50-hp. four-cylinder gasoline engine. The exciting generator and air compressor are also driven by this engine. Current from the 2-volt generator is carried by heavy buses to the brush units located on either side of the car. The brush unit assembly (Fig. 5) consists of two sets of

Fig. 4—The Self-Propelled Type of Detector Car

eight solid phosphor-bronze brushes mounted about 3 ft. apart and rigidly connected by a cast frame. These brushes are supplied with individual springs which insure a pressure of 50 lb. per brush. Flanged wheels located at the ends of the brush holder engage the rail when the brushes are down and, due to their angular mounting, cling to the gage side regardless of varying track conditions. The brush units are applied to the rail by air-operated pistons electrically controlled from the operator's switchboard. A safety switch is connected to the gear shift of the motor-car drive so that the brushes will lift if the driver attempts to back up while they are on the rail. It was found that brush contact resistance was greatly reduced by the use of water on the rail. Provision is therefore made to carry a sufficient supply of water for this purpose and the brush control switch applies the water to the rail automatically when the brushes are lowered. Mounted between the brushes is a separate carriage upon which is supported the pick-up unit. This carriage with its eight wheels and swivel mounting holds the pick-up unit at a fixed distance above the rail and rides over burns and flat spots without interruption. Initially driver burns and flat spots gave indications which tended to slow up the testing. This source of trouble was materially reduced by the eight-wheel mounting. Recently a new pick-up device of radically different

Fig. 5—Main Current Brush Unit

design has been on test on five of the cars. It eliminates the indications due to flat spots and all but the largest of the burns.

The operating room is located at the back end of the car and contains the recording table and control board (Fig. 6) as well as amplifying equipment and batteries.

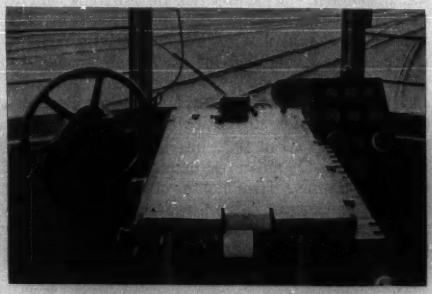

Fig. 6—Recording Table and Control Board

The output from the pick-up unit is brought up to a four-tube amplifier which is properly shielded and protected from shock. Three recorder pen relays are connected to the output of the amplifier and adjusted to respond to different values of plate current, thus giving an indication of the size of the defect detected. Other relays are provided to operate paint guns so that the fissure will be automatically marked by a spot of paint. The record (Fig. 7) is made up of nine lines; the three

pens on either side (numbered 1, 2, 3) give the relative size of the fissure while the lines in the center record the angle bars. Joint cut-outs mounted on the brush holders engage the angle bars and cut out the fissure pens and paint gun at the joint, at the same time operating the joint pens. The line to the extreme right is a land-mark pen operated from the motor-car driver's switch-board. He presses the button whenever a mile post, bridge, signal tower or other land-mark is passed and the operator at the recording table stamps in the proper designation.

Operation of Car

In operation, the car passes along the track at approximately 6 mi. per hr. with from 2,000 to 3,000 amperes flowing through each rail, the current value depending upon the weight of rail being tested. The operator, seated at the recording table in the rear of the car, has the record before him and a clear view of the tested track. When the car passes over a fissure an indication appears on the record and the defect is automatically marked with a spot of paint. The operator notes the record, sees the paint mark on the rail, and stopping the car, backs it up to the paint mark

Fig. 7—Sample of Record Made by Detector Car

for a hand test of the suspected spot. To make the hand test, 1,500 amperes is introduced into the rail by means of air-operated contactors and the unit I. R. drop is measured with a galvanometer. By means of this test the size of a transverse fissure can be determined within a few per cent. The detector car not only locates transverse fissures, but split heads (Fig. 8), horizontal fissures (Fig. 9), compound fissures, pipes, cracked webs, broken bases, and other defects. When a defect is located by the car, the first question asked by the railroad representative is, will it be safe to leave the rail in track until there is a lull in the day's traffic, or must the next train be tied up while the rail is replaced? The answer to this question lies in the result of the hand test. When the hand test shows no transverse crack, the rail is marked for removal at the next opportune time, but if a large transverse fissure is indicated the railroad takes no chance, removing the rail at once.

Although the testing speed of the car is 6 mi. per hr., there are many elements which enter in to reduce the total mileage tested per day.

1. Traffic delays due to clearing for trains frequently consume considerable time, but are unavoidable.

2. When a fissure is located, the railroad representative always makes a record of the manufacturer, the year rolled, ingot letter, and heat number of the rail. If the rail has been in track for several years or has been oiled by the track oiler, it is often difficult to locate and decipher the heat numbers and it is not at all uncommon on some roads to have from five to fifteen minutes wasted in obtaining the rail data.

3. A repeat run over a suspected rail takes 30 sec-

Fig. 8—A Split Head

onds and from two to three minutes are necessary for a hand test.

Testing Procedure

The testing procedure on the ten cars now in service is practically identical, but the railroads differ somewhat in their method of conducting the test. A survey of the various methods employed indicates that the following procedure results in a minimum of time lost.

The railroad has one man on the car and one on a

Fig. 9—A Horizontal Fissure

hand car which follows the detector car. In addition there are, of course, the conductor and flagman. When an indication is recorded the car is stopped, a check run made over the suspected rail, and the car backed up for a hand test. The operator examines the rail and if this visual inspection discloses exterior evidence of a split head, horizontal fissure, or other defect he gives the word to the railroad man, who makes a red paint mark

on the tie at the defective spot and chalks the identification number on the rail. If no visible defect is noted, the hand test is made and the rail is marked as before. The car then proceeds with the testing while the man on the hand car stops to take down all of the data together with the location of the rail, and then continues on after the test car. In this way the maximum time taken is the two or three minutes required for hand testing.

Conclusion

Since the first detector car was placed in service two years ago, many interesting fissure facts have been accumulated.

1. Rails have been found containing many defects as, for example, one which had 24 transverse fissures.

2. Fissures have been found in rails 28 years old; and also in rails 5 months old.

3. Many miles of track have been found free of fissures; other short stretches have had 20 or 30 fissures per mile.

While these isolated cases are of interest, the most important point is that the transverse fissures are now being located and removed from track before they can cause any damage, whereas prior to two years ago their presence was not known until they had grown to the surface or broken the rail.

Inclined Catenary Calculations

BY B. M. PICKENS[1]

Associate, A. I. E. E.

Synopsis.—*This paper gives a simple method of determining the general U and S curves for a catenary system. They are both obtained from the same equation and give the general shape the contact wire must take in order to lie in a horizontal plane.*

The specific shape the contact wire will take on any curve is found by adding together multiples of the U and S curves. Methods of obtaining the multipliers for track curves of varying radii and for constant radius curves are given. The method used postulates a smooth continuous curve for the contact wire from span to span lying in a horizontal plane.

The contact wire does not necessarily have the same shape as the track curve. For a given catenary system the shape taken is determined only by the relative location of the points of messenger support. The condition of design necessary to give a contact wire curve which has practically the same shape as the track curve, is shown.

THE use of "inclined catenary" on curves in railway catenary construction has become quite general in the United States on all systems having a single contact wire whether simple or compound catenary. The calculations involved in determining the dimensions of the inclined catenary are not as simple as for tangent catenary and are not generally known. For the inclined catenary it is necessary to determine the dimensions of a space figure while tangent catenary can be considered as lying wholly in a plane. Also there are more variables affecting the shape of the inclined catenary. A fairly simple method of solution can be followed if the projections of the inclined catenary on horizontal and vertical planes are considered.

The contact wire will lie wholly in a horizontal plane

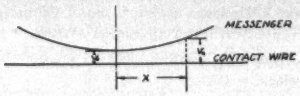

FIG. 1—VERTICAL PROJECTION OF MESSENGER AND CONTACT WIRE

and will be supported from the messenger by hangers inclined to this plane. If the hangers are equally spaced each one will carry the same portion of the contact wire weight since there can be no transfer of weight from one to another without a vertical dimension to the contact wire. Since each hanger carries an equal load to the messenger the vertical load on the messenger is the same as on tangent catenary, neglecting the variation in hanger weights, and the vertical projection of the messenger will be the same as for a tangent catenary messenger of the same system in a span of the same length.

The shape of the vertical projection can be determined with sufficient accuracy if it is considered to be a parabola. The curve actually lies somewhere between a catenary and a parabola. If all the weight of the system were concentrated in the messenger it would take a catenary shape, if in the contact wire, a parabolic shape. The parabola has the simplest formula and is generally used for catenary calculations of this type. If a system of axes is taken as shown in Fig. 1 with the origin located on the vertical projection of the contact wire directly below the low point of the messenger, then the vertical projection v_n of any hanger x feet from the origin will be

$$v_n = v_o + W x^2/2 M \qquad (1)$$

where

v_o = vertical projection of shortest hanger
W = weight of system per ft. lb.
M = messenger tension lb.

Equation (1) gives either the tangent hanger length or the vertical projection of any curve hanger.

If the horizontal projections of the contact wire and messenger are now considered it can be shown[2] that their shapes are mutually reflected at some scale depending on their tensions. An axis can be drawn between the projections so located that the perpendicular distance from any point on the axis to the projection of the contact wire is to the perpendicular distance

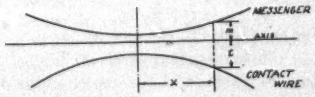

FIG. 2—HORIZONTAL PROJECTION OF MESSENGER AND CONTACT WIRE

from the same point, to the messenger projection, as the messenger tension is to the contact wire tension. This is shown in Fig. 2 where

$$t/m = M/T$$

T = contact wire tension

If the horizontal projection of any hanger is h

$$t + m = h$$

and

$$h = t\,(1 + T/M) \qquad (2)$$

Since the relationship between the horizontal pro-

1. Gen. Engg. Dept., Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

2. *Catenary Design for Overhead Contact Systems*, H. F. Brown, A. I. E. E. TRANS. 1927, p. 1082.

jections of messenger and contact wires is known, it is only necessary to determine the shape of one of the wires in order to find the hanger lengths. It is convenient to work with the contact wire projection and determine the values of t for various hangers. The curve hanger length will be the square root of the sum of the squares of the vertical and horizontal hanger projections.

Curve hanger length

$$g = \sqrt{h^2 + v^2} \tag{3}$$

In Fig. 3 the relationship between the values of t at three adjacent hanger locations at any place in the span is

$$t_{n+1} = t_n + (t_n - t_{n-1}) + \Delta \tag{4}$$

The horizontal load f_n applied to the nth hanger by the change in direction of the contact wire is directly proportional to change Δ and the contact wire tension T and inversely proportional to the hanger spacing s.

$$f_n = \Delta\, T/s \tag{5}$$

The slope of the hangers is determined by the horizontal and vertical loads applied at the lower end. The horizontal load on the nth hanger is f_n and the vertical load w is the weight of s feet of the contact

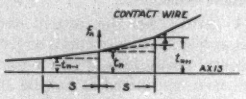

Fig. 3—Horizontal Projection of Three Adjacent Hanger Locations at any Place in the Span

wire or wires. Usually half the weight of the hanger is also considered as part of the vertical load.

From Fig. 4 it will be seen that

$$f_n = w\, h_n/v_n \tag{6}$$

Combining equations (2), (5), and (6)

$$\Delta = s\, w\, t_n\, (1 + T/M)/v_n\, T \tag{7}$$

and substituting (7) in (4)

$$t_{n+1} = t_n\, [2 + s\, w(1 + T/M)/v_n\, T] - t_{n-1} \tag{8}$$

if we change equation (1) to

$$v_n = v_o\, (1 + x^2\, W/2\, v_o\, M)$$

and substitute in (8)

$$t_{n+1} = t_n\, [2 + c/(1 + x^2/c_1)] - t_{n-1} \tag{9}$$

where

$$c = s\, w\, (M + T)/v_o\, T\, M$$

and

$$c_1 = 2\, v_o\, M/W$$

also

$$x = \text{value of } x \text{ for } n\text{th hanger.}$$

The values of c and c_1 are constant for any given system.

Equation (9) is a general equation which gives the relationship between the values of t for any three adjacent hangers. The equation holds for any type of span, uniform curve, reverse curve or changing curve as no assumptions were made concerning the nature of the curve. It gives the general shape the contact wire must take in order to lie in a horizontal plane. If the relationship between any two adjacent values of t in a span is known, the value for the adjoining t can be determined and all other values of t in the span can be found by applying (9) progressively. From equation (9) two general shape curves can be plotted for any given system. Assume that the values of t adjacent to and equally spaced from the origin, are equal, then a curve can be calculated which will have

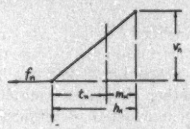

Fig. 4—Cross Section of Catenary at Nth Hanger

a U shape. Also, if it is assumed that the values of t are equal but of opposite sign another curve develops which has an S shape. It is convenient to select a catenary system in order to plot and show the use of these curves.

	Wgt. per ft.	Tension
Messenger bronze	0.744	3820
Hangers	0.080	
Auxiliary wire 2/0 copper	0.403	1200
Contact wire 4/0 bronze	0.641	2500
Weight of system per ft	1.868	

Hanger spacing s = 15 ft.
Length v_o = 0.5 ft.
Weight w = 15 (0.641 + 0.403 + 0.040) = 16.26
Weight W = 1.868

From these data the values of the constants c and c_1 can be calculated.

$$c = \frac{15 \times 16.26\ (3820 + 3700)}{0.5 \times 3820 \times 3700} = 0.260$$

$$c_1 = \frac{2 \times 0.5 \times 3820}{1.868} = 2048$$

For the system selected equation (9) can be written

$$t_{n+1} = t_n\, [2 + 0.260/(1 + x^2/2048)] - t_{n-1}$$

the value of the quantity in the brackets depends only on x and can be solved for the value of x for each hanger location. If we assign a value of + 1 to t_1 and + 1 to t_{-1} then we can calculate all other values of t in the span.

$$t_2 = 1 \times 2.2530 - 1 = 1.253$$
$$t_3 = 1.253 \times 2.2081 - 1 = 1.770$$
$$t_4 = 1.770 \times 2.1540 - 1.253 = 2.56$$
$$t_5 = 2.56 \times 2.111 - 1.770 = 3.63$$

and the other values for t (U) can be obtained in a similar manner.

The values for $F(x)$ for each hanger location for a 330-ft. span are given in Table I, together with the values of t for the U and S curves. The S curve is obtained by assigning to t_1 a value of $+1$ and to t_{-1} a value of -1.

$$t_2 = 1 \times 2.253 + 1 = 3.253$$
$$t_3 = 3.253 \times 2.2081 - 1 = 6.18$$
$$t_4 = 6.18 \times 2.154 - 3.253 = 10.15$$
$$t_5 = 10.15 \times 2.111 - 6.18 = 15.24$$

and the other values of $t(S)$ can be obtained in a similar manner and are given in Table I.

TABLE I

Hanger location	Ft. from low point	$F(x)$	$t(U)$	$t(S)$	$0.25\,t(S)$
1	7.5	2.253	1.00	1.00	0.25
2	22.5	2.208	1.25	3.25	.81
3	37.5	2.154	1.77	6.18	1.55
4	52.5	2.111	2.56	10.15	2.54
5	67.5	2.081	3.63	15.24	3.81
6	82.5	2.060	4.99	21.65	5.41
7	97.5	2.046	6.64	29.36	7.34
8	112.5	2.036	8.59	38.40	9.60
9	127.5	2.029	10.84	48.74	12.18
10	142.5	2.024	13.41	60.50	15.13
11	157.5	2.020	16.36	73.90	18.48
			19.60	88.50	22.13

In order to have curves of about the same scale it is convenient to plot 0.25 of the $t(S)$ values. The values given in Table I are for one half of each curve. It is not necessary to plot both halves as the U curve is symmetrical about the t axes and the S curve to the

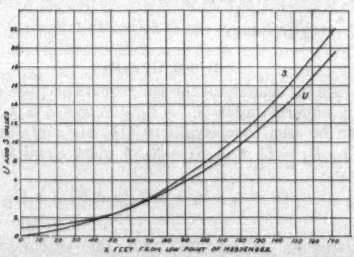

FIG. 5—U AND S CURVES FOR CATENARY SYSTEM

left of the origin has the same values as to the right but they are negative. Fig. 5 shows the U and S curves plotted from the values given in Table I.

Where two spans of inclined catenary join, the contact wire must have a common tangent since the wire is continuous from span to span and there is no attachment to the contact wire to cause a change in direction where pull-offs are not used. This is one of the conditions for which the hangers must be designed. In the solution it is, therefore, necessary to know the slope of the general curves U and S. Since equation (9) is not in a form that can be differentiated the simplest method of obtaining the slope is to do it graphically. dU/dx and dS/dx curves are given in Fig. 6. These curves were obtained graphically from the U and S curves of Fig. 5.

The values of t given for either the U or S curves satisfy equation (9). Also any multiple or any portion of either of the curves satisfies the equation or the two curves may be added together to give a new curve which satisfies the equation. It follows that a general equation for t may be written

$$t = a\,U + b\,S \tag{10}$$

where a and b are constants to be determined and the

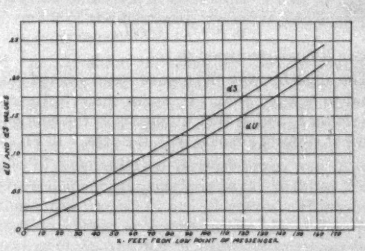

FIG. 6—dU AND dS CURVES FOR CATENARY SYSTEM

values of U and S vary with the distance x from the origin and are obtained from curves which can be drawn for any given catenary system. The slope of this curve at any point will be

$$dt = a\,dU + b\,dS \tag{11}$$

the values of dU and dS are obtained from the dU and dS curves plotted for the system.

In a catenary system where a constant length of shortest hanger is used on tangent and the vertical projection of the shortest curve hanger is made equal to this length, the vertical height of any messenger support will be determined by the span length and will be the same on both tangent and curve. Where pull-offs are not used the only connection to the catenary is at the point of messenger support; since the vertical height of this support is the same on both tangent and curve it follows that the shape of the catenary is influenced only by the relative location of the points of support in a horizontal plane and the degree of the track curve influences the catenary only in as much as it causes greater or lesser angles between the lines connecting the points of support. The catenary is designed to follow the path of the center of the pantograph as closely as possible and the only way its shape can be varied is to change the support locations. The support locations must be such that they will bring the contact wire very close to the path of the center of the pantograph. In Fig. 7, A, B, and C are points on the path of travel of the center of a pantograph shoe, located at supports 1, 2 and 3. The perpendicular

distance e from B to the line $A\,C$ can be obtained from field measurements or track curve characteristics. Super-elevation and height of contact wire should be taken into account in determining e.

The catenary should be designed to pass through points A, B, and C. Then if the curve taken by the contact wire is similar to the track curve the contact wire will fall on the path of the center of the pantograph. The angle between lines $A\,B$ and $B\,C$ is α. This is a measure of the change in direction of the track in the spans. The change in direction of the *axes* of the catenary in the two spans may be something different. It is shown in Fig. 7 as β. In the same figure

$$\alpha = e/L_1 + e/L_2 \tag{12}$$

since e is small compared to L.

The angle between the axes of spans 1-2 and 2-3, β, is equal to $\alpha + \theta_1 + \theta_2$. It is also equal to the sum of the slopes δ_1 and δ_2 that a tangent to the contact wire curve at support 2 makes with the axes. In Fig. 7

$$\theta_1 = (t_2 - t_1)/L_1$$
$$\theta_2 = (t_2 - t_3)/L_2$$

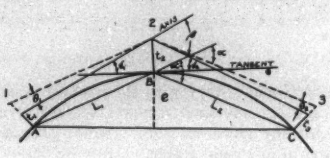

FIG. 7—ANGULAR RELATION BETWEEN CATENARY SUPPORTS ON CURVES

approximately,
and

$$\beta = \alpha + \theta_1 + \theta_2 = \delta_1 + \delta_2$$

which can be changed to

$$\delta_1 + \delta_2 = (e + t_2 - t_1)/L_1 + (e + t_2 - t_3)/L_2 \tag{13}$$

the slope of the t curve for span 1-2 at support 2 is

$$\delta_1 = d\,t_1 = a\,d\,U_2 + b\,d\,S_2 \tag{14}$$

and for span 2-3 at support 2 is

$$\delta_2 = d\,t_2 = a'\,d\,U_2 - b'\,d\,S_2 \tag{15}$$

where a and b are constants for span 1-2 and a' and b' are the constants for span 2-3.

Other equations which may be written for these two spans are

$$t_1 = a\,U_1 - b\,S_1 \tag{16}$$
$$t_2 = a\,U_2 + b\,S_2 \tag{17}$$
$$t_2 = a'\,U_2 - b'\,S_2 \tag{18}$$
$$t_3 = a'\,U_3 + b'\,S_3 \tag{19}$$

where the sub numbers are used to indicate the support. In equations (13) to (19) inclusive there are nine unknowns and seven equations. However, t_1 can usually be determined or made equal to 0 by starting from a tangent section of track and t_3 can be roughly calculated from the value e at support 3. A small error in the value of t_3 does not affect the values of a, b, and t_2 very much. So it is possible to begin at one end of a curve and work through it one span at a time using the values of a, b, and t_2 from each set of calculations and discarding the a', b' and assumed t_3 values. In this way the t_3 of one set of calculations is the t_2 of the next set. Equations (13) to (19) may be solved to determine the values of a, b, a', b' and t_2.

$$b' = \frac{K + K_1 - Z_1 - Z_2 - K_2}{Y_2\,(Y_1 - Z) + Y_3} \tag{20}$$

$$b = U_2\,(k_2 - k_1)/Y - b'\,Y_2/Y \tag{21}$$
$$a' = k_2 - b'\,r_2 \tag{22}$$
$$a = k_1 + b\,r_1 \tag{23}$$
$$t_2 = U_2\,k_2 - b'\,Y_2 \tag{24}$$

where

$$K = Y_1\,U_2\,(k_2 - k_1)$$
$$K_1 = d\,U_2\,(k_2 + k_1)$$
$$Z_2 = (e - t_2)/L_2$$
$$Z_1 = (e - t_1)/L_1$$
$$K_2 = U_2\,k_2\,Z$$
$$Z = (L_1 + L_2)/L_1\,L_2$$
$$Y = r_1\,U_2 + S_2$$
$$Y_1 = (r_1\,d\,U_2 + d\,S_2)/Y$$
$$Y_3 = r_2\,d\,U_2 + d\,S_2$$
$$Y_2 = r_2\,U_2 + S_2$$
$$S_1/U_1 = r_1$$
$$S_3/U_3 = r_2$$
$$t_1/U_1 = k_1$$
$$t_3/U_3 = k_2$$

UNIFORM CURVES

Where the spans have the same length and the curve has a uniform radius $L_1 = L_2 = L$ and $t_1 = t_2 = t_3$. It can be seen from equations (16) and (17) that if $t_1 = t_2$, $U_1 = U_2$ and $S_1 = S_2$, b must equal 0, also $b' = 0$.

$$t_1 = t_2 = t_3 = a\,U_1 = a\,U_2 = a\,U_3$$

and

$$t = a\,U \tag{25}$$

for a uniform curve

$$\delta_1 = \delta_2 = \beta/2 = \alpha/2$$
$$\alpha = 2\,e/L = 2\,a\,d\,U_2$$
$$a = e/L\,d\,U_2$$

for a uniform curve

$$e = L^2/2\,R$$

therefore

$$a = L/2\,R\,d\,U_2 \tag{26}$$

The values of L, R and $d\,U_2$ are all known so that a can be determined.

$$t = L\,U/2\,R\,d\,U_2 \tag{27}$$

where $d\,U_2$ is the slope of the contact wire curve at the support.

Shape of Curve

The path of the center of a pantograph on a curve of constant radius and super-elevation will be a circle. If the contact wire follows this path it would exert an equal force on all hangers in the span

$$f_1 = f_2 = f_3 = f\,n$$

since the vertical load on all the hangers must be equal for a horizontal contact wire it follows that all the hangers must have the same slope if the contact wire is to have a circular shape. The horizontal curve pull exerted by the contact wire on each hanger is

$$f = s\,T/R \tag{28}$$

where R is the radius of the curve and the vertical load on each hanger is W.

In Fig. 8 all the hangers are drawn with the same slope. The horizontal deflection of the messenger

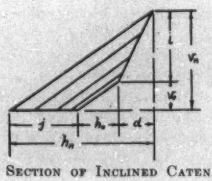

Fig. 8—Cross Section of Inclined Catenary Taken at Center of Span

depends on the curve pull of the contact wire and will be

$$d = f\,x^2/2\,M \tag{29}$$

and the vertical sag of the messenger will be

$$i = W\,x^2/2\,M \tag{30}$$

the horizontal deflection of the contact wire will be

$$u = x^2/2\,R \tag{31}$$

if the section of contact wire in a span is considered to be a parabola. (The difference between a circle and a parabola in the range of span lengths and radii encountered is very small.) In Fig. 8

$$w/f = v_o/h_o = v_n/h_n = (v_n - v_o)/(h_n - h_o)$$
$$i = v_n - v_o$$
$$d + j = h_n - h_o$$
$$w/f = i/(d + j) \tag{32}$$

If equations (28), (29), (30) and (31) are substituted in (32) and for W its equivalent $w/s + w_m/s$ where w_m is the weight of s ft. of messenger plus ½ the weight of one hanger, the result is that

$$M/T = w_m/w \tag{33}$$

which indicates that with a given set of wires the ratio of messenger to contact wire tension must equal the ratio of their weights if the contact wire is to follow a parabolic curve which closely approximates the circular track curve. For any other ratio of tensions the contact wire will follow some other curve which deviates from the track curve as the difference in the ratios increases. However, for most catenary systems this deviation is small. It can easily be checked by comparing the difference in the values of t at the support and center of the span with the curve middle ordinate for the same span.

If the condition of equation (33) is fulfilled in designing a catenary system the slope of any hanger on a regular curve will depend on the radius of the curve and the contact wire tension. A chart can be drawn up giving the vertical projections of the hangers and the angles of inclination for various curves, and from this chart regular curve hanger lengths can be taken directly without the use of U and S curves. However, for transition curves or curves where the radius is not known or varies, the hanger lengths must be determined by some other method and that outlined previously may be used.

It also follows that where the contact wire curve is a parabola the U curve is a parabola. Its equation is

$$U = 1 + x^2/p \tag{34}$$

where

$$p = 2\,v_o\,M/W$$

and the slope curve $d\,U$ is a straight line

$$d\,U = 2\,x/p \tag{35}$$

Equations (34) and (35) may be used for plotting the U and $d\,U$ curves where the messenger and contact wire tensions have the same ratio as their weights but for the S and $d\,S$ curves the method previously outlined should be used.

Discussion

For discussion of this paper see page 646.

Design of Catenary System for Cleveland Union Terminal

BY N. F. CLEMENT[1] and E. E. RICHARDS[2]
Associate, A. I. E. E. Non-member

Synopsis.—*The article presents a brief description of the new electrification at Cleveland, Ohio, and the methods used in designing the overhead catenary system.*

The tangent chord compound catenary is used, the system being quite heavy. The main line construction weighs 3.855 and the yard construction 2.321 lb. per ft. dead load.

The main line system is composed of a main messenger, an auxiliary messenger, and two contact wires hanging side by side. The yard system has a main messenger, and two contact wires, one above the other.

The parabola formulas were used for the calculations, and where feasible, were made up in the form of charts, illustrations of which appear in the text.

* * * * *

I—General Description

THE electrification of the Cleveland Union Terminal at Cleveland, Ohio, extends about eleven miles east of the Public Square, where the station is located, and about six miles west of the same point.

At both Collinwood on the east and Linndale on the west, there were large yards and roundhouses, which made these two locations strategic points for changing from steam to electric operation.

An inspection shed for electric locomotives is located at each of these points, together with the necessary power change and engine standing tracks. At Collinwood, in addition, there is located a repair shop fully equipped to handle all repairs necessary on the electric locomotives.

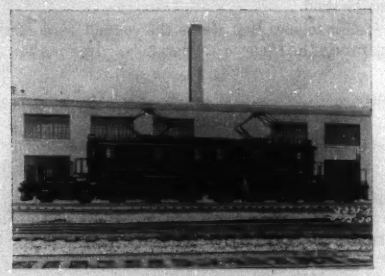

Fig. 1—Passenger Locomotive

Two main line tracks are electrified from Linndale to Collinwood, and with the crossovers, and yard tracks, the total electrified mileage is about 59 track miles.

The 3,000-volt d-c. supply is furnished by two substations, each approximately midway between an end of the electrification and the Cleveland station. There are also six circuit breaker houses which act as tie points where the overhead systems are joined through circuit breakers to a common bus.

The electric locomotives weigh about 200 tons. They are capable of pulling a 15-car pullman train up the maximum grade. About 20 of these will be in active service, with two more as spares. The type of locomotive is shown in Fig. 1.

Passenger stations are located at the Linndale terminal, and at the Cleveland Public Square. In addition, on the east side, there is a station at Superior Avenue, about half way between Cleveland and Collinwood.

II—Description of Catenary System

The tangent chord type of construction is used, as the weighty system necessary for the carrying of heavy currents does not readily lend itself to the inclined type of construction.

The main line construction consists of a ⅞-in. primary or main messenger, a 4/0, 19-wire hard drawn copper auxiliary messenger suspended from the primary messenger by hangers, and two 4/0 grooved bronze-contact wires clipped to the auxiliary messenger.

The primary messenger has a 19-wire core of high strength bronze, and a 16-wire outer layer of copper, the total conductivity being equal to 350,000 cir. mils of copper. The minimum breaking strength is 39,300 lb.

The hangers, of No. 1 hard drawn copper wire, are spaced, normally, every 20 ft. and the contact wires are clipped to the auxiliary messenger in 20-ft. spans. The two contact wire spans overlap, which results in a clip at each quarter point between hangers.

This main line construction is used over the two main line tracks from one end of the line to the other, except through the Cleveland station, where messenger construction is not used.

For dead ending the main messenger, a yoke set is used, with two strings of three insulators each, in parallel. The 10-in., 18,000-lb. insulators give, with this arrangement, a strength of 36,000 lb., and a wet flash-over voltage of 115 kv. It will be noticed that this

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

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dead end develops the full strength of the cable attached to it. This principle has been adhered to, as far as possible, in the whole design.

The auxiliary messenger and the two contact wires are dead ended with a separate assembly similar to the above. While the mechanical strength developed by this dead end is greater than that required for these three wires, the electrical strength is the same as that

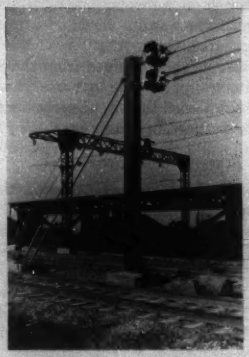

Fig. 2—Main Line Dead-End Construction

of the primary messenger dead end, and the whole arrangement has a more symmetrical appearance than if different types of dead end assemblies were used. Fig. 2 shows some main line dead end assemblies.

For yards and crossovers, the auxiliary messenger is dispensed with. The two 4/0, grooved bronze contact wires are placed one above the other, the upper one supporting the lower one by clips, it in turn being suspended by means of hangers from a ⅝-in. bronze messenger. This yard messenger has a conductivity of 31,700 cir. mils copper equivalent and a minimum breaking strength of 28,300 lb.

The dead-end assembly for the yard messenger consists of two 12-in. insulators, in series, the assembly having a rating of about 24,000 lb. tensile strength. The same arrangement is used for the two contact wires. Turnbuckles are used on both ends of yard messenger and yard contact wire systems. Fig. 3 shows yard dead-end construction.

In the station area, tandem contact wires only, are used. Two 4/0, grooved bronze-contact wires, one above the other, as in yard construction, are suspended in approximately 70-ft. spans. This construction was necessary in order to keep the line down to around 16½ ft. above the rail as most of the station area is covered by buildings, with an overhead clearance of 17 ft.

Each main line catenary is divided into five sections electrically, by four air gaps, or isolated sections. Standard main line dead ends are inserted in the main line, using, however, four insulators instead of six. Another short section of main line construction is run parallel with the main catenary, 14 in. away from it. The pantograph rides on this section and off the main catenary for a distance of about 150 ft. Separate feeders are run to the isolated sections. Two sections are on the east and two west of Cleveland station.

At the station area where the tandem construction is used, the main messenger and auxiliary are dead-ended on both sides of the station, and one 1,500,000-cir. mil cable for each main line track is carried through a duct line from one dead end to the other, to supply the necessary conductivity.

Several different types of catenary structures are used. Where more than two tracks are to be spanned, as in the yards and on the Cuyahoga Viaduct, bridges, made up of latticed columns and truss are used. Center poles, formed of an *H* beam column and a cross arm of two channels are used on most of the double track section. For single yard tracks and sidings, bracket poles, of the same type of construction as the center poles, are used. There are also certain other special types of structures, made necessary by unusual physical conditions along the right-of-way. The pull-off poles and anchor poles are *H* beams.

III—Design of Catenary System

A. Location and Height of Structures.

The maximum span on tangent track was fixed at 300 ft. This gives a sag of approximately 5 ft. at 60 deg. fahr. with the assumed weights. The suspension assembly, of two 10-in. suspension insulators with clevis, eyebolt and messenger clamp, is about 21 in.

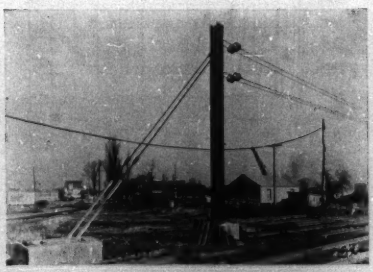

Fig. 3—Yard Line Dead-End Construction

long. The center hanger was chosen at 6 in., and the distance between the auxiliary messenger and the contact wires was assumed to be 3 in. The sum of these distances amounts to about 7 ft. 6 in., which must be added to the normal contact wire height, to give the height of the structures. Due to the great number of low clearance overhead street bridges, it was decided to use a rather low normal contact wire height, 18 ft. 6 in.

at 60 deg. fahr. This, then, gives the height of structure, for a 300 ft. span, of 26 ft.

On curves, the contact wire was not allowed to be more than 12 in. from the inclined center line of the track. Assuming that the wire would be 12 in. outside of the center line at the ends of the span, and 12 in. inside at the center of the span, the maximum allowable

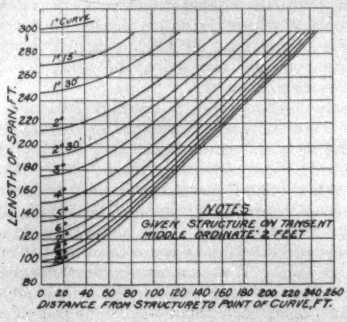

Fig. 4—Spacing Chart for Locating Structures

middle ordinate is 2 ft. Two charts, for spacing structures on curves, were made up. These are shown in Figs. 4 and 5. These are similar to the charts used by the designers of the Illinois Central catenary system.

At crossovers and turnouts, the structures were located, as far as other conditions would permit, to act

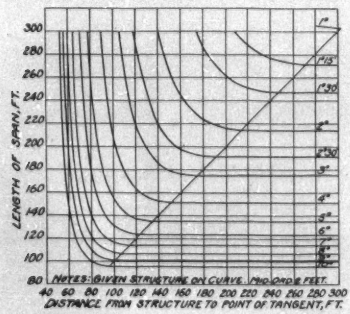

Fig. 5—Spacing Chart for Locating Structures

as the pull-off for the crossover or turnout catenary thus eliminating a separate pull-off pole.

At the same time that the structures were located on the track plans, a profile was drawn on cross section paper. The track profile was put on first, with vertical lines representing the structures spaced according to the plans. The contact wire was then drawn in parallel to the track rail at 18 ft. 6 in. elevation, except where low clearance necessitated lower elevations. A set of celluloid curves was made, representing the messenger curve at 0 deg., 60 deg. and 120 deg. fahr. These were used to draw the messenger profile, thus obtaining the height of all structures.

When approaching low clearance overhead street bridges, the line was gradually lowered so that the messenger took its natural curve under the bridge, avoiding upthrust on the suspension insulators attached to the bridge. The natural curve used in these cases was the 0 deg. curve.

The structure heights obtained from the profile were assembled into groups varying by 6 in. The foundation depths were also varied by 6-in. steps so that the structures could be designed in groups varying by one ft.

B. Sag and Tension Calculations.

The parabolic formulas were used in the sag and tension calculations, as the parabola of the simple curves, gives probably the closest approximation to the true curve of the messenger.

The A. E. R. E. A. sag and tension chart was drawn for a 300-ft. span. This chart was modified slightly from the A. E. R. E. A. chart in that the load lines were made curved instead of straight, thus allowing the use of a uniform scale for sag.

On account of the decision to use a non-corrosive messenger, it was found that a large vertical movement could not be avoided. The messenger finally selected has a 3 ft. 10½-in. sag at 0 deg., 4 ft. 10½-in. at 60 deg., and 6 ft 1-in. at 120 deg. fahr. in a 300-ft. span.

The equivalent span was found for the main and yard lines, using Idail's formula:

$$L_{eq} = L_{av} + 2/3\ (L_{max} - L_{av})$$

where

L_{eq} = equivalent span length
L_{av} = average span length
L_{max} = maximum span length

The main line equivalent length was 260 ft., while the yard lines had values ranging from 120 to 280 ft. As it was impracticable to string every yard line at a different tension basis, the yard line values were divided into two sets, those having equivalent span lengths up to 200 ft. using 160 ft., and from 200 ft. up using 240 ft.

The A. E. R. E. A. charts were then drawn up using the equivalent spans as a basis. The equivalent span for the auxiliary and contact wires was 20 ft. From these charts, curves of tension against temperature were plotted. These stringing charts were made up for all the wires; main messenger, yard messengers, auxiliary messenger, and contact wire. The main messenger chart also had curves of sag against temperature, so that the messenger could be strung by either the sag stick method or the dynamometer method. Figs. 6 and 7 show respectively, the sag and tension chart and the stringing chart, for the main messenger.

C. Design and Spacing of Hangers.

After consideration of the various methods of spacing hangers, a system was worked out which is simple, goodlooking, and easy to lay out in the field. The end hanger is always 10 ft. from the suspension clamp. Then, working toward the center, there are as many 20-ft. spaces each side of the center as will go evenly. The space left in the center of the span, if any, is divided up into from one to three spaces, all equal, varying from 10 ft. 6 in. to 19 ft. 6 in. in length. The chief engineer of the construction company designed an instrument which rides along on the messenger and marks the spaces, so that no tape lines were necessary. The marking was done from both ends toward the center, so that the variations in the actual length of span from the length shown on the plans were taken in the center spaces.

A large parabola, representing the messenger at 60 deg. fahr. was drawn. Over the tracing was laid a piece of cross section cloth, with the vertical and horizontal scales to which the parabola was drawn, marked on it. By moving this cross section cloth so that the horizontal scale was always in the same relation to the parabola, as the auxiliary messenger was to the primary messenger, (as indicated by the profile), the hanger lengths could be read off directly to the nearest half inch.

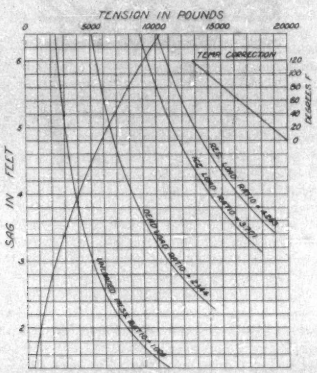

Fig. 6—Sag and Tension Chart for Main Messenger

The hanger data were made up in diagram form as shown in Fig. 8. These sheets were bound up in books, covering different sections of the line.

D. Steadies and Pull-Offs.

It was decided to use a steady arm (Fig. 9) at all structures on tangent track. At those structures which were followed immediately by a curve or a spiral, a pull-off attachment was added to the steady arm. This is merely a clamp which grips the steady arm pipe, and has two rods with clips on one end to hold the contact wires. These were necessary because of the low auxiliary messenger tension. If the line has a slight change of alinement, as on the beginning of a spiral, and the contact wires are not held, they pull toward the inside of the curve enough so that the auxiliary messenger gathers slack from both sides of the steady arm, and a large horizontal sag occurs in it at the point where it is clamped by the steady arm.

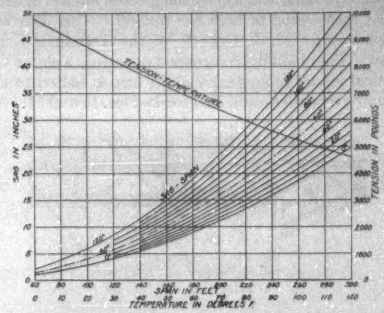

Fig. 7—Stringing Chart for Main Messenger

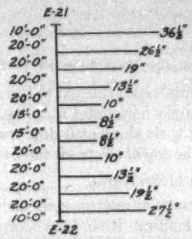

Fig. 8—Hanger Data Sheet

Hanger lengths
1-210-ft. span
Between structures Nos. E-21 and E-22

Fig. 9—Steady Arm

Two types of pull-off arms are used. One clamps the auxiliary and two contact wires and is for main line catenary. The other takes two contact wires, one above the other, as in yard construction. This yard

pull-off has the clamp detachable from the arm, so that it can be used alone where a short pull-off is needed. The clamp has an additional fitting for gripping the top contact wire or the auxiliary messenger of an adjacent line when pulling off a turnout.

Much time was saved in erection, and much more will be saved in maintenance, by using chain instead of cable and Crosby clips to hold the pull-off. This idea was originated by M. J. Griffith, of the Terminals Company. The chain link is dropped down into a slot in the vertical leg of an angle, which in turn is bolted to the column or bracket as illustrated in Figs. 10 and 11.

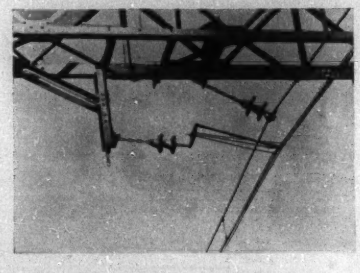

Fig. 10—Main Line Pull-Off

The steady arms were attached to the structures so as to be approximately horizontal at 0 deg. fahr., and the pull-offs so that their slope at 0 deg. would be parallel to a line across the top of the track rails.

E. Attachments to Structures.

Nearly all brackets, angles, etc., for supporting the catenary and holding it in the correct horizontal relation to the track are fastened to the structures with bolts. This facilitates shifting them to conform with changes in track alinement. The steady arm and pull-off attachments are designed to permit a vertical adjustment of one foot.

A convenient chart was prepared to quickly determine the angle of a suspension string supporting a turnout messenger. From this angle is calculated the offset from the center line of the main track at which the supporting bracket is to be placed.

IV—Data for Erection

The data supplied the contractor consisted, first, of drawings showing a side elevation of each catenary structure. On these elevations was shown the location and type of each catenary attachment. The contractor was next supplied a set of tables giving the distance down from the bottom of the truss or cross arm at which the steady arm or pull-off angle was to be bolted. This list also showed the number of links of ½-in. chain at the different suspension locations.

The next information furnished the contractor was the wiring diagrams and stringing charts. The wiring diagrams were simply track plans on which single lines representing the catenary systems were drawn in colored inks. They also showed the location of dead ends, pull-offs, etc.

Following the stringing of the messengers came the hangers. Hanger books, as previously described, were made up, giving the spacing and length of the hangers. These were given to the contractor along with a list showing the length of steady arms and pull-offs.

In many cases special conditions required entirely special equipment or arrangement of fittings. Letter size drawings were made of each special case and furnished in book form to the contractor's engineers.

The contractor's storekeepers and material men were furnished with books containing letter size drawings of every assembly used on the system, whether standard or special.

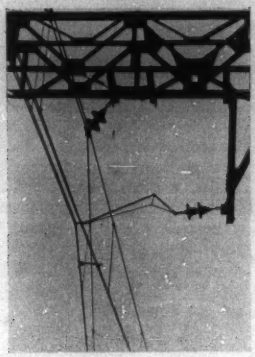

Fig. 11—Yard Line Pull-Off

After the erection was completed a final alinement of the overhead system was made, using the pantograph of a locomotive as a guide. The pantograph was marked with white lines one foot on each side of the center line, and the catenary was adjusted so that the contact wires were within these lines at all points.

Discussion

INCLINED CATENARY CALCULATIONS

(Pickens)

DESIGN OF CATENARY SYSTEM

(Clement and Richards)

W. B. Potter: Mr. Pickens has presented an excellent paper on a method of calculating the mechanical details of inclined catenary. This type of construction is undoubtedly neat in appearance. It is my opinion, however, that it does not provide for the same flexibility on curves as the chord system. The essential feature of an overhead contact line is to provide for the collection of current without arcing at the collector. Arcing more than mechanical wear reduces the life of the contact line and the collector surfaces. The contact between the line and collector is best assured by such flexibility of the line and freedom from hard spots so as not to cause any jumping of the collector.

H. F. Brown: Mr. Pickens in his paper has not only presented a valuable addition to the meager literature existing on inclined catenary design, but has simplified somewhat the calculations required not only in the development of the shape factors, but in their application to the track alinement.

It is of interest to note that equation (33) of Mr. Pickens' paper is the same as the fundamental relationship proposed by Mr. Jorstad,* for a simplified design for inclined catenary. This relationship, however, is difficult to maintain, taking into account the various weights and tensions of the contact and messenger members as determined by the conductivity requirements, and the height limitations of the supporting structures, so that very often deviations from the simple parabolic shape must occur.

It may be said that for short, symmetrical spans (up to say 150 to 175 ft.) of fairly light catenary, the error using a parabolic shape factor alone is not very serious; but for longer spans, transitional spans, asymmetrical spans, and with heavier messenger and contact members, the deviation becomes more noticeable and more accurate calculations should be employed. Mr. Pickens' paper outlines a somewhat simplified means for such calculations.

One factor, which is not brought out by either Mr. Pickens, or my paper[2], and which may cause a large error in determining the true shape, is that in the method of design it is assumed the hanger rods are all parallel, whereas they are really installed nearly radial to the contact wire. The error involved on long rods, may become a considerable amount, necessitating a slight trimming of the rods, or a slight change in their location along the contact wire to fit the actual distance between the messenger and the contact, to make a smooth curve.

It is of interest to note that in general the catenary system which lends itself most readily to the inclined type on curves, is the lighter type used on the high-voltage railroad electrifications. There are very good examples of this design on the recently installed Reading electrification in Philadelphia, on the Pennsylvania Railroad electrification between Wilmington and Trenton and on the older Broad Street-Paoli installation; on the Great Northern (where the spans are fairly short); on the Norfolk & Western, on the Virginian and on the New Haven, where this construction originated.

The railroads which have recently electrified using the lower voltages, such as the Illinois Central, the Cleveland Union Terminal, and the Delaware, Lackawanna and Western, have not used this type of construction with the heavier type of catenary which was required.

The paper on *Design of Catenary System for Cleveland Union Terminals* just presented is valuable for the graphical methods outlined and for the description of the methods used during construction.

One thought occurs in connection with the design of the heavier types of catenary, such as the one described, especially those systems using a bronze messenger for conductivity (or for resisting corrosion). On spans of 250 to 300 feet, the temperature movements of the center of the span are much larger than in the case where steel messengers are used. If the catenary is designed for level contact wire at 60 degrees, this means that at the lowest temperatures there will be an appreciable raising of the contact wire in the center of the span, which, with the increased tension also due to temperature, in the contact wire or wires, will exert considerable compression in the long hangers adjacent to the support. When the hangers are of small diameter, as on the system just described, such hangers are liable to become bowed or even permanently bent or "dog legged," to the extent that they will not resume their former straight line appearance when the temperature rises, thus detracting somewhat from the general good appearance of the rest of the design.

The suggestion is offered for consideration in such cases, that a larger diameter rod, having greater resistance to bending under compression be used, or that possibly, if light rods are desirable, the catenary design be based on a normal tension of say 30 deg. fahr. with level wire at that temperature instead of at 60 deg., thus allowing a greater deviation from the normal under maximum temperature conditions (when the tensions are always lower in all members) but keeping the deviation under minimum temperature conditions, to values which will not over stress any of the lighter members.

On the New Haven Railroad the smallest hanger rods are not less than ⅝ in. in diameter if of bronze, and ½ in. if of steel, and the spacing is somewhat closer than on the more recent installations. I believe, however, that both the larger size and frequent spacing has been of extreme value under abnormal temperature as well as broken wire conditions, as affording a better equalization of both tension and compression stresses between the contact wire and the messenger.

The use of chains is somewhat novel for suspensions and pull-off members, and it is admitted that it gives great flexibility of adjustment. Twenty-five years of operation of the overhead system on the New Haven have shown that parts in moving contact with each other and exposed to the weather and corrosion, may wear very thin in a few years of such exposure, coupled with vibration, and that such parts should be reduced to a minimum. It will be of interest to see whether the chains now in use on the Cleveland overhead are still there in 1955, or have been replaced by single members of the proper length.

N. Litchfield: It is of interest to note that the use of the so-called "curved" or "inclined" catenary was originated by the engineers of the New Haven Railroad in 1909. The methods of calculation employed were described in a paper presented before the Franklin Institute in 1914. These methods enabled them to calculate the length of the trolley hangers, and have them manufactured and delivered ready for use without further adjustment.

Mr. Brown stated in his paper,[2]—

"On this type of compound catenary spans were fairly short, being but 150 ft. for the maximum condition. The calculation involving the inclined rods on curves was therefore comparatively simple, as the errors made in assuming the hangers had the same inclination throughout the span, were very small. These calculations were based on the theory that the contact wire had a parabolic shape, with the horizontal 'sag' equal in value to the middle ordinate of the circular curve of the track for that span. The horizontal force or 'curve pull' was assumed to be uniform for each hanger rod. Both of these assumptions are fallacious, but do not introduce appreciable errors until it is attempted to apply this method of design to single catenary having much longer spans, and on sharper curves."

After the New Haven electrification, when other electrifications were encountered having long spans and sharp curves, it became necessary to modify the original method of calculation. This modification, applying particularly to transition curves, was worked out by Messrs. H. S. Richmond and Philip Goldstone in 1913-14 for the consulting engineers for the Norfolk & Western Railway on the electrification of the Elkhorn Grade. It was later used by them on other electrifications and has become practically the standard method in general use. Inasmuch as Messrs. Richmond and Goldstone never presented their theory in any paper before a technical society, it seems proper at this time to record their connection with the development of this art.

H. S. Richmond: Through the ability to calculate the horizontal offsets of the messenger suspensions and the lengths of hangers on irregular curves and transitions as well as on so-called regular curves, a great economy was effected; as previously a great deal of the calculation was done in a physical manner

1. *Railway Inclined Catenary Standardized Design*, O. M. Jorstad, A. I. E. E. Trans., 1927, p. 1125.

2. *Catenary Design for Overhead Contact Systems*, H. F. Brown, A. I. E. E. Trans., 1927, p. 1082.

and on top of the work train, an expensive method of calculation. It has indeed been found that calculations may be made with such refinement that positions of insulators may be determined and steel supports fabricated with attachments provided in advance to receive the suspensions, provided careful track information is obtained. This, however, must be specially obtained for the purpose as, generally, theoretical map location is not sufficiently accurate.

In the opinion of the writer, equation (33) is not of significance as the U-curve is practically quite a close approximation to the circle and most track curves are not actually very uniform. We have found it convenient, by abbreviated methods, to rapidly perform the calculations without going to the trouble of investigating the track curvature for regularity.

The suggested relation expressed in formula (33) $\frac{M\,W}{T\,W_m} = 1$ imposes an arbitrary limitation on design which is not desirable. In the example offered $\frac{M\,W}{T\,W_m} = 1.4$. Assuming that the trolley tension chosen is a desirable one, to lower the messenger tension to bring about the proposed relationship would result in an excessive sag and an uneconomical spacing of supporting structures. This would work directly toward excessive cost since the cost of the individual structure is very nearly a constant, irrespective of the spacing.

The theorems presented will warrant the careful study of the designer who decides to adopt this type of construction.

J. B. Cooley: Mr. Pickens' paper furnishes a valuable text for anyone desiring to design an overhead contact system employing inclined catenary. I believe that it is the only paper of its kind which not only furnishes a method of computing the general shape curves U and S, but also supplies us with a definite means for using them in determining the characteristics of a catenary system.

I have been designing inclined catenary for a number of years and would like to offer the following amendment to Mr. Pickens' solution. The method I am about to describe is, I believe, much shorter than that of the paper, and although not quite as exact, it is within the bounds of practicability.

The U and S curves are combined to form a T curve, a transition curve. This is a curve having value of the horizontal distance between messenger and contact (called the offset), zero at some point X distance from the low point of the messenger; real value at any other point. With point X established, an infinite number of curves may be drawn, all having the general equation $t = C\,T$, where the multiplier C depends on the value of t at any point, such as the offset at a support. Therefore, in any span of catenary, two transitions may be computed, the sum of which will give the complete catenary for the span. If t_1 and t_2 are the offsets at supports 1 and 2, then one transition will have zero value at support 1, value t_2 at support 2; the other will have zero value at support 2, value t_1 at support 1.

Since the distance X from low point to support where value equals zero varies as the span lengths vary; it would seem necessary to compute a T curve for every half span. This is not, however, the case, as it has been found that if distance X is so chosen that it represents the mean of the span lengths encountered in any design, and a T curve computed, this curve may be used for any other distance X without appreciable error. The equation for any span of catenary may then be written

$$t = C\,T + C'\,T$$

and

$$C = \frac{t_2}{T_2} \qquad C' = \frac{t_1}{T_1}$$

Where T_2 signifies value of base curve at support 2, T_1 value at support 1.

The several equations listed by Mr. Pickens to find the values of t for a catenary system having no pull-offs, may be condensed to one equation.

$$t_3 = \frac{e\left(\frac{1}{L_1} + \frac{1}{L_2}\right) - t_1\left(\frac{1}{L_1} - \frac{p}{T_1}\right) - t_2\left(\frac{1}{L_2} - \frac{p}{T_2}\right)}{\frac{2}{T_2}\left(\frac{d\,T}{d\,x}\right)_2 - \left(\frac{1}{L_1} + \frac{1}{L_2}\right)}$$

Where e, L_1, L_2, t_1, t_2, t_3 refer to the same variables as indicated in the paper, T_1, T_2, T_3 are values of the transition curve at support 1, 2, 3; $\left(\frac{d\,T}{d\,X}\right)_2$ is the slope of the transition curve at support 2; and p is the slope of the transition curve at point X where T equals zero. To solve start at the beginning of the curved track and assign values to $t_1, t_2, t_3, t_4, \ldots\ldots t_n$. These are found by assuming a regular curve as described in the paper. Insert assigned values of t_1 and t_3 in above equation, and solve for t_2. Proceed to next structure, and use solved value of t_3, assigned value of t_4, and solve for t_5, etc. If any solved value of t differs from assigned value by more than 10 per cent, the equation for the preceding structure must be recomputed by inserting the corrected value and solving again.

I would like to say in reference to equation (33) of the paper that a catenary system bearing those relations will only simplify computations if the railroad curvature is of a regular degree. It has been my experience that most railroad curves are compounded with a spiral at each end, and the exact radius at any point is difficult to determine. Thus the method of base curves for determining the characteristics of any catenary system must be employed, and any advantage or reduction of labor in computation, by using equation (33) is lost.

B. M. Pickens: In connection with equation (33,) it should be noted that where materials of the same density are used in the messenger and contact wires, the equation requires that the same average unit stress be used in each. This gives a picture of the limitations imposed by the equation.

In those systems where it is desired to make use of high-strength messengers and an auxiliary wire at low tension as part of the contact system, it is impractical to use this relationship. If the conditions of equation (33) were fulfilled, advantage could not be taken of the high-strength messengers available and the sag would be excessive. However, in the case of a simple catenary using a composite messenger and bronze contact wire the unit stresses can be made the same without imposing any undesirable limitations on the design. If these conditions are fulfilled the design of the curve catenary on uniform curves is very simple. This is, of course, only of advantage where there are uniform curves.

N. F. Clement: There has been doubt expressed from many sources as to the advisability of using chain to connect the pull-off arms to the structures.

As far as I know there is no installation using chain in this manner, so we had no experience or precedent to guide us in its use. However, there is a number of points which seemed to us to make it worth while to take the step.

In the first place the cost of installation is small. Then the cost of replacement and adjustment is very much less than that of the clip and cable system. This latter is important in view of the fact that there are apt to be many changes in track layout the first few years of operation over a new track system.

Of course it was known there would be wear due to vibration, but to just what extent it was impossible to determine without

extensive tests. The chain system is very easily inspected and excessive wear or corrosion can be recognized at once.

The most wear will come between the link held in the slot of the angle on the structure, and the next one out from it. In case the rest of the chain is sound it will be possible to pull the pull-off up one link thus making it practically as good as new with no expenditure for material and very little for labor.

I am not inclined to agree with Mr. Brown on the matter of stiff end hangers. We knew at the start there would be some bowing of end hangers and we let them bow.

Due to our large vertical movement, about 2 ft. 2 in., and consequent tendency for the contact wires to rise at the structures, a stiff hanger would probably cause a hard spot. Furthermore, it would push the auxiliary down toward the contact wires with danger of having the pantograph hit the lower hanger clamp.

With loop hangers conductivity is lost as the loop lifts off the primary messenger. There is also a liability of arcing at this point. Flexible connections are expensive and as a rule not lasting.

I will admit there is a question about the economy of using chain on the pull-offs. There is of course no doubt about the saving in material and installation cost at the beginning. We can only estimate the life and replacement costs, and time only will show how correct that estimate was.

Some Experiments with Arcs Between Metal Electrodes*

BY G. M. SHRUM† and H. G. WIEST, Jr.‡

Non-member Non-member

Synopsis.—The paper describes experiments on the arc between metal electrodes in argon and other gases, in which the current was controlled by means of a kenotron. By varying the temperature of the cathode in the kenotron, it was found possible to obtain volt-ampere characteristics of the discharge for the transition from glow to arc or the reverse. It was also observed that in the arc stage, for at least smaller currents, the spectrum of the discharge is characteristic of the cathode material.

Introduction

THE phenomena associated with the arc discharge between cold metallic electrodes are unusually complex, and, in general, are not well understood. The growing use of the arc commercially, and especially in industrial welding operations, warrants a further study of the fundamental process underlying this particular type of discharge. In this report we have considered chiefly arcs between iron and copper electrodes separated by only a few millimeters of air, or some inert gas. The gas pressures varied from one mm. of Hg. to one atmosphere.

One group of experiments included arcs controlled by means of external series resistance, in which currents ranging from one to twenty amperes were used. Later it was decided to extend the investigation to the discharges which precede the arc as the current is gradually increased from zero. In this region the negative characteristic is usually very steep and it was necessary to work with much higher line voltages and limit the current through the discharge by means of a kenotron. A galvanometer and bridge circuit, such as used in this laboratory, was arranged whereby the volt-ampere characteristics could be obtained for currents ranging from 1×10^{-6} to 100×10^{-3} amperes. It was necessary to modify this method in order to obtain the volt-ampere characteristic of the transition region between the glow and the arc discharge. The galvanometer and bridge circuit were replaced by an oscillograph, by means of which a record of the voltage and current was obtained simultaneously. It was thus possible to determine volt-ampere characteristics for discharges where the current varied continuously from 50×10^{-3} to 2 amperes.

A combination of the above methods thus enabled us to study the characteristics of discharges for currents varying from 1×10^{-6} to 20 amperes. In this range of currents, the discharge passes by continuous gradations from the stage of normal cathode glow through that of the abnormal cathode glow into the low-voltage, high-current type which constitutes an arc. It was found possible, as shown in this paper, to obtain the continuous transition not only from the glow to the arc, but also in the reverse direction.

Arcs with Current Limited by External Resistance

Apparatus. For this work a 250-500-volt d-c. generator was used, the current through the arc being limited by means of an external series resistance. The early experiments indicated that reproducible results could be obtained only if the shape, size, and condition of the surface of the electrodes were kept constant. Some preliminary experiments indicated that electrodes 1/8 in.—3/16 in. diameter and 2 in. long were very satisfactory as they could be mounted in a holder and when necessary, replaced very easily. This effected a considerable saving in electrode material, since, owing to the heating and consequent melting, new electrodes were required for each experiment. They were prepared by cutting from longer rods of the material and thus required no further shaping.

Characteristics of Arcs in Various Gases It was found that the values obtained for the arc voltage for a given current, electrode material, and arc length, were in good agreement with values given by Seeliger, Nottingham and others. Some typical data are given in Table I. The arc voltage given depends, of course, upon the arc

TABLE I
ARC CHARACTERISTICS FOR CLEAN ELECTRODES IN VARIOUS GASES AT ATMOSPHERIC PRESSURE

Electrode material	Gas	Arc voltage	Current in amperes	Arc length mm.	Remarks
Fe	Air	32	2	1	Very steady.
Fe	Argon	24	2	1	Very unsteady. Cathode spot wanders.
Fe	N_2	30	2	1	Unsteady.
Fe	CO_2	29	2	1	Very steady.
Fe	H_2	—	2	1	Would not strike.
W	Air	40	2	1	Voltage fluctuates.
W	Argon	—	2	1	Arc would strike but could not be maintained.
W	N_2	22	2	1	Fairly steady.
W	CO_2	37	2	1	Fairly steady.
Cu	Air	25	2	1	Steady.
Cu	N_2	38	2	1	Fairly steady.
Cu	CO_2	30	2	1	Steady.
Cu	Argon	22	2	1	Fairly steady.
Cu	N_2	—	2	1	Would not strike

*The experiments were conducted at the Research Laboratory of the General Electric Co., Schenectady, N. Y.

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length which was not determined very accurately in this part of the work.

It was not always possible to duplicate the readings for air, owing, possibly, to variations produced by the chemical action at the molten surfaces of the electrodes. The results given in Table I are for gases at atmospheric pressure and for electrodes which had been thoroughly cleaned before striking the arc. It will be noted that in all cases the arc was steady in air and CO_2, while it would either not strike or was unsteady in H_2, Ar and N_2. This phenomenon was particularly true for iron electrodes. At pressures lower than one atmosphere, the arc was less stable for all gases. Later it was found that by pretreatment of the electrodes, stable arcs could be maintained in all these gases, even at pressures of the order of 2 cm. of Hg.

Experiments with Oxidized Electrodes. It was found that electrodes which had been used for arcs in air and later used in argon gave very steady arcs. This phenomenon was investigated and it was found to be due to the oxidation of the electrodes when used in air. The same degree of stability was obtained by oxidizing the ends of the electrodes by other methods. Thus when the tip of the electrode was melted in an oxidizing flame, or oxidized by electrolysis, it served equally well. It was found too that it was necessary to treat the cathode only, in order to produce a stable arc. Thus a clean iron anode and an oxidized cathode gave a steady arc in argon and N_2, and even in H_2 for a short time, until the oxide was reduced. The combination of an oxidized anode and a clean cathode, however, gave an unstable arc. It appears that in air both electrodes must become oxidized since either the anode or the cathode serves later when used as a cathode in argon to give a steady arc. It is true, however, that an electrode which has been used as an anode in air serves to make a better cathode than one which has been used as the negative electrode. It was found too that Fe electrodes coated with either the red or black oxide of iron make good cathodes, while other oxide coatings on the cathode, including barium, calcium, and thorium, were found to be very effective, although for some purposes they would not be satisfactory as some of them tend to lower the potential drop across the arc.

With the treated cathodes it was observed that as the arc was struck, the cathode spot was established at the point where the electrodes had been in contact. As the arc was established, there seemed to be no wandering of the cathode spot, whereas with the clean electrodes, it wandered so rapidly over the surface of the cathode that the arc soon broke. A possible explanation is that the oxide prevents the wandering of the cathode spot, and in order to test this point, a flat plate was fastened on the end of a rod and used as cathode. One part of this plate was oxidized, while the remainder was thoroughly cleaned. When the arc was struck on the oxidized surface, it remained steady, but when struck on the clean surface, it wandered very rapidly, but never crossed over on to the oxidized portion, thus supporting the explanation that the oxide prevents the wandering of the cathode spot.

It is known that in air the cathode spot wanders less on carbon than on other materials, and this has been ascribed to the roughness of the carbon surface. This, however, cannot be the explanation of the phenomena observed here, because clean roughened surfaces showed little improvement over similar surfaces polished.

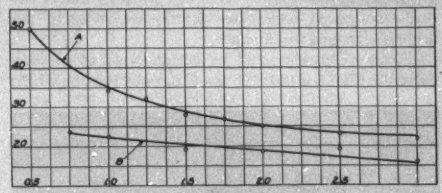

Fig. 1a—Ordinate—Volts
Abscissa—Amperes

Effect of moist argon (curve *A*) as compared with dry argon (curve *B*) on volt-ampere characteristic of arc between iron electrodes.

Then again iron anodes from arcs in air are often very smooth, and it has been shown that these make excellent cathodes.

When the cathode spot is prevented from wandering, the temperature rises locally on the cathode surface and a steady arc results. An explanation of the process whereby the oxide prevents the wandering of the cathode spot is probably to be found in the observations that oxygen on the surface of metals, such as tungsten,

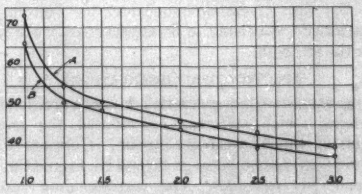

Fig. 1b—Ordinate—Volts
Abscissa—Amperes

Effect of moist air (curve *A*) as compared with dry air (curve *B*) on volt-ampere characteristic of arc between iron electrodes

molybdenum, and tantalum, decreases both thermionic emission and secondary electron emission due to positive ion bombardment. Probably the effect of oxygen on iron is similar to that observed for the other metals mentioned.

Experiments with Water Vapor and CO. The volt-ampere characteristic of an iron arc was taken in commercial argon, dried by passing it over P_2O_5, and also when the same gas had been bubbled through a flask of water, thus saturating it with water vapor before

entering the arc chamber. It will be noted from Fig. 1A that the voltage drop across the arc for the moist argon was from 15 to 20 per cent greater than for dry argon.

Fig. 1B represents the corresponding results for dry and moist air. The difference is not quite as great as it is in argon, but otherwise the effect is about the same. At low current values the arc in moist argon was much more stable than in dry argon, and thus curve *A* (moist) for Fig. 1A was extended to lower currents than those given for curve *B* (dry). This was probably due to the oxidation which took place at the electrodes in the argon containing water vapor.

If this difference could be extrapolated to welding currents, then it is fairly obvious that any coating which

FIG. 2—SPECTRA OF ARCS WITH VARIOUS CATHODE MATERIALS

would supply water vapor to the arc would tend to raise the arc drop and thus, for the same current, increase the rate of melting.

In the case of carbon monoxide, the volt-ampere characteristic was found to be approximately the same as for the arc in air.

SPECTROSCOPIC OBSERVATIONS

Some experiments were carried out with arcs in air to determine which electrode contributed the greater amount of vapor to the arc stream. Currents of 1.5 amperes were passed between Fe and Ni, Fe and Cu, and Cu and Ni electrodes. The corresponding spectra taken with a Hilger constant deviation glass spectrograph are shown by Fig. 2. On this plate, *a* and *c* represent the spectrum of the nickel, while *b*, *d*, and *g* are for pure iron. An iron anode with nickel cathode is represented by *e*, while *f* is the spectrum for a nickel anode and iron cathode. An iron cathode and nickel anode are represened by *h*, while *i* gives the spectrum for a nickel cathode and iron anode. Thus it will be seen that while there are a few faint lines due to the anode, *the spectra in each case are typical of the cathode material.*

At the comparatively low currents at which these spectra were obtained, little or no fusion of the cathode occurred, but evidently enough vapor was emitted from the cathode spot to produce the characteristic spectrum of the metal.

Spectrograms were taken of the *normal and abnormal cathode glow* and also of the anode glow for an iron arc in commercial argon (14 per cent N_2). The negative glow for both the normal and abnormal arc showed the argon lines and the negative bands of N_2, while in the anode glow it was the positive bands of N_2 which predominated. Similar experiments in H_2 showed the Balmer lines very *strongly in the negative glow, while the secondary or molecular spectrum predominated in the anode glow.*

These results indicate that for a short gap such as used in these experiments, the degree of excitation at the cathode is much more intense than that at the anode. The difference in the types of spectra observed for the glow discharge and the arc is in agreement with the observation that for the normal and abnormal glow the cathode drop is a hundred volts and higher, while for the arc the cathode drop is 30 volts or lower.

THE TRANSITION BETWEEN THE GLOW AND THE ARC DISCHARGE BY GALVANOMETER-BRIDGE METHOD

Apparatus. A diagram of the apparatus used is shown in Fig. 3. *K* was an 8-in. Pyrex bulb which served as an arc chamber. The electrodes were supported by molybdenum rods R_1 and R_2 sealed in ground glass stoppers S_1 and S_2. In order to study the region of the abnormal cathode fall, it was necessary to limit the area of the cathode exposed to the discharge. Thus the cathode *C* was made up of two quartz protecting tubes C_2 and C_3. The exposed surface of the cathode, *C*, had an area of 0.22 cm. and was slightly

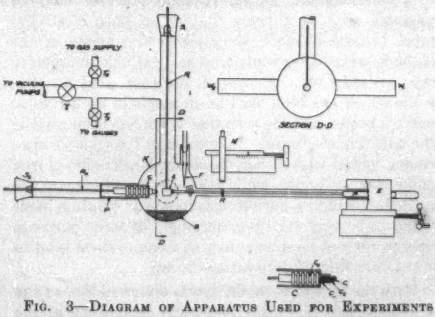

FIG. 3—DIAGRAM OF APPARATUS USED FOR EXPERIMENTS ON ARCS BETWEEN METAL ELECTRODES IN GASES AT VARYING PRESSURES

convex on the side towards the anode. The threaded stem of the cathode fitted closely into the quartz tube C_2 which in turn was ground to fit the opening in the larger piece of quartz C_3. The large copper piece C_4 was arranged with disks for cooling purposes. The length of the gap was determined by means of a traveling microscope, *M*, in conjunction with the knife edge, *R*, attached to the rod of the soft iron plunger, *P*. Glass beads, *B*, in the bottom of the flash served to prevent drops of molten metal from cracking the glass. *F* was

a heavy tungsten filament which was used for cleaning up traces of oxygen in the argon.

The electrical circuit is shown diagrammatically in Fig. 4. For these experiments the switches, S_1 and S_2, were closed. When the kenotron, U, was required, the switch, S_3, was in position 1, while with external resistance for limiting the current, S_3 was at position 2. B represents an air circuit breaker, R_1 a potentiometer resistance, R_4 a megohm resistance for protecting the galvanometer G, and R_2 a current limiting resistance. A was a multiple range ammeter, while V was a high

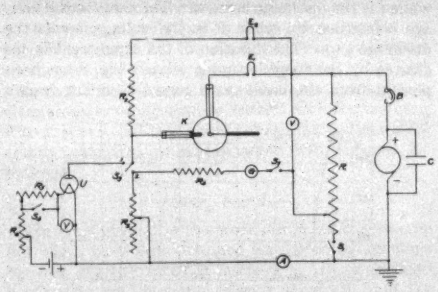

FIG. 4—ELECTRICAL ARRANGEMENT FOR EXPERIMENTS ON ARCS

resistance voltmeter,* in some cases an electrostatic voltmeter. For currents from 50×10^{-3} to 5 amperes, a UV-214 kenotron was used, while the lower currents were obtained by using a smaller three element tube with grid and plate tied together. This gave a low saturation voltage, and consequently a lower line voltage was required, which in turn helped to keep the leakage currents very small. The 2-μf. condenser, C, across the terminals of the generator served to eliminate disturbances in the voltage wave. The resistance, R_2, and the oscillograph elements E_1 and E_2 were not used in this part of the work.

Observations. Voltage readings were taken for various types of electrodes for currents varying from 1×10^{-6} amperes to 2 amperes. (With the apparatus as arranged, the leakage currents were such as to make it impossible to extend the readings to lower current values.) By this method it was possible to trace the development of the discharge from the corona or brush discharge through the normal glow and into the region of the abnormal cathode fall. Fig. 5 shows a plot on semi-log paper of the results obtained for copper electrodes with a separation of 2 mm., in commercial argon at 2-cm. pressure. It will be noted that the volt-ampere characteristic is negative for the brush and normal glow discharge, while it is positive for the abnormal glow discharge. The maximum voltage reached in the abnormal glow varied somewhat, but was of the order of 500 volts, while the corresponding current was approximately 100 milliamperes. As the current was increased further, the voltage across the gap decreased, and this continued until a drop of 30-50 volts was reached when the discharge had all the characteristics of an arc. It was almost impossible to obtain reproducible results in this transition region between the abnormal glow and the arc, due, mainly, to the excessive heating of the cathode and of the protecting refractory material. At high temperatures the discharge characteristics were modified by the nature of the refractory material. A number of high temperature refractory materials was tried, including oxides of magnesium, aluminum, and thorium, but generally it was found that clear fused quartz was the most satisfactory.

Very often the arc would strike at the junction between the quartz and the metal, and when this happened, the quartz fused and decomposed very rapidly.

The real difficulty is the problem of dissipating from such a small cathode surface the energy corresponding to voltages up to 500 volts and to currents of the order of 100 milliamperes. If larger cathodes are used, then correspondingly greater currents have to be used in order to bring the current density up to the required value for the abnormal glow.

This phase of the problem was partially solved,

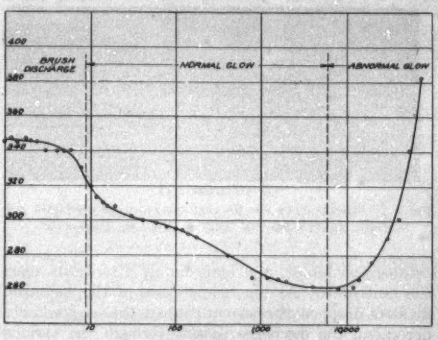

FIG. 5—ABSCISSA—LOG (MILLIAMPERES)
ORDINATE-VOLTS

however, by using an oscillograph in conjunction with a large kenotron for limiting the current.

THE STUDY OF THE TRANSITION BETWEEN THE GLOW AND THE ARC DISCHARGE BY MEANS OF AN OSCILLOGRAPH

Apparatus. The arrangement of apparatus and electrical connections are shown in Figs. 3 and 4. The current was supplied by a 4,000-volt d-c. generator. As a kenotron, a high-voltage, water-cooled radiotron UV-214 was used. This tube was rated for a maximum peak plate current of 7.5 amperes. The volt-ampere

*Figure shows two voltmeters, V. The text refers, of course, to the instrument across the arc.

characteristic for plate currents less than 2 amperes was fairly flat for plate voltages beyond 1,500 volts.

In Fig. 4 the switches S_1 and S_2 were open, and S_3 at position 1. E_1 represents the current element of the oscillograph and E_2 a very high sensitivity voltage

FIG. 6—VOLT-AMPERE CHARACTERISTIC OF DISCHARGE BETWEEN COPPER ELECTRODES IN ARGON AT 2-CM. PRESSURE

Oscillogram for transition from arc to glow for copper electrodes in argon at 25-cm. pressure

(*io* and *Vo* represent zero lines for current and volts respectively

element. R_2 was used as a multiplier for the voltage element. The temperature of the kenotron filament was adjusted so as to give a plate current of approximately two amperes when the switch S_4 was closed. The procedure was to strike the arc, then start the

FIG. 7—OSCILLOGRAM FOR TRANSITION FROM ARC TO GLOW FOR IRON ELECTRODES IN ARGON AT 2-CM. PRESSURE

oscillograph motor, and immediately afterwards open the switch S_4. As the temperature of the kenotron filament dropped, the current through the arc gradually decreased, the discharge passing through the various low current stages. The resistances R_5 and R_6 served to control the initial and final currents through the kenotron filament, and thereby the rate at which the arc current changed.

Transitions from the glow to the arc were obtained by starting with the very low current discharge and with S_4 open. As the oscillograph motor was started S_4 was closed, and the filament temperature increased rapidly, causing a corresponding increase in the arc current. As the current in the discharge increased, the latter passed through the glow stages and finally into an arc.

Results. At pressures of 25 cm. of argon, it was found that the transition from the arc to the glow was very abrupt. It was noted also that the transition from the glow to the arc was practically identical with that from the arc to the glow, taking place at practically the same values for the voltage and current. Since the time taken for the filament to arrive at the temperature corresponding to a *particular value of the discharge current must be different in the two cases*, it would appear that *the transition is governed more by the current density at the cathode than by the temperature.* Fig. 6 is an oscillogram record of the transition from the arc to the glow for copper electrodes, 1 mm. separation, in argon at 25 cm. pressure. The zero line for current is represented by i_0. while i is the discharge current. The zero for the voltage is marked V_0, while V is the voltage across the discharge gap. The duration of the experiment is indicated by the 60-cycle timing wave. Fig. 8 has been plotted from the oscillogram record. Similar results

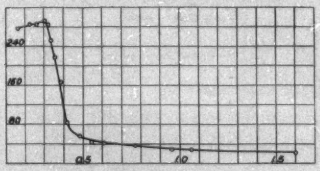

FIG. 8—VOLT-AMPERE CHARACTERISTIC FOR CASE OF OSCILLOGRAM IN FIG. 6

were obtained for the transition from the glow to the arc.

At low pressures the transition is much more gradual. Fig. 7 represents a typical oscillograph record of the transition from the arc to the glow for iron electrodes in argon at 2-cm. pressure for a gap length of

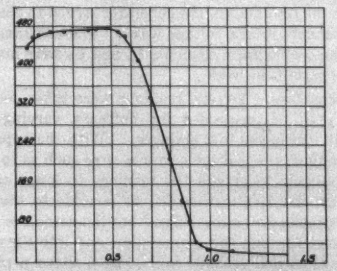

FIG. 9—VOLT-AMPERE CHARACTERISTIC FOR CASE OF OSCILLOGRAM IN FIG. 7

2 mm. Fig. 9 represents the corresponding volt-ampere characteristic as plotted from the oscillogram. It will be noted that the transition is quite gradual at these pressures, and especially at the short gap lengths. From observations on gaps of varying lengths, it appears that the arc persists down to lower current values as the length of the gap is increased, while the

voltage peak for the abnormal glow remains at about the same current density, causing a much more abrupt transition. Thus either high pressures or longer gap lengths tend to produce sudden transitions. It should be noted that for the shorter gap length the abnormal voltage rises to higher values than for the longer gaps. This is in agreement with the low value observed for the maximum abnormal voltage at high pressures. At pressures lower than 2 cm. the oscillogram records were very erratic, probably due to the much higher abnormal cathode voltages, which caused excessive heating and bombardment of the refractory material.

The voltage oscillations in the transition region are probably caused by the steep negative volt-ampere characteristic of the discharge at this point. It is possible that a kenotron with a much flatter space charge characteristic would tend to control such fluctuations. Also it was found that these voltage fluctuations were greatly enhanced when the argon was replaced by hydrogen.

Conclusion

The chief interest in this work lies in the characteristics which have been obtained for the transition stage between the two types of discharges. If, as Seeliger[1] notes, the arc discharge always develops from the glow or Townsend discharge, then this transition stage precedes every arc. Many workers, however, have doubted the existence of such a transition region, and it has usually been assumed that the transition from arc to glow is so abrupt that the different stages cannot be obtained. Stark[2] studied this transition and came to the conclusion that as the glow spreads over the cathode surface, the latter gradually gets hot enough at some point to start an arc. Seeliger[3] investigated experimentally the development of the arc from the glow discharge. He used a special discharge tube with the cathode a mercury surface at the end of a small quartz tube, the discharge tube being filled with a rare gas. Wehrli[4] also has investigated the transition from the glow to the arc in the case of arcs between tungsten electrodes in pure nitrogen.

Our results obtained with the oscillograph indicate that at low pressures the abnormal cathode fall rises to very high values and then as the current is increased, the drop across the electrodes decreases and this continues until the discharge eventually has all the characteristics of an arc. The volt-ampere characteristic for the transition stage is very steep. Our results do not support the assumption that it is a temperature effect at the cathode and the resulting thermionic emission which causes the glow discharge to go over into an arc. The results obtained indicate rather that it is really a matter of current density at the cathode. Some experiments were tried in which the exposed area of the cathode was increased, and it was found that much higher currents were required before the arc could be developed. This was true in spite of the fact that the electrode temperature was occasionally high enough to cause fusion of the metal.

It was found to be very difficult to prevent the cathode spot from forming on the insulating material which was used to limit the area of the cathode spot. The explanation for this is not clear, although it may be due to a contamination of the adjacent metal surface of the cathode by material sputtered from the insulator under the positive ion bombardment of the abnormal cathode drop. In this connection it may be interesting to note that Oliphant[5] found for helium positive ions that the secondary electron emission was much greater for contaminated surfaces than for clean surfaces. He also found that it was less for hot surfaces than for coldones.

In a very interesting paper, Güntherschulze[6] has discussed the distribution of energy between the anode and cathode in the glow discharge. He found for the abnormal glow discharge that the number of positive ions required to liberate one electron from the cathode rapidly decreased with increasing cathode fall and for a given cathode fall was greater for iron than any of the other metals investigated.

It may be pointed out that an extension of this study of the transition from the glow to the arc and from the arc to the glow under various conditions seems to be a line of investigation which should lead to interesting results. An arrangement permitting the control and accurate measurement of the temperature of the electrodes, especially that of the cathode, together with oscillograph records of the transition stages, should lead to a better understanding of the actual mechanism by which electrons are emitted at the cathode.

In conclusion, the authors wish to express their appreciation of the interest shown in this investigation by Dr. S. Dushman and Mr. C. G. Found.

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Discussion

S. Dushman: One of the most important results which follows from this investigation is the conclusion that the transition from the glow to the arc, or vice versa, is governed more by the current density at the cathode than by the temperature of the latter. It is of interest to discuss this conclusion from the point of view of prevalent theories on the phenomena occurring in the transition from glow to arc and on the origin of electrons at the cathode of an arc discharge.

As is shown in Figs. 8 and 9, the transition from glow to arc is accompanied by a decrease in voltage drop between the electrodes. This is accounted for by the change in cathode drop. For the glow, this drop is of the order of 100 volts or higher, and there is built up a positive ion space charge at the boundary between the glow and the cathode dark space. At the immediate surface of the cathode itself, the space charge is negligible and the electron emission is limited by the rate at which positive ions strike the cathode. However, when the arc forms, a re-

distribution occurs of potential in the space between the electrodes so that the cathode drop decreases to the order of the ionizing potential of the metal vapor.

Owing to the fact that the cathode is now emitting all the electrons that are required for the arc current, an electron space charge is built up at the surface of the cathode, while the positive ion space charge region is formed at a distance from the cathode which is of the order of a mean free path.

In consequence of this there exists a gradient at the cathode of an arc which is of the order of 10^5 to 10^6 volts per cm. It is this gradient which is held responsible, according to Dr. Langmuir's theory, for the emission of electrons from the cathode; for as pointed out by Dr. Compton the theory of pure temperature emission will not account for current in such arcs as those between electrodes of copper, iron or mercury.

However, it would appear from the observations on cold cathode emission and as a consequence of the theoretical considerations of Fowler that the gradients of the order of 10^6 volts per cm. are not quite high enough to pull electrons out of cold metals and therefore it is necessary to look for still another process by which electrons may be emitted. Mr. Found's theory that a great part of the emission is due to metastable atoms certainly appears very plausible, especially in view of the results which he has obtained in a study of arcs in neon, mercury, and other gases. Also the results obtained in the present investigation on the part played by current density in governing the transition from glow to arc would not be in contradiction with such a theory, although it is rather difficult at present to work out the quantitative relations.

C. G. Found: The paper by Shrum and Wiest is a valuable contribution to the study not only of arcs but also of general discharges in gases, particularly the transition stages from the normal to the abnormal glow and from the abnormal glow to the arc. The transition from the normal to the abnormal glow is accompanied by an increase in cathode drop, which causes the positive ions to strike the cathode with increasing velocity and thereby the secondary electron emission from the cathode is increased. The transition from the abnormal glow to the arc is most interesting, and, I believe, indicative of the processes responsible for this transition. As pointed out by the authors, a thermionic emission does not account for the experimental results, and Dr. Dushman has mentioned the unsatisfactory results of assuming the cathode current to be due to the electrostatic field.

In order to account for the development of the arc on a basis of secondary electron emission by positive ions, it is necessary, from space charge consideration, to assume a high efficiency of secondary electron emission. Moreover, comparing Figs. 8 and 9 it is noted that at the higher pressures although the abnormal cathode drop is lower, the arc develops more rapidly at a lower current than at the lower pressure where the abnormal cathode drop is larger.

Since the efficiency of secondary emission by positive ions increases with voltage, it would appear the opposite would be true and the arc would strike at a lower current when the cathode drop was greater. I would like to suggest a mechanism of electron production at the cathode which is not directly dependent on the voltage distribution, namely, the secondary electron emission by metastable gas atoms. Results of investigations of discharges in neon show that the efficiency of secondary emission by metastable atoms is very high and that this process is responsible for the low arc-back voltage in neon.

The number of metastable atoms reaching the cathode is independent of the voltage distribution and determined only by their rate of production and their rate of diffusion. As the current in the abnormal glow is increased more and more metastable atoms are produced and gradually the secondary emission due to them exceeds that due to positive ions.

The formation of metastable atoms is produced mainly by collision of the second kind between excited and normal atoms. Since the number of such collisions increases with pressure, the concentration of metastable atoms is higher at the higher pressures and thus the secondary electron emission from this source is greater and the arc starts at a lower current at the higher pressure in agreement with the observations given in Figs. 8 and 9.

J. Slepian: A study of the transition between the glow and arc discharge such as the authors have made is particularly interesting and important because it is able to subject the various theories of the cathode of the arc to quite searching tests. It was a study of this transition in connection with the early development of the autovalve arrester which first made me doubt the universal validity of the thermionic emission theory of the cathode at a time when this theory was generally accepted. The authors also find that the transition is independent of the temperature of the cathode and thus add another nail to the coffin of the thermionic theory.

The results obtained by the authors would seem to be highly significant also in relation to the theory of the cathode of an arc which is generally accepted today, namely, the high field theory proposed by Langmuir. According to this theory, in the electric arc, electrons are drawn from the cold cathode by the very intense electric field produced by positive space charge very close to the cathode. To produce the large space charge necessary for the maintenance of so intense a field, very high current densities are necessary, of the order of thousands of amperes per square centimeter. Such current densities are observed usually at the cathodes of arcs and thus the theory seems plausible. However, if arcs should be found with current densities at the cathode of much smaller magnitude, then the high field theory becomes impossible, for these arcs at least.

Hence the finding of a continuous transition between glow and arc with cold cathodes as observed by the authors is very important, because in these transition states the voltage is much less than the normal glow voltage, and at the same time the current density is much less than the thousands of amperes per cm.^{2} usually found in an arc. Since the voltage is below that of the glow, some ionizing agent must be active other than those which are normally effective in the glow, namely ionization by collision and freeing of electrons from the cathode by positive ion bombardment and impact of excited molecules. This ionizing agent cannot be an electric field of sufficient intensity to draw electrons out from the cathode, because the current density is not great enough to support a space charge corresponding to so large a field.

Some time ago, I suggested that this new ionizing agent might reside in the gas next to the cathode, where there is a very great energy input density, and might be akin to thermal ionization of the gas. I say akin, because thermal ionization refers to the ionization existing in a gas in thermodynamic equilibrium, whereas the gas next to the cathode is in a state in which it probably cannot be described as having a definite temperature at all. If the gas next to the cathode was sufficiently ionized, the current to the cathode could be carried by positive ions entirely without requiring the liberation of electrons from the cathode. Hence, the need of an intense electric field at the cathode would be obviated, and the requirements of current density at the cathode would be less drastic.

Because of the importance of these transition states between glow and arc with cold cathodes, it is necessary to subject the demonstration of their existence given by the authors to the closest scrutiny to make sure that no valid objections can be raised. While ready, myself, to accept their existence, I do see one possible objection which I bring up hoping that the authors may be able to continue their experiments and set it aside. It is this. May not the transition states observed by the authors be only apparently equilibrium transition states? May they not consist of very rapid alternations between the glow state, and the high current density arc state? These alternations might take place with such high frequency as to escape detection on the oscillograph used by the authors.

An Improved Arc Welding Generator

BY J. H. BLANKENBUEHLER[1]
Non-member

Synopsis.—*The welding operator may encounter trouble in starting the arc or in having the arc extinguish itself too readily. Those causes of trouble which are attributable to the welding generator are analyzed and explained in terms of generator design. Development of means for eliminating the generator characteristics causing these troubles, is outlined and the results obtained are shown by oscillographic records. Other benefits accruing from this improvement are mentioned.*

GENERALLY speaking, an electric welding operator has two kinds of trouble when using an inferior welding generator, *i. e.*, the electrode may adhere firmly to the parent metal when touched to it and during welding the arc may be extinguished by the slightest disturbance.

The sticking of the electrode to the parent metal is caused by a fusing together of the molten end of the electrode and the molten parent metal. When an arc is first struck with a new electrode on a cold plate, neither should be melted until an arc is drawn. They can be melted, however, at the point of contact, if the electrical resistance of the contact and the short-circuit current of the welding generator create enough heat. This fusing together of the contacted metals immediately forms a lower resistance contact, which decreases the heating at that point, permits the molten metal to freeze and prevents the electrode from melting loose. This analysis of the process of electrode sticking indicates one requirement of a good arc welding generator, that is, to have a low ratio of short-circuit current to welding current.

However, from a study and tests of a number of different types of single-operator inherently-regulating arc welding generators it was observed that different generators with an identical ratio of short-circuit current to welding current showed entirely different arc striking characteristics. Oscillograph films of the current from these generators at short circuit revealed the difference. The generator with which it was most difficult to keep the electrode from sticking showed a large surge of current at short circuit. All of the generators tested, showed relatively large surges, and one produced surges more than ten times the final stable short-circuit current. Fig. 1, curve *A*, shows a surge from a generator which was reported to be a reasonably good welder. This figure is plotted to scale from an oscillograph picture taken during the short circuit of the welding generator. Curve *B* of Fig. 1, shows the short-circuit current characteristics of an improved arc welding generator, that is, one without any surge at short circuit.

The operator's other annoyance, instability of the arc, is explained by the fact that there is probably a minimum value below which the current in any particular arc cannot drop without extinguishing the arc. This minimum current is dependent on the length of the arc, the size and material of the welding electrode and the material being welded, the magnetic conditions about the arc, the presence of flux or stabilizing gases, the velocity of the air blowing on the arc and a number of other factors. Assuming this minimum current to exist, it can then be said that any disturbance at the arc, which reacts on the generator so as to allow the current to drop below the minimum stable arc value, will extinguish the arc.

It has been found that one of two different generators with identical volt-ampere characteristics may produce a very unstable arc even though the other be very stable. The explanation is that the inferior welding

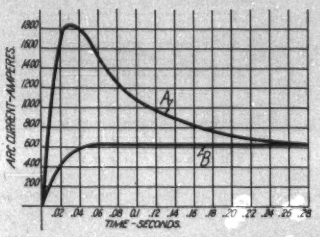

FIG. 1—WELDING CURRENT AT SHORT CIRCUIT

generator allowed the arc current to drop below the minimum stable value during arc disturbances. Oscillograph films of the current from these generators showing the change of current caused by an instantaneous increase in load resistance revealed the difference. The generator, with which it was difficult to maintain the arc, allowed the current to droop during the change considerably below the final value which the current assumed after the change. Fig. 2, curve *A*, shows this droop. If the current i on this curve is less than the minimum current for the arc, generator *A* would permit the arc to break. The stability of the arc supplied by generator *B*, which does not allow the current to droop below the final value I_2 is entirely dependent upon the slope of the volt-ampere curve of the generator. That is, if the volt-ampere curve of generator *B* as shown in Fig. 3 causes I_2 to be so much less than I_1 as to be below the minimum stable current, the arc will break even though there be no transient droop below I_2. At normal arc voltages the volt-ampere curves of practically all commercial welding

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generators give too low a change of current per volt change in arc length to render this condition important. Therefore a generator which has a short-circuit characteristic as shown in Fig. 1, curve *B*, and an arc lengthening characteristic as shown on Fig. 2, curve *B* should and does produce a very flexible and easily managed arc.

All single-operator inherently-regulating arc welding generators are commercially used with considerable reactance in their welding circuits, which reactance tends to prevent changes in the welding current. The reactance required to eliminate current surge at short circuit and to eliminate current droop when the arc is lengthened, with a generator of this type and of a size comparable to an ordinary generator of equivalent output, is undesirably large and gives off a large amount of power as heat. This power lost in the reactor must come from the supply line through the motor and generator, thus increasing the input to the set for a given output which raises the cost of welding. Since the motor and generator must also transmit this extra power, both must be larger than necessary. In some

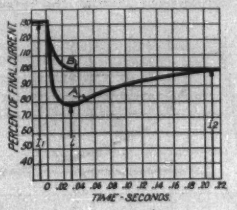

Fig. 2—Change of Current from Single-Operator Welding Generator with an Increase in Voltage from 15 to 25 Volts

commercial sets this reactor loss at rated output is 40 per cent of the output of the set and still it does not possess the ideal characteristics outlined above. Elimination of current surge at short circuit and current droop when the arc is lengthened requires that the reactance delay the change of current long enough to allow the generator to change its voltage to suit the new conditions. Due to various retarding forces in the generator, this may be a considerable time. If the change of current is delayed too long however, the conditions at the arc may have undergone a second change before the welding generator has adjusted itself to the first change.

The approximate constant-current volt-ampere characteristic desired for a single-operator welding generator can be most efficiently obtained with a separately excited differentially compound wound generator. The voltage of this type of generator follows the arc voltage with any particular separately excited field current because of the variation of the current in the series field. Thus, for a certain separately excited field current, there is a definite no-load voltage induced in the armature by the flux produced by the field magnetomotive force. Any current flowing in the armature circuit produces a proportionate amount of differential series ampere turns, thus decreasing the total field ampere turns and, therefore, the flux and the voltage. This type of generator gives a series of volt-ampere curves as shown in Fig. 3, one curve for each field current.

When the arc is struck, the generator's voltage must change from the no-load voltage to zero. This change

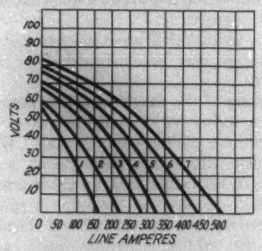

Fig. 3—Voltage Ampere Curves

7½-kw., 25-volt, 1,750-r. p. m., differentially compound-wound separately-excited generator

is produced by the current in the series field rising enough to reduce the total field ampere turns to that value which will only induce enough voltage to force the short circuit current through the resistance of the circuit. As the series ampere turns increase, the flux should decrease from the no-load value to the short-circuit value. It is difficult, however, to make this flux change rapidly since all the coils linking this flux have induced in them a voltage which produces a current causing a magnetomotive force opposing the change of flux. This action

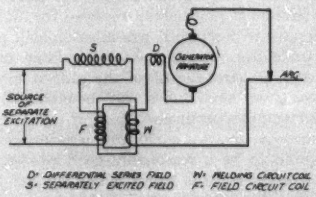

Fig. 4—Wiring Diagram

delays the change of the voltage from no load to zero and the current flowing through the short circuit assumes a value higher than the ultimate value until the flux change is completed. This produces the surge shown in curve *A*, Fig. 1. The elimination of the surge requires that the flux shall have changed to its final value by the time the current builds up to its final value. Reactance in the welding circuit works toward this end by delaying the rise of current. Enough reactance would delay the rise of current long enough to

allow the flux change to be completed, but this reactor would cause too much power loss and would be undesirably large.

Instantaneous lengthening of the arc during welding requires a change in the generator almost the reciprocal in all respects to the change at short circuit. Lengthening the arc requires an increase in generator voltage which requires that the flux raise from one value to a higher value. This change is also delayed by all the coils linking the flux, and since the current at any instant is equal to the voltage at that instant divided by the circuit resistance, the current will droop below its ultimate value until the voltage reaches its final value. This gives the effect shown in curve *A*, Fig. 2. As in the case of elimination of surge at short circuit, elimination of this droop in current with a rise in arc voltage requires that the flux shall have changed to its final value by the time the current has dropped to its final value. As before, reactance in the welding circuit works toward this end by delaying the drop of current. Enough reactance to give the droopless curve *B*, Fig. 2, would be undesirably large and would cause a prohibitive power loss.

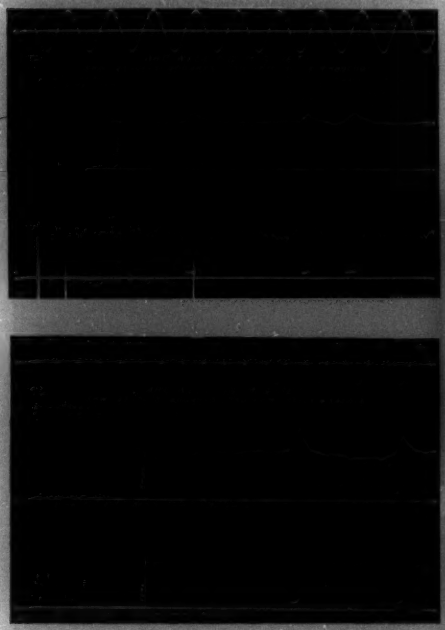

Fig. 5—Manual Welding, Horizontal T Weld, No. Mag. Blow, 1/2-In. Plate, 5/32-In. Rod, 170 Amperes (Approx.)

Top, with transformer
Bottom, with reactor

Both difficulties, evidently require the same cure, that is, to increase the speed of change of flux by removing or neutralizing the forces acting to retard its change. These forces are magnetomotive forces produced by currents induced in any conductor linking the changing flux. Investigation of the relative effects of the various sources of delay in all types of generators, showed the field coils to be the worst offenders, since the voltage induced in these coils increased or decreased the field current in surprising amounts with the resultant effect of greatly retarding the flux change. Elimination of this effect required the prevention of the development of this voltage in the field coils or the neutralization of its effect by introducing an equal and opposite voltage in the field circuit. This neutralizing voltage must, of course, be approximately equal to the voltage induced in the field coils at every instant so that the field current cannot change and retard the flux change.

Fig. 6—Manual Welding, Overhead T Weld, No Mag. Blow, 5/32-In. Rod, 170 Amperes (Approx.), 1/2-In. Plate

Top, with transformer
Bottom, with reactor

The solid iron parts of the generator's magnetic circuit and the rivets in the laminated field poles would have

induced in them currents opposing any flux change. Tests on a generator with complete and perfectly laminated magnetic circuits showed that the retarding effect of these factors is negligible except when the iron parts are worked at very high flux densities which are seldom reached in industrial generators and never obtained at any possible arc voltages in welding generators.

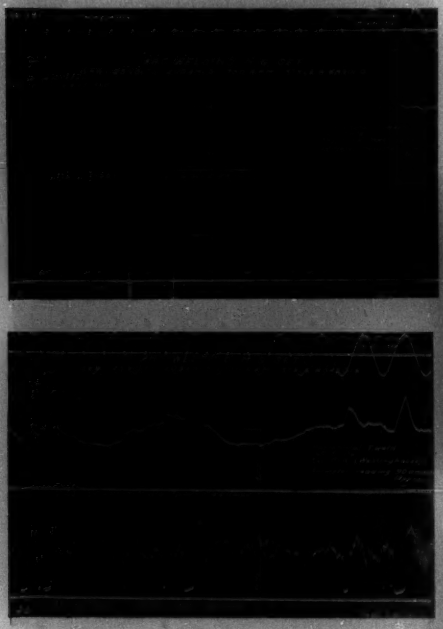

Fig. 7—Manual Welding, Horizontal T Weld, 1/8-In. Plate, 3/32-In. Rod, 90 Amperes (Approx.)

Top, with transformer
Bottom, with reactor

The logical method of producing this neutralizing voltage is to use a transformer with the primary in the welding circuit and the secondary in the field circuit, as shown by Fig. 4. It is quite obvious that the transformer can be proportioned so as to cause the voltage induced in the secondary by a current change in the primary to be the same as the voltage induced in the field coils by the same current change in the differential series coils. Since the series field and the primary of the transformer are in series, and carry the same current, the change in both occur at the same time, and the voltages in the secondary of the transformer, and the field coils will neutralize at every instant.

Another feature of this system lies in the fact that by making the proportions between the primary and secondary of the transformer such that the voltage induced in the secondary is greater than the voltage induced in the field coils by a change in welding current, then the field current can be changed to introduce a magnetomotive force promoting the flux change. This force can be used to offset any retarding forces other than in the field coils.

While the description of the functioning of this device has only considered its results when used with a separately-excited differentially-compoundwound generator, nevertheless the scheme can be used with any type of welding generator, whether separately excited or self excited, to neutralize the retarding effect of the generator field coils.

As previously stated the voltage induced in the secondary is about equal to that induced in the field coils. Its actual value is dependent on the number of

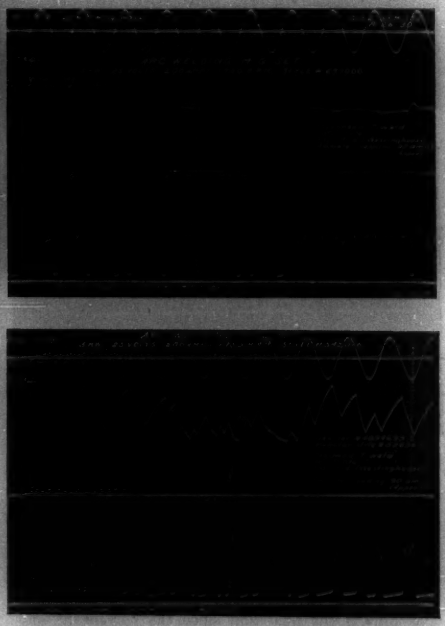

Fig. 8—Manual Welding, Overhead T Weld, 1/8-In. Plate, 3/32-In. Rod, 90 Amperes (Approx.)

Top, with transformer
Bottom, with reactor

turns in the field coils and the rate of change of the flux linking these coils. This voltage will reach a maximum value several times that of the excitation voltage. However, this is nothing unusual in electrical machine design, since much higher voltages are encountered every time the field circuits of d-c. motors or generators are opened. Considered from a safety standpoint,

the welding operator is in no danger of being shocked by the secondary voltage, because the welding circuit is electrically distinct and completely insulated from the secondary and the excitation circuit.

Curves *B* of Figs. 1 and 2 are plotted to scale from oscillograms taken on generators with this device, and curves *A* of the same figure show the action of the same generators with the reactor previously used.

Figs. 5, 6, 7 and 8 show oscillograms of the arc voltage and current during manual welding under various conditions with a 200-ampere welder. Each figure shows welding under identical conditions with the same welding generator both with the above device and with the reactor previously used. The reduction of current surge when the arc is short-circuited by a globule of metal is obvious, since in one case it is limited to the value shown by the volt-ampere curve of the generator, and in the other case it greatly exceeds this value.

A secondary benefit obtained from this device, in addition to the ease of striking and maintaining the arc, is a decrease in the porosity of the weld. Tests show that sudden increase of the current flowing in molten iron such as occurs in the molten globule of weld metal when it short circuits the arc, will cause sudden vaporization of parts of this metal. These vaporized parts expand tremendously and subsequent cooling of the globule leaves it full of holes where this vaporization has occurred and the gases have not been released. This sudden vaporization will also give the effect of explosion of the globule and parts of it will be blown away from the arc, thus decreasing the efficiency of deposit. Minimizing the increase of the arc current when the molten globule short circuits the arc, which occurs from 10 to 25 times per second, will, therefore, give a more dense deposit and allow a larger percentage of the electrode to be deposited as useful weld metal.

Summary

The use of a d-c. transformer in the welding and excitation circuits of any single-operator inherently-regulating arc welding generator will (if properly designed with relation to the field circuit constants) produce ideal arc characteristics.

1. The arc obtained with such a combination of equipment is very easy to strike because all tendency for the electrode to freeze to the work is eliminated. This is due to the elimination of the usual short-circuit current surge of several times the normal welding current value.

2. The arc will not fail without apparent reason when disturbances take place in the arc circuit such as momentary slight increases in arc length which follow after each globule of molten metal leaves the electrode and is deposited on the work. This is due to the elimination of the usual current dip or drop when the arc is momentarily lengthened.

3. The deposited metal is surprisingly free from porosity. This is due to the elimination of current surges above the desired value which over-heat the welding materials and produce excessive amounts of occluded gas.

As a result of the first two features a novice can readily strike and maintain an arc on his first trial, whereas with other types of single-operator machines he will require considerable practise and coaching before he acquires the skill and technique of striking the arc, to say nothing of learning to weld satisfactorily.

Discussion

For discussion of this paper see page 688.

The Neutralized Welder—A Means of Controlling Transients

BY F. CREEDY*
Member, A. I. E. E.

THE characteristics of d-c. welders are of several different types. In the first place there are machines with straight line characteristics. Such machines usually have both a shunt and separately-excited coil, and by varying the currents in these two coils by means of the rheostat, the slope of the characteristic curve can be altered and the curves made to cross. In this way the welder may be adjusted so as to work at various currents. All these machines, however, have serious disadvantages, because the short-circuit current must be materially larger than the welding current and hence the ill-effect of shortening the arc or accidentally touching the work with the electrode are very much accentuated. Another type is the plain shunt machine working in the vertical portion of the characteristic. In this machine the short-circuit current is if anything less than the welder current, which, while better than having it larger than the welding current, is still not precisely what is required. In order to prevent the machine demagnetising

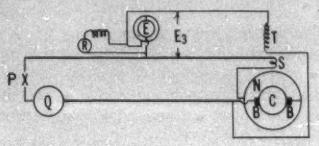

FIG. 1

when it is short-circuited, it is usual in plain shunt machines to use a certain amount of resistance in series with the arc circuit. While this has the advantage as pointed out below of steadying the operation of the set, it gives rise to a great reduction of efficiency, which should be avoided if possible. A combination of these two types may be made in which the machine regulates for almost exactly constant current over the range required during welding. This may take several forms and one of these has finally seemed most suitable for practical purposes. This is shown in Fig. 1. In this C represents the armature of the welding dynamo, which is supplied with a neutralizing coil N, for reasons which will be described below. On the commutator of this armature rest brushes B B and the neutralizing coil N is connected to a series coil S in such a manner that its ampere turns are opposite to those of the shunt coil T. The shunt coil T is connected across the brushes in series with the exciter E also shown as a neutralized machine, though this is less essential, the field of this exciter being regulated by means of the resistance R. Finally connected across the brushes is the arc P and the steadying reactance Q. The operation of such a circuit may readily be understood as follows: If we call E_1 the voltage across the main commutator and E_3 that across the exciter, then the current in the shunt coil T will be

$$\frac{E_1 + E_3}{R_3} = \frac{E_1}{R_3} + \frac{E_3}{R_3}$$

where R_3 is the resistance of T.

Thus such a circuit is equivalent to the two circuits shown in Fig. 6 in which T_1 is connected directly across the main commutator, the current in it being $\frac{E_1}{R_3}$, and V is connected across the exciter, the current in it being E_3/R_3. The ampere turns in the single coil T are clearly the same as those in the two separate coils T_1 and V, and hence the single coil is adopted as being the simpler. It is not difficult to prove that such an arrangement may have a constant current characteristic.

Let

K = armature volts per ampere turn (assuming no saturation during welding)
R = external resistance taking the place of the arc
r_1 = armature resistance
n_3 = shunt turns
n_1 = series turns
r_3 = shunt resistance
e_1 = commutator volts
e_3 = exciter volts
i_1 = line current
i_3 = shunt current

$$i_1 = \frac{K\left(\frac{n_3}{r_3}(e_1 + e_3) - n_1 i_1\right)}{r_1 + R}$$

Terminal voltage $e_1 = R\, i_1$.

$$i_1 = \frac{K\left(\frac{n_3}{r_3}(R\, i_1 + e_3) - n_1 i_1\right)}{r_1 + R}$$

Solving for i_1

$$i_1 \left\{ (r_1 + R) - K\left(\frac{n_3}{r_3} R - n_1\right) \right\} = K \cdot \frac{n_3}{r_3} \cdot e_3$$

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$$i_1 = \frac{K \cdot \frac{n_3}{r_3} \cdot e_3}{r_1 + K\, n_1 + R\left(1 - \frac{K \cdot n_3}{r_3}\right)}$$

If $\left(1 - \frac{K\, n_3}{r_3}\right) = 0$ or $K_{n3} = r_3$

$$\text{giving } i_1 = \frac{e_3}{r_1 + K\, n_1} \qquad (1)$$

variations in the external resistance or load R make no difference to the current i_1, or in other words the machine has a constant current characteristic.

Examining equation (1) above, we see that the line current, although it is independent of the external resistance or arc current, is directly proportional to e_3, that is, the voltage across the exciter. By varying this voltage, for instance by means of the field rheostat R in the exciter circuit, we change from one constant current characteristic to another. For each particular voltage across the exciter the machine regulates automatically for a particular current, which will be adapted to some particular size of electrode, and this regulation is effected solely by manipulating the single rheostat R.

We may generalize the above investigation a little.

We may write:

$$i_1\,(r + K\, n_1) + e_1\left(1 - \frac{K\, n_3}{r_3}\right) = K \cdot \frac{n_3}{r_3} \cdot e_3$$

remembering that $e_1 = R\, i_1$.

Consider how this may be plotted on a sheet of coordinate paper with e_1 as ordinate and i_1 as abscissa. For different values of e_3 the exciter voltage, it represents a family of parallel straight lines whose slope depends on the value of the coefficients of i_1 and e_1. For $r_3 = K\, n_3$ these lines become parallel to e_1, that is, we have a constant current characteristic as shown above. For other values of r_3 we have other slopes for the curves. In the ordinary operation of the machine, r_3 would not be varied, the operator would choose the slope of curve he preferred and stick to it for all sizes of electrode, still using only one rheostat for adjustment.

A study of the curves will make it clear how the constant current machine works with only one rheostat. We only obtain the constant current characteristic for a particular value of r_3 the resistance of the shunt circuit. Thus any variation of this means a departure from the constant current characteristic. Hence, the circuit in addition to its improved welding characteristic, has the further advantage of simplicity, since it requires only one rheostat. This is an immense point from the practical welder's point of view. For very low currents, below about 70 or 80 amperes, the striking voltage becomes rather low if the same number of turns is used on the series coil as with the higher currents.

In order to maintain a sufficiently high striking voltage for the arc, an extra tap in the series winding may be provided for use with currents below about this value, there being in all three welding terminals, A, B, and C. A and B are used for currents from say 80 to 300 amperes and A and C from 0 to 80 amperes. A single pole two-way switch can, of course, be used to change from B to C.

The first step in the investigation, therefore, consists of the simplification of the methods of excitation and elimination of unnecessary circuits. The circuit chosen has the practical advantage that the number of turns on the shunt field winding is approximately halved, and the size of the wire doubled, which conduces very strongly to reliability and long life.

Stability

Arc welders differ from one another in a quality which is vaguely called stability or steadiness. With some types of welder the arc apparently has much less tendency to splutter or fail than with others. These variations in characteristics are known to be due to the transient effects which occur during the change of circuit conditions and whose magnitude varies in different types of apparatus. Is it possible to define clearly what we mean by stability, and to reduce it to calculation?

What Occurs During "Spluttering." In order to throw light on the question of stability it will be worth while to discuss in considerable detail what happens in the welding dynamo during a "splutter" on the arc. This paper is naturally confined entirely to the electromagnetic occurrences, which are best studied from the mathematical point of view. A good beginning has been made in this direction by Mr. K. L. Hansen. (A. I. E. E. Trans., Vol. 39, p. 1357.)

Suppose that welding is proceeding with a constant current, $i_1\,s$ say, and that some change in the circuit conditions occurs which we may represent by a change in the total resistance. For instance, the operator may momentarily shorten the arc or cause it to touch the work. If, for instance, the operator brought the electrode into permanent contact with the work, the current would change from the steady value of $i_1\,s$ to another steady value of $i_1\,p$. It is possible, as mentioned above, so to regulate the welding dynamo that it operates on a practically constant current, that is, so that the short-circuit current is practically the same as the welding current. In this case $i_1\,s = i_1\,p$, and as this is the simplest case for purposes of explanation, this is discussed first. The question to be investigated is: what happens during the change from $i_1\,s$ to $i_2\,p$? As is well known, owing to the presence of certain transient voltages in the circuit, the instantaneous currents during welding do not coincide with the steady currents, as shown by the characteristic curve, but on the other hand a sudden short circuit involves a momentary kick of current, which may be considerably greater than the steady short-circuit current. Can the value of this momen-

tary kick of current be calculated to find out what factors influence it, so as to be able to reduce it to a minimum? It is clear that only by reducing this kick to a minimum can we get the steadiest possible operation. A study of the differential equations of the circuit shows that any change in circuit conditions sets up a transient wave of very low-frequency alternating current, superposed on the continuous current and rapidly dying away in accordance with a logarithmic decrement. The nature of this change is shown in Fig. 2 where on the left is shown the constant welding current $i_1 s$ and on the right the constant short-circuit current $i_1 p$. If, at the point C the arc is shortened or momentarily short-circuited, a kick is produced as shown, the current increasing to a value perhaps twice as great as the permanent short-circuit value, then decreasing to a value less than the permanent short-circuit value before it becomes steady. Theoretically several such waves might exist. Practically, the depression which follows the first kick has often been observed on the oscillograph, but owing to the slight unsteadiness which always exists in practise, preventing the current from showing us a perfect straight line on the oscillograph, the remaining waves of very small

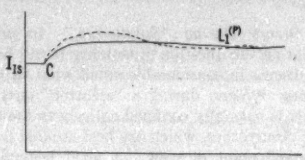

FIG. 2

amplitude are difficult to detect. Such a wave may be represented by the formula $B_2\ (e^{-pt} \sin q\,t)$. Thus the wave is enveloped by the curves of logarithmic decrement $B_2\ (\pm\ e^{-pt})$ and touches it at the various points where $\sin q\,t = \pm 1$. Clearly the value of the current kick depends on the value of B_2, p and q.

These may be worked out and p and q represented by the following formulas (which neglect armature resistance).

$$p = \frac{r_1 n_1 \left(L o + \frac{n_1^2}{\rho} \right)}{2 L o \frac{n_1^2}{\rho}} \left\{ -1 \pm \sqrt{1 - \frac{K}{r_1} \cdot \frac{4 L o \frac{n_1^2}{\rho}}{\left(L o + \frac{n_1^2}{\rho} \right)^2}} \right\}$$

(See Appendix for definitions of symbols.)

For certain values of the constants the expression under the radical can clearly be negative, giving an oscillatory solution. Put $\frac{n_1^2}{\rho} = L_{11}$ and examine the expression

$$\frac{4 L_{11} L o}{(L_{11} + Lo)^2}$$

It is clearly zero if $L o = 0$ or if $L o = \infty$ and must therefore, have a maximum for some intermediate value of $L o$.

Differentiating and equating to zero we get

$$L o^2 - L_{11}^2 = 0$$

$$L_{11} = \pm L0$$

If we put $L_{11} = L o$ the maximum value of the expression is unity.

That this expression should be a maximum, that is, that the external self induction should be equal to that due to the main field, is clearly the worst condition for producing oscillations. If the external self induction is much *less* (an unpractical condition, since it does not permit of the current being kept steady) or much *more* (as it should be) then oscillations are impossible. Assuming this worst condition, then, we shall obtain oscillations if $\frac{K}{r_1} > 1$.

But this is the condition studied in the appendix in the case where no armature self induction exists, and it is there found to mean that the $I^2 R$ loss in the field is *greater* than the power liberated by the decrease of the flux, so that some must be supplied by the rotation volts.

Oscillations can therefore always be prevented by increasing the resistance of the field.

But the condition $\frac{K}{r_1} = 1$ is that which gives rise to a constant current characteristic in a machine with shunt, separately excited, and reverse series coil. Hence we should endeavour to keep this quantity about unity, neither decisively greater nor decisively less, and rely for the prevention of oscillations on making the external self induction decisively greater than the internal. This may be done in two ways either, by making the external self induction large, or, the internal self induction *small* the latter being by far the better plan. Or of course, both may be adopted.

Summarising the investigation given in full in the Appendix, the main conclusions are as follows:

In the welding generator, two distinct problems exist.

1. To keep the current constant during rapid changes of conditions. This is solved by the use of reactance.

2. To design the machine so that the volts generated may vary rapidly to meet changes of conditions without much change of current.

It is essential to recognize that there are two distinct problems, and not one, and that it is only the second of these which requires the use of a special generator. Every change in voltage means a change in field strength.

1. Rapid changes require that field variations can take place with only a small inflow or outflow of magnetic energy to the field, that is, that *the magnetic energy stored in the field shall be as small as possible.*

2. For this purpose the generator should be designed as follows:

(a) With low saturation. This will be secured in any case, owing to the need for a high voltage on open circuit.

(b) The shortest possible air gap involving semi-closed rotor slots and laminated pole tips.

3. Low magnetic energy means very rapid increment or decrement of flux to meet changed conditions, as proved in the appendix.

4. It also means absence of oscillations if used in conjunction with a large external self-induction.

5. It is proved below that oscillations are due primarily to the fact that the magnetic energy stored in the field is about equal to that in the reactance. This condition should be avoided, and the energy stored in the reactance should be much greater than that stored in the field.

6. To store a large amount of energy, the reactance should have many turns, and a fairly long air gap, (it may of course be made of cast iron, or cast steel).

7. It is a common practise to use the reactance of the armature circuit due to the cross flux, for this purpose. This is bad practise for the following reasons:

(a) The necessity for a short air gap has just been pointed out in the interests of low stored field energy.

(b) If there are many armature turns with a short air gap, the cross flux will produce high saturation, tending to increase the stored field energy.

(c) If the pole arc is wide this cross flux will create sparking.

(d) If we reduce the pole arc to avoid this, we reduce the reactance formed by the cross flux and increase the reluctance of the field.

(e) A concentrated interpole winding diminishes the effectiveness of the machine as a reactance.

8. It is better therefore, to use a neutralizing winding or distributed interpole, completely abolishing the cross flux, and to have the reactance in separate form. The endeavor to combine the conditions for maximum reactance, and the opposite conditions for minimum field energy, means that neither set of conditions can be met satisfactorily. Only separate pieces of apparatus can satisfy both in the best possible manner.

9. This is true, even with a purely separately excited machine. If in addition there is a shunt coil, placing the reactance external to the tapping points of the shunt circuit, has further advantages. It shields the shunt winding from the effect of the rapid oscillations of the arc.

10. Owing to lag, caused by the self-induction of the main field, connecting the shunt winding directly across the arc, does not enable it to follow rapid variations, though it can follow fairly slow ones. It is therefore better to rely on the reactance to deal with rapid variations, and shield the shunt winding from them.

11. The lower the field energy, and the higher the field resistance the more rapid are the variations which can be followed.

12. The higher field resistance also tends to check oscillations but, it means more power required for excitation, and it is desirable in a machine with a shunt circuit, that the field resistance should be such as to give the constant current condition. It is therefore better to rely on other means, namely—

Low field energy, and high external reactance to

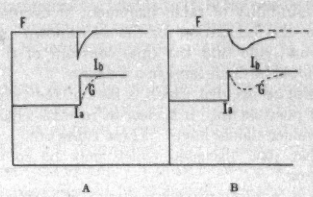

Fig. 3

avoid oscillations, and *not* to connect the shunt circuit directly across the arc. This last is perhaps a point of secondary importance, as to which some difference of opinion might be allowed.

13. Leakage self-induction in the shunt or separately excited circuit, prevents rapid response by permitting the shunt current to vary at a different rate from the armature current. Such leakage is the *sole* cause of these variations.

14. If there is no leakage between the series coil and the shunt or separately excited coil, all currents must vary at exactly the same rate so "sluggishness" is impossible.

The Effect of Field Leakage in the Welder

It may be of interest to discuss further the effect of field leakage in the welder. It has been previously known that currents in the separately excited coil produce a damping effect, and since this prevents rapid variation leakage has been thought advantageous. Their action may be most easily understood by assuming that the steady currents flow at all times and transient currents are superposed on them whenever the flux alters.

Let Fig. 3 refer to a separately excited machine in which a constant voltage is applied to the separately excited coil and flux change is due entirely to changes in the current in the series coil. If the series current suddenly changes from $I\,a$ to $I\,b$, then for the case of no leakage, the change in ampere turns is instantly masked by transient currents in the separately excited coil whose current is shown at F so that at the first instant there is no resultant change of flux. These transient currents die away at a rate which may be made very great by sufficiently reducing the energy stored in the main field. The resultant curve of the ampere turns is shown at G in Fig. 3A. Using leakage the damping currents will not be so great *and will die away more slowly* as shown in the curve F of Fig. 3B the resultant ampere turns being shown by the dotted curve G of Fig. 3B thus giving rise to an oscillation of a detrimental nature. The physical meaning of this damping current is that even if the series ampere turns made a sudden change, the flux cannot change until *time* has been allowed for the necessary energy to flow in and the less the energy the less the time.

The desirability of rapid increment or decrement is of course, the foundation for the well known and perfectly sound rule that the time constant of the field circuit should be made large in a welder.

It cannot be decided which is best of the two types of welder possible until it is clear as to what characteristics a welder should have. These types are:

1. That just discussed which may be called the "series type."

2. That in which there is a shunt coil having a variable instead of a constant e. m. f. impressed on it, the variations of the flux being due to the variations in the current in this shunt coil rather than to variations in the main current which in this type of welder might be maintained absolutely constant without impairing its operation.

In the writers view the conditions to be met with by a welder are as follows:

Current:

1. Welding current must not rise too much even on short circuit or it will cause "spluttering."

2. Current must not fall too rapidly with increasing arc length or bad welding will result as well as excessive voltage required to maintain the arc.

Voltage:

3. Voltage must as rapidly as possible assume a fixed minimum value as soon as the electrodes are separated and never fall below it or the arc will go out. This minimum voltage is about 38.5 (according to Mrs. Ayrton) for the carbon arc and probably 15 to 20 for the iron arc.

4. Voltage must be capable of *rapid* increase above this value to take care of variations of arc length. On open circuit it must be capable of being doubled or more.

Frequency. There are two types of variations high frequency and relatively low frequency.

In the writers opinion the reactance takes care of the high frequency variations and the machine of the relatively low frequency variations that no practical reactance could deal with.

The conditions connected with the current mean that the welding current should be kept nearly constant a slight reduction only being allowed to permit the reactance to produce a voltage tending to keep the arc going. Hence a small change in current should produce a comparatively large change in voltage. The voltage generated by the machine should follow that required by the arc as rapidly as possible.

In order to obtain a large change of voltage with a small change of current, and particularly if there are independent changes of voltage and current it is better to have some means of changing the voltage (such as a shunt coil) independently of the current.

Without a shunt coil there is only one machine voltage corresponding to every current. Every time the current assumes a particular value the voltage assumes its corresponding value. Thus a welder with shunt coil, separately excited, and reverse series coil such as that discussed above is likely to be more sensitive and responsive than one without shunt coil if there are no countervailing disadvantages.

It seems in fact, inconsistent to take elaborate precautions to reduce current variations by using a reactance and then rely on them to produce voltage variations.

In spite of the above *prima facie* advantages the use of the shunt coil has fallen into disrepute. A common opinion is that "it is notoriously sluggish" and one cannot help agreeing to this for machines constructed as usual. But this is due to the fact that in machines of the usual construction the shunt coil has a heavy magnetic leakage, combined with high stored magnetic energy, and if this magnetic leakage and high stored energy can be eliminated the use of such a coil can be rehabilitated. This will make a better welder than one without a shunt coil.

Leakage in the separately excited coil as pointed out has not the same effect as leakage in the shunt coil, and in fact may even be beneficial by reducing damping currents. (However, resistance seems preferable for this purpose, as is the common practise.)

Returning therefore to the machine with shunt coil instead of or as well as the separately excited coil, it is clear that variations of the current in it will tend to produce damping currents in the series winding. But the series winding already has a very large self-induction connected in series with it, so that any tendency it may have to damp out variations of flux due to variations of shunt current must be small. If, however, there is a distinct separately-excited winding it will

tend to damp out variations due to the shunt winding as already discussed. It is to this cause the difference between the two oscillograms of Figs. 4 and 5 is attributed. The oscillogram in which there was a distinct separately excited winding (Fig. 5) shows larger current rise on short circuit than Fig. 4 the one in which the arrangement of Fig. 1 is used, which obviates these damping currents altogether.

It is true also that damping currents will be induced in such a shunt coil by variations of the series ampere turns just as if the coil were separately excited. But such damping currents may be reduced to a minimum by keeping the welding current nearly constant which

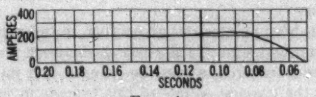

Fig. 4

can be better effected with this type of machine than with the series type. As stated above if the field energy is small they die away so rapidly that they cause little harm. This can be shown from the differential equations.

If the shunt coil however is to produce an armature voltage proportional to the e. m. f. applied to its terminals the *shunt current must be proportional to this e. m. f.* that is, the self induction of the shunt circuit (due to the main and the leakage flux) should be reduced to a minimum while the resistance remains high. The self-induction due to the main flux as already shown should be minimized by reducing the field energy and it is also important to eliminate the leakage flux.

The use of series resistance to eliminate high frequency oscillations is the common practise, and it is clear that it is effective by increasing the rate of increment or decrement of the currents and fluxes. But this is simply because adequate precautions are not taken in the welder of ordinary design to reduce the field energy and when it is reduced to a minimum the decrement or increment of the currents and fluxes is quite sufficiently rapid without the use of series resistance. The welder described above has no series resistance, and gives an over-all efficiency between the watts input on the a-c. side as registered on the a-c. wattmeter and the actual energy going into the arc of 60 per cent. This includes all the a-c. losses, all the d-c. losses, exciter losses, and losses in the reactance and cables. It seems therefore, that series resistance is wholly needless if the welder is properly designed.

Stability Ratio

It is now possible to answer the question asked above, as to how it is possible to define clearly what is meant by stability and to reduce it to calculation and measurement.

It is proposed that the stability of all welding apparatus should be defined in terms of what may be called the *stability ratio*, and that the stability ratio should be defined and measured by the following standard test.

1. Load the welder up to its maximum load, say 30 volts, 200 amperes on a resistance.

2. Arrange a switch so that the resistance may be instantly short-circuited, thereby instantly short-circuiting the terminals of the machine through the steadying reactance.

3. Measure the maximum current kick produced, (say 600 amperes by way of example) when the short-circuiting switch is closed. The *stability ratio* is the ratio of this maximum kick to the normal full load current. In the example given, where the kick is 600 amperes and the full load current 200 amperes, the stability ratio is three. When this stability ratio has a small value the welder will operate steadily.

There is no serious difficulty in measuring this kick by well-known methods, with the instruments ordinarily found in a test room. In order to do this, remove the glass front from an ammeter, preferably of the moving coil or dynamometer type and whose moving system has a low moment of inertia. Hold the needle by means of a pencil point or otherwise at approximately the value which the current is expected to have, say at about 600 amperes. Short-circuit the machine and if the current is less than the value indicated by the pointer, it will continue to rest against the pencil point which prevents it from moving back to zero. If the current is greater, the needle will momentarily flutter away from it to a greater or less extent. If there is no flutter it indicates that the test should be repeated with the needle held opposite a lower reading on the scale, say 500 amperes, and if the flutter is vigorous it indicates that the needle should be held opposite a higher reading on the scale, say 700 amperes, the test being repeated until a point is found where the needle only just shows a faint indication of movement. By this means, using only stand-

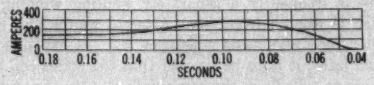

Fig. 5

ard d-c. instruments, the instantaneous short-circuit current may be measured with very fair accuracy.

In Figs. 4 and 5 oscillograms are shown of the neutralized welder switched on from open circuit to full load (a severer test than that stated above). Fig. 4 shows the machine arranged with shunt and separate excitation combined as in Fig. 1 while Fig. 5 shows it with shunt circuit and separately-excited circuit independent. The reason for the difference between these two oscillograms is suggested in the paper.

It is desirable to describe in detail the methods of construction adopted to take practical advantage of the theoretical results just obtained. In order to reduce

the magnetic leakage between the shunt and series circuit, it is necessary to reduce the self induction of the armature circuit to a minimum. The most effective means of doing this is by making use of a neutralizing winding connected in series with the armature circuit, which may be constructed in one of several manners, as for instance those shown in Figs. 6 and 7. Fig. 7 shows a welding dynamo with a uniform airgap all round the circumference. The armature A is

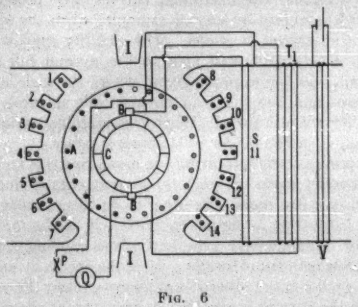

Fig. 6

fitted with a commutator C on which rest brushes B B, and is surrounded by a stator having in the figure shown, 24 slots D. These slots contain a neutralizing winding N, connected in series with the armature, the connections not being shown, and such that the ampere conductors in its slots are nearly equal and opposite to those of the armature, being made in most cases slightly greater in order to produce the effect of an interpole. In addition to the neutralizing winding, certain of the slots contain a series winding S S, also connected in series with the armature, and a shunt winding T connected across the terminals of the armature through a suitable regulating resistance R. In addition to this there is frequently a separately exciting winding V, which is excited from some external source and controlled by a regulating resistance R_1. As explained in detail later, it is to be desired that the series coil and the shunt coils lie in the same slots, so that the mutual induction between them may be as high as possible, though since both of these excite the same magnetic circuit, it must in any case be high. The series winding is so connected as to oppose the action of the shunt winding and the separately-excited winding, where this exists. The armature circuit is then completed through the steadying reactance Q and the arc P.

Another construction of machine is that shown in Fig. 6, in which the machine is of the ordinary salient-pole type, fitted with interpoles and having slots made in the pole face. In this machine A, B B and C represent the armature, brushes, and commutator as before, while D represents the slots, which again contain a number of ampere conductors almost equal and opposite to those in the armature, but slightly greater as a rule. The interpole in such a machine need not be wound, as the ampere conductors, due to the winding in the slots will be sufficient to excite it. Alternatively, the ampere conductors in the slots may be as exactly as possible equal and opposite to those in the armature, while a small number of ampere turns are placed on the interpole, in order to produce the commutating flux. The use of a neutralizing winding permits the airgap to be reduced to a value similar to that used in induction motors, thus minimizing the field energy. The various field windings, S, T, and V are the same as before, similarly connected, but they are now, of course, wound on the salient poles in the form of a spool in the usual manner.

It has been pointed out above that in order to eliminate transient e. m. f.'s the leakage between the shunt and armature circuits must be reduced to a minimum. By far the greatest source of this leakage in machines of ordinary construction, is the armature flux, which is eliminated by the use of a neutralizing winding, and when this has been eliminated the remaining leakages are of much less importance. However, in order still further to reduce the magnetic leakage between the

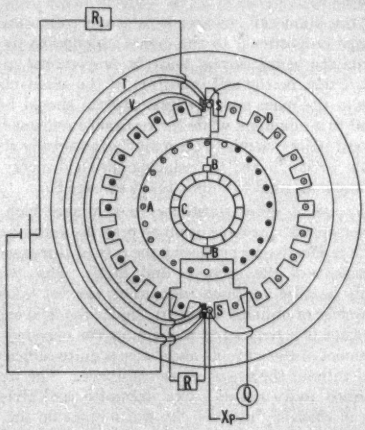

Fig. 7

series and shunt coil, the series and shunt coils may be wound in alternate layers, making up a combined coil containing both windings. Since the number of turns on the shunt coil will usually be large, and that on the series coil small, it will be desirable, to wind the series coil with a number of conductors in parallel. Suppose, for example, that the number of turns on the shunt coil is 2,400 and that on the series coil six, the shunt coil might for instance conveniently be arranged with 80 conductors per layer and 30 layers.

It is not necessary that there should be a layer connected to the series winding adjacent to each layer of the shunt winding. It will be sufficient, for instance, if a layer of the series winding be interleaved with, say, every 10 layers of the shunt winding. In order to bring this about, the series winding may be wound in two parallels, each parallel containing six turns and being disposed in a single layer. This is shown in Fig. 8, in which *S S* show the two layers of the series winding which are in parallel and *T T T* show three sections of the shunt winding, each containing 10 layers and 80 conductors per layer, between which the layers of the series winding are placed. Where there is a separately exciting winding it may take the place of half the shunt windings in the manner shown dotted at *V*. Where the shunt and series windings are contained in slots as in Fig. 7, it is desirable though not essential that the series and shunt windings should be contained in the same slots as shown in Fig. 7, where again *S S* represent the series winding and *T* the shunt winding, a portion of which may be replaced by a separately exciting winding shown at *V*. It is further desirable that the whole of the shunt windings should be carried in a single slot per pole, in which case the winding *T*, Fig. 7 should be regarded as being the whole of the shunt winding, corresponding to a particular pole. In a machine with slotted stator the number of shunt turns may be very much less than in a machine of the ordinary type, and consequently, the number of layers of the shunt winding measured circumferentially will be small. Hence it is not difficult to bring the series and shunt windings into very close proximity.

Of the various arrangements possible, that shown in Fig. 7 in which the series windings occupy one side and the shunt windings the other, is somewhat preferable to one in which the series windings occupy the top and the shunt winding the bottom, in cases where the circumferential width of the slot is less than its depth.

The usual plan of operation on a-c. circuits is, of course, to couple the welding dynamo to a standard a-c. motor forming a motor-generator set, and this is the source of much of the first cost of the set. To eliminate this, the next step is to combine the welding dynamo and a-c. motor into one structure in which the magnetic circuit carries two distinct fluxes of different pole number. The a-c. motor may be either of the synchronous or induction type, though the latter is usually adopted in the sizes here described. To explain the principle of the machine clearly it will be convenient to consider the synchronous type first. Consider a machine carrying four windings—two on the stator and two on the rotor, which we may call *a*, *b*, *c*, *d*. Suppose, for instance that of the rotor windings *c* and *d*, *c* is arranged for six-pole and *d* is a two-pole winding, and of the two stator windings *a* and *b*, *b* is arranged for two poles and *a* is a *six-pole* field winding. Suppose direct current to circulate in the two-pole field winding *d* on the rotor and suppose the machine to be running at the synchronous speed of this two pole winding, namely 36,00 r. p. m. on 60 cycles. Clearly the two-pole winding can induce no voltage in the rotor winding *c* since it revolves with it and hence does not cut it. Similarly, if direct current circulates in the field winding *a* on the stator, clearly this direct current can induce no e. m. f. in the other winding *b* on the same core, since both windings are fixed. This winding *a* induces e. m. f.'s only in winding *c* and winding *d* induces e. m. f.'s only in winding *b*. The flux due to winding *d*—stationary with respect to the rotor—will revolve in respect to the stator, while that due to winding *a* cuts the rotor and is stationary with respect to the stator.

If winding *b* is supplied with 60-cycle single-phase or polyphase current the machine will work as a synchronous motor, and if winding *c* is joined to a commutator a continuous e. m. f. will be produced across that commutator.

Winding *d* cannot induce any e. m. f.'s in winding *a* since the two are wound for dissimilar numbers of poles, and for the same reason winding *a* cannot induce any e. m. f.'s in winding *d*.

In practise the use of two windings on the rotor of such a machine is inconvenient. This is rendered unnecessary if the single rotor winding attached to the commutator is given a pitch which is equally adapted to operate on six or two poles, such, for instance, as the full two-pole pitch which is also equally well suited to six poles, for in both cases north and south poles are diametrically opposite. If this winding is tapped off at points differing by two pole pitches of the six-pole field, that is, at three points 120 deg. apart, and these points brought out to sliprings, no e. m. f.'s will be induced by the six-pole flux between any two of these sliprings. If, therefore, a pair of them is short-circuited and direct current inserted between the short-circuited pair and the remaining ring, a two-pole excitation of the rotor may be brought about without requiring the use of a separate winding, and in this way the six-pole and two-pole windings may be combined. Means exist for combining the two separate stator windings also, but it is thought sufficient to consider that separate d-c. and a-c. windings are employed on the stator, as is actually the case.

The alternating side of the machine may also be caused to operate as an induction motor if the points which before were connected to sliprings are short-circuited together. Under these circumstances currents of slip frequency will flow through the short-circuited connections and the sliprings may be eliminated. In this case, of course, the short-circuited points need not be limited to three so long as care is taken that only points two pole pitches apart of the six-pole field are connected. This type of machine, owing to the absence of sliprings and of the necessity of direct current

excitation of the rotor, is the most suitable for small sizes.

In the induction type of machine it is no longer true to say that one flux is stationary with respect to the rotor and the other with respect to the stator. The flux which is now generated by the alternating current winding on the stator is in slow rotation with respect to the rotor also. Of course, the machine is not limited to two and six poles, but can be built for many other ratios of poles, though for welding purposes this ratio is convenient.

It will next be of interest to describe the practical

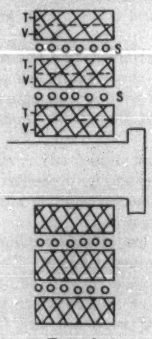

Fig. 8

construction of a machine arranged for two and six poles, and converting from three-phase to direct current.

A machine constructed as in Fig. 7 may readily be modified so that it acts as a converter from alternating current to direct current as well as a welding dynamo pure and simple. This modification is shown in Fig. 9 where the windings S, T, and V of Fig. 7 occupy slots 1, 5, 9, 13, 17 and 21, the welding dynamo being a six-pole machine. The neutralizing winding N occupies the rear portions of the slots, being arranged as a two-circuit drum winding, while those portions of the slot next to the airgap contain the alternating current windings L arranged for two poles, which may, for instance, be of the standard three-phase kind. The armature A, the commutator C and the brushes $B\,B\,B\,B\,B\,B$ are shown in the figure (though not lettered) there being equalizers F joining points on the commutator one-third of the circumference apart in order to form circuits for the current induced by the a-c. windings, while at the same time acting as equalizers for the welding dynamo. For the sake of clearness the rotor winding is shown as a ring winding. If the brushes are, connected, for instance, in series with the tappings Y_1 Y_2 on the interpole winding, it will act purely as a neutralizing winding, but if they be connected as shown to certain other tappings, Y_1' Y_2' the m. m. f. of the neutralizing winding will have a certain component along the axis of the field circuit, and hence this winding will act both as a neutralizing winding and a series coil combined. The shunt winding T is connected in series with an exciter E (of the same type as shown in Fig. 1) across the main commutator. The coil T is thus excited by the voltage of the main machine and the exciter in series. The circuit is then completed through the arc P and the steadying reactance Q.

It is sometimes convenient in order to improve the starting characteristics to connect a part of the field winding T into the a-c. winding. For this purpose the winding T should be made up of three coils a, b, and c, each spanning one-half of the circumference, the coil a lying, for instance, in slots 1 and 13; the coil b in c and 21, and the coil c in 17 and 5. These are all connected in series to form the shunt coil T. To employ them for starting purposes, they may be disconnected from the direct current side of the machine and the ends short-circuited. If taps are brought out from between the individual coils, they will form a mesh as shown in Fig. 10 and this may be connected into the star point of the a-c. winding, where it will be equivalent to an added number of turns and will improve the starting characteristics. Instead of the whole of the shunt coil being so treated, only a portion may be used for this purpose, the remainder being open-circuited during the process of starting. In many cases the short

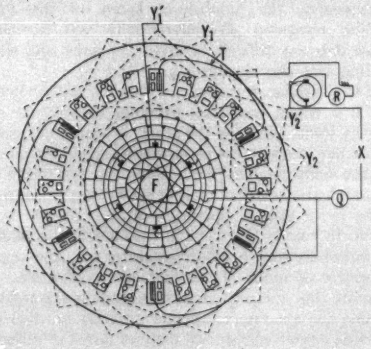

Fig. 9

circuit between the coils A and C may be omitted, the shunt coil being connected to the d-c. circuit in the normal manner and the ends of the three-phase winding simply tapped on to connections intermediate between the coils, A, B, and C, whether these form the whole or only part of the shunt winding.

Such a machine is usually arranged to operate at 3,400 r. p. m. on 60 cycles since this is the most economical speed. In Fig. 11 which shows a cross section of this type of machine are shown what are known as "fan risers." These consist of a number of blades

attached, say, to every tenth segment, and at a high speed of rotation this produces a very powerful current of air which is extremely effective in cooling the commutator, so much so in fact, that in spite of its carrying a current of 300 amperes, the temperature rise is seldom more than 20 deg. fahr. to 40 deg. fahr.

The starting gear is in many cases that appropriate to a squirrel-cage induction motor. For sizes suitable for most welding operations three-phase star-delta

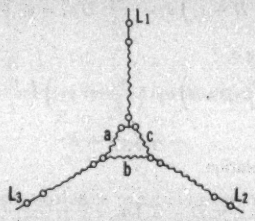

Fig. 10

starters are employed, somewhat modified where the connections of Fig. 10 are employed, while in sizes exceeding this one may employ an auto-transformer starter. The apparatus completed and ready for operation is seen in Fig. 12. Owing to the short distance between bearings the machine may be readily designed with vertical shaft. In this set as will be seen the ammeter and voltmeter are at the top coming next the rheostats (two in this case). Below this comes

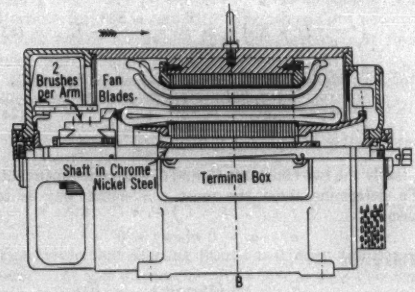

Fig. 11

the star-delta starter and then comes a single-pole two-way switch arranged to change the tap on the series winding, as desired to weld, from 0-80 amperes or from 80-200 amperes. Below this are the two welding terminals and then the large cast-iron reactance. The small exciter is seen mounted on top of the generator.

Appendix I

A mathematical study of a series of simple cases of the welding generator is given in this appendix, each one elucidating some particular aspect of the general problem and by means of such studies we endeavor to obtain a clear physical grasp of the whole of the phenomena.

In the first place, Hansen treats the arc as if it were a resistance R in his paper referred to above.

The short period oscillations in voltage across the arc are due in the main to varying length l, if the mean current is nearly constant. It is well known that the voltage across an alternating (iron) arc is nearly constant during a half period, while the current rises from zero to a maximum and falls to zero again and then abruptly reverses when the current changes sign. This should be sufficient proof that the voltage across the arc is nearly independent of the current.

Thus instead of representing the arc by resistance R

Fig. 12

it will be represented either by a constant e. m. f. E or when we wish to study its variations by an e. m. f. $E_0 + E_1 \sin pt$. Clearly the sine wave could be replaced by any Fourier's series, but for the sake of simplicity this has not been used. The type of circuit to be studied will be that previously described in which the shunt is connected across the main commutator and exciter in series. By choosing this circuit we obtain differential equations of the second order only, instead of those of the third order, obtained when there are three independent circuits, as Hansen points out, and this is a very great simplification.

It is generally admitted that the resistance of the armature circuit (unlike that of the field circuit) plays no essential part in the phenomena. Accordingly this

is assumed zero throughout for the sake of simplicity.

If

$$K = \frac{\text{Total armature condrs.} \times \text{speed (r.p.s.)}}{\rho \times 10^8} = \frac{Z\,S}{\rho \times 10^8}$$

Let ρ be the reluctance of the main magnetic circuit (assumed constant).

i_3 the shunt current

i_1 the armature current

n_3 the number of shunt turns

n_1 the number of series field turns

$K\,(n_3\,i_3 - n_1\,i_1)$ the e. m. f. generated in the armature.

r_0 the shunt field resistance per turn (so that $r_0\,n_3$ is the resistance of the shunt)

$a_3 = n_3\,i_3$

$a_1 = n_1\,i_1$

L_{33} coefficient of self induction of the field circuit due to the main flux

L_{31} coefficient of self induction of the field circuit due to the field leakage flux

L_{11} coefficient of self induction of series field circuit due to the main flux

Lo coefficient of self induction of the series field circuit due to the field leakage flux

M coefficient of mutual induction between field and armature circuit

e_1 exciter volts (assumed constant).

Then the main flux will be

$$\phi = \frac{n_3\,i_3}{\rho} - \frac{n_1\,i_1}{\rho} = \phi_3 - \phi_1 \text{ say} \tag{1}$$

The e. m. f. induced in the shunt circuit.

$$n_3\frac{d\,\phi}{d\,t} = n_3\frac{d}{d\,t}\left(\frac{n_3\,i_3}{\rho} - \frac{n_1\,i_1}{\rho}\right) = L_{33}\frac{d\,i_3}{d\,t} - M\frac{d\,i_1}{d\,t} \tag{2}$$

Thus

$$L_{33} = \frac{n_3^2}{\rho} \text{ and } M = \frac{n_3\,n_1}{\rho}$$

The e. m. f. induced in the series field circuit will similarly be

$$n_1\frac{d\,\phi}{d\,t} = n_1\frac{d}{d\,t}\left(\frac{n_3\,i_3}{\rho} - \frac{n_1\,i_1}{\rho}\right) \tag{3}$$

$$= M\frac{d\,i_3}{d\,t} - L_{11}\frac{d\,i_1}{d\,t} \tag{4}$$

Thus

$$L_{11} = \frac{n_1^2}{\rho}$$

Let us assume first of all that the magnetic leakage is zero then the differential equation of the main circuit is—

$$\frac{n_1}{\rho}\,\frac{d\,(a_3 - a_1)}{d\,t} + K\,(a_3 - a_1) = E \tag{5}$$

and that of the field circuit

$$\frac{n_3}{\rho}\,\frac{d\,(a_3 - a_1)}{d\,t} + r_0\,a_3 = e_1 + E\,. \tag{6}$$

In symbolic form this may be written

$$\left(\frac{n_1}{\rho}\,D + K\right)a_3 - \left(\frac{n_1}{\rho}\,D + K\right)a_1 = E \tag{7}$$

$$\left(\frac{n_3}{\rho}\,D + r_0\right)a_3 - \frac{n_3}{\rho}\,D\,a_1 = e_1 + E \tag{8}$$

Eliminating a_1

$$\frac{n_3}{\rho}\,D\left(\frac{n_1}{\rho}\,D + K\right)a_3 - \left(\frac{n_3}{\rho}\,D + r_0\right)\left(\frac{n_1}{\rho}\,D + K\right)a_3$$

$$= -\,K\,(e_1 + E) \tag{9}$$

or on cancelation

$$\left(\frac{n_1}{\rho}\,D + K\right)a_3 = \frac{K\,(e_1 + E)}{r_0} \tag{10}$$

the solution of which is

$$a_3 = C_3\,\epsilon^{-\frac{K\rho t}{n_1}} + \frac{e_1 + E}{r_0} \tag{11}$$

Eliminating a_3

$$-\left(\frac{n_3}{\rho}\,D + r_0\right)\left(\frac{n_1}{\rho}\,D + K\right) + \frac{n_3}{\rho}\,D\left(\frac{n_1}{\rho}\,D + K\right)$$

$$= r_0\,E - K\,(e_1 + E) \tag{12}$$

or on cancelation

$$\left(\frac{n_1}{\rho}\,D + K\right)a_1 = \frac{K\,(e_1 + E)}{r_0} - E. \tag{13}$$

The solution of which is

$$a_1 = C_1\,\epsilon^{-\frac{K\rho t}{n_1}} - \frac{E}{K} + \frac{e_1 + E}{r_0} \tag{14}$$

Clearly the two constants C_3 and C_1 are not independent. To determine them the most convenient way is to assume

$$a_3 - a_1 = 0 \text{ when } t = 0$$

This gives

$$C_3 - C_1 = -\frac{E}{K} \text{ and } \frac{d\,(a_3 - a_1)}{d\,t} = -\frac{K\,\rho}{n_1}\,(C_3 - C_1)$$

$$= \frac{E\,\rho}{n_1}$$

Substituting in equation (6)

$$\frac{n_3}{n_1}\,E + r_0\,C_3 + e_1 + E = e_1 + E$$

$$C_3 = -\frac{n_3}{n_1}\,\frac{E}{r_0} \tag{15}$$

$$C_1 = \frac{E}{K} - \frac{n_3}{n_1}\frac{E}{r_0} \tag{16}$$

Thus finally

$$a_3 = \frac{e_1}{r_0} + \frac{E}{r_0}\left(1 - \frac{n_3}{n_1}\epsilon^{-\frac{Kpt}{n_1}}\right) \tag{17}$$

$$a_1 = \frac{e_1}{r_0} + \frac{E}{r_0}\left(1 - \frac{n_3}{n_1}\epsilon^{-\frac{Kpt}{n_1}}\right) = \frac{E}{K}\left(\epsilon^{-\frac{Kpt}{n_1}} - 1\right). \tag{18}$$

$$a_3 - a_1 = \frac{E}{K}\left(1 - \epsilon^{-\frac{Kpt}{n_1}}\right). \tag{19}$$

The steady condition being as one would expect that

$$K(a_3 - a_1) = E \tag{20}$$

$$a_3 = \frac{e_1 + E}{r_0} \tag{21}$$

It will be worth while to devote some considerable attention to working out a clear physical interpretation of these equations.

In the first place all terms of second order cancel the equations reducing to the first order. Consequently all the currents (or ampere turns) a_3, a_1, and $a_3 - a_1$ vary together *at exactly the same rate* which is measured by

$$\epsilon^{-\frac{Kpt}{n_1}}$$

This corresponds physically to the fact that there is only one flux, the main flux of the machine which can clearly only vary at one rate. Thus so long as the magnetic leakage is zero, the "sluggishness" of one coil (which means essentially that the current in it rises more slowly than in the others) is impossible.

Substituting the above values in the equations themselves we get

$$\frac{n_1}{\rho}\frac{d(a_3 - a_1)}{dt} + K(a_3 - a_1) = E \tag{22}$$

$$E\epsilon^{-\frac{Kpt}{n_1}} + E\left(1 - \epsilon^{-\frac{Kpt}{n_1}}\right) = E \tag{23}$$

This shows that starting from $t = 0$ when the flux is zero. *The rate of increase of the flux is such that that portion of the terminal E M F which cannot be supplied by the rotation voltage is supplied by the transformer voltage induced by the flux variation.*

$$\frac{n_3}{\rho}\frac{d(a_3 - a_1)}{dt} + r_0 a_3 = e_1 + E \tag{24}$$

$$\frac{n_3}{n_1}E\epsilon^{-\frac{Kpt}{n_1}} + e_1 + E - \frac{n_3}{n_1}E\epsilon^{-\frac{Kpt}{n_1}} = e_1 + E. \tag{25}$$

The field current may be considered as made up of two components, the steady component $\frac{e_1 + E}{r_0}$ *regarded as always flowing and whose resistance drop balances the applied E M F and a transient component flowing in the opposite direction, whose resistance drop balances the transformer voltage induced by the variation of the main field.*

The series field ampere turns (a_1) *and therefore the armature current takes up such a value as to balance the field ampere turns* (a_3) *except for such portion as is needed to produce the rotation voltage necessary to balance the resultant of the terminal voltage and the induced or transformer volts.*

The short-circuit conditions ($E = 0$) are worth a brief examination. We then have

$$a_3 = a_1 = \frac{e_1}{r_0}, a_3 - a_1 = 0 \text{ at every instant.}$$

That is to say the flux is infinitesimally small being just enough to urge a sufficient current round the (zero) resistance of the short-circuited circuit to produce a number of ampere turns in the series field coil exactly equal and opposite to the shunt field ampere turns. Since there is no field at any instant, there is no stored magnetic energy and no reason why the currents should not instantaneously assume their final values.

If at the moment of short circuit the flux has a particular value $\phi = \frac{A}{\rho}$ and the ampere turns therefore a particular value A, then since at the instant of short circuit the only e. m. fs. in the armature circuit are—

1. The rotation voltage.
2. The transformer voltage due to the decrement of the flux. At the moment of short circuit the flux still retains the value $\phi = \frac{A}{p}$ and hence its rate of change must instantly assume a value such that the transformer voltage can balance the rotation voltage according to the equation.

$$\frac{n_1}{\rho}\frac{d(a_3 - a_1)}{dt} + K(a_3 - a_1) = 0$$

or

$$\frac{d(a_3 - a_1)}{dt} = -\frac{K\rho}{n_1}(a_3 - a_1)$$

of which the only solution is

$$a_3 - a_1 = (C_3 - C_1)\epsilon^{-\frac{Kpt}{n_1}}$$

This is the physical explanation of the value of the exponent of ϵ and why it does not contain the shunt resistance. The actual energy of the field is absorbed

by the shunt resistance at the rate determined as above.

$$\frac{K\rho}{n_1} = \frac{Z\,s}{n_1} \times 10^{-8}$$

This becomes greater and the decrement therefore faster, the fewer the turns n, which for a given flux and therefore a given $Z.s$ means the less the reluctance. Hence a low field reluctance means a machine capable of rapid adjustment. The power represented by the product of the rotation volts and armature current goes to maintain the flux by transformer action and is ultimately dissipated in resistance or pumped into the exciter.

Self Induction in the Field Circuit

The Cause of Sluggishness. Suppose there is a leakage self induction L_{31} in the field circuit $\left(\text{so that } \frac{L_{31}}{n_3}\frac{d\,a_3}{d\,t} = L_{31}\frac{d\,i_3}{d\,t}\right)$.

The equations will now be

$$\frac{n_1}{p}\frac{d\,(a_3 - a_1)}{d\,t} + K\,(a_3 - a_1) = E \tag{28}$$

$$\frac{n_3}{p}\frac{d\,(a_3 - a_1)}{d\,t} + r_0\,a_3 + \frac{L_{31}}{n_3}\frac{d\,a_3}{d\,t} = e_1 + E. \tag{29}$$

As before the variables are separable and equation (5) may be directly solved for $(a_3 - a_1)$ giving

$$a_3 - a_1 = (C_3 - C_1)\,\epsilon^{-\frac{K\rho t}{n_1}} + \frac{E}{K} \tag{30}$$

and

$$C_3 - C_1 = -\frac{E}{K} \text{ if } a_3 - a_1 = 0 \text{ when } t = 0$$

$$\frac{L_{31}}{n_3}\frac{d\,a_3}{d\,t} + r_0\,a_3 = e_1 + E - \frac{n_3}{n_1}E\,\epsilon^{-\frac{K\rho t}{n_1}} \tag{31}$$

General solution

$$a_3 = C_3\,\epsilon^{-\frac{r_0 n_3 t}{L_{31}}} \tag{32}$$

Particular integral

$$a_3 = \frac{e_1 + E}{r_0} - \frac{\frac{n_3}{n_1}E}{r_0 - \frac{L_{31}}{n_3}\frac{K\rho}{n_1}}\,\epsilon^{-\frac{K\rho t}{n_1}} \tag{33}$$

Finally

$$a_3 = C_3\,\epsilon^{-\frac{r_0 n_3 t}{L_{31}}} + \frac{e_1 + E}{r_0} - \frac{\frac{n_3}{n_1}E}{r_0 - \frac{i_{31}}{n_3}\frac{K\rho}{n_1}}\,\epsilon^{-\frac{K\rho t}{n_1}} \tag{34}$$

$$a_3 - a_1 = \frac{E}{K}\left(1 - \epsilon^{-\frac{K\rho t}{n_1}}\right) \text{ or } a_1 = a_3 - \frac{E}{K}\left(1 - \epsilon^{-\frac{K\rho t}{n_1}}\right)$$

C_3 may be independently determined by assuming that for instance when $t = 0$; $a_3 = \frac{e_1 + E}{r_0}$ when we have in general

$$a_3 = \frac{e_1 + E}{r_0} - \frac{\frac{n_3}{n_1}E}{r_0 - \frac{L_{31}}{n_3}\frac{K\rho}{n_1}}\left(\epsilon^{-\frac{K\rho t}{n_1}} - \epsilon^{-\frac{r_0 n_3 t}{L_{31}}}\right)$$

Let us restate the equations (5) and (6) above in terms of the currents. First substitute $i_{33} = \frac{n_3}{n_1}i_3$.

We obtain

$$L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + K\,n_1\,(i_{33} - i_1) = E \tag{36}$$

$$\frac{n_3}{n_1}L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + r_0\,n_1\,(i_{33}) = e_1 + E$$

$$L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + r_1\,n_1\,i_{33} = \frac{n_1}{n_3}(e_1 + E) \text{ if } r_1 = -\frac{n_1}{n_3}r_0. \tag{37}$$

By the general theory of linear differential equations the general solution of these equations is the general solution of the equations in which the terms independent of i_1 and i_{33} are put equal to zero as below

$$L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + K\,n_1\,(i_{33} - i_1) = 0 \tag{38}$$

$$L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + r_1\,n_1\,i_{33} = 0 \tag{39}$$

together with a particular integral usually giving the steady state of the original equations.

From the physical point of view, this means that the steady state may be regarded as always in existence, the transient effects being superadded at the moment of switching on or off and dying to zero after a few seconds.

If therefore these transient effects are studied by *themselves* it is possible to obtain a better physical grasp of them than if the study is complicated by a superposed steady state. Equations (38), (39) represent the case in which *both* armature and shunt field are short-circuited. We suppose that at $t = 0$ a certain initial value of $n_1\,(i_{33} - i_1) = A$ say exists and study in what manners this dies way to zero. Clearly if the shunt coil is short-circuited it cannot matter how many turns it contains, and hence the substitution $i_{33} = \frac{n_3}{n_1}i_3$ has been introduced equivalent to assuming that it contains the same number of turns, n_1 as the series coil.

Solving the above equations we have from (38)

$$i_{33} - i_1 = A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

$$\frac{d\,(i_{33} - i_1)}{d\,t} = -\frac{K\,n_1}{L_{11}}\,A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

From (39)

$$r_1\,n_1\,i_{33} = L_{11}\,\frac{K\,n_1}{L_{11}}\,A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

$$i_{33} = \frac{K}{r_1}\,A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

$$i_1 = \left(\frac{K}{r_1} - 1\right) A\,\epsilon^{-\frac{Kn_1t}{L_{11}}} \tag{40}$$

It has before been pointed out that with a short-circuited armature the rate of decrement of the flux must be such as to induce an e. m. f. in the series coil equal and opposite to the rotation volts.

If however the armature circuit be opened, the field circuit has a natural rate of decrement of its own, determined by solving equation (39) for the case when $i_1 = 0$.

This gives us

$$i_{33} = C\,\epsilon^{-\frac{r_1\,n_1\,t}{L_{11}}} \tag{41}$$

Now if the natural rate of decrement of the field is equal to that determined by short-circuiting the armature then

$$\frac{r_1\,n_1}{L_{11}} = \frac{K\,n_1}{L_{11}} \text{ or } \frac{K}{r_1} = 1 \tag{42}$$

Under these circumstances, as the above equation (40) shows $i_1 = 0$ even though the armature be short-circuited. In other words the natural rate of decrement of the field produces an e. m. f. in the series coil, which just balances the rotation volts without the need for any armature current to forcibly modify the rate of decrement.

From the point of view of power and energy the natural rate of decrement of the flux is such that the energy released by the falling of the flux exactly supplies that converted into heat by the field resistance.

If the natural rate of decrement of the field is *less* than that determined by the armature we have—

$$\frac{K}{r_1} \pm 1$$

Total energy stored in flux

$$w = 1/2\,L_{11}\,(i_{33} - i_1)^2$$

Power liberated by its decrease

$$\frac{d\,w}{d\,t} = (i_{33} - i_1)\,L_{11}\,\frac{d\,(i_{33} - i_1)}{d\,t} = -\,K\,n_1\,A^2\,\epsilon^{-\frac{2Kn_1t}{L_{11}}}$$

$I^2\,R$ loss in field

$$i_{33}^2\,r_1\,n_1 = \frac{K^2\,n_1}{r_1}\,A^2\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

If $\frac{K}{r_1} > 1$ clearly the $I^2\,r$ loss in the field is greater than the power liberated by the flux. Some therefore must be supplied by the rotation volts. This is confirmed by the fact that the armature current

$$i_1 = \left(\frac{K}{r_1} - 1\right) A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

and the rotation volts

$$K\,n_1\,(i_{33} - i_1) = K\,n_1\,A\,\epsilon^{-\frac{dKn_1t}{L_{11}}} \tag{43}$$

are in the same direction, or the rotation volts generates power. If the $I^2\,R$ loss in the field is *less* than the power liberated by the flux, some of this power must be devoted to motoring the machine. In this case $\frac{K}{r_1} < 1$ and i_1 reverses its sign flowing *against* the rotation volts. Thus the machine is a motor.

Returning to equations (5) and (6)

The steady state solutions are—

$$a_3 - a_1 = \frac{E}{K} \text{ or } n_1\,(i_{33} - i_1) = \frac{E}{K} \tag{44}$$

$$a_3 = \frac{e_1 + E}{r_0} \text{ or } n_1\,i_{33} = \frac{n_1}{n_3}\,\frac{e_1 + E}{r_0} \tag{45}$$

$$a_1 = \frac{e_1 + E}{r_0} - \frac{E}{K} \text{ or } n_1\,i_1 = \frac{n_1}{n_3}\,\frac{e_1 + E}{r_0} - \frac{E}{K}\,. \tag{46}$$

these being arrived at on the assumption that when $t = 0$

$$a_3 - a_1 = 0$$

Now

$$a_3 - a_1 = n_1\,(i_{33} - i_1)$$

If therefore we put

$$n_1\,A = -\frac{E}{K}$$

we may suppose that the steady state solutions exist at all instants (including $t = 0$) but that superposed on these there is a transient solution whose value is equal and opposite to that of the steady solution when $t = 0$. In the resultant therefore, we have $a_3 - a_1 = 0$ at $t = 0$ gradually approximating to the steady state as the transients die away.

The value of A given above gives us

$$a_3 = -\frac{E}{r_1}\,\epsilon^{-\frac{Kn_1t}{L_{11}}} \tag{47}$$

on which we superpose a "steady" solution $\frac{e_1 + E}{r_0}$ to get

$$a_3 = \frac{e_1}{r_0} + \frac{E}{r_0}\left(1 - \frac{n_3}{n_1}\,\epsilon^{-\frac{Kn_1t}{L_{11}}}\right) \text{ remembering}$$

$$\text{that } r_1 = \frac{n_1}{n_3}\,r_0 \tag{48}$$

and

$$a_1 = -\left(\frac{K}{r_1} - 1\right)\frac{E}{K}\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

on which we superpose

$$\frac{e_1 + E}{r_0} + \frac{E}{K}$$

$$a_1 = \frac{e_1}{r_0} + \frac{E}{r_0}\left(1 - \frac{n_3}{n_1}\,\epsilon^{-\frac{Kn_1t}{L_{11}}}\right) + \frac{E}{K}\left(\epsilon^{-\frac{Kn_1t}{L_{11}}} - 1\right)$$

$$L_{11} = \frac{n_1^2}{\rho} \text{ so that } \frac{K\,n_1\,t}{L_{11}} = \frac{K\,\rho\,t}{n_1}$$

thus making the identity of the two formulas obvious.

From the point now reached it is not difficult to understand why the presence of leakage self induction in the shunt coil, should permit the current in it to have a different rate of decrement from that of the main flux.

Clearly it is only the main flux of the machine which is determined by the condition that the transformer e. m. f. in the series coil due to its decrement shall balance the rotation voltage on short circuit. If there is an additional flux which produces no rotation voltage —namely, the flux of leakage self induction this condition does not apply to it, and there is no reason why it should not be determined by the further condition (usual in inductive circuits) that the energy released by the decrement of the whole flux (main flux and leakage flux) shall be equal to that transformed into heat by the resistance of the circuit.

Self-Induction in the Armature Circuit

The Cause of Oscillation. The equations now become (ignoring the steady state as before)

$$\frac{n_1^2}{\rho}\,\frac{d\,(i_{33} - i_1)}{d\,t} + K\,n_1\,(i_{33} - i_1) - L_0\,\frac{d\,i_1}{d\,t} = 0 \tag{51}$$

$$\frac{n_1^2}{\rho}\,\frac{d\,(i_{33} - i_1)}{d\,t} + r_1\,n_1\,i_{33} = 0 \tag{52}$$

or in symbolic form

$$\left(\frac{n_1^2}{\rho}\,D + K\,n_1\right)i_{33} - \left\{\left(\frac{n_1^2}{\rho} + L_0\right)D + K\,n_1\right\}\,i_1 = 0$$

$$\left(\frac{n_1^2}{\rho}\,D + r_1\,n_1\right)i_{33} - \frac{n_1^2}{\rho}\,D\,i_1 = 0$$

$$\left\{L_0\,\frac{n_1^2}{\rho}\,D^2 + r_1\,n_1\left(L_0 + \frac{n_1^2}{\rho}\right)D + r_1\,n_1\,K\,n_1\right\}\,i_{33} = 0$$

The solution of this is

$$i_{33} = C_1\,\epsilon^{p_1t} + C_2\,\epsilon^{p_2t} \text{ with}$$

$$p = \frac{-r_1\,n_1\left(L_0 + \frac{n_1^2}{\rho}\right) \pm \sqrt{n_1^2\,r_1^2\left(L_0 + \frac{n_1^2}{\rho}\right)^2 - 4r_1Kn_1^2L_0\frac{n_1^2}{\rho}}}{2\,L_0\,\frac{n_1^2}{\rho}}$$

or

$$p = \frac{-\,r_1\,n_1\left(L_0 + \frac{n_1^2}{\rho}\right)}{2\,L_0\,\frac{n_1^2}{\rho}} \times$$

$$\left\{-1 \pm \sqrt{1 - \frac{K}{r_1}\,\frac{4\,L_0\,\frac{n_1^2}{\rho}}{\left(L_0 + \frac{n_1^2}{\rho}\right)^2}}\right\}$$

This equation is fully discussed above.

Returning to the differential equation—

$$\frac{n_1^2}{\rho}\,\frac{d\,(i_{33} - i_1)}{d\,t} + K\,n_1\,(i_{33} - i_1) - L_0\,\frac{d\,i_1}{d\,t} = 0$$

Consider three successive instants assuming that the solution is oscillatory.

$$i_{33} - i_1 \text{ a maximum } \frac{d\,(i_{33} - i_1)}{d\,t} = 0. \tag{1}$$

At this instant the transformer voltage due to the main flux is zero, but the flux is not. Hence the rotation voltage exists and in fact is a maximum. The current i_1 is not zero, and hence the rotation voltage generates power which stores energy in the external self induction.

$$i_1 \text{ a maximum } \frac{d\,i_1}{d\,t} = 0. \tag{2}$$

At this instant the rotation voltage is balanced by the transformer voltage due to the variation of the main field, and since by hypothesis $\frac{K}{r_1} > 1$ it is still generating power to increase the field energy.

$$i_{33} - i_1 = 0 \text{ rotation voltage zero.} \tag{3}$$

At this instant an exchange of energy takes place between the reactance and the main field that in the reactance say, falling off, and that in the main field increasing towards a maximum, when the cycle repeats.

Briefly the rotation voltage supplies energy to the field and the external reactance which is partly stored and partly consumed by the field resistance and the

energy oscillates between these two. If so large a proportion of the energy of the circuit is contained in the external reactance that any variations in the field energy produce but a slight effect on it oscillations cannot occur.

The reason why in a shunt machine the self induction should be outside the shunt tappings rather than inside, as when it is formed by the cross flux of the armature, can be explained without differential equations.

If the shunt coil be (1) connected directly across the arc, all variations in the latter directly affect the shunt coil.

Whereas (2) if a self induction is interposed between them the shunt is shielded from these oscillations.

Suppose for instance, the voltage across the arc be represented by $E_0 + E_1 \sin p\,t$ then in case (1) a partly alternating e. m. f. is impressed on the shunt coil giving rise to an alternating component of current therein, *which may lag approximately 90 deg. behind* $E_1 \sin p\,t$. and produce a correspondingly lagging component of rotation voltage which will by no means balance $E_1 \sin p\,t$.

Thus unless the variations are so slow and the resistance of the shunt circuit so high that this lag does not exist, connecting the shunt circuit across the arc does *not* permit it to follow rapid variations of the latter. It is better therefore, to rely on the self induction to take up very rapid variations and to shield the shunt coil from them.

But the less magnetic energy stored in the field the easier is it for the machine to follow rapid variations, and it should be remembered that the variations in the arc are probably not as rapid as those of an ordinary alternating current.

Discussion

For discussion of this paper see page 688.

A New System for D-C. Arc Welding

BY S. R. BERGMAN*

Associate, A. I. E. E.

Synopsis.—*A short description is first given of a d-c. generator having characteristics suitable for arc welding. Curves showing performance under steady conditions of load are shown. It is pointed out that for quick changes of load transients appear detrimental to arc welding. In order to eliminate these transients, an external inductive device called a transforming reactor is introduced in the arc welding circuit. This transforming reactor has a dual purpose. First, it depresses the transients as is shown by oscillograms, resulting in quick recovery of the voltage and a small short-circuit current. Thus it is shown that under arc welding load the generator performs in accordance with the curves corresponding to a steady load. Second, the transforming reactor introduces reactance in the arc circuit which, experience has shown, is desirable in order to make the arc stable and therefore easy to handle.*

The system for d-c. arc welding which is described consists, therefore, of two units,—a generator and a transforming reactor, forming a combination capable of producing uniform welds of high strength.

ARC welding, particularly when applied to ferrous metals, has, during the last decade, developed into an art of great importance in industry. As this art has developed, the requirements for a proper power supply have been studied with the view of obtaining uniform and great strength of the welds as well as securing ease in welding. To meet this demand, a motor-generator set has been developed and, in addition, an auxiliary device of inductive character to aid the generator in producing ideal welding conditions.

The requirements for the generator may be briefly stated as follows:

1. The open-circuit voltage was chosen to range from 70 to 90 volts. Experiments have shown that it is desirable to have this voltage above 70. With less voltage available, the arc has a tendency to break as it is elongated, due to unsteady welding, or the blowing from influences of external magnetic fields. This voltage is also required in order to break through scale on the iron. Tests have also shown that there is no advantage derived from a voltage higher than 80 volts, although a margin of 10 volts above this is advantageous to compensate for the voltage drop in extra long leads.

2. The arc voltage runs from 14 to 25 volts depending upon the strength of the welding current. It is thus required that the voltage of the generator should inherently drop from an open-circuit voltage of 70 to 80 volts to an arc voltage of 14 to 25 volts. The regulation of the generator is shown in Fig. 1, giving approximately a straight-line relation between voltage and current.

The arrangement of the generator is shown in Fig. 2. The principle involved is that of a dual magnetic circuit. The armature is wound for two poles and the field contains four poles. In general, the armature should be wound for half the number of poles contained in the field. In d-c. machines of standard design, adjacent poles have opposite polarity, but in this machine, the poles are paired in groups of the same polarity. Thus, there is a group of two north poles followed by a group of two south poles, etc.

In order to establish a working theory, assume that the flux distribution is such as shown in Fig. 2. There exist two fluxes ϕ_m and ϕ_c displaced 90 electrical deg. from each other. The flux ϕ_m will be designated the main flux and the flux ϕ_c the cross-flux.

The load current of the armature is taken from the two brushes A and B, placed in neutrals located between

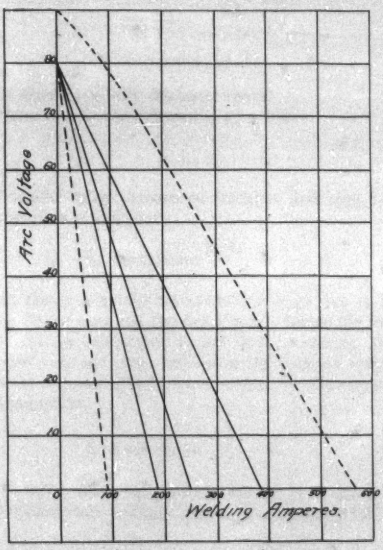

FIG. 1—CHARACTERISTICS OF GENERATOR

poles of opposite polarity. In addition, there is an exciting brush, E. This machine is self-excited, the excitation being taken from brushes A and E, as shown in Fig. 2. The main magnetic circuit is so designed that magnetic saturation exists, whereas the cross-magnetic circuit is designed without any saturation. As soon as the armature is loaded, there is set up an armature reaction, R, which may be resolved in two

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components; one, $R\,M$, in the direction of the main flux and the other, $R\,C$, in opposition to the cross-flux. Since the main magnetic circuit is saturated, the component, $R\,M$, can not force any more flux through this circuit and the main flux remains practically constant.

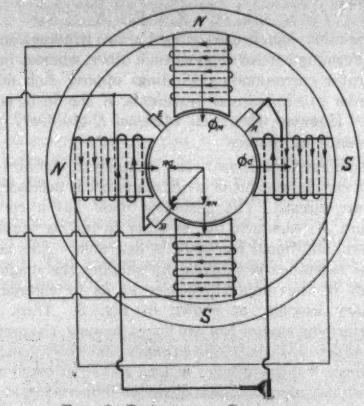

Fig. 2—Principle of Generator

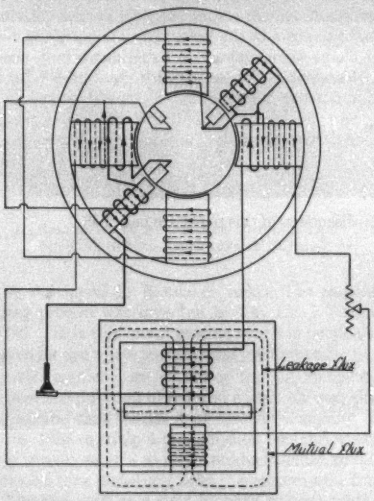

Fig. 3—Generator and Transforming Reactor

The component, $R\,C$, however, blows out the cross-flux which thus decreases as the load increases. The machine voltage $A\,B$ is the algebraic sum of $A\,E$ and $E\,B$. Thus, $A\,B = A\,E + E\,B$. The voltage $A\,E$ is induced by the main flux and remains practically constant since the main flux is constant, and the speed is constant. The voltage $E\,B$, however, decreases with the load since the cross-flux decreases as described. As a matter of fact, the cross-flux passes through zero, and is finally reversed.

If it is desired to weld with smaller currents, a series winding is placed on the cross-poles (Fig. 2) supporting the armature reaction. Since it is desirable to be able to obtain a large number of different welding currents, the series winding is provided with taps. To secure a close adjustment of the welding current, a field rheostat is inserted in the cross circuit. To obtain good commutation, commutating poles are added.

A set of welding currents may now be obtained as shown in Fig. 1, which particular curves apply to a machine rated at 300 amperes. The rated current range of this machine is from 90 to 375 amperes although a minimum current of 75 amperes, and a maximum current of 450 amperes is obtainable.

In actual welding the current is never constant, because the circuit is frequently short-circuited by small globules of metal and because the electrode is not fed at exactly the proper rate, particularly in hand welding.

Fig. 4

Under such conditions, transient currents and potentials appear in the generator and the regulation deviates from the curves shown in Fig. 1, which correspond to static conditions. In striking the arc, the current overshoots its steady value, and in breaking the arc, a certain time interval is required for the voltage to return to its normal open-circuit value. It is desirable that the short-circuited current should not be excessive, and it is also desirable to have the voltage build up rapidly.

In order to cope with these transient conditions, a special inductive device which may be called a transforming reactor was developed as shown in Fig. 3. This device contains two windings, a primary winding connected in series with the arc, and a secondary winding connected in series with the shunt field of the cross-magnetic circuit. These two windings are connected in such a way that the direct current flowing through them will magnetize in the same direction.

The function of the transforming reactor is to provide a magnetic coupling external to the generator between the arc circuit and the field circuit, which will be opposite in effect to that between the two circuits within the generator. Suppose that the arc is suddenly

broken; then the ampere turns of the arc circuit, both in the generator and in the transforming reactor, suddenly disappear and since the secondary winding attempts to maintain the mutual flux, the secondary current has a tendency to increase, counteracting the tendency of the generator magnetic circuit to make it decrease. Thus, the transforming reactor tends to keep the cross-shunt field current constant, regardless of changes in the arc-current. The result of this is to make the voltage build up more rapidly to its normal value when the arc is broken and to limit the peak value of current when a short circuit is made.

To illustrate the principle of this generator in conjunction with the transforming reactor, an oscillogram is shown in Fig. 4, showing what happens when the arc is short-circuited and when the short circuit immediately thereupon is opened. The steady value of the short-circuited current is I, and the transient value, i, is quite small. When the short circuit is removed, it will be noted that the voltage of the generator nearly instantaneously builds up to the open-circuit voltage, V.

This oscillogram proves that the transforming reactor fulfills its purpose in practically eliminating the transients, making the generator, during sudden changes of arc current, follow the regulation curves obtained under steady conditions.

This device should, in order to function ideally, be designed in such a manner that the mutual induction between its two windings is the same as the mutual induction existing between the cross-magnetic shunt winding and the armature winding with its series winding.

Let N_1 = total effective number of turns on the cross axis of the generator carrying the welding current.

N_2 = total number of cross-shunt field turns.

n_1 = number of turns on the transforming reactor carrying the welding current.

n_2 = number of turns on the transforming reactor in series with the cross-shunt field.

F = cross-flux per ampere turn of the generator.

f = mutual flux per ampere turn of the reactor.

The design of the transforming-reactor should then satisfy:

$$f \times n_1 \times n_2 = - K \times F \times N_1 \times N_2$$

The minus sign is used, since the two windings on the transforming reactor support each other, whereas in the generator corresponding windings oppose each other. In order to eliminate all transients, K should be equal to 1. However, fractional values of K give very good transient performance.

Experience has shown that in order to stabilize the arc, a certain amount of series reactance is necessary in the arc circuit. The generator itself has a certain amount of reactance, but in order to further stabilize the arc, additional reactance is desirable. This additional reactance is supplied by shaping the magnetic circuit of the inductive device so as to enlarge its primary leakage, as shown in Fig. 3. Thus, the transforming reactor has two fluxes, namely, the mutual flux which links both the primary and the secondary windings, and the primary leakage flux interlinked with the primary alone. These fluxes are indicated in Fig. 3.

This generator is self-excited, requiring no exciter. It has inherent regulation suitable for arc welding. The transforming reactor serves the dual purpose of eliminating transients in the generator and supplying reactance to the welding circuit. Experiments have shown that this combination welds with great ease both for hand and automatic welding. It lays down the metal with ease and accuracy under all conditions met in welding, producing welds of uniformity and strength.

Discussion

For discussion of this paper see page 688.

Resistance Welding of Motor Frames

BY MALCOLM THOMSON*

Associate, A. I. E. E.

Synopsis.—Cost reduction calls for a continual revision of methods. The making of motor frames using rolled plate in place of castings formerly used, the object being to reduce cost is one such revision. The method of manufacture is briefly outlined including the rolling, welding, sizing, and other operations. The electric resistance welding method is emphasized and results of electrical tests on equipment are outlined. The oscillograph records give the character of a resistance flash weld in some detail.

The conclusion is reached that not only is there a saving in cost of frames, but also a reduction in machining costs. The result is a better product at less cost.

* * * * *

THE continual demands of modern industry have caused and are causing a continual revision of methods employed. Perhaps no more interesting application of electricity to shop manufacture occurs than in its use in various forms of electric welding. The methods in most common use are the arc methods, the carbon arc and the metallic arc, and the resistance method. The carbon arc process was invented by DeMeritens in France in 1881 and the metallic arc process by Slavianoff in Russia in 1890. The resistance welding process was invented by Dr. Elihu Thomson in 1886. It is this latter process which is particularly adapted for use where there is mass production.

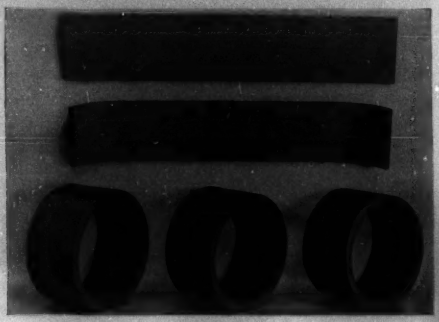

FIG. 1—ROLLED-PLATE AUTOMOTIVE-MOTOR FRAME. VIEWS SHOWING SUCCESSIVE STAGES IN FORMING

In the manufacture of motor frames, competition brings in the question of reduced costs. It was, therefore, natural that the General Electric Company took steps in 1928 to equip a modern shop for fabrication of motor frames, as well as other products which could be fabricated from structural steel. A section of a building formerly used as a steel foundry was set aside for this purpose. The manufacture of rings for frames of d-c. motors is briefly outlined as follows:

The steel for the rings is ordered from the mills in carload lots of the proper width and length, the length being figured so as to include the necessary amount for welding and sizing operations. The steel which has been found most suitable for magnet frames is a basic open hearth process manufacture of low carbon content. The slabs of steel are unloaded at one end of the department, and are of various sizes in accordance with different sizes of magnet frames. The slabs are passed to a 1,200-ton hydraulic press which crimps or curls the ends to avoid flats and waste of stock at portions which cannot be rolled.

A view showing the steel as purchased and also illustrating the various steps in fabrication is given in Fig. 1. A typical ring in bending rolls is shown in Fig. 2.

Attention is called to the vertical plate which insures accurate alinement of the ends of the ring during rolling process. The rolls have electric motor drive for raising and lowering rolls, as well as motor drive for rolls. The end section is pivoted and controlled by an air cylinder which is shown in the figure. This feature is for quick removal of rings.

FIG. 2—ROLLING MACHINE FOR FABRICATING AUTOMOTIVE-MOTOR MAGNET FRAMES

From the rolls the rings pass by a conveyor which has

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considerable storage capacity to a specially designed resistance flash welder of the Thomson-Gibb make. Fig. 3 shows this welder in action. The rings are held with powerful clamps actuated by hydraulic cylinders against special aluminum bronze die blocks. The

Fig. 3—Specially Designed Thomson-Gibb Resistance Flash Welder in Operation

operator separates the ends of the rings slightly, then starts slowly feeding the jaws of the machine together with a hand valve control of a hydraulic push up cylinder.

The current is turned on by a push button work-

Fig. 4—Electric Hoist Removing Ring from Welder

ing a small contactor which in turn controls a large contactor, the latter closing the connection of the primary of the transformer to the power lines. As the ends of the ring meet, at first feeble then violent flashing occurs. A predetermined amount is flashed off, averaging slightly less than an inch total. The flashing heats the abutting faces to the melting point of steel and the area immediately behind to a welding heat mainly by conduction, but partly by resistance. At the end of the flashing period, the full pressure of the hydraulic ram or about 100,000 lb. is applied, suddenly forcing the molten faces together. The result is a perfect and homogeneous weld, all burnt or oxidized metal being forced out. This welder while rated at 12 sq. in. maximum, cross section is used on sections up to 18 sq. in. The welder has a 750-kva. water-cooled transformer. The open-circuit voltage at welding jaws is 12 volts.

The demands on welding transformers call for different features of design other than those used with the usual types. The service is intermittent but large amounts of current are necessary, particularly at the final or upset period of the welding cycle. Due to the severe nature of the service, accessibility is an important feature, hence the coils are usually assembled on a straight core section, then a "U" shaped section is

Fig. 5—Arc Welding, Building 30, River Works

placed in position and fastened to the straight section with bolts and straps. This allows for more easy repair in case of trouble. On 60-cycle supply, the core is worked at a flux density of 60,000 to 80,000 lines per sq. in.

The secondary is usually of cast copper with cooling tubes for water cast in and of single turn type. In larger transformers usually several cast copper sections are connected in parallel. The ends of the copper secondary are machined, one end being fastened solidly to the fixed platen. The other end is bolted to a flexible lead made up of many thin sections of copper strip, this lead being connected to movable platen. The welding dies are located on top of the platens and are of hard-rolled copper or special wear-resisting material, depending on the nature of service. The work is very solidly clamped to the die blocks because of the push-up pressure required. The cast copper is worked at a current density of 1,800 to 2,200 amperes per sq. in., the flexible leads at 2,000 to 2,400 amperes and the joints at 400 to 600 amperes per sq. in. The

dies are worked at about 3,000 amperes per sq. in. current density. Wherever possible dies and platen are water cooled internally. The two sides of the secondary are kept as close together as possible and the length of this circuit kept at a minimum to avoid any unnecessary secondary voltage drop.

Primary coils of the larger type transformers are

Fig. 6—Hydraulic Press for Fabricated Automotive-Motor Magnet Frames

usually built up of copper strip insulated between layers by a sleeve of cloth or layer of asbestos paper. The coils are impregnated and baked; then wound with tape and finally painted.

Voltage regulation on the primary is obtained either by use of a separate auto-transformer or by bringing out taps, a regulating switch being employed. Usually there are from five to ten points of voltage regulation, the lowest being about 50 per cent of the highest. Since on production work the breaking of the current may occur several times a minute, contactors used to

Fig. 7

switch on the power supply must be able to break full load of welder without undue arcing.

The transformers are usually designed for one particular job; that is, a transformer designed for flash welding of steel would not be suitable for fast upsetting, and one designed for steel not suitable for welding copper or aluminum.

The transformer in the Thomson-Gibb machine described was designed especially for welding of rings up to 12 sq. in. cross section, but as previously stated, is used on somewhat heavier sections also.

Readings taken while welding section of 18 sq. in. show that there is a primary current of about 900 amperes taken during the flashing period with a peak load of 2,250 amperes at the final or completion period of weld when the ring is forced together. The primary voltage is 236. Current is at 60 cycle frequency. At high point of weld, readings taken show 461,250 volt-

Fig. 8

amperes and 252,000 watts which gives a power factor of about 55 per cent. The average during flashing was 126,000 watts; with the secondary open, 1,400 watts.

Oscillograph records have been made on this welder which are reproduced in Figs. 7 to 11 inclusive. Fig. 7

Fig. 9

shows conditions at start of flash, first a brief interval of short circuit, then open circuit followed by beginning of contact of pieces. This represents a period of feeble flashing. As the flashing becomes more violent, conditions are as in Fig. 8. Toward the end of the flashing period, as shown in Fig. 9, when flashing appears continuous, the wave form still shows no continuous flow of current, but rather closely compact intervals of short circuit and open circuit. This picture also shows the upset or final push-up section of the weld. Figs. 10 and 11 show views of wave analysis at early stages and near final stage of flashing period.

After the welding operation is completed, the ring is removed by an electric hoist shown in Fig. 4. The upset and flash resulting from the welding operation show clearly in this view. This is removed, the operation being done in a vertical motor-driven broach which trims outside and inside while the metal is still red hot. The rings then pass by a conveyor similar to that pre-

Fig. 10

viously shown, to an electric furnace. The electric furnace, (shown in Fig. 5) while primarily used for heating rings for sizing, also is advantageous in that it gives a strain anneal. The rings on the floor are complete rings ready for shipment. Those in the front rows have feet welded on.

The rings are heated to about 1,400 deg. fahr. in the furnace and then pass down a slide to an eight-cylinder hydraulic press located in the floor as shown in Fig. 6. Part of the flooring has been removed to show construction of the press. The press is so constructed that each cylinder has a removable head portion so that various sizes of rings may be accommodated. The rings are lowered horizontally into the press by means of an air-operated table and then squeezed to vary within 1/16 in. to 3/32 in. of correct size, depending on size of ring.

The ring is now complete with the exception of feet. The feet are made from steel bent into a U form in the hydraulic press, then gas cut apart. Each U makes one pair of feet and by inverting the U's, the right and left are made. It is the practise to cut all four feet at once using a double torch machine for this purpose.

The sized rings pass by conveyor to arc welding stations where the feet are tack welded, then finished welded into place. The completed frames are then sand-blasted, after which they are shipped to the motor building for machining and assembly. Some of the rings are for automotive equipment and do not have feet. These latter rings have dropped forged arms and bearing retainers welded in place by metallic arc process.

Fig. 11

By the use of electric welding of two kinds, it has been possible to produce frames 25 to 50 per cent cheaper than corresponding steel castings with added advantage of much greater uniformity in dimensions and structure. This, in turn, reflects in lowering machining costs and better motors.

Discussion

For discussion of this paper see page 688.

Electric Resistance Brazing

BY SAMUEL MARTIN, JR.*
Non-member

Synopsis.—*This paper describes in general the equipment for and process of making brazed joints. The lead soldered joint has been in use for a long time and has done its work well but because of certain inherent weaknesses, it is slowly being replaced by the brazed joint. The electric brazing equipment is listed and a short description of it given. The preparation of various sizes and shapes of work is taken up in detail and a short discussion of the different types of joints is given. Three kinds of brazing mediums are discussed and applications of each are mentioned. A description of the actual brazing process follows and the paper is concluded with a list of advantages of the brazed joint over the lead-soldered joint, including: increased factor of safety, saving in time, reduced cost, reduced size, and increased accessibility of joints for inspection.*

* * * * *

ALTHOUGH the lead solder joint will probably not be entirely superseded for a long time; it is, due to its inherent weaknesses, slowly being replaced by a new type of brazed joint. The lead solder joint is not all that could be desired as far as mechanical strength is concerned; when heated, it breaks apart at relatively low temperatures; and when a joint is made, its resistance is not always a known quantity. Such is the problem confronting the electrical industry if it desires to continue to set the pace for the present industrial age. A joint with high mechanical strength, high conductivity and high heat resisting qualities is needed.

Electric resistance brazing has been developed to meet these needs. This process is simple to accomplish, utilizing the high resistance of carbon blocks to the flow of electricity. The pieces to be joined are clamped between a pair of carbon blocks and current is passed through them and the joints. The carbon electrodes heat up quickly and the heat passes from them into the parts being joined, raising their temperature to the melting point of the brazing alloy. The alloy, in the form of a thin strip for convenience, is then applied to the contact surfaces of the parts being brazed. It melts and flows through the joints, uniting the parts into a solid mass.

The equipment required for making this type of joint is simple, consisting of an air-cooled transformer, primary and secondary leads, the latter being connected to the tongs for holding the carbon blocks, and a foot switch. The two most useful sizes of transformers are:

1. Five-kva., weighing approximately 45 lb. and capable of brazing conductors, cables, etc., up to 300,000 cm. in cross-section.

2. Ten kva., weighing approximately 95 lb. and capable of brazing conductors, bars, cables, etc., between 300,000 and 2,000,000 cm. in cross-section.

The five-kva. transformer is wound to give a single secondary voltage of 8 volts, while the 10-kva. transformer is wound to give 8, 10, and 12 volts on the secondary. Fig. 1 shows a complete 10-kva. equipment ready for connection to a 220-volt, single-phase, 60-cycle power line. The primary taps to give the different secondary voltages can be clearly seen. The momentary overload of 400 per cent can be carried by either of the transformers.

There are many variables which affect the operating characteristics of the transformer. Consider first that the transformers have a turn ratio of 220 to 12, 10 or 8. For the 12-volt secondary, the turn ratio is 18.4; for the 10-volt secondary, it is 22; and for the 8-volt secondary, it is 27.5. For simplicity, consider one tap only, the 12-volt one. For every ampere in the primary, there are 18.4 amperes in the secondary. Then, for 100 amperes in the primary, there are 1,840 amperes in the secondary. With comparatively high current and low voltage, the reactance of the leads to the tongs has quite an effect on the power factor. The load, which is the resistance of the carbon blocks plus that of the piece being brazed, also affects the power factor. For any given length of lead, 7-ft. ones are recommended for use with the equipment, the kind of carbon block, the kind of joint and the amount of contact between the carbon blocks and the joint affect the power factor, high resistance tending to raise it, and low resistance tending to lower it.

For comparison, the following test data were obtained for (A) no joint between the carbon blocks, and, (B) a joint between the carbon blocks:

(A) Short circuit through 1⅜-in. soft carbon blocks:

Volts	Amperes	Watts	Volt-amperes	Power factor (%)
222	0	0		
212	118	15,600	25,000	62.4
210	120	16,000	25,200	63.5

(B) Brazing terminal using 1½-in. hard carbon blocks:

Volts	Amperes	Watts	Volt-amperes	Power factor (%)
237	0	0		
236	40	6,800	9,440	72.2
235	58	10,000	13,600	73.5

As the weights of the equipment indicate, the outfits are portable, thereby making it possible to apply the tongs to joints in any position. It is sometimes convenient to mount the 10-kva. transformer on a small truck supported by casters.

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For joints larger than 2,000,000 cir. mils. cross-section, stationary machines or presses have been adopted, using much larger transformers.

For different types of joints, various styles of tongs have been developed. Fig. 2 shows one of these tongs.

The preparation of the work for the brazing process is important. The parts to be brazed should be as flat as

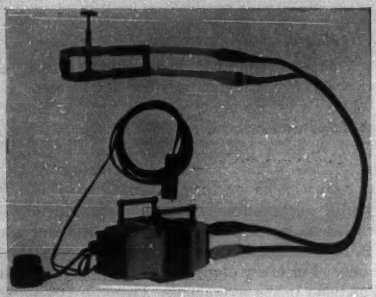

Fig. 1—10-Kva., 8—10—12-Volt Brazing Transformer, Complete with Leads, Tongs, Foot-Switch and Plug

possible so that the carbon blocks will come in intimate contact with the full surface of the material. This will prevent arcing and hot spots occurring between the carbon blocks and the metal parts. Round bars, rods, solid wire, etc., should be flattened with a hammer or machining process to insure good contact. If the round bar or irregular shaped piece cannot be flattened as mentioned above, the carbons may be cut to fit the

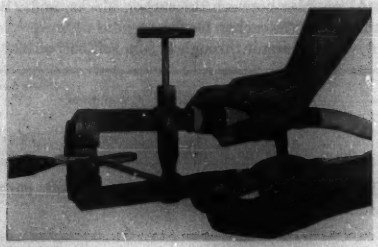

Fig. 2—Application of Tongs to Brazing Cable to Flat-Plate Terminal

piece. Flat plates replace tubular terminals as shown in Fig. 3. If the flat plate terminal or bus bar is more than 1 in. wide, it is slotted to prevent excessive heat radiation. The brazing operation is then performed on each individual section as a separate operation. Stranded cable is prepared in a slightly different manner. It is either wrapped with thin copper strip or inserted in a thin copper tube and then flattened before clamping in the tongs. If it is desired to make a very flat joint, the thin copper strip should be wound around the strands rather loosely. The wrapping or tubing should extend on the cable ½ to 1 in. beyond the portion being brazed.

Copper braid is prepared in a somewhat similar manner as shown in Fig. 3. Here a copper clip about 0.02 in. thick and slightly wider than the braid is used. The copper clip should extend on the braid about ½ in. beyond the contact surface. When several flat strips of braid are used in parallel, a thin piece of brazing alloy about ½ in. square is usually placed in the end of alternate braids throughout the group and one clip is put over the end of the group before clamping in the tongs.

Depending on the size of the cable, the insulation on it should be removed for a distance of ½ in. to 2 in. from

Fig. 3—Comparison of a Brazed Flat-Plate Terminal and Soldered-Tubular Terminal

the joint. The bare cable should be wrapped with damp asbestos at the end of the insulation to protect it from over-heating during the brazing process.

Lap joints on cables, conductors, etc., are made by lapping pieces to be brazed about 1½ times the thickness of the conductor. If it is desired to make a butt or scarf joint on rectangular conductors, a clamp can be made for holding the two ends together, applying the carbon blocks to the edges of the conductor and feeding the alloy in from the flat side.

In soft soldering, it is very essential that the surfaces be clean and free from oxides and other impurities It is not so essential to clean the surfaces for brazing, but certain oils and varnishes, however, are not attacked by the flux used in brazing and should be removed. The flux most generally used in brazing is fused borax. In some cases, it is more satisfactory to use potassium acid

fluoride in paste form. The fused borax lends itself very readily in a dry powder form to being shifted from a small shaker or applied by a small power bellow. It can also be mixed in water or mixed with vaseline and applied to the joints. Any of the above methods of applying the flux is more economical than throwing it on the joint with the hand.

Depending upon the joint to be made, there are three different silver alloys used as brazing medium. A comparison of these alloys can be obtained from the following table:

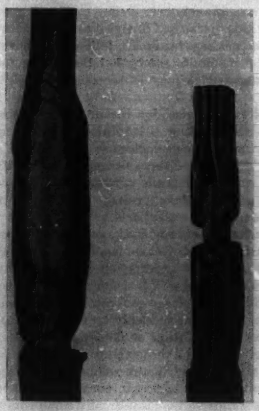

FIG. 4—COMPARISON OF SOLDERED AND BRAZED JOINTS SHOWING POSSIBILITY OF INSPECTION OF JOINT

Per cent silver	Melting point deg. cent.	Melting point deg. fahr.
10%	830	1526
30%	735	1355
50%	725	1337

The three different silver solders have their particular applications, depending on the permissible temperature determined by the physical conditions of the joints. For example, to prevent over-heating of very small copper wire, it is essential to keep the brazing temperature to a minimum and the 50 per cent alloy is used. On larger cables, terminals, etc., the 30 per cent alloy is used, because it is less expensive, while on the heavy bus bar plates, etc., where there is no insulation, the 10 per cent silver solder is satisfactory.

When the copper bars have been prepared as described, the tongs are tightly clamped on the joints by turning the handwheel. The flux is then applied to the joints and the current is turned on by pushing the foot-operated switch applying power to the primary of the transformer. The carbon blocks heat quickly and if the parts to be brazed are small, the metal will reach a brazing temperature in a few seconds, but if the parts to be brazed are large, more time will be required and the current should be applied intermittently to permit the heat to distribute itself throughout the mass, thus insuring a well-brazed joint.

The borax will melt when the brazing temperature has been reached, and a strip of silver solder is applied at the line of contact between the parts being joined. As the alloy melts, it will follow the borax through the joints. When the borax rises to the surface of the molten alloy, it is an indication that the heat is sufficient and the current can be turned off. Care should be taken not to remove the tongs or to release the pressure on them until the joint has cooled to a dull red.

The economics of brazing are sound. Many cost studies have been made on various size cables, plates, bars, etc. For comparison, consider only three cases, (A) terminal on 180,000-cir. mils cable, (B) terminal on 1,500,000-cir. mils. cable, and (C) twelve ⅛-in. by ¼-in. conductors fastened to a 770,000-cir. mils cable.

Case	Preparation time	Brazing or soldering time	Finishing time	Cost of alloy or solder	Cost of labor at 60c./hr.	Total cost
A-B	3.41	3.17	1.2	0.027	0.077	0.104
A-S	3.41	5.17		0.025	0.085	0.110
B-B	10.90	7.28	4.0	0.225	0.220	0 445
B-S	10.90	17.97		0.247	0.288	0.535
C-B	5.35	15.57		0.225	0.209	0.434
C-S	5.35	24.80		0.46	0.302	0.762

In the column headed "Case," the B is for brazing and the S for soldering. The preparation time includes skinning, binding, and forming of the cable. The finishing time is that taken for cleaning. The above costs are figured for the most favorable conditions for soldering. If labor were figured at 90c. per hr., the saving on the brazed joint would be much greater than that indicated.

Copper bus bars, conductors, cables, etc., are of particular interest to the electrical industry. When the silver solder is applied to a joint formed of these parts, it dissolves a thin film of the metal surface and forms a new alloy having a melting point higher than that of the original solder. This alloy is very rich in copper which makes it an excellent conductor. Since the melting point of the new alloy formed is higher than that of the silver solder, a higher temperature is always required to open such a joint than to make it.

There are several distinct advantages of joints made with silver solder or other high melting point alloys over the ordinary soft solder joints. If the lap joints are designed to have a contact area of at least 1½ times

the cross section of the conductor, the factor of safety of such a joint is very high for it will carry as much overload as the conductor itself. Time is saved in making a joint by the resistance brazed method, some joints being made in from ½ to ⅓ the time required to make a lead-soldered joint on the same size conductor. As a result of the saving in time and the small amount of alloy used in making the brazed joint, it is invariably less expensive to make. As illustrated by Fig. 4, brazed joints are considerably smaller and are easily accessible to inspection, since all of the work is open. When a tubular terminal is soldered to a cable, it is impossible to inspect the joint thoroughly, but a cable brazed to a flat terminal can easily be examined to see that there are no defects.

The mechanical strength, durability and high electrical conductivity of brazed joints marks a step forward in the electrical industry.

Discussion

AN IMPROVED ARC WELDING GENERATOR

(BLANKENBUEHLER)

THE NEUTRALIZED WELDER—A MEANS OF CONTROLLING TRANSIENTS

(CREEDY)

A NEW SYSTEM FOR D-C. ARC WELDING

(BERGMAN)

RESISTANCE WELDING OF MOTOR FRAMES

(THOMSON)

ELECTRIC RESISTANCE BRAZING

(MARTIN)

A. Churchward: My study and interpretation of these papers lead me to believe that the constant potential type of welding machine is the one which we would all be putting out as single arc units if its design and construction were not so expensive and the inherent losses on short leads great. It therefore seems logical to assume that we are all trying our utmost to obtain the same results, as nearly as possible, that the constant potential 60-volt generator gives.

These inherent losses disappear entirely when long leads are used, such as specified in the latest Navy Specifications, but I still do not see why the single operator is being used at so much higher voltage than the constant potential, namely 60 volts. Of course this may be due to the inherent characteristics of the bucking series type of machine. I would state that I used the transformer reactor early in 1924 on a self-excited drooping characteristic welding generator, and my tests confirm Mr. Blankenbuehler's results.

D. W. Ver Planck: It is interesting to consider a method of analyzing the current and voltage transients which occur in self-excited arc-welding generators of the type described by Mr. Bergman. The results which can be obtained are of value in showing how the transients vary with the design of the machine and in predetermining approximately the transient characteristics when the design has been decided upon.

To bring the problem within the range of simple analysis it is necessary to make certain assumptions regarding magnetic saturation which fortunately are fulfilled closely in actual machines. They are:

1. The cross magnetic circuit of the generator and the magnetic circuit of the transforming-reactor have no saturation.

2. The main magnetic circuit of the generator is so highly saturated that the main flux is independent of changes in excitation on the main axis.

These assumptions make it possible to replace the generator for purposes of analysis by an equivalent linear electrical network. In this network the armature generated voltage is separated into two parts: a constant positive part equal to the open-circuit voltage of the machine; and a variable negative part which is directly proportional to the change from no load in the cross axis net excitation.

The behavior under various transient conditions of the generator equivalent circuit together with the transforming reactor can then be quite readily determined by the methods of operational circuit analysis.

Applying this method of analysis to finding the voltage transient at the welding terminals following the sudden opening of a steady short circuit shows that the transient consists of the following parts, which may be seen in Fig. 4 of Mr. Bergman's paper:

1. A voltage impulse due to the sudden decay of the flux which links the welding circuit alone, largely in the leakage paths of the transforming-reactor.

2. A sudden rise of the voltage, exclusive of the impulse, which is sometimes called the "instantaneous recovery voltage." This instantaneous recovery voltage is shown by the analysis to consist of the following three parts:

a. The voltage generated in the armature by rotation through short-circuit flux, which voltage is numerically equal to the total resistance drop in the welding circuit caused by short-circuit current.

b. An additional voltage generated in the armature by rotation through flux which suddenly appears in the generator as a result of the redistribution of field flux linkages between the transforming-reactor and the generator.

c. A voltage induced in the welding circuit by transformer action as the field flux begins to change toward its final open-circuit value. This voltage is normally small compared to parts a and b.

3. From the instantaneous recovery voltage the terminal voltage approaches its final open-circuit value exponentially with time. The analysis shows that the time constant of this part of the transient is equal to the total self inductance of the cross field circuit divided by the total resistance of this circuit; that is, the cross field circuit time-constant.

Thus the analysis shows to what particular quality of the machine each part of the transient is due and hence it suggests to the designer the way in which to emphasize or suppress any part of the transient.

The current transient following the making of a short circuit can be handled in a similar fashion, though with somewhat less ease.

One of the most important features of Professor Creedy's paper is the demonstration that the steady-state and some of the transient characteristics of welding generators can be determined analytically. This should go a long way toward dispelling the mystery which seems to shroud welding transients.

However, all of the transient problems have not been cleared up yet by any means. So far all that has been shown is what happens after a single change in terminal conditions such as the sudden change from one arc voltage to another, or the sudden opening of the circuit. While this information is very valuable, it must be remembered that during actual welding the terminal conditions are always fluctuating so that the generator operates continually under a series of transient conditions. The treatment of this problem would of course involve the variable resistance of the arc, making the solution very difficult.

While Professor Creedy's analysis leads to the same results as the equivalent circuit method, I believe that the latter is considerably easier to handle. Also the equivalent circuit

method should lead to the definition of certain machine constants, the knowledge of which would completely specify the steady-state and transient characteristics of the generator.

Such machine constants for welding generators would be analogous to the various steady-state and transient reactances used in synchronous machine theory. Their use would make it no longer necessary to solve the differential equations for every welding transient problem. And more important than this, the general use of such constants should do much toward making it easier for the welding engineers to tell the generator engineers what kind of machines to build.

C. J. Holslag: I was the first to discover in 1915 this transformer action during the making and breaking of the arc in a welding generator. This effect is very simple, despite the elaborate papers concerning it. When you touch the electrode to the work, the armature reaction increases the field current to double, triple, or four times its normal value and the effect is cumulative because the armature current increases, etc., until they all come back to rest at the steady value.

If an attempt is made to strike the arc during this period, all is well, but by waiting a small fraction of a second, the opposite effect occurs, *i. e.*, field current is cut in two or even reversed by the transformer action of the laminated field pole with series and shunt winding, or even the armature current alone. This action is very detrimental because a good fraction of a second is taken by the field to recover. During this period, the arc generally goes out and the operator has to "pick" again and again until he gets the right combination to hold it. The whole problem is the above analysis of the interaction of field and armature current. There are many ways to overcome this, once the problem is made clear. The simple way to do it is to use a compound exciter of approximately twice the voltage induced in the field circuit. This prevents the flux dropping below the knee with accompanying recovery.

J. F. Lincoln: The systems employing a transformer in the circuit of a welding set, or the so-called transformer reactor described in the papers by Messrs. J. H. Blankenbuehler and S. R. Bergman are old. They were first described in 1926, being later described by this company, in 1928, and used by another manufacturer for about a year in so far as the transformer and the wiring scheme are concerned.

There is no doubt that this transforming reactor will overcome some of the difficulties which are found in the usual set if no reactor were used. It is extremely doubtful that it is as good either from the point of view of cost, or operation, as the usual well-known reactance coil which has been used for a number of years. The company with whom the writer is connected came to that conclusion, after very complete tests, at least two years before it was ever used by anyone else in this country.

S. S. Scott: The fabrication of heavy frames by the use of resistance welding known commercially by the term "flash welding" as described by Mr. Thomson is used extensively by the automobile body builders, especially for welding 20 gage sheets together in 120-in. widths, thus affording a large saving by purchasing same in small sizes.

The initial cost of these resistance welders is high but in quantity lots the fabricated product can readily pay off the first cost within a short period. For instance, one automobile body builder purchased a 1,500-kva. flash welder to take in 120-in. width sheets of 20 gage material for $75,000 and spent $35,000 on developing same and the following year purchased a similar machine for $75,000 and spent $20,000 for development.

Attention is called to Mr. Thomson's description of the flash welder under discussion, which is a 750-kva. water-cooled transformer and the open-circuit voltage at welding jaws is 12 volts. We had considerable experience in my former company with a 500-kva. flash welder which had an open-circuit voltage of 7 volts at the welding jaws. This machine had a capacity rating of 12 sq. in. maximum, cross section. By using this machine for flash welding large cylindrical rings, 12 sq. in. in cross section, we were able to reduce the cost of welding 80 per cent.

Experiments have proven the fact that transformers designed for flash welding are usually designed for one particular job as it is proven that a transformer designed for flash welding of steel is not suitable for fast upsetting and one designed for steel not suitable for welding alloys of copper or aluminum.

Mr. Samuel Martin, Jr.'s paper is of particular importance to the welding field today as it brings forth the efficiency to which the portable electric resistance brazing machines have been perfected.

In visiting a number of plants throughout the country in the past few years, it was of interest to see how this phase of brazing has been advanced, especially so amongst the small industrial companies.

Mr. Martin refers to a comparison of the three different silver alloys used as a brazing medium, which is of interest to the field and I wish to add the following:

a. Where the design of the joint will allow, tinned surfaces are brazed together using no flux. This applies to many types of joints, tinned tap straps to winding conductors, etc.

b. For butt brazing use the standard silver alloys.

c. For brazing cable, strips, and braids to flat terminals use standard silver alloys.

d. For all bar work or where joints from raw copper can be made without considering insulation use 10 per cent silver alloy.

e. Use alloy consisting of 90 per cent lead and 10 per cent tin, 0.005 in. thick for solidifying braids and similar work.

With reference to flux used for brazing, we might add:

a. For lead 90 per cent, tin 10 per cent—no flux is required. For pure tin leaf use lactic acid. For silver alloys use fused borax.

Sometimes it is beneficial to use borax dry or as a paste with vaseline, according to the nature of the work.

It is of interest to note in Mr. Martin's paper that for every ampere in the primary lines there are 18.4 amperes in the secondary.

The load, which is the resistance of the carbon blocks plus that of the piece being brazed, has a great deal to do in affecting the power factor. Thus, for heavy brazing units, carbon having resistance of 0.0007 ohms per cu. in. should be used. On large joints time is required for the uniform distribution of heat. High-resistance carbon blocks heat much faster than low-resistance carbon and care is required to prevent the burning of the joint surfaces.

The cost data for brazing, as shown by Mr. Martin, are very interesting and verify tests made recently, as the time required to braze a joint is in practically every case less than that required to solder or rivet and solder.

Many types of joints now standard practise reduce the time formerly required for leading from 50 to 75 per cent. With proper equipment and well trained operators brazed joints can invariably be made for less cost than the original type of leaded joint.

J. C. Lincoln: Mr. Bergman in Fig. 2 of his paper shows a method of obtaining a voltage control for the welding machine by reducing, and in some cases, reversing the flux in the horizontal path in the magnetic circuit. In other words, the path marked RC. At the same time he saturates the part of the magnetic circuit which he calls RM so that the armature reactance does not increase the flux in this magnetic circuit.

Some 25 or 30 years ago I used the same scheme in a somewhat similar form to obtain a compounding effect for a plating machine with the use of a shunt winding only. As is well known, the use of a series coil on a plating machine is objectionable because there is a chance that the counter electromotive force of the material in the plating vats may discharge through the series coil of the plating generator and reverse the polarity. This possibility, which is a real objection to a compound-wound generator does not exist in a shunt-wound generator.

In Fig. 1, the direction of rotation is indicated, also the polarity of the main pole of the bi-polar structure. The polarity of the field from the armature ampere turns is also shown. There is a slot in the middle of the pole faces as indicated. The shunt field instead of being connected between the positive and negative brushes as usual, is connected between the third brush and the positive brush. As the load comes on armature reaction tends to increase the flux into the armature on the upper part of the north pole in the field and the lower part of the south pole in the field.

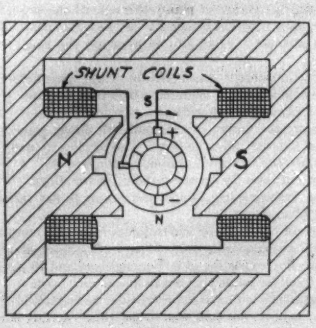

Fig. 1

Magnetic circuit was designed so that this increase of flux could take place and as a result the voltage between the third brush and the positive brush increases as the load comes on, the increase being proportional to the load. By this means a compound effect was obtained similar to what would be obtained with a series coil. This machine and others like it were put into, and so far as I know, are still in commercial operation.

is approximately 500 volts, the arc voltage is 15 or 20 volts so there is no possible chance of overshooting in welding made by apparatus of this sort, but it also is true that as a general thing, welds made with apparatus of this sort are poor. Therefore, it seems to me that it is clear that the elimination of overshooting does not necessarily produce a high-grade weld.

These three papers show different types of welding machines which have been designed to use ordinary bare wire and which satisfactorily meet the conditions which they were designed to take care of.

Fig. 2 is a typical oscillogram when bare wire or thinly coated wire is used on the electrode. It is here seen that very frequently the voltage drops to zero and this occurs whenever the arc is short-circuited by the globule of metal dropping off the electrode to the work. It is this condition of continuous and frequent short circuit which the welding machine of the present day has to take care of. It is further true that welds made with bare wire or thinly coated wire have satisfactory tensile strength but do not have ductility which compares with the ductility of the parent metal.

Recently there have been introduced electrodes with heavy coatings in which the weld metal has a tensile strength equal to or higher than the tensile strength of welds made with bare wire and in which the ductility of the weld metal approaches that of the parent metal. This high ductility of the weld metal makes possible the distortion of the welds without breaking; makes possible a slight stretch of the welds in order to relieve any locked up strains and in general presents the advantages of ductile steel as compared with non ductile cast iron in any structure.

A very interesting thing to note is that the oscillogram of a weld made using heavy-coated electrode is materially different from that using bare wire. (Fig. 3.) It will be noted in this oscillogram that there are no periods of short circuit, and the metal apparently goes across the arc in small particles. It certainly indicates the fact that the arc is not short-circuited during the ordinary welding operation.

I am calling attention to this characteristic, because as I see it the welding machines of the future will have to meet the

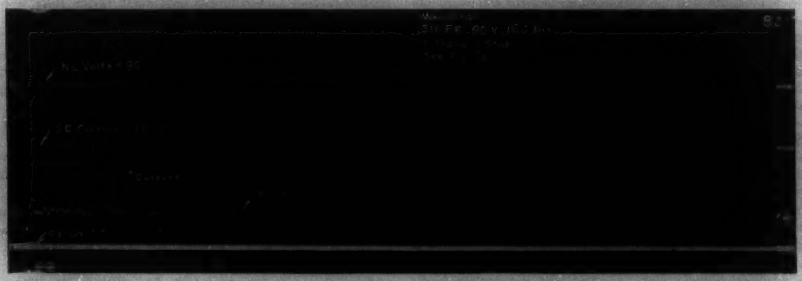

Fig. 2

The statement is made by Mr. Blankenbuehler that a deposited metal is free from porosity when the welding machine is so built there is no overshoot in the current. The question of eliminating porosity in welding is of great importance and I am asking what evidence Mr. Blankenbuehler has for the statement (3) in his summary.

In this connection I would call attention to the fact that a great deal of welding is done on street railways using a resistance connected to the trolley wire. In this case the supply of voltage

conditions shown in oscillogram No. 37 rather than the conditions shown in oscillogram No. 82, and the machines which are discussed at the conventions five years from now will be machines designed to meet the conditions shown on oscillogram No. 37 rather than the conditions shown on oscillogram No. 82.

Mr. Thomson in his paper makes the following statement: "A predetermined amount is flashed off, averaging slightly less than an inch total." The cuts seem to indicate that the rings are bent up so that it would not be necessary to flash off

an inch of metal in order to get the full cross section and the question I am asking is—why is it necessary to flash off an inch of metal when apparently half of this amount would be just as effective?

Again on page 684 it is stated that the electric furnace is advantageous in that it gives a strain anneal. I would assume that a weld of this kind made in soft steel would not have locked up strains of such an amount set up so that strain anneal would be necessary. The question is whether the furnace would be used for strain anneal if it were not necessary to size the rings.

J. H. Blankenbuehler: My answers to Mr. J. C. Lincoln's questions follow.

Laboratory tests, in which current was passed through a molten pool of metal at such a value as to keep the pool molten, then caused to increase instantly to a higher value, and then turned off, definitely showed that current surges caused the deposit to be porous.

The welding done on street railways using a resistor connected to the trolley wire is not usually of the best quality because of two factors; first, the polarity is the reverse of that recommended for good welding practise and, second, with such a high-voltage supply the operator finds it easier to hold a longer arc than is consistent with good quality. The presence or absence of current surges in this case are not determining factors.

F. Creedy: It is very gratifying to me to note that engineers are beginning to appreciate the importance to good welding of eliminating transients. When I first drew the attention of American firms to this in 1927 (based on work done in 1924, 5, 6), I found practically no one who appreciated the point except Mr. Ludwig of the Westinghouse Company and Professor C. A. Adams. I note the remarks of Mr. J. F. Lincoln in the discussion, but I certainly got no favorable reaction from his firm at that time.

Like Mr. Churchward I do not see why single arc welders should have a much higher open-circuit voltage than 60 volts although I note that several makers advocate 70, 80, and 90 volts.

I quite agree with Mr. D. W. Ver Planck that the study of welding transients is only begun and there are a great many problems which have not yet been cleared up. Perhaps at a later date he will make a mathematical contribution. In making this analysis, however, it is a mistake to limit oneself to the use of the transforming reactor either of the Bergman or Blankenbuehler form. The essential reason for these transients is that magnetic energy has to flow in or out whenever the fluxes increase or decrease. To design a machine on conventional lines with very high-stored magnetic energy and then use a powerful transforming reactor to produce a forced flow of energy can hardly, one would think, be the final solution. With regard to the remarks of those who claim to have used such a transformer at an early date, it should be pointed out, that it is not enough merely to place a series transformer with primary in the welding circuit and secondary opposing the transient voltages in the exciting circuit but this transformer *must be so proportioned* that it changes the constants of the circuit, so that the solutions of the differential equations on which the current depends become approximately logarithmic instead of oscillatory. If wrongly proportioned it might actually make matters worse instead of better.

M. Thomson: Mr. Lincoln has asked the question as to why we flash off as much as an inch of stock. The answer is that in forming of rings for magnet frames there is a sort of a "V" at the joint, and also sheared stock is rather irregular, so it is therefore necessary to flash off considerable metal to make proper fit between the faces to be welded. It is also necessary to heat the stock sufficiently to insure a sound weld. We find that with perfectly fitted faces and very high-current density the amount of flash can be reduced by at least one-half, but cost of such a joint is higher than the one we use.

Fig. 3

The other question asked by Mr. Lincoln, was whether furnace would be used for strain anneal, if it were not necessary to size the rings. While the rings are heated to 1,400 deg. fahr. primarily for sizing operation, we make it a practise to give a strain anneal to all our fabricated motors, whether or not sized, the practise having been extended because the strain annealed steel has been found to be better magnetically. Not all the strains in the stock are from welding. The rolling and forming of the plate itself leave strains which it is desirable to relieve.

As to when it pays to invest in a flash welder; how many pieces must one make to justify the expense? The whole question is one of economics and must be figured for each job. The original cost of equipment, depreciation, power, labor costs, number of pieces made and all such factors enter into the problem. However, on frames of the type we build, I would estimate that about two hundred frames a week would justify the expense of resistance welding equipment.

The Conductivity of Insulating Oils—II

BY J. B. WHITEHEAD*

Fellow, A. I. E. E.

Introduction

THE conductivity of poorly conducting or "insulating" liquids, although long recognized as ionic in character,[1] is nevertheless highly anomalous and is in no case subject to exact expression in terms of empirical laws. On the first application of a constant difference of continuous potential the current has a high initial value which decreases slowly with time, reaching a constant value only after hours or even days. There is no discharge curve of corresponding magnitude. The phenomenon suggests the somewhat similar dielectric absorption in solids, but is obviously of different character and origin. The decaying current-time curve is characteristic of all liquid dielectrics. By various measures of refinement and purification it is possible to reduce greatly both the initial and the final conductivities[2, 3, 4] but even under the most careful conditions of preparation and experiment the phenomenon is present in pronounced degree. In fact, it tends to disappear only in liquids of higher conductivity, being then masked by conduction of more familiar types.

The final constant current often shows saturation characteristics similar to those of ionized gases. This, through the work of Jaffé,[2] Schröder[4] and Mie,[5] has led to the generally accepted conclusion that conduction in such liquids obeys the corresponding laws of gases. The value unity, of the Langevin[6] constant, necessary for agreement with experiment, leads Jaffé to conclude that hexane, pentane and certain weak solutions in these liquids, behave like very dense gases. It is also usually assumed that this type of agreement with gas theory is limited to highly refined liquids, as studied by Jaffé and Schröder under repeated distillation and careful elimination of influence from the electrodes, walls of containing vessels, etc. The saturation characteristic of the final current obtains also, however, for transformer oil,[7] and other refined insulating oils of engineering practise. The range of values of final conductivities with saturation characteristic is thus from 10^{-18}, the lowest value given by Jaffé, to 10^{-13} mhos per cu. cm. corresponding to a none too good transformer oil.[6] The ionic mobilities in these cases are of the same order of magnitude, ranging from 5×10^{-4} in hexane[2] to 1×10^{-4} in transformer oil,[7] these values being thus of the same order as those of electrolytic ions in aqueous solutions. The source of the ionization in these residual conducting states is not clearly defined. It has been variously attributed to external radio active influence,[1] to gases adsorbed in the electrodes,[4, 8] and to electrolytic impurities for the simplest liquids. Walden[9] has suggested an unrecognized type of dissociation arising in the complexity of the organic molecular structure, which seems more probable in the case of the insulating oils.

Little attention has been given the *initial* high values of conductivity other than to record the decay of its value with time. This decay is generally attributed to the presence of impurities or dissolved electrolytes which are being swept out by the applied electric field. This cannot be the complete explanation, however, since the effect is conspicuous in the most carefully refined liquids, and in all cases the liquid soon resumes its initial condition, permitting repetition. In fact there is much evidence that this decaying conductivity is superposed on that due to dissolved impurities, the clean-up process being only initial and partial, because the ratio of initial to final conductivity is increased on purification, and becomes less in the presence of increased extraneous conductivity (see below).

The early high value of conductivity is of great practical importance in the case of insulating oils in its bearing on the energy loss under alternating electric stress. These carefully refined oils have remarkably good insulating properties; their final conductivities are in the range 10^{-13} to 10^{-15} mhos per cu. cm., being better in this particular than many of the liquids used in the experiments referred to above, and in most cases showing pronounced residual saturation characteristics. The initial values of conductivity are from three to ten or more times the final values. The initial values determine the alternating loss at commercial power frequencies. Tank,[10] using the Helmholz pendulum in a point-by-point method, measured the initial conductivity of several commercial oils over the first 0.25 sec. following the application of potential difference. Within this range he found the conductivity to be constant and of proper value to account for the loss at 50 cycles. We have found a similar result for a poor oil, but a quite different result for a carefully purified oil. We have in fact shown that a brief initial dielectric absorption may be present and that by taking due account of this the alternating losses are completely accounted for by the characteristics as observed under continuous potential.

The Experiments

Continuous current-time records, beginning within 0.01 sec. after the application of voltage have already been reported by the author,[7] using the string galvanometer. They revealed pronounced variations in the shapes of the curves for different oils, also saturation, unequal potential gradient, and other anomalous properties. In the present work we have extended the studies to two other oils and have made corresponding observations with continuous and alternating voltage, measuring the losses, thus permitting correlation of properties under the two conditions.

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1. For references see Bibliography.

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31-40

The two oils now studied were two samples of the same insulating oil prepared by the refiner especially for this work. One sample (B-4) had suffered slight deterioration by heating to 80 deg. cent. in the presence of air. The other was freed of air by slow spraying into a 0.1 mm. vacuum. The measurements were made on films 1 mm. thick between coaxial cylindrical electrodes equipped with guard rings and protective screening, and immersed in constant temperature vacuum tanks.[16] Observations were made within the temperature range 30 to 60 deg. and at 500, 1,000, and 1,500 volts, continuous and alternating; frequency, 60 cycles. The short-time current curves were obtained with the amplifier-oscillograph and rapid action switch as developed by Waldorf[11] for this work. The high sensitivity Schering bridge for the alternating loss measurements is due to W. B. Kouwenhoven and A. Banos, Jr.[12] It measures energy loss to within 1×10^{-6} watts, at all values down to 1×10^{-6} watts, and at power factors down to 0.00007 $\pm$ 0.000005.

Fig. 1—Typical Decaying Charging Current of Insulating Oils

Interval: 0.002—24 seconds

The Results

Fig. 1 shows the decay in current in oil B-4 at 60 deg. during the first 24 sec. following the application of 1,500 volts. This curve is generally typical of the oils of this class. They differ, however, as to the absolute values and also as to the initial rates of decay. Fig. 2 shows the first 0.03 sec. after the application of voltage; in this record the initial low peak is the trace, at greatly reduced sensitivity, of the geometric charging current and its merging into the subsequent constant conduction current. The complete history in semi-logarithmic coordinates of the change of current in the same oil from an initial value sensibly constant over a period of one sec., is shown in Fig. 3 followed by a slow decay to an approximately constant value reached only after 40 min. The left hand portion of the curve of Fig. 3 was taken photographically and the right hand portion was obtained with a high sensitivity Weston d'Arsonval galvanometer. The initial constant current of 77.5 $\times 10^{-8}$ amperes corresponds to a conductivity of 15×10^{-14} mhos. per cu. cm., and the final constant current of 18.8×10^{-8} amperes to a conductivity of 3.6×10^{-14} mhos. per cu. cm.

Fig. 2—Initial Constant Conduction Current in Certain Oils

Interval: 0.002—0.03 second

As among different oils there is a noticeable difference in the type of variation of the initial current with the electric intensity. Fig. 4 taken from the earlier paper,[7] shows this relation for a pure transformer oil (A) (final conductivity 3×10^{-16} mhos per cu. cm.) and for the same oil slightly deteriorated (B) (final conductivity 2.4×10^{-15} mhos per cu. cm.), as result of standing 6 months exposed to air. In the former the initial current increases more rapidly than the applied voltage, in the latter in accordance with Ohm's law, for the range 500 to 1,500 volts. The final constant current in this oil showed clear saturation characteristics. Similar results for the oils B-4 and B-6 of the present studies are shown in Figs. 5 and 6, in which the tendency to saturation for the final current is evident for both oils.

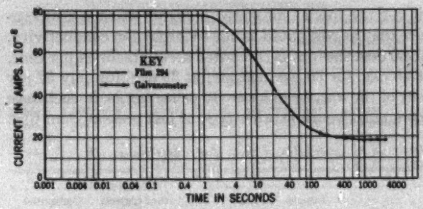

Fig. 3—Charge Current Curve Specimen *B* 4 at 1,500 Volts and 59.7 Deg. Cent.

The influence of temperature on initial and final currents is seen in Figs. 7 and 8. The increase with temperature in both oils is very much greater for the final than for the initial conductivities. In Figs. 9 and 10 the ratio of the initial to final current is shown as related to applied voltage and to temperature. In both oils this ratio increases with the applied voltage and more rapidly for the purer oil. In both oils at constant voltage the ratio decreases with increasing temperature, and more slowly in the purer oil B-6. These facts are accounted for by the initial-current-voltage relation; by the tendency to saturation described above, and particularly by the increase of conductivity with increase of temperature, in its tendency to mask the decay of current with time. There is here clear indication that the initial and final conductivities are of different origin.

In Table I is given a series of values of energy loss measurements with corresponding values of power factor, for oils B-4 and B-6 at 1,500 volts, 60 cycles, and at temperatures 30 deg., 45 deg., and 60 deg. cent. Corresponding values of the initial constant values of cur-

rents as taken from the oscillograms are also given, and in the last column the value of the energy loss as computed from the initial conductivity, *i. e.*, assuming that the loss is all of $I^2 r$ type. It is seen that there is excellent agreement for oil B-4 at all temperatures between the loss as measured and as so computed. For

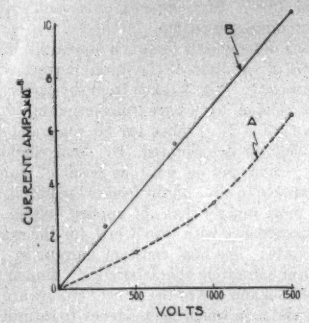

FIG. 4—VARIATION OF INITIAL CURRENT AT 22 DEG. CENT. WITH VOLTAGE TRANSFORMER OILS "A" AND "B"

oil B-6 the loss as measured is higher at each temperature than as computed as due to initial d-c. conductivity. At 60 deg. the loss as computed accounts for 84 per cent, at 45 deg. for 80 per cent and at 30 deg. for only 67 per cent of the measured loss. In other words, in each case there must be some other type of loss present than that

TABLE I—OIL SPECIMEN B 4

Temperature (deg. cent.)	Loss in microwatts 1,500 v., 60 ~	Power factor	Initial constant current in amperes 1,500 volts, d-c.	Computed d-c. loss microwatts
30	94	0.00015	6.5×10^{-8}	97
45	407	0.00065	26.9×10^{-8}	404
60	1031	0.00165	68.3×10^{-8}	1025

due to conduction alone. In order to investigate this question further additional current time curves were taken at 30 and 45 deg. increasing the sensitivity of the amplifier oscillograph to the maximum, and reducing the time interval between voltage application and exposure to the minimum, as permitted by the apparatus. The results at 30 deg. and 1,500 volts for both charge and discharge, as plotted from the oscillogram, are shown in Fig. 11. A record at 45 deg. showing brief initial decay followed by constant value is seen in Fig. 12. These curves show that within 0.004 sec. following the application of voltage, or of short circuit following charge, there is a brief initial condition of decay of both charge and discharge currents of approximately the same type and magnitude and so suggesting the phenomenon of dielectric absorption in solids. It is not accounted for by the transient due to the normal circuit constants for this is negligible when the record begins. Further it is greater at 30 deg. than at 45 deg. and it does not appear on the records at 60 deg., indicating its low value and brief duration. It is thus apparently related to the increasing viscosity of the oil at lower temperatures. In Figs. 13 and 14 these initial quasi-absorption currents are shown, as plotted from the records, in their relation to the subsequent longer, but still brief, constant values, followed by slow decay to much lower ultimate possibly constant values. As seen the initial absorption is more pronounced for 30 deg. than for 45 deg. These curves should be compared with Fig. 3 for an oil in which the absorption is completely obscured by high conductivity.

There remains the question whether the brief initial absorption shown in Figs. 11 to 14 is sufficient to

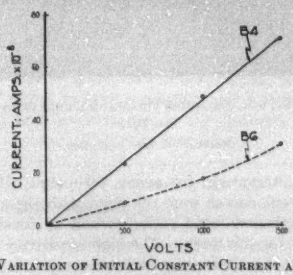

FIG. 5—VARIATION OF INITIAL CONSTANT CURRENT AT 60 DEG. CENT. WITH VOLTAGE

account for the excess of the measured a-c. loss over that computed from the relatively long period of the initial constant conductivity following the brief initial absorption. The answer is that it is sufficient. The absorption portions of both charge and discharge curves

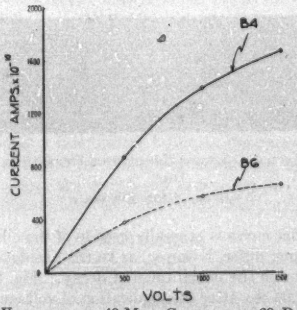

FIG. 6—VARIATION OF 40-MIN. CURRENT AT 60 DEG. CENT. WITH VOLTAGE

obey very closely a negative exponential relation with the time, as shown by the empirical equations given in Fig. 11. While the values of the ordinates of the curves in the oscillograms are small they nevertheless permit the reproduction of Fig. 11 to within an accuracy of

about five per cent, for those taken at 30 and 45 deg.; reproduction was not attempted for the record at 60 deg., owing to the extremely small values of current. If now using the method of von Schweidler[13,14] the total loss be computed as the sum of the losses due to absorp-

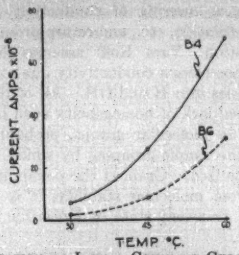

Fig. 7—Variation of Initial Constant Current at 1,500 Volts with Temperature

tion and to subsequent constant conductivity, we find the very excellent agreement between the losses as measured under alternating stress, and as computed from the d-c. absorption and conductivity given in Table II. The constant conductivity following the

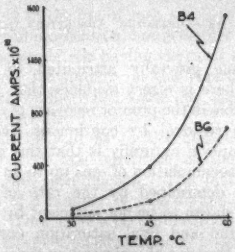

Fig. 8—Variation of 40-Min. Current at 1,500 Volts with Temperature

absorption, while of only brief duration, say about one sec. or more, is nevertheless of relatively quite long duration as compared with the period of the alternating circuit, and so it may be considered as the final constant value of von Schweidler's analysis. The initial absorption is absent in specimen B-4, being apparently masked by the greater initial conductivity which in this case is sufficient to account for all loss.

It has been suggested that the oscillations of polar molecules may account for a portion of dielectric loss even at frequencies as low as 60 cycles. Evidence to the contrary has recently been presented by Hamburger,[15] who concludes that for normal temperatures and materials this type of loss is negligible for frequencies less than 10,000 cycles. The present experiments are in accord with this view since the entire loss is accounted for by the anomalous conductivity as observed with continuous voltage.

Discussion

The fact that the initial absorption and conductivity as measured determine dielectric loss under alternating stress gives to the short-time characteristics great practical importance. For this means that they are probably also related to the commonly observed rate of deterioration of the insulating properties of these oils. A considerable proportion of the initial conductivity as ordinarily found is apparently electrolytic and due to impurities, to the products of slow oxidation, to inert suspended matter, and to residual moisture. All of these effects may be largely removed by filtration, distillation, and evacuation, and the prolonged application of electric stress further contributes to the clean up. However, even after the most careful purification by the methods mentioned, an initial absorption and a conductivity much greater than the final conductivity remain.

The question then arises, can the ultimate application of the most refined methods of purification yield a liquid which, in accordance with the idea of Kohlrausch,

TABLE II—OIL SPECIMEN B 6

Temp. (deg.) cent.	Loss in microwatts 1,500 volts 60 ~	Power factor	Initial constant current in amperes 1,500 volts d-c.	Computed losses in microwatts: From initial constant current	From absorption	Total
30	47	0.00007	2.10×10^{-8}	31.5	15.2	46.7
45	165	0.00024	8.83×10^{-8}	132.5	32.8	165.3
60	544	0.00079	30.6×10^{-8}	459.0		

shows no conductivity, or even a liquid with a minimum conductivity which is constant from the first application of electric stress? The answer to this question seems to be definitely, no. First as regards the final steady current: While Jaffé concludes from his studies on

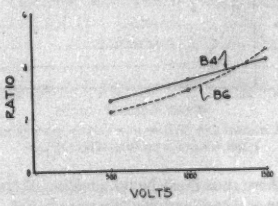

Fig. 9—Ratio of Currents at 60 Deg. Cent. Initial Constant to 40 Min. vs. Voltage

hexane and pentane, that in these liquids there is no dissociation and inherent conductivity, he was nevertheless unable to eliminate all conductivity, and only able to account for a part of that remaining as due to radio active influence. Schröder[4] concludes that in

ethyl ether there is a residual volume ionization. This however is questioned by Fassbinder,[8] who concludes that ethyl ether has no dissociation, but that the conductivity observed is the result of absorption at the electrodes, with resulting dissociation. Carvallo[9] concludes that sulfureous anhydride has no inherent dissociation and that the conductivity which can not be eliminated is due to photoelectric influence of

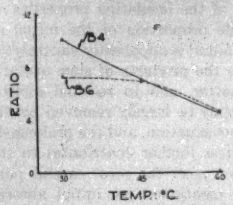

Fig. 10—Ratio of Currents at 1,500 Volts: Initial Constant to 40 Min. vs. Temperature

physical, rather than chemical type. Thus all of these investigations in which the utmost care was taken to free the liquid from impurities indicate the impossibility of completely suppressing electric conductivity, which in each case is attributed to the influence of some extraneous cause impossible to eliminate.

A somewhat more satisfactory explanation is offered by P. Walden[9] who points out that the purest sulfuric acid conducts better than some water salt solutions

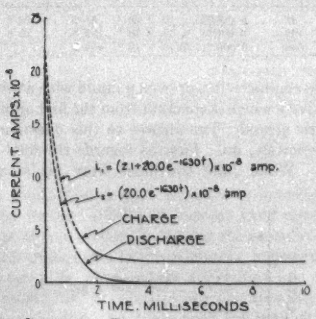

Fig. 11—Charge and Discharge Currents Specimen *B* 6 at 1,500 Volts and 30.3 Deg. Cent.

and that there thus must be no question of the possible dissociation and conductivity of a pure liquid. Further, that sulfur dioxide, while subject to electric purification as regards conductivity, is nevertheless decomposed, with resulting increase of conductivity at moderate values of electric stress. Regarding this as evidence of chemical instability, he notes that in a number of dissociating liquids containing several CO-, OH-, and CN- groups, the values of mean conductivity are in the same sequence as the dissociating power. The same is true of solvents which form multi-valued molecules and which tend to polymerization and dissociation. Mixtures of pure liquids sometimes show higher and sometimes lower conductivities, than those of the compounds. Some conducting liquids yield with difficulty to a lowering of conductivity by electric clean-up, distillation, etc., and return promptly to their initial condition. Vant Hoff[9] assumes that perfectly pure water possesses a conductivity due to dissociation of its molecules into H and OH. All of these matters point to a real lack of homogeneity even in the purest liquids and to molecular groups, probably transient, other than the simple grouping by which the liquid is usually recognized. Granted the possibility of such a departure from molecular stability, it is not difficult to picture a transient state of ionization resulting in conductivity.

As regards the initial conductivity, as stated, its excess over the final value is accentuated in the purest

Fig. 12—Dielectric Absorption in Oil

liquids. While generally attributed to electrolytic impurities, there is direct evidence that this view is scarcely tenable in the prompt recovery of conductivity following its removal by continuous electric stress. A first assumption naturally is that this conductivity is due to an accumulation of ions up to an equilibrium condition as determined by the rate of formation. This view is shown to be not sufficient by Schröder in that the initial and final saturation current values indicate that the type of ionization in these cases would be swept out far more rapidly than is indicated by the experimental current curves. It appears more likely that the initial conductivity is due to the action of the electric field on the complex molecular groupings suggested by Walden, drawing them off more slowly than the simpler type of ion, owing to their larger size. Whitehead and Marvin[7] have shown the accumulation of space charge of a very sluggish type throughout the volume of transformer oil under electric stress, and it appears probable that the resulting polarization also contributes to the slow decay from the initial conductivity. This view is also supported by the initial current-voltage curves of the present experiments in which the current increases more than in proportion to the voltage, indicating a cumulative or secondary ionization effect. It is probable that with increasing field strength a greater and greater proportion of the complex molecular groups are split up into ions.

Insulating oils such as described in this work may clearly be included in the picture suggested above. Their final conductivities compare favorably with those of the purest simple liquid. Their molecular complexity is well known and hence a large initial current is to be expected. As regards the question of losses, since they are due to the initial absorption and conductivity, improvement in insulating properties

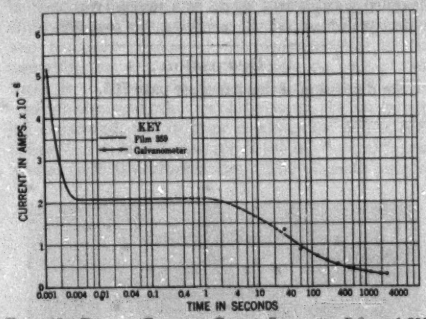

Fig. 13—Charge Current Curve Specimen *B* 6 at 1,500 Volts and 30.0 Deg. Cent.

and probably in the life of such oils is to be looked for first by careful purification, but generally by the experimental selection of oils having the most stable molecular structure; for stability in this regard should mean lower initial conductivity, lower loss, slower deterioration, and longer life.

Little can be said as to the dielectric absorption noted, owing to our ignorance of the fundamental nature of this phenomenon. The picture presented however is of a number of ions always present in the free state. They have a certain latitude of movement, restricted, however, in some way by a viscous structure of the liquid. On the application of electric stress there is a general motion, limited, but resulting in some type of permanent polarization, which is the origin of the current on discharge.

Conclusions

1. Certain high-grade insulating oils have electrical characteristics similar in type and comparable in magnitude to those of the carefully refined poorly conducting liquids, used in physical research. Conductivities of the order 10^{-16} mhos/cu. cm. are common.

2. In the purest state they show a definite though brief dielectric absorption. There are in fact three definite stages of conductivity: A. An initial brief, rapidly descending value, with corresponding curves for charge and discharge current, thus indicating the presence of dielectric absorption. The duration of this phase is of the order of 0.004 sec. B. An approximately constant value of conductivity within the interval 0.004 to 1 sec. C. A very slowly decaying conductivity, reaching a constant value only after 30 min. or more.

3. The initial dielectric absorption increases with decrease of temperature and is thus associated with the viscosity. It is difficult to measure at 60 deg. or in an oil slightly deteriorated. In these cases the absorption, if present, is obscured by the greater value of conductivity of phase B.

4. The dielectric loss under 60 cycles alternating stress is definitely determined by the continuous current characteristics within the interval 0 to 0.1 sec. The loss at the higher temperatures and for oils having high initial conductivity is completely accounted for by the conductivity of phase B. For the purest and best oils below 60 deg. the alternating loss is completely accounted for by the conductivity of phase B plus the loss computed from the curve of dielectric absorption pertaining to phase A. At 30 deg. this second component may represent 35 per cent of the total loss.

5. The initial absorption, conductivity, and subsequent decay, are not completely accounted for by residual impurities, as frequently stated. It is highly probable that they are in part due to the variety of molecular complexes and groupings occurring in such hydrocarbons, resulting in a heavy slow moving type of ion, which is subject to wide variation in properties with the value of the electric intensity and with other external influence. Thus these ions which give rise to the initial stages of conductivity are probably quite

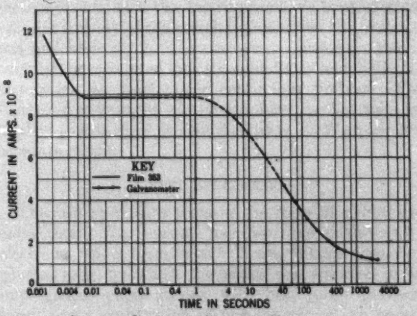

Fig. 14—Charge Current Curve Specimen *B* 6 at 1,500 Volts and 45.0 Deg. Cent.

different from those involved in the final conductivity, and which have been used by Jaffé and others for drawing analogies between the conductivity of a liquid and of a gas. There is in fact no immediate evidence that this latter type of ion is involved in the alternating loss at commercial frequencies.

The experimental results reported have been taken partly from a research on the fundamental properties of insulating oils, supported by the Engineering Founda-

tion, and partly from a research on the properties of impregnated paper, supported by the Utilities Research Commission of Illinois, both in the Johns Hopkins University. Grateful acknowledgment is made of the interest and careful work of Professor W. B. Kouwenhoven, and Drs. R. H. Marvin and S. K. Waldorf, and Mr. A. Banos, Jr.

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Discussion

For discussion of this paper see page 705.

Fundamental Properties of Impregnated Paper

BY J. B. WHITEHEAD* and W. B. KOUWENHOVEN†

Fellow, A. I. E. E. Member, A. I. E. E.

Introduction

THE excellent insulating properties of impregnated paper and their adaptability to the special requirements of the insulation of high-voltage underground cables are well known. However, the complexity of the component parts and of the processes involved in their combination has so far prevented a complete understanding of underlying electrical processes, and also complete control of the characteristics of the assembled material. The immediate purpose of this paper is to report the first results of a study of the separate electrical behavior of the paper, the impregnating compound, and of their combination, and of such indications as may be present of the relations of fundamental to final characteristics. The ultimate purpose of this study and those to follow is the hope to point out methods of control and improvement.

Fig. 1—Constant Temperature Drying, Impregnating, and Measuring Tanks

Experimental Method

In the equipment shown in Fig. 1, three tanks were provided for the test specimens. In one of these the paper was studied, in another the oil, and the third used for studies of impregnated paper. The temperature of the test specimen in each tank was determined by resistance thermometers and was automatically controlled for each specimen independently of the others to within 0.1 deg. cent. The method of temperature control is described in a paper by Kouwenhoven and Waldorf.[1]

Electrical measurements were made under both alternating and continuous voltage. Dielectric loss, power factor and capacitance, as related to temperature between 30 deg. and 60 deg. cent. were measured at alternating, 60-cycle voltage of 500, 1,000, and 1,500 volts effective. These quantities were determined with a modified Schering bridge, whose phase angle sensitivity is $\pm 5 \times 10^{-6}$ and which has permitted measurements of power factor as low as 0.00007. The special features of this bridge, its shielding and other conditions necessary to obtain this degree of accuracy are described in a separate paper by Kouwenhoven and Banos.[2]

The measurements under continuous voltage were the short and long time values of charge and discharge currents and the dielectric absorption, irreversible absorption, and final conductivity deducible therefrom. The ranges of voltage and temperature were the same as those explored in the alternating measurements. For the short time values of charge and discharge current beginning within one millisecond of the application of voltage, or of short circuit, the amplifier oscillograph as developed by Waldorf[3] was employed. This instrument increases the normal sensitivity of the electro-magnetic oscillograph 10^5 times, giving a deflection on the photographic film of 1 mm. per 4×10^{-8} amperes. The measurements with this instrument have been of great value in a study of the correlation between the continuous and alternating characteristics. Dielectric loss is completely accounted for by conductivity and dielectric absorption as read under continuous voltage. The further development of the amplifier oscillograph, as applied to this work, is described by S. K. Waldorf[4] in a separate paper.

Test Specimens

The test specimens were in the form of cylindrical capacitors. The central member, the high-voltage electrode, was a brass tube 5.08 cm. outside diameter, 61 cm. long, on which the paper was spiralled as in impregnated paper cables, with butt joints between tapes and a registration of one-quarter of the width. The outer electrodes were of 0.4 mm. sheet lead; the main electrode had a length of 25.4 cm. with a guard electrode of 7.6 cm. at each end. The main or measuring electrode and its lead were completely shielded (see Fig. 2). The entire test capacitor was suspended from the cover of the vacuum tank and all connections were brought through this cover.

A second capacitor of approximately equal dimensions was used for the oil. In this case, however, the inner

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1. For references see Bibliography.

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electrode and guards were rigidly attached to the central cylinder with bakelite spacing rings. The nominal capacitor dimensions were the same for both types, as follows: outside diameter of inner cylinder 5.08 cm., inside diameter of outer or measuring electrode 5.385 cm., total thickness of dielectric of oil, of paper, or of impregnated paper 1.52 mm. After separate measurements on paper and oil the oil was drawn into

Fig. 2—Specimen Mounted for Test

the heated vacuum chamber containing the paper sample, impregnating the same.

Materials and Preparation

A high-grade wood pulp paper 0.1 mm. thick and 2.54 cm. wide was used, 15 layers making up the specimen. The samples were dried and evacuated at 0.1 mm. Hg. (105 deg. cent.). They were brought to the same condition as regards dryness, as indicated by a standard value of final conductivity of 1.6×10^{-15} mhos per cu. cm. (105 deg. cent.). The two specimens used in the present tests, designated as A-2 and A-3, proved to have closely similar characteristics throughout all tests.

Oils

Two samples of high-grade cable oil, which will be designated as B-4 and B-6 were used. Both were from the same manufacturer, and from the same batch. The oil was received in air tight containers; nevertheless, it contained considerable air.

Oil B-4 was allowed to deteriorate slightly by raising its temperature to 70 and 80 deg. for the time intervals indicated in Fig. 3, showing the history of this treatment and its results. The entrapped air was largely though not completely removed before heating, and the deterioration, therefore, was doubtless due to oxidation. It will be noted that the results of this deterioration were a substantial increase in dielectric loss and in final conductivity. The final long time conductivity of B-4 at 45 deg. cent. was 0.70×10^{-14} mho. As shown below its value at four milliseconds after application of voltage was 4.84×10^{-14} mho. Oil B-4 was used to impregnate paper sample A-2 and the finished product was called C-1. The pressure at the time of impregnation was less than 1 mm. Hg. at 60 deg. cent. The impregnation of the paper sample A-3 took place at 0.5 mm. pressure (60 deg. cent.). The final conductivity of oil specimen B-6 at 45 deg. cent. was 0.21×10^{-14} mho, and at four milliseconds its value was 1.43×10^{-18} mho.

Experimental Results

The results are represented in a series of curves as follows: The alternating current results give the loss-voltage, loss-temperature, power-factor-voltage and capacitance-temperature relations for the specimens B-4, A-2, and C-1 in Figs. 4 and 5 and for B-6, A-3, and C-2 in Figs. 6 and 7. The comparison of the two oil samples as regards continuous current-temperature and current-voltage relations is found in Figs. 8 and 9 respectively. Figs. 10, 11, 12, and 13 give the continuous current-temperature and current-voltage data for the specimens A-3 and C-2.

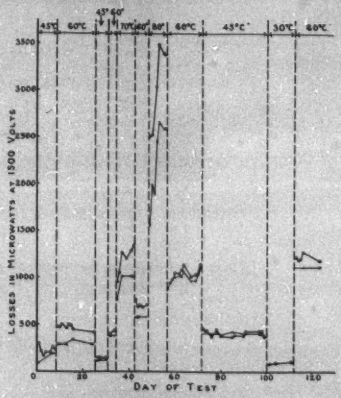

Fig. 3

Oils: A-C. Behavior. As between the two oils, as a result of the deterioration the loss and power factor in B-4 are about twice those in B-6. This ratio also extends to the observations under continuous voltage, as shown in Figs. 8 and 9. In each case there is a sharp rise of loss with increasing temperature. The power-factor-voltage curves are flat at 30 and 45 deg. but show a slow linear rise at 60 deg., being much the same in both

oils. As regards apparent capacitance, determined by alternating measurements, B-4 shows a slightly higher value and both show a capacitance decreasing with increasing temperature.

Papers: A-C. Behavior. Both papers, A-2 and A-3, show much the same characteristics throughout. The losses, closely proportional to the square of the voltage, decrease with increasing temperature. The power-factor-voltage curves are flat throughout, decreasing with increasing temperature. The capacitance increases

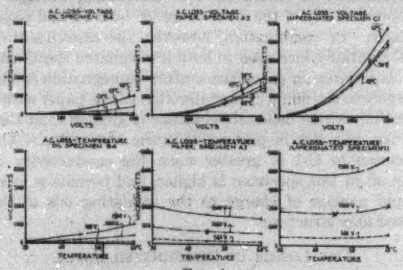

Fig. 4

with temperature. Thus the variation of both loss and capacitance with temperature is in opposite sense to that found in the oils. The decrease of loss in paper with increasing temperature may be ascribed to a decrease in dielectric absorption. The high resistance conducting films of residual moisture take on increased conductivity with increasing temperature (Evershed effect). There is

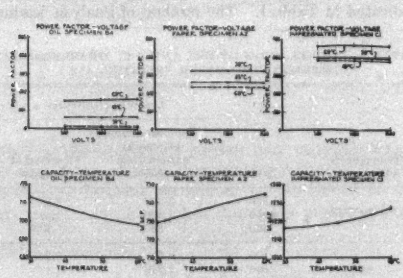

Fig. 5

thus a lower $I^2 r$ loss in the passage over them of the charging current of the main dielectric structure. At the same time there is a corresponding increase in number and of surface of these conducting filaments which means an increased effective capacitance as shown in the curves. Note, also, in support of this suggestion the decrease of both four milliseconds and eight milliseconds charging current readings with increasing temperature, (Fig. 12) with corresponding increase of final values of charging current, the former indicating decreased absorption and the latter increased conductivity.

Impregnated Paper: A-C. Behavior. As between the two impregnated specimens C-1 and C-2 there are some noteworthy differences. In each the power factor is constant with respect to the voltage and the losses increase approximately as the voltage squared. In C-1, impregnated with deteriorated oil, the loss decreases with increasing temperature from 30 deg. up to 40 deg. but increases thereafter, (Fig. 4) and the loss tempera-

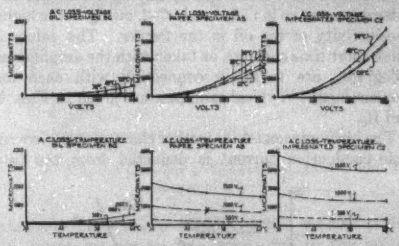

Fig. 6

ture curves have a "U" shape at all three voltages. In C-2, on the other hand, the loss decreases with increasing temperature at all voltages. In both oils the losses increase rapidly with increase in temperature, (Figs. 4 and 6) but in B-4 the greater value of the loss is sufficient to offset the corresponding decrease in loss in the paper. In B-6 the magnitude of the loss is not sufficient to offset the opposite type of variation in the paper over the temperature range studied.

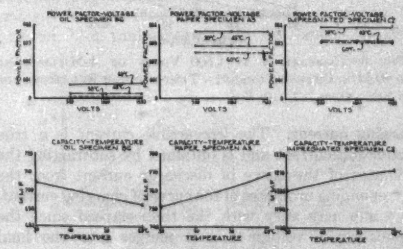

Fig. 7

It is interesting to note that the measured loss in the impregnated specimen C-1 is 1.9 times the sum of the losses in the separate samples of the oil B-4 in paper A-2, throughout the voltage and temperature range studied. In sample C-2, the ratio varies from 1.4 to 1.58. It is evident that the increase in loss following impregnation is due to the large increase in the dielectric absorption resulting in the combination of the two dielectrics, oil and paper. In this combination the oil penetrates the interstices between the paper fibers,

and as the oil has some conductivity it constitutes an electrode surface and conducting filaments thereto, for the deeper lying dielectric structure of the fibers, thus increasing the absorption current. For example, the value of the current at four milliseconds after the application of 1,500 volts on paper sample A-2 at 30 deg. was 55×10^{-8} amperes, on impregnated specimen C-1, 130×10^{-8} amperes, on paper sample A-3, 54×10^{-8} amperes, and on impregnated sample C-2, 96×10^{-8} amperes (see Figs. 10 and 11). Thus the charging current in each case is increased by impregnation, and considerably more so in C-1 than in C-2, owing to the higher conductivity of the oil in the former. The values of these short time currents, as taken with the amplifier oscillograph, are taken in connection with each loss measurement. Typical examples are given in Figs. 14 and 15.

The phenomenon is still further clarified if the irreversible conduction current is deducted from the total

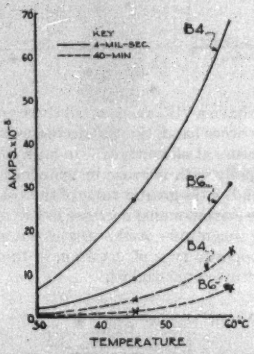

FIG. 8—COMPARISON AT 1,500 VOLTS OF 4-MILLISECOND AND 40-MIN. CHARGE CURRENT—TEMPERATURE RELATIONS FOR OIL SPECIMENS B 4 AND B 6

charging current. The irreversible current is a true conduction current and is obtained by subtracting the ordinates of the curve of discharge current from the corresponding ordinates of the curve of charging current. Its value may vary with the time elapsed since the application of voltage and it merges into the final conduction current. Since it is a conduction current, having no corresponding discharge current, the irreversible current contributes to the loss, but plays no part in dielectric absorption. Deducting the irreversible current in each of the above cases, the four millisecond and eight millisecond charging currents on samples C-1 were 83×10^{-8} and 48×10^{-8} amperes with ratio 1.73, giving a rough measure of the slope of the current-time curve. The corresponding figures for C-2 were 50 and 23, with ratio of 2.17. These values obtain for 60 deg. cent. and 1,000 volts. We note from these figures that in impregnated specimen C-2 the short time absorption current is not only lower in value, but also decreases more rapidly than in the case of C-1. Therefore the dielectric absorption is less in C-2. Thus the well-known fact that the higher conductivity in an oil leads to greater values of dielectric loss is attributable in part to an increase in dielectric absorption, and in part to increased irreversible conduction current.

A further result of the penetration of the conducting oil into the structure of the paper is found in the large increase in capacitance. As shown in Figs. 5 and 7, the capacitance as measured with alternating voltage is approximately of the same value for both oil and paper singly. In combination, however, the capacitance is substantially increased in both impregnated specimens. This increase in capacitance after impregnation is due largely to the filling of the interstices in the paper with a medium whose dielectric constant is greater than that of air, but also in part to dielectric absorption. The increase in C-1 is greater since the conductivity of the oil in this specimen is higher, and permits a more rapid passage of charge to the insulating oils of the wood fiber structure.

THE LOSSES IN IMPREGNATED PAPER

The measured values of loss in all the samples studied whether of oil, of paper, or of impregnated paper, may be computed in each case from the fundamental absorption and conduction characteristics of the respective specimens, as determined from the short time charge and discharge currents measured with continuous potential. The accuracy of this agreement between alternating current measured and direct current computed values for the impregnated specimen C-2 is indicated in Table I. The method of analysis leading

TABLE I—CORRELATION OF D-C. AND A-C. MEASUREMENTS IMPREGNATED PAPER SPECIMEN C-2

Temperature (deg. cent.)	Volts	Losses in microwatts: Computed from d-c. charge "current-time" relation	Losses in microwatts: Observed at 60 cycle
60	1,500	3,003	3,046
	1,000	1,290	1,341
	500	335	333
45	1,500	2,925	3,107
	1,000	1,388	1,378
	500	356	344
30	1,500	3,690	3,816
	1,000	1,770	1,691
	500	448	424

to this agreement has already been briefly described in a paper by one of the authors.[5] It is proposed to discuss the method in greater detail as related to the present work in a separate paper.

The general nature of the relation between direct current and alternating current characteristics may be briefly stated as follows: The total alternating loss in

insulation showing dielectric absorption is due to three factors, namely, reversible dielectric absorption, irreversible absorption, and true conductivity. The relative importance of these three quantities as determining alternating loss may vary widely. Dielectric absorption, with the sustained values of charging current which it causes, results in a component of alternating current in phase with the electromotive force, thus causing loss. The magnitude of this component depends on the initial value of the continuous current curve and on the rate at which it decreases. Both these quantities depend in an indeterminate way on the ratios of resistivity to specific inductive capacity in the two media oil and paper. The irreversible absorption current has no corresponding component on discharge and under alternating stress thus results in a true alternating conduction current and loss. The amount of this component of the loss may be quite considerable, and inas-

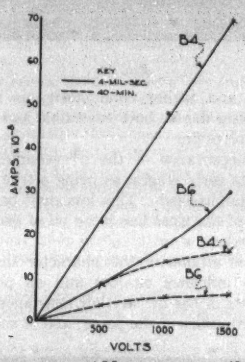

FIG. 9—COMPARISON OF 4-MILLISECOND AND 40-MIN. CHARGE CURRENTS OIL SPECIMENS B 4 AND B 6 VARIATION WITH VOLTAGE AT 60 DEG. CENT.

much as the irreversible current may be much higher for short time intervals than for long intervals, it is the short time interval, as related to the period of the alternating circuit, which is important in determining the loss. The irreversible absorption current merges eventually into the final conduction current. The amount of loss due to this final value is usually very small and its contribution to the loss negligible in the better grades of insulation. (See Table II.)

As an example of the relative values of the losses due to these several causes, refer to Table II, in which the losses at 1,500 volts and 60 deg. due to the several causes are separated for the oil alone, the paper alone, and for the impregnated paper, and for both the oils studied.

TABLE II—MEASURED A-C. LOSSES AND CALCULATED D-C. LOSSES IN MICROWATTS 1500 VOLTS—60 DEG. CENT.

Specimen	Measured a-c. losses at 60 cycles	Losses computed from continuous current records: Reversible absorption	Initial Irreversible	Final conductivity	Total
Oil B-4	1031	6*	1025	229	1031
Paper A-2	1465	1410	80	0.59	1490
Impregnated paper C-1	4892	3451	1350	3.4	4801
Oil B-6	544	85*	459	100	544
Paper A-3	1555	1410	75	0.65	1485
Impregnated paper C-2	3046	2478	525	2.1	3003

*Estimated.

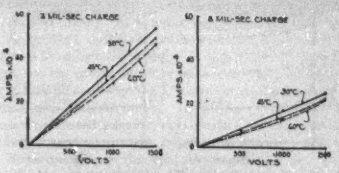

FIG. 10—CHARGE CURRENT—VOLTAGE RELATIONS PAPER SPECIMEN A 3

Note particularly that in the oil B-4 99.5 per cent of the total losses are conduction losses, due to the initial value of the irreversible charging current. There is practically no absorption, and final conductivity is submerged in initial conductivity, which is over four times as great as the final value. In the case of the papers the loss is almost entirely due to reversible absorption, that due to the initial irreversible current being only of the order of 0.5 per cent, with still smaller values for the loss due to final conductivity. After impregnation, however, the loss due to irreversible current rises from 0.5 per cent to about 30 per cent in specimen C-1, impregnated with the oil B-4 of higher conductivity, and from 0.5 per cent to 17.4 per cent in specimen C-2, impregnated with the better oil, B-6.

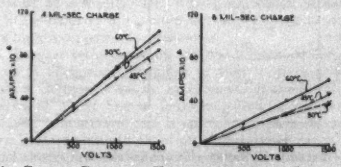

FIG. 11—CHARGE CURRENT—VOLTAGE RELATIONS IMPREGNATED PAPER SPECIMEN C 2

The remaining components of the losses in the impregnated samples are due to absorption, that in C-1 being greater than in C-2, for reasons already described above.

D-C. BEHAVIOR

The d-c. characteristics of oil, paper, and impregnated paper, and their separate influence, have already been described in some measure. Figs. 8 to 13 also present several additional matters of interest. The short time characteristics of the two oils, as related to temperature,

are shown in Fig. 8. As indicated the higher final conductivity of B-4 is reflected in about the same ratio, in the values of charging current at 4 milliseconds after the application of voltage. The variation with voltage as shown in Fig. 9 reveals a straight linear relation between current and voltage for the oil B-4, but with a tendency for a greater rate of increase for oil B-60. We have noticed this tendency also in other oils, and this is ascribed to some type of secondary ionization, which is masked as the conductivity is higher, as in

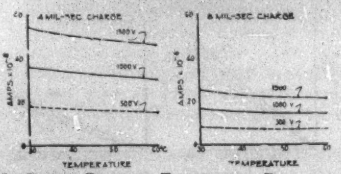

Fig. 12—Charge Current—Temperature Relations Paper Specimen A 3

B-4. Note that the long time values of charging current in both oils show pronounced approach to saturation values conforming to the behavior of the purest dielectric liquids reported in the literature of physical research.

As regards the paper, it may be noted (see Figs. 10 and 12) that the initial values of current decrease slowly with increasing temperature, while the opposite is the case of the final conductivity of dry paper.

For the impregnated paper (see Figs. 11 and 13) it is of interest to note the greater influence of temperature on the eight millisecond values as compared with the four millisecond values. The later values increase steadily with increasing temperature. This is due to the greater influence after the longer interval of the

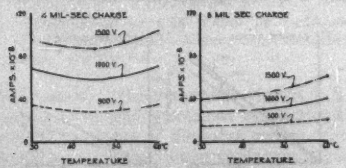

Fig. 13—Charge Current—Temperature Relations Impregnated Paper Specimen C 2

temperature increase of the conductivity of the oil. The importance of the early values of charging current in causing the loss is clearly evident because for C-2 we find the loss decreasing with increasing temperature.

Acknowledgments

The authors wish to acknowledge the help given by Messrs. S. K. Waldorf and A. Banos, Jr., in all phases of the work. Grateful appreciation is also expressed to the Utilities Research Commission of Illinois, whose financial support has made possible the initiation and active prosecution of the project.

Conclusions

1. The total dielectric losses in impregnated paper have been separated into three components, absorption, irreversible, and final conductivities, due to fundamental characteristics, which have been studied by means of continuous current measurements.

2. The relative importance of the several components has been studied for two grades of impregnating oil as applied to the same paper.

3. The importance of the initial conductivity of the oil, as distinguished from the final conductivity, is

Fig. 14

emphasized and higher conductivity is reflected in increased losses due to both reversible and irreversible dielectric absorption.

4. The importance of the irreversible absorption current in its early stages, as being a true conduction current, is emphasized. This loss may be as high as 30 per cent of the total loss in an oil of excellent insulating properties.

5. Further studies of this character should clarify the relative influence on loss and on power factor characteristics of the density, air resistance, thickness, and other properties of the paper, and of such chemical

Fig. 15

and physical properties of oil as may be correlated with conductivity.

The work is being continued.

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Discussion

THE CONDUCTIVITY OF INSULATING OILS—II

(WHITEHEAD)

FUNDAMENTAL PROPERTIES OF IMPREGNATED PAPER

(WHITEHEAD & KOUWENHOVEN)

H. H. Race: The main conclusions of Dr. Whitehead's paper are based upon the results given in Table II which shows measurements for one oil at three temperatures. The second column gives the dielectric loss as measured with a 60-cycle alternating voltage. The fifth column gives the loss as calculated from the first horizontal portion of the due current-time curve. Except at the highest temperature these two calculations do not agree and the author states that the difference is accounted for by an initial exponential characteristic of the d-c. current-time curve during a very short time after the application of potential. I should like to ask Dr. Whitehead whether he has any physical picture of a mechanism which would explain the characteristic that he has observed.

Because of the high resistivity of good oils, the currents to be measured are very small and the author deserves considerable credit for developing sensitive apparatus for determining the short-time d-c. characteristics. His conclusions, if correct, will have a far reaching effect in the development of the theory of liquid dielectrics.

For this very reason, the data deserve very critical analysis. In studying the oscillograms presented in the paper, the question arose as to whether the initial exponential portion might not be an oscillograph error rather than a characteristic of the oil. The reasons for thinking this might be true are as follows:

1. The d-c. data at 45 deg. cent. are shown by the oscillogram of Fig. 12. The measured current is so small that the deflection is only about three times the width of the line. The record for 30 deg. cent. is not shown but the deflection would be only about 25 per cent of that shown in Fig. 12 due to the increased resistivity of the oil and would be correspondingly difficult to measure accurately.

2. The transient term of the deflection of a standard galvanometer element as a function of time is of the form

$$S = (A\,\epsilon^{iat} + B\,\epsilon^{-iat})\,\epsilon^{\frac{-pt}{2m}}$$

where

$$a = \sqrt{\frac{k}{m} - \left(\frac{\rho}{2m}\right)^2}$$

ρ = frictional damping constant
m = mass of moving element
k = restoring force

For critical damping, $a = 0$ so that the transient term is a pure exponential, of the form

$$S = (A + B)\,\epsilon^{\frac{-pt}{2m}}$$

which has a time constant, $(\rho/2m)$.

When such an element is underdamped the transient term is oscillatory with a frequency of $(a/2\pi)$. If very slightly underdamped (a) might be very near zero so that the element might overshoot once and then approach the final value from above. This seems to be at least a possible explanation for the observed curve since the oscillogram shows that the element required about 0.5 millisecond to reach a maximum deflection after the low sensitivity shunt was removed and the exponential portion of the curve lasted only for about another millisecond.

3. Again, the natural frequency of a high sensitivity oscillograph galvanometer varies from 1,000 to 3,000 cycles per second which is of the same order as the time constant of 1,630 which was calculated from the observed curves.

4. If the oil used by Dr. Whitehead is similar to that tested by us, the data shown in Fig. 19 of my paper (*Some Electrical Characteristics of Cable Oils*) indicate that the 60-cycle a-c. loss would be greater than the d-c. loss at low temperatures. A 60-cycle curve would be similar to our curve (1) but below it and would probably depart from the d-c. locus at about 50 deg. cent. This offers a possible alternative explanation of the differences between a-c. and d-c. loss observed by Dr. Whitehead.

The short-time d-c. characteristic can be used to distinguish between the mechanisms of space charge formation and polar molecule orientation as explanations of dielectric loss. Therefore, it would be very valuable if Dr. Whitehead could extend both the frequency and temperature ranges of his measurements.

Wm. A. Del Mar: In a paper[1] which I presented to the A. I. E. E. in 1922, in collaboration with Mr. C. F. Hanson, a connection was shown between the resistivity of petrolatum compound and the power factor of paper impregnated therewith. This curve was plotted through 18 experimental points and connected them quite satisfactorily. In the discussion of this paper Mr. Hanson presented a similar curve with over 40 points, showing a similar correspondence between resistivity and power factor. Subsequent experimental work enabled us to obtain curves with several hundred points lying very close to them. The curves were plotted from a formula based on the assumption that the entire dielectric loss was ohmic in character. The resistivities were all measured with direct current after one minute electrification.

Dr. Whitehead now tells us that we should not use the resistivity measured after one minute, but after about 1/10th second electrification. If his conclusions are correct, there must be a definite relation between the resistivity after 1/10th second and after one minute electrification for a given oil. This, however, is not borne out by the data on oils, *B*-4, and *B*-6, in the Whitehead-Kouwenhoven paper which are said to be identical except in regard to aging.

The resistivity-power factor relation which holds so well for petrolatum does not hold for cylinder-oil-rosin type compound. Possibly this may be due to different ratio between the 1/10th second and one minute electrification resistivities. Whatever the explanation, it is an undoubted fact that to obtain a given power factor the resistivity of this type of compound does not have to be as high as where petrolatum is used. As resistivity measurements made after 1/10th second electrification are difficult to make, whereas those made after one minute electrification present no difficulty, Dr. Whitehead would be doing a service to the industry if he would correlate these tests with the object of improving power-factor control.

C. L. Dawes: The paper by Messrs. Whitehead and Kouwenhoven is particularly valuable in that it gives a definite explanation, substantiated by experiment, of the so-called *V*- or *U*-curves of power loss in impregnated-paper insulation as a function of temperature. As is well known, these characteristic curves are frequently obtained with impregnated paper, either when in test samples or in cables. In this type of characteristic the minimum value of power occurs usually in the region between 30 deg. cent. and 50 deg. cent. Many explanations of this characteristic have been advanced. Hochstädter probably came nearest to giving the correct explanation, in view of the experimental results obtained by the authors of this paper. (See London *Electrician*, Vol. 89, August 18, 1922, page 180.) He attributes the *U* or *V* curves to the combination of a component having a drooping loss-temperature characteristic and a component having a rising loss-temperature characteristic just as the authors do. However, instead of attributing the drooping characteristic to the paper alone as has been done by the authors, he attributes it to hysteresis loss in the insulation when it is in the more viscous or the solid state which occurs at the lower

1. *Effects of the Composite Structure of Impregnated Paper Insulation on its Electrical Properties*, A. I. E. E. TRANS., Vol. XLI 1922, p. 563.

temperatures. When the insulating compound becomes more fluid at the higher temperatures, the resistance loss predominates and this increases rapidly with the temperature. Hochstädter found that *V* curves were obtained with a mixture of beeswax and rosin alone; the oils alone tested by the authors do not give *V* curves, but have continually rising power-temperature characteristics such as Hochstädter obtained with beeswax and rosin at higher temperatures. The difference at the lower temperatures between Hochstädter's results and those of the authors is probably due to the much greater viscosity of the beeswax and rosin at these lower temperatures. Also, Hochstädter attributed the rising characteristic almost entirely to resistance or conduction losses and this also appears to agree with the results obtained by the authors, as may be seen by a study of their Table II. Thus in general the authors and Hochstädter appear to be in rather close agreement. The only difference of any moment arises from Hochstädter's attributing the drooping hysteresis loss characteristic to the more viscous condition of the impregnating compound while the authors find it to be due to the loss in the paper alone.

It follows that to obtain the *U* curve, the impregnating oil must have low loss relative to that of the paper. Thus it may be inferred that when impregnated paper has a *U* curve of power loss, a relatively low-loss impregnating compound is implied.

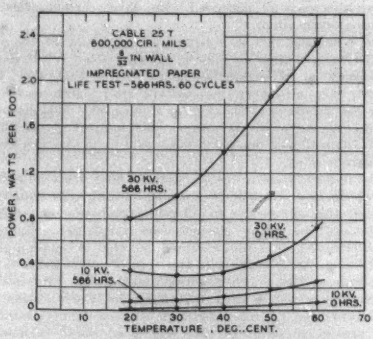

FIG. 1—EFFECT OF LIFE TEST ON POWER-TEMPERATURE CHARACTERISTIC OF CABLE

In a number of tests of impregnated paper samples which has been conducted at the Harvard Engineering School, this *U*-shaped curve has not as yet been obtained. That is, the power loss has always increased with temperature. In order to make comparison with the samples tested by the authors, I have assumed that the loss in their Figs. 5 and 7 increases as the square of the voltage gradient. Under these conditions our samples showed slightly less loss for any given temperature than those of the authors. This might indicate that a *U* curve should have been obtained with our samples, but such was not the case. Perhaps the power loss in our paper was very low.

Frequently, however, we have obtained these *U* curves with cables. Fig. 1, herewith, shows four power-loss-temperature curves taken at 60 cycles for a 600,000-cir. mil, 6/32-in. (1.76-mm.) wall paper single-conductor cable. Two of the curves were taken at 10 and 30 kv. before the cable was put on test. Two more curves were obtained at these same two voltages after the cable had been subjected to 566 hours of accelerated life tests with weekly cycles of voltage gradient variation (from zero to 225 volts per mil average) and temperature variation (20 deg. cent. to 60 deg. cent.).

The curves taken at 10 kv. show no indication of the *U* characteristic. The initial curve at 30 kv., however, has a decided *U* characteristic, whereas the curve taken at the conclusion of the life test rises rapidly and has no *U* characteristic. This undoubtedly indicates that under the time and cyclic applications of stress and temperature the impregnating compound underwent deterioration. This resulted in the compound having higher relative loss than the paper and thus eliminating the *U* curve obtained with it initially, and a continually rising characteristic resulted. It would have been interesting to know the extent to which the characteristic of the paper itself had been affected by the life test.

It is well known that at the higher temperatures the power-factor curve, when plotted as a function of voltage, usually is not flat. It may increase or it may decrease with voltage gradient. Such results have been obtained by many investigators and also at Johns Hopkins University. (For example see Whitehead and Hamburger, *Impregnated Paper-Insulation*, TRANS. A. I. E. E., January 1928, Vol. 47, p. 314.) However, the authors state that for both specimens of impregnated paper the power factor is constant with respect to the voltage. This is substantiated by their Figs. 5 and 7 to the accuracy which can be obtained from their very small scale. I am asking the authors if they have any explanation as to why their values of power factor at 45 deg. and 60 deg. cent. are constant, as this is not generally true.

I believe that the data and the curves which the authors present would be much more useful if they were more fundamental in character and did not involve the geometry of their apparatus. For example, microwatts per cubic cm. is much more fundamental and much more useful for comparative purposes than the total microwatts of their particular samples, which they give in the paper; likewise, dielectric constant is much more fundamental and useful than the capacitances of their particular samples.

The authors have, however, made a distinct contribution to our knowledge of the effects of the components of the dielectric loss in impregnated-paper insulation, such for example as the determination of the direct and alternating current losses in the paper and oil individually and when in combination.

S. K. Waldorf: Dr. Race has suggested that the small decay in the charge current records on the oscillograms, an example of which is given in Fig. 12 of Dr. Whitehead's paper, may be due to an over-travel of the oscillograph vibrator, rather than due to absorption in the oil. By recording the response of the apparatus to d-c. impulses, it has been shown that the oscillograph vibrator used was over-damped rather than under-damped, and did not over-travel. Additional tests with an air condenser at high voltage, and with a mica condenser and resistance at low voltage, have proved the reliability of the amplifier-oscillograph. The tests are described in greater detail in a paper by the writer, which is to be presented at a meeting of the Institute in the near future.

Further, discharge records on oils have been obtained indicating the same decaying current on discharge, verifying the presence of reversible dielectric absorption in the oils.

Dr. Race has also raised the question as to the accuracy of the measurement of oscillograms. It is our practise to measure recorded deflections to a tenth of a millimeter, which gives a value of current accurate to within about 5 per cent or better on oscillograms having the smaller deflections. With larger deflections, the accuracy of such measurements is of the order of 1 per cent and better. It has been found by taking additional oscillograms that the results are reproducible, and the probable accuracy of the measurements thereby improved.

J. B. Whitehead: Dr. Race has asked what is meant by "dielectric absorption" in connection with these studies. Ever since the discovery of the residual charge of a Leyden jar physicists, and more recently engineers, have speculated as to the nature of this phenomenon. The term dielectric absorption is commonly used to describe it, evidently in the idea that the

residual charge is in some way absorbed in the dielectric. Maxwell has shown beyond question that a mixture of different solid dielectrics will account for the phenomenon of absorption. He also recognizes that a similar phenomenon might arise in other causes. This has been clearly shown in many experiments since his time, as for example, in the anomalous conductivity of liquids, *i. e.*, a conductivity which may vary with the time and show departures from Ohm's law. Solid complex dielectrics often show evidence of this type of conductivity, which causes them to show departures as regards dielectric absorption from the simple conception of Maxwell.

While all liquids, even the purest, show anomalous conductivity, such as mentioned, it has always been supposed that they do not show the phenomenon of residual charge or dielectric absorption. We have shown, however, in our paper the presence of a small and short-lived dielectric absorption in an insulating oil, which is sufficient to account for the excess of dielectric loss, as measured, over the maximum loss which may be accounted for by conduction. My answer to Dr. Race, therefore, is that in our experiments we have encountered in an oil the same phenomenon that in the case of solids is usually described as dielectric absorption.

There has been a noticeable tendency recently to attribute the excess of measured loss under alternating stress, over that which may be accounted for by conduction, to a molecular frictional loss in terms of the Debye theory of the oscillation of dipolar molecules. There is no question that this theory offers a convincing picture and appears to account satisfactorily for dielectric losses within the upper ranges of frequency and for highly viscous media. For low frequencies, say 60 cycles, and for viscosities ordinarily encountered in insulating oils, no convincing evidence has been presented that the observed losses contain any appreciable component of the Debye type. It is the suggestion of my paper that for low frequencies and low viscosities, the greater portion, if not all, the loss may be accounted for by the initial or short time conductivity, including dielectric absorption.

Answering Mr. Del Mar's comments, we have found as a usual thing that as the conductivity of an oil increases, say due to oxidation or the presence of impurities, the difference between the initial short-time conductivity and the long-time conductivity becomes less and less. The permanent or final conductivity then masks the effect of the initial condition, which apparently is due to different causes. In such a high conductivity oil, the initial and final conductivity being the same, the loss and hence the power factor is completely accounted for by the conductivity measured at one minute or later.

It is difficult to compare Mr. Del Mar's results on petrolatum with our studies in oil, because he has correlated the conductivity of the petrolatum with the power factor of the impregnated paper. The separate influences of both oil and paper on the impregnated product are so complex that the facts before us do not seem to warrant discussion. I may say, however, that so far we have not been able to find any direct correlation between the one-minute conductivity of an oil and the subsequent properties of impregnated paper. It is our hope that through the more intimate study of the fundamental properties of both oil and paper that we may be able eventually to indicate those fundamental properties which do have most important bearing on the final behavior.

W. B. Kouwenhoven: Professor Dawes data in regard to the *U* loss-temperature characteristics of impregnated paper are interesting. In Fig. 4 of our paper we show a *U*-shaped curve for specimen C-1. Specimen C-2 has a falling loss-temperature characteristic for the range plotted in Fig. 6. However, we made loss measurements of this specimen at temperatures above 60 deg. cent. We found that the minimum loss occurred at a temperature between 60 deg. and 75 deg. cent. and, therefore, this specimen also has a *U* characteristic when the temperature range is extended. In general we believe that for a given sample of paper the minimum loss occurs at higher and higher temperatures as the quality (lowness of loss) of the impregnating oil is improved.

In the impregnated specimens C-1 and C-2 the power-factor voltage relations are so nearly constant that they appear in Figs. 5 and 7 as straight lines. The actual values of power factor at various voltages and temperatures from which these curves were plotted are given in the table below:

Voltage	Specimen C-1 at temperature cent.			Specimen C-2 at temperature cent.		
	30 deg.	45 deg.	60 deg.	30 deg.	45 deg.	60 deg.
500	0.00399	0.00385	0.00461	0.00389	0.00315	0.00305
1,000	0.00397	0.00382	0.00465	0.00389	0.00315	0.00306
1,500	0.00394	0.00377	0.00463	0.00388	0.00315	0.00306

A study of this table indicates that the values in most cases vary slightly with voltage. It must also be kept in mind that the voltage gradient in our specimens at 1,500 volts applied is 25 volts per mil. At low gradients there is in good impregnated specimens very little change of power factor with voltage.

Traffic Control by Light Beams
For Major-Minor Intersections

BY R. C. HITCHCOCK[1]
Member, A. I. E. E.

Synopsis.—*The object of this traffic controller is to keep the signal green on the major street, and to give the green signal to the minor street only when requested. The minor street vehicle can always get the green signal after a short wait, while the major street receives green light for as much extra time as the minor street traffic permits.*

Two experimental horizontal beam installations, which accomplish the above object, are now in operation in Wilkinsburg, Pennsylvania. The focussed light beam, phototube unit, relay panels, and associated circuits are described in this paper.

* * * * *

THE installations described were developed by the Westinghouse Research Laboratories to obtain working information on traffic control systems and field experience on phototubes illuminated at a distance from light sources. They are experimental and the equipment is not available in commercial form.

When traveling on a major highway, it is annoying, as well as wasteful of time, to stop at a red traffic signal, when there is no car waiting to use the green light which has been flashed on the minor highway. To keep the light green all the time on the major, and give the right-of-way to the minor street only when necessary[2] a light beam can be focussed on a phototube across the minor street, so that when a car intercepts the beam the traffic light would be flashed green on the minor street.

The first installation was in East Liberty, Pennsylvania, at the corner of Stanton and Highland Avenues. The second installation was at the corner of Coal Street and South Avenue, the latter being parallel to the William Penn and Lincoln Highways, in Wilkinsburg, Pennsylvania. South Avenue carries a fraction of the heavy main highway traffic. This installation is illustrated in Fig. 1.

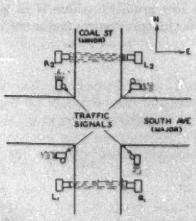

FIG. 1—SECOND INSTALLATION SOUTH AVE., AND COAL ST.

The next step was to make an installation on the main highway itself. This third controller is at the corner of Penn and Trenton Avenues, Wilkinsburg, Pennsylvania, Penn Avenue at this point carrying all the traffic of the William Penn and Lincoln Highways. This intersection was selected because of the heavy traffic on Penn Avenue, the density often reaching one thousand cars per hour.

In some localities having wide boulevards, independent and combining streams of traffic can often be allowed to move continuously. For instance, a right

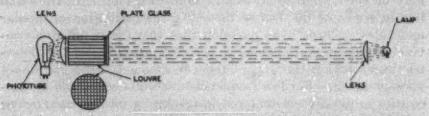

FIG. 2—OPTICAL SYSTEM, LENSES AND SECOND INSTALLATION LOUVER

turn is sometimes permitted on a red signal, when the turn is made into a street having four traffic lanes. The turning vehicle keeps to a curb lane, and the through vehicle takes the adjacent lane. The majority of Pittsburgh streets are narrow, consequently all turns are made only on the green signal.

The system to be described is designed to handle only cross traffic on major-minor intersections. One reason for following this idea is the large number of potential installations. Probably there are more major-minor intersections than there are of the major-major type. Another strong reason is that the installation is fairly foolproof, a regular timer motor being provided to control the intersection if anything should go wrong in the automatic control.

Devices are in use which are actuated by the traffic on both intersecting streets, and no specified program is used. Such equipment has to be very carefully designed and should be inspected frequently, for if failure occurs in any unit, the signals may remain as they were at the time of failure. With the major-minor device to be described, the failure of any of several units causes the vehicle controlled device to be automatically disconnected and the timer motor runs on the predetermined cycle.

TYPICAL TRAFFIC CYCLE

Using a one minute cycle, a typical timing for a

1. Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.
2. The first work on this device was done by Dr. P. Thomas of the Westinghouse Research Laboratories.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

major-minor street is 42 sec. major street green, 3 sec. amber, 12 sec. minor green, and 3 sec. amber. Thus the ratio of green light for major and minor streets is 42/12 = 3.5:1, yet this timing is frequently used at an intersection where the traffic ratio is from 10:1 to 50:1, where 1,000 cars per hour and more are passing on the major street. The reason for the short major cycle is to allow the minor street

FIG. 3—THIRD INSTALLATION, WM. PENN AND LINCOLN HIGHWAYS, WILKINSBURG, PA.

cars to enter after a maximum waiting time of 45 sec. This is done with the new automatic controller, while allowing the major street as much more than a 42-sec. green signal period as the minor traffic permits. The object of the major-minor street traffic controller is to keep the major street traffic moving with a continuous green signal, and, following the cycle just mentioned, to allow minor street traffic to gain a green signal after a maximum wait of 45 sec.

This has worked out quite well in practise—sometimes the major street gets a green signal for as much as five minutes, but after the first 45 sec., the controller allows a minor street vehicle to get the right of way in 5 sec., or whatever time delay is set to allow for major street turns.

Recently a movement has been made in Pennsylvania to eliminate the use of traffic lights on main highways, especially where they pass through small towns. The regular cycle timer undoubtedly works a hardship on the major traffic lane by flashing a red signal when there is no need for it. However, the use of a minor street controlled traffic signal always gives the major street a certain minimum green signal, and at the end of this minimum time the signal is not changed until a minor street vehicle requests it by intercepting the light beam.

LIGHT BEAM DIRECTION

The first installation used vertical light beams, but the grease and dirt which fell on the part of the system placed below street level[3] suggested a change in light beam direction. In the second and third installations, the light beams are horizontal and the cleaning problem is therefore decreased. Since the second installation was put in, several details have been improved and up to the present time it has been working for over five months with only three minor interruptions. The third installation has been operating for over a month, with no interruptions.

One interesting problem presented by the horizontal device, was that cars turning from the main street would cross the minor street light beam, and if no provision were made to allow for this, the light would be flashed green on the side street, with no cars waiting for it. The solution was to introduce a time delay, requiring a car to stop in the beam for several seconds before any action took place. This was so timed that cars making the turn would not stay long enough in the beam to cause operation. It is usual for a driver to stop as he drives up to a red light, and it is felt that it is only right for a minor street driver to wait for his turn of the green light. The usual time that the car must wait is 3 to 5 sec.

RELIABILITY

Probably the main advantage of both the original and revised forms of the controller is its reliability. If any of several minor accidents should happen, the regular predetermined program period is followed. Some of these possible minor accidents are; burning out of the lamp sending the light beam across the street,

FIG. 4—CAR INTERCEPTING LIGHT BEAM THIRD INSTALLATION

a blown fuse in any of the four control boxes, a collision between a car and one of the control boxes, or dirt collecting on any of the glass parts which transmit the light from the lamp to the receiving phototubes.

If any of the above things should happen, the major and minor streets divide the cycle and the control is the same as a regular program traffic timer. When the device is operating properly, the major street receives

3. For this reason the first installation has been discontinued.

as much extra green light as the minor street traffic allows.

After five months' operation only three interruptions have occurred—two of these due to dirty lenses and resulting from the apparatus going on "program" operation; and the third due to a loose connection which resulted in one relay box being out of service for about two hours.

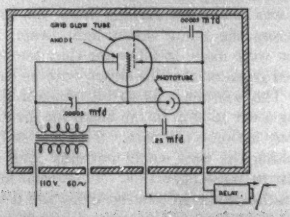

Fig. 5—Phototube-Grid Glow Tube Circuit

One of the lamps burned out after 4⅓ months, and after 4½ months one of the original glow tubes was replaced. As will be shown in explaining the next installation, data are now available on getting more light across at the lower filament voltages, which will increase the life. Glow tubes with longer life are now

Fig. 6—Mounted Phototube, and Grid Glow Tube with Adjusting Condenser

being made. Just before a grid glow tube fails, its upper or lower operating limit, as shown by the variable condenser setting, increases or lowers, respectively. Grid glow tubes sometimes fail glowing, or fail to glow, depending on the construction. For the circuit shown in Fig. 5 it is an advantage to have the tube fail glowing, so that the program goes on the regular program cycle, and this type of tube is used. However, by making inspections and recording the upper and lower condenser limit, the failure can be predicted a few days before it happens. It is, of course, quite easy to replace tubes.

Third Installation

Penn Avenue carries through traffic on U. S. routes 22 and 30, the William Penn and Lincoln Highways. At the intersection of Penn Avenue with Trenton Avenue the third installation was put in working order on

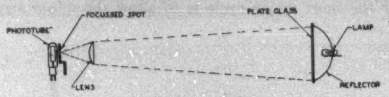

Fig. 7—Optical System, Reflector Lens and Focused-Spot Third Installation

September 16, 1930. An illustration of the intersection, showing by circles the location of the control boxes, is given in Fig. 3. Fig. 4 shows one light unit, and its receiver, with a car intercepting the beam. The centerline of the light beam was placed low enough to work with any car now on the market.

The grid glow phototube unit is shown in Fig. 5.[4] The use of paraffin-impregnated cork washers and a clamping device[5] was found to keep the tube bases free

Fig. 8—Rear View of Reflector Unit, Lamp Mounting, Rheostat and Switch

from moisture and dirt. The wiring, as shown in Fig. 5 is inside a small casting, the upper panel being of moldarta, and screwed to the casting with a paraffin-cork gasket to keep it tight. The transformer and

4. The theory and operation of grid glow tube circuits has been covered by articles by Mr. D. D. Knowles in: *Elec. Journal*, Apr. 1928, p. 176, *Elec. Journal*, Feb. 1930, p. 116; *Elec. Jour.* Apr. 1930, p. 232; *Electronics*, July 1930, p. 183.

5. E. H. Vedder, *Elec. Jour.*, Apr. 1930, p. 224.

condenser are enclosed. The no-load loss of the transformer provides sufficient heat to prevent condensation.

Tests were made of the phototube-grid glow tube unit, in a humidifier, and even with the humidity at 100 per cent the unit ran satisfactorily for several hours. In getting 100 per cent humidity water was sprayed over the unit, at times, and still the paraffin-cork washers and gaskets held their seal.

The glass of the phototube is painted black with the exception of the front. This keeps daylight out of the tube so that adjustments may readily be made with the back of the control box open. A shield in front of the phototube, has a small hole on which the light source is focussed. Most of the extraneous light is kept out, by this focussed hole. In the second installation, a louver was used to keep out the extraneous light. The complete optical system of the third installation is shown in Fig. 7.

The light system used in the second installation is shown in Fig. 2. A lens focusses an image of the lamp filament on a plate glass disk, the light rays being practically parallel. To prevent light from any other source from entering the phototube, a louver is provided. This is a square sectioned honeycomb, which passes light parallel to its axis, but stops light which enters at an angle of a few degrees. For convenience in mounting the louver, the lens in front of the phototube is placed with its plane side facing the lamp. The phototube plate is covered by light from the lens, space being saved by moving the tube closer to the lens than would be possible if the light were brought to a spot on the phototube plate.

It is difficult to figure the efficiency of these optical systems, but experimental results show that a fairly good parabolic reflector is better than any reasonably priced lens of similar diameter. The rear view of the lamp unit box, showing the reflector, is given in Fig. 8. The lamp socket has two knurled clamp nuts which allow motion of the lamp socket in and out, and a large hole in the support permits lateral motion. A resistor is provided in series with the primary of the transformer so that the lamp can be run at the desired voltage.

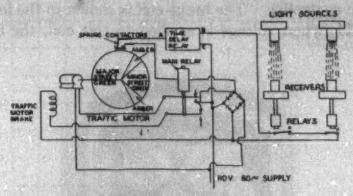

Fig. 9—Schematic Diagram Third Installation

The schematic wiring of the third controller is shown in Fig. 9 and the detailed wiring in Fig. 10. As shown in Fig. 9 the major street signal is green, the main relay is pulled up, power is off the motor, and the brake is on. When a car crosses the light beam momentarily as in turning from the major street into the minor, the light controlled relays pull up, and the circuit wires marked *A* and *B* of the time delay relay are open circuited. However, unless the car remains in the beam for the specified time delay there is no other action in the controller. If, though, the car does wait in the beam

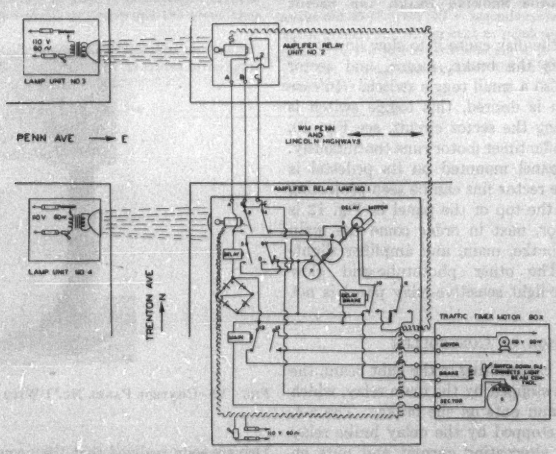

Fig. 10—Detailed Wiring, Third Installation, Penn and Trenton Avenues

for three seconds, circuit $A\ B$ is opened and then the traffic motor starts up, and the cycle begins, giving amber then green to the minor street.

In Fig. 10, the traffic motor is shown at the right, and has the usual sector and brake. Fig. 11 shows a simplex traffic timer motor as adapted for use in the automatic controller. The brake coil is shown to the left of Fig. 11, when voltage is applied to this coil, the eddy

Fig. 11—Traffic Motor Showing Brake and Sector

currents induced in the disk cause it to slow down. A small panel contains the brake, motor, and sector binding posts, and also a small toggle switch. In case the regular program is desired, this toggle switch is pressed down, opening the sector circuit, see Fig. 10, so that the motor traffic timer motor runs continuously.

The main relay panel mounted on its pedestal is shown in Fig. 12, the rectox fins can be seen projecting from the back. At the top of the panel in Fig. 12 is the time delay motor, next in order come the main delay, delay motor brake, main, and amplifier (light-sensitive) relays. The other phototube-grid glow tube unit and single-light sensitive-relay panel is not shown.

Operation of Controller

When there are no cars waiting in the light beam, the traffic timer motor is stopped by the main relay, which takes off the power and puts on the brake. The a-c. delay motor, too, is stopped by the delay brake relay which takes off the alternating current and puts on direct current.

If either light relay is pulled up due to a car intercepting the beam, two connections are made; (1). the delay relay pulls up, transferring the main relay control to segment 8 of the delay motor; (2). the delay brake relay opens, releasing the delay motor brake and putting alternating current on the delay motor. If a car stops in the beam, segment 8 makes contact for 5 seconds, and as it opens the main relay also opens, takes off the brake and starts up the traffic timer motor.

If the car does not wait in the beam, but just intercepts the beam momentarily, as in making a turn, the delay relay opens, the main relay connection is transferred from the delay motor segment 8 and delay relay contact 6 D to the delay relay contact 6 E, and the timer motor is not started up.

Control boxes 1 and 2 have only one fuse system, that of box No. 1. If the power goes off, or a fuse blows, the main relay has no power to pull up, and when it is in the released position the traffic timer motor is supplied with voltage, and runs on the regular cycle.

Reception by Public

A yellow line was painted on the street, and in the third installation the right hand control box was painted yellow and an illuminated sign reading, For Green Light Stop Opposite Yellow Box, was provided. For those drivers who never seem to stop for yellow lines on the street, or read signs, there soon appeared a group of self-appointed "traffic directors" who, living in the neighborhood, knew how the device operated—and the motorists were instructed to pull up opposite the yellow box.

Due to the newness of the device, traffic during certain hours of the day, has increased on Trenton Avenue.

Fig. 12—Control Panel No. 1 With GS Unit on Pedestal

Drivers soon realized that they could get a green signal on the minor street by driving up and intercepting the light beam. Even so, the major street always gets its regular quota of the green signal.

Short time statistical studies indicate that the time of red light flashed on Penn Avenue using the automatic controller, is from 50 to 70 per cent as much as was flashed when using the regular cycle timer.

ACKNOWLEDGMENTS

The writer wishes to express his appreciation of the cooperation of the Wilkinsburg officials, especially Burgess W. M. Turner and Chief of Police W. R. Bishop.

In the Westinghouse Research Laboratories, much credit is due to Dr. P. Thomas for his original work on the system. Thanks are due to Mr. D. D. Knowles and Dr. E. D. Wilson for furnishing appropriate grid glow and phototubes. The writer also wishes to express his thanks to Mr. E. F. Critchlow for his valuable assistance in this work.

Discussion

F. Hamburger, Jr.: I have been very much interested in this method of traffic control for major—minor intersections. Particularly the presentation showing the time saved in this method as compared to straight automatic control in a predetermined cycle. I notice however that the paper states that a 12-second green is allowed for minor street traffic. For light traffic this is, I think, still too long a time; possibly 6 seconds would normally be sufficient and a saving of time would result.

I should like to ask whether it is possible with this control to set the time to, for example, 6 seconds for the minor green and provide a step-up feature or time extension to permit a longer minor street green in the event that several cars follow each other on the minor street. This system would still further decrease the wasted time.

R. C. Hitchcock: The installation described is at a corner where street cars make turns. The 12-second allowance permits an automobile, following a street car, to pass before the light flashes red again. If no street cars were present, a shorter fixed green time could undoubtedly be used to advantage. Experimental circuits are being devised to proportion the green time to the vehicle density.

Recently a statistical study was made of the relative time lost when using different traffic light cycles.* The curves in the figure show the time wasted for two fixed program controllers, the theoretical best controller, and the present automatic (light beam actuated) controller. The abscissa is the traffic ratio, and the ordinate is the time wasted in car seconds per hour.

The average automatic cycle wastes 62 per cent as much time

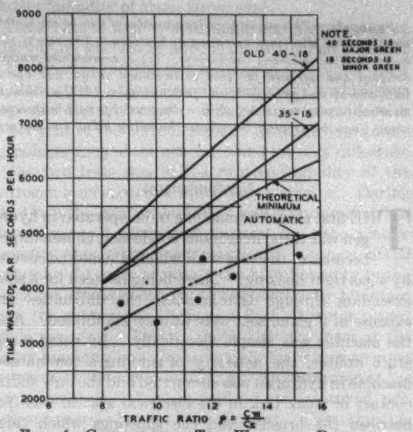

FIG. 1—COMPARISON OF TIME WASTED BY DIFFERENT CONTROLLERS

as the old 40-18 cycle (40 = seconds major green; 18 = seconds minor green), and 80 per cent as much as a theoretically perfect cycle. In the present installation, at a traffic density ratio of 15:1 the average saving, over the original cycle, is 3,100 car seconds per hour.

*Electric Journal, May 1931, page 318.

Commutation and Current Collection in Hydrogen

BY ROBERT M. BAKER[1]
Associate, A. I. E. E.

Synopsis.—This paper describes a number of tests on commutator machines and slip-rings operating in hydrogen. The results seem to warrant the following conclusions: (1) A commutator machine designed and adjusted for good commutation in air will operate satisfactorily and give good brush life in hydrogen. (2) If a commutator machine must spark in hydrogen, the brush life may be increased many times by keeping the relative humidity below ten per cent. (3) The contact resistance of carbon brushes on a commutator may be lowered materially by the introduction of hydrogen. (4) Carbon or graphite brushes cannot be operated satisfactorily on plain tool-steel slip-rings running in hydrogen, and (5) The contact drop between a carbon or graphite brush and a brass slip-ring may be ten times as high when the ring is running in air as when it is running in hydrogen.

Introduction

THE first electrical machine to be operated in hydrogen was the synchronous condenser, chosen largely because of the ease with which it could be covered by a gas tight housing. There being no need for a shaft extending through the housing, the difficulties and expense of a gland seal were at once eliminated. Also, the machine was simple electrically. By using a separate exciter, the necessity of running a commutator machine in hydrogen was eliminated and the only sliding contact or weak link in the electrical system was that between the brushes and the slip-rings which carry current for the rotor. The next advance will probably be the application of a direct-connected exciter to one of these machines and this will require the exciter machine to be operated in hydrogen. If it is proved that commutation can be satisfactorily accomplished in hydrogen it may not be long before the hydrogen-cooled synchronous converter will make its appearance. Therefore it would seem that much depends upon the accumulation of knowledge on the general subject of commutation and current collection in hydrogen. It represents a field which until just recently had received no attention and whether or not there will arise any new and serious difficulties is yet to be seen.

This paper is not intended as a complete treatise of the subject but is only a beginning. It describes a few tests made on commutator machines and slip-rings running in hydrogen. Although not extensive these tests establish a few facts quite conclusively.

Commutation Tests

The set-up used for these tests is shown in Fig. 1. The machines are respectively;—a one-hp. d-c. driving motor and a five-hp. d-c. generator which was operated with its armature short-circuited. Electrical connections through the bed-plate facilitate metering, and make it possible to regulate the degree of commutation on the generator by shunting current from the commutating pole winding. The tank, shown suspended, fits into a mercury seal in the bed-plate, and has a window in each end for viewing the commutators of the respective machines. The gas connections are also in the bed-plate, the gas being carried in at one corner and out at another. Upon starting each new test it was necessary to pass considerable hydrogen through the tank to wash out the air, but after a good hydrogen purity was once established it was sufficient to merely maintain a small positive pressure in the tank. No quantitative data were taken on the brushes of the driving motor and the commutation on this machine was maintained good throughout the tests.

Fig. 1—Commutation Test Set

A hygrometer installed in the tank after the first two tests showed the existence of some rather surprising conditions. For instance, it was thought, at the time the first two tests were made, that the machines were running in a dry hydrogen atmosphere. The hygrometer showed that this was not true. The upper curve of Fig. 2 shows how the humidity in the tank varied during one test which was run under full load and good com-

1. Research Engr., Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

mutating conditions. Hydrogen was passed into the tank during this test at the rate of about 10 or 15 cu. ft. a day. The gas in the supply line was found to be quite dry and it apparently absorbed moisture after passing into the tank. It was found that by leaving the inside of the tank and the machines exposed to the air for only a short time during a shutdown, it was possible to reduce quite materially the initial value of relative humidity in the tank. The lower curve of Fig. 2 was taken after an exposure of only a few hours.

TABLE I—CALCULATED BRUSH LIFE OF GENERATOR BRUSHES

			Calculated brush life in days			
Test No.	Duration of test—days	Atmosphere	Brush No. 1	Brush No. 2	Brush No. 3	Brush No. 4
1	15.0	Air	588.0	229.0	812.0	362.0
2	6.41	Hydrogen	94.4	9.2	128.0	10.6
3	6.78	"	222.0	205.0	282.0	161.0
4	7.0	"	43.7	59.9	69.3	45.0
5	15.0	"	1,875.0	1,770.0	2,000.0	2,310.0

Brush life is calculated on the assumption of one inch allowable brush wear. Brushes Nos. 2 and 4 are the two positive brushes of the generator and are on opposite brush arms.

Table I gives the results of five brush life tests made on the short-circuited generator. All tests were made

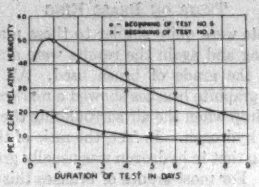

FIG. 2—HUMIDITY OF GAS IN TANK VS. DURATION OF TEST

with full-load current in the brushes but the generator was not always adjusted for the same grade of commutation. Tests, Nos. 1, 2, 3 and 4 were made with 30 per cent of the current shunted from the commutating pole winding of the generator. The sparking at the trailing edge of the brushes was quite severe, and naturally resulted in rather short brush life even under the most favorable conditions. Test No. 5 was made with almost sparkless commutation.

The most interesting comparison is that between tests Nos. 1 and 2, which were made under identical conditions except that the former was made in air and the latter in hydrogen. In air the minimum calculated brush life was almost a year whereas in hydrogen it was only a little more than a week. Furthermore the damage to the commutator during the test in hydrogen was considerable. The trailing edges of the commutator bars were burned away, leaving a bright metallic surface. In some cases the depression was as much as 1/32 in. This first test in hydrogen was quite unique in that it showed such a great difference in wear between the positive and negative brushes. Both tests described were made before the hygrometer was installed and consequently the relative humidity of the medium in which they were run is not known. It is very likely however, that the test in hydrogen was started with a relative humidity of about 50 per cent and ended with a relative humidity of about 20 per cent.

Tests Nos. 3 and 4 were made for the purpose of determining whether or not the unusually bad behavior of the brushes and the commutator in test No. 2 could be attributed to the high relative humidity which was suspected during this test. These tests were made with 30 per cent of the current shunted from the commutating pole winding of the generator and the only difference in the two tests was in the relative humidity of the hydrogen atmosphere in which they were run. During test No. 3 the relative humidity was maintained at less than 10 per cent; whereas during test No. 4 the relative humidity was maintained practically constant at 50 per cent. In order to obtain the high humidity in the latter test it was necessary to bubble the dry hydrogen from the supply line through a flask of water placed inside the tank. Table I shows that the wear at the higher humidity was approximately four times as great as it was at the lower humidity. An inspection of the commutator gave further evidence of improved performance during the low-humidity test. Upon shutting down test No. 3 the commutator appeared dark and uniformly smutted, but apparently no damage had been done. A light application of fine sand paper showed the commutator to be in perfect condition. This was not true of the commutator at the end of test No. 4. It was apparent immediately upon shutting down this test, that the commutator had been badly burned. The bright copper was showing on the trailing edge of the commutator bars just as it was after test No. 2, and the roughness could not be removed by any reasonable amount of sanding. It is interesting to note that brushes Nos. 1 and 4 which were on the side of the machine nearest the moisture supply, show somewhat shorter life than brushes Nos. 2 and 3 which were on the other side of the machine. In conclusion it may be said that if a commutator machine is running in hydrogen and sparking badly, the presence of moisture is detrimental to both the brushes and the commutator.

So far, all the tests described were made with the generator adjusted to give severe sparking at the brushes, the idea being to determine what would happen if this condition should arise in service. The various troubles encountered under this severe condition should not lead the reader to believe that commutation in hydrogen is especially difficult or that it will usually be associated with troubles such as roughened commutator and rapid brush wear. To show that operation in hydrogen may be quite satisfactory when the commutating conditions are normal it is only necessary to refer to test No. 5, the results of which are tabulated in the last line of Table I. This test was made while the generator was delivering full load current, but

the shunt had been removed from the commutating pole winding. It was during this test that the data for the upper curve of Fig. 2 were taken. The minimum brush life of almost five years and the good condition of the commutator at the end of this test would seem to indicate that as long as a machine is designed and adjusted for good commutation in air, it will behave quite satisfactorily in hydrogen. This last test was repeated several times and the same good results were always obtained.

Slip-Ring Tests

The apparatus used in these tests consisted of a collector or set of slip-rings driven by an adjustable speed d-c. motor and so arranged that the space around the rings and motor could be filled with hydrogen. The arrangement was similar to that shown in Fig. 1 except that the d-c. generator was replaced by a set of

Fig. 3—Brush Faces Showing Damage Done by the Formation of Hard Spots

slip-rings. Each ring was 7/8 in. wide and 9½ in. in diameter and was trimmed with one brush 5/8 in. wide by 1¾ in. thick by 2 in. long. Direct current was supplied to the brushes and rings by an arc-welding set. Electrical leads, brought out through the bed-plate made it possible to measure the contact drop of any brush.

A test consisted simply of allowing a brush to operate on a slip-ring in air or in hydrogen and observing the results. Most of the tests were made with a current density of about 40 amperes per sq. in. and a peripheral ring speed of 2,500 ft. per min. The duration of a test was determined entirely by the behavior of the brush and ring. Some tests lasted only a day whereas others were continued for more than a week.

Tests on Steel Ring

The first tests were made on a tool steel ring trimmed with different grades of carbon and graphite brushes. The results of these tests were always the same. Lumps of hard material, sometimes 0.02 or 0.03 in. in diameter, were formed in the brush face, and immediately began to score the ring. Upon analysis, the hard material proved to be cementite (Fe_3C) which had apparently been formed by a direct combination of iron and carbon in the presence of localized high temperatures in the contact. Fig. 3 shows two brushes, each of which was operated for about 24 hours with a density of 40 amperes per sq. in. on a tool-steel ring in hydrogen. The brush on the right is a fairly hard carbon brush whereas the one on the left is a very soft graphitic brush. Some of the hard particles still remain in the face of the soft graphitic brush although most of them have fallen out. The difference in the appearance of these two brush faces is undoubtedly due to the relative ease with which the hard particles could burrow through the material of the respective brushes.

In an attempt to prevent the formation of hard spots in the brush face, a helical groove[2] was cut in the face of the tool steel ring. With this change it was found that a number of brush grades which failed on the plain ring could be operated without difficulty. The formation of hard particles in the brush face was eliminated and the ring acquired a good polish.

Tests on Brass Ring

These tests were quite similar to those made on the tool-steel ring and again the results were almost independent of the grade of brush used. All grades of carbon and graphite brushes showed a marked decrease in contact drop when the air surrounding the rings was replaced by hydrogen. In fact, the ratio of the contact drop in air to that in hydrogen was usually greater than ten to one. For most grades of brushes this reduction was permanent although other grades, after running for a while, began to smut the ring, and when this occurred the contact drop increased to about the same value it would have in air. A tabulation of contact drop values in air and in hydrogen for three grades of brushes is

TABLE II—CONTACT DROPS IN AIR AND IN HYDROGEN

	Contact drop in air		Contact drop in hydrogen	
Brush grade	Brush pos.	Brush neg.	Brush pos.	Brush neg.
A	1.34	0.88	0.18	0.17
B	1.30	0.92	0.08	0.11
C	1.18	1.05	0.08	0.10

Contact drops given in volts.
Grade A—Graphitic.
Grade B—Similar to A.
Grade C—Carbon brush slightly graphetized.

given in Table II. The time required for the contact drop to change from the high to the low value or vice versa was quite variable. In some tests an hour or more was required for the change, whereas in others the change was almost instantaneous.

No attempt will be made to explain the phenomenon

2. This device is described in *Application of the Helical Groove to Slip-Rings and Commutators*, by G. M. Little, see page 718.

disclosed by the above tests, but it was noticed that the results were peculiar to the particular brass ring on which the tests were made. Similar tests were made on the brass ring of another test set and it was impossible to even approach the low values of contact drop obtained in the first brass-ring tests. The only explanation which can be found for this disagreement in behaviour is the difference in the compositions of the two brass rings. This would also explain the fact that the steel slip-ring in the earlier tests showed only a slight decrease in contact drop when hydrogen was introduced.

When it was first discovered that a very low contact drop could exist between a carbon brush and a brass slip-ring, it was realized that such an occurrence on the commutator of a d-c. machine might prove disastrous. As a result the contact drop of one of the brushes on the generator used in the commutation tests was measured both in air and in hydrogen. The average contact drop in hydrogen was found to be only about twenty per cent less than the contact drop in air, and on the small machine this was not enough to make any visible change in the grade of commutation obtained.

Conclusions

It is realized that the tests reported in this paper are not very complete but from the results of the tests described the following tentative conclusions may be made:

1. A commutator machine designed and adjusted for good commutation in air will operate satisfactorily and give good brush life in hydrogen.

2. If a commutator machine must spark in hydrogen the brush life may be increased many times by keeping the relative humidity below 10 per cent. This will also prevent damage to the commutator.

3. The contact resistance of carbon brushes on a commutator may be materially lowered by the introduction of hydrogen.

4. Carbon or graphite brushes cannot be operated satisfactorily on plain tool-steel slip-rings running in hydrogen.

5. The contact drop between a carbon or graphite brush and a brass slip-ring may be ten times as high when the ring is running in air as when it is running in hydrogen.

Discussion

For discussion of this paper see page 722.

The Application of the Helical Groove to Slip-Rings and Commutators

BY G. M. LITTLE[1]
Member, A. I. E. E.

Synopsis.—*An outline is given of experiments showing that a sliding-brush-slipring contact, even with well seated brushes, only makes electrical contact at a comparatively small number of minute spots at any given instant.*

Differences are shown in the properties of the contact film formed on a grooved and on a plain slip ring.

Commerical applications of the helical grooves are mentioned.

It is shown that a helical groove cut in the face of a slip-ring or commutator has the following effects:

(1) It provides a means for forcing a more uniform distribution of current over the brush face, and between parallel connected brushes.

(2) It permits the escape of air trapped under a brush.

(3) It prevents local spot heating and glowing under overload conditions.

(4) It raises the contact drop of the brush, producing better commutation and cooler operation.

(5) In some cases, an increased brush life.

* * * * *

THE performance of brushes on slip-rings and commutators is dependent on many more or less obscure conditions. Some of these are fairly well understood and are under control, so that for the most part it is pretty well known what current density, spring pressure, peripheral speed and grade of brush are best for a given brush application. There are many purely mechanical conditions that are undesirable such as loose bars, worn bearings, poorly balanced armatures, rough tracks (in traction service), friction, vibration, and loose connections, and there are dirt, oil, acid fumes and many other obstacles to good brush performance.

Even though all of the above mentioned external conditions are satisfactory, there are still difficulties in transmitting current to and from the stationary brushes and the rotating slip-ring or commutator. It is usually found that the current is not divided evenly between parallel connected brushes and this overloads some of them.

The present article is the result of an investigation of some of the causes of trouble that exists right at the sliding contact between a brush and a rotating slip-ring or commutator.

Much is said in this article about overloads and the concentration of current, at the brush face, into one or more spots which get very hot. It is a most plausible supposition that under the *normal* running condition of a brush, which is well fitted to a slip-ring (or commutator), that the current flows across the sliding contact *at all times* through a multitude of tiny contacts distributed over the *entire polished face of the brush*. There is good evidence that this is only partly so. It is shown that with a well fitting brush riding on a smooth slip-ring, there is only a comparatively small number of minute contact points or groups of points concerned in carrying the current, *at any given instant*. Under light pressure there may be from 3 to 6 active current carrying spots, while under increased pressure, 50 to 100 spots may be shown to be active.

The demonstration consisted in discharging a large condenser across the contact between the brush and the slip ring. This discharge, estimated to be at least 3,000 amperes, was over in only a few millionths of a second, and was so rapid that, though the slip-ring was revolving at 1,500 r. p. m. the pattern burned on the face of the ring and on the face of the brush consisted of a group of well defined minute spots, and the distribution of these spots over the brush face was found never to be the same in successive experiments. These spots constitute

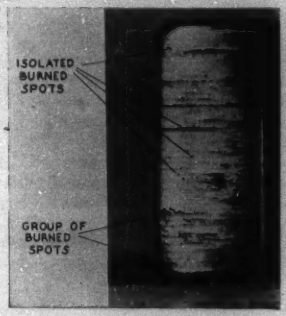

FIG. 1—SPOTS PRODUCED ON BRUSH FACE BY THE DISCHARGE OF A LARGE CONDENSER (3,000 AMPERES)

Apparently only a very minute portion of the brush face area is carrying current at any given instant

less than 3 per cent of the total area of the polished brush face. (Fig. 1).

It is probable that under normal running operating conditions the current at each successive instant of time finds a different group of contact spots to carry it across the sliding contact. These groups shift around with incredible rapidity as was shown by a method suggested by Mr. R. M. Baker, who proposed the use of a double filament oscillograph to record simultaneously but separately the current flowing through the two halves of a divided brush. In the oscillogram (Fig. 2) the

1. Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

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current is seen in some instances to drop to zero in one half while at the same instant the other half is seen to be carrying all of the load (oscillogram made by Mr. Mebbs and the writer).

It was observed that if the current density was very high, a brush was likely to glow and throw off white hot streamers. This glowing manifested itself as a small stationary or slow moving white hot spot in the polished surface of the brush. This of course could not be seen directly, but it heated the surrounding brush material for 1/8 of an in. in all directions to a good red color and if the small white hot spot was near to the edge of the brush the red glow of the surrounding brush material was readily seen.

A study of this glowing, confirmed in the writer's mind the correctness of the usual explanation of the cause which was based on the negative temperature coefficient of carbon. If a brush is overloaded and one part of the brush face should by chance be a little warmer than another part, the current will tend to

Fig. 2—Oscillogram Showing Fluctuations in the Current between the Leading (No. 1) and the Trailing (No. 2) Halves of a Divided Brush

crowd into this warmer area raising its temperature more and more until a low-resistance hot spot is developed, which then carries substantially all of the current.

It was observed that a hot spot of this kind will remain in one location on the brush face for many hundred revolutions of a commutator or slip-ring, during which time all sparking of the brush will be stopped. This tendency of a spot to take all the current and to hold to one position on the brush face is further promoted by the local expansion of the hot carbon. This expansion causes the hot spot to become slightly raised above the polished level surface and the brush rides on this "hill" until it is worn down. During this time most of the spring pressure is concentrated on the minute raised area.

The local temperature rise due to friction is thus probably a large contributing cause for bringing about the stationary hot-spot condition.

A very convincing observation of this action was made on a badly overloaded glowing brush riding on a slip-ring. The expansion at the hot spot was so great that it raised the brush off of the slip-ring noticeably and it was possible to see through the gaps on either side of the hill.

Fig. 3 shows the surface of a carbon brush that had been glowing. The light colored spots, around which are seen semi-circular cracks, show where the current had concentrated. This had raised the local temperature so high that it had changed the dark colored carbon into the lighter colored graphite.

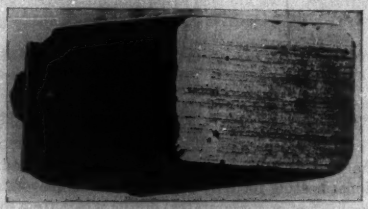

Fig. 3—Face of a Brush That Had Been "Glowing" and Throwing off "Streamers"

Note the light colored graphite spots, the semi-circular cracks, and the black cavities left when a graphitized spot falls out

When the cracks go clear around, a little white hot semi-spherical particle falls out and is thrown off by the rapid rotation as a brilliant streamer.

Narrow crosswise grooves in the surface of a rotating commutator, produced by undercutting the mica, are not able to break this up, because a hot spot has sufficient area that it can bridge the small gaps without interference to the current flow. Also if the current was broken, there would not be enough time for a spot to cool appreciably before it again made contact. When a brush is badly overloaded the hot-spot temperatures

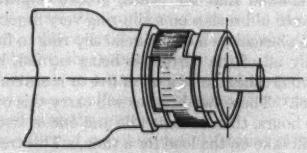

Fig. 4—The "Straight Around" Offset Groove

may rise to 2,000 deg. cent., sufficient to graphitize the minute areas affected.

With these observations in mind it occurred to the writer that if the current passing through the hot spot could be blocked and some other portion of the brush face, or even another parallel connected brush, could be forced to take the load for a portion of each revolution of the commutator, during which period the hot spot would have an appreciable time to cool; perhaps a more uniform distribution of current under the brush face and

between parallel connected brushes would result, together with an absence of local spot heating and glowing.

Two schemes were evolved, (1) a row of side-by-side, straight around, offset grooves (see Fig. 4) and (2) the helical groove, (see Fig. 5). Both schemes were tried and gave similar results but the helical groove was easier and cheaper to make and could, if necessary, be cleaned more readily. In both schemes a given spot on the face

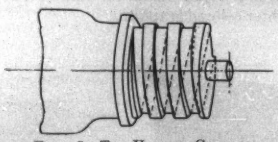

Fig. 5—The Helical Groove

It forces a more uniform current distribution between parallel connected brushes and diminishes any tendency to glow

of the brush can carry current and be subjected to frictional heating for only a portion of each revolution of the slip-ring or commutator, during which time some other spot or group of spots is forced to carry the load. Any air trapped under the brush face which is a contributing factor in uneven current distribution is permitted to escape as are any loose abrasive, solid particles of brush or ring material. Some years ago Messrs. Shand, Wagner, and Eschholz demonstrated that, in the case of parallel-connected metal graphite brushes running on a slip-ring, air was trapped under the brush face. This was illustrated by drilling a hole down through the brush and connecting it to a manometer tube. Eight or ten in. of water pressure was sometimes found. Crosswise grooves were placed in the face of the slip ring to let this air escape and it was found that more uniform distribution of current followed. These experiments did not reach a commercial success because of the tendency of the grooves to fill with dirt.

Tests showed that the helical groove improved the performance of brushes on a slip-ring very much. It is not at all unusual on a commercial slip ring to find that practically all of the current is being carried, by only one or two brushes out of a set of five or more connected in parallel. These two brushes will carry this overload for some hours, then become idle and one or two of the others will take on the load for a time. This treatment works a serious injury to the flexible copper connections, the shunts, oxidizing them and raising the resistance of the joint between the shunt and the brush. The overloaded brushes waste away much faster than those which are carrying their correct rated load.

Tests were then made to determine what effect grooving a slip-ring had on the contact resistance. Use was made of two copper slip-rings, one plain and one grooved, mounted on the same shaft. Each was equipped with one positive and one negative brush and the rings were so arranged that they could be operated either in series or in parallel.

At the start with clean sand-papered rings and brushes, the contact resistance on both the rings was extremely low but the grooved ring brush contact was almost always much lower than that of the plain ring. The resistance of both rings then slowly rose, until finally the resistance of the grooved ring and brushes became about 20 per cent higher than that of the plain ring and its brushes; this sometimes took 20 or 30 hours.

At the beginning of this test, it would appear that no film or polish having a high resistance was present and the bare surface of the grooved ring had the lower contact resistance even though the available contact area was much smaller, because the pressure per square inch exerted by the spring on this smaller area was raised, in this particular instance, to about three times that on the plain ring. As the test proceeded, there was apparently a very slowly formed high-resistance film or polish, on the sliding surfaces, and this film on the grooved ring and its brushes finally became higher in resistance than that which formed on the plain ring and its brushes. With certain brushes, (such as Stackpole L21) running on a plain and a grooved ring connected in parallel, and supplied with a constant current it was usual to find, after the day's run, that the warm grooved ring had the higher resistance and was taking only 5 amperes while the warm plain ring was taking 10 amperes. On shutting down for the night and starting again in the morning, cold, both rings were found to have acquired a very

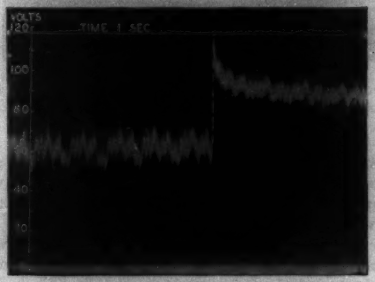

Fig. 6—Voltage Change on a Plain Slip-Ring as Current is Suddenly Stepped Up from 1 to 7 Amperes

Note that the voltage is very high during the first fiftieth of a second. By means of the helical groove, the current carrying time of a given spot can be limited to less than 1/50 of a second. This keeps the temperature down and the resistance up, giving a desirable higher contact drop.

high resistance, and usually the grooved ring was initially the lower of the two. This grooved ring would start off with ten amperes while the plain one took only five. This condition often lasted for hours though finally a return to the former state always occurred as the rings became warm.

While the difference in the character, between the slowly formed film of the grooved ring, from that on a

plain ring is probably the main reason for the difference in the contact resistance (the 20 per cent increase), there is another well demonstrated cause:

An oscillograph shows that on a plain ring a sudden forced increase in current density at the contact between a brush and a ring causes a large, but rapidly diminishing, increase in the contact resistance (see Fig. 6). It would appear that with a suddenly applied current an appreciable time is required for the temperatures of the current carrying contact spots to rise, during which time the resistance (owing to the negative temperature coefficient) remains comparatively high. The helical groove scheme furnishes means of control whereby the duration of time that any given spot on a brush face can carry a heating current and maintain frictional contact may be limited to 0.02 of a second or less; and from an inspection of the oscillogram as given in Fig. 6 it appears reasonable to believe that this should result in an increase in the contact resistance.

This temperature lag effect on the resistance of the sliding brush contact is shown if a 25-cycle a-c. current is used. The repeated changes from 0 to the maximum current value and back occur so quickly that the temperature of the current carrying contact points does not change appreciably and as a consequence the voltage and current are in phase and the resistance remains constant, as is shown by an oscillogram in Fig. 7. This is further confirmed by slowing down the frequency so much, (to 3 cycles per minute) that a very appreciable period of time lapses while the current is passing through one cycle. The oscillogram then shows that the voltage curve is distorted and the resistance changes enormously, as in Fig. 8, as the current values slowly change.

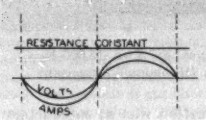

FIG. 7

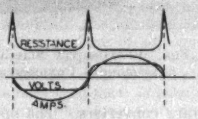

FIG. 8

FIG. 7—CURVE (FROM OSCILLOGRAM) SHOWING THAT VERY RAPID RECURRING FLUCTUATIONS IN CURRENT DENSITY ARE NOT FOLLOWED BY FLUCTUATIONS IN TEMPERATURE AND RESISTANCE

FIG. 8—CURVE (FROM OSCILLOGRAM) SHOWING THAT SLOW RECURRING FLUCTUATIONS IN CURRENT DENSITY ARE FOLLOWED BY CORRESPONDING CHANGES IN TEMPERATURE AND RESISTANCE

LABORATORY TESTS ON SLIP-RINGS

A test[2] of five parallel-connected, metal-graphite brushes on a plain bronze slip-ring gave frequent exhibitions of one or two brushes taking all of the current, while later the same brushes run on a grooved ring were free from this trouble.

Another test showed that with a grooved and a plain ring in series, and with only one brush on each ring, the plain ring brush sparked badly, the grooved ring brush operated perfectly, while the set was subjected to a heavy overload.

Still another test was made on plain steel rings operating in an atmosphere of hydrogen. Concentration of current and local overheating caused pitting of the brush face and the formation of extremely hard particles (apparently iron carbide) which became imbedded in the face of the brush and rapidly scored the surface of the ring. Grooving this same ring did away with the scoring of the ring and the coarse pitting of the brush face.

LABORATORY TESTS OF THE HELICAL GROOVE ON COMMUTATORS

The laboratory investigation of the application of a helical groove to a commutator has disclosed some interesting information.

One of the first trials was made on a defective d-c. 2,000-ampere, 500-volt generator.

This machine would carry ½ or ¾ load very satisfactorily, but on full load trouble would develop in a few hours time, and on load and a half, the glowing, sparking, and streamers would develop within ten minutes.

After the commutator was grooved ⅜ in. pitch, ⅛ in. groove, the machine carried full load indefinitely (over a week) with no glowing and stood the 50 per cent overload for 3 hours; also it was noted the grooved commutator ran 10 deg. cent. cooler than the plain commutator. Later this same grooved commutator machine was operated at full load and 8 out of the 44 brushes were removed; no distress was observed. This suggested the possibility of building shorter and less expensive commutators.

A long (200 hr.) test on a plain, and later, grooved ⅛-in. ridge, ⅛-in. groove d-c. traction motor, 40-amperes, 500-volts, gave an improvement in sparking, a 7 deg. lower commutator operating temperature, and left the commutator in better condition at the end of the test. The current distribution between the parallel connected brushes was more even.

A short test on a large a-c. single-phase railway motor showed a more uniform distribution of current between the brushes on the same arm.

One odd and possibly useful feature of the helical groove is, that if sparking occurs at one spot only in the revolution of the commutator, *e. g.*, at a high or a low bar or a flat, the fact is shown by the formation of a peculiar pattern burnt on the face of the brush and this can serve to locate the offending bar by rotating the commutator until the burned pattern rests accurately on the ridge. This is illustrated in Fig. 9.

It appeared from the above data that grooving a commutator probably does much more than simply prevent the formation of hot spots.

The cooler operation suggested that the wasteful circulating current that occurs under a brush face during

2. Taken at the laboratory of the Stackpole Carbon Co.

ordinary commutation had probably been reduced, and this could only be explained by supposing that in some way the contact resistance had been raised.

The observed reduction in the sparking at the trailing edge of brushes on a grooved commutator, may be explained by the increased contact resistance, which served to cut down the volume of circulating current which the spark had to handle, but there is another explanation that may be even more plausible.

It is common belief that an arc (a spark) can be formed in air, only, if the temperature of the separating contacts is high enough to ionize the air and make it a conductor.

As the helical groove on a commutator prevents any large rise in temperature of contact spots under the brush face, it is very probable that the small arc-starting spots at the trailing edge of a brush and under the brush face are, to some extent, prevented from reaching the temperature necessary to ionize the air and start the spark.

The depth of the groove has little or no effect on the performance, though it is reasonable to expect that a deep groove might promote cooler operating due to the increased radiating surface and the larger amount of cooling air that is dragged across the brush face.

The width of the groove and the pitch of the thread are being studied and it would appear that to secure the best results in different kinds of service, these dimensions will be varied considerably.

When a helical groove is cut in a commutator or slip ring the available sliding contact area is reduced, and with no change in the spring pressure, the pressure per unit area is raised.

If this increase in the unit area pressure is prevented by lowering the spring pressure on the grooved ring brush, a rise in the contact resistance is more easily and surely shown. It is possible to lower the spring pressure on a grooved ring to a greater extent than can be done on a plain ring without causing sparking but this must not be done if there is any vibration.

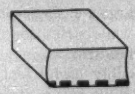

Fig. 9—Pattern Burned on a Brush Face if There is Sparking at One Spot on a Grooved Commutator, *e. g.*, at a High Bar

The appearance presented by a grooved commutator while it is rotating, conveys an impression that there must be a powerful action tending to drag the brushes toward one end of the shaft, but no such tendency exists as the brushes slide on the tops of the ridges.

When grooving a commutator, it is very desirable to stone the surface after the groove has been cut in and the edges slightly rounded, as any burr left is disastrous to brush life and sand papering alone is not effective. For subsequent polishing, a very fine sand paper backed by a wooden block, is best.

Commercial Applications to Slip-Rings

The above slip-ring tests were so convincing that when the information was given to one of the public service companies, they decided to cooperate in an experiment on the rings of one of their 13,000-ampere

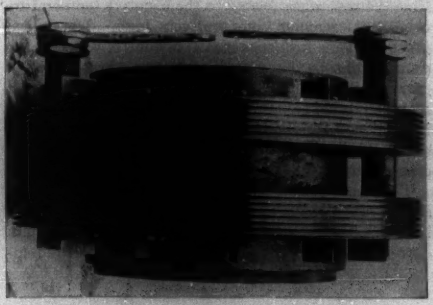

Fig. 10—Grooved Slip-Rings on Two 15,000-Kva. Synchronous Condensers (Hydrogen Enclosed)

rotary converters. The operation was so satisfactory that they are now having another machine grooved and put into service. Another public service company is putting in operation, a large synchronous condenser equipped with hydrogen enclosed steel slip-rings with helical grooved surfaces.

Fig. 10 shows a ⅛-in. wide groove ⅛-in. wide ridge, ½-in. pitch ring for a 15,000-kva. synchronous condenser.

Discussion

COMMUTATION AND CURRENT COLLECTION IN HYDROGEN

(Baker)

THE APPLICATION OF THE HELICAL GROOVE TO SLIP-RINGS AND COMMUTATORS

(Little)

T. T. Hambleton: It will be of interest in connection with Mr. Baker's paper to comment briefly on similar work the General Electric Company has done.

In the summer of 1929 the General Electric Co. had undertaken to equip a 3,000-kw., 10,000-ampere, 300-volt, 60-cycle converter with a gas tight enclosure for a trial of operation in hydrogen. This machine was erected in the customer's plant early in the year 1930. But for various reasons, among them being load requirements, operation in hydrogen was not tried till November 1930. Several trial runs each of several days duration, have since been made in hydrogen. The results show a reduction of windage loss of approximately 25 kilowatts, or eight-tenths of one per cent increase in efficiency at full load and increased cooling efficiency.

Means were provided for observing from outside the enclosure, the temperatures of main and commutating field coils and com-

mutator. The results obtained as compared with operation in air check fairly well with what would be expected based on the thermal characteristics of hydrogen.

It was found when operating in hydrogen, that the rating of the converter could be increased 25 to 30 per cent with approximately the same ultimate temperatures in current carrying parts as were reached at the lower rating when operating in air. The same quantity and temperature of cooling water was maintained in these tests.

The reduction in commutator temperature was much greater than in the other parts. Its temperature rise at full load in hydrogen was just 50 per cent of that measured when running in air. This is accounted for by the large reduction in brush contact voltage drop and possibly some reduction in friction together with the higher efficiency of cooling medium. The appearance of the commutator was much better than is customarily seen in air; the surface being highly polished and entirely free from any trace of film or deposit. Commutation was very good both in air and hydrogen. No material difference could be noted.

The one condition which will determine whether or not hydrogen cooling can be applied to large converters relates to a-c. brush wear. A certain rate of a-c. brush wear has been accepted in the past where machines have been serviced and cleaned on an average of once per week. But when a converter is enclosed in hydrogen, this running period must be lengthened to two or three months in order that the cost scavenging gas and replacement of hydrogen may be warranted.

It is probable that the rate of a-c. brush wear acceptable with open converters will not be permissible with this greatly lengthened period between cleanings. The problem of a-c. brush dusting is therefore the principal one to be solved. Its solution may be found in some entirely different grade of brush; in some arrangement of gas filter to remove the dust; in isolating and providing suitable insulation for the a-c. brush gear; or in devising some arrangement for blowing out and cleaning the surface of the insulation by means of a hydrogen blast which can be applied without the necessity of opening the enclosure.

W. C. Kalb: Mr. Little has presented a very valuable contribution to the literature on current collection in rotating electrical machinery. His conclusions appear sound from a theoretical viewpoint and are well supported by laboratory tests. It will be interesting to note the results obtained from more extensive application under operating conditions, especially on commutators.

The fact that the current passing between a brush and the slip-ring or commutator is carried at a given instant by a few points of small area, has been well established. It is probable, however, that many engineers have not realized the extremely high-current density momentarily existing at these points, nor the current density which must be reached before glowing is obtained. A test made by the writer a few years ago indicates that the local current density when glowing occurs, must be well over 1,000 amperes per sq. in.

Two graphitized brushes of approximately half and half lampblack and petroleum coke composition were mounted on a bronze slip-ring 12¾ in. in diameter and driven at a surface speed of 2,430 ft. per min. The brushes were 2½ in. by 1 in. by 1 in. with the contact face reduced to an area 1 in. by ¼ in. To insure firm contact, a tension of 18 lb. per sq. in. was applied to this reduced area. Contact drop of positive plus negative brushes was measured with varying current density and the results obtained are indicated by Fig. 1. It will be noted that this curve is of conventional form up to 360 amperes per sq. in., when the rate at which contact drop increases began to accelerate. The curve, however, continues regular in form up to 780 amperes per sq. in., when it took a sudden drop and readings from that point on through still higher current densities were very irregular. While visible glowing was observed in only four instances, at current densities of 820, 830, 850 and 860 amperes per sq. in., it seems probable that above 780 amperes per sq. in. glowing was taking place under the brush face practically all the time. This accounts for the erratic character of the readings at the very high-current densities.

The increasing increment in contact drop above 360 amperes per sq. in., contrary to the commonly accepted theory that contact drop tends to reach a fixed upper level, is probably due to

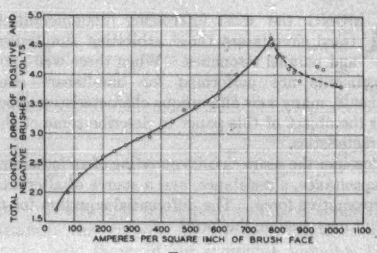

Fig. 1

the difference in coefficient of expansion of the bronze ring and the carbon brush. On this particular test the arc of contact was not large enough to observe such an effect but on another test with a larger arc of contact it was observed that at the very high-current densities, when the rate of increase in contact drop began to accelerate, a line of light could be seen between the ring and the central portion of the brush face. The brush was bearing only at the extreme leading and trailing edges, indicating that the expansion of the ring had given it a longer radius than the brush face and probably accounting for the unusual increase in contact drop.

Robert M. Baker: I wish to thank Mr. Hambleton for his discussion which I feel is a valuable addition to my paper. Mr. Hambleton does not specify the actual decrease in contact-drop of the commutator brushes in hydrogen, although in view of the results of slip-ring tests reported in my paper this decrease may have been as great as 90 per cent.

It is my belief that the problem of slip-ring brush dusting can be solved most easily by resorting to an entirely different grade of brush.

Studies in Non-Linear Circuits

BY CHAUNCEY GUY SUITS*

Non-member

Synopsis.—*The series circuit consisting of an inductancy L, capacitance C, resistance R, and a source of alternating electromotive force, where the circuit elements are all linear, is regarded as a special case of a more general type of circuit in which L, C, and R may be functions of the current or o her variables. The practical importance of these non-linear circuits is noted and some of their properties are reported. In the series circuit with a linear resistance, capacitance, and an iron core inductance it is shown that the abrupt rise in current at a certain critical voltage is characterized by resonance in the sense that*

$$L\frac{di}{dt} = -\frac{1}{c}\int i\,dt.$$

In this non-linear resonance however the above condition is satisfied in but a limited portion of the cycle. During this portion of the cycle the peak current may be calculated from Ohm's law.

The series circuit with linear resistance, capacitance, and an iron core reactor with superimposed steady magnetizing force, is shown to exhibit a pulsation oscillation which is explained on the basis of the positive impedance-current characteristic of such reactors.

Parallel resonance for the non-linear case of an iron core reactor is reported, and it is shown that in conjunction with series resonance the unique characteristic is obtained wherein the r. m. s. current decreases with increasing applied voltage.

The abrupt volt-ampere characteristic of one of these circuits is combined with the power amplifying property of the saturable core reactor to produce a sensitive relay of unique actuating characteristics.

AMONG the most interesting fundamental electrical circuits are those exhibiting simple series and parallel resonance. When these well-known experiments are performed for non-linear[1] circuit elements, many new and unique characteristics appear. It is the object of this paper to describe some of these characteristics.

Consider the series circuit consisting of an inductance, a capacitance, a resistance, and a source of alternating electromotive force. The differential equation for this circuit may assume the form[2]

$$L(i)\frac{di}{dt} + \frac{1}{C(i)}\int i\,dt + R(i)\,i = E\sin(\omega t + \psi)$$

where the applied voltage is sinusoidal. Thus in general the coefficients $L(i)$, $C(i)$, and $R(i)$ may be functions of the current of any form whatsoever.

The familiar simple series circuit where the inductance, capacitance, and resistance are essentially independent of current is in this sense a special case of a much more generalized type of phenomena described by the equation above. The linear circuit, partly because it lends itself to a simple mathematical treatment capable of adequately representing the physical functions encountered in many practical cases, has received more thorough study. The mathematical impedimenta need not however intimidate the experimental approach, for many of the non-linear circuits may be demonstrated with the simplicity characteristic of the more familiar cases.

The importance of the study of these circuits rests to some degree upon the fact that a great many practical cases, as for example power circuits in which ferro-magnetic materials are used, are by no means well approximated by linear circuits and behave under certain conditions in a manner which by comparison is regarded as anomalous, when in reality they are a fundamentally different and unfamiliar phenomena. The large number of cases that have been reported in recent years[3] of various types of instability in power circuits incorporating capacitances can probably be analyzed in terms of the elementary circuit types here described, although that has not been attempted in this paper.

Many non-linear circuit elements are familiar; for example, resistances are available which increase or decrease with increasing current, instantaneously or with thermal lag, or have abrupt discontinuities; inductive impedances are available which increase or decrease with increasing current, while capacitances are known with very complicated volt-ampere characteristics. In the interests of clarity only one non-linear circuit will be described in detail; this is the simple series circuit consisting of a linear resistance, linear capacitance, and iron core inductance.[4] The special properties of other fundamentally important non-linear circuits will be summarized, and a single technical application will be noted.

The impedance-voltage characteristic for an iron core inductance obtained for sinusoidal applied voltage has the general form of Fig. 1, where r. m. s. values are plotted. When the current is very small, the "effective

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1. "Non-linear" will be used throughout to indicate a non-linear volt-ampere characteristic for alternating current.

2. The $L(i)$, $C(i)$, and $R(i)$ may be given explicit expression where experimental data are available. In the case of the closed iron-core inductance, for example, this has been done in the appendix.

3. K. E. Gould, A. I. E. E. JOURNAL, November 1927, p. 1160.
L. F. Blume, A. I. E. E. TRANS., 1914, p. 735.
G. Faccioli, A. I. E. E. JOURNAL, 1922, p. 351.
O. G. C. Dahl, A. I. E. E. TRANS., 1925, p. 792.
R. C. Clinker, A. I. E. E. TRANS., 1914, p. 723.

4. This case was probably first discovered by Martienssen, (*Phys. Z.*, 1910, p. 448), and was later studied by Biermanns (*Arch. f. El.*, 1915, p. 345) and Margand (*Rev. Gen. d'El.*, 1921, p. 635).

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

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impedance"[5] increases with increasing current; for a larger value of current the impedance is largely independent of current, until saturation effects appear where the impedance decreases as the current increases. In

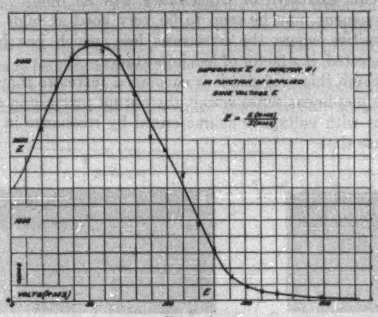

FIG. 1

the simple series circuit here described the current will always be held to values which are sufficiently large to cause the impedance to be either independent of current or decreasing with increasing current.

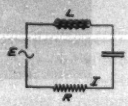

FIG. 2

If a sinusoidal alternating voltage is applied to the series resistance-capacitance-inductance circuit of Fig. 2, the interesting volt-ampere characteristic of Fig. 3

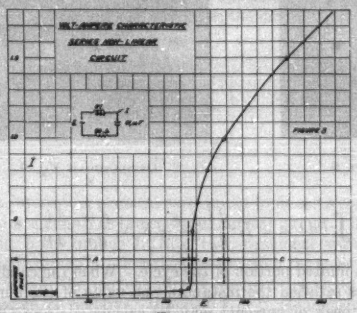

FIG. 3

may be obtained. It may be seen that the current in the circuit rises very rapidly at a certain voltage, and continues to rise with increasing voltages, although less rapidly. This sequence of events is shown strikingly by the film record of Fig. 3, where the current I in the circuit is shown with the applied voltage E. The current rises many fold, in response to a voltage change which is imperceptible on the film. The film further illustrates the voltage E_c across the capacitance and the voltage E_L across the inductance when the current is building up. The voltages and current in the circuit may be studied more closely from the oscillograms numbered from 1 to 9, which were taken for points of the curve of Fig. 3. For the region (*a*), Fig. 3, of small applied voltage, the current through the inductance is not sufficient for saturation, and because of proper choice of circuit constants the inductive impedance is large compared to the capacitative or resistive impe-

TABLE I

E r. m. s.	I peak amperes	E volts	R ohms
118	0.90	95.3	105
121	1.30	123	95
127	1.64	158	97
138	2.0	187	93
179	2.90	258	89

D-c. resistance for the circuit, exclusive of capacitance = 38 ohms.

dances. Nearly all of the applied voltage is across the inductance; the current and voltage relations are approximately those of an inductive circuit. The oscillograms numbered 1 to 3 inclusive are characteristic of this region. For numbers 1 to 3 are shown the applied voltage E, current I, inductive voltage drop E_L, and capacitative drop E_c for $E = 110$ volts r. m. s. It may be seen that the current lags behind the applied voltage E (approximately in phase with E_L) by about 90 deg., and that the capacitative drop is small compared to the inductive drop for all portions of the cycle. As the voltage is further increased, saturation effects appear, the effective inductance decreases, approaching

FIG. 3-A

a high-current condition. This further increases the current, decreasing the inductance, resulting in the abrupt increase in current with voltage, characteristic of the region (*b*). The oscillograms numbered 4, 5, and 6 have been taken for the applied voltage $E = 118$ volts r. m. s.; this is the steepest portion of the volt-ampere characteristic. Of particular interest is the oscillogram numbered 6. It may be seen that the capacitative voltage E_c is equal and opposite to the inductive voltage drop E_L throughout the interval

5. See Appendix.

indicated by the arrows. (Shown in phase, for convenience in comparison.) Moreover, the peak of current occurs at this point in the cycle.

Let us examine the condition for resonance in the analogous linear circuit. The differential equation is:

$$L\frac{di}{dt} + \frac{1}{c}\int i\,dt + Ri = E\sin(\omega t + \psi) \qquad (1)$$

For resonance

$$L\frac{di}{dt} = -\frac{1}{c}\int i\,dt \qquad (2)$$

Table I are shown the values of current and voltage measured from the oscillograms (for five points of the curve of Fig. 3), and the calculated resistance at the peak of current. The measured d-c. resistance for the circuit exclusive of capacitance was 88 ohms. The agreement may be seen to be very good at high flux densities where the hysteresis loss, which has been neglected, is a factor of less importance.

From this evidence it is concluded therefore that for the non-linear series circuit the abrupt increase in current with voltage is characterized by resonance in the sense that

Oscillogram No. 1

Oscillogram No. 2

Oscillogram No. 3

Oscillogram No. 4

Oscillogram No. 5

Oscillogram No. 6

Oscillogram No. 7

Oscillogram No. 8

Oscillogram No. 9

that is, the inductive term is equal and opposite to the capacitance term. When this is true the current i is limited only by resistance and is given by

$$i = \frac{E}{R}\sin\omega t \qquad (3)$$

From oscillogram No. 6 it may be seen that for the *non-linear* circuit the condition (2) is indeed satisfied, but for a limited portion of the cycle. When the circuit resistance is calculated from the instantaneous voltage and the current peak (7) good agreement with the measured values of resistance may be obtained. In

$$\frac{1}{c}\int i\,dt = -L\frac{di}{dt} = -KN\frac{d\phi}{di}\frac{di}{dt} \qquad (4)$$

for a limited portion of the cycle. When the condition (4) is satisfied the instantaneous value of the current (the peak of current) may be calculated from Ohm's law and is limited only by resistance. Resonance in non-linear circuits may thus be regarded as partial; because it is characteristic of these circuits, the terminology *"non-linear resonance"* is suggested.

If the applied voltage is now further increased the current continues to increase (region c), although less

rapidly. Oscillograms 7, 8, and 9 characterize this region. If the voltage is now decreased, this volt-ampere curve may be retraced exactly. Through the series of events described the peak of current has advanced from a position of lagging 90 deg. (referred to the peak of voltage) to a position of leading by approximately 15 deg., and may with further increase in applied voltage lead by a larger angle.

For some values of applied voltage the non-linear circuit is analogous to the linear simple series resonance

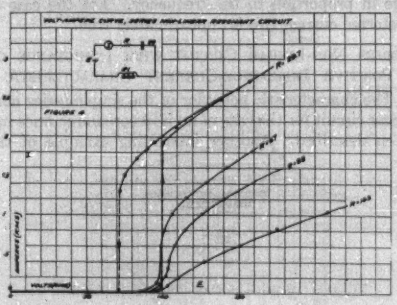

Fig. 4

in that the fundamental components of voltage and current have the same angular position. Incidentally the r. m. s. values of the inductive and capacitative voltage drops are of the same order of magnitude.

A complete family of curves for different values of resistance is given in Fig. 4. It may be seen that the non-linear properties of the circuit increase as the series resistance decreases, but that there is a critical value beyond which a hysteresis effect appears. The critical value of resistance is 67 ohms for the circuit for which these data were obtained.

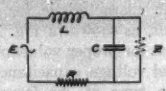

Fig. 5

An interesting classical experiment in linear resonant circuits is known as the constant current circuit. Thus in Fig. 5 the branch L, R, C is in resonance with the applied voltage E. For the case in which the resistance R is small, the current in any element (capacitative, inductive, or resistive), shunting L or C, is independent of the magnitude of the shunt impedance Z. The phase of the current in the shunt branch is displaced 90 deg. referred to the applied voltage E.

An experiment which is interesting because of analogy with the above is that of the constant current shunt to a non-linear resonance circuit. Thus in Fig. 6 is shown a variation of current with resistance when the resistance is shunting the capacitance of a non-linear circuit. It may be seen that for the range of resistance between 0 and 40 ohms the current decreases with increasing resistance; for values of resistance between 40 and 140 ohms the current increases approximately linearly with increasing resistance; and for values of resistance higher than 140 ohms the current decreases with increasing resistance.

It would be a great convenience for many purposes to be able to calculate the "critical voltage" for which the rapid rise in current takes place. There is at present no exact means of making this computation. An empirical method will be given however for determining the critical voltage for non-linear resonance. Reference to the family of characteristics of Fig. 4 will show that this quantity is essentially independent of the resistance in the circuit. In Fig. 7 is given the family of volt-ampere characteristics for the case of changing capacitance. The slight dependence of critical voltage on capacitance may be observed.

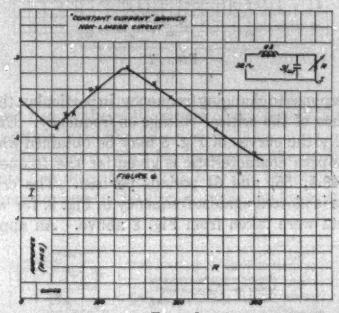

Fig. 6

This voltage therefore depends primarily upon the characteristics of the iron core reactor. The essential correctness of this conclusion is made plausible by a consideration of the enormous range of inductance in reactors of this type. The maximum and minimum values of $\frac{d\phi}{di}$ (proportional to instantaneous inductance, where ϕ is magnetic flux and i is current) differ by a factor of several hundred, in practical cases where the saturation portion of the magnetization curve is included. One may expect therefore that several-fold changes in capacitance or resistance might be comparatively unimportant, as is indeed true. It is found experimentally that the critical voltage for resonance is such that the peak ampere-turns reach approximately the knee or saturation portion, of the (alternating) B-H characteristic for the iron. Since commercial transformers are usually designed so that the peak ampere-turns, for the specified voltage, reach the knee of the magnetization

curve, the critical voltage for a series circuit using this transformer winding is approximately the rated voltage of the winding. This makes the practical design of a series resonant circuit comparatively simple; a 110-volt transformer coil will resonate at approximately 110 volts when used in this circuit.

An important characteristic of the non-linear circuit

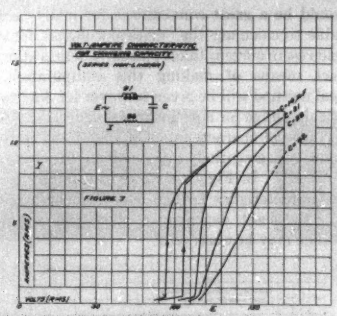

Fig. 7

is its response to changing frequency, in particular, the dependence of the critical voltage on frequency. Experimentally the curves of Fig. 8 may be obtained where a volt-ampere characteristic is given for the frequencies 35.2, 43.1, 55, and 61 periods per sec. The critical voltage is plotted against frequency in Fig. 9, for which the data were taken from Fig. 8 above. An approxi-

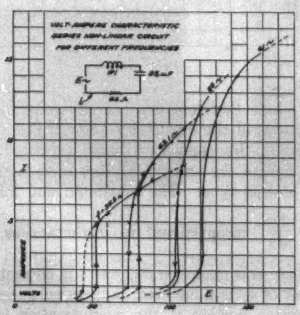

Fig. 8

mately linear variation with frequency may be observed. That this must be so may be inferred from the following consideration: It has been shown that the critical voltage for resonance depends, to the practical exclusion of all other factors, upon the properties of the iron, and in particular upon producing a peak magnetizing force (proportional to the current) for which saturation becomes rapid. The maximum value E_L of the voltage across a coil is given by

$$E_L = K \mu f I \quad K = \text{geometric factor}$$

$$\mu = \frac{d\phi}{di}, \ \phi \text{ flux}, \ i \text{ current}$$

$$f = \text{frequency}$$

$$I = \text{current (maximum)}$$

For the special case of μ constant it is clear that to maintain I at any fixed value one must decrease voltage in proportion to the frequency. For the case of distorted current this is approximately true.

Series Non-Linear Circuit with Inductance of Positive Impedance-Current Characteristic

The characteristic behavior of the simple series non-linear circuit for the case of an iron core inductance has been described. Another interesting example of the

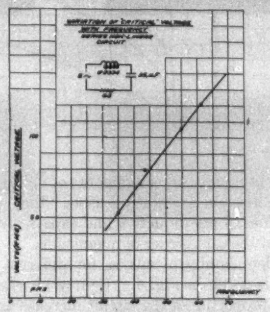

Fig. 9

series circuit is that in which the iron core reactor has superimposed a steady magnetizing force. Such reactors[6] have as a distinguishing feature a positive impedance-current characteristic for some values of alternating current, as shown in Fig. 10. Experimentally one finds that for proper values of steady magnetizing force, applied voltage, capacitance, and resistance for the circuit, a type of low-frequency oscillation appears. The film record of Fig. 11 illustrates this condition. The topmost tracing for each film is that of the voltage applied to the circuit, while the lower is that of the current. The voltage applied to the circuit is being continuously increased throughout the record. The pulsation, which begins at approximately 20 volts peak, persists with but small change in pulsation frequency until the applied voltage has risen to 70 volts peak, where a permanent type of oscillation sets in. This behavior may be described as a type of resonance, in which the rising resonant current increases the impedance of the reactor to the point where the circuit is detuned from its resonant condition. The

6. A. Boyajian, A. I. E. E. Journal, 1924, p. 958.

current then drops to a low value and this sequence of operations repeats itself. The frequency of pulsation of the r. m. s. current depends to the greatest degree upon the steady magnetizing force in the reactor and the voltage applied to the circuit, and as before, the capacitance and resistance in the circuit are factors of smaller importance.

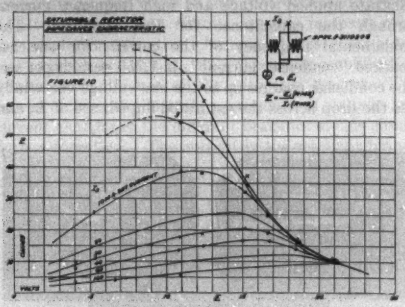

Fig. 10

Series-Parallel Non-Linear Circuit

Another type of resonant circuit with non-linear characteristics, which represents many interesting features, is the case of parallel resonance. Parallel resonance is difficult to demonstrate alone for non-linear elements, but lends itself very well to an experiment in which parallel and series resonance are combined. In the circuit of Fig. 12 are shown resistance R_1 in series with the reactor L_1 in series with the shunt branch, including the reactor L_2, resistance R_2, and capacitance C. When the voltage applied to this circuit is increased continuously the volt-ampere characteristic of Fig. 13 may be obtained. We will presume that the reactor L_2 is made up of more turns than the reactor L_1; that is, it is designed so that a higher voltage is necessary to produce saturation effects. For low values of applied voltage the impedance of the reactor L_1 is high. All the voltage to a good approximation is across the reactor L_1, the voltage across the capacitance is small, and the reactor L_2 is unsaturated and its impedance is high. The portion of the volt-ampere characteristic (*a*) is thus traversed, which is entirely analogous to the simple series resonance circuit first described. When, however, non-linear resonance is initiated, the voltage across the capacitance rises and eventually becomes sufficiently great to saturate the reactor L_2. Thus the shunt branch R_2, L_2, C becomes resonant and, analogous to linear resonance, the impedance between its terminals becomes high, causing the reactor L_1 to desaturate and the series current I to fall.

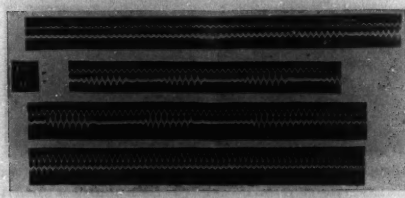

Fig. 11

This behavior is characteristic of region (*b*) in Fig. 13, where the r. m. s. current decreases with increasing applied voltage. This is the only circuit known to the author, with the exception of thermionic devices, that has this property. If the voltage applied to the circuit is further increased, the point will eventually be reached where the reactor L_1 again saturates, resulting in the second rise in the volt-ampere characteristic shown by the portion (*c*) of Fig. 13.

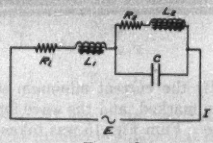

Fig. 12

A film record, Fig. 14, illustrates the sequence of events just described. The topmost tracing is that of voltage applied to the circuit, while the lower tracing is for the current in the series branch. The interesting portions of the record have been indicated on the film. At (*a*) the series circuit becomes non-linear resonant characterized by the high peak current, resistance limited. The current wave form is symmetrical to the point (*b*), where a phase shift of harmonics occurs. At (*c*) the current is very unsymmetrical with a single large peak and two small peaks, indicating the presence

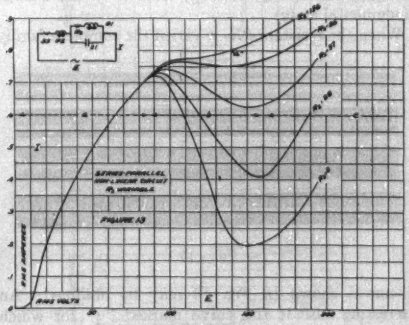

Fig. 13

of even harmonics. Between (*c*) and (*d*) the r. m. s. current is decreasing with increasing applied voltage, reaching a minimum at (*d*); the parallel branch is resonant here, with the current (not shown) reaching a maximum at (*d*). At (*d*) the series reactor L_1 again saturates, resulting in the second rise in r. m. s. current shown between (*d*), (*e*), and following points. In the circuit for which this record was taken, $R_1 = 0$ (nominally), $R_2 = 30$ ohms, $C = 21\ \mu F$, L_1 = reactor No. 1, L_2 = reactor No. 2.

When the series resistance for the parallel circuit is removed (all constants as above except $R_2 = 30$

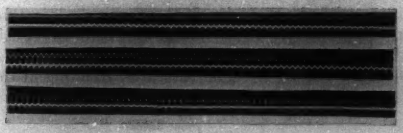

Fig. 14

ohms (nominal)) the current minimum shown at (*d*), Fig. 14, is more marked, and the wave form distortion is more extreme. Film Fig. 15 was taken in the same manner as Fig. 14 with the exception that $R_2 = 0$. The minimum current at (*d*) is very striking. At (*e*) the r. m. s. current has increased but slightly over the value at (*d*). At (*f*) the series reactor again saturates, the series current rises to a high value and is predominantly third harmonic. No adequate theory of this behavior is yet available. The harmonic content is changing between (*f*) and (*g*), tending toward a large fundamental at (*g*).

When the resistance R_2 is varied the complete family

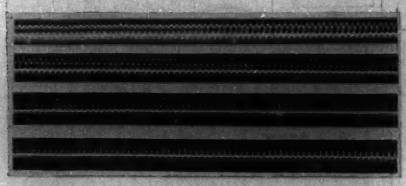

Fig. 15

of curves shown in Fig. 13 may be obtained. An interesting case is the curve marked "*a*" for which through a certain range of voltage, which in practical cases may be as large as 30 per cent, the r. m. s. current is independent of the applied voltage. It is also possible to so adjust the constants of the circuit that the peak current in the circuit is similarly independent of voltage, and constant in magnitude and phase throughout wide limits. The oscillograms of Fig. 15A illustrate the latter case. Here the applied voltage is shown as the topmost tracing, while the series current is shown in the lower tracing. The circuit constants for this case are: $R_1 = 12.9$ ohms, $R_2 = 164$ ohms, L_1 = reactor No. 2, L_2 = reactor No. 1, and $C = 21\ \mu f$.

When the natural frequencies of the parallel branch R_2, L_2, C lie above the frequency of the applied voltage E, the series parallel circuit operates in the manner analogous to the arc circuit for the production of high frequency oscillations. The oscillograms of Fig. 16 illustrate applied voltage and high frequency current through the capacitance for this condition. The fundamental frequency of the current for one case (marked "condenser current") is 1,750 periods per sec. The condenser charges up to the line voltage vectorially less the drop across the saturated impedance of L_1 and

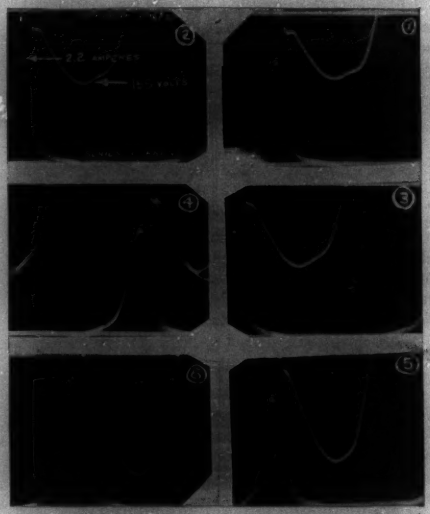

Fig. 15-A

discharges in damped oscillations during each half cycle. The fundamental frequency of the damped oscillations may be varied through wide limits, and in the laboratory frequencies as high as 3,000 periods per sec. have been produced from a 60-cycle source.

Alternating Current Relays Using Non-Linear Circuits

A relay has in general two characteristic properties: (a) a non-linear actuating characteristic and (b) capability of power amplification. Practical mechanical relays have in addition a hysteresis cycle, which is very undesirable in many applications. Thus a mechanical relay which is designed to actuate at two volts will not release until the applied potential drops to approximately one volt. The use of the series non-linear circuit as a relay is suggested by the simi-

larity of the volt-ampere curve to the characteristic of an ideal relay. The resonant circuit has no power amplifying properties of its own, but the saturable core reactor[7] is available for this purpose, and the two may be combined as shown in Fig. 17. In this circuit the voltage across the capacitance in the resonant circuit is used to actuate the saturating reactor, which in turn controls a load through its alternating current circuit. In this resonant relay the non-linear actuating characteristic is contributed by the resonance circuit, while the power amplification results from the use of the saturable reactor. The curves of Fig. 18 illustrate a sequence of voltage and current changes in the different branches of this circuit during operation. The current in the resonant branch R_1, X_1, X_2 is indicated by I_1; the direct current in the primary of the saturating reactor is indicated by I_2;

Fig. 16

and the load current in the a-c. side of the saturating reactor is indicated by I_3. The over-all operating characteristics of the device as a relay are indicated by the curve I_3 E_1. These curves are a very close approximation to the characteristics of an ideal relay. It may be seen that the ratio of the "on" to "off" load current may be made as large as desired and is a function only of the design of the saturating reactor and is governed by the same considerations as in all circuits where these types of reactors are used to control power.

The *amount of power* which may be controlled in a relay of this type is of importance in certain applications. For a saturable core reactor the power amplification factor is a function of the maximum and minimum values of impedance required, or in terms of load current, the on to off current ratio. Some theoretical considerations indicate that the power amplification of a saturable reactor is quite unlimited; for reactors which have been produced thus far however, an amplification of 1,000 for an incandescent lamp load has not been exceeded. This present limit is easily removed by cascading the saturable reactors. The method is illustrated in Fig. 19, where three stages of amplifica-

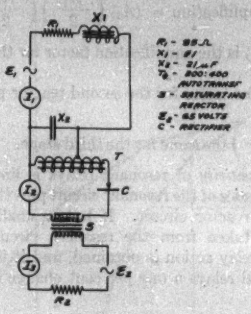

Fig. 17

tion are shown. It should be emphasized that there is no limit to the number of stages that may be cascaded in this manner, and hence no limit to the power amplification that may be obtained. In the laboratory experiments thus far it has not been necessary to use more

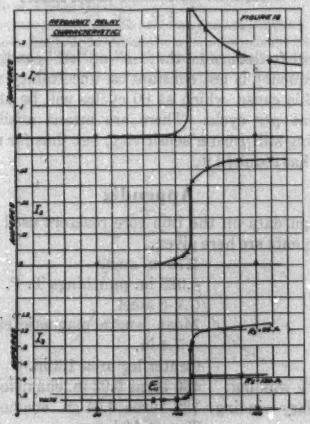

Fig. 18

than two stages. A power amplification of 10,000 times is easily obtainable with very inefficient equipment in two stages. If the power amplification factor for the reactor, and the efficiency of the rectifier, are known, it is easily possible to calculate the total amplification that may be obtained in cascade. Let us assume for the reactors of Fig. 19 an amplification factor of 100 for

7. A. Boyajian, *loc. cit.*

a given $\left(\frac{\text{on}}{\text{off}}\right)$ impedance ratio, and that the efficiency of the rectifier is 50 per cent for the load we require. Then

$$\text{Total amplification} = (A_1)\left(\frac{A_2}{2}\right)\left(\frac{A_3}{2}\right) = \frac{10^6}{4}$$

where (A_1) is the amplification factor for the first stage, $\left(\frac{A_2}{2}\right)$ the factor for the second reactor plus rectifier, and $\left(\frac{A_3}{2}\right)$ the same for the third stage.

The *sensitivity* of resonant relays is limited by the characteristics of the resonant circuit plus the associated saturable reactor circuit. If but a small amount of power is taken from the resonant circuit the most sensitive relay action is obtained, and it is found that in practical relays a one per cent change in voltage is

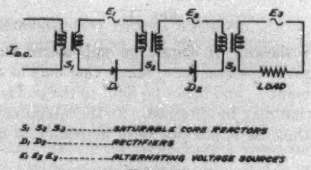

FIG. 19

sufficient to determine the "on" or "off" condition. If the frequency is not constant an uncertainty will exist from this cause, which at 60 cycles will be about one per cent per cycle. There are means available for obtaining still greater sensitivity, but thus far no application has required it.

Appendix

Design data on the iron core reactors used in these experiments are here given.

Core:	1.75 in. stack
	0.025 in. punchings T. P. 41
	CX-5 steel
	Mean length magnetic circuit = 7.4 in.
	Center leg area 1.3 in.
Primary winding:	480 turns 0.02 in. enameled wire
Secondary winding:	132 turns 0.04 in. enameled wire
	Tap at 44th turn
Rating:	110-volt 100-watt 50 to 140-cycle
	10/20/30-volt secondary

The iron core reactors used in these experiments have the following specifications:

Reactor No. 1: 110-volt primary winding (480 turns)
Reactor No. 2: 30-volt secondary (132 turns)

The magnetization curve for the iron used in this type of transformer is included in Fig. 20.

IMPEDANCE

The term "impedance" or "effective impedance" must be properly defined for this special case. The instantaneous value of the potential difference e between the terminals of a coil is given by

$$e = K N \frac{d\phi}{dt} = K N \frac{d\phi}{di}\frac{di}{dt} = L(i)\frac{di}{dt}$$

where K is a constant, ϕ is the magnetic flux, N the number of turns, and i is the current. Leakage flux and ohmic resistance will be neglected. The term $\frac{d\phi}{di}$ varies with the magnitude of the current in a

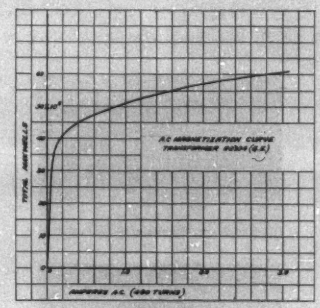

FIG. 20

complicated manner and has a hysteresis cycle; it is similar to permeability. The inductance defined by

$$L(i) = K N \frac{d\phi}{di}$$

varies in a similar manner with the current. The value of the "impedance" Z defined by

$$Z = \frac{E}{i}$$

where E is the r. m. s. alternating voltage between the terminals of a coil and i is the r. m. s. current through the coil, will therefore depend on the wave forms of E and i. In the data that are included here Z has been measured with applied sinusoidal 60-cycle voltage and distorted current; the difference between this value and Z measured for other wave forms may be quite large.

Discussion

Ernst Weber: The author considers the phenomena in a simple non-linear circuit having resistance, capacitance, and an iron-core inductance in series. Oscillograms show distortions in the waves of the current and the voltage drops across the reactor and capacitance, and on page 726 the author suggests the new terminology "non-linear resonance" to account for these irregularities. However, a fundamental analysis will show that this phenomenon is quite in accord with ordinary series resonance

of the double energy linear circuit, and differs only in that it exists for only a small part of the period.

Instead of giving "effective" magnetizing curves with a-c. excitation, it would have been more advantageous to give the actual hysteresis curve for one cycle of the magnetizing current. For example, in the sketch in Fig. 1 let flux Φ be plotted against the magnetizing current i_m and let the arrows indicate the directional variation of the flux Φ with the increase and decrease of i_m.

A well-known method is to replace the characteristic of a non-linear inductance, as in the case of iron cores, by a series of broken lines as indicated in Fig. 2. During the increase of i_m in a section, approximately given by a straight line characteristic, the flux Φ is proportional to i_m, so that we may write

$$\Phi_\nu = K_\nu i_m + C_\nu \tag{1}$$

Then the induced e. m. f. equals

$$e_{L\nu} = \frac{d\,\Phi_\nu}{d\,t} = -K_\nu \frac{d\,i_m}{d\,t} \tag{2}$$

and from a comparison with the usual equation for the self-inductive e. m. f.,

$$e_L = -L\frac{d\,i}{d\,t}$$

we note that

$$L_\nu = K_\nu \tag{3}$$

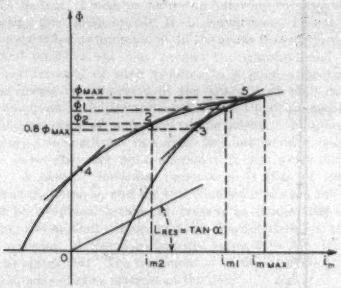

FIG. 1

K_ν is therefore defined as the inductance of this section, and is evidently a constant parameter of this section.

In the case of a double-energy linear circuit, the resonance condition is given by

$$\omega\, L_{res} = \frac{1}{\omega\, C}, \qquad L_{res} = \frac{1}{\omega^2\, C} \tag{4}$$

If C and ω are constants, there is only one definite value of L_{res} which satisfies this condition. Instead of changing the inductance mechanically or externally in order to obtain resonance as in the linear double-energy circuit, this variation of inductance in the non-linear circuit is performed by the action of the iron core itself. It is apparent that any one of these sectionally defined inductances L_ν might coincide with the resonance inductance L_{res} according to equation (4), and that this coincidence would then hold for only a small part of the period. Furthermore, the increase and decrease of i_m produces two different values of Φ for the same i_m, so that for a sinusoidal alternating voltage as used by the author in taking the oscillograms, the increase and decrease of voltage produces two different values of i_m at the same voltage. Hence we should expect to find two different resonance regions in the current curve and this is borne out by the oscillograms on page 726 of the author's paper. The fact that this resonance condition exists for a short period of time rather than for just an instant (corresponding to an exact mathematical solution) is due partly to the inaccuracy of the oscillogram recorder, and partly due to the form of the resonance curve itself as indicated in Fig. 3, where the change in current is plotted against the variation of inductance. Thus this so-

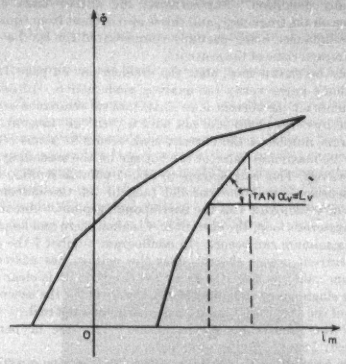

FIG. 2

called "partial resonance" is only another aspect of the well-known phenomenon of series resonance. At the same time the above analysis accounts for the author's successful computation of the resistance from the oscillogram data.

If the author states that no exact means are available to compute the so-called critical voltage at which the current rise is very rapid, suggesting the presence of a third harmonic, he is right in that an exact theory of ferromagnetic materials has never as yet

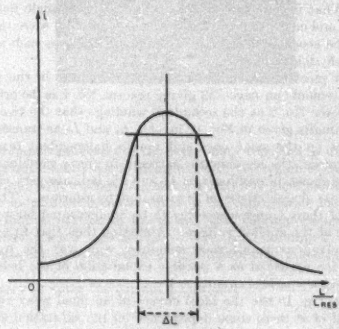

FIG. 3

been advanced. But with good engineering accuracy we can compute this critical voltage, since we know that for series resonance the equation

$$L_\nu = \frac{1}{\omega^2\, C} \tag{4a}$$

must be satisfied. Considering L_ν as the slope of a straight line in Fig. 1 the parallel tangents on the hysteresis loop give the

two positive values of Φ and i_m at which resonance occurs, (points 1 and 2 in Fig. 1). Knowing Φ it is not difficult to calculate the "critical voltage."

The author finds that the critical voltage is in the region of the "saturation" portion of the magnetization curve. This is natural, since it is in this region that the greatest change of L occurs and therefore the most probable region for resonance under any condition. Furthermore, the author finds a dependence on the frequency, and this is also evident from equation (**4a**), its influence being partially compensated for by the non-linear characteristic of the iron core.

It may be shown now, that the oscillograms on page 726 of the author's paper verify the analysis made above. In oscillogram number 1 the current is so small that no resonance occurs, since the hysteresis loop does not have a "critical" tangent. In oscillogram number 4 the current peak occurs at about 80 per cent of the maximum value of the voltage on the ascending part of the wave. This corresponds to about point 3 in Fig. 1 (of this discussion), and the parallel tangent on the descending portion of Φ indicates 4 as the corresponding point. According to the hysteresis loop, the current at 4 is about zero and hence no second maximum can occur. In oscillogram number 7 the peak of the current occurs shortly after the voltage has passed its maximum value, giving the point 5 in Fig. 1. It is clear that this is a single point since the parallel tangent on the ascending portion of the hysteresis loop is "imaginary" and not real.

Quite the same explanations hold in the case of parallel resonance; I cannot find any new difficulty.

Before concluding I would like to ask the following questions:

1. In the measurement of the effective impedance and effective current, was proper account taken of the fact that in circuits with iron-core inductances the transients are of long duration, sometimes several minutes?

2. What is meant by the following statement on page 727 ". . . but there is a critical value of resistance beyond which a hysteresis effect appears." It would seem that the hysteresis effect appears at any value of the resistance, the amount of the hysteresis effect being dependent upon the maximum value of the current i_m, which in turn depends upon the resistance.

3. What is the exact significance of a phase angle between voltage and current of two different non-sinusoidal wave shapes, when the accepted definition of the phase angle in such a case has no physical meaning?

4. It must be assumed from various remarks in the paper (see statements on page 732 giving reactor No. 1 as the primary and reactor No. 2 as the secondary winding) that the two non-linear circuits given in Fig. 12 involve L_1 and L_2 as transformer windings on the same core and not as independent reactors. Furthermore, the asymmetric currents in these combined circuits, as shown in oscillograms 14 and 15, indicate very clearly transients, characteristic of a saturated transformer. The decrease of these transients seems to be compensated for by the increase of the applied voltage. Assuming then that L_1 and L_2 are involved as transformer windings why was the mutual inductance neglected as a possible explanation of the behavior of this series—parallel circuit?

5. In Fig. 18 are the ideal curves of an ideal relay readily obtained or is there some dependence on the saturation curves of the different transformers?

In conclusion, may I suggest as a reference for the graphical treatment of non-linear double-energy circuits the work by R. Rüdenberg entitled "Elektrische Schaltvorgänge" (*Electrical Switching Phenomena*), published by J. Springer, Berlin.

B. L. Robertson: The study of very elementary systems in mechanics or electrical engineering usually involves the solution of the linear type differential equation, either by differential equation processes or by means of the operational calculus. A great number of such problems has been worked out but the cases treated have been mostly those in which the coefficients have been constant to start with, or are assumed constant in order to simplify the labor in obtaining the answer. Such an assumption, if its limitations are properly known, may be entirely satisfactory since the answer is understood as an approximation and can be altered later to suit actual conditions.

The vast majority of transient systems, however, contain variable parameters and hence must be represented by the general form of differential equation given by Mr. Suits in his second paragraph of the paper, the coefficients of the derivatives being functions of the dependent variable. The first of such transient systems which comes to mind is that discussed in the paper, the series electrical circuit with considerable iron as part of the magnetic path.

Inductance is usually defined and treated as bearing a linear relation to the current flowing through the circuit, and most textbooks make no mention of the fact that circuit constants are not always constant but really are variable. One notable exception is the discussion in the chapter on Networks With Variable Parameters given in Dr. V. Bush's book Operational Circuit Analysis. The treatment of variable circuit quantities would appear of next interest in the field of transient studies and the paper by Mr. Suits is therefore quite welcome.

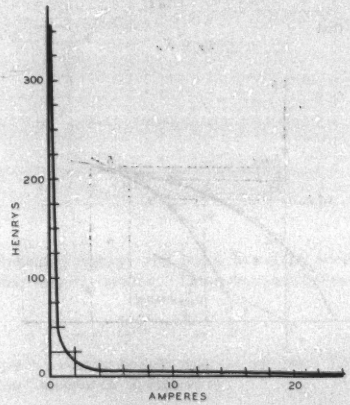

Fig. 4—Inductance Curve from the Magnetization Curve of Fig. 20

To show the variation in inductance for the transformer used in the analysis which the paper describes, an inductance curve was plotted from the magnetization curve given by Fig. 20 of the paper. The inductance is expressed as

$$L(i) = K N \frac{d\phi}{di}$$

$$= 480 \times 10^{-8} \frac{d\phi}{di} \text{ henrys}$$

This equation assumes that all of the flux links all of the turns, which is of course not strictly true but is a pretty good approximation for a transformer. $\frac{d\phi}{di}$ is the slope of the saturation curve. The variation in inductance is shown by Fig. 4 of this discussion, the range being from 360 to 1.8 henrys.

The inductance can be expressed to a very good degree of accuracy in terms of the current and this relation substituted in

the original differential equation for the circuit. That is, this particular problem, being one of the simplest of those containing variable circuit quantities, is capable of solution by methods not too involved.

The impedance-voltage curve, Fig. 1, for the transformer, is of considerable interest. It was taken experimentally and hence actually shows what the transformer will do under different impressed voltages, but the first (rising) portion of the curve does not appear to check with the magnetization curve. The inductance, if following that given by Fig. 4 of the discussion, would show the greatest impedance at zero voltage, the impedance curve from then on being of the same shape as that illustrated in Fig. 20.

The answer probably lies in the fact that the magnetization curves for iron do not rise as abruptly as shown in Fig. 20 but have a kink in them at the start. Such a kink would immediately explain the curve presented in the paper because the inductance would begin at a lower value than 360 henrys, rise to a value considerably higher than that, and then fall away as before. The impedance curve would follow it although not proportional to it. I would like to ask Mr. Suits if this is not the case with the iron he has been using.

It might be pointed out that although the characteristics for permeability, inductance, and impedance for such a magnetic circuit as a transformer possess the same general shape, these quantities are distinct physically, and analytically do not bear a linear relation to each other.

C. G. Suits: I wish to make the following comments on the interesting discussion by Mr. B. L. Robertson. Linear circuits may be regarded as understood, in the sense that given sufficient time and patience one may predict the transient and steady-state performance of such systems to a good approximation. The comparative simplicity of the analysis of such cases has doubtless been an important incentive to the attainment of the present high state of perfection of the science. Non-linear circuits are on the contrary in a much less happy state; except for simple cases they are not understood in the sense alluded to above. A case of particular importance is that of the ferromagnetic inductance, and for this element it is even impossible, except by graphical means, at present to predict the current, knowing the applied voltage and the characteristics of the magnetic material. I therefore heartily agree with Mr. Robertson when he states that "The treatment of variable circuit quantities would appear of next interest in the field . . ."

The case of the inductance of an iron-core reactor which Mr. Robertson has discussed is particularly interesting. Consider the hysteresis loops of Fig. 5, (this discussion) which are reproduced from a paper by Lehde, *Rev. Sci. Inst.*, Jan. 1931. The slope of the hysteresis curve is proportional to the incremental (or instantaneous) value of the inductance. The alternating *B-H* characteristic (similar to Fig. 20 of the paper) may be obtained by drawing a curve through the tips of the hysteresis loops. It is true that the initial portion of the *alternating B-H* characteristic has the "kink" noted by Mr. Robertson. However, this initial inflection is never traversed in a-c. circuits except during the first cycle for a completely demagnetized specimen. The rising portion of the impedance-voltage curve of Fig. 1 (of the paper) can therefore not be ascribed to this cause. The reason for this effect will be apparent from a consideration of the *shape* of the hysteresis loops. Thus, at sufficiently small magnetizing forces the loop (as loop I) comprises steep portions (of high incremental inductance) and less steep portions (of relatively low incremental inductance) of approximately like extent. The effective inductance *averaged* over a cycle will therefore assume a mean value lying between these limits. For greater magnetizing forces the *shape* of the loop changes (as loop II) in such a manner that the effective inductance which is similarly averaged over a cycle now comprises a larger proportion of incremental values of high inductance.

Several conclusions may be drawn from the situation thus presented. The effective impedance of an iron-core reactor operating in the hysteresis region is determined not only by the maximum value of the incremental inductance, which is given closely by the alternating *B-H* curve, but by the *shape* of the hysteresis loops, which cannot be inferred from the alternating *B-H* characteristic. It is interesting to note that a type of saturation and hence low incremental inductance occurs at low as well as high magnetizing forces; the change in $\frac{d\Phi}{di}$ takes place discontinuously in the hysteresis region and smoothly at large values of H.

Mr. Weber points out that when a hysteresis loop is traversed there are in general two values of incremental inductance which might be available for non-linear resonance. This is true for the hysteresis region but is not true for the saturation region where the flux is a single valued function of the magnetizing force.

It is further stated that "with good engineering accuracy we can compute this critical voltage . . .," and, "Knowing Φ it is not difficult to calculate the critical voltage."

Fig. 5

The important result of the experiment reported in the paper is that quasi-resonance is identified with the region of rapid rise in current with voltage in the sense that the incremental value of the inductance becomes equal to the capacitative impedance for a limited portion of the cycle. But this knowledge is not sufficient to allow this critical voltage to be calculated. For to calculate the resonant value of the incremental inductance it is necessary to know the current $\left(\text{assuming } \frac{d\Phi}{di} \text{ to be related to the current in a known analytic manner}\right)$ in the circuit. The current, however, depends upon the capacitance, the frequency, the resistance, the magnetic constants, and the applied voltage E in a manner which expressed analytically is a solution of

$$L(i)\frac{di}{dt} + \frac{1}{c}\int i\,dt + Ri = E\sin(\omega t + \psi) \qquad (1)$$

Solutions of (1) for the case of the closed iron-core inductance have never appeared. That is not surprising in view of the fact

that the simpler case where $c = \infty$ is yet unsolved. I therefore cannot agree with Mr. Weber's statements quoted above.

In the conclusion of his discussion, Mr. Weber asks five questions which are herewith answered.

1. Yes.

2. The hysteresis effect referred to in the quotation is the hysteresis which appears in the volt-ampere curve of Fig. 4 (of the paper) and not the hysteresis loops of the *B-H* characteristic as assumed by Mr. Weber. In view of the confusion which exists it would doubtless be desirable to give a different name to the former effect. Possibly the statement: "but there is a critical value of resistance beyond which the effective current becomes a double valued function of the applied voltage," removes the ambiguity.

3. There are at least two ways in which the conventional concept of a phase angle as used for sinusoidal wave forms may be used for non-sinusoidal cases. One may without ambiguity refer to the "phase of the zero of voltage," or the "phase of the peak of voltage," see page 726 of the paper. It is equally useful to refer to the phase of the fundamental components of non-sinusoidal wave forms always described in the paper.

4. The reactors of Fig. 12 of the paper have separate magnetic circuits as shown in the diagram. I find no basis either in Fig. 12 or in the appendix for the assumption that L_1 and L_2 have a common magnetic path.

5. The considerations which govern the design of the transformer (by no means a necessary part of the circuit) in the resonant relay of Fig. 17 (of the paper) are essentially no different from those found in any power circuit. Thus for the case in point the magnetizing current of the transformer T should be a predetermined small percentage of the maximum (ohmic) load current.

Magnetic-Circuit Units as Adopted by the I. E. C.

BY ARTHUR E. KENNELLY*

Fellow and Past President, A. I. E. E.

Synopsis.—The decisions of the International Electrotechnical Commission (I. E. C.) at Oslo in July 1930, with respect to international magnetic units and unit names, are briefly reviewed.

So far as concerns the c. g. s. magnetic units, the decisions appear likely to meet with general favor; but the series is incomplete and might advantageously be supplemented by at least one name—that for the c. g. s. unit of reluctance.

The series of units in the practical or volt-ampere-ohm system is also incomplete; but apparently cannot be completed satisfactorily without first deciding two questions: (a) whether the system should be rationalised and (b) whether the m. k. s. or the c. g. s. system should be endorsed. It is recommended that these questions be left for the present undecided, until they have been widely discussed, and illustrative literature developed in aid of international decision.

C. G. S. Magnetic Units

At its Plenary Meeting in Oslo, last July, the International Electrotechnical Commission (I. E. C.) adopted resolutions concerning c. g. s. magnetic units, which should enable various ambiguities and inconsistencies to be eliminated in future from international electrotechnical literature of the magnetic circuit.

Firstly, it reaffirmed the *maxwell* as the name of the c. g. s. magnetic unit of flux ϕ, as adopted at Paris in 1900.

Secondly, it unanimously supported the hypothesis that, for electrotechnical purposes, permeability μ_0 of free space is a quantity having certain physical dimensions, although numerically equal to unity in the c. g. s. magnetic system. Hence, even in a vacuum, or free space, magnetic flux density $\mathcal{B}$ is a different physical quantity from magnetising force $\mathcal{H}$; so that there is both need and justification for their two quantity names (magnetising force and flux density), their two c. g. s. unit names (*oersted* and *gauss*) and their two symbols ($\mathcal{H}$ and $\mathcal{B}$). It was decided to apply the name *oersted* to the c. g. s. magnetic unit of $\mathcal{H}$ and the name *gauss* to the c. g. s. magnetic unit of $\mathcal{B}$. The absolute permeability μ of a magnetisable substance was decided to have the same unknown dimensions as space permeability μ_0; but the relative permeability of the substance μ/μ_0 was held to be nondimensional, or a mere numeric.

Thirdly, it adopted the name *gilbert* for the c. g. s. unit of magnetomotive force $\mathcal{F}$.

It may not be amiss to consider the changes that these I. E. C. resolutions probably import into electrotechnical usage.

By far the most important result of the new I. E. C. resolutions is that, if followed, the indefiniteness and ambiguity which have often characterised the literature of the magnetic circuit, on questions of $\mathcal{H}$ and $\mathcal{B}$, can be eliminated. The term *gauss* for instance, can only be used for flux density $\mathcal{B}$.

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

I. E. C. Resolutions in Engineering Work With C. G. S. Magnetic Units

In order to conform with the new I. E. C. rulings concerning c. g. s. magnetic units, it seems to be indicated that:

1. When $\mathcal{H}$-$\mathcal{B}$ curves, or "saturation curves," are drawn in the c. g. s. system, for a sample of magnetic material, such as dynamo steel, the abscissas of $\mathcal{H}$ should be in *oersteds* and the ordinates of $\mathcal{B}$ in *gausses*.

2. When $\mathcal{H}$-μ curves, or "force-permeability" curves are drawn in the c. g. s. system, for a sample of magnetic material, the abscissas should be in *oersteds* and the ordinates in units of *absolute permeability* μ. These units of μ have no name at present, and their physical dimensions in terms of length, mass and time, have not been assigned. If however, the same curves are to be interpreted as *relative permeability* curves, the ordinates become expressible in μ/μ_0, and are mere numbers.

3. When $\mathcal{F}$-ϕ curves, or "excitation curves," are drawn in the c. g. s. system, for any given magnetic circuit, the abscissas of $\mathcal{F}$ should be in *gilberts* and the ordinates of ϕ should be in *maxwells*.

I. E. C. Resolutions and Text-Book Literature

The equation for magnetisation of materials in the current literature of various countries is well known to be:

$$\mathcal{B} = \mathcal{H} + 4\pi\mathcal{I} \qquad \text{gausses} \quad (1)$$

where $\mathcal{H}$ is the uniform magnetising force acting upon uniform isotropic material, $\mathcal{I}$ is the uniform intensity of magnetisation thereby established in the material along the direction of $\mathcal{H}$, and $\mathcal{B}$ is the uniform flux density, also established in that direction. From a logical standpoint, and in conformity with the I. E. C. ruling, the above equation should be interpreted as:

$$\mathcal{B} = \mu_0\mathcal{H} + 4\pi\mathcal{I} \qquad \text{gausses} \quad (2)$$

where μ_0 is the permeability of free space (*gausses per oersted*) and is numerically equal to unity in the C. G. S. magnetic system.

If this interpretation is not read into equation (1), we have for the case of non-magnetic material, such as a vacuum,

$$\mathcal{I} = 0 \qquad (3)$$

and $$\mathcal{B} = \mathcal{H} \tag{4}$$

or flux density $\mathcal{B}$ is physically, as well as numerically, equal to the magnetising force $\mathcal{H}$. If that were the case, one of the two quantities magnetising force and flux density would be superfluous, and either $\mathcal{B}$ or $\mathcal{H}$ could serve for both. The dislocation in electrotechnical literature involved by such a hypothesis would apparently be much greater than that entailed by these new I. E. C. resolutions.

If, however, the world's physicists should determine and declare at some future time, that the permeability μ_0 of free space is a mere numeric without dimensions; so that $\mathcal{B}$ and $\mathcal{H}$ are physically identical, it would be incumbent upon electrotechnicians to reconsider these I. E. C. resolutions.

The I. E. C. Resolutions and Magnetic Observatory Measurements

In the determination and recording of the earth's magnetic field at the various magnetic observatories of the world, measurements are ordinarily made of the "field intensity," in its vertical and horizontal components. Since these measurements are made in air, which is a very feebly magnetic gaseous medium, it may be claimed that the quantity measured is a magnetic flux density $\mathcal{B}$ and not a magnetising force $\mathcal{H}$; although the numerical difference between the two is for ordinary purposes insignificant.

These component flux-density measurements are commonly expressed in *gammas* or γ; where $1\ \gamma = 10^{-5}$ *gauss*. The Greek letter gamma seems to have been chosen[1] on account of its resemblance to G, the initial letter of the name Gauss. The I. E. C. endorsement of the name *gauss* as the c. g. s. flux-density unit, thus conforms with this magnetic observatory usage.

Steps Necessary to Complete the Series of Names Internationally Assigned to the C. G. S. Magnetic Units

Although the I. E. C. Resolutions of 1930, in pursuance of the Paris Congress Resolutions of 1900, provide names for the most important units of the C. G. S. magnetic system; yet there remain certain gaps in the terminology of the series, as the accompanying Table shows.

The question arises as to whether new internationally selected names are desirable to fill in the blanks of the Table. Opinions differ, not merely in different countries; but also in one and the same country. They vary over the whole range from the one extreme that all should be named, to the other extreme that none should be named; but that in each case the phrase c. g. s. unit of . . . should be used. It should, however, be pointed out that in the latter case, since there are two c. g. s. systems, the magnetic (with $\mu_0 = 1$ and $\kappa_0 = 1/v^2$) and the electric (with $\kappa_0 = 1$ and $\mu_0 = 1/v^2$), it is desirable to remove ambiguity, by using the somewhat lengthy phrase—c. g. s. magnetic unit of . . . .

1. Bibliography 24.

Table I
C. G. S. Magnetic Units and Their Names

	Quantity	Symbol	Name	Date of International Adoption
1	Magnetomotive force	$\mathfrak{F}$	gilbert	1930
2	Magnetising force	$\mathcal{H}$	oersted	1930
3	Magnetic flux	ϕ	maxwell	1900 and 1930
4	Magnetic flux density	$\mathcal{B}$	gauss	1900 and 1930
5	Magnetic permeance	$\mathcal{P}$		
6	Permeability (absolute)	μ		
7	Magnetic reluctance	$\mathcal{R}$		
8	Magnetic reluctivity	ν		
9	Magnetic pole strength	m		
10	Intensity of magnetisation	$\mathfrak{I}$		

In the writer's opinion, at least one of the blanks in Table I should be filled in, by the international adoption of the name for the c. g. s. unit of magnetic reluctance $\mathcal{R}$. It is often desirable to develop the complete expression for magnetic Ohm's law $\phi = \mathfrak{F}/\mathcal{R}$, with names for all three units. With the name of the reluctance unit adopted, it would not be necessary to adopt a special name for the unit of reluctivity ν. Just as the c. g. s. unit of resistance R under the provisional name *abohm*, gives rise to the name for the c. g. s. unit of resistivity ρ under the derivation *abohm-cm.*, so any name for the c. g. s. magnetic unit of reluctivity can be derived by adding (– cm.) to the adopted c. g. s. magnetic unit of reluctance.

Since reluctance is magnetomotive force per unit of flux, a temporary name for the unit of reluctance, until international action may have been taken, is the *gilbert per maxwell*. In the same way, those who for any reason may prefer not to use the I. E. C. name of *oersted* for $\mathcal{H}$ may correctly employ the longer term—*gilbert per cm.*, and likewise those who prefer not to use the I. E. C. name *gauss* for $\mathcal{B}$ may correctly employ the longer term—*maxwell per sq. cm.*

The Practical System of Magnetic Units

By its resolution to adopt the "practical" unit of magnetic flux, (10^8 *maxwells* or a *maxwell-eight*), under the title *pramaxwell*, together with the availability of the prefix *pra* for converting c. g. s. magnetic unit names into corresponding practical unit names, the I. E. C. has sanctioned the use of practical magnetic units, in response to a proposition from the Italian National Committee, at Bellagio, in 1927.

On this basis, a conducting loop has one volt induced in it, when it is threading magnetic flux uniformly at the rate of one *pramaxwell per second*.

It is important to consider the possibilities presented by this international recognition of a practical magnetic unit. As early as 1891, a committee of the American

Institute of Electrical Engineers recommended for international consideration a series of magnetic units in the practical system; but the recorded opposition of the Chicago Congress, in 1893, to the suggestion, caused the plan to be set aside in favor of international magnetic units in the c. g. s. magnetic system. After nearly forty years, the I. E. C. action of 1930 has restored a practical magnetic unit to good standing, if it can justify itself in the opinion of electrotechnicians throughout the world. The immediate advantage of the *pramaxwell* is, of course, that in the fairly numerous working formulas of electrical engineering connecting induced voltages with magnetic flux, the numerical coefficient 10^8 can ordinarily be eliminated and dispensed with by expressing the flux in *pramaxwells* instead of in *maxwells*. Some consider that this is a sufficient justification for the adoption of the *pramaxwell*, without extending the practical system any further. On the other hand, a single isolated magnetic unit in the practical system has less chance of proving its utility, than if it forms a member of a practical series.

What advantages might be secured from the adoption of a complete series of magnetic units in the practical system?

As an answer to this question, it has been pointed out by various writers, that if the magnetic units in the practical system were suitably adopted, the practical system would be converted into an absolute system independent of the parent c. g. s. system. Instead of defining the *volt* as 10^8 c. g. s. magnetic units of e. m. f., the *ohm* as 10^9 c. g. s. m. u. of resistance, and so on, these various practical units would be the respective fundamental units of e. m. f., resistance, etc., in the comprehensive electrodynamical system to which they all belonged. At present, this desirable condition does not hold; because the *volt*, *ohm* and *ampere*, etc., may be regarded as belonging to any one of a series of systems, such as the quadrant-eleventh-gram-second system of Maxwell, the meter-kilogram-second system of Giorgi, or the cm-gram-seven-second system[2] of Dellinger, Bennett, Karapetoff, and Mie. In each of these cases, the *watt* is the mechanical power required to accelerate the unit of mass with the unit of acceleration: *i. e.*, the eleventh-gram accelerated one quadrant per second^2, or the kilogram one meter per second^2, or the gram-seven(ten metric tons)one centimeter per second^2. Consequently if the *pragauss*, as the practical unit of flux density were internationally adopted as one *pramaxwell per square quadrant*, (10^{-10} gauss) the Maxwell system would be endorsed, if per square meter (10^4 gauss), the Giorgi system would be endorsed, and if per square centimeter (10^8 gauss), the Dellinger-Bennett system would be endorsed. It is evident that while the adoption of the *pramaxwell* is a relatively simple matter, and does not involve entanglement with any particular dynamical system, the adoption of the *pragauss* as the *pramaxwell* per unit area, would logically involve association with a particular dynamical system.

The adoption of the *pragilbert* as the practical magnetic unit of magnetomotive force involves another question; namely, whether the practical system should be "rationalised" or not?

Rationalised Units

Up to the present point of development, the practical volt-ampere-ohm system of units is neutral, being neither rationalised nor unrationalised; but the adoption of either the practical unit of m. m. f. or of the practical unit of flux density, would logically determine which of the two conditions should prevail. It was Heaviside who first drew attention[3] to the fact that the units in the c. g. s. system were so defined and adopted that a large number of formulas dealing with electrogeometrical problems of the rectilinear type, such as the capacitance of a slab condenser, prominently included the spherical factor 4π, whereas corresponding problems of a spherical type, such as the capacitance of a spherical condenser, wherein such a factor might be expected to occur, did not include it. He proposed to rectify this inconsistency by revising and rationalising the fundamental definitions of the units and their dependent formulas; but the method which he advocated for doing this would have involved changing the magnitudes of the units. It would have been necessary to recalibrate all or nearly all working standard ohms, voltmeters, ammeters, etc.—a manifestly forbidding proposition. It has since been pointed out, however, by various writers,[4] that the advantages of rationalisation can be secured without changing any existing practical units or standards, by taking the factor 4π out of the unit definition of m. m. f. and inserting it into the unit definitions of permeance and reluctance, thereby keeping the unit of flux unchanged. This would, however, involve other corresponding changes in fundamental formulas.

Doubtful Advantage of Attempting to Rationalise the C. G. S. System Now

At the present time, it seems very doubtful whether attempts could usefully be made to rationalise the c. g. s. system by taking the 4π out of m. m. f. and putting it into permeance. The Paris Congress of 1900 opposed the plan, and the I. E. C. Conference of 1930 endorsed the *gilbert* as the simple unrationalised c. g. s. unit of m. m. f. at the value of the 4πth of an abampere-turn (0.07958), or the $10/(4\pi)$ of an ampere-turn (0.7958). These are good reasons for leaving the c. g. s. system in its classical unrationalised state.

2. Bibliography 8, 10, 13, 14, 15, 17, 18.

3. Bibliography 3.

4. See discussion by Bennett of bibliography 25, Trans. A. I. E. E., April 1930, Vol. 49, p. 497.

Reasons for Rationalising the Practical Magnetic Units

Although the c. g. s. magnetic units remain unrationalised, there is no reason why the practical magnetic units should not be rationalised if it were agreed to be advisable to do so internationally.

Table II presents a few magnetic-circuit formulas, under both unrationalised and rationalised forms, in parallel columns, for comparison. It will be seen that in case No. 1, which is the fundamental formula of the c. g. s. system, representing the repulsive force between two equal positive poles, the unrationalised formula is simpler than the rationalised formula. In all the other cases, however, the rationalised formula is the simpler and more logical.

Table II
Magnetic—Circuit Unit Systems

Unrationalised	Rationalised
1. The force between two like poles, each of strength m at distance r, in a medium of permeability μ, is: $F = \frac{m^2}{\mu r^2}$	$F = \frac{m^2}{4 \pi \mu r^2}$ (5)
2. The magnetomotive force of a coil of N turns, carrying a current I, is: $F = 4 \pi N I$	$F = N I$ (6)
3. A magnetic pole of strength m would emit a magnetic flux ϕ of: $\phi = 4 \pi m$	$\phi = m$ (7)
4. The mechanical work done by an active coil of steady m. m. f. $\mathfrak{F}$ during a movement which changes by ϕ units the total extraneous flux linked with it, is: $W = \frac{\mathfrak{F} \phi}{4 \pi}$	$W = \mathfrak{F} \phi$ (8)
5. The work done in carrying a magnetic pole of strength m once around a current of strength I, is: $W = 4 \pi m I$	$W = m I$ (9)
6. The tension or tractive force between opposed parallel pole faces per unit of active surface area, carrying uniform flux density $\mathcal{B}$, if μ_0 is the permeability of free space, is: $f = \frac{\mathcal{B}^2}{8 \pi \mu_0}$	$f = \frac{\mathcal{B}^2}{2 \mu_0}$ (10)
7. The magnetic energy per unit volume of a medium of permeability μ, carrying uniform flux density $\mathcal{B}$, is: $w = \frac{\mathcal{B}^2}{8 \pi \mu}$	$w = \frac{\mathcal{B}^2}{2 \mu}$ (11)
8. The hysteretic work per unit of volume and per cycle, of a magnetisable material subjected to cyclic magnetisation, is: The area of the Ewing loop in $\mathcal{H}$ $\mathcal{B}$ units, divided by 4π. $w = \frac{1}{4 \pi} \int \mathcal{H} \cdot d\mathcal{B}$	The area of the Ewing loop in $\mathcal{H}$ $\mathcal{B}$ units. $w = \int \mathcal{H} \cdot d\mathcal{B}$ (12)

A great advantage of the rational series is that its unit of m. m. f. is the current-turn itself and not the 4πth part, or 0.0795775th of a current-turn in any system. In the practical volt-ampere-ohm system, the rationalised unit of m. m. f. becomes the ampere-turn, which is self explanatory and for which no special name is needed. Otherwise a *pragilbert* based on the gilbert, would be the 4πth part of one ampere-turn and would be unrationalised. If therefore the ampere-turn came to be adopted as the practical unit of m. m. f., the practical magnetic system would logically become rationalised. In view of what has been shown by Heaviside, Lorentz, and other authors, as to the advantages of rationalised units, it is the opinion of the present writer that they should meet with general favor, and ultimately become internationally adopted. It appears that this would also involve the rationalisation of the electrostatic units, as an automatic and equally advantageous consequence. Thus, the total electrostatic flux in a condenser would become numerically equal to its charge in *coulombs*.

It may be noted that in the books and papers that have already been published in the m. k. s. and c. c. g. s. systems, their units are rationalised.

Need for Dissemination of Literature on Serviceability of Practical Magnetic Units; but no Need for Action being taken on Them in Near Future

Table III presents a series of rationalised magnetic units in the m. k. s. and c. g. s. s. practical systems, columns IV and VI give provisional names of the units; while columns V and VII give the number of the basic c. g. s. magnetic units in each practical unit. Thus there are 10^4 c. g. s. units of flux density or gausses in the *pramaxwell per square meter*, and 10^8 gausses in the *pramaxwell per square centimeter*.

Before attempting to secure international action leading to a choice between the m. k. s. and c. g. s. s. practical systems of magnetic units, it is desirable that their relative advantages should be generally discussed and more widely known. For this purpose, it would be useful to have papers or primers published indicating the formulas and treatment of practical problems in each. In the writer's opinion, their respective advantages are fairly well balanced, so long as the formulas and problems relate to simple magnetic and electrostatic circuits. In more general cases, however, the unit of mass of 10^7 grams in the c. g. s. s. system places it at a great disadvantage compared with the unit mass of 10^3 grams or one kilogram in the m. k. s. system.

For the present, it seems desirable to allow time for the practical units to develop along the lines of individual preference. It is evident from Tables I and III that magnetic-circuit computations can continue to be carried on without difficulty in the classical c. g. s. system, especially if an international unit of reluctance is added to the list. Writers desiring to use either of the two rival practical systems have sufficient units already available for ordinary use, although some of the names are lengthy, provisional, and open to criticism.

TABLE III
MAGNETIC UNITS IN THE RATIONALISED M. K. S. AND C. G. S. S. SYSTEMS

I	Unit II	III Symbol	$L = 1$ m $M = 10^3$ gm. M. K. S. (Giorgi) IV	V C. G. S. U.	$L = 1$ cm. $M = 10^7$ gm. C. G. S. S. (Dellinger-Bennett) VI	VII C. G. S. U.
1	Magnetomotive force	$\mathfrak{F}$	ampere-turn	$4\pi/10$	ampere-turn	$4\pi/10$
2	Flux	ϕ	pramaxwell	10^8	pramaxwell	10^8
3	Magnetising force	$\mathcal{H}$	ampere-turn/m	$4\pi/10^3$	ampere-turn/cm	$4\pi/10$
4	Flux density	$\mathcal{B}$	pramaxwell/sq. m	10^4	pramaxwell/sq. cm	10^8
5	Space permeability	μ_0	henry/m	$10^7/4\pi$	henry/cm	$10^9/4\pi$
6	Permeance	$\mathcal{P}$	henry	$10^3/4\pi$	henry	$10^9/4\pi$
7	Space reluctivity	ν_0	yrneh-m	$4\pi/10^7$	yrneh-cm	$4\pi/10^9$
8	Reluctance	$\mathcal{R}$	yrneh	$4\pi/10^3$	yrneh	$4\pi/10^9$
9	Inductance	$\mathcal{L}$	henry	10^3	henry	10^9
10	Magnetisation	$\mathfrak{J}$	pramaxwell/sq. m	$10^4/4\pi$	pramaxwell/sq. cm	$10^8/4\pi$
11	Pole strength	m	pramaxwell	$10^3/4\pi$	pramaxwell	$10^8/4\pi$
12	Magnetic moment	$\mathfrak{M}$	pramaxwell-m	$10^{10}/4\pi$	pramaxwell-cm	$10^8/4\pi$

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Discussion

S. L. Gokhale: The paper under discussion divides itself naturally into two parts.

1. A summary of what the I. E. C. decided during the last meeting at Oslo, July 1930.
2. The author's suggestions and recommendations as to what should be done next.

With reference to the first item, it is gratifying to note, that in supporting the hypothesis, that the permeability of space is a dimensional quantity, the I. E. C. has qualified the recommendation with the limiting clause, that the hypothesis is accepted for "electrotechnical purposes." To the proposition thus stated, there can be no objection even by one who believes space permeability to be non-dimensional. It is, in my opinion, the best possible compromise under the circumstances.

For many years, the scientists dealing with electromagnetic phenomena have been divided into two groups, according to their conception of the nature of magnetic field. According to magnetostatic conception, which is generally accepted by physicists, permeability is a non-dimensional quantity.

According to the magnetodynamic conception which is almost unanimously accepted by the electrical engineers, permeability is a dimensional quantity. No crucial test to judge the validity of these two conceptions has yet been formulated, and no rational

decision is possible for the present. A technical decision based on majority of opinion could have easily been reached, but such a decision would have been of no more value than that of the classical decision against the Copernican theory. The resolution of the I. E. C. has carefully avoided a similar error by accepting the magnetodynamic conception "for electrotechnical purposes," leaving physicists free to retain their faith in the opposite conception until a rational decision can be reached.

According to the author, the I. E. C. resolution carries with it certain implications, some of which have been explicitly stated in the paper. There is one implication, however, which seems to have been overlooked by the author and which might well be stated in explicit terms. According to the magnetodynamic conception, B_0, that is induction in non-ferro-magnetic space, is physically not identical with H though the two quantities might be made numerically equal by choosing such units for B and H as will secure the equality. According to the magnetostatic conception, B_0 and H are physically identical and cannot be made unequal except by an arbitrary fiat, such as by dislocation of the 4π from the equation $F = 4\pi N I/10$ to the equation $\mu_0 = 4\pi/10$. The I. E. C. has not made this change, which if it had made it, would have been contrary to the spirit of compromise which has guided its decision regarding the dimensional character of permeability.

The question of the change of position 4π, naturally leads to the question of reasons for and against the change. The reasons favoring the change are:

a. That the 4π in the equation $F = 4\pi N I/10 L$, is irrational.

b. That the transference of the 4π from the above equation to the equation for space permeability $\mu_0 = 4\pi/10$ leads to simplification of computation of magnetic circuits.

The reasons opposing the change are:

c. That the 4π in the equation $F = 4\pi N I/10 L$ is rational; on the contrary its presence in the equation $B_0 = 4\pi H/10$ would be irrational.

d. That the transference of the 4π from the equations for F and H to the equations for B_0 and μ_0, does not lead to any simplification in computation except in the few cases where the magnetic circuit is purely ferromagnetic, not partly or wholly non-magnetic. In practical work the magnetic circuits under computation are generally partly iron and partly air; the transference of the 4π cannot therefore lead to the desired simplification to an extent sufficient to justify the change.

e. The insertion of the 4π into the equation $B_0 = H$, so as to change it to the form $B_0 = 4\pi H/10$ is a direct repudiation of the alternative hypothesis that permeability is a non-dimensional quantity. Such repudiation is contrary to the spirit of compromise which has thus far guided the decision of the I. E. C., and is unjustifiable until the dimensional character of permeability is conclusively proved.

As to the arguments (a) and (c), it is easy to ascertain which of the two views is rational. The equation for m. m. f. as a function of ampere-turns expresses the relation known by its appropriate name, "the law of circuitation of magnetic force." Circuitation is a process of going round a circuit, and involves the factor π in some form. The presence of the π in the equation $F = 4\pi N I/10$ is therefore perfectly rational. It is the equation $B_0 = \mu_0 H = 4\pi \cdot H/10$, that the π is really irrational for there is no circuitation involved in the relation of B_0 to H, even when B_0 and H are supposed to be quantities of dissimilar kinds.

As to simplification, as one of the expected results of the rationalization, it is easy to see that much of the simplification claimed by the advocates of the rationalized units is an illusion. For example, in Table II, item 6, the two equations are:

"Unrationalized" "Rationalized"

No. 6..... $f = B^2/8\pi\mu_0 \ldots f = B^2/2\mu_0$

Apparently the factor 4π has been dropped out of the rationalized equation; it is necessary to note, however, that the value of μ_0 is not the same in the two cases; on substituting the two values of μ_0 we have

"Unrationalized" "Rationalized"

No. 6a..... $f = B^2/8\pi_x 1 \ldots f = B^2/2_x 4\pi$

so that when it comes to numerical computation, there is no advantage gained by the use of the rationalized formulas. One type of computation which confronts a practical designer is the number of ampere-turns necessary to produce a specified flux, in a specified region of the magnetic circuit which is partly iron and partly air, such as the field of an electric motor or generator. In such cases, if the usual "unrationalized" formula be used, the 4π enters into the computation of the ampere-turns for the iron part, but not for the air part; if on the contrary the rationalized formula be used, the same factor 4π enters into the computation for the air part. The only important case within my scope in which the 4π is effectively suppressed is that of a transformer without leakage. The advantage so gained in this one case, is certainly not worth the dislocation of the 4π and the inconvenience it involves, irrespective of the theoretical reasons for and against this change.

One part of the author's argument is not clear to me, and might be equally confusing to others besides myself. A little further explanation will be appreciated. Referring to the equation

$$B = H + 4\pi J \tag{1}$$

the author says

"from a logical standpoint, and in confirmity with the I. E. C. ruling, the above equation should be interpreted as

$$B = \mu_0 H + 4\pi J \tag{2}$$

"If this interpretation is not read into equation (1), we have for the case of non-magnetic material, such as air or vacuum,

$$B = H \tag{4}$$

or flux density B is physically as well as numerically equal to the magnetizing force H."

As I understand the situation, the interpretation suggested is perfectly legitimate provided its use is limited to "electrotechnical purpose." That is all that is implied in the I. E. C. ruling, and nothing more. The ruling does call on the advocates of magnetostatic conception to abandon that conception as if it were conclusively disproved, and to accept the doctrine that permeability of space is a dimensional quantity for all purposes as if it were proved.

Physicists who believe that B and H are physically identical—and of these I am one—prefer to use separate symbols and names of units, because, as they believe, B and H are two different aspects of the same physical entity, *viz.*, magnetic force. In magnetic material this force cannot be measured directly, that is, statically, by the magnitude of the mechanical force on a magnet pole. It has, therefore, to be measured indirectly for which there are two practical methods available namely, the kinetic and the kinetostatic. In the kinetic method a pole is supposed to have traveled along the line of the force, the amount of work done during this displacement being used as a measure of the force

$$\text{force} = d\,w/d\,l.$$

When thus measured, the force is called magnetizing force expressed symbolically by H, and measured in terms of the two quantities in terms of which it is measured, that is gilberts (work) per centimeter (movement). This method gives the value, not of the magnetic force at a point, but the mean value of the force for all points along a short length $(d\,l)$ of the path.

Another indirect method of measuring the force is kinetostatic, that is in terms of the static force (electrostatic force) generated by kinetic action (translatory movement) of the magnetic lines of force. The e. m. f. generated by the translation of the lines is a measure of the rate of translation of the lines

$$E = d\,\phi/d\,t$$

or

$$B = \frac{d\,\phi}{d\,s} = \frac{E\,.\,d\,t}{d\,s}$$

Thus we have a second method of measurement and, therefore, also a second unit. This method gives also, not the value of the force at a point, but the mean value of the force for all points of a surface transverse to the line of the force. It is just as we measure grain by the bushel or by the pound. The mere fact that we find it convenient and in a sense necessary to use two methods of measurement does not prove that the entity measured and expressed in two ways is therefore not identical in its physical nature. Such is my understanding. If there be an error I would highly appreciate correction.

C. C. Whipple: In equation (1) $\beta = H + 4\pi J$ which has been used for sometime past, the ideas presented are that the total magnetic density in a body is partly supplied directly by the magnetizing force (H) and partly by the so-called intensity of magnetization, within the material. The difficulty with equation (1) is not with the ideas presented, but with the mathematical exactness of the expression. In equations such as this, it is usually accepted that the units on each side of the equation mark, must be the same. Here is where the difficulty has been with equation (1).

If the "hypothesis" suggested by the I. E. C. is generally agreed to and adopted, as it will no doubt be, then "Permeability μ_0 of free space is a quantity having certain physical dimensions, although numerically equal to unity in the c. g. s. magnetic system." There then follows the equation (2) $\beta = \mu_0 H + 4\pi J$ in which the difficulty referred to above does not exist, provided the dimensions of μ_0 are so defined that the product $\mu_0 H$ has the same dimensions as β.

Certain workers in magnetics have in the past written equation (1) as $B = H + \beta$. This simplification is very useful in discussing magnetic properties. In this equation β was referred to as the intrinsic or metallic flux density. This symbol has been used by Dr. Steinmetz and Evershed.

In this connection equation (2) will now be used and it would be advisable at this time for the A. I. E. E. to consider a symbol for the two right terms of the equation, in order that the literature of the future may be uniform in this respect. If the Greek β is retained for the left hand member, I should like to suggest the following symbols which if agreed upon should be adopted for future use:

$$\beta = \beta_0 + \beta_m \quad \text{or} \quad \beta = \beta_0 + \beta_i$$

In this case β_0 is associated with μ_0—free space—and β_m or β_i with the magnetic material present, as the intrinsic or metallic flux density.

To some this point may seem of little importance, but when one considers the confusion which has resulted from the lack of proper consideration to nomenclature in the past, it would seem at once advisable at this stage to adopt some uniform symbols for the terms referred to. If no uniform symbols are adopted by the A. I. E. E., the individual workers will adopt their own, with resultant confusion in the literature.

Personally I would favor the symbol β_m for the term $4\pi J$. If at some later date the units are rationalized the 4π will drop out and J will stand alone.

A. E. Kennelly: It is gratifying to observe that the I. E. C. decision to accept the working hypothesis of dimensions for permeability meets with Mr. Gokhale's approval, although he confesses the opinion that in reality "B and H are physically identical." It is through friendly acquiescences in a convention of this kind that we electrotechnicians can hope to secure international agreement. As he points out, the convention that μ has dimensions involves a change from equation (1) to equation (2) if we are to avoid confusion in electrotechnical books.

As regards the question of "rationalization" in practical magnetic units, that is a matter for discussion and international decision. In the author's opinion, the question is important and should be carefully considered before taking a decision which may commit electrotechnicians for many decades. Such papers and textbooks as have already appeared in the Giorgi and Dellinger-Bennett systems have, however, employed rationalized magnetic units, and there are strong arguments for them.

There is much to be said in favor of Mr. Whipple's plea for a special symbol of metallic flux density.

Mercury-Arc Rectifier Research

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Non-member Associate, A. I. E. E.

1. Introduction

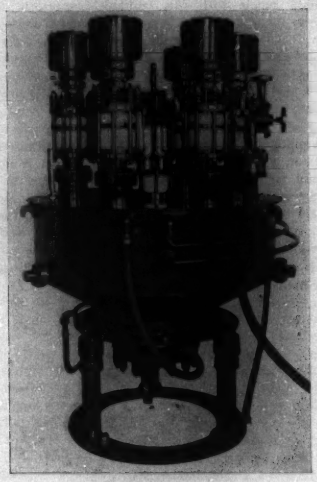
EXPERIMENTAL research during the last three years has removed much of the mystery which shrouded the mercury-arc rectifier, and made it a piece of standard electric apparatus comparable to the motor and transformer. This paper gives a brief account of some of these investigations.

The method has been experimental, guided by modern theory. The effect of different factors upon the operation of the rectifier has been observed, and the results interpreted in the light of theory. Much remains to be learned, especially in the field of high current and high voltage; but the boundary line between knowledge and ignorance is fairly well marked.

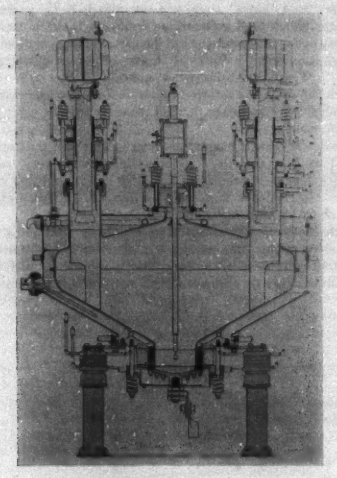
These investigations have greatly expanded the range of current and voltage that can be handled by a rectifier and have indicated the path for still further progress. No theoretical limit to either current or voltage is in sight at present.

2. Operating Conditions

Frequent references to normal structure and operating conditions will be necessary. Figs. 1 and 2 show, for purposes of illustration, typical General Electric rectifiers of early and more modern types respectively. The differences are mostly in the neighborhood of the anodes, and will be discussed in the following pages. One conspicuous change, however, may be dismissed with a very brief statement, *viz.*, the change from mercury-cooled iron anodes supported by porcelain bushings, to graphite anodes with micalex bushings. This change is an important mechanical simplification, but has no great operating significance, under present conditions, except the more certain maintenance of good vacuum. The micalex seal, a section of which is shown in Fig. 2c, is made from a mixture of mica and lead borate glass, fused and molded under high pressure, yielding a vacuum-tight seal of high mechanical and dielectric strength. This allows the clamped seal (Fig. 2c) to be metal-metal instead of metal-porcelain, thus avoiding differential lateral expansion. In the latest experimental models this metal-metal seal is welded.



A



B

Fig. 1—Typical Iron-Anode Rectifier

(A) assembled, (B) section

The normal operating requirements of a rectifier are very simple:

1. It must *start easily, i. e.*, it must begin rectifying

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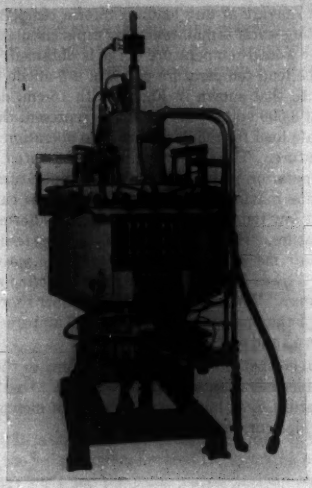
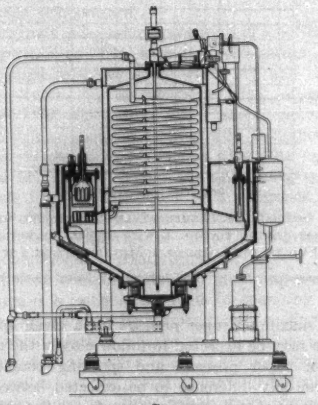
whenever the a-c. crest voltage exceeds the d-c. line voltage by a small margin, of the order of 20 volts.

2. It must *not arc back, i. e.*, allow current to pass in the reverse direction.

3. The *voltage drop* between anode and cathode,



A

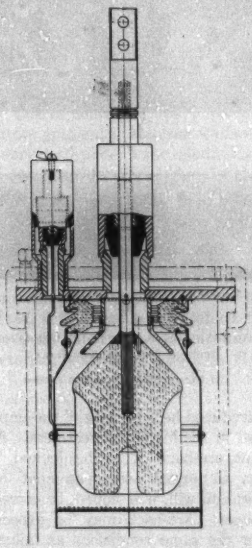


B

when current is flowing, must be low enough to meet the efficiency requirements. The practical limits of voltage drop, which are approximately 20 volts minimum and 40 volts maximum, are generally satisfactory for line voltages of 500 or more, but are too high for 250-volt operation. The hot-cathode rectifier promises to fill this gap (see section 10).

4. *Voltage regulation* will be required in the future. This is a circuit function rather than a rectifier function in the case of a simple rectifier, and can be accomplished by transformer compounding. But grid-controlled rectifiers of the type shown in Fig. 2, called thyratrons, can be made to give any desired regulation without compounding. This is discussed in section 9.

The arc-back requirement has received most attention up to the present, because the greatest electrical sin of today is interruption of service. When a rectifier



C

FIG. 2—TYPICAL RECTIFIER OF RECENT DESIGN, WITH GRAPHITE ANODES, SUBDIVIDING GRILLES, AND CONTROL GRIDS

(A) assembled, (B) section
(C) Section of anode housing in (B), showing micalex seal and graphite anode

arcs back it is a two-way conductor instead of one-way, and hence constitutes a short circuit of the a-c. supply line as well as the d-c. line or network which it feeds. The harm done by such short circuits, assuming they are cleared by circuit-breakers, is mainly interruption of service. Rectifiers are built to withstand arc-backs of less than one-second duration without injury, the only symptoms being a slight pitting of the anode tip which has performed the forbidden function of acting as cathode, and a slight evolution of gas resulting from this melting. Figs. 3A and 3B show photographs of iron and graphite anodes respectively that have been oper-

ated for long periods. The iron anodes are seen to be somewhat pitted, but the graphite anodes show scarcely any roughening.

The gas evolution accompanying arc-back, if excessive, delays resumption of service until the pumps can restore the necessary vacuum. Fast pumps and well-degassed anodes have reduced this time to the order of one minute, and experiments now in progress are expected to reduce it to one second or less.

This brief discussion of what arc-back is and does

A

B

Fig. 3—Typical Anodes After Long Operation, Showing Pitting Due to Arc-Back

(A) Iron anodes; (B) carbon anodes

has been introduced here because of the important role it has played in the history of rectifiers. Any misimpression that arc-back is to be looked upon with tolerance as a normal accompaniment of mercury rectifier operation will, it is hoped, be corrected in the following pages. Arc-back may be expected in the future under the same conditions as failure of other electrical apparatus, *viz.*, when the rectifier is loaded beyond its capacity, or in case of mechanical failure of some of its parts, such as vacuum pumps or water-conditioning apparatus. An important conclusion from the above discussion is that when such overloads or failures do cause arc-back, the rectifier as a sealed-up unit is not injured, and only the offending mechanical part needs repair.

3. Control of Mercury Vapor Pressure

The most important single factor in mercury-arc rectifier operation is the control of the mercury vapor pressure. This pressure depends on the temperature of the cooling water. The problem for investigation divides itself into two parts: (a) What is the best cooling-water temperature, assumed uniform, and the permissible temperature limits? (b) If the water temperature is not uniform, what parts should be controlled?

(*a*) *Optimum Temperature and Temperature Range.* The optimum temperature may be determined empirically, by gradually increasing the load current, at a given water temperature, until arc-back occurs, then repeating at a higher temperature. Eventually a limiting temperature is reached, above which the rectifier will not operate at any load. Uniform temperature of the cooling water is maintained by rapid circulation. In this way a load-temperature curve is obtained. Fig. 4 gives the load-temperature curve of a 6-anode rectifier similar to that shown in Fig. 2, with 15-cm. diameter arms, at 3,000 volts. The ordinates represent the maximum safe load for reliable short time operation at each temperature. The important fact to be noted is that, while operation is possible at any temperature below a definite maximum, the load that can be carried is small at low temperature and increases uniformly with temperature. Hence: (1) *The best operating temperature is as near the maximum as can safely be maintained.* (2) *For a given load there is a definite operating range of temperature.* For example, for the rectifier represented in Fig. 4 the operating range for 1,500 amperes load is between 42 and 60 deg. cent.

It is seen that there are two limitations to the range of reliable operation at a given voltage. One is high temperature, *i. e.*, high vapor pressure of mercury; the other is high current.

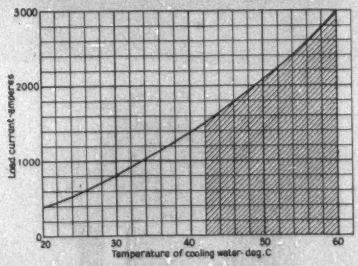


Fig. 4—Load Vs. Temperature Curve of Six-Anode Rectifier Similar to Fig. 2, with 15-Cm. Diameter Arms, at 3,000 Volts, for Loads of 1 Min. Duration (of Fig. 7.)

The operating range for 1,500 amperes is indicated by the shaded area

The maximum *vapor pressure* for a given operating voltage may be expected to follow closely the relation between vapor pressure and sparking potential; in fact, the only difference to be expected between arc-back potential and sparking potential is the change in the surface condition of the negative electrode, due to the presence, in and just below the surface, of gas molecules, which have penetrated the surface in the form of positive ions. Such a composite surface is known to emit electrons more easily than a pure metal surface, and may be expected to allow a spark discharge

at lower voltage. Hence the arc-back voltage should be very close to the sparking potential between electrodes of low electron work-function.

This conclusion appears to be justified by the results of a large number of tests which we have made of sparking potentials. Typical results are shown in Figs. 5 and 6. Fig. 5 gives the sparking potential of an idle iron anode in a rectifier similar to Fig. 1, having 15-cm. diameter arms, with and without current to an adjacent

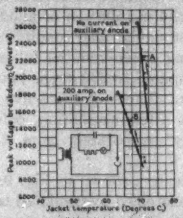


FIG. 5—SPARKING POTENTIAL AS FUNCTION OF MERCURY VAPOR PRESSURE, FOR AN IDLE ANODE IN A RECTIFIER SIMILAR TO FIG. 1

Note that the sparking potential is lowered by the flow of current in an adjacent arm

anode. It is seen that the presence of ions due to current in the adjacent arm lowered the sparking potential by 50 per cent for the same vapor pressure; or, conversely, lowered the vapor pressure for the same sparking potential by 50 per cent (vapor pressure doubles for each 10 deg. temperature).

Fig. 6 shows the sparking potentials of a hot-cathode mercury-vapor rectifier, with the cold barium-coated cathode used, first, as anode (curve *A*), and then as cathode (curve *B*), showing the effect of cathode material. For comparison, the curve of actual arc-back voltage of a similar rectifier with 15-ampere load is shown in curve *C*. The fact that the arc-back voltages are slightly higher than the cold sparking potentials may be attributed to a difference in the cleanness of the anode surfaces in the two tubes. There is no evidence in the case of curve *C* of a lowering of spark potential due to positive ions.

The points to be observed in Figs. 5 and 6 are, (1) that there is a fairly sharp limit to operating vapor-pressure, as expressed by temperature of cooling water, above which spark discharge will take place between the electrodes even when idle; (2) the evidence available indicates that the operating arc-back potential, for light loads, does not differ greatly from the sparking potential but that with the intense ionization accompanying heavy loads it may fall to about half the sparking potential.

The *current limitation* is of a different nature. It will be shown later (section 6) that the limiting load current at any temperature can be predicted from the volt-ampere characteristic. This characteristic changes from negative to positive as the load increases, and at a definite load rises abruptly. It is at this point that arc-back occurs.[1] The steep rise is associated with the approach to complete ionization of all the gas in the anode arms. The load-temperature curve thus acquires a rational meaning, *viz.*, the temperature determines the vapor pressure of mercury in the anode arms, and *the maximum load current is that which utilizes all the molecules of this vapor as positive ions to neutralize the electron space charge.* This fundamental limitation to the current-carrying capacity of an arc was predicted by Langmuir and Mott-Smith,[2] and verified experimentally by them for a small mercury arc in a quartz capillary tube. It has been found to hold for all the rectifiers tested.

The vapor pressure in the anode arms is not completely determined by the temperature of the cooling water in the case of continued heavy loads, because of the temperature gradient in the iron wall and the iron-water boundary, and under some conditions a pressure gradient in the mercury vapor due to convection of mercury spray and its vaporization in the arc stream. Differences as great as 10 deg. cent. between water temperature and equivalent vapor pressure have been observed and even greater differences may exist. Hence, when the water temperature is adjusted to give the maximum safe vapor pressure in order to carry the maximum load, if this load is maintained the pressure will rise above the safe value. This results in a load-time characteristic such as is shown in Fig. 7, which

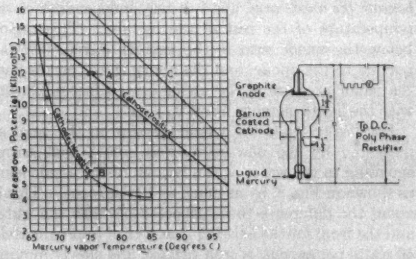


FIG. 6—SPARKING POTENTIAL AND ARC-BACK POTENTIAL IN A HOT-CATHODE MERCURY-VAPOR RECTIFIER, SHOWING EFFECT OF ELECTRODE MATERIAL

Curve A, sparking potential, carbon negative; curve B, sparking potential, barium-coated cathode negative; curve C, arc-back potential of similar rectifier with 15-ampere load

represents the behavior of the same rectifier, the characteristics of which were shown in Fig. 4, at 60 deg. cent. and 3,000 volts.

(*b*) *Vapor Pressure and "Control Point."* What part of the cooling water determines the vapor pressure in the anode arm? To answer this question a series of

1. The mechanism of this arc-back is discussed in section 6.
2. Langmuir and Mott-Smith, *G. E. Rev.*, *27*, 770, 1924.

simultaneous measurements was made of vapor pressure in the anode arms and temperature in different parts of the tank, during normal low-voltage operation. The vapor pressure was measured by a method originally suggested by Langmuir. A copper block containing a thermometer was pressed against the outer wall of the glass anode insulator, and slowly cooled until, at a temperature t_1, a thin film of condensed mercury appeared on the inner glass wall. The block was then slowly heated until the film disappeared at t_2. The

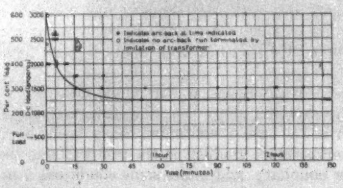


Fig. 7—Load-Time Curve at 3,000 Volts of Same Rectifier as in Fig. 4, at 60 Deg. Cent.

Abscissas represent observations of the length of time each load could be carried without arc-back

mean of the "dew-points" t_1 and t_2 was taken to represent the pressure in the arm. Since the dew-point readings require time, only the steady-state condition for each load was observed.

It was found that, with light loads, *the dew-point corresponds very closely to the temperature of the part just beneath the anode arm*, and is nearly independent of the temperature of the rest of the rectifier. This region below the anode arm is the nearest mercury-wetted portion to the anode, and the above result means that the vapor pressure in the anode arm is in equilibrium with the liquid mercury below it.

For heavy loads the vapor pressure in the arm, as measured by the dew-point, rises above that corresponding to the water at the bottom of the anode arm, as shown in Fig. 8, by as much as 10 deg. cent. This is about the difference to be expected between the water and the iron, for the existing heat flux of approximately 40 watts per sq. cm. of iron surface, and slowly moving water.

4. Anode Heaters

The limits of vapor pressure are almost certain to be momentarily exceeded if liquid mercury comes in contact with hot anodes, or any hot surfaces near the anodes. In some of the early models it was observed, when they were opened, that the anodes and their housings were thickly coated with small drops of mercury, which had condensed as the rectifier cooled. These rectifiers were known to arc back frequently within the period between 20 min. and 2 hr. after starting, which was about the range of time required, under varying loads, for the anodes and housings to become hot enough to vaporize mercury rapidly, and thus lead to excessive local pressure, with resultant arc-back. Heaters were therefore installed in the anodes, as shown in Fig. 9A, to maintain the anodes and housings at a temperature higher than the body of the rectifier, so that condensation could not occur.

The result was the entire disappearance of the 20-min. to 2-hr. arc-backs, and, in fact, of nearly all arc-backs. Fig. 10 shows the load record (interruptions indicated by small black squares) of a typical station containing four rectifiers, for several months preceding and following the installation of anode heaters. It is seen that, under similar load conditions, the number of arc-backs during seven months following installation was five as compared to 200 in the two months preceding installation. Three of these five occurred in the first three days after installation, before the mercury had completely evaporated from the anodes and housings.

The anode heaters shown in Fig. 9A were awkward and had to be energized by insulated transformers. In the newer rectifiers the heaters are around the anode necks, as shown in Fig. 9B, and are thus at tank potential and may be operated in series or parallel. The heaters may perhaps be omitted entirely if, as indicated in Fig. 2, the level of the cooling water is above the top of the anode chambers, and its volume and flow is so controlled that the hottest water is always on top, and loss of heat from the anode terminals is prevented.

5. Dimensions Vs. Current-Capacity

Since the current-capacity depends but little upon temperatures other than those near the anode arms, it may be anticipated that the important dimensions will be those of the arms. This has been found to be true. The first tests, however, gave the surprising result that the capacity is nearly independent of the dimensions of the arms, for geometrically similar plain cylindrical arms

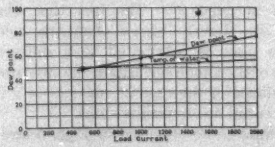


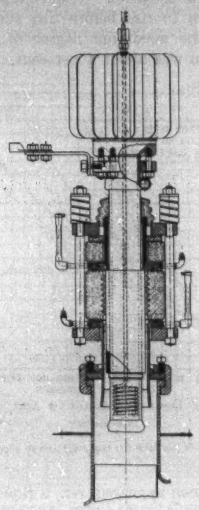
Fig. 8—Dew-Point of Mercury in Anode Arm, as Function of Load

Note that for small loads the dew-point coincides with the temperature of the water just below the anode arm, but rises above it with heavy load

like Fig. 1. Typical results of these tests are shown in Fig. 11. The reason that the current does not increase with diameter is that the maximum safe operating temperature decreases as the diameter of the arm increases. Thus the total amount of vapor in the arm, and hence the current capacity, should remain about constant, in agreement with observation.

The behavior is very different when the anode arm is

subdivided so as to be equivalent to a group of small arms. If the part of the arm just below the anode is divided by metal vanes into narrow parallel channels, the limiting vapor pressure is determined by the dimensions of these channels and their distance from the anode, and is independent of the diameter of the arm. Under these conditions *the current capacity is proportional to the square of the diameter, i. e., to the cross section of the anode arm.* This is shown in Fig. 12, which gives the observed maximum loads, of one minute duration, for arms of different diameters, similar to that shown in Fig. 2. The mode of operation of these subdivided arms will be discussed in Section 8.



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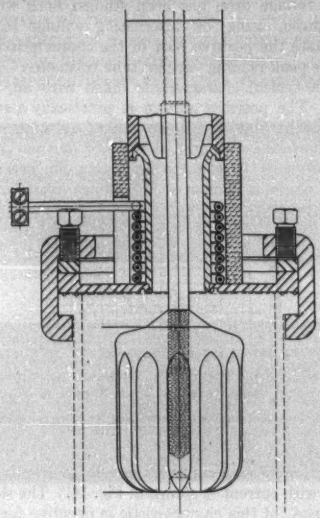

B

Fig. 9 (A) Internal Immersion Type Heaters Installed in the Iron Anodes of Early Rectifiers
(B) External Sheath-Wire Heaters Around Anode Necks

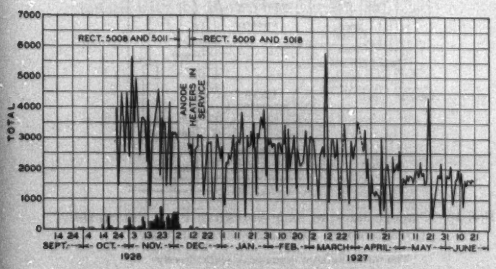

Fig. 10—Load Record of Typical Station Containing Four Rectifiers Before and After the Installation of Anode Heaters

Each small black square indicates an arc-back. The time of installation is indicated by arrow. Note that the number of arc-backs fell from 100 per month to essentially zero

6. Voltage Drop. Positive Resistance

The pulse of a rectifier is the voltage drop between cathode and anode when the anode is positive. It is a valuable index of health, especially in studying or conditioning a new rectifier. For example, an exceptionally low voltage-drop tells that the vapor pressure is abnormally high, and arc-back may be expected with continued load; a high voltage-drop indicates excessive load, which must be reduced or the vapor pressure raised if reliable operation is to continue. If one anode

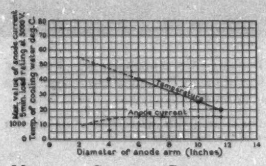

Fig. 11—Maximum Current Capacity and Maximum Operating Temperature as a Function of Diameter of Anode Arms

For plain cylindrical arms of type shown in Fig. 1

has a higher voltage-drop than the others, it contains foreign gas, due either to leakage or incomplete exhaust; if lower, its grid has probably been injured during assembly or exhaust.

The voltage drop has been studied both with the oscillograph, using low alternating voltage so as to accentuate the positive part of the characteristic, and with the peak-reading vacuum tube voltmeter. Fig. 13 shows a typical characteristic taken with an oscillograph. The positive portion is practically a straight line, showing that the voltage drop at constant current, under normal conditions, is constant and not a function of time.[3] It is, however, a function of current. If characteristics similar to Fig. 13 are taken with different values of load current, the voltage drop is found to vary with current as shown in Fig. 14. The slope or "resistance" of this characteristic is negative for small currents, zero for moderate currents, and positive for large currents.

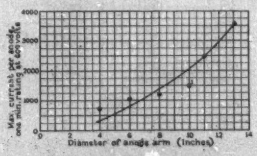

FIG. 12—CURRENT CAPACITY VS. DIAMETER OF ANODE ARMS FOR SUBDIVIDED ARMS OF TYPE SHOWN IN FIG. 2

The solid line represents the theoretical limiting value of 4 amperes per cm.^{2} (The point marked (1) is from a test at 3,000 volts)

The negative portion of the characteristic is well known. The positive portion is observable only with low-pressure arcs, and is due to the approach to complete utilization of all the gas. This effect was first observed by Langmuir and Mott-Smith,[4] who pointed out that the number of ions needed to neutralize space charge is proportional to the current; and that the maximum number of ions cannot exceed the number of molecules of the gas. When all the gas has been ionized, a further increase in current can be obtained only by increasing the speed of the electrons, which means increased arc-drop. The voltage may be expected to begin to rise before this state is reached. In our tests the maximum degree of ionization for stable operation was about 4 per cent. At this point a new phenomenon appeared, *viz.*, a rapid reduction of pressure in the anode arm, which appears to be due to a pumping action not thoroughly understood at present. The pressure falls during the current cycle, causing the voltage drop to increase, often to very high values.

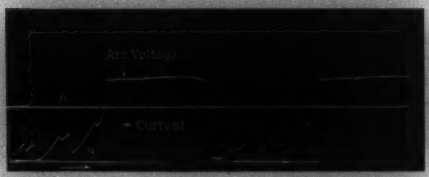

FIG. 13—TYPICAL OSCILLOGRAM OF VOLTAGE BETWEEN ANODE AND CATHODE IN SINGLE-PHASE RECTIFIER WITH LOW ALTERNATING VOLTAGE

The voltage-drop is essentially independent of time but is a function of current as shown in the next figure

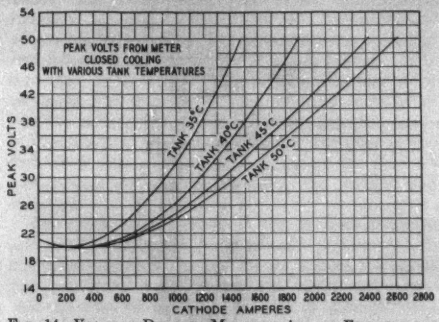

FIG. 14—VOLTAGE DROP IN MERCURY ARC AS FUNCTION OF CURRENT

The resistance is positive for currents above a certain value

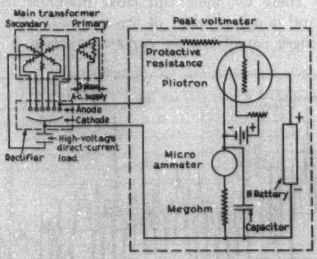

FIG. 15—CIRCUIT SHOWING SELF-ADJUSTING PEAK VOLTMETER CONNECTED TO MERCURY-ARC RECTIFIER

The microammeter reads peak volts directly

When volt-ampere characteristics are obtained for different vapor pressures, it is found that the current at which the characteristic becomes positive is proportional to the vapor pressure, as may be seen in Fig. 14. This is to be expected from the point of view expressed above that positive resistance is due to full utilization of the mercury vapor.

3. Under the *abnormal* conditions of extreme overload the vapor pressure, and with it the voltage drop, may vary with time during the current cycle.

4. Langmuir and Mott-Smith, loc. cit.

At operating voltages oscillograms are inconvenient and the voltage-drop is too small to be read accurately. In order to observe the voltage drop under operating conditions, a peak-reading vacuum-tube voltmeter was adapted to be self adjusting and direct reading, by the insertion of a high resistance in multiple with a con-

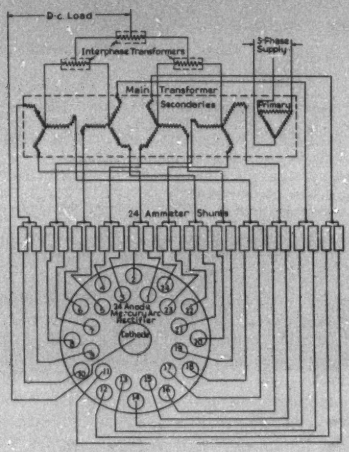

FIG. 16—CONNECTIONS FOR TESTING DIVISION OF CURRENT BETWEEN ANODES OPERATING IN PARALLEL

denser in the cathode circuit (Fig. 15), and was used in all tests. It was found that *arc-back usually occurs when the voltage drop exceeds about 50 volts.* Hence the maximum load which a rectifier can carry may be predicted from the intersection of its volt-ampere characteristic with the 50-volt line (see Fig. 14).

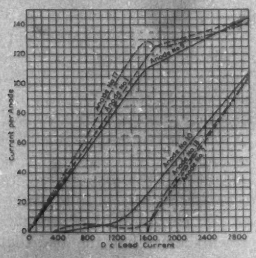

FIG. 17—DIVISION OF CURRENT BETWEEN INNER AND OUTER ANODE FOR THREE TYPICAL PAIRS OF PARALLEL-CONNECTED ANODES

The reason for these high current or *high arc-drop* arc-backs is not fully understood, but is believed to be high local gas pressure, caused by one or perhaps both of the following series of events:

(a) Excessive voltage drop is usually accompanied by surges which excite oscillations of the transformer system, sometimes of sufficient voltage to arc over insulation. Such oscillations may be seen in the oscillograms of Figs. 18 and 19. It seems probable that the surges are caused by rapid variations of vapor pressure, due to the following mechanism:

Ions which strike the metal walls with energy greater

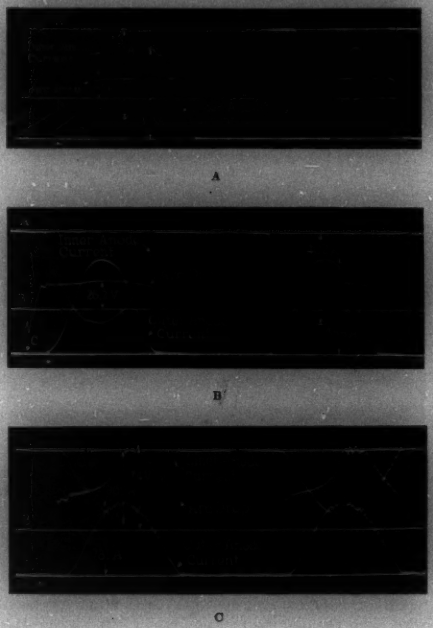

FIG. 18—OSCILLOGRAMS SHOWING THE CURRENT TO THE TWO ANODES (DEFLECTION DOWN FOR INNER ANODE, UP FOR OUTER) AND THE VOLTAGE DROP, FOR A PAIR OF PARALLEL ANODES

Trace A = current to inner anode
Trace C = Current to outer anode
Trace B = voltage between cathode and anode
(A) Total load 250 amperes; (B) total load 1,500 amperes; (C) load 3,000 amperes. Note that the anodes share the current equally at large load. The voltage drop is normal with 250 and 1,500-ampere load, but very high at 3,000-ampere load

than 22 volts[5] penetrate the walls, and remain occluded for a considerable time, constituting a large reservoir of stored gas that diffuses out slowly. The disappearance of vapor in this way may take place so rapidly that the reduction in pressure cannot immediately be compensated by an inflow of vapor. This causes the voltage drop to rise still higher, until a point is reached where

5. Hull, A. I. E. E. TRANS., Vol. 47, 1928, p. 755.

the stored gas becomes unstable. The increasing energy of impact of the ions begins to heat the walls sufficiently to liberate the stored gas, which quickly restores the vapor pressure, probably to a higher value than normal. The cycle is then repeated. There is evidence from other experiments that the amount of gas thus stored and liberated may far exceed the normal quantity of vapor in the anode arm. Thus if the vapor pressure were previously near the maximum safe value, it will be raised momentarily above this value, and arc-back will result.

(b) A sudden evolution of gas in the anode arm, sufficient to cause arc-back, may result from the formation of a cathode spot on the metal wall near the anode. There is abundant evidence that such cathode spots do develop when, and only when, the voltage-drop exceeds about 40 volts.

For example, if a rectifier is "baked out" by current under such conditions that the voltage-drop is high, the walls are found to be scored by numerous bright, narrow channels, frequently forked, running from the bottom of the anode arm to a point opposite the anode

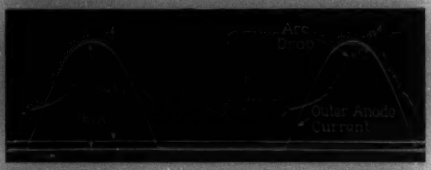

Fig. 19—Oscillogram of Current and Voltage of One Set of Single Anodes, in the Same Circuit as the Pairs of Anodes of Fig. 18

Total load 1,500 amperes. Note excessive voltage drop, and compare with Fig. 18, B and C.

tip. Each of these channels represents the path of a rapidly moving cathode spot. If the voltage has been very high, the housing will be found melted where the cathode spot has stood still opposite the anode tip.

As a second example, a special hot-cathode metal rectifier was made, the central portion of which consisted of an iron tube 2.5 cm. in diameter and 30 cm. long interposed between cathode and anode. The tube was so constructed that the formation of a cathode spot near the anode could be observed visually. It was found that such a spot invariably formed when the voltage between cathode and anode exceeded 40 volts.

An example of the absence of such cathode spots, when the voltage-drop is kept low, is furnished by a new mercury-arc rectifier which was exhausted with great care. The current and temperature were so regulated that the voltage-drop, as continuously observed by a peak-reading voltmeter, never exceeded 30 volts. It was then opened and found to be entirely free from scorings.

The formation of a cathode spot on the wall of the rectifier requires the maintenance of two arcs in series. The insulated metal tank must be anode for the mercury arc starting from the cathode, which requires that it shall be at least 10 volts positive with respect to the cathode. The current then passes by conduction through the iron to a point near the anode, where an iron arc plays from wall to anode, which requires about 30 volts. The minimum voltage for the two arcs in series is therefore about 40 volts, in agreement with observation.

7. Parallel Operation in Positive Resistance Range

If the positive resistance shown in Fig. 14 represents a true instantaneous characteristic of the arc, it should be possible to operate such arcs in multiple, for large currents, without stabilizing resistance or reactance. This was tested in a 24-anode rectifier similar to the 12-anode rectifier illustrated in Fig. 1, except that the anode arms (15 cm. diameter) were 30 cm. longer than shown in Fig. 1. The anodes were arranged in two concentric rings of twelve each. They were connected in pairs to the terminals of a 12-phase transformer, each pair consisting of one anode of the outer group and one of the inner group, as shown in Fig. 16. The length of the arc path to the outer anode was 15 cm. longer than that to the inner, the total lengths of the arms, from the main body of the rectifier to the anode tip, being 45 cm. and 60 cm. for the inner and outer arms respectively. This seemed a sufficiently unfavorable condition for division of current between the anodes to constitute a crucial test. The individual anode leads contained no reactance or resistance except a 0.002-ohm ammeter shunt for measuring the current.

From the characteristic given in Fig. 14 we should expect one anode of each pair to carry all the current for light loads, for which the characteristic is negative, and for a small portion of the positive range, up to the current value (cf. Fig. 14) at which the voltage-drop is equal to the voltage-drop for zero current of the other anode of the pair; here the current will start to flow to the second anode. At loads greater than this both anodes of each pair will carry current, and will share it more and more equally as the load increases.

Fig. 17 shows the currents actually observed for three pairs of anodes, which are typical of all. They conform completely to the values predicted from the characteristic arc-drop curve. Up to a certain load one anode of each pair carries all the current. This limiting load represents about half the load that one anode alone can safely carry, so that there is no danger or disadvantage in having one anode do all the work. Above this limiting load both anodes of each pair share the current. It will be noted that the inner anode of each pair picked up the load first in every case, and carried slightly more current at all loads than the outer one, corresponding to its lower resistance, the two currents being approximately in the inverse ratio, 45/60,

of the lengths of their individual arc paths. Thus the current divides according to resistance as in any ohmic circuit.

As additional evidence of the approximately equal division of current, the rectifier was tested for current-capacity, first with one set of anodes alone and then with the two sets in parallel. *It was found that the rectifier would carry twice as much current with the parallel anode connection as with either set of 12 anodes alone.*

The division of current between parallel anodes was also observed oscillographically. Six pairs of anodes were used, each pair connected as in Fig. 16, in a standard six-phase double wye circuit. Fig. 18A shows the current and voltage for one pair when the total load was 250 amperes. Current starts first to the inner anode (trace *A*, deflection down) while that to the outer anode (trace *C*, deflection up) does not start until the middle of the cycle. In Fig. 18B, where the total load is 1,500 amperes, current starts in the two anode arms almost simultaneously, and they share the load in about the inverse ratio of their lengths. The low voltage-drop of 26.2 volts maximum shows that the arms are not overloaded. With 3,000 amperes (Fig. 18C) the two anodes share the current almost equally. But the voltage-drop rises during the cycle to a maximum value of 74 volts, and exhibits the oscillations characteristic of overload.

For comparison, a similar test was made with a single set of six anodes, the outer row of anodes being used. Fig. 19 shows the current and voltage drop for one of these anodes at a total load of 1,500 amperes. The voltage-drop shows the same features as in Fig. 18C, rising to an ever higher value, *viz.*, to 94.5 volts. It is to be noted that this high voltage drop occurred at 3,000 amperes in the case of parallel anodes (Fig. 18C) whereas it occurs at 1,500 amperes for the single set of anodes (Fig. 19). The voltage drop is normal at 1,500 amperes (Fig. 18B) in the parallel case, *i. e.*, when two anodes instead of one are connected to each transformer terminal. This evidence, therefore, agrees with that of the load limit tests in showing that the *load capacity is doubled by doubling the number of anodes and using them in pairs, each pair being in multiple without stabilizing reactance.*

8. Subdivision of Anode Arms

The tests described in the last section indicate that if any number n of adjacent anodes are grouped in parallel as a unit, without stabilizing resistance or reactance, the current which they can safely carry is n times that of a single anode. Combining this result with that of section 5, where it was shown that the current capacity of a small anode in a small cylindrical housing is equal to that of a large one, we have a means of increasing the capacity of a rectifier almost without limit, *viz.*, by equipping it with a large number of small anodes, and using for each phase as many of these anodes in parallel as are needed to supply the required load.

The group of parallel-connected anodes may obviously be connected together inside the rectifier instead of outside. This is equivalent to combining them into one large anode, with a number of parallel paths leading to it, each path similar in dimensions to one of the single anode arms. This is the principle of subdivided arms referred to in section 5. The parallel paths are most conveniently formed by subdividing the arm by partitions. Fig. 2B shows one type of subdivision, in which the partitions are concentric cylinders.

This principle of subdivided arms has been very carefully tested, using partitions of different forms and dimensions, and anode arms from 15 to 33 cm. diameter. The results show that with a given "mesh" of subdividing partition the current that can be carried is proportional to the total cross section of the arm, as it should be. The maximum operating vapor pressure, and hence the current per unit cross section, depend upon the dimensions of the partitions.

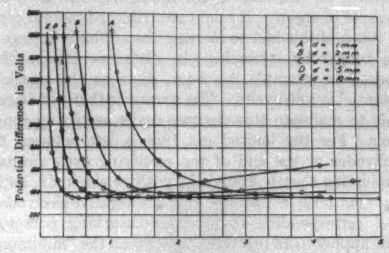

Fig. 20—Sparking Potential Vs. Pressure for Air, at Different Electrode Distances *d*.

Note that for low pressures the sparking potential is higher the lower the pressure and the smaller the distance

Grid and partitions somewhat similar to those described here have been used in rectifiers in the past, but their function and mode of operation have never been clearly demonstrated. It is believed that the tests described above constitute a firm basis for the use of such partitions. Their mode of operation is described in the following section.

9. Grids and Their Uses

Grids may serve two purposes:

(a) Grids or grilles like those described in the last section, consisting of parallel passages longer than they are wide, may perform the function described above of enabling a rectifier to operate at higher vapor pressure than would be possible without these grilles. To call them anti-arc-back devices is not quite appropriate, since the rectifier will operate as reliably without as with them, but at a lower temperature and correspondingly smaller current. It is not the purpose of this

article to discuss the mechanism of arc-back, many points of which are still obscure, but the following brief picture will make clear the function of the grille in increasing the possible operating pressure. For this purpose arc-back potential will be identified with sparking potential of the gas, which appears to be justified by the tests described in section 3.

The sparking potential of a gas decreases with decreasing pressure, as is well known, to a minimum value, and then increases rapidly with further decrease of pressure. Fig. 20 shows typical voltage-pressure curves for air. The minimum sparking potential occurs at pressures between 1 mm. and 1 cm. for the different gases, at electrode distances less than 1 cm., and its value varies from about 100 to 400 volts. The "tungar" rectifier, which operates near this minimum, has an arc-back potential of 175 volts, and hence a maximum operating d-c. potential of about 75 volts.

It will be noted in Fig. 20 that, at pressures above that corresponding to the minimum, the sparking potential is higher the greater the distance. The tungar operates on this side of the minimum, *viz.*, at 5 cm. pressure of argon. It might seem that advantage could be taken of this fact to increase the arc-back voltage of the tungar by increasing the distance between electrodes; for example, at 2-cm. distance the sparking potential is 1,800 volts. The answer is that it is impossible to maintain this distance under rectifying conditions. For the ionized gas that is left between the electrodes at the end of the rectifying cycle forms a conducting sheath around the positive electrode, building it out until its distance from the negative electrode is that corresponding to minimum sparking potential. This applies to all pressures higher than the "minimum" and all reasonable electrode distances.

It is evident therefore that high-voltage rectification can be accomplished, if at all, only in the range of pressure below the minimum point. In this range *the sparking potential is higher the smaller the distance between electrodes.* Here also the ionized gas tends to distort the electric field, in such manner as to *lengthen* the distance between electrodes and thus lower the sparking potential. But the possible lengthening is limited to the dimensions of the space around the anode. Thus with an anode arm 30 cm. in diameter the effective sparking distance between anode and wall is of the order of 60 cm. At this distance the pressure of mercury vapor corresponding to a sparking potential of 6,000 volts is approximately 0.003 mm. *The function of the grille is to decrease this distance* and thus allow a higher operating pressure. For example, by placing a grille 5 cm. below the anode the distance is decreased from 60 cm. to 5 cm., and the pressure thus increased from 0.003 mm. to about 0.040 mm.

The form of the grid is important, if it is to perform this function of shortening the distance between anode and body of the rectifier. Tests have shown that wire grids, unless their mesh is very fine, are not effective in shortening the distance, since the ions diffuse freely through the holes. On the other hand, partitioned passages or grilles like those described above are effective because they do not allow ions to diffuse through, provided the length of the passages is large compared to their width. Schottky[6] has shown that for a circular passage of radius r the fraction of ions which reach a distance l from the mouth is $\exp \frac{-2.40\,l}{r}$. Hence if the length of the passage is twice its diameter, only e^{-10} or $\frac{1}{22000}$ of the ions that enter will get through.

It is thus seen that a grille of this type, while allowing the arc to pass through, will not permit ions from the main body of the rectifier to diffuse through it during the time when the anode is negative. Under these conditions the distance which determines the operating vapor pressure is that between grille and anode. This theory is in agreement with our experiments.

(b) Grids may be used to control the starting of an arc. Rectifiers equipped with such grids are called *thyratrons.* Any type of grid can perform this function, provided it is insulated, and the voltage that must be applied to it to allow or prevent starting of the arc depends upon its location and fineness. For example, with the combination shown in Fig. 2C, of an insulated wire grid close to the anode and an uninsulated grille below, the arc will not start at all at practical operating voltages unless the grid is made positive with respect to the body of the rectifier. On the other hand, grids may be made so coarse that the arc will start easily with zero or positive grid voltage, but may be prevented from starting by making the grid sufficiently negative.

The characteristics and applications of thyratrons have been dealt with elsewhere.[7] One application, however, which is especially useful in rectifier operation, should be described here, *viz.*, the use of the grids for obtaining automatic voltage regulation. The principle of this regulation is that the *phase* of the voltage applied to the grids is automatically advanced as the load increases. Thus with light load the phase of the grid voltage lags considerably behind that of the corresponding anode, so that the starting of the anode current, which cannot take place until the grid becomes positive, occurs relatively late in each cycle. As the load increases the phase of the grid voltage is automatically advanced, causing the current to start earlier in the

6. W. Schottky, *Zeit. f. Phys.*, *31*, 181, 1924.

7. A. W. Hull, "Hot-Cathode Thyratrons," *G. E. Rev.*, *32*, 213-23, 390-99, 1929; D. C. Prince, "The Inverter Using Thyratron Tubes," *G. E. Rev.* *31*, 347, 1928; O. W. Pike, "Grid-Regulated Heavy-Current Tubes," *Electronics*, May, 1930; C. A. Sabbah, "The Series-Type Inverter," *G. E. Rev.* (to appear soon).

cycle and *flow for a longer time in each cycle*, thus compensating for its *lower average value* resulting from the lower line voltage.

The circuit for accomplishing this phase advance is shown in Fig. 21 and the resulting regulation in Fig. 22. The grid voltage is derived from a current transformer in the line, so connected that its variable output voltage

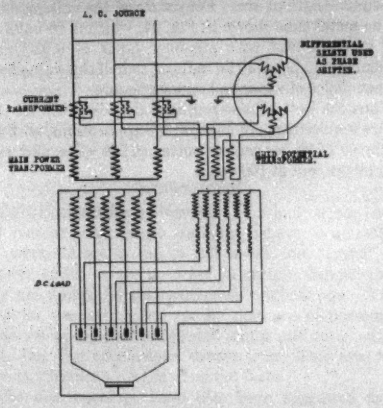

Fig. 21—Circuit for Obtaining Voltage Regulation by Advancing the Phase of Grid Voltage

is added to a 90-deg. lagging constant voltage derived from a selsyn transformer. The constants are so adjusted that at light load the anode current starts late in the cycle, advancing with load until at the desired overload it starts at the earliest possible starting time, namely, the time at which the anode voltage rises above the rectified d-c. line voltage. A further advance in

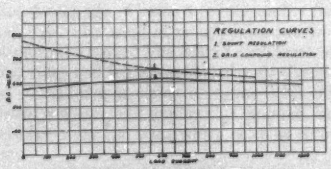

Fig. 22—Voltage Regulation Obtained with the Circuit Shown in Fig. 21

The unregulated characteristic is shown by the broken line, Curve 1

phase cannot further increase the time of current flow per cycle, and therefore the output voltage must fall with increase of load beyond this value.

10. Hot-Cathode Rectifiers

An extended discussion of this subject would be premature, since the largest hot-cathode installations at the time of writing are about 1,000 kw. Hot-cathode rectifiers and thyratrons are available in capacities up to 400 kw. at 8,000 volts. Fig. 23 shows a group of these tubes, in glass bulbs, and Fig. 24 a developmental model of hot-cathode metal thyratron for moderate voltage, with current rating of 100 amperes average, 600 amperes maximum.

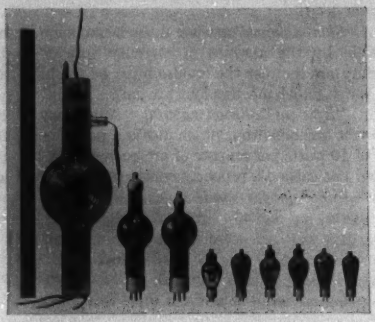

Fig. 23—Typical Hot-Cathode Mercury-Vapor Tubes

There are many features which make hot-cathode rectifiers attractive, two of which deserve special mention. The first is ease of control of the mercury-vapor pressure. A single drop of mercury suffices to furnish the required amount of vapor. If this drop is located

Fig. 24—Hot-Cathode Metal Thyratron, Rated 100 Amperes Average, 600 Amperes Maximum

in *the coolest part of the tube*, which may be remote from the arc, then the temperature of this part will completely determine the vapor pressure throughout the tube. The rest of the walls may be as hot as desired, since no liquid mercury ever comes in contact with them. For example, the rectifier shown in Fig. 24 operates

satisfactorily with its walls at 400 deg. cent. This makes it possible to build hot-cathode mercury-vapor rectifiers much smaller than mercury-arc rectifiers of similar capacity.

A second advantage of hot-cathode rectifiers is low voltage drop, with resulting high efficiency at low operating voltage. Three factors contribute to this low voltage drop:

(a) *Short distance* between cathode and anode, made possible by the absence of mercury spray. In the mercury-arc rectifier the anodes must be shielded from this spray, which means a long arc path.

(b) *"Efficient" cathode material.* A mercury arc has a 10-volt cathode drop, which means a power consumption of 10 watts per ampere of arc-current. A barium-coated hot cathode, properly constructed to avoid loss of heat by radiation, requires but 2 watts per ampere.

(c) *Choice of gas.* A hot-cathode rectifier may utilize any monatomic gas or metal vapor whose pressure can be maintained at the proper value. There are many practical vapors such as cadmium, sodium, caesium, with ionizing potentials lower than mercury, which may be expected to give a lower voltage drop for the same length of arc. For example, the voltage drop in the metal tube shown in Fig. 24, which is 10 volts at 100 amperes load with mercury vapor of 0.030-mm. pressure, may possibly be reduced to half this value by a proper choice of vapor and vapor pressure.

Thus the hot-cathode rectifiers promise to fulfill the future requirements of low-voltage applications, with an efficiency of 95 per cent or better at 250 volts, and well over 90 per cent at 100 volts.

Induction Motor Slot Combinations

Rules to Predetermine Crawling, Vibration, Noise and Hooks in the Speed-Torque Curve

BY GABRIEL KRON[1]

Associate, A. I. E. E.

Synopsis.—*A theory of coexisting stationary and rotating slot openings is developed with the aid of "revolving permeances," and the magnitude and speed of the parasitic fluxes due to the slot openings and the fundamental current density wave are found. It is shown that the interaction of the fundamental flux and a parasitic flux of the same order of magnitude as the fundamental flux causes the vibrations, objectionable noises, and crawlings of induction motors. The existence of these ruinous irregularities depends only in the difference of slots, the number of poles, and in some cases on the critical speeds of the rotor for circular or tortional vibrations, and is independent of the number of phases or the type of winding.*

Three rules are developed for the determination of destructive vibrations and noise, eight rules for establishing crawling speeds, and eight rules for finding hooks in the speed-torque curves of induction motors.

Introduction

SINCE induction motors were first built it has been noticed that some motors behave erratically; some do not go up to full speed, but "crawl" at a lower speed or are so noisy in certain regions that they are useless for all commercial applications. This behavior was attributed to the presence of harmonic fluxes set up by the distributed stator and rotor windings, but the conclusions drawn from their analysis were at variance with the observed facts.

The slot openings have also been suggested as a possible cause of these irregularities. However, in the analysis of slot openings either one of the surfaces was assumed to be smooth (Chapman, Krondl) or emphasis was laid only on the change in the fundamental flux due to the presence of slots (Adams, Carter, Lehmann).

In the meantime engineers arrived at certain empirical rules for the selection of rotor slots and by using these they were reasonably safe from unpleasant surprises. Such rules as the avoidance of less than 20 per cent difference in the number of stator and rotor slots or the use of twice a prime number for rotor slots had passed by word of mouth from one designing generation to the next. The lack of confidence in these empirical rules had led manufacturers to the wasteful method of building a series of rotors with different slots, when a new line of motors was required, in order to determine by actual tests which of these rotors was the best.

In spite of the importance of the problem and the large amount of available test data, the results of only *one* series of tests are available in engineering literature. Dr.-Ing. W. Stiel published in 1921 carefully measured speed-torque curves of a three-phase, four-pole, 24-slot stator provided with ten different rotors having 18, 19, 20, 22, 25, 27, 28, 29, 41 and 42 slots. The six published speed-torque curves are represented in Figs. 2-7. Stiel did not try to explain the interesting results of his tests, he only suggested empirical rules from them for the selection of rotor slots. Punga, Ottenstein, and Heubach also contributed experimental data in their attempt to solve the riddle of slot combinations.

In the following pages the writer investigates the causes of the erratic behavior of these and other motors tested by him or by others and sets up rules to predetermine them. Many of the noisy regions, hooks and crawlings had been foretold by him from these rules and those were all found at the preassigned speeds without exceptions.

The slot openings are analyzed by a new method. The idea of "revolving permeances" is believed to be novel. By assuming a two pole flux as the first harmonic and the fundamental flux with p pairs of poles as the pth harmonic, the confusing use of "subharmonics" and "fractional harmonics" is avoided. With this notation and with the use of the general term of Fourier's series the rules and the formulas for speeds are expressed in simple forms easily remembered.

I—Harmonic Fluxes Due to Slot Openings

I—Assumptions

As a first approximation, the theory of induction motors assumes a fundamental sine wave current-density distribution in space and in time. This is assumed to produce a sine wave magnetomotive force which in conjunction with a uniform permeance along the air-gap produces a sine wave flux-density distribution in time and in space. Refinements are usually introduced by assuming non-sinusoidal distribution of current density in time (time harmonics) or in space (space harmonics), still assuming constant permeance along the air-gap.

In the following an investigation will be made to determine (1) what fluxes exist in the presence of a fundamental sinusoidal current-density and a non-

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

sinusoidal air-gap permeance and (2) how these fluxes interact.

The following assumptions are made besides the usual ones:

1. The slots are replaced by *effective* slots having rectangular shape. (Fig. 1a)
2. The lines of flux are parallel in the air.

II—The Revolving Permeances

First it is assumed that both the stator and rotor teeth are removed. The permeance of the resulting gap $\mathcal{P}_o$ is constant and is represented on Fig. 1b by unit height.

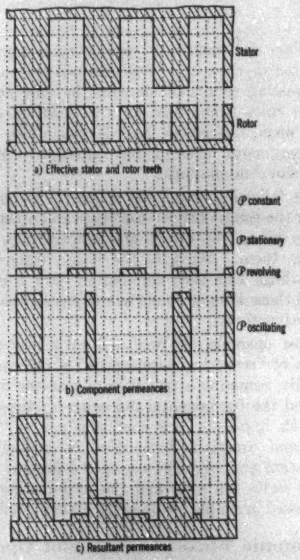

Fig. 1—Permeances Due to Slot Openings

a. Effective stator and rotor teeth
b. Component permeances
c. Resultant permeance

Let the stator teeth be introduced. At certain places the length of the gap is decreased and the permeance increased. This change of permeance can be considered as if an additional permeance $\mathcal{P}_s$ consisting of rectangles had been introduced with the stator teeth. If the number of stator teeth is G_1, this permeance $\mathcal{P}_s$ can be analyzed into a constant term and a series of harmonic permeances, $\pm A_k \cos k\, G_1\, x$ (See Appendix), all being the integer multiples of G_1. The largest of these is $A \cos G_1\, x$, the harmonic having G_1 pairs of poles.

Let now the stator teeth be removed and the revolving rotor teeth, G_2 in number, be introduced. This is equivalent to introducing an additional revolving permeance $\mathcal{P}_r = \Sigma\, B_k \cos (k\, G_2\, x - v\, k\, G_2\, \omega\, t)$ where v is the ratio of the rotor speed to the two pole synchronous speed S. (S is 3,600 with 60 cycles.) The speed of each of these permeances is $v\, k\, G_2 / k\, G_2 = v$. The largest permeance is $B \cos (G_2\, x - v\, G_2\, \omega\, t)$, the one with G_2 pairs of poles.

If the stator teeth are *not* removed when the rotor

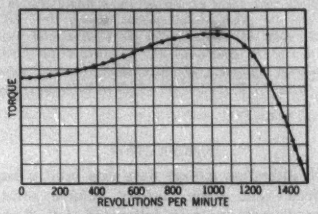

Fig. 2—Slot Ratio 24/18—4-Pole—Three-Phase (Stiel)

Other smooth speed-torque curves tested:
Slot ratio 24/18—4, and 8 pole—two and three phase
Slot ratio 24/19—4, 6 and 8 pole—two and three phase
Slot ratio 24/20—6, and 8 pole—two and three phase
Slot ratio 24/22—4, 6 and 8 pole—two and three phase
Slot ratio 24/28—8 pole—three phase

teeth are introduced, at those points along the circumference where no stator teeth exist, the additional permeance due to the rotor teeth is $\mathcal{P}_r$, same as above, but at those points where stator teeth do exist, the additional permeance is larger than $\mathcal{P}_r$ by an amount $\mathcal{P}_o$. This permeance $\mathcal{P}_o$, due to the coexistence of slots, is much larger than $\mathcal{P}_s$ or $\mathcal{P}_r$. In decreasing the air-

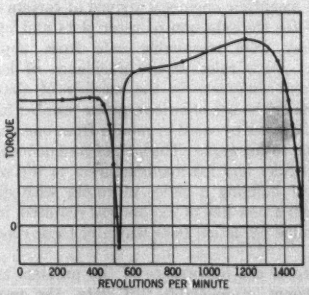

Fig. 3—Slot Ratio 24/19—4-Pole—Three-Phase (Stiel)

gap, $\mathcal{P}_s$ or $\mathcal{P}_r$ change very slightly, but $\mathcal{P}_o$ increases very rapidly.

As the rotor revolves, each part of $\mathcal{P}_o$ describes a peculiar cycle. The width of each rectangle increases from zero to a definite value, then it moves along the periphery, soon decreases and finally disappears. In spite of its complicated behavior it can easily be expressed in a Fourier's series by noting that if two series representing $\mathcal{P}_s$ and $\mathcal{P}_r$ are multiplied together the

product represents the oscillating permeance $\mathcal{P}_o$. Hence

$$\mathcal{P}_o = \sum C_k \cos [(k_1 G_2 \pm k_2 G_2) x \pm v k_2 G_2 \omega t]$$

The largest among these revolving double-infinity harmonics are

$$\mathcal{P}_o = C_1 \cos [(G_1 - G_2) x + v G_2 \omega t] + C_2 \cos G_1 x + C_3 \cos (G_2 x - v G_2 \omega t)$$

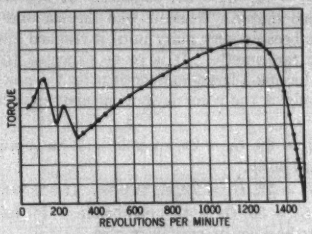

FIG. 4—SLOT RATIO 24/20—4-POLE—THREE-PHASE (STIEL)

With decreasing air-gap $\mathcal{P}_o$ increases rapidly and it may reach a value of the same order of magnitude as the constant gap. It is this very permeance which causes all the trouble.

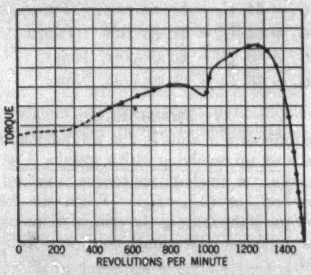

FIG. 5—SLOT RATIO 24/27—4-POLE—THREE-PHASE (STIEL)

Among the permeances only the largest ones, $\mathcal{P}_o$ with $(G_1 - G_2) = \pm D$ pairs of poles, $\mathcal{P}_o$ with G_1 and $\mathcal{P}_r$ with G_2 pairs of poles will be considered.

III—FLUXES

It is shown in the Appendix that whenever a revolving current density with "p_1" pairs of poles exists along an air-gap having a constant permeance plus a revolving permeance with "p_2" pairs of poles, it produces a revolving m. m. f. space distribution with "p_1" pairs of poles. This m. m. f. produces with the permeance "p_2" two revolving fluxes having $(p_1 + p_2)$ and $(p_1 - p_2)$ pairs of poles.

If $p_2 = p_1 = p$, the m. m. f. has in addition an oscillating term (under the assumptions). One revolving flux is produced with $2\,p$ pairs of poles and a single-phase flux with p pairs of poles in addition to the revolving fundamental flux.

Among the fluxes only those produced by $\mathcal{P}_o$ can effectively interact with the fundamental. The fundamental m. m. f., $M \cos (p\,x - \omega\,t)$ and $\mathcal{P}_o = C \cos (D\,x + v\,G_2\,\omega\,t)$ produce fluxes $F \cos [(p \pm D)x \pm (v\,G_2 \pm 1)\,\omega\,t]$ with $(p \pm D)$ pairs of poles revolving with a speed $\pm S\,(v\,G_2 \pm 1)/(p \pm D)$. When $p \pm D = p$ or $p \pm 1$, powerful disturbing effects occur.

It should be noted that no assumptions had been made as to the number of phases or types of winding, hence all the rules in part I are independent of them.

IV—FIRST RULE

If the difference of slots is 1 or $2\,p \pm 1$, a flux is produced differing by two poles from the fundamental flux. An unbalanced force moves with a speed $(G_1\,v)\,S$ or $[(G_1 \pm 2\,p)\,v \pm 2]\,S$ with respect to the rotor. When the speed of this unbalanced force approaches the critical speed of the rotor, noise and vibration occur accompanied by a diminution of torque or even crawling.

It has been shown by Fritze that whenever two fluxes differing by two poles exist, an unbalanced force moves with the difference of their frequencies. In these motors one of the fluxes is the fundamental, the other is a flux of the same order of magnitude as the fundamental, consequently the noise at the critical speed is so objectionable that whenever it occurs in the working

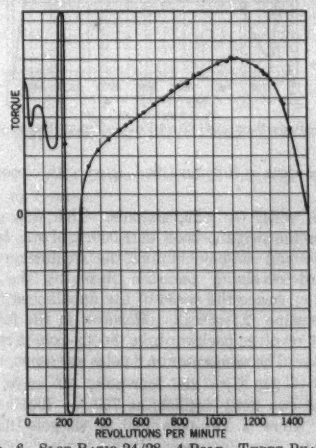

FIG. 6—SLOT RATIO 24/28—4-POLE—THREE-PHASE—UNSKEWED (STIEL)

range, the motor is usually useless for all purposes. The chances that the unbalanced force (which revolves sometimes G_1 times as fast as the rotor) reaches the critical speed before the rotor reaches its working speed, are greater with smaller p, larger G_1 and larger

rotors. Experiments also show that the severeness of the vibrations and the diminution of torque are proportional to the rotor current.

Experimental evidence

1. $G_1 = 24$, $G_2 = 19$, four-pole, three-phase, 50 cycle. Stiel shows its speed-torque curve on Fig. 3. At 500 r. p. m. its noise was an "indescribable wild roar." It also crawled at that speed.

The writer tested a similar motor on 60 cycles and the shrill noise and vibration occured in the neighborhood of 1,000 r. p. m. With decreasing frequency the speed of vibration slightly increased, just as required by the formula. The diminution of torque was not noticeable.

This same motor was tested on two-phase and single-phase showing exactly the same behavior as on three-phase.

As the closed rotor slots were gradually opened no change in the intensity of the noise was noticeable. This fact clearly shows that the assumption of rectangu-

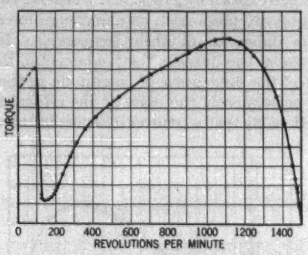

Fig. 7—Slot Ratio 24/42—4-Pole—Three-Phase (Stiel)

lar rotor slots is fully justified, even with closed rotor slots.

2. $G_1 = 24$, $G_2 = 19$, six-pole. This was noisiest again in the neighborhood of 1,000 r. p. m. In both four-pole and six-pole motors the frequency of the backward revolving parasitic fluxes is the same, $(1 + 22\,v)$, hence the speed of the unbalanced force is also the same $(2 + 20\,v)$. (However in the four-pole motor it revolves backward, in the six-pole it revolves forward.) The same rotor on 8 poles was quiet.

3. $G_1 = 24$, $G_2 = 25$, four-pole, 50 cycle. Stiel describes two noisy regions, at 750 (or 700) and 900 r. p. m. Here a two-pole and a six-pole flux are produced. The frequency of the forward revolving six-pole flux is $(1 + 22\,v)$ and of the fundamental $(1 - 2\,v)$, hence the frequency of the unbalanced force is $24\,v$. The frequency of the forward revolving two-pole flux with respect of the rotor is $(1 - 26\,v)$, hence the frequency of the unbalanced force is again $24\,v$. The two unbalanced forces are identical. The motor did not crawl like the 19-slot rotor at 500 r. p. m.

4. $G_1 = 24$, $G_2 = 27$, four-pole. Stiel gives its speed-torque curve as Fig. 5. Noise and diminution of torque occurred at 1,000 r. p. m.

5. $G_1 = 24$, $G_2 = 29$, four-pole. Stiel found it useless due to noises.

6. $G_1 = 54$, $G_2 = 53$, six-pole. Stiel mentions that this motor is known to behave as the 24/25 combination.

7. $G_1 = 72$, $G_2 = 57$, 16-pole. It was used on a two-speed motor. When it was thrown over to low speed it became very noisy until it reached low speed. Then it ran quietly. It is to be expected that with motors having a large number of poles the noise would occur only if the rotor is moving above synchronous speed.

8. $G_1 = 36$, $G_2 = 29$, six-pole. It has been known to be very noisy.

9. $G_1 = 72$, $G_2 = 79$, eight-pole. It was very noisy.

10. $G_1 = 36$, $G_2 = 45$, eight-pole. It has been known to be noisy on various frames.

11. $G_1 = 48$, $G_2 = 43$, four-pole. Heubach mentions that it was noisy and vibrated badly.

12. $G_1 = 46$, $G_2 = 41$, four-pole, single-phase. (Heubach.) Noisy.

13. $G_1 = 60$, $G_2 = 65$, six-pole, two-phase. (Punga.) Noisy.

14. $G_1 = 54$, $G_2 = 47$, six-pole. (Punga.) Noisy.

15. $G_1 = 54$, $G_2 = 59$, six-pole. (Punga.) Noisy.

16. $G_1 = 36$, $G_2 = 41$, six-pole. Noisy.

17. $G_1 = 36$, $G_2 = 43$, six-pole. Noisy.

V—Second Rule

If the difference of slots is $2\,p$, a fundamental parasitic flux is set up revolving with a speed $s\,(1 \pm v\,G_2)/p$ rev. per min. A single-phase torque is set up with a frequency $v\,G_2$ or $(2 \pm v\,G_2)$. When the frequency of this torque is the same as the critical frequency of the rotor for torsional vibrations, noise and vibration is produced.

Rumbling noise is also set up when the speed of the parasitic flux is low. The cause of this rumbling noise requires more investigation.

Experimental evidence

1. $G_1 = 24$, $G_2 = 18$, six-pole. It was quiet below synchronous speed. When driven backward, it had a rumbling noise in the neighborhood of 200 r. p. m. When driven beyond synchronous speed as a generator, it became very noisy around double synchronous speed.

2. $G_1 = 24$, $G_2 = 28$, four-pole. It was noisy at all speeds, with the strongest rumbling noise on both sides of 140 r. p. m. It was tested on one-, two-, and three-phase.

3. $G_1 = 48$, $G_2 = 60$, 12-pole. It was very noisy at 95 r. p. m.

4. $G_1 = 36$, $G_2 = 30$, six-pole. It was known to be noisy.

5. $G_1 = 24$, $G_2 = 22$, two-pole. It was very noisy at low speeds.

VI—Third Rule

If the difference of slots is p, a single-phase flux with p pairs of poles exists (see Appendix) which sets up a single-phase torque with a frequency $2\,v\,G_2$ or $(2 \pm 2\,v\,G_2)$. When the frequency of this torque is the same as the critical frequency of the rotor for torsional vibrations, noise, and vibrations are produced.

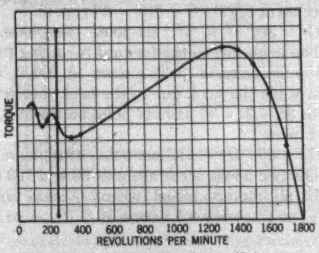

Fig. 8—Slot Ratio 24/28—4-Pole—Three-Phase—Skewed

Experimental evidence

1. $G_1 = 48$, $G_2 = 46$, four-pole. It was very noisy at all speeds.

2. $G_1 = 36$, $G_2 = 34$, four-pole. It was noisiest around 1,000 r. p. m. showing also signs of vibration.

3. $G_1 = 24$, $G_2 = 22$, four-pole. Stiel found this motor quiet. The writer also found a similar motor quiet, but when it was driven above synchronous speed it became very noisy in the neighborhood of 2,400

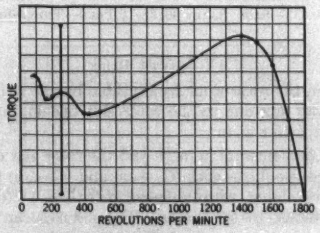

Fig. 9—Slot Ratio 24/28—4-Pole—Two-Phase—Skewed

r. p. m. This combination has been known to be noisy below synchronous speed also.

4. $G_1 = 24$, $G_2 = 28$, eight-pole. Severe vibrations occurred above 1,800 r. p. m.

VII—Fourth Rule

If the difference of slots is $2\,p$, at a rotor speed $(2/G_2)\,S$ the speed of the parasitic flux with p pairs of poles becomes that of the fundamental, the motor crawls and behaves just like a reaction machine. When G_2 is greater than G_1 this locking occurs in the working range; when smaller, the locking occurs at a negative speed.

Experimental evidence

1. $G_1 = 24$, $G_2 = 28$, four-pole, one-, two-, and three-phase. Figs. 8 and 9. The rotor was skewed one slot pitch, still the variation of torque while locked was considerable. (The synchronous speed was watched in all examples through a stroboscope.) Stiel did not know of the possibility of the existence of a superimposed reaction machine, so he plotted the vertical line as a hook in the speed-torque curve (Fig. 6). His motor was unskewed.

2. $G_1 = 48$, $G_2 = 60$, 12-pole (Fig. 10). The crawling occurred at $(2/60)\ 3{,}600 = 120$ r. p. m. It was skewed.

3. $G_1 = 36$, $G_2 = 42$, six-pole. Stiel mentions that this motor behaves as the 24/28 combinations. In both cases locking occurs at one-seventh synchronous speed and this fact gave the erroneous idea that they

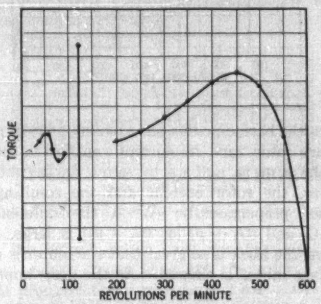

Fig. 10—Slot Ratio 48/60—12-Pole—Three-Phase—Skewed

were caused by a powerful seventh harmonic of the stator m. m. f. wave.

4. $G_1 = 24$, $G_2 = 20$, four-pole. When driven backward or when the leads were reversed under load, the motor crawled at $(2/20)\ 3{,}600 = 360$ r. p. m.

5. $G_1 = 24$, $G_2 = 18$, six-pole. When driven backward it crawled at $(2/18)\ 3{,}600 = 400$ r. p. m.

6. $G_1 = 48$, $G_2 = 58$, 10-pole (Fig. 11). It crawled at $(2/58)\ 3{,}600 = 124$ r. p. m.

7. Any reaction machine is a particular case where $G_1 = 0$, $G_2 = 2\,p$, number of poles $= 2\,p$. The parasitic flux has $p - 2\,p = -\,p$ pairs of poles. When the speed of the rotor is $(2/2\,p)\,S = S/p$ r. p. m., the fundamental and the parasitic flux lock.

VIII—Fifth Rule

When the rotor has more than $(G_1 + p)$ slots, the flux due to the stator slot opening produces a hook at $S/(G_1 + p)$ r. p. m. When the slots are more than $(G_1 - p)$, a hook occurs at $S/(G_1 - p)$ r. p. m. if driven backward.

The fundamental m. m. f. $A \cos (p\,x - \omega\,t)$ with p

pairs of poles and the stator slot opening $\mathcal{P}_s = B \cos G_1 x$ with G_1 pairs of poles produce two fluxes $C \cos [(G_1 + p) x - \omega t] + D \cos [(G_1 - p) x + \omega t]$ with $(G_1 + p)$ and $(G_1 - p)$ pairs of poles, the first revolving forward with a uniform speed $S/(G_1 + p)$ the second revolving backward with a uniform speed $S/(G_1 - p)$. The hook in the speed-torque curve is due to induction motor effect.

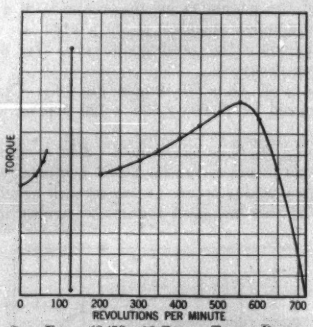

FIG. 11—SLOT RATIO 48/58—10-POLE—THREE-PHASE—SKEWED

When the harmonic flux has as many pairs of poles as there are rotor bars, no current can flow in the rotor, since the e. m. fs. add up to zero. As the difference increases, the rotor current and the resulting hook increases proportionally. When the difference between G_2 and $(G_1 + p)$ [or $(G_1 - p)$] is large, say 80 per cent, the hook is so great that the motor is useless, unless the motor is skewed. As the skew approaches

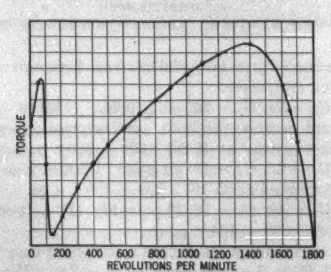

FIG. 12—SLOT RATIO 36/62—4-POLE—THREE-PHASE

$p/(p + G_1)$ times the circumference, the hook disappears proportionally.

The greater number of hooks and crawlings which are observed in practise belong under this rule.

Experimental evidence

1. $G_1 = 24$, $G_2 = 42$, four-pole, unskewed. (Fig. 7.) A hook occurs at $3{,}600/(2 + 24) = 115$ r. p. m. Since the difference of slots is large and the motor is unskewed, the motor is useless.

2. $G_1 = 24$, $G_2 = 41$, four-pole, unskewed. Stiel mentions that this motor behaved as the 24/42 combination, and due to the hook it was also useless.

3. $G_1 = 36$, $G_2 = 62$, four-pole, unskewed. (Fig. 12.) The hook occurs at $3{,}600/(2 + 36) = 95$ r. p. m.

4. $G_1 = 36$, $G_2 = 58$, six-pole, unskewed. (Fig. 13.) The hook occurs at $3{,}600/(3 + 36) = 92$ r. p. m.

5. $G_1 = 12$, $G_2 = 19$, two-pole, 100-cycle. (Fig. 14.) The hook occurs at $6{,}000/(1 + 12) = 460$ r. p. m.

6. $G_1 = 48$, $G_2 = 60$, 12-pole. (Fig. 10.) The hook occurs at $3{,}600/(6 + 48) = 67$ r. p. m.

7. $G_1 = 36$, $G_2 = 57$, six-pole, two-phase. (Fig. 15.) The hook due to the forward revolving flux with $3 + 36 = 39$ pairs of poles occurs at $3{,}600/39 = 92$ r. p. m., while the hook due to the backward revolving flux with $36 - 3 = 33$ pairs of poles occurs at $3{,}600/33 = 109$ r. p. m.

8. $G_1 = 36$, $G_2 = 41$, six-pole, two-phase. (Fig. 16.) The two hooks occur again at 92 and 109 r. p. m.

9. In all motors tested where G_1 was less than $(G_1 + p)$ no trace of hook occurs at $S/(G_1 + p)$ r. p. m.

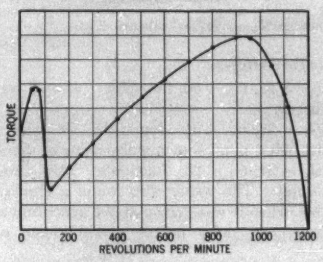

FIG. 13—SLOT RATIO 36/38—6-POLE—THREE-PHASE

IX—SIXTH RULE

When the rotor slots are divisible by the number of poles the flux due to the rotor slot opening causes a hook at S/G_2 r. p. m.

The fundamental m. m. f. and the rotor slot opening produce two fluxes with $(G_2 + p)$ and $(G_2 - p)$ pairs of poles revolving with a speed $S\,(v\,G_2 \pm 1)/(G_2 \pm p)$ with respect to the stator. These fluxes can induce e. m. fs. in the stator winding wound for p pairs of poles only if $(p \pm G_2)$ is an odd multiple of p. That is, when $p + G_2 = p\,(1 + 2\,k)$, or when $G_2/2\,p = k =$ integer. Since these fluxes stand still with respect to the stator at S/G_2, a hook occurs at that speed due to induction motor effect. Since the rotor slot openings are usually small, these hooks are small and do not cause crawling.

*Experimental evidence**

1. $G_1 = 24$, $G_2 = 20$, four-pole, two-phase. (Fig. 17.) A hook occurs at $3{,}600/20 = 180$ r. p. m.

*See also the seventh rule.

2. $G_1 = 24$, $G_2 = 20$, four-pole, three-phase. (Fig. 4.) A hook occurs at $3{,}600/20 = 180$ r. p. m.

3. $G_1 = 24$, $G_2 = 28$, four-pole, two-phase. (Fig. 9.) A hook occurs at $3{,}600/28 = 129$ r. p. m.

4. $G_1 = 24$, $G_2 = 28$, four-pole, three-phase. (Figs. 6 and 8.) A hook occurs at $3{,}600/28 = 129$ r. p. m.

5. All motors tested where G_2 is not divisible by p do not show any hook in the neighborhood of S/G_2 r. p. m.

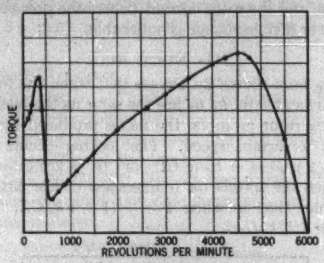

Fig. 14—Slot Ratio 12/19—2-Pole—Three-Phase—100 Cycle

6. In all self-starting synchronous motors $G_1 = 0$, $G_2 = 2\,p$. Here $p - 2\,p = -\,p$ is an odd multiple of p. Since the slot opening (the gap between the poles) is large, crawling may occur in the neighborhood of $S/2\,p$ = one-half synchronous speed.

II—Harmonic Fluxes due to Winding

X—Harmonic M. M. Fs.

So far it has been assumed that the air-gap is slotted and only the fundamental m. m. f. acts. Now it will be assumed that the air-gap is smooth and the harmonic m. m. fs. due to the distribution of winding also act.

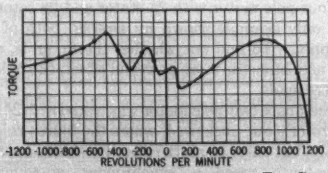

Fig. 15—Slot Ratio 36/57—6-Pole—Two-Phase

It is well known that in a full-pitch stator winding with integer number of slots per pole phase group and with p pairs of poles the m. m. f. wave contains harmonics with $p \pm k\,\mathcal{P} = n = p \pm k\,2\,p\,\psi$ pairs of poles (ψ = number of phases, $\mathcal{P}$ = number of pole-phase groups) revolving with a speed S/n. It is also known that if a *squirrel-cage rotor* with G_2 slots is cut by a revolving flux with n pairs of poles the produced m. m. f. wave and flux have $(n \pm k\,G_2)$ pairs of poles revolving with a speed $S\,(1 \pm v\,k\,G_2)/(n \pm k\,G_2)$ with respect to the stator.

In a three-phase stator the *largest* forward revolving harmonic flux has $7\,p$ pairs of poles, in a two-phase motor it has $5\,p$ pairs of poles. In the rotor the largest harmonic m. m. fs. and fluxes have n, $(n + G_2)$ and $(n - G_2)$ pairs of poles and their magnitude is inversely proportional to the number of their poles.

XI—Seventh Rule

Whenever G_2 is divisible by the number of poles $2\,p$, a small hook occurs at $S/7\,p$ r. p. m. in a three-phase motor, at $S/5\,p$ in a two-phase motor and also at S/G_2 in both types, due to "internal concatenation" (cascade) effect of the harmonic fluxes.

If the harmonic flux due to the stator winding is small, say less than 8 per cent of the fundamental flux, the harmonic rotor current due to it is so small that the produced harmonic torque is negligible, provided the fluxes due to this harmonic rotor current can not induce an e. m. f. in the stator winding. If an e. m. f. can be induced in the stator winding, the stator acts as a short-circuited secondary to the harmonic rotor current, the rotor current is much larger and the resulting hook is noticeable.

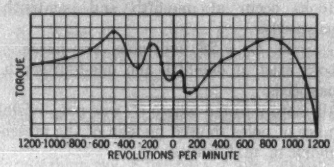

Fig. 16—Slot Ratio 36/41—6-Pole—Two-Phase

Let the largest stator harmonic have $(p + 2\,p\,\psi)$ pairs of poles. When they cut the rotor, the rotor current produces fluxes with $(p + 2\,p\,\psi \pm G_2)$ pairs of poles. Now a flux can induce an e. m. f. in a winding wound for p pairs of poles only if the flux is an odd multiple of p, that is, if it has $p\,(1 + p\,k)$ pairs of poles. Hence, $(p + 2\,p\,\psi \pm G_2)$ is equal to $p\,(1 + p\,k)$ only if $G_2/2\,p = k$, that is if the number of rotor slots is divisible by the number of poles.

At $S/(p + 2\,p\,\psi)$ r. p. m. the stator harmonic flux stands still with respect to the rotor and the superimposed harmonic torque is zero. At S/G_2 r. p. m. all the rotor fluxes stand still with respect to the stator, the rotor current is small and the superimposed harmonic torque is again zero. At the neighborhood of these two speeds hooks occur in the speed-torque curves.

In three-phase motors the hook at S/G_2 is larger than the other as a consequence of the sixth rule. In two-phase motors however, the hook caused by the fifth harmonic is larger and under load it may even cause crawling.

Experimental evidence

1. $G_1 = 24$, $G_2 = 20$, four-pole, three-phase. (Fig. 4.) Small hooks occur at one-seventh and one-tenth synchronous speed.

2. $G_1 = 24$, $G_2 = 28$, four-pole, three-phase. (Figs. 6 and 8.) Hooks occur at one-seventh and one-fourteenth synchronous speeds.

3. $G_1 = 24$, $G_2 = 20$, two-pole, three-phase. (Fig. 19.) Hooks occur at 1/7 and 1/20 synchronous speed.

4. $G_1 = 24$, $G_2 = 18$, six-pole, three-phase. (Fig. 18.) The hooks at one-sixth and one-seventh synchronous speeds merge into one.

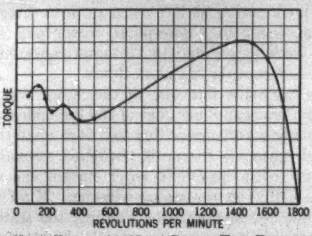

Fig. 17—Slot Ratio 24/20—4-Pole—Two-Phase—Skewed

5. $G_1 = 24$, $G_2 = 20$, four-pole, two-phase. (Fig. 17.) Hooks occur at one-fifth and one-tenth synchronous speeds.

6. $G_1 = 24$, $G_2 = 28$, four-pole, two-phase. (Fig. 9.) Hooks exist at one-fifth and one-fourteenth synchronous speeds.

7. All motors tested where G_2 was not divisible by the number of poles showed no sign of the presence of the harmonic m. m. f. waves.

XII—Eighth Rule

Whenever G_2 is divisible by the number of poles $2\,p$, a hook occurs at S/G_2 r. p. m. due to the internal

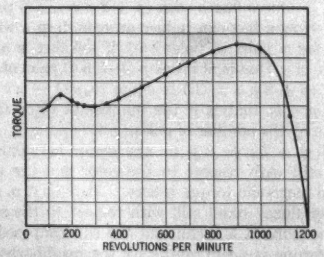

Fig. 18—Slot Ratio 24/18—6-Pole—Three-Phase

cascade connection of the *fundamental* flux with the rotor harmonic fluxes. The smaller $G_2/2\,p$ is, the greater the hook, since the magnitude of the rotor harmonic fluxes is greater. When $G_2/2\,p = 1$ or $G_2 = 2\,p$, the produced harmonic flux $(p - G_2) = -\,p$ is as large as the fundamental flux and crawling occurs. Crawling may occur even if $G_2/2\,p$ is an integer close to unity.

This phenomenon is known in the case of $G_2 = 2\,p$ as the "Görges effect" and it corresponds to a polyphase motor with single-phase rotor.

It should be noted that the hooks at S/G_2 r. p. m. are attributed to three different causes. They are attributed to the fluxes produced by (1) the fundamental m. m. f. and the rotor slot opening (this effect is noticeable especially in self-starting salient-pole synchronous motors), (2) the fundamental m. m. f. and the rotor currents (Görges effect), (3) the harmonic m. m. fs. and the rotor current (where the presence of a seventh or fifth harmonic is noticeable.)

XIII—Ninth Rule

It has been shown by Dreese that when the forward revolving stator m. m. fs. are the same as the backward revolving rotor m. m. fs. the fluxes produced by them lock at a certain speed. The locking occurs when $(p + k_1\,2\,p\,\psi) = -\,(p \pm G_2)$ or $G_2 = 2\,p\,(1 + k_1\,\psi)$ and when the speeds (or frequencies) of the fluxes are the same, that is when $1 = -\,(1 \pm v\,k_2\,G_2)$. From this $v = S(2/k_2\,G_2)$.

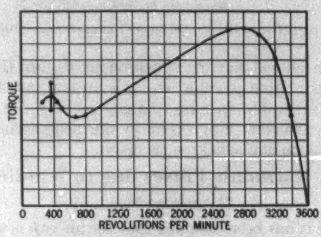

Fig. 19—Slot Ratio 24/20—2-Pole—Three-Phase—Unskewed

It is shown in the fourth rule that locking occurs at $S(2/G_2)$ when $G_2 - G_1 = 2\,p$. In the case of a three-phase four-pole motor with 24 stator and 28 rotor slots (in the only example shown by Dreese) both of these rules apply. To find out whether the m. m. fs. or the permeances have the more powerful effect, the following tests were made.

A three-phase two-pole motor was tested with 24 stator and 20 rotor slots, unskewed, where the fourth rule does not apply, but only the ninth rule. By slowly approaching one-tenth speed a synchronous crawling was established after much trial and was watched through a stroboscope. But with slight variation of load (Fig. 19) the motor stepped out of synchronous speed, although it was not at all skewed. (In a skewed 24/20-two-phase four-pole motor this crawling could not be noticed. Fig. 17.)

Another three-phase 12-pole motor was tested with 48 stator and 60 rotor slots, where the ninth rule does not apply, but only the fourth rule. Although this motor has a full skew, it crawled at one-fifth synchronous speed if started under load and it crawled through a large variation of load as shown in Fig. 10.

XIV—Tenth Rule

Whenever a harmonic flux larger than say 8 per cent of the fundamental is produced by the winding, a hook occurs in the speed-torque curve by induction motor effect irrespective of the number of rotor slots.

In windings with integral number of slots per pole phase group, the only harmonic which causes a hook in the speed-torque curve irrespective of the number of rotor slots is the backward revolving third harmonic (with $3\,p$ pairs of poles) of the two-phase induction motor.

Experimental evidence

1. $G_1 = 36, G_2 = 57$, six-pole, two-phase. (Fig. 15.) The backward revolving third harmonic is pronounced, but no trace of the forward revolving fifth harmonic was found, since 57 is not divisible by 6.

2. $G_1 = 36, G_2 = 41$, six-pole, two-phase. (Fig. 16.) Here also the third harmonic is present, but the fifth harmonic is absent.

XV—Summary of Rules

A. Vibration and noise are liable to be present as follows:

1. When the slots differ by one or by the number of poles plus or minus one, transverse vibration and noise may occur.
2. When the slots differ by half the number of poles, torsional vibration and noise may occur.
3. When the slots differ by the number of poles, torsional vibration and noise may occur and also rumbling noises unaccompanied by critical vibrations.

The chances that the noise occurs in the working range are greater with smaller number of poles, higher speed and with smaller transverse or torsional critical speeds. When these noises do occur the motor is practically useless.

B. Crawling may be present as follows:

1. When the slots differ by the number of poles and the rotor has more slots than the stator, synchronous crawling (reaction machine effect) occurs at a speed $S\,(2/G_2)$. When the rotor has less slots the synchronous crawling occurs at a negative speed.
2. When the rotor slots are greater than $(G_1 + p)$ by, say, 80 per cent, crawling occurs at $S/(G_1 + p)$.
3. When the rotor slots are greater than $(G_1 - p)$ by, say, 80 per cent, crawling occurs at $S/(G_1 - p)$ if driven backward.
4. When the rotor slots are divisible by the number of poles, a two-phase motor may crawl at one-fifth synchronous speed it if is started under load.
5. A two-phase motor may crawl at one-third synchronous speed if it is driven backward under load.
6. When $G_2/2\,p$ is an integer equal to or close to unity, crawling may occur at S/G_2 r. p. m.
7. When the vibrations under A occur at low speeds, the motor crawls.
8. When the harmonic flux due to the winding is large, say larger than 10 per cent of the fundamental, the motor may crawl.

C. Smaller irregularities, hooks in the speed-torque curve, may appear as follows:

1. When the rotor slots are greater than $(G_1 + p)$, a hook occurs at $S\,(G_1 + p)$ r. p. m.
2. When the rotor slots are greater than $(G_1 - p)$, a hook occurs at $S/(G_1 - p)$ r. p. m. if it is driven backward.
3. When the rotor slots are divisible by the number of poles, a hook occurs at S/G_2 r. p. m.
4. When the number of rotor slots is divisible by the number of poles, a hook occurs at one-seventh synchronous speed in a three-phase motor and at one-fifth synchronous speed in a two-phase motor. In both cases the hook is accompanied by another hook at S/G_2 r. p. m.
5. When $G_2 = 2\,p\,(1 + k_1\,\psi)\,2\,p$, a small synchronous locking (synchronous motor effect) occurs at $S\,(2/k_2\,G_2)$ r. p. m.
6. Any two-phase motor has a hook at one-third synchronous speed if it is driven backward.
7. When a winding introduces a harmonic flux larger than, say, 8 per cent of the fundamental, it always produces a hook.
8. When the vibrations under A occur at higher speeds, proportionally smaller hooks occur at those speeds.

It should be emphasized that these rules are not necessarily complete. They only cover the cases known to the writer. The series of rules for smaller noises, which make the motor useless only for certain applications are not studied at all in this paper, neither are the variations of torque during starting and at very low speeds.

Acknowledgment

The writer acknowledges the helpful suggestions of Prof. E. E. Dreese during the investigation made in the Research Laboratories of the Lincoln Electric Co. The writer also wishes to thank Mr. J. F. Lincoln for permission to publish these rules.

Nomenclature

p = number of pairs of poles of the stator winding
G_1 = number of stator slots
G_2 = number of rotor slots
D = difference of slots = $G_1 - G_2$
k = any positive integer
S = synchronous speed with two poles
v = ratio of rotor speed to two-pole synchronous speed
β = ratio of width of effective slot to tooth pitch
$\mathcal{P}_c$ = gap permeance without teeth

$\mathcal{P}_s$ = stationary permeance due to additional stator teeth

$\mathcal{P}_r$ = revolving permeance due to additional rotor teeth

$\mathcal{P}_o$ = Oscillating permeance due to coexistence of slots.

Appendix

Permeance Due to Revolving Teeth

Calculations for the width and length of an effective tooth can be found in the writings of Carter, Lehmann and others.

Let the length of the whole air-gap be 2π radians, the number of slots be G and the ratio of the slot width to tooth pitch be β. The equation of the permeance curve of *one* revolving tooth (Fig. 20) is $f(x) = \mathcal{P}$ and

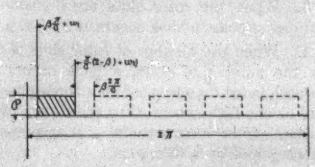

Fig. 20—The Revolving Permeance

the curve extends between the limits $2\pi/G + \omega t$ and $\pi(2-\beta)/G + \omega t$.

The coefficient of the cth harmonic *sine* term is

$$a_c = \frac{1}{\pi}\int_{\beta\pi/G+\omega t}^{\pi(2-\beta)/G+\omega t} \mathcal{P}\sin c x\, d x$$

$$= \frac{2\mathcal{P}}{c\pi}\sin\frac{c\pi}{G}(1-\beta)\sin\left(\frac{c\pi}{G} + c\,\omega t\right)$$

The coefficient of the cth harmonic *cosine* term is

$$b_c = \frac{1}{\pi}\int_{\beta\pi/G+\omega t}^{\pi(2-\beta)/G+\omega t} \mathcal{P}\cos c x\, d x$$

$$= \frac{2\mathcal{P}}{c\pi}\sin\frac{c\pi}{G}(1-\beta)\cos\left(\frac{c\pi}{G} + c\,\omega t\right)$$

The cth harmonic of the permeance curve is

$$\mathcal{P}_c = \frac{2\mathcal{P}}{c\pi}\sin\frac{c\pi}{G}(1-\beta)\cos\left(c x - c\,\omega t - \frac{c\pi}{G}\right)$$

There are G teeth displaced in space by $2\pi/G$ radians, hence the cth harmonic of the permeance curve is displaced by $2c\pi/G$ radians. When there are n sine waves displaced by α_1 the *magnitude* of the resultant is the magnitude of one component increased by $\sin n(\alpha/2)/\sin(\alpha/2)$, the *phase angle* of the resultant is $-(n-1)\alpha/2$.

Hence

The *phase angle* of the resultant is $(-c\pi + c\pi/G)$

The *magnitude* of the resultant is increased by $\sin(c\pi/\sin(c\pi/G)$.

The numerator is always zero, hence the expression is zero unless the denominator is also zero or $c/G = k$ = positive integer. (Fourier's series is valid only for positive harmonics.) Differentiating $\sin k G\pi/\sin k\pi = 0/0$, it gives $G\cos k G\pi/\cos k\pi$. Substituting,

$$\mathcal{P}_c = \pm\frac{2\mathcal{P}}{k\pi}\sin k\pi(1-\beta)\cos(k G x - k G\omega t)$$

$$= \pm A\cos(k G x - k G\omega t)$$

positive if k is even, negative if k is odd

a. The constant term is one-half of the found value if $k = 0$ is substituted. Constant permeance = $\mathcal{P}(1-\beta)$.

b. If $\omega = 0$ the *stationary* permeance is $\pm A\cos k G x$.

Flux Due to Current-Density Distributions

Let a sinusoidal current-density distribution with p pairs of poles be along the air-gap. Its equation is $i = A\sin(p_1 x - \omega t)$.

The magnetomotive force acting along a closed path through the origin and a variable point x (Fig. 21).

$$\text{M. m. f.} = \int_0^x i\, d x = \frac{A}{p_1}[\cos\omega t - \cos(p_1 x - \omega t)]$$

The magnetomotive force acting across the air-gap at any point x is

$$(\text{M. m. f.})_{xb} = (\text{M. m. f.})_{xbo} - (\text{M. m. f.})_{ao}$$

$$= (A/p_1)[\cos\omega t - \cos(p_1 x - \omega t)] - M \quad (1)$$

Let the permeance at any point x be

$$\mathcal{P} = C + D\cos(p_2 x + v G_2\omega t)$$

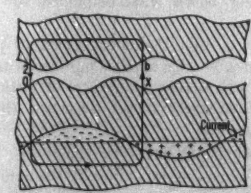

Fig. 21—M. m. f. Due to Current

Then the flux density at any point k is

$$B = (A C/p_1)\cos\omega t - C M - (A D/2 p_1)\cos[(p_1 - p_2)x + (v G_2 + 1)\omega t] + (A D/2 p_1)\cos[p_2 x + (v G_2 \pm 1)\omega t] - (A D/2 p_1)\cos[(p_1 + p_2)x + (v G_2 - 1)\omega t] - M D\cos(p_2 x + v G_2\omega t) \quad (2)$$

But the space integral of the flux density from zero to 2π has to be equal to zero. This is possible only if in equation (2) the terms not containing x are zero. Two cases have to be distinguished.

1. When p_2 is different from p_1, the first two terms of equation (2) are zero, $(A C/p_1)\cos\omega t - C M = 0$. From this $M = (A/p_1)\cos\omega t$. If this value of M is substituted into equation (1)

$$(\text{M. m. f.})_{xb} = -\cos(p_1 x - \omega t).$$

That is, the m. m. f. wave is the same as the current density wave.

2. When $p_2 = p_1$, $p_1 - p_2 = 0$ and the first three terms of equation (2) are zero. Finding the value for M and substituting into equation (1), it gives

$$(\text{M.m.f.})_{ab} = (A\,D/2\,C\,p_1)\cos(v\,G_2+1)\,\omega\,t - \cos(p\,x - \omega\,t.)$$

That is, the m. m. f. wave has also an oscillating component besides the fundamental. If the value of M is substituted into equation (2) the resultant flux density is

$$\begin{aligned} B = {} & (A\,D^2/4\,C\,p)\,\{\cos[\,p\,x + (2\,v\,G_2 + 1)\,\omega\,t] \\ & \qquad + \cos(p\,x - \omega\,t)\,\} \\ & - (A\,C/p)\cos(p\,x - \omega\,t) \\ & \qquad - (A\,D/2\,p)\cos[2\,p\,x + (v\,G_2 - 1)\,\omega\,t] \end{aligned}$$

The first term represents a *single-phase* flux with p pairs of poles.

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Discussion

C. R. Boothby: Mr. Kron has given a very comprehensive set of rules by which irregularities in the speed-torque curves of induction motors may be predicted. He also gives a number of rules by which noise and vibration may be predicted.

The first inspection of these rules indicates that some difficulties would be encountered with almost any practicable slot combination. That the difficulties are not as great as one might interpret them, is amply illustrated by the speed-torque curves given in the paper. In many of the speed-torques shown, while indicating appreciable hooks, would not be unsatisfactory for most commercial applications. Combinations which are noisy at low speeds often are very quiet at normal operating speeds. In many of such cases the motor speeds up sufficiently fast that this noise is not objectionable.

It is, of course, desirable to avoid any combination which shows undesirable characteristics as to either noise or hooks. However, in some cases this is not possible due to other limitations imposed upon the designer. In such cases it is necessary to resort to other measures in order to reduce or eliminate the undesirable characteristic.

A hook in the speed-torque curve, when it occurs at a low speed, can be reduced to an inappreciable magnitude by skewing the rotor one pole pitch of the harmonic field causing the hook. In cases where two or more hooks exist at widely different speeds, it is obvious that they cannot both be eliminated by this method. A disadvantage resulting from skewing the rotor is the reduction in the break-down torque. This effect will be noticed in polyphase motors as well as single phase. It is due, in part at least, to an increase in leakage reactance. When the pull-off torque of a motor is low due to the presence of hooks it may be increased by making the rotor resistance larger. In the case of polyphase motors, torque at start and at low speed is increased, thus reducing the proportion of parasitic torque to fundamental field torque. The magnitude of the parasitic torque will not be reduced. In slip-ring motors that start with a high resistance in series with the rotor, hooks will, in general, be of such magnitude as to be negligible.

In the case of a single-phase induction motor started as a repulsion motor, the portion of the speed-torque curve below the switch operating speed is of no importance. In such cases slot combinations may be used which would be unsatisfactory for polyphase or split-phase motors. Such a case is the combination where the rotor and stator slots are divisible by the number of poles. Motors with this combination are often found to operate very satisfactorily.

It has been found that the synchronous motor effect described in rule VII can be reduced by skewing the rotor. However, it is seldom that it can be reduced to an extent as to make such a combination satisfactory.

H. C. Specht: The question of noise and cusps in the speed-torque curves of induction motors has been under discussion among designing engineers for many years and even today, we do not have a clear conception of all the factors involved. The writings of Dr. Stiel, Fritze, and Chapman, published about ten years ago, have added materially to the theory of this subject. Since that time, little has been added to the known theories.

Mr. Kron, in his paper, tries to present a clear picture by introducing a theory of revolving permeances. Whether or not this theory is more useful in simplifying this very complicated problem is somewhat doubtful. Moreover, the author considers the two-pole flux as the fundamental or first harmonic, regardless of the number of poles. This conveniently eliminates the use of sub-harmonics, *i. e.*, harmonics of a fractional order. However, on the other hand, this is apt to create some confusion in the minds of those accustomed to the conventional idea of harmonics.

When analyzing the phenomena of cusps in the speed-torque curve, a distinction should be made between induction motor action and synchronous motor action caused by the interlocking at certain speeds, including standstill. This latter mentioned action can occur only when the rotor slots become located symmetrically with respect to the stator slots, *i. e.*, when the resultant torque of all the rotor teeth under each pole is zero. By way of example may be mentioned the four-pole, 24-28 slot motor discussed in Prof. Dreese's paper. Mr. Kron in his paper, has given some more cases where such interlocking takes place at certain speeds, but does not discuss the case where interlocking at standstill occurs.

Mr. Kron gives ten rules for determining which of various slot combinations should be avoided. But when all these rules are applied, there seems to remain hardly any combinations which avoid undesirable cusps and noise. This is rather dis-

courging to a young designing engineer, leaving him wondering why someone does not tell him what combinations would be suitable to use. As a matter of fact, there are hardly any slot combinations which do not show, at certain speeds, at least small cusps, although these may not be at all objectionable. The main problem, the one in which we are most interested today, is the building of motors which shall be as quiet as it is possible for them to be made, and which, moreover, are free from objectionable cusps.

In the earlier days, designers followed the practise of Tesla and Lamme in using a prime number of rotor slots. Application of this rule generally resulted in motors relatively free from dead points or positions of low torque. However, Dr. Stiel, Fritze and others have shown later that a prime number of rotor slots results in an unbalanced pull on the rotor and, therefore, a tendency towards noise and vibration. The modern art of design, therefore, avoids the use of prime numbers. If the number of rotor slots be even, especially if divisible by the number of poles, quieter operation will result than if the number were prime. However, the danger of obtaining cusps and low-torque points at starting is much greater. However, an increase of rotor slots, a proper skew, and sufficient rotor resistance will render the cusps and low-torque points harmless.

In addition to making the number of rotor slots divisible by the number of poles, it is advisable to make the difference between rotor and stator slots greater than the number of poles. For example, 36 stator slots and 48 rotor slots have proven quite satisfactory for four and six poles. But according to Mr. Kron's rules, this combination should be avoided. There are also other slot combinations which give better results than indicated by the author's rules. Possibly he bases his theories on motors which are not skewed. Normally, the rotors of all induction motors are, as they should be, skewed for the sake of quiet operation.

The author's rule number five seems too general. It does not hold for all cases where the number of rotor slots is more than the number of stator slots plus or minus the pole pairs. The slot combinations covered by rules 2, 4, 6, 7, and 8 are identical. Why not combine the cusps due to the various causes under one heading.

In the Appendix, the author gives some mathematical equations but he does not show how these can be applied to calculate crawling effects and the magnitude of cusps. Moreover, he does not show very clearly how his ten rules were derived from his mathematical work. If this were done, there is certainly no doubt but what the reader would profit a great deal more from the valuable data given by the author.

It is a well-known fact that the factor which has the most important bearing upon quiet operation is the slot combination. Nevertheless, there are many other factors which should be taken into consideration such as, flux density, length of air gap, skew, mechanical construction, etc. Before the question of cusps and noise is fully solved, much more work must be done along these lines.

L. A. Doggett: Mr. Kron deserves a great deal of credit for attacking this elusive problem. His analysis leads to a set of rules. It is hoped that Mr. Kron will go one step further and give us a few tables, recommending the number of slots for the more common cases. On trying to do this for the case of a two-pole three-phase machine, it was found that all possible values of rotor slot numbers were eliminated by one or more of Mr. Kron's ten rules. In the case of the four-pole three-phase machine, there were very few slot numbers that survived Mr. Kron's rules of elimination. Incidentally most of the rotor slot numbers, recommended by authors of design books, are eliminated by one or more of Mr. Kron's rules.

Although perfectly obvious and long well known, one more rule should be added for completeness. The number of rotor slots should not be equal to the number of stator slots, *i. e.*, $G_1 \neq G_2$.

Gabriel Kron: In this paper no attempt was made to recommend satisfactory slot combinations, only to determine what combinations are objectionable. The available experimental data are yet too meager to establish *all* the possible causes of irregular behavior, hence any rule recommending certain combinations may prove disastrous for some designer. The writer has in mind Stiel's recommendation of the 4-pole, 24-slot stator with a 22-slot rotor that Stiel found quiet and free from hooks. The writer also found a similar combination quiet, but a 4-pole motor with 36-stator and 34-rotor slots started up as a fire-syren. This experience resulted in rule number three and in the mathematical development in the second half of the Appendix. Later it was found that other companies had similar troubles with the 4-pole 24/22 combination itself. The rules should not be considered complete since there are other causes of noise and crawlings besides those considered.

As Mr. Boothby indicates, a large part of the irregularities, covered by rules VI-X, are insignificant, while the larger irregularities, covered by rules I-V, can be at least partially skewed out. However, when skewing is resorted to, the fundamental flux decreases and the pull-out torque is lowered, since the harmonic flux is not damped out.

Mr. Specht doubts the usefulness of introducing a new method of reasoning or assuming new units. The use of the "revolving permeances" the assumption of a two-pole flux as a first harmonic, the speed of the two-pole flux as unit speed, etc., can be appreciated, especially when one actually has to analyze a large number of seemingly contradictory experimental data for which no explanation has yet been given. Of course, after the explanation is found, any other reasoning or any other assumption may be used to get the same results. The writer acknowledges that it involves a certain amount of drudgery to set up the equations of the m. m. fs. permeances and fluxes for different slot-combinations, to follow the inter-relation of fluxes and to see the rules clearly, but there would be much more confusion if the cosine terms would involve fractions, or still worse, if a pictorial representation of the possible fluxes would be attempted for each m. m. f. wave with each possible slot combination and for each possible position of the rotor.

The rules have been worked out for unskewed motors. The first part of the Appendix gives the equation for the speed and the number of poles of the revolving permeances. The determination of the actual values of the fluxes and hooks is a subject which this paper does not touch at all. The writer was chiefly interested in the different *causes* of hooks, hence the rules are grouped according to causes and not according to slot combinations. Rules II and V refer to a difference of 2 p slots, while rules VI-VIII refer to rotors whose number of slots are divisible by 2 p, hence the possible slot combinations are not identical in these two cases, as assumed by Mr. Specht. Rule number five is general. In at least fifty different unskewed slot combinations this rule has been found to hold true without a single exception.

Prof. Doggett finds that few slot combinations survive the rules. Experimental data collected since the paper was submitted show that there are no slot combinations whose speed-torque curves do not have a considerable number of hooks. However, usually one has to worry about only the first five rules, covering ruinous irregularities caused by the fundamental flux. As far as the writer is aware, not one single rule recommended in standard design books is based on the analysis of the problem, only on experience. The problem of cogging, which is worse when $G_2 = G_1$ has not been mentioned in the paper due to the lack of experimental data.

It seems that before any definite rules can be arrived at for the selection of good slot combinations, more experimental data must be collected and published by manufacturing companies and engineering colleges.

The Pulling into Step of a Salient-Pole Synchronous Motor

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Synopsis.—A synchronous motor started as an induction motor must pass through an electro-mechanical transient after the field circuit is supplied with electrical excitation in order to change its action from that of an induction motor to that of a synchronous motor. The calculation of this problem has been difficult because the differential equation of motion is of a non-linear type of which no analytical solution has been made.

The integraph at the Massachusetts Institute of Technology has given a new and direct method of solving equations of the type encountered in these pulling-into-step transients. In this paper are compiled the results of many integraph solutions which were made after combining the factors that influence the performance of such machines into ratios and relative factors so that the results will apply to all practical cases. From the solutions several generalizations are made which can be directly applied to determine whether or not any type of salient-pole synchronous motor will synchronize properly.

These solutions take into account inertia, saliency, incompleteness of amortisseur windings, load, switching angle, and field current, and assume negligible armature resistance, negligible rotor leakage reactance, no saturation, constant load torque near synchronous speed, and negligible electrical transients. Oscillograph tests of a 160-hp. motor and a sample calculation are given.

I. Introduction

INDUCTION-MOTOR action is used to start synchronous motors and from the highest speed attainable from the induction action the mechanical and electrical torques, resulting from the application of field current, tend to accelerate and synchronize the rotor, that is, pull it into step. Steinmetz[1] in a discussion on this subject said, "....the least known or available in the literature are the phenomena of pulling into step..(of a) synchronous motor in that range of speed where it is not an induction motor anymore and where it is not yet a synchronous motor." Lamme,[2] about this same time, said of the problem, "It is difficult to 'see' just what is going on in the motor at the instant it pulls into synchronism."

Exact mathematical analysis of the pulling-into-step phenomena of even the simplest types of synchronous motors requires the solution of a non-linear differential equation. This difficult problem has been solved by approximate methods from a consideration of energy and by step-by-step analyses. Carr,[3] Cotton,[4] Dreyfus,[5] Böhm,[6] Fraenckel,[7] Lindstrom,[18] and others have published important papers treating this subject.

The advent of the integraph at the Massachusetts Institute of Technology several years ago made it possible for the first time to obtain rapidly solutions of non-linear differential equations of the type encountered in describing the motion of a synchronous motor during the pulling-into-step process. The results of investigations,[8-9] which utilized the integraph have recently appeared treating the round-rotor type of motor.

Solutions of the pulling-into-step transients of the salient-pole motor are given in this paper.

Before starting the mathematical solution of the pulling-into-step problem a discussion of the various factors that influence the phenomena will be given.

II. Factors Influencing Pulling-Into-Step Characteristics

Salient-pole synchronous motors are usually started as induction motors using amortisseur windings in the pole faces as the induction-motor rotor circuits. The field winding is commonly short-circuited through a resistance before the field current is applied.

Prior to the application of the field the speed of the motor will reach equilibrium at such a speed that the average induction-motor torque just balances the shaft load torque, and the windage and friction torques. An induction motor with a polyphase rotor having a uniform air gap will run at a constant speed if the load is constant. A salient-pole motor does not run at a constant speed for two reasons: first, it does not have a balanced polyphase amortisseur winding and secondly, the reluctance of the air gap is not uniform. Under these conditions the torque developed by the amortisseur winding due to induction motor or generator action does not depend upon the slip alone but also upon the angular displacement between the amortisseurs and the revolving stator field. Thus as the rotor slips the induction motor torque pulsates giving alternately an excess and a deficiency of torque which in turn causes variations in the slip. The salient-poles, due to their reluctance effect, always tend to place themselves so that the rotating magnetic field will be directly opposite them. As the rotor slips this reluctance torque tends alternately to accelerate and retard the rotor, causing slip variations. Summarizing—a salient-pole motor runs at such an average slip that its average torque equals the load torque. The degree of variation of the slip depends upon the closeness with which the amortisseur winding approaches a polyphase winding, upon the moment of inertia of the rotating masses, and upon the size of the reluctance torque.

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1. For references see Bibliography.

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In case the shaft-load torque is relatively small, the motor may pull into synchronism and run as a "reluctance motor" with neither field current nor induced currents in the amortisseur bars. This interesting case is not considered in this paper since it occurs only when the shaft load is relatively small.

After a synchronous motor reaches its maximum induction-motor speed the field circuit is connected to an exciter or to a supply of direct current. The motor either pulls into synchronism or continues to slip but with violent current power and slip pulsations. The object of this paper is to demonstrate a means of computing these phenomena and to give a method of predicting the necessary conditions for satisfactory pulling into step.

A consideration of pulling-into-step problems in general for salient-pole motors shows that there are seven important factors which influence the phenomena. These will now be discussed separately.

1. The amount of load torque that is on the shaft is an important factor. When the machine is operating as an induction motor a considerable slip is required to supply a large shaft load. Consequently more kinetic energy is required to raise the speed of the rotor to synchronism than is required for smaller values of shaft load. The source of this energy is the excess of the sum of the induction and synchronous torques over that required to supply the load. The fact that a motor with a large shaft load may not pull into step is recognized by all.

2. Another factor is the maximum synchronizing torque available after the field is connected. The field current and the maximum synchronizing torque are nearly proportional for most motors. The greater this maximum torque the greater is the chance for pulling into step. The synchronizing torque also is indirectly effective through its effect on the natural frequency of mechanical oscillation. This effect is taken into account mathematically in a dimensionless equation of motion by the relative damping factor, k, which varies inversely as the square root of the maximum synchronizing torque (see equation (3) and the paragraph following it).

3. The amount of inertia of the rotor and the connected rotating masses determines the torque required to accelerate these parts from subsynchronous to synchronous speed during the synchronizing period. It also affects the damping in the same manner as does the maximum synchronizing torque by affecting the natural period of the angular oscillation.

4. A very important factor is the effectiveness of the amortisseur windings and the field circuit as an induction-motor rotor. For a given load torque the steepest slip-torque characteristic brings the rotor the closest to synchronous speed and it follows, for the same reason as stated in factor 1, the chance for pulling into step is more favorable. Besides making the slip smaller at the beginning of synchronizing period the greater induction-motor torque damps out the angular oscillations more quickly and thus tends to prevent these from becoming violent. These amortisseur requirements for good synchronizing qualities may conflict with those for high-starting torques.

5. Another factor is the angle between the terminal voltage and that induced by the d-c. field current which exists when the exciter is connected. If the exciter is connected when the angle is such as to cause generator action, the slip at first increases and the machine begins to depart further from synchronism, thus increasing the difficulty of subsequent synchronization.

6. The reluctance torque is another factor. This torque is a function of angular displacement as has already been discussed and causes the slip to vary before the field is connected. Afterwards it aids the motor to pull into step in some cases and influences somewhat the form of the angular transients.

7. The degree of unbalance in the induction motor torque due to the incompleteness of the amortisseur winding influences the pulling into step of a salient-pole synchronous motor by causing the initial slip to vary as previously discussed and by causing the damping to be a function of the angular displacement as well as of the slip during the transient.

It should be pointed out that factors 6 and 7 are not independent of each other. For instance the damping from a complete amortisseur winding is a variable with angle because of the variable air gap. The conclusions that are made in the paragraph preceding equation (4) state that neither of these factors greatly influence the criterion for pulling into step.

These seven major factors all appear in the terms of the differential equation of torques and solutions of this equation show their influence.

III. Differential Equation of Motion

The differential equation of motion of a salient-pole synchronous motor is generally given in approximately the following form, assuming negligible armature resistance, negligible rotor leakage reactance, no saturation of the magnetic materials, constant load torque near synchronous speed, and negligible electrical transients. The last assumption is justified by the high speed of the electrical transients in comparison with the relatively slow electro-mechanical transients.

$$P_i \frac{d^2\theta}{dt^2} + P_d (1 - b \cos 2\theta) \frac{d\theta}{dt} + P_n \sin 2\theta + (P_m \sin \theta) 1 = P_L \qquad (1)$$

θ is the angle between the terminal voltage in one phase and the voltage induced in the same phase by the d-c. excitation. It is the mechanical angle (measured in electrical degrees) between the axis of a field pole and the axis of a corresponding imaginary field pole which would induce a voltage, with the armature open-circuited, at the terminals in phase with the applied voltage. θ would be zero in a lossless synchronous

motor operating at no load. It can be taken as positive for either motor or generator action without altering the equation. In what follows θ is taken as positive for motor action. The units used by the authors are stated in part V, although any other consistent set of units can be used.

The first term, $P \dfrac{d^2 \theta}{d t^2}$ represents the torque that results from acceleration of the rotor and the connected rotating masses of the load, where P_j is a coefficient proportional to the moment of inertia.

The second term is proportional to the slip since the slip is proportional to $\dfrac{d \theta}{d t}$. In contrast with the ordinary induction motor term for balanced polyphase rotor circuits the coefficient for this term is a function of angle. By methods such as employed by Park,[10] Linville,[11] or Putman[12] it can be shown that this function can be closely approximated by the form $(1 - b \cos 2 \theta)$. This variation of the damping or induction motor effect is due to electrical unbalance of the rotor caused by incomplete amortisseur windings, by the field circuit, and by the non-uniform air gap. The double angle appears because of the symmetry about each pole instead of each pair of poles.

$P_R \sin 2 \theta$ is the reluctance torque term. This torque is due to the tendency of the salient-poles to place themselves in alinement with the maximum magnetomotive force of the stator rotating magnetic field. The double angle appears here as in the damping term and for the same reason, namely, because the poles react individually and not in pairs as there is no d-c. excitation. The two-reaction theory[13] gives this term in units of synchronous torque as:

$$P_R = \frac{3 (x_d - x_q)}{(2\, x_d\, x_q)} \frac{V^2}{1000} \text{ kw.}$$

where x_d is the synchronous reactance in the direct axis and x_q is the synchronous reactance in the quadrature axis, both per phase. V is the applied stator voltage per phase.

The synchronizing effect of the field current is expressed by $(P_m \sin \theta)\, \mathbf{1}$ in equation (1) where Heaviside's unit function, **1**, states the boundary conditions. It signifies the sudden appearance of the term $(P_m \sin \theta)$ at zero time. Before the exciter is connected there is no synchronizing torque except that due to reluctance. After the exciter is connected the synchronizing torque immediately appears neglecting a short electrical transient. The unit function is zero before and unity after $t = 0$, which is taken as the time of closing the exciter circuit. The numerical value of P_m is determined by $P_m = \dfrac{3\, E\, V}{x_d} \dfrac{\text{kw.}}{1000}$ (for a three-phase machine) which shows that it is proportional both to the terminal voltage and the field current. It is inversely proportional to the direct synchronous reactance. E is the induced electromotive force per phase in the armature due to the direct-current field excitation.

Equation (1) will be reduced to a more convenient form by dividing by P_m and by changing the time variable by the following relationship. Let $t = \lambda\, a$ where λ is a new variable and "a" is a constant to be determined. The derivatives with respect to the new variable λ in terms of those with respect to the original variable t are:

$$\frac{d}{d \lambda} = a \frac{d}{d t} \qquad \frac{d^2}{d \lambda^2} = a^2 \frac{d^2}{d t^2}$$

Replacing t in equation (1) by λ gives:

$$\frac{P_j}{a^2 P_m} \frac{d^2 \theta}{d \lambda^2} + \frac{P_d}{a P_m} (1 - b \cos 2 \theta) \frac{d \theta}{d \lambda} + \frac{P_R}{P_m} \sin 2 \theta + (\sin \theta)\, \mathbf{1} = \frac{P_L}{P_m} \qquad (2)$$

The undetermined factor "a" is selected so that for any values of the coefficients in equation (1), the acceleration term of equation (2) has a unit coefficient. To do this equate the coefficient of this term to unity and solve for "a".

$$\frac{P_j}{a^2 P_m} = 1 \qquad \therefore \qquad a = \sqrt{\frac{P_j}{P_m}}$$

This value of "a" substituted into equation (2) gives:

$$\frac{d^2 \theta}{d \lambda^2} + k (1 - b \cos 2 \theta) \frac{d \theta}{d \lambda} + \frac{P_R}{P_m} \sin 2 \theta + (\sin \theta)\, \mathbf{1} = \frac{P_L}{P_m} \qquad (3)$$

where $k = P_d / \sqrt{P_j P_m}$; k is called the "relative damping coefficient" and is a very significant quantity. All machines having any given value of k respond with similar electro-magnetic transients when similar disturbances are imposed upon them, regardless of the size of the machines.

All motors, large or small, with large or small inertias, swing with the same period of oscillation in terms of the new variable, λ, which it will be noticed is dimensionless; λ is converted to time in seconds for any numerical case by multiplying by "a", *i. e.*, $t = \lambda\, a$.

In other words changing the variable from t to λ stretches or shrinks the time scale so that all motors having equal relative damping coefficients, values of b, and P_R/P_m have similar solutions of the same period. This transformation does not reduce the generality of the equation but does reduce the number of coefficients and thus aids materially in obtaining a solution covering all cases.

The M. I. T. integraph solves the differential equation of motion (3) for both the initial steady-state solution and the transient. The methods that it uses to achieve this result have been adequately described elsewhere.[8, 9, 14–17] However in this problem the integraph was used in a way slightly different from that previously described because of advantages that could be gained. Appendix I gives a brief description of the method of solving the pulling-into-step problem using the newer method. This method is also included in this paper so that those who are not familiar with this method of solving differential equations will be able to realize the directness and novelty of the integraph solutions.

IV. Solutions of the Differential Equation of Motion and Their Classification

The solutions assume that the motor is running in a steady-state condition as an induction motor at the time the exciter switch is closed. This means that the differential equation must be solved for a steady-state condition without the synchronizing term. As this induction-motor steady state is itself an oscillation due to the variable induction-motor torque and variable-reluctance torque, the initial conditions for the transient caused by applying the field current depend upon the part of the oscillation at which this switching occurs. As stated in the assumptions no allowance is made in the solutions for the time lag introduced by the electrical transients. These are usually over by the time the rotor has changed its angular position significantly because the angular transients are relatively slow compared to the electrical. However, the electrical transients in the stator and rotor certainly do influence the electrical torques and this effect is undoubtedly appreciable in large motors. Likewise no allowance is made for differences of the induction motor effects before and after applying the field current. The damping characteristic will be radically changed if the external impedance in the field circuit is changed very much during the switching operations. Usually the external impedance is considerably reduced when the exciter is connected, resulting in a beneficial effect.

The integraph made plots of angle against slip which satisfied the requirements of the differential equation of motion. Fig. 1 shows the form of these solutions. The curve $A\,B\,J$ is the steady-state induction motor solution with no field excitation. The variation of the slip is due to the disturbing effects of unbalance in the rotor winding and the salient poles. Its magnitude depends upon k, $\frac{P_R}{P_m}$, b, and $\frac{P_L}{P_m}$.

Consider as an example the case where the field switch is closed at an angle of 120 degrees. The motor action of the synchronous torque is very large at that point and the slip decreases very rapidly as indicated by the steepness of the curve from B to C. At about the point C the sum of the synchronous and reluctance torques is equal to the load torque, the only remaining torque being that due to induction motor action which continues to accelerate the rotor until the point D is reached at which the sum of the slightly decreased synchronous and reluctance torques and the induction motor torque is just equal to the load torque. At D then the speed (and slip) are constant, but not at their synchronous values and θ therefore continues to increase, with a resulting decrease of net accelerating torque due to further loss of synchronizing and reluctance torques. Obviously the machine cannot synchronize at this minimum of slip and the rotor must slide along at least one pole of the armature field before again having an opportunity to pull into step.

The slip reaches a maximum in the region of zero angle. From this angle on, the motor action of the synchronous torque increases rapidly and the slip decreases correspondingly reaching zero at F. At that moment the motor is running at synchronous speed and is exactly in the situation of the same synchronous motor

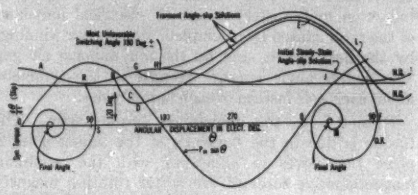

Fig. 1—Typical Integraph Angle-Slip Solutions of the Equation of Motion of a Synchronous Motor During Pulling-Into-Step Transients

whose rotor is displaced from its steady-state normal position at M. The slip-angle curve from this point on is the curve of the free damped oscillation with constant load. This curve is a sort of spiral in the slip-angle plane turning around the final steady-state angle at M.

If the switching angle is more favorable, the slip will decrease to zero immediately and spiral about the steady-state operating angle, following a curve such as $R\,S$.

Should the relative damping coefficient be small, the switching angle unfavorable, and the initial slip large, the motor may not pull into step on the second possible pole but continue to slip many poles with violent variations in the slip. The motor may synchronize after several swings or possibly not at all. In either case the operation is not satisfactory.

For the classification of the integraph solutions for this paper the criterion of pulling into step was whether or not the motor synchronized within the first two opportunities as shown in the two characteristic swings, Fig. 1, that have been described. A solution such as curve G-E (Fig. 1) is not satisfactory although it may show synchronization within the next 360 deg.

When a machine is running as an induction motor, it is difficult to connect the exciter to the field at a given angle. For this analysis only the worst angle of switching will be considered, since synchronous motors are ordinarily designed so as to be capable of synchronizing when the field switch is closed independent of the slip and the angular position of the rotor. A series of solutions has definitely shown that the most unfavorable switching angle was always very close to 180 deg. At its exact value the slip-angle solution departs farthest from synchronous speed. (See solution $H\ E\ L$, Fig. 1). This curve is tangent to the initial steady-state induction-motor angle-slip solution and thus does not intersect the initial steady-state solution at two values of angle as does the solution marked $G\ E$.

All other factors being constant, the larger the value of k, the more favorable are the chances of pulling into step. This is because a large damping or induction motor effect, besides bringing the rotor to a higher initial speed, helps to damp out the violent angular oscillations and thus aids the synchronizing processes. In fact if k is sufficiently large a motor may synchronize even if the load ratio is unity.

The procedure for the integraph solutions was as follows. Numerical values of b, $\frac{P_R}{P_m}$, and $\frac{P_L}{P_m}$ were selected so that they were in the range of values that apply to practical cases. Then an integraph solution for the worst angle of switching (such as H, Fig. 1) was made with a numerical value of k. If the slip-angle curve showed that the motor did not pull into step (such as $H\ L$ in Fig. 1) this fact was noted in a table such as Table I, column 5. A larger value of k was then used for the next integraph solution. In this way a series of solutions was made until a value of k was reached which showed that pulling into step just resulted. The first seven rows of Table I were made in this manner. The values of the constants were

$$P_L/P_m = 0.5$$
$$P_R/P_m = 0.25$$
$$b = 0.5$$

The critical value of k was in this way found to be between the values of 0.063 and 0.065.

The same procedure was then followed for a smaller value of load ratio and then for a larger value of load ratio.

Then other values of P_R/P_m and b were used in making integraph solutions. The most important results of these are given in Table I and are plotted in Fig. 2. It is important to notice the last column of this table which shows that the ratio of P_L/P_m to k for the critical value of k is *independent of* P_R/P_m and b within limitations on the solutions imposed by the errors of the integraph. This relation allows an important conclusion to be made, namely: *The motor represented by differential equation* (1) *will pull into step if the coefficients* (P_L/P_m and k) *satisfy the relationship*

$$P_L/P_m < 7.6\,k \tag{4}$$

The number 7.6 is the slope of the line in Fig. 2, which has been arbitrarily placed so that it favors the points for the larger load ratios.

A limitation to the practical application of this relationship between coefficients necessary for satisfactory pulling-into-step characteristics will be noticed in the next section. This limitation is imposed by a linear slip-torque assumption. The results of this paper, therefore, are optimistic when they are applied to a motor which has at any time during the pulling-into-step transient a slip such that the slip-torque characteristic is curved. During the angular oscillations the maximum transient slip is about double the initial slip. Consequently, if double the initial slip has such a value that it corresponds to operation on the curved portion of the slip-torque curve a margin of safety should be included.

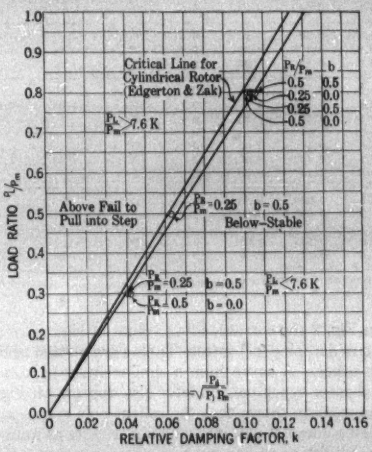

FIG. 2—CHART SHOWING DATA OF TABLE I

The region below the line represents the conditions for favorable pulling-into-step characteristics

The reluctance torque does not influence the value of the critical ratio to any considerable extent. This torque makes the initial slip more unfavorable than it would be if it were not present (as in round rotor machines) but a greater synchronizing effect is exerted by the joint action of the reluctance and synchronizing torques. These two opposite influences of the reluctance torque tend to cancel and its net effect, therefore, on the ability of the motor to synchronize is inappreciable.

The degree of unbalance of the damping winding

apparently also has a rather minor effect on the critical ratio. This is undoubtedly due to the fact that the rotor swings through such a large range of values of angle that only the average damping is of importance.

TABLE I—TABLE OF RESULTS FROM INTEGRAPH SOLUTIONS

$\frac{P_L}{P_m}$	k	$\frac{P_R}{P_m}$	b	O X	N. G. O. K.	Critical k_0	$\frac{P_L/P_m}{k_0}$
0.5	0.028	0.25	0.5	O			
0.5	0.038	0.25	0.5	O			
0.5	0.043	0.25	0.5	O			
0.5	0.055	0.25	0.5	O		0.64	7.80
0.5	0.063	0.25	0.5	O			
0.5	0.065	0.25	0.5	X			
0.5	0.082	0.25	0.5	X			
0.3	0.03	0.25	0.5	O			
0.3	0.04	0.25	0.5	O		0.41	7.3
0.3	0.042	0.25	0.5	X			
0.8	0.10	0.25	0.5	O			
0.8	0.105	0.25	0.5	X		1.04	7.7
0.8	0.11	0.25	0.5	X			
0.8	0.10	0.25	0	O			
0.8	0.105	0.25	0	X		1.05	7.6
0.8	0.11	0.25	0	X			
0.8	0.10	0.50	0	O			
0.8	0.105	0.50	0	X		1.03	7.75
0.8	0.11	0.50	0	X			
0.3	0.040	0.50	0	O			
0.3	0.043	0.50	0	X		0.42	7.15
0.8	0.105	0.50	−0.5	O			
0.8	0.11	0.50	−0.5	X		1.06	7.55
Edgerton and Zak—Cylindrical rotor motor							8.0

In Fig. 2 the corresponding values of load ratio and relative damping factor from Table I are plotted against each other. A curve from a similar study of a cylindrical-rotor synchronous motor also appears on Fig. 2. A comparison shows that the salient-pole synchronous machine does not pull into step under quite as unfavorable conditions as the cylindrical-rotor motor. However the difference is not great and no accurate comparison between the two types of motors can be made on the basis of these curves.

V. Practical Formula Giving Conditions for Pulling Into Step

The relation between the coefficients given in equation (4) which is necessary for satisfactory pulling-into-step characteristics is easily converted into a more usable form by substituting into it the values of the coefficients in terms of the $W R^2$ and the electrical constants as indicated in the following tabulation:

$$P_i = 18.5 \frac{(W R^2) f}{p^2} 10^{-6} \text{ kw/elect. deg./sec.}^2$$

(See Appendix II for derivation of P_i)

$(W R^2)$ = moment of inertia in lb. ft.^{2}
f = frequency in cycles per second
p = number of poles

$$P_d = \frac{P_L}{360 f s} \text{ kw/elect. deg./sec.}$$

P_L and s are taken from the near-synchronous straight-line slip-torque curve
P_L = load torque in synchronous kilowatts
s = the per unit slip

$$P_m = \frac{3 E V}{x_d (1000)} \text{ kw. (for a three-phase machine)}$$

where
V = applied stator voltage per phase
E = induced stator e. m. f. per phase due to the field current.
x_d = the direct-axis synchronous reactance per phase.
n = synchronous speed in r. p. m.

Substituting the values of P_i and P_d into equation (4) and solving for the slip gives the convenient relationship

$$s < \frac{590}{n} \sqrt{\frac{P_m}{(W R^2) f}} \tag{5}$$

The amortisseur windings must furnish sufficient torque to raise the average speed of the rotor to such a value that the slip satisfies the inequality (5) if the motor is to pull into step.

Another convenient form for the criterion (5) is obtained by solving for the maximum synchronizing torque and thus determining the necessary field current to pull a motor into step.

$$P_m > 2.86 f (W R^2) n^2 s^2 \times 10^{-6} \tag{5a}$$

The field current should be such that the maximum synchronous torque is greater than the indicated function of the frequency, moment of inertia, speed, and slip in order for synchronism to always result.

VI. Example

Pulling-Into-Step Tests on a 160-Hp. Salient-Pole Synchronous Motor

Oscillographic tests were made in the laboratory of the Massachusetts Institute of Technology of the pulling-into-step transients of a 160-hp. synchronous motor. This motor drives two synchronous generators, one rated at 50 kw., 60 cycles, 230 volts, the other at 50 kw., 24 cycles, 230 volts. An exciter is directly connected to the shaft. A photograph of the set is shown in Fig. 3.

The electrical connections as used in the tests are shown in the wiring diagram, Fig. 4. Auto-transformers supply the motor with about half voltage (1,150 volts) for starting. The 24-cycle generator was used as a shaft load with its armature connected to properly adjusted resistors. To measure angular displacements the 60-cycle generator was excited and its open-circuit terminal voltage was connected in series with a voltage from a potential transformer connected to the terminals of the motor. This potential trans-

former had such a ratio that the two voltages were equal in magnitude. The difference between these two voltages is a function of the angular displacement, since it is the vector difference between a voltage having a fixed relation with the excitation voltage, and the terminal voltage. The oscillograph recorded this voltage and it is termed "angle voltage" on the oscillograms. The poles of the 60-cycle generator rotor are

Fig. 3—160-Hp. Salient-Pole Synchronous Motor that was Used for Pulling-Into-Step Tests

32 electrical deg. in advance of the poles of the motor rotor.

The exciter was separately excited and its terminal voltage was opposed by a storage battery so that the resulting voltage at synchronous speed was equal to zero. In this way it was possible to record slip on the oscillograph. The commutator ripples were made insignificant by means of a choke coil. As a result the time constant of the circuit was about 1/10 of a second so that the current in the vibrator did not follow the slip exactly.

The polyphase power input to the stator of the motor was recorded by a Westinghouse instantaneous polyphase-power element. Armature current, field current

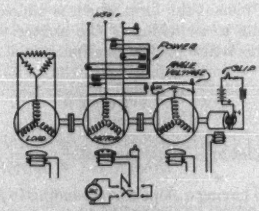

Fig. 4—Wiring Diagram of Connections Used for Oscillographic Tests

and terminal voltage were also put on the oscillograms. The field current was held at a value which induced e. m. f. of about 1,150 volts (line to line) for all the oscillographic tests.

Oscillogram 1, Fig. 5, shows a satisfactory pulling-into-step transient. The initial slip has an average value of about 0.0149. The switching angle is about 40 deg. (generator). The oscillographic record of slip shows that the speed rises quickly to synchronous and then oscillates about that value.

Oscillogram 2, Fig. 6, gives a very interesting picture of the transients in a motor which is just on the verge of being satisfactory. The average initial slip for this transient is 0.0145 and the switching angle is 95 deg.

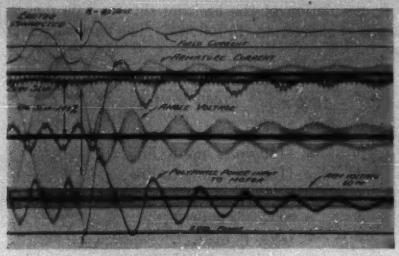

Fig. 5—Oscillogram No. 1

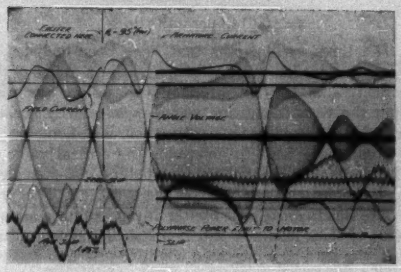

Fig. 6—Oscillogram No. 2

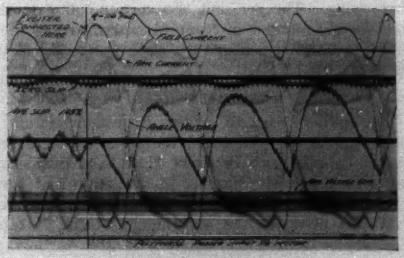

Fig. 7—Oscillogram No. 3

(motor). The speed decreases, soon after the exciter is connected, then it increases to exact synchronous speed and runs there for nearly a second. The angular displacement at this point is such as to give unstable equilibrium. The rotor finally swings around again before reaching its steady-state operating angle.

Oscillogram 3, Fig. 7 shows the transients in a motor

which does not pull into step for seven or eight swings (only four shown on the oscillogram). The initial average slip is 0.0149 and the switching angle is 110 deg. (motor). During the oscillations following switching each oscillation brings the average speed closer to synchronous speed. The current and power oscillate violently at a low frequency.

A calculation of the limiting slip for this motor by means of expression (5) follows. The necessary factors are,

$$P_m = \frac{3\,V\,E}{1000\,x} = \frac{1150 \times 1150}{1000 \times 21.4} = 61.5 \text{ kw.}$$

since x = 21.4 ohms per phase
V = 1150 volts line to line
E = 1150 volts line to line
$W\,R^2$ = 2,762 lb. ft.^{2} (total of all three machines)
f = 60 cycles per sec.
n = 720 r. p. m.

$$s < \frac{590}{720}\sqrt{\frac{61.5}{2762\ \ 60}}$$

$$s < 0.0143 \text{ per unit slip}$$

The three oscillograms that have been included in this paper are for values of slip slightly in excess of the critical value, but for switching angles more favorable than the worst condition which explains why oscillogram No. 1 showed satisfactory synchronization.

Preliminary tests to determine the critical value of slip for 100 per cent satisfactory pulling-into-step were made before the oscillograms were taken. The procedure was to switch the exciter at random for a certain value of shaft load. If the motor always synchronized then the load was increased slightly. As this was repeated, eventually a load was found where for some switching angles the motor did not pull into step the first swing. The per unit slip corresponding to this load for the motor that has been described was about 0.014 to 0.015 which is very close to the calculated value of 0.0143.

ACKNOWLEDGMENT

The authors wish to express their appreciation to those who have helped during the course of this investigation, especially to Professor Lyon, Dr. Bush, Professor Dahl and Mr. Hazen. Messrs. Alger and Terry of the General Electric Co. furnished data for the motor that was used for the experimental tests. Messrs. Kingsley and Levine assisted with the oscillograph testing and Messrs. Bower and Gillon aided in operating the integraph.

Appendix I

Outline of Integraph Method of Solving Differential Equation of Motion

Equation (3) is put into an integral form by transposing all terms except the acceleration term and then integrating both sides twice with respect to λ.

$$\theta = \int_0^\lambda\int_0^\lambda \frac{d^2\theta}{d\lambda^2}\,d\lambda\,d\tau = \int_0^\lambda\int_0^\lambda \left\{ \frac{P_L}{P_m} - k(1 - b\cos 2\,\theta) \frac{d\theta}{d\lambda} - \frac{P_R}{P_m}\sin 2\,\theta - (\sin\theta)\,1 \right\} d\lambda\ \ d\lambda \qquad (6)$$

Rewriting in a slightly different form by factoring out $(1 - b\cos 2\,\theta)$

$$\theta = \int_0^\lambda\int_0^\lambda \left\{ \frac{\dfrac{P_L}{P_m} - \dfrac{P_R}{P_m}\sin 2\,\theta}{(1 - b\cos 2\,\theta)} + \frac{-\sin\theta}{1 - b\cos 2\,\theta}\,1 - k\frac{d\theta}{d\lambda} \right\} (1 - b\cos 2\,\theta)\,d\lambda\ d\lambda \qquad (7)$$

To solve equation (7) on the integraph it is necessary to have plots of the various terms (see Fig. 8).

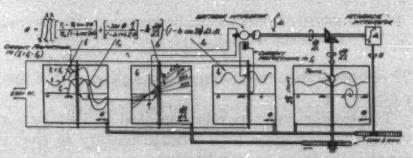

Fig. 8.—Schematic Diagram of the Integraph Illustrating Methods Employed in Solving the Characteristic Equation of Motion of the Salient-Pole Synchronous Motor

Let f_1 be a plot of the following term as a function of angle, θ.

$$f_1 = \frac{\left(\dfrac{P_L}{P_m} - \dfrac{P_R}{P_m}\sin 2\,\theta\right)}{(1 - b\cos 2\,\theta)}$$

Let f_2 be a plot of the term which is added at a time corresponding to switching of the exciter to the field circuit. This is also plotted as a function of angle.

$$f_2 = \frac{-\sin\theta}{1 - b\cos 2\,\theta}$$

Let f_3 be a plot of $-k\dfrac{d\theta}{d\lambda}$ against $\dfrac{d\theta}{d\lambda}$. This is a straight line having a slope corresponding to k.

Let f_4 be a plot of the term which is multiplied into the sum of the preceding functions. It is also plotted as a function of the angle.

$$f_4\,(1 - b\cos 2\,\theta)$$

Equation (7) now can be rewritten

$$\theta = \int_0^\lambda\int_0^\lambda (f_1 + f_2 + f_3)\,f_4\,d\lambda\ d\lambda.$$

$f_1 + f_2$ is put on a platen (see Fig. 8) on the integraph and this platen is back-coupled to the output of

the second integrator. This output is the angle θ which is the variable against which f_1 and f_2 are plotted; f_3 is put on a platen which is back-coupled to the output of the first integrator. The output of this is equal to $d\,\theta/d\,\lambda$ as can be shown by differentiating equation (7) with respect to λ.

The potentiometers over these two platens are connected in series so that the resulting voltage is equal to $(f_1 + f_2 + f_3)$.

A third platen has a plot of f_4 upon it and since f_4 is a function of angle the platen is back-coupled to the output of the second integrator which is proportional to θ in the same manner as the platens upon which f_1 and f_2 are plotted.

Fig. 8 shows these plots upon the platens and the method of back-coupling them so that their displacements are proportional to the angular displacement and the slip.

The platen on the extreme right records the slip as a function of the angular displacement. Characteristic solutions are shown in Fig. 1 and are explained in the paper.

The procedure to find the initial steady-state induction motor solution was to start the integraph (with the synchronizing term (f_2) out) and let it run until the slip for each corresponding angle repeated itself every 360 electrical deg. This process is exactly the same as the operation of the motor inasmuch as an identical transient condition in both the motor and integraph must be run through before the final steady-state running condition is reached. The person operating the integraph soon learns to guess quite closely to the final value and this reduces the time necessary to grind out this preliminary steady state for the initial boundary conditions.

After the steady state is reached the pulling-into-step transient is obtained by following the curve $f_1 + f_2$ on the left-hand platen at a time corresponding to the switching angle that is under consideration. The sketch on this platen in Fig. 8 shows the approximate form of the curves at the most unfavorable angle.

Appendix II

Derivation of P_j, the Inertia Coefficient

Inertia torque $= J\,\alpha$ in lb. ft.

where $J = \dfrac{W\,R^2}{g}$ = the moment of inertia in poundals

α = the acceleration in mechanical radians per sec.

Inertia power $= \dfrac{2\,\pi\,n}{60}\,J\,\alpha$ in lb ft. per sec.

n = speed in r. p. m.

If the acceleration is to be expressed in electrical deg. per sec. $(d^2\,\theta/d\,t^2)$ the conversion factors to give proper mechanical units are

$$\alpha = \frac{2\,\pi}{180\,p}\,\frac{d^2\,\theta}{d\,t^2} \text{ mechanical radians per sec.}^2$$

since $2\,\pi$ mechanical radians $= 180\,p$ electrical deg. The expression for inertia power is now written in terms of power in kilowatts, electrical deg. and sec. (at synchronous speed).

Inertia power in kw.

$$= \frac{2\,\pi\,n}{60}\left(\frac{W\,R^2}{g}\right)\frac{2\,\pi}{180\,p}\,\frac{0.746}{550}\,\frac{d^2\,\theta}{d\,t^2}$$

$$= P_j\,\frac{d^2\,\theta}{d\,t^2}$$

where

$$P_j = 0.154\,\frac{n\,(W\,R^2)}{p} \times 10^{-6} \text{ or } 18.5\,\frac{f\,(W\,R^2)}{p^2} \times 10^{-6}$$

(kw. per electrical deg. per sec.^{2})

since $n = \dfrac{f\,120}{p}$ r. p. m.

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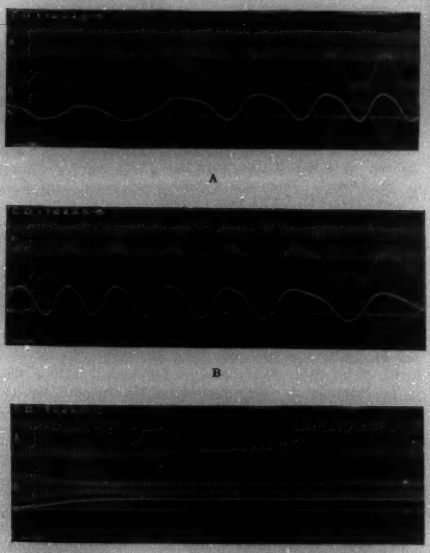
Discussion

D. W. McLenegan and I. A. Terry: This paper presents the solution of a problem of considerable complexity and is another illustration of the value of the M. I. T. integraph in solving problems which are very difficult to treat analytically. The



A

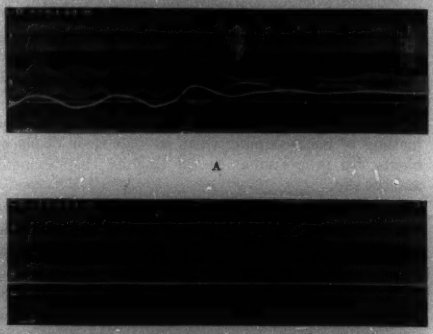
B

C

Fig. 1.—Synchronizing of 75 Hp., 900-R. P. M., Salient-Pole Synchronous Motor with Normal Field Time Constant. Load 1.2 Times Normal

Curve *A*: Applied stator voltage
Curve *B*: Motor stator current
Curve *C*: Motor field current

practical value of the solution cannot be over emphasized. As the solution. In the differential equation (1) describing the rotor motion the synchronizing effect of the field current was expressed as ($P_m \sin \theta$) 1 which assumes that the normal torque-angle characteristic of the motor exists at the instant the excitation is applied to the motor. This is not true since there is a the field of application of synchronous motors is broadened, a clearer understanding of the complete theory must be available. For example, a synchronous motor driving a centrifugal pump must usually pull into step under full load but there is a relatively small inertia in the rotating system. On the other hand, a synchronous motor driving a sintering fan must pull into step



A

B

Fig. 2.—Synchronizing of 75-Hp., 900-R. P. M. Salient-Pole Synchronous Motor with Half Normal Field Time Constant. Initial Load 1.2 Times Normal

Curve *A*: Applied stator voltage
Curve *B*: Motor stator current
Curve *C*: Motor field current

under full load and in addition there is a large inertia in the rotating system. Since to synchronize, the motor must develop sufficient torque during one-half of a slip cycle to supply the load and in addition accelerate the inertia from the slip speed to full speed, it is evident that a synchronous motor which will develop normal torque at 95 per cent speed may be suitable for driving a centrifugal pump or other loads with a small inertia, but quite unsuitable for driving a sintering fan or other apparatus with a relatively large inertia. There are other cases where the torque of a motor at 95 per cent speed does not represent a true criterion for the ability to attain synchronism. As yet, there is not an Institute or even a commercial standard which truly represents the ability of a motor to synchronize under given conditions. This paper furnishes a theoretical basis for the ultimate formulation of such a standard.

One of the most surprising results of the studies is the relatively unimportant part played by the reluctance torque in the synchronizing of a motor. The authors have shown that a salient-pole machine can synchronize from a slip which is less than

$$\frac{590}{n}\sqrt{\frac{P_m}{(W\,R^2)\,f}}$$

whereas it has been shown in bibliography 9 that a round rotor machine can synchronize from a slip less than

$$\frac{620}{n}\sqrt{\frac{P_m}{(W\,R^2)\,f}}$$

so that a salient-pole machine with a given average induction motor torque at 95 per cent speed and a given value of P_m has a slightly smaller ability to pull into synchronism than a round rotor machine with similar characteristics. Since a salient-pole machine has a greater synchronous pull-out torque with a given excitation and a greater synchronizing torque with very low

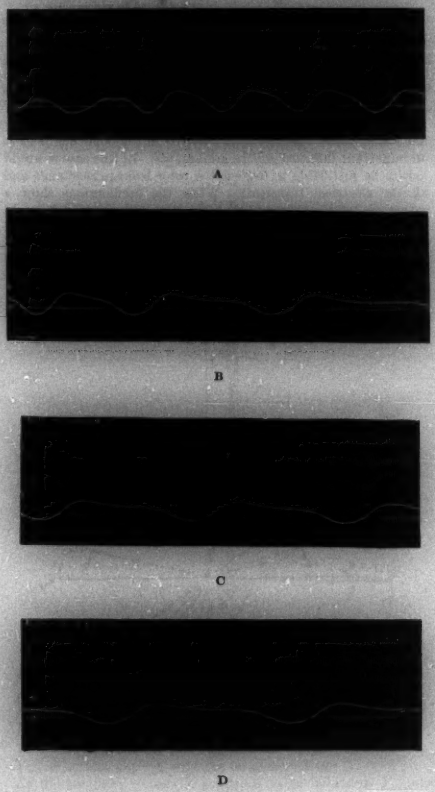


Fig. 3—Attempt to Synchronize 75-Hp., 900-R. P. M. Salient-Pole Synchronous Motor with Half Normal Field Time Constant

Curve A: Applied stator voltage
Curve B: Motor stator current
Curve C: Motor field current

excitations than a round rotor machine, it seems that it should also give a greater synchronizing or pull-in torque at normal excitation. We would like the authors to comment on this apparent inconsistency.

In solving engineering problems certain assumptions must always be made to simplify the analysis. The accuracy with which the assumptions are realized determines the accuracy of definite time required to build up the current in the field winding. For present day industrial motors the short-circuit time constant of the field winding is frequently around a quarter to a half second which means that this time will have elapsed before 0.64 P_m will be available. The slip from which the motor must synchronize may represent a transient in the armature current of the same order. In the machine tested by the authors the short-circuit time constant of the field winding is much shorter than the time of the armature current transient.

To determine the effect of the field time constant upon the pull-in characteristics tests have been made upon a 75-hp., three-phase, 60-cycle, 900-r. p. m., 0.80-power factor synchronous motor direct-connected to a heavy fan. The total WR of the rotating system was 3,870 lb. ft. sq. The motor was allowed to attain its maximum speed as an induction motor with various fan loads and then field current was applied under different conditions. It was found that for fan loads near the critical value (above which the motor could not be synchronized) the shorter the field time constant, the better the ability to synchronize although at lighter loads the data are not conclusive. The oscillogram shown in Fig. 1 was taken with the field winding excited at the time indicated by an abrupt change in the field current, from a constant potential of 125 volts with sufficient external resistance in the circuit to permit normal full load field current to flow under steady-state conditions. The short-circuit time constant of the field winding was 0.122 seconds. The initial slip was 0.021 with an observed load of 1.2 times normal. The motor slipped 10 poles before synchronism was attained. The oscillogram shown in Fig. 2 was taken with the field winding excited at the time indicated from a constant potential of 250 volts with sufficient external resistance in the circuit to permit normal full load field current to flow under steady-state conditions. The short-circuit time constant of the field winding was therefore half of the previous value. The initial slip was 0.02 and the observed load 1.2 times normal. Only two poles were slipped before synchronism was attained. The angle at which the field current was switched was slightly less favorable in the second case than in the first, but even then the shorter time constant permitted more rapid synchronization. These oscillograms were taken on a motor with a much shorter field time constant than armature current transient at the time application of field current. In case of a machine where the reverse is true then it appears that the effect of the time constant would be still greater. In general, there should be two results. One would be to shift forward (in time) the optimum angle of application of field voltage, and the other would be to make a slight reduction in the slip from which a motor may synchronize.

Another oscillogram, shown in Fig. 3, was taken on the same motor with the field excited from a constant potential source of 125 volts with sufficient external resistance to permit half of normal full load field current to flow under steady-state conditions. The switching angle was very unfavorable and the motor failed to synchronize. This oscillogram, together with the first one mentioned, therefore, afford a good means of checking the authors' equation (5). For the first oscillogram the observed slip from which the motor synchronized was 0.021 and the switch angle was quite favorable. From the authors' equation (5) the slip should be less than 0.0144 if the machine were to synchronize during the first two opportunities from an initial position of the worst switching angle. In the last mentioned oscillogram where the motor did not synchronize, the equation (5) gives the slip from which the motor would synchronize as 0.010. Table I shows a comparison between test values and values calculated from equation (5) for four different types of synchronous motors.

The conclusions this tabulation justifies are:

TABLE I

Motor rating	P_m	Total WR^2	Observed slip	Calculated slip	Comment
75 hp.-900 r. p. m.	112 kw.	3,870	0.021	0.0144	Normal-time constant synchronism attained
75 hp.-900 r. p. m.	112 kw.	3,870	0.020	0.0144	Half-time constant synchronism attained
75 hp.-900 r. p. m.	56 kw.	3,870	0.023	0.0102	Half-time constant (half excitation) motor failed to synchronize
1,700 hp.-720 r. p. m.	2,600 kw.	28,600	0.020	0.032	Normal-time constant synchronism attained
2,800 hp.-600 r. p. m.	4,240 kw.	71,700	0.030	0.033	Normal-time constant synchronism attained
125 hp.-277 r. p. m.	150 kw.	6,375	0.040	0.0422	Normal-time constant synchronism attained

A motor may pull into step from a slip greater than that indicated by the formula, due to ideal timing of the instant at which field is applied; the formula will always give values such that if the slip calculated from it is attained, the motor will pull into step.

V. Bush: The excellent work presented in this paper appeals to me very strongly, especially as it was carried out by means of an integraph which is admittedly not particularly convenient nor accurate. This device at the Massachusetts Institute of Technology is really a first model, and we now have under construction a much better piece of equipment. This new apparatus will be capable of solving total differential equations of very considerable complexity and with much better precision than the present machine. I have every confidence that this new tool, put in the hands of Mr. Edgerton and the group which is interested in machine transients, will produce still further results along these lines, and may make it possible to include some of the effects which have necessarily been omitted in the present analysis.

The new machine for solving differential equations will shortly be described in some publication. I wish to emphasize again at this time that the Massachusetts Institute of Technology is anxious to cooperate in the use of these devices with any one having a serious research problem involving the solution of differential equations on which the instruments can be used to advantage.

Another very interesting matter in this paper is the new stroboscope, and I trust that this may have a separate presentation. It is interesting to note that the light intensities produced are very large indeed. Mr. Edgerton used a stroboscope which had a period of illumination of only about 10 microseconds occurring once each cycle, and yet was able to obtain sufficient light for photography. The instantaneous candle power of the source is hence enormous, as may readily be computed. This appears to have interesting possibilities for further development.

Ernst Weber: The paper deals generally with synchronous motors having damper windings so that an interaction of asynchronous and synchronous torques results when the field current is suddenly applied to the pole winding. It is known that the behavior of salient-pole machines is generally dependent on the materials used for the poles. Machines with solid iron poles and those with laminated poles show important differences especially with regard to mechanical oscillations. For this reason it would be pertinent to inquire as to the pole design features of the machine used in the tests described.

It is surprising to note that the typical integraph angle-slip solutions, given in Fig. 1, include only curves beginning with a decrease in slip, whereas oscillograms numbers 2 and 3 first show an important increase in slip, the transients being taken for switching angles of 95 and 110 degrees which are supposed to give good results. This might indicate some reaction between the establishment of the magnetic field in the poles and the damper winding. It is also possible (assuming the machine to have solid poles) that the eddy currents in the poles might have an important influence on the oscillations. In this case it would hardly be justifiable to neglect the rotor leakage reactance and all the electrical transients. When the oscillograms show a rather uncertain agreement with the integraph solution it would be necessary to include an additional term in the mechanical differential equation to account for the effect of the eddy currents. A second differential equation should include the effect of the electrical transients which have a relatively long duration compared with the variation of field current in the pole winding as shown in the oscillograms on page 775.

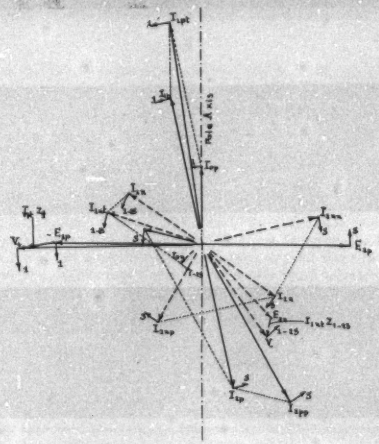


Fig 4

The work performed and presented in this paper is of the greatest value as a step forward in the graphical solution of nonlinear differential equations of complicated forms. In the special case of the application, however, the underlying theory of the phenomenon seems to be incomplete.

P. Fourmarier: Independently of a very exact calculation, it can be shown, by drawing the vector diagram of the synchronous motor running as an induction motor, that the induction motor power term is of the general form:

$$P_d\,[1 - b\cos(2\,\theta + \beta)]\,\frac{d\,\theta}{d\,t}$$

where β is a small angle.

Let us draw that diagram for a two-pole synchronous motor; in that case, the time diagram coincides with the space diagram.[1] All vectors are referred to the stator; the velocity of the rotor vectors is indicated with respect to the rotor; the velocity of the stator vectors is indicated with respect to the stator. The rotor vectors represent the values of the different variables in the middle bar of the pole.

The subscript 2 indicates the rotor vectors; the subscript 1, the stator vectors.

The notations in the diagram are listed below:

E_{1p} = e. m. f. induced by the positively rotating air-gap flux
E_{1n} = e. m. f. induced by the negatively rotating air-gap flux
I_{2pp} = positively rotating component of current due to E_{1p}
I_{2np} = negatively rotating component of current due to E_{1p}
I_{2pn} = positively rotating component of current due to E_{1n}
I_{2nn} = negatively rotating component of current due to E_{1n}
I_{2p} = total positively rotating rotor current
I_{2n} = total negatively rotating rotor current

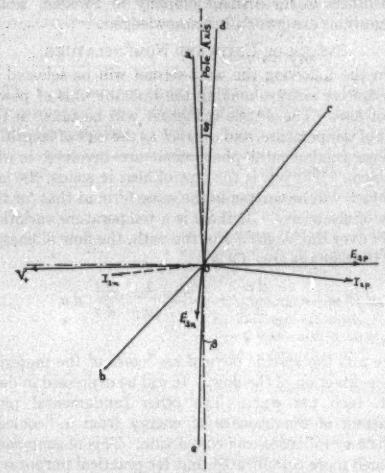


FIG. 5

I_{op} = positively rotating magnetizing stator current
I_{on} = negatively rotating magnetizing stator current
I'_{pt} = total positively rotating stator current
I_{1nt} = total negatively rotating stator current
V_+ = total positively rotating terminal voltage
V_- = total negatively rotating terminal voltage

As the terminal voltage is supposed to be balanced, V_- must be equal to zero.

Power. The total power supplied to the shaft can be divided into two principal parts:

a. The constant powers including:
1. The total positive power P_p [vectors $E_p I_p$]
2. The total negative power P_n [vectors $E_n I_n$]

b. Two periodical powers P_{pn} and P_{np} of double frequency, caused by the voltages and currents rotating in opposite directions. Their average value is zero but their instantaneous value is given by the product of the vectors by the cosine of the angle between them. [vectors $E_p I_n$ and $E_n I_p$]

Simplified Diagram. For small slips (pulling into step) the diagram can be shown to reduce to Fig. 5 as the rotor bars reactance and the stator winding resistance are neglected.

1. See Bibliography 12.

We can derive from that diagram the positions of E_{1p} corresponding to the maxima of unbalanced powers:

a Position of E_{1p} for maximum $E_{1p} I_{2n}$
b Position of E_{1n} for maximum $E_{1n} I_{1p}$
c Position of E_{1p} for maximum $E_{1n} I_{1p}$
d Position of E_{1p} for maximum total unbalanced power
e Position of V_+ for maximum total unbalanced power.

It must not be forgotten that the powers, being of double frequency, have to be added on a special double frequency diagram. The calculations show that β and δ are very small. We can thus represent this unbalanced power by

$$P_1 \cos (2\,\theta + \beta_1) \frac{d\,\theta}{d\,t}$$

where β_1 is a small angle.

Effect of the Field Winding. The field winding can be considered as a single-phase winding 90 deg. ahead of the bar winding; its unbalanced power can be represented by a similar expression

$$P_2 \cos (2\,\theta + \beta_2) \frac{d\,\theta}{d\,t} \qquad (2)$$

This unbalanced power is in general small as compared with the first one, which means that the sum of expressions (1) and (2) can be written in the form

$$P_d\, b\, (\cos 2\,\theta + \beta) \frac{d\,\theta}{d\,t}$$

where β is a small angle.

Harold E. Edgerton: It is a source of satisfaction to us to know that our efforts to solve the pulling-into-step problem of the salient-pole synchronous motor have resulted in a practical form which is of use to those who are concerned with the application of synchronous motors to drive difficult loads.

Messrs. Terry and McLenegan ask us to discuss the comparison that they show between our studies of the cylindrical and salient-pole synchronous machines. As we pointed out in the paper our accuracy is hardly sufficient to make a comparison of these equations, however, we are certain that the reluctance effect does not have a pronounced effect on the critical value of average initial slip for the worst switching angle. The reluctance torque has two main effects, one which is beneficial and the other detrimental. The beneficial effect of the reluctance torque is to help the motor pull into synchronism. The detrimental effect is the influence of the reluctance torque in causing speed variations so that the initial condition of slip is worse than the average for the unfavorable switching condition. For some favorable switching conditions the salient-pole synchronous motor will pull into step easier than the cylindrical-rotor type. The initial variations of slip that are caused by the reluctance torque are very noticeable in the oscillograms, Figs. 5, 6, and 7.

The discussion and the tests given by Messrs. Terry and McLenegan showing the influence of the electrical transients upon the pulling-into-step of a motor are interesting. We hope to obtain some integraph solutions of these transients in future studies. The tabulated comparisons show no disagreement. However, these tests are not conclusive since the switching conditions were uncertain.

There is a great variety of synchronous-machine dynamic problems that the integraph can solve. In some cases, such as in this problem, a general form of solution can be obtained by taking ratios of factors and by changing variables. In other problems the number of variables and factors are not easily simplified and a general solution for all cases is not possible. The integraph solutions are then only made for a specific set of parameters that fit some particular set of conditions that are to be investigated.

Soon a new mechanical integraph will be completed by Dr. Bush and those working with him. It has remarkable accuracy and will handle sixth order differential equations.

Steady Flow of Heat in Large Turbine-Generators

BY C. RICHARD SODERBERG[1]

Member, A. I. E. E.

Synopsis.—*The present paper represents an attempt at a reasonably complete solution of the heat flow in large turbine generators, where the temperature drops through the various iron paths are of consequence. The solution of two-dimensional problems of heat flow is obtained by means of "equivalent thermal circuits." The approximations involved in this method are discussed in two appendixes where exact solutions are developed. The theory has been applied to a number of large turbine generators with satisfactory agreement between measured and calculated temperatures. The application of the results to actual machines will be made the subject of a subsequent paper.*

* * * * *

Introduction

THE predetermination of the temperature in electrical machines involves three distinct problems:

1. The magnitude and distribution of the losses.
2. The flow of the cooling medium through the machine, and its cooling effect.
3. The flow of heat from the source of generation to the cooling medium.

All of these problems present great difficulties when accurate solutions are wanted, although the electrical industry has managed fairly well, as indicated by the comparatively close agreement between calculations and test results in modern machines. Agreement, however, between calculations and tests is not always a perfectly reliable criterion of the correctness of the underlying theory, because in machines designed on the basis of the performance of nearly similar units a very rough theory may give results in excellent agreement with tests. The same theory applied to a machine of different proportions or size may fail altogether.

The present paper is restricted to the third of the above problems and represents an attempt at a reasonably complete solution for the heat flow in large machines with deep slots, where the temperature drops through the various iron paths are of consequence. The analysis proceeds from a simple conception of the general aspect of the heat flow, by progressive stages of elaboration, to what is believed to be a fairly correct picture of the complicated phenomena involved. The more elaborate mathematical deductions are given in the appendixes. It is important to remember that the final accuracy of a temperature calculation depends primarily on the correctness of the loss distribution, and the thermal properties of the materials, upon which the calculation has been based. Several details of the following analysis have been carried to a degree of refinement which may be considered impractical for the ordinary types of machines. An approximation, however, is justified only when the degree of approximation is known, and this has not always been the case with the approximate methods commonly used.

Only the conditions of constant heat flow are considered. The problems encountered in predetermining the flow of heat before a stationary temperature is reached will be discussed in a later paper.

During the preparation of this paper the writer received many valuable suggestions from Dr. J. Wennerberg and others of the ASEA Company in Sweden, which suggestions are gratefully acknowledged.

System of Units and Nomenclature

In the following the watt-second will be selected as the unit of energy, making the *watt* the unit of power or of loss. The degree *centigrade* will be taken as the unit of temperature, and the *inch* as the unit of length.

Two fundamental phenomena are involved in the problem. The one is the flow of heat in solids, the law of which will be written in the same form as that for the flow of electricity. If there is a temperature variation of $d\,t$ over the length $d\,x$ of the path, the flow of energy $d\,P$ through the area $d\,a$ is

$$d\,P = -\frac{d\,t}{\rho\,\dfrac{d\,x}{d\,a}} = -\frac{1}{\rho}\,\frac{d\,t}{d\,x}\,d\,a \qquad (1)$$

Here ρ is the *specific thermal resistance* of the material in the direction of the flow. It will be expressed in deg. cent. inch per watt. The other fundamental phenomenon is the transfer of energy from a "cooled" surface by radiation and convection. This phenomenon is much more complicated, but for practical purposes it is permissible to assume that the flow of energy per unit area is proportional to the difference of temperature between the cooled surface and the cooling medium. The law will be written in the form

$$d\,P = -\frac{1}{C}\,t\,d\,a \qquad (2)$$

where C is defined as the *surface resistance* at the cooling surface, expressed in deg. cent. inch^2 per watt. *Emissivity* is the usual name given to the quantity $1/C$.

The conception of surface resistance may also be extended to the case of heat flow across comparatively thin layers of insulation, where the temperature gradient may be considered constant. Thus, the flow of heat across an insulation layer of thickness $\Delta\,x$ may be expressed by an equation of the same form as (2), where $C = \rho\,\Delta\,x$ may be defined as the surface resistance of the insulation layer.

The expression $d\,P/d\,a$ will be denoted by q and

1. Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

referred to as the *surface intensity* of the flow. When losses are generated in a solid, the loss per unit volume dP/dV will be denoted by p and referred to as the *volume intensity* of the loss.

The following list gives the general symbols, which are used in the paper, with the exception of occasional symbols which are always defined when used. Each symbol may be appended by one or several of the following indexes:

Index	Significance
1	Copper in slot
2	Iron in teeth
3	Airgap surface
a	Cooling medium
c	Iron in core back of teeth
i	Slot insulation
x	Axial direction
y (or r)	Radial direction

The sign * written thus, C^*, will be used to designate a "corrected" or "slightly different" value of the symbol in question.

Symbol	Units	Significance
a	inch^2	Area of cross section
b	inch	Width of copper in slot b_1, mean thickness of tooth b_2. Equivalent width of cooling surface b_a ("cooled width")
C	$\frac{\text{deg. cent. inch}^2}{\text{watt}}$	Surface resistance. Of insulation C_i, at cooling surface C_a, equivalent surface resistance for flow through tooth C
f (and k, m, n, o)		Numerical constants
h	inch	Depth of copper h_1, height of tooth h_2,
l		Half spacing of radial vent ducts l_1, half axial length of iron package l_2
P	watt	Loss
p	$\frac{\text{watt}}{\text{inch}^3}$	Volume intensity of loss
Q		Number of slots
q	$\frac{\text{watt}}{\text{inch}^2}$	Surface intensity of loss
R		Special cylinder function
r	inch	Radius, radial coordinate
t	deg. cent.	Temperature
x	inch	Axial coordinate
X and Y		Numerical constants (for expressions $m\,\alpha\,l$ and $n\,\alpha\,l$)
y	inch	Radial coordinate
α (and m, n)	$\frac{1}{\text{inch}}$	Argument constants (in expression $e^{\alpha x}$)
θ		Taper constant
κ		General numerical constant
ρ	$\frac{\text{deg. cent. inch}}{\text{watt}}$	Specific thermal resistance
τ	inch	Slot pitch

The significance of the various combinations of indexes will be self evident in most cases. b_{ax2} is the "cooled width" of the tooth in the x-direction (in the radial ducts), C_{ax2} is the corresponding surface resistance, ρ_{x1} is the specific thermal resistance of the copper in the axial direction, and so on. The order of the letters in the indexes just given will be retained throughout.

The General Problem

The present analysis applies to an arrangement of a general character, intended to embody the principal features of a rotor and a stator of a modern machine with radial ventilation. This arrangement is a slotted iron core with windings in the slots. In the core part the heat is dissipated from cooling surfaces in the airgap, at the back of the core, and in radial ducts. The latter divide the core into short, axial segments, which may be considered insulated from each other.

Thus, axial flow of heat takes place through the windings only. The general solution of the temperature

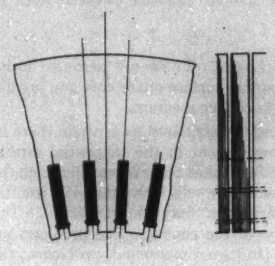


Fig. 1—Typical Arrangement of Stator

problem involves a solution for unit or several of the core part (a vent package), a solution for the end windings, and a solution for the axial distribution of temperature in the windings. The present discussion will be restricted to the first and most important part of the problem. The dissipation of heat from the end-windings is not always easy to determine, but the difficulties are chiefly due to lack of data on the thermal properties of the insulation and the behaviour of the cooling medium. The axial distribution of the temperature of the windings is a comparatively simple one-dimensional problem, which has received considerable attention in the past.[2]

The structure to be studied is thus a short section of the core, it may be either a stator section, Fig. 1, or a rotor section, Fig. 2 and Fig. 3. It is of comparatively short axial extension so that the temperature of the copper may be considered constant in the axial direction. In order to maintain the general nature of

2. For an interesting discussion of the general subject of the flow of heat in electrical machines, and a particularly complete bibliography, see Edoard Roth: "Introduction a L'Étude Analytique de l'Echauffement des Machines Electriques."

the problem the existance of the following losses will be assumed: 1. Copper Losses. These are generated inside the slot insulation and flow through the insulation layer into the tooth, whence they continue radially and axially to the cooling medium. 2. Iron Losses. These are generated in the core material itself whence they flow axially and radially to the cooling medium.

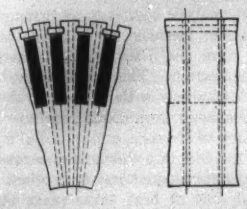


Fig. 2—Typical Arrangement of Rotor Axial Flow of the Cooling Medium Through Central Hole in Rotor Core

3. Surface Losses. These are assumed to be generated at the airgap surface of the core and are dissipated directly to the cooling medium.

Considering the structure as a whole there is no appreciable flow of heat in the tangential direction. A considerable flow takes place across the teeth, but since their dimensions are relatively small the resulting temperature variations may be neglected. The same results apply to the copper. The problems may thus be reduced to two-dimensional problems, the main directions of flow being axial and radial. Due to the

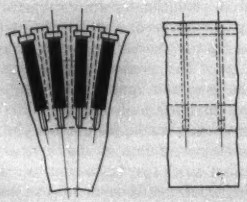


Fig. 3—Typical Arrangement of Rotor Axial Flow of the Cooling Medium Through Vent Ducts at the Bottom of Slots

symmetry of the arrangement, the axial flow of the iron package, and of the copper, divides along the central plane of the package. The division of the radial flow in the teeth and in the copper is somewhat more difficult to establish since it depends upon the relative intensity of the losses, and the cooling effects of the tooth region and at the back of the core.

Geometrically, the structure may be divided into the *tooth-region* and the *core*, bounded by a cylindrical surface through the bottom of the slots. In an arrangement of the type shown in Fig. 3 the cooling effect at the bottom of the slot is very pronounced, so that a considerable portion of the losses in the tooth-region will flow towards the back of the core. In the arrangement of Figs. 1 and 2 this tendency will be less pronounced, but in general, the radial flow will always divide in the tooth region. Before proceeding with the discussion of these problems the general method of analysis will be reviewed and applied to a few simple problems, the solutions of which will be used in the general solution of the problem.

The Thermal Circuit

Consider a plate, Fig. 4, of thickness b, length l and height h, in which are generated the losses P watts with uniform intensity at all parts of the plate volume. These losses are carried to the cooling medium at the

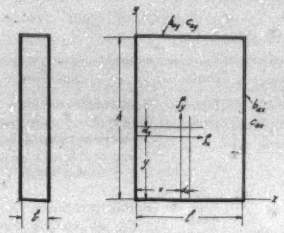


Fig. 4—Simple Two-Dimensional Problem

surfaces $x = l$ and $y = h$, all other surfaces being completely insulated. The effective cooling surfaces at these edges will be taken as $b_{ax}\,h$ and $b_{ay}\,l$, and the corresponding surface resistances as C_{ax} and C_{ay}, respectively.

Assuming at first that the surface $y = h$ also is heat insulated, a simple one-dimensional problem is obtained. The temperature of the plate is then easily found to be

$$t = P\left[\frac{C_{ax}}{b_{ax}\,h} + \frac{\rho\,x}{b\,h}\,\frac{l}{2}\left(1 - \frac{x^2}{l^2}\right)\right] \quad (3)$$

The mean temperature is, consequently

$$t_m = P\left[\frac{C_{ax}}{b_{ax}\,h} + \frac{\rho\,x}{b\,h}\,\frac{l}{3}\right] \quad (4)$$

This shows that the obstruction to the flow of heat from the mean temperature to the temperature of the cooling medium can be regarded as two thermal resistances in series $\frac{C_{ax}}{b_{ax}\,h}$ at the cooling surface, and $\frac{\rho\,x}{b\,h}\,\frac{l}{3}$ in the plate. The latter is clearly the thermal resistance of a bar of area $b\,h$ and length $l/3$ (see Fig. 5). Simi-

larly, the obstruction to a one-dimensional flow in the y-direction may be regarded as a combined thermal resistance of $\frac{C_{ay}}{b_{ay}\,l} + \frac{\rho\,y}{b\,l}\,\frac{h}{3}$.

The obstruction to the two-dimensional flow may now

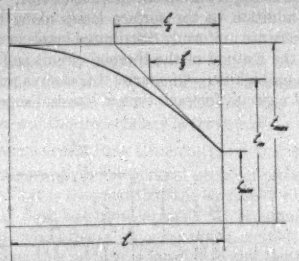


FIG. 5—TO ILLUSTRATE PARABOLIC TEMPERATURE DISTRIBUTION IN ONE-DIMENSIONAL FLOW OF DISTRIBUTED LOSSES

be regarded as the combination in parallel of these two groups of resistances. The *thermal circuit* of Fig. 6 may thus be used to illustrate the passage of the losses P from the mean temperature t_m to the temperature of

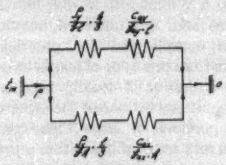


FIG. 6—THERMAL CIRCUIT FOR RECTANGULAR PLATE WITH DISTRIBUTED LOSSES

the cooling medium. In solving for the mean temperature from this circuit introduce the quantities

$$\left.\begin{aligned} C_x &= \frac{2\,l\,C_{ax}}{b_{ax}} + \frac{2\,l^2\,\rho_x}{3\,b} \\ C_y &= \frac{2\,h\,C_{ay}}{b_{ay}} + \frac{2\,h^2\,\rho_y}{3\,b} \\ C &= \frac{C_x\,C_y}{C_x + C_y} \end{aligned}\right\} \quad (5)$$

which all have the dimensions of surface resistance. Then the mean temperature becomes

$$t_m = \frac{P}{2\,h\,l}\,C \quad (6)$$

The surface resistance C represents the insulating properties of an imaginary insulation layer which is placed on the two sides of the plate and which presents the same thermal resistance as the combined paths of the plate itself. C_x and C_y have the same significance for the one-dimensional flow in the x and y directions, respectively. The temperature at an arbitrary point (x, y) of the plate is easily obtained if the temperature variation is assumed to be parabolic. If it is found that

$$t = t_m\,\frac{\dfrac{2\,l\,C_{ax}}{b_{ax}} + \dfrac{l^2\,\rho_x}{b}\left(1 - \dfrac{x^2}{l^2}\right)}{C_x}\,\cdot\,\frac{\dfrac{2\,h\,C_{ay}}{b_{ay}} + \dfrac{h^2\,\rho_y}{b}\left(1 - \dfrac{y^2}{h^2}\right)}{C_y} \quad (7)$$

It must be emphasized that this conception of the two-dimensional flow is a very rough approximation of the actual circumstances. As shown in Appendix I, however, it does give quantitative results which are sufficiently accurate for practical purposes.

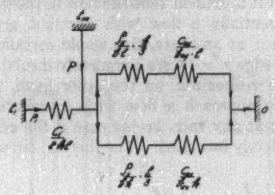


FIG. 7—THERMAL CIRCUIT FOR RECTANGULAR PLATE WITH DISTRIBUTED LOSSES AND INCOMPLETELY INSULATED SIDES

Consider next the case where the sides of the plate are only partially insulated, the outside of the insulation being held at a constant temperature t_1, which is higher than the temperature of the plate itself. A certain amount of heat will now flow inward through the insulation layer where it will "mix" with the heat generated in the plate. The problem is now complicated by the fact that the intensity of flow across the insulation is not constant at all points. The total flow of heat across the insulation layer will be proportional to the average drop of temperature $t_1 - t_m$, but the contribution to the volume intensity of loss in the plate by the flow through the insulation layer will not be constant. The error in the mean temperature is small, however, as shown in Appendix I. Neglecting this error the problem may be represented by the thermal circuit shown in Fig. 7. P_1 is the loss flowing into the plate across the insulation, and C_i is the surface resistance of the insulation layer on the sides. With the notations given in (5) it is obvious from Fig. 7 that

$$t_m = \frac{P + P_1}{2\,h\,l}\,C \quad (8)$$

and

$$t_1 = \frac{P_1}{2\,h\,l}\,C_i + \frac{P + P_1}{2\,h\,l}\,C \quad (9)$$

In order to facilitate a comparison with the exact solution this result will be written in a different form by introducing the temperature

$$t_1' = t_1 + \frac{P}{2\,h\,l}\,C_i \tag{10}$$

whence, for the mean temperature of the plate

$$t_m = t_1' \frac{C}{C + C_i} \tag{11}$$

The temperature t_1' corresponds to the case when all the losses enter the plate from the outside through the insulation on the sides.

The thermal circuits shown so far cover cases of distributed losses. Similar circuits may be constructed for other methods of supplying the losses. Suppose, for example, that the losses P watts are introduced into the plate in Fig. 4 through the edge $y = o$. The exact solution for this case has not been investigated by the author, but it is evident that the flow in the y-direction will be essentially a flow with constant gradient of temperature; at any rate, this would certainly be the case if the edge $x = l$ were completely insulated. The flow in the x-direction, on the other hand, might be expected to approach a flow with distributed losses; this would, at any rate, be the case if the edge $y = h$ were completely insulated. The thermal circuit for the

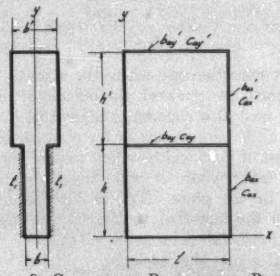


Fig. 8—Compounded Rectangular Plate

mean temperature of the plate would then take the form shown in Fig. 6 with the modification that the resistance $\frac{\rho_y}{b\,l}\,\frac{h}{3}$ is changed to $\frac{\rho_y}{b\,l}\,\frac{h}{2}$. Moreover, the temperature variation in the y-direction will be essentially linear so that the solution of the problem may be approximated by the equation

$$t = t_m \frac{\dfrac{2\,l\,C_{ax}}{b_{ax}} + \dfrac{2\,l^2\,\rho_x}{b}\left(1 - \dfrac{x^2}{l^2}\right)}{\dfrac{2\,l\,C_{ax}}{b_{ax}} + \dfrac{2\,l^2\,\rho_x}{3\,b}}\,.$$

$$\frac{\dfrac{2\,h\,C_{ay}}{b_{ay}} + \dfrac{2\,h^2\,\rho_y}{b}\left(1 - \dfrac{y}{h}\right)}{\dfrac{2\,h\,C_{ay}}{b_{ay}} + \dfrac{h^2\,\rho_y}{b}} \tag{12}$$

where t_m is computed from the thermal circuit.

If, in addition to the surface losses along the edge $y = o$, there are uniformly distributed losses in the plate volume, the method of the thermal circuit suffers from a certain ambiguity. Of course, it is always possible to construct a circuit for each type of loss and superimpose

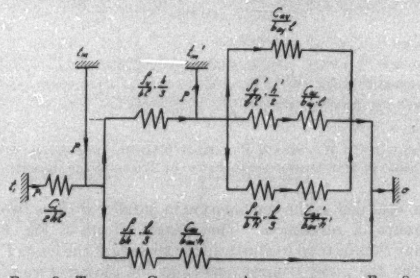


Fig. 9—Thermal Circuit for Arrangement in Fig. 8

the temperature fields thus obtained, but this method is somewhat too complicated for practical purposes. Whenever more than one type of loss is to be introduced into the same circuit it is usually necessary to make certain approximations for one or the other of the losses. The usual proportions of large turbo generators are such that the errors are negligible from a practical point of view.

To illustrate this situation consider the problem shown in Fig. 8. The lower plate is identical in all respects with that shown in Fig. 4. The insulation on the sides has the surface resistance C_i and heat is flowing into this insulation from the temperature t_1. The sides of the upper plate are completely insulated. It is joined to the lower plate at the edge $y = h$ without affecting the cooling conditions for the lower plate at that edge. The losses in the lower and upper plates are P and P', respectively. The losses entering the lower plate through the insulation on the sides will be denoted by P_1.

If the losses P' may be considered concentrated along the edge $y = h$ of the upper plate, the thermal circuit of Fig. 9 will take care of the problem adequately. Obviously, the circuit is correct for the case that $P' = o$ as well. On the other hand, if the losses P' are evenly distributed throughout the volume, the circuit of Fig. 9 may be in error, depending on the importance of the paths of flow in the y-direction. If the conditions in the lower plate are such that only a small

percentage of the flow is diverted in the y-direction, the thermal circuit of Fig. 9 will be more correct if the thermal resistance $\frac{\rho_y'}{b' l} \frac{h'}{2}$ be changed to $\frac{\rho_y'}{b' l} \frac{h'}{3}$ and the temperature variation in the upper plate be considered parabolic.

It is necessary, therefore, to exercise caution in applying the method of the equivalent thermal circuit. However, as shown in the following, the practical significance of this uncertainty is very small.

The Temperature Rise Constants of the Machine

Returning now to the general problem presented by Figs. 1 to 3 it is assumed that the surface of division of the radial flow has been located. It is assumed, furthermore, since this always occurs in actual arrangements, that this surface is located in the tooth region, which it divides into two parts. The criterion for locating the surface of division is that the temperatures for the two regions "match" at this surface.

Considering first the inner part of the tooth-region (nearest the bottom of the slot) it is evident that the arrangement resembles that of Fig. 8. The lower plate corresponds to the tooth with the surface of division of the radial flow at the edge $y = o$. The upper part corresponds to the core. In constructing an equivalent thermal circuit for this system an uncertainty with regard to its construction will exist. This uncertainty is of very small practical consequence, however, as shown by the following reasoning. In the stator the radial component of the flow of heat in the tooth is usually very small so that the influence upon the tooth-region of the core is also small. Assuming the core losses to be uniformly distributed throughout the volume, the thermal circuit of Fig. 9, modified as suggested in the preceding paragraph, will be nearest to actual conditions. In the rotor, no core losses exist and the radial flow is large, so that the circuit of Fig. 9 is the correct one to use. It is evident, therefore, that a calculation of the conditions in the inner part of the tooth-region and the core is possible. The general process of analyzing the circuit thus decided upon will be obvious from the following discussion for the outer part.

It is a fortunate coincidence in the typical turbo generator that the surface of division of the radial flow in the tooth region is located near the bottom of the slots. Moreover, the error committed by placing it at the bottom of the slot is never very large and is on the side of higher temperatures. For this reason it is the outer part of the tooth region that is of greatest interest, and its solution leads to a set of quantities which may be referred to as the *temperature-rise constants of the machine*. Thus, the above discussion for the inner region merely serves as a check on the matching of the temperatures at the assumed position of the boundary.

The arrangement shown in Fig. 10 is a simplified tooth-region for a stator or a rotor of very large diameter. This may be referred to as two "coupled" plates of homogenous materials. The plate with index 1 represents the copper and has a thickness b_1 a length l_1 (one-half the vent-pitch) and a height h_1. Its specific thermal resistances are ρ_{x1} and ρ_{y1}, respectively, and the cooling conditions at the edges $x = l_1$ and $y = h_1$ are given by the cooling surfaces $b_{ax1} h_1$, and $b_{ay1} l_1$ and the surfaces resistances C_{ax1} and C_{ay1} respectively. The cooling surface at $x = l_1$ corresponds to the exposed part of the coil in the vent-duct, that at $y = h_1$ corresponds to the slot wedge. The plate with index 2 represents the iron, and the various quantities carry index 2; their significance is similar to that of the other plate. The insulation layer between the two plates is assumed to have a surface resistance of C_i. The following losses are considered:

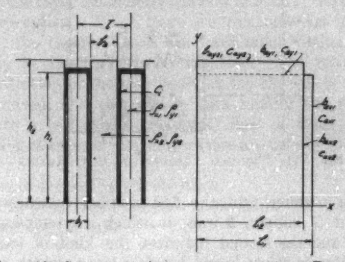


Fig. 10—Simplified Arrangement of Slot Region

1. Copper Losses. These are expressed either by the volume intensity p_1 in the copper volume $b_1 h_1 l_1$ or by the surface intensity q_1 on the air gap surface τl_1. The former is a function of x and y. If the mean value is denoted by $\bar{p}_1$ it is obvious that

$$q_1 = \frac{b_1 h_1}{\tau} \bar{p}_1 \tag{13}$$

2. Iron Losses. These are expressed either by the volume intensity p_2 in the iron volume $b_2 h_2 l_2$ or by the surface intensity q_2 on the air gap surface τl_1. The former is a function of x and y. If the mean value of p_2 is denoted by $\bar{p}_2$ it is obvious that

$$q_2 = \frac{b_2 h_2 l_2}{\tau l_1} \bar{p}_2 \tag{14}$$

3. Surface Losses. (Pole-face losses.) These are assumed to be uniformly distributed over the tooth area $b_2 l_2$ in the air gap, and are expressed by the equivalent surface intensity q_3 on the air-gap surface τl_1.

The principal object of the analysis is to determine the mean temperatures t_{1m} and t_{2m} of the copper and the iron. Assuming that the loss intensities p_1 and p_2

are uniform throughout the volumes the construction of the equivalent thermal circuit for the arrangement is comparatively simple. The results are shown in Fig. 11, which will thus give the solution of our problem with a degree of accuracy which will be discussed later. The solution will be put in the following form:

$$t_{1m} = C_{11}\, q_1 + C_{12}\, q_2 + C_{13}\, q_3$$
$$t_{2m} = C_{21}\, q_1 + C_{22}\, q_2 + C_{23}\, q_3 \qquad (15)$$

The constants C_{11}, C_{12}, etc., expressed in deg. cent inch² per watt, will be defined as *the temperature rise constants of the machine.* The indexes are written in

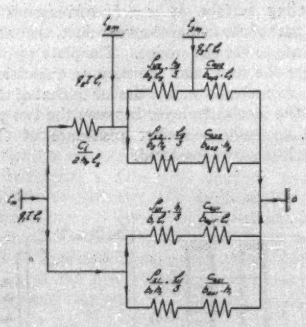


FIG. 11—THERMAL CIRCUIT FOR SLOT REGION

such a manner that the first number indicates the element (1 copper, 2 iron) to which the temperature refers, and the second indicates the kind of loss (1 copper losses, 2 iron losses, 3 surface losses) which cause the temperature rise. The thermal properties of plate 1 will be given by the surface resistances

$$C_{x1} = \frac{2\, l_1\, C_{ax1}}{b_{ax1}} + \frac{2\ .\ l_1^2\, \rho_{x1}}{3\, b_1}$$

$$C_{y1} = \frac{2\, h_1\, C_{ay1}}{b_{ay1}} + \frac{2\ .\ h_1^2\, \rho_{y1}}{3\, b_1} \qquad (16)$$

$$C_1 = \frac{C_{x1}\, C_{y1}}{C_{x1} + C_{y1}}$$

and those of the plate 2 by the surface resistances

$$C_{x2} = \frac{2\ l_2\ C_{ax2}}{b_{ax2}} + \frac{2\ l_2^2\ \rho_{x2}}{3\, b_2}$$

$$C_{y2} = \frac{2\, h_2\, C_{ay2}}{b_{ay2}} + \frac{2\ h_2^2\ \rho_{y2}}{3\, b_2} \qquad (17)$$

$$C_2 = \frac{C_{x2}\, C_{y2}}{C_{x2} + C_{y2}}$$

The significance of these quantities is obvious from the above discussion of the two-dimensional problems. In addition, we introduce the dimensionless ratios

$$\left[\kappa_1 = \frac{C_1}{C_i + \frac{l_2}{l_1} C_1 + \frac{h_1}{h_2} C}\right.$$

$$\left.\kappa_2 = \frac{l_1}{l_2}\ \frac{C_i + \frac{l_2}{l_1} C_1}{C_i + \frac{l_2}{l_1} C_1 + \frac{h_1}{h_2} C_2}\right] \qquad (18)$$

Then the temperature rise constants of the machine are given by

$$C_{11} = \frac{\tau}{2\, h_2}\ \kappa_1 \left[\frac{h_2}{h_1} C_i + C_2\right]$$

$$C_{12} = C_{21} = \frac{\tau}{2\, h_2}\ \kappa_1\, C_2$$

$$C_{22} = \frac{\tau}{2\, h_2}\ \kappa_2\, C_2$$

$$C_{13} = C_{31} = \frac{\tau}{b_{ay2}}\ \frac{C_2}{C_{y2}}\ \kappa_1\, C_{ay2}$$

$$C_{23} = C_{32} = \frac{\tau}{b_{ay2}}\ \frac{C_2}{C_{y2}}\ \kappa_2\, C_{ay2} \qquad (19)$$

The distribution of the copper and iron losses to the various paths is shown in Fig. 12 and Fig. 13. Of the copper losses in the slot, q_1, the portion $m\, q_1$ flows across the slot insulation to the tooth, the remainder $(1 - m)\, q_1$ flows directly through the copper to the

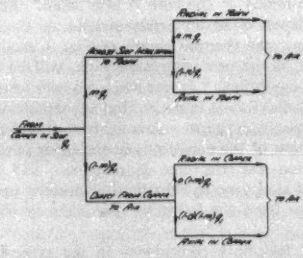


FIG. 12—DISTRIBUTION OF FLOW OF COPPER LOSSES

cooling medium. Of the former portion $n\ m\ q_1$ flows in the radial direction of the tooth, and $(1 - n)\ m\ q_1$ flows in the axial direction. Of the latter portion $\sigma\ (1 - m)\ q_1$ flows in the radial direction. The iron losses in the tooth are divided into $n\ q_2$ flowing in the radial direction, and $(1 - n)\ q_2$ in the axial direction. The constants m, n and o are found to have the values

$$m = \kappa_1 \left[\frac{l_2}{l_1} - \frac{q_2}{q_1} \frac{h_1}{h_2} \frac{C_2}{C_1} - \frac{q_2}{q_1} \frac{2\,h_1}{b_{ay2}} \frac{C_2}{C_1} \frac{C_{ay2}}{C_{y2}} \right]$$

$$n = \frac{C_2}{C_{y2}} \left[1 - \frac{q_2}{q_1\,m + q_2} \frac{2\,h_2}{b_{ay2}} \frac{C_{ay2}}{C_{x2}} \right]$$

$$o = \frac{C_1}{C_{y1}} \tag{20}$$

Errors Due to the Approximations

The solution given in the preceding paragraph is usually sufficiently accurate for practical problems, but it is of importance, nevertheless, to know the approximate errors that are committed by neglecting some of the features of the actual problem. Therefore, the present section will be devoted to a discussion of the influence upon the solution of the following factors: 1. The inaccuracy of the approximate solution of the two-dimensional problem with distributed losses; 2. The taper of the teeth; and 3. The variation of the volume intensity of the copper and iron losses.

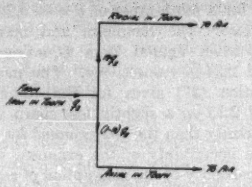


Fig. 13—Distribution of Flow of Iron Losses

The Inaccuracy of the Approximate Solution of the Two-Dimensional Problem

The solution given by the thermal circuit of Fig. 6 for the system shown in Fig. 4 gives too high a value of t_m, because the interference of the radial and axial flow has been neglected. It is obvious that the error is greatest when the flow is of the same intensity in the two directions. The error must also depend upon the relative magnitude of the temperature drops in the plate and at the cooling surfaces, because the approximate solution becomes exact when the former are zero. The result obtained in Appendix I-A agrees with this reasoning. It is shown there that a nearly exact value of t_m is obtained if the surface resistance C is replaced by a corrected value

$$C^* = \left[1 - \frac{1}{5} \frac{2\,\rho_x\,l^2}{3\,b\,C_x} \frac{2\,\rho_y\,h^2}{3\,b\,C_y} \right] C \tag{21}$$

In extreme cases of very deep stator teeth the ratios $(2\rho_x l^2)/(3\,b\,C_x)$ and $(2\rho_y h)^2/(3\,b\,C_y)$ may have the value 0.65 and 0.45, respectively, so that an error in mean temperature of six per cent is possible from this source. This may mean 1 deg. cent. for the iron temperature and 1.5 deg. cent. on the copper temperature. The errors in maximum temperature may be twice as large.

This conclusion applies to cases in which the slot-insulation is very heavy (C_i large), so that the copper losses enter the tooth at uniform density. In actual cases, particularly for low-voltage machines and for the rotor, where this is not the case, there is an additional error. It is shown in Appendix I-B, that when this additional effect is included, the error in mean temperature may amount to 7 per cent. A "semi-exact" solution is also given, whereby it is possible to reduce this error to 1 or 2 per cent.

All of the above errors are on the "safe side," that is, the calculated temperatures are larger than the actual. We may conclude, therefore, that this group of errors may be neglected for the ordinary types of machines.

The Influence of the Taper of the Teeth

The taper of the teeth may influence the results in two ways. When, as in the rotor, there are no iron losses, the taper will modify the flow of heat in the tooth to some extent. When there are iron losses, as in the stator, the tapered tooth will have much higher flux density at the tip than at the root. The resulting concentration of the iron losses at the tip will be very pronounced, and this will in turn modify the radial (and to a less extent the axial) flow of heat. The resulting changes in the temperature are usually greater than those due directly to the taper, but both types of influences are comparatively small.

It has been found convenient to define the taper of the tooth by the ratio

$$\theta = \frac{\pi\,h}{b\,Q} \tag{22}$$

where b and h are the mean thickness and the height of the tooth, respectively, and Q is the number of tooth pitches on the circumference. The ratio θ will be called the *taper constant*. This quantity represents the ratio of the depth of the tooth to the equivalent diameter[3] of the core at the middle of the tooth, and is found to appear in a simple manner in relations expressing the influence of tooth taper. θ is reckoned positive when the flow of heat is towards the wider part of the tooth (rotor), and negative when the flow is towards the narrow part of the tooth (stator). $\theta = 1$ represents a rotor tooth which goes to a point at the bottom, $\theta = 0$ a tooth with parallel sides, and $\theta = -1$ a stator tooth which goes to a point at the tip (see Fig. 15). Obviously, the conception of taper is easily extended to any cylinder problem.

In order to investigate the influence of the taper upon the flow of heat in the tooth when no readjustment of the losses takes place, consider the arrangement shown in Fig. 14 and assume that the thickness of the plate varies directly with y, with a mean value of b. Assume

3. The diameter for zero slot-width ($2r_m$ in Fig. 14.)

that the loss intensity in the plate varies inversely with the thickness, so that the loss per unit height of the plate is constant. This assumption will be the nearest to the actual case of the rotor, when the copper losses enter the tooth across the insulation with approximately constant intensity at all points. This problem is discussed at length in Appendix II-A. It is shown that for moderate values of the taper constant ($|\theta| = 0.3$) the simple expression for the mean temperature can be

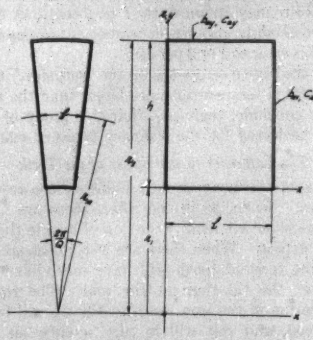


FIG. 14—SIMPLE CYLINDER PROBLEM

retained if the specific thermal resistance ρ_y is given a corrected value, determined by the equation

$$\rho_y^* = \frac{1}{\sqrt{1+\theta}}\,\rho_y \tag{23}$$

In this expression θ is to be given a positive sign for a rotor tooth and a negative sign for a stator tooth. This simple correction gives a remarkably close agreement with the accurate solution even when θ is greater than 0.3, although such cases are rare in actual practise. When $\theta = 0.3$, $\rho_y^* = 0.88\,\rho_y$ for the rotor, and $\rho_y^* = 1.2\,y$ for the stator.

To establish the influence of the taper upon the distribution of the iron losses, we assume that the same flux flows through each section of the tooth, and that the loss intensity varies with the nth power of the flux density. If the loss intensity at the mean flux density is denoted by p_{2m} it is easily seen that

$$p_2 = \frac{p_{2m}}{\left[1 - \theta\left(1 - \frac{2y}{h}\right)\right]^n} \tag{24}$$

From this expression, the mean loss intensity is found to be

$$\bar{p}_2 = k_n\,p_{2m} \tag{25}$$

where

$$k_n = -\frac{1}{2\,\theta\,(n-2)}\left[\frac{1}{(1+\theta)^{n-2}} - \frac{1}{(1-\theta)^{n-2}}\right] \tag{26}$$

For the present purpose it is more convenient to replace the function (24) by a parabola. It is found that a fair agreement is obtained by putting

$$p_2 = \frac{1 + 3\,k_2\left(\frac{y}{h_2}\right)^2}{1 + k_2} \tag{27}$$

where

$$k_2 = \frac{1}{3}\left[\left(\frac{1-\theta}{1+\theta}\right)^n - 1\right] \tag{28}$$

It should be noted that θ is to be entered in equations (26) and (28) with a negative sign. By these relations the iron losses have been divided into a constant part $\bar{p}_2/(1 + k_2)$ and a parabolically varying part $[\bar{p}_2/(1 + k_2)]\cdot[1 + 3\,k_2\,(y/h_2)^2]$. The reason for selecting this particular type of division is found in the next paragraph where the influence of irregular distribution of the iron losses is discussed.

For the comparatively high inductions which are the rule in modern turbo-generators the losses vary with the 3rd or 4th power of the induction. Thus, if $n = 4$ we obtain, for a tooth-taper of $\theta = -0{,}20$, $k_n = 1{,}09$ and $k_2 = 1.35$. The minimum and maximum loss intensities, when figured from equation (24), are $0.44\,\bar{p}_2$ and $2.24\,\bar{p}_2$ respectively. The more approximate equation (27) gives for the same quantities $0.43\,\bar{p}_2$ and $2.15\,\bar{p}_2$, a slightly less sharp variation of the loss intensity than the one assumed for $n = 4$. In view of the uncertainty of the exponent n, however, and the uncertainty of the assumption of constant flux, there is no justification for a more accurate value of the constant k_2. The distribution thus obtained will be qualitatively correct, and the general magnitude of the influence upon the mean temperature will be sufficiently exact.

The Influence of the Variation of the Intensity of the Losses upon the Mean Temperature

The influence upon the temperature of irregular distribution of the losses is intimately connected with

FIG. 15—TO ILLUSTRATE TAPER CONSTANT θ

the specific thermal resistances of the copper and the tooth material. Therefore, this problem demands for its solution a discussion of the entire system of Fig. 10 as a whole, with particular emphasis given to the interaction of the two plates upon one another. In order to bring out the essential features, the investigation may be restricted to a straight tooth, and in addition, $h_1 = h_2 = h$ and $l_1 = l_2 = l$. These simplifications will not influence the main points of the results.

The copper losses are assumed to be made up of two parts, the losses due to the d-c. resistance, and the eddy current losses. The latter vary nearly parabolically with y, and their magnitude is a fraction k_1 of the d-c. losses, so that

$$p_1 = \frac{\bar{p}_1}{1 + k_1}\left[1 + 3\,k_1\left(\frac{y}{h}\right)^2\right] \qquad (29)$$

The quantity $1 + k_1$ is the ordinary eddy factor. Similarly, as shown in the preceding paragraph, the iron losses can be put

$$p_2 = \frac{\bar{p}_2}{1 + k_2}\left[1 + 3\,k_2\left(\frac{y}{h}\right)^2\right] \qquad (30)$$

The specific thermal resistance of the iron in the teeth does not vary a great deal from machine to machine. In general we may put for 0.014-in. thick enameled sheets $\rho_{2x} = 35$ and $\rho_{2y} = 1.25$ deg. cent. inch per watt. The specific thermal resistances of the copper are always the same in the axial direction, but in the radial direction are greatly dependent on the insulation between strands. For a typical 13,000-volt machine we may put $\rho_{1x} = 0.11$ and $\rho_{1y} = 60$ deg. cent. inch per watt. Thus, the essential features of the problem will be covered if we put $\rho_{1x} = 0$ and consider the effect of varying ρ_{1y} between zero and infinity.

$$\rho_{x1} = 0;\ \rho_{1y} = 0$$

In this case the copper temperature will remain practically constant, and the temperature drop across the insulation will vary with the iron temperature. Since the variation in intensity of the copper losses will not influence the temperatures at all in this case, only the variation of the iron losses need be considered. Since the losses in the teeth are concentrated towards the tip it is obvious that the mean temperature will be lowered. The amount will depend upon the relative magnitude of ρ_{2y} and C_i.

If the quantity $(2\,\rho_{y2}\,h^2)/(3\,b_2\,C_i)$ is small (< 0.5) the corrected mean iron temperature may be shown to be

$$t_{2m}^* = t_{2m} - 0.4\,\Delta\,t_2 \qquad (31)$$

where

$$\Delta\,t_2 \leq \frac{\tau}{2\,h}\ \frac{2\,h^2\,\rho_{2y}}{3\,b_2}\ \frac{k_2}{1 + k_2}\,q_2 \qquad (32)$$

The decrease in the maximum iron temperature is found to be $0.75\ \Delta\,t_2$.

Thus the difference between the maximum and mean temperature will be changed by $(0.75 - 0.40)\ \Delta\,t_2 = 0.35\ \Delta\,t_2$. For a typical turbo-generator for 13,000 volts, the temperature $\Delta\,t_2$ is rarely above 5 deg. cent. Therefore, the correction in mean temperature of the iron is of the order of 2 deg. cent. If the quantity $(2\rho_{y2}\,h^2)/(3\,b_2 C_i)$ is large, the correction is a smaller part of $\Delta\,t_2$, and the temperatures must be computed by more complicated expressions.

It is worth noting that the direct influence of the tooth taper in obstructing the radial flow, and the indirect influence in redistributing the iron losses, compensate each other, so that the net effect for the stator is usually very small.

$$\rho_{x1} = 0;\ \rho_{y1} = \infty$$

This case is usually nearer to the actual conditions. The copper temperature will now remain constant in the axial direction, but it will vary in the radial direction, where it will tend to adjust itself to the iron temperature. The influence upon the mean temperature, beyond that due to the change in the mean iron temperature, is very small. The maximum copper temperature will be affected to a greater degree. If we put

$$\Delta\,t_1 = \frac{\tau}{2\,h}\,C_i\,\frac{k_1}{1 + k_1}\,q_1 \qquad (33)$$

it can be shown that the corrected maximum copper temperature may be put

$$t_1^*{}_{max} = t_{1m} + (t_{2\,max} - t_{2m}) - (0.35\,\Delta\,t_2 + \Delta\,t_1) \qquad (34)$$

Thus, the difference between maximum and minimum temperature of the copper will be the difference between the mean temperature of the iron at the root of the tooth and the mean iron temperature, less $(0.35\ \Delta\,t_2 + \Delta\,t_1)$. The former temperature difference may amount to 5 deg. cent. in a typical machine, the latter may be slightly larger. Thus, according to (34) the copper temperature may actually be lower at the bottom of the slot than at the top. The corrections in this equation are limiting values, however, which are usually not reached. We are justified, however, in concluding that the copper temperature in a typical turbo-generator is very nearly constant for the entire depth of the slot.

Conclusion

It is shown in the above that the conception of an equivalent thermal circuit is helpful in deriving approximate solutions for the complicated phenomena encountered in predetermining the flow of heat in large turbo-generators. The degree of complexity to which it is necessary to go will depend on the individual arrangement of the machines. In general, it is obvious that a simple circuit, emphasizing the essential features of the flow, is to be preferred to a more accurate, but more complicated circuit. It is believed that with this method the temperature can be predicted to within a few per cent of the iron temperature; the uncertainty of losses and thermal properties are usually so great that additional accuracy is chimerical.

The results obtained above have been applied to a number of turbine-generators of the general type of construction referred to. The agreement between measured and calculated temperatures is satisfactory in all cases, considering the unavoidable uncertainty in the thermal properties of the materials involved. The application of the results to actual machines will be made the subject of a separate paper in the future.

Appendix I

Two-Dimensional Problems

A.

This analysis applies to the system shown in Fig. 4 and has for its object a comparison of the approximate solution, given in the paper, and the exact solution. Considering the heat balance for an element $dx \,.\, dy \,.\, b$ of the plate, the following differential equation is easily obtained for the temperature t

$$\frac{1}{\rho_x}\frac{\partial^2 t}{\partial x^2} + \frac{1}{\rho_y}\frac{\partial^2 t}{\partial y^2} + p = 0 \qquad (35)$$

This partial differential equation is to be solved for the following boundary conditions

$$\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0 \quad \left(\frac{\partial t}{\partial x}\right)_{x=l} = -\frac{\rho_x b_{ax}}{b\, C_{ax}}\,(t)_{x=l}$$

$$\left(\frac{\partial t}{\partial y}\right)_{y=0} = 0 \quad \left(\frac{\partial t}{\partial y}\right)_{y=h} = -\frac{\rho_y b_{ay}}{b\, C_{ay}}\,(t)_{y=h} \qquad (36)$$

The loss intensity p is assumed to be constant throughout the volume.

It is obvious that the approximate solution becomes exact for two limiting conditions: a, when the problem is one-dimensional, and b, when the specific thermal resistances ρ_x and ρ_y are zero. It is easily seen that the essential aspects of the problem are embodied in the numerical constants

$$f_x = \frac{\rho_x b_{ax}\, l}{b\, C_{ax}} \text{ and } f_y = \frac{\rho_y b_{ay}\, h}{b\, C_{ay}} \qquad (37)$$

The approximate solution becomes exact if one or both of these constants are zero. One may be justified in concluding from this that the most unfavorable case for the approximate solution occurs when $f_x = f_y =$ the maximum value that may occur in practise.

The differential equation (35) may also be written

$$\frac{\partial^2 t}{\partial\left(x\sqrt{\frac{\rho_x}{\rho_y}}\right)^2} + \frac{\partial^2 t}{\partial y^2} + p\,\rho_y = 0 \qquad (38)$$

from which it is evident that the difference in specific thermal resistance may be obviated by changing the x-dimension of the plate in the ratio $\sqrt{\rho_x/\rho_y}$. Naturally, the cooling properties of the surface $y = h$ must be modified in a corresponding manner. This shows, however, that there is no loss of generality involved in putting $\rho_x = \rho_y = \rho$. Therefore, the following discussion will be restricted to the differential equation

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + p\,\rho = 0 \qquad (39)$$

with the boundary conditions

$$\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0 \quad \left(\frac{\partial t}{\partial x}\right)_{x=l} = -\frac{f_x}{l}\,(t)_{x=l}$$

$$\left(\frac{\partial t}{\partial y}\right)_{y=0} = 0 \quad \left(\frac{\partial t}{\partial y}\right)_{y=h} = -\frac{f_y}{h}\,(t)_{y=h} \qquad (40)$$

The classical problem solved by Fourier[4] corresponds to the case when $p = 0$. The solution

$$t = A \cos m\,x \cos n\,y \qquad (41)$$

satisfies the corresponding Laplaces' equation; it will also fit the boundary conditions if m and n are roots of the equations

$$m\,l \tan m\,l = f_x$$
$$n\,h \tan n\,h = f_y \qquad (42)$$

These equations have series of successive roots m_i ($i = 1, 2 \ldots$) and n_j ($j = 1, 2 \ldots$). The general solution for the case that $p = 0$ may thus be written

$$t = i \sum_1^\infty j \sum_1^\infty A_{ij} \cos m_i\,x \cos n_j\,y \qquad (43)$$

This solution may also be used for the case that $p \neq 0$. (Poissons' equation) by determining the constants A_{ij} appropriately.

Introducing (43) into (39) we obtain

$$p\,\rho = i \sum_1^\infty j \sum_1^\infty A_{ij}\,(m_i^2 + n_j^2) \cos m_i\,x \cos n_j\,y \qquad (44)$$

giving the relation which the constants A_{ij} must satisfy. Multiplying this expression by $\int_0^l \cos\, m_i\, x\, d\, x$ and $\int_0^l \cos\, n_j\, y\, d\, y$, integrating, and using the expressions (42), it is found that[5]

$$A_{ij} = p\,\rho\,\frac{M_i N_j}{m_i^2 + n_j^2} \qquad (45)$$

where

$$M_i = \frac{4 \sin m_i\, l}{2\, m_i\, l + \sin 2\, m_i\, l} \qquad N_j = \frac{4 \sin n_j\, h}{2\, n_j\, h + \sin 2\, n_j\, h} \qquad (46)$$

For the present purpose the maximum and mean temperatures are of greatest interest. These are found to be

$$t_{max} = i \sum_1^\infty j \sum_1^\infty A_{ij} \qquad (47)$$

$$t_m = i \sum_1^\infty j \sum_1^\infty A_{ij}\,\frac{\sin m_i\, l}{m_i\, l}\,\frac{\sin n_j\, h}{n_j\, h} \qquad (48)$$

This result will be referred to as the "exact solution."

4. "The Analytical Theory of Heat," Freeman's translation, page 311.

5. M. Roth's treatise, previously referred to, contains a useful diagram for determining the roots of the equations (42) and tables of the functions M_i and N_j.

The approximate solution given by (5) to (7) may now be written in such a form that a direct comparison with the exact solution is facilitated. The surface resistances C_x, C_y and C are found to be

$$C_x = \frac{2\rho_x l^2}{b}\left(\frac{1}{f_x}+\frac{1}{3}\right) \quad C_y = \frac{2\rho_y h^2}{b}\left(\frac{1}{f_y}+\frac{1}{3}\right) \quad (49)$$

$$C = \frac{C_x C_y}{C_x + C_y}$$

When $\rho_x = \rho_y = \rho$, the maximum and minimum temperatures become

$$t_{max} = p\rho \frac{l^2\left(\frac{1}{f_x}+\frac{1}{2}\right) \quad h^2\left(\frac{1}{f_y}+\frac{1}{2}\right)}{l^2\left(\frac{1}{f_x}+\frac{1}{3}\right)+h^2\left(\frac{1}{f_x}+\frac{1}{3}\right)} \quad (50)$$

$$t_m = p\rho \frac{l^2\left(\frac{1}{f_x}+\frac{1}{3}\right) \quad h^2\left(\frac{1}{f_x}+\frac{1}{3}\right)}{l^2\left(\frac{1}{f_x}+\frac{1}{3}\right)+h^2\left(\frac{1}{f_x}+\frac{1}{3}\right)} \quad (51)$$

This result will be referred to as the "first approximation."

A somewhat closer approximation may be obtained by coordinating the solution (7) with the differential equation (35). For example, $\partial^2 t/\partial y^2$ may be evaluated from (7) and introduced into (35) which then becomes an ordinary differential equation of the form

$$\frac{d^2 t}{d x^2} = -p\rho\left[M + N\left(\frac{x}{l}\right)^2\right] \quad (52)$$

where M and N are two numerical constants determined by

$$M = \frac{C_y - \frac{\rho l^2}{3b}}{C_x + C_y} \text{ and } N = \frac{\rho l^2}{b\,(C_x + C_y)} \quad (53)$$

This equation is to be solved for the boundary conditions

$$\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0 \quad \left(\frac{\partial t}{\partial x}\right)_{x=l} = -\frac{f_x}{l}\,(t)_{x=l} \quad (54)$$

The result is

$$t = p\rho\left[M\frac{l^2 - x^2}{3} + N\frac{l^4 - x^4}{12\,l^2} + \frac{1}{f_x}\left(M + \frac{N}{3}\right)\right] \quad (55)$$

This gives for the corrected maximum and mean temperatures

$$t_{max}^* = t_{max}\left[1 - \frac{1}{6}\,\frac{\rho^2 l^4}{b^2 C_x C_y}\right]$$

$$t_m^* = t_m\left[1 - \frac{4}{45}\,\frac{\rho^2 l^4}{b^2 C_x C_y}\right] \quad (56)$$

where t_{max} and t_m are the values obtained from the solution given by (7). An alternative correction is obtained by starting out from the approximate value for $\partial^2 t/\partial x^2$. The result will differ from the above only to the extent that l will be replaced by h. The alternative corrections will be alike, therefore, when $l = h$. It is obvious on the other hand, that the approximate solution (7) is most inaccurate when $l = h$, because when $h = 0$ or $l = 0$, it becomes exact. This circumstance suggests as a reasonable correction the geometrical mean between the above and that which is obtained by solving a differential equation in y. This gives

$$\left[t_{max}^* = t_{max}\left[1 - \frac{1}{6}\,\frac{\rho_x l^2}{b\,C_y}\cdot\frac{\rho_y h^2}{b\,C_y}\right]\right.$$

$$\left. = t_{max}\left[1 - \frac{3}{8}\,\frac{f_x f_y}{(f_x+3)\,(f_y+3)}\right]\right]$$

$$\left[t_m^* = t_m\left[1 - \frac{4}{45}\,\frac{\rho_x l^2}{b\,C_x}\cdot\frac{\rho_y h^2}{b\,C_y}\right]\right.$$

$$\left. = t_m\left[1 - \frac{1}{5}\,\frac{f_x f_y}{(f_x+3)\,(f_y+3)}\right]\right] \quad (57)$$

This result will be referred to as the "second approximation."

Table I gives a comparison of these three solutions for the case that $p\rho l^2 = 1$, $h = l$, and the values 0.5, 1.5, 3, 5 and ∞ of the constants $f_x = f_y$.

In the problems encountered in turbo-generators the constants f_x and f_y rarely exceed 3 and 2, respectively. Thus, the first approximation gives an error in the mean temperature which does not exceed 4 per cent, and an error in maximum temperature which does not exceed 7.5 per cent. Usually the errors are much smaller, however. The second approximation will give both maximum and mean temperatures within one per cent.

B.

The two-dimensional problem, which is discussed in the preceding paragraphs, is a special case of the general tooth problem, Fig. 4, when the slot insulation is completely insulating ($C_i = \infty$). We now consider the case when the insulation of the sides is not complete, but restrict the discussion to the case when the external temperature of this insulation, the copper temperature t_{1m}, is constant. The differential equation is now found to be

$$\frac{1}{\rho_x}\,\frac{\partial^2 t}{\partial x^2} + \frac{1}{\rho_y}\,\frac{\partial^2 t}{\partial y^2} - \frac{2}{b\,C_i}\,(t - t_{1m}) + p = 0 \quad (58)$$

The boundary conditions are to remain the same as

TABLE I

	Maximum temp.						Mean temp.				
		1st approx.		2nd approx.				1st approx.		2nd approx.	
$f_x = f_y$	Exact t_{max}	t_{max}	Error per cent	t_{max}^*	Error per cent	Exact t_m	t_m	Error per cent	t_m^*	Error per cent
0.5	1.327	1.339	0.9	1.329	+ 0.2	1.161	1.167	0.5	1.162	+0.1
1.5	0.652	0.681	4.5	0.653	+ 0.2	0.490	0.500	2.0	0.489	−0.2
3	0.479	0.521	8.8	0.472	− 1.5	0.319	0.333	4.4	0.317	−0.6
5	0.408	0.459	12.5	0.392	− 3.9	0.249	0.267	7.2	0.246	−1.2
∞	0.296	0.375	26.7	0.234	−21.0	0.140	0.167	19.3	0.133	−5.0

for the simpler problem, (**40**). By putting

$$t_{1m}' = t_{1m} + \frac{p\,b}{2}\,C_i \tag{59}$$

and

$$\alpha = \sqrt{\frac{2\,\rho_y}{b\,C_i}} \tag{60}$$

the equation may be written

$$\frac{\partial^2 t}{\partial\left(x\sqrt{\frac{\rho_x}{\rho_y}}\right)^2} + \frac{\partial^2 t}{\partial_y^2} - \alpha^2\,(t - t_{1m}') = 0 \tag{61}$$

showing that: I. The loss intensity of the plate p may be taken into account by adding $(p\,b/2)\cdot C_i$ to the temperature t_{1m}. The addition is clearly the mean drop through the insulation due to the losses p. 2. The specific thermal resistances may be made alike by stretching the x-dimension in the ratio $\sqrt{\rho_x/\rho_y}$. There will be no loss of generality, therefore, in making $\rho_x = \rho_y$, so that the following discussion will be applied to the differential equation

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} - \alpha^2\,(t - t_{1m}') = 0 \tag{62}$$

The exact solution will again be written in the form

$$t = i\sum_1^\infty j\sum_1^\infty A_{ij}\cos m_i\,x\cos n_j\,y \tag{63}$$

where m_i and n_j are the series of roots of the equations (**42**). It is easily found by a reasoning almost identical with that used above that

$$A_{ij} = t_m'\,\frac{\alpha^2}{m_i^2 + n_j^2 + \alpha^2}\,M_i\,N_j \tag{64}$$

Where M_i and N_j are determined from (**46**). The exact solution thus gives the temperature as a function of the temperature t_{1m}'. When $C_i = \infty$ this solution becomes identical with that for the simple problem.

Another solution which is not exact, but which is very nearly exact for most cases, is obtained in the following manner. The differential equation (**62**) is easily shown to be satisfied by the solution.

$$t = t_{1m}' + A\cosh m\,\alpha\,x\cosh n\,\alpha\,y \tag{65}$$

where m and n ω now are two numerical constants satisfying the relation

$$m^2 + n^2 = 1 \tag{66}$$

This solution also satisfies the boundary conditions $(\partial\,t/\partial\,x)_{x=0} = 0$ and $(\partial\,t/\partial\,y)_{y=0} = 0$. The remaining boundary conditions $(\partial\,t/\partial\,x)_{x=l} = (-f_x/l)\,.\,(t)_{x=l}$ and $(\partial\,t/\partial\,y)_{y=h} = (-f_y/h)\,.\,(t)_{y=h}$ can not be fully satisfied, however, unless $t_{1m}' = 0$, which is a trivial solution. But it is possible to let the *mean* temperature at the cooling surfaces satisfy the boundary conditions. This should give fairly exact results, at least as far as the mean temperature is concerned. Mathematically, this means that the boundary conditions are written in the form.

$$\int_0^l\left(\frac{\partial\,t}{\partial\,y}\right)_{y=h}d\,x = -\int_0^l\frac{f_y}{h}\,(t)_{y=h}\,d\,x$$

and (**67**)

$$\int_0^h\left(\frac{\partial\,t}{\partial\,x}\right)_{x=l}d\,y = -\int_0^h\frac{f_x}{l}\,(t)_{x=l}\,d\,y$$

The integrated equations will not contain x or y, and will give the two equations required for determining m (or n) and A. Carrying out the integration, and writing for the arguments

$$X = m\,\alpha\,l \text{ and } Y = n\,\alpha\,h \tag{68}$$

it is found that

$$A = -\,t_m'\,\frac{1}{\sinh X\sinh Y}\,\frac{X\,Y}{F} \tag{69}$$

where F is the function

$$F = \frac{X^2}{f_x} + \frac{X}{\tanh X} = \frac{Y^2}{f_y} + \frac{Y}{\tanh Y} \tag{70}$$

The flow of heat will thus arrange itself in such a manner that the quantity F obtains the same value whether evaluated from X or from Y. Equations (**66**) and (**70**) may thus be used for determining the quantities m and n.

The curves of Fig. 16 may be used for determining F. These curves give the value of the quantity F as a function of z ($= X$ or Y) and the parameter f ($= f_x$ or f_y). m and n are first given values satisfying the equation (**66**). The corresponding values of X and Y give two

values of F; if these are alike the assumed values of m and n are correct; if not the procedure is repeated with a new combination. Assume that this process has been carried out and write the solution

$$t = t_m' \left(1 - \frac{X\,Y}{F} \frac{\cosh X \dfrac{x}{l} \cosh Y \dfrac{y}{h}}{\sinh X \sinh Y} \right) \tag{71}$$

The mean temperature is given by a particularly simple expression

$$t_m = t_m' \frac{F-1}{F} \tag{72}$$

Thus, to evaluate the mean temperature, it is sufficient to know the value of the quantity F. This solution will

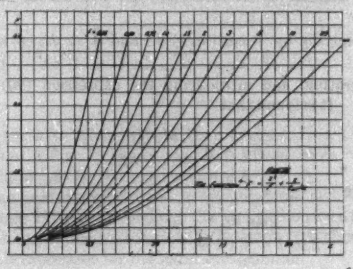


FIG. 16

be referred to as the "semi-exact" solution.

When the arguments X and Y are small the function $z/\tanh z$ may be expanded into a series

$$\frac{z}{\tanh z} = 1 + \frac{z^2}{3} - \frac{z^4}{45} + \ldots \tag{73}$$

If terms of higher order of z than the second are neglected, it is seen that

$$F - 1 = \left[\frac{1}{f_x} + \frac{1}{3} \right] X^2 = \left[\frac{1}{f_y} + \frac{1}{3} \right] Y^2 \tag{74}$$

Now, introducing the values of f_x, f_y, it is found that

$$m^2 = \frac{C}{C_x} \tag{75}$$

$$n^2 = \frac{C}{C_y} \tag{76}$$

and

$$F - 1 = \frac{C}{C_i} \tag{77}$$

The mean temperature now becomes

$$t_m = t_m' \frac{C}{C + C_i} \tag{78}$$

This value is identical with that obtained by the approximate solution (11), and which has been referred to as the first approximation.

The semi-exact solution thus deteriorates into the ordinary approximate solution when C_i is very large. For this reason it is an improvement upon the approximate solution only when the quantities X and Y are large.

Table II has been calculated to illustrate this condition. This table gives the maximum and minimum temperature of the iron when $t_m' = 1$, $l = h$, $f_x = f_y = 3$, and for the value 0.5, 1.0, 2.0 and 3.0 for the argument $\alpha\, l = \alpha\, h$. For this symmetrical case it is evident from (66) that $m = n = 1/\sqrt{2}$. $f_x = f_y = 3$ may be regarded as a typical case for the stator; the value of $\alpha\, l = \alpha\, h$ does not usually exceed unity.

If the third term of the series (73) is retained it is found that the approximate solution for the mean temperature is still correct in form when the following corrected values for the quantities C_x and C_y are used

$$C_x^* = C_x \left[1 - \frac{1}{5} \frac{C}{C_i} \left(\frac{2\,\rho_x\, l^2}{3\, b\, C_x} \right)^2 \right];$$

$$C_y^* = C_y \left[1 - \frac{1}{5} \frac{C}{C_i} \left(\frac{2\,\rho_y\, h^2}{3\, b\, C_y} \right)^2 \right] \tag{79}$$

where C_x, C_y, and C are the ordinary surface resistances given by equations (5). This result also shows that when C_i is large compared with C the semi-exact solution becomes the ordinary approximate solution. It is an interesting fact that this correction is of the same form as that obtained for the simple two-dimensional problem (57).

Appendix II

CYLINDER PROBLEMS

A.

Consider a two-dimensional problem of the same type as the one given in Fig. 4, but with varying thickness of the plate (Fig. 14). As before, it is assumed that

TABLE II

	Maximum temp.						Mean temp.				
		Semi-exact		1st approx.				Semi-exact		1st approx.	
$\alpha\, l = \alpha\, h$	Exact t_{max}	t_{max}	Error per cent	t_{max}	Error per cent	Exact t_m	t_m	Error per cent	t_m	Error per cent
0.5	0.110	0.111	0.9	0.120	9.1	00.73	0.077	5.5	0.077	5.5
1.0	0.347	0.360	3.8	0.391	12.7	02.37	0.246	3.8	0.250	5.5
2.0	0.744	0.763	2.6	0.893	20.0	05.38	0.558	3.7	0.572	6.3
3.0	0.917	0.927	1.1	1.171	27.7	07.10	0.728	2.5	0.750	5.6

the edges $x = l$ and $r = r_1$, and the two sides, are insulated. The ordinary cylinder problem with distributed losses is then obtained. The differential equation is now found to be, when $\rho_x = \rho_y = \rho$.

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \rho = 0 \ldots \quad (80)$$

The difficulties in obtaining an exact solution of this problem are connected with the variation of p with r, and with the boundary conditions at the surface $x = l$. As mentioned before, the case when p varies inversely as r is particularly interesting but, unfortunately, a simple exact solution can be obtained only for the case when p is constant. The "cooled width," b_{ax}, varies inversely with the radius on stator problems, but on rotor problems it is usually constant. An exact solution is possible only in the former case. Therefore, the exact solution will be derived for the case where p is constant and b_{ax} varies inversely with the radius. The boundary conditions will then be

$$\left.\begin{aligned} &\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0 \qquad \left(\frac{\partial t}{\partial r}\right)_{r=r_1} = 0 \\ &\left(\frac{\partial t}{\partial x}\right)_{x=l} = -\frac{f_x}{l}(t)_{x=l} \qquad \left(\frac{\partial t}{\partial r}\right)_{r=r_2} = -\frac{f_r}{h}(t)_{r=r_2} \end{aligned}\right\} \quad (81)$$

where $$f_x = \frac{\rho\, b_{ax}\, l}{b\, C_{ax}}$$

and $$f_r = \frac{r_m}{r_2}\,\frac{\rho\, b_{ay}\, h}{b\, C_{ay}} \quad (82)$$

The solution will contain products of the Bessel's functions $J_p\,(n\,r)$ and $K_p\,(n\,r)$[6] It will be found convenient to introduce the special functions

$$R_0\,(n\,r) = (n\,r_1)\begin{vmatrix} J_0\,(n\,r), & K_0\,(n\,r) \\ J_1\,(n\,r_1), & K_1\,(n\,r_1) \end{vmatrix}$$

and $$R_1\,(n\,r) = n\,r_1)\begin{vmatrix} J_1\,(n\,r), & K_1\,(n\,r) \\ J_1\,(n\,r_1), & K_1\,(n\,r_1) \end{vmatrix} \quad (83)$$

When $r = r_1$ it is found that $R_0 = 1$ and $R_1 = 0$, a property which these functions have in common with $\cos n\,(r - r_1)$ and $\sin n\,(r - r_1)$, respectively, and whose values they approach when r is very large compared with $r_2 - r_1 = h$.

The expression

$$t = A \cos m\,x\, R_0\,(n\,r) \quad (84)$$

satisfies the equation (80) when $p = 0$; the boundary conditions (82) will also be satisfied if m and n are roots of the equations

$$m\,l \tan m\,l = f_x \quad (85)$$

$$n\,h\,\frac{R_1(n\,r_2)}{R_0\,(n\,r_2)} = f_r \quad (86)$$

These equations have series of roots m_i ($i = 1, 2 \ldots$), and n_j ($j = 1, 2 \ldots$), so that the solution can be written as a series

$$t = i \sum_1^\infty j \sum_1^\infty A_{ij} \cos\,(m_i\,x)\, R_0\,(n_j\,r) \quad (87)$$

Introducing this series into (80), the following relation is obtained

$$i \sum_1^\infty j \sum_1^\infty A_{ij}\,(m_1^2 + n_j^2) \cos\,(m_i\,x)\, R_0\,(n_j\,r) = p \quad (88)$$

This enables us to determine the coefficients A_{ij} by multiplying by $\int_0^l \cos\,(m_i\,x)\,d\,x$ and $\int_{r_1}^{r_2} r.\ R_0\,(n_j\,r)\,d\,r$, integrating, and introducing the expression (90) and (91). It is found that

$$A_{ij} = p\,\frac{M_i\,N_j}{m_1^2 + n_j^2} \quad (89)$$

where

$$M_i = \frac{4 \sin m_i}{2\,m_i\,l + \sin 2\,m_i\,l} \quad (90)$$

$$N_j = \frac{2}{n_j\,r_2}\,\frac{R_0\,(n_j\,r_2)}{R_0^2\,(n_j\,r_2) + R_1^2\,(n_j\,r_2) - \dfrac{r_1^2}{r_2^2}} \quad (91)$$

Fig. 14, with which the above solution is associated, represents the conditions in a rotor tooth, or a stator core, where the heat flows towards the wider section. It is evident, however, that the solution will remain valid even for a stator tooth, where the flow is in the opposite direction. The notations of Fig. 14 may still be used if $r_2 < r_1$. Put in general

$$r = r_m - \frac{h}{2} + y = h\,\frac{1 - \theta\left(1 - \dfrac{2\,y}{h}\right)}{2\,\theta} \quad (92)$$

where

$$\theta = \frac{h}{2\,r_m} = \frac{\pi\,h}{b\,Q} \quad (93)$$

is the *taper constant*. The distinction between a rotor tooth and a stator tooth is that θ is positive for the former, and negative for the latter. The functions $R_0\,(n\,r)$ and $R_1\,(n\,r)$ may now be considered as functions of $n\,y$ and θ instead. The values of r_1 and r_2 in terms of θ and h are

$$r_1 = h\,\frac{1 - \theta}{2\,\theta} \quad \text{and} \quad r_2 = h\,\frac{1 + \theta}{2\,\theta} \quad (94)$$

6. This notation is used by Jahnke and Emde in "Funktionentafeln," from which the numerical values have been taken.

Fig. 17 shows the function $R_0\left(n\,h\,\frac{1+\theta}{2\,\theta}\right)$ for different values of $n\,h$ and θ. Fig. 18 shows the corresponding values of $R_1\left(n\,h\,\frac{1+\theta}{2\,\theta}\right)$. It should be noted that when $\theta = 0$ the two functions become $\cos n\,h$

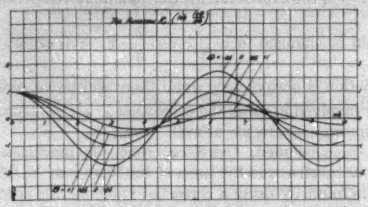


Fig. 17

and $\sin n\,h$; when $\theta = 1$ they become $J_0(n\,h)$ and $J_1(n\,h)$. Fig. 19 shows the three lowest roots of the equation (**86**) for the same values of θ.

The maximum and mean temperatures are of the greatest interest in making the comparisons with the more approximate solutions. In order to obtain a simple expression for the latter the definition

$$t_m = \frac{\int_0^l \int_{r_1}^{r_2} t\,.\,r\,.\,d\,x\,.\,d\,r}{\int_0^l \int_{r_1}^{r_2} r\,.\,d\,x\,.\,d\,r} \tag{95}$$

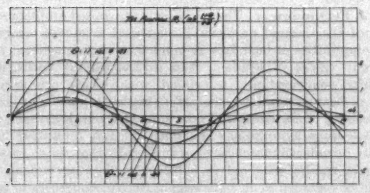


Fig. 18

will be used. It is found that

$$t_{max} = i \sum_1^\infty j \sum_1^\infty A_{ij} \tag{96}$$

$$t_m = i \sum_1^\infty j \sum_1^\infty A_{ij} \frac{\sin m_i\,l}{m_i\,l} \frac{R_1\,(n_j\,r_2)}{\frac{r_m}{r_2}\,.\,n_j\,h} \tag{97}$$

In establishing the approximate solution for the tapered tooth it is assumed that the radial and axial flow take place independently of each other, so that only the quantities entering in the radial flow will be corrected for the taper. Referring again to Fig. 14 assume that the edge $x = l$ is insulated and determine the solution for the corresponding one-dimensional problem. It is easily found that

$$t = p\,\rho\left[\frac{r_2^2 - r^2}{4} + \frac{r_1^2}{2}\log\frac{r_1}{r_2}\right] + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{98}$$

whence

$$t_{max} = p\,\rho\left[\frac{r_2^2 - r_1^2}{4} + \frac{r_1^2}{2}\log\frac{r_1}{r_2}\right] + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{99}$$

and

$$t_m = \frac{\int_{r_1}^{r_2} t\,r\,.\,d\,r}{\int_{r}^{r_2} r\,d\,r} = p\,\rho\left[\frac{r_2^2 - 3\,r_1^2}{8} + \frac{r_1^4}{2\,(r_2^2 - r_1^2)}\log\frac{r_2}{r_1}\right] + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{100}$$

Now replace r_1 and r_2 by their expressions in h and θ (**94**).

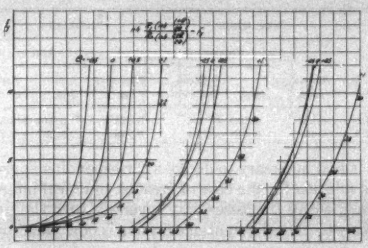


Fig. 19—The Three Lowest Roots to the Equation

Assuming that θ is small ($\theta < 0.3$) we expand the logarithmical terms in series and neglect higher powers of θ than the second. Then

$$t_{max} = \frac{p\,\rho\,h^2}{2}\left(1 - \frac{2}{3}\,\theta\right) + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{101}$$

$$t_m = \frac{p\,\rho\,h^2}{3}\,(1 - \theta) + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{102}$$

The approximation will be somewhat closer for comparatively large values of θ if $1 - \frac{2}{3}\,\theta$ is replaced by $1/(1 + \frac{2}{3}\,\theta)$ and $1\text{-}\theta$ by $1/(1 + \theta)$. The result then

becomes

$$t_{max} = \frac{p\,b}{2}\left[\frac{2\,hC_{av}}{b_{av}} + \frac{\rho\,h^2}{b}\,\frac{1}{1+\frac{2}{3}\,\theta}\right] \qquad (103)$$

$$t_m = \frac{p\,b}{2}\left[\frac{2\,h\,C_{av}}{b_{av}} + \frac{2\rho h^2}{3\,b}\,\frac{1}{1+\theta}\right] \qquad (104)$$

This shows that the simple solution for a straight tooth may be applied to the tapered tooth if the specific thermal resistance in the radial direction is multiplied by a factor, the value of which depends upon the taper constant. Table III gives a comparison of the solution thus obtained with the exact solution for an arrangement where $f_z = 3, f_r = \frac{r_m}{r_2}\,3 = \frac{3}{1+\theta}, p\,\rho\,h^2 = 1$, $h = l$, and the values -0.5 and $+0.5$ for θ. It is obvious from this comparison that even for these extreme conditions the above method of correcting the specific thermal resistance gives results which are sufficiently accurate for practical purposes. The correction for tooth taper eliminates a part of the error due to the inaccuracy of the solution for the straight tooth, so that the errors are actually smaller than those which are obtained when $\theta = 0$.

TABLE III

	Max. temp.			Mean temp.		
		Approx.			Approx.	
θ	Exact t_{max}	t_{max}	Error per cent	Exact t_m	t_m	Error per cent
−0.5	0.581	0.590	+ 1.5	0.421	0.444	+ 5.5
+0.5	0.402	0.448	+ 11.4	0.248	0.267	+ 7.7

It is now necessary to derive a corresponding approximate solution for the case of greatest practical interest, namely, where the losses vary inversely with the tooth thickness. The differential equation is now found to be

$$\frac{1}{\rho_z}\,\frac{\partial^2 t}{\partial x^2} + \frac{1}{\rho_y}\,\frac{\partial^2 t}{\partial r^2} + \frac{1}{\rho_y}\,\frac{1}{r}\,\frac{\partial t}{\partial r} + \frac{r_m}{r}\,p = 0 \qquad (105)$$

The boundary conditions are to be identical with those given in (82). Assuming that the flow is entirely radial, so that $\partial^2 t/\partial x^2 = 0$, this equation yields the solution

$$t = p\,\rho_y\left[r_m\,h + r_1\,r_m \log\frac{r}{r_2}\right] + p\,\frac{b\,h\,C_{av}}{b_{av}} \qquad (106)$$

Replacing r_1 and r_m by their expressions in h and θ and neglecting powers above the second in θ,

$$t_{max} = \frac{p\,b}{2}\left[\frac{2\,h\,C_{av}}{b_{av}} + \frac{h^2\,\rho_y}{b}\left(1 - \frac{\theta}{3}\right)\right] \qquad (107)$$

In evaluating the mean temperature the expression $t_m = \frac{1}{h}\int_{r_2}^{r_1} t\,dr$ should be used rather than (95), which was introduced in order to simplify the exact solution. Then

$$t_m = \frac{p\,b}{2}\left[\frac{2\,h\,C_{av}}{b_{av}} + \frac{h^2\,\rho_y}{b}\left(1 - \frac{\theta}{2}\right)\right] \qquad (108)$$

In this case it is found that the accuracy for comparatively large values of θ is improved if the factor $1 - \theta/2$ is replaced by $1/\sqrt{1+\theta}$. The correction factor $1 - \theta/3$ in the expression for the maximum temperature should be changed to $1/\sqrt{1 + \frac{2}{3}\,\theta}$, but since the correction is always very small there will be no serious error involved in using the correction $1/\sqrt{1+\theta}$ for the maximum temperature as well.

It is concluded, therefore, that for machines of normal construction the taper of the teeth may be taken into account by using a corrected value for the specific thermal resistance in the radial direction. The value of this corrected resistance is

$$\rho_y^* = \rho_y\,\frac{1}{\sqrt{1+\theta}} \qquad (109)$$

B.

The above problem is only a special case of the general problem of the tooth region. This problem may also be solved by a semi-exact method similar to that developed in Appendix I-B, where cylinder function of the type (83) would appear. The results are too complicated to be of practical use however. Besides, the influence of ordinary tooth taper is so small that the results obtained in the preceding section are sufficiently accurate for all practical purposes.

Discussion

C. J. Fechheimer: Those who have attempted to obtain solutions of the thermal problems in electrical machines recognize the complexities of heat flow in more than one dimension, with different values of conductivities in the various paths, with irregular configurations, associated with which are varying rates of heat generation, etc. Most engineers have considered the problem unmanageable, if not confined to one dimension, and the necessary assumptions imposed by that limitation have usually introduced inaccuracies of such order that the calculated and test results were not in sufficiently close agreement. It should be regarded as an achievement for Mr. Soderberg to have succeeded in placing before us a solution of this intricate problem. By modifying his boundary conditions, he has simplified his solution, and then has put it into workable form by substituting equivalent electrical circuits for the heat circuits.

It is of interest to obtain a picture of the equations for the equivalent electrical circuit from another standpoint. The two equations which Mr. Soderberg has given the number 15 are:

$$t_1\,m = C_{11}\,q_1 + C_{12}\,q_2 + C_{13}\,q_3$$
$$t_2\,m = C_{21}\,q_1 + C_{22}\,q_2 + C_{23}\,q_3$$

$t_1\,m$ is the copper temperature; $t_2\,m$ the tooth temperature; q_1, q_2 and q_3 are respectively the loss intensities in the copper, teeth and surface. C_{11}, C_{12}, etc. are "influence factors." That is, C_{11} is the coefficient by which the copper loss intensity is multiplied to give the temperature rise of the copper due to the loss

therein; C_{12} is the coefficient by which the tooth loss intensity is multiplied to give the temperature rise of the copper due to the tooth loss. Thus, C_{12} shows the influence of iron loss upon copper temperature, etc. In the second equation, the copper loss intensity q_1 is multiplied by C_{21} to give the temperature rise of the iron due to the loss in the copper. It is of interest to note that $C_{12} = C_{21}$ (equation 19).

These equations will be recognized as embodying the principles of superposition and of reciprocity. According to the first, the temperature at any part of a system due to one source of heat, acting alone, may be added to the temperature in the same part due to another source of heat also acting alone, and the sum is the temperature at that point if both sources supplied heat simultaneously. The fact that $C_{12} = C_{21}$, involving the reciprocity principle, shows that the temperature rise in the first due to a given loss in the second equals the temperature rise in the second due to the same loss in the first. These are well-recognized principles.

Use of these principles may also be made in testing electrical machinery, particularly when the machine is too large to test at full load. Thus, the usual temperature measurements may be made, say on the armature, with the machine operating without load at normal voltage on open circuit, and a second run made at short circuit with normal current. The full load temperature rise is then the sum of the open- and short-circuit temperature rises. For example, if detectors are placed between coils in the slot, on open circuit they read (nearly) the temperature of the copper due to iron loss, and on short circuit the temperature of the copper due to the copper loss. The full load temperature rise is the sum of these two rises.

It is evident that the influence factors, C_{11}, C_{12}, etc. may be obtained from test. If run on open circuit, $q_1 = 0$, and if we neglect, q_2, (the surface losses)

$$C_{12}' = \frac{t_1 m}{q_2}.$$

The detector is then used to obtain the copper temperature rise, $t_1 m$. Also from the open-circuit test,

$$C_{22} = \frac{t_2 m}{q_2},$$

some suitable means being employed to measure the mean tooth temperature rise, $t_2 m$. On short circuit, $q_2 = 0$, and

$$C_{11} = \frac{t_1 m}{q_1};$$

also

$$C_{21} = \frac{t_2 m}{q_1}.$$

The value of C_{21} thus obtained may then be compared with C_{12} from the open-circuit test.

The method was suggested by the writer in an article in the *Sibley Journal* in December, 1912, and has since been proposed by others.*

K. K. Paluev: It might be of interest to the Institute that a short-circuit table such as is used for the analysis of short-circuit characteristics of transmission systems can be very beneficially employed for solving some of the thermal problems similar to those mentioned by Mr. Soderberg. I had occasion to make use of such a table several years ago when it was necessary to determine temperature distribution between a number of bodies (see 1-2-3, etc. of Fig. 1), each generating a different amount of heat. It was necessary to solve this problem for a considerable number of numerical examples where the number of the bodies in the structure and the ratio A/B were varying through a wide range. It was found that an analytical solution was too complicated and laborious. In its place the following method was used:

Thermal resistance from each of the bodies to the external equi-potential surface S was determined on the assumption that the dotted vertical lines in Fig. 1 served as boundaries made up of perfect heat insulators. Then the thermal resistance between each adjacent pair of the bodies was determined, in which case the boundaries indicated by vertical lines were omitted. This permitted the formation of a network of resistances as shown in Fig. 1. This network was set up on the short-circuit table, then voltage was applied between S and each of the points, 1, 2, 3, etc. in turn. In each case the total current at the point of application (1, 2, 3, etc.) was made proportional to the heat dissipated by the respective body. The currents found in each branch (R_1, R_2, etc.) for all n readings were added together, giving quantities proportional to the total amount of heat flowing from each body directly to the equi-potential surface S. The product of this current and the corresponding resistance (R_1, R_2, etc.) gave quantities proportional to the temperature differences between the respective bodies and the surface S.

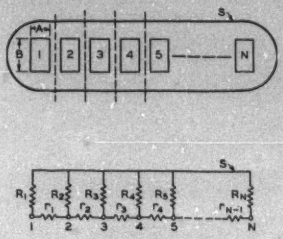


FIG. 1

D. S. Snell: The complexity of the solution given in this paper for the problem of stationary heat flow in a single tooth segment of a turbine-generator well illustrates the complex nature of the problem of temperature calculation in electrical machinery, and much credit is due the author for his unique and thorough treatment of this difficult problem. As pointed out in the paper, however, the correctness of any solution of this problem is, in the last analysis, wholly dependent upon the accuracy of the values of the constants used, and, it must be conceded, our present knowledge of several of these constants,—for example, the heat transfer coefficients for the air gap surfaces of the rotor and stator, is somewhat incomplete. Hence an approximate solution may, in actual practise, give as good results as a more involved one.

The assumption made by the author that the plane of division of radial heat flow in the tooth region is at the bottom of the slot, while probably justifiable for the types of turbine-generators considered, *i. e.*, with radial ventilating ducts in both stator and rotor, may give temperatures very much higher than the actual for turbine-generator rotors of the solid forged type, having ventilating channels below the coil slots. In Fig. 2 is shown the temperature distribution in a slot and tooth of a rotor of the latter type, as calculated by a method given by Dr. H. F. Shait in *Archiv fuer Elektrotechnik*, v., 22, p. 402, which shows the plane of division of radial heat flow to occur nearly half way up the tooth.

*"The Temperature Rise on Electrical Machines," by Herbert Jehle, *E. T. Z.*, Aug. 14, 1930.

"Temperature Rise of Electrical Machine and Comparison between Single and Double Rating," by Jutaro Takedchi, *Journal of the Institute of Elec. Eng. of Japan*, Aug., 1926.

An application of the method just mentioned shows that in the rotor of a large turbine-generator a very small part of the total loss passes directly to the air gap or ventilating channel, hence negligible error would be introduced in calculating the rotor temperature if all of the loss were assumed to pass first to the tooth and thence to the gap and channel. By the use of this assumption and the further one that the loss is uniformly distributed in the tooth, an approximate method of calculating temperatures in the solid forged type of rotor has been developed which is illustrated in Fig. 3. In this method the position of the plane of division of radial heat flow (designated as the "neutral" plane) is first assumed and the various thermal resistances are calculated. The tooth temperatures on either side of this neutral plane are then determined, and if these are found to be equal, the position of the plane as assumed may be considered correct. If these temperatures are not in agreement, a new position of the neutral plane is assumed and the calculation repeated. Thus by successive approximations the neutral plane is finally located, and the temperature rise of the winding determined.

The equations from which the rotor temperature may be calculated by the above approximate method are given below, and an application of this method to three different turbine-generators is given in Table I.

TABLE I

Kva. rating r. p. m.	67,000 1,500	43,750 1,800	62,500 1,800
f	0.655	0.565	0.615
T_g	9.6	20.	18.5
T_p	7.1	6.6	5.6
R_m	2.23	2.35	2.57
R_s	1.66	1.96	1.96
R_t	0.37	0.34	0.34
R_n	1.98	3.2	2.91
R_r	0.43	0.52	0.51
R_f	12.12	10.41	11.6
R_{im}	6.55	7.76	7.15
R_{in}	12.3	10.2	11.37
T_1	63.6	78.1	82.1
T_2	65.9	79.7	82.8
T_1'	36.0	47.2	49.5
T_2'	36.6	47.0	49.0
T_0 (calc.)	64.4	78.9	82.4
T_0 (test)	63.5	79.0	80.0

Temperature Equations

Let:

f = fraction of winding above neutral plane
N_1 = no. of field turns per slot
d = section area of conductor, sq. in.
I = current per conductor, amperes
u = resistance of a copper bar, 1 in. by 1 in. by 12 in., at temp. of ingoing air
k = temperature coefficient of copper at temp. of ingoing air
T_1 = mean temp. rise of winding, for section above neut. plane
T_2 = mean temp. rise of winding, for section below neut. plane
T_0 = mean temp. rise of winding, for entire slot
T_m = mean temp. rise of tooth, for section above neut. plane
T_n = mean temp. rise of tooth, for section below neut. plane
T_1' = tooth temp. rise at neutral plane, air gap side
T_2' = tooth temp. rise at neutral plane, channel gap side
T_g = mean temp. rise of air in air gap
T_p = mean temp. rise of rotor surface from pole-face losses
(All temperatures in deg. cent.)

Also, let:

$$Q = \frac{N_1 I^2 u}{24\, d};\ D = f\,Q;\ E = k\,f\,Q;\ F = (1-f)\,Q;\ G = (1-f)\,k\,Q$$

$$H = \frac{R_m}{3};\ J = \frac{R_n}{3};\ R_g = R_m + R_t + R_s;\ R_c = R_n + R_r + R_f$$

$$M = R_g + R_{im} - H;\ N = R_c + R_{in} - J$$

Then:

$$T_1 = \frac{T_g + T_p + D\,M}{1 - E\,M};\ T_2 = \frac{F\,N}{1 - G\,N};\ T_0 = f\,T_1 + (1-f)\,T_2$$

And

$$T_1' = T_g + T_p + R_g\,(D + E\,T_1);\ T_2' = R_c\,(F + G\,T_2)$$

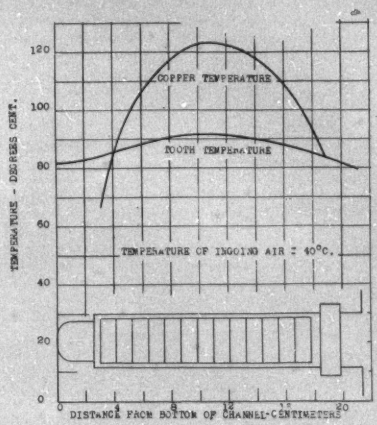


Fig. 2—Calculated Temperature Distribution in Rotor Slot and Tooth for a Four-Pole 90,000-Kva. R. P. M. Turbine-Generator at Full Load

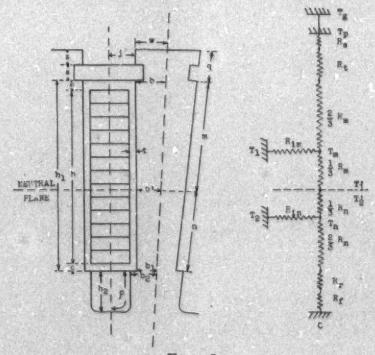


Fig. 3

Thermal Resistances

(All values in deg. per watt for 1 in. axial length, for ½ slot and tooth. Dimensions from Fig. 3 in inches)

Tooth:

Let λ = thermal conductivity of rotor steel, watt per cu. in. per deg. cent.

$$m = f\,h + a;\; n = (1 - f)\,h + c;\; r = \frac{b - b_1}{h_1}$$

Then

$$R_m = \frac{\theta}{\lambda\, m\,(b + b')};\; R_n = \frac{\Delta}{\lambda\, n\,(b_1 + b')}$$

$$\theta = \frac{m^2}{2} + \frac{b'\,m}{r} - \frac{(b')^2}{(r)^2}\log_e\left(1 + \frac{m\,r}{b'}\right)$$

$$\Delta = \frac{n^2}{2} - \frac{b'\,n}{r} - \frac{(b')^2}{(r)^2}\log_e\left(1 - \frac{n\,r}{b'}\right)$$

Insulation:

Let λ_i = thermal conductivity of slot insulation, watt per cu. in. per deg. cent.

$$m' = f\,h + .5\,a;\; n' = (1 - f)\,h + .5\,c$$

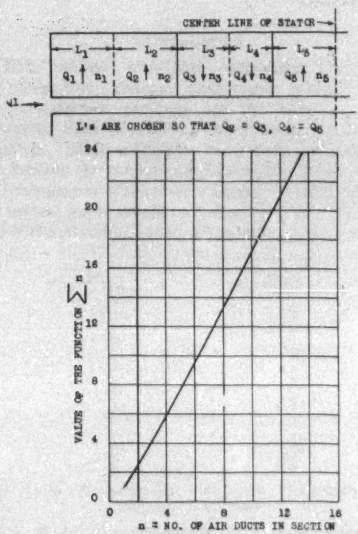


FIG. 4

Then

$$R_{im} = \frac{t}{m'\,\lambda_i};\; R_{in} = \frac{t}{n'\,\lambda_i}$$

Tooth Tip and Wedge:

$$W = 0.5\,(b + 2\,j + s + w);\; h' = \frac{s\,j + .5\,q\,(b + w)}{W};\; R_t = \frac{h'}{\lambda\, W}$$

Tooth Root:

Let $p = \dfrac{1}{2}$ perimeter of channel wall, in.

$$\text{Then } R_r = \frac{h_2}{\lambda\,(b_2 + p)}$$

Air gap surface:

Let

α_s = heat transfer coefficient for rotor surface, watt per in.^{2} per deg. cent.

S = no. of rotor coil slots

D_r = rotor diameter, in.

Then

$$R_s = \frac{2\,S}{\alpha_s\,\pi\,D_r}$$

Ventilating Channel Surface:

Let

α_m = mean heat transfer coefficient for channel, watt per in.^{2} per deg. cent.

ψ = a factor (usually between 0.7 and 0.8 for large generators) to eliminate channel air temperature from equations.

Then

$$R_f = \frac{1}{\alpha_m\,\psi\,p}$$

Surface temperature rise from pole-face losses:

$$T_p = \frac{\text{Pole-face loss in watts}}{\alpha_s\,\pi\,D_r\,x\ \text{rotor length}}$$

Mean temperature rise of air in air gap:

Referring to Fig. 4, the value of T_g for a typical turbine-generator may be approximated from the following equations:

Let:

Q = cu. ft. of air per min. to or from a given stator section

n = no. of air ducts in section of core of length L

F = kw. loss to stator air ducts in length L of inlet section

U = kw. loss to air gap from rotor and from tips of stator teeth, per in. length of air gap

V = kw. loss given to volume of air Q_i before entering gap

$\Sigma\, n$ = a function given by the curve in Fig. 4.

Then

$$T_g = \frac{R_1\,L_1 + R_2\,L_2 + R_3\,L_3 + R_4\,L_4 + R_5\,L_5}{L_1 + L_2 + L_3 + L_4 + L_5}$$

Where:

$$R_1 = \frac{1852}{Q_1}\left(V + \frac{L_1\,U}{2\,n_1}\sum n_1\right)$$

$$R_2 = \frac{1852}{Q_2}\left(F_2 + L_2\,U + \frac{L_2\,U}{2\,n_2}\sum n_2\right)$$

$$R_3 = \frac{1852}{Q_3}\left(F_3 + \frac{L_3\,U}{2\,n_3}\sum n_3\right)$$

$$R_4 = \frac{1852}{Q_4}\left(F_4 + \frac{L_4\,U}{2\,n_4}\sum n_4\right)$$

$$R_5 = \frac{1852}{Q_5}\left(F_4 + L_4\,U + \frac{L_5\,U}{2\,n_5}\sum n_5\right)$$

C. Richard Soderberg: Referring at first to the discussion by Mr. Fechheimer, the principles of superposition and reciprocity are direct consequences of the simplification of the problems into thermal circuits. This fact was probably not emphasized sufficiently in my paper, and I fully agree with Mr. Fechheimer on the practical importance of these principles.

Mr. Palueff's reference to the use of short-circuit tables for the solution of problems of this type is very interesting. I am wondering, however, whether the type of problem to which Mr. Palueff is referring, involving a large number of similar bodies, would not be more simply treated by considering the aggregate as a homogeneous body. The errors by such a simplification are usually very small, and the numerical work is very greatly simplified.

Referring to the discussion by Mr. Snell, I wish to emphasize that, while most turbine-generators can be treated with sufficient accuracy by assuming the division of the radial flow to occur at the bottom of the slots, this restriction of the results obtained in the paper is not implied. The procedure outlined by Mr. Snell is mentioned in the paper; the methods outlined apply particularly to the case that the division of the radial flow occurs in the slot region, which is practically always the case. In an arrangement of the type shown in Fig. 3, (of the paper) where there are ventilating ducts at the bottom of the slot, the division of the radial flow occurs at about the middle of the tooth; when there are no axial ducts, the division occurs nearer the bottom of the slots. The formulas given by Mr. Snell will be found to give practically identical results with those obtained by the formulas given in the paper. The excellent agreement between calculated and tested values, given in Table I by Mr. Snell, is perhaps somewhat better than our usual experience, but it is my firm belief that even the temperature problem can be reduced to reliable theoretical treatment when more experience with rational methods of calculation has been obtained.

I fully agree with Mr. Snell that many niceties in the theoretical solutions may be over-shadowed by the uncertainties of the constants entering the solution, but it is in new, radical departure from standard practise that these may become important. For example, assume that it were possible to reduce the thermal resistance of the slot insulation of large turbo-rotors to one-half of the present usual value. Maintaining the rise of the copper the same, the mean rise of the teeth would increase by something of the order of 50 per cent above the present value. It would then become much more important to calculate this drop accurately and the error due to the neglection of tooth taper, for example, would begin to be serious. Thus, the justification for many of the details of the present treatment should be looked for in the treatment of new, abnormal constructions, and not in the application of the results to routine design practise, where much simpler methods may yield satisfactory results.

Effect of Transient Voltages on Power Transformer Design—III

Non-Resonating Auto-Transformer

BY K. K. PALUEFF*

Member, A. I. E. E.

Synopsis.—*Transient voltage phenomena within auto-transformers differ from those in transformers, because in the former there are two points of the winding connected to independent transmission lines. This allows independent variation of the potential of these points.*

When a surge comes to an auto-transformer from one line only, the other line acts practically as a "solid ground" for internal oscillations of the windings, provided it is long enough.

Transient voltage stresses between turns and between coils, therefore, are higher in auto-transformers than in transformers of the same voltage rating.

The non-resonating auto-transformer has been developed for commercial use. In mechanical construction it is essentially the same as the non-resonating transformer.

The presence of two independent lines connected to an auto-transformer permits several conditions of surge excitation. This allows corresponding variations in the design of shields.

* * * * *

Introduction

THE general principles controlling the high-frequency voltage phenomena within auto-transformer windings are the same as those governing these phenomena in ordinary transformer windings. These principles have been presented to the Institute in various communications mentioned in the bibliography of the present paper. On this account, the present paper should be looked upon as a continuation of the preceding papers of the author, and is written on the assumption that the reader is thoroughly familiar with the above mentioned papers.

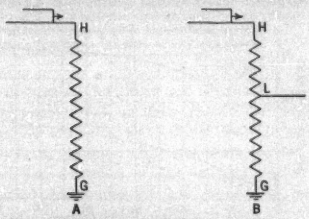


Fig. 1—A. Transformer with Line Connected and Neutral Grounded

Fig. 1—B. Auto-Transformer with Two Lines Connected and Neutral Grounded

The justification for writing this paper lies in the fact that in certain respects the high-frequency characteristics of an auto-transformer are essentially different from those of a straight transformer.

Effect of Transmission Line Surge Impedance

In a grounded transformer one end of the high-voltage winding is connected to the transmission line, and the other end is grounded (see Fig. 1). In an auto-transformer, in addition to the above, some intermediate point of the high-voltage winding is connected to some other transmission line. It is this feature that makes the oscillation of an auto-transformer winding essentially different from that of a transformer winding.

In this paper the line that brings a given disturbance to an auto-transformer is called the "primary line," while the other line is called the "secondary line."

Furthermore, the transmission line connected to the highest point of the auto-transformer winding is termed the "high-voltage line," while the other line is termed the "low-voltage line."

The surge impedance of a transmission line generally does not exceed a few hundred ohms, while the impedance of a transformer to a surge is many times higher. On this account, the impedance of the primary line has practically no effect on the internal oscillation of windings, while the impedance of the secondary line dominates the phenomena.

As was pointed out before,[1] if a transformer neutral is connected to ground through a resistance of the order of a few hundred ohms, the transformer behaves under finite traveling wave excitation as produced by lightning, as if it were solidly grounded, because the surge impedance of the transformer is many times higher than the neutral resistance. Should the transformer neutral be connected to an infinite transmission line instead of being grounded through a resistance, the transformer will again act as if it were solidly grounded, provided the surge impedance of the line is of the same order of magnitude as the neutral resistance it replaces, *i. e.*, a few hundred ohms.

It is quite obvious that if the transmission line is connected not to the neutral point but to any other point of the winding, with the neutral left isolated,

*Research and Designing Engineer, General Electric Co., Pittsfield, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

1. See Bibliography 4.

the transformer will behave as if that were solidly grounded, provided there is a sufficient part of the high-voltage winding remaining between the primary and secondary transmission lines. If the secondary transmission line is connected only a few turns below the high-voltage line terminal, then the "surge impedance" of these few turns may be very small, and the transformer will act practically as a small inductance inserted

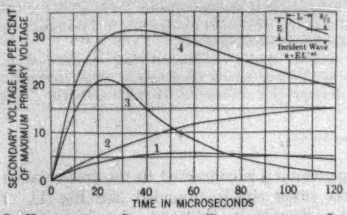


FIG. 2—VOLTAGE AT SECONDARY TERMINAL WITH INCIDENT WAVE APPLIED TO HIGH-VOLTAGE TERMINAL

1 and 3—Length L of incident wave 20 microseconds
2 and 4—Length L of incident wave 100 microseconds
1 and 2—High effective auto-transformer inductance
3 and 4—Low effective auto-transformer inductance
Long secondary line (see Fig. 4). Surge impedance of both lines 300 ohms
auto-transformer ratio 2:1
High effective inductance covers range from 20,000 kva. 220,000 volts Y to 1,000 kva. 66,000 volts Y, and low effective inductance from 80,000 kva. 220,000 volts Y to 10,000 kva. 66,000 volts Y

between the two lines. For this reason, even a relatively short traveling wave may bring the potential of the secondary transmission line appreciably above ground. This potential depends on the surge impedance of the two lines, the value of the effective inductance of the part of the winding between the two lines, terminal conditions of any other winding on the same core, and the shape of the incident wave.

The statement that the secondary line acts as a "solid ground" for internal oscillation is therefore only approximately correct. The potential of the secondary line rises in accordance with known law. In most cases this rise is very slow in comparison with the frequency of internal oscillations. It is for this reason that a long secondary line can be looked upon as a solid ground.

The curves of Figs. 2, 3 and 4 were obtained by use of the complete formula which is rather complicated, and will be published in the near future. Very nearly the same results may be obtained by assuming that the auto transformer acts as an air inductance coil connected between high- and low-voltage lines, the value of its inductance being equal to the short-circuit inductance between series and common windings. The effective surge impedance of primary line is its actual surge impedance, the effective surge impedance of the secondary line in its actual surge impedance multiplied by square of ratio of primary winding turns to secondary winding turns.

The complete phenomenon can, then, be divided into two parts: first, the gradual rise and fall of voltage on the secondary line, or the slow transient; and second, the internal oscillations within the windings, or the fast transient.

ORDINARY AUTO-TRANSFORMER

It can be seen that with the disturbances coming in from the high-voltage transmission line, with the low-voltage line connected and neutral grounded, the internal oscillations in an auto-transformer will be as if the end of the series winding were solidly grounded, and the common winding short-circuited.

It is also obvious that should the disturbance come in from the low-voltage line, with the high-voltage line connected, these oscillations would be as if both ends of the winding were solidly grounded.

The curves of Fig. 2 were prepared to determine more precisely the potential that is apt to appear on the secondary line in practise.

They show that under conditions commonly found, the potential of the secondary line remains a small fraction of the potential of the primary line.

There are four possible conditions of surge excitation of an auto-transformer when it is connected to both its lines:

A. Traveling wave arrives from high-voltage line.
B. Traveling wave arrives from low-voltage line.

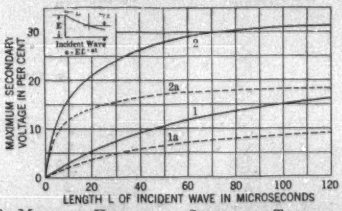


FIG. 3—MAXIMUM VOLTAGE AT SECONDARY TERMINAL WITH INCIDENT WAVE APPLIED TO HIGH-VOLTAGE TERMINAL

1 and 1A—High effective auto-transformer inductance
2 and 2A—Low effective auto-transformer inductance
1 and 2—Maximum secondary voltage expressed in per cent of maximum primary voltage
1A and 2A—Maximum possible secondary voltage expressed in per cent of primary line flashover voltage for very short wave. Obtained from curves 1 and 2 by taking impulse ratio of primary line insulation into account
Circuit constants the same as in Fig. 2

C. Two traveling waves of same polarity arrive simultaneously from both lines.

D. Two traveling waves of opposite polarity arrive simultaneously from both lines.

The first two cases are the most common. The last two are possible, practically speaking, in two cases only: first, when the lines are parallel, and second, when lightning storms take place directly over the auto-transformer.

It is believed that condition "D" can be satisfied only in case the same lightning discharge creates two traveling waves—one on one line by direct stroke, and another on the other line by induction.

For these reasons it appears that the probability of an auto-transformer being struck from both sides simultaneously, with waves of maximum possible amplitude, is very small.

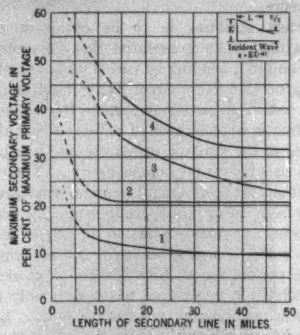


FIG. 4—EFFECT OF LENGTH OF SECONDARY LINE ON MAXIMUM SECONDARY VOLTAGE

1 and 3—Length of incident wave 20 microseconds
2 and 4—Length of incident wave 100 microseconds
1 and 2—High effective auto-transformer inductance
3 and 4—Low effective auto-transformer inductance
Incident wave applied to high-voltage terminal
Circuit constants the same as in Fig. 2 and Fig. 3

Fig. 5 illustrates the initial and the maximum voltage to ground produced in an auto-transformer winding by a rectangular traveling wave under conditions A, B, C, and D. It is assumed that the wave is not shorter than half the duration of the natural period of the fundamental harmonics of the respective windings. This condition is generally fulfilled in service.

Condition "A." The important conclusion that can be drawn from these diagrams is the fact that the insulation between coils and turns in the series part of the high-voltage winding is stressed to a much higher degree when the low-voltage line is connected than when it is disconnected.

It also follows that since the series winding contains only part of the total turns of the high-voltage winding, the stresses between turns are higher in the entire series winding than in the winding of a transformer of the same voltage rating. This occurs because the full voltage of the traveling wave appears across the series part of the high-voltage winding, not only during the initial voltage distribution but also during the oscillations that follow. The next important feature is that, owing to induction, relatively higher voltages are induced in the portions of the common and of the other windings adjacent to the series winding.

This increase in the voltage stresses between turns of an auto-transformer over those experienced in a straight transformer of the same voltage rating obviously becomes higher as the voltage ratio of the auto-transformer approaches unity. This is true because in case the rated voltages of an auto-transformer are in ratio of say 2:1, the series winding, which is one-half of the entire high-voltage winding, is subjected to the full voltage of the traveling wave on the high-voltage line, and its turn stresses are roughly twice the stresses that would be found in the high-voltage winding were the low-voltage line disconnected from the auto-transformer. However, in the case of an auto-transformer with the ratio of the two rated voltages equal to 1.1:1, the series winding is 9 per cent of the entire winding, and therefore the high-frequency voltage stresses between turns of the series winding will be very much higher than in an auto-transformer of 2:1 ratio. It should be noted however, that they will not be exactly 5½ times as high, because the voltages of internal oscillations do not increase strictly in an inverse proportion to the turns.

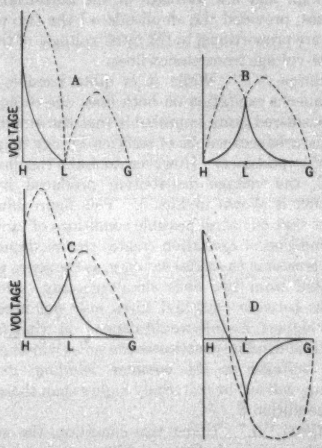


FIG. 5—SOLID LINES SHOW INITIAL VOLTAGE DISTRIBUTION, AND DOTTED LINES MAXIMUM VOLTAGE TO GROUND DUE TO SUBSEQUENT INTERNAL OSCILLATIONS

H, *L*, and *G* are high-voltage, low-voltage and ground terminals. Incident waves assumed to be proportional to rated terminal voltages.

Depending on the auto-transformer ratio the voltage to ground throughout the common winding under this mode of excitation may be substantially higher or lower than when the low-voltage transmission line is disconnected. The stresses between coils and turns in the common winding, adjacent to the series winding, may be of the same order or even greater.

Condition "B." The series winding is again subjected to the full voltage of the traveling wave, this time coming in from the low-voltage line. This means that should the number of turns of the series winding be substantially less than that of the common winding, the transient voltage stresses between turns of the series winding will be substantially greater than in the common winding.

In case the auto-transformer ratio approaches unity, it is obvious that exceedingly high stresses can be created in the series winding and, through induction, in the other windings adjacent to the series winding.

Voltage to ground at various points of the common winding may be higher under this type of excitation than under condition "A," but this depends on the ratio of the auto-transformer and the relative position of the windings.

Voltage to ground at various points of the series part of the winding will be either lower or equal to that caused by the previously described condition of excitation, depending again on the ratio of the auto-transformer. It becomes lower as the difference between the rated high and low voltages of the auto-transformer increases, provided the amplitudes of the two incident waves are proportional to the rated voltages of the high- and low-voltage transmission lines.

Condition "C." While it is quite possible to get simultaneous excitation on both lines due to lightning, it is considered quite improbable that the amplitude of both disturbances will be of sufficiently high magnitude to be of importance. However, to make the discussion general, the voltage distribution produced by such excitation is shown in Fig. 5. This figure illustrates the fact that out of all possible conditions of excitation, this condition of excitation creates the maximum voltages *to ground* in the series winding, as the series winding is excited from two ends simultaneously. However, voltages between coils and turns may not necessarily be the highest, as this would depend on what fraction of the total winding constitutes the series winding.

The voltages in the common winding, generally speaking, will not be materially higher than those found under condition B.

Condition "D." Under this condition, the voltages *across* the series winding are higher than under any other condition. While the voltages of various points of the series winding to ground may not be the highest, the voltages between coils and turns are the highest, and therefore the voltages induced in other windings, particularly parts adjacent to the series winding, are also the maximum. Voltages in the common winding are of the same order of magnitude as those found under condition B, except for the part of the common winding adjacent to the series winding, where they may be higher on account of induction, as above mentioned.

Fortunately, it appears most improbable that under this condition of excitation the amplitude of both traveling waves should approach extremely high values.

Non-Resonating Auto-Transformer

It was shown earlier in this paper that the internal transient voltages in auto-transformers may equal or considerably exceed those found in transformers.

The successful operation of all non-resonating transformers (totaling over 1,500,000 kva.) put into service during the last few years made it very desirable to extend the principle of electrostatic shielding to auto-transformers.

If perfect shielding is to be obtained, practical considerations make it essential that the potential of only *one* end of a winding be varied while the other end remains grounded either directly or through an impedor.

In the case of an auto-transformer, the neutral end of the winding is kept at ground potential, but there are *two* points in the winding instead of one, the potential of which can be varied independently. They are the line terminals of the series and common windings.

As has been shown, there are four possible conditions for transient voltage excitation of these terminals. This fact makes it impractical to obtain shielding which would be perfect for all four conditions of excitation. Fortunately, it is reasonable to consider only the first three conditions as important and probable. For these three conditions, the shielding methods described below give non-resonating characteristics to auto-transformer windings in a sufficient degree to make their use highly beneficial.

It was shown that under conditions experienced in service, the secondary transmission line connected to the auto-transformer may be considered as being at ground potential in regard to the internal voltage oscillations of the windings. Since traveling waves of very high magnitude are found to be relatively short, the point of the auto-transformer connected to the secondary transmission line may be considered as being solidly grounded during the period of internal oscillations.

Taking the above into consideration, it was found that a non-resonating auto-transformer can be produced without any radical change in the principal mechanical features from those used in a concentric winding core-type transformer.

It further was found that an auto-transformer can be made non-resonating to the various degrees of perfection that appear to be suitable for different service conditions.

In construction of windings and shields, the non-resonating auto-transformer is similar to the non-resonating transformer, (see Fig. 6).

The series and common windings are concentric with the tertiary winding. One of the most advantageous features of this construction is the absence of reentrant windings, which are undesirable but practically unavoidable in interleaved construction, (see Fig. 7). In addition to other reasons, reentrant windings are undesirable because of extremely non-uniform voltage distribution throughout the winding. This results in high local stresses, especially at the points of discontinu-

ity, where the transient voltages of adjacent parts of different windings are generally in phase opposition.

There is a considerable number of methods which can be used in shielding auto-transformers. Six of them are shown in Fig. 8. On the left side of the figure are shown the connections of the shields to the windings; on the right side are shown the initial and maximum voltage distributions produced in service under four conditions. These conditions are:

Fig. 6—20,000-Kva. (Output) 215,000- to 107,500- to 11,000-Volt Non-Resonating Auto-Transformer

W. Surge comes from the high-voltage line with the low-voltage line disconnected.

X. Surge comes from the high-voltage line with the low-voltage line connected.

Y. Surge comes from the low-voltage line with the high-voltage line connected.

Z. Surge comes from the low-voltage line with the high-voltage line disconnected.

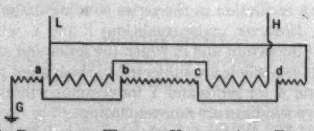


Fig. 7—Reentrant Winding Used in Auto-Transformer of Interleaved Construction. Points of Discontinuity at *a*, *b*, *c*, and *d*

The dotted lines in all of these diagrams indicate the maximum voltage to ground produced by transient voltages. The solid lines indicate the initial or electrostatic voltage distribution. Where two lines coincide only the solid line is shown.

It should be noted that under most of the methods of shielding, the internal voltage distribution created by transient voltages coincides with that of the standard acceptance induced voltage test. This is a very important feature, as it makes the standard induced voltage test a real test of ability of the insulation to withstand the stresses to which it can be subjected in service. As was pointed out before, this is not true for either

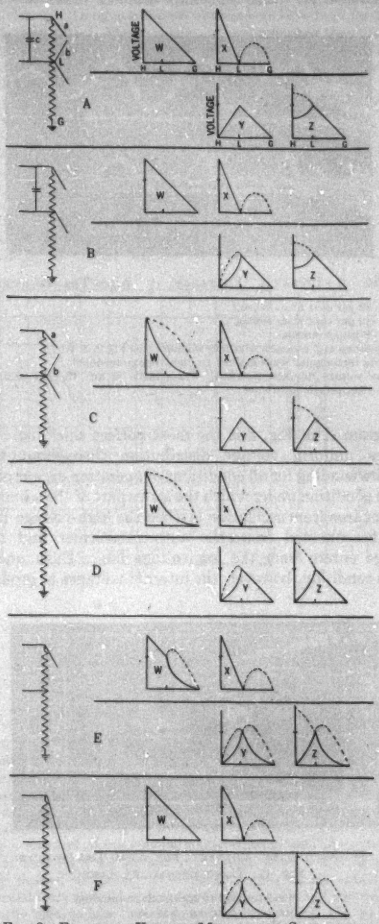


Fig. 8—Effect of Various Methods of Shielding on Voltage Distribution in Auto-Transformers

transformers or auto-transformers of conventional design.

In some of the cases shown, where only partial shielding is employed, the voltages produced by oscillation exceed those produced by the standard induced voltage

test (see *C*, *D*, and *E* of Fig. 8), to a lesser degree however than that found in ordinary auto-transformers. The insulation strength in this part of the winding should therefore correspondingly exceed that required for the standard induced voltage test.

Method A. The shielding method illustrated by

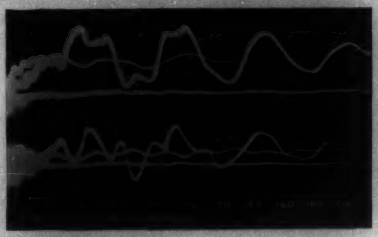


FIG. 9—INTERNAL VOLTAGES IN AUTO-TRANSFORMER

1—32 per cent from neutral
2—12 per cent from neutral
a—Without shields
b—Series and common windings shielded (see Fig. 8 *B W*)
Long rectangular wave applied to high-voltage terminal
Low-voltage line disconnected. Tests made in air, thereby creating some electrostatic unbalance

diagram *A* of Fig. 8 is the most perfect shielding. It gives uniform voltage distribution throughout the entire winding for all conditions of operation except one. The condition under which the series part of the winding is not free from oscillation is when the high-voltage line is disconnected from the auto-transformer and the surge enters from the low-voltage line. Even under this condition, however, the internal voltages to ground

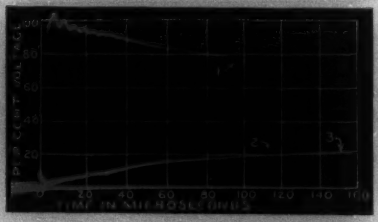


FIG. 10—SECONDARY VOLTAGE FOR AUTO-TRANSFORMER OF FIG. 6. LONG SECONDARY LINE

1—Wave applied to high-voltage terminal
2—Measured secondary voltage
3—Calculated secondary voltage

do not exceed the equivalent values of voltage produced during the standard induced voltage test.

Since this condition of operation is very unusual, the shielding, as illustrated in this diagram, can be considered as being complete for all practical purposes.

Capacitor *c* consists of two concentric electrodes with a suitable number of herkolite cylinders and oil ducts between them.

Method B. This method is similar to method A, except that shield *b*, as shown in diagram *A*, is omitted. When this method is used, under conditions W and X the windings are non-resonating; while under conditions Y and Z the series winding oscillates but within the limits of stresses obtained during the induced voltage test.

Method C. This method is similar to method A, except for the omission of capacitor *c*.

The capacitor is desirable only when the low-voltage line is either disconnected or too short to act as an effective ground to the low-voltage terminal for disturbances entering from the high-voltage line (condition W).

The omission of the capacitor results in oscillations in the series and common windings under condition W,

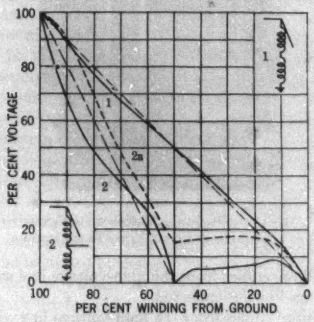


FIG. 11—MEASURED VOLTAGE DISTRIBUTOIN IN AUTO-TRANSFORMER OF FIG. 6

1—Initial voltage distribution and maximum voltage to ground, secondary line disconnected
2—Initial voltage distribution, secondary line connected
3—Maximum voltage to ground, secondary line connected

and some oscillation in the series winding under condition Z. However, under conditions X and Y, which are the most important and in which the series and common windings as a whole are subjected to the highest stresses (condition X for series and Y for common winding), the respective windings are non-resonating.

Method D. Without much increase in local stresses, method C can be simplified by the omission of shield *b*. This results in oscillation of the series winding when a surge comes from the low-voltage line. Otherwise it gives results as satisfactory as are obtained by the previous method.

Method E. When it is desired to make only the series winding non-resonating, which is the most common case since disturbances in the series winding are higher than those in the common winding, shielding method illustrated in diagram *E* is employed.

Method F. When the condition of operation shown

under W dominates, a longer shield than that employed in method E may be used. This modifies the results as shown in diagram *F*.

Figs. 9, 10, and 11 show oscillograms and voltage distribution obtained in a non-resonating auto-transformer built for commercial use and shown in Fig. 6.

Conclusions

Ordinary Auto-Transformer

a. With only one transmission line connected to an auto-transformer, a traveling wave produces internal oscillations similar to those produced in a transformer.

b. With both transmission lines connected to their respective terminals, a traveling wave impressed from one line creates two kinds of voltage transients in the windings. The first is relatively slow, and is responsible for the rise of potential on the secondary line. The second is relatively fast, and consists of internal free oscillations in the windings.

c. For the internal free oscillation, a secondary line of sufficient length acts as a "solid ground."

d. Internal stresses, particularly those between turns, are higher in an auto-transformer than in a transformer of the same voltage rating. This becomes more so as the auto-transformer voltage ratio approaches unity.

Non-Resonating Auto-Transformer.

a. Auto-transformers can be built which are non-resonating for usual conditions of operation.

b. The windings of non-resonating auto-transformers are uniform and free of breaks.

c. Non-resonating auto-transformers are similar to non-resonating transformers in general appearance and construction.

Acknowledgment

The credit for establishing some of the rigorous mathematical relations used in this paper is due to Mr. J. H. Hagenguth. The general assistance of Mr. W. A. McMorris in the preparation of this paper is also acknowledged.

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2. *Effect of Transient Voltages on Power Transformer Design*, K. K. Palueff, A. I. E. E. Trans., Vol. 48, July 1929, p. 681.

3. *Lightning Studies of Transformers by the Cathode Ray Oscillograph*, F. F. Brand and K. K. Palueff, A. I. E. E. Trans., Vol. 48, July 1929, p. 998.

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Discussion

F. J. Vogel: Mr. Palueff's use of the term "non-resonating" indicates that it should be defined as a transformer in which means have been provided to reduce oscillations rather than a transformer in which there are no oscillations. It is obviously incorrect to speak of varying degrees of non-resonance unless the term is defined as above.

In the description of the ordinary auto-transformer, condition *A*, the important conclusion is drawn that the insulation between coils and turns in the series part of the high-voltage winding is stressed to a much higher degree when the low-voltage line is connected. Further, this is true regardless of ratio, true whether due to initial distribution or oscillation, and as the author explains, will be roughly stressed in the order of the ratio of the transformer. Designers of auto-transformers have been familiar with these facts. Experience has always indicated that practically the entire surge voltage is across the series part of the winding whether the low-voltage line is connected or not and good insulation practise has been established on this basis.

In connection with Mr. Palueff's statements regarding interleaved windings, Fig. 1 (of this discussion) shows the initial distribution and oscillations within the type of winding most commonly employed. Note that all the stress is distributed over the series part of the winding. According to Mr. Palueff, point *b c* is a point of discontinuity, where high voltages prevail. This is not the case. The relation between the voltage stress at point *a c* and the insulation at that point is obviously the same regardless of whether or not the winding is of interleaved type or not, assuming, of course, that the same factors of safety are used.

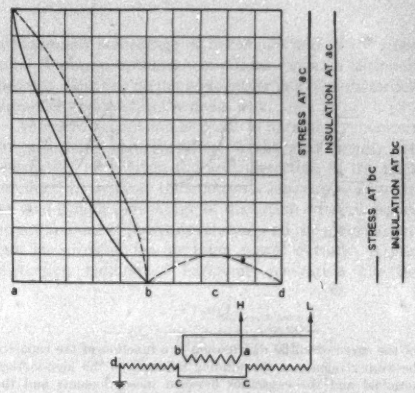

Fig. 1

The normal graded insulation at point *b c* is approximately 50 per cent of that at point *a c*, while the corresponding stress is a much smaller proportion, so that instead of the insulation being very highly stressed at this point, there is a very large factor of safety.

In Mr. Palueff's Fig. 8A, an arrangement of shielding is shown which purports to show a uniform distribution of potential in the series part of the winding under all conditions of excitation. It is doubtful if the method of shielding shown will bring about this desired condition. It could be approximately obtained if excited from the high-voltage line by a single shield *a* connected to the high-voltage line lead, as he has shown in Fig. 8B (*X*). However, if shield *b* is used as shown in Figs. 8A and 8C, this shield will be at practically ground potential, when the surge enters from the high-voltage line, and when the low-voltage line is connected, for reasons previously explained. The part of the winding influenced by this shield will then have a large capacity to ground, and will approach ground potential very rapidly. The result of this will be to distort the distribution as I have shown from curve Fig. 2A to Fig. 2B and permit oscillations as indicated. Similarly, if a surge comes in on the low-voltage

line, shield a will be at practically ground potential, and the part of the winding influenced by this shield will approach ground potential very rapidly. The result of this will be to distort the distribution as I have shown from curve Fig. 2c to Fig. 2D.

In Fig. 8E, covering the case of the single high-voltage shield, sketch W indicates a much lower voltage gradient than sketch X, sketch W being the case for the low-voltage line disconnected and sketch X with the low-voltage line connected. Since the slope

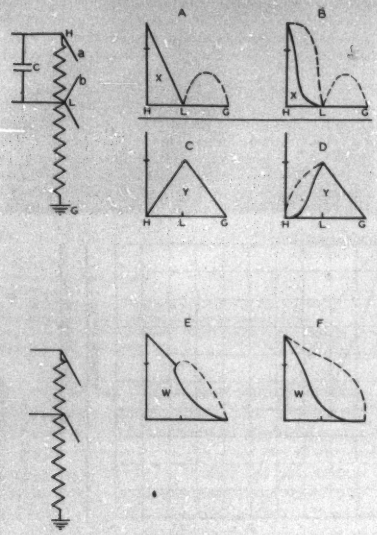

Fig. 2

of the curve of initial distribution is a function of the capacity between elements of the winding adjacent to the high-voltage terminal and the capacities between these elements and the ground and shield, it is obvious that the initial slope of this curve cannot be affected by the connection of the low-voltage lead. Mr. Palueff's sketch W, Fig. 8E, which I have shown as Fig. 2E, should be modified as indicated in Fig. 2F.

K. K. Palueff: I shall attempt to answer Mr. Vogel's criticisms in the order he presented them.

The term 'non-resonating" is proper for the transformers bearing that name, as these transformers do not oscillate. In case of auto-transformers, a variety of shielding methods (Fig. 8) is possible which reduces oscillations very materially. Some of these methods (for example W, X, and Y of Fig. 8A) eliminate oscillations under operating conditions in the winding subjected to the surge, and in such cases the auto-transformers are non-resonating.

I believe that Mr. Vogel's remark regarding effect of connecting secondary line is referred to initial distribution only and he neglects to consider the subsequent oscillation.

Regarding points of discontinuity, Mr. Vogel informs us that the distribution of windings shown in his Fig. 1 is most commonly used, rather than that shown in Fig. 7. Apparently, he has in mind small units, for this arrangement of windings is not practical for large auto-transformers.

The insulation allowances shown in Fig. 1 of the discussion may be sufficient when operating with the low-voltage line connected, but with the low-voltage line being short-circuited or disconnected the oscillations are similar to those in a transformer, and the stresses at b_C will be much higher than shown.[2,3,4] The insulation must be sufficient for both conditions of operation.

The initial distribution shown by the solid line in Fig. 1 of Mr. Vogel's discussion is much better than the curve he found to be representative of his transformers, and published in Fig. 3 on page 62 of the A. I. E. E. Trans., Vol. 50, March 1931. Correspondingly, the maximum voltage to ground shown by the dotted line is also too optimistic. Fig. 3 herewith compares his present data with that shown in his paper referred to above.

The voltage distribution shown in Fig. 8A is correct. If Mr. Vogel would make a calculation for the shielding method shown, he would find that it is an easy task to balance the effects of shields a and b to give the desired results.

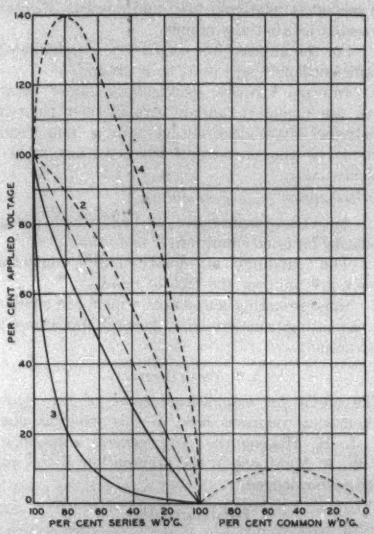

Fig. 3—1 and 2—Initial Distribution and Maximum Voltage to Ground Respectively, as Shown in Fig. 1 of Mr. Vogel's Discussion

3—Initial Distribution Curve Published by Mr. Vogel (See Reference in Discussion)

4—Maximum Voltage to Ground, Estimated from Curve 3

Sketch X under Fig. 8E is correct as shown. This condition is more important than that shown under W, since in the former case the entire impulse voltage appears across the series winding. Mr. Vogel is right in his correction of sketch W. I agree that the corrected distribution is better than that originally shown, since it does not have such high concentration of voltage near point L.

Harmonic Generation by Means of Grid-Circuit Distortion

BY F. E. TERMAN,[1] D. E. CHAMBERS,[2] and E. H. FISHER[3]

Associate, A. I. E. E. Associate, A. I. E. E. Non-member

Synopsis.—*The usual vacuum tube harmonic generators are fundamentally high distortion amplifiers operated under conditions of grid bias and plate voltage which, to distort the input voltage wave, utilize the non-linear relation between plate voltage and plate current. This paper gives the result of an investigation into the possibilities of using the non-linear grid voltage-current characteristic to produce harmonics. Grid distortion can be utilized by operating the tube so that the grid goes positive for a part of each fundamental frequency cycle and then inserting in series with the grid lead an impedance across which the grid current impulses produce a voltage drop rich in harmonics, which is then amplified in the plate circuit.*

The grid circuit distortion harmonic generator has the advantages of producing the harmonic in one circuit and developing the power in another, combined with the very high distortion properties of the grid circuit. The result is that the grid-distortion harmonic generator develops from 100 to over 1,000 times as much power on the higher harmonics as does the usual plate dist rtion generator. Thus, when using a suitable parallel resonant circuit as the distorting grid impedance, a steady output of 135 volts effective has been obtained on the 22nd harmonic from a radio receiving tube When the grid impedance consists of a large inductance it is possible to generate simultaneously at least 1/20th milliwatts of power on every harmonic up to the 50th. The grid circuit input power required to give these results is several hundred milliwatts in the case of the parallel resonant circuit, and considerably less when a grid inductance is used.

The paper gives the mechanism of operation, illustrated with oscillograms and harmonic spectra, and includes a discussion of the effect of circuit conditions on the operation of the grid-distortion generator.

Introduction

FREQUENCY multiplication by the use of harmonic generators has an important place in communication. Thus in the type *B* carrier-current equipment of the Bell System the carrier frequencies of the different channels are harmonics of a single master frequency. Again, it is common practise to control the frequency of a radio transmitter by means of a piezoelectric crystal, and where the transmitted frequency is too high to be obtained directly from such a crystal it can be derived indirectly from a lower frequency by making use of harmonic generators. Another example of the practical value of the harmonic generator in communication technique is furnished by the broadcasting of the same chain program from a number of stations all operating on a common carrier frequency, as is being done by the British Broadcasting Company. Synchronization is maintained between the carrier waves of the stations in this English chain by deriving each carrier from a 1015.6 cycle current transmitted over wires and run through a series of 10-frequency doublers.

The vacuum-tube harmonic generators of present practise are fundamentally amplifiers operated under conditions of input voltage and grid bias such that the output voltage is a distorted replica of the input wave, and accordingly contains harmonics of the a-c. voltage that is applied to the grid. Harmonic generators of this type have the disadvantage of using the plate circuit of the tube to produce both the distortion and the power output. As a consequence it is impossible to adjust for a maximum percentage of harmonics and at the same time to obtain optimum conditions for large harmonic power. The two do not go together in a high-distortion amplifier operated in the usual way.

An improvement can be made in harmonic generators by producing the distortion in the grid circuit, and selectively amplifying the grid distortion in the plate circuit. In this way the grid conditions can be adjusted to give a high percentage of distortion without regard to the amount of power developed, and the plate current can be adjusted for optimum power output. A grid distortion method of harmonic generation has the

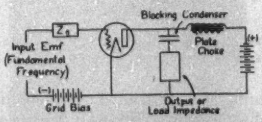

Fig. 1—Basic Circuit of the Grid-Distortion Harmonic Generator

Showing the distorting grid impedance Z_g

additional advantage that the grid circuit is inherently capable of producing greater distortion than is the plate circuit. That is to say, the voltage-current characteristic of the grid circuit is more curved and contains a sharper bend than does the corresponding plate voltage-current relation.

The Grid-Distortion Harmonic Generator

Grid distortion can be utilized to produce harmonics by means of the arrangement shown in Fig. 1, in which an impedance Z_g has been placed in series with the grid. Operation of the tube so that the grid goes somewhat positive at each positive crest of the input voltage

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2. General Electric Co., Schenectady, N. Y.
3. Research Assistant, Stanford University, Cal.

Presented at the Pacific Coast Convention of the A. I. E. E., Portland, Oregon, September 2-5, 1930.

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produces impulses of grid current which flow through the grid impedance Z_g and develop a voltage drop across it. Since the impulses of grid current represent a highly distorted current wave rich in harmonics, the voltage drop across Z_g will also be distorted, particularly if the added grid impedance Z_g offers a high impedance to the higher harmonics contained in the grid current impulses. These harmonic voltages developed across the grid impedance Z_g are amplified in the plate circuit of the tube, and under favorable conditions can develop a surprising amount of harmonic power in the output impedance shown in Fig. 1.

Analysis of Harmonic Generator Outputs

Analysis of the output voltage of the harmonic generators investigated was made possible by the "power circuit harmonic analyzer" which is part of the equipment of the Ryan High Voltage Laboratory. This piece of apparatus, which was developed by the American Telephone and Telegraph Company for inductive interference investigations, operates by selecting the desired harmonic component with suitable

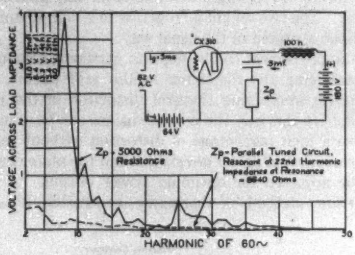

Fig. 2—Harmonic Spectrum Produced by Plate-Distortion Harmonic Generator with 60-Cycle Input

tuned circuits and measuring its potential with a vacuum-tube voltmeter. This analyzer has a frequency range up to the 50th harmonic of 60 cycles, and measures potentials down to 0.5 millivolt with good accuracy.

Harmonic analysis of the voltage developed across a resistance load by a plate-distortion type of harmonic generator gives frequency spectra of which Fig. 2 is typical. In the special case shown the grid bias and input voltage were adjusted to give a maximum output on the 22nd harmonic. For this particular curve the grid was allowed to become positive for a portion of each cycle. If the input voltage had been reduced sufficiently to keep the grid negative at all times the result would have been a harmonic spectrum similar to that of Fig. 2, but with all amplitudes somewhat reduced.

It is characteristic of all plate-distortion harmonic generators to develop a relatively large power output on the low order harmonics, particularly the second, while at the same time producing practically nothing on the high harmonics. This is clearly evident in Fig. 2. Replacing the load resistance by a circuit tuned to a high harmonic does not appreciably increase the output even on the resonant frequency, as is made apparent by the dotted line in Fig. 2, which is for a resonant circuit load tuned to the 22nd harmonic.

The possibilities of harmonic generation by grid

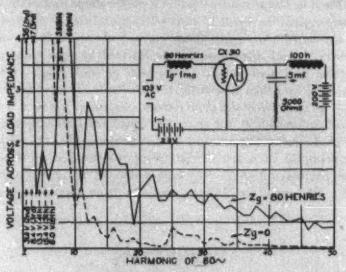

Fig. 3—Harmonic Spectrum Produced by Grid-Distortion Harmonic Generator

When the distorting grid impedance is an 80-henry inductance and the fundamental frequency is 60 cycles

distortion are shown by comparing Fig. 3 with Fig. 2. The results of Fig. 3 were obtained by using a distorting-grid impedance consisting of a large inductance, as shown by the circuit diagram on this figure, and by adjusting grid bias and input voltage to develop maximum voltage on a high order harmonic. For purposes of comparison, the harmonic spectrum developed by the plate-distortion generator with resistance load is

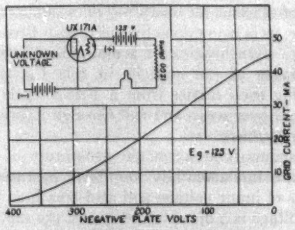

Fig. 4—Circuit and Typical Calibration Curve for Oscillograph Work Using the Inverted Vacuum Tube

replotted on this figure in dotted lines. The effect of the grid-circuit distortion introduced by the added grid inductance is primarily to increase, without producing a significant change in the lower order output, the voltage output on the high order harmonics to many times the value given by the plate-distortion method. Interpreted in terms of power, the distorting grid inductance produces an improvement in power output

on harmonics above the 20th ranging from 100 to 1,000 times, and develops a workable output (exceeding 1/20 milliwatt) on every harmonic up to the 50th.

By means of a parallel resonant distorting-grid impedance it is possible to accentuate the power output on a given harmonic. Replacing the resistance load by a suitable tuned circuit serves to increase the se'ective effect even more. An idea of what can be done by means of a tuned grid-harmonic generator is given by

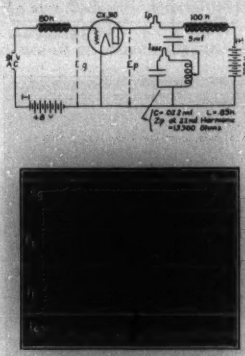

FIG. 5—CIRCUIT, TEST CONDITIONS, AND SERIES OF OSCILLOGRAMS

Showing performance of grid-distortion harmonic generator when using a distorting grid impedance consisting of an inductance

the oscillograms of Fig. 6. Here the 22nd harmonic voltage developed across the tuned circuit is 135 volts (which is the plate-filament voltage stepped up 5.6 times), and the 22nd harmonic circulating current in the tuned load circuit is 25 milliamperes. This represents several thousand times as much power as can be developed on this harmonic by plate distortion under comparable conditions.

The use of a grid-distorting impedance consisting of a resistance was found to give little if any more harmonic power than no grid distortion at all. The principal effect of a grid resistance is to eliminate to a large extent the power loss in the grid circuit when the grid goes positive, by limiting the grid current.

The results presented in Figs. 2, 3, and 6 are typical examples selected from a large number of data. These figures serve as an index of what can be expected from grid- and plate-distortion harmonic generators under practical conditions.

THE MECHANISM OF THE GRID-DISTORTION HARMONIC GENERATOR

The rather complex action taking place in grid-distortion harmonic generators was investigated by means of oscillograms. In order to do this it was necessary to take oscillograms of rather high potentials, such as grid-filament and plate-filament voltages, without drawing any current, which was very satisfactorily accomplished by using an inverted vacuum tube.[4]

The inverted vacuum tube is essentially an ordinary triode with the functions of the grid and plate interchanged. That is, the grid is the positive anode, and the plate is kept at a negative potential and is the control electrode. Operated in this inverted manner, the tube acts as a voltage-reducing power amplifier. Any voltage applied to the negative plate causes a proportional variation in the grid current, and, since the plate of a small tube is capable of handling hundreds of volts, the inverted tube is ideally suited to oscillographic work in high-voltage low-current circuits. Fig. 4 gives the connection used for taking oscillograms with the inverted vacuum tube, and shows a typical calibration curve.

The series of oscillograms given in Fig. 5 shows the action taking place in the grid-distortion harmonic generator when the distorting grid impedance is an inductance, and the plate load is an impedance tuned to

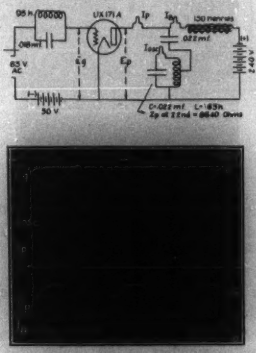

FIG. 6—CIRCUIT, TEST CONDITIONS, AND SERIES OF OSCILLOGRAMS

Showing performance of grid-distortion harmonic generator when using a distorting grid impedance tuned to the 22nd harmonic. The 22nd harmonic component of I_{osc} amounts to 25 milliamperes

the 22nd harmonic. The effect of the grid inductance is to cause the grid-filament voltage to be very irregular even though the input voltage is a pure sine wave. The distorted grid voltage is characterized by a sudden change of grid potential as the grid goes negative, accompanied by an oscillation which is probably due to the distributed capacity of the grid inductance. The plate current wave is seen to be highly distorted, with sudden rises and falls which indicate the presence

4. "The Inverted Vacuum Tube, a Voltage Reducing Power Amplifier," by F. E. Terman, *Proceedings I. R. E.*, 16 p. 447, April 1928.

of high frequency components. The response obtained from this current in the circuit tuned to the 22nd harmonic is seen to be quite appreciable.

The series of oscillograms in Figs. 6, 7, 8A, and 8B shows the action taking place in the harmonic generator when the distorting-grid impedance is a resonant circuit tuned to a harmonic. The results obtained with this type of distortion depend very considerably upon the circuit adjustments. In particular, best results are obtained when the plate current flows throughout the cycle, but at the same time the plate voltage must approach zero when the grid is positive.

The oscillograms in Fig. 6 show the operation of the tuned grid impedance under the most favorable conditions. It is seen that when the grid potential goes negative an oscillation is set up in the grid tuned circuit. This oscillation is amplified in the plate circuit, producing a very considerable 22nd harmonic circulating current in the tuned load impedance. This circulating current is a true harmonic of the fundamental frequency even when the resonant circuits are slightly detuned,

Fig. 7—Series of Oscillograms Similar to Fig. 6

Except that the circuit conditions are such as to allow the plate current to become zero for part of the cycle

because the action taking place repeats exactly at each cycle of fundamental frequency. The circulating current is not a pure wave of 22nd harmonic, but rather is modulated somewhat at 60 cycles and harmonics thereof, giving sideband components on the harmonics near the 22nd.

Adjusting the tuned grid-harmonic generator to give a continuous flow of plate current throughout the entire period when the grid is negative makes it possible for the amplified grid oscillation to supply energy to the load during the entire existence of the oscillation. The effect of allowing the plate current to go to zero for part of the cycle is shown in Fig. 7, and is seen to cause a very great reduction in the harmonic output.

Conditions most favorable for a large harmonic power output are obtained by using a small plate blocking condenser, a very high impedance in the plate choke, and a tube with an amplification constant such that satisfactory grid distortion can be secured with a C bias that does not entirely cut off the plate current. The poor results of Fig. 7 are caused by a negative grid bias that is unnecessarily great, complicated by insufficient inductance in the plate reactor. Both of these factors combine to reduce the portion of the time the plate current flows.

The use of a small blocking condenser is necessary in

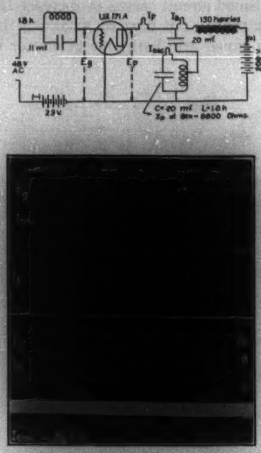

Fig. 8A—Circuit, Test Conditions, and Oscillograms

Showing performance of harmonic generator with and without a distorting-grid impedance tuned to the 6th harmonic. In the former case the 6th harmonic component of I_{osc} is 54 milliamperes

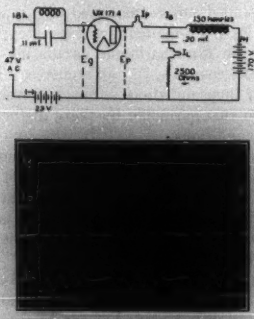

Fig. 8B—6th Harmonic Generation

Under conditions similar to those in Fig. 8A except that a resistance load is used

order to make the plate voltage drop to a very low value when the grid goes positive, and to cause the plate potential to rise rapidly when the grid again goes negative. The small blocking condenser can do this because it charges and discharges rapidly due to its small

capacity. The blocking condenser capacity giving optimum results with a given set of circuit conditions is rather critical. Values approximately the same as the plate circuit tuning condenser were found to be the best. A large condenser keeps the plate voltage high even when the grid is considerably positive. Under these conditions, with grid and plate both at positive potentials of considerable magnitude, secondary electron emission takes place at the grid, which results in a negative grid-filament tube resistance, and consequent self-oscillation at a frequency determined by the circuit constants. This condition can ordinarily be detected by the existence of an abnormally low or even negative grid current, a consequence of secondary electron emission at the grid.

Characteristics of the Tuned Grid-Harmonic Generator

Harmonic generation by the use of a suitable parallel resonant circuit for a distorting grid impedance will generate low order harmonics with great success, as is evident from the series of oscillograms in Fig. 8A, which shows sixth harmonic generation. The sixth harmonic circulating current in the tuned plate circuit in this case amounts to 54 milliamperes, while the harmonic voltage across the tuned circuit is 117 volts, including a step-up ratio of 2 to 1, representing 530 milliwatts. It would be a relatively simple matter to combine low loss tuned circuits with a ratio of transformation of approximately ten, thus giving 585 volts effective from the generator instead of 117, and this output could be obtained from a radio receiving tube!

The advantages of using grid distortion instead of plate distortion in the production of harmonics are very vividly shown by the two oscillograms in Fig. 8A which contrast the current in the tuned plate circuit ($I_{ac.c.}$) when the tuned grid impedance is present, and when it is removed. With grid distortion the 6th harmonic current circulating in the output impedance is strong and steady, enduring from cycle to cycle with little change of amplitude, while plate distortion gives an output that, in addition to being weak, is of irregular amplitude. The current ratio in the two cases is about 10 to 1, representing a power ratio of 100 to 1.

The effect of replacing the tuned plate impedance by a load resistance is seen in Fig. 8B. It is apparent that this change does not alter the harmonic output very greatly, as is to be expected because the harmonic is actually generated by the grid in both cases.

The amount of harmonic voltage produced in the grid circuit between grid and filament is roughly proportional to the grid current and to the inductance used in the tuned grid circuit. Whatever harmonic voltage is produced in this way at the grid is then amplified in the plate circuit and produces a harmonic output that depends upon the extent to which the load impedance is able to absorb the available power. The way these relations work out in a practical case is shown in Fig. 9. In particular, it is to be noted that when the plate load impedance is varied by changing the circuit resistance, there is a very definite condition for maximum power output, as is always the case in amplifier operation.

The resistance of the tuned grid circuit should be low enough to prevent the harmonic currents in the circuit from dying out before the cycle of fundamental voltage is completed. In Fig. 6 it is evident that the grid oscillation is almost damped out before it is extinguished by the grid going positive. A lower tuned circuit resistance would have caused the grid oscillation to maintain itself better, and would have resulted in a higher harmonic output, while higher tuned circuit resistances would have reduced the output very considerably. Contrasting with this is the situation

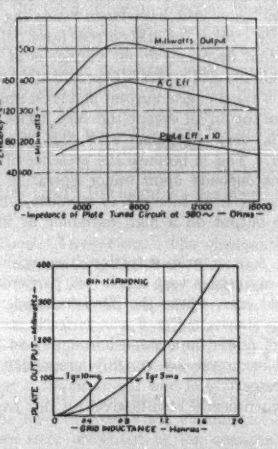

Fig. 9—Performance of Tuned Grid-Harmonic Generator Under Varying Circuit Conditions

presented in Fig. 8, where the oscillation in the tuned grid impedance is substantially unchanged in amplitude for the fundamental cycle. Here the tuned circuit resistance could be increased considerably before the damping of the grid oscillation would be sufficient to alter extensively the harmonic output.

In view of the large harmonic power developed by the tuned grid generator it is interesting to compute the efficiencies involved. This has been done in Fig. 9 for a number of conditions involving the sixth harmonic. The plate efficiency is the ratio of harmonic power to the plate battery power, and is seen to be about the same as when the tube is used as a distortionless audio frequency amplifier. The "a-c. efficiency" is the ratio of the harmonic power output to the fundamental frequency power supplied to the grid circuit of the harmonic

generator, and can of course exceed 100 per cent. When the "a-c. efficiency" is 100 per cent it means that the harmonic power output is equal to the input power required by a stage of harmonic generator. It is accordingly apparent from Fig. 9 that a grid distorting generator will put out an amount of power on the sixth harmonic that is easily sufficient to operate directly the next stage of generation.

Miscellaneous Comments

The only disadvantage of the grid-distortion method of harmonic generation compared with the plate-distortion method lies in the fundamental frequency power required as a result of the grid current that must flow to produce grid distortion. With tubes of the size used in this study the input power was in the order of a few hundred milliwatts for a tuned grid impedance, and much less for a grid impedance consisting of a large inductance. Since the grid current flows only when the fundamental frequency input voltage is at or near its crest value, the grid power is approximately equal to the product of the d-c. grid current and crest input voltage.

The output on all except the very lowest order harmonics is very irregular with the plate-distortion generator. This is because the presence of the higher harmonics depends upon minor operating conditions which change from moment to moment sufficiently to vary the output on a high harmonic as much as 50 per cent.

In the case of grid-distortion harmonic generators this unsteadiness is very much less, particularly in the tuned grid case, where the 22nd harmonic under the conditions shown in Fig. 6 was perfectly steady. The amount of power on a given harmonic and the steadiness of this power are roughly proportional to each other, irrespective of how the harmonic is generated.

The experimental results given in this paper are all for a fundamental frequency of 60 cycles, but they can be applied to other frequencies by keeping all circuit reactances and resistances constant as the frequency is changed. Thus the results obtained with an 80-henry distorting grid inductance at a fundamental frequency of 60 cycles are the same as would be obtained at a 1,200-cycle fundamental frequency with 4 henrys distorting inductance. At radio frequencies certain factors such as tube capacities and plate to grid feedback must be taken into account in making such transformations. The use of a neutralized grid-distorting harmonic generator is very desirable when handling high frequencies.

Conclusions

The results of this study show that the grid-distortion method of harmonic generation is very definitely superior to the usual plate-distortion method in respect to: (1) power that can be obtained on a desired harmonic, particularly when this is a high order harmonic; (2) power that can be obtained simultaneously on a large number of harmonics; and (3) steadiness of output on a particular harmonic frequency. The power developed by the grid-distortion generator at a given harmonic ordinarily is at least 10 times the maximum value obtainable from the plate-distortion method and in some cases will be over 1,000 times as great.

These advantages of the grid-distortion harmonic generators are gained at the cost of a small power loss in the grid circuit of the generator. This power consumption will ordinarily be several hundred milliwatts or less, and must be supplied by the fundamental frequency input to the generator.

The authors wish to take this opportunity to express their appreciation of the cooperation and assistance rendered by Professor J. S. Carroll of the Ryan Laboratory.

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1931

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its forty-seventh Annual Report, for the fiscal year ending April 30, 1931. A general balance sheet showing the condition of the Institute's finances on April 30, 1931, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year, more detailed information having been published from month to month in the Institute JOURNAL and its successor, ELECTRICAL ENGINEERING.

Director's Meetings.—The Board of Directors held seven meetings during the year, four in New York, one at Toronto, Ont., Canada, one at Philadelphia, Pa., and one at Pittsburgh, Pa. The Executive Committee acted upon various matters during intervals between Board meetings.

Information regarding the more important activities of the Institute which have been under consideration by the Board of Directors, the committees, and the various officers is published each month in the section of ELECTRICAL ENGINEERING devoted to "News of Institute and Related Activities."

President's Visits.—President Lee attended the two national conventions and four District meetings held since the beginning of his administration, and visited a number of the Sections.

The following is a list of places visited in addition to many trips to Institute headquarters in New York: Regina, Sask., Canada; Vancouver, B. C., Canada; Portland, Ore. (Pacific Coast Convention); San Francisco, Calif.; Los Angeles, Calif.; Salt Lake City, Utah; Denver, Colo.; Omaha, Nebr.; Washington, D. C. (A. E. C. Meeting); Pittsburgh, Pa. (District Meeting); New York, N. Y. (including Winter Convention); Louisville, Ky. (District Meeting); Philadelphia, Pa. (District Meeting); Rochester, N. Y. (District Meeting); and Toronto, Ont., Canada.

In May and June, President Lee will visit other Sections and attend the Summer Convention in Asheville, N. C.

Meetings.—Three national conventions and five District meetings were held during the year, and a brief report on each is given below.

Annual Meeting.—The Annual Business Meeting was held at the Royal York Hotel, Toronto, Ont., Canada, on Monday morning, June 23, 1930, during the annual Summer Convention. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1930, was presented, and the Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1930.

Summer Convention.—The forty-sixth annual Summer Convention was held at Toronto, Ont., Canada, June 23-27, 1930. Forty-five papers were presented at eight technical sessions, and printed copies of fifteen technical committee reports were available. The Annual Conference of Officers, Delegates, and Members, under the auspices of the Sections Committee and the Committee on Student Branches, was held on Monday, June 23, and Tuesday, June 24, and fifty-one Section delegates, eight District Secretaries, and eight Counselor delegates were present. The entertainment features included golf and tennis tournaments, a dinner, and an all-day trip to the Welland Canal, with optional additional trips to Queenston and Niagara Falls. The Lamme Medal, awarded by the Institute, was presented to Rudolf E. Hellmund at the dinner. More than 1,100 members and guests attended the convention.

Pacific Coast Convention.—The nineteenth Pacific Coast Convention was held in Portland, Oregon, September 2-5, 1930. At five technical sessions, seventeen papers were presented. Ten technical papers by students were presented at two additional sessions. The program included various social events and trips. The attendance was more than 300.

Winter Convention.—The nineteenth Winter Convention was held in New York, January 26-30, 1931. Fifty papers were presented at ten sessions. At an evening session, the Edison Medal was presented to Dr. Frank Conrad, and a lecture on "Your Nimble Servant—The Electron," was given by Dr. Karl Taylor Compton, President of Massachusetts Institute of Technology. Numerous inspection trips, a smoker, and a dinner-dance were held. Attendance about 2,000.

District Meetings.—

District No.	Location	Dates	Papers	Attendance
1	Springfield, Mass.	May 7-10	24	360
2	Philadelphia, Pa.	Oct. 13-15	22	500
4	Louisville, Ky.	Nov. 19-22	18	300
2	Pittsburgh, Pa.	Mar. 11-13	16	500
1	Rochester, N. Y.	Apr. 29-May 2	22	370

Papers were presented at District meetings by students as follows: Springfield 6, Louisville 8, and

Rochester 8. A Conference on Student Activities was held at each of the five meetings listed above.

Sections.—Practically all Sections have been active, and many of the meetings have been especially effective. The programs have included a wide variety of subjects, and much attention has been given to the desirability of increasing the participation of Section members in the meetings.

The Memphis and San Antonio Sections, organized early in the fiscal year, have been very active. The Florida Section was organized March 27, 1931.

The four technical groups of the New York Section and the Power Group of the Chicago Section have been highly successful in their activities.

The New York Section organized three classes in effective speaking, with a total enrolment of more than 90, which met weekly for three months, and the results were highly satisfactory to those participating.

The two lecture courses, "Engineering Economics" at Armour Institute of Technology, and "Recent Developments in Electron Physics and Chemistry" at the University of Chicago, conducted through the coöperation of the Chicago Section and the Western Society of Engineers, have aroused so much interest that the Chicago Section has requested its members to indicate their preferences in a list of seven lecture courses and four courses in class instruction which are under consideration for next year.

President Lee visited a considerable number of Sections (see names under heading "President's Visits"), and will visit others later.

Under the plan for Institute Lecturers to visit Sections, approved by the Board of Directors in June, 1930, Past-President William McClellan visited the Atlanta, Cincinnati, Dallas, Houston, Louisville, Memphis, Oklahoma City, and San Antonio Sections in February and March, and Past-President A. E. Kennelly visited the Akron, Erie, Niagara Frontier, and Schenectady Sections in March.

Some of the Vice-Presidents visited all or nearly all of the Sections within their Districts.

The addresses of S. P. Grace, Assistant Vice-President, Bell Telephone Laboratories, Inc., have continued to be very popular, the average attendance per meeting at the ten meetings reported to headquarters being more than 3,200.

Many of the Sections have continued their close coöperation with neighboring Student Branches, with excellent results.

Student Activities.—In many respects, the student activities have shown remarkable improvement in each of the last two years. Nearly all Branches have been active during the past year, and the number of those depending to a considerable extent upon students for their programs was greater than ever before.

New Branches were organized at the University of British Columbia, Harvard University, University of Illinois, and Rice Institute.

The keen interest of Counselors and Branch Chairmen in the District Conferences on Student Activities has been continued.

Reports on such conferences were given at many Branch meetings. Many student conventions have been held, some sponsored by Sections and some by Districts.

Students gave talks at Branch meetings on a wide variety of subjects, including many of the latest technical developments, summer experiences, current events, and biographies. Many Branches held general discussions of their activities.

The suggestions made by Professor W. H. Timbie, Chairman, Committee on Student Branches, that the Branches have talks by students upon the life and work of Faraday, and that they purchase and present to their institutions pictures of Faraday, have already been carried out by about one-third of them.

A considerable number of the Branches were visited by the Vice-Presidents in their Districts.

The Constitution and By-laws were amended to change the provisions for Student enrolment and to provide for admitting Enrolled Students to the Institute as Associates, upon the expiration of their term of enrolment, without the payment of the usual entrance fee.

As a result of a recommendation of the Counselor Delegates at the Summer Convention in Toronto that the Institute make available to high school students information regarding the field of electrical engineering, and later more detailed recommendations of a subcommittee of the Committee on Student Branches, the Board of Directors authorized the publication of a pamphlet for this purpose, and the material is being prepared.

Section and Branch Statistics.—

	For fiscal year ending			
	April 30 1925	April 30 1927	April 30 1929	April 30 1931
SECTIONS				
Number of Sections	49	52	54	59
Number of Section meetings held	386	431	460	491
Total attendance	49,029	60,708	73,254	108,523
BRANCHES				
Number of Branches	82	95	100	109
Number of Branch meetings held	548	842	940	1,137
Total attendance	27,603	42,650	47,408	51,807

Meetings and Papers Committee.—The arrangement of technical programs for national conventions, coöperation with District meeting program committees, and the reviewing of technical papers have comprised the more important activities of the Meetings and Papers Committee for the year 1930-1931. During the year, the committee has reviewed 277 papers; of these 175 have been recommended for publication in the TRANSACTIONS, others are awaiting scheduling for future meetings, a number were recommended for ELECTRICAL

ENGINEERING publication only, and a few were returned to the authors. The committee members have, furthermore, assisted the Committee on Award of Institute Prizes in the selection of papers deserving of final consideration for national Institute paper prize awards.

Three national conventions and five District meetings were held during the year. 229 papers were presented, including 15 annual reports of technical committees. This represented an increase of 83 papers over the number which was presented during the previous year or an increase of more than 50 per cent. Many excellent contributions to the theory and practise of electrical engineering in its various fields are contained in these papers. Details regarding attendance and the number of papers presented at each meeting are given above in the reports on the various conventions and meetings.

The policy of requesting discussors to submit their discussion in writing was inaugurated as a trial at the Summer Convention, 1930. Discussors were requested to submit their remarks in writing within two weeks after the close of the meeting. Results attained indicate that discussion prepared in writing is more comprehensive than discussion given extemporaneously. The adoption of this policy also saves the expense of engaging a reporter, and results in a saving of time in the handling of discussion. The plan met with favor and, as a result, it was continued for all of the subsequent meetings throughout the year.

There is at all times an abundance of material for meetings and publication. To a considerable extent the selection of this material should be based on the breadth of the interest in the subjects on the part of the whole membership as well as on the intensity of interest by special groups in papers of limited appeal. In order that the committee may in the future better balance the meeting programs from this point of view, the committee adopted a resolution, April 10, 1931, requesting the Board of Directors to have a survey made of the membership for the purpose of ascertaining the principal fields of electrical interest of each member.

The technical program of the 1931 Summer Convention is completed; the program for the Pacific Coast Convention has taken on definite form; and consideration has also been given to appropriate subjects for technical sessions at the 1932 Winter Convention.

Publication Committee.—The country-wide survey referred to in last year's report having been completed and a careful analysis made of the various opinions expressed in the numerous replies received, the Publication Committee prepared a report which reviewed the situation, and in which recommendations were made that the present policy of publishing the TRANSACTIONS be continued but that certain important changes be made in the JOURNAL.

This report was presented to the Board of Directors at their May, 1930, meeting, but action was deferred pending its consideration at the Conference of Officers, Delegates, and Members at the Summer Convention, held in Toronto, June 25, 1930. At this conference the recommendations of the Committee were discussed with great interest and the report was endorsed with a recommendation that the Board of Directors adopt it as a statement of the Institute's policy regarding publications.

The general policy as recommended by the Publication Committee in its report may be briefly summarized as follows:

TRANSACTIONS.—Continue the present policy of publishing the TRANSACTIONS quarterly and printing papers in full together with discussion thereon.

MONTHLY JOURNAL.—

1. Publish interpretive abstracts of all papers presented at conventions and District meetings, and papers selected from those presented at Section and Branch meetings; and print in full only a limited number of outstanding papers which are of general interest to the engineering profession.
2. Print all annual reports of technical committees in full in both the JOURNAL and TRANSACTIONS.
3. Print in full special articles on timely scientific subjects of particular interest to engineers.
4. Print articles and reviews on recent developments in electrical engineering and related fields, constituting a review of the world-wide achievements in engineering and allied arts and sciences.
5. Record more adequately the outstanding activities of Sections and Branches.
6. Publish important electrical items from the Engineering Index to the extent of possibly four pages of the JOURNAL per month.

The report, including these recommendations, was formally approved by the Board of Directors at the meeting held in Toronto, June 25, 1930, with the understanding that the recommended changes become effective January 1, 1931. At a subsequent meeting, the Board approved the recommendation of the Publication Committee that the title of the monthly publication be changed from JOURNAL to ELECTRICAL ENGINEERING.

The Publication Committee and the National Secretary, after giving careful consideration to the necessary rearrangement and additions to the editorial staff with a view to building up an organization fully capable of successfully executing the recommendations embodied in its report to the Board of Directors, published the first issue of ELECTRICAL ENGINEERING in January, 1931.

This monthly official organ of the Institute which has a much broader scope than the JOURNAL previously published has been favorably received by the membership at large. This is indicated by the many comments received commending the Publication Committee and the editorial staff on their work.

Because of the many questions arising in connection with the new publication policy and because of many other matters coming up from time to time relative to

all of the Institute's publications, the Committee has been meeting with the editorial staff on an average of once a month during the past year for the purpose of giving careful consideration to the various problems as they present themselves.

Coordination Committee.—The Committee corresponded with the officers of all Districts and Sections, and obtained their views regarding national and District meetings during the year 1932, resulting in recommendations to and adoption by the Board of Directors, in January and March, 1931, of a complete schedule of national and District meetings to be held during 1932.

The Committee considered and made recommendations upon various matters referred to it by the Board of Directors and the National Secretary.

Standards Committee.—Three new Standards have been added to the A. I. E. E. series during the past year, as follows: Constant Current Transformers, Air Circuit Breakers, and Switchboards and Switching Equipment for Power and Light. Four other Standards have been revised, *i. e.*, Transformers, Automatic Stations, Insulators, and Recommended Practise for Electrical Installations on Shipboard. Three reports on proposed Standards were likewise issued. Recognizing the economic importance and the demand on the part of users to obtain in service, under appropriate conditions, outputs greater than the test rating, as defined in the A. I. E. E. Standards, there was approved and issued a pamphlet, dealing with "Recommendations for the Operation of Transformers." Progress reports of further development of Standards through the agency of some of the Institute's technical committees indicated that work on two proposed Standards is practically completed and that the development of a series of test codes is likewise well under way.

Working under the procedure of the American Standards Association, the Institute has continued its sole or joint sponsorship in some twenty projects and is, in addition, represented on thirty additional undertakings going forward under the sponsorship of other organizations. A number of the Sectional Committees have submitted reports on proposed American Standards which the Institute has approved as a sponsor, notably the Sectional Committee on Insulated Wires and Cables and that on Scientific and Engineering Symbols and Abbreviations. The Sectional Committee on Electric Welding has been organized and there have been consolidations and realignments of other Sectional Committees in the interest of progress and economy of time. The Sectional Committee on Rotating Electrical Machinery reported recently that the task of revising and coördinating the five Institute Standards before them is now nearing completion and as a result of their work the five Standards will be consolidated and eventually issued in a single pamphlet. The Sectional Committee on Electrical Definitions, of which the Institute is sole sponsor, and whose undertaking is of such proportions as to require the organization of fifteen subcommittees, including three regularly organized Sectional Committees, has been making rapid progress and expects to issue its first report, containing from 2,500 to 3,000 definitions in the fall of 1931.

Committee on Safety Codes.—The most important subject within the Committee's field this year was the recommendation for changes in the National Electrical Code, made to the Electrical Committee of National Fire Protection Association, which Committee met on February 17, 18, and 19, in New York City. The recommendations which had previously had the consideration of Articles Committees were available in printed form, and copies were forwarded to all members of the Committee on Safety Codes, and opinions obtained as to whether the contained matter warranted a meeting for its consideration. The Chairman, and Vice-Chairman as alternate, attended the meeting of the Electrical Committee as representatives of the Committee on Safety Codes.

Committee on the Engineering Profession.—In August, 1929, the President of the Institute was authorized to appoint a "Committee on the Engineering Profession" to consider various matters affecting the status of the engineering profession, to coöperate with the American Engineering Council Committee on Engineering and the Allied Technical Professions, and to assume the duties of the A. I. E. E. Committee on Licensing of Engineers, which was at that time discontinued.

The Board of Directors at the October, 1931, meeting authorized the President to enlarge the Committee to include members located in many parts of the United States, and the personnel was promptly increased to fourteen. A list of the members may be found on page 53 of the January, 1931, issue of ELECTRICAL ENGINEERING.

This Committee has several very important fields of activity, but has, during the past year, devoted its principal efforts to coöperation with other organizations in the development of suitable provisions for the licensing of engineers, as proposed laws and amendments on this subject in various states have demanded a large amount of attention.

U. S. National Committee of the I. E. C.—The Seventh Plenary Meeting of the International Electrotechnical Commission was held June 27-July 9, 1930, with thirty-two delegates representing the United States National Committee, and about 300 representing other countries. The opening plenary meeting was held in Copenhagen, the meetings of the advisory (working) committees were held in Stockholm, and the final plenary meeting to confirm the actions taken by the advisory committees was held in Oslo.

At meetings of seventeen of the advisory committees, during the week beginning June 30, the following important actions were taken:

1. The official adoption by the I. E. C. of names for magnetic C. G. S. units, as follows: magnetic flux, maxwell; flux density, gauss; magnetic field intensity, oersted; magnetomotive force, gilbert.

2. Approval of a revised standard specification for rating of electrical machinery, containing new material relating to tolerances and revised values of temperature rise.

3. Approval of revision of Publication 35, dealing with graphical symbols for heavy-current systems, and of graphical symbols for radio communication.

4. Approval of document on Steam Turbines comprising two parts:

I. Specifications for Condensing Steam Turbines.

II. Rules for Acceptance Tests of Condensing Steam Turbines.

5. The methods of sampling of insulating oils proposed by the U. S. National Committee were accepted for adoption under the six months' rule.

6. Specifications for two classes of a-c. watthour meters and two classes of current and voltage transformers where especially intended for use with watthour meters were adopted.

7. Bases for computing and units for expressing waterpower resources for statistical use and for the purpose of making comparisons between different rivers, river basins, regions, and countries were adopted.

8. In connection with oil switches and circuit breakers, definitions or rules for the following items were agreed upon: operating duty, series of operations, standard series of interrupting operations, standard series of circuit making tests, recovery voltages, making and breaking current (for determining the performance of the circuit breaker), short-time current, power factor, and breaking power and breaking capacity.

In addition to the above, much progress was made in the work of the Commission which does not lend itself to being summarized in any brief way. The vitality, practical usefulness, and predominating influence of the I. E. C. in its field were clearly demonstrated anew.

Professor Enstrom of Sweden was elected President of the I. E. C. in succession to Professor Feldman.

It was decided to hold the next I. E. C. Plenary Meeting in Czechoslovakia in 1934.

Since the I. E. C. meeting, the U. S. National Committee has held several meetings, and is going forward diligently in preparation for meetings of various advisory committees, two of which will be held in the fall: Magnetic Units, in London, September 1931, of which Dr. A. E. Kennelly is Chairman; and Electric Traction Equipment, Brussels, October 1931. It is also proposed to convene meetings of several of the advisory committees during the year 1932.

In accordance with the statutes, the U. S. N. C. was reorganized following the I. E. C. Meeting. It is lending coöperation to the plans which are being developed by the Electrical Advisory Committee for the establishment of an electrical standards committee in the United States.

U. S. National Committee of the International Commission on Illumination.—The primary aim of the Committee during the past year was to arrange for creditable representation of American interests, developments, and opinions at the sessions of the International Commission which will be held in Great Britain, September 2-19, 1931.

The U. S. Committee is particularly charged with four important divisions of the Commission's work: (1) motor-vehicle lighting, (2) factory and school lighting, (3) aviation lighting, and (4) applied lighting practise in fields not otherwise specifically covered. The last two are new assignments, and in these there has been the keenest interest.

A special coördinating committee was formed to serve as a general clearing house for American contributions to the congress and commission sessions outside the scope of the secretariat committees.

Other international organizations sought the assistance of the commission on two important questions of standardization: (1) lamp bases and sockets, and (2) problems of photometry.

Technical Committees.—Reports of technical committees embracing an outline of the year's work and a summary of progress in the industry will be presented at the annual Summer Convention and printed in ELECTRICAL ENGINEERING and the TRANSACTIONS.

Membership.—The work of the Membership Committee has been carried on this year in much the same manner as in previous years, except that the practise of sending names of candidates furnished by the membership at large to headquarters was restored. This practise seems to be favored by the majority of the Section Membership Committee Chairmen.

A number of the Section Membership Committees have improved their organizations for membership work during the year.

The change in the By-laws admitting Enrolled Students to Associate membership without admission fee has had a good effect in increasing the number of such students applying for admission. In accordance with the established policy of the Institute, emphasis has continued to be placed on the qualifications of candidates.

In the absence of specific assignment to any one of the duty of promoting transfers to the higher grades, the Chairman of the Membership Committee has assumed the initiative in this activity by suggesting to the District Vice-Chairmen of the Committee that they undertake this work.

The additions to and deductions from the membership are shown in the following table. The total membership as of April 30, is 18,334.

	Honorary	Fellow	Member	Associate	Total
Membership on April 30, 1930	10	702	3,557	13,734	18,003
Additions:					
Transferred		56	334		
New members qualified		4	126	1,552	
Reinstated			13	113	
Total	10	762	4,030	15,399	20,201
Deductions:					
Died		6	34	61	
Resigned		3	43	422	
Transferred			46	344	
Dropped		2	59	847	
Membership on April 30, 1931	10	751	3,848	13,725	18,334

Deaths.—The following deaths have occurred during the year.

Fellows: Percy E. Hart, Charles D. Knight, C. Wellman Parks, George W. Patterson, William N. Ryerson, Robert J. Scott.

Members: Daniel Adamson, Harry B. Alverson, Arthur N. Anderson, Charles K. Badger, Seymour D. Benedict, Miles W. Birkett, William H. Bristol, Peter A. Coghlin, Edward G. Connette, Edwin F. Creager, Huibert Doyer, Frederick P. Fish, Frank R. Ford, Charles H. Gaffeney, Theodore I. Jones, John H. Lendi, Stanley L. B. Lines, John J. Lyng, Fred D. Lyon, Andrew S. MacDonald, Elwood F. McLaughlin, Paul H. Reinholdt, Andrew L. Riker, Edward P. Roberts, John F. Schoemaker, Elmer A. Sperry, Thomas Sproule, Edward J. Stanton, Ivan H. Summers, Harold L. Turner, Harold M. Walmsley, William F. Willmann, B. C. Wolverton, Beverly L. Worden.

Associates: Nathan B. Ambler, Koh-ichi Arai, Elwood W. Beck, Hyla H. Betts, Alexander L. Black, Edmund H. Bru, Waldo C. Bryant, Edgar E. Campbell, Louis M. Campfield, Voris O. Chewning, William B. Clarkson, Howard R. Connell, G. Stanley Covey, Donald K. Crawford, Alfred H. Crossman, William E. Davis, Samuel W. Dean, Eugene L. Delafield, William K. Detlor, Cyrus L. Doub, McKee Duncan, Leonard W. Egan, Joseph S. Fogerty, Howard H. Fuller, Harold W. Geis, Harold R. Goss, Hans Grabow, Eugene A. Hart, W. L. Henry, David C. Hershberger, Thornton L. High, Charles F. Hill, Benjamin C. Howard, Otto E. Huebner, Harry J. Hunsicker, Oscar C. Larson, J. H. Livsey, John Lundie, Francis MacLehose, Albert B. Mathis, Edmund D. McCarthy, William F. McKnight, Alvin Meyers, William H. Moses, Charles A. Mudge, Irvine M. Noble, Joseph C. Norton, Fenton L. Osgerby, Frederick Platt, Edward B. Plenge, Edward D. Priest, Joseph B. Rawlings, William A. Rosenbaum, Walter H. Seaver, Erik G. Sohlberg, Edward J. Stertman, Ralph E. Thurston, Harold W. Wikle, Robert M. Wilson, Thor Wollebak, Thorsten Zweigbergk.

Board of Examiners.—The Board of Examiners held nine meetings during the past year, averaging about three and one-quarter hours. A total of 3,773 cases were considered, divided as shown in the following table:

Applications for Admissions			
Recommended for grade of Associate	1,129		
Not recommended	32	1,161	
Recommended for grade of Member	130		
Not recommended	33	166	
Recommended for grade of Fellow	3		
Not recommended	1	4	
Recommended for enrollment as Students		2,042	
Applications for Transfer			
Recommended for grade of Member	322		
Not recommended	31	353	
Recommended for grade of Fellow	44		
Not recommended	3	47	
		3,773	

Institute Prizes.—Four National Prizes and fifteen District Prizes for papers were awarded during the year, as announced through the Institute JOURNAL. These prizes were for papers presented at Institute meetings during 1929. The National Prizes were presented at the 1930 Summer Convention and the District Prizes at various meetings in the respective Districts. The National Prizes consist of $100.00 each and a certificate, while the District Prizes consist of $25.00 cash and a certificate.

Scholarships.—The governing bodies of Columbia University have placed at the disposal of the Institute each year a scholarship in electrical engineering for each class. The awards are made annually by an Institute committee. Each scholarship pays $350.00 toward annual tuition, with provision for reappointment.

Complete details governing prizes and scholarships may be obtained by applying to the National Secretary of the Institute.

Edison Medal.—The Edison Medal, founded by associates and friends of Thomas A. Edison, is awarded annually by a committee consisting of twenty-four members of the Institute "for meritorious achievement in electrical science, electrical engineering, or the electrical arts." The medal for 1930 was awarded to Dr. Frank Conrad, "for his contributions to radio broadcasting and short-wave radio transmission." The medal was presented at the Winter Convention of the Institute, January 28, 1931.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of civil, mining, mechanical, and electrical engineers, awarded the twenty-seventh medal to Admiral David Watson Taylor.

Lamme Medal.—The Lamme Medal was founded as a result of a bequest of the late Benjamin G. Lamme, Chief Engineer of the Westinghouse Electric and Manufacturing Company, who died on July 8, 1924. The bequest provides for the award by the Institute of a gold medal (together with a bronze replica thereof) annually to a member of the A. I. E. E. "who has shown meritorious achievement in the development of electrical apparatus or machinery" and for the award of two such medals in some years if the accumulation from the funds warrants.

The Lamme Medal Committee of the Institute awarded the third (1930) Lamme Medal to Dr. William James Foster, "for his contributions to the design of rotating alternating current machinery." Arrangements are being made for the presentation of the medal at the annual Summer Convention at Asheville, N. C., June 22-26, 1931.

Commission of Washington Award.—The Washington Award for 1931 was made to Dr. Ralph Modjeski, "for preëminent service in advancing human progress."

The award is made annually "to an engineer whose work in some special instance, or whose services in general have been noteworthy for their merit in promoting the public good," by a committee composed of nine representatives of the Western Society of Engineers and two each from the A. S. C. E., the A. I. M. E., the A. S. M. E., and the A. I. E. E.

Employment Service.—The Institute coöperates with the national societies of civil, mechanical, and mining engineers in the operation of the Engineering Societies Employment Service with its main office in the Engineering Societies Building, New York. Offices are operated in Chicago and San Francisco also, with the coöperation of the Western Society of Engineers in the former city, and the California Section of the American Chemical Society and the Engineers Club of San Francisco in the latter.

The service is supported by the joint contributions of the societies and their individual members who are benefited. As in the past it consists principally in acting as a medium for bringing together the employer and the employee. In addition to the publication of the Employment Service announcements monthly in ELECTRICAL ENGINEERING, weekly subscription bulletins are issued for those seeking positions.

American Engineering Council.—Since its organization about ten years ago, American Engineering Council has experienced a steady, normal growth. It includes in its membership twenty-six national, state, and local engineering societies which have a total membership of about 60,000.

The Council represents its constituents at conferences, conventions, and congressional hearings upon matters involving engineering and on important national committees. It also extends valuable individual service to its member organizations. Several notable surveys on matters of concern to the entire engineering profession have been conducted.

Through a standing Committee on Engineering and Allied Technical Professions, the Council endeavors to secure information upon and to improve the general status of the engineering profession.

Other standing committees are Constitution and By-laws, Finance, Membership and Representation, Public Affairs, Publicity and Publications, and Regional Activities.

The Council has many special committees, including those on: Airports; Airport Drainage and Surfacing; Air Transport Service in Foreign Commerce; Bridges; Communications; Competition of Governmental Agencies with Engineers in Private Practise; Dams; Flood Control; Government Reorganization; Man-Hour Information; Oil Pollution of Streams; Patents; Power; Program for Research; Reforestation; Representation; Street and Highway Safety; Street Signs, Signals, and Markings; Water Resources and Control; Weather Bureau; and Who's Who in Engineering.

In recent months, the Council, through its headquarter's staff and the A. S. C. E., A. S. M. E., and A. I. E. E., rendered assistance to the President's Emergency Committee for Employment. Some of the other important problems which have recently been considered are: the use of engineering works as means of alleviating the effects of the economic depression; methods of balancing the forces of production, distribution, and consumption; government coöperation with engineers; methods of making the work of the U. S. Weather Bureau of more use to engineers; flood control; studies of earthquakes; prompt publication of water supply and other data of the U. S. Geological Survey; government regulation of communication systems; pollution of navigable waters; single court of patent appeals; aircraft equipment tests; Federal relations to education; bridge legislation; Muscle Shoals; Federal Department of Public Engineering Works; and Ship model towing basin.

The Council publishes the American Engineering Council Bulletin monthly except July and August.

United Engineering Trustees, Inc.—This organization, formerly United Engineering Society, was set up by the four national societies of civil, mining, mechanical, and electrical engineers to hold in trust and to administer for them the Engineering Societies Building, in which their headquarters are located, certain funds, and the Library. Its charter gives it broad powers for the advancement of the engineering arts and sciences.

Extracts from the annual report of United Engineering Trustees, Inc., were published in the April, 1931, issue of ELECTRICAL ENGINEERING.

Engineering Foundation.—A department of United Engineering Trustees, Inc., was established in 1914 by the national societies of civil, mining, mechanical, and electrical engineers "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." It was conceived by Ambrose Swasey, of Cleveland, Ohio, and he has made three gifts towards its endowment. The fund has been generously increased through the gifts of Edward D. Adams and others, and also through a bequest of the late Henry R. Towne.

Appropriations have been made for various research projects and coöperation has been extended in others.

Engineering Societies Library.—The Library is administered as a free public library under the direction of the Library Board of United Engineering Trustees, Inc., this Board being composed of representatives of the national societies of civil, mining, mechanical, and electrical engineers.

The Library contains nearly 200,000 books and pamphlets. It receives regularly about 1,200 technical periodicals in many languages, and about 800 additional publications issued irregularly.

A staff of technically trained searchers and translators is maintained. The staff is prepared to furnish the following types of service: photoprints, abstracts, translations, bibliographies, searches, etc. Special arrangements have been made for lending books.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years. A complete list of representatives is published in the September and January issues of ELECTRICAL ENGINEERING.

Finance Committee.—During the year the Committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins & Sells, certified public accountants, have audited the books, and their report follows.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON,

National Secretary.

May 19, 1931.

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS

OFFICES IN THE PRINCIPAL CITIES OF THE UNITED STATES OF AMERICA
—AND IN—
LONDON, PARIS, BERLIN, SHANGHAI, MANILA, MONTREAL, HAVANA, MEXICO CITY

CABLE ADDRESS "HASKSELLS"

NEW YORK CENTRAL BUILDING
75 EAST 45TH STREET
NEW YORK

May 16, 1931.

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

We have audited your accounts for the year ended April 30, 1931, and submit the following exhibits and schedule:

Exhibit

A—Balance Sheet, April 30, 1931.
Schedule
1—Reserve Capital Fund—Securities.

B—Summary of Income and Surplus for the Year Ended April 30, 1931.

WE HEREBY CERTIFY that in our opinion Exhibits A and B set forth the financial condition of the Institute at April 30, 1931, and the results of its operations for the year ended that date.

Yours truly,
HASKINS & SELLS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

EXHIBIT A. BALANCE SHEET, APRIL 30, 1931

ASSETS

REAL ESTATE:			
One-fourth interest in Land, Building, and Equipment of United Engineering Trustees, Inc.,—25 to 33 West 39th Street (depreciation carried on books of the United Engineering Trustees, Inc.)			$496,948.48
EQUIPMENT:			
Library—Volumes and fixtures		$37,796.37	
Works of art, paintings, etc.		3,001.35	
Office furniture and fixtures	$31,226.33		
Less reserve for depreciation (including $10,431.79 funded)	15,510.78	15,715.55	
Total equipment			56,513.27
WORKING ASSETS:			
"Transactions," etc.		$ 5,354.90	
Text and cover paper		1,904.89	
Badges		1,734.31	
Total working assets			8,994.10
CURRENT ASSETS:			
Cash		$ 9,151.39	
Accounts receivable:			
Members—For dues		27,317.95	
Advertisers		1,632.84	
Miscellaneous		2,064.30	
Accrued interest on investments		3,349.96	
Total current assets			43,516.44
FUNDS:			
Reserve Capital Fund:			
Cash	$ 5,621.75		
Securities—Schedule 1	214,010.50	$219,632.25	
Life Membership Fund:			
Cash	$ 6,559.99		
Chicago, Burlington & Quincy Railroad 4% bonds, 1958, registered, face value, $5,000.00	4,868.75		
Accrued interest	33.33	11,462.07	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 1,295.61		
New York City 4½% corporate stock, 1957, par value, $2,000.00	2,204.05		
New York Telephone Company 4½% bond, 1939, registered, face value, $1,000.00	878.75		
Accrued interest	67.50	4,445.91	
Mailloux Fund:			
Cash	$ .17		
New York Telephone Company 4½% bond, 1939, registered, face value, $1,000.00	1,000.00		
Accrued interest	22.50	1,023.67	
Lamme Medal Fund:			
Cash	$ 280.28		
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series C bond, 1995, face value, $4,000.00	4,330.00		
Accrued interest	100.00	4,710.28	
Depreciation of Furniture and Fixtures Fund:			
Cash	$ 366.79		
Cleveland Union Terminals Company 5% sinking fund series B gold bonds, 1973, registered, face value, $4,000.00	4,010.00		
Fidelity Union Title and Mortgage Guaranty Company first mortgage certificates (on property, Nos. 75-79 Prospect Street, East Orange, N. J.) 5½%, due 1933	1,000.00		
United Gas Improvement Company $5.00 preferred stock, 20 shares	1,995.00		
Consolidated Gas Company of New York $5.00 cumulative preferred stock, 30 shares	3,060.00	10,431.79	
Total funds			251,704.97
TOTAL			$857,677.26

LIABILITIES

CURRENT LIABILITIES:		
Accounts payable	$11,556.34	
Dues received in advance	2,489.70	
Entrance fees and dues advanced by applicants for membership	275.00	
Subscriptions for "Transactions" received in advance	303.00	
Total current liabilities		$ 14,624.04
FUND RESERVES (not including depreciation reserve):		
Reserve Capital Fund	$219,632.25	
Life Membership Fund	11,462.07	
International Electrical Congress of St. Louis—Library Fund	4,445.91	
Mailloux Fund	1,022.67	
Lamme Medal Fund	4,710.28	
Total fund reserves (not including depreciation reserve)		241,273.18
SURPLUS, Per Exhibit "B"		601,780.04
TOTAL		857,677.26

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Summary of Income and Surplus For the Year Ended April 30, 1931

Exhibit B

Income:			
Dues		*$233,169.04	
Students' fees		10,423.50	
Entrance fees		12,229.00	
Transfer fees		1,975.00	
Advertising		75,439.78	
"Electrical Engineering" subscriptions		8,863.34	
"Transactions" subscriptions		12,340.10	
Sales—Miscellaneous publications		10,553.96	
Sales—Badges	$ 3,233.75		
Less cost	2,895.13	338.62	
Interest on securities in Reserve Capital Fund		11,130.66	
Interest on securities in Depreciation Reserve Fund		210.84	
Interest on bank balances		951.20	
Total			$377,625.04
Expenses:			
Publications:			
"Electrical Engineering"	$93,761.05		
"Transactions"	33,428.73		
Technical papers	16,320.94		
Year book	9,700.86		
Miscellaneous	4,556.13	$157,767.71	
Institute meetings		19,239.40	
Administrative expenses		63,363.08	
President's special appropriation		639.89	
Board of Directors, traveling expenses		4,422.15	
National nominating committee, traveling expenses		948.97	
Institute representatives, traveling expenses		13.25	
Sections		35,371.30	
Geographical districts' expenses:			
Traveling expense:			
Executive committees	$ 2,873.88		
Vice-Presidents	1,350.82		
District best paper prizes	175.00		
District prizes for initial paper	150.00		
Institute lecturers	464.25	4,913.95	
Branches		10,493.86	
Membership		8,903.67	
Forward		$306,077.23	$377,625.04

*Includes $60,015.00 allocated to "Electrical Engineering" subscriptions.

Total Income (Forward)		$377,625.04
Expenses (Forward)	$306,077.23	
Finance	419.74	
Code Committee	60.00	
Technical Committee	205.88	
Headquarters Committee	240.48	
Standards	7,602.13	
American Standards Association	1,500.00	
International Electrotechnical Commission	1,207.25	
Engineering Societies Library, maintenance	10,297.48	
Engineering Societies Employment Service	204.37	
United Engineering Trustees, Inc., assessment	5,713.56	
American Engineering Council	17,607.50	
Institute prizes	418.00	
Edison Medal Committee	63.92	
Edison Statue	500.00	
United States National Committee of International Commission on Illumination	300.00	
Total		352,417.54
Net Income		$ 25,207.50
Surplus Credits:		
Increase in value of equity in building	$ 3,595.88	
Adjustment of Life Membership Fund Reserve—Excess of fund over required amount as determined by the Institute in accordance with the by-laws	995.65	
Total		4,591.53
Gross Surplus for the Year		$ 29,799.03
Surplus Charges:		
Uncollectible dues and members' charges written off	$ 9,159.32	
Provision for depreciation of furniture and fixtures	2,260.87	
Loss on disposal of office fittings	94.36	
Decrease in value of library	3,765.86	
Total		15,280.41
Surplus for the Year		$ 14,518.62
Surplus, May 1, 1930	$595,882.42	
Less transferred to Reserve Capital Fund in accordance with resolution of the Board of Directors	8,621.00	587,261.42
Surplus, April 30, 1931		$601,780.04

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Reserve Capital Fund—Securities

April 30, 1931

Exhibit A.
Schedule No. 1.

	Face Value or Number of Shares	Book Value
Railroad Bonds:		
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series C bonds, due 1995, registered	$ 8,000.00	$ 8,940.00
Central of Georgia Railway Company 5½% refunding and general mortgage series B bonds, due 1959, registered	5,000.00	5,283.75
Chicago and Erie Railroad Company 5% first mortgage gold bond, due 1982, registered	1,000.00	1,105.00
Chicago, Burlington & Quincy Railroad Company 5% first and refunding mortgage series A gold bond, due 1971, registered	1,000.00	1,010.00
Chicago, Terre Haute & Southeastern Railway Company 5% first and refunding mortgage gold bonds, due 1960, registered	8,000.00	7,940.00
Florida East Coast Railway Company 5% first and refunding mortgage series A gold bonds, due 1974, registered	10,000.00	9,818.75
Great Northern Railroad Company 5½% general mortgage series B gold bonds, due 1952, registered	10,000.00	9,847.50
New York Central Railroad Company 5% refunding and improvement mortgage series C bonds, due 2013, registered	6,000.00	5,742.50
Pennsylvania Railroad Company 4½% general mortgage series A gold bonds, due 1965, registered	5,000.00	5,130.00
St. Louis, San Francisco Railway Company 5% prior lien mortgage series B bonds, due 1950, registered	6,000.00	5,497.50
Southern Railway Company 5% first consolidated mortgage gold bonds, due 1994, registered	1,000.00	980.00
Total		$ 61,295.00
Public Utility Bonds:		
American Telephone and Telegraph 5% sinking fund gold debentures, due 1960, registered	$15,000.00	$ 14,625.00
Consolidated Gas Company of New York 5½% gold debentures, due 1945, registered	5,000.00	5,187.50
Duquesne Light Company 4½% first mortgage series A gold bonds, due 1967, registered	3,000.00	2,970.00
New York Telephone Company 6% gold debenture bonds, due 1949, registered	5,000.00	5,525.00
Northern States Power Company 5½% first lien and general mortgage gold bonds series B, due 1950, registered	10,000.00	10,400.00
Ontario Power Service Corporation, Limited, 5½% first closed mortgage sinking fund gold bonds, due 1950, registered	5,000.00	4,711.00
Pacific Gas & Electric Company 5½% first and refunding mortgage series C gold bonds, due 1952, registered	5,000.00	5,137.50
Philadelphia Company secured 5% series A gold bonds, due 1967, registered	10,000.00	10,000.00
Shawinigan Water and Power Company 4½% first mortgage and collateral trust sinking fund series A gold bonds, due 1967	5,000.00	4,581.25
Texas Electric Service Company 5% first mortgage gold bonds, due 1960, registered	4,000.00	3,910.00
United Light and Power Company 5½% first lien and consolidated mortgage gold bonds, due 1959	5,000.00	4,975.00
Total		$ 72,022.25

	Face Value or Number of Shares	Book Value
Total Railroad Bonds (Forward)		$ 61,295.00
Total Public Utility Bonds (Forward)		72,022.25
Industrial Bonds:		
American Smelting and Refining Company 5% first mortgage series A gold bonds, due 1947, registered	$ 9,000.00	$ 9,085.00
Bethlehem Steel Company 5% purchase money and improvement mortgage sinking fund gold bonds, due 1936, registered	5,000.00	5,033.75
Fidelity Union Title and Mortgage Guaranty Company first mortgage certificates (on property Nos. 75-79 Prospect Street, East Orange, N. J.) 5½%, due 1933	14,000.00	14,000.00
International Match Corporation 5% convertible gold debentures, due 1941	3,000.00	2,880.00
New York Steam Corporation 6% first mortgage gold bonds, due 1947, registered	10,000.00	10,837.50
United States Rubber Company 5% first and refunding mortgage series A bonds, due 1947, registered	2,000.00	1,915.00
Western Electric Company 5% debentures, due 1944, registered	10,000.00	9,818.75
Youngstown Sheet and Tube Company 5% first mortgage sinking fund series A gold bonds, due 1978, registered	20,000.00	20,275.00
Total		$ 73,845.00
Stocks:		
Commonwealth Edison Company	12 shares	$ 2,892.00
Public Service Corporation of New Jersey, $5.00 preferred	30 "	2,958.75
United Gas Improvement Company, $5.00 preferred	10 "	997.50
Total		$ 6,848.25
Total		$214,010.50

Burn-Off Characteristics of A-C. Low-Voltage Network Cables

By GEORGE SUTHERLAND[1] and D. S. MacCORKLE[1]
Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis.—This paper describes tests made to determine the characteristics of the clearance of faults in several types of a-c. low-voltage network copper-conductor cables installed in accordance with present standards of underground construction, and includes a discussion of test results together with other relative data studied. The investigation included the burning clear of various types of faults in buried non-magnetic sheath cables, and non-metallic sheath and lead-covered cables for duct installation. A comparison is made of the burn-off characteristics of the several cable constructions and their installations.

The amount of short-circuit current and length of time required to clear the different types of faults in No. 4/0, 350,000- and 500,000-cir. mil cables have been determined by test and by calculations in some instances. Conclusive data were not obtained to clear up all present conflicting opinions related to the burn-off characteristics of a-c. low-voltage network cables. However, a study of the subject matter would seem to establish at least a partial basis in regard to burn-off characteristics for determining the conductor size of low-voltage network mains and the spacing of network transformers. Desirable developments for a-c. network cables indicated by the results of this investigation are emphasized.

* * * * *

I. Introduction

OPERATION of underground, low-voltage network systems, designed to supply continuous electric service, has proved the value of data derived from a thorough investigation into the inherent characteristics of a given design to clear faults. The maximum reduction of service outages, prevention of manhole fires, explosions and other system disturbances due to the occurrence of faults are therefore receiving increased consideration of operating and design engineers.

The clearing of faults in the cables of a network depends upon; (1) the relative rates of generation and dissipation of heat in the fault and in the cable conductors joining the fault; (2) physical conditions at the fault location; and (3) the effect of gases generated if an arcing condition is established.

The heat generated in the fault is a function of the contact resistance and the square of the short-circuit current flowing into the fault. The heat is dissipated principally by conduction and radiation, previous to the existence of an arc. The clearance of the fault at its initial location will depend usually upon its inability to dissipate the heat generated, provided that the relative amounts of heat generated and dissipated in the conductors joining the fault are such that the latter conductors will not fail first. The thermal capacity of the fault, relative to the heat generated, is in this case less than that of the cable conductors. In this class, a high fault resistance is associated with a low rate of heat dissipation. As will be seen from the discussion in this paper, particular consideration has been given to this class of faults, because most faults occurring in practise first exist as a fault of this class.

If the relative rates of generation and dissipation of heat in the cable conductors joining the fault are such that the fusing temperature is attained in the conductors before the fault itself burns clear, clearing will take place at a location other than the initial location of the fault. Relative to the amounts of heat generated in the fault and in the conductors, the thermal capacity of the fault is greater than that of the cable conductors.

The cables used generally in the past for low-voltage underground network systems have been rubber or paper insulated and lead covered. Operating experience with low-voltage (115/199, 120/208 and 115/230 volts) underground systems of standard construction, and previous tests, prove that faults between the lead sheath and a conductor have cleared instantly or within a few seconds with little injury to the faulty cable and generally no serious injury, if any, to adjacent cables. Previous tests have shown also that a copper-to-copper fault, in network systems of phase-to-phase voltages of the order of 208 volts, formed by loosely wrapping the conductors with binding wire, or by a spike as large as 3/8 of an inch in diameter, being driven into stranded 500,000-cir. mil conductors will usually clear in a short interval of time (less than 30 seconds) with a total minimum value of current of approximately 2,500 amperes or greater (within certain current limits depending on cable construction). The quick clearance of the faults depends to a great extent on the relative positions of the cables in the duct at the fault location. It seems questionable at present as to whether the greater number of explosions and manhole fires occurring in network systems have been due to ignition of sewer and escaped illuminating gases or gases generated from cable materials subjected to short-circuit conditions. One preliminary study[2] has indicated that most explosions occurring during a five-month period of careful observation were probably due to the volatilization of cable or joint insulation and rapid combustion due to the intense heat of the electric arc.

The use of cables suitable for burying in the ground

1. Both of the New York and Queens Electric Light and Power Company.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

2. "Study on Failures in A-C. Low-Voltage Network Cables," by J. M. Comly, Brooklyn Edison Company, Inc., and J. A. McHugh, The New York Edison Company.

31-66

without conduit for low-voltage network distribution is a new field of distribution engineering in this country. The burn-off characteristics of this cable, made up in several types of construction and installed under several proposed conditions, were unknown previous to the making of tests described herein. Before the installation of any buried cables in a network system, even as an experimental investigation, it was desirable that tests be made to determine the clearing of faults which might occur at rare intervals in such a system.

II. Scope

An attempt has been made to determine the burn-off characteristics of cables installed under actual service conditions. A total of 86 tests was made to check some of the results of previous investigations, and in addition, to determine the advantages or disadvantages of new cable constructions installed under several conditions.

Copper conductor cables with various insulations and coverings were used for test. It is recognized that other pure metals or metal alloys may have better burn-off characteristics than copper. Cables with conductors of metal other than copper constitute a field for a separate investigation, and are not further discussed in this paper.

It has been the purpose of this paper to study the burn-off characteristics of the several types of copper-conductor cable under consideration with special emphasis on the following:

1. The amount of current and length of time required to burn clear copper-to-copper and ground faults representative of those occurring in practise in the types

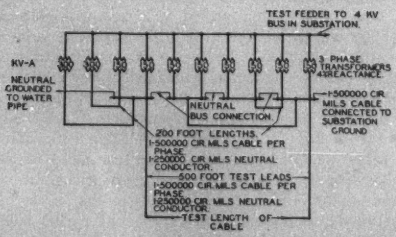

Fig. 1—Circuit Diagram—Tests Made May 19, 1929

and sizes of cables proposed and commonly used for a-c. underground low-voltage network distribution.

2. A comparison of the self-clearing of three-phase, phase-to-phase, phase-to-neutral, and ground faults in the several types of cables tested.

3. The relative burn-off characteristics of multiple-conductor and single-conductor buried cables.

4. The effect on burn-off characteristics of the spacing of buried cables.

5. The amount of flame, smoke and gas due to the cable insulation and covering burned under short-circuit conditions.

6. The advantages, if any, of paper insulation compared with rubber insulation.

7. The burn-off characteristics of non-metallic sheath cables for duct installation compared with lead-covered cables.

8. The burn-off characteristics of lead-covered cables in fibre duct, tile duct, and iron pipe.

9. A comparison of the explosibility of gases generated in the duct line under the conditions of Item 8.

III. Test Equipment

Provision was made for isolating a 13,200-volt feeder connected to an isolated substation bus section to which

Fig. 2—Water-Cooled Resistors Removed from Barrels

a-c. 60-cycle power was supplied. The voltage was reduced to 4,000 volts, phase-to-phase, through a 4,800-kva. oil-insulated, self-cooled transformer. Between the 4,000-volt bus and the 4,000/230-volt network transformers, the secondary terminals of which were connected to the low-tension test circuit, were connected three single-phase ten per cent voltage regulators. The distance from the 4,000-volt substation bus to the network transformers was approximately 300 circuit-feet. The network transformers, consisting of two 300-kva. and eight 150-kva. transformers, were connected for some of the tests, as shown in Fig. 1, so that 900 kva. capacity supplied power to each side of the fault in the test cable. This test set-up is nearly equivalent to a network layout for city blocks, 200 by 1,000 ft., having a 300-kva. transformer installed at each intersection and a fault at the center of the long side of the block.

Cable reel reactors, water-cooled wire resistors, (Fig. 2) and spaced lengths of cable were used in several combinations to limit the short-circuit current to the desired values. High-speed graphic ammeters, mounted on a temporary wooden panel, were used for recording current values. The secondary bus voltage of the transformers was determined by means of indicating voltmeters.

Each test length of buried cable was located approximately 18 in. beneath the surface of the ground. When the cables were spaced, the neutral weatherproof, braided conductor was placed at one side of the trench and the three cables for buried installation were laid in parallel positions. The length of buried cables subject

to burn-off was approximately 17 ft. Various joints and service taps, simulating field conditions were made in the lengths of cable under test.

The cables for duct installation were pulled into four-inch ducts between distribution boxes, made for test purposes and located approximately 75 ft. apart. These boxes were approximately 24 in. wide and 28 in. in depth and length. They were equipped with standard reinforced concrete covers. Standard three-way joints were made in the boxes at one end of the test length. At the other end the cables were not spliced, but were pulled directly through the boxes (Fig. 10).

The various types of faults and cable constructions tested will be described under the heading, Discussion and Analysis of Test Results.

IV. Test Procedure

All signals for control operations were given by the blowing of a whistle, a code having been agreed upon for each operation.

The graphic meter charts were placed in operation before the oil circuit breakers in the test feeder were closed so that a complete record of current values was obtained. The secondary bus voltage of the network transformers was read from the indicating meters in order to detect any trouble that may have developed in the test equipment, and to provide comparative data for an analysis of results. The bus voltage in the substation was also noted throughout each test.

Careful observation was made of the flames, smoke and explosion, if such occurred.

Each test length of cable was carefully inspected after the tests were completed to determine the condition of the insulation and coverings, and the location where clearing took place.

Samples of gas, generated in representative tests on cables in ducts, were collected by means of high percentage rubber tubing and inverted bottles of water during the arcing conditions. Analyses of these samples were made for comparative purposes.

The short circuit was energized in all tests, except two, until the fault was cleared.

V. Discussion and Analysis of Test Results

After making several tests in which cable reel reactors were used to limit the short-circuit current, it was suggested that the highly inductive circuit might possibly aid the sustenance of the arc. Accordingly, tests were made using long cable leads to limit the current. Results indicated that the high inductance of the test circuit first used did not affect the arc at the low test voltage, 133/230 volts, and with those values of current used. The ratio of calculated resistance to the calculated reactance of the test circuit varied between the approximate limits of 0.20 and 0.70. This question was not given any further consideration in the remaining tests conducted.

Tests on Multiple-Conductor Buried Non-Magnetic Sheath Cables. Eleven tests were made on five-conductor cables constructed as indicated in Fig. 3A. Three tests were made on five-conductor cables constructed as indicated in Fig. 3B. As shown in both figures, the insulating material was a rubber compound. In both constructions, the maximum separation of phase conductors was 0.3 in., the minimum separation, 0.2 in.

A heavy iron clamp, squeezing the conductors (No. 4/0 phase conductors, No. 2/0 neutral) together, the insulation and covering having been removed throughout a length of approximately six inches, was used in making the faults in four tests. This produced a fault having a thermal capacity, relative to the amount of heat generated, greater than that of the cable conductors. Consequently, at values of total initial short-circuit current of approximately 4,000 amperes these faults did not clear; but the insulation was charred throughout the test length between the joints, and failure tended to take place at the joints where two or more conductors joined the one at fault.

At values of total current, approximately 11,000

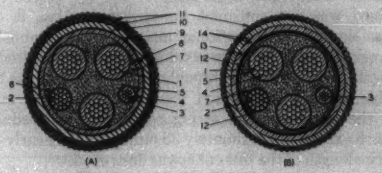

Fig. 3—Two Types of Construction of Five-Conductor Buried Non-Magnetic Sheath Cables

Note:

1—No. 4/0 copper conductor, 19 strands
2—No. 2/0 copper conductor, 19 strands
3—No. 6 copper conductor, 7 strands
4—Special rubber compound
5—Rubber-filled cotton tape
6—Weatherproof cotton braid
7—Impregnated jute filler
8—Saturated asbestos braid
9—Paper separator
10—Sisal braid
11—Helically applied jute wrap
12—Impregnated fabric tape
13—Lapped bronze tape
14—Impregnated fibre tape

amperes flowing into the fault, there was such concentration of heat at the edge of the clamp that fusing of the conductors took place, causing failure before the insulation between conductors in the adjacent lengths of cable was destroyed enough to allow a progression of the fault. Clearing of the fault took place at this value of current in approximately 35 seconds.

In one test, a pneumatic cable stabber was used to produce a fault in an energized test length of five-conductor cable, Fig. 3A. The blade of the stabber severed the cable except for a small section of the covering on one side. The total values of initial short-circuit current in the three-phase conductors and neutral were 2,000, 1,280, 1,600 and 800 amperes, respectively. The fault cleared instantly, approximately an inch of the blade being destroyed by the arc.

A fault was also produced in the test length of cable mentioned in the foregoing paragraph by driving a bullpoint into it. Clearance took place instantly, the total initial values of short-circuit current in the phase

conductors being 1,900, 2,200 and 2,500 amperes. It is reasonable to assume that a fault produced by driving a pick axe into the cable would clear in a similar manner.

Another fault was made in a test length of cable of the construction indicated in Fig. 3A by baring the conductors and arranging a 2 by 4-in. plank so as to force the conductors together when the plank was struck with a sledge hammer. The insulation between conductors to be short-circuited was removed throughout a length of approximately two inches. This method produced a contact between conductors and a phase-to-phase fault of this type cleared in 20 seconds with a total value of current varying from 0 to 2,480 amperes. A second test on the same cable with a two-phase-to-neutral fault, made in the same manner, failed to clear at the point of fault with a total value of current of about 3,800 amperes. It seems that a point contact was formed in the first case, which was cleared easily with the low value of current. But, in the second case, a fault of greater contact surface than that of a point contact must have been formed; and the current obtained was not sufficient for clearing. Also, it was possible that the collection of molten copper established a metallic path between conductors, which resulted in burning the insulation throughout the length of the test cable, making subsequent dielectric breakdown easy. In the latter test, an arcing condition was produced which existed approximately 35 minutes until nearly the entire length of the test cable was destroyed.

Two tests on multiple-conductor cables were made with the conductors grounded to a water pipe which was connected to the city mains. A fault made by grounding one of the phase conductors and the neutral on the water pipe cleared in approximately two minutes at the location of the fault with a total value of current, remaining nearly constant, of 3,750 amperes. There was considerable arcing and emission of steam due to water, from the hole burned in the pipe, coming in contact with the arc. The fault was entirely submerged in water before clearing took place. The other phase conductors were not involved in the fault. In the second test in which one phase conductor was grounded to the water pipe, the maximum current obtained during the test had a total value of 2,900 amperes. This fault did not clear due to the low value of current (approximately 400 amperes) that resulted during the first 9.5 minutes of the eleven-minute period that the circuit breaker of the test feeder was left closed.

In the majority of the tests, the fault was made up by baring the conductors for a length of about six inches, forcing them together, and taping tightly overall with friction tape to hold the conductors in contact and completely covering the bared portion of the conductors. This method gave a contact length between conductors of approximately three inches. This is probably the worst type of fault that would occur initially in the low-voltage mains of a network system, except when a fault is produced by mistake in making connections to the network switch (or similar operations), or when the insulation and covering may be charred throughout a considerable length of cable by low values of short-circuit current, allowing the conductors to come in contact. Unless otherwise stated, faults mentioned in the discussion which follows will be assumed to be of this type (three-inch contact fault).

A three-phase-to-neutral fault in five-conductor cable of the construction indicated in Fig. 3A burned for 42.5 minutes with a total initial value of current of 3,200 amperes. Another burned for 35 minutes with a total initial value of current of approximately 3,900 amperes.

Five burn-off tests were made on four-conductor buried cable containing three No. 4/0 phase conductors and one No. 2/0 neutral; and two burn-off tests were made on three-conductor buried cable containing three No. 4/0 phase conductors, with a single-conductor, No. 2/0 weatherproof braided cable as a neutral. Test results obtained with these two latter types indicate that the burn-off characteristics are the same as for the five-conductor buried cables tested under the same conditions. The latter statement is supported by the following data. A test length of four-conductor, paper-insulated cable with a three-phase-to-neutral fault burned for 36 minutes with an initial value of total short-circuit current of 6,200 amperes; and cleared in the splice box at one end of the test length, and at the point where the test leads were connected at the other end. Similarly, a test length of the three-conductor cable burned for 23 minutes with an initial value of total short-circuit current of 4,800 amperes.

The four-conductor cable was insulated with sulfinated oiled paper. The three-conductor cable was insulated with a rubber compound of the same composition as that of the insulation of the five-conductor cable, Fig. 3A. The separation of conductors in both constructions was the same as that of the five-conductor cable.

From the tests made on the multiple-conductor cables, the data obtained indicate that, with values of total initial short-circuit current of 6,200 amperes and less flowing into the fault, self-clearing of the fault is very unsatisfactory; and when an arcing condition has been established, the fault tends to progress from its initial location, completely destroying the cable and leaving molten metal in the earth where the cable was buried. The destruction of the cable produced by such an arcing condition is indicated in Fig. 4.

For values of total initial short-circuit current between 4,000 and 6,500 amperes clearance under such arcing conditions usually takes place at joints where two or more conductors join the one at fault. Without actual test data available, it is difficult to predict what minimum value of current is required to clear an arcing fault in the multiple-conductor cable; but, it is evident

from the data obtained that such value of current is higher than the corresponding value for single-conductor buried cable, as shown from information which follows.

With the cables buried approximately 18 in. beneath the ground surface and with no protective covering, arcing faults produced flames of a maximum height of nearly 5 ft. above the ground. Smoke and volatilized

Fig. 4—Remains of Four-Conductor Buried Cable After Test

Total value of current varied from 0 to 6,200 amperes during period of 33 minutes, clearing fault

materials were blown from the holes in the soil formed by escaping gases.

Single-Conductor Buried Non-Magnetic Sheath Cables. Following are the constructions of single-conductor buried non-magnetic sheath cables tested for burn-off characteristics and the number of tests made on each.

	Construction	No. of Tests
1.	No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound One wrap of double-faced rubber-filled cotton tape Two wraps of tarred jute applied in reverse directions served with asphalt compound One treated burlap wrap, in reverse direction to last jute wrap Two wraps of heavy asphalted jute Soapstone finish	3
2.	No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound One wrap of single-faced rubber-filled cotton tape Serve of heavy asphaltum compound One wrap of asphalted jute Two rubber-filled fiber armoring tapes Serve of heavy asphaltum compound One wrap of jute saturated with asphalt compound Soapstone finish	3
3.	No. 4/0 copper conductor, 19 strands 110 mils of special rubber compound One wrap of single-faced rubber-filled cotton tape 5 mils of bronze tape Serve of tar pitch Two wraps of fiber tape Serve of tar pitch One wrap of jute Special flame-proof compound Soapstone finish	5
4.	No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound 125 mils of 60 per cent rubber compound	
5.	No. 4/0 copper conductor, 19 strands 80 mils of special intermediate rubber compound One wrap of single-faced rubber-filled cotton tape Serve of asbestos base caulk Impregnated asbestos braid Serve of asbestos base caulk Two wraps of non-hydroscopic kraft armor Serve of asbestos base caulk One wrap of asphalted jute Serve of special finishing compound Soapstone finish	5
6.	No. 4/0 copper conductor, 19 strands 20 mils of vulcanized sulfinated oiled paper One wrap of single-faced rubber-filled cotton tape Remainder of covering, same as No. 5	12
7.	No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound One wrap of single-faced rubber-filled cotton tape 95 mils of 40 per cent rubber compound One wrap of impregnated burlap tape Serve of special asphalt compound One wrap of impregnated burlap tape Serve of special asphalt compound One wrap of impregnated duck tape Serve of special finishing compound Soapstone finish	3
8.	No. 4/0 copper conductor, 19 strands 80 mils of 30 per cent rubber compound One wrap of single-faced rubber-filled cotton tape Serve of special asphalt compound One wrap of tar paper tape Serve of special asphalt compound One impregnated burlap tape Serve of special asphalt compound One impregnated burlap tape Serve of special asphalt compound One impregnated duck tape Serve of special finishing compound Soapstone finish	3

The greater number of these tests were made with the three No. 4/0 phase cables and a No. 2/0 weatherproof braided neutral cable placed at random in the trench and buried. These tests were made in both wet and relatively dry soils. The fault was not submerged in water in any of these tests.

The results indicate that, with the No. 4/0 single-conductor buried cables spaced approximately 1.5 to 2 in., cover to cover, a total minimum initial value of current of approximately 6,000 amperes, in the test circuit used, is required to clear satisfactorily the three-inch contact fault. (Fig. 5). The clearing time required with this value of current may generally be expected to be in the order of one minute.

With total values of initial short-circuit current

Fig. 5—Single-Conductor Buried Cables Showing Fault Cleared at Initial Location

Total value of current of 5,800 amperes cleared fault in 40 seconds

somewhat less than 6,000 amperes and with the cables buried together at random, it was found that long time arcing conditions were generally produced. The clearing action under such conditions was usually found to be the same as that experienced with the multiple-conductor buried cables. When no protective covering was used with the lower values of current, flames broke through the ground to various heights, the maximum of which was ten feet. These flames were burning gases from the cable coverings and insulation and continued to burn as long as the arc produced such gases.

In order to obtain some indication as to the flames and smoke due to burning of the insulation and coverings and the effect of the gases, so produced, on the clearing action of the fault, two tests were made on No. 4/0 bare conductors buried in the ground. In one test, the conductors were buried in a relatively dry mixture of clay and sand with a spacing between conductors of 1 to 1.5 in. A three-phase-to-neutral fault cleared in 6 min. and 20 sec., approximately one foot on each side of the location of the initial fault, with a total initial value of current of 5,000 amperes. A second test was made with the conductors spaced and buried in relatively dry loam soil containing some cinder and waste material. A three-phase-to-neutral fault cleared in six and one-half minutes, at points approximately 18 in. on each side of the location of the initial fault, with a total initial value of current of 5,000 amperes. No smoke or flame broke through the ground during either of these tests.

It seems that the conductors at the fault location, in these two tests, must have been heated to a fused state and an arc was formed which must have aided in clearing away the metallic path between conductors. In the case of insulated conductors under the same conditions, the insulation would probably be carbonized at this value of current which would aid the existence of the fault. Therefore, because of the carbonization of the insulation and covering of a completed cable under the same test conditions, gases are not generated in any appreciable quantity in the presence of the arc. Hence, it seems that the gases are not present in sufficient quantity to aid in extinguishing the arc by their deionizing[3] effect.

When an arcing condition exists, causing a progression of the fault, clearing will usually take place at the point where two or more cables join the one at fault. If the joints of all conductors are enclosed in one junction box filled with an asphaltic compound, the compound is ignited when exposed to the arc and aids in sustaining it. When each joint is enclosed in a separate junction box as indicated in Fig. 6, clearing may be expected to take place at such a location.

Spacing the cables in the trench will reduce the period of burning considerably, since this condition makes it more difficult for the arc to subsist; and also, it aids in preventing the spread of single-phase faults to the other

Fig. 6—Method of Making Service Connections to Buried Cables

phases. It was found that the separation between cable coverings must be at least 1.5 to 2 in. to improve appreciably burn-off characteristics.

A 2 by 10-inch creosoted yellow pine plank located and buried about two inches above the cables prevented flames from bursting through the surface of the

3. *Arcs in Low-Voltage A-C. Networks*, by J. Slepian and A. P. Strom, see page 847.

ground. Such a covering in a network system has the additional advantage of protecting the buried cables against injury from the tools of workmen.

Some tests were made with the cables buried in relatively dry soil, and other tests were made under the same conditions except that the soil was water-soaked. No difference in burn-off characteristics under these two conditions was observed.

As accurately as could be determined from the test results, the amount of current and time required for clearing was the same for the several constructions tested.

Tests on samples of constructions, 3, 5, and 6 show that these three buried non-magnetic sheath cables contained less inflammable material than that in the

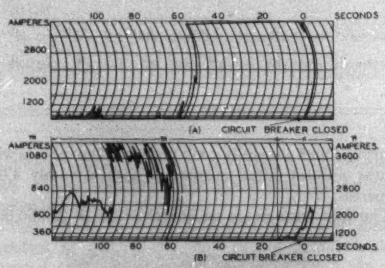

Fig. 7—Typical Charts Showing Variation of Current During Part of Short-Circuit Period

other makes tested, and therefore produced less smoke and flame under short-circuit conditions.

As accurately as could be observed, tests made on constructions 5 and 6 indicated that the vulcanized sulfinated oiled paper insulation of the latter cable possessed no advantage in burn-off characteristics over the rubber insulation of the former, the coverings being the same in the two cables.

There are not sufficient test data on construction 3 to determine whether or not the bronze tape has any effect upon the burn-off characteristics. It seems probable that the bronze tape in the covering would help to prevent the spreading of single-phase faults into other phases, if the heat of the burning conditions is not too intense. The melting point of bronze is approximately 900 deg. cent.

Representative test data supporting the foregoing discussion on burn-off characteristics of single-conductor buried cables are included in Table I.

In Fig. 7 are shown two sections of meter charts, indicating typical current variations during the short-circuit period.

Single-Conductor Cables for Duct Installation. Following are the cable constructions for duct installation which were tested for burn-off characteristics and the number of tests made on each.

Construction	No. of Tests
A. Non-Metallic Sheath Cables	
1. No. 4/0 copper conductor 19 strands	
80 mils, 30 per cent A. S. T. M. rubber compound	
One single-faced rubber-filled cotton tape	
Weatherproof cotton braid	4
2. No. 4/0 copper conductor, 259 strands	
15 mils of felted asbestos	
Three varnished cambric tapes	
30 mils of felted asbestos	
45-mil asbestos braid	2
3. No 4/0 copper conductor, 19 strands	
80 mils of intermediate rubber compound	
One double-faced rubber-filled cotton tape	
Serve of asbestos base caulk compound	
60-mil, saturated asbestos braid	
Serve of asbestos base caulk compound	
70-mil, saturated sisal braid	
Flameproof moisture resistance compound	2
4. No. 4/0 copper conductor, 19 strands	
5 mils of vulcanized sulfinated oiled paper	
One double-faced rubber-filled cotton tape	
(Remaining construction same as construction 3)	1
B. Lead-Covered Cables	
1. No. 4/0 copper conductor, 19 strands	
80 mils, 30 per cent A. S. T. M. rubber compound	
One single-faced rubber-filled cotton tape	
80 mils of lead	6
2. 350,000 cir. mils, copper conductor, 37 strands	
95 mils of oiled paper	
95 mils of lead	4
3. 500,000 cir. mils, copper conductor, 37 strands	
95 mils of oiled paper	
95 mils of lead	7

In each of these tests, three single-conductor phase cables of the construction under test and the bare neutral conductor were pulled into the 75-ft. duct length installed approximately two and one-half ft. beneath the surface of the ground. No. 4/0 neutral conductors were used with the 350,000- and 500,000-cir. mil phase conductors, and a No. 2/0 neutral with the No. 4/0 phase conductors. In none of these tests was the fault submerged in water.

The results obtained from tests on the non-metallic sheath cables do not disclose any particular advantage of any one of these types, as compared to the others. The amount of smoke, flame and disturbance produced in each case indicated the same relation to the clearing value of current required. The tests were insufficient in number to supply conclusive data on which to base a thorough comparison with lead-covered cables

TABLE I—REPRESENTATIVE DATA FROM BURN-OFF

Duration of short-circuit current	Current flowing into fault-amperes											
	One side of fault from bus No. 1				Other side of fault from bus No. 2				Total			
Min.—sec.	A	B	C	Neutral	A	B	C	Neutral	A	B	C	Neutral
5—24	2,120	2,290	2,130	0	2,050	2,240	2,060	0	4,170	4,530	4,190	0
27—36	0–1,300	0–1,850	0–1,980	0–670	0–1,270	0–1,170	0–1,200	0–600	0–2,570	0–3,020	0–2,680	0–1,270
3— 0	0	0	0	0	0	0	0	0	0	0	0	0
36— 0*												
Momentary	2,580	2,440	2,500	0	2,560	2,440	2,640	0	5,140	4,880	5,140	0
10— 0	0–1,000	0–1,050	0–800	0	0–800	0–800	0–800	0	0–1,800	0–1,850	0–1,600	0
10—30	0–1,000	0–1,050	0–800	0	0	0	0	0	0–1,000	0–1,050	0–800	0
3— 0	0	0	0	0	0	0	0	0	0	0	0	0
23—30												
0—40	2,520	3,060	2,930	400	2,980	3,120	2,760	0	5,500	6,180	5,690	400
5—20	0	0	0	0	0	0	0	0	0	0	0	0
6— 0												

1—See Fig. 1

However, in addition to valuable data obtained, the proper lines of development in such cable constructions are indicated.

A four-inch fiber duct was used in all of the tests involving No. 4/0 phase conductors.

The results obtained from the tests made on No. 4/0

FIG. 8—SINGLE-CONDUCTOR CABLES IN DUCT SHOWING FAULT CLEARED AT INITIAL LOCATION

Total value of current of 6,000 amperes cleared fault instantly

rubber insulated, lead covered cables were essentially the same as those obtained on the No. 4/0, non-metallic sheath types. A total minimum initial value of current of approximately 6,000 amperes is required to clear the three-inch contact fault at its initial location in any of those types of No. 4/0 cables when tested under the conditions described, Fig. 8. Results indicate that the clearing time would rarely exceed one minute for this value of current. When the fault failed to clear at the initial location, the cables between the distribution boxes were for the most part destroyed, Fig. 9. The greater separation of cables in the distribution boxes, and the tension at the duct edge due to the short length of suspended cable must have aided clearing which generally took place at these points with values of total initial current of approximately 4,000 amperes.

The four tests made on construction B, 2, 350,000-

FIG. 9—REMAINS OF SINGLE-CONDUCTOR CABLES AND DUCTS AFTER TEST

Left—Total value of current varied between 0 and 5,000 amperes for a period of 11.5 minutes

Right—Total value of current varied between 0 and 3,600 amperes for a period of 21.5 minutes

cir. mil paper insulated, lead-covered cable, indicate that the three-inch contact fault in cables of this conductor size and type requires a total minimum initial value of current of approximately 8,000 amperes at 133/230 volts for clearing at the initial location; and

TESTS MADE ON SINGLE-CONDUCTOR BURIED CABLES

Duration of short-circuit current	Bus voltage							Test remarks	Description of cables and fault (See page 5 for cable construction)	Results
	Bus No. 1			Bus No. 2						
Min.—sec.	A–B	B–C	C–A	A–B	B–C	C–A				
5—24	226	228	226	229	226	228	Max.	Current nearly constant	Construction No. 6	Fault was cleared at point one foot from joints at one end and just beyond joints at other end of test length. Cables destroyed and flames broke through ground to height of 3 feet between points of clearance.
27—36	199	201	200	215	212	214	Min.	Period of arcing	Phase conductors and neutral short-circuited. Taped fault.	
3— 0								Time allowed for fault to re-establish.		
36— 0										
Momentary	235	235	238	224	233	234	Max.	Initial current decreased rapidly	Construction No. 5	Clearance took place at point one foot from the initial fault on one side and at a point three feet from the initial fault on the other side. Flames broke through ground to a height of 3 feet.
10— 0	220	218	220		222	222	Min.	Period of arcing	Phase conductors and neutral short-circuited. Taped fault	
10—30								Fault was cleared on side fed from Bus 2		
3— 0								Time allowed for fault to re-establish.		
23—30										
0—40	214	214	214	214	212	212	Max.	Current nearly constant	Construction No. 5	Fault was cleared at initial location.
5—20	172	172	172	194	192	192	Min.	Time allowed for fault to re-establish.	Phase conductors and neutral short-circuited. Taped fault.	
6— 0										

that the clearing time is in the order of one minute or less. These tests were also made with the cables installed in four-inch fiber duct.

Test results show no difference in burn-off characteristics of 500,000-cir. mil rubber-insulated, lead-covered cables in fiber and tile ducts; and the data

FIG. 10—DISTRIBUTION BOXES AT ONE END OF DUCT SECTION

Asbestos board partitions removed. Cables were subjected to values of current below the clearing values of 3-inch contact fault

obtained indicate no difference in burn-off characteristics of 500,000-cir. mil cable in fiber duct and iron pipe. However, there are not enough data to state conclusively the difference in burn-off characteristics between the two latter constructions. These test results indicate that a total minimum initial value of current of approximately 10,000 amperes at 133/230 volts will satisfactorily clear the three-inch contact fault in 500,000-cir. mil cables in approximately 1.25 min. or less. Figs. 10 and 11 show the condition of cables subjected to short-circuit currents less than the minimum clearing value of the three-inch contact fault, except Test 83, Fig. 11, in which test the total short-circuit current was approximately 10,000 amperes.

No attempt was made in these tests to determine the

FIG. 11—500,000-CIR. MIL PAPER-INSULATED LEAD-COVERED CABLES REMOVED FROM DUCTS AFTER TEST

Note cables of Test No. 83—total value of current of 10,400 amperes cleared three-inch contact fault in 1.25 minutes. Remaining cables shown were subjected to total initial values of current between 7,300 and 9,000 amperes

maximum values of current at which arcing faults in the several cable constructions might be extinguished.

The clearing of only one ground fault in lead-covered cable, conductor-to-sheath, was tested. This fault was made by driving a screw into the cable and was cleared instantly. A study of available operating and test data shows that ground faults between the conduc-

tor and sheath of lead-covered cables have cleared in all instances. The extensive tests made by the Siemens-Schuckert Company[4] show that such faults will clear in a 243/420-volt network system.

The moving of cables, installed in ducts, due to magnetic forces under short-circuit conditions depends upon the relative sizes of the cables and duct. In a smaller duct the cables have less chance to separate and therefore the arc may take longer to be started and the fault may progress throughout a greater distance. As the duct size for a given group of cables is decreased, a condition is reached finally where the construction simulates multiple-conductor cables.

Rust, grease, and foreign material on the inside surface of an iron pipe, being an insulator, might in many cases prevent grounding of a phase conductor, the insulation and coverings being burned away, if the arc has not previously burned this coating on the duct wall. However, no data have been obtained which prove the practical aspect of this statement.

It has been suggested that the resistance of the cupric oxide formed on the bare conductors, after the insulation and coverings have been burned away, may aid in keeping the initial fault localized. Tests have been made on a copper conductor, producing a coating of the oxide varying in thickness throughout as wide a range as can be obtained by heating the conductor in air. The breakdown voltage of the oxide film varied from a minimum of 15 volts to a maximum of 38 volts. It is probable that the bare conductors under short-circuit conditions do not establish continuous electrical contact throughout the duct, but that the conductors may be forced together either by gravity, magnetic forces or both so as to establish a new fault.

The effect of molten metal in the bottom of the duct upon time of burn-off depends upon the relative positions of the cables. In a fiber duct this may establish a conducting path between two or more of the circuit conductors. In an iron pipe, the same conditions might result, or, in addition, a ground might be established to one or more of the phase conductors, irrespective of the neutral cable construction. At values of current somewhat less than the minimum clearing values, it was noted that the lead sheaths of the lead-covered cables in many cases were mostly melted off the phase conductors. This was due to heat, produced partly by induced currents in the sheath and also by the wattage loss in the conductor.

Samples of gases were collected from the duct line during two of the tests in fiber duct, and one in tile duct. An analysis was made of these gases, the results of which are shown in the following table:

TABLE II—ANALYSIS OF GASES COLLECTED FROM DUCTS AND DISTRIBUTION BOXES DURING TESTS

Constituent gases evolved	Relative Percentage of Each Gas				
	Test No. 81 east end D. B.[1]	Test No. 84 east end D. B.[1]	Test No. 84 center of duct line	Test No. 85 east end D. B.[1]	Test No. 85 west end D. B.[1]
Kind of duct	Fiber	Fiber	Fiber	Tile	Tile
Carbon dioxide	1.0	2.4	7.7	7.7	7.1
Unsaturated hydrocarbons	0.4	0.8	1.8	1.9	2.1
Oxygen	20.3	16.6	11.8	13.0	13.2
Carbon monoxide	0.7	2.5	3.3	3.3	3.5
Nitrogen	77.6	77.7	75.4	74.1	74.1

1. D. B. = distribution box

It is evident that the principal source of the oxygen and nitrogen shown in the above results was the air. The results therefore show that an amount of the order of 50 per cent of the gases generated from the cable materials was combustible. There is very little difference in the content of the gaseous mixture given off under short circuit conditions in tile and fiber ducts. The fiber duct itself contains considerable volatile matter, but this does not alter to any marked extent the gaseous products under short-circuit conditions. The results also show that the gas at the center of the duct line was nearly the same as that in the distribution boxes.

Since the samples of gases collected during these tests were from paper-insulated cables, it was desirable to obtain an analysis of the gases given off from rubber-insulated cables. This was done in the laboratory by wrapping a resistance wire about a 500,000-cir. mil, rubber-insulated cable, from which the lead sheath had been removed, and collecting the gases given off when the insulation was heated in this manner in open air. An analysis of the gases given off from paper-insulated cable under the same conditions was also made. The results of these analyses are shown in the following table.

TABLE III—ANALYSIS OF GASES COLLECTED IN LABORATORY

Constituent gases evolved	Relative percentage of each gas	
	Rubber insulation	Oiled-paper insulation
Carbon dioxide	10.9	10.3
Unsaturated hydrocarbons	6.4	6.9
Carbon Monoxide	9.2	9.5
Oxygen	10.9	10.6
Nitrogen	62.6	62.7

Also, in consideration of these results, it should be borne in mind that the principal source of the oxygen

4. "New Developments in the Operation of Polyphase Low-Voltage Network Systems," by Hans Besold and Otto Mueller, *Elektrotech. Zeitsch.*, July 3, 1930, p. 953.

and nitrogen was the air. Approximately 60 per cent of the remaining gases was combustible. It seems probable that some of the unsaturated hydrocarbons burned, forming carbon dioxide and water. The results indicate that there is no appreciable difference in the inflammability or explosibility of the gases given off from rubber and oiled paper-insulated cables under simulated short-circuit conditions.

The unsaturated hydrocarbons and the carbon monoxide are the constituents in the gaseous mixture which produce burning and explosion when they unite with oxygen. The force of explosion depends upon the amount of these gases which collects in the duct line before ignition. Should there be air currents in the duct line so that the gases may escape as they are generated, the violence of the explosions may be considerably reduced with probable elimination of explosions altogether.

The occurrence of explosions during the tests did not prove conclusively the relative characteristics of the different sizes and types of cables as regards explosions occurring during short-circuit conditions, due to varying test conditions. However, these results do support the indications of one preliminary study on underground cable failures, namely, that explosions were mostly due to the volatilization of cable or joint insulating materials. The covers were in place during all the tests on the different types of No. 4/0 cables; but the wind condition varied producing stronger air currents through the ducts in some tests than in others. The distribution box covers were partly broken in some tests, leaving holes through which gas may have escaped. Also, the covers were left ajar during some of the tests because of hazards introduced. In those tests in which it is known that gases had not collected in the duct line during previous tests, explosions occurred from 5.5 to 15 minutes after the initial short circuit.

From observation of the tests on cables installed in fiber ducts, there was more smoke produced during the tests on the non-metallic sheath types than during the tests on the lead-covered cable. Under the same conditions, there seemed to be little difference in the amount of smoke produced from the four non-metallic sheath constructions. Inspection of the fiber ducts after some of the tests on the non-metallic sheath types, however, showed that a considerable amount of the duct material had been burned.

In several of the tests on 500,000-cir. mil cables, the lead sheaths were bonded together in the distribution boxes at one end of the test cables, 75 ft. in length. Omission of the bonds at the other distribution box did not produce pitting of the sheath under short-circuit conditions. No voltage due to sheath currents was indicated by voltmeters connected between the lead sheaths, and between the lead sheaths and neutral. However, the distance from the point of bonding to a possible fault location in the usual system layout would be considerably greater, should bonding be done only in manholes at street intersections. It was not possible to obtain conclusive data regarding this question.

Representative data obtained from tests on cables for duct installation are included in Table IV.

Calculated Clearing Time of Faults in Cables. As stated in the foregoing, a fault in cables may be one of the two general classes:

1. A fault having a thermal capacity, relative to the heat generated, less than that of the cable conductors.
2. A fault having a thermal capacity, relative to the heat generated, greater than that of the cable conductors.

Because of the extremely variable character of faults of the first class, it is impracticable to attempt to calculate the clearing time unless the available short-circuit current and the fault resistance are known. The three-inch contact fault, previously described, belongs to this class. Calculations based upon approximate voltage and current readings indicate that its resistance varied in those No. 4/0 conductor cables tested between the limits of 0.004 and 0.02 ohm. The wide variation was due in part to the change of the fault between the time of making it and the time it was tested, and in part to the difficulty of producing the same contact surface between conductors in each test.

In connection with faults of the second class, it is possible to make calculations,[5] within certain current limits, based upon the physical, thermal, and electrical characteristics of the cable, to determine the time required for fusing the conductors joining the fault. In these calculations, it is assumed that the cable insulation and covering remain practically intact so that their heat dissipating characteristics are not altered during the fusing period, and that the fault does not progress from its initial location.

As seen from the representative test data included herein, the short-circuit current remained nearly constant as long as an arcing condition was not established. Taking a constant value of current and assuming the temperature coefficient of resistance to remain constant between the cold temperature of the conductor (in the order of 20 deg. cent.) to the fusing temperature of the conductor, fusing curves can be determined as shown in Fig. 12. Fusing curves may be obtained by similar calculations, assuming a constant voltage applied to a given length of cable. As neither a constant-voltage nor a constant-current condition results when a fault occurs in a network, average values of watts per foot per thousand circular mils have been plotted against fusing time. For a given circuit condition, approximate relations between initial watts, average watts and fusing current may be established.

5. "Current Capacity of Wires and Cables," by George E. Luke, *Electric Journal*, Vol. 20, p. 127. *Underground Alternating Current Network Distribution for Central Station Systems*, by A. H. Kehoe, A. I. E. E. Trans., Vol. 43, p. 853, Fig. 27.

TABLE IV—REPRESENTATIVE DATA FROM BURN-OFF TESTS MADE

Duration of short-circuit current	Current flowing into fault-amperes											
	One side of fault from bus No. 1				Other side of fault from bus No. 2				Total			
Min.—sec.	A	B	C	Neutral	A	B	C	Neutral	A	B	C	Neutral
0— 5	2,200	2,560	2,440	0	2,320	2,440	2,540	0	4,520	5,000	4,980	0
8— 5	0–2,080	0–2,080	0–2,080	0–1,190	0–1,940	0–1,980	0–2,080	0–1,060	0–4,020	0–4,060	0–4,160	0–2,250
3—30	0	0	0	0	0–1,940	0–1,980	0–2,080	0–1,060	0–1,940	0–1,980	0–2,080	0–1,060
3—20	0	0	0	0	0	0	0	0	0	0	0	0
15— 0												
Momentary	2,520	2,760	3,260	2,400	2,880	3,220	3,120	2,440	5,400	5,980	6,380	4,840
5— 0	0	0	0	0	0	0	0	0	0	0	0	0
5— 0												
2— 0	2,120	2,260	2,200	0	2,040	2,160	2,010	0	4,160	4,420	4,210	0
13— 0	0–2,080	0–2,080	0–2,080	0–1,200	0–1,680	0–2,080	0–1,960	0–1,180	0–3,760	0–4,160	0–4,040	0–2,380
3— 0	0	0	0	0	0	0	0	0	0	0	0	0
18— 0												
2—30	2,110	2,280	2,190	0	2,040	2,150	1,980	0	4,150	4,430	4,170	0
15— 0	0–1,720	0–2,080	0–2,080	0–1,120	0–1,940	0–2,280	0–1,880	0–1,140	0–3,660	0–4,360	0–3,960	0–2,260
3— 0	0	0	0	0	0	0	0	0	0	0	0	0
20—30												
0—32	4,080	4,100	3,800	0	4,150	4,080	3,540	0	8,230	8,100	7,340	0
5—28	0	0	0	0	0	0	0	0	0	0	0	0
6— 0												
4—16	4,320	4,320	4,350	0	4,500	4,500	4,200	0	8,820	8,820	8,550	0
2—58	4,320	3,000	4,350	0–1,700	0–4,200	1,800	3,500	0–3,200				
2— 8	4,320	4,300	0	0	0–4,200	4,200	0	0–3,200				
9—18	0	0	0	0	0–1,200	0–1,200	0	0–1,000	0–1,200	0–1,200	0	0
9—20	0	0	0	0	0	0	0	0	0	0	0	0
28— 0												
1—16	5,100	5,000	5,100	0	5,300	5,470	5,100	0	10,400	10,470	10,200	0
0—11	0	0	0	0	0	0	5,100	0	0	0	5,100	0
10—33	0	0	0	0	0	0	0	0	0	0	0	0
12— 0												

Note: Tests were made with dry ducts only.

Calculations show the fusing times of paper and rubber-insulated cables of the same conductor size, covering and thickness of insulation to be so nearly the same that the difference may be neglected.

In the few tests in which an arcing fault was not produced and in which the fault was of the second class, the calculated fusing time checked with time determined by test within a maximum error of ten per cent. Sufficient test data were not obtained to examine thoroughly the accuracy of the three curves and their applicable portions. It seems probable that the portions of the curves shown in solid line might be applicable for checks against test results. The curves at least show an interesting theoretical comparison.

VI. Conclusions

A study of test results and other data leads to the following conclusions:

1. Faults included in the class of point contacts and occurring in single-conductor a-c. low-voltage network cables, operating at voltage values of the order of 120/208 volts, will usually clear within a period of several seconds if a minimum value of current of approximately 3,000 amperes is available, irrespective of

ON SINGLE-CONDUCTOR CABLES FOR DUCT INSTALLATION

Duration of short-circuit current	Bus voltage: Bus No. 1			Bus voltage: Bus No. 2			Test remarks	Description of cables and fault (see page 7 for cable construction)	Results
Min.—sec.	A–B	B–C	C–A	A–B	B–C	C–A			
0— 5	232	232	234	232	234	232	Max. Current nearly constant	No. 4/0 non-metallic sheath cables in 4" fiber duct. Cable construction, A, 2	Fault was cleared in boxes at each end. Cables between boxes destroyed. Fiber duct burned.
8— 5	230	232	230	224	226	222	Min. Period of arcing		
3—30							Cleared on side fed from bus No. 1	Phase conductors and neutral short-circuited. Taped fault.	Explosion blew covers of boxes at one end of duct 20′ above ground. Heavy smoke and flames from boxes at each end.
3—20							Time allowed for fault to re-establish.		
15— 0									
Momentary	214	214	214	214	212	212	Max.	No. 4/0 non-metallic sheath cables in 4" fiber duct. Cable construction, A, 1.	Fault was cleared instantly at initial location.
5— 0	170	176	176	190	200	190	Min. Time allowed for fault to re-establish.		
5— 0								Phase conductors and neutral short-circuited. Taped fault.	
2— 0	228	229	228	228	228	228	Max. Current nearly constant	No. 4/0 non-metallic sheath cables in 4" fiber duct. Cable construction, A, 1.	Cables between boxes destroyed. Fiber duct burned. Fault was cleared in boxes at each end. Heavy black smoke from boxes. Flames from one box.
13— 0	201	204	201	216	216	216	Min. Period of arcing		
3— 0							Time allowed for fault to re-establish.	Phase conductors and neutral short-circuited. Taped fault.	
18— 0									
2—30	228	228	226	228	228	228	Max. Current nearly constant	No. 4/0 lead covered cables in 4" fiber duct. Cable construction, B, 1	Cables between boxes destroyed. Fiber duct burned. Fault was cleared in boxes at each end. Heavy black smoke from boxes. No flames.
15— 0	200	203	200	216	214	214	Min. Period of arcing		
3— 0							Time allowed for fault to re-establish.	Phase conductors and neutral short-circuited. Taped fault.	
20—30									
0—32	218	219	218	218	217	230	Max. Current nearly constant	350,000-cir. mil. lead-covered cables in 4" fiber duct. Cable construction, B, 2.	Fault was cleared at initial location.
5—28	158	166	163	204	208	208	Min. Time allowed for fault to re-establish.		
6— 0								Phase conductors and neutral short-circuited. Taped fault.	
4—16				234	236	236	Max. Current nearly constant	500,000-cir. mil. lead-covered cables in 4" tile duct. Cable construction, B, 3.	Fault was cleared in box at one end of duct length and at point between initial fault and box at other end of duct. Conductors welded together at initial fault. Heavy black smoke from boxes. Explosions at both ends, covers of boxes at one end blown 20 feet above ground.
2—58				198	200	200	Min.		
2— 8							C phase conductor was cleared.		
9—18							Side of fault fed from bus No. 1 was cleared.	Phase conductors and neutral short-circuited. Taped fault.	
9—20							Time allowed for fault to re-establish.		
28— 0									
1—16				234	238	238	Max. Current nearly constant	500,000-cir. mil. lead-covered cables in 4" iron pipe. Cable construction, B, 3.	Fault was cleared at initial location.
0—11				198	200	200	Min.		
10—33							Time allowed for fault to re-establish.		
12— 0								Phase conductors and neutral short-circuited. Taped fault.	

the conductor size. Tests and operating data show that ground faults between the copper conductor and the sheath of lead-covered cables have been cleared in all cases with voltage values of 115/199 to 243/420 volts, inclusive.

2. Copper-to-copper faults, in single-conductor a-c. low-voltage (120/208 volts) network cables, made by the taping the bared conductors overall and thereby obtaining a contact length of approximately three inches between conductors will clear at the location of the fault within a period of the order of one minute (usually less) provided the following *minimum* values of short-circuit current flowing into the fault are maintained during the period of short circuit:

TABLE V

Conductor size	Constant values of short-circuit current, amps.: Per conductor in test circuit	Total
No. 4/0	3,000	6,000
350,000 cir. mils	4,000	8,000
500,000 cir. mils	5,000	10,000

3. The amount of current and length of time required to burn clear a given type of fault in the buried

non-magnetic sheath cables, dropped into the trench at random, are the same as those for that type of fault in the same sizes and general types of cables installed in four-inch non-metallic ducts. A spacing of approximately two inches between adjacent coverings of single-conductor non-magnetic sheath cables buried in relatively compact soil improves the burn-off characteristics.

4. The time required to fuse a-c. low-voltage network cables with nearly constant values of current can be calculated accurately enough for practical purposes provided that the total time does not exceed eight minutes.

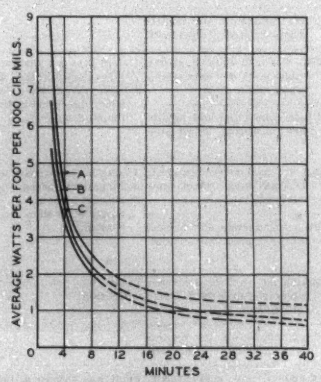

FIG. 12—FUSING CURVES FOR LOW-VOLTAGE CABLES

A—No. 4/0 conductor, 80 mils of paper insulation, 80 mils of lead
B—350,000-cir. mil conductor, 95 mils of paper insulation, 95 mils of lead
C—500,000-cir. mil conductor, 95 mils of paper insulation, 95 mils of lead

5. The burn-off characteristics of multiple-conductor buried non-magnetic sheath cables make this construction undesirable for low-voltage a-c. network distribution. The better burn-off characteristics of the single-conductor buried cables are attributed to the wider separation of conductors and to the coverings impregnated with flameproof compound.

6. As accurately as could be observed in tests on buried cables, vulcanized sulfinated oiled paper compared to rubber insulation improves burn-off characteristics very little. Analyses show no appreciable difference between oiled paper and rubber insulations in the explosibility and inflammability of gases produced from lead-covered cables. With either insulation, explosions due to gases evolved from the cable materials by short circuits in present standard cables installed in standard four-inch duct systems may occur if the fault exists longer than five minutes, provided explosive gases do not already exist in the duct line.

7. Sufficient data are not available from tests thus far made to determine whether or not single-conductor non-metallic sheath cables, insulated with materials now used and installed in one duct, have better burn-off characteristics than lead-covered cables under the same conditions.

8. Sufficient data have not been obtained to determine conclusively the difference in the effect on burn-off characteristics of single-conductor cables in iron pipe and the same cables in fiber or tile duct.

9. Analyses of gases produced under short-circuit conditions show that the materials of fiber duct in standard use do not increase the inflammability or combustibility of the gases.

The foregoing conclusions are based upon operating data and results of tests made on cables buried in relatively dry and wet soil and installed in relatively dry ducts. The results of these tests indicate the desirable burn-off characteristics of low-voltage cables for underground network systems. The development of new insulating materials for cables having these characteristics will require further research.

ACKNOWLEDGMENT

The writers wish to acknowledge their indebtedness to those persons who made several helpful suggestions in connection with this study, and also, to the several manufacturing companies for their advice and cooperation in the supply of cable for some of the tests made.

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Discussion

W. R. Bullard: I believe the most important conclusion which has been drawn in this paper, is that single-conductor buried cable of the non-metallic armored type has appreciably better burn-off characteristics than the other types of construction tested. However, a very important feature of such construction is that with such cable the necessity for good burn-off characteristics might be expected to be very largely eliminated. That is, with single-conductor, non-metallic sheathed cable buried with a separation of one or more inches, the possibility of the occurrence of short circuits would be very remote. An insulation failure would, of course, result in the flow of some leakage current to ground, but it would be very unusual for such leakage current to reach sufficient magnitude to constitute a short circuit.

This type of construction, therefore, offers excellent possibilities for increased reliability of service for a-c. network systems. Unfortunately, one important factor connected with such construction is unknown. This is the useful life of the non-metallic sheathed cable. It is to be hoped that this feature will be diligently investigated and that the development will progress to the point where it will be known with certainty just what may be expected from the different types now on the market under various local installation conditions.

There are now in service or being installed several low-voltage distribution systems employing this type of construction, notably Mr. Sutherland's system and one in Santiago, Chile. The latter has a nominal service voltage of 220/380 volts, three-phase, four-wire. The experience gained from the operation of such systems will, of course, be very helpful in obtaining the necessary information for the further development of a-c. networks in general.

One further point which should be noted in connection with this paper is that the conditions of test were, in general, more severe than conditions ordinarily encountered in practise. That is, the tying together of the conductors produces considerably lower fault resistance than is ordinarily encountered under actual operating conditions. Numerous tests made under actual service conditions have shown considerably better burn-off characteristics than those listed in the paper, even in the case of systems utilizing voltages of the 220/380 class.

Wm. A. Del Mar: The non-metallic type of trench cable has undoubtedly come to stay in connection with the secondary network system. Every time that a new type of cable has appeared, the question of its permanence has been raised, and has usually stayed with us, in one form or another. It is, therefore, natural that it should be raised in connection with this type; —all the more so, since the essential peculiarity of the type is the omission of the very covering which has been relied upon to protect the insulation from disintegrating agencies, namely, the lead sheath. There are about as many types as there are manufacturers, so that experience with one type is no criterion of the performance of any other. Some of these types undoubtedly possess the characteristics that make for permanence, while others may not.

Some of this cable is purchased on specifications which merely describe one type or another to the exclusion of all others, while some is purchased on the basis of performance tests without obvious reference to the type. Unfortunately there are nearly as many opinions about such tests as there are performance type specifications. It is about time that some coordination be effected in this field.

Looking over these performance tests, we find tests to assure adequate resistance to the following agencies of deterioration:

(a) Water, with special reference to prevention of water-logging of rubber insulation.

(b) Acids, with special reference to installations in marshy land.

(c) Alkali, with special reference to installations in alkaline soil.

(d) Abrasion, with special reference to handling.

(e) Impact, with special reference to damage by excavators.

(f) Shear, with special reference to handling.

(g) Dielectric stress, with special reference to corona-formation, a matter of no interest in connection with secondary networks, but important for series lighting cables.

(h) Bending, with special reference to terminal and splicing conditions.

(i) Burning, with special reference to entering houses, and arc extinction.

Some of these agencies of deterioration obviously are of little significance, but too much stress cannot be laid on protection from water as rubber will not last in contact with soil water.

The protective materials used in the various types of cable are cellulose (in the form of paper, jute, cotton, sisal, asbestos reinforcement, etc.) asbestos, semi-solid hydrocarbons (such as asphalt, oxidized and vulcanized oils, pitch, etc.), rubber, waxes, and oils.

The protective structures used are tapes, braids, wrappings, and extruded sheaths.

The problem of the operating engineer is to give proper weight to the various requirements, the devising of tests should be a joint effort of the operating and manufacturing engineers, and the designs of cables to meet these tests should be left to the cable maker.

B. M. Jones: A large part of the test data in this paper bears out conclusions that the Duquesne Light Company arrived at as a result of tests made early in 1928. It was interesting to us that these tests by Messrs. Sutherland and MacCorkle showed no material difference in the time or currents required to burn clear of fault in fiber, clay, or iron ducts.

I would like to call attention to one particular feature which should not be overlooked in planning and designing a low-voltage a-c. network, and that is the providing of capacity to burn clear a fault in a weak spot on the network under three extreme emergency conditions, which are—(1) the short circuit being of the most severe type of copper-to-copper contact; (2) the fault occurring when one particular high-tension supply feeder is out of service thus reducing the short-circuit capacity available at the weak spot in the network; (3) the fault occurring at this weakest spot.

Assuming 6,000 amperes required to burn clear a fault of various sizes of cables, and with a transformer bank out of service thus leaving a weak spot in the network, to which spot 6,000 amperes flows from one direction and 3,500 amperes from another direction, the fault would burn clear from the end supplied with 6,000 amperes and would not burn clear from the end supplied with 3,500 amperes.

Obviously, it would be necessary to increase the short-circuit current flowing from the weak side by increasing the transformer capacity in that direction, which would cost a considerable amount of money in some cases and this additional capacity would be required solely and only for the purpose of increasing the short-circuit amperes to burn clear a fault upon the simultaneous happenings of *three extremely remote contingencies*.

While the demands are for better and better service particularly in the business areas that are suitable for low-voltage a-c. networks; costs must be recognized and the day has arrived when the engineer must justify his expenditures as well as his design.

Most of the engineers and operators realize that electrical equipment in general is supplied with a "back-up unit," and there is a generator ready for service in case the largest one fails, a spare transformer bank or unit, an automatic circuit breaker to back up another one and relay schemes to back each other up, but there is not another back-up for the first back-up; so the providing for three extremely remote happenings, if it costs any money, is very questionable.

The engineers and operators considering a low-voltage a-c. network must make recommendations as to how much of a gamble they shall take on having these three extremely remote contingencies happen at the same time, or how much money they are willing to spend to be ready to handle these rare simultaneous occurrences, and of course the management's sanction should be obtained on either of these two schemes.

J. M. Comly and J. A. McHugh: During a six months' period 29 cases of fires and explosions caused by faults on a-c. low-voltage network mains cables were reported and of these none were found to give evidence of the presence of sewer or illuminating gases. Five cases involved explosions, nineteen cases involved sustained fires, and five cases involved only brief fires.

Four of the five brief fire cases showed negligible burning of insulation. One case resulted in considerable damage to insulation of No. 6 B&S cable used as main.

Seventeen cases of sustained fire showed from 3 to 3,400 ft. of cable insulation damaged for an average of approximately 420 ft. Most of these faults developed under dry conditions. In the two cases where 2 ft. or less of insulation was damaged the faults were submerged. The five failures involving explosions showed from 180 ft. to 560 ft. of cable insulation damaged for an average also of approximately 420 ft. In four of the five cases the underground system was dry in the vicinity of the explosion.

A comparison of the reported clearing conditions and the calculated available short-circuit currents indicates a considerable variation in fault resistance and this appears to be supported by the observed physical effects. Fourteen of the twenty-nine cases involved failure of network switches or fuses to clear.

Arcs in Low-Voltage A-C. Networks

BY J. SLEPIAN* and A. P. STROM*
Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The extinction of a-c. arcs at current zero is reviewed, and arc reignition characteristic and circuit reignition characteristic are defined. From a study of arc reignition characteristics of short arcs remote from insulation it is concluded that such arcs are incapable of interrupting practical low-voltage a-c. network circuits. The extinction of arcs in practical network cables is then ascribed to the deionizing action of gas blasts coming from decomposing adjacent insulation. Experiments with arcs in cables, and arcs between parallel plates remote from insulation, and closely bounded by insulation, confirm this view.

Inorganic insulating materials may also assist in arc extinction by generating gas blasts by their decomposition. Of the various inorganic materials tried, boric acid was the most effective.

Charring of organic insulation may be expected to cause it to lose its arc extinction aiding characteristic.

* * * * *

I. Introduction

THE great success of the low-voltage a-c. cable network described by Kehoe,[1] which is finding increasing application as a system of distribution in metropolitan areas is dependent fundamentally upon the capacity for self clearing of faults on the network which this system enjoys. The tying together of the secondaries of many transformers into a network ensures that at any point of fault there will be a sufficiently large current to melt apart any contacting metallic conductors and to start an arc. The low a-c. voltage of the network ensures that the arc will be extinguished quickly at a normal zero of current. Thus the success of the system depends upon the properties of the arc which occurs at a cable fault. A close study of these arcs and how they are extinguished would seem to be worth while, particularly as the extension of this system to higher voltages which seems necessary for economy and proper voltage regulation where loads are very heavy, leads to conditions in which the extinction of the arc is far less certain than in lower voltage systems.

II. The Extinction of A-C. Arcs

Arc Reignition and Circuit Reignition Characteristics.

Very considerable advance has been made in recent years in the theory of extinction of a-c. arcs. The continuance of the a-c. arc is believed to depend on the repeated reignition of the arc for each new half cycle, after a current zero. Just after a current zero the arc space is losing ionization and recovering the ability to withstand voltage. On the other hand voltage tending to break down the arc space again is building up across the arc space terminals, following some natural transient of the external circuit. If the dielectric strength of the deionizing arc space, as measured in volts which the space can withstand, grows faster than the voltage which the external circuit impresses upon the terminals, then the arc will be extinguished. If the voltage supplied by the external circuit builds up faster than the dielectric strength of the arc space, then the arc will be reignited.

From the standpoint of arc extinction, therefore, the arc is best described in terms of a curve between the recovered dielectric strength of the arc space measured in volts, and the time after current zero measured in microseconds. Such a curve may be called the "arc reignition characteristic." Likewise, from the standpoint of arc extinction, the external circuit is best described in terms of a curve between the voltage which would appear across the arc terminals if the current remained zero after the particular current zero under consideration, and the time after that current zero measured in microseconds. Such a curve may be called the "circuit reignition characteristic." The arc is extinguished at a current zero if the arc reignition characteristic lies completely above the circuit reignition characteristic. The arc is reignited if the circuit reignition characteristic is anywhere above the arc reignition characteristic.

The circuit reignition characteristic may be calculated according to well-known electrical engineering principles. It depends upon circuit constants, *i. e.*, magnitudes and interconnection of reactances, resistance, capacitances, etc., and upon circuit conditions existing at the current zero, *i. e.*, excitation and phase position of synchronous machines, momentary currents in windings, momentary charges upon conductors, etc. It is independent of the arc itself except in so far as the arc voltage previous to the current zero may have influenced circuit conditions. Circuit reignition characteristics, for various simple cases have been published.[2–6]

The arc reignition characteristic cannot be calculated even approximately except in a few special simple cases in the present state of the science. It depends on conditions internal to the arc space, such as nature and geometrical arrangement of electrodes, the nature of the gas, presence of magnetic fields, and gas blasts. It does not depend upon the external circuit except in so far as the current previous to the current zero may be influenced and thus cause the conditions internal to the arc space to be altered.

The arc reignition characteristic is determined fundamentally by the manner of disappearance of the ions in the arc space immediately after current zero.

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31-54

Ions disappear from the space, (1) by direct recombination in the arc space itself, (2) by discharge to the electrodes, (3) by recombination on surrounding walls or in surrounding cooler gases, (4) by recombination in cooler gases mixed turbulently into the arc space by gas blasts. (2) appears to be most important in short arcs, and (3) in longer arcs. The importance of (4) in practical devices such as fuses and oil circuit breakers where intense gas blasts are generated by the decomposition of material under the heat of the arc has only recently begun to be appreciated.[2]

III. The Short Arc Without Gas Blast

The arc reignition characteristic for a short arc in the absence of gases coming from decomposing neighboring materials, and with what were believed to be relatively cool, non-vaporizing copper electrodes has been given by Slepian,[3] and is reproduced as curve *A* in Fig. 1. Noteworthy in this curve is the extremely rapid, almost instantaneous recovery by the arc space of a dielectric strength of 240 volts, and the slower further recovery of dielectric strength. The theory

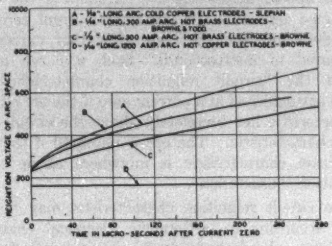

Fig. 1—Arc Reignition Characteristics

offered by Slepian also predicted that the immediately recovered dielectric strength would be independent of the arc length and arc current, but would depend on the nature of the (incoming) cathode, and the gas at the cathode. The slope of the arc reignition characteristic would depend on arc length and current if ions disappeared from the arc space by means other than direct recombination in the arc path, and if these means were affected by current and arc length.

Browne and Todd[7] have found similar arc reignition characteristics for short arcs with metal electrodes but with the metal at the arc terminals hot, and probably vigorously boiling. Their results for brass electrodes are given in curves *B* and *C*, in Fig. 1. The immediately recovered dielectric strength was 230 volts, and the slope of the characteristic for the 1/16-in. 300-ampere arc was nearly the same as that of the cold electrode copper arc given by Slepian. The slope was found to decrease with increase of arc length, however, as shown by curve *C*, Fig. 1, and also to decrease with increase of arc current, although complete arc reignition characteristics were not given for larger currents.

Curve *D*, Fig. 1 gives unpublished results of a limited number of tests made by Browne for a heavier current stationary arc with copper electrodes. Here is noted the extremely low value of the immediately recovered dielectric strength, about 160 volts, and the extremely small slope of the characteristic for the first few hundred microseconds.

IV. Extinction of Arcs in A-C. Networks

According to the tests of Kehoe[1] the arc following a fault in an a-c. network cable will always be extinguished at 220 volts, r. m. s. At 1,000 volts r. m. s. currents up to 500 amperes and at 2,000 volts r. m. s. up to 100 amperes were also successfully interrupted by the arc. In the tests at higher voltages, however, the currents were limited by resistances so that it may not be concluded that extinction of the arc would have taken place in a reactive circuit of like current. At 220 volts, however, we may conclude that the arc would be extinguished under all circuit conditions.

Similar results have been obtained by Besold and Mueller[8] although here again, the currents were apparently limited by resistance so that it is not certain that the results will apply also for reactive circuits.

These results are entirely compatible with the earlier data on arc reignition characteristics such as curves *A*, *B*, and *C* of Fig. 1. For example, on curve *C*, the immediately recovered dielectric strength of the arc space is 230 volts r. m. s. However, on curve *C*, dielectric strength of 311 volts is recovered in 60 microseconds, and since, in practical a-c. network circuits it will take longer than this for maximum open-circuit voltage to build up from arc voltage after current zero, the arc will not reignite after current zero.

The more recent results of Browne, however, indicated in curve *D*, and relating to heavier current arcs between copper electrodes, and which should be more directly applicable to cable fault arcs, are not compatible with these results. According to curve *D*, arcs with currents exceeding 1,200 amperes should not be extinguished with more than 125 or 130 volts r. m. s. in a low power-factor circuit. We are then driven to the conclusion that in a-c. cable fault arcs additional deionizing agents must exist which are not active in ordinary short arcs between metal electrodes.

These additional deionizing agents must come from the insulation used in cables, and the authors believe that it comes from the intense blasts of gas directed through the arc from the decomposing insulation adjacent to the arc. That gas blasts through an arc are strongly deionizing has been long known, and gas blasts coming from decomposing insulation are believed to be responsible for the effectiveness in circuit interruption of the expulsion fuses and the oil circuit breaker. The cable fault arc is *in* the insulation in the same way and sense as the oil circuit breaker arc is *in* the oil. In both cases the arc is in a gas pocket or bubble, but this gas and the arc in it are not in a quiescent state

but are being most violently stirred by the turbulent mixing into it of large volumes of fresh, relatively cool, unionized gas coming from the decomposing insulation or oil. This turbulent inpouring of fresh gas causes the extinction of the arc in circuits of higher voltage than would otherwise be possible.

The gases coming from the thermal decomposition of the insulation in cables would generally be regarded as a nuisance. Since they are inflammable, under certain circumstances, they may even constitute an explosion hazard. Nevertheless they are probably necessary in the functioning of present day low-voltage a-c. networks.

V. Tests on Arcs in Cables

While the ideas expressed in the preceeding sections were developing, it seemed worthwhile to get some first hand knowledge of the characteristics of arcs in cables. Accordingly copper-to-copper short circuits were made in short lengths of oil impregnated paper insulated cable. The cable used was 600-volt, three-conductor, 500,000-cm. lead-covered cable, manufactured by the Habirshaw Cable Co. The arcs were started in several different ways, namely, (1) by twisting together two strands of the conductors to be short-circuited; (2) by driving a nail through the stranded conductors; (3) by tying the conductors together with No. 20 bare copper wire. The tests were made in a 550-volt r. m. s., 60-cycle circuit in which the current was limited only by the reactive impedance of the transformers and cables. Currents varied from 3,000 to 10,000 amperes. In some of these tests the short circuit was made near one end of the cable, power being supplied from the opposite end. In others, the short circuit was made near the center of the cable section, power being supplied from both ends of the cable, in order to eliminate the strong magnetic blowout effect that is present with the first method of short-circuiting. In most of these tests the cable was drawn into a 3¾-in. fibre duct, the ends of which were closed by packing with rags.

It was found that in general the arcs would be extinguished in one to fifteen half cycles. Occasionally, however, the arc would not be extinguished, and in such a case provision had been made to throw in various values of resistance in shunt with the arc to see what influence would be exerted on the extinction of the arc. When the arc persisted it was found that resistance in shunt with even as low a value as one ohm had no influence on the continuance of the arc.

The arc voltage was found to be usually of the order of 100 volts when the short circuit was made 5 or 6 in. away from the insulation, and about 350 volts when the arc occurred near the insulation. The first of these figures is approximately what would be expected for arcs of corresponding length in air between copper electrodes, but the second is very much too high for an arc drawn in air and unexposed to any gas blasts. This arc voltage was high enough to raise the power factor of the circuit considerably, which favored the arc extinction. A study of the oscillograms shows that in some cases power factors of from 0.3 to 0.6 were reached.

In general it was found that when the arc was closely bounded by insulation, the arc would be promptly extinguished, but in those tests where the insulation had been carefully cleaned away about six or more inches from the arc, then the arc would persist. As previously mentioned, it was also found that the arc voltage was about three to four times as large when the arc was closely bounded by insulation, as when remote from insulation.

These results speak eloquently of the large part the insulation plays in the extinction of the arcs, and show how persistent the arcs are when not subjected to the influence of the insulation.

VI. Tests on Short Arcs Free from Insulation

It seemed worth while to obtain some data on the extinction of short arcs between copper electrodes free from insulation, for currents larger than those used

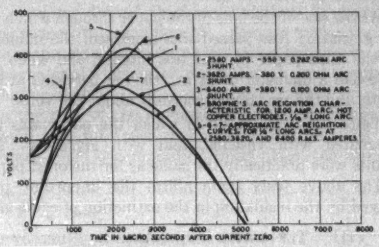

Fig. 2—Circuit Reignition Characteristics on Tests with Hot Copper Electrodes

by Browne. Accordingly, a pair of copper plates ¼ in. thick and 4 in. by 27 in. were used, spaced apart ¼ in. To keep the gas pressure between the plates low, ½-in. diameter holes, spaced 1.41 in. between centers were punched in the plates.

The arc was started by a No. 20 copper fuse wire drawn through one pair of holes. The current was obtained from transformers and was limited only by air-cored reactors where it was not the short-circuit current of the transformers.

Tests were made at various voltages and currents, and values of resistance which when in shunt to the arc would just cause it to go out were determined. With these values of resistance the circuit reignition characteristic should just be tangent to the arc reignition characteristic. Numerous circuit reignition characteristics were calculated, and several are shown in Fig. 2. In this figure, the curve of Browne from Fig. 1 is redrawn. Also, probable arc reignition characteristics for the larger currents and spacing are indicated by drawing tangents from the 160-volt point on the

voltage axis to the various circuit reignition characteristics.

Arc voltages in these tests were found to vary from 40 to 100 volts.

These tests show decisively that the simple short arc between copper electrodes free from insulation is inadequate to meet the fault clearing requirements of low-voltage a-c. networks.

VII. Tests with Short Arcs Bounded by Organic Insulation

Further tests were made with the electrode arrangement of the preceding section, but with oil soaked paper clamped between the copper plates. A ⅛-in. hole was cut in the paper through which the fuse was passed for starting the arc.

At 550 volts the arc was extinguished for all values of current tried, up to 11,000 amperes. Similar results were obtained as the voltage was increased up to 750 volts. At 1,150 volts, the arc persisted. The arc voltage was found to vary from 300 volts at low currents to 600 volts for high currents.

As the size of the hole in the paper through which the arc initiating fuse passed was increased, the influence of the oiled paper became less intense, and when the hole had a diameter of ⅜ in., the arc persisted at 550 volts, and the arc voltage was only about 200 volts.

Tests were also made in which the paper was impregnated with vaseline, and with glycerine. The results were practically the same as for the oiled paper.

The above tests again affirm the important part played by the insulation in the extinction of cable arcs.

VIII. Tests with Short Arcs Bounded by Inorganic Insulations

As has been mentioned before, the gases resulting from the decomposition of organic insulation, and which are effective in extinguishing the arc, are in themselves combustible, and therefore under certain circumstance may constitute an explosion hazard. It seemed worth while, therefore, to make some preliminary tests using materials which would give off incombustible gases such as water or carbon dioxide.

The following materials were tried:

Material	Decomposition products: Gases	Decomposition products: Solid
Boric Acid, H_3BO_3	H_2O	B_2O_3
Gypsum, $CaSO_4 \cdot 2H_2O$	H_2O, SO_2, O_2	CaO
Borax $Ha_2B_4O_7 \cdot 10H_2O$	H_2O	$Na_2B_4O_7$
Sodium Bicarbonate, $NaHCO_3$	H_2O, CO_2	$NaOH$
Magnesium Carbonate $M_3CO_3 \cdot 3H_2O$	H_2O, CO_2	MgO
Basic Magnesium Carbonate $3MgCO_3Mg(OH)_2 \cdot 3H_2O$	H_2O, CO_2	MgO
Magnesium Oxide, MgO		MgO
Ammonium Carbonate $(NH_4)_2CO_3H_2O$	H_2O, CO_2, NH_3	
Aluminum Ammonia Sulfat. $Al_2(SO_4)_3(NH_4)_2SO_4 \cdot 24H_2O$	H_2O, SO_2, NH_3	Al_2O_3

The various materials in the form of powder were packed between the copper plates described in sections VI and VII, and a fuse wire pulled through a pair of holes in the two plates. Tests were then made, as in preceding sections. The arc was permitted to make as large a hole as it would in the closely packed powdered material.

Before comparing the results obtained with the various materials, it will be well to consider the characteristics which are found for arcs subjected to deionization from decomposing bounding walls. Fig. 3, which gives results for basic magnesium carbonate, is typical for the relation between the volts which can be interrupted by a given length of arc, and the current. As the current is increased from small values, the volts which can be interrupted decrease, reach a minimum and then increase again. This relation is rather to be expected if it is reflected that for very large currents the gas blast will be very intense, and that for very small currents the arc will be particularly sensitive to deionizing influences.

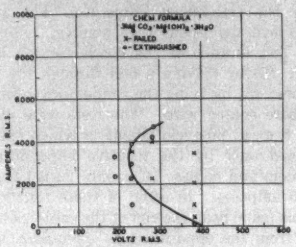

Fig. 3—Basic Magnesium Carbonate as Arc Extinguisher

Used between parallel copper plates spaced ¼ in. apart, without arc shunting resistance

Hence starting with low voltages, all currents can be interrupted by the arc. Above a certain voltage, however, (the minimum voltage on the curve of Fig. 3) there will be two limits of current, such that between these limits the arc will not be extinguished. For values of current less than the lower limit and for values of current greater than the larger limit, the arc will interrupt the circuit, but for intermediate values of current, the arc will persist. Such relations as these are found generally in expulsion fuses, and oil circuit breakers.

If now, the voltage of the circuit is kept constant and only the circuit reignition characteristic is altered, similar results are obtained. Below a certain "speed" of circuit all currents are interrupted, whereas for higher "speeds" there will be an intermediate range of current for which the arc will persist.

This is illustrated in Fig. 4, which shows the results of tests using gypsum powder. All these tests were

at 550 volts, with varying values of resistance shunting the arc giving different circuit reignition characteristics. For shunting resistances less than 16 ohms, the arc interrupted all values of current, but for resistance greater than 16 ohms there would be an intermediate range of current for which the arc would persist. When the arc was unshunted the arc would persist for currents from 2,800 amperes down to 400 amperes, the smallest tried, although undoubtedly for a sufficiently small

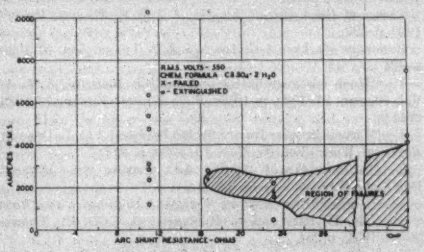

FIG. 4—GYPSUM AS AN ARC EXTINGUISHER

Used between parallel copper plates spaced ¼ in. apart

current the arc would have been extinguished. For currents larger than 4,100 amperes the arc always interrupted the circuit.

As it so happens that most of the data for the different powders were obtained at 550 volts with varying resistance shunts, we may compare their efficacy by plotting in one figure curves similar to that of Fig. 4. This has been done in Fig. 5. No curve is given for boric acid, as the arc interrupted all values of current even when unshunted, except for one test which did not repeat itself. The curve for gypsum is taken from Fig. 4, and as is seen consists of the boundary of the region of arc persistency. Because of the smallness of the number of test points in Fig. 4, the location of this boundary is rather vague, so that although a definite curve is shown in Fig. 5, no accurate quantitative value should be attached to it. Similar remarks apply to the curves for the other powdered materials, which were obtained from test data similar to that shown in Fig. 4. For the other materials only the lower branch of the curve is shown, as the other branch if it existed, occurred at values of current greater than those available for these tests. For comparison, the curve for air is also included in Fig. 5.

From all these data we conclude that boric acid made the best arc extinguishing powder being as good as oiled paper over the range of these tests. Next, although considerably inferior, is gypsum. Then far inferior, though better than plain air, are the rest of the materials tried.

IX. REQUIREMENTS OF GAS-GENERATING ARC-EXTINGUISHING SUBSTANCES

In considering the various substances which may be placed adjacent to an a-c. arc to cause its extinction by the generated gas blast, one would expect that that substance would be best which generated the largest amount of gas in a given volume, or which generated the largest amount of gas for a given energy input from the arc. However, the results of the preceding sections show that some other factor must also be taken into account. For example, boric acid and oiled paper, which proved to be the best among the substances tried, are certainly inferior to ammonium carbonate, (NH_4) (CO_3H_2O), one of the worst of the substances tried from the standpoint of gas generation per unit volume or per unit energy.

This other factor probably relates to the state of the surface of the bounding material just after the moment of arc extinction. The gases generated from the substance, may ensure that the arc will not reignite in the arc space itself, but that with the arc not reigniting, the substance itself must be able to stand the voltage without breaking down. Organic insulation, in general, seems to meet this requirement. Some of the insulation close to the arc is decomposed into gases, but the surface of the material which is left remains clean, and has good insulating quality.

In the case of the ammonium carbonate, however, because this substance melts below the temperature at which rapid decomposition takes place, the surface

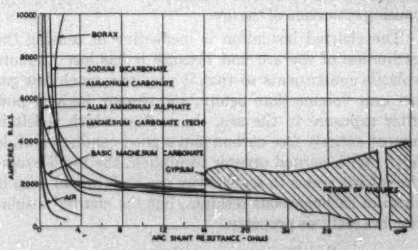

FIG. 5—POWDER FILLERS AS ARC EXTINGUISHERS

Used between parallel copper plates spaced ¼ in. apart r. m. s. volts 550

of the material which is left after a half cycle of arcing undoubtedly consists of a molten film of the ammonium carbonate. Furthermore, molten ammonium carbonate is highly conducting. Just after current zero, the arc is undoubtedly extinguished momentarily, and open-circuit voltage momentarily appears across the electrodes. This voltage causes a considerable current to flow in the molten surface film, which is brought immediately to a violent boil and gas evolution, and the resulting tearing of this film while carrying current will restart the arc.

Boric acid also has its melting point below that at which rapid decomposition takes place, but the electrical resistivity of the molten boric acid is very high. Hence, just after the arc extinction at current zero, when open-circuit voltage is restored, the current sent through the molten surface film will be small, even for voltage gradients as high as those used in the tests, namely 550 volts r. m. s. across ¼ in. The current will be so small, that the power will be insufficient to keep the boric acid film hot, and it will recongeal, thus permanently opening the circuit.

While organic insulation in general has the property of being left with a surface of good dielectric quality after exposure to an arc, there is a circumstance important in practical a-c. network operation under which the insulation may be expected not to have this property. This is when the insulation has been subjected to a moderately high temperature so that it is more or less completely charred before the arc begins to act upon it.

Such a circumstance may arise for example when a very low-resistance fault forms at a point in a network where there is available only a small short-circuit current, which may take a very long time to melt apart the low-resistance fault. Because of this long time, the whole cable leading to the fault may be heated and the insulation charred. By the time the arc forms, the organic insulation is no longer in condition to cause its extinction and the arc burns along the cable until it reaches some point where the insulation, due to lower temperatures, is still in condition to be effective in causing extinction of the arc.

The charred insulation is ineffective in causing the extinction of the arc first because it has lost its more volatile constituents so that it produces much less gas per unit volume than before, and second, because now after exposure to the arc, the surface which is left is almost entirely free carbon and a good conductor while hot. With normal organic insulation, the rapid evolution of gas seems to scour away the surface and keep it clean of carbonaceous residues, but the charred insulation has lost this property.

X. Summary and Conclusions

1. Short arcs between copper electrodes and not adjacent to insulation are inadequate to meet the fault clearing requirements of low-voltage a-c. networks of more than 130 volts.

2. Actual low-voltage networks of higher voltage than 130 volts depend for the extinction of arcs formed at faults upon the strongly deionizing action of gas blasts coming from decomposing adjacent insulation.

3. The extinction of arcs in cables is strongly affected by the degree of remoteness of the insulation from the arc. In a ⅛-in. hole in oiled paper insulation, a ¼-in. long arc will interrupt current at 750 volts, whereas in a ⅜-in. hole it will fail to interrupt current at 550 volts.

4. Inorganic insulating materials giving off noncombustible gases are also effective in aiding arc extinction. Of the various materials tried, boric acid was the best.

5. Charring of organic insulation may be expected to cause it to lose its arc extinction aiding properties.

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Discussion

W. R. Bullard: The statement at the end of the paper is interesting in that the arc extinguishing features of network cables would be destroyed by the pre-carbonization of the cable insulation. This fact was discovered about four years ago during some tests made by the Cia Cubana de Electricidad in Havana, Cuba under the direction of the writer and was subsequently published in the *Electrical World*. I feel that most of the few sustained arcs which have occurred in actual operation of a-c. networks have probably been due to this particular phenomenon.

The portion of this paper dealing with the arc extinguishing properties of non-inflammable gases is very interesting and it is appropriate to inquire whether practical application may not be found for the knowledge thus gained. Intensive study in this direction would seem to be in order since the elimination of inflammable gas production during short-circuit conditions would be highly desirable, whereas our present knowledge based on the work outlined in this paper would indicate that the production of gases of some kind is necessary for proper arc extinguishing characteristics.

W. G. Dow: Three of the arc reignition characteristics shown in Fig. 1 of the paper converge to a reignition voltage of 230 at the extreme left. In contrast with this, the tests described in our paper see page 854 indicate convergence to a value between 340 and 380 volts. The regularity with which the short-time reignition voltage repeated itself in our work was in fact very striking. There are several possible explanations for the difference between the 230-volt figure in one paper and the 340- to 380-volt values in the other.

1. Slepian and Strom's arc reignition characteristics should correspond, from the method used in their determinations, to the lower envelope curves of Figs. 15 and 16 in our paper. Thus while their work makes no distinction between various individual restriking voltages, it does give rise to curves which mark the boundary between the region on the graph in which reignition cannot occur (below our bottom envelope) and that in which it may occur (above our bottom envelope).

Our work gives rise to a lower envelope curving *upward* toward its left end; their arc reignition characteristics show no such behavior. It seems to us that their method of test would not permit discovery of such an upward curvature. Their arc reignition characteristic is drawn tangent to all of a family of circuit reignition characteristics having voltage and resistance values such as to just fall short of producing reignition. The resistance shunt type of circuit which they used produces a circuit reignition curve which is always rising. No such curve could ever be tangent to that part of an arc reignition characteristic which might lie to the left of a minimum such as our lower envelope has at about 25 microseconds.

Tests of the type used by Slepian, Strom, and Browne[1] in locating the extreme left end of their arc reignition characteristics would be made with relatively low-circuit voltages (below 300 volts peak) and high-shunt resistances. Now if the true curve actually has such a minimum point as our lower envelope exhibits, the peak voltage of the test circuit might be considerably lower than the short-time reignition voltage, and yet be sufficient to cause reignition at from 20 to 30 microseconds after the current zero. If the shunt resistance is adjusted until reignition just does occur, it will still be occurring at or near the minimum point on the curve. The results of the test would in that case have little or no relation to the true value of reignition voltage required at one or two microseconds after the current zero.

In brief, if the true arc reignition characteristic does have a minimum value at a time later than zero:

(a) The method of test used by Slepian and Strom may not discover such a minimum.

(b) The value of short-time reignition voltage derived from their tests (230 volts) will be considerably too low.

2. In determining the family of circuit reignition curves to which their arc reignition curve is tangent, no account was taken of the distributed capacity, nor of the possibility that the current might stop prior to reaching its zero value. In order to operate on moderate voltage circuits with the large currents used in their tests, the series inductance must have been rather small. This would tend to minimize the effect of the arc-failure current being greater than zero. A very small value of distributed capacity, however, increased the probability of a dip to a negative glow before voltage recovery.

In general, both of the effects just mentioned, if considered, would tend to shift all of the arc reignition characteristics determined by Messrs. Slepian, Strom, and Browne[2] toward the right.

3. All of the arc reignition characteristics appearing in the paper by Messrs. Slepian and Strom result from tests using currents of 300 amperes or more. In many cases the arcs were in rapid motion. In our work the currents were between 25 and 30 amperes, and the arc relatively stationary. These rather striking differences may of course provide the true explanation for the differences between the two sets of arc reignition characteristics. In fact, it seems rather surprising that the two sets of tests do not give more divergent results than they do.

Perhaps the most interesting part of their work is that relating to the effects of the presence of impregnated paper and of boric acid in accelerating deionization. I would like to point out in this connection something that is very commonly lost sight of: namely, that the presence of any surface whatever, liquid or solid, conducting or non-conducting, is a great aid to recombination of electrons and positive ions wherever the ion concentration is at all great. Recombination in the volume of a gas not adjacent to a surface is always much less important than that at any surface. Ions disappear at the walls of an ionized region much as water flows over the edge of a dam; that is, practically every ion or electron that reaches the walls in the course of its random motions at once ceases to play any further role as a current carrier.

The existence of turbulent motion due to evolved gases will of course freshly expose various parts of the ionized region to surfaces, as well as exerting a cooling action which makes more difficult the production of new ions to replace those that recombine.

1. See discussion by T. E. Browne, Jr. page 868.

2. *Loc. cit.*

Reignition of Metallic A-C. Arcs in Air

BY S. S. ATTWOOD,* Member, A. I. E. E.
W. G. DOW* Associate, A. I. E. E.
and
W. KRAUSNICK† Member, A. I. E. E.

Synopsis.—Developments made in circuit breakers in the last two years have emphasized the necessity for obtaining experimental evidence of the current-voltage-time relationships that exist during the period when the a-c. arc between metallic electrodes passes through its cyclic current zero. Twenty-nine cathode-ray oscillograms of these relationships are presented.

During the current zero period the arc electrode voltage is determined by the circuit constants and rises until the electrode voltage reaches the breakdown or reignition value, which is determined by the deionizing influences at work while the arc is extinguished. Alteration of the circuit constants permits a variation in the rate of voltage rise with a consequent change in the reignition voltage.

Permanent extinguishment of the arc occurs when the gap breakdown voltage has risen, due to deionizing influences, to a value that cannot be reached by the electrode voltage controlled by circuit constants.

The action of a circuit breaker in extinguishing an arc is greatly influenced by the presence of adjacent load circuits and by the presence of distributed inductance and capacity in the connecting lines.

Introduction

RECENT work on the general subject of arc extinction as related to switch and circuit breaker action has emphasized the need for more accurate information concerning the conditions existing in an a-c. arc during the period of current zero. In particular, experimental evidence has been needed to show the rate at which the arc voltage rises toward a reignition value in the new circuit direction. This paper presents a study of the voltage and current relationships near the cyclic current zero in arcs between metallic electrodes, at moderate values of current and voltage, based on some two hundred cathode-ray oscillograms. The cathode-ray oscillograph is necessary inasmuch as the voltage may change at the rate of several hundred millions per second.

Records were taken with various circuit conditions, and with different electrode materials, shapes, and spacings. The electron beam in the oscillograph was initiated by a surge circuit controlled by a rotating contact synchronously driven from the same voltage source as that which supplied the arc. The most consistent results were obtained with an arc between copper plates 8 in. sq. and ¾ of an in. apart. This arrangement gives an arc of constant length and one which reaches its current zero at nearly the same part of each half cycle. For most of the work the power source was a transformer supplying 440 volts at 60 cycles. However, for certain circuit arrangements requiring high reignition voltages a 660-volt supply was used. The currents used were 25 amperes or less.

This investigation shows that when a small value of arc current is reached the current breaks sharply to substantially zero, where it remains until the discharge starts again with the opposite polarity. During this period the voltage across the arc electrodes rises according to the usual circuit equations until breakdown occurs, yielding first a glow discharge in the arc gap and later an arc with the new polarity. Variation of the circuit constants offers the opportunity to alter the rate of the voltage rise. It is found that a voltage rising slowly must rise to a relatively large value before breakdown occurs. The curve of breakdown voltage against time has been discussed by Dr. Slepian from a theoretical basis.[1] This study presents the experimental evidence that permits this curve to be found and shows the modifications that must be made in it.

Moreover these oscillograms show the influence that nearby loads exert upon arc extinction in circuit breakers and emphasize the necessity for further study of deionizing agents in the arc gap and of the influence of long lines with distributed constants upon arc extinction.

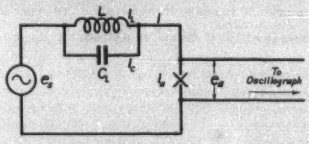

FIG. 1—CIRCUIT FOR UNSHUNTED ARC

Arc Current—Voltage Relations

I. Unshunted Arc

The arc current in the tests made was limited by an air core series reactor of sufficient size to make the current lag nearly 90 deg. behind the supply voltage. Fig. 1 shows the circuit composed of the transformer of voltage e_s connected through the reactor L, with its small (but not negligible) distributed capacitance C_L, to the arc. The arc voltage e_a was measured by the oscillograph. The time relationships existing between the supply voltage e_s, the arc voltage e_a and the arc current i_a are shown qualitatively in Fig. 2. Except

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1. *Extinction of a Long A-C. Arc*, J. Slepian, A. I. E. E. TRANS., Vol. 49, 1930, p. 421.

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at instants near the current zero the current is practically a sine wave nearly 90 deg. behind the supply voltage. Throughout this part of the cycle the arc current i_a is practically the same as the current in the inductance, i_L.

Eventually the arc current reduces sinusoidally to a small value (order of 0.1 ampere) I_1 at which it suddenly fails, falling very sharply to practically zero, where it remains until the arc again is made with current in the opposite sense. In some cases this transition period to the arc in the new direction has lasted for 600 microseconds, which is 7 per cent of one-half cycle at 60 cycles. Throughout this period the supply voltage is practically constant ($e_s = E$).

The current i_L in the inductance cannot drop instantaneously and therefore surges into its own distributed capacitance. From $t = 0$ to $t = t_1$ the voltage e_a may be computed by the usual circuit equations, inasmuch as the arc current is practically zero. See Appendix A.

$$e_a = E - E_m \cos(\omega t + \theta) \qquad (1)$$

where

$$\omega = \frac{1}{\sqrt{L C_L}} \qquad E_m = \sqrt{(E - E_1)^2 + \left(\frac{I_1}{\omega C_L}\right)^2}$$

E = the maximum of the supply voltage.
E_1 = the arc voltage when the arc fails.
I_1 = the arc current when the arc fails.
e_a = the arc voltage during the period of $t = 0$ to $t = t_1$

$$\theta = \tan^{-1} \frac{I_1/\omega C_L}{E - E_1} = \text{angle from arc failure to negative maximum.}$$

Particular note should be made of the fact that both the magnitude and the phase position of the negative voltage maximum are determined largely by the magnitude of the arc current when the arc fails. The frequency of the oscillation represented by this equation was of the order of 30,000 cycles per sec. in the circuits used. During this period from $t = 0$ to $t = t_1$ the current has changed from an extremely small negative value (much less than the 0.1 ampere arc-failure current) through zero at the zero of voltage to a small positive value. The current appears to change at approximately a uniform rate throughout this interval.

At time t_1 the voltage across the electrodes no longer rises. In fact it drops quickly to a voltage e_g, where it remains practically constant for a period of from zero to as long as 500 microseconds. The steady voltage e_g is indicative of the existence of a glow state (high-voltage small current arc). The glow voltage is of the order of 350 volts. During the glow the current though still very small rises at a considerably greater rate than during the preceding period. Eventually the glow current rises to a value sufficient to maintain an arc proper (low-voltage, relatively high-current phenomenon). This change occurs at $t = t_2$. At this moment the electrode voltage drops with great rapidity from the glow value of about 350 to the arc value of about 75, the latter depending on the length of the arc. Throughout the remainder of the half cycle the arc voltage remains constant, but the arc current exhibits its sine character.

Typical oscillograms illustrating this circuit condition for arcs between iron, aluminum, and copper electrodes respectively are shown in Figs. 3A, B, and C. In these three figures the starting glow voltages are 360, 310, and 340. The breakdown voltages preceding the glows are 380, 320, and 380 occurring in 24, 27, and 20 microseconds respectively. There appears to be far less difference between these kinds of electrodes, at least, than between various circuit conditions for only one electrode material.

It has been shown that the negative electrode voltage dip is due to the sharp break in the negative arc current and depends for its magnitude upon the value of the current at the moment it fails. It is possible for the

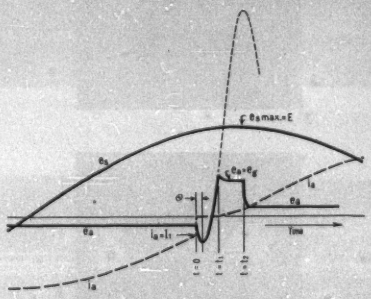

Fig. 2—Unshunted Arc Current and Voltage

current to fail earlier in the cycle at a larger negative value. In this case the negative electrode voltage may drop very sharply, in practically linear manner with time, to a sufficient value to produce a negative glow which lasts for a few microseconds and dies out. After this the electrode voltage follows the usual cosine curve given by equation (1). Fig. 3D illustrates the two types of negative voltage dip that may occur. This particular oscillogram is one of an 0.0096 μf. capacity shunted across the arc, but similar negative dips with the unshunted arc have been observed frequently.

The sudden changes of the volt-ampere relationships in the electrical discharge that is passing at low-current values are best understood in terms of the volt-ampere characteristics of the arc and glow discharge in air at atmospheric pressure. Compton[2] gives the type of characteristic shown in Fig. 4A, based on the use of direct current over long time intervals. The oscillo-

2. *The Physical Nature of the Electric Arc*, Compton, A. I. E. E. Trans., 1927, p. 868.

grams presented in this paper clearly show the existence of both the arc and glow states at low arc current values, but they indicate that the shape of the volt-ampere characteristic is appreciably different from that shown in Fig. 4A.

In the a-c. arc, with conditions changing with great

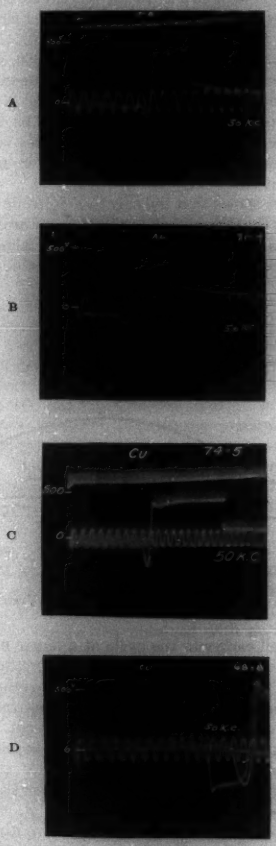

FIG. 3—VOLTAGE OSCILLOGRAMS FOR *Fe*, *Al* AND *Cu*, UNSHUNTED ARCS. 3D ILLUSTRATES NEGATIVE DIP (ROUND CURVE), AND NEGATIVE GLOW (PEAKED CURVE)

rapidity, the arc current may fail at larger or smaller values, the larger values producing a negative glow, the smaller values failing to do so. A positive arc voltage produces a very small current growing with the voltage until the glow voltage is reached, when the current rises at practically constant voltage. This condition continues until the arc current has reached a value sufficient to maintain an arc proper, whereupon the voltage drops sharply (about 400 to 75) and the current returns to its sinusoidal variation at constant arc voltage. Fig. 4B is a cyclogram of arc voltage against arc current illustrating this point. This cyclogram was taken with a 4,000 ohms shunt across the arc, but it is representative of the unshunted arc case. It should be noted in the cyclogram that the curve has a different shape for the two polarities in the alternating-current case, while an extension of Fig. 4A to include negative

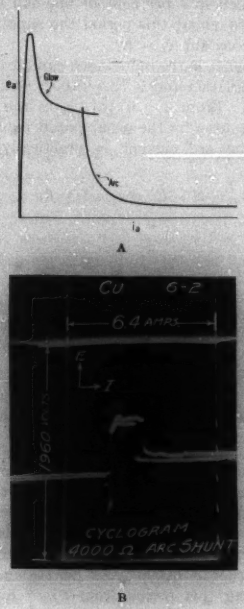

FIG. 4—VOLTAGE CURRENT CYCLOGRAMS

A. Direct current and long time (Compton)
B. A-c. short-time oscillogram

currents for the direct-current situation would be a perfect reflection of the curve as shown but with reversed polarity.

II. RESISTANCE SHUNTED ARC

The circuit used for the resistance shunted arc is shown in Fig. 7A. The electrode voltage during the arc current zero for a resistance shunted arc must be written in two forms, depending upon whether the resistance R is greater or less than the critical resistance R_c, which is equal to $\frac{1}{2}\sqrt{\frac{L}{C_L}}$. See Appendix B.

For resistances larger than the critical resistance the electrode voltage becomes

$$e_a = E - e^{-\frac{1}{2RC_L}t}\left[(E - E_1)\cos\omega t + \frac{1}{\omega C_L}\left(-I_1 + \frac{E - E_1}{2R}\sin\omega t\right]\right. \qquad (2)$$

$$\omega = \sqrt{\frac{1}{LC_L} - \frac{1}{4R^2C_L^2}}$$

$$R > \frac{1}{2}\sqrt{\frac{L}{C_L}}$$

See Appendix B-I.

Figs. 5A, B and C illustrate this condition. For the first two of these cases with an infinite resistance equation (2) reduces to the simpler form of equation (1). Equation (2) holds for Fig. 5C inasmuch as the 10,000-ohm arc shunt is greater than the critical resistance of 6,125 ohms.

The circuit conditions back of all of the oscillograms of Fig. 5 were identical except for the shunt resistance, and the maximum value of the line voltage varied only slightly from 975. The series reactor possessed an inductance L of 0.066 henrys and a distributed capacity C_L of 440 $\mu\mu$f. This distributed capacity may easily be calculated by applying equation (1) to one of the curves of Fig. 5B. From the magnitude and phase position of the negative dip maximum ω may be computed and hence C_L found. The arc was struck between the centers of two 8 in. by 8 in. parallel copper plates spaced ¾ of an in. apart. The arc was semi-enclosed to prevent effects from stray air currents.

For resistances smaller than the critical resistance the electrode voltage takes the form

$$e_a = E - e^{-\frac{1}{2RC_L}t}\left[(E - E_1)\cosh\beta t + \frac{-I_1 + \frac{E - E_1}{2R}}{\beta C_L}\sinh\beta t\right] \qquad (3)$$

$$\beta = \sqrt{\frac{1}{4R^2C_L^2} - \frac{1}{LC_L}}$$

$$R < \frac{1}{2}\sqrt{\frac{L}{C_L}}$$

See Appendix B-II.

Figs. 5D, E, F, G and H for shunts of 4,000, 2,000, 1,200, 800 and 500 ohms respectively may be represented by equation (3).

For purposes of computation equation (4) may be used in place of equation (3) from which it has been reformed.

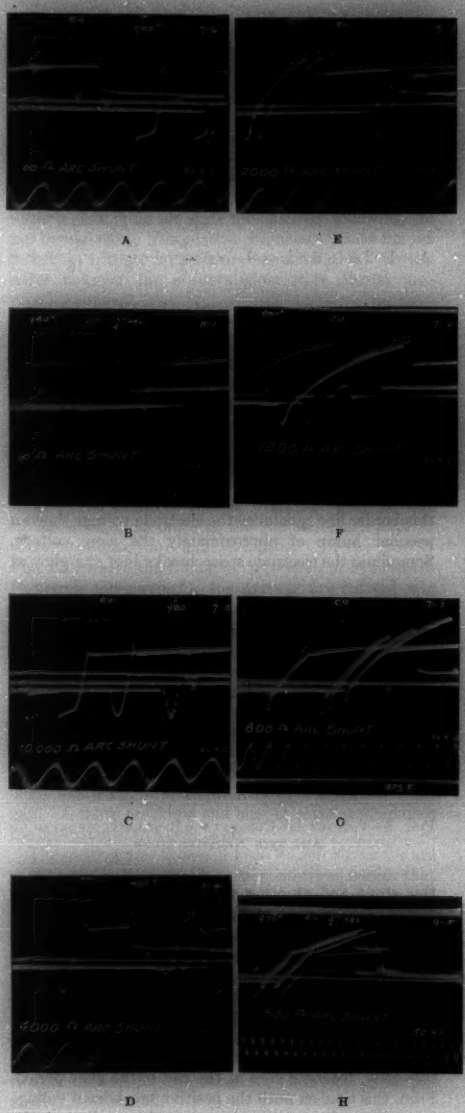

Fig. 5—Voltage Oscillograms for Resistance Shunted Arc. 25-Ampere, 660-Volt, ¾-in. Arc Between Flat Copper Electrodes

$$e_a = E - \frac{1}{1-\Delta_1}\left\{ e^{-\frac{Rt}{L}(1+\Delta_2)}\left[(E-E_1)\left(1-\frac{\Delta_1}{2}\right)-I_1 R\right] - e^{-\frac{t}{RC_L}\left(1-\frac{\Delta_1}{2}\right)}\left[(E-E_1)\frac{\Delta_1}{2}-I_1 R\right]\right\} \quad (4)$$

where

$$\alpha = \frac{4 R^2 C_L}{L}$$

$$\Delta_1 = 1 - 2 R C_L \beta = 1 - \sqrt{1-\alpha} = \frac{\alpha}{2} + \frac{\alpha^2}{8} + \frac{\alpha^3}{16} + + +$$

$$\Delta_2 = \frac{2\,\Delta_1}{\alpha} - 1 = \frac{\alpha}{4} + \frac{\alpha^2}{8} + + +$$

As t increases the second term in the large bracket becomes very small, and when $t > \frac{\log_e 10}{\beta}$ the error introduced by its omission is less than one per cent.

The 500-ohm shunt of Fig. 5H is the lowest resistance at which it was possible to operate the arc. At and near this border line resistance the electrode voltage shows a peculiar hump at approximately the glow voltage. Sometimes the electrode space breaks into the glow at

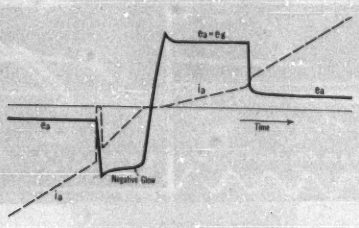

FIG. 6—VOLTAGE AND CURRENT FOR RESISTANCE SHUNTED *Cu* ARC WITH NEGATIVE GLOW

this stage and at other times the voltage rises to a higher breakdown point, then drops to glow or arc value.

The arc current curve takes the general form shown in Fig. 2 if the electrode voltage goes through its round negative dip. However, if the sharp voltage drop leading to a negative glow occurs (Fig. 6), the current drops to zero simultaneously but at once rises to a value sufficient to sustain a negative glow, after which it drops gradually throughout the existence of the negative glow, reaching zero at the end of the glow period. From this time on until the positive breakdown voltage is reached the current is extremely small and the voltage equations given hold.

The reactor current can be obtained by mechanical integration from the electrode voltage curve, provided the reactor distributed capacity is relatively small.

$$E - e_a = L\frac{di}{dt}, \text{ so that}$$

$$i = \int (E - e_a)\,dt + K$$

Subtracting the current through the shunt resistance gives the arc current.

$$i_a = i - i_n = \int (E - e_a)\,dt + K - \frac{e_a}{R} \quad (5)$$

The effect of the arbitrary constant may be eliminated

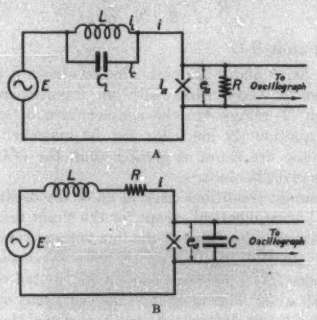

FIG. 7—CIRCUITS FOR RESISTANCE AND CAPACITY SHUNTED ARCS

A Arc shunted by resistance
B Arc shunted by capacity

by choosing the arc current as being zero when the arc voltage is zero.

III. CAPACITY SHUNTED ARC

The circuit used for the capacity shunted arc is shown in Fig. 7B. The small distributed capacity of the series reactor C_L may be neglected in comparison with the capacity across the arc C, inasmuch as the latter is far the greater.

The electrode voltage is given by equation (6), for the case of $\frac{R^2}{4\,L^2} < \frac{1}{LC}$. See Appendix C.

$$e_a = E + e^{-\frac{R}{2L}t}\left[-(E-E_1)\cos\omega t + \frac{I_1 - \frac{RC}{2L}(E-E_1)}{\omega C}\sin\omega t\right] \quad (6)$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4\,L^2}} \qquad \frac{R^2}{4\,L^2} < \frac{1}{LC}$$

The studies presented in this paper involved a series resistance so low that it might be neglected. Making $R = 0$ equation (6) reduces to the form shown in equation (7), which is identical with equation (1)

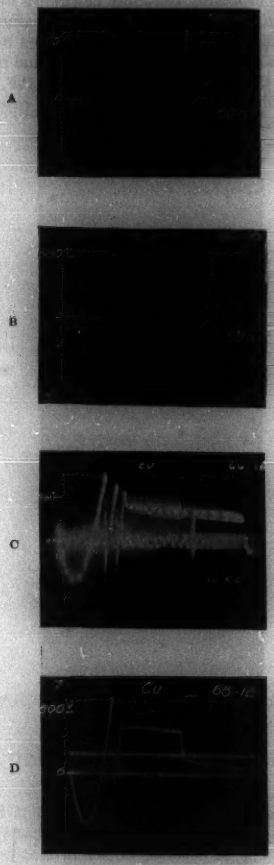

FIG. 8—VOLTAGE OSCILLOGRAMS FOR CAPACITY SHUNTED Cu ARC

providing the capacity of the coil is now replaced with that placed across the arc.

$$e_a = E - E_m \cos(\omega t + \theta)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\theta = \tan^{-1} \frac{I_1}{\omega C (E - E_1)} \tag{7}$$

$$E_m = \sqrt{(E - E_1)^2 + \left(\frac{I_1}{\omega C}\right)^2}$$

The electrode voltage, with capacity shunt, may exhibit the rounding negative dip or the sharp drop that leads to the negative glow, as shown in Figs. 8A and B. In these figures is shown also the possibility of having either a glow or an arc state immediately following the gap breakdown. The general character of these curves is similar to those for resistance shunted arcs. In the latter however the positive glow state almost universally appears.

All four electrode voltages shown in Fig. 8 were obtained by using a capacity of 0.0125 μf. Figs. 8C and D illustrate a state that exists only with a relatively large capacity. In these figures it is seen that the voltage following the initial breakdown may break to a negative value or may develop into a sawtooth wave. Since the electrode and the capacity are shunted, the extremely rapid drop in voltage indicates a short time discharge of the condenser, or spark through the arc space. The negative voltage overshoot may be due to the small inductance of the condenser arc path or to some internal arc ionic condition. In the case of Fig. 12D, where the timing is stretched out sufficiently to permit measurement, the average arc current was found to be 13 amperes during the first spark and approximately 8 amperes during each of the succeeding sparks.

Following the spark the electrode voltage, starting from approximately zero, rises again according to equation (7) but at a steeper slope because the initial current I_1 in the series reactor path has grown considerably since the first breakdown. Spark after spark may follow until the series reactor current has grown to a point where a glow or an arc may be maintained in the gap. A sharp crackling and hissing noise accompanies the operation of this type of arc.

The oscillograms shown in Figs. 9A and B give the arc current and voltage for identical conditions, except that it was necessary to introduce a small coil into the arc path in order to measure the current. The capacity was 0.0125 μf. The oscillograms were taken only a few seconds apart. The current picture shows clearly the initial break in the current, the prolonged current zero period and the spark that occurs whenever the electrode voltage drops suddenly. The current curve has been offset from the zero line. The arc current curve has approximately the same slope as the series reactor current for the condition of the arc short-circuited, as it should.

Figs. 10A and B are oscillograms of a cyclogram and an arc voltage taken a few seconds apart under the same conditions as those of the preceding figure. The

new point to be observed is the loop on the cyclogram which represents the current-voltage relation during the transition from the glow to the arc. Probably the loop should be pointed in the current direction; the rounding may be partially due to the inductance of the small coil inserted in the arc path for measurement purposes. The records on Fig. 11A, taken at intervals of a few seconds, show the current and voltage with the same capacity (0.0125 μf.) when a sawtooth occurs.

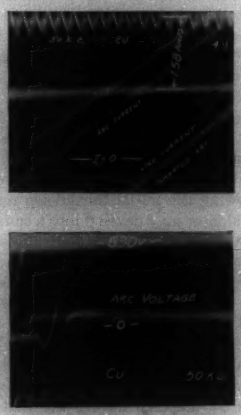

FIG. 9—CURRENT AND VOLTAGE OSCILLOGRAMS FOR 0.0125 μf SHUNT

Fig. 11B is the complex type of cyclogram that may be expected in this case.

The effects of varying the capacity shunted across the arc are shown in Fig. 12. Below 0.01 μf. the gap goes through the glow to reach the arc stage, but for larger capacities the sawtooth replaces the glow interval and the voltage changes require longer and longer periods. For the larger capacities the negative voltage dip is missing, not because the current continues until I_1 is zero but because the term $\frac{I_1}{\omega C}$ in equation (7) becomes negligibly small compared to $E - E_1$.

ARC-FAILURE CURRENT VALUES

A knowledge of the numerical values of arc-failure currents is desirable. Now as soon as the arc stops momentarily at the end of a half-cycle the voltage recovery curve starts sharply downwards (increasing negative values) with a slope given by

$$\left(\frac{d\,e_a}{d\,t}\right)_{t=0} = \frac{I_1}{C} \tag{8}$$

as shown in the appendixes.

Theoretically this provides a measure of I_1, which is the arc-failure current for any particular record. Practically the slope is changing with considerable rapidity at this moment, so that it is impossible to obtain an accurate measure of the current by this means.

The trace ordinarily continues downward to a negative maximum which provides a good measure of the arc-failure current. Equation (9), obtained by equating the derivative of equation (4) to zero and rearranging, shows the relation between I_1 and the extreme of the negative dip, for the logarithmic case with a resistance shunt.

FIG. 10—CYCLOGRAM AND VOLTAGE FOR 0.0125 μf SHUNT

$$I_1 = \frac{1 - \frac{\Delta_1}{2}}{R}\left[(E - E_1) - (E - e_m)\,T^{\frac{\Delta_1}{2}}\right] \tag{9}$$

where

$$T = e^{2\omega t_m} = \frac{\frac{2\,I_1\,R}{\Delta_1} - (E - E_1)}{\left(\frac{I_1\,R}{1 - \frac{\Delta_1}{2}}\right) - (E - E_1)} \tag{10}$$

e_m = negative maximum voltage.
t_m = time at which it occurs.

$$\Delta_2 = \frac{1}{2\,R\,C_L\,\beta} - 1 = \frac{1}{\sqrt{1-\alpha}} - 1$$

$$= \frac{\alpha}{2} + \frac{3\,\alpha^2}{8} + \frac{5\,\alpha^3}{16} + +$$

Δ_1, β and α have the meanings previously assigned.

$$R < \frac{1}{2}\sqrt{L/C_L}$$

Equation (10) can readily be put into a form giving a direct solution for I_1 in terms of t_m, but in the records

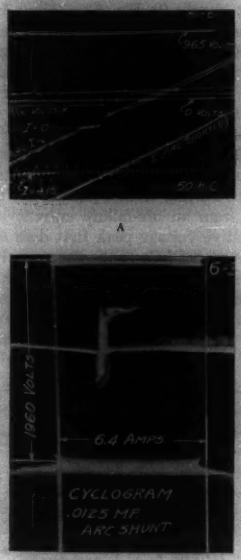

FIG. 11—CYCLOGRAM, CURRENT AND VOLTAGE FOR 0.0125 μf SHUNT

obtained t_m could not be accurately measured. It is easy, however, to measure e_m closely. I_1 is then obtainable by a trial and error solution of equation (9), in which $T^{\frac{\Delta_2}{2}}$ is only slightly sensitive to changes in I_1. The greatest value of $T^{\frac{\Delta_2}{2}}$ for all of the records studied was about 1.2, obtained in one of the 4,000-ohm cases; its minimum was of course unity.

For very small values of α, such as occur in the present study when R is 500 ohms, Δ_1 and Δ_2 can be neglected entirely, giving the still simpler form of equation (11).

$$I_1 = \frac{1}{R}\,(e_m - E_1) \tag{11}$$

For the harmonic case with a resistance shunt equation (12), obtained in a manner similar to equation (9), provides the best approach to I_1.

$$I_1 = -\sqrt{\frac{C_L}{L}\left[(E-e_m)^2\,e^{\frac{t_m}{RC_L}} - (E-E_1)^2\left(1-\frac{1}{\alpha}\right)\right]} + \frac{E-E_1}{2\,R} \tag{12}$$

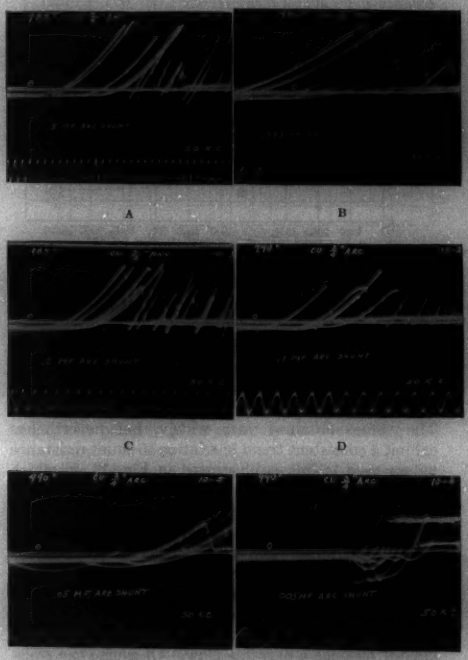

FIG. 12—VOLTAGE OSCILLOGRAMS FOR CAPACITY SHUNTED ARC. 25-AMPERE, 660-VOLT, ¾-IN. ARC BETWEEN FLAT COPPER ELECTRODES

where

$$\frac{t_m}{2\,R\,C_L} = \frac{I_1}{I_1 - \dfrac{(E-E_1)\,\alpha}{2\,R}}$$

$$R > \frac{1}{2}\sqrt{\frac{L}{C_L}}$$

As in the preceding case, a trial solution is required if t_m cannot be accurately measured.

For the capacity shunted arc, $E - e_m$ becomes E_m in equation (1), which is very readily solved for I_1.

Of course if the arc-failure current is large enough so that the negative maximum that it would naturally reach exceeds the reignition voltage, a negative glow forms, and the normal extreme indicated by the circuit equations is never reached. This makes impossible an accurate determination of the arc-failure currents

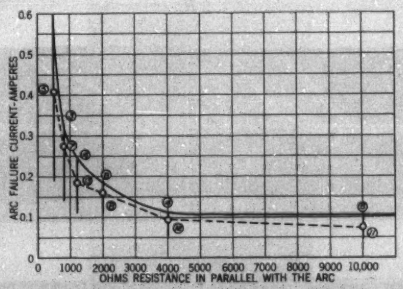

Fig. 13—Arc-Failure Currents for Resistance Shunts

for such cases; all that can be said is that they exceed a certain value.

Complete calculations have been made of the arc-failure currents from those traces that did not indicate a negative glow, the determinations being based on the extreme negative dip of the voltage recovery curve. The values obtained vary widely, but nevertheless exhibit a consistent trend in relation to shunt resistance values used. This is illustrated in Fig. 13, in which a heavy vertical line extends from the extreme lower to the upper value of arc-failure current observed with each resistance condition studied. The solid curve marking the upper ends of these lines merely indicates the boundary between currents that will and those that will not produce a negative glow.

Dotted lines join points that have been plotted to indicate the average of those arc-failure currents that did not produce a negative glow. Of course the omission of the glow cases prevents the averages from being of great significance, since every omission indicates an arc failure at a current near to or above the maximum of those contributing to the average. There is nevertheless a sufficiently definite trend to make the plot of interest. The figure adjacent to each point is the number of records contributing to the average, and at the top of each heavy vertical line is another figure which stands for the number of records made indicating a negative glow.

Fig. 13 indicates that the resistance acts in some way to affect the most likely value of arc-failure current, and to change the likelihood of very small or very large values. A low resistance is favorable toward arc failure at a relatively high value of arc current. A large capacity seems to have a similar tendency.

The maximum arc-failure current evaluated, considering all the observations with resistance and with capacitance in parallel with the arc, was less than 0.6 ampere. Some that resulted in a negative glow may have been slightly greater. The minimum determined was about 0.03 ampere. Values less than 0.05 ampere were very unusual.

Arc Leakage with Resistance Shunts

A peculiar behavior of considerable interest is exhibited most clearly in Fig. 5G. There is a sudden change in the slope of the voltage rise curve just before it reaches zero on the return from the negative peak. This is quite pronounced in the 1,200-ohm and 800-ohm records, and actually exists to a greater or less extent in all of the logarithmic cases except that for 4,000 ohms. In order to analyze it carefully the extreme left record of Fig. 5E has been reproduced as the solid line in Fig. 14. The lower dotted line represents the theoretical curve resulting from equation (3) that should be expected from an arc-failure current sufficient to give the negative voltage peak that does occur. The oscillographic record follows the theoretical curve very closely until shortly after the negative peak is reached, then for a short time the potential between the electrodes decreases much more rapidly than it would according to circuit theory. This is indicated by the departure of the voltage trace from the lower line.

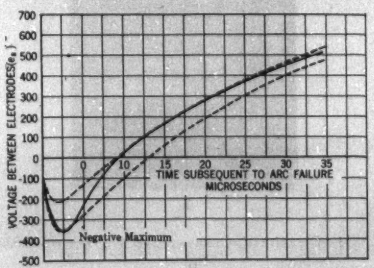

Fig. 14—Analysis of Voltage Curve from Fig. 5E to Show Effect of the Arc-Leakage Current

As soon as zero voltage is reached it again follows a theoretical curve closely until reignition. The new curve has the same decrement and final asymptote as the original one, but differs in reference to initial conditions.

This indicates that for a few microseconds after the negative peak an appreciable "leak" of current takes place between the electrodes, without, however, resulting in a negative glow at its characteristic voltage. No such behavior as is illustrated in Fig. 14 appeared to take place in the capacity shunt observations.

Reignition Voltage as Related to Circuit Constants

One of the primary objectives of this investigation has been the determination of the nature of the relationship that exists between the value of voltage required for reignition and the time taken to reach that voltage. A program to meet this requirement necessitated modifying the circuit constants in such a way as to cause the voltage to rise at widely different rates. This was done by the use, first of resistance, then capacity, in parallel with the arc. The results of the resistance tests appear in Fig. 5, and those with the capacity in Fig. 12.

In making the tests for Figs. 5 and 12 considerable pains were taken to maintain the arc length, current, and power consumption constant. In spite of this quite a variation in reignition voltage appeared, especially in the case of slowly rising voltages. The results of all the tests that were made on the ¾-in. 25-ampere 660-volt circuit arc are combined in Figs. 15 and 16. Each point on these graphs represents a reignition voltage, plotted against time measured from the instant at which the arc current failed.

Referring back to Fig. 14, it is seen that after zero voltage the recovery curve with a resistance shunt follows a theoretical trace dependent on the logarithmic decrement determined by circuit constants, on the asymptote determined by line voltage, and on the time at which zero voltage occurs. The dotted lines in Fig. 15 are such curves calculated from equation (4).

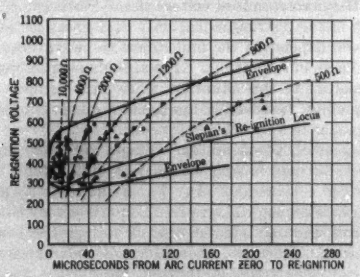

Fig. 15—Reignition Voltage—Time Values for Resistance Shunts

25-ampere, 660-volt, 3/4-in. arc between flat copper electrodes

In placing them the zero voltage time was selected so to make the curve lie as close as possible to all of the observed reignition points.

In Fig. 16, representing the capacity shunt condition, the dotted lines that appear for each value of capacity used are theoretical voltage rise curves calculated from equation (7). The values of arc failure currents used in the calculations were chosen to produce curves best fitting the observed reignition points. These currents may be regarded as representative arc-failure currents for each set of circuit conditions. The occasional points at some distance from the curves merely indicate arc-current failure at some value considerably different from the representative value chosen for the curve. For the 0.05, 0.2, 0.33 and 0.5 μf. capacity conditions the most representative arc-failure currents were so small, relative to other terms in the equation, that they had a negligible effect on the positions of the curves.

A small group of points to the extreme left of both figures, between 330 and 380 volts, and between one

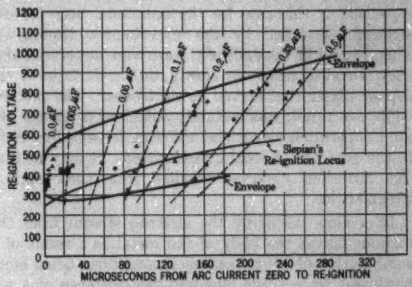

Fig. 16—Reignition Voltage—Time Values for Capacity Shunts

25-ampere, 660-volt, 3/4-in. arc between flat copper electrodes

and two microseconds, is taken from ignition points of the negative glow. These fit very well into the general appearance of the rest of the diagram; this tends to discount any supposition that would make the change in polarity of importance in determining the short-time re-striking voltage.

Closely related to the requirement of a generally higher re-striking voltage for smaller resistances or larger capacities there developed a consistently increasing difficulty in maintaining the arc for any great length of time. After only a few seconds of operation with small resistances or large capacities, and at some apparently accidentally determined half-cycle, the required re-striking voltage continued indefinitely to exceed the recovering voltage, hence the arc ceased.

The upper and lower envelope curves that appear on Figs. 15 and 16 provide a demarcation of the zone within which reignition occurred. It is interesting to note that the reignition locus obtained by Slepian using an indirect method is in general similar in shape to the upper envelope, but starts at a short-time value of 235 volts and is correspondingly low in value throughout its length. This locus is plotted on Figs. 15 and 16 for comparison.

The general features of these locus figures may be summed up as follows:

(1) For a very short time delay, the reignition voltage is quite definite in value, and is about the same as the glow voltage, but perhaps a trifle higher.

(2) As the voltage rise is delayed, the reignition voltage value varies between wide limits, the maximum rising considerably above the glow voltage, the minimum not changing greatly.

(3) There is no very marked difference between the reignition voltage required for a given time delay produced by resistance and that produced by capacity. The envelope curves in the two diagrams are identical.

It seems difficult to account for the great variation in late reignition voltages in terms of the various deionization processes that have been supposed to exist; especially is this true for the occasional very low restriking voltages. In the experiments leading to Figs. 15 and 16 the ends of the arc were not stationary; they traveled, relatively slowly, on the surfaces of the copper plates that served as electrodes. The motion, while not rapid, was certainly sufficient to expose the arc to operation on surfaces differing from one moment to the next due to local irregularities. It may be that the variations observed resulted from these surface irregularities.

In a glow discharge the electron stream from the cathode leaves the body of the metal as a result of a combination of several causes, among them being bombardment by positive ions and the existence of a potential gradient due to a space-charge adjacent to the metal surface. The severity of the ion bombardment is of course closely related to the potential gradient at the cathode and to the space-charge. It seems reasonable to suppose that when some critical gradient at the surface is reached, electrons are suddenly released freely enough to support a glow discharge. In this case the reignition voltage might be expected to depend somewhat on surface irregularities, surface temperature, and space-charge distribution in the arc path, all of these being variable elements.

Application to Power Systems

The facts that have been presented are of interest in a consideration of conditions under which circuit breakers on electric power systems must operate. Manufacturers have observed that circuit breaker test installations in which reactance coils are used to limit short-circuit current impose an actual circuit breaker duty more severe than that resulting from the same current interruption requirement in normal service on a power system. The difference is attributed to the introduction of a considerable capacity in parallel with the arc by the lines and apparatus that limit the short-circuit current, and to the production of a condition roughly equivalent to a parallel resistance by the presence of load circuits. Both situations delay the voltage rise toward reignition, thereby permitting a longer time for recovery of dielectric strength in the arc region. This of course favors an early extinguishing of the arc.

A single-phase short circuit is usually interrupted by an oil switch in which there are two breaks in series in each of the two line wires involved. There will then be four arcs in series in which reignition must occur if the arc is to persist after the end of any given half-cycle. Due to the variability of the reignition voltage, it is not to be supposed that all four arcs will reach re-striking voltage at exactly the same instant, but as soon as one breaks down the others will follow immediately due to the increased voltage imposed on them. If the electrostatic fields in the circuit breaker are such as to produce an approximately even distribution of the total voltage between the four breaks, the over-all reignition voltage will be approximately four times that for any one arc.

In oil circuit breakers the arc is drawn in oil. After a very short time it is playing in a bubble of gas formed by the decomposition of oil. The reignition and glow voltages cannot be expected to be the same in these gases as in air, but they should be of the same order of magnitude. The total reignition voltage can then be represented in the manner of Figs. 15 and 16 except with voltages in general about four times as great.

The calculation of the voltage rise curves for a transmission circuit is complicated by the necessity of considering distributed capacity and inductance in their relation to the time required for the propagation of an electrical impulse along a line. Thus it requires about 50 microseconds for an impulse to traverse 10 miles of line; therefore it would seem that only the capacity of that part of the line less than 10 miles from the circuit breaker can effect the voltage across the arc during the first 50 microseconds of voltage rise. Furthermore, the effect of the distributed capacity of a given section of line will depend on the inductance to be traversed by the charge in reaching it. The authors plan to present an analysis of this situation in the near future. For the present, however, a few general observations can be made as to what is likely to occur.

In general, power system voltages are much higher than those used in securing the results that have been described. Equations (**1**), (**2**), (**3**) and (**4**) all show that these higher voltages result in a tendency toward a correspondingly greater rate of voltage rise toward reignition. However, the values of inductance and capacity that apply to transmission circuits are also much larger than those used here. Rough estimates using characteristic values of circuit constants make it seem probable that the net result is a delay of reignition by periods of time of the order of magnitude of those illustrated in the oscillograms.

There will of course be great differences in the rates of voltage recovery according to the location of a circuit breaker in a transmission system. There seems no doubt, for example, but that the voltage rise in a circuit breaker just off a generating station bus will take place very much more rapidly than in one located at the load end of a 100-kv. line of considerable length.

The current density in an arc is usually approximately constant. Changes in total current result in an in-

crease or decrease in the cross sectional area of the arc stream and in the area of the cathode spot from which the electron stream emerges. As yet no reason is apparent for believing that the decreasing current in an arc of 1,000 amperes r. m. s. value will cease at a minimum value any higher than the 0.05 to 0.6 ampere found here to apply in the case of a 25-ampere arc. The larger arc current will produce a higher electrode temperature, and this might easily favor a lower arc-failure current. In an application to practical problems the arc-failure current is of much less importance on high voltage than on moderate voltage circuits so far as its effect on the rate of voltage recovery is concerned.

The arc studied in this investigation was ¾ in. long. When interrupting a short circuit of any considerable magnitude a circuit breaker arc will be maintained to a considerably greater length than this, and while it lasts each half-cycle will end with a considerably greater electrode spacing than did the preceding one.

The voltage required for reignition in the case of extremely rapid voltage recovery is approximately the glow voltage. With a moderately short electrode spacing this is much higher than the normal arc voltage that exists during the greater part of each half-cycle. Most of the potential difference between electrodes in the glow occurs in the cathode drop associated with the positive ion space-charge adjacent to the cathode. This cathode drop is of course not affected by increased electrode spacing. As the electrodes separate the glow voltage increases, but not nearly in proportion to the change in spacing, as the increase takes place entirely in the region between the cathode drop and the anode.

As the cathode drop in the normal arc is a much smaller fraction of the total than is that in the glow, the arc voltage may be expected to increase much more rapidly than the glow voltage as the electrodes spread. By the time the separation is several inches the two may be approximately equal. It is very unlikely, however, that the short-time reignition voltage will ever be less than the normal arc voltage. If it were to be so, a complete half-cycle of glow, at low current and glow voltage, would immediately precede final arc extinction. Such behavior in a circuit breaker would be readily observable in the voltage and current traces of an ordinary oscillographic record, and no tests indicating it have come to the authors' attention. Low-voltage arcs between copper rods have, however, been observed to extinguish immediately following a half-cycle at glow rather than at arc voltage, the extinction resulting from gradual electrode separation.

In any case, the short-time reignition voltage will be greater for each successive half-cycle as the electrodes separate. In addition to producing this change, the greater spacing permits readier access of relatively cold oil vapors and in this and other ways accelerates the deionization that is taking place in the arc region prior to reignition. This acceleration appears as an increase of slope in the reignition locus. Thus the reignition locus for each half-cycle should be steeper than the previous one as well as starting at a higher value.

In Fig. 17 an illustrative set of expected reignition loci for three successive half-cycles during a circuit breaker operation has been sketched qualitatively. Each one of these represents what must inevitably be true, that at every instant of time following each current zero there is some definite voltage that would cause reignition, and that this voltage increases with the passage of time. The reignition zones for a ¾-in. arc in air, bounded by envelope curves of Figs. 15 and 16, indicate the limits within which a reignition locus representing the behavior at the end of that particular half-cycle corresponding to a ¾-in. separation must lie. It will be observed that the reignition curves have been

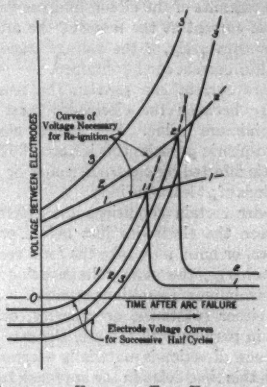

Fig. 17—Reignition Voltage—Time Values for Capacity Shunts with Increasing Electrode Spacing

represented in Fig. 17 as increasing both in short-time value and in steepness for successive half-cycles in accordance with the preceding discussion.

The lower family of curves of Fig. 17 represents typical voltage recovery curves, of the capacity shunt type, for successive half-cycles, numbered to correspond with the proper reignition locus. Each curve starts at a lower point on the graph than its predecessor because of the greater arc length and consequent greater normal arc voltage. The shapes of these curves are all alike, however, as they depend only on circuit constants external to the active breaker.

As soon as the electrode separation results in a reignition locus not intersected by the corresponding voltage rise curve, as illustrated by the No. 3 pair of Fig. 17, the circuit breaker duty is completed; the arc will not re-strike. It is evident that in any situation where the circuit constants result in a very rapid

voltage recovery, there must be compensation for this by a correspondingly rapid deionization in the region between the electrodes if the arc is not to be drawn out to an extreme length. Also, on the whole, a parallel resistance condition should be more favorable than a parallel capacity, as it would not demand ultimately so steep a slope of the reignition locus to result in a failure of the curves to intersect.

Summary

A series of oscillographic records of voltage and current in an a-c. arc between copper electrodes, taken near the moment of current reversal, has indicated the following points:

1. For a brief but measurable period at the end of each half-cycle the current between the electrodes is essentially zero.

2. The values of electrode potential difference during this time can be accurately predicted if the electrical constants of the circuit and the values of arc voltage and current at the moment the arc stops are known; or conversely, if the voltage trace is known, the arc-failure current can be obtained.

3. Before current can pass in the new direction the voltage between the electrodes must rise to a definite minimum value. For copper electrodes in air at atmospheric pressure this is about 300 volts, and is very little affected by electrode shape or placement up to spacings of at least one inch.

(4) Under certain conditions the electric current flow between the electrodes just before or after the current zero, or both, may take the form recognized as a glow discharge, the glow voltage being about the same as the minimum reignition potential.

(5) If either resistance, or capacity, or both, are connected in parallel with the arc, the rate of voltage rise in the new direction is materially decreased. Associated with this time delay in the approach to reignition potential an increase in the voltage required for reignition usually occurs. The change, however, is not consistent from one half-cycle to the next, due to variable factors whose nature is not yet definitely determined.

(6) As applied to circuit breakers on power transmission and distribution systems the equations for electrode voltage during the period of zero arc current involve consideration of distributed inductance and capacity, and perhaps more important, the time of propagation of the electrical impulses between points concerned on the circuit.

(7) The rate of voltage rise toward reignition in power system circuits may be expected to be greatly affected by the position of the circuit breaker and fault relative to the inductance and distributed capacity of the system.

(8) The presence of load circuits paralleled with the arc adjacent to its input side may be expected to act in the same general manner as a resistance in parallel with the arc, tending to cause a delay in the rise toward reignition, with a consequent lessening of the expected severity of circuit breaker duty.

(9) Increasing length of arc due to progressive electrode separation may reasonably be expected to result in a greater initial value, and greater steepness, of the reignition locus, and in a simultaneous lowering of the voltage available for reignition at any specified moment following the current zero. The arc extinguishes when at the end of some half cycle the voltage curve fails to cross the reignition locus.

Acknowledgment

The authors wish to acknowledge the kindness of Dr. J. Slepian for suggesting the desirability of this study, and the loan by the Detroit Edison Company of a cathode-ray oscillograph with which many of the oscillograms were taken. The remainder were taken with a General Electric Company cathode-ray oscillograph.

Appendix A

(See Fig. 1)

Arc Voltage During Arc Current Zero. (No Arc Shunt)

$$E = e_a + L\frac{d\,i_L}{d\,t} = e_a + \frac{q_c}{C_L}$$

$$i_L + i_c = 0$$

$$\frac{d\,i_L}{d\,t} = -\frac{d\,i_c}{d\,t}$$

$$\frac{q_c}{C_L} + L\frac{d^2\,q_c}{d\,t^2} = 0$$

$$q_c = A\cos\omega\,t + B\sin\omega\,t$$

$$\omega = \frac{1}{\sqrt{L\,C_L}}$$

$$e_a = E - \frac{1}{C_L}(A\cos\omega\,t + B\sin\omega\,t)$$

$$i_L = -i_c = -\frac{d\,q_c}{d\,t} = A\,\omega\sin\omega\,t - B\,\omega\cos\omega\,t$$

Initial Conditions:

At $t = 0$ $e_a = E_1$ and $i_L = I_1$

$$E_1 = E - \frac{1}{C_L}A \qquad A = C_L(E - E_1)$$

$$I_1 = -B\,\omega \qquad B = -\frac{I_1}{\omega}$$

Hence $e_a = E - (E - E_1)\cos\omega\,t + \dfrac{I_1}{\omega\,C_L}\sin\omega\,t$

$$e_a = E - E_m\cos(\omega\,t + \theta)$$

where $\omega = \dfrac{1}{\sqrt{L\,C_L}}$

E = the maximum of the supply voltage.

E_1 = the arc voltage when the arc fails.

I_1 = the arc current when the arc fails.
e_a = the arc voltage during the period $t = 0$ to $t = t_1$ (Fig. 2)

$$\theta = \tan^{-1} \frac{I_1/\omega C_L}{E - E_1}$$

$$E_m = \sqrt{(E-E_1)^2 + \left(\frac{I_1}{\omega C_L}\right)^2} \quad \Big\| \quad I_1 = [\sqrt{E_m^2 - (E-E_1)^2}]$$

$$\omega C_L \left(\frac{d e_a}{d t}\right)_{t=0} = \frac{I_1}{C_L}$$

Appendix B

(See Fig. 7A)

$$E = L \frac{d i_L}{d t} + R i = \frac{q_c}{C_L} + R i$$

$$i = i_L + \frac{d q_c}{d t} = \text{line current}$$

$$\frac{d i}{d t} = \frac{d i_L}{d t} + \frac{d^2 q_c}{d t^2}$$

$$E = \frac{q_c}{C_L} + R\left(i_L + \frac{d q_c}{d t}\right)$$

$$O = \frac{1}{C_L} \frac{d q_c}{d t} + R\left(\frac{d i_L}{d t} + \frac{d^2 q_c}{d t^2}\right)$$

$$O = q_c + \frac{L}{R} \frac{d q_c}{d t} + L C_L \frac{d^2 q_c}{d t^2}$$

If $q_c = e^{mt}$

$$1 + \frac{L}{R} m + L C_L m^2 = 0$$

$$m = -\frac{1}{2 R C_L} \pm \sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}$$

I. If $\frac{1}{4 R^2 C_L^2} \Big\langle \frac{1}{L C_L} \Big\| R \Big\rangle R_c \Big\| R_c = \frac{1}{2} \sqrt{\frac{L}{C_L}}$

$$q_c = e^{-\frac{1}{2RC_L} t} [A \cos \omega t + B \sin \omega t]$$

where $\omega = \sqrt{\frac{1}{L C_L} - \frac{1}{4 R^2 C_L^2}}$

$$e_a = E - \frac{q_c}{C_L} = E - \frac{1}{C_L} e^{-\frac{1}{2RC_L} t} [A \cos \omega t + B \sin \omega t]$$

$$i_c = \frac{d q_c}{d t} = -C_L \frac{d e_a}{d t}$$

$$= e^{-\frac{1}{2RC_L} t} [-A \omega \sin \omega t + B \omega \cos \omega t]$$

$$- \frac{1}{2 R C_L} e^{-\frac{1}{2RC_L} t} [A \cos \omega t + B \sin \omega t]$$

Initial Conditions:

At $t = 0$ $\quad e_a = E_1$ and $i_c = -I_1$

$$E_1 = E - \frac{1}{C_L} A \qquad A = C_L (E - E_1)$$

$$-I_1 = B \omega - \frac{1}{2 R C_L} A = B \omega - \frac{E - E_1}{2 R}$$

$$B = \frac{1}{\omega} \left(-I_1 + \frac{E - E_1}{2 R}\right)$$

Hence

$$e_a = E - e^{-\frac{1}{2RC_L} t} \Big[(E - E_1) \cos \omega t$$

$$+ \frac{1}{\omega C_L} \left(-I_1 + \frac{E - E_1}{2 R}\right) \sin \omega t \Big]$$

This reduces to the value given for e_a in Appendix A when $R = \infty$. I_1 = the arc current at the breaking point.

$$\left(\frac{d e_a}{d t}\right)_{t=0} = \frac{I_1}{C_L}$$

II. If $\frac{1}{4 R^2 C_L^2} \Big\rangle \frac{1}{L C_L} \Big\| R \Big\langle R_c \Big\| R_c = \frac{1}{2} \sqrt{\frac{L}{C_L}}$

Let $\beta = \sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}$

$$q_c = e^{-\frac{1}{2RC_L} t} [A \cosh \beta t + B \sinh \beta t]$$

$$e_a = E - \frac{q_c}{C_L}$$

$$= E - \frac{1}{C_L} e^{-\frac{1}{2RC_L} t} [A \cosh \beta t + B \sinh \beta t]$$

$$i_c = \frac{d q_c}{d t} = -C_L \frac{d e_a}{d t}$$

$$= e^{-\frac{1}{2RC_L} t} [A \beta \sinh \beta t + B \beta \cosh \beta t]$$

$$- \frac{1}{2 R C_L} e^{-\frac{1}{2RC_L} t} [A \cosh \beta t + B \sinh \beta t]$$

Initial Conditions:

At $t = 0$ $\quad e_a = E_1$ and $i_c = -I_1$

$$E_1 = E - \frac{1}{C_L} A \qquad A = C_L (E - E_1)$$

$$-I_1 = B \beta - \frac{1}{2 R C_L} C_L (E - E_1)$$

$$B = \frac{1}{\sqrt{\frac{1}{4 R^2 C_L^2} - \frac{1}{L C_L}}} \left[-I_1 + \frac{E - E_1}{2 R}\right]$$

Hence

$$e_a = E - e^{-\frac{1}{2RC_L}t}\left[(E-E_1)\cosh\sqrt{\frac{1}{4R^2C_L^2}-\frac{1}{LC_L}}\,t + \right.$$

$$\left. + \frac{-I_1 + \frac{E-E_1}{2R}}{C_L\sqrt{\frac{1}{4R^2C_L^2}-\frac{1}{LC_L}}}\sinh\sqrt{\frac{1}{4R^2C_L^2}-\frac{1}{LC_L}}\,t\right]$$

$$\left(\frac{de_a}{dt}\right)_{t=0} = \frac{I_1}{C_L}$$

Appendix C

(See Fig. 7B)

$$E = L\frac{di}{dt} + Ri + \frac{q}{C}$$

$$E = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}$$

$$O = L\frac{d^3q}{dt^3} + R\frac{d^2q}{dt^2} + \frac{1}{C}\frac{dq}{dt}$$

If $q = e^{mt}$

$$O = Lm^3 + Rn^2 + \frac{1}{C}m$$

$$m = O$$

$$O = Lm^2 + Rm + \frac{1}{C}$$

$$m = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

I. If $\frac{R^2}{4L^2} < \frac{1}{LC}$

$$q = CE + e^{-\frac{R}{2L}t}[A\cos\omega t + B\sin\omega t]$$

where $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

$$i = \frac{dq}{dt} = e^{-\frac{R}{2L}t}\left[\left(\omega B - \frac{R}{2L}A\right)\cos\omega t - \left(\omega A + \frac{R}{2L}B\right)\sin\omega t\right]$$

$$e_a = \frac{q}{c} = E + \frac{1}{C}e^{-\frac{R}{2L}t}[A\cos\omega t + B\sin\omega t]$$

Initial Conditions:

At $t = O$ $e_a = E_1$, $i = I_1$ and $q = CE_1$

$$CE_1 = CE + A \qquad A = -C(E - E_1)$$

$$I_1 = \omega B + \frac{R}{2L}C(E - E_1)$$

$$B = \frac{1}{\omega}\left[I_1 - \frac{R}{2L}C(E - E_1)\right]$$

Hence

$$e_a = E + e^{-\frac{R}{2L}t}\left[-(E-E_1)\cos\omega t + \frac{1}{\omega C}\left(I_1 - \frac{R}{2L}C(E-E_1)\right)\sin\omega t\right]$$

$$\left(\frac{de_a}{dt}\right)_{t=0} = \frac{I_1}{C}$$

II. If $R = O$,

$$e_a = E - \left[(E-E_1)\cos\omega t - \frac{I_1}{\omega C}\sin\omega t\right]$$

$$i = I_1\cos\omega t + \omega C(E-E_1)\sin\omega t$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$e_a = E - E_m\cos(\omega t + \theta)$$

where $E_m = \sqrt{(E-E_1)^2 + \left(\frac{I_1}{\omega C}\right)^2}$

$$\tan\theta = \frac{I_1}{\omega C(E-E_1)}$$

$$I_1 = \omega C\sqrt{E_m^2 - (E-E_1)^2}$$

which is the same as the case of Appendix A.

$$\left(\frac{de_a}{dt}\right)_{t=0} = \frac{I_1}{C}$$

Discussion

T. E. Browne, Jr.: The investigation described in this paper is of extreme interest to those of us who are working on the problem of arc extinction, and a very welcome contribution to our knowledge of this subject. These oscillograms are, so far as I know, the first to be obtained showing directly on a time axis just what occurs to the current and voltage of a short a-c. arc during the elusive current zero period. Theories have, in the past, been only rough and approximate because of our lack of means for accurately checking them experimentally. The paper gives a commendably thorough interpretation and analysis of these important oscillograms, and of their relation to the problems of circuit interruption at moderate voltages.

Although many of the facts presented had previously been learned by less direct methods, there are several distinctly new contributions. Some of these are in confirmation of previous theoretical deductions, while others have not hitherto been suspected. In the first place, it is definitely shown that there does exist, even for the fastest practical circuits, a measurable period of time during which the current through the arc is sensibly zero and the voltage across the arc terminals is controlled almost entirely by the transient characteristics of the circuit in which the arc is playing. It is also shown, however, that during this period

there may be a small "leakage" current of sufficient magnitude to measurably alter the circuit transient. The existence of such a current has already been postulated in connection with the theory of the formation of a space charge sheath at the cathode.[1] It is also revealed that the circuit transient depends to some extent on the arc failure current, which is largely a random quantity, but is in turn partially determined by the circuit constants. The occurrence of a "negative glow" preceding reversal of arc voltage is also recorded for the first time.

It may be of interest to call attention to the points of agreement and also of apparent disagreement between the results of these experiments and the results of an earlier but somewhat similar investigation[2] using a cathode-ray oscilloscope. In the former case, photographic records of individual arc reignitions could not be obtained, but by photographing the figures produced on the fluorescent screen by a large number (900) of retracings of the dynamic volt-ampere characteristic (or "cyclogram") of a steadily running arc, records of the average type of voltage and current variation near current zero were obtained for short arcs under a variety of conditions. With plane copper electrodes, three distinct types of arcs having different restriking characteristics were observed. They are as follows: (1) the stationary arc, with terminals remaining in one position on the electrodes long enough to produce melting and boiling of the electrode surfaces, with resulting burring and pitting; (2) the "wandering" arc, which moves more or less discontinuously from one adjacent spot to the next, leaving a marked trail but not causing appreciable burning away of the electrode surfaces; and (3) the so-called "cold-cathode"[3] or Stolt[4] arc in which the arc terminals are moved over the electrodes with such velocity that no burning occurs, even with very large currents.

The first type of arc was very difficult to obtain with clean electrodes and currents less than 100 amperes. It is believed, however, that this is the normal type of copper-electrode arc for currents of several hundred amperes or more, where considerable melting and burning away of the electrodes is generally observed. Such an arc is therefore of primary practical interest since it is the most likely to occur in the operation of ordinary contactors and circuit breakers. In the earlier investigation referred to, its reignition voltages appeared to vary widely between values slightly in excess of the glow voltage and values very much less than this. No indication of the existence of an actual glow was observed.

The third type of arc can be obtained only by very rapidly rotating the electrodes mechanically or the arc magnetically. This is a special case. It has been shown[5] to possess the desirable characteristics of requiring comparatively high reignition voltages (more than three times glow voltage observed), even in the fastest circuits obtainable, and of normally breaking down to an unusually long-lived glow at constant voltage before transition to the arc.

It is clearly the second type, or wandering arc, which the authors of this paper have used in their experiments. The peculiar tendency of the voltage of the intermediate glow discharge to increase substantially with current immediately following its initial formation, as shown by the oscillograms and cyclograms of Figs. 3, 5, and 10, produce a repeated volt-ampere characteristic, which appeared to Mr. Todd and myself to defy any reasonable explanation at the time of the previous investigation. It is gratifying to see this puzzling case completely analyzed here, but the peculiar variation of the glow voltage still lacks explanation. The fact that the reignition and glow voltages measured by the authors are somewhat higher than those found in the earlier tests is probably due to the additional voltage required by the ¾-inch positive column, which cannot be neglected for the longer electrode separation. The non-occurrence of a "negative glow" in the former experiments with the wandering arc may be due to different circuit characteristics, which did not produce quite as high "negative dip" voltages, or to the higher arc current (92 amperes), which may have the same effect.

Perhaps the most interesting feature of the paper from a practical standpoint is the evidence presented in regard to the variation with time after current zero of the voltage required to reignite the arc. The most noticeable feature of Figs. 15 and 16, which show this relation, is the extreme variability of this voltage from one-half cycle to the next under the same arc and circuit conditions. This fact agrees with past experience and observation[2] though it is still not fully understood. On the basis of present theory,[1] it would seem to represent variation in the minimum thickness of the space charge sheath at the incoming cathode, which can be caused by varying contour and condition of the electrode surfaces, as suggested by the authors, and perhaps also by varying density of ionization next to the cathode due to turbulence of the arc gases. In consequence of this variation, the arc space obviously cannot be said to have, in general, a definite breakdown voltage at a given instant of time. Therefore, we can only say that there is a certain probability that a given voltage will cause reignition of the arc when applied after a given interval following a current zero. This probability may be expected to be a function, not only of the voltage and time coordinates of the graphs, but also of the length of time during which a voltage in excess of the minimum breakdown value is applied. Thus, a rapidly rising transient may be expected to reach a higher voltage before breakdown than will a transient passing through the region of high breakdown probability (bounded by the envelopes in the figures) at the same time but rising at a slower rate. Probability of breakdown with a given voltage applied after a given interval and for a given length of time may also be expected to vary somewhat with the area of the space charge sheath, and therefore with the arc current, since the likelihood of the random occurrence of a "thin spot," having a breakdown voltage near to the possible minimum, increases with the area of the sheath. From this it follows that lower and more consistent breakdown voltages are to be expected when using larger currents than the 25 amperes employed in these experiments; and this is borne out by experience in the laboratory.

In the "indirect" method by which the so-called "Slepian's focus," shown in the figures, was obtained, the rate of rise of circuit voltage to different peak values was also controlled by resistance shunts, (as in Fig. 15), but the final adjustment of these was such that at each circuit voltage the arc would just fail to reignite after one-half cycle. The transients corresponding to these critical circuit constants were plotted by means of an approximate equation, and the curve shown was drawn as their envelope. These applied voltage curves were probably quite flat as they approached the region of high probability of breakdown, and so would not be likely to rise much above the lower boundary of the region without causing restriking of the arc. Also, the currents used were of the order of hundreds of amperes. Consequently, their envelope might be expected to correspond very closely to the lower branch of the envelope determined by the authors of this paper. It is believed that such a curve gives the most accurate practical measure of the dielectric strength, or the minimum breakdown voltage, of an arc space obtainable, provided that the critical voltage recovery curves are accurately plotted. Unfortunately, the particular locus shown was obtained for an arc of the third type mentioned above (high-speed cold-cathode arc as used in the deion-air circuit breaker)[5] and so cannot be directly compared with the results given here. The above method applied to a 1/16-in., 300-ampere arc of the first, or stationary type (normal for large currents) yields a curve,

1. *Extinction of an A-C. Arc*, J. Slepian, Trans. A. I. E. E., Vol. 47, 1928, p. 1398.
2. "Restriking of Short A-C. Arcs," F. C. Todd and T. E. Browne, Jr., *Phys. Rev.*, Aug. 15, 1930, p. 732.
3. J. Slepian, *Phys. Rev.*, 27, 1926, p. 407.
4. H. Stolt, *Ann. d. Physik*, 74, 1924, pp. 80-104.
5. *Theory of the Deion Circuit Breaker*, J. Slepian, Journal of the A. I. E. E., Feb. 1929.

which, though also not directly comparable, is very similar in shape to the lower envelope of these figures, but lies about 150 volts below it.

Part of this discrepancy is very probably due to the distinctly different reignition characteristics already described and also to the difference in arc lengths in the two cases. However, it also seems from this work that our curves are too low, since in plotting the individual voltage rise curves of which it is the envelope, no account was taken of either the negative dip, due to finite arc failure current and arc voltage at current failure or of the reactor distributed capacity, both of which tend (as shown by the exact equations given) to cause the maximum value of the recovered voltage to exceed that given by the simple equation which was actually used. We may conclude, then, that the actual values of voltage which may be successfully withstood by a very short arc space between hot copper electrodes following the zero of a large current lie on a curve that probably falls somewhere between our curve, shown in Mr. Strom's paper, and the lower envelope of the region determined by Attwood, Dow, and Krausnick.

Before concluding, I wish to emphasize the need of caution in attempting to generalize about the behavior of short arcs from the results of these limited experiments. In our work in the laboratory, it has been found that the characteristics of short a-c. arcs, particularly with regard to extinction, may depend to a very large extent upon such conditions as current magnitude, arc length, electrode material, and gas medium,[6] as well as upon the conditions of motion or lack of motion mentioned above.

With regard to the attempt of the authors of this paper to apply their results to the action of circuit breakers in high-voltage power circuits, I believe a further work of caution is needed. It is undoubtedly true that the curves of their Fig. 17 do give a rough qualitative picture of the race between recovered voltage and recovered ability to withstand voltage, which, as Dr. Slepian[1] and others have already pointed out, determines the extinction or reignition of an a-c. arc in any practical circuit. However, it must be kept in mind that the few hundred volts which can be withstood by the space adjacent to the cathode in the length of time usually available is inadequate to account for the interruption of power circuits involving thousands of volts, even when four breaks in series are considered, and electrostatic unbalance[7] is neglected. It is clear, then, that a very large part of the high dielectric strength recovered by long a-c. arcs at current zero must reside in the main body of the arc path, where the mechanism of deionizatin, and therefore both the manner of recovery with time and the effect on this recovery of possible influencing factors may be entirely different from those relating to the region next to the cathode. This is, of course, especially true of oil circuit breaker arcs where both arc and interrupted gradients are many times those for an arc in air. Although in recent years much progress has been made in determining the factors that improve the interrupting ability of long a-c. arcs and in applying this knowledge to practical circuit-breakers, there is still no experimental information or exact theory with regard to the variation of dielectric strength of a long a-c. arc with time during the first few microseconds of the current zero period. Consequently, we should be careful to keep our minds open on this point until information is obtained that can be directly applied.

S. S. Attwood: The "leakage" current occurring during the interval which we described as the "current zero period" was only a simple fraction of an ampere. Had this current disturbed the circuit transient, our equations would hardly have held true. On the contrary the voltage equations proved to be correct to a surprising degree of accuracy. The slight slope of the current (so called current-zero) in Fig. 9A shows this leakage.

The arcs discussed in this paper were purely of the type which Mr. Browne describes as "wandering," inasmuch as they were moving slowly over the electrodes and the oscillograms were taken at any convenient current-zero period. However, the authors have since taken further oscillograms of arcs at the end of the first half cycle, when the arc life was perhaps only one-quarter cycle. The oscillograms are practically identical with those shown in the paper. In this short arc-life the wandering was very small.

Cathode-ray oscillograms recently taken of a 90-ampere "cold-cathode" arc moving at a considerable speed indicated a reignition voltage of about 1,000 volts and the existence of a prolonged glow at this voltage, which is about three times the glow values given in the paper, and is in entire agreement with Mr. Browne's observations. This high reignition and glow voltage value certainly distinguishes the cold cathode from the wandering arc.

It is suggested that reignition variability is due to variability of cathode space-charge sheath. Alternately, the authors would like to suggest the presence of electrode impurities as a likely cause of variability. It has been brought to our attention that the presence of certain foreign elements, purposely incorporated in the electrodes, tends to insure restriking of the arc. The wandering arc should be peculiarly sensitive to this influence. Impurities alone may be enough to account for the irregularity.

6. "Extinction of Short A-C. Arcs Between Brass Electrodes," T. E. Browne, Jr. and F. C. Todd, *Phys. Rev.*, Aug. 15, 1930, p. 726.

7. *Extinction of a Long A-C. Arc*, J. Slepian, JOURNAL OF THE A. I. E. E. April 1930, p. 810.

The Primary Network

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Synopsis.—The primary network is similar to the low-voltage network in so far as the interconnection of the secondary mains is concerned. Two transformation steps are required in the primary network described here, and only one in the low-voltage network. Nevertheless, the primary network has economic possibilities which should be considered for areas of medium load density.

A preliminary study of the so-called primary network indicated that the principle had possibilities worthy of exploitation. It was, therefore, decided that such a network would be installed if at all feasible to obtain operating experience with the system.

The design work for this initial installation is complete and construction work has started.

It is recognized that there is a number of problems to be worked out more thoroughly. As experience in operation is obtained, and with intensive study of the requirements it is expected that improvements will be made and the designs materially simplified.

Even with the limitations existing at the time of writing it appears that the primary network offers important advantages over the radial system for certain types of load.

* * * * *

Introduction

THIS paper is presented to describe a system of electric distribution which offers great possibilities in the effort of the engineers of the power industry toward rendering better service to the consumer at a lower cost.

A network may be said to exist when the secondaries of two or more transformers not at the same location are connected in parallel to supply a load. This definition may be applied to low-voltage or high-voltage networks.

Low-voltage or secondary a-c. networks have been applied to areas of high-load density (10,000 kva. per sq. mi. and above) and the principles of design have been described in previous papers. The elimination of the substation is a major source of economy in this system. In the radial system two conversion steps were necessary, first, from the generator voltage (11-13 kv.) to distribution voltage (4 kv.) and second, from distribution voltage to a utilization value (110-120 volts). The secondary network permits this conversion to be made in one step (from 11-13 kv. to 110-120 volts) thus eliminating the substation.

The secondary network as applied to areas with load densities of 10,000 kva. per sq. mi. and above, are generally underground systems. Residential and commercial areas (6,000 kva. per sq. mi. and below) are usually fed by the overhead radial systems.

In 1927 there was suggested a method of distribution utilizing the general principles of the high-tension network. This scheme utilized a number of substations of moderate size (5,000 kva.) interconnected on the 4-kv. side forming a primary network.[1]

This principle of interconnecting 4-kv. mains, and certain principles used in the design of secondary networks have been combined in a new design of primary network fed at transmission voltage. This design is described below, together with an outline of the advantages of the system.

*Byllesby Engineering & Management Corp., Chicago, Ill.

1. "Serving a Medium Voltage Network," by D. K. Blake, *Electrical World*, March 5, 1927.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

The Primary Network Compared with the Radial System

A typical radial distribution system is shown in Fig. 1. There are four elements in this system (a) the transmission feeder from the source of supply to the substation, (b) the substation, (c) the distribution feeders, and (d) the mains. The feeders in Fig. 1 are shown terminating at feeder points, and mains are shown radiating from one feeder point.

The substation is located as nearly as possible to the load center of the area served, and each feeder terminates at the approximate load center of its own area, the mains radiating from these load centers.

Fig. 2 shows diagrammatically a primary network. The transformers are distributed over the area to be

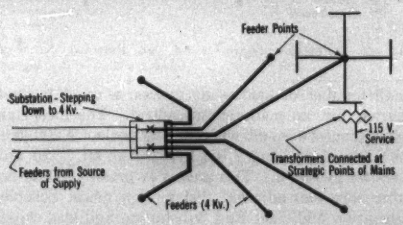


Fig. 1—Typical Distribution System Showing High-Voltage Supply Feeders and Mains with a Distribution Transformer Connected

served, each transformer being located at an intersection of primary mains with the transformer intersection constituting a unit. If regulators are required they will be installed between the transformer breaker and the breakers controlling the mains.

The area to be served is supplied from a number of these small and simple transformer and switching units rather than from a large substation. The transmission supply should preferably be by three or more feeders to reduce the spare capacity required when one transmission feeder is out of service. This is similar to the principle of design used in low-voltage networks.

There are, therefore, three elements in the primary network scheme, (a) the transmission feeders from the source of supply, (b) the transformers and switches,

and (c) the primary mains. Compared with the radial scheme the elements are the same except that the primary feeders have been eliminated. This feature, however, is only one of the items which permit a saving in initial cost over the radial system.

Cost Estimates of the Squirrel Hill Area

The initial study of a primary network was made for a section of Pittsburgh known as Squirrel Hill. This section embraces residences principally. There are a

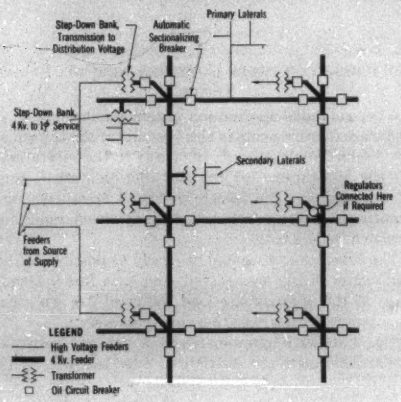


Fig. 2—Schematic Diagram of the Primary Network

small number of apartment houses, a small business district, but no manufacturing load. The area comprises about 2.0 sq. mi. with a winter peak of 4,200 kva., the load density being 2,100 kva. per sq. mi. The area is shown in Fig. 3. This area is at present fed by circuits from two adjacent distribution areas, Schenley Substation and East End Substation, and was chosen for the initial economic study; first, because the area had relatively definite boundaries, with no existing substation within its limits, and second, the growth of load indicated extension of existing radial system or adoption of a new system.

Three transmission feeders are available within the area. (See Fig. 4).

The estimates for the scheme of extension of a radial system are based on the standard practise for the Pittsburgh district. The substation location was chosen after an economic study of a number of available sites permitted by zoning ordinances and is reasonably near the load center of the area.

The substation proposed is a semi-outdoor type 22/4 kv. full automatic similar to those used on the Duquesne Light Company system.

The initial installation would consist of two 4,000-kva. transformer banks, a 22-kv. tie breaker, three 300-ampere 4-kv. regulated feeders and other necessary equipment. The building would provide space for an ultimate of two 6,000-kva. transformer banks and five 300-ampere regulated feeders.

The primary feeders would be run as shown in Fig. 3 in existing duct lines to their feeding points with sectionalizing switches as indicated.

The primary network includes the same area as covered by the radial system and is shown in Fig. 5.

The transformer and switching units of submersible type are to be in underground vaults located in the street.

Transformers are 1,500-kva. 22/4-kv. subway type with four 2½ per cent taps below and two 2½ per cent taps above on the high-voltage side, and a three-position oil immersed disconnecting and grounding switch on the 22-kv. side. The transformer oil circuit breaker is integral with the transformer. Lead-covered cable is used for all 4-kv. work in the vault.

The oil circuit breakers are 7,500 volt 600 amperes having a rupturing capacity of 14,000 amperes 2OCO

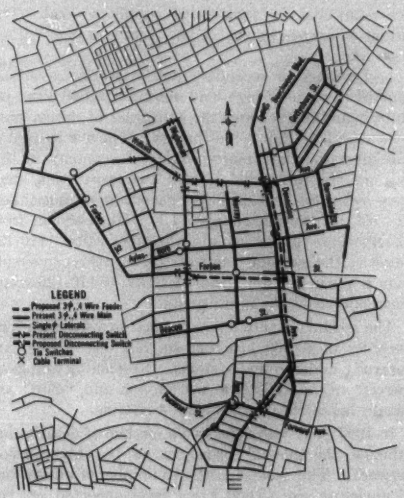


Fig. 3—Squirrel Hill Area

Radial primary distribution system which ordinarily would be used to supply area

at 4,000 volts. They will be arranged to reclose once on a fault, the breakers at each end closing at different times so that the equivalent of two reclosings is obtained. The breaker with its control mechanism and relays will be mounted in a submersible type box equipped with wiping sleeves for lead cable.

Duct connections are provided between existing manholes and the transformer vault. Cable is run in

pipe from the vault to the several poles at the intersection terminating in disconnecting potheads.

Table I shows comparative costs of radial and primary network schemes. The operating capacity of each scheme is 7,500 kva. If the cost units per kva. of capacity available for the radial scheme with substation are represented as 100 per cent, a corresponding

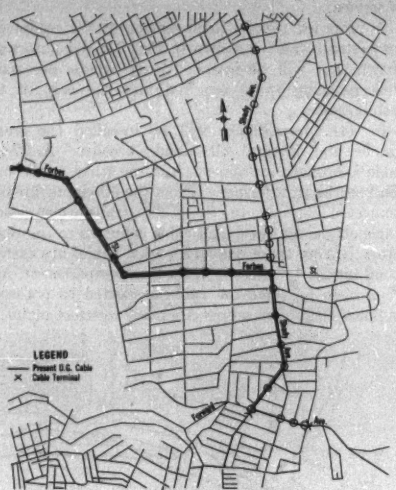


FIG. 4—SQUIRREL HILL AREA

Transmission feeders available

cost of the primary network is 56 per cent, or as expressed in the table the radial system is 78 per cent more costly than the network. If system voltage variation permits, regulators will, of course, be omitted in the network scheme because of the better inherent regulation due to the network.

Primary feeder and duct costs being common to both schemes they are omitted from the comparison.

Since cost data applying to one locality may not apply in other areas because of difference in labor costs, construction conditions and limitations, accounting procedure, etc. costs submitted are comparative and based upon experience in substation, transmission, and distribution construction in this territory.

The distribution capacity in the Squirrel Hill area is sufficient to carry the load for the present but since the study showed important savings, it was decided to make a similar study of the Verona-Oakmont district as immediate relief is needed for this district. Up to the present time this area has had adequate distribution and substation capacity. A brief description and comparative estimates for the two types of systems is given in the following section.

COST ESTIMATES OF THE VERONA-OAKMONT AREA

The Verona-Oakmont district shown in Fig. 6 is located about ten miles northeast of Pittsburgh. The area covered is 4.7 sq. mi. with a load of 2,000-kva. The load density is 425 kva. per sq. mi. The area is residential and light commercial load is supplied at present at 2,300 volts. The load in this area has only recently grown to an amount to justify a changeover to 4,000 volts. The present substation is to be abandoned for distribution purposes and the area converted to 4 kv.

For a radial system a new substation was proposed having two 3,000-kva. transformer banks. The station would be rated at 3,750 kva.[2] Two 4,000-volt regulated feeders are planned. Fig. 7 shows the proposed feeder runs.

In the primary network three 1,500-kva. transformer points are planned, located as shown in Fig. 8. With

TABLE I—SQUIRREL HILL COST COMPARISON

INITIAL STEP

Load Density 2,100 kva. per sq. mi.

	Radial	Network
Load	4,200 kva	4,200 kva.
Transf. capacity installed	8,000 kva	9,000 kva.
Transf. capacity with spare capacity out of service	5,000 kva	7,500 kva.
	Costs in per cent	
Real estate	18	
Substations	21	26.7
Electrical equipment	67	56
Transmission	1.2	1.5
Distribution	18.4	3.4
Ducts	4.7	10.8
Changes in 4-kv. feeders & mains		1.1
Total initial step	130	100
SECOND STEP		
Load	7,500 kva	7,500 kva.
Transformer capacity installed	12,000 kva	9,000 kva.
Transformer capacity with spare capacity out of service	7,500 kva.	7,500 kva.
	Costs in per cent	
Substation increase	28	None
Distribution increase	20.5	None
Total second step	178	100

Note: Radial system (initial step) will have two 4,000 kva., three-phase, transformers. With one out of service, the other will carry 25 per cent overload through peak or 5,000 kva., which is satisfactory for initial step and allowing for growth.

Network will have six 1,500-kva., three-phase banks and when one transmission line goes out, two 1,500 kva., banks go out also, leaving 6,000 kva. in service which, with 25 per cent overload capacity, gives 7,500 kva. capacity to carry 4,200 kva. initial load.

one feeder out of service the capacity is 3,750 kva. based upon 25 per cent overload on the remaining transformers.

The cost estimates are shown in Table II.

The network scheme has been estimated in three ways: (a) all equipment in underground vaults, (b) equipment in small buildings above ground, (c) outdoor

2. 125 per cent load on remaining bank with one bank out.

installation (equipment above ground on concrete pads). Calling method (c) 100 per cent, we find that the radial scheme is 35 per cent more costly, with the building type installation, and the underground vault at 7 and 18 per cent more costly respectively than the outdoor transformer installation. In areas of this type vaults will not ordinarily be necessary.

It was decided that one of each of the above types of

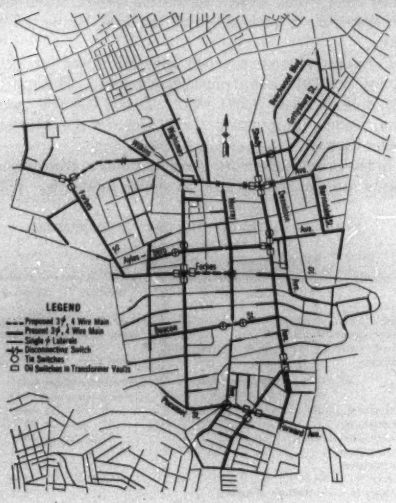


FIG. 5—SQUIRREL HILL AREA

Diagram of mains for primary network showing transformer points

installation be used in this initial primary network to form the three transformer points proposed for this area.

GENERAL DESCRIPTION OF TRANSFORMER AND SWITCHING UNIT

The transformer switching unit consists of (a) the transformer and (b) the oil circuit breakers and accessories. If system regulation is such as to make it necessary, a third element (c) the regulators, may be added.

These elements may be placed:

(a) In underground vault
(b) In small buildings
(c) On concrete pads surrounded by a fence.

Underground vaults may be used in congested areas where space above ground is expensive or cannot be obtained, or where zoning ordinances prohibit.

Small buildings may be used in the less congested and medium grade residential areas.

In lower grade areas the equipment may be placed on concrete pads and surrounded by a fence.

In the underground vault, it is essential that the equipment be subway type. Subway type transformers are now available, and regulators can readily be converted to subway type. Since there are no oil circuit breakers of the type desired that are submersible, it is proposed to mount standard breakers in welded steel submersible cubicles. Equipment used in buildings or on concrete pads may be standard for that type of service.

A proposed general arrangement of the equipment for a street vault is shown in Fig. 9. Regulators are shown, but will be installed only where necessary, thus reducing space required and the cost of the installation.

In one compartment will be installed the transformer and integral oil circuit breaker. A 22-kv. cable is brought in from ducts in the street to the pothead on the transformer. From the oil circuit breaker connected directly to the 4-kv. transformer leads, single-conductor 4-kv. lead cable connects to the regulators. From these regulators 4-kv. cables are carried to oil circuit breakers in the other compartment, and from these breakers the cable is carried to potheads on poles where connections are made to street mains.

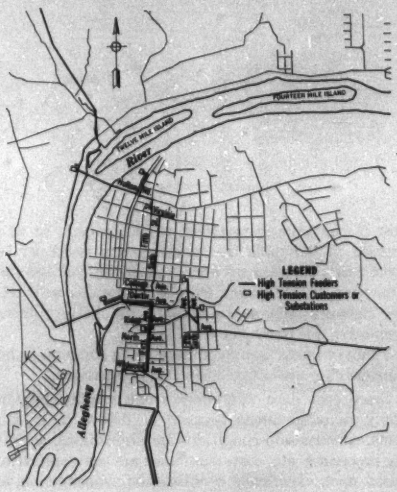


FIG. 6—VERONA—OAKMONT AREA

Transmission feeders available

It will be noted that the principle of barrier construction is used here whereby the transformer and switching equipment are separated. This principle was described in a paper by the authors as applied to the low-voltage network.[3]

3. *Low-Voltage A-C. Networks*, by Stanley and Sinclair, A. I. E. E. TRANS., January 1930, p. 265.

A removable hatch is provided over each compartment for installation and removal of equipment. Manhole covers are placed in each hatch for normal access.

A small power transformer bank (three 1½ kva.) is located in the transformer compartment for circuit breaker operation and for relay potential.

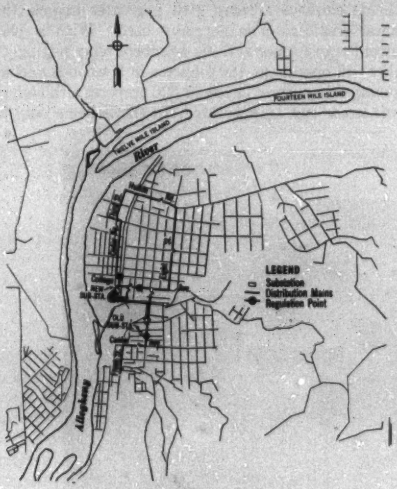


FIG. 7—VERONA—OAKMONT AREA

Radial primary distribution system which ordinarily would be used to supply area

Such a vault (1,500 kva.) capacity requires a floor space approximately 12 ft. by 30 ft., and a depth of 13 ft.

RELAY PROTECTION

When the primary network was first considered no scheme for protection had been studied, and it was felt that it would require a complicated combination of relays or the possible development of a new relay. However, when the matter was investigated it was found that a very simple protection scheme could be used and lend itself to ready expansion without the necessity of changing relay settings.

Transformer Protection. A network relay similar to the one used on the low-voltage a-c. network will be used to protect the transformer bank breakers. This relay would open the breaker on any type of high-tension fault, and also if the supply line is de-energized the magnetizing current for the bank in combination with the charging current of the line will be sufficient to open the breaker. It is also possible with this relay to have the breaker close when the high-tension line is reenergized. Therefore, the type of protection used on this breaker will be identical to that of the low-voltage network protector. In addition to this protection, consideration is being given to tripping this breaker by means of a contact-making

TABLE II—VERONA–OAKMONT COST COMPARISON
Load Density 425 kv-a. per sq. mi.

	Radial	Network		
		U. G. Vault	Bldg. type	Outdoor
Load	2,000 kv-a.	2,000 kv-a.	2,000 kv-a.	2,000 kv-a.
Proposed inst. capacity	6,000 kv-a.	4,500 kv-a.	4,500 kv-a.	4,500 kv-a.
Operating capacity*	3,750 kv-a.	3,750 kv-a.	3,750 kv-a.	3,750 kv-a.
		Costs in per cent		
Real estate	17.0		2.8	2.8
Substation bldg. or vault	4.2	24.5	13.0	5.8
Equipment	95.0	71.0	66.3	66.3
Transmission	2.5	3.3	4.4	4.4
Distribution	1.6	13.2	13.2	13.2
Duct connections	14.5	5.7	7.5	7.5
Total	135	118	107	100

*Based on 125 per cent load on transformers for five hours in case of feeder or transformer failure.

Note. Cost of regulators included in above estimates.

thermometer placed on the transformer. In case of excessive overload which would be likely to damage the transformer, the thermometer when it reaches a predetermined value would energize the trip circuit, thus

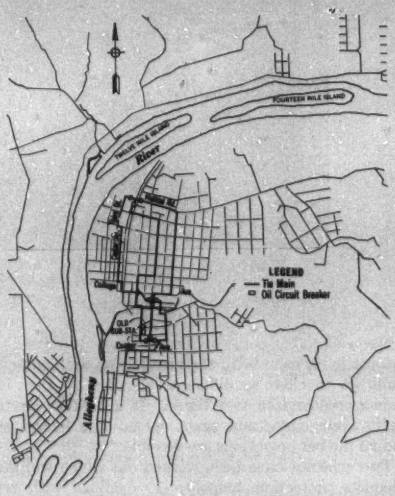


FIG. 8—VERONA—OAKMONT AREA

Diagram of mains for primary network showing transformer points

disconnecting the transformer from its load. This might be advisable in the event that two high-tension feeders should be out of service during a peak-load period, but would permit the one remaining bank to

carry the load for as long a period as possible without damaging the transformer. In the majority of cases relief could probably be given this one unit before the temperature would reach the tripping value. Under this scheme no other overload protection would be used for the transformer breaker.

Bus Protection. Bus differential will be used at each transformer installation to protect the bus within the intersection so that any type of fault will open all the breakers and lock them open. The overload relay for this protection will be installed in the transformer breaker cubicle.

Tie Main Protection. A detailed study has been made to determine the type of protection best suited to protect all tie mains. All breakers are arranged for automatic reclosing with the breakers on each end of a main set to close at different intervals, so that the main is reenergized two times. If a breaker on the faulty main closes and again opens, it is locked out and no further operations are possible.

Two schemes have been worked out which will give adequate protection, namely: (1) overload with definite time setting and (2) overload with inverse time characteristic.

1. Overload with Definite Time Setting. In this scheme the relays are set to operate at 150 to 200 per cent of the transformers full load current, namely 300 to 400 amperes. The time settings are staggered so that a breaker on one end of a main will open in 0.6 sec. and on the other end in 1.1 sec. This scheme has one disadvantage that breakers other than those connected to the faulty main open, although there is no interruption to service on other mains since none of the breakers set at 1.1 sec. have sufficient current after the 0.6 sec. breakers open to operate except the one directly connected to the fault.

2. Overload with Inverse Time Characteristic. This scheme employs a relay with a greater inverse time characteristic than is normally used. With a 400/5 current transformer and the 4-ampere relay setting, the relay will operate at 480 amperes in 9 seconds and will operate on a current as low as 320 amperes although the time would not be accurate. In this scheme only the two breakers connected to the faulty line operate. However, if corner transformers are installed in which tie mains are run in only two directions, proper selection cannot be made when this particular transformer is out of service. In general, it may not be advisable to use this design but rather run a third main to some adjacent intersection. This scheme appeared to have certain advantages over scheme (1) and was adopted.

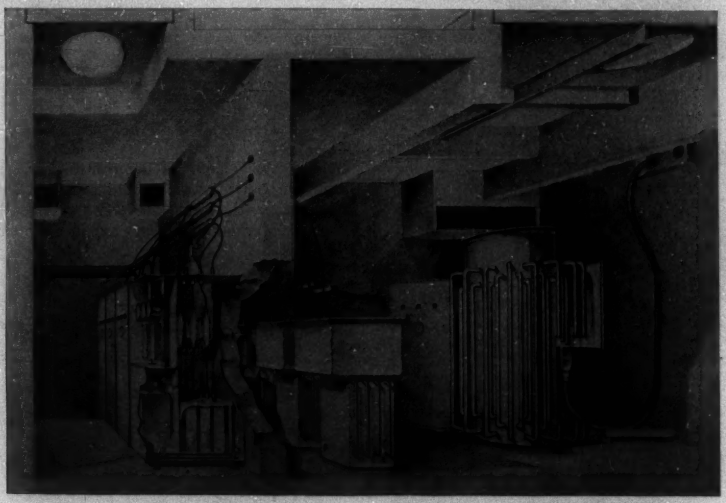


Fig. 9—Transformers, Regulators, and Oil Circuit Breaker Cubicles in an Underground Vault

Note the barrier wall construction. The same general arrangement will be used in building jobs

Application

Short-circuit studies were made of the network layout for three-phase and phase-to-ground faults at all locations along the tie mains, with all transmission feeders in service and with each one out to determine the definite requirements of a relay.

Fig. 10 shows a typical example of the fault currents with a fault on one of the mains adjacent to the College Avenue and 6th St. intersection. In this case oil circuit breakers Nos. 1 and 5 must open to clear the fault. No. 5 has 3,960 amperes which from the curve will open in 0.5 sec. No. 1 has only 500 amperes and it would take 7.3 sec. for it to open while Nos. 6 and 8 are

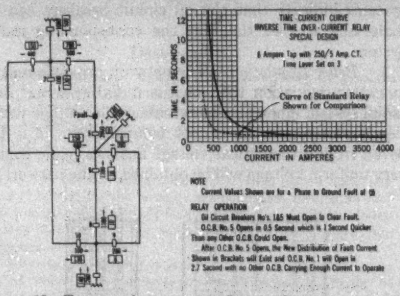


FIG. 10—TYPICAL ANALYSIS OF RELAY OPERATION ON TIE MAIN FAULT

carrying 1,100 amperes and would open in 1.5 sec. However, after No. 5 opens in 0.5 sec. which is 1.0 sec. sooner than Nos. 6 and 8 can open, a redistribution of current exists as is shown by the blocked figures. No. 1 is the only one now carrying enough current (780 amperes) to open and it will open in 2.7 sec. thus clearing the fault. Therefore, the fault is cleared without interruption to tie mains other than the one in fault, and the fault is cleared entirely in 3.2 sec. Faults at other locations show the same general conditions and in all cases only service on the faulty tie main is interrupted.

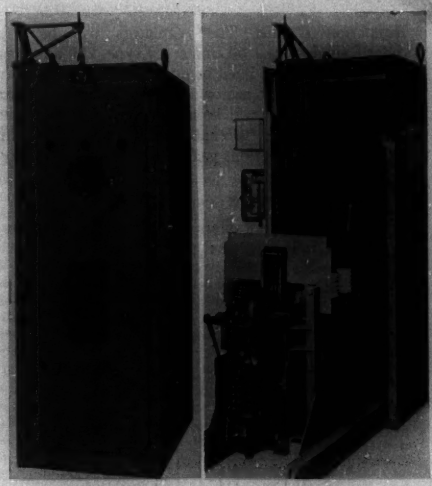
Similar studies have been made on layouts using various numbers of transformers, and with various length mains, and in all cases the results are comparable to the Verona-Oakmont study, so it is believed that no trouble should be experienced in relaying the primary network particularly if two-way feed transformer installations can be eliminated.

TRANSFORMER DETAILS

The transformer will be 1,500-kva. three-phase subway type 22,000/4,000-volt Y with four 2½ per cent taps below and two 2½ per cent taps above 22,000 volts, and to have an inherent impedance of ten per cent. The 22,000-volt cable supplying the transformer will terminate in an oil tight pothead with a three-position switch so that the cable may be either disconnected from the transformer, connected or grounded, and is protected by an interlocking mechanism so if the transformer is energized from either side it will not be possible to operate the switch. This pothead or oil tight chamber is mounted on the transformer as an integral part of the case. On the 4,000-volt side the leads will be brought out through oil tight bushings so constructed that an oil circuit breaker cubicle can be bolted to the transformer case making a water-tight connection.

OIL CIRCUIT BREAKER DETAILS

The oil circuit breakers are 600 amperes 7,500 volt and will rupture 14,000 amperes with the standard duty cycle. They are mounted on a small truck and placed in a small subway type cubicle in which all relaying equipment, current transformers, motor-operated mechanism, etc., are mounted. Space is not allowed for inspection of the breakers while in the cubicle but they are easily disconnected and rolled out. Each unit is self-contained, and at the time of installation it is only necessary to connect the cables to the wiping sleeves. A number of arrangements for the cubicle design has been worked out. Figs. 11 and 12 show the simplest design which requires cable connections on both sides, and this design will be used in the Verona-Oakmont installation. Fig. 13 shows the outline of this cubicle. Another design considered is shown in Fig. 14 using bus



FIGS. 11 AND 12—OIL CIRCUIT BREAKER CUBICLE

Submersible type showing door removed, truck pulled out, relay panel swung open and barriers removed.
Only one current transformer is shown in place

connections between cubicles. A vertical truck type connection is shown in Fig. 15 in which the leads are disconnected when the oil switch is lowered.

REGULATION

The percentage regulation for the network is very low so that in general it does not appear necessary to use regulators unless there is considerable variation in the

supply voltage of the transmission lines. To prevent the problems which arise with normal parallel regulator operation, it will be necessary to control each intersection by a contact-making voltmeter set so that each intersection will maintain the same constant voltage.

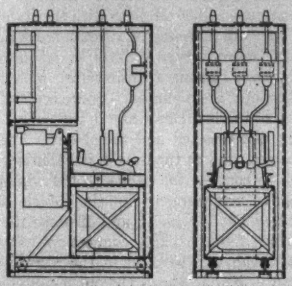


FIG. 13—OIL CIRCUIT BREAKER CUBICLE

Outline of cubicle similar to Figs. 11 and 12 except arranged for different oil circuit breaker

Regulation Details. The regulators will be the standard single-phase oil immersed type, 200 amperes 2,300 volt, 5 per cent buck or boost and equipped with wiping sleeves. The motor, control, and transformer for the motor circuit will be mounted in a box on the side of the regulator, so that the unit is complete in itself, with all connections gasketed to make it subway type.

CONCLUSIONS

The primary network offers the following advantages over the usual radial system:

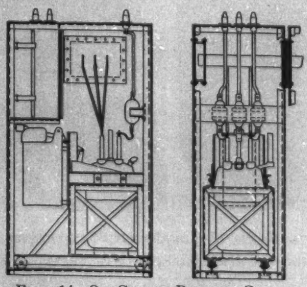


FIG. 14—OIL CIRCUIT BREAKER CUBICLE

Outline showing bus connections between cubicles

1. Material Reduction in Investment. Table I. indicates that the radial system is 78 per cent more costly than the network. Or, expressing it in the reverse order, the network costs about 56 per cent of the radial system now in use. Even under adverse conditions such as in areas of load densities as low as 425 kva. per sq. mi., the cost runs as much as 35 per cent higher for the radial system.

2. Greater Copper Economy. Reduced copper sizes are possible on the mains because of the smaller currents and shorter distances of feed, the results of the network principle.

3. Reduced Duty on Oil Circuit Breakers. Short-circuit studies of the two areas studied and of theoretical systems indicate that the oil circuit breakers can be materially smaller than for the corresponding radial system.

4. Better Regulation. The network principle results in better regulation than on the radial system. Any radial system inherently has considerable voltage variation which regulators cannot compensate. It is believed from a theoretical voltage analysis made that a very uniform voltage will be obtained on the network.

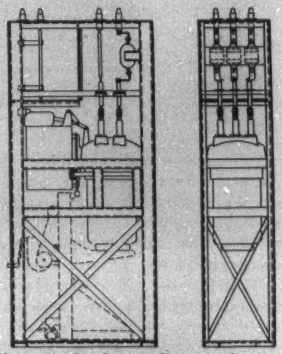


FIG. 15—OIL CIRCUIT BREAKER CUBICLE

Outline showing vertical lift type trunk breaker

5. Small and Standardized Transformer Units. Transformer units (transformers, switches, regulators, if used, and auxiliary equipment), can be manufactured in units, stocked if necessary, and installed when required.

6. Capacity may be Installed in Small Increments. The radial system requires a substation and distribution system to be built well in advance of the load growth. Long range estimates are necessary for load growth, in both direction and magnitude. In the primary network transformer units are installed *where and when needed.* Only as the load appears is the system expanded. Long range predictions are unnecessary.

7. Investment for Load Growth in Small Increments. The investment is not made in large steps far in advance of the load growth. (See 6).

8. Reduced System Losses. Calculations indicate that the total losses in the primary network are less than for the radial system. One item of saving for example, is the complete elimination of feeder losses as there are no feeders.

9. Area of Outage Reduced. With the designs considered each section will have a capacity of 600-700 kva. The radial circuit has a capacity of 2,100 kva. A failure in the case of the primary network therefore, involves less than one-third the load of the radial system.

10. No 22-kv. breakers. The high-tension cables run directly to the network transformers and hence no breakers are needed.

Discussion

E. R. Hendrickson: Although the area affected by a failure on the type of system described by Mr. Stanley is somewhat reduced, still the length of interruption to any individual or group of individual customers would not be materially different from that of the usual radial system with proper emergency switching centers.

Because of the increasing dependence being placed upon continuity of service by the average medium class residential customer who is now using electrically operated oil burners, clocks, etc., it is becoming of very much more importance to give him continuous service.

In order to provide this type of service, it appears that some form of low-voltage secondary network must eventually be used for at least the better class of residential and apartment house loads. I would therefore like to ask whether or not, in general, it is considered that the primary network as described would represent the correct solution for service to a territory of low or medium load density, where because of the importance of continuity of service or future growth in load, a secondary network would be required in the future, and if so, how would the secondary network be superimposed on the primary network system?

In many cases in high-class residential territory it is not possible to install overhead transmission feeders to the various transformer stations. Have any studies been made which would indicate that the primary network is the most economical solution if underground transmission feeders are to be installed to each transformer station instead of only extending them to a central substation?

H. Richter: The primary network system that is described in the 1927 *Electrical World* article mentioned in this paper has been publicly recommended to the industry on several occasions. That system utilizes substations each of about 5,000 kva. capacity. Westinghouse engineers worked on such a scheme for two mid-western cities in 1924 and 1925, and again in 1927 and 1928 for two cities in the east. The 1924, 1925, and 1927 projects were broached primarily to improve service in medium load density and high-class residential areas, but the 1927 study had in view mainly a possible saving due to eliminating the usual type of step-down substation.

In every case the method of relaying using the inverse time limit overload relays thus far available, and advocated in the *Electrical World* article, was dismissed as not likely to give sufficient continuity of service from a 4-kv. network. It was early realized that this particular type of system would certainly not be less expensive than the radial system and might become quite complicated to operate. For these reasons publicity did not seem warranted. Up to the present time there has been no known application of the scheme.

The primary network system evolved here in Pittsburgh, on the other hand, reduces the transformer installations to relatively smaller sizes. This permits the equipment to be installed in manholes, where there is no real-estate item, or on small lots away from the more expensive locations and less likely to be affected by zoning ordinances. A considerable reduction in rupturing capacity of the oil-circuit breakers is another important result, and a fault on a 4-kv. tie line shuts down service to a much smaller number of consumers. The Pittsburgh type primary network is thus practical, and the comparisons in the paper show distinct savings over the radial system for two specific installations.

With regard to the method of relaying, it will be observed from Fig. 10 that the characteristic curve of the standard inverse time over-current relay becomes quite flat beyond 1,000 amperes and that the selectivity therefore is impaired beyond that point. It was only when the improved characteristic shown in Fig. 10 was suggested for the Pittsburgh primary network that satisfactory relaying appeared likely. Even this curve flattens beyond 1,500 amperes, and the useful part, which is the inverse portion, is still fairly restricted. Calculations on an average system of this type indicate that short-circuit currents falling outside of the useful portion of this curve may be encountered. This is especially true when the system is enlarged or the unit transformer capacity increased. To furnish the better selectivity at these higher currents the relay should have the inverse time characteristic over a greater range.

This more ideal relay characteristic, however, will give no better performance at corner transformers where tie mains branch in only two directions than the relay of Fig. 10. The method using over-current relays with definite time setting was devised to avoid the need of bringing in a third tie at such corners and of depending on the inverse time characteristic for proper selectivity throughout the system.

The two 1,500-kva. three-phase transformers manufactured in Sharon for the Verona application are the largest subway type transformers thus far produced. The regulators are likewise of special interest in that they constitute the first subway type in this country as far as is known. Stable operation of the regulators at different transformer installations can be assured by the simple method of using a bucking characteristic with increase of load.

Prior to the work on the Pittsburgh type primary network the only form of oil circuit breaker of relatively high rupture capacity in use in manholes was a strictly subway type breaker. Several years ago, in conjunction with proposed underground loop feeders to supply low-voltage networks in four cities, standard indoor type B-16 and B-20 oil circuit breakers in submersible welded steel cubicles were offered. This appeared to be the best way to combine accessibility, safety against water penetration, and interchangeability of parts, with reasonable cost. The B-13 breakers with type CFO motor-operated mechanism, in subway type cubicles as shown in Fig. 12, comprise the first application of this idea.

If no regulators are required all of the oil circuit breakers associated with a transformer might be mounted on the transformer tank. This would make a complete factory-built set, conserve space and cable connections, and permit the handling and installation of the apparatus as a single unit. This construction would be particularly applicable to outdoor or indoor installations, but might also be feasible for manholes under some circumstances.

W. R. Bullard: The paper by Messrs. Stanley and Sinclair emphasizes certain economies and advantages of a particular type of overhead distribution system. This type of system has a place in the art for particular conditions, such for instance, as those encountered in the particular situation described in the paper. One of the particular conditions referred to is the presence, in the area considered, of a main supply operating at 22 kv. Another is the requirement for an unusually high grade of service.

I feel, however, that the picture is incomplete without further mention of another much simpler type of system which I believe to be basically more economical than the one described. This is the type utilizing generating station or transmission terminal substation voltage (6.6 kv., 11 kv., 13.2 kv. and even higher) as the primary voltage for supplying the distribution transformers

directly. This system avoids entirely the use of the intermediate transformation and thus usually effects a very considerable saving in investment and energy losses over any system using an intermediate voltage for the distribution primary. Indeed, it has been shown repeatedly that the elimination of the intermediate transformation usually provides considerably greater possibilities of economy than any of the other plans for reducing investment in distribution systems which have recently been studied.

To date, systems of this type serving overhead districts have had radial primary and secondary circuits. Reasonable reliability of service can be provided by other means than the duplication of supply units. One method is the use of higher grades of construction in order to reduce the number of system faults. The use of the higher primary distribution voltages usually carries with it, for other reasons, the use of higher safety factors and better quality of construction. Therefore, radial systems of the type mentioned, while possibly not having quite the high grade of service reliability of the 4-kv. network, are in successful operation and are giving entirely satisfactory service under average conditions encountered in certain urban, suburban, and rural districts where overhead service is normally required.

Under conditions where greater service reliability may be required, it appears that the network principle could be applied to the higher voltage primaries in much the same manner as described in this paper, except for the absence of the intermediate transformation. However, the cost and complexity of sectionalizing devices would be greater in the case of the higher voltages, and a much simpler solution presents itself. This is the probability of the development in the near future of an overhead type of low-voltage a-c. network. Thus the installation of radial systems utilizing the higher voltage primaries is a natural step toward the development of the most economical type of system giving highest quality of service, viz., the low-voltage a-c. network with radial primaries utilizing generating station or transmission terminal substation voltage.

There are certain difficulties encountered in connection with the installation of overhead, open-wire primaries utilizing the higher voltages, such as governmental objections to overhead high-voltage lines in the streets, clearance required from shady trees, etc., but these difficulties are being in many cases entirely overcome, and since such systems are in line with the present positive drift to higher primary voltages, I confidently expect that they will find a large use in the future. I also am confident that such systems offer appreciably greater possibilities for economy than those involved in the system described by Messrs. Stanley and Sinclair.

A. H. Sweetnam: In 1930, The Edison Electric Illuminating Company of Boston was faced with the problem of relieving its a-c. substation in Allston of approximately 3,400 kva. of load before the fall of 1931, as the capacity of this station with the largest transformer bank out of service, was only 13,600 kva., and it was estimated that the 1931 peak load would be about 17,000 kva.

A preliminary study indicated that by building a new substation in Brookline, it would be possible to pick up about 4,500 kva. of load in an area of approximately 4.4 sq. mi. (load density 1,020 kva. per sq. mi.) by installing short 4,000-volt radial feeders out to the load centers of several of the existing circuits in that immediate vicinity.

It was proposed, therefore, to install initially two 7,500-kva. transformer banks in the new station and supply them by taps from two existing transmission lines in the vicinity.

A location for the new substation was chosen, and a careful study was made for the purpose of obtaining the cost of this station and of the necessary conduit and cable work in the street to bring the transmission lines into the station and to take the new 4,000-volt feeders out to their respective load centers.

About the time that the preliminary plans for the new substation was completed, attention was directed to the possibility of serving this same area by means of a 4,000-volt a-c. network, and, accordingly, with the assistance of Mr. D. K. Blake of the General Electric Company, a preliminary study of the existing conditions was made which indicated such an appreciable saving in initial and ultimate investment, by the installation of an a-c. network in this territory, not to mention the other inherent advantages offered by this system of distribution, that it was decided to obtain a detailed estimate of cost for such an installation.

Very careful estimates were made of the relative costs of supplying the area in Brookline by means of the 4,000-volt network and by a new substation. This economic study was extended to cover the ultimate load requirements of the territory to be served, and it was very clearly indicated that the capital investment showed a reduction of at least 20 per cent through the installation of the network system. Corresponding figures for the initial installation showed even greater savings.

The reduction in the ultimate investment did not include savings in favor of the network obtained through being able to add relatively small increments of capacity, thus closely following the requirements of the load, as against large increment additions inherent in the development of a substation.

In general, the principle of the 4,000-volt a-c. network is to distribute the transformer capacity in relatively small units over the area to be supplied, rather than to concentrate it in one central location, as in the case of a substation.

With this general idea of the network as a guide, the particular problem consisted of creating this new system of distribution from the existing 4,000-volt radial circuits.

The problem resolved itself into three major parts:

1. The location and equipment of the vaults.
2. The transmission line extensions to the vaults.
3. The cutting over of the existing radial circuits to form a 4-wire network between vaults.

After careful consideration of this immediate problem and of the larger problem of the more general application of this network system, it was decided to proceed with the network installation and to standardize on the 1,500-kva. unit and its equipment installed in underground vaults approximately 30 ft. long, 10 ft. wide, and 10 ft. deep. Each vault will contain one 1,500-kva., three-phase, self-cooled transformer rated 13,800 volts primary and 4,330 volts secondary, with one full capacity tap for 5 per cent above normal brought out to a ratio adjuster. The transformer will also have the necessary equipment for load ratio control, designed to automatically change the ratio in eight equal steps over a 10 per cent range, two steps being above the tap in use and six steps below. Provision will also be made so that blower equipment may be added to increase the rating of these transformers to 2,000 kva.

All apparatus will be entirely metal clad making it impervious to moisture and permitting actual flooding of the vault without damage.

The primary and secondary windings of the transformer will both be Y-connected with the neutral of the 4-kv. winding solidly grounded, and the neutral of the 13.8-kv. winding arranged so that it may be operated either solidly grounded or isolated.

The transformer and submersible metal-clad switching equipment will be designed for assembly and operation as a unit, thereby standardizing the vault construction and simplifying and hastening the installation of new equipment when needed.

It was not necessary to do much rephasing on the network because the single-phase loads on the existing radial circuits were already balanced between the three phases.

Lightning arresters are to be installed on all standpipe poles where the 4-wire mains change from overhead to underground,

and on all overhead taps of any considerable length which connect directly into an underground main.

Voltage regulation will be ideal due to the fact that vaults will be located at load centers and will be relatively close together.

All circuit breakers being used in the network vaults are of ample rating (36,000 amperes at 4,000 volts) to handle any short-circuit currents which may be developed by the network, up to an installation of capacity sufficient to supply load of a maximum density of 20,000 kva. per square mile.

To those who are faced with the problem of keeping pace with the ever increasing short-circuit duties on substation circuit breakers, this feature of the network will have a particularly strong appeal.

A study of the above factors shows quite definitely such a decided superiority of the network over the radial system of distribution with regard to economy, simplicity, continuity of service, voltage regulation, standardization of equipment, provision for load growth, et cetera, and as it is so readily adaptable to areas of any load density, that whenever it becomes necessary to make any extensive changes in the existing distribution system, or plan a new system, careful consideration should be given to the possibility of installing a 4,000-volt network.

L. G. Smith: I desire to stress one point of economic consideration, of which I am sure the authors are aware, but which, in my opinion, is not sufficiently emphasized in the paper by virtue of the fact that the relative economics of a conventional radial system as compared with a primary network system may depend upon this one point.

In making an economic comparison of a primary network with a conventional design of radial system undoubtedly the cost of transmission to the substations will be somewhat higher in the primary network system. Of course, this depends to a considerable extent upon local conditions including the capacity per transmission circuit and the general layout of the system. The substation costs, moreover, will probably be somewhat higher for the primary network system due to the fact that submersible equipment for manhole installations will be more expensive and since the transformer capacities are smaller, a higher cost per kva. of transformer capacity should result. Naturally, there is one feature of saving in the substations with the primary network systems, namely, the saving in feeder regulators. In the primary network system, the distribution costs from the substation to the feeder point are entirely eliminated because the substations are placed at the feeder points. Other items of cost, such as primary mains, distribution transformers, secondaries, and customers' services, will be the same in either type of system. With the conventional radial type of system, assuming the cost of transmission, substation and distribution to the feeder points at 100 per cent, this cost will be divided approximately as follows:

Transmission (underground system)	33 per cent
Substations	40 per cent
Distribution feeders	27 per cent

The savings obtained by eliminating the distribution feeders may be sufficient to overbalance the increased cost of transmission and substations. From this it can be seen that the balance of economy may not be very great in either direction and as a matter of fact, local conditions may have a very material effect upon any economic studies.

In addition to the above points the primary network system presents another distinct economic advantage over the conventional radial system, that is, in the cost of spare capacity. With a primary network due to the fact that substation units are added in very small capacity steps, it is possible to follow the load growth very closely and maintain a reasonably small percentage of spare capacity. With a radial system this is not true. If, for example, a 20,000-kva. substation is shown by study to be of economical ultimate size, the initial step may consist of two 10,000-kva. transformer banks and four 13.2-kv. cables giving a sustained capacity of 10,000 kva. In view of the fact that the initial load may be well under 10,000 kva. the spare capacity during the initial life of the substation may be well over 50 per cent. This materially increases the cost per kva. of transmission and substations during the early stages of the substation development. As a matter of fact the costs do not decrease to the ideal values until the ultimate development is reached, at which time a new substation has to be started. Therefore, the cost per kva. for substations and transmission capacity cannot be determined for the ideal basis in an economic study but an average cost must be used which covers the average cost of sustained peak capacity per kva. over the period of the growth of the substation from its initial construction to its ultimate development.

In view of the fact that the cost per kva. of substation and transmission capacity may be 4 or 5 times the cost per kva. for the ultimate development, it can be seen that the characteristic of the primary network system of being able to closely follow the load with installed capacity may be the feature which determines its economic advantage. As I see it, one of the most important features in the design of a power system is the maintaining of adequate reliability to service by means of spare capacity at a minimum cost.

R. T. Henry: Some of the claims made in this paper for the primary network scheme are very optimistic. There is danger in attempting to draw general conclusions from comparisons in specific cases which involve such unusual conditions as the two cases described. It should be noted that both of these cases involve very small areas where transmission circuits already existed and that practically no expense is included for real estate for the network scheme. In such comparisons the investment already made in transmission circuits should be recognized as a part of the total investment if not of the immediate investment required.

In considering the primary network for a larger area, it should be noted that the ideal scheme would provide a different transmission circuit for each network unit. Obviously, this cannot be realized in any ordinary case and it becomes necessary to connect two or more units to each transmission circuit. Two units connected to the same transmission circuit must not be adjacent nor even alternate units if satisfactory diversity is to be obtained. They must be separated in every direction by at least two units connected to different transmission circuits. It can readily be seen that, when this idea is applied to a large area, it results in a perfect maze of transmission circuits and becomes very difficult and costly.

In addition to the difficulty involved in the transmission circuits, as described above, the problem of relaying an extensive primary network is rather difficult. The inverse time relays may operate satisfactorily on feeder faults if each unit has four feeders connected and all transformers are in service. However, with a transformer out of service there is very real danger of undesired operations, particularly if less than four feeders are connected to the unit whose transformer is out of service. This is still further complicated by inequalities in the length of different feeders which cannot be avoided in practical cases.

About two years ago the Buffalo General Electric Company was faced with the problem of designing and building a distribution system covering a territory of about 65 square miles. The load involved was in excess of 100,000 kw. In this case it was necessary to provide transmission circuits to serve the new distribution system. The primary network scheme was very carefully and thoroughly studied at that time in comparison with several other schemes. These studies resulted in the adoption of a radial scheme, a description of which will probably be published in the near future. This scheme has practically all of the advantages of the primary network scheme without its disadvantages and careful comparative estimates indicated that it would cost at least 10 per cent less than the primary network scheme for the territory involved.

Referring to the conclusions in the paper in the same order in which they are presented, we have the following discussion to offer:

1. Material Reduction in Investment. It is very doubtful if network substation locations can be secured in every case, nor even in most cases, without any cost for real estate. Certainly this is too much to expect in cities.

The figures presented indicated a considerable saving in the cost of electrical equipment in favor of the primary network scheme. This should not and need not be the case in a properly designed radial substation. In many cases, if not in most cases, the cost of transmission is a considerable portion of the total cost and should certainly be recognized in making such comparisons.

The studies made in Buffalo covering a larger territory and including transmission indicate that the total cost of the primary network scheme would be greater by at least 10 per cent than the cost of the radial scheme adopted.

2. Greater Copper Economy. For the conditions encountered in Buffalo, there would have been practically no reduction in the cost of distribution feeders in the network scheme over the cost for the radial scheme adopted. This is largely due to the irregularities encountered and to the fact that a large proportion of the investment in feeders is in the branches which are the same in either case.

3. Reduced Duty on Oil Circuit Breakers. It is easily possible in a properly designed radial system to keep the duty on oil circuit breakers as low or even considerably lower than in the primary network scheme. In the radial scheme adopted in Buffalo the maximum short-circuit current at the 4-kv. bus is 65,000 kva. This value will not be exceeded even with the ultimate development of the scheme.

4. Better Regulation. While the voltage level in the network scheme may be more uniform than in a radial scheme with long feeders, it should be noted that in the primary network scheme, it is practically impossible to apply load compensation without involving instability between regulators. Since the voltage drop in distribution transformers is as great or greater than the total voltage drop in the primary circuits, this is a very serious objection. In the radial system adopted in Buffalo the feeders are very short and load compensation is provided to overcome the voltage drop in the distribution transformers as well as in the primary circuits.

5. Small and Standardized Transformer Units. A properly designed radial system permits of standardization at least as readily as the primary network scheme or any other scheme. The initial installation in the new distribution system in Buffalo consists of 24 standard substations, each containing three standardized units making a total of 72 standardized units. This entire installation is covered by one set of drawings.

6. Capacity May be Installed in Small Increments. The radial system adopted in Buffalo provides for the addition of capacity in small increments "where and when needed." Long range predictions of load growth are quite as unnecessary in this scheme as in the primary network scheme.

7. Investment for Load Growth in Small Increments. Same as No. 6.

8. Reduced System Losses. The claim of "complete elimination of feeder losses as there are no feeders" is hardly justified as the feeders are not eliminated but simply called "tie lines." Here, too, the principal losses are in the branches rather than in the main part of the feeder. Studies indicated that there was practically no difference in the losses in the radial system adopted in Buffalo from those in a primary network system for the same territory.

9. Area of Outage Reduced. The area affected by an outage is no greater in the radial system adopted in Buffalo than in the primary network system described in the paper.

10. No. 22 Kv. Breakers. The high-tension cables are connected direct to the substation transformers without high-tension breakers in the radial system adopted in Buffalo as well as in the network system described in the paper.

H. L. Wallau: The relative cost submitted by the authors of this paper is the reverse of that obtained by me some six years ago when I had occasion to make a study along similar lines.

The comparison made by me was between a 12,000-kva. radial substation and four 3,000-kva. substations with 4,600-volt lines networked. The equipment estimate was, of course, based on the types available at that date. The study, however, included the total transmission required from the generating plant to the stations to be served, land, buildings, substation, and distribution equipment for a load of 9,000 kva., 3,000 kva. of spare transformer capacity being available in each case. Transmission was at 11 kv. underground and regulation was by means of feeder, not bus regulators. Under these conditions the relative investments arrived at were 100 per cent for the radial type *vs.* 106 to 118 per cent for the networked type.

The figures obtained by the authors are considerably lower for the primary network. This is accounted for in part by the use of existing facilities, and in part by the use of bus regulation. The installation now being made seems entirely justified for the conditions existing. Whether or not it can be laid down as a general principle that this type of distribution for territories of the load densities under consideraton will always be justified is open to question.

In Table II there appears a figure which seems doubtful. The relative investment in real estate is given as 17 per cent for a radial substation and as only 2.8 per cent for the networked stations of the surface type. This latter figure is but 16 per cent of the former and to be comparable must cover the cost of three distinct sites. Is it not possible that the real estate investment in but one such site was included through an oversight?

Assuming the correctness of the figures as given could not the three independent surface installations of each type be concentrated at one location? Using the same types of construction and equipment, exclusive of network interconnections, relative investments of the following order, on the basis of three independent primary feeds, would seem indicated:

	Bldg. type	Outdoor type
Real estate	2.8	2.8
Structures	13.0	5.8
Equipment	66.3	66.3
Transmission*	3.8	3.8
Distribution	1.6	1.6
Duct Connections	14.5	14.5
	102.0	94.8

*Increased 50 per cent for a third primary feeder.

If these assumptions are correct, they would indicate that a modification in the radial type substation design might yield economies in excess of those shown by the authors, for the primary network type, in the Verona-Oakmont district.

The development of a-c. networks is progressing. Nevertheless, many d-c. networks still exist involving large investments which it may not be advisable to retire. If the d-c. substation and its numerous low-voltage feeders could be eliminated, being replaced by high-voltage a-c. feeders and relatively inexpensive d-c. conversion sources of moderate capacities for installation at points corresponding to those at which network transformers are now installed, the picture would be quite different.

With the continued development of the thyratron, it seems not unreasonable to visualize a three-wire d-c. network fed at street intersections from manholes or vaults through tubes. The supply transformers would be single-phase high voltage, *e. g.*, 11- to 33-kv., two pairs of tubes, one on each side of the

three-wire circuit, smoothing reactances, and an over-current, reverse-current d-c. circuit breaker, completing the main installation. Grid current at suitable voltage would be obtained from an extra winding on the transformers. Individual installations would be balanced across the three phases of the primary supply cables and these in turn so interlaced that the loss of one or more primary cables (dependent upon the extent and capacity of the network) would in no way interrupt service to the consumers supplied therefrom.

It is possible that this development may be realized in the comparatively near future. Three years ago Mr. A. W. Hill, of the General Electric Company, in a paper presented before the New York Section, stated that "cathodes with a normal emission of 10,000 amperes appear quite practical."

G. M. Miller: From the load densities found in the Verona-Oakmont District it seems questionable whether the network class of service is necessary. Should the primary network prove to be as practical and satisfactory as seems entirely possible it may be that the customers in the lower density areas will receive better service than those in the older heavily-loaded areas served by the radial feeder and large transformer substation scheme.

The use of small transformers for stepping from the transmission to the distribution voltage is a decided advantage and permits the installation of additional transformer stations at locations where the load is increasing. Possible mistakes in location are avoided by this scheme whereas with large banks of transformers or large capacity substations it is difficult to select the right location due to the shifting of load centers. In some cases the trend of load growth changes and large investments cannot be recovered. Improved regulation and copper economies are possible with a primary network which have not been accomplished in any other way.

Experience with relaying schemes and the general arrangement of the network will no doubt lead to improvements which will reduce cost of equipment. Improvement in design and methods of installation may further reduce the over-all costs so that the primary network will be much less expensive than the present radial scheme and the class of service will be superior to that given at present.

C. T. Sinclair: Mr. Hendrickson raises the question as to whether or not the primary network is the correct solution for areas of low and medium load density where a high standard of service is required and where a low-voltage network will be needed in the future. In the present system, the radial distribution system has been considered adequate to fit these areas. In most low and medium load density areas it possibly will be many years before a low-voltage network will be economically justifiable. It is our thought that in these areas a higher standard of service may be rendered by the primary network than by the usual radial system. The primary network is less costly than the present radial systems for the Pittsburgh area and our experience thus far indicates that it may be expected to render a higher grade of service.

In converting the usual radial system to a network there is quite frequently a serious economical handicap in scrapping or at least removing equipment already in operation. With the primary network it does not appear that a conversion would be so difficult. The first step would obviously be to network the secondaries, using the present 4-kv. mains as primaries. The additional capacity beyond this point would be provided by feeders into the secondary grid supplied by the base supply voltage (22 kv. in the Pittsburgh case).

In starting the low-voltage network a minimum of three feeders available for supply is very desirable in order to reduce the percentage of spare capacity in the system. By way of illustration, if only two feeders were used 100 per cent spare capacity must be provided in the system for the system must be designed on the assumption that the one feeder is out. With the two-feeder system either feeder with transformer banks must be capable of supplying the entire load. With the three-feeder system 50 per cent spare capacity is required on the same assumption, and with four feeders 25 per cent. Thus it is apparent that with more feeders available the percentage spare capacity is reduced. The primary network in the conversion scheme mentioned above has certain advantages in this respect in so far as each sectionalized primary main is essentially a feeder and a number of these mains will be available by the very nature of the network. This will especially be true when a given area has reached the load density at which a low-voltage network appears economically feasible.

We are in complete accord with Mr. Bullard's statement as to the desirability of using generating station or transmission terminal substation voltage where possible. We are confident that at this time it would be impractical to operate an 11-, 13-, or 22-kv. system for our distribution areas. The necessity to work this voltage under congested conditions is extremely difficult. In some of the areas underground construction would make it impossible to operate such a distribution system. As Mr. Bullard has pointed out, there are certain governmental objections and in certain areas it is practically impossible to spread this higher voltage throughout the entire district. The extensive joint use system introduces another factor difficult of solution.

For certain types of distribution areas, for example rural distribution, the higher voltages discussed by Mr. Bullard certainly offer material advantages. The primary network, however, is not offered as a solution to this problem.

It is very interesting to note that Mr. Sweetnam has likewise found material economies in the adoption of the primary network and his discussion needs no elaboration here. The important conclusion to be drawn from his discussion is that the primary network is readily adaptable to areas differing in type from Pittsburgh.

Mr. Smith refers to the reasonably small percentage of spare capacity. This advantage is true in both the primary network and the low-voltage network. It is possible to add load in small increments when and where desired. We would be inclined to term this extra capacity needed "over capacity" rather than "spare capacity," as he terms it. Spare capacity is always provided to take care of emergency and abnormal conditions whereas this over capacity is required in the usual system by the very nature of the large units involved in the usual radial system. As Mr. Smith points out, the initial load on a 10,000 kva. bank may be during the initial life of the substation well under 50 per cent (or, as he expresses it, the spare capacity may be well over 50 per cent). As he points out, the cost per kva. cannot be determined on an ideal basis in an economic study but average costs must be used considering the period of the growth of the substation.

The difficulties of relaying and regulation, mentioned by Mr. Henry, have not been met with thus far in the Pittsburgh network after five months of operation and several series of tests. The fact that this time is too short to make any definite statements is recognized but our studies combined with the little operating experience have led us to believe that these problems are no more difficult than those met with in other cases in the electric light and power industry. We feel sure that the scheme described by Mr. Henry is very satisfactory and will give a good account of itself.

Mr. Wallau raises the question as to the investment figure of 17 per cent given in our table and notes that it is high compared to the cost for the network stations. The 17 per cent represents the actual value of the property now in our possession which was to be utilized for the construction of the substation and is not an estimated figure but actual value. Radial substation locations are chosen after an economic study which, speaking generally, balances feeder costs against property costs. In other

words, it is possible to locate a station away from the load center and buy cheap property. This, however, means a longer average feeder length and each substation location may be chosen after this economic study is made, taking into consideration all factors involved. The substation location chosen in this report for a radial substation is the best location for the radial system considered. The reason that the real estate item in the network analysis is low is because of the fact that this substation can be located either underground entirely, where no property is purchased or on a small and inexpensive lot.

We feel confident that in our particular case the three independent units could not be located at one spot as suggested by Mr. Wallau as this would result in costs largely as described for the radial type system. The use of the thyratron for supplying d-c. networks seems to have certain possibilities. It is also interesting to note that the application of d-c. transmission to the primary network appears to have possibilities. This, however, is looking rather far in to the future.

It is interesting to note that Mr. Miller feels the importance of being able to add transformer capacity when and where the load is increasing. He points out that this largely eliminates the possibility of mistakes in choosing large substation locations.

The authors of this paper do not contend that primary network should be applied to all overhead distribution systems. It is felt, however, that this system provides economies and operating possibilities for the areas described that are superior to other systems now in use. Our operating experience thus far has justified our analysis. Additional studies made indicate that the scheme can very likely be applied to a large proportion of our load.

The Philadelphia A-C. Network System

Development and Operation of a Network System Having Fused Secondary Mains and 2,300-Volt Loop Circuit Supply

BY H. S. DAVIS[1] and W. R. ROSS[2]
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The object of this paper is to outline the engineering plan of the low-voltage a-c. network system in Philadelphia, Pa. and to summarize operating experience over a five-year period. The development of this system and the operating experience will be of interest, since several basic features, which contribute toward the successful results, are different from the basic plan in other types of network systems.

The system consists of sectionalized primary loop feeders, supplying a fused secondary network. At present there are twenty-four feeders in operation, with a total normal operating capacity of 32,540 kva.; the coincident feeder peak to date being 27,600 kva.

The conclusions in the paper are essentially as follows:

1. The system plan initially adopted has been followed without fundamental change, and has proven amply flexible to meet changed load conditions as the area develops.

2. Operating experience has been very satisfactory, under all conditions, including those at time of faults on the network or elsewhere on the system.

Introduction

TEN years ago many of the central station companies were experiencing considerable difficulty in determining upon a distribution system which would best provide for load growth in areas of higher load densities. A number of individual companies inaugurated studies to determine the practicability of a secondary a-c. network system to supplement or supplant the d-c. network and radial a-c. distribution systems. Although the principle of multiple primary feeders supplying a secondary network underlies all network systems, the methods and apparatus actually used have varied to such an extent, due to local conditions and standards, that a number of different types of network systems is in use.

The system described herein was found by a study of all types of secondary network systems to be best suited to conditions in the downtown business district of Philadelphia from the standpoints of reliability, simplicity, flexibility, and economy.

General Description

The first network was established in the early part of 1926 as an integral part of a ten-year program of changing the downtown business district from direct current to alternating current.

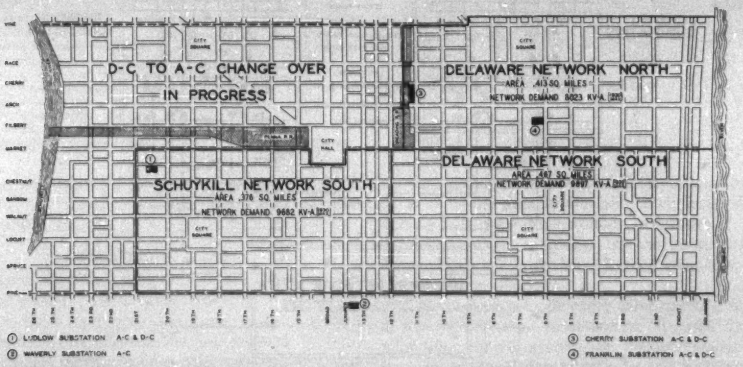


Fig. 1—Philadelphia Downtown Business District Showing A-C. Network Areas

The downtown business district, covering an area of 1.6 sq. mi. (Fig. 1), consists largely of office buildings, banking institutions, hotels, apartment houses, theatres, railway terminals, high-class merchandizing establishments, etc.

The 1930 peak demand of this area was 72,820 kva.

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31-56

of which 27,600 kva. was carried on the a-c. network system and 6,000 kw. on the d-c. system. The remaining load is generally in such large blocks and of such a character as to generally warrant service at a higher voltage and is usually supplied from dual radial nominally 2,300-volt or 13,200-volt feeders.

The entire network and the greater part of the radial feeder load in this area, as well as some of the radial feeder load in areas adjacent thereto, is supplied from two generating stations through four substations located as shown on Fig. 1.

The present a-c. network system comprising an area of 1.26 sq. mi., consists of three separate networks, each having a load of approximately 9,000 kva. Each network area is supplied by eight 2,300-volt loop feeders from two substations. Each such pair of substations is fed from the same generating station, by a number of 13,200-volt feeders. The feeders to the same substation are connected to different generating station bus sections which are separated from each other by barrier walls, as illustrated in Fig. 2. The generating stations, Schuylkill and Delaware, are located approximately one and one-half miles from the network area.

Voltage regulation on the network system is secured by means of induction type regulators on the low-voltage side of the substation transformer banks or on the individual loop feeders. The use of bus regulation is gradually being extended to all substations feeding the network system.

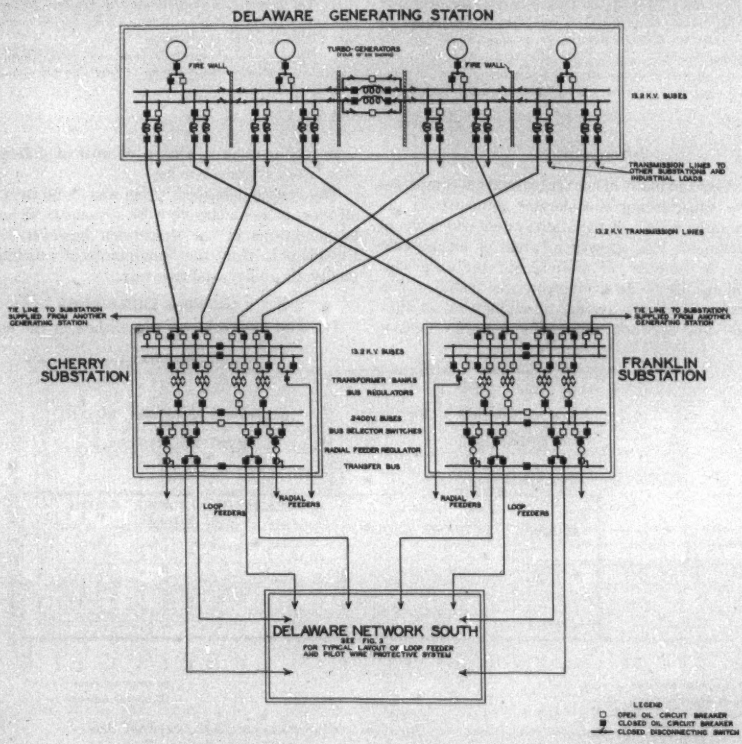

Fig. 2—Schematic Diagram Showing Typical Method of Supply to Network Area

No auto transformers are required on the secondary network because the system conforms to accepted voltage standards for both light and power service; therefore, all parts of the system can be paralleled without the use of translating devices.

Statistics pertaining to the network load at the time

of the 1930 peak, number of loop feeders, length of feeders, number of transformers and oil circuit breakers are presented by areas in Table I.

The secondary a-c. network system is a gridiron of secondary mains, two-phase, five-wire 115/230 volts,[3] fed from transformer banks located at each main street intersection and which are supplied from a number of interlaced loop primary feeders.

The secondary mains and transformer leads are provided with copper-link fuses for automatically sectionalizing parts of the system in case of trouble.

The loop primary feeders consist of several unit sections which are connected by means of 15-kv. automatic oil circuit breakers operated by means of a balanced pilot wire control system. Practically all the cable comprising the loop feeders, together with the majority of the stepdown transformers have been purchased and installed for possible ultimate 13.2-kv. operation. Oil circuit breakers are rated at 400 amperes and 15 kv.

TABLE I—DATA PERTAINING TO THE PHILADELPHIA A-C. NETWORK SYSTEM

	Delaware north	Delaware south	Schuykill south	Totals December 1930
1. Area in square miles operating network	0.413	0.467	0.376	1.256
2. Maximum demand in kva. (network only)	8,023	9,897	9,682	27,602
3. Number primary loop feeders	8	8	8	24
4. Total normal capacity of loop feeders	10,620 kva.	9,600 kva.	12,320 kva.	32,540 kva.
5. Average normal load per loop feeder	1,003 kva.	1,237 kva.	1,210 kva.	1,150
6. Total length of loop feeder cable	70,400 ft.	99,800 ft.	92,400 ft.	262,600 ft.
7. Average length per loop feeder	8,800 ft.	12,475 ft.	11,550 ft.	.....
8. Total length of loop feeder cable between substation and first transformer bank	17,160 ft.	36,698 ft.	30,240 ft.	84,098 ft.
9. Total length of loop feeder cable between substation and boundary of network area		21,292 ft.	6,805 ft.	28,097 ft.
10. Number of unit sections on loops	57	70	72	199
11. Average loop section length in feet	1,230 ft.	1,430 ft.	1,280 ft.	1,320 ft.
12. Number of primary oil circuit breakers	49	62	64	175
13. Installed transformer capacity kva	12,500	13,600	15,400	41,500
14. Number of transformers (100 kva.)	125	136	154	415
15. Number of transformer manholes	41	54	56	151
16. Ratio normal loop feeder capacity to transformer capacity	85%	71%	80%	78%
17. Ratio maximum coincident feeder demand to transformer capacity	64%	73%	63%	66%

Operating Results

Operating experience with this network system over a five-year period has been satisfactory.

To date there have been two major cases of system disturbances which affected the a-c. network system; in one instance service to one entire network area was interrupted for a short time and in the other case the continuity of the network service was unaffected.

Case 1. Due to trouble in one of the generating stations, power supply to the two substations feeding a 5,000-kva. network load was lost. One of the two substations involved had "tie lines" to a third substation which was fed from another generating station. By closing the tie line oil circuit breakers, this substation picked up the entire load of the network and carried it until conditions were restored to normal.

Case 2. Two substations *A* and *B* were jointly carrying the load of one of the network areas totaling approximately 6,000 kva. The 13.2-kv. feeders to substation *A* dropped out successively, so that the entire transmission supply was lost. The remaining substation *B* continued to feed all of its regular load, and all the network load without noticeable drop in voltage. In addition, the feed back through the network and the loop feeders energized the 2,300-volt bus in substation *A* which in turn picked up a part of the load on a limited number of important outgoing radial 2,300-volt feeders.

Twenty-two primary cable and three transformer failures have occurred, and in every case the oil circuit breakers and fuses have isolated the faulty section without interruption to service to any other unit section or impairing service to any customer.

The operation of secondary fuses in well-established network areas has demonstrated that faults on secondary mains are quickly and selectively isolated, consequently with a minimum amount of damage.

In two cases of trouble on the secondary transformer leads the faults have burned clear within a few inches of point of origin.

An inspection is made every three months by a group of five men in order to determine the condition of the pilot wire control system, oil circuit breakers, transformers, sectionalizing boxes, etc. It is expected that the rate of inspection may be reduced. The periodic field inspection consists of:

1. Testing transformers, oil circuit breakers and sectionalizing fuse boxes for water tightness. Work involved in making the equipment water-tight consumes the major part of the inspection time.

2. Testing the pilot wire system (conductors, current transformer, and trip coils) for (a) grounds, (b) open and short circuits.

3. Inspection of trip coil settings and operating mechanism for sensitivity.

4. Examination of sectionalizing boxes for—(a) blown fuses, (b) open secondary cable mains.

3. *Two-Phase, Five-Wire Distribution*, by P. H. Chase, A. I. E. E. Trans., Vol. XLIV, p. 737.

5. Minor repairs and adjustments.

Open breakers on the loop circuits are checked in two ways. By recording the hourly ammeter readings on each loop feeder, and by alternately opening the loop feeder ends, twice each week, and checking the approximate total currents under these test conditions. The current readings on each loop end often indicate if a unit section is open and its approximate location.

Basis of System Design

Reliability, flexibility, simplicity, and economy were the four outstanding requisites upon which the system was designed and the basis upon which extensions have been made.

The detail engineering plan was based upon the requirements:

1. That the network system as a whole shall have sufficient spare capacity to assure against overloading any equipment during period of unit section or transformer outage.

2. That adequate network voltage shall be maintained under all conditions, including short-time (approximately 30 minutes) loss of one of the two substations supplying a given area. With the loss of one substation feeding a network area, it is possible to carry the entire load on the remaining substation by taking advantage of short-time overload rating on all substation and network equipment.

Development of Detail System Design

With the method of protection employed and the interlacing of loop feeders, operating experience has shown that it is possible to load the network transformers to 80 per cent of the manufacturer's nameplate rating with the assurance that there will be sufficient capacity available in the event of a failure occurring in a loop unit section at time of the network peak.

Loop feeders are normally operated at 80 per cent of the rated equipment capacity. The loading on some of the individual loop feeders may exceed the 80 per cent value for periods of relatively short duration, *i. e.*, at the time of the network peak. Conditions of this nature are infrequent and are permissible in view of the available reserve capacity in all parts of the system. With the loss of a unit section of loop feeder, the feeder continues to function, though the load on the respective ends of the loop feeder may be unequal, depending upon the location of the unit section isolated by the operation of the pilot-wire protection system. In the event of a failure in either the first or the last unit section, the remaining operating end carries approximately 75 per cent of the total load carried on the respective ends prior to the fault on the open end. The load dropped by the faulty loop, is absorbed by the adjacent interlaced loop feeders.

Coincident with the studies undertaken to determine the possibilities of establishing an a-c. network system, it was recognized that an "a-c. calculating board" upon which actual alternating-current conditions of operation could be simulated, would prove of material assistance in securing the desired information with the maximum degree of accuracy, in the minimum space of time. Accordingly a board was designed and constructed by the Philadelphia Electric Co.,[4] upon which various conditions of operation, kinds of faults, possible changes in the fundamental plan, etc., were studied. This proved to be a most valuable aid in the solution of problems involving the magnitude of currents flowing into faults of various kinds; the amount of circulating current caused by phase angle or voltage differences between substations, the effect of motor starting currents, together with the permissible loading of loop feeders under given fault conditions.

Having determined upon the operating conditions that would be encountered, complete laboratory tests were conducted to determine the ability of the protective scheme to function properly at time of a fault.

Primary Feeders

The primary supply to the network system is in the form of three-conductor, 350,000 cir. mils, paper-insulated, lead-covered cables. Each loop feeder extends from a substation through a number of intersections throughout the network area and returns to the same substation. Each feeder is routed through widely separated parts of the network area and interlaced with other feeders.

The primary feeders at present are operating at 2,300 volts, three-wire, two-phase.

Since the initial operation of the network system in 1926, there have been 22 primary cable failures approximating one failure per 34 unit sections per year, or 12 failures per 100 miles of operating cable per year. The relatively high rate of cable failures has been largely due to external mechanical injury.

Balanced Pilot Wire System of Protection

Each primary loop feeder is composed of a series of unit sections, each of the unit sections being provided with sectionalizing oil circuit breakers. The present average length of unit sections is approximately 1,300 ft. The arrangement of substation equipment, composition of unit sections and protective scheme is shown in schematic form in Fig. 3. The balanced pilot wire system is fully described in an article by Mr. P. H. Chase.[4]

Each unit section is protected by a pilot wire control circuit connecting current transformers and trip coils in the circuit breakers at each end of the unit section. Current transformers in the secondary leads of the distribution transformers are also connected into the control circuit. The distribution transformers are thus included within the protected section of the feeder.

Each unit section of feeder may be considered as

4. "An Alternating-Current Network," by P. H. Chase, *Electrical World*, Vol. 88, No. 13, Sept. 25, 1926.

three branches around the point where the distribution transformer bank is connected to the feeder. Current transformers in each of these three branches are connected through the pilot wire to obtain a three-way current balance, corresponding to the current balance in the three branches of the unit section. Trip coils in the oil circuit breakers are connected across the pilot wire system at each end of the unit section.

Under normal operating conditions, the relative polarity of the current transformers is always such as to cause a circulating current to flow in the pilot wire circuits, with zero or negligible current in the trip coils. In the event of a fault anywhere in a unit section, the relative polarity of the associated current transformers change, so that current flows through the trip coils and the oil circuit breakers at each end of the faulty unit section are tripped open. The back feed from the low-voltage network blows the 1,500-ampere fuses in the transformer secondary leads and the unit section is completely isolated.

The protective system for each unit feeder section is responsive only to fault conditions within the section. There may be reversals of current in other unit sections, but the relative polarities of the current transformers are always such that a current balance in the control circuit is maintained. There may also be a heavy current flow in other unit sections, but the impedance of the trip coils is sufficiently high, relative to that of the pilot cable, to prevent current of operating value from flowing through them.

It will be observed that only one transformer bank and associated primary cable is isolated in case of a fault on the primary loop feeder. The other transformer banks remain in service, being supplied from the two parts of the loop feeder which have been separated by the isolation of the faulty unit section. Due to this feature the occurrence of a fault causes a minimum of disturbance to the remainder of the system.

The pilot cable which parallels the primary loop feeder is a four-conductor No. 4, rubber-insulated, and lead-covered cable. All branches of the pilot wire control system are connected through manhole type junction boxes provided with test clips to facilitate testing and measuring load on transformer banks.

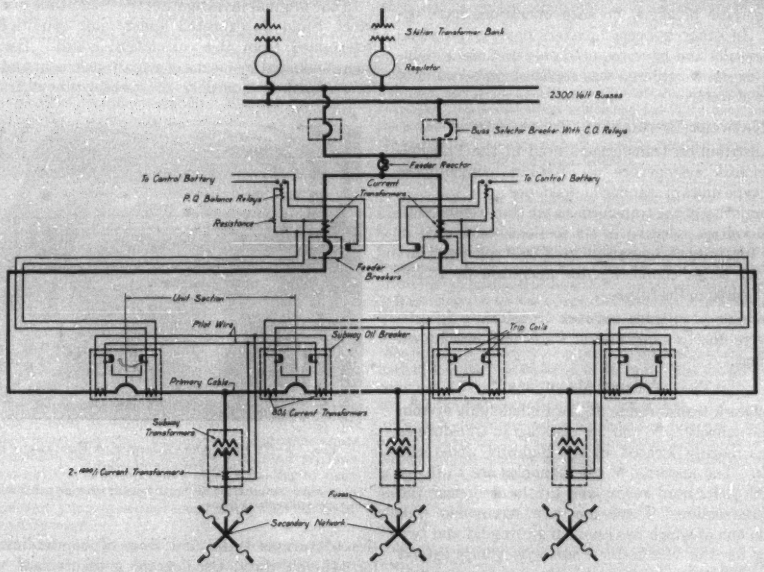

FIG. 3—SINGLE LINE SCHEMATIC DIAGRAM PILOT WIRE PROTECTIVE SYSTEM

Field inspections to date have disclosed three cases of failure in the four-conductor cable of the pilot wire system; two cases of incorrect cable connections in the junction boxes and a defective current transformer. In no case did the six irregularities cause operation of the loop feeder oil circuit breakers.

PRIMARY OIL CIRCUIT BREAKERS

The oil circuit breakers used for sectionalizing the primary loop feeders are of the subway type, rated at

15,000 volts and 400 amperes, with an interrupting capacity of 30,000 amperes at 2,300 volts on standard 2 OCO basis. Each oil circuit breaker contains six bushing type current transformers, and six 5-ampere trip coils. Connections are made to the oil circuit breakers with single-conductor cables which in turn are spliced to the three-conductor primary cables. Wiping bushings are provided for extension of current transformer and trip coil leads, in lead-covered cables, for connection to the balanced pilot wire protective system.

On the first consignment of oil circuit breakers, difficulties were experienced with tripping out due to vibration. Since the proper corrections have been made to these breakers, no such operations have occurred. In other respects breakers have given satisfactory results and have required only minimum maintenance except where one was replaced on account of entrance of water.

Network Distribution Transformers

The distribution transformers used in the Philadelphia network system are all 100-kva. single-phase subway type units of standard impedance.

The majority of the transformers are constructed with the high-voltage winding in six sections, connected in parallel for present operation at 2,300 volts and arranged for series connection for operation at 13,200 volts, if desired in the future.

There have been three network transformer failures which were due to admission of water and breakdown of windings.

Transformer Manholes

All network transformers on the Philadelphia system are of the subway type and are installed in transformer manholes usually located in the highway under the sidewalk. The majority of the manholes are 15 ft. by 9 ft. with 8 ft. head room, and are located near the street intersection. There are two removable slab openings, one of which has an open grating lid and two openings for the ventilating chimneys, which extend from the street surface to the bottom of the manhole at each end.

Each transformer manhole usually contains from one to four 100-kva. transformers, one primary oil circuit breaker, two pilot cable junction boxes and four low-voltage sectionalizing boxes, illustrated in Fig. 4. Barrier walls are not used to isolate the primary oil circuit breakers from the transformer but all cables, except the control cables, are fireproofed.

Secondary Network

The cables on the 115/230-volt, five-wire, two-phase secondary network are 350,000 cir. mils, 600 volt paper-insulated and lead-covered. Single-conductor cables from the single polarity five-way fused sectionalizing boxes which are installed at every main street intersection may be spliced to single or multiple conductor secondary mains. Three-conductor cable mains of the former d-c. network were used in many instances for one phase, and two single-conductor cables installed to complete the two-phase main system. The present practise is to install single-conductor cable for the phase wires and a bare No. 2/0 neutral wire. Both lighting and power loads are supplied from the same set of secondary mains.

Two-phase motors of various sizes up to 40-hp. are taken on any part of the secondary network system when well established, without requiring special starting current limiting devices.

Secondary Fuse Protection

In the original investigation the magnitude of current which could be expected under fault conditions was determined from the calculating board. Tests were then conducted by means of a spare generator and transformer bank to determine the current-time characteristics of various types and sizes of copper-link fuses for 115-volt duty. Oscillograph measurement of the current and time of blowing of the fuses mounted in different types of boxes, with fuse contact blocks of different sizes and under different temperature conditions indicated that the fuse sizes determined upon gave the necessary selectivity.

Fig. 4—Typical Arrangement of Equipment in Transformer Manhole

Left to right—transformer, oil circuit breaker, pilot cable junction boxes, secondary sectionalizing boxes

These copper-link fuses are installed in five-way single polarity sectionalizing boxes, to which the four secondary mains and the transformer leads are connected. The mains are protected with 1,250- and the transformer leads with 1,500-ampere rated capacity fuses.

Operating experience with a fused a-c. secondary network system in well-established areas has indicated that fusing of mains and secondary transformer leads is desirable and that the fuse sizes used are satisfactory. In establishing the secondary network, old d-c. mains of various types and sizes were used. The different condi-

tions existing on the secondary system and in the supply to the network during the construction periods make it impossible to definitely analyze or summarize the types of secondary failures and the operation of fuses at times of faults. There have been no cases of fuses blowing when a fault has not existed on the particular section. When fuses have not blown the faults have cleared themselves without any damage other than destruction of the cable immediately adjacent to the point of failure.

Discussion

H. Richter: The paper states that there is a number of types of a-c. low-voltage network systems in use for underground distribution in this country. There are really only two types, the Philadelphia system, which uses loop feeders and fuses for protection on the network side of the transformers, and that which employs radial feeders and automatic network protectors for protection. There are four variations of the automatic protector scheme, but no differences fundamentally. Whether the secondaries are two-phase or three-phase is incidental.

In 1924, when the two types of systems were in their infancy, a few individuals predicted that both would be made to give satisfactory service, despite dire misgivings from many quarters. This paper, together with the extensive literature on the automatic protector system, proves that the *predictions* were correct.

As of January 1930, the two systems served peak loads of 28,000 kva. and 330,000 kva., respectively. In terms of operating companies that employ the two systems as the present standard, the ratio is 1 to about 55. Loads that are among the most important and exacting in the world, as in the skyscrapers in New York City, depend entirely on the automatic protector system. This system is being installed where direct current with storage battery was the rule.

The network system described in the paper seems to have a disadvantage as regards length of time for the transformer secondary fuses to clear primary faults. These fuses take at least twice as long to clear as would automatic protectors, and frequently much more than this. Primary cable and transformer faults predominate, and it is very desirable to isolate every fault as quickly as possible to prevent the trouble spreading by overstressing the other parts of the system. The automatic protector, I believe, answers this requirement much better than the transformer secondary fuses.

One object of the sectionalized loop feeder scheme with one transformer bank per feeder section is to keep down the spare transformer capacity required to carry the network after a primary fault is isolated. The paper states that this permits loading the transformers to 80 per cent of their rating at time of peak load. In the automatic network protector system, 16 parallel radial feeders would correspond, with respect to the approximate total length, to the 8 loop feeders in each of the three network divisions in the Philadelphia network. Under the same assumption of a single primary fault, tripping out an entire radial feeder would require no more total spare transformer capacity than with the sectionalized loop scheme, and possibly less might be tolerated. On the other hand, more copper is needed in the loops.

The two-phase five-wire secondary network utilizes accepted voltage standards for power as well as light. But it is also true that the fear of having to use considerable booster auto-transformer capacity for the prevalent three-phase system has long since faded away. Extensive experience has proven that auto-transformers constitute but a small part of the total expense of a well-designed automatic protector system employing 120/208- or 115/199-volt secondaries. The minor difficulties in fitting standard devices to the non-standard voltages in this type of secondary system comprise just one of the routine matters encountered in underground distribution practise. The effort to spread the use of translators for 115/230-volt three-phase networks has likewise fallen down, so there is no need to compare the two-phase network with that scheme.

To sum up, it appears that the two types of a-c. underground network system are, in general, giving reliable service; but the automatic protector system is probably cheaper in first cost and maintenance and conserves the investment by more promptly isolating primary faults.

H. S. Davis and W. R. Ross: Mr. Richter points out the difference in the two prevailing types of a-c. network systems and indicates the relative amount of load connected to each. Admittedly, there are a predominance of a-c. network systems which employ the low-voltage automatic a-c. network protector. However, in spite of the larger number of systems of this type, little or no thorough-giving information has been placed at the disposal of the industry as to actual operating performance. The object of our paper was to present operating data secured over a five-year period of actual operation of the Philadelphia a-c. networks. This operating experience has fully demonstrated the soundness of the sectionalized loop-feeder plan. We believe the highly satisfactory performance has resulted largely from the simplicity of the equipment utilized.

Of the various devices available to interrupt circuits in the 115/230-volt class, it is believed that the link fuse is one that can be depended upon reliably to interrupt the circuit. Further, it is our opinion that the slight additional time required for the fuse, as compared with the network protector, to interrupt fault currents of small magnitude, does not nearly outweigh the advantage of having this dependable simple interrupting device.

Regarding the matter of speed of isolation of primary faults, Mr. Richter overlooks the fact that under the loop scheme the primary sectionalizing breakers operate in about ten cycles and thus isolate the faulty section from the remainder of the primary feeder, then the final isolation of the fault, from the network, is accomplished by the fuses interrupting only the fault current fed back through one network transformer bank.

To meet conditions such as the complete outage of a feeder or all the feeders due to the loss of a substation, the parallel radial feeders scheme requires as high a degree of interlacing as does the section sectionalized loop-feeder plan. Unless this is done, an equivalent grade of service cannot be afforded without materially increasing the total feeder or transformer capacity, or both.

No attempt was made to stress the relative merits of the five-wire, 115/230-volt, two-phase, network system. Mr. Richter concedes that minor difficulties are experienced from time to time in fitting standard devices to operate satisfactorily on non-standard voltage systems. However, to quote Mr. Richter, "this has become more or less one of the routine matters encountered in underground distribution practise." It is obvious that standard voltages have very definite advantages and afford greater ease of operation from a regulation standpoint than can be considered where the secondary network consists of four-wire, three-phase, 120/208- or 115/199-volt secondary mains.

In summing up his remarks, Mr. Richter makes the general statement that the automatic network low-voltage protector system is probably cheaper in first cost and maintenance. Exception is taken to these very general statements in that information at hand does not bear out this contention. Comparisons of operating maintenance costs on systems using the network protectors and the maintenance cost of operating loop-sectionalizing devices, such as those in operation in Philadelphia, indicate that there is little, if any, saving in one system over the other.

Present Day Practise in Grounding of Transmission Systems

Second Report of Subject Committee on Grounding*

Introduction

IN 1923 the Protective Devices Committee through its Subcommittee on Grounding issued a report on the grounding practise of that day. Last year it was felt that there had probably been sufficient change since that time to justify a second report of the same nature, and the Joint Interconnection Subcommittee of the Power Generation, Protective Devices, and Transmission and Distribution Committees undertook to work up the report which is submitted herewith.

In preparing this report the general scheme of the first report was followed, the information received from the participating utilities being given without any attempt to draw conclusions as to preferred practise.

Some of the salient points of comparison between ungrounded, solidly grounded, resistance grounded, reactance grounded, and tuned reactance grounded systems, such as drop in voltage, kilowatt demand, steady-state voltage stresses and apparatus cost can be fairly well calculated, but there is still too much uncertainty regarding other factors, such as transient voltage stresses, possibilities of double faults, effects on relaying, to allow of any clear-cut preference for any one scheme of grounding over the others. However, in order to assist the reader in applying the data to his own conditions, the committee felt that a short review of the fundamentals of grounding might not be out of place.

Definitions

For the purposes of discussion the Committee has set up the following definitions:

A. Solidly Grounded

A station is considered solidly grounded when the neutral of every generator or transformer bank in that station is connected solidly and permanently to earth through a low resistance connection. A system is solidly grounded when all of its major generating stations or transmission substations are solidly grounded.

B. Resistance Grounded

A system is considered resistance grounded when a resistor is interposed between the neutral and earth.

C. Reactance Grounded

A system is considered reactance grounded when an air core or iron core reactor is interposed between the neutral and earth; if grounded through a grounding transformer, or if only a portion of the generator or transformer capacity at any given location is solidly grounded.

D. Tuned Reactance Grounded

A system is considered tuned reactance grounded when the aggregate reactive current passed by the neutral reactors or their equivalent substantially equals the aggregate charging current to ground of the system when one conductor is grounded, as with the Petersen coil, Bauch transformer, and the dissonance coil.

E. Ungrounded

A system is considered ungrounded when there are no conductive paths between the neutral and earth, except perhaps through potential transformers.

Selection of Grounding Method

There are several factors which influence the choice of grounding method, and the most important of them are listed below:

1. Apparatus insulation.
2. Fault-to-ground current.
3. Stability.
4. Relaying.
5. Arcing grounds.
6. Double faults.
7. Lightning protection.
8. Inductive coordination.
9. Continuity of service.
10. Adaptability to interconnection.
11. Operating procedure.
12. Equipment cost.

1. Apparatus Insulation

Transmission line insulation is principally determined on the basis of keeping down flashovers due to lightning and there is practically no difference in this insulation regardless of method of grounding. On transformers having voltage ratings lower than 115 kv., both ends of the high-voltage winding are insulated according to the rated voltage of the transformer, hence there is no difference in transformer costs regardless of how the neutral is grounded. For rated circuit voltages, 115 kv. and above, a saving in the cost of the transformers can be effected by the use of transformers having the insulation graded to various extents. The maximum saving is reached on transformers for solidly grounded neutral service, particularly where the rated circuit voltage is

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high. Smaller savings are effected if the transformer is only partially graded as when reactance or resistance grounding is employed. No savings are available on oil circuit breakers except for 230-kv. service where under favorable conditions a special breaker with reduced insulation may be used for solidly grounded neutral service. While the level of insulation will be determined largely by lightning surges, undoubtedly switching surges and surges resulting from system disturbances contribute to insulation failures so that maintenance charges should be less over a long period with a grounding system giving fewer and smaller surges.

In cable systems, it is the practise of some companies when using multi-conductor cables to provide insulation between conductor and sheath in accordance with the type of grounding used, that is, the maximum insulation is used on isolated-neutral systems and the minimum on solidly-grounded systems. For the higher voltages, type *H* cable is now quite commonly used. On account of its fundamentally correct design, for any given voltage, type *H* cable with a total insulation equal to only the conductor insulation of belted cable is generally considered to have an equal factor of safety. However, the present practise is to use a slightly higher value of insulation than this in such cable.

2. Fault-to-Ground Current

The fault-to-ground current varies very widely, depending upon the type of ground connection used. The maximum current is obtained when the neutral is solidly grounded and the minimum current when a tuned reactance is used. In general, there are no cost differences for the different methods of grounding because equipment must be applied on the basis of the worst possible conditions, so that circuit breaker interrupting capacities are usually fixed by line-to-line short circuits, and this is also true of generator and transformer bracing. The heavier short-circuit currents cause greater and more widespread drops in system voltage, possibilities of system instability and greater damage to equipment at the point of fault with consequently increased time of outage. However, the practical development of the high-speed, high-voltage breaker and associated relay systems gives promise of relieving this situation.

3. Stability

The introduction of high-speed breakers and relays has made it possible to meet practically all stability problems under conditions of single-phase faults to ground. The fact, however, that high-speed clearing of faults to ground is sometimes necessary for the maintenance of stability with a solidly grounded neutral, should not be held as a reason for penalizing this method of grounding with the extra cost of high-speed breakers. Due to the fact that line-to-line faults are sufficiently numerous that they must be taken into account, high-speed breakers would be installed in any case, irrespective of the method of grounding. Where resistance grounding is used, assistance in maintaining stability is afforded by the power loss in the resistor (provided it is located at the sending end), but this assistance decreases with decreasing breaker and relay time. It is of no value for line-to-line faults but may be of considerable help in the case of a two-conductor-to-ground fault.

4. Relaying

Relaying for ground faults is simplest on a system grounded at one point because the residual currents flow only in one direction. On systems grounded through a tuned reactance, the fault current itself is practically zero although there is considerable current in the reactor itself, depending upon the extent of the system. The distribution of fault current, however, is not affected by the location of the fault. The relaying when necessary is accomplished by short-circuiting the tuned reactor and then relaying in the usual manner. The total current is then comparable to that obtained with a system grounded at one point. Relaying is extremely difficult on ungrounded neutral systems, as the charging current and its distribution change radically when the line connections are changed.

5. Arcing Grounds

By arcing grounds is meant the tendency for the creation of a high-frequency oscillation at the point of fault and building up the voltage to several times normal to ground on successive restriking. This condition is apparently caused by the action of the line capacity to ground in maintaining a direct-current potential between successive arcs.

6. Double Faults

Faults to ground on different phases may occur on any system, but since an excessive dynamic voltage to an upper limit of 1.73 times normal appears on two of the three conductors all over the system in the case of a single-phase ground on the systems grounded through high impedance, the possibility of a second fault appearing as a result of the first fault is increased.

7. Lightning Protection

Lightning arresters have been developed to the point where it is now possible to obtain a protective ratio of approximately 2.5 to 1, that is, the maximum voltage during surges can be limited to 2.5 times the normal dynamic voltage. However, the arrester must be applied on the basis of withstanding the maximum dynamic voltage which may exist from line to ground and in the case of ungrounded systems, or systems grounded through a high impedance, the arresters must be applied on the basis of withstanding 1.73 times normal phase-to-ground voltage without the flow of power current. This results in a decrease in the protective value of the arrester and an increase in its cost. The stress on the neutral end of the transformer winding due to a surge is higher when the neutral is grounded through a reactor than when solidly grounded or grounded through an ideal resistor. This requires that

the neutral end of the transformer and the reactor be correspondingly strengthened or some form of protective device, such as a lightning arrester or capacitor, be placed across the reactor.

8. Inductive Coordination

Under normal operating condition, power line phase currents and voltages are usually balanced as far as practicable. However, it is impossible to obtain an absolute balance of these factors. Any residual current or voltage, however small, will operate through the earth when the power system neutral is grounded. These residuals may be manifested in adjacent communication circuits as noise or troublesome induced voltage. When the system neutral is grounded, it is, therefore, necessary to maintain closer balances of both lines and equipment than would otherwise be necessary. It may be necessary also to reduce generated harmonics of the fundamental frequency or render them innocuous by means of suitable filters.

The magnitude of voltage induced into a communication circuit from a ground fault on a power circuit is dependent upon the coupling between the power and the communication circuit and the magnitude of the ground fault current. Under certain conditions there may be fundamental frequency voltage induced in the communication circuit from the unbalanced voltages on the power circuit at the time of fault. These are usually unimportant compared to the effects induced from the currents. Manifestations in the communication circuit are dependent both upon the magnitude and duration of the abnormal voltage. On grounded neutral systems large fault-to-ground currents are most likely to occur if the system is solidly grounded. The introduction of any impedance in the neutral lessens the magnitude of this current, and the higher the impedance naturally the lower the magnitude of the current. The use of high neutral impedance puts serious limitations on relaying, both as to selectivity and as to the total time required for clearing the fault. This, of course, may have adverse reaction from the inductive coordination standpoint, as would also any tendency to increase the frequency of occurrence of double faults.

If double faults occur on a system with isolated or high impedance neutral the ground fault current may be greater than the ground fault current for a single fault on a solidly-grounded neutral system. However, the probability of double faults occurring at such points as to lead to the largest induced voltages is less than the similar probability of a single fault on a solidly-grounded neutral circuit.

9. Continuity of Service

With any type of system grounding except ungrounded systems of small extent, or systems grounded through tuned reactance, a momentary flash to ground on one conductor requires that the circuit be isolated in order to clear the fault. This means that where continuity of service is essential a second circuit must be available to maintain the supply. However, even on systems where a flashover to ground is not accompanied by power-arc, a second circuit would probably be required to take care of faults due to causes other than flashovers.

10. Adaptability to Interconnection

Systems grounded solidly, through a resistance or through a reactance are usually adaptable to interconnection. A system grounded through a tuned reactance, however, cannot be tied in conductively with systems using another method of grounding, because the tuning is destroyed and arc suppression cannot be obtained. Ungrounded neutral systems are subject to the same difficulty, in that connection to a grounded system destroys their characteristic.

11. Operating Procedure

The operating procedure for solidly grounded, resistance grounded, and reactance grounded systems is the procedure which is well known in this country today, that is, automatic isolation of lines and switching in and out of lines at will. With the system grounded through tuned reactance, when lines are switched in and out of service, the taps on the coils must be changed so as to compensate for the charging current to ground. This need not be done for each change in transmission line set-up, but the compensation must be maintained within reasonable limits.

12. Equipment Cost

The solidly-grounded system requires no neutral impedance device and therefore effects a saving in this regard. In addition there may be a saving in apparatus insulation. In the case of 230-kv. transformers, the saving resulting from the use of graded insulation is considerable with smaller savings down to 115 kv. For lower voltages than this, there is no saving. The resistance and reactance grounded systems are probably next lowest in cost. In the case of high-voltage systems a partial grading may be possible with these methods of grounding with some saving in cost. The tuned reactor coil may under certain circumstances cost less than a resistor or reactor, but grading of the transformer insulation is not possible. The lightning arrester costs on the resistance, reactance, and ungrounded systems will be more than on a solidly-grounded system of equal voltage.

Summary of Grounding Practise

For the purpose of obtaining a cross-section of present day practise in grounding system neutrals a questionnaire was submitted to thirty-nine representative utilities throughout the States and thirty-two replies were received. A list of the participating companies is given at the end of the paper. The reference numbers used in the tables and figures do not correspond to this list.

The Committee deemed it desirable to analyze and

condense the data received rather than publish them in their entirety, and this has been done as far as possible in the form of tables and figures. In these by a "system" is meant a certain number of circuit miles conductively connected, that is, connected by metallic ties. One company, therefore, may operate several systems, and on the other hand a system may overlap two or more properties. This is necessary because in the problem of grounding the mileage of the conductively connected transmission lines is of importance and not the total mileage owned by any one operating company.

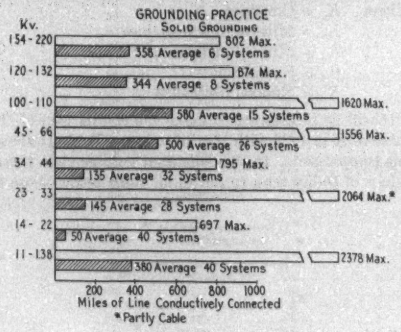

FIG. 1

METHODS OF GROUNDING

The methods of grounding adopted by the companies reporting are given in Figs. 1 to 4, which show the longest single system and the average length of all the systems. Where there are only two or three systems of one type, they have been reported in full, since an average would not be representative. In these charts the Committee accepted the reporting companies' classifications of their grounding methods, which may not agree with the definitions set up in the first part of this report.

The information contained in the figures has also been given in Table I, which shows the reported total mileage of transmission lines in the various voltage classes.

It will be noticed from the charts that 93 per cent of the reporting systems of 110 kv. and above are classified as solidly grounded, showing a marked preference for this type of grounding. One company reports solidly-grounded neutrals on 132- and 66-kv. systems except at three substations, where from 53 to 75 per cent of the transformer capacity is grounded. These three sub-

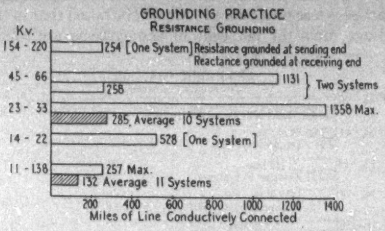

FIG. 2

stations are by definition reactance grounded, but because of the preponderance of solidly-grounded substations on the systems they are reported as solidly grounded. Another company grounds only one-third of the transformer capacity at one station, but the entire bank capacity at other grounding locations.

One company reports using a resistance at the gen-

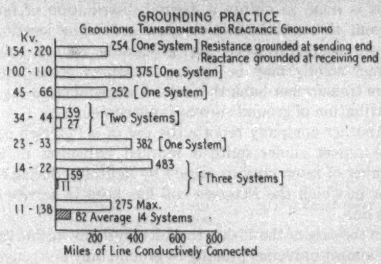

FIG. 3

erating end of its 220-kv. system and a reactance at the receiving end. This arrangement tends to decrease the angular swing between the two ends in case of a fault to ground and thus improves the stability of the system. Another company is using an impedance

TABLE I

System kv.	Miles reported	Solidly grounded		Resistance grounded		Reactance grounded		Ungrounded	
		Miles	Per cent	Miles	Per cent	Miles	Per cent	Miles	Per cent
154–220	2,402	2,148	89.4	127	5.3	127	5.3	None reported	
120–132	2,752	2,752	100	None reported		None reported		None reported	
100–110	9,075	8,700	96.0	None reported		375	4.0	None reported	
45–66	17,265	13,000	75.3	1389	8.0	252	1.5	2624	15.2
34–44	4,556	4,320	94.9	None reported		66	1.4	170	3.7
23–33	8,022	4,060	50.6	2850	35.5	382	4.8	730	9.1
14–22	3,796	2,000	52.7	528	13.9	553	14.6	715	18.8
11–13.8	8,991	5,320	59.2	1452	16.2	1148	12.8	1071	11.8
Total all systems	56,859	42,300	74.5	6346	11.1	2903	5.1	5310	9.3

grounding device in its most recent 220-kv. transformer installation. There appears to be a tendency in very high-voltage systems to limit the ground current by some kind of impedance—resistance, reactance or partial grounding of the total transformer capacity.

On systems of 66 kv. and below, the solid grounding practise is less pronounced. It will be noted that in this voltage range 51 per cent of the systems are solidly

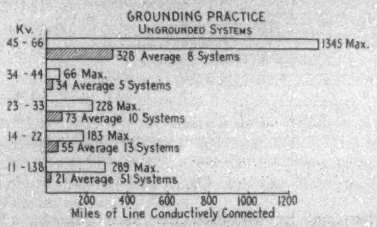

FIG. 4

grounded, 9 per cent are grounded through resistance, 8 per cent are grounded through either reactors or grounding transformers, 32 per cent have free neutrals.

Two companies report cases of reactance grounded substations in parallel with solidly-grounded substations. This is done to obtain a desired distribution of fault current through the two or more grounding locations. It will be readily seen that grounding a small transformer solidly may be the equivalent of grounding a large transformer bank through an impedance as far as distribution of ground current is concerned.

Another company reports the use of a Petersen coil. The report shows quite a marked reduction in the number of interruptions caused by lightning during the year in which the Petersen coil has been in service on this line.

In the case of the 11-kv. to 14-kv. systems we find that it is almost universal practise to ground only one generator on each bus section. Four companies report grounding the generator bus sections through grounding transformers, and two use grounding reactors.

It has been generally accepted as a result of experience that on ungrounded systems the possibility of obtaining dangerous voltages becomes more prevalent as mileage and voltage increase. This was universally recognized and led to different methods of grounding. In America solid grounding was resorted to in the majority of cases, and resistance was inserted in the neutral only where the fault current was likely to be excessive.

Regarding the value of resistance used no very definite practise was followed except to keep it high enough to achieve the desired result in limiting current and low enough to pass adequate current for relaying. For the same degree of protection it may be assumed that the grounding resistance should vary inversely as the charging current. A formula based on this premise was published in the First Committee Report, *viz.*:

$$K = (L_a + 5 + 25\,L_b) \times f \times R$$

Where K = Constant
L_a = Miles of overhead lines
L_b = Miles of underground cable
f = Frequency
R = Neutral resistance

It will be interesting to make the comparison of the value of K in 1923 with that of today. Table II shows this comparison. In the generated voltage class the K values of today tend to fall in all cases below those of

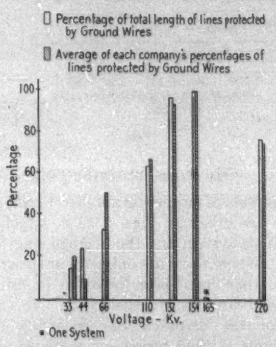

FIG. 5

1923, while in the higher voltages they have remained about the same except as regards the maximum. It is significant that this maximum value of 3.4×10^6 occurs on a system in which the company is contemplating reducing the value of neutral resistance. Even so, it is well within the 6.2×10^6 value, which according to the first report, Petersen states will prevent arcing grounds.

TABLE II

	1923 Values				1930 Values			
		Value of $K/10^6$				Value of $K/10^6$		
Note	No. of systems	Min.	Average	Max.	No. of systems	Min.	Average	Max.
A	8	0.008	1.04	3.55	13	0.0069	0.116	0.93
B	7	0.022	0.52	2.00	12	0.0134	0.495	3.4

Note *A* = Systems at generated voltage.
Note *B* = Systems above generated voltage

Ground Wire Protection

Twenty-nine companies reported their practise as regards the use of ground wires, and this information is given in Table III.

TABLE III—GROUND WIRE PROTECTION

Ref. No.	Volts	Total miles	Ground wires: Miles protected	Ground wires: No.	Ground wires: Diam. inches	Ground wires: Mat.	% of lines with G. W.	Remarks
1	132	278	278	1	1/2	*C W*	100	*C W* = Copper-weld
2	132	218	218	1	1/2	*C W*	100	
	66	22	22	1	3/8	*C W*	100	
	66	143	143	2	5/16	*G S*	100	*G S* = Galvanized steel
3	66	258	258	1	7/16	*C W*	100	
4	110	375	0				0	
	66	252	0				0	
	33	382	0				0	
5	66	294	294	1	11/32	*C W*	100	
6	110	26	0				0	
	66	130	0				0	
7	154	97	97	1	3/8	*G S*	100	
	110	418	418	1	3/8	*G S*	100	
	66	217	150	1	3/8	*G S*	69	
	44	605	60	1	3/8	*G S*	10	
8	44	302	0				0	
9	110	1,190	734	1 & 2	3/8	*G S*	62	
	66	370	292	1 & 2	3/8	*G S*	79	
	44	865	0				0	
	38	495	0				0	
10	110	122	122	2	1/2	*G S*	100	
	44	379	0				0	
	33	19	0				0	
11	154	88	88	2	7/16	*G S*	100	
	110	1,620	1,620	2	1/2	*G S*	100	
	44	1,781	949	1 & 2	3/8	*G S*	53	
12	66	1,177	94	1	1/4	*G S*	8	
	33	359	40	1	1/4	*G S*	11	Located below conductors
13	110	458	449	2	3/8	*C W*	98	
	66	1.130	845	1 & 2	3/8	*C W*	75	
	33	194	0				0	
14	120	560	560	1	7/16	*C u*	100	*C u* = copper
	40	8		1	5/16	*C u*		3,000 ft. from each end
15	132	116	116	1 & 2	7/16	*C u*	100	
	110	287	287	1	5/8	*G S*	100	
	66	197	197	1	5/16	*C W*	100	
	33	497	58	1	5/16	*G S*	12	
16	66	100	100	1			100	
17	110	282	282	1	3/8	*G S*	100	
	66	1,254	1,254	1	3/8	*G S*	100	
	33	228	228	1	3/8	*G S*	100	
18	132	397	397	2	3/8	*C W*	100	
19	220	352	352	2	1/2	*A 1*	100	*A 1* = Aluminum
	132	165	165	2	5/8	*C W*	100	
	66	1,143	168	2	9/16	*C W*	15	
	33	369	58	1	3/8	*G S*	16	
20	66	433	433	1	3/8	*A 1*	100	
	33	140	140	1	3/8	*G S*	100	
21	132	145	66	2	3/8	*G S*	45	
	66	1,935	1,053	1	3/8	*G S*	55	
	33	100	24	1	3/8	*G S*	24	
22	110	383	0				0	
	66	830	0				0	
	44	63	0				0	
	33	97	0				0	
23	110	628	377	2	5/16	*G S*	60	
	66	566	0				0	
	33	150	0				0	
24	100	810	810	2	3/8	*G S*	100	
	66	1,662	0				0	
25	220	254	254	2	1/2	*G S*	100	
	110	481	481	2	7/16	*G S*	100	
	66	694	492			*G S*	71	
	33	129	30				23	
26	220	420	0				0	
	165	398	0				0	
	110	1,968	348	1	3/8	*G S*	18	
	60	2,602	0				0	
	44	168	0				0	
27	220	802	802			*G S*	100	
	66	1,838	0				0	
	33	111	0				0	
29	132	874	874	1 & 2	7/16	*A 1*	100	
	33	1.379	0				0	

In order to show the over-all practise in the use of ground wires for the protection of transmission lines in a more concise manner, Fig. 5 has been prepared. This figure has been worked up on two percentage bases. In the first case the total length of all lines reported in a given voltage class was taken as 100 per cent and the mileage of lines protected by ground wires was drawn as a percentage of that total. This gives the practise of the country as a whole. In the second case, the total mileage in a given voltage class of each individual company was taken as 100 per cent, and the mileage of protected lines was expressed as a percentage of this total (eighth column in Table III). The average of these percentages in any voltage class is the value given in the chart. This tends to show the opinion of transmission engineers better than the first value perhaps, because the short systems are given the same weight as the long systems.

Tower Footing Resistance

In protecting a line against direct strokes of lightning the performance of the ground wire depends on its receiving the direct stroke and conducting the energy to ground without causing a flashover to the line conductors. This depends upon the ability of the ground wires to remain at a low potential, and this again is dependent to a large extent on the value of the tower footing resistance.

From the replies to the question regarding tower footing resistance it is apparent that there is very little accurate data available, and that at this time no average figures of any value can be given, either on the basis of general territory or nature of soil. One company reports a range of 1 ohm to 58 ohms over a very restricted territory. The extremes reported by the seven companies replying to this question are 0.05 ohm and 300 ohms and the average 22.5 ohms.

Simultaneous Faults at Different Locations

In reply to the question: "Do you have evidence of simultaneous faults at different locations?" twenty-five replies were received. Of these sixteen companies reported no evidence; two reported less than 1 per cent; three reported 1 per cent; one reported 5 per cent, and three gave replies as noted below. In all cases the evidence was taken from relay operations or from automatic oscillographs.

One company had trouble on a 300-mile 66-kv. system until it was grounded. The neutral was found to shift to 80 per cent of the line-to-ground voltage at the middle of the section when one conductor was grounded.

One company reports no evidence of simultaneous faults on its 132-kv. and 220-kv. systems. One 66-kv. and two 22-kv. systems show 1 per cent or less; one 66-kv. system 15 per cent, eight 26-kv. and three 13-kv. systems (mostly overhead) show 33 per cent, as indicated by relay operations. The last two percentages include double faults at the same physical location as well as double faults at different locations, the former very largely predominating.

One company operating an ungrounded system reports conclusive evidence of arcing grounds without stating any figure for this type of fault.

Types of Faults

There are four possible types of faults on a system:

a. One conductor to ground.
b. Two conductors to ground.
c. Phase-to-phase short circuit.
d. Three-phase short circuit.

In reply to the question regarding the division of the faults into these four classes twenty-seven replies were received. These are given in Table IV as a percentage of the total number of faults. It must be borne in mind that these figures include both overhead and underground lines.

TABLE IV

	One cond. to ground	Two cond. to ground	Ph. to Ph. sh. circuit	Three-ph. sh. circuit
Maximum	97	45	54	36
Average	69	14	11	6
Minimum	30	0	0	0

Sufficiency of Ground Current

Regarding the sufficiency of ground current for relaying purposes, all companies except two report sufficient current for relaying purposes. These two exceptions encounter trouble at times of light load in the matter of selectivity.

The loss of synchronous load caused by ground faults is rare. Seven companies report having this occasionally. One company is making changes in its relay equipment to rectify this condition.

Recent or Contemplated Changes in Method of Grounding

In reply to this question, some companies have described certain changes in their system layouts which they are contemplating or have recently completed to correct faults which they have been experiencing. The changes which seem pertinent to the subject under report are given below.

Company No. 1. The neutral resistor on the 11-kv. underground system has been reduced from three ohms to one and one-half ohms to prevent occurrence of simultaneous faults on this system.

Company No. 7. This company formerly operated certain lines ungrounded, but in order to eliminate arcing grounds and simultaneous faults, it is now grounding everything solidly including the 11-kv. circuits.

Companies Nos. 9 and 11. The tendency of these two companies has been to reduce the number of grounding points on their systems so as to decrease the magnitude

of the ground currents. This is to reduce the duty on equipment and the possibility of telephone interference.

Company No. 12. The 33-kv. lines of this company were originally fed through delta-connected transformers. These have been replaced by auto transformers with grounded neutrals. This change has resulted in eliminating about 60 per cent of the trouble on these lines.

Company No. 18. The resistance in the 25-kv. neutral has been decreased from 28.8 ohms to 19 ohms and at one of the stations to 14 ohms. This was made possible by the installation of higher capacity breakers.

Company No. 19. One of the original 66-kv. lines of about 30 circuit miles was delta connected and ungrounded. In extending this system it was necessary to use underground cable, which was not suitable for ungrounded operation, and this fact, together with the advantages of the grounded system in eliminating arcing grounds and limiting transient voltages, led to a decision to change the entire system to star connection with the neutral grounded.

The original 26-kv. systems were solidly grounded through the wye-connected transformers at the supply end, but resistors of 75 ohms have been installed as an inductive coordination measure. However, secondary advantages have been obtained as a result of lower fault currents, which produce less general disturbance to the system and are less destructive at the point of fault and to apparatus. The relay system is somewhat more expensive and complicated but is proving quite reliable in operation.

The only other change in method of grounding was the adoption of the policy of grounding one piece of equipment on each side of certain bus sectionalizing breakers. The object of this change is to permit retention of a fixed neutral in the good section of a bus should sectionalizing breaker open. It is expected that this will result in fewer insulator failures on the operating bus section at times of fault, but to date little operating experience has been secured.

Company No. 23. Grounding transformers have been installed recently on a formerly ungrounded 60-kv. delta-connected system to relieve telephone interference and arcing grounds resulting from line-to-ground faults. Results have been satisfactory.

Company No. 25. In the generating stations, the neutral grounding resistances are being reduced as the growth of the low-tension distribution systems require. The value of resistance is selected on the basis of minimum allowable relay currents, and the maximum allowable resistance which will eliminate arcing grounds.

Company No. 29. This company's 33-kv. system was formerly solidly grounded, but now five points are grounded through 50 ohms resistance and the other points are left ungrounded. This step was taken to eliminate telephone interference.

Company No. 31 This company had the 13.2-kv. system neutral grounded through resistors which would limit the ground current to 1,000 amperes. The method of operation was changed so that the sources of generation were segregated and synchronized at the load. This permitted short-circuiting the neutral resistors and going over to solid grounding, the only tie between generator bus sections being at remote locations. Only one generator neutral on any bus section is grounded. The new method of operation has resulted in a greatly reduced number of simultaneous faults.

Participating Companies

Alabama Power Company and Associates.
Buffalo, Niagara and Eastern Power Corporation.
Cleveland Electric Illuminating Company.
Commonwealth Edison Company.
Connecticut Light and Power Company.
Consolidated Gas, Electric Light and Power Company of Baltimore.
Detroit Edison Company.
Duquesne Light Company.
Georgia Power Company.
Houston Lighting and Power Company.
Memphis Power and Light Company.
Metropolitan Edison Company and Associates.
Mississippi Power Company.
Montana Power Company.
New York Edison Company and Associates.
New England Power Association.
Northern States Power Company.
Ohio Edison Company and Associates.
Oklahoma Gas and Electric Company.
Pacific Gas and Electric Company and Associates.
Pennsylvania Power and Light Company.
Philadelphia Electric Company.
Public Service Electric and Gas Company.
Public Service Company of Northern Illinois and Associates.
Puget Sound Power and Light Company.
Southern California Edison Company.
South Carolina Power Company.
Tennessee Electric Power Company.
Texas Power and Light Company.
Union Electric Light and Power Company and Associates.
Washington Water Power Company.
West Penn Power Company.

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Discussion

E. C. Stone: A comparison of the first report on grounding published about eight years ago with the present report shows no radical changes in practise, but indicates some interesting trends. This report covers 57,000 miles of transmission line as compared with 31,000 miles covered in the previous report.

In the following table is a comparative summary of the mileages of lines grounded by the various methods:

	Present report		1923 report	
Method of grounding	Mileage	Per cent of total	Mileage	Per cent of total
Ungrounded	5,310	9.3	6,385	20.4
Solidly grounded	42,300	74.5	18,375	58.5
Resistance grounded	6,346	11.1	6,448	20.8
Reactance grounded	2,903	5.1	100	0.3
Total	56,859	100.0	31,308	100.0

It is interesting to note that with the exception of approximately 5 per cent of the total mileage which is reactance grounded, all of the increase in mileage covered by the second report is solidly grounded.

The present report clearly indicates a better understanding of the principles of system neutral grounding, of the fields for application of the various methods and of the results to be expected. Thus it has been found that the ground current in cases of a system fault must be large enough to prevent arcing grounds and to provide for positive selective relay operation. The points at which the neutral is grounded must be sufficient in number and so located to permit of adequate current flow wherever on the system a fault may be located. On the other hand the neutral ground current must be limited to values which will not cause undue damage at the point of fault and which will not produce system instability. The limit is of course a function of the maximum time a fault current is permitted to remain on the system. Larger currents are permissible where high-speed circuit breaker tripping is employed.

Effects on communication circuits are a function both of the wave form and the magnitude of the fault current.

It seems to the writer that the present report should definitely differentiate between systems operated at generated voltage and those operated at higher than generated voltage, just as the first report did. Analysis of the data presented in the present report shows that of the 36,000 miles of line reported on as operating at 34,000 volts or above, approximately 34,000 miles or 90 per cent is solidly grounded, while for the 21,000 miles operated below 34,000 volts only 55 per cent is solidly grounded. There are definite reasons for this difference in practise. On the high-voltage lines solid grounding gives greater benefits from the insulation standpoint while at the same time the ground currents in relation to system capacity are relatively low and these systems are in major part outdoor overhead systems so that the physical damage from ground fault currents generally is reduced. On the other hand on the lower voltage systems the fault ground currents are relatively large and the insulation benefits from solid grounding comparatively small. These systems consist largely of underground cable and indoor substations, so that damage from large fault ground current becomes more serious.

As stated above the present report shows lower average impedance in the grounding circuit for systems operated at generated voltage. It is believed that further analysis of the data on these systems will show that while the over-all grounded neutral impedance may be lower than before, ground fault currents are still limited in value by the sectionalizing of the generating station buses so that a larger element of line impedance than before is introduced into the ground fault circuit.

The problem of double simultaneous faults at different parts of a system is of particular importance because of the difficulties in many cases of providing relaying which will isolate such faults without interruption to service. In an ungrounded system simultaneous faults are naturally charged to arcing grounds. In a properly grounded system arcing grounds should be impossible so that simultaneous faults may be assumed to be the result of the rise in voltage on two-phase when the third-phase is grounded. It seems fair to raise the question, particularly on moderate voltage systems, whether the relatively small lowering of fault voltages by reduction in value of ground impedance will correct the situation and whether the insulation required for normal operation should not be sufficient to withstand these excess voltages resulting from ground currents for the short periods of time required for relay operation. The conclusion is drawn that the solution is to be found not in reducing the impedance of the neutral ground circuit but rather in raising the system insulation to a level which will permit it to withstand the overvoltages prescribed by our standard A. I. E. E. tests.

Another subject discussed in the report which is of major importance is the impedance of grounds which are used in connection with lightning protection. The evidence of operating experience seems to point very clearly to the fact that much lightning protective apparatus has had its effectiveness much reduced or destroyed by the unsatisfactory grounds used in connection with it. With the latest types of lightning arrester now available it is beginning to look as though the neck of the bottle in lightning protection lay in the impedance of the grounding circuits of the protective equipment.

The problem of system grounding is only one phase of the greater general problem of reducing insulation failures which is facing power engineers today. We may be optimistic on this subject, however, because of the extensive research in progress and the definite advances being made.

C. A. Powel: Mr. Stone's comparison of the present report with the first report is particularly valuable and raises some points which might well be taken into consideration in future reports. It is hoped that the next report will include comparisons based on more precise methods of defining the degree of grounding so as to facilitate the classification of the various systems.

Reactance of Transmission Lines With Ground Return

BY J. E. CLEM[1]

Member, A. I. E. E.

Synopsis.—*In this paper there is presented an analysis of the circuit consisting of any number of line conductors in parallel with a return through the earth and any number of ground wires in parallel. In the development of this analysis certain assumptions have been made which tests indicate do not detract at all from the engineering usefulness of the result but which simplify the formulas very much. The general formulas resulting from this analysis are condensed in Table A-1.*

For the application of this method to the calculation of ground faults it is necessary to know what value of earth conductivity applies. Considerable data in regard to this for various localities have been collected and are given in Table C-2. For convenience in making the calculations curves are given for various values of the earth conductivity over the range as indicated by test and experience. These curves are given as Figs. C-2 and C-3. Occasionally preliminary calculations are made in which very approximate reactance values are used. In such cases a line reactance of 0.8 ohms per mi. is used, and a zero phase sequence reactance of 2.6 ohms per mi. could be used with the same degree of accuracy. With one ground wire the zero phase sequence reactance would be about 2.0 ohms per mi. and with two ground wires about 1.6 ohms per mi.

In the application of this method it should be noted that the values of a total impedance should be used and that it is necessary to know the contact resistance between conductor and earth at each end of the circuit. When a fault occurs somewhere along the line the contact resistance may be very high especially if no ground wire is used. When a ground wire is used the effective fault resistance will be relatively low even though the contact resistance of the tower footing to ground at that particular tower is high, since the fault current can pass to earth at several towers.

Introduction

As electric power systems continue to increase in size and as demands for more nearly perfect service become more insistent, increasing attention must be given to the provision of arrangements whereby faulty portions of the system may be promptly isolated. Predictions concerning the stability of systems are largely based upon a knowledge of the flow of synchronizing power between machines during fault conditions, and the application of suitable relays is determined largely upon knowledge of current flow during fault conditions. The faults which must be considered are generally those involving ground. Calculations involving unbalanced faults are greatly simplified by the use of the symmetrical component method introduced by Fortescue[2] and applied by MacKerras[3] and others. The successful use of this method requires a more accurate knowledge of the zero phase sequence impedance of transmission lines than has heretofore been available.

The zero phase sequence reactance of transmission lines is the reactance of the three-line conductors in parallel with ground return. In 1927 the writer developed equations for the impedance of a circuit consisting of any number of line wires in parallel and a return path through ground or through ground and any number of ground wires in parallel. The application of these equations is contingent upon a knowledge of the impedance of a single conductor with ground return. The impedance of such a circuit varies with several factors, chief of which is the distribution of the current in the return path below the surface of the earth. As this is largely dependent upon soil conditions, it will obviously vary with the locality and terrain. Hence, field tests have been made to obtain experimental data for use in predicting ground impedance, and to check the theoretical analysis previously prepared. This paper presents not only the analytical study, but also its confirmation by field test data, plus a suggested method of predicting the impedance of the fundamental circuit of conductor with ground return.

A. Circuit Analysis

The analysis of the circuit is very simple although it involves such factors as the self inductance of the earth path and the mutual inductance between a wire and the earth which have usually been considered as fictitious quantities. However, by proper manipulation the equations can be made to take on a real and definite form. For the analysis as given here certain simplifying assumptions are made. It is assumed that the line wires are transposed, so that the same current flows in each of the conductors. It is assumed also that the spread of the current in the earth is such that its mutual effect on all the line conductors or ground wires is the same within the usual horizontal spread of line conductors. The tests herein reported show that the error from this source is small and these assumptions, therefore, do not detract from the engineering usefulness of the results.

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In Fig. A-1 let

m = number of line wires.
n = number of ground wires.
g = subscript indicating ground.
o = subscript indicating ground wire.
w = subscript indicating line wire.
t = subscript indicating auxiliary test wire.
L = $j\,w\,\mathcal{L}$ self inductance, as indicated by subscripts.
M = $j\,w\,\mathfrak{M}$ mutual inductance, as indicated by subscripts.
r = resistance, as indicated by subscripts.
E_{mn} = circuit voltage.
E_t = voltage induced on test wire.
I = current as indicated by subscripts—lack of subscript indicates total current.

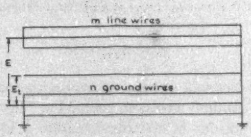

FIG. A-1

Impedance of Circuit. The impedance of the circuit in this analysis is defined as the ratio of the applied voltage, E_{mn}, to the total current I. The current flows from the generator out along the line wires to the point of fault, and flows back to the generator through the ground and ground wire in parallel. The effect of the current in the ground is simulated by a pseudo conductor embedded in the earth as shown in Fig. A-2.

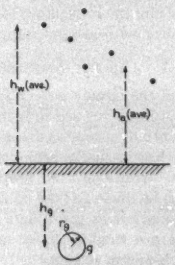

FIG. A-2

This artificial earth conductor is assumed to have a self inductance of its own and to have mutual reactance with the real conductors of the circuit. The depth of this earth conductor below the surface of the earth and its effective diameter are not of interest at the present time.

Starting from the generator the voltage of the circuit is made up of the voltage drop from the generator through the ground to the point of fault and the voltage drop along one of the line conductors from the point of fault back to the generator. The voltage drop in any conductor of a circuit is made up of the voltage drop produced by the current in the conductor and its own self inductance and the voltage drop induced in the conductor by the currents in other conductors through the mutual inductance. Writing this in the form of an equation we get:

$$E_{mn} = I_g\,(r_g + L_g) + I_o\,M_{og} - I\,M_{wg} + \frac{I}{m}\,(r_w + L_w) + \frac{m-1}{m}\,I\,M_{ww} - I_o\,M_{ow} - I_g\,M_{wg} \quad (1)$$

But $I_g = I - I_o$ so that this expression becomes

$$E_{mn} = I\,(r_w + L_w + r_g + L_g - 2\,M_{wg}) - \frac{m-1}{m}\,I\,(r_w + L_w - M_{ww}) - I_o\,(r_g + L_g - M_{wg} - M_{og} + M_{ow}) \quad (2)$$

In this expression the self inductance involving the earth is usually said to be impossible of separate determination. If it is combined with the mutual inductance between the earth and the line wire, or ground wire, reactance quantities are obtained which are real and can be determined by measurement. The self inductance of a conductor is a characteristic of the conductor itself and does not depend in any way upon other conductors in the neighborhood. For mutual inductance another conductor is required. If the current returns through the other conductor the voltage drop in the first conductor is $I\,(L - M)$. This quantity $(L - M)$ is that which is usually known as reactance. It can usually be determined by test or calculations either for a one way conductor of a circuit alone or for both in series.

If $(r_g + L_g)$ is combined with M_{wg} a measurable reactance results and the introduction of $(r_o + L_o) - (r_o + L_o)$ to combine with $-M_{og} + M_{ow}$ will give measureable reactances; or if $(r_g + L_g)$ is combined with M_{og} another measurable reactance results and the introduction of $(r_w + L_w) - (r_w + L_w)$ will combined with $-M_{wg} + M_{ow}$ to give other measurable reactances. In the form of an equation this can be expressed by

$$r_g + L_g\,M_{wg}\,M_{og} + M_{ow} = \frac{r_g + L_g}{2} - M_{wg} + \frac{r_w + L_w}{2} + \frac{r_o + L_o}{2} - M_{og} + \frac{r_g + L_g}{2} - \frac{r_o + L_o}{2} + M_{ow} - \frac{r_w + L_w}{2} = \frac{Z_{wg} + Z_{og} - Z_{ow}}{2} \quad (3)$$

Let

$$r_w + L_w - 2\,M_{wg} + L_g + r_g = Z_{wg}$$

$$r_w + L_w - M_{ww} = \frac{Z_{ww}}{2}$$

When these expressions together with that derived in (3) are introduced into equation (2) it becomes:

$$E_{mn} = I\,Z_{wg} - I_o\,\frac{Z_{wg} + Z_{og} - Z_{ow}}{2} - \frac{m-1}{2\,m}\,I\,Z_{ww} \tag{4}$$

Let $$M_{ow} = \frac{Z_{wg} + Z_{og} - Z_{ow}}{2} \tag{5}$$

and then the total circuit impedance is given by

$$\frac{E_{mn}}{I} = Z_{mn} = Z_{wg} - \frac{I_o}{I}\,M_{ow} - \frac{m-1}{2\,m}\,Z_{ww} \tag{6}$$

The indulgence of the reader is asked in keeping the terminology of M as it is defined in (5), as a form of mutual impedance, and appears in (6) distinct from that appearing in equation (1). This is an easy matter because the meaning of M as defined by (5) is the only one used in the final equations and in the calculations. The other meaning of M, which is the mutual inductance between two conductors, is only used in the course of the development.

Current in Ground Wires. The over-all voltage drop from the generator through the ground to the point of fault outside of the earth and along the ground wire to the point of fault must be equal so that

$$I_g = (r_g + L_g) - I\,M_{wg} + I_o\,M_{og}$$

$$= \frac{I_o}{n}\,(r_o + L_o) + \frac{n-1}{n}\,I_o\,M_{oo} - I\,M_{ow} + I_g\,M_{og} \tag{7}$$

As before $I_g = I - I_o$ so that

$$I\,(r_g + L_g - M_{wg} - M_{og} + M_{ow})$$
$$= I_o\,(r_o + L_o + r_g + L_g - 2\,M_{og})$$
$$- \frac{n-1}{n}\,I_o\,(r_o + L_o - M_{oo}) \tag{8}$$

From which in the same manner as before and letting

$$r_o + L_o - M_{oo} = \frac{Z_{oo}}{2}$$

$$I\,\frac{(Z_{wg} + Z_{og} - Z_{ow})}{2} = I_o\,Z_{og} - \frac{n-1}{2\,n}\,I_o\,Z_{oo} \tag{9}$$

which gives

$$\frac{I_o}{I} = \frac{M_{ow}}{Z_{og} - \dfrac{n-1}{2\,n}\,Z_{oo}} \tag{10}$$

Voltage Induced on Test Wire. In Fig. A-1 the voltage to ground E_t which can be measured on the test wire must be equal to the voltage drop along the ground plus the voltage induced along the test wire. This test wire was introduced to obtain additional data for checking the analysis and it can also be considered to simulate a communication circuit on the same supports as the power line.

$$E_t = I_g\,(r_g + L_g) + I_o\,M_{og} - I\,M_{wg}$$
$$+ I\,M_{wt} - I_o\,M_{to} - I_g\,M_{tg}$$

As before $I_g = I - I_o$ so that

$$E_t = I\,(r_g + L_g - M_{wg} - M_{tg} + M_{wt})$$
$$- I_o\,(r_g + L_g - M_{og} - M_{tg} + M_{ot}) \tag{12}$$

Which expression reduces to

$$E_t = I\,M_{wt} - I_o\,M_{ot} \tag{13}$$

In Table A-1 the equations which apply to 1, 2, or 3 line wires in parallel and 0, 1, 2, 3 ground wires are given. Numerical subscripts refer to number of line wires and number of ground wires respectively, and in the order given. Thus $Z_{3,2}$ means the impedance of the circuit with ground return when three line wires are in parallel and there are two ground wires. The impedances then are defined as follows:

I_o = total return current in ground wires.

I = total fault current.

Z_{wg} = average impedance, line wire with ground return.

Z_{og} = average impedance, ground wire with ground return.

Z_{tg} = average impedance, auxiliary wire with ground return.

Z_{ww} = average impedance, line wire with line wire return.

Z_{wo} = average impedance, line wire with ground wire return.

Z_{oo} = average impedance, ground wire with ground wire return.

Z_{tw} = average impedance, auxiliary wire with line wire return.

Z_{to} = average impedance, auxiliary wire with ground wire return.

The zero phase sequence impedance enters into the calculations of ground fault currents as a quantity which operates on the zero phase sequence current. This current is equal and in the same direction in all three line wires and is one-third of the ground fault current. Accordingly the zero phase sequence impedance of a transmission line is the quantity obtained by multiplying by 3 the value of impedance given in column 3 of Table A-1 depending upon the number of round wires. This is indicated in column 4 of Table A-1.

B. Tests

Tests have been made on four power systems to check this analysis. Design data on these systems are given in Table B-8.

On three of the systems tests were made before and after a ground wire was installed; and on the fourth, a line was tested which had ground wires over part of its length. The test data are given in Tables B-3 to

TABLE A-1—IMPEDANCE OF CIRCUIT WITH GROUND RETURN CURRENT IN GROUND WIRE RETURN

No. gd. wires	1 One line wire	2 Two line wires	3 Three line wires	4 Zero phase sequence impedance
0	$Z_{10} = Z_{wg}$	$Z_{20} = Z_{wg} - \frac{Z_{ww}}{4}$	$Z_{30} = Z_{wg} - \frac{Z_{ww}}{3}$	$3 \times Z_{30}$
1	$Z_{11} = Z_{wg} - \frac{I_o'}{I} M_{ow}$	$Z_{21} = Z_{11} - \frac{Z_{ww}}{4}$	$Z_{31} = Z_{11} - \frac{Z_{ww}}{3}$	$3 \times Z_{31}$
	$\frac{I_o'}{I} = \frac{M_{ow}}{Z_{og}}$	Same as Col. 1	Same as Col. 1	.....
2	$Z_{12} = Z_{wg} - \frac{I_o''}{I} M_{ow}$	$Z_{22} = Z_{12} - \frac{Z_{ww}}{4}$	$Z_{32} = Z_{12} - \frac{Z_{ww}}{3}$	$3 \times Z_{32}$
	$\frac{I_o''}{I} = \frac{M_{ow}}{Z_{og} - \frac{Z_{oo}}{4}}$	Same as Col. 1	Same as Col. 1	.....
3	$Z_{13} = Z_{wg} - \frac{I_o''}{I} M_{ow}$	$Z_{23} = Z_{13} - \frac{Z_{ww}}{4}$	$Z_{33} = Z_{13} - \frac{Z_{ww}}{3}$	$3 \times Z_{33}$
	$\frac{I_o''}{I} = \frac{M_{ow}}{Z_{og} - \frac{Z_{oo}}{3}}$	Same as Col. 1	Same as Col. 1	.....

B-7. The tests are as follows. (See Fig. A-1)

Test No. 1 One wire out—ground return
Test No. 2 Two wires out—ground return
Test No. 3 Three wires out—ground return
Test No. 4 One wire out—ground and one wire return
Test No. 5 One wire out—ground and two wires return
Test No. 5A Two wires out—ground and one wire return
Test No. 6 One wire out—one wire return
Test No. 7 One wire out—two wires return

In tests 4-5-5A one or more line wires were used as a ground wire.

In Table B-1 there is given a comparison of the calculated and tested results for tests made before the installation of the ground wire and in Table B-2 the same comparison is given for the tests made after the installation of the ground wire.

For Table B-1 the known values are considered to be the impedance of one conductor with ground return, indicated as Z_{wg}, and obtained from tests 1-2-3; and also the impedance of one conductor with line wire return, indicated as Z_{ww}, and obtained from tests 6-7.

In Table B-2 the known values are Z_{wg}, obtained from the tests without the ground wire, and Z_{ww} obtained from tests 6-7. Similar known impedances with the ground wire Z_{og}, Z_{oo}, and Z_{ow} are found by calculation to obtain Z_{oo} and Z_{ow}, and by a combination of calculation and reference to the tested values of Z_{wg} to obtain Z_{og}.

For the calculation of those tests when one of the line wires is in parallel with the real ground wires Z_{og}, Z_{oo}, and Z_{ow} must be modified. The modified values as used in the calculation are not given in the tabulation but may be obtained easily. For instance, if there was one real ground wire (Z_{og}) and one line wire used as a ground wire (Z_{wg}) then the effective ground wire impedance (Z_{og}') used in the calculation would be

$$Z_{og}' = \frac{Z_{og} + Z_{wg}}{2}$$

and the other necessary quantities are obtained in the same manner.

During the tests after the ground wire was installed on the Turner-Logan line a lineman was stationed on one of the towers near the middle of the line to read the current in the ground wire. The results indicated that the current in the ground wire was the same at the middle of the line as at the end. Data are given in Table B-9. In tests 4, 5, and 5-A one or more of the line wires used as an additional ground wire was in parallel with the actual ground wire so that in Table B-5 the ground wire current given is the sum total, while in Table B-9 the current in the actual ground wire itself is recorded.

C. Predetermination of Ground Return Impedance

The main problem in the calculation of ground fault currents and the division of the return current between the ground wires and ground is the predetermination of the impedance of the fundamental circuit consisting of the conductor with ground return.

It seems that for many years it was customary to consider the earth as a perfectly conducting plane. All the reactance was assigned to the conductor and the corresponding distance to the "neutral plane" was

TABLE B-1—COMPARISON OF TESTS AND CALCULATIONS
Tests Made Before Ground Wire Installed

Test		Pa. Pr. & Lt. Co. 220 Kv. Wallenpaupack—Siegfried	Appa. Elec. Pr. Co. 132 Kv. Turner—Logan	Alabama Pr. Co. 110 Kv. North Auburn—Newann	Consumers Power Co. 140 Kv. Saginaw—Flint
1-2-3	By test, Z_{wg} =	17.00 + j 95.53	14.00 + j 57.55	25.55 + j 120.3	16.42 + j 60.13
6-7	By test, Z_{ww} =	16.2 + j 107.0	22.29 + j 66.36	30.91 + j 114.7	26.65 + j 73.8
4	Calculated, I_e =	44.7	41.5	51.7	76.4
	Tested =	44.9	41.2	51.6	74.3
	Calculated, Z =	13.08 + j 76.75	12.84 + j 47.45	20.33 + j 87.76	15.27 + j 51.39
	Tested =	12.84 + j 77.88	14.14 + j 47.40	22.20 + j 86.50	16.9 + j 51.6
	Calculated, E_t =	23.78	14.37	30.80	
	Tested =	23.75	14.35	30.70	
5	Calculated, I_e =	61.4	58.7	68.3	55.9
	Tested =	61.9	58.2	66.5	53.4
	Calculated, Z =	11.57 + j 69.80	12.62 + j 43.25	18.65 + j 77.30	14.66 + j 46.98
	Tested =	10.86 + j 69.48	13.83 + j 43.23	19.65 + j 77.00	16.51 + j 47.3
5A	Calculated, I_e =	44.7	41.5	51.7	76.4
	Tested =	44.9	41.3	51.4	74.8
	Calculated, Z =	9.03 + j 50.50	7.27 + j 30.96	12.60 + j 59.08	8.61 + 32.94
	Tested =	8.93 + j 49.70	8.71 + j 30.77	15.23 + j 57.80	9.43 + 33.3

Calculations are made on the basis of the data obtained in Tests 1-2-3 and Tests 6-7.

TABLE B-2—COMPARISON OF TESTS AND CALCULATIONS
Tests Made After Ground Wire Installed

Test No.		Pa. Pr. & Lt. Co. 220 Kv. Wallenpaupack	Appa. Elec. Pr. Co. 132 Kv. Turner—Logan	Alabama Pr. Co. 110 Kv. Martin Dam—North Auburn	Consumers Pr. Co. 140 Kv. Saginaw—Flint
	By test and calc. Z_{wg} =	17.09 + 95.53	14.00 + j 57.55	14.88 + j 45.30	22.2 + j 60.13
	By test and calc. Z_{ww} =	16.2 + j 107.0	22.11 + j 66.78	12.76 + j 45.32	31.6 + j 73.2
	" " " " Z_{og} =	43.78 + j 102.2	21.31 + j 58.54	32.53 + j 48.05	77.4 + j 64.5
	" " " " Z_{oe} =	70.0 + j 119.6		48.06 + j 50.18	
	" " " " Z_{ow} =	43.1 + j 107.1	29.31 + j 72.05	30.41 + j 44.63	86.8 + j 81.7
1-2-3	Calculated, I_o =	60.1	35.7		43.9
	Tested =	58.0	41.6		42-38-32
	Calculated Z =	15.13 + j 85.54	12.93 + j 49.69	9.53 + j 32.97	19.38 + j 50.67
	Tested =	15.14 + j 86.85	14.28 + j 49.57	13.53 + j 33.10	21.2 + j 55.3
	Calculated Z_t =		16.40		
	Tested =		16.49		
4	Calculated I_e =	74.7	55.7		
	Tested =	74.6	57.9		
	Calculated Z =	12.05 + j 70.86	12.62 + j 44.76	8.71 + j 28.16	
	Tested =	11.83 + j 73.60	13.04 + j 44.75	10.37 + j 29.87	
	Calculated Z_t =		11.47		
	Tested =		11.71		
5	Calculated I_e =	93.9	66.3		
	Tested =	94.3	62.2		
	Calculated Z =	9.67 + j 62.56	12.03 + j 42.00	10.41 + j 26.20	
	Test =	10.67 + j 67.52	12.81 + j 42.35	8.33 + j 27.27	
5A	Calculated I_e =	74.7	55.7		
	Tested =	85.5	60.6		
	Calculated Z =	8.00 + j 44.11	7.10 + j 28.07	5.52 + j 16.72	
	Tested =	8.00 + j 47.00	7.85 + j 28.02	7.10 + j 18.23	

calculated. This gave effective conductor heights up to several thousand feet and a wide variation was found. This assumption always seemed too much in error to the writer because no good method of predicting the impedance of the circuit could be built up from it.

The calculation of the impedance of the fundamental circuit made up of a conductor with ground return requires a study of the distribution of the current in the ground. A solution to this problem has been given by Carson in the *Bell System Technical Journal* for

TABLE B-3—TEST DATA
Pennsylvania Power & Light Co.
Wallenpaupack—Siegfried, 220-Kv. Line
Tests Made Before Ground Wire Installed

Test No.	Total volts E	Resistence component R	Reactive component X	Total amperes I	Ground amperes I_g	Ground wire amperes I_o	Line ground wire amperes I_o'	Aux. wire volts E_t
1	9,747	1,712	9,593	100	100			4,318
2	7,059	1,362	6,918	100	100			4,339
3	6,010	1,107	5,906	100	100			
4	7,680	1,269	7,588	100	55.2	44.8		2,375
5	7,008	1,086	6,948	100	38.1	61.9		
5A	5,052	893	4,970	100	55.7	44.6		
6-7	10,820	1,620	10,700	100				
Tests made after ground wire installed								
1	8,787	1,466	8,643	100	42.0	58.0		3,442
2	6,143	1,129	6,040	100	42.0	58.0		3,440
3	5,230	993	5,130	100	41.9	59.1		
4	7,453	1,183	7,360	100	25.1	35.2	39.4	2,027
5	6,833	1,067	6,752	100	22.1	36.6	57.7	
5A	4,770	800	4,700	100	31.9	46.1	39.4	
6-7	10,620	1,692	10,480	100				

TABLE B-4—TEST DATA
Pennsylvania Power & Light Co.
Wallenpaupack—Tower 19-5-220 Kv.
Tests Made Before Ground Wire Installed

Test No.	Total volts E	Resistance component R	Reactive component X	Total amperes I	Ground amperes I_g	Ground wire amperes I_o	Line ground wire amperes I_t	Aux. wire volts E
1	8,313	8,055		100	100			7,745
2	7,985	7,928		100	100			7,687
3	7,880	7,825		100	100			
4	3,015	751	2,920	100	18.4	94.1		1,457
5	2,316	468	2,268	100	9.6	95.2		
5A	2,234	651	2,139	100	18.8	94.3		
6-7	3,109	515	3,065	100				
Tests made after ground wire installed								
1	2,118	474	2,064	100	42	58		
2	1,320	394	1,258	100	42.2	57.8		
3	1,075	358	1,015	100	42.1	57.9		
4	1,933	415	1,888	100	30.9	42.6	26.6	1,967
5	1,878	370	1,841	100	24.3	33.3	42.3	
5A	1,196	300	1,157	100	30.9	42.3	26.8	
6-7	3,102	499	3,064	100				

TABLE B-5—TEST DATA
Appalachian Electric Power Company
Turner—Logan 132-Kv. Line
Tests Made Before Ground Wire Installed

Test No.	Total volts E	Resistance component R	Reactive amperes X	Total amperes I	Ground current I_g	Ground wire I_o	Aux. volts E_t
1	5,955	1,380	5,790	100	100		2,461
2	4,149	851	4,060	100	100		2,473
3	3,608	670	3,545	100	100		
4	4,949	1,414	4,740	100	58.1	41.2	1,435
5	4,530	1,363	4,323	100	40.8	58.2	
5A	3,198	871	3,077	100	58.8	41.3	
6-7	7,001	2,229	6,636	100			
Tests made after ground wire installed							
1	5,180	1,408	4,985	100	59.6	41.3	1,646
2	3,379	867	3,262	100	60.2	41.8	1,652
3	2,833	715	2,742	100	59.9	41.5	
4	4,662	1,304	4,475	100	40.9	57.9	1,171
5	4,425	1,281	4,235	100	50.8	68.2	
5A	2,908	785	2,802	100	40.9	60.5	
6-7	7,039	2,211	6,678	100			

TABLE B-6—TEST DATA
Alabama Power Co. 110 Kv.
Martin Dam—North Auburn

Test No.	Total volts E	Resistance component R	Reactive component X	Total amperes I	Ground amperes I_g	Ground wire amperes I_e	Aux. wire volts E_t
1	3,567	1,353	3,300	100	100		
2	2,415	1,040	2,179	100	100		
3	2,020	923	1,797	100	100		
4	3,163	1,037	2,987	100	65.1	35.5	
5	2,853	833	2,727	100	47.0	49.9	
5A	1,973	755	1,823	100	65.4	35.1	
6-7		1,276	4,532	100			
North Auburn—State Line							
1		15,493		100	100		
2		15,786		100	100		
3		15,880		100	100		
4	6,590	2,413	6,132	100	21.5	89.2	
5	5,257	1,542	5,024	100	11.2	92.8	
5A	4,867	1,983	4,445	100	23.4	89.8	
6-7		1,825	6,602	100			
North Auburn—Newnan (with ground wire)							
1	12,130	2,488	11,880	100	100		
2	9,215	1,828	8,980	100	100		
3	8,470	1,541	83,20	100	100		
4	8,936	2,220	8,650	100	47.7	51.6	3,070
5	7,940	1,965	7,700	100	29.1	66.5	
5A	5,975	1,523	5,780	100	47.6	51.4	
6-7	11,530	3,091	11,470	100			

Oct. 1926. With some modification the expression developed by Carson is

$$Z = r_w + x_w + Z_g$$

Z = total impedance of circuit
r_w = resistance of conductor
x_w = reactance of conductor
Z_g = impedance of ground return.

The reactance of the conductor x_w is calculated as if the return conductor was below the surface of the ground a distance equal to the height of the conductor, in other words it is the one way reactance of the conductor with image return.

TABLE B-7
Consumers Power Co. 140 Kv.
Data Summary
Saginaw—Flint
Tests Made Before Ground Wire Installed

Test	Volts	Watts	Amperes	R	X	Z
1	12,250	629.3	196.3	16.32	60.20	62.40
2	8,237	365.3	191.4	9.92	41.65	42.80
3	7,020	280	192	7.48	35.5	36.27
4	11,020	696	203	16.9	51.6	54.3
5	9,840	637	196	16.51	47.3	50.1
6	14,260	876	183.2	26.1	73.3	77.8
7	11,850	821.3	200.5	20.4	55.5	59.1

Tests made after ground wire installed

Test No.	Volts	Kilowatts	Amperes	R	X	Z
1	11,870	854.4	200.7	21.2	55.3	59.2
2	7,925	539.2	202.4	13.2	37.0	39.2
3	6,580	432.0	199.2	10.9	31.2	33.0
4	10,550	812.8	199.7	20.4	48.8	52.9
5	9,865	777.6	200.3	19.4	45.4	49.3
6	14,425	1,040.4	181.9	31.6	73.6	79.3
7	14,370	1,382.4	241.6	23.7	54.6	59.5

$$X_w = 0.2794 \log \frac{4h}{d} + 0.0303 \text{ ohms per mi.—natural logs} \quad (14)$$

The impedance of the earth Z_g, depends upon a qualtity ϕ, which for 60 cycles is defined as

$$\phi = 4196\, h \sqrt{\lambda} \quad (15)$$

$2h$ = distance from conductor to its image
d = diameter of conductor
λ = conductivity of the earth.

for any value of ϕ the corresponding value of Z_g for 60 cycles is given by

$$Z_g = 0.243\,(P + j\,Q) \quad (16)$$

and the values of P and Q in ohms per mi. are obtained from Table C-1.

In Table C-1 are given values of the earth resistance "P" in ohms per mi. and the earth reactance "Q" in ohms per mi. as a function of the quantity ϕ and the angle θ. These values were calculated by W. B. Jordan using Carsons formula and cover the range of earth conductivity "λ" as indicated by tests. The values for $\theta = 0$ are used when the self impedance of conductor and earth return is to be calculated. The values for other values of θ are used when it is desired to make calculations involving the mutual impedance in a more rigorous manner than in section A of this paper. For instance, refer to Fig. C-1 and see that

$$\text{Cos}\ \theta = \frac{h + H}{R}, \ \text{Sin}\ \theta = \frac{D}{R} \quad (17)$$

The mutual impedance M is then

TABLE B-8—LINE DATA

System	Pennsylvania Pr. & Lt.	Appalachian Elec. Pr. Co.		Alabama Pr. Co.		Consumers Pr. Co.
Line tested	Wallenpaupack-Siegfried	Turner-Logan	Martin Dam-North Auburn	No. Auburn Line	State Line-Newnan	Saginaw-Flint
Voltage rating	220 kv	132 kv	110 kv	110 kv	110 kv	140 kv.
Length, mi	64.77	41	26.9	40.4	30.1	43.25
Conductor						
Size	ACSR-795,000 cir. mil	ACSR-336,400[3]	ACSR-397,500 cir. mil	ACSR-397,500 cir. mil	ACSR-397,500 cir. mil	Copper 3/0
Arrangement	Horizontal	2 circuit V	Horizontal	Horizontal	Horizontal	2 circuit V
Separation, ft		23-28				16 approx.
Spacing, ft	24.04-H	13-V	17-H	17-H	13[4]-H	12.04-V
Average height, ft	55.1	56.5[1]	43.5	43.1	44.2[4]	38.97[1]
Ground wire						
Number	2[5]		2	2[5]	none	1
Size	ACSR-184,000 cir. mil	ACSR-177,000 cir. mil	ACSR 1/0	ACSR 1/0		Copperweld 3/8
Diameter	0.707					
Spacing, ft	24.04		17	14		
Average height, ft	65.5	94.5	52.7	52.3		74.59

1. To bottom conductor.
2. 19.2 mi. from Wallenpaupack and 4.54 mi. from Siegfried.
3. 5.7 mi. ACSR-292,000 cir. mil.
4. 9.35 mi. at 14 ft. spacing and 43.1 ft. height.
5. Insulated ground wires.

$$M = 0.2794\, L_{ag} \frac{R}{S} + M\, Q \qquad (18)$$

$$M_g = 0.243\, (P + j\, Q) \qquad (19)$$

For the mutual impedance of the earth return M_g, P and Q are obtained for the proper angle θ and a value of ϕ calculated from the expression

$$\phi = 2098\, R \sqrt{\lambda} \qquad (20)$$

In order to apply this method it is necessary to have some idea of the values of the conductivity of the earth since that is the final variable. In Table C-2 the few values of the earth conductivity λ as determined from the tests of this paper are given together with many more values obtained from other sources. The values given apply to specific tests and specific regions but can be used to estimate the value of λ to be used for other places. The striking thing about these data is the wide variation in λ for a relatively small variation in ohms per mile impedance.

TABLE B-9
Appalachian Electric Power Company
Turner-Logan 132-Kv. Line
Comparison of Current in Ground Wire at Middle and End of Line

Test No.	Ground wire current Middle	End
1	37.7	41.3
2	40.2	41.8
4	58.5	62.2
5	58.9	56.3
5A	79.9	81.7

Fortunately it is not necessary to know λ with any great degree of accuracy to determine the impedance of a transmission line with earth return within a reasonable degree of accuracy. An error of 1,000 per cent (10-1) in the value of λ will give about 30 per cent error in the earth reactance, which reduces to about 13 per cent error in the reactance of the circuit of conductor with ground return. The relative error dwindles further when the line reactance is in series with transformer and generator reactance.

In Table C-3 is given calculated values of earth reactance and resistance of the earth return path at 60 cycles. These values are shown in the curves in Figs. C-2 and C-3. These curves cover the range of earth conductivity well beyond that found in tests as indicated by Table C-2. To use the curve it is necessary to select the most probable value of earth conductivity

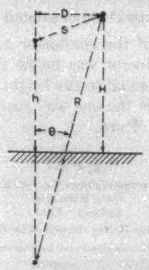

Fig. C-1

for the line in question and then read off the unit values r_g and x_g for the proper value of average height of line.

Example. As an illustration of the application of the method calculations will be made on the 25-cycle 220-kv. line recently installed in Ontario.[4] This line is 230 miles long, two circuit with 150 ft. separation, horizontally arranged conductors, and has two ground wires. The conductor is 795,000 cir. mils A. C. S. R.

4. *The 220,000-Volt System of the Hydro-Electric Power Commission of Ontario*, E. T. J. Brandon, A. I. E. E. Trans., Oct. 1930.

and the ground wire is the same as the steel core of this cable. The height of the conductor at the tower is 60 ft. and the sag is 36 ft. The conductor spacing is 25 ft. 3 in. and the ground wires are 13 ft. 4 in. above and spaced 26 ft. 6 in. In the calculations the average height of the conductor will be taken as 38 ft. It appears from the meager information available to the writer that the conductivity of the earth is poor and accordingly it will be taken as 1×10^{-14}.

In Fig. C-4 the line and dimensions are indicated. In some of the calculations the three conductors and the two ground wires of each circuit are grouped together and considered as a single conductor in effect. Fig. C-5 shows the equivalent circuit. When this is done the self impedance of the equivalent conductor is the impedance of three line, or two ground, conductors in parallel; the mutual impedance is unchanged except that it is based upon the logarithmic average of the spacings.

1. Auxiliary Quantities

The ordinary impedance between the line wires, between ground wires, and between line wires and ground wires is needed. By the customary methods there are found to be, for 230 mi. of line

$$\left.\begin{aligned} Z_{ww} &= 54.3 + j\,158.3 \\ Z_{oo} &= 1820 + j\,179.6 \\ Z_{ow} &= 937 + j\,161.2 \end{aligned}\right\} \text{All wires in same circuit}$$

$$\left.\begin{aligned} Z_{ww}' &= 54.3 + j\,194.3 \\ Z_{oo}' &= 1820 + j\,219.8 \\ Z_{ow}' &= 937 + j\,206.6 \end{aligned}\right\} \text{Between circuits}$$

By test

$$Z_{ww} = 63.3 + j\,158.1$$

The difference in resistance between test and calculations indicates that there is some extraneous resistance somewhere. This extra resistance should also appear in the other measurements. However, in the calculations which follow no allowance for this extraneous resistance nor for the resistance to ground at the end points has been made. In actual practise an allowance should be made for these factors as indicated by experience or definite knowledge of the conditions.

2. Impedance:—One Conductor—Ground Return

TABLE C-1—VALUES OF P AND Q FOR CARSON'S FORMULA

	P				Q			
ϕ	$\theta = 0°$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$	$\theta = 0°$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
0.00010	0.39268	0.39268	0.39268	0.39269	4.9132	4.9132	4.9132	4.9131
0.00011	67	68	68	69	4.8655	4.8655	4.8655	4.8655
0.00012	67	68	68	69	4.8220	4.8220	4.8220	4.8219
0.00013	67	67	68	68	4.7820	4.7820	4.7820	4.7820
0.00014	67	67	68	68	4.7449	4.7449	4.7449	4.7449
0.00015	66	67	67	68	4.7104	4.7104	4.7104	4.7104
0.00016	66	67	67	68	4.6782	4.6782	4.6782	4.6782
0.00018	66	66	67	68	4.6193	4.6193	4.6193	4.6193
0.00020	65	66	67	68	4.5666	4.5666	4.5666	4.5666
0.00025	64	65	66	67	4.4551	4.4550	4.4550	4.4550
0.00030	63	64	65	66	4.3639	4.3639	4.3639	4.3639
0.0004	61	62	63	65	4.2201	4.2201	4.2201	4.2200
0.0005	58	60	62	64	4.1085	4.1085	4.1085	4.1085
0.0006	56	58	60	63	4.0174	4.0174	4.0174	4.1073
0.0007	53	56	58	62	3.9404	3.9403	3.9403	3.9403
0.0008	51	54	57	61	3.8736	3.8736	3.8735	3.8734
0.0009	49	52	55	59	3.8147	3.8147	3.8147	3.8146
0.0010	0.39246	0.39250	0.39253	0.39258	3.7621	3.7620	3.7620	3.7620
0.0011	44	48	52	57	3.7145	3.7144	3.7144	3.7143
0.0012	42	46	50	56	3.6710	3.6709	3.6709	3.6707
0.0013	39	43	48	55	3.6310	3.6309	3.6309	3.6308
0.0014	37	41	47	53	3.5939	3.5939	3.5938	3.5938
0.0015	35	39	45	52	3.5595	3.5594	3.5594	3.5593
0.0016	32	37	43	51	3.5272	3.5272	3.5271	3.5270
0.0018	28	33	40	49	3.4684	3.4683	3.4683	3.4682
0.0020	23	29	37	46	3.4157	3.4157	3.4156	3.4155
0.0025	0.39211	19	28	40	3.3043	3.3042	3.3041	3.3040
0.0030	0.39199	0.39209	20	35	3.2132	3.2131	3.2130	3.2129
0.004	77	0.39189	0.39203	23	3.0696	3.0695	3.0694	3.0692
0.005	53	69	0.39187	0.39210	2.9583	2.9581	2.9580	2.9577
0.006	30	49	70	0.39198	2.8674	2.8672	2.8670	2.8667
0.007	0.39107	28	53	86	2.7905	2.7903	2.7901	2.7897
0.008	0.39083	0.39108	37	75	2.7240	2.7238	2.7235	2.7231
0.009	61	0.39088	20	62	2.6653	2.6651	2.6647	2.6643
0.010	0.39038	0.39068	0.39103	0.39151	2.6129	2.6126	2.6122	2.6117

TABLE C-1—VALUES OF P AND Q FOR CARSON'S FORMULA—*Continued*

	P				Q			
ϕ	$\theta = 0°$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$	$\theta = 0°$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
0.010	0.39038	0.39068	0.39103	0.39151	2.6129	2.6126	2.6122	2.6117
0.011	0.39015	48	0.39088	39	2.5655	2.5661	2.5647	2.5642
0.012	0.38992	28	71	27	2.5222	2.5218	2.5214	2.5207
0.013	70	0.39008	54	15	2.4824	2.4820	2.2815	2.4809
0.014	47	0.38989	38	0.39102	2.4456	2.4452	2.4447	2.4440
0.015	24	68	21	0.39090	2.4114	2.4109	2.4103	2.4096
0.016	0.38902	50	0.39004	77	2.3793	2.3788	2.3782	2.3775
0.018	0.38857	0.38910	0.38972	54	2.3209	2.3203	2.3197	2.3188
0.020	0.38812	0.38870	0.38939	0.39029	2.2687	2.2681	2.2673	2.2664
0.025	0.38701	0.38772	0.38856	0.38969	2.1583	2.1575	2.1566	2.1554
0.030	0.3859	0.3868	0.3877	0.3891	2.0683	2.0673	2.0662	2.0648
0.04	37	48	61	79	1.9268	1.9255	1.9241	1.9222
0.05	0.3816	29	45	66	1.8175	1.8160	1.8142	1.8118
0.06	0.3795	0.3910	29	54	1.7286	1.7268	1.7247	1.7218
0.07	74	0.3792	0.3813	41	1.6539	1.6518	1.6493	1.6460
0.08	54	73	0.3797	29	1.5894	1.5870	1.5842	1.5804
0.09	34	55	81	16	1.5327	1.5301	1.5269	1.5228
0.10	0.3714	0.3737	0.3765	0.3803	1.4824	1.4794	1.4759	1.4713
0.11	0.3695	19	49	0.3791	1.4370	1.4338	1.4299	1.4249
0.12	76	0.3702	34	78	1.3957	1.3922	1.3881	1.3826
0.13	57	0.3685	18	65	1.3579	1.3542	1.3497	1.3438
0.14	38	67	0.3703	52	1.3231	1.3191	1.3143	1.3080
0.15	20	50	0.3688	40	1.2908	1.2866	1.2815	1.2747
0.16	0.3602	33	72	27	1.2608	1.2563	1.2509	1.2437
0.18	0.3566	0.3600	42	0.3701	1.2063	1.2013	1.1953	1.1873
0.20	0.3531	0.3567	0.3612	0.3675	1.1580	1.1526	1.1459	1.1371
0.25	0.3448	0.3488	0.3539	0.3612	1.0572	1.0506	1.0426	1.0319
0.30	0.3368	0.3413	0.3468	0.3548	0.9766	0.9690	0.9597	0.9472
0.40	0.3222	0.3270	0.3332	0.3422	0.8534	0.8439	0.8323	0.8163
0.50	0.3089	0.3139	0.3204	0.3300	0.7617	0.7506	0.7369	0.7180
0.60	0.2967	0.3016	0.3082	0.3180	0.6897	0.6773	0.6618	0.6402
0.70	0.2854	0.2903	0.2967	0.3065	0.6312	0.6176	0.6006	0.5766
0.80	0.2750	0.2797	0.2858	0.2953	0.5825	0.5679	0.5494	0.5235
0.90	0.2654	0.2697	0.2755	0.2846	0.5410	0.5255	0.5060	0.4782
1.00	0.2564	0.2604	0.2658	0.2742	0.5052	0.4890	0.4685	0.4392
1.1	0.2480	0.2516	0.2565	0.2644	0.4740	0.4572	0.4358	0.4051
1.2	0.2401	0.2433	0.2478	0.2548	0.4465	0.4292	0.4070	0.3752
1.3	0.2327	0.2355	0.2394	0.2458	0.4200	0.4042	0.3815	0.3487
1.4	0.2257	0.2282	0.2315	0.2370	0.4000	0.3819	0.3587	0.3251
1.00	0.2564	0.2604	0.2658	0.2742	0.5052	0.4890	0.4685	0.4392
1.1	0.2480	0.2516	0.2565	0.2644	0.4740	0.4572	0.4358	0.4051
1.2	0.2401	0.2433	0.2478	0.2548	0.4465	0.4292	0.4070	0.3752
1.3	0.2327	0.2355	0.2394	0.2458	0.4220	0.4042	0.3815	0.3487
1.4	0.2257	0.2282	0.2315	0.2370	0.4000	0.3819	0.3587	0.3251
1.5	0.2192	0.2192	0.2240	0.2287	0.3802	0.3618	0.3382	0.3040
1.6	0.2130	0.2146	0.2169	0.2207	0.3623	0.3437	0.3198	0.2849
1.8	0.2015	0.2024	0.2036	0.2058	0.3309	0.3121	0.2878	0.2522
2.0	0.1913	0.1913	0.1916	0.1922	0.3045	0.2856	0.2611	0.2251
2.5	0.1695	0.1681	0.1661	0.1631	0.2536	0.2348	0.2106	0.1749
3.0	0.1521	0.1494	0.1458	0.1400	0.2169	0.1988	0.1754	0.1411
4.0	0.1260	0.1217	0.1159	0.1066	0.1678	0.1514	0.1303	0.1000
5.0	0.1075	0.1023	0.0959	0.0839	0.1364	0.1218	0.1038	0.0763
6.0	0.0936	0.0878	0.0809	0.0697	0.1148	0.1019	0.0854	0.0623
7.0	0.0828	0.0771	0.0699	0.0587	0.0991	0.0874	0.0728	0.0526
8.0	0.0742	0.0687	0.0615	0.0507	0.0871	0.0765	0.0634	0.0456
9.0	0.0672	0.0619	0.0549	0.0445	0.0776	0.0681	0.0562	0.0403
10.0	0.0614	0.0562	0.0495	0.0397	0.0700	0.0612	0.0505	0.0361

$J = P + jQ$. $Z = J$ D abohms per cm.

To convert to practical ohms per mi., multiply by $5280 \times 12 \times 2.539954 \times 10^{-9} = 1.609315 \times 10^{-4}$.

For a 60-cycle wave, $Z = 0.24268\ (P + jQ)$ ohms/mi.

Data calculated from formulas given by John R. Carson, "Wave Propagation in Overhead Lines with Ground Return," reprint B-219 of the Bell Telephone Laboratories, November, 1926.

Data calculated by W. B. Jordan, April, 1930.

TABLE C-2—DETERMINATION OF λ

System or location	Reactance per mile Line	Reactance per mile Earth	$\times 10^{-14}$ λ
P P & L	0.976	0.499	1.8
Consumers	1.069	0.321	42.7
Appalachian	0.970	0.434	3.3
Alabama	0.984	0.724	0.065
Cross Keys[1]			42
Glens Falls[1]			175
Massilon[1]			36
Cross Keys[2]			33
U. S.–Canad [2]			200 to 0.1
Vicinity Chicago[4]			9
Eastern Texas			125
West Virginia			55
Penn.—Maryland			0.5
New Jersey			9
New Jersey			30
Ohio			100
Utah			30
Minn			10
Indiana			30
Maryland			8
New York			15
Penn			1
Penn			10
Wisconsin			1
Tenn			1
Ga			0.3
Ill			50
Near Ottawa			5
Oklahoma			100
Alabama			5

1. Gilkeson and Bowen, A. I. E. E. Trans., Oct. 1930, p. 1375.
2. E. P. Peck, Discussion A. I. E. E. Trans., Oct. 1930, p. 1379.
3. E. B. King, Discussion, A. I. E. E. Trans., Oct. 1930, p. 1379. Values given are average of more than 30 tests and the statement is made that 75 per cent of the values of λ are in the range from 100×10^{-14}, to 10×10^{-14}.
4. This and all the values following given through the courtesy of the joint D & R Committee sponsored by the A. T. & T. and NELA.

Let Z_{w-g} = Impedance single conductor (with image return); generator to fault, one way only. Calculated by the usual method using a spacing of $2h$.

$$Z_{w-g} = 27.1 + j\,89.3$$

For the impedance of the ground return (Z_{g-w}) a little consideration is needed because the line operates at 25 cycles while the curves in Fig. C-2 and C-3 are for 60 cycles. The curves can be entered with an effec-

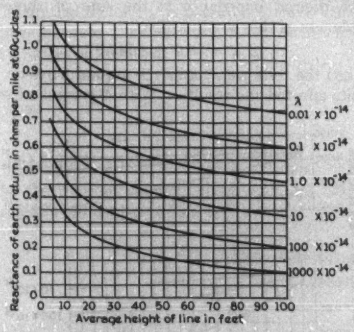

Fig. C-2—These Curves Give the Reactance of the Return Path Through the Earth for Different Values of the Earth Conductivity. Calculations Based on Carsons Analysis.

TABLE C-3

Values of Resistance and Reactance at 60 Cycles of Return Path through the Earth for Various Heights of Line Conductors and Earth Conductivity

λ = 0.01				λ = 0.1				λ = 1.0			
φ	h	x	r	φ	h	x	r	φ	h	x	r
0.0002	4.77	1.10822	0.09529	0.0005	3.77	0.99705	0.09527	0.002	4.77	0.82892	0.09519
0.0004	9.53	1.02413	0.09528	0.001	7.54	0.91299	0.09524	0.004	9.53	0.74493	0.09507
0.0006	14.30	0.97494	0.09527	0.002	15.07	0.02892	0.09519	0.006	14.30	0.69586	0.09496
0.0008	19.07	0.93005	0.09525	0.003	22.61	0.77978	0.09513	0.008	19.07	0.67106	0.09474
0.0012	28.60	0.89088	0.09523	0.004	30.15	0.74493	0.09507	0.012	28.60	0.61209	0.09463
0.0016	38.13	0.85598	0.09521	0.005	37.68	0.71792	0.09502	0.016	38.13	0.57741	0.09441
0.002	47.66	0.82892	0.09519	0.006	45.22	0.69586	0.09496	0.02	47.66	0.55057	0.09419
0.0025	59.58	0.80189	0.09516	0.008	60.29	0.67106	0.09485	0.025	59.58	0.52378	0.09392
0.0030	71.50	0.77978	0.09513	0.010	75.36	0.63410	0.09474	0.030	71.50	0.50194	0.09365
0.0040	95.33	0.74493	0.09507	0.012	90.44	0.61209	0.09463	0.040	95.33	0.46760	0.09312
0.0050	119.16	0.71792	0.09502	0.014	105.51	0.59350	0.09452	0.050	119.16	0.44107	0.09261

$\phi = 4196\, h\, \sqrt{\lambda}$.

TABLE C-3—*Continued*

λ = 10				λ = 100				λ = 1000			
φ	h	x	r	φ	h	x	r	φ	h	x	r
0.005	3.77	0.71792	0.09502	0.02	4.77	0.55057	0.09419	0.05	3.77	0.44107	0.09261
0.01	7.54	0.63410	0.09474	0.04	9.53	0.46760	0.09312	0.1	7.54	0.35975	0.09013
0.02	15.07	0.55057	0.09419	0.06	14.30	0.41906	0.09210	0.2	15.07	0.28103	0.08569
0.03	22.61	0.50194	0.09365	0.08	19.07	0.38572	0.09110	0.3	22.61	0.23700	0.08173
0.04	30.15	0.46760	0.09312	0.12	28.60	0.33871	0.08921	0.4	30.15	0.20710	0.07819
0.05	37.68	0.44107	0.09261	0.16	38.13	0.30597	0.08741	0.5	37.68	0.18485	0.07496
0.06	45.22	0.41906	0.09210	0.2	47.66	0.28103	0.08569	0.6	45.22	0.16738	0.07200
0.08	60.29	0.38572	0.09110	0.25	59.58	0.25656	0.08368	0.8	60.29	0.14136	0.06674
0.10	75.36	0.35975	0.09013	0.3	71.50	0.23700	0.08173	1.0	75.36	0.12260	0.06222
0.12	90.44	0.33871	0.08921	0.4	95.33	0.20710	0.07819	1.2	90.44	0.10836	0.05827
0.14	105.51	0.32109	0.08829	0.5	119.16	0.18485	0.07496	1.4	105.51	0.09707	0.05477

$\phi = 4196\, h\, \sqrt{\lambda}$.

tive height $h' = \sqrt{\frac{25}{60}}$. The value read from the curves must also be changed from 60 cycles to 25 cycles by direct proportion.

$$h' = 38 \times 0.646 = 24.5$$

Read the curves for $\lambda = 1 \times 10^{-14}$ and $h = 34.5$ for the unit values of r_g and x_g at 60 cycles, from which the impedance of 230 mi. of line at 25 cycles is found to be

$$Z_{g-w} = 9.1 + j\,60.6$$

The desired impedance is the sum of these two impedances as they are in series,

$$Z_{wg} = 36.2 + j\,149.9$$

When the tests were made the ground wires on both circuits affected the results. With 4 ground wires the

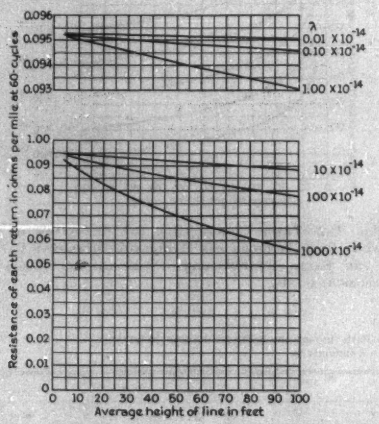

FIG. C-3—THESE CURVES GIVE THE RESISTANCE OF THE RETURN PATH THROUGH THE EARTH FOR DIFFERENT VALUES OF THE EARTH CONDUCTIVITY. CALCULATIONS BASED ON CARSONS ANALYSIS

expression for the desired impedance is from Table A-1

$$Z_{14} = Z_{wg} - \frac{M_{gw}^2}{Z_{gg} - \frac{3}{8} Z_{gg}}$$

and the proper average values of impedances must be inserted into the expression. For the ground wire on the same circuit as the conductor the self impedance of the ground wire should be based on its image return using a spacing of 2 H, but the same ground impedance as for the conductor should be used, so that

$$Z_{gg} = 919 + j\,166.2$$

For the other two ground wires the calculations should be made upon the basis of the image distance from the ground wire on one circuit to the conductors of the other circuit. The angle θ, Fig. C-1, is about 60 deg. but the value ϕ is low enough so that the curves can be used, and then

$$Z_{gg}' = 919 + j\,162.4$$

For the calculation,

$$\text{Average } Z_{gg} = 919 + j\,164.3$$
$$\text{Average } Z_{gc} = 1220 + j\,206.4$$
$$\text{Average } Z_{gw} = 937 + j\,183.9$$

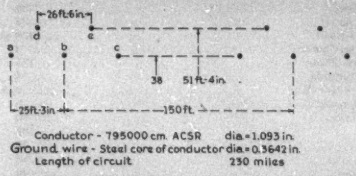

FIG. C-4

With these values there results

$$Z_{14} = 36.2 + j\,149.9 - (9.1 + j\,65.7) \times 0.263$$
$$= 33.8 + j\,132.6 \text{ calculated}$$
$$= 51.2 + j\,138.2 \text{ tested}$$

The calculations may also be made by the more rigorous method of the appendix. In which case the impedance is given by the following expression which is obtained from equation (**21**) of the appendix:

$$Z_{14} = A_a - \frac{I_3}{I_a} B_{a3} - \frac{I_4}{I_a} B_{a4}$$

(See Appendix for usage of A and B)

$A_a = Z_{wg} = 36.2 + j\,149.9$—Self impedance of line wire.

$A_3 = A_4 = 464 + j\,117.7$—Self impedance of two ground wires in parallel.

FIG. C-5

$B_{a3} = 9.1 + j\,74.0$ —Mutual impedance as indicated.

$B_{a4} = 9. + j\,52.8$ —Mutual impedance as indicated.

$B_{34} = +. + j\,52.6$ —Mutual impedance as indicated.

From equation (**21**) the values of currents are found as

$$\frac{I_3}{I_a} = \frac{B_{a3}}{A_3} \frac{1 - \frac{B_{a4}}{B_{a3}} \frac{B_{34}}{A_4}}{1 - \frac{B_{34}^2}{A_3 A_4}}$$

$$\frac{I_4}{I_a} = \frac{B_{a4}}{A_4} \frac{1 - \dfrac{B_{a3}}{B_{a4}} \dfrac{B_{34}}{A_3}}{1 - \dfrac{B_{34}^2}{A_3 A_4}}$$

$$\frac{I_3}{I_a} = 0.01549 \qquad \frac{I_4}{I_a} = 0.1102$$

It should be noted that $I_3 = I_a \dfrac{B_{a3}}{A_3}$ and $I_4 = I_a \dfrac{B_{a4}}{A_4}$ very closely for the spacings of this line.

$$Z_{14} = 33.8 + j\,132.4$$

This is the same as by the previous calculation and the total ground wire current is 0.265 as compared with 0.263.

3. Impedance:—Three Conductors in Parallel—Ground Return

By the approximate method from Table A-1

$$Z_{14} = 33.8 + j\,132.6$$

$$\frac{1}{3} Z_{ww} = 18.1 + j\,52.8$$

$$Z_{34} = 15.7 + j\,79.8 \quad \text{calculated}$$
$$= 34.6 + j\,87.4 \quad \text{tested}$$

By the rigorous method from equation (**21**) or (**23**)

$$Z_{34} = \frac{A_a + 2\,B_a}{3} - \frac{I_3}{I} B_{a3} - \frac{I_4}{I} B_{a4}$$

There are two values of B_a that might be used. Since the line conductors are transposed the same current flows in each and the effect of the ground is the same on each of the three conductors, and so

$$B_a = j\,230 \times 0.1165 \operatorname{Log} \frac{2\,h}{S} + B_{g\text{-}w}$$

$$= j\,10.1 + 9.1 + j\,60.6$$
$$= 9.1 + j\,70.7$$

and then

$$Z_{34} = 15.7 + j\,79.6$$

There are those who would prefer to use the image distance in calculating $B_{g\text{-}w}$. The average image distance is 85.45 ft. and from which

$$B_a = 9.1 + j\,70.7$$

which gives the same result.

4. Impedance:—Two Circuits in Parallel.

For this we need to know the impedance of one circuit out and the other return, neglecting earth and ground wires. It is calculated by the usual methods but with three conductors in parallel and transposed.

$$Z_{ww}' = 18.1 + j\,88.8 \text{ (3 in parallel)}$$

and then, by the approximate method using Table A-1

$$Z_{34} = 15.7 + j\,79.8$$

$$\frac{1}{4} Z_{ww}' = 4.5 + j\,22.2$$

$$Z_{64} = 11.2 + j\,57.6 \text{ calculated}$$
$$= 28.1 + j\,62.6 \text{ tested}$$

By the rigorous method of the Appendix, from an adaption of equation (**23**)

$$Z_{64} = \frac{A_a + B_a + B_a'}{6} - \frac{I_3}{I} B_{a3} - \frac{I_4}{I} B_{a4}$$

The mutual factor B_a' is based on the average image distance between the line wires of one circuit and the ground wires of the other.

$$B_a' = 9.0 + j\,52.6$$

and then

$$Z_{64} = 11.2 + j\,59.0$$

5. Impedance:—Out One Circuit—Return Other.

By the rigorous method using equation (**27**) with conductors grouped as in Fig. C-5

$$Z_5 = A_1 - 2\,B_{12} + A_2 - 2\,\frac{I_3}{I_1}(B_{13} - B_{14})$$

$$\frac{I_3}{I_1} = \frac{B_{13} - B_{14}}{A_3 - B_{34}}$$

$A_1 = A_2 = \dfrac{A_a + 2\,B_a}{3} = 18.1 + j\,97.1$ self impedance, 3 line wires in parallel

$A_3 = A_4 = 464 + j\,117.7$ self impedance, 2 ground wires in parallel

$B_{12} = B_a' = 9.0 + j\,52.6$ mutual impedance, as indicated

$B_{13} = B_{a3} = 9.1 + j\,74.0$ mutual impedance, as indicated

$B_{14} = B_{a4} = 9.0 + j\,52.8$ mutual impedance, as indicated

$B_{34} = B_{34} = 9.0 + j\,52.6$ mutual impedance, as indicated

and then

$$\frac{I_3}{I_1} = 0.0461$$

$$Z_5 = 18.2 + j\,89.0 - 0.98$$
$$Z_5 = 18.2 + j\,88.0 \text{ calculated}$$
$$= 46.6 + j\,94.0 \text{ tested}$$

By the approximate method this was calculated as Z_{ww}' under paragraph (4) preceding.

6. Impedance:—Out on One Circuit—Part Way Back on Other.

For this test the fault was located 131.9 mi. from the

generating end. For this condition there are in effect two impedances in series, so that

$$Z_6 = \frac{131.9}{230} Z_{34} + \frac{98.1}{230} Z_5$$

$$Z_6 = 16.8 + j\,83.2 \text{ calculated}$$
$$= 33.0 + j\,91.7 \text{ tested}$$

7. Impedance:—Fault on One Circuit—(Fig. D-1)

For the values of the constants on this line the following equations hold very closely. This is based on an observation made in regard to I_3 and I_4 in the calculations of condition (2)

$$\frac{I_2}{I_1} = 1 + \frac{Z_5}{A_1 - B_{12} - (B_{13} B_{14}) \dfrac{B_{13} - B_{14}}{A_3}}$$

$$\frac{E_1}{I_1} = A_1 + Z_5 - \frac{B_{13}^2 + B_{14}^2}{A_3} + \frac{I_2}{I_1}\left(B_{12} - 2\,\frac{B_{13} B_{14}}{A_3}\right)$$

When the substitutions are made there results

$$Z_7 = 7.43 + j\,45.4 \text{ calculated}$$
$$= 22.7 + j\,41.3 \text{ tested.}$$

The writer wishes to thank the Pennsylvania Power and Light Company, the Consumers Power Co., the Appalachian Electric Power Co., and the Alabama Power Co., for their assistance in making the tests for the confirmation of the analysis; also the Hydro-Electric Power Commission of Ontario for permission to use its test data for the sample calculation.

D. Appendix

General Solution. For those who desire a more general solution the following is offered. Unfortunately this solution requires one assumption and that is that the earth is homogeneous so that the earth conductivity is everywhere uniform. Consider in Fig. A-1 a system of "m" line wires (indicated by number subscript) and "n" ground wires (indicated by letter subscript). The circuit voltage of the line wires is E_1, E_2, etc. and of the ground wires is 0. Then

$$\left.\begin{aligned}
E_1 &= I_1 A_1 + I_2 B_{12} + I_3 B_{13} + \ldots \\
&\quad + I_a B_{1a} + I_b B_{1b} + \ldots \\
E_2 &= I_1 B_{12} + I_1 A_2 + I_3 B_{23} + \ldots \\
&\quad I_a B_{2a} + I_b B_{2b} + \ldots \\
E_3 &= \text{etc.} \\
\text{etc.} &=
\end{aligned}\right\} \quad (21)$$

$$\left.\begin{aligned}
0 &= I_1 B_{a1} + I_2 B_{a2} + I_3 B_{a3} + \ldots \\
&\quad + I_a A_a + I_b B_{ab} + \ldots \\
0 &= I_1 B_{b1} + I_2 B_{b2} + I_3 B_{b_g} + \ldots \\
&\quad I_a B_{ab} + I_b A_b + = \\
0 &= \text{etc.} \\
\text{etc.} &=
\end{aligned}\right\} \quad (22)$$

The solution of this set of equations gives the current distribution among the line and ground wires and the net impedance voltage for any circuit configuration.

The process of solution is as follows. Solve those equations (22) involving the ground wire currents in terms of the line wire currents. Substitute these values of ground wire currents into the equations (21) for the line voltage and then solve for the line wire currents.

In this set of equations

A = self impedance of conductor with ground return.
$A = Z_w + Z_g$
Z_w = self impedance of conductor with image return.
Z_g = Carsonian self impedance of the earth return.
B = mutual impedance between pairs of conductors with ground return.
$B = M_w + M_g$
M_w = mutual impedance of a pair of conductors with image return.
M_g = Carsonian mutual impedance of a pair of conductors with earth return.

The quantities A and B are to be taken for the full

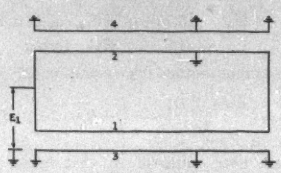

Fig. D-1

length of the line. The application of the equations requires that the average height and spacing of the conductors be known; and that the contact resistance between earth and wire be known, and properly inserted into the expressions.

For any given configuration of line wires and ground wires the results as obtained by this method can be transposed into those obtained in section A by assuming transportation of line conductors and that θ is small enough that Z_g can be used for M_g.

If it is assumed that the line wires are transposed so that they each carry the same current the zero phase sequence reactance is

$$\left.\begin{aligned}
Z_o &= A + 2\,B && \text{no ground wire} \\
&= A + 2\,B - 3\,\frac{I_4}{I_1} B_4 && \text{1 ground wire} \\
&= A + 2\,B - 3\,\frac{I_4 B_4 + I_5 B_5}{I_1} && \text{2 ground wires} \\
&= A + 2\,B - 3\,\frac{I_4 B_4 + I_5 B_5 + I_6 B_6}{I_1} && \text{3 ground wires}
\end{aligned}\right\} \quad (23)$$

In these equations I_4, I_5, etc. are the ground wire currents and are obtained from the equations (22) by determinant solution. The quantities B_4, B_5, etc. are the average mutual impedances between each of the ground wire and the line wires. If an average value for mutual impedance between line wires and ground wires were used these equations would, as just mentioned, reduce to those developed under section A.

Parallel Circuits. The solution for parallel circuits will be indicated in a general way only. In Fig. D-1 a fault is indicated on one circuit. In this analysis the line conductors are considered to be transposed so that the quantities A and B involving the line conductors and ground wires may be taken as average values. Note that A and B represent circuit values and not unit values. The method of solution is as follows.

At left of fault

$$\left.\begin{aligned} E_1 &= I_1 A_1 + I_2 B_{12} + I_3 B_{13} + I_4 B_{14} + E \\ E_2 &= I_1 B_{12} + I_2 A_2 + I_3 B_{23} + I_4 B_{24} \end{aligned}\right\} \quad (24)$$

$$\left.\begin{aligned} 0 &= I_1 B_{13} + I_2 B_{23} + I_3 A_3 + I_4 B_{34} \\ 0 &= I_1 B_{14} + I_2 B_{24} + I_3 B_{34} + I_4 A_4 \end{aligned}\right\} \quad (25)$$

At right of fault

$$\left.\begin{aligned} E_1' &= I_1' A_1' + I_2' B_{12}' + I_3' B_{13}' + I_4' B_{14}' \\ E_2' &= I_1' B_{12}' + I_2' A_2' + I_3' B_{23}' + I_4' B_{24}' \end{aligned}\right\} \quad (26)$$

$$\left.\begin{aligned} 0 &= I_1' B_{13}' + I_2' B_{23}' + I_3' A_3' + I_4' B_{34}' \\ 0 &= I_1' B_{14}' + I_2' B_{24}' + I_3' B_{24}' + I_4' A_4' \end{aligned}\right\} \quad (27)$$

$$E = E_1' - E_2'$$

Solve equations (27) for the ground wire currents I_3' and I_4'. Substitute these values in equations (26)—and note that $I_1' = I_1$, and $I_2' = -I_1$. The value of E is now found.

Now solve equations (25) for the ground wire currents I_3 and I_4. Substitute these together with the value of E in equations (24) and then solve for I_1 and I_2. The zero phase sequence impedance is $\dfrac{E_1}{I_1 + I_2}$ not forgetting that $E_2 = E_1$.

Discussion

P. A. Jeanne: The method of developing the formulas for zero sequence impedance given by the author in the main body of the paper, involves initially an arbitrary separation of the impedance of a circuit formed by an overhead wire with ground return into two parts, one assigned to the overhead portion and the other to the underground portion of the circuit. In the end it is then necessary to rearrange or combine the terms to form impedances of complete ground return or metallic circuits so that the terms in the formulas will be measurable quantities. It therefore seems to me that the method outlined in the appendix, which only involves the use of complete ground return circuit impedances, is the preferable and more direct. The Joint Subcommittee on Development and Research of the N. E. L. A. and Bell System has been using formulas developed by this latter method.

With regard to the influence of earth resistivity, I should like to point out that although large variations in this quantity produce relatively small variations in the zero sequence impedance and even less in total fault current it does affect ground wire currents quite appreciably; for example, taking a 220-kv. line with two high conductivity (186,000-cir. mil ACSR) ground wires, a change in earth resistivity from 1,000 to 100,000 ohms per cm. cube (conductivity of 10^{-12} to 10^{-14} abmhos per cm. cube) changes the zero sequence impedance about nine per cent, while it changes the ground wire current about 26 per cent on the assumption of the same fault current in the two cases.

F. J. Grueter: One of the simplifying assumptions made by Mr. Clem in developing his method of calculating zero sequence impedance is that of uniform current along the length of the ground wire, and he cites some tests on the Turner-Logan line which agree with this assumption. This condition holds only when the ground wire is terminated in impedances to ground which are small compared to the resistance to ground of the tower footings. Such an assumption is valid when the ground wires are connected to station ground at both ends and the fault is outside of the section of line for which the zero sequence impedance is being determined.

Recently the Joint Subcommittee on Development and Research of the N. E. L. A.-Bell System made a series of tests to determine the variation of ground wire current along the length of the line. Fig. 1 gives some of the results of these tests which were made on a 220-kv. transmission line, equipped with two 200,000-cir. mil ACSR ground wires in a region where the earth

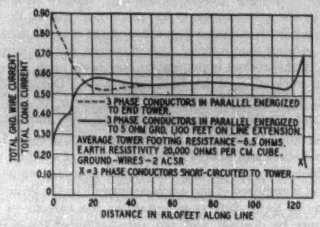

FIG. 1—VARIATION OF GROUND-WIRE CURRENT ALONG A TRANSMISSION LINE WHEN ENERGIZED WITH GROUND-RETURN CURRENT

resistivity was about 20,000 ohms per cm. cube, (earth conductivity of 5×10^{-14} abmhos per cm. cube) and where the average tower footing resistance is about 6.8 ohms. At one end of the line, where the ground wires were terminated, the three-phase conductors were multiplied and energized to ground, while at a point about 23 miles distance, the conductors were short-circuited together and to ground through a tower. The ground wires extended about three miles beyond this point of grounding.

The ground wire current observed was a stepped function, due to the lumped leakage of the ground wires at the towers. The smooth curves on the plot were drawn giving averaged results and the general shape of these curves, including the overshoot, has been checked from theoretical considerations. The solid line presents the ground wire currents observed when the conductors were energized to a 5-ohm ground about 1,100 ft. beyond the end of the line. The dotted line gives the values when the line is energized directly to the end tower footing. At the energized end, the increase or decrease of ground wire currents, over the value at the middle of the line, persists for a considerable distance. Due to the continuation of the ground wires beyond the point of fault, the change in ground wire current at the fault end was observed over considerably shorter distances.

It should be pointed out that the distribution of the currents in the ground wire at the energized end is not representative of the condition which exists when a ground wire is connected to the

station ground of low resistance, relative to the tower footing resistance. Under the latter arrangement the current in the ground wire entering the station would be equal to that in the middle of the line. Under these conditions, the ground wire current, with the exception of the case of a fault very close to the substation, will be of substantially constant value practically throughout the length of the line. In the case where the ground wire terminates on a tower not connected to the station structure, the solid curve of the slide would be typical of the resulting conditions.

The results of tests which are referred to here are a part of a study which has been conducted by the Joint Subcommittee on the general subject of the magnitude and distribution of currents induced in grounded conductors due to their presence in a magnetic field produced by current in a ground return circuit. The effects of these currents in grounded conductors on the zero sequence impedance of power circuits and of their effects on the voltages induced in nearby communication circuits have also been studied and it is expected that this information will be made generally available to the industry in a short time.

George Wascheck: In determining the zero phase sequence impedance of transmission lines Mr. Clem treats the current in the ground wires as a constant quantity throughout the length of the line. Where ground wires are connected to a station ground of low resistance compared to tower grounds, and the fault is distant from the station it is quite accurate for the calculations of the zero phase sequence impedance to assume the current in the ground wire to be constant along the line. However, with a fault within a few miles of the station the constant value of ground wire current may not exist for any appreciable length. A theoretical investigation has been made, in conjunction with work of the Joint Subcommittee on Development and Research of the N. E. L. A. and Bell System, of the possible effect of this condition on the zero phase sequence impedance.

For the solution of the impedance of such a system, in which the ground wires and towers assume the form of a ladder network (tower distances and ground resistances assumed constant) an evaluation is required of the average current in the ground wires. By means of finite difference equations the current in the nth section may be derived and the summation of the currents in all the sections becomes a geometrical series which may readily be evaluated. This, divided by the number of sections, becomes the average or equivalent constant current to be inserted in the circuit equations.

The resulting formula for the zero phase sequence impedance has been put into the following form, consisting of three terms:

$$Z_O = Z_P - n\frac{Z_{PG}^2}{Z_G} + n\,Z_G\left[1-\frac{Z_{PG}}{Z_G}\right]^2\left[\frac{m_1^{2K}-1}{K\;m_1^{2K-1}(m_1^2-1)}\right]$$

where:

Z_O = zero phase sequence impedance.

Z_P = grounded impedance of one phase wire with the other phase wires carrying equal currents, and in the absence of ground wires.

Z_G = series impedance of ground wires with earth return. (Equal and constant currents in all ground wires, no current in phase wires).

Z_{PG} = mutual impedance between ground wires and phase wires.

R = resistance of tower to ground.

s = length of section between towers in miles.

K = number of sections.

n = number of phase wires.

$$m_1 = 1 + \frac{s\,Z_G}{2\,R} + \frac{1}{2}\sqrt{\frac{4\,s\,Z_G}{R} + \left(\frac{s\,Z_G}{R}\right)^2}$$

The formula is based on the assumption of solidly grounded ground wires at Station C and a fault to a tower with the line extending indefinitely beyond, although the formula for any combination of the end groundings is determinable.

It is seen that the first term is the zero phase sequence impedance in the absence of ground wires, the second, the additional term that would be obtained if the ground wires were solidly grounded at each end of the circuit involved, and the third, the contribution of the varying ground wire currents near the point of grounding.

An application of this formula was made to a three-phase transmission line of flat construction of 28.5-ft. spacing with two ACSR ground wires of the same spacing placed 13.9 ft. symmetrically above the phase wires. The three factors in the above equation were evaluated for assumed tower ground resistances of 5 and of 50 ohms each. The results may be seen in Fig. 2. A is the zero phase sequence impedance per mile with no ground wire, B the term to be subtracted vectorially with constant current assumed in the ground wire (the ends solidly grounded), and C

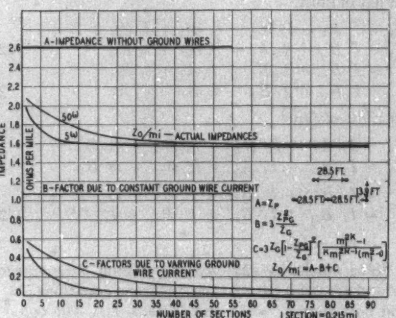

FIG. 2—ZERO-PHASE SEQUENCE IMPEDANCES

(Variation of zero-phase sequence impedance per mile and its components *versus* circuit length for two values of tower ground resistances. Ground wire solidly grounded at station O. Fault taken to any tower with line indefinitely long beyond used transmission line configuration shown below with phase wires 70 ft. above ground. Frequency 60 cycles per sec.)

the correction due to the varying currents in the end sections near the fault. Parts A and B are constant but C is variable with length and with tower ground resistance. The curve marked $Z_O/Mi.$ is the vector sum of all three components and is the actual zero phase sequence impedance per mile. Thus, with a fault five sections from the station (about one mile) the zero phase sequence impedance per mile with 5-ohm grounds is 1.74 ohms per mile; with 50-ohm grounds 1.96 ohms per mile. This is in contrast to 1.56 ohms per mile should the station ground wire current have been assumed. The following table shows a comparison between that of the net zero phase sequence impedance per mile against that assumed with the constant current of solidly grounded ground wires.

It is to be observed that the error is quite small for a fault distant from the station and that for the two cases illustrated the effect of increased current in the ground wire near the fault is to increase the zero phase sequence impedance. It appears that the additional impedance of each tower to ground over that were the fault tower solidly grounded is sufficient to offset the reduction in impedance to be expected by the larger ground wire current.

No. of sections	Length miles	Zo per mi. with constant ground wire current		Zo per mi. allowing for varying current in ground wire		Per cent increase in magnitude	
		5 ohms or 50 ohms		5 ohms	50 ohms	5 ohms	50 ohms
2	1/2	1.56/75.6		1.92/67.4	2.05/67.8	23.0	31.5
5	1	"		1.74/68.4	1.96/67.4	11.5	25.5
9	2	"		1.65/71	1.86/67.6	5.8	19.2
14	3	"		1.61/72	1.77/68.2	3.2	13.5
23	5	"		1.59/74	1.67/69.5	1.9	7.0
47	10	"		1.58/74.5	1.61/72.2	1.3	3.2

H. M. Trueblood: Carson's expression for the self-impedance of a ground-return circuit is

$$Z_{iw} + j\,2\,\omega \log \frac{\rho''}{a} + 4\,\omega \int_0^\infty \left(\sqrt{\mu^2 + j} - \mu\right) e^{-2h'\mu}\, d\,\mu$$

Mr. Clem speaks of the second term in this formula as representing the reactance of the wire with image return. More fully expressed, it is the reactance, external to the wire, of the complete ground-return circuit *on the hypothesis that the earth is of infinite conductivity*. On this hypothesis, the third term of the formula would vanish, and there would be neither flux nor current in the body of the earth. If the wire also were of infinite conductivity, the second term would represent the entire impedance, since Z_{iw}, which is the internal impedance of the wire, would in that case likewise be zero. All flux would thread between the surface of the wire and that of the earth, there being none within the material of either, and, of course, no resistance anywhere.

Should we then regard the first and third terms of the formula as "correction" terms, introduced to take care of departures from the ideal condition in which both the wire and the earth have infinite conductivity? This is a permissible point of view, but the third term does not strictly represent an "impedance of the earth," in the sense in which Z_{iw} represents an impedance of the wire, *viz.*, its *internal* impedance. The third term, as Carson points out, formulates the effect of the finite conductivity of the earth. This effect does not consist solely in causing flux and current to appear within the body of the earth; it consists also, in part, in changing the amount and distribution of the *external* flux, *i. e.*, the flux between the surfaces of the wire and the earth. The third term, therefore, is not directly or simply identified with an impedance associated with the earth.

However, it is entirely possible to assign a definite meaning to the expression "internal impedance of the earth" and the conception is of some value in the physical visualization of a circuit consisting of a straight long horizontal wire with ground return. In such a circuit, the internal impedance of the earth, per unit length of the circuit, may be taken as the electric force, parallel to the wire, at the earth's surface directly below the wire. This is identical with the voltage induced per unit length in a second grounded conductor lying on the earth's surface directly below the first wire, when unit current flows in the ground-return circuit of which the first wire is one side.

With this understanding, the *total* impedance of a ground-return circuit can be written

$$Z_{iw} + j\,\omega\,\Phi_e + Z_{ie}$$

in which Z_{iw} is the internal impedance of the wire, identical with the first term of Carson's expression, Z_{ie} is the internal impedance of the earth and $j\,\omega\,\Phi_e$ is the voltage drop due to the flux threading between the surface of the wire and the surface of the earth. The last two terms are not the same respectively as the last two terms of Carson's formula, although, of course, the *sum* of the last two terms is the same in both expressions.

This expression is of exactly the same form as appears in writing the total self-impedance of a circuit consisting of two *wires*. It is true, of course, that Z_{ie}, the internal impedance of the earth, depends on the height of the wire above the earth, whereas, with wires, we commonly think of the internal impedance as a property of the wire alone, at a given frequency. This difference is apparent rather than real, however, as is obvious if we think for a moment of such problems as that of the proximity effect in two closely adjacent wires at high frequencies, where the current distribution, and hence the internal impedance, of either wire, is affected by its separation from the other. The ground-return circuit problem, in fact, is really nothing but a case of proximity effect, as can be seen by first imagining two wires forming a "metallic" circuit, and then allowing the radius of one to increase indefinitely while the distance between the surfaces remains constant. The wire with indefinitely expanding radius ultimately represents the earth, and the "proximity effect" consists essentially in the resulting non-uniform current distribution in this conductor, and associated phenomena. Cases can be easily imagined in which a noticeable proximity effect would be maintained in the wire whose radius remains constant, thus making the analogy between the two-wire circuit and the wire-earth circuit complete. In other cases, also, the usual simple conception of internal impedance becomes impossible and must be generalized; for example, in circuits of which railroad rails form a part.

C. F. Wagner: Mr. Clem's paper is particularly valuable because it collects together a large mass of data corroborating Carson's formulas for the calculation of ground return circuits. His Table C-2 enumerates the available data for ground conductivity which forms a basis for the calculation of the ground return impedances.

I have been using Carson's formulas, also, for this purpose but the form is somewhat different from that which Mr. Clem presents. This is given in the April, 1931 issue of *The Electric Journal*, Vol. 28, No. 4, pages 241-244.

S. Whitehead and P. D. Morgan: The present paper is a valuable contribution to the important subject of the impedance of a line when one or more phases is connected to earth as may occur under fault conditions. The author has performed a real service to engineering in publishing Table C1 by means of which Carson's formula may be readily computed for any special case. The analysis employed by the author in treating the effect of earth wires in parallel with the earth differs from ours in that the partial impedance coefficients the L's and M's of the first part of the A's and B's of the Appendix are eliminated by expressing them in terms of complete coefficients relating to go and return circuits, *i. e.* the Z's. We, on the other hand (*J. I. E. E.*, March 1930, page 367 *et seq*) have used the partial coefficients as determinable quantities by relating them to the plane of the earth as assumed neutral plane. The present author's method is superior to ours in that the application of formulas such as Carson's or Pollaczek's is simpler.

In the case of Rudenberg's formula for the impedance of line and earth return we may put—

$$\text{Impedance} = \left[\pi^2 f + \omega j \left[1 + 2 \log h \left(\frac{2}{y\,K\,h}\right)\right] + R + 2\,\omega j \log h \frac{h}{r}\right] 10^{-9} \text{ ohms/cm.}$$

Where

f = frequency

$\omega = 2\,\pi\,f$

$K = 2\pi\sqrt{f/u}$

u = specific resistance of the earth in abohms

h = height of conductor in cm.

r = over-all radius of conductor in cm.

y = 1.781

R = resistance of conductor in abohms per cm.

The first two terms may be taken as referring to the earth and the second to the conductor with the earth as a neutral plane. In such a case our method and that of the author lead to identical results. Our method is, however, easier to apply to a multiple earthed earth wire or an underground cable where the current in the earth wire or cable sheaths varies along the line. Both the methods suffer from the disadvantage that they may be rigorously applied only to concentrated currents such as in a conductor since otherwise the distributive and commutative laws of the coefficients no longer hold. Fortunately, however, the aerial field of the earth current is usually negligible while the earth field of the aerial conductors is rapidly attenuated and may also be neglected. We used Rudenberg's formula on account of simplicity but for most lines it agrees fairly closely with those of Carson and Pollaczek.

The theory of Rudenberg is a simplification of the more rigorous methods of the other two authors and its errors appear when the induced voltage at a distance from the power line is required, the Rudenberg theory giving too concentrated a current distribution in the earth. For the case of the English "grid" lines examined by us (*ibid.* pp. 406-7) it has been verified that the present author's methods give the same results as ours and this will be the case for nearly all power lines, divergence is only to be anticipated for lines at a great height from the ground or for frequencies considerably above those used for normal power transmission. It would appear, therefore, in view of the extensive tests made by the author in America and ourselves in England that the calculations of the earth or zero-phase impedance may be approached with confidence by engineers.

We wish, however, to draw attention to the fact that the earth wire current may vary considerably along the line with a high-conductivity earth wire such as is used in England unless the fault resistance to the earth wire is the same as the "earth plate" resistance, at the fault. In one of our tests there was only an additional 1¼ ohms fault resistance to earth as compared with the resistance to the earth wire but theory and experiment showed that the earth wire current fell from 70 per cent to about 30 per cent from the end to the middle of the line. The effect on the impedance is however, usually negligible as shown in our paper (*ibid.*). The same effect occurs with cables and may produce large voltage rises at the bonds adjacent to the fault.

In our paper the actual fault impedance but not the zero-phase impedance for a three-phase line was given. This omission was rectified in the discussion (*J. I. E. E.*, June 1930, page 779) where it is also mentioned that short-circuit calculations may be much simplified in practise by treating all the impedances as pure reactances, that is, neglecting the phase angles. The errors introduced by this on fairly short lines would probably not exceed about 10 per cent so that this approximation may often be useful.

In our tests the value adopted for the resistivity of the earth was based partly on measurements of the voltage induced in neighboring telephone lines which are very sensitive to the magnitude of the effective resistivity. A certain amount of agreement was observed with surface resistivity measurements and in considering similar tests made elsewhere in Europe it appears possible that some correlation between the a-c. resistance and the known electrical properties of various geological strata might be obtained. Has the author been in a position to attempt such a correlation and, if so, has he come to any conclusion as to the feasibility of this?

Finally we should like to mention, as in our paper, that a second earth wire placed below the conductors appears to have considerable advantages notably from the standpoint of reducing inductive interference from short-circuit currents.

Simultaneous Faults on Three-Phase Systems

BY EDITH CLARKE*

Associate, A. I. E. E.

Synopsis.—*The method of symmetrical components now so extensively used to determine short-circuit currents and stability limits during transient conditions for three-phase transmission systems when a fault involving one or more of the three conductors occurs at any one point of the system, has been extended to apply to three-phase systems during simultaneous faults at two or more points of the system.*

A general equivalent circuit is developed to replace, in the positive phase diagram, two simultaneous faults involving any combination of the six conductors. An approximate equivalent circuit to be used with the d-c. calculating table when resistance is neglected is also given.

Special equivalent circuits are employed to replace two simultaneous faults and the lines upon which they occur, when the lines are unloaded feeders radiating from a common point or lines of equal impedances bussed at both ends.

The methods and formulas given in this paper were developed in answer to such questions as the following:

1. Which is a more severe shock to a system, a double line-to-ground fault on one circuit or two single line-to-ground faults occurring simultaneously on two separate circuits?

2. Do simultaneous double line-to-ground faults which involve the same phases, a and b, on two circuits, result in more or less ground current than faults which involve phases a and b on one circuit and phases b and c on the other?

* * * * *

WHEN double circuit towers carry two three-phase circuits, disturbances may involve one or more conductors of each circuit. From published records[1] of the number of flashovers on double circuit towers which have tripped out both circuits, and from opinions expressed by operating engineers of various power companies who have been consulted, it seems reasonable to conclude that in the neighborhood of 20 per cent of the faults on double circuit towers involve conductors of both circuits. In addition there are instances where faults in substations have involved conductors of circuits not on the same towers.

It seems worth while therefore, to have in convenient form, methods for calculating short-circuit currents, and of determining the stability limit of a system when faults occur simultaneously at two separate and distinct points of the system. The general case will cover simultaneous faults at any two points of the system, involving one, two or three conductors at each point, while short circuits on parallel circuits on the same tower will be a special case in which the two points of fault are symmetrical with respect to the system, although they will not be symmetrical with respect to ground unless the faults are on the same phase or phases in both circuits.

Mr. C. L. Fortescue has shown[2] that any system of three vectors may be replaced by three sets of balanced components. The fundamental equations expressing actual currents and voltages in terms of their symmetrical components, and expressing the symmetrical components of current and voltage in terms of the actual currents and voltages respectively are given in Appendix A.

SINGLE FAULT

When one or more of the conductors of a three-phase circuit becomes grounded, the voltages to ground on the three conductors and the currents in the three lines are no longer balanced. If the three unbalanced voltages to ground, V_a, V_b and V_c, at the point of fault, and three line currents flowing into the fault, I_a, I_b and I_c, are resolved into their symmetrical components, it will be found that there are enough relations existing between these six components to allow the positive component of voltage at the fault, V_{a1}, to be expressed in terms of the positive component of current in the fault, I_{a1}, and the zero and negative phase impedances, Z_0 and Z_2 respectively, viewed from the fault. To do this requires five equations.

Since there is no generated zero nor negative phase sequence voltage, and the positive direction for all component currents is taken towards the fault,

$$0 - V_{a0} = I_{a0} Z_0 \tag{1}$$

$$0 - V_{a2} = I_{a2} Z_2 \tag{2}$$

At a point of fault there are certain relations between the positive, negative, and zero components of current which flow into the fault, and also between the positive, negative, and zero components of voltage at the fault. These relations between the components of current and voltage, depending upon the type of fault, provide additional equations connecting the unknowns. For all types of fault there will be three independent equations connecting the components of current and voltage at the point of fault. These equations are tabulated in Table I, and the manner of their derivation shown in Appendix B.

From five equations with six unknowns, the four unknowns V_{a0}, V_{a2}, I_{a0} and I_{a2} may be eliminated, and V_{a1} expressed in terms of I_{a1} thus:

$$V_{a1} = K I_{a1} \tag{3}$$

where K is a function of Z_0 and Z_2 depending upon the type of fault. For line-to-ground faults $K = Z_0 + Z_2$, for line-to-line faults $K = Z_2$, and for double-line-to-ground faults $K = \dfrac{Z_0 Z_2}{Z_0 + Z_2}$.

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*Central Station Engg. Dept. General Electric Company, Schenectady, N. Y.

1. For references see Bibliography.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

Equation (3) will be satisfied, and the positive phase current and voltage distribution may be determined if the fault in the positive phase diagram is replaced[3] by the shunt impedance K, the value of K being determined by Z_0, Z_2 and the type of fault.

Two Simultaneous Faults

The method of procedure outlined above for determining the positive phase current and voltage distribution when a fault involves one or more of the three conductors at a single point, may be followed for simultaneous faults at two distinct points.

If the two points of fault are C and D and the conductors at C are a, b and c and those at D are α, β and γ (a and α being conductors of the same phase, as are b and β, and c and γ), the six components of voltage and current at C will be V_{a0}, V_{a1}, V_{a2}, I_{a0}, I_{a1} and I_{a2}, and those at D will be $V_{\alpha 0}$, $V_{\alpha 1}$, $V_{\alpha 2}$, $I_{\alpha 0}$, $I_{\alpha 1}$ and $I_{\alpha 2}$. It will be shown that there are ten independent equations connecting these twelve unknowns. It is proposed to eliminate the eight unknowns $V_{\alpha 0}$, V_{a0}, V_{a2}, $V_{\alpha 2}$, I_{a0}, $I_{\alpha 0}$, I_{a2} and $I_{\alpha 2}$ and to reduce the number of equations to two, expressing the positive components of voltage, V_{b1} and $V_{\alpha 1}$, in terms of the positive components of current, I_{a1} and $I_{\alpha 1}$, and the known zero and negative impedances.

Zero Phase System. With a single fault, the zero phase current in the fault varies directly as the zero phase voltage to ground at the fault. When there are two simultaneous faults, the zero phase current in either fault depends upon the zero phase voltages at both points of fault. In order to readily express the two zero phase voltages in terms of the two zero phase currents, the zero phase impedance diagram[4] will be simplified, remembering that all points of zero potential for zero phase voltage may be considered bussed at the same point, S. Equivalent circuits to replace two parallel transmission lines with mutual impedance between them in the zero phase system are given in Appendix C. When there are more than two parallel lines it may not be possible to accurately represent them by a simple equivalent circuit in the zero phase system but an approximate equivalent circuit may usually be obtained. When the zero phase system can be represented by an equivalent impedance diagram, it is always possible to reduce it to an equivalent Y, connecting the two points of fault, C and D, and the zero potential point, S. In some systems this may be done by $\Delta - Y$ or $Y - \Delta$ transformations,[5] but in complicated systems it may be necessary to use an a-c. or d-c. calculating table to determine the branches of the equivalent Y or Δ connecting the points C, D, and S.

Determination of Equivalent Y for the Zero Phase Impedance Network by Means of the Calculating Table. For an exact determination, the use of an a-c. calculating table such as the M. I. T. Network Analyzer[6] is necessary, in which case the currents and voltages measured as described below would have to be in vector form, requiring the use of a watt-meter as well as an ammeter and voltmeter. For many purposes the resistance in the networks can be neglected and the impedances considered to consist of reactance alone. In this latter case the d-c. calculating table may be used. The general procedure in either case is as follows:

(a) Set up the zero phase impedance network on the calculating table. Apply a voltage to ground, V_c, at point C with point D ungrounded, but all zero potential points, S, grounded. Measure the total ground current, I_s. Current I_s at C is the same as the total ground current I_s at S.

(b) Apply a voltage to ground, V_c, at C with point D grounded but all zero potential points, S, ungrounded. Measure the current I_d at D.

(c) Apply a voltage to ground, V_d, at point D with point C ungrounded, but all zero potential points, S, grounded and measure the current I_s.

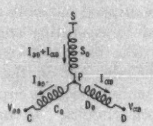

Fig. 1—Equivalent Y for Zero Phase Sequence Network

If the branches of the equivalent Y are C_0, D_0, and S_0, then

$$\frac{V_c}{I_s} = C_0 + S_0 \qquad \textbf{(a)}$$

$$\frac{V_c}{I_d} = C_0 + D_0 \qquad \textbf{(b)}$$

$$\frac{V_d}{I_s} = D_0 + S_0 \qquad \textbf{(c)}$$

From equations (**a**), (**b**) and (**c**) the branches of the Y are determined.

Fig. 1 represents the equivalent Y for the zero phase network connecting C, D and S with its branch impedances, C_0, D_0 and S_0. Let the positive direction for zero phase currents be taken towards the faults C and D. Then since there is no generated zero phase voltage, by superposing the voltages due to the two component currents, the following equations are obtained:

$$0 - V_{a0} = - V_{a0} = I_{a0}(C_0 + S_0) + I_{\alpha 0} S_0 \qquad (5)$$

$$0 - V_{\alpha 0} = - V_{\alpha 0} = I_{\alpha 0} S_0 + I_{a0}(D_0 + S_0) \qquad (6)$$

Equations (**5**) and (**6**) express the zero components of voltage at the two points of fault in terms of the two zero phase currents flowing into the faults and the branch impedances of the equivalent Y.

Rewriting equations (**5**) and (**6**) to express the cur-

rents I_{a0} and I_{a0} in terms of V_{a0} and V_{a0}, or from Fig. 1 directly:

$$I_{a0} = -V_{a0}\frac{D_0 + S_0}{Z_{00}} + V_{a0}\frac{S_0}{Z_{00}} \tag{7}$$

$$I_{a0} = V_{a0}\frac{S_0}{Z_{00}} - V_{a0}\frac{C_0 + S_0}{Z_{00}} \tag{8}$$

where $Z_{00} = C_0 D_0 + C_0 S_0 + D_0 S_0$ (9)

Equations (7) and (8) are not independent of equations (5) and (6). There are four unknowns, V_{a0}, V_{a0}, I_{a0} and I_{a0} in the zero phase system and two independent equations connecting them.

Negative Phase System. In the negative phase systen, just as in the zero phase system, there is no generated voltage, and the positive direction for negative phase currents is taken from the neutrals of the machines towards the faults. The neutrals of generators and loads are points of zero potential for the negative phase system. The negative phase network[4] of a system may be reduced to an equivalent Y or Δ connecting the points of fault C and D and the points of zero potential, which may be considered bussed at a common point, S. It is important to note that S, the point of

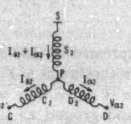

FIG. 2—EQUIVALENT Y FOR NEGATIVE PHASE SEQUENCE NETWORK

zero potential for zero phase voltage is not necessarily the same point of the actual system as S, the point of zero potential for negative phase voltage. Neither are the two P points identical. The same letters are used, however, to obtain symmetry in form for the zero and negative phase current and voltage equations.

From Fig. 2 which represents the equivalent Y for the negative phase network connecting C, D and S, the two negative phase voltages may be expressed in terms of the two negative phase currents and the impedances of the Y, thus:

$$-V_{a2} = I_{a2}(C_2 + S_2) + I_{a2}S_2 \tag{10}$$

$$-V_{a2} = I_{a2}S_2 + I_{a2}(D_2 + S_2) \tag{11}$$

Rewriting equations (10) and (11) to express the currents I_{a2} and I_{a2} in terms of V_{a2} and V_{a2}, or from Fig. 2 directly:

$$I_{a2} = -V_{a2}\frac{D_2 + S_2}{Z_{22}} + V_{a2}\frac{S_2}{Z_{22}} \tag{12}$$

$$I_{a2} = V_{a2}\frac{S_2}{Z_{22}} - V_{a2}\frac{C_2 + S_2}{Z_{22}} \tag{13}$$

where $Z_{22} = C_2 D_2 + C_2 S_2 + D_2 S_2$ (14)

Equations (12) and (13) are not independent of equations (10) and (11). There are four unknowns V_{a2}, V_{a2}, I_{a2} and I_{a2} in the negative phase system, and two independent equations connecting them.

Relations between Components of Current and of Voltage at Two Points of Fault. At each point of fault, the relations between the components of current flowing into the fault, and between the components of voltage to ground at the fault are independent of the rest of the system. Table I gives three equations connecting the components of current or of voltage at a point of fault for various types of fault. When there are two points of fault, there will be three equations connecting components of current or voltage at each point. By replacing a, b and c in Table I by α, β and γ respectively, the three equations connecting components of current or voltage at the second point of fault, D, are obtained.

Simultaneous Equations. Three equations expressing relations between the components of current or voltage at each of the two points of fault, two equations connecting zero phase currents and voltages and two connecting negative phase currents and voltages give the ten equations needed to eliminate the eight unknowns V_{a0}, V_{a0}, I_{a0}, I_{a0}, V_{a2}, V_{a2}, I_{a2} and I_{a2} so that the positive components of voltage V_{a1} and V_{a1} may be expressed in terms of the positive components of current, I_{a1} and I_{a1}, and the known zero and negative phase impedances of the system. Since the ten equations are linear the two resulting equations can be put in the form:

$$V_{a1} = k\,I_{a1} + m\,I_{a1} \tag{15}$$

$$V_{a1} = n\,I_{a1} + l\,I_{a1} \tag{16}$$

where k, l, m and n depend upon the branch impedances of the equivalent Y's replacing the negative and zero phase networks, and the particular combination of conductors involved in the simultaneous faults. It should be noted that k, l, m and n do not involve positive phase impedances.

Equations giving k, l, m and n for faults which involve various combinations of the six conductors are derived in Appendix D and tabulated in Table II.

Positive Phase System. The positive phase system differs from the negative and zero phase systems because positive phase sequence voltages are generated at various points of the positive phase system. In general there will be as many separate positive phase generated voltages as there are separate generators or synchronous motors on the system. When the operating conditions just previous to the occurrence of the faults are known, these generated voltages are determined in magnitude and in phase. For steady state calculations, the excitation voltage or voltage behind synchronous reactance is required, while for transient calculations, the voltage behind transient reactance should be known. However, in either case the internal generated voltages are determined from the given operating conditions.

TABLE I

Relations between the symmetrical components of current in the fault and between the components of voltage to ground at the fault for various types of fault.

The operators a and a^2 have been defined in equations (**13a**) and (**14a**).

Case	Type of fault	Phases involved	Current relations	Voltage relations
A	(a) Line to ground	a	$I_{a0} = I_{a1}, I_{a2} = I_{a1}$	$V_{a1} = -(V_{a0} + V_{a1})$
	(b) Line to ground	b	$I_{a0} = a^2 I_{a1}; I_{a1} = a I_{a0}$ $I_{a2} = a I_{a1}; I_{a1} = a^2 I_{a2}$	$V_a = -(a V_{a0} + a^2 V_{a2})$
	(c) Line to ground	c	$I_{a0} = a I_{a1}; I_{a1} = a^2 I_{a0}$ $I_{a2} = a^2 I_{a1}; I_{a1} = a I_{a2}$	$V_{a1} = -(a^2 V_{a0} + a V_{a2})$
B	(a) Line to line	b, c	$I_{a0} = 0, I_{a2} = -I_{a1}$	$V_{a2} = V_{a1}$
	(b) Line to line	a, c	$I_{a0} = 0$ $I_{a2} = -a I_{a1}; I_{a1} = -a^2 I_{a2}$	$V_{a2} = a V_{a1}; V_{a1} = a^2 V_{a2}$
	(c) Line to line	a, b	$I_{a0} = 0$ $I_{a2} = -a^2 I_{a1}; I_{a1} = -a I_{a2}$	$V_{a2} = a^2 V_{a1}; V_{a1} = a V_{a2}$
C	(a) Double line to ground	b, c	$I_{a1} = -(I_{a0} + I_{a2})$	$V_{a0} = V_{a1}, V_{a2} = V_{a1}$
	(b) Double line to ground	a, c	$I_{a1} = -(a I_{a0} + a^2 I_{a2})$	$V_{a0} = a^2 V_{a1}; V_{a1} = a V_{a0}$ $V_{a2} = a V_{a1}, V_{a1} = a^2 V_{a2}$
	(c) Double line to ground	a, b	$I_{a1} = -(a^2 I_{a0} + a I_{a2})$	$V_{a0} = a V_{a1}; V_{a1} = a^2 V_{a0}$ $V_{a2} = a^2 V_{a1}; V_{a1} = a V_{a2}$
D	(a) Three-phase	a, b, c	$I_{a0} = 0$	$V_{a1} = 0, V_{a2} = 0$
	(b) Three-phase to ground	a, b, c		$V_{a1} = 0, V_{a0} = 0$ $V_{a2} = 0.$

At point C there is a positive phase voltage, V_{a1}, to neutral and a positive phase current, I_{a1}, flowing into the fault. At point D there is a positive phase voltage, V_{a1}, to neutral and a positive current, I_{a1}, flowing into the fault, the relations between voltages and currents being given in equations (**15**) and (**16**).

EQUIVALENT CIRCUITS REPLACING TWO SIMULTANEOUS FAULTS IN POSITIVE PHASE SYSTEM

Equations (**15**) and (**16**) may be written

$$V_{a1} = (k - n) I_{a1} + \frac{m + n}{2} (I_{a1} + I_{a1}) + \frac{m - n}{2} (I_{a1} - I_{a1}) \qquad (17)$$

$$V_{a1} = (l - m) I_{a1} + \frac{m + n}{2} (I_{a1} + I_{a1}) + \frac{m - n}{2} (I_{a1} - I_{a1}) \qquad (18)$$

The last term of (**17**) and of (**18**) may also be written

$$\frac{n - m}{2} (I_{a1} - I_{a1})$$

Case I. m and n equal.

When m and n are equal (**17**) and (**18**) become

$$V_{a1} = (k - m) I_{a1} + m (I_{a1} + I_{a1}) \qquad (19)$$

$$V_{a1} = (l - m) I_{a1} + m (I_{a1} + I_{a1}) \qquad (20)$$

The relations expressed in (**19**) and (**20**) are satisfied by substituting for the faults a Y network having the branch impedances $(k - m)$, $(l - m)$ and m connecting the points C, D and ground as shown in Fig. 3. In this case the positive phase voltages and currents of the system may be determined as in any other balanced load distribution problem: that is, by calculation, or by means of a calculating table, the equivalent Y replacing the two faults in the positive phase network.

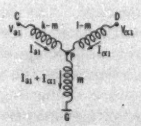

FIG. 3—EQUIVALENT Y REPLACING TWO FAULTS IN POSITIVE PHASE SEQUENCE NETWORK FOR SPECIAL CASE OF $m = n$

Case II. m and n unequal.

When m and n are not equal, the positive distribution of currents and voltage due to the faults can not be obtained by inserting an equivalent Y or Δ between the points, C, D and ground.

A. General Equivalent Circuit. Fig. 4 shows an equivalent circuit which may replace the two faults in the positive phase system. The equivalent circuit consists of a Y having branch impedances $k - n$, $l - m$ and $\frac{m + n}{2}$ connected between points C, D and F,

TABLE II

Values of k, l, m and n to be substituted in equations (15) and (16) for simultaneous faults at two points of the system. The operators a and a^2 have been defined in equations (13a) and (14a), and Z_{00} and Z_{22} in equations (9) and (14). For the sake of brevity let,

$$Z_{cs} = C_0 + S_0 + C_2 + S_2$$
$$Z_{ds} = D_0 + S_0 + D_2 + S_2$$
$$Z_{cds} = (C_0 + C_2)(D_0 + D_2) + (C_0 + C_2)(S_0 + S_2) + (D_0 + D_2)(S_0 + S_2)$$

Case E. Single Line-to-Ground Faults at Two Points.

(a) Phases a and α.

$$k = Z_{cs} \qquad m = S_0 + S_2$$
$$n = S_0 + S_2 \qquad l = Z_{ds}$$

(b) Phases b and α.

$$k = Z_{cs} \qquad m = a\,S_0 + a^2\,S_2$$
$$n = a^2\,S_0 + a\,S_2 \qquad l = Z_{ds}$$

(c) Phases c and α.

$$k = Z_{cs} \qquad m = a^2\,S_0 + a\,S_2$$
$$n = a\,S_0 + a^2\,S_2 \qquad l = Z_{ds}$$

Case F. Line-to-Line Faults at Two Points.

(a) Phases b, c and β, γ.*

$$k = C_2 + S_2 \qquad m = S_2$$
$$n = S_2 \qquad l = D_2 + S_2$$

(b) Phases a, c and β, γ.

$$k = C_2 + S_2 \qquad m = a^2\,S_2$$
$$n = a\,S_2 \qquad l = D_2 + S_2$$

(c) Phases a, b and β, γ.

$$k = C_2 + S_2 \qquad m = a\,S_2$$
$$n = a^2\,S_2 \qquad l = D_2 + S_2$$

Case G. Double Line-to-Ground Faults at Two Points.

(a) Phases b, c and β, γ.*

$$k = \frac{Z_{22}(C_0 + S_0) + Z_{00}(C_2 + S_2)}{Z_{cds}} \qquad m = \frac{Z_{22}\,S_0 + Z_{00}\,S_2}{Z_{cds}}$$

$$n = \frac{Z_{22}\,S_0 + Z_{00}\,S_2}{Z_{cds}} \qquad l = \frac{Z_{22}(D_0 + S_0) + Z_{00}(D_2 + S_2)}{Z_{cds}}$$

(b) Phases a, c and β, γ.

$$k = \frac{Z_{22}(C_0 + S_0) + Z_{00}(C_2 + S_2)}{Z_{cds} + 3\,S_0\,S_2} \qquad m = \frac{a\,S_0\,Z_{22} + a^2\,S_2\,Z_{00}}{Z_{cds} + 3\,S_0\,S_2}$$

$$n = \frac{a^2\,S_0\,Z_{22} + a\,S_2\,Z_{00}}{Z_{cds} + 3\,S_0\,S_2} \qquad l = \frac{Z_{22}(D_0 + S_0) + Z_{00}(D_2 + S_2)}{Z_{cds} + 3\,S_0\,S_2}$$

(c) Phases a, b and β, γ.

$$k = \frac{Z_{22}(C_0 + S_0) + Z_{00}(C_2 + S_2)}{Z_{cds} + 3\,S_0\,S_2} \qquad m = \frac{a^2\,S_0\,Z_{22} + a\,S_2\,Z_{00}}{Z_{cds} + 3\,S_0\,S_2}$$

$$n = \frac{a\,S_0\,Z_{22} + a^2\,S_0\,Z_{00}}{Z_{cds} + 3\,S_0\,S_2} \qquad l = \frac{Z_{22}(D_0 + S_0) + Z_{00}(D_2 + S_2)}{Z_{cds} + 3\,S_0\,S_2}$$

Case H. Three-Phase Faults at Two Points.

$$k = 0 \qquad m = 0$$
$$n = 0 \qquad l = 0$$

Case I. Line-to-Line Fault at C and Single Line-to-Ground Fault at D.

(a) Phases b, c and α.

$$k = C_2 + S_2 \qquad m = -\,S_2$$
$$n = -\,S_2 \qquad l = Z_{ds}$$

(b) Phases a, c and α.

$$k = C_2 + S_2 \qquad m = -\,a^2\,S_2$$
$$n = -\,a\,S_2 \qquad l = Z_{ds}$$

(c) Phases a, b and α.

$$k = C_2 + S_2 \qquad m = -\,a\,S_2$$
$$n = -\,a^2\,S_2 \qquad l = Z_{ds}$$

*See "Equivalent Y vs. Equivalent Δ" page 927.

TABLE II—*Continued*

Case J. Double Line-to-Ground Fault at C and Single Line-to Ground Faults at D.

(a) Phases b, c and α.

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}} \qquad m = -\frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{Z_{cs}}$$

$$n = -\frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{Z_{cs}} \qquad l = Z_{ds} - \frac{(S_0 - S_2)^2}{Z_{cs}}$$

(b) Phases a, c and α.

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}} \qquad m = -\frac{a S_0(C_2 + S_2) + a^2 S_2(C_0 + S_0)}{Z_{cs}}$$

$$n = -\frac{a^2 S_0(C_2 + S_2) + a S_2(C_0 + S_0)}{Z_{cs}} \qquad l = Z_{ds} - \frac{S_0^2 + S_0 S_2 + S_2^2}{Z_{cs}}$$

(c) Phases a, b and α.

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}} \qquad m = -\frac{a^2 S_0(C_2 + S_2) + a S_2(C_0 + S_0)}{Z_{cs}}$$

$$n = -\frac{a S_0(C_2 + S_2) + a^2 S_2(C_0 + S_0)}{Z_{cs}} \qquad l = Z_{ds} - \frac{S_0^2 + S_0 S_2 + S_2^2}{Z_{cs}}$$

Class K. Three-phase Fault at C and Single Line-to-Ground Fault at D.

(a) Three-phase fault not involving ground.

$$k = 0 \qquad m = 0$$

$$n = 0 \qquad l = D_0 + S_0 + D_2 + \frac{C_2 S_2}{C_2 + S_2}$$

(b) Three-phase fault involving ground.

$$k = 0 \qquad m = 0$$

$$n = 0 \qquad l = D_0 + \frac{C_0 S_0}{C_0 + S_0} + D_2 + \frac{C_2 S_2}{C_2 + S_2}$$

and between F and ground an impedance, $\frac{m-n}{2}$ or $\frac{n-m}{2}$, paralleled by an adjustable voltage, V_s. Equations (17) and (18) will be satisfied if current $(I_{a1} - I_{a1})$

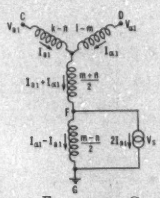

FIG. 4—EXACT GENERAL EQUIVALENT CIRCUIT REPLACING TWO FAULTS IN POSITIVE PHASE SEQUENCE NETWORK

is made to flow through the impedance $\frac{m-n}{2}$, or if current $(I_{a1} - I_{a1})$ flows through the impedance $\frac{n-m}{2}$. If the voltage, V_s, is adjusted in phase and magnitude until the current through it to ground is double I_{a1}, the current entering the fault at C, then the current $(I_{a1} - I_{a1})$ will flow in the impedance $\frac{m-n}{2}$. If the impedance $\frac{n-m}{2}$ is used, V_s must be adjusted until the current through it is double I_{a1}, the current entering the fault at D.

B. Approximate Equivalent Circuit for Use on the D-C. Calculating Table. In short-circuit studies it is customary to neglect capacitance and resistance, and to consider all generators operating with no load excitations. The system can then be represented on a d-c. calculating table.

A study of Table II shows that with resistance and capacitance neglected, k and l have no real components, but are positive reactive impedances larger in magnitude than m and n. When m and n are equal, they also have no real components and are positive reactive impedances; but when m and n are unequal they have real components which are equal and opposite, while their reactive components are equal in magnitude and of the same sign.

The error made by neglecting the real components of

m and n will ordinarily be less than the error made by neglecting line resistances. When the real components of m and n are neglected, $\frac{m-n}{2} = 0$, and the general equivalent circuit in Fig. 4 becomes a Y connecting C, D, and G, with branch impedances $(k-n)$, $(l-m)$ and $\frac{m+n}{2}$, see Fig. 5. The branches $k-n$, and

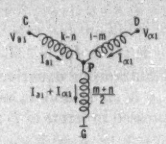

Fig. 5—Approximate Equivalent Circuit Replacing Two Faults in Positive Phase Sequence Network

$(l-m)$ will be positive reactive impedances and therefore can be represented on the d-c. calculating table, while $\frac{m+n}{2}$ may be either positive or negative. If positive it can also be represented on the d-c. table.

It will be remembered that the branch impedance $\frac{m+n}{2}$ is connected to ground as are the neutrals of the generators in the positive phase diagram. This impedance is therefore in series with the generator reactance when there is a single generating source and may be combined with it. When $\frac{m+n}{2}$ is negative and it is not possible to combine it with any other reactance, this branch of the Y between P and G may be set to zero and the distribution of currents obtained, these currents to be increased by the ratio $\frac{X_p}{X_p + \frac{m+n}{2}}$, where X_p is the equivalent impedance between generator neutrals and P, and is found by dividing generator voltage by total ground current when P is grounded.

C. Special Equivalent Circuits. Lines upon which the faults occur, together with the faults may be represented by a particular equivalent circuit when the faults occur; (1) on unloaded feeders radiating from a common point and (2) on lines of equal impedances bussed at both ends.

When the equivalent circuit represents the lines upon which the faults occur as well as the faults, the sum of the currents in the faulted lines, the total ground current, and the positive phase current and voltage distribution for the rest of the system may readily be obtained on the calculating table. Then the division of positive phase currents between the faulted lines, and the positive phase voltages at the points of fault may be determined by calculation.

(1) *Equivalent circuit for simultaneous faults on unloaded feeders radiating from a common point*

Let the fault points C and D be on unloaded feeders radiating from a common point H, then referring to Fig. 6A, let

V_h = positive phase voltage at H.
V_{a1} = positive phase voltage at C.
V_{a1} = positive phase voltage at D.
Z_{ch} = positive phase impedance between H and C.
Z_{dh} = positive phase impedance between H and D.
I_h = positive phase current at H.
I_{a1} = positive phase current at C.
I_{a1} = positive phase current at D.

Then

$$I_h = I_{a1} + I_{a1} \tag{21}$$

$$V_h - V_{a1} = I_{a1} Z_{ch} \tag{22}$$

$$V_h - V_{a1} = I_{a1} Z_{dh} \tag{23}$$

subtracting (23) from (22)

$$-V_{a1} + V_{a1} = I_{a1} Z_{ch} - I_{a1} Z_{dh} \tag{24}$$

subtracting (16) from (15)

$$V_{a1} - V_{a1} = (k-n) I_{a1} - (l-m) I_{a1} \tag{25}$$

adding (15) to (22) and (24) to (25)

$$V_h = (Z_{ch} + k) I_{a1} + m I_{a1} \tag{26}$$

$$0 = (Z_{ch} + k - n) I_{a1} - (Z_{dh} + l - m) I_{a1} \tag{27}$$

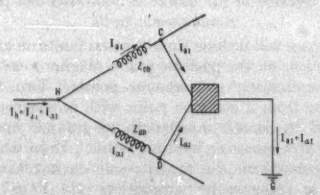

Fig. 6A—Unloaded Feeders with Faults at C and D

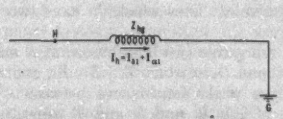

Fig. 6B—Equivalent Impedance Replacing the Two Faults and Feeders of Fig. 6A

Solving equations (26) and (27) for I_{a1} and I_{a1}:

$$I_{a1} = V_h \frac{Z_{dh} + l - m}{(Z_{ch} + k)(Z_{dh} + l) - m\,n} \tag{28}$$

$$I_{a1} = V_h \frac{Z_{ch} + k - n}{(Z_{ch} + k)(Z_{dh} + l) - m\,n} \tag{29}$$

adding (28) and (29) and replacing $I_{a1} + I_{a1}$ by I_h

$$I_h = V_h \frac{Z_{ch} + k + Z_{dh} + l - (m + n)}{(Z_{ch} + k)(Z_{dh} + l) - m\,n} \tag{30}$$

Substituting V_h from (30) in (28) and (29)

$$I_{a1} = I_h \frac{Z_{dh} + l - m}{Z_{ch} + k + Z_{dh} + l - (m + n)} \tag{31}$$

$$I_{a1} = I_h \frac{Z_{ch} + k - n}{Z_{ch} + k + Z_{dh} + l - (m + n)} \tag{32}$$

From equation (30)

$$\frac{V_h}{I_h} = \frac{(Z_{ch} + k)(Z_{dh} + l) - m\,n}{Z_{ch} + k + Z_{dh} + l - (m + n)} \tag{33}$$

but $\frac{V_h}{I_h} = Z_{hg}$ = impedance between point H and ground, which is the equivalent circuit sought.

Therefore, two unloaded feeders radiating from H with two simultaneous faults may be replaced by a single lumped impedance, Z_{hg}, whose value is given by equation (33). The total ground current and the positive phase current and voltage distribution for the system exclusive of the feeders themselves may then be obtained. Knowing I_h and V_h, the positive phase currents in the feeders, I_{a1} and I_{a1}, may be obtained from equations (31) and (32), and the voltages at the points of fault, V_{a1} and V_{a1}, from equations (22) and (23).

(2) Equivalent circuit for two lines which have two points in common or are bussed at both ends and two simultaneous faults

This case will include simultaneous faults on any two lines which, on the positive phase diagram, have their ends terminating at common points. Two feeders radiating from a common point with impedance loads replaced by shunt impedances to ground, have the ground as a common terminal point. Even when the lines upon which the faults occur do not have two points in common by appropriate $\Delta - Y$ or $Y - \Delta$ transformations it is often possible to represent them by two equivalent lines which do have two points in common.

Fig. 7A represents two lines bussed at R and T with faults at C and D, in which Z_{cr}, Z_{cl}, Z_{dr} and Z_{dl} represent positive phase impedances between C and R, C and T, D and R, and D and T respectively, and V_r, V_l, V_{a1} and V_{a1} the voltage at R, T, C and D. I_r and I_l represent the currents at R and T respectively, positive direction being towards the faults and I_{a1} and I_{a1} the currents flowing into the faults from C and D.

From Fig. 7A

$$I_r + I_l = I_{a1} + I_{a1} = I_g = \text{total ground current}$$

Let I_r divide into I_x and $I_r - I_x$, and I_l divide into I_y and $I_l - I_y$.

Let $I_{a1} = I_x + I_y$ and $I_{a1} = I_r + I_l - I_x - I_y$.

Let $Z_{cr} = a$, $Z_{cl} = b$, $Z_{dr} = c$, $Z_{dl} = d$ and $a + b + c + d = S$.

Then

$$V_{a1} = V_r - a\,I_x \tag{34}$$

$$V_{a1} = V_l - b\,I_y \tag{35}$$

$$V_{a1} = V_r - c\,(I_r - I_x) \tag{36}$$

$$V_{a1} = V_l - d\,(I_l - I_y) \tag{37}$$

Substituting the above values for I_{a1} and I_{a1} in (15) and (16)

$$V_{a1} = (k - m)(I_x + I_y) + m\,(I_r + I_l) \tag{38}$$

$$V_{a1} = (n - l)(I_x + I_y) + l\,(I_r + I_l) \tag{39}$$

From the six simultaneous equations (34)—(39) the four unknowns V_{a1}, V_{a1}, I_x and I_y will be eliminated and V_r and V_l expressed in terms of I_r and I_l.

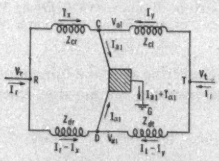

FIG. 7A—TWO LINES BUSSED AT BOTH ENDS WITH TWO SIMULTANEOUS FAULTS

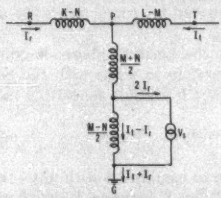

FIG. 7B—EXACT EQUIVALENT CIRCUIT REPLACING THE TWO FAULTED LINES OF FIG. 7A

Substituting (38) and (39) in (34) and (37) respectively, and transposing:

$$V_r = m\,I_r + m\,I_l + (a + k - m)\,I_x + (k - m)\,I_y \tag{40}$$

$$V_l = l\,I_r + (d + l)\,I_l + (n - l)\,I_x + (n - l - d)\,I_y \tag{41}$$

Subtracting the sum of equations (35) and (36) from the sum of (34) and (37); and the sum of (34) and (39) from the sum of (36) and (38):

$$0 = c\,I_r - d\,I_l - (a + c)\,I_x + (b + d)\,I_y \tag{42}$$

$$0 = (m - c - l)\,I_r + (m - l)\,I_l + (a + c + k + l - m - n)\,I_x + (k + l - m - n)\,I_y \tag{43}$$

Expressing I_x and I_y from (42) and (43) in terms of I_r and I_l, and replacing $a + b + c + d$ by S.

$$I_x = \frac{\left[\begin{array}{l}[(b + c + d)(l - m) + c\,(b + d + k - n)]\,I_r \\ + [b\,(l - m) - d\,(k - n)]\,I_l\end{array}\right]}{S\,(k + l - m - n) + (a + c)\,(b + d)} \tag{44}$$

$$I_y = \frac{\left[\begin{array}{c}[a\,(l-m) - c\,(k-n)]\,I_r \\ +[(a+c+d)(l-m)+d(a+c+k-n)]I_t\end{array}\right]}{S\,(k+l-m-n) + (a+c)\,(b+d)} \quad (45)$$

Substituting (44) and (45) in (40) and (41)

$$V_r = \frac{\left[\begin{array}{c}S\,(k\,l-m\,n) - a\,c\,(m+n) + c\,(a+b+d)\,k \\ +\,a\,(b+c+d)\,l + a\,c\,(b+d)\end{array}\right]}{S\,(k+l-m-n) + (a+c)\,(b+d)}\,I_r$$

$$+\frac{S\,(k\,l-m\,n) + b\,c\,m + a\,d\,n + c\,d\,k + a\,b\,l}{S\,(k+l-m-n) + (a+c)\,(b+d)}\,I_t \quad (46)$$

$$V_t = \frac{S\,(k\,l-m\,n) + a\,d\,m + b\,c\,n + c\,d\,k + a\,b\,l}{S\,(k+l-m-n) + (a+c)\,(b+d)}\,I_r$$

$$+\frac{\left[\begin{array}{c}S\,(k\,l-m\,n) - b\,d\,(m+n) + d\,(a+b+c)\,k \\ +\,b\,(a+c+d)\,l + b\,d\,(a+c)\end{array}\right]}{S\,(k+l-m-n) + (a+c)\,(b+d)}\,I_t \quad (47)$$

(*a*) *General Case.* A comparison of the coefficient of I_t in (46) with that of I_r in (47) shows that they are not identical, and therefore a simple impedance Y or Δ can not replace the lines with the two simultaneous faults in the general case where m and n are unequal and there is no fixed relation between the impedances Z_{cr}, Z_{cd}, Z_{ct} and Z_{dt}.

The general equivalent circuit or the approximate equivalent circuit may be used to replace the two lines with the two simultaneous faults, rather than just to replace the two faults, if it is found advantageous to do so. To obtain this circuit, equations (46) and (47) may be written:

$$V_r = K\,I_r + M\,I_t \quad (48)$$

$$V_t = N\,I_r + L\,I_t \quad (49)$$

If k, l, m and n in Figs. 4 and 5 are replaced by K, L, M and N as defined in equations (46)—(49) and and I_{s1} and I_{a1} by I_r and I_t the desired circuits are obtained. The general equivalent circuit is shown in Fig. 7B.

(*b*) *Special Case.* The coefficients of I_t and I_r in equations (46) and (47) respectively will be identical if $b\,c\,m + a\,d\,n = a\,d\,m + b\,c\,n$, that is if $m = n$ or $a\,d = b\,c$. If either of these conditions is satisfied the two lines with the two simultaneous faults may be replaced by an equivalent Y having the branch impedances $K - M$, $L - M$ and M.

Equivalent circuit for two transmission lines of equal impedances bussed at both ends and two simultaneous faults

Since $Z_{dr} = Z_{cr} = a$ and $Z_{dt} = Z_{ct} = b$, c and d in equations (46) and (47) may be replaced by a and b respectively and the equations rewritten thus

$$V_r = \left[\frac{a}{2}\,I_r\right.$$

$$\left.+\frac{a\,b\,(k+l+m+n) + 2\,(a+b)\,(k\,l-m\,n)}{4\,a\,b + 2\,(a+b)\,(k+l-m-n)}(I_r + I_t)\right] \quad (50)$$

$$V_t = \frac{b}{2}\,I_t$$

$$+\frac{a\,b\,(k+l+m+n) + 2\,(a+b)\,(k\,l-m\,n)}{4\,a\,b + 2\,(a+b)\,(k+l-m-n)}(I_r + I_t) \quad (51)$$

Equations (50) and (51) will be satisfied if the two lines and the two simultaneous faults are replaced by a Y between the points R, T and G, Fig. 8B, the branch impedances of the Y being $\frac{a}{2}$, $\frac{b}{2}$ and

$$\frac{a\,b\,(k+l+m+n) + 2\,(a+b)\,(k\,l-m\,n)}{4\,a\,b + 2\,(a+b)\,(k+l-m-n)}.$$

In problems where the division of currents between the two lines and the voltages at the points of fault are required they may be determined by calculations from the voltages and currents at the ends of the lines. If the positive phase current and voltage distribution has been determined with the equivalent Y replacing the

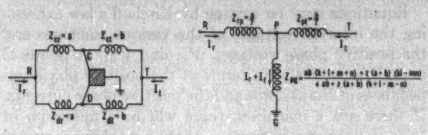

Fig. 8A—Special Case of Fig. 7A when Lines have Equal Impedance and Fault Points are Equidistant from One End

Fig. 8B—Exact Equivalent Circuit for Fig. 8A

lines and the two simultaneous faults, then V_r, V_t, I_r and I_t will be known. I_x and I_y may be obtained by substituting the values of I_r and I_t in equations (44) and (45); V_{s1} may be obtained from equations (34) or (35); and V_{a1} from (36) or (37).

Equivalent Y vs. Equivalent Δ. In the zero and negative phase systems, equivalent Y's were used to replace the network connected between the two fault points and the points of zero potential, assumed bussed at S. Equivalent Δ's might have been used through this paper. The choice of equivalent Y's and impedances has given, in the majority of cases, equations of simpler form for the constants k, l, m and n than would have been obtained with equivalent Δ's and admittances. In the case of double line-to-ground faults, however, the solution with equivalent Δ's and admittances is a simple one and the form of the resulting equations of special interest when the faults are on the same phases. Let the admittances of the Δ replacing the equivalent Y for the zero phase system be Y_{0cd}, Y_{0ca} and Y_{0da} and those for the negative system be

Y_{2cd}, Y_{2ce} and Y_{2de}, the first subscript indicating the sequence and the second and third the terminal points.

Replacing[8] the impedances of the Y's by the admittances of the Δ's in equations (**13d**) and (**14d**) of Appendix D:

$$I_{a1} = V_{a1}(Y_{0ce} + Y_{2ce} + Y_{0ed} + Y_{2ed}) - V_{a1}(Y_{0ed} + Y_{2cd})$$

$$I_{a1} = -V_{a1}(Y_{0ed} + Y_{2cd}) + V_{a1}(Y_{0de} + Y_{2de} + Y_{0cd} + Y_{2ed})$$

These equations are satisfied if the zero and negative phase networks are connected in parallel between the points C, D and G of the positive phase system, points C in the three systems being connected, also points D, and the points of zero potential of all three systems. With these connections on the calculating table, the zero and negative phase currents for phase a as well as the positive, may be read directly for all parts of the system.

Analytical Solution of Positive Phase Network

When a calculating table is not available, or when greater accuracy is required than can be secured from either the d-c. table or the a-c. network analyzer, the distribution of positive phase currents and voltages may be determined by calculation.

Equations may be written by Kirchoff's law expressing the internal voltages of the various machines and the positive phase voltages, V_{a1} and V_{a1}, at points C and D respectively, in terms of the positive phase impedances of the systems and the positive phase currents. If there are n machines, there will be a minimum of $(n + 2)$ unknown currents, *i. e.*, the n machines currents and the two fault currents; and a minimum of $(n + 2)$ voltage equations, *i. e.*, the n machine internal voltage equations and the two fault voltage equations. Each loop in the system introduces an additional unknown current, but also an additional equation, the voltage drop around the loop being zero. In these $(n + 2)$ equations the n machines internal voltages are known, the $(n + 2)$ currents and the two fault voltages are unknown. With equations (**15**) and (**16**), (which are independent of the positive phase sequence impedances and the internal voltages of the machine, but depend upon the types of fault and the zero and negative phase impedances) there will be $(n + 4)$ equations and $(n + 4)$ unknowns. It is possible, therefore, to solve for the positive currents in terms of the internal generated voltages and the positive, negative, and zero phase impedances of the system. Knowing the positive currents, the positive voltages for the system may be obtained.

Solution at Positive Phase Network by Superposition

To avoid the additional equations and unknowns due to loops in the positive phase system, the following equations may be used:

Positive Phase Sequence Equations Expressing Positive Voltages in Terms of Positive Currents and Impedances*

$$E_1 = I_1 Q_{11} + I_2 Q_{12} + \ldots\ldots + I_n Q_{1n} + I_{a1} Q_{1c} + I_{a1} Q_{1d}$$
$$E_2 = I_1 Q_{21} + I_2 Q_{22} + \ldots\ldots + I_n Q_{2n} + I_{a1} Q_{2c} + I_{a1} Q_{2d}$$

$$E_n = I_1 Q_{n1} + I_2 Q_{n2} + \ldots\ldots + I_n Q_{nn} + I_{a1} Q_{nc} + I_{a1} Q_{nd}$$
$$V_{a1} = I_1 Q_{c1} + I_2 Q_{c2} + \ldots\ldots + I_n Q_{cn} + I_{a1} Q_{cc} + I_{a1} Q_{cd}$$
$$V_{a1} = I_1 Q_{d1} + I_2 Q_{d2} + \ldots\ldots + I_n Q_{dn} + I_{a1} Q_{dc} + I_{a1} Q_{dd}$$

where $E_1, E_2 \ldots\ldots E_n$ are known internal voltages on machines 1, 2 and n, and I_1, I_2 and In are the corresponding currents. V_{a1} and V_{a1} are the positive phase voltages at the points of fault C and D respectively, and I_{a1} and I_{a1} are the corresponding currents flowing into the faults. Positive direction for machine currents is taken from the generator neutrals. Positive direction for faults current, I_{a1} and I_{a1}, is taken from the system into the faults.

Also

$Q_{11} = \dfrac{V_1}{I_1}$ where V_1 is the internal voltage of machine I, when no fault currents and no machine currents are flowing except I_1.

$Q_{1n} = \dfrac{V_1}{I_n}$ when no fault nor machine currents are flowing except I_n.

$Q_{cd} = \dfrac{V_{a1}}{I_{a1}}$ when no machine currents are flowing and no fault current except I_{a1}.

$Q_{nn} = \dfrac{V_n}{I_n}$ when V_n is the internal voltage of machine, or when no fault currents and no machine currents are flowing except I_n.

The Q's associated with I_{a1} and I_{a1} in the above equations will be negative, due to the arbitrary assumptions for positive direction of current flow.

When there is a point of zero potential on the positive phase system at which no voltage is generated, such as the neutral of a shunt impedance load, there will be a return path for the currents when one current only is flowing, and the Q's may readily be obtained on the calculating table. When there is no such point it will be necessary to ground one machine, No. 2, to provide a return path for the currents when the only current flowing is the one under consideration. The voltage equation $E_2 = I_1 Q_{a1} + \ldots\ldots + I_{a1} Q_{2d}$ will then become indeterminate but may be replaced by the current equation $I_1 + I_2 \ldots\ldots + I_n = I_{a1} + I_{a1}$. The current distribution will first be determined with machine No. 2 grounded ($E_2 = 0$), and then with all machines except No. 2 grounded. By superposing the two sets of currents the distribution of positive phase currents will be determined.

*These equations were suggested by Mr. R. H. Park.

Solution of Negative and Zero Phase Networks

When the positive phase voltages at the points of fault and the positive phase currents flowing into the faults have been determined, four of the eight unknowns V_{a0}, V_{a0}, V_{a2}, V_{a2}, I_{a0}, I_{a0}, I_{a2}, and I_{a2} may readily be obtained from the relations given in Table I. The four remaining unknowns, two of which will be in the zero phase and two in the negative phase system, may be obtained from equations (5), (6), (10) and (11), or from (7), (8), (12) and (13).

In general, the voltages at the two points of fault of either system will not be in phase with each other nor with the reference voltage of the positive phase system. The currents in the network of either system may be determined by superposition, *i. e.*, by adding vectorially the currents due to the fault voltage at C with point D grounded, and the currents due to the fault voltage at D with C grounded.

When an a-c. calculating table is available the current and voltage distribution in the zero phase system may be obtained by applying voltages to ground, V_{a0} and V_{a0}, at points C and D respectively of the zero phase network; and in a similar manner the negative phase current and voltage distribution will be determined by applying voltages V_{a2} and V_{a2} at point C and D of the negative phase system.

System Currents and Voltages

By use of the fundamental symmetrical component equations given in Appendix A, the positive, negative, and zero components of currents and voltages may be combined to give actual currents and voltages over the entire system.

Three Simultaneous Faults

The method used for determining currents and voltages when simultaneous faults occur at two separate and distinct points of the system may be extended to apply to three or more simultaneous faults.

With three simultaneous faults on the system, three independent equations may be written expressing the three zero phase voltages at the points of fault in terms of the three zero phase currents flowing into the faults. The equations given above for use in determining the positive phase currents may be used to advantage in this connection; or if three equivalent Y's for the zero phase system are drawn, each preserving the identity of the points of zero potential considered bussed at a common point, S, and two of the three fault points, the three equations each expressing one zero phase fault voltage in terms of the three zero phase currents flowing into the faults may be written by inspection.

In like manner the negative phase system will provide three independent equations connecting negative phase fault voltages and currents.

At each of the three points of fault there will be three independent equations connecting components of current or of voltage.

Nine equations expressing relations between the components of current or voltage at the three fault points, three equations connecting zero phase currents and voltages, and three connecting negative phase currents and voltages, provide fifteen equations. The unknown zero and negative phase currents and voltages may be eliminated from these equations, the number being reduced to three, and the three positive phase voltages to ground at the fault points expressed in terms of the three positive phase currents flowing into the faults and the negative and zero phase impedances of the system.

In the positive phase system there will be one more unknown voltage and one more unknown current than with two simultaneous faults, but there will be two additional voltage equations, one involving positive phase currents flowing into the faults and negative and zero phase impedances, and the other involving positive phase line currents and positive phase impedances. A complete determination of the positive phase currents, therefore, is possible, and from these, the currents and voltages over the entire system may be obtained.

Generalizing from the above, it may be seen that n simultaneous faults would be solved in a similar manner.

Single-Pole Switching

In the past there has been some discussion as to the advisability of switching out one phase and retaining the two good phases when a ground occurs on one phase only of a three-phase system. When grounds occur at two separate and distinct points of the system but each involves one conductor only, it may be desirable to investigate the possibility of simultaneous single-pole switching.

When a conductor is disconnected from the system by opening the breakers at its terminals, the current in it and in the other two conductors and the three voltages to ground at the terminals of the line are the same as they would be if the conductor were open at one point only, provided there is no ground on the conductor and capacitance is negligible. For convenience, when a conductor is open at both ends it will be considered open at one point only, the point having any convenient location along the line. This will not change the currents in the conductors nor the voltages at the ends of the section under consideration. The voltage to ground of the open conductor, however, will depend upon the location of the opening.

When one conductor of a three-phase transmission line is open the voltages to ground on the two sides of the opening in general will be different, and a voltage will exist across the opening. Fig. 9 represents three conductors a, b, and c with conductor a open at point C. Let e_a, e_b, e_c be the differences in voltage on the two sides

of the opening for conductors a, b, and c respectively, and i_a, i_b and i_c the line currents at point C.

(a) conductor a open.

Fault conditions: $e_b = e_c = 0$, and $i_a = 0$

From (**1a**), (**10a**)—(**12a**)

$$i_{a1} = -(i_{a0} + i_{a2}) \tag{52}$$

$$e_{a1} = e_{a0} = e_{a2} \tag{53}$$

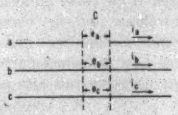

FIG. 9—THREE-PHASE LINE WITH ONE CONDUCTOR OPEN SHOWING SERIES VOLTAGES USED IN ANALYSIS

(b) conductor b open.

Fault conditions: $e_a = e_c = 0$, and $i_b = 0$

From (**2a**) and (**10a**)—(**12a**)

$$i_{a1} = -(a\, i_{a0} + a^2\, i_{a2}) \tag{54}$$

$$e_{a1} = a\, e_{a0} = a^2\, e_{a2} \tag{55}$$

(c) conductor c open.

By analogy from (b)

$$i_{a1} = -(a^2\, i_{a0} + a\, i_{a2}) \tag{56}$$

$$e_{a1} = a^2\, e_{a0} = a\, e_{a2} \tag{57}$$

It is of interest to note that equations (**52**)—(**57**) are identical in form with the equations given in Table I, C for double line-to-ground faults. In a similar manner it may be shown that when two conductors are opened the resulting equations between components of series voltages and between components of line currents will be identical in form with those given in Table I, A for single line-to-ground faults.

Series Voltages and Series Impedances. It should be noted that e_a, e_{a0}, e_{a1} and e_{a2} are series voltages. Due to the opening, a series voltage, e_{ac}, appears in the zero phase network at point C and a line current, i_{a0}, which meets a series impedance z_0. Likewise the series voltage, e_{a2}, appears in the negative phase network at C and a line current, i_{a2}, which meets the series impedance z_2. Expressed in equations:

$$-i_{a0} = \frac{e_{a0}}{z_0} \tag{58}$$

$$-i_{a2} = \frac{e_{a2}}{z_2} \tag{59}$$

where z_0 and z_2 are the series impedances for the zero and negative phase networks respectively, viewed from the point C.

Single-Pole Switching on One Circuit. It has been shown[8] that the effect on the positive phase currents and voltages of the system of opening a single conductor may be represented by inserting the zero and negative phase series impedance circuits viewed from the opening in parallel across the opening in the positive phase impedance diagram; and if two conductors are opened, by inserting the zero and negative phase series impedance circuits in series. This is seen by adding (**58**) and (**59**) and substituting (**52**) and (**53**) in the sum, then transposing:

$$e_{a1} = \frac{z_0\, z_2}{z_0 + z_2}\, i_{a1} \tag{60}$$

and when two conductors are open, by analogy from a single line-to-ground fault, it follows that

$$e_{a1} = (z_0 + z_2)\, i_{a1} \tag{61}$$

Single-Pole Switching on Two Circuits Simultaneously. The relations between the components of line current and the components of series voltages at a point where a conductor is open are independent of the rest of the system. These relations are given in equations (**52**)—(**57**) for conductors a, b, and c. By substituting α, β and γ for a, b, and c respectively, the relations existing at a second point of opening are obtained.

In both the zero and negative phase networks there will be two series voltages introduced by the opening. If the zero and negative phase networks are reduced to their simplest form consisting of three branches each, as shown in Figs. 10A and 10B, the identity of the lines in which the conductors are open being preserved, equations connecting components of current and voltage may be written thus

$$-e_{a0} = i_{a0}\,(c_0 + s_0) + i_{a0}\, s_0 \tag{62}$$

$$-e_{a0} = i_{a0}\, s_0 + i_{a0}\,(d_0 + s_0) \tag{63}$$

$$-e_{a2} = i_{a2}\,(c_2 + s_2) + i_{a2}\, e_2 \tag{64}$$

$$-e_{a2} = i_{a2}\, s_2 + i_{a0}\,(d_2 + s_2) \tag{65}$$

It is seen that equations (**62**)—(**65**) have the same form as (**5**), (**6**), (**10**), and (**11**), and since equations (**52**)—(**57**) have the same form as the equations of Table I,

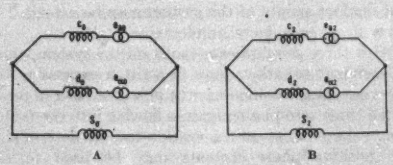

FIG. 10A—ZERO PHASE SEQUENCE SERIES EQUIVALENT CIRCUIT FOR TWO LINES WITH CONDUCTORS OPEN AT TWO POINTS

FIG. 10B—NEGATIVE PHASE SEQUENCE SERIES EQUIVALENT CIRCUIT FOR TWO LINES WITH CONDUCTORS OPEN AT TWO POINTS

C for double line-to-ground faults, it follows by analogy from the work on simultaneous faults that the positive series voltages e_{a1} and e_{a1} at points C and D respectively, when one conductor is open at each point may be expressed in terms of the positive phase line currents i_{a1} and i_{a1} thus:

$$e_{a1} = k'\, i_{a1} + m'\, i_{a1} \tag{66}$$

$$e_{a1} = n'\, i_{a1} + l'\, i_{a1} \tag{67}$$

where k', l', m' and n' are functions of c_0, d_0, s_0, c_2, d_2 and s_2, the branches of the simplified circuits representing the zero and negative phase networks, and are given in Table II, G if k', l', m' and n' are substituted for k, l, m, n and c_0, d_0, s_0, c_2, d_2 and s_2 for C_0, D_0, S_0, C_2, D_2 and S_2 respectively.

Equivalent Circuits Replacing Openings in Two Transmission Lines in Positive Phase System. The effect of an opening in a conductor in the positive phase system is to introduce a series voltage opposing the flow of line current. When there are two openings, two series voltages will be introduced, one at each opening, the relations between these series voltages and the line currents being given by equations (**66**) and (**67**). Equivalent circuits may replace the two openings just as they may replace the two simultaneous faults, but since these equivalent circuits are to be inserted in series with the lines in which the conductors are open they will depend upon the end connections of the lines.

When the lines are bussed at one end, and m' and n' are equal, an equivalent Y having the branch impedances $k' - m'$, $l' - m'$ and m' may be inserted by opening the lines at the common point and connecting the branch $k' - m'$ to the line in which the opening occurs at C, the branch $l' - m'$ to the line with the opening at D and the branch m' to the common point. When m' and n' are not equal the general equivalent circuit or the approximate equivalent circuit may be inserted in the same manner. These circuits may be obtained from Figs. 4 and 5 if k, l, m, n, I_{a1} and I_{a1} are replaced by k', l', m', n', i_{a1} and i_{a1}, respectively.

When the lines are not bussed at either end the positive currents may be determined analytically as under two simultaneous faults, remembering that the voltages at C and D are series voltages and the currents are line currents. In the special case where m' and n' are equal the openings in the two conductors may be replaced by a four terminal mesh network similar to that shown in Fig. 17B, Appendix C. If terminals 1 and 2 are connected across the opening at point C and 3 and 4 across the opening at point D, the impedances of the six branches may be determined from equations (**1c**) Z_a, Z_b and Z_{ab} being replaced by k', l', and m' respectively.

When the lines are bussed at both ends a single impedance may replace the lines with the openings in the positive phase system. Fig. 11.

Equivalent circuit replacing in the positive phase system two lines bussed at both ends each having one conductor open. Let the lines with the openings at C and D, having positive phase impedances between terminals Z_c and Z_d, respectively, be bussed at points R and T. Let the sum of the currents entering the lines at R and leaving them at T be I_r, and let V_r and V_t be the voltages at R and T, respectively, Fig. 11A.

Then

$$I_r = i_{a1} + i_{a1} \tag{68}$$

$$Z_{rt} = \frac{V_r - V_t}{i_{a1} + i_{a1}} = \text{equivalent impedances} \tag{69}$$

$$V_r - V_t = Z_c\, i_{a1} + e_{a1} \tag{70}$$

$$V_r - V_t = Z_d\, i_{a1} + e_{a1} \tag{71}$$

Substituting (**66**) and (**67**) in (**70**) and (**71**), respectively,

$$V_r - V_t = (Z_c + k')\, i_{a1} + m'\, i_{a1} \tag{72}$$

$$V_r - V_t = n'\, i_{a1} + (Z_d + l')\, i_{a1} \tag{73}$$

Solving (**72**) and (**73**) for i_{a1} and i_{a1}

$$i_{a1} = \frac{Z_d + l' - m'}{(Z_c + k')(Z_d + l') - m'\,n'}\,(V_r - V_t) \tag{74}$$

$$i_{a1} = \frac{Z_c + k' - n'}{(Z_c + k')(Z_d + l') - m'\,n'}\,(V_r - V_t) \tag{75}$$

Adding (**74**) and (**75**) and substituting in (**69**)

$$Z_{rt} = \frac{(Z_c + k')(Z_d + l') - m'\,n'}{Z_c + Z_d + k' + l' - m' - n'} \tag{76}$$

Equation (**76**) gives the equivalent impedance, Z_{rt}, which replaces the two lines bussed at both ends each having one conductor open. Fig. 11B.

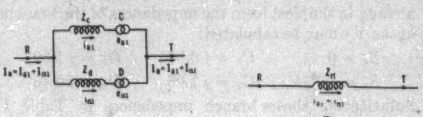

FIG. 11A AND FIG. 11B—POSITIVE PHASE SEQUENCE EQUIVALENT CIRCUITS REPLACING TWO LINES BUSSED AT BOTH ENDS, EACH HAVING ONE CONDUCTOR OPEN

SHORT-CIRCUIT AND STABILITY PROBLEMS

Methods have been given for determining the currents and voltages in all parts of the system during simultaneous faults when there are any number of connected machines. For practical problems it is not ordinarily required to know the exact currents and voltages in all parts of the system

For short-circuit studies when a high degree of precision is not required, resistance and capacitance are neglected, and all generated voltages are assumed equal and in phase.

In stability studies one equivalent machine is often used to replace several machines or groups of machines, so that the number of machines involved in the calculations is small. It is necessary to know the generated voltages on all machines or equivalent machines in order to calculate the power which each sends out or receives, but it is not required to know the currents if power is calculated from voltages, angular displacements, and impedances. In order to determine the stability[7,8] of a system during transient conditions the transfer[8] impedance between the various machines before, during, and after the disturbance must be known,

and also the driving point[8] impedance of each machine when resistance is taken into account.

The way in which the analytical methods developed in the preceding part of the paper may be applied to actual problems is shown in the two examples given below.

Problem I. Short-Circuit Study. Simultaneous double line-to-ground faults occur on two circuits; phases a and c are involved at point C, and phases b and c (β and γ) at point D. Find the currents in the six conductors in "per unit"[9] neglecting resistance and assuming no-load excitations on all synchronous machines.

Fig. 12A gives the simplified positive phase diagram of the system. Since the currents in the faulted lines are the only ones required, the rest of the system has been reduced to an equivalent generator viewed from H, with transient reactance $x_d' = 0.34$ and excitation $E_{a1} = 1.00$. Negative phase impedances are taken equal to positive phase impedances, Fig. 12C. The zero phase impedance between C and the zero potential point, S, is 0.904 and between D and S is 1.80, and there is no mutual between the two circuits, Fig. 12B.

Since the zero and negative phase diagrams are already in simplest form the impedances of the branches of the Y's may be tabulated:

$$S_0 = 0 \qquad C_0 = j\,0.904 \qquad D_0 = j\,1.80$$
$$S_2 = j\,0.34 \qquad C_2 = j\,1.06 \qquad D_2 = j\,0.75$$

Substituting these branch impedances in Table II, G (b), k, l, m and n will be calculated.

$$Z_{22} = j\,1.06 \times j\,0.34 + j\,1.06 \times j\,0.75 + j\,0.34 \times j\,0.75 = -\,1.410$$

$$Z_{00} = j\,0.904 \times j\,1.80 = -\,1.627$$

$$Z_{cds} = j\,1.964 \times j\,2.55 + j\,1.964 \times j\,0.34 + j\,2.55 \times j\,0.34 = -\,6.54$$

$$3\,S_0\,S_2 = 0$$

$$k = \frac{-\,1.410 \times j\,0.904 - 1.627 \times j\,1.40}{-\,6.54} = j\;0.543$$

$$l = \frac{-\,1.410 \times j\,1.80 - 1.627 \times j\,1.09}{-\,6.54} = j\;0.660$$

$$m = \frac{a^2 \times j\,0.34 \times (-\,1.627)}{-\,6.54} = 0.0732 - j\,0.0423$$

$$n = \frac{a \times j\,0.34 \times (-\,1.627)}{-\,6.54} = -\,0.0732 - j\,0.0423$$

In the positive phase diagram the equivalent circuit for two unloaded feeders radiating from the same point H, may be used and Z_{hg} calculated from equation (**33**).

$$Z_{hg} = \frac{(j\,1.06 + j\,0.543)\,(j\,0.75 + j\,0.660) + 0.007}{j\,1.06 + j\,0.543 + j\,0.75 + j\,0.660 + j\,0.846} = \frac{-\,2.253}{j\,3.098} = j\,0.727$$

$$I_h = I_{a1} + I_{a1} = \frac{E_{a1}}{j\,0.34 + j\,0.727} = \frac{1.00}{j\,1.067} = -j\;0.937$$

$$V_h = E_{a1} - (j\,0.34) \times (-\,j\,0.937) = 1. - 0.3186 = 0.681$$

From (**31**) and (**32**)

$$I_{a1} = -\,j\,0.937\,\frac{j\,0.75 + j\,0.66 - (0.0732 - j\,0.0423)}{j\,3.098} = 0.022 - j\,0.439$$

$$I_{a1} = -\,j\,0.937\,\frac{j\,1.06 + j\,0.543 - (-\,0.0732 - j\,0.0423)}{j\,3.098} = -\,0.022 - j\,0.498$$

From equations (**22**) and (**23**)

$$V_{a1} = 0.681 - (0.0221 - j\,0.439)\,(j\,1.06) = 0.216 - j\,0.023$$

$$V_{a1} = 0.681 - (-\,0.0221 - j\,0.498)\,(j\,0.75) = 0.308 + j\,0.016$$

From Table I, C(b) and C(a)

$$V_{a0} = a^2\,V_{a1} = -\,0.128 - j\,0.1755$$
$$V_{a2} = a\,V_{a1} = -\,0.088 + j\,0.1985$$
$$V_{a0} = V_{a2} = V_{a1} = 0.308 + j\,.016$$

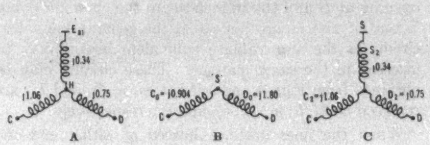

FIG. 12

A—POSITIVE PHASE SEQUENCE DIAGRAM FOR PROBLEM I
B—EQUIVALENT Y FOR ZERO PHASE SEQUENCE DIAGRAM
C—EQUIVALENT Y FOR NEGATIVE PHASE SEQUENCE DIAGRAM

From Fig. 12B and equations (**5**) and (**6**)

$$I_{a0} = -\,\frac{V_{a0}}{j\,0.904} = 0.194 - j\,0.142$$

$$I_{a0} = -\,\frac{V_{a0}}{j\,1.80} = -\,0.009 + j\,0.171$$

From Fig. 12C and equations (**12**) and (**13**)

$$I_{a2} = -\,(-\,0.088 + j\,0.1985)\,\frac{j\,1.09}{-\,1.410} + (0.308 + j\,0.016)\,\frac{j\,0.34}{-\,1.410} = -\,0.150 - j\,0.142$$

$$I_{a2} = -\,(-\,0.088 + j\,0.1985)\,\frac{j\,0.34}{-\,1.410} - (0.308 + j\,0.016)\,\frac{j\,1.40}{-\,1.410} = 0.032 + j\,0.327$$

Substituting the component currents in equations (**1a**)—(**3a**), the line currents are determined.

$$I_a = 0.066 - j\,0.723$$
$$I_b = 0$$
$$I_c = 0.515 + j\,0.298$$
$$I_\alpha = 0$$
$$I\beta = -\,0.729 + j\,0.303$$
$$I\gamma = 0.701 + j\,0.210$$

If the approximate equivalent circuit, Fig. 5, is used in the positive phase diagram, Fig. 13 would result.

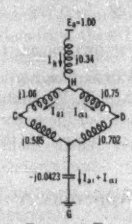

FIG. 13—APPROXIMATE POSITIVE PHASE SEQUENCE DIAGRAM FOR PROBLEM I

From Fig. 13, $I_h = -\,j\,0.935$, $I_{a1} = -\,j\,0.438$, $I_{c1} = -\,j\,0.497$. These values check those obtained by the exact method very closely, and the line currents calculated by using the approximate equivalent circuit would differ very little from those given above.

Problem II. Transfer Impedances (for Stability Study). Find the transfer impedance between the generator G. and the equivalent motor, M, in Fig.14A with simultaneous line-to-ground faults at points C and D, conductor b being grounded at C and α at D. Find also the transfer impedance when conductors b and α are open. Reactances are given in per unit on the generator base. Transformers are Δ-Y, solidly grounded on the high sides. The four transmission lines are on double-circuit towers, which are not on the same right-of-way, and have light steel ground wires. Power is delivered at two points, B and E, to a system which has been replaced by an equivalent motor, M, and two impedances.

The negative phase reactances of the system have been indicated on Fig. 14A when they differ from the positive phase values. The zero phase diagram has been constructed, Fig. 14B, using the average zero phase line and mutual reactances given in Appendix C.

(a) *Transfer impedance between G and M with fault on.* The negative and zero diagrams reduced to equivalent Y's are shown in Figs. 14C and 14D, respectively.

Tabulating the branch impedances of these equivalent Y's:

$C_0 = j\,0.175$ $\quad D_0 = j\,0.175$ $\quad S_0 = j\,0.0066$

$C_2 = j\,0.16$ $\quad D_2 = j\,0.16$ $\quad S_2 = j\,0.13$

Substituting these values in Table II, E(b).

$$k = j\,0.4716$$
$$l = j\,0.4716$$
$$m = -\,0.1067 - j\,0.0683$$
$$n = 0.1067 - j\,0.0683$$

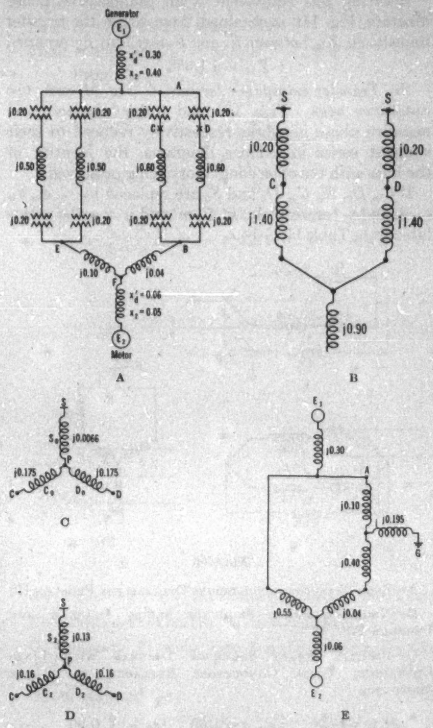

FIG. 14

A—SIMPLIFIED SYSTEM DIAGRAM FOR PROBLEM IIA

B—ZERO PHASE SEQUENCE DIAGRAM

C—EQUIVALENT Y FOR ZERO PHASE SEQUENCE DIAGRAM

D—EQUIVALENT Y FOR NEGATIVE PHASE SEQUENCE DIAGRAM

E—EQUIVALENT POSITIVE PHASE SEQUENCE DIAGRAM WITH LINES AND FAULTS REPLACED BY EQUIVALENT Y

The two faults with the lines upon which they occur may be replaced by an equivalent Y, Fig. 8B, having the branch impedances

$$Z_{rp} = \frac{a}{2} = j\,0.10$$

$$Z_{tp} = \frac{b}{2} = j\,0.40$$

$$Z_{pq} = \frac{-.16 \times j.8066 + 2 \times j1.00 \times (-0.2063)}{-0.64 + 2 \times j\,1.00 \times j\,1.080} = j\,0.195$$

Inserting this equivalent Y in the positive phase diagram, Fig. 14E is obtained, from which the transfer impedance, Z_{12}, between E_1 and E_2 is readily determined

$$Z_{12} = j\,1.052$$

(b) *Transfer impedance berween G and M with two conductors open.* Figs. 15A and B show the zero and negative phase networks respectively reduced to their simplest series impedance diagrams, the identity of the lines with the open conductors being preserved.

If C_0, D_0, S_0, C_2, D_2 and S_2 are replaced by c_0, d_0, s_0, c_2, d_2 and s_2 respectively, k', l', m' and n' may be calculated from Table II, G(b).

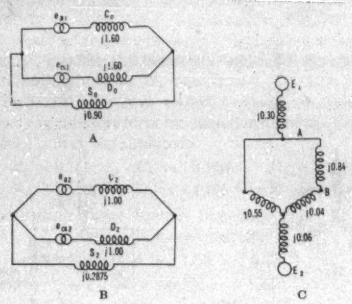

FIG. 15

A—ZERO PHASE SEQUENCE SERIES DIAGRAM FOR PROBLEM IIb

B—NEGATIVE PHASE SEQUENCE SERIES DIAGRAM FOR PROBLEM IIb

C—POSITIVE PHASE SEQUENCE DIAGRAM WITH LINES CONTAINING OPEN CONDUCTORS REPLACED BY SINGLE IMPEDANCE

$$c_0 = j\,1.60 \qquad d_0 = j\,1.60 \qquad s_0 = j\,0.90$$
$$c_2 = j\,1.00 \qquad d_2 = j\,1.00 \qquad s_2 = j\,0.2875$$

$$Z_{00} = c_0\,d_0 + c_0\,s_0 + d_0\,s_0 = -5.44$$
$$Z_{22} = c_2\,d_2 + c_2\,s_2 + d_2\,s_2 = -1.575$$
$$Z_{cds} = j\,2.60 \times j\,2.60 + j\,2.60 \times j\,1.1875 + j\,2.60 \times j\,1.1875 = -12.935$$
$$3\,s_0\,s_2 = 3 \times j\,0.90 \times j\,0.2875 = -0.776$$
$$Z_{cds} + 3\,S_0\,S_2 = -13.71$$
$$k' = j\,0.799$$
$$l' = j\,0.799$$
$$m' = 0.0093 - j\,0.1086$$
$$n' = -0.0093 - j\,0.1086$$

Substituting these values of k', l', m', n' in equation (76) the value of the impedance, Z_{ri}, to replace the two lines with the two open conductors may be obtained. Z_c and Z_d are the positive phase reactances of the two lines.

$$Z_c = Z_d = j\,1.00$$

$$Z_{ri} = \frac{j\,1.799 \times j\,1.799 + 0.0119}{j\,3.598 + j\,0.217} = j\,0.840$$

If the impedance Z_{ri} is inserted in the positive phase diagram to replace the two lines with two conductors open, Fig. 15C is obtained, from which the transfer impedance, Z_{12}, between E_1 and E_2 may be calculated.

$$Z_{12} = 0.798$$

ACKNOWLEDGMENT

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Appendix A

Fundamental Symmetrical Component Equations:[2]

$$I_a = I_{a0} + I_{a1} + I_{a2} \tag{1a}$$
$$I_b = I_{a0} + a^2\,I_{a1} + a\,I_2 \tag{2a}$$
$$I_c = I_{a0} + a\,I_{a1} + a^2\,I_{a2} \tag{3a}$$
$$V_a = V_{a0} + V_{a1} + V_{a2} \tag{4a}$$
$$V_b = V_{a0} + a^2\,V_{a1} + a\,V_{a2} \tag{5a}$$
$$V_c = V_{a0} + a\,V_{a1} + a^2\,V_{a2} \tag{6a}$$
$$I_{a0} = \frac{1}{3}\,(I_a + I_b + I_c) \tag{7a}$$
$$I_{a1} = \frac{1}{3}\,(I_a + a\,I_b + a^2\,I_c) \tag{8a}$$
$$I_{a2} = \frac{1}{3}\,(I_a + a^2\,I_b + a\,I_c) \tag{9a}$$
$$V_{a0} = \frac{1}{3}\,(V_a + V_b + V_c) \tag{10a}$$
$$V_{a1} = \frac{1}{3}\,(V_a + a\,V_b + a^2\,V_c) \tag{11a}$$
$$V_{a2} = \frac{1}{3}\,(V_a + a^2\,V_b + a\,V_c) \tag{12a}$$

where

$$a = -\frac{1}{2} + j\,\frac{\sqrt{3}}{2} = 1.00\,\underline{/120^\circ} \tag{13a}$$

$$a^2 = -\frac{1}{2} - j\,\frac{\sqrt{3}}{2} = 1.00\,\underline{/240^\circ} = 1.00\,\underline{/-120^\circ} \tag{14a}$$

Notation

I_a, I_b and I_c are the three line currents at any point of the system.

V_a, V_b and V_c are the three voltages to ground at any point of the system.

I_{a1}, I_{b1} and I_{c1} are the positive phase sequence currents in the three conductors. By definition these currents are equal in magnitude and I_{a1} leads I_{b1} by 120 deg. and I_{c1} by 240 deg.

I_{a2}, I_{b2} and I_{c2} are the negative phase sequence currents in the three conductors. By definition these currents are equal in magnitude and I_{a2} leads I_{c2} by 120 deg. and I_{b2} by 240 deg.

I_{a0}, I_{b0} and I_{c0} are the zero phase sequence currents in the three conductors. By definition these currents are equal in magnitude and in phase.

Notation for components of voltage corresponds to that for components of currents.

Appendix B

Relations Between the Components of Current and of Voltage at a Single Fault for Various Types of Fault

Let I_a, I_b and I_c be the currents flowing *into* the fault from lines a, b and c respectively, and V_a, V_b and V_c the voltages to ground of phases a, b and c respectively, all at point of fault, C.

When a ground occurs on a three-phase system the conductors which become grounded have zero voltage to ground and the conductors which are not involved have zero current flowing into the fault. When the fault is between conductors and not to ground, voltages to ground on the faulted conductors are equal and the sum of the currents flowing into the fault is zero. Thus three equations may be written in terms of actual currents or voltages, which when used with the fundamental equations (**1a**)—(**12a**) of Appendix A permit zero and negative phase currents and voltages to be expressed in terms of positive phase currents and voltage, respectively.

In these fundamental equations phase a is taken as reference phase, and the currents and voltages of phases b and c are expressed in terms of the symmetrical components of phase a.

Negative and zero phase voltages and currents will be expressed in terms of positive phase voltages and currents respectively for typical fault conditions.

A. Line-to-ground Fault.

Fault on phase b, Fig. 16A

Fault conditions: $I_a = I_c = 0$ and $V_b = 0$.

From (**5a**): $V_{a1} = -(a V_{a0} + a^2 V_{a2})$

From (**7a**), (**8a**) and (**9a**):

$$I_{a0} = \frac{I_b}{3}, I_{a1} = \frac{a I_b}{3}, I_{a2} = \frac{a^2 I_b}{3}$$

$$\therefore I_{a0} = a^2 I_{a1} \text{ or } a I_{a0} = I_{a1}$$
$$\text{and } I_{a2} = a I_{a1} \text{ or } a^2 I_{a2} = I_{a1}$$

B. Line-to-Line Faults.

Fault on phases a and b, Fig. 16B

Fault conditions: $I_c = 0$, $I_b = -I_a$ and $V_a = V_b$.

From (**7a**): $I_{a0} = 0$

From (**8a**): $I_{a1} = \dfrac{I_a(1-a)}{3}$

From (**9a**): $I_{a2} = \dfrac{I_a(1-a^2)}{3}$

$$\therefore I_{a2} = \frac{1-a^2}{1-a} I_{a1} = -a^2 I_{a1} \text{ or } -a I_{a2} = I_{a1}$$

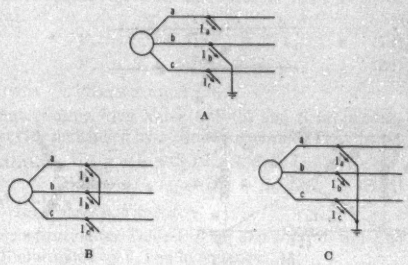

Fig. 16

A—Currents Flowing Into Single Line-to-Ground Fault

B—Currents Flowing Into Line-to-Line Fault

C—Currents Flowing Into Double Line-to-Ground Fault

From (**4a**) and (**5a**):

$$V_a - V_b = (1-a^2) V_{a1} + (1-a) V_{a2} = 0$$

$$\therefore V_{a2} = -\frac{1-a^2}{1-a} V_{a1} = a^2 V_{a1} \text{ or } V_{a1} = a V_{a2}$$

C. Double Line-to-Ground Fault.

Fault on phases a and c, Fig. 16C

Fault conditions: $V_a = V_c = 0$ and $I_b = 0$.

From (**2a**) $I_b = I_{a0} + a^2 I_{a1} + a I_{a2} = 0$

$$\therefore I_{a1} = -(a I_{a0} + a^2 I_{a2})$$

From (**10a**), (**11a**), (**12a**):

$$V_{a0} = \frac{V_b}{3}, V_{a1} = \frac{a V_b}{3}, V_{a2} = \frac{a^2 V_b}{3}$$

$$\therefore V_{a0} = a^2 V_{a1} \text{ or } V_{a1} = a V_{a0}$$
$$\text{and } V_{a2} = a V_{a1} \text{ or } V_{a1} = a^2 V_{a2}$$

D. Three-Phase Fault.

(a) Conductors not grounded.

Fault conditions: $V_a = V_b = V_c$ and $I_a + I_b + I_c = 0$

From (**7a**): $I_{a0} = 0$

From (**11a**): $V_{a1} = 0$

From (**12a**): $V_{a2} = 0$

(b) Conductors grounded.

Fault conditions: $V_a = V_b = V_c = 0$

From (**10a**): $V_{a0} = 0$

From (**11a**): $V_{a1} = 0$

From (**12a**): $V_{a2} = 0$

Appendix C

Zero Phase Equivalent Circuits for Two Parallel Transmission Lines

It will be assumed that the conductors of all circuits are completely transposed, so that the same impedance is offered to zero phase currents by each of the three conductors of an any circuit.

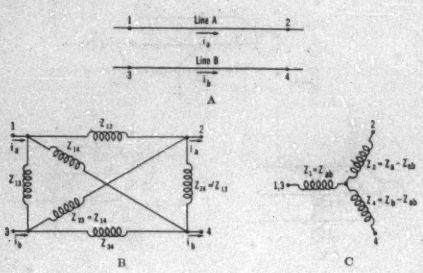

Fig. 17

A—Parallel Transmission Lines Having Impedance Z_a and Z_b and Mutual Z_{ab}

B—Equivalent Circuit to Replace the Two Parallel Lines Between Terminals 1, 2, 3 and 4

C—Equivalent Circuit to Replace Two Parallel Lines Bussed at One End but Not at the Other

1. Two Parallel Transmission Lines.

(*a*) *General Case.* Any two parallel transmission lines or portions of parallel transmission lines having impedances Z_a and Z_b and mutual reactance, Z_{ab}, between them may be replaced at their terminals by an equivalent circuit consisting of a six-branch network directly connecting the four terminals.[10] If 1 and 2 are the terminals of line *A*, and 3 and 4 the terminals of line *B*, see Fig. 17, then the six impedances of the equivalent circuit are:

$$Z_{12} = \frac{Z_a Z_b - Z_{ab}^2}{Z_b} = \text{impedance between points 1 and 2.}$$

$$Z_{34} = \frac{Z_a Z_b - Z_{ab}^2}{Z_a} = \text{impedance between points 3 and 4.}$$

$$Z_{13} = Z_{24} = \frac{Z_a Z_b - Z_{ab}^2}{Z_{ab}} = \text{impedance between points 1 and 3 and between points 2 and 4.}$$

$$Z_{14} = Z_{23} = -\frac{Z_a Z_b - Z_{ab}^2}{Z_{ab}} = \text{impedance between points 1 and 4 and between points 2 and 3.} \qquad \textbf{(1c)}$$

Equations, (**1c**), were derived by analogy from similar equations for the equivalent circuit of a two winding transformer given by Mr. George A. Campbell in "Cisoidal Oscillations," p. 890, Proc. A. I. E. E., 1911.

(*b*) *Two Lines bussed at both ends.* If two parallel lines are bussed at both ends the equivalent circuit of Fig. 17B reduces to a single impedance branch Z_e where

$$Z_e = \frac{1}{\dfrac{1}{Z_{12}} + \dfrac{1}{Z_{14}} + \dfrac{1}{Z_{23}} + \dfrac{1}{Z_{34}}} = \frac{Z_a Z_b - Z_{ab}^2}{Z_a + Z_b - 2Z_{ab}} \qquad \textbf{(2c)}$$

If $Z_a = Z_b$

$$Z_e = \frac{Z_a + Z_{ab}}{2} \qquad \textbf{(3c)}$$

Neglecting Resistance

$$X_e = \frac{X_a + X_{ab}}{2} \qquad \textbf{(4c)}$$

(*c*) *Two Lines bussed at one end but not at the other.* If the lines are bussed so that points 1 and 3 are together Fig. 17B becomes a Δ connecting points 1, 2 and 4, which may be converted into a *Y*, Fig. 17C having branch impedances Z_1, Z_2 and Z_4 where

$$\left.\begin{aligned} Z_1 &= Z_{ab} \\ Z_2 &= Z_a - Z_{ab} \\ Z_4 &= Z_b - Z_{ab} \end{aligned}\right\} \qquad \textbf{(5c)}$$

This case may be extended to include parallel lines which are bussed through unequal impedances. On the zero phase diagram of an actual system which includes two parallel lines it is usually possible to find a point where the parallel lines are bussed at one end or the other through impedances. Such a point may be the ground or any branch point of the system, or it may be obtained by Δ-*Y* or *Y*-Δ transformations.

If the impedance from the branch point 5 to the terminals 1 and 3 of the lines *A* and *B* are Z_α and Z_β respectively, an equivalent *Y* may be used between points 5, 2 and 4, having the branch impedances Z_5, Z_2 and Z_4, where

$$Z_5 = Z_{ab}$$

$$Z_2 = Z_a + Z_\alpha - Z_{ab}$$

$$Z_4 = Z_b + Z_\beta - Z_{ab}$$

These above relations follow directly from (**17c**) if Z_α and Z_β are the impedances from the junction point

and therefore include the external impedances Z_a and Z_β.

Mutual Reactance Between Two Parallel Transmission Lines

From (4c)

$$X_{ab} = 2\,X_e - X_a \tag{6c}$$

Equation (6c) gives the mutual reactance X_{ab} in terms of X_a, the reactance of one line alone, and X_e the reactance of the two lines in parallel. X_{ab} may be expressed in terms of X_1, the positive phase reactance of one line, by substituting for X_a and X_e their values in terms of X_1.

For lines without ground wires or with light steel ground wires.[7]

If $X_a = 3.5\,X_1$ and $X_e = 2.5\,X_1$ then

$$X_{ab} = 1.5\,X_1$$

For heavy copper ground wires.

If $X_a = 2\,X_1$ and $X_e = 1.5\,X_1$ then

$$X_{ab} = X_1$$

These approximate values for mutual reactance may be used in determining zero phase equivalent circuits for two parallel transmission lines when extreme accuracy is not required.

Appendix D

Relations Between Positive Phase Components of Voltage at the Faults and Positive Phase Currents Flowing Into the Faults for Two Simultaneous Faults

Table I expresses negative and zero phase voltages at the point of fault, and currents flowing into the fault in terms of the positive phase voltage at the fault and the positive phase current into the fault when grounds occur on one or more of the three conductors of a three-phase system, taking phase a as reference phase. Similar relations for a second point of fault are obtained by substituting α, β, γ for a, b, and c in Table I. These two sets of relations from Table I substituted in combinations of equations (5)—(14) allow V_{a1} and $V_{\alpha1}$ to be expressed in terms of I_{a1} and $I_{\alpha1}$.

Case E. Single Line-to-Ground Faults at Two Points.

(a) corresponding phases a and α.

Adding equations (5) and (10) and (6) and (11) and substituting from Table I, A (a):

$$V_{a1} = (C_0 + S_0 + C_2 + S_2)\,I_{a1} + (S_0 + S_2)\,I_{\alpha1} \tag{1d}$$

$$V_{\alpha1} = (S_0 + S_2)\,I_{a1} + (D_0 + S_0 + D_2 + S_2)\,I_{\alpha1} \tag{2d}$$

(b) Phases b and α.

Multiplying equations (5) by a and equation (10) by a^2 and adding: adding equations (6) and (11); substituting from Table I, A (b) and A (a):

$$V_{a1} = (C_0 + S_0 + C_2 + S_2)\,I_{a1} + (a\,S_0 + a^2\,S_2)\,I_{\alpha1} \tag{3d}$$

$$V_{\alpha1} = (a^2\,S_0 + a\,S_2)\,I_{a1} + (D_0 + S_0 + D_2 + S_2)\,I_{\alpha1} \tag{4d}$$

(c) Phases c and α.

Multiplying equation (5) by a^2 and equation (10) by a and adding; adding equations (6) and (11); substituting from Table I, A (c) and A (a):

$$V_{a1} = (C_0 + S_0 + C_2 + S_2)\,I_{a1} + (a^2\,S_0 + a\,S_2)\,I_{\alpha1} \tag{5d}$$

$$V_{\alpha1} = (a\,S_0 + a^2\,S_2)\,I_{a1} + (D_0 + S_0 + D_2 + S_2)\,I_{\alpha1} \tag{6d}$$

Case F. Line-to-Line Faults at Two Points.

(a) Corresponding phases b, c and β, γ.

Replacing negative phase currents and voltages in equations (10) and (11) by their values in terms of positive phase currents and voltages from Table I, B(a):

$$V_{a1} = (C_2 + S_2)\,I_{a1} + S_2\,I_{\alpha1} \tag{7d}$$

$$V_{\alpha1} = S_2\,I_{a1} + (D_2 + S_2)\,I_{\alpha1} \tag{8d}$$

(b) Phases a, c and β, γ.

Substituting from Table I, B (b) and B (a) in equation (10) multiplied by a^2, and in equation (11):

$$V_{a1} = (C_2 + S_2)\,I_{a1} + a^2\,S_2\,I_{\alpha1} \tag{9d}$$

$$V_{\alpha1} = a\,S_2\,I_{a1} + (D_2 + S_2)\,I_{\alpha1} \tag{10d}$$

(c) Phases a, b and β, γ.

Substituting for Table I, B (c) and B (a) in equation (10) multiplied by a, and in equation (11):

$$V_{a1} = (C_2 + S_2)\,I_{a1} + a\,S_2\,I_{\alpha1} \tag{11d}$$

$$V_{\alpha1} = a^2\,S_2\,I_{a1} + (D_2 + S_2)\,I_{\alpha1} \tag{12d}$$

Case G. Double Line-to-Ground Faults at Two Points.

(a) Corresponding phases b, c and β, γ.

Adding equations (7) and (12), and (8) and (13), and substituting from Table I, C (a).

$$I_{a1} = V_{a1}\left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}}\right) - V_{\alpha1}\left(\frac{S_0}{Z_{00}} + \frac{S_2}{Z_{22}}\right) \tag{13d}$$

$$I_{\alpha1} = -V_{a1}\left(\frac{S_0}{Z_{00}} + \frac{S_2}{Z_{22}}\right) + V_{\alpha1}\left(\frac{C_0 + S_0}{Z_{00}} + \frac{C_2 + S_2}{Z_{22}}\right) \tag{14d}$$

Expressing V_{a1} and $V_{\alpha1}$ in terms of I_{a1} and $I_{\alpha1}$ by solving equations (13d) and (14d) and making use of equations (9) and (14):

$$V_{a1} = \frac{[Z_{22}\,(C_0 + S_0) + Z_{00}\,(C_2 + S_2)]\,I_{a1} + [Z_{22}\,S_0 + Z_{00}\,S_2]\,I_{\alpha1}}{(C_0 + C_2)\,(D_0 + D_2) + (C_0 + C_2)\,(S_0 + S_2) + (D_0 + D_2)\,(S_0 + S_2)} \tag{15d}$$

$$V_{\alpha1} = \frac{[Z_{22}\,S_0 + Z_{00}\,S_2]\,I_{a1} + [Z_{22}\,(D_0 + S_0) + Z_{00}\,(D_2 + S_2)]\,I_{\alpha1}}{(C_0 + C_2)\,(D_0 + D_2) + (C_0 + C_2)\,(S_0 + S_2) + (D_0 + D_2)\,(S_0 + S_2)} \tag{16d}$$

(b) Phases a, c and β, γ.

Multiplying (7) by a and (12) by a^2 and adding: adding (8) and (13): substituting from Table I, C (b) and C(a):

$$I_{a1} = V_{a1}\left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}}\right) - V_{a1}\left(\frac{a\ S_0}{Z_{00}} + \frac{a^2 S_2}{Z_{22}}\right) \tag{17d}$$

$$I_{a1} = -V_{a1}\left(\frac{a^2 S_0}{Z_{00}} + \frac{a\ S_2}{Z_{22}}\right) + V_{a1}\left(\frac{C_0 + S_0}{Z_{00}} + \frac{C_2 + S_2}{Z_{22}}\right) \tag{18d}$$

Solving (17d) and (18d) for V_{a1} and V_{a1}, remembering that $a \times a^2 = 1$ and $a + a^2 = -1$:

$$V_{a1} = \frac{[Z_{22}\,(C_0 + S_0) + Z_{00}\,(C_2 + S_2)]\,I_{a1} + (a\,S_0\,Z_{22} + a^2\,S_2\,Z_{00})\,I_{a1}}{(C_0 + C_2)\,(D_0 + D_2) + (C_0 + C_2)\,(S_0 + S_2) + (D_0 + D_2)\,(S_0 + S_2) + 3\,S_0\,S_2} \tag{19d}$$

$$V_{a1} = \frac{(a^2\,S_0\,Z_{22} + a\,S_2\,Z_{00})\,I_{a1} + [Z_{22}\,(D_0 + S_0) + Z_{00}\,(D_2 + S_2)]\,I_{a1}}{(C_0 + C_2)\,(D_0 + D_2) + (C_0 + C_2)\,(S_0 + S_2) + (D_0 + D_2)\,(S_0 + S_2) + 3\,S_0\,S_2} \tag{20d}$$

(c) Phases a, b, and β, γ.

This case may be solved by analogy from (b), a replacing a^2, and a^2 replacing a in equations (19d) and (20d).

Case H. Three-Phase Faults at Two Points.

From Table I, D.

$$V_{a1} = 0 \tag{21d}$$

$$V_{a1} = 0 \tag{22d}$$

Case I. Line-to-Line Fault at C and Single Line-to-Ground Fault at D.

(a) Phases b, c and α.

Substituting from Table I, B (a) and A (a) in equations (10) and in (6) plus (11)

$$V_{a1} = I_{a1}\,(C_2 + S_2) - I_{a1}\,S_2 \tag{23d}$$

$$V_{a1} = -\,I_{a1}\,S_2 + I_{a1}\,(D_0 + S_0 + D_2 + S_2) \tag{24d}$$

(b) Phases a, c and α.

Substituting from Table I, B (b) and A (a) in equation (10) multiplied by a^2, and in equation (6) plus (11):

$$V_{a1} = I_{a1}\,(C_2 + S_2) - I_{a1}\,a^2\,S_2 \tag{25d}$$

$$V_{a1} = -\,a\,S_2\,I_{a1} + I_{a1}\,(D_0 + S_0 + D_2 + S_2) \tag{26d}$$

(c) Phases a, b, and α.

By analogy from (b) replacing a by a^2, and a^2 by a.

Case J. Double Line-to-Ground Fault at C, Single Line-to-Ground Fault at D.

(a) Phases b, c and α.

Adding equations (7) and (12) and substituting from Table I, C (a):

$$I_{a1} = V_{a1}\left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}}\right) - V_{a0}\,\frac{S_0}{Z_{00}} - V_{a2}\,\frac{S_2}{Z_{22}} \tag{27d}$$

Transposing and simplifying equations (8) and (13) to express V_{a0} in terms of V_{a0} and I_{a0} and V_{a2} in terms of V_{a2} and I_{a2}: then replacing V_{a0} and V_{a2} by V_{a1} from Table I, C (a) and I_{a0} and I_{a2} by I_{a1} from A (a):

$$V_{a0} = V_{a1}\,\frac{S_0}{C_0 + S_0} - I_{a1}\,\frac{Z_{00}}{C_0 + S_0} \tag{28d}$$

$$V_{a2} = V_{a1}\,\frac{S_2}{C_2 + S_2} - I_{a1}\,\frac{Z_{22}}{C_2 + S_2} \tag{29d}$$

Substituting (28d) and (29d) in (27d), and replacing Z_{00} and Z_{22} by their values given in (9) and (14):

$$I_{a1} = V_{a1}\left(\frac{1}{C_0 + S_0} + \frac{1}{C_2 + S_2}\right) + I_{a1}\left(\frac{S_0}{C_0 + S_0} + \frac{S_2}{C_2 + S_2}\right) \tag{30d}$$

Transposing (30d)

$$V_{a1} = \frac{(C_0 + S_0)\,(C_2 + S_2)}{C_0 + S_0 + C_2 + S_2}\,I_{a1} - \frac{S_0\,(C_2 + S_2) + S_2\,(C_0 + S_0)}{C_0 + S_0 + C_2 + S_2}\,I_{a1} \tag{31d}$$

Adding equations (28d) and (29d), and replacing $(V_{a0} + V_{a2})$ by $-\,V_{a1}$ from Table I, A (a) and V_{a1} by its value from (31d)

$$V_{a1} = -\,\frac{S_0\,(C_2 + S_2) + S_2\,(C_0 + S_0)}{C_0 + S_0 + C_2 + S_2}\,I_{a1} + \left(D_0 + D_2 + S_0 + S_2 - \frac{(S_0 - S_2)^2}{C_0 + S_0 + C_2 + S_2}\right)I_{a1} \tag{32d}$$

(b) Phases a, c and α.

Multiplying equation (7) by a, equation (12) by a^2; adding and substituting from Table I, C (b):

$$I_{a1} = V_{a1}\left(\frac{D_0 + S_0}{Z_{00}} + \frac{D_2 + S_2}{Z_{22}}\right) - a\,V_{a0}\,\frac{S_0}{Z_{00}} - a^2\,V_{a2}\,\frac{S_2}{Z_{22}} \tag{33d}$$

From equations (8) and (13) and Table I, C (b) and A (a)

$$V_{a0} = a^2 V_{a1} \frac{S_0}{C_0 + S_0} - I_{a1} \frac{Z_{00}}{C_0 + S_0} \quad (34d)$$

$$V_{a2} = a V_{a1} \frac{S_2}{C_2 + S_2} - I_{a1} \frac{Z_{22}}{C_2 + S_2} \quad (35d)$$

Substituting (34d) and (35d) in (33d) and transposing:

$$V_{a1} = \frac{(C_0 + S_0)(C_2 + S_2)}{C_0 + S_0 + C_2 + S_2} I_{a1}$$

$$- \frac{a S_0 (C_2 + S_2) + a^2 S_2 (C_0 + S_0)}{C_0 + S_0 + C_2 + S_2} I_{a1} \quad (36d)$$

Adding (34d) and (35d), replacing $(V_{a0} + V_{a2})$ by $- V_{a1}$ from Table I, *A* (a) and V_{a1} by its value from (36d):

$$V_{a1} = - \frac{a^2 S_0 (C_2 + S_2) + a S_2 (C_0 + S_0)}{C_0 + S_0 + C_2 + S_2} I_{a1}$$

$$+ \left(D_0 + D_2 + S_0 + S_2 - \frac{S_0^2 + S_0 S_2 + S_2^2}{C_0 + S_0 + C_2 + S_2} \right) I_{a1} \quad (37d)$$

(c) Phases *a*, *b* and *α*.

By analogy from (36d) and (37d) replacing *a* by a^2, and a^2 by *a*.

Case K. Three-Phase Fault at C and Single Line-to-Ground Fault at D, .

(a) Three-phase fault not involving ground.

Substituting $I_{a0} = 0$ from Table I, *D* (a) in equations (5) and (6), and $V_{a2} = 0$ in equation (10):

$$- V_{a0} = I_{a0} S_0 \quad (38d)$$

$$- V_{a0} = I_{a0} (D_0 + S_0) \quad (39d)$$

$$I_{a2} = - I_{a2} \frac{S_2}{C_2 + S_2} \quad (40d)$$

Substituting (40d) in (11) and simplifying

$$- V_{a2} = I_{a2} \left(D_2 + \frac{C_2 S_2}{C_2 + S_2} \right) \quad (41d)$$

adding (39d) and (41d) and substituting from Table I, *A* (a):

$$V_{a1} = I_{a1} \left(D_0 + S_0 + D_2 + \frac{C_2 S_2}{C_2 + S_2} \right) \quad (42d)$$

From Table I, *D*.

$$V_{a1} = 0 \quad (43d)$$

(b) Three-phase fault involving ground.

Substituting $V_{a0} = 0$, and $V_{a2} = 0$ from Table I, *D* (b) in (5) and (10) and expressing I_{a0} and I_{a2} in terms of I_{a0} and I_{a2}.

$$I_{a0} = - I_{a0} \frac{S_0}{C_0 + S_0} \quad (44d)$$

$$I_{a2} = - I_{a2} \frac{S_2}{C_2 + S_2} \quad (45d)$$

Adding (6) and (11), replacing I_{a0} and I_{a2} by their values given in (44d) and (45d), and substituting from Table I, *A* (a) and simplifying:

$$V_{a1} = \left(D_0 + \frac{C_0 S_0}{C_0 + S_0} + D_2 + \frac{C_2 S_2}{C_2 + S_2} \right) I_{a1} \quad (46d)$$

$$V_{a0} = 0 \quad (47d)$$

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Discussion

M. MacFerran: Miss Clarke's paper is of great practical value because, as is pointed out in the introduction, simultaneous faults do occur quite frequently especially when multicircuit tower lines are in use. For example, on the system of the Southern California Edison Co., Ltd., there are many tower lines carrying from two to twelve circuits, and there have been several cases in which trouble on such multicircuit structures has involved two lines going to entirely different parts of the system. Another very interesting case of simultaneous faulting occurred at a substation which was being operated with buses sectionalized, one section being on one major station, the other on another. Trouble originating on one bus spread to the other, thus producing a condition quite similar to that covered by the numerical example in the paper.

From the standpoint of one who is faced with the task of calculating simultaneous faults, the outstanding features of Miss Clarke's paper are first, the derivation of the values of *k*, *l*, *m*, and *n* for all conceivable combinations of faults; and second, the development of the approximate equivalent fault wye which can be used with a high degree of accuracy even when *m* is not equal to *n*. These two features make it possible to work out the complete story of any type of simultaneous fault in a quite reasonable length of time, and therefore enable one to give due attention to such faults in setting relays or analyzing their performance. This in turn represents an important step forward in protection engineering.

H. W. Bibber: While Miss Clarke's paper deals very exhaustively with problems involving simultaneous faults as they are

met in cases of greatest practical importance,—*i. e.*, where one fault is on one circuit and another is on a different circuit of the same *grounded* system—it seems possible that there may be cases where a solution is desired for simultaneous faults on an *ungrounded* system.

Perhaps because ungrounded systems are more common in Europe, the special case of simultaneous faults on a single-circuit line of an isolated neutral system has been treated to a limited extent in the European technical press,[8] but Miss Clarke has presented a much more general method which can be extended to cover such a special case.

Employing methods similar in nature to those which Miss Clarke uses for simultaneous faults on grounded systems, to solve for currents and voltages in isolated neutral systems gives interesting results.

On an isolated system, it is obvious that both of two simultaneous faults must involve ground or there will be no zero phase currents.

The equivalent circuit for the zero phase system if both faults involve ground will be a simple loop which will usually be easily obtainable by calculation. Should complication of the network make the use of a calculating table advisable, only one reading is necessary, that to determine $(C_o + D_o)$, following Miss Clarke's nomenclature.

The significant fact regarding the zero phase sequence currents at the fault points in an isolated system, is that one is the negative of the other. Since the zero phase circuit is a simple loop there will be a single independent voltage equation that can be written. The two relations to be noted are therefore, (cf. equations (7) and (8).

$$I_{ao} = -I_{ao}$$

$$I_{ao} = \frac{V_{ao} - V_{ao}}{C_o + D_o}$$

As far as the negative phase system is concerned, the grounding of the system is immaterial, so that an equivalent Y can be obtained for the isolated system as described by Miss Clarke. Equations (10) to (14) inclusive of her paper hold for this case.

As she has shown, there will be six equations connecting the components of current and the components of voltage to ground at the fault points, and in addition to the two equations connecting negative phase currents and voltages, such as (12) and (13), there are the two equations connecting the zero phase currents and voltages I have previously given. This provides ten simultaneous equations so that eight unknowns can be eliminated and the positive components of current, I_{a1} and I_{a1} may be expressed in terms of each other, the positive components of voltage, V_{a1} and V_{a1}, and the known zero and negative phase impedances. Here as in Miss Clarke's paper, the ten equations are linear and we may write:

$$I_{a1} = A\,V_{a1} + B\,V_{a1}$$

$$I_{a1} = F\,I_{a1}$$

where A, B, and F depend on the branch impedances of the equivalent Y replacing the negative phase network $(C_o + D_o)$, and the type of faults. Values of A, B and F, can be derived for faults which involve various combinations of conductors, and tabulated in a form similar to Table II of Miss Clarke's paper.

In the time that has been available I have not been able to find any simple general equivalent circuit which will replace the two simultaneous faults in the positive phase system. For the special cases of two unloaded feeders radiating from a single point and two lines bused at both ends I have worked out equivalent impedances to replace the faults.

[8] "Double Ground Faults on a Single-Circuit Line Fed from Two Points in the Light of Symmetrical Components," by G. Oberdorfer, *Wissenschaftliche Veroffentlichen aus dem Siemens*—Konzern IX Band 1930.

For other than the special cases mentioned, it is evident that the method of analytical solution described by Miss Clarke may be used for isolated systems, until such time as simple equivalent circuits may be brought forth. The two zero phase current and voltage relations I stated earlier as holding for the isolated neutral case, are substituted for equations (15) and (16). There will then be $(n + 4)$ equations with $(n + 4)$ unknowns when there are n machines in the system, and a complete solution can be obtained.

Although the case of two simultaneous faults on a single-circuit line of a *grounded* system has not been specifically mentioned by Miss Clarke, it is evident that her general methods hold for such a case, as C and D are any points at all on a system.

Viewed from the many different angles presented by different types of systems and fault conditions, it seems that Miss Clarke has pointed the way to the solution of almost any problem involving simultaneous faults that might arise.

W. C. Hahn: The material in this paper will be needed for the general case of simultaneous faults or open circuits on three-phase systems. There are three special cases, however, where the procedure can be simplified in making use of either the a-c. or d-c. calculating table. These are the cases of simultaneous three-phase, line-to-line, or double line-to-ground faults occurring on the same phases.

For simultaneous three-phase faults, of course, all that is necessary is to apply the several faults simultaneously on the usual set up.

For the case of simultaneous line-to-line faults on the same phase, such as discussed in Case F, Appendix D, sub-head "a," it is necessary to set up the positive and negative phase sequence diagrams simultaneously. Connections between the two networks are made to join the corresponding fault positions. The short-circuit plug is then applied at the neutral point of the negative phase sequence set up. A little consideration will show that this satisfies all the conditions given in Table I, providing all the faults are between the same two phases of the system.

The case of simultaneous double line-to-ground faults such as is given in Case G, Appendix D, sub-head "a," can also be represented by setting up the positive, negative and zero phase sequence diagrams on the calculating table. In this case connections between the networks are made between corresponding fault positions in all three networks.

It will be noted that these simplifications cannot be used for simultaneous line-to-ground faults because in this case, it is necessary to insert, in the connection between the positive and zero sequence setup a negative phase sequence voltage corresponding to that fault. It may be possible to find an equivalent circuit for the negative phase sequence network, which will satisfy this condition, but it appears that such a procedure may be more complicated than following that given in the paper.

It is also not feasible to use these simplifications for simultaneous faults on different phases when using the d-c. calculating table, because from Table I, it will appear that in at least one of the connections between the networks, a phase shift of 120 deg. must be introduced both in current and in voltage. While it is possible to produce the equivalent of this shift on the d-c. calculating table, by various reconnections of the networks, it is believed that the method outlined in the paper is simpler and occasions less confusion.

H. L. Hazen: Until rather recently unsymmetrical short circuits on power system networks have been unmanageable in any but the simplest cases. Each of the single unsymmetrical faults, as Miss Clarke has pointed out, has previously been reduced to an equivalent circuit, representing the fault, which can be applied to the ordinary positive phase sequence diagram, thus reducing the problem to the relatively simple study of a

balanced polyphase system. This brings it within the scope of calculating-table attack, rendering solutions practicable even with large and complicated networks.

Miss Clarke in the present paper has extended this treatment to cover the much more complicated case of two simultaneous unsymmetrical faults, treating not only each of the various kinds of unsymmetrical faults but most of the combinations in which these can occur; and, which is very important from the system engineer's point of view, she has put it in a form well suited to calculating-table treatment. This she has done in a very clear and complete manner.

As she has pointed out, either the d-c. calculating table or a-c. network analyzer can be used for the calculations; the former where the relatively rough approximations involved in its use are satisfactory; the latter where the desire for more accurate results dictates the use of the a-c. device. The a-c. network analyzer of course enables one to deal with all qualities in complex form, to use actual values of individual generator excitations, and thus to obtain results which are a good approximation to the truth.

In making calculating-table solutions of various sorts involving equivalent circuits one not infrequently encounters the serious difficulty that the real portions of certain impedances are negative. These impedances therefore are not physically realizable as such by any simple means. For practical purposes this limitation can be overcome with relative ease in the following way: insert in place of the unrealizable impedance element an adjustable voltage which, in the alternating-current case, is variable in both phase and magnitude. With voltmeter, ammeter and wattmeter measure the voltage, current, and power of this element. By a cut-and-try method adjust this voltage until the desired vector relation $V = I Z$ exists as shown by the instrument readings. Provided the network as a whole is dissipative—which must be the case where the equivalent circuits are a representation of actual physical circuits—there is only one vector value of the inserted voltage which will satisfy this condition. Practically this process is not difficult to carry out, as has been demonstrated by its use on the M. I. T. Network Analyzer, and it removes what otherwise might be an embarrassing restriction upon certain calculating-table solutions.

Edith Clarke: It is gratifying to learn from Miss Macferran's discussion that an operating engineer has found the paper of practical value.

Mr. H. W. Bibber by extending the methods of the paper to the case of simultaneous faults on *ungrounded* systems, has materially enlarged its scope.

Mr. W. C. Hahn in his discussion emphasizes the important fact that simultaneous line-to-line or double-line-to-ground faults on the same phases may be treated most simply as special cases. I am glad Mr. Hahn has brought out this point, for although the case of simultaneous double line-to-ground faults on the same phases has been covered as a special case in the paper, no reference to this was made in Table II where the constants k, l, m and n are tabulated.

Under the heading "Equivalent Y vs. Equivalent Δ" (page 927 of the paper), it is shown that positive sequence currents and voltages at the fault points are satisfied if the zero and negative sequence networks are connected in parallel between the fault points of the positive sequence system, corresponding fault points in the three networks being connected, and also the points of zero potential for the three systems.

In a similar manner from equations (7d) and (8d), by expressing I_{a1} and I_{a1} in terms of V_{a1} and V_{a1} and replacing the impedances of the equivalent Y by the admittances of the equivalent Δ, a proof may be obtained of Mr. Hahn's statement that simultaneous line-to-line faults on the same phases may be solved on the a-c. or d-c. calculating table by setting up the positive and negative sequence networks and joining corresponding fault points and points of zero potential in the two systems.

It is interesting to note from Mr. Hazen's discussion that circuits in which the real portions of certain impedances are negative (negative resistances) may now be represented on the M. I. T. Network Analyzer with the same degree of precision as the familiar positive resistances.

Power Equipment at New KDKA Station

BY R. L. DAVIS[1]

Associate, A. I. E. E.

THE new KDKA Station at Saxonburg, Pa., has been designed both for broadcasting and experimental work dealing with the development of large high-power transmitters. On this account the equipment is considerably larger than is found in any ordinary broadcasting station, the main rectifier being the largest high-voltage unit ever constructed for such use. The control and transfer features of the various power units are also considerably different than would be required for a conventional broadcasting transmitter, and designed not only with a view to safety and convenience for normal operation, but to facilitate experimental work as well.

The equipment and power apparatus has been laid out to provide the necessary special power supplies to two complete and independent high-power broadcasting transmitters, one being operated normally at 980 kilocycles, using the familiar signature of KDKA, while the other is planned for higher frequency work. Both of these equipments are designed to operate separately but for experimental purposes arrangements are made to operate either transmitter from the various power supplies of the other, thus providing the greatest variation possible in the range of power circuits as well as providing virtually duplicate equipment throughout for the main broadcasting transmitter. All of the power equipment, except transformers and reactors which are mounted outdoors, is arranged on the ground floor of the main station building, thus leaving the second floor entirely free for the radio apparatus and control boards. A view of the station building is shown in Fig. 1.

To meet possible future requirements of high power transmitters which quite likely may employ tubes much larger than any now known, it was deemed necessary to provide a plate voltage supply of 900 kw. at 30 kv. of high-voltage direct-current energy for the standard broadcast transmitter. Similarly, the filament heating requirements, were set at 3,000 amperes, 40 volts, and the grid voltage supply at 3,000 volts direct current. In addition to the foregoing, a moderate amount of power at 10 to 15 kv. and still smaller amounts at 3 kv. and 400 volts are needed for the various intermediate amplifiers and the master oscillator.

Because of the probable limitation of the high-frequency tubes it was not considered necessary to provide as large a main plate power supply system for the short-wave transmitter and therefore the second rectifier unit was designed for only 450 kw. at 22 kv. However, since either plate supply system may be used with either transmitter, the maximum plate power can also be used for any high-frequency experimental work that may require it. Approximately the same power supplies are needed for the intermediate amplifiers and master oscillator of both the high-frequency and the standard broadcast transmitter and therefore these were designed in duplicate to provide the greatest interchangeability of equipment. Filament heating requirements for the high-frequency transmitter were set at 2,000 amperes.

Since all high quality broadcasting equipment required pure direct current for grid plate and filament supplies, and because all commercial power circuits are alternating current, it can readily be recognized that the problem of converting the normal three-phase 60-cycle supply to meet the special needs of broadcasting is one of greatest importance in radio station design.

The two main rectifiers are of the polyphase type and operate directly from the 2,300-volt three-phase station

FIG. 1—STATION BUILDING

supply. The 900-kw. rectifier employs six 190-kva. single-phase transformers to provide the proper voltage and phase relations for rectification into the desired direct-current energy. Each of these six rectifier plate transformers has double high-voltage windings and these are connected to form four stars, the branches of the stars being displaced in phase by special connections so as to produce twelve-phase rectification. Each of the stars is connected to multi-anode mercury cathode rectifier tubes and the direct current output of the four tubes is connected in series so that while the rectification takes place in three-phase units the result is the same as if a single high-voltage multi-anode tube of twelve phases were employed. Connecting four stars in series in this manner also requires only a moderate inverse peak voltage per tube in spite of the unusually high d-c. voltage developed and by keeping the inverse voltage well within the conservative upper limit for mercury vapor tubes, conditions most suitable for a long trouble-free life are obtained. So far no noticeable troubles due

1. Westinghouse Electric and Mfg. Co., Chicopee Falls, Mass.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

31-67

to flashbacks and similar disturbances have been experienced with the tubes now in service at the new station.

The smaller, or 450-kw. rectifier, is similar to the larger one except only three single-phase plate transformers giving six-phase rectification are employed, this being considered sufficient for the voltage and power desired. These plate transformers are each provided with four high-voltage windings and four stars are formed which are connected to a similar number of rectifier tubes, as in the case of the twelve-phase rectifier, but only six-phase rectification is obtained as no effort was made to provide the special phase relations between the various stars as in the case of the larger unit.

sides of the delta-connected bank, stable phase relations of the desired displacement can be obtained. Also, the windings are more nearly the same voltage so that only a slight excess kva. capacity and proper taps are required to make all six transformers exactly alike, which permits connection of a single unit into either star or delta groups, an important feature in providing for reserve or spare equipment.

To smooth the rectifier output into pure continuous current, untuned filter circuits made up of large iron cored reactors and banks of high-voltage condensers are connected between the rectifier tubes and radio equipment. The rectifier plate transformers and filter reactors constitute quite a percentage of the total bulk of

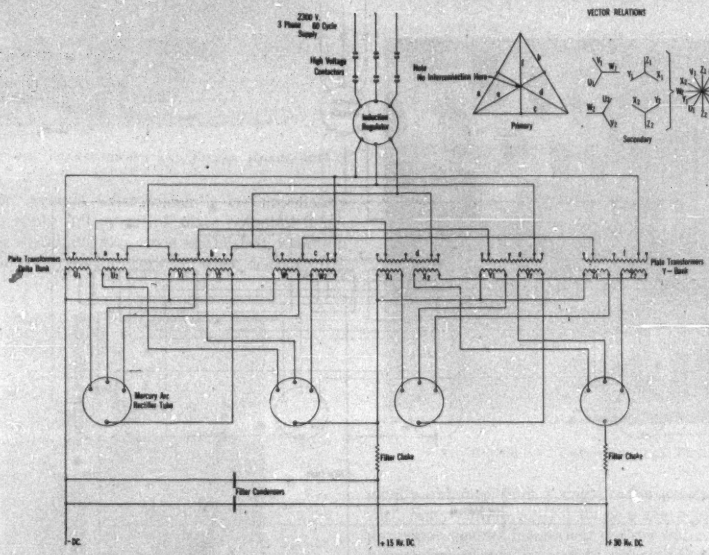

FIG. 2A—SCHEMATIC DIAGRAM OF RECTIFIERS 900 KW., 12 PHASE, 30 KV.

Connection of the transformer banks and associated apparatus for the main rectifiers is shown schematically in Fig. 2. Reference to this diagram indicates the special connections employed on the supply side of the plate transformers in the 900-kw. rectifier to provide the twelve-phase relations for the rectifier tubes. It might be thought that a simple star connection could be employed for three of the transformers to furnish the 30-deg. phase displacement between the star and delta banks. However, consideration shows that star-star connections are particularly bad for rectifiers because of the floating neutral and should never be employed, but by carrying the phase windings of the star across, like the medians of a triangle, and tying them into the

power equipment and are mounted out-of-doors in the rear of the building, thereby saving considerably in its size and cost. Fig. 3 shows the arrangement of this apparatus within the enclosure at the rear of the main building.

In order to control the d-c. voltage output developed by the rectifier, three-phase induction regulators are employed with which it is possible to vary the applied a-c. voltage from 0 to 4,600, the regulators being designed to give 100 per cent range both above and below the line potential. This is a greater range of voltage control than is usually provided for most radio station rectifiers, but because of the desirability, for experimental work, to have continuous variation over the

entire range of voltage without steps, it seemed advisable to install sufficient regulator capacity for this purpose. Induction regulators of the size needed, if made self-cooling, would occupy considerable floor space but since water-cooling facilities were also required for the radio tubes, it was possible to employ water-cooled regulators with considerable advantage, both in initial cost and space saved. Motor operation is provided for both rectifier regulators which makes it possible to control the high-voltage direct current supplied to the radio equipment from any one of several push-button stations located conveniently in the main transmitter and control rooms. A simple position switch on the regulator rotor, operating signal lights in the main transmitter room, gives an approximate indication of what voltage will be developed by the rectifier before the power contactors are closed. Actual power control of the rectifier is handled by air-break contactors, similar to those used in mine-hoist control, operating directly in the 2,300-volt supply to the induction regulator and plate transformers. Additional push-buttons and signal lights at the various stations around the transmitter room enable control of the power contactors and indicate their condition of operation. The contactor control circuits are interlocked for safety to the operators through switches on doors giving access to high-voltage circuits. Alternating- and direct-current overload relays are provided to shut down the equipment in event of trouble necessitating the removal of plate voltage. An oil circuit breaker, manually operated from the main switchboard, is also furnished. This circuit breaker is equipped with dash-pot overload relays set to trip on continued or extreme overload conditions.

The tubes used in the main rectifier are of the glass envelope mercury cathode type, somewhat similar to ones formerly used for operating d-c. series arc light circuits, except that they are larger and have three main anodes. Two small sustaining arc anodes and one starting mercury anode are also provided in each tube. One of these tubes is shown in Fig. 4. In operating, the tubes are fully immersed in oil, each mounted in a separate oil tank in a cradle by means of which it can be readily removed from the oil. All necessary connections are made when the cradle slides into place, thus permitting tube changes to be conveniently made with minimum effort. For interchangeability, tubes of the same rating are used in both main rectifiers. A view of the four tube tanks of the 900-kw. rectifier is shown in Fig. 5.

Automatic tilting mechanisms for starting the sustaining arcs are provided. This permits the rectifier

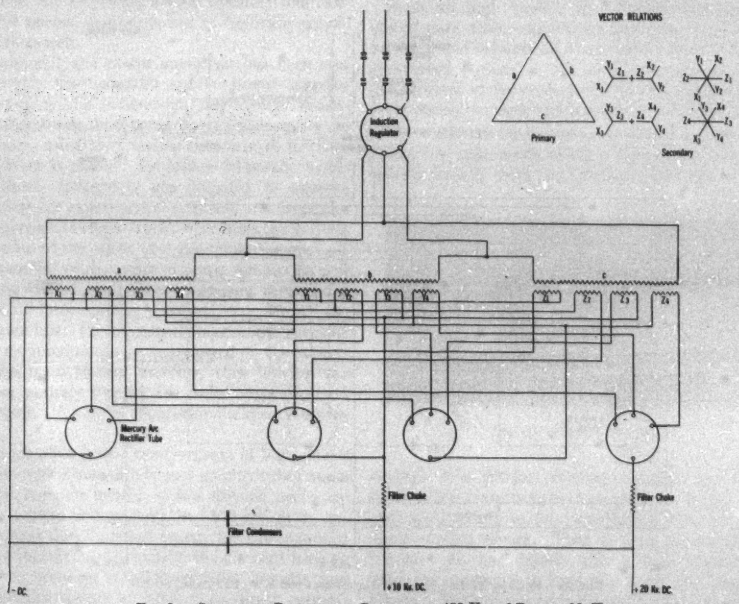

Fig. 2b—Schematic Diagram of Rectifiers 450 Kw., 6 Phase, 22 Kv.

to be placed in operation from a remote push-button station with no more attention from the operator than the switching on of the filament heating circuit of hot cathode tubes. The starting mechanism functions in

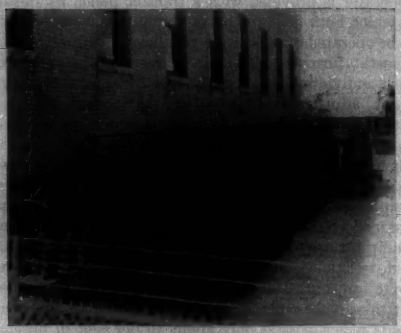

Fig. 3—Plate Transformers and Filter Reactances

one to two seconds, after which it is immediately possible to apply full operating plate potential if desired. The sustaining arcs, once established, maintain the tubes in readiness for operation when desired with-

Fig. 4—High-Voltage Mercury Cathode Tubes

out further use of the starting mechanism unless the power supply circuit is interrupted. Very little energy, about 120 watts per tube, is required to operate the sustaining arcs. This is low compared to filament type tubes since the mercury cathode automatically provides emission as needed at the hot spot in the mercury pool and therefore the power for the sustaining arc does not need to maintain the maximum emission at all times. Mercury pool rectifier tubes such as these described, while not commonly employed heretofore for radio power apparatus, appear to have a number of desirable characteristics, particularly in installations of such size that the use of filament tubes would require a great number of them both in series and parallel to furnish the equivalent amount of high-voltage direct-current energy.

In all but very small telephone transmitters, direct current is required for filament heating. With even the largest sets now commercially built this can be

Fig. 5—900-Kw. Rectifier Tube Tanks

readily obtained from a single motor-generator set and with fairly simple control to give the necessary tube protection against insufficient cooling water, this latter being accomplished by merely stopping the whole unit or preventing its operation when there is not enough cooling water circulating through the tube jackets. Also, gradual application of filament voltage, which is essential in the starting of large tubes, is obtained by merely having the machine connected with the tubes when building up.

To operate the new Saxonburg broadcast transmitter in this manner, would require a 3,000-ampere generator. While high-current low-voltage machines are available, these are generally unsatisfactory for radio station use, being built primarily for electrolytic or battery charging work where ripple is not important. It being considered inadvisable to develop a special generator for this installation, it was felt best to arrange for the operation of standard generators in parallel. Although this is common practise in power work, it complicates

the filament supply problem by making necessary the operation of all machines in unison to obtain the control advantages of a single unit system.

In view of the filament requirements of both low- and high-frequency transmitters, a total of six 1,000-ampere generator sets was installed, three being normally assigned to the low-frequency set and two to the high frequency equipment, leaving one in reserve. A view of the filament generator installation is shown in Fig. 6. Each generator is conservatively rated at 1,000 amperes at 40 volts and will carry 1,200 amperes for several hours without excessive heating. The generator design includes special features such as large air gaps, skewed slots and a high number of commutator segments which have been found very effective in reducing commutator noise to a sufficiently low value such that no filter is required when these generators are employed in radio broadcasting station operation. Excitation is obtained from a separate d-c. circuit provided in the station for

Fig. 6—Filament Motor-Generator Sets

such service, and while interpoles are used, no series field is employed, the regulation of the machines being satisfactory, and parallel operation much simpler, without any compounding. A line-start type 75-hp. motor drives each generator and operates directly from the 2,300-volt three-phase station supply with only a simple magnetic oil contactor for control. Although the generators, when in parallel, are operated in unison, the motors are started individually so as to reduce the peak power demands as much as possible.

While the filament generators were balanced to operate very smoothly it was felt advisable to insure that no vibration from these large machines was transmitted to the building structure and radio equipment and special provision was made to accomplish this. Each motor-generator is mounted on a concrete base and the whole assembly supported on cork pads. The area of the supporting cork is so designed that compression stress is carried to a point where maximum resilience is obtained, thereby producing the greatest absorption of vibration produced in the machines.

The distribution of a filament current is accomplished over a heavy copper bus structure, over 4,000 lb. of copper being used in its construction. Each filament generator is provided with a transfer panel, by means of which both field and output circuits of all generators to be operated in unison are paralleled. A master field rheostat furnishes the main filament voltage control with individual rheostats on each machine to make slight adjustments for equal distribution of load. A transfer bus is used so that machines may be grouped as desired and any machine removed from service without handicapping the station operation. Suitable low-water protection is accomplished by interlocking the main exciter circuits with the water relays so that no filament voltage can be built up until the tube water conditions are satisfactory. A special starting step of field resistance is also interlocked with the filament excitation control so that a damaging current rush does not take place when excitation is applied to cold filaments which possess only about 10 per cent of normal operating resistance.

Besides the six motor-generator units installed for filament heating service, there are four 10-kw. 3,000-volt generators which are arranged to furnish the plate circuit energy for the smaller intermediate amplifiers and also provide grid bias voltages for the radio tubes. In receivers and small transmitters, rectifiers can be used very conveniently to supply the grid voltages involved. However, in large broadcasting transmitters, particularly those employing low power modulation with linear amplification in the power amplifier stages, rectifiers are not well adapted for this service since they are inherently incapable of passing current in opposition to their generated voltage, which is an essential requirement in the operation of broadcasting transmitter power amplifiers. While a relatively large rectifier can be used with a load resistance to permit the passage of grid current, this is rather wasteful in any except small sets. Direct-current generators are, however, particularly well adapted for grid circuit service since the generator internal resistance is low and good voltage stability is maintained regardless of the fluctuation of grid current due to modulation. This latter characteristic is particularly important. The tubes used in the high power amplifier require grid potentials at approximately 3,000 volts and since the intermediate plate power circuits require voltages of the same order, similar motor-generator sets can be used advantageously for both services. Doing so permits the greatest of interchangeability and allows the minimum spare equipment for a given installation.

Of the four 3,000-volt generator units now installed, it is planned to operate one for plate power to the main broadcast transmitter and one each for grid potential for the broadcast and high-frequency transmitters. This allows one for reserve since the high-

frequency transmitter is equipped with its own self-contained six-phase intermediate plate rectifier that will be used for normal operation. The intermediate plate supply generators develop 3,000 volts using two commutators in series and are provided with filters to reduce the commutator ripple. The driving motors are of the line-start type for simplicity of control and generator excitation is taken from the station supply.

To make more simple the problem of interchanging connections, all the generators have one terminal permanently connected to the filament bus or ground circuit. This is possible since the change of polarity necessary when changing from plate to grid voltage service is very easily accomplished by merely reversing the excitation. Even with this simplification the providing of the necessary interconnections of eight terminals to four different circuits by conventional disconnect switches would be very difficult. A cable and jack board was considered but thought to be unsafe at the voltages involved because of the danger from cable insulation failure and uninserted live plugs. However, it was possible to provide all the necessary connections through the use of a bar and plug board without cables and virtually eliminate the live plug danger. This was accomplished by a special design of dead front panel with vertical and horizontal interconnecting bars in the rear to which the transmitter circuits and generator terminals are connected, the desired combinations

Fig. 7—Main Station Control Board

being effected by plugging through from the front of the panel. In this manner all cables were eliminated and by careful design the plugs were rendered safe on withdrawal, since they become dead before being exposed sufficiently to allow the operator's coming in contact with current carrying parts.

Adequate control of the various intermediate amplifier stages in the transmitter requires the opening and closing under load of 3,000-volt d-c. circuits and a special 3,000-volt 2-ampere contactor was developed for this service. This contactor has two air breaks in series, equipped with magnetic blowouts, and is arranged with special mechanism to provide rapid contact travel and wide openings. A severe test successfully withstood by one of these units consisted in operating it once a second for over 200,000 times, breaking 3,000 volts, 5 amperes, direct current supplied to a load consisting of a resistor and a 14-henry choke connected in series.

The control scheme, in so far as possible, has been arranged to make it unnecessary for the operator super-

Fig. 8—Substation Feeding Station

vising the equipment to leave his position in the main transmitter room in the normal course of operation. This is accomplished by the remote control of all driving motors for generator sets and rotating equipment being centralized in push buttons on the main station switchboard, at which one of the operators is located. Similarly, the control of the power supplied to the radio equipment is effected through remote control of high-voltage contactors from the main station switchboard together with field rheostats conveniently grouped there. Relays and interlocks are provided so that only the proper sequence of operation is possible. These also function in event of severe or abnormal conditions to remove the power from the circuits in trouble including all those following in the pyramid of amplifiers. Signal lights adjacent to the push buttons indicate the state of circuit relays and response of contactors being controlled. Where circuits carrying dangerous voltages are involved, two signal lights of distinctive colors are provided to indicate open or closed conditions, as is common practise with circuit breakers. Meters enable the operator to obtain directly, or through the use of multi-point switches, the essential conditions of all important transmitter power circuits. Besides the signal lights and push-button control position on the main board, auxiliary positions are located at convenient places about the transmitter itself and in the audio control room which permits the operator on duty there to control the equipment should occasion arise. This latter is an aid to safety and is facilitated by the operating portion of the control room being elevated so as to overlook the main transmitting floor.

One of the panels on the main station control board, a view of which is shown in Fig. 7, provides control of the outdoor station which furnishes the power used. All controls and meter leads from the circuit breakers and instrument transformers are brought under ground 700 ft. to the main building so that it is possible for the operator to have control of the power line supply circuits and equipment without the delay that would be occasioned if it were necessary for someone to go to the substation structure for the operation of circuit breakers. The substation is fed by a 25,000-volt loop and the circuit breakers are arranged with relays which in event of trouble will automatically disconnect whichever end of the loop becomes faulty and enable service to be continued without interruption.

Three 500-kva. single-phase 25,000 to 2,500-volt transformers are employed to step down the power service to potentials suitable for use in the radio station. Power purchased is metered on the high-voltage side of the transformers through the use of outdoor metering equipment.

From the low-voltage side of the transformer bank power is carried over two three-phase 2,500-volt armored cables laid directly in the ground without special duct. The two cables are tied permanently in parallel but in case of failure one could be quickly disconnected allowing limited operation on the remaining circuit.

Circuit breakers such as those in the substation are normally tripped and closed by the use of 110-volt d-c. power. While only a small amount of current is needed for tripping, the closing operation requires nearly 100 amperes for coils when wound for 110 volts. To deliver this amount of current to the substation would require a large battery to be installed for this purpose alone, either in the radio station or in a special building at the substation structure. Moreover, if the battery were located in the radio station the distance would require large cables to prevent undue voltage drop. A special 110-volt battery being very undesirable, the necessity for one was eliminated by employing the 400-volt storage battery for this service. Special 400-volt coils were used in the circuit breakers which reduced the current required so that no special cables were necessary to maintain the proper voltage at the circuit breakers notwithstanding the considerable distance involved.

Besides the major power units required, particularly for the radio equipment, there is a considerable amount of conventional equipment such as exciters, tube-cooling water pumps, blowers, battery-charging equipment, as well as station service transformers for the small power units, lights and heating, which hardly need description. Two six-cell 1,600-ampere-hour storage batteries are furnished for the filament supply of the audio and synchronizing equipment in addition to the 400-volt battery already mentioned which is used for plate supply to equipment employing small tubes. Whereas one battery will suffice for plate service, since it can readily be floated or charged while in service by the employment of suitable filters on the charging equipment, two filament batteries are advisable, each capable of handling the entire load for a day, since it is very difficult to remove the noise developed by charging equipment where large currents are involved, and considerable of the amplification follows the tubes operated in this manner. These large batteries also supply certain essential indicating lamps pertaining to the substation circuit breaker control which require continuous illumination even in the event of power failure.

Although the water-cooling system may not be considered as part of the electrical power equipment, certain features of it are rather interesting. In many broadcasting stations the water directly cooling the radio tubes is in turn cooled by a system of radiators and blowers. This method if employed at KDKA's new station would require a very large and expensive installation because of the much greater dissipation involved. Instead of employing such an installation advantage is taken of the natural high level of the ground water by excavating a small pond from which sufficient water can be obtained for cooling the tube water by means of heat interchangers.

While commercial broadcasting stations as yet do not require equipment as large as that installed in KDKA's new plant, nevertheless it is only by actual construction and use of such apparatus that the development of commercial stations to give safe and economical operation can well be carried out. It is also hoped that data can be obtained which will be of value not only in the specialized field of radio engineering but to electrical science as a whole.

Discussion

R. L. Young: The paper mentions that the capacity of the power plant is considerably in excess of present use. Is the whole station on the same basis and what is its maximum rating as ratings of stations now go?

Also what is the approximate commercial cost of a station of this power?

R. L. Davis: Practically all of the power equipment at the KDKA Station has installed capacity much in excess of the present power output licensed by the Federal Radio Commission in order that experimental work involving wide variations in current and voltages may be carried out. A maximum radio-frequency power output of three to four hundred kilowatts will be possible with the station operating at full capacity of the power supply unit. This is approximately six to eight times the rated capacity of the largest stations now licensed commercially in the United States.

Inasmuch as the equipment installed at KDKA involves so many experimental features, the commercial cost of a station equipped in this manner will not compare favorably with one designed solely for commercial operation. No estimates have been made on the commercial station of this power. A number of comparative figures on the cost of broadcasting stations has recently been published.*

* See June issue of *Electronics* "The Dollar Cost of Broadcasting Stations" compiled for the National Advisory Council on Radio in Education.

Automatic Power Plants for Telephone Offices

By R. L. YOUNG[1] and R. L. LUNSFORD[2]
Member, A. I. E. E. Member, A. I. E. E.

Synopsis.—The nature of power requirements for telephone offices is discussed, with emphasis on continuity of service. Automatic controls are indicated because of their more exact performance, with consequent reduction in variations and in interruptions and their saving in maintenance, particularly with 24-hour operation demanded. Developments are traced, showing an increasing trend toward automatic regulation and control of main power supplies, ringing and other signaling energy sources.

The development of "unit type power plants" for telephone offices is discussed and information is given on a number of standardized plants which operate upon a full automatic or a semi-automatic basis. These furnish power supply for manual, toll, and telegraph central offices, for magneto offices and for manual and dial system private branch exchanges also for small dial system central offices.

Favorable operating experience points the way toward further introduction of automatic devices which will place most telephone power plants, except those in the larger dial system offices, in a position to operate themselves over considerable intervals of time.

THE nature of telephone service is such that continuity is regarded as important, perhaps the key-note guiding developments, and equipment is provided to furnish service 24 hours each day in the year and year after year. In times of storm or emergency when other public services may be prevented from functioning the telephone service will often be particularly important and may have to carry heavier loads.

Reliability

In order to maintain service the first essential is to have the necessary power available; and in this respect the telephone power plant has made a creditable record for reliability. It has survived fires, floods, and lightning. The basement of one metropolitan office, for example, was completely flooded by a broken street water main which resulted in the burning up of the regular 2,200-volt a-c. switching equipment, but the 240-volt d-c. service which had been provided there as an emergency continued to hold in under water and the drive motor equipment on upper floors in this case automatically transferred to this source, while the storage battery as usual provided the required low-voltage direct current. In another case an open-tank storage battery continued the office in operation though submerged by rising flood water from a nearby river. In another metropolitan office lightning struck the power service on a pole across the street, turning the transformer vault in the telephone building into a blazing furnace. The fire walls and steel door held, permitting continued operation of the power plant upon an emergency gas engine in another part of the basement.

An attempt has been made to estimate the percentage of the time during which the power plant is available for use. Complete failures such as would result from inability to provide a direct current from the battery or generators are of rare occurrence. Partial outages resulting from failure of the ringing current will become embarrassing if continued for any substantial time, such as for several minutes. Inquiry has been made as to both types of interruptions in a metropolitan division having 23 offices in 1910, 63 at present and an average of 42 offices in use during the past 20 years. The reported time out for the main power supply in all offices during 20 years averaged two seconds per year per office. The loss of ringing current on the other hand, including both complete and partial failures to certain groups of equipment, was about two minutes per year per office. These suspensions of ringing current did not interfere with conversations in progress, most of them merely resulting in a slight delay in getting through new calls. Taking the larger failure figure the power plant service is good more than 99.9996 per cent of full time.

Requirements

In order to secure reliability it is necessary that the main source of immediate telephone power may be of a type not subject to interruption or failure and that the various machines and important accessories be provided in duplicate. More than one source of primary power is also needed unless there is storage of reserve power in sufficient quantity to bridge over the times during which interruptions may occur. Provision for quick change from regular to reserve equipment is necessary. The main source of d-c. supply must be controlled within close voltage limits for certain types of plants, while the several kinds of signaling machines must be operated within established voltage and frequency limits.

These requirements point to the use of automatic regulating equipment and automatic operating arrangements to as large an extent as practicable, particularly since they must be maintained effective both day and night. For the main d-c. power a regulated reserve storage battery best meets conditions. No equipment is infallible, being subject to wear, deterioration, and accident; neither is a human attendant perfect. By care in selection of equipment, by provision of spare parts, and by sufficiently frequent testing to bring out any defects which may develop before they cause a service interruption, it is believed that a greater degree of reliability can be secured by automatic means than is possible by manual.

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Historical Background

The use of automatic power plants, though recent, is, of course, not new in the electrical art, and our friends in the light and power industries, including also the electric railway field, have made very substantial progress within recent years. In fact, it would seem that a high degree of perfection has been obtained with their automatic substations and automatic hydroelectric plants.

This excellent public service power supply is used as the primary source in practically all telephone offices built within recent years, except in a few repeater installations in isolated districts. This power, however, can not be applied to telephone circuits as received, but is converted into direct current and alternating current of various voltages and frequencies, although in panel dial offices the outside service is used directly to operate small motors driving switching equipment.

Early Automatic Plants for Central Offices

Among the earliest installations of automatically-operated power equipment in regular telephone central offices were some in Pennsylvania where signaling hydrometers controlled automatic solenoid-operated switches to connect automatic starting mercury-arc rectifiers. A single battery equipment was put into service at Berwyn in April, 1917, and during the same year an installation was arranged for Donora in the Pittsburgh district, and one for Wayne with certain improvements and additions to the automatic circuit to take care of two batteries.

These plants had the batteries usual at small manual central offices, a larger 24-volt No. 1 main battery, and a smaller 24-volt No. 2 battery connected in series to give 48 volts for toll transmission. Since drain on the latter was relatively light, several days' reserve was economically provided. Either of the batteries could secure the services of the single charging rectifier if idle when a charge was needed. However, if the main No. 1 battery discharged to a predetermined point, while the rectifier was charging No. 2, the rectifier immediately switched over, and after the No. 1 charge was completed again took up the interrupted charge on No. 2. If, on the other hand, No. 2 arrived at the charging stage while the rectifier was busy on No. 1, it had to wait, but an alarm lamp visible to a switchboard operator was lighted. Alarms were given when for any reason either battery discharged 50 per cent of its capacity, also when the rectifier did not start the charge promptly after one of the hydrometers indicated that a charge was required. Some of these plants are still in satisfactory operation, one being at Oakmont near Pittsburgh. These ingenious control circuits were suggested by engineers of the Bell Telephone Company of Pennsylvania, the immediate urge being elimination of overtime attendance for charging Saturday afternoon and Sunday, since batteries were not large enough to carry the offices safely over this period.

The Modern Automatic Plant

Much of the same general arrangement can be seen in the most recent plants designed for small offices, which will be described in detail later, in which the signaling hydrometer has been replaced by an ampere-hour meter with operating and alarm contacts while the mercury arc rectifier has been replaced by a hot cathode tube type which is self-starting without any mechanical equipment other than that necessary to turn on the power. The newer rectifier is lower in cost, and two or more are provided as necessary to avoid switching between batteries.

The ampere-hour meter is more sturdy, is cheaper than the earlier control device and is equipped to provide some overcharge each cycle, which can be adjusted to the amount needed by the operating cycle used. This eliminates the periodic overcharges formerly given under manual supervision.

In the case of larger automatic plants it is necessary to have a number of the machines controlled automatically in their several characteristics as well as to provide equipment to start, stop or continue them in operation as required. It is believed that it will be of interest to mention some of the equipment developed to accomplish this. In most cases it was found that regulating equipment commercially available was not suitable for the special conditions involved in the telephone power plant and the modifications made will be described.

Regulation of Main Central Office D-C. Voltage

Storage Batteries and Charging Machines

With the more extended use of panel type dial equipment by the Bell System inaugurated about a dozen years ago,[3] it appeared that voltage control of the main central office batteries by use of counter e. m. f. cells then employed was not economical in large sizes. The plan at that time was to provide several counter cells, one or more of which was normally in circuit to reduce the battery voltage to required limits. During discharging of the battery, the counter cells were cut out, while for charging additional counter cells were introduced.

The Floating System and Manual Regulation

A duplicate battery system was developed and additional "emergency cells" of the regular type were provided.[4] The duplicate batteries were so connected with switches that either one could be disconnected from the circuit for charging as might be necessary and afterwards reconnected without interrupting the circuit, the remaining battery, in the meantime, being floated at a constant voltage from charging generators which

3. Semi-mechanical at Newark in 1915; full mechanical at New York toll tandem, 1920, local offices at Omaha, 1921, New York, 1922.

4. U. S. Pat. 1,468,096, Sept. 18, 1923.

were also connected to the office load. This eliminated the increased voltage upon the office circuits during freshening charges so no counter cells were needed, while the emergency cells were also available for connection in the office supply circuit to maintain the entire battery voltage at its usual value during an emergency discharge.

This system used the economical plan of "floating" the storage batteries, preferably as near to 2.15 volts per cell as the arrangements provided allowed, at which value they would absorb just the proper amount of current to keep them permanently in good condition. With an office load fluctuating widely during a day's time it was, of course, found difficult to maintain a reasonably constant voltage and trickle current into the battery by manual adjustments of the charging generator output, so various systems of automatic regulation were studied. The storage battery voltage is inherently critical at this point, a 24-volt battery changing more than 1.5 volts or 6 per cent as the current is increased from zero to 2 per cent of normal charging rate.

Office Load Curves

The difficult conditions of variable current which must be met by either manual or automatic control are shown in Fig. 1 by load curves representing several types of offices.

Group *A* is a typical small manual office with 24-volt currents from 5 to 130 amperes, the peak being at 11 a. m. with an auxiliary peak at 8 p. m. caused by residential social calls. The 48-volt load for toll transmission is very light. Automatic power plants for loads of this character are in satisfactory operation in both manual and dial offices, the larger sizes, however, not being available yet as coded plants.

Group *B* shows a typical panel dial unit in a residential district, in which it will be noted that the current demands as far as voltage is concerned are quite reversed. In such an office the main operating power is taken at 48 volts, while the local talking or transmission load is at 24 volts.

Group *C* shows an entirely different load, a through repeater central office with nearly constant currents to light the filaments and supply the plate circuits. Most of such repeaters are kept in operative condition continuously. Later, descriptions will be given of automatic plants to supply load of this type with current at regulated voltage.

Group *D* shows the load curves read on a large metropolitan panel dial power plant in a business district with more than 2,000 amperes combined peak in the daytime reduced almost to 400 at night, the latter load being accounted for largely by float-charging of storage batteries at private branch exchanges. This plant at the time was supplying three offices carrying two-thirds of a million daily calls, not including interconnecting calls at tributary private branch exchanges.

Group *E* shows the 24-volt load of the New York Long Distance Office with switchboards distributed over 10 floors supplied by a power plant in two sections, one section installed first now being assigned largely to supply repeaters nearby. It is commonly

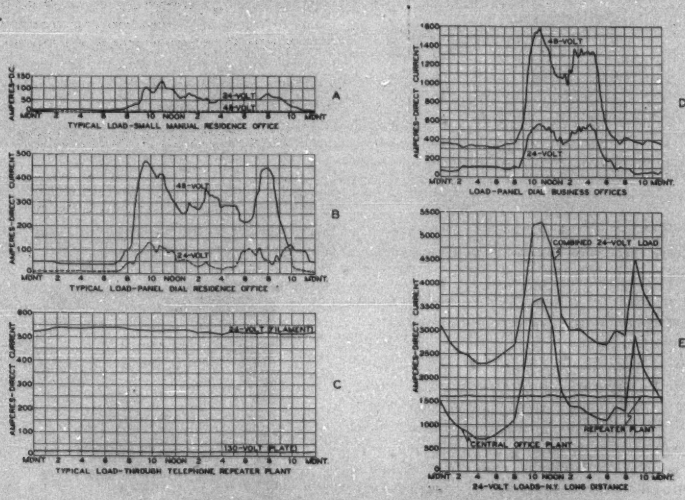

Fig. 1—Telephone Office Load Variations in 24 Hours

known that the power used to carry voice currents over a telephone line is very small but it is not always appreciated that considerable equipment must be operated in the central office to generate, supervise, and control this. The peak load of an ordinary day for long distance is about 5,300 amperes at 24 volts with another 4,500-ampere peak in the evening when social long-distance traffic is encouraged by lower rates. Over 10,000 amperes in 24-volt generators and a reserve battery of 34,880-ampere hours weighing 111 tons are now provided, also 1,200 amperes additional in 130-volt generators. In addition to the reserve batteries,

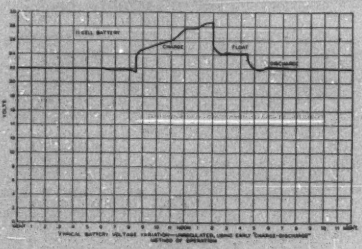

Fig. 2—Typical Battery Voltage Variation

Unregulated, using early "charge-discharge" method of operation

reliability is insured by duplicate a-c. feeders from one generating system and by several d-c. feeders from a different generating system.

Over 1,000 motor hp. is required to drive these charging machines and a vacuum tube system to handle tickets, part of this being reserve. The New York Long Distance Building, including the addition now under construction, also contains a toll and some local offices and is believed to be the heaviest powered building in the entire city, with a-c. and d-c. services provided for a total office and building connected load estimated at 10,000 hp. in 1934.

For the load illustrated by groups *D* and *E* automatic voltage regulating equipment is in use, but automatic power plants are not yet developed, results on smaller installations being awaited.

Voltage Regulation by Manual Control

Under the system of battery and generator operation formerly in common use in manual central offices a range of about 7 volts or 28 per cent was not uncommon when no effort to regulate was made, as shown in Fig. 2.

More than a dozen trials of the continuous floating method of regulating voltage manually were put into operation during the year 1919 in New York, Boston, Philadelphia, and Chicago, following earlier tests. The regulation normally obtained with frequent manual adjustments reduced the variation to 2 volts, equaling 8 or 9 per cent as shown in the recording voltmeter chart, Fig. 3.

Improvement with the Cam Type Regulator

Several types of commercial automatic regulators were tried but were not found satisfactory. Because of the time lag of the storage battery in arriving at a stable voltage after the input current was changed, regulators which worked upon a rapid basis such as be the well-known solenoid type gave rise to severe and continuous hunting. A Wheatstone bridge type of voltage recorder modified to act as a regulator, in conjunction with a separate motor-driven rheostat, was found to be the most satisfactory. Following this a so-called "cam type" automatic voltage regulator was developed and has given very satisfactory results over several years. It is usually adjusted to maintain the voltage within one-half volt total variation or slightly less on the 24-volt battery, holding voltage within 1 per cent of mean value as shown in Fig. 4 which covers a section of chart on the earliest experimental model.

This is close enough for the purpose, and does not involve the more or less continuous operation which may result from a narrow setting which responds to minor battery voltage changes caused by fluctuating loads. A duplicate installation of the latest regulators of this type, together with their switching equipment to control generators on two batteries is shown in Fig. 5 from

Fig. 3—Voltage of Continuously Floated Battery

With intermittent manual adjustment of generator, or "hand regulation"

which the regulator will be seen to consist essentially of three parts:

(1) A sensitive solenoid contacting device working in conjunction with (2) a motor-driven cam delay equipment which at intervals is in position to energize contactors to control (3) a drive motor geared to a rotary face plate resistance box.

These are mounted in a steel cabinet with doors on the front and on the back, and arranged for assembly in the power board lineup or upon a separate framework.

The cam motor is usually of the synchronous clock type taking very small energy and giving a uniform speed. The contacting cam is so shaped and positioned that no contact is made when the battery voltage is correct but when the voltage is high or low, contact for a second, more or less, is made at intervals of ten seconds.

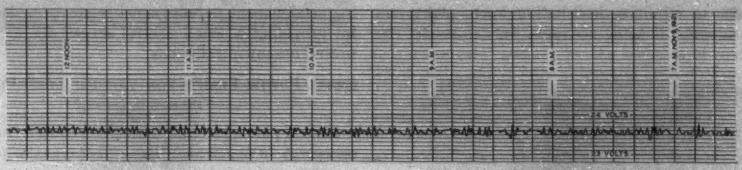

Fig. 4—Voltage Chart Early Model Rotating Cam-Type Regulator

Automatically adjusting a motor-driven generator field rheostat. variation ± 0.15 volts

The duration of contact varis direectly with the amount that the main battery voltage deviates from normal, so for slight deviations very short corrective impulses are given. When variable 24-volt and 48-volt loads are taken from the same string of battery cells a regulator is used across each section. This device for the first time satisfactorily solved the problem, which had engaged our attention for several years,[5] of regulating battery voltage closely and automatically.

Typical Unit Type Power Plants for Telephone Offices Arranged for Automatic or Semi-Automatic Operation

As previously outlined the need for automatic equipment in telephone and telegraph power plants to meet the increasingly more stringent requirements for local and long distance communication led to the development of several small power plants which in normal operation are fully or partly automatic and require practically no attendance other than routine maintenance. The following descriptions and illustrations briefly picture some of these plants. Larger plants using the same general principles are on trial and under development.

Plants for Small Private Branch Exchanges

For many years storage batteries located at private branch exchanges have been charged over cable pairs or by local equipment upon an unattended basis, current being supplied at approximately constant rate. This system has certain limitations, however, not being well adapted to considerable variations in load caused by unusual business conditions or seasonal variations, and it is difficult to select a charging rate which will meet all conditions without being excessive during part of the time.

With the introduction of the dial private branch exchange operated mechanically without an attendant, the power requirements as compared with manual operation were increased and more complicated arrangements were needed for automatic ringing, for providing tones and other signals and in larger units for charging the battery economically under the load variations likely to be encountered.

In selecting equipment for this service advantage was taken of recent developments, particularly the small sealed-in type batteries, while dry-disk type chargers using the copper-oxide rectifying principle, and further development of the earlier hot cathode tube rectifier, placed at the disposal of telephone engineers charging equipment better suited to automatic operation than the motor-generator sets and mercury arc rectifiers previously available.

Fig. 5—Cam-Type Automatic Voltage Regulators for Large Machines and Batteries

Assemblies of automatic plants developed for small dial private branch exchanges are illustrated in Fig. 6 and Fig. 7.

These plants consist essentially of two steel cabinets, the lower cabinet containing the battery equipment while the upper cabinet mounts the control equipment

5. U. S. Pat. 1,586,569, June 1, 1926.

including fuses, meters, switches, relays, ringing machine and also automatic control for maintaining the voltage range within six volts.

Rectifiers on the small plants are inside but on the larger plant are mounted on top of the upper unit as shown in Fig. 7. Motor-generators instead of rectifiers are used in d-c. districts.

In order to minimize installation work, plants are shipped as units, it being necessary only to mount the

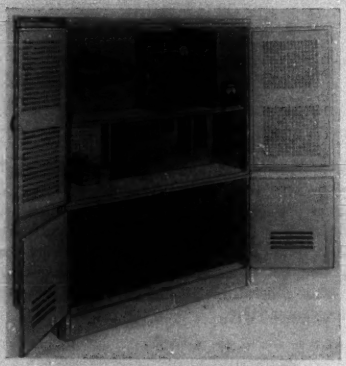

Fig. 6—Power Plant for Small Private Branch Exchange

Dial type No. 102-B, 44-50 volts, 2.5 amperes

battery and charging equipment and connect the leads furnished with terminals attached. A plug and flexible cord are provided for connection to any nearby receptacle providing commercial power. The active cells of the battery are shipped, *charged and filled* ready for interconnection, and are equipped with varicolored wax ball charge indicators which show at a glance the approximate condition of cell charge. The 32-45-volt plants use 18 cells and the 44-50-volt plant 23 cells with four counter e. m. f. cells.

Charging equipment is of capacity to carry a large percentage of the busy hour load and return to the battery during the light load period whatever discharge has been taken, plus additional current to offset battery losses. In these plants the rectifiers operate continuously. Voltage regulation is obtained automatically by means of counter cells cut in or out by solenoid type switches under the control of a voltmeter relay.

Use of one or two 3-ampere hot cathode type Tungar rectifiers allows a flexible arrangement to meet economically load conditions from the initial to the ultimate stage of plant growth.

Ringing current is generally obtained over cable pairs from the nearest central office with tones provided from local relay equipment. Where such supply is not satisfactory a llams inverted rotary converter operating on the battery is provided as shown, this also furnishing tones.

This machine has automatic control, starting when a receiver is lifted from the switchhook and stopping after completion of the call. It is equipped with jack and plug connections for ready replacement by a spare machine when maintenance is required. This mounting arrangement permits some economy to be realized by providing a small central stock where there are several nearby installations.

Small Power Plants for Vacuum Tube Plate and Filament Supply

Several plants have been designed to supply economical energy to plates and filaments of repeater tubes and to other equipment, when such energy is not available from regular central office power plants. Unit plants up to 80, 200 and 800 milliamperes, respectively, at 130 volts are described, followed by larger plants from 5 to 20 amperes. A small 24-volt plant up to 8 amperes is covered, which is supplemented by a line of larger low-voltage plants for various purposes.

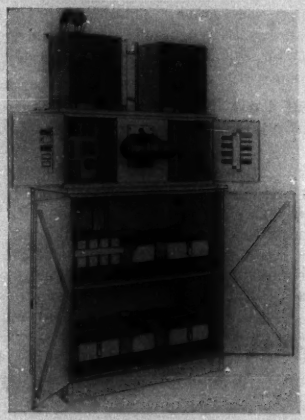

Fig. 7—Power Plant for Larger Dial Private Branch Exchange

No. 102-B, 44-50 volts, 2.5 and 5 amperes

In order to match more nearly in appearance the repeater equipment with which they are usually associated the small plants are arranged for mounting on floor-supported vertical channels.

In the illustrations shown the equipment is mounted open upon shelves or panels. In addition to this arrangement, enclosing sheet steel covers are being prepared for several of these plants for use in locations where it is desired to provide further protection or to improve the appearance of the plant.

130-Volt Plants

The 603-A 50-80-milliampere 130-volt plant, not illustrated, consists of a copper-oxide rectifier, filter section and reserve batteries of block type dry cells automatically connected into circuit upon failure of the regular power service. An unusual arrangement is employed for this purpose, the dry cells being across the supply rectifier in series with a copper-oxide rectifier disk assembly which acts as a valve because of

Fig. 8—Small Plant for Supplying Vacuum Tube Plates

No. 604-A, 125-135 volts, 50 to 200 milliamperes

its pronounced unidirectional current characteristic. Under normal conditions a small current trickles through the dry cells, this having the desirable effect of tending to overcome local action, but being too small to electrolyze seriously the water in the cells. If the voltage drops by failure of the primary power source, the dry cells automatically pick up the load by sending current through the Rectox valve in the reverse direction at which time it interposes very little resistance.

The next larger, 50-200-milliampere 130-volt plant shown in Fig. 8 consists of two 66-cell couple-type enclosed glass jar storage batteries, a half-wave Tungar rectifier and the necessary control equipment. The voltage at the bus bars is automatically controlled by means of a voltmeter relay adjusted for 126-134 volts. Automatic voltage control of the output is obtained with the aid of tapped resistors, alternately cut in and out of the charging circuit by a voltmeter relay, this varying the trickle charge current going into the battery to maintain its voltage within the stated limits.

A 200-800-milliampere, 130-volt plant coded 604-B operates in substantially the same manner as the 604-A plant differing only in the use of more equipment to care for the additional load.

Type 400 plants from 5 to 20 amperes at 130 volts are under development, the 5-ampere rating now being available and so arranged that it can be extended later to a 20-ampere capacity by the addition of equipment which will make it fully automatic both during normal floating and during replacement of a charge following an emergency discharge. In the larger plants one battery is connected across the generator, and another battery across the load, with a choke coil between, providing a very effective noise filter to reduce any tendency toward disturbance of the telephone circuits by the charging equipment.

Small 24-Volt Plant

The 10-ampere plant now available is shown in Fig. 9. It employs an 11-cell sealed glass jar storage battery and

Fig. 9—Small Power Plant for Supplying Vacuum Tube Filaments

No. 602-A, 20-28 volts, 10 amperes

a 12-ampere 75-volt Tungar rectifier. Automatic control equipment is mounted on unit panels and the batteries on shelves.

To provide close voltage regulation when required another bay of equipment may be added, consisting of a motor-driven rheostat, automatic control equipment and a filter comprising a choke coil and electrolytic

condensers.[6] The filter is necessary to prevent noise and crosstalk which would otherwise result from introduction of the regulating resistance. A finely adjusted voltmeter relay, connected across the points to be regulated, actuates the control circuit of the motor rheostat, through other relays and contactors, to drive the rheostat in either direction automatically keeping the voltage within the desired range.[7] The voltmeter type of relay has been used extensively for voltage regulation and for alarm purposes and many thousands of them are in service in the Bell System.

Suitable visual and audible alarms are included as a part of all these power plants to notify the maintenance forces of the failure of a fuse, a Tungar bulb, the primary source of power or occurrence of high- or low-voltage at the equipment supplied. As with the other plants, batteries have charge indicators to simplify maintenance.

Fig. 10—Low Voltage Plant for Small Central Offices

No. 203-A, 44-50 volts, 15, 30, or 50 amperes

Among considerations in the mechanical design of the equipment for unit type plants generally were ease of access for adjustment, additions of other units to care for growth, and applicability to telephone equipment of a variety of types.

6. For discussion of condenser and power plant filters see H. O. Siegmund "The Aluminum Electrolytic Condenser," *Bell System Tech. Jl.*, p. 41, Jan. 1929, or *Trans. Amer. Electrochemical Soc.*, p. 203, 1928, *Bell Laboratories Record*, p. 276, April 1927, together with U. S. Pat. 1,725,377, Aug. 20, 1929.

Also R. L. Young "Power Plants for Telephone Offices," *Bell System Tech. Jl.*, p. 708, Oct. 1927.

7. For use of these relays in early automatic control system for telephone batteries see U. S. Pat. 1,520,793, Dec. 30, 1924.

Larger Low-Voltage Plants Controlled by an Ampere-Hour Meter

To meet the requirements for automatic battery supply plants larger than the sizes already described, the following have been developed to furnish from 5 to 100 amperes at nominal voltages of 24, 34, and 48. These plants are coded as the 200 type.

The larger the telephone system served the more necessary it is to furnish a reliable and continuous source of electric energy and the more savings may be gained in the efficient operation of the power equipment, especially the batteries. A considerable improvement in the life of the battery equipment together with a reduction in charging energy may be obtained by accurately measuring the amount of discharge and providing for return to the battery of this amount plus sufficient to make up the internal losses. Investigation has shown that for larger plants of the size herein considered, particularly when the load varies from day to day, the ampere-hour meter gives good results. The 200-type plants, therefore, have been designed on this basis, the meter automatically starting or stopping the chargers at predetermined points on the cycle of battery discharge and charge. An assembly of this plant for one of the lower current ratings is illustrated by Fig. 10.

The Ampere-Hour Meter and Alarms

The features included in ampere-hour meters for telephone plant control include start-charge, stop-charge and alarm contacts and, of course, the adjustable overcharge shunt usually provided. Addition of a thermocouple was also found necessary to increase light-load accuracy. The very great variations in discharge current with small drains over prolonged periods at night gave a service cycle not usual in commercial work and with the ordinary type meter resulted in discharges not compensated for during the shorter, heavier current charging period.

The meter controls the power supply to rectifier chargers through contactors. If, due to power failure, bulb failure or otherwise, the meter continues to record a discharge for a certain period after the charge is supposed to start, an alarm consisting of lamp and bell is operated. If the office is unattended this alarm as well as others for blown fuses, failure of ringing current, etc., is transmitted over cable pairs to the nearest attended office. Independently of alarm calls, however, it will ordinarily be the practise to have a maintenance man visit the unattended office periodically to inspect equipment and perform routine maintenance.

Batteries

On the 203-A, 44-50-volt plants the battery consists of 23 cells and four counter e. m. f. cells, while for the 205-A, 45-50-volt plants it consists of 24 cells and five counter e. m. f. cells. The counter e. m. f. cells together with a voltmeter relay and control equipment maintain the bus bar voltage automatically within proper range.

Two types of batteries with only one capacity in each type have been selected; a 3-cell sealed glass jar pasted-plate unit of about 105-ampere-hour capacity, and a larger sealed glass jar single cell of 480-ampere-hour capacity. From one to four sets of batteries in parallel may be used, providing a flexible arrangement to handle growth in economical steps.

Chargers

The 12-ampere Tungar rectifier including two, three or four in parallel meets varying load conditions from the initial to the ultimate growth. Where only d-c. service is available, a motor-generator set will be furnished.

Ringing Equipment

Two types of ringing equipment are available, namely "a-c.—d-c." and "superimposed," the latter required in connection with four-party selective ringing.

The a-c.—d-c. ringing current is obtained from a small ringing machine mounted on the power board, operating under the control of the switchhook as previously described, generating alternating current which is passed through a battery to add a d-c. component, this being helpful in tripping off the automatic ringing when the called subscriber answers.

In the superimposed ringing system, alternating current from a machine is sent through two batteries oppositely poled to furnish both positive and negative superimposed current.

Unit Type Ringing Plants

It is, of course, essential to have a continuously and automatically controlled supply of signaling equipment for ringing subscribers' bells and furnishing various signals to properly supervise and complete telephone calls. Individually coded ringing and signaling power plants have been standardized as described and illustrated below for use with the battery supply plants previously covered.

Small Ringing and Signaling Plants, Including One Plant Which Also Supplies Battery Current

To meet a considerable demand for a small, compact, economical, and automatic ringing and signaling plant suitable for use in outlying districts often served by magneto type central offices, the 800-type plants were designed. The equipment is arranged for mounting upon a standard floor-supported channel rack the assembly of which is illustrated by Fig. 11.

The capacities and voltages of power plants of this series are as follows, some being automatic and some being arranged for manual transfer to the reserve source.

The 801-A plant, and the 801-C plant illustrated, intended principally for magneto offices having carrier system installations, supply ringing power at a 20-cycle frequency from a motor-driven magneto during normal operation. The circuits are so arranged, however, that they will be automatically connected to an emergency vibrating interrupter operated from dry cells in the event that the primary source of power should fail.

Fig. 11—Small Plant for Supplying Ringing and Transmitter Current

No. 801-C, 75-110 volts, 20 cycles, 0.19 amperes a-c.
8.2-9.2 volts, 20 or 40 ampere-hours d-c.

When the 801-C plant is used in a magneto type office power supply for the operator's telephone circuits is provided by a small 8-volt storage battery continuously supplied by a Rectox rectifier, making this a double-current plant.

Referring to Fig. 11 it will be noted that the battery equipment for the interrupter consists of dry cells and radio block type *B* batteries. The case on the lowest shelf houses the 8-volt battery supply unit.

The 802-A plant supplies power at 135 cycles from

Type No.	Capacity	Nominal a-c. voltage	Nominal frequency	Service
801-A	0.19 ampere	75–110	20	Ringing
801-C	0.19 ampere	75–110	20	Ringing
	20 or 40 ampere-hours	8.2–9.2 (d-c.)		Transmitter battery supply
802-A	40 watts	35	135	Composite telegraph-telephone ringing
802-B	0.015 ampere	6	1000	Toll line signaling
	0.025 ampere	4.25	1000	Toll line signaling

24-volt battery-driven motor-generator sets for signaling on long telephone toll lines which are composited to provide channels for telegraph circuits. Similarly, the 802-B plant provides signaling current at 1,000 cycles, sometimes referred to as "voice-frequency ringing" for use on long toll lines.

Large Ringing Plants

In addition to the ringing power plants just described there are others with increased capacities ranging from one to eight amperes. This series is illustrated by one of the smaller sizes in Fig. 12.

Fig. 12—Power Plant for Ringing and Coin Control

No. 803-A, 105-100-85-77 volts, 1 to 8 amperes, 20 cycles a-c. also + 110 and − 110 volts, 0.25 to 0.50 ampere d-c.

The larger plants of this group also supply positive and negative 110-volt direct current for use in collecting and refunding coins in connection with coin box service. This current is obtained from a double-current generator with an associated transformer equipped with several secondary taps. Two generators with their control equipment are supplied for each central office, one of which is ordinarily driven from an a-c. motor and the other from a battery motor, the former, under normal conditions, being continuously operating. Each machine has directly coupled to it by means of the shaft and gears equipment to supply tones and signals used in the supervision of telephone calls and to provide interrupted current for automatic "machine ringing." Fig. 12 illustrates the segmental drum interrupters used for many years, and also shows the radio-frequency choke coils, contained in rectangular boxes, provided to prevent interference from the current interruptions.

Fig. 13 pictures one of the newer machines equipped with the recently developed tone alternator and mercury type drum interrupters. This machine is shown provided with both line and battery-driven motors which have been used under some conditions.

Automatic Transfer to Reserve

In order to insure practically continuous ringing service the circuit for these machines is arranged to automatically transfer the office load from the line-driven to the battery-driven generator in case an interruption of any sort in the ringing current is encountered, alarms indicating such a condition. If the interruption has been caused by a failure of the commercial source of power to the driving motor, the transfer to the battery-driven set will remain effective only while the commercial power is off, automatically restoring to normal operation upon the return of this power. This conserves the battery energy and insures the most efficient operation of the plant.

The transfer feature uses a relay permanently connected across the output leads of the line-driven generator. Several 12-pole double-throw contactors are provided in the leads from the two generators to the load which is connected to the mid-point of these solenoid-operated switches.

Fig. 13—Ringing Motor-Generator Set for Large Offices

Including tone alternator and mercury interrupters

In case the ringing current is interrupted or reduced materially in voltage, the above mentioned relay releases and closes a circuit to the automatic starter of the battery motor, starting the reserve set. The voltage on the reserve generator builds up very rapidly and at a predetermined point operates the relay again which closes a circuit to the operating coils of the contactors, these making the proper transfer of the office leads to the emergency machine. The operation, from failure of ringing current on one generator to its complete restoration on the other, is accomplished in from 5 to 10 seconds. This transfer merely delays such calls as are awaiting ringing service at the time and,

since transfers are infrequent, has a negligible reaction upon the service as a whole. Transfers for routine maintenance are made without the delay.

Automatic Voltage and Speed Control

A solenoid-type voltage regulator of the continuous vibrating type which opens and short-circuits a resistance in the generator field circuit has been provided for several years past to control automatically one of four a-c. voltages which are available. It indirectly controls the other three and also the two d-c. voltages. This regulator takes care of variations in load and in supply voltage conditions.

Fig. 14—Ground Potential Compensator for Telegraph Offices

No. 705-A. 0-115 volts positive or negative, 5 amperes

When driven by an induction motor inherent speed regulation is sufficient to give satisfactory frequency. When, however, the generator is driven by a d-c. motor operating upon the reserve storage battery with variable voltage, a centrifugal governor is provided which operates rapidly to open and then short-circuit a resistance in series with the motor field, the relative intervals open and closed being such as to maintain substantially constant speed.

In addition to the automatic battery, ringing, and signaling plants covered above it may be of interest to describe a rather unusual plant provided for use in some telephone offices to maintain continuous "grounded telegraph" service under very unfavorable ground voltage conditions.

Ground Potential Compensator for Telegraph Circuits in Telephone Offices

A plant known as a "ground potential compensator" is illustrated by Fig. 14. It consists essentially of two motor-generator sets, one line-operated for regular use and the other for reserve driven by a motor operated from the battery. The control equipment is mounted on impregnated asbestos panels supported by a framework upon which the motor-generator sets are mounted at the rear of the panels. Up to the present time it has not been considered necessary in this service to arrange for automatic transfer from one set to the other, reliance being placed upon alarms and the manual transfer of the sets since the offices where these sets are in use are continuously attended. The compensator is capable of neutralizing automatically ground potentials between zero and + 115 or − 115 volts.

The object of this plant is to compensate[8] for the varying ground potentials sometimes encountered between telegraph central offices when of a magnitude to interfere seriously with telegraph transmission. A geological formation with a rather high ground resistance between stations, especially when this condition occurs near a parallel run, d-c. street railway, accounts for the most serious fluctuating ground potentials.

The armature of the operating generator is connected in series with the ground lead of the telegraph batteries and the field excitation is controlled indirectly by means of the ground potential fluctuations between the local ground and a ground of approximately zero potential with respect to the distant telegraph station or office.

Such a neutral ground is usually obtainable within a few miles of the affected office over a pilot wire the current in which operates a sensitive pilot relay which causes positive or negative battery as required to flow through the fields of the generator to buck or boost the local ground. In order to make the pilot relay exceedingly sensitive, an auxiliary circuit is added which causes it to vibrate at from 100 to 200 cycles per second.

Operating Experience with Automatic Power Equipment

No trouble of consequence has been reported on automatic starting or voltage regulating equipment in two or three years' operation.

With regard to the dry type copper-oxide rectifiers used in small offices the failures thus far reported appear to be less than 1 per cent. The oldest rectifiers, however, have been in service for less than four years so it is possible that aging may in time increase the mortality, though there is little sign of this at present. Taps to restore output in the event of loss by aging are provided.

The supervisory indicator ball system provided in the batteries has been found helpful in warning of

8. U. S. Pat. 1,126,956, Feb. 2, 1915.

abnormal conditions before the reserve capacity has become exhausted.

The larger automatic plants where the charge is controlled by an ampere-hour meter have had some trouble reported in about 10 per cent of the installations where the meters have been slowed up by dirt or mercury dross accumulating inside the sealed mercury chambers, so that the meters did not record the small discharges at night and, therefore, put back insufficient quantities during the shorter charging periods to keep the batteries properly charged. In most cases upon which records are at hand the difficulties have been discovered and corrected before the batteries ran down completely. In one division upon which detailed data are available six such troubles developed on a total of 68 installations made during a three-year period. All these troubles were discovered and corrected in time to prevent loss of service.

The trial installations of the automatic transfer of ringing machines operated satisfactorily on a few actual occasions of power failure and on numerous tests during some three years, with no failures yet recorded in the comparatively small number of offices so equipped.

One trial installation in a large building with fully automatic central power plant for private branch exchanges, using a motor-generator set, has been in operation for over one and one-half years without any failures. Two similar trials in central offices have been in operation for well over a year without failures reported. As these installations are still considered experimental they have not yet been coded and are not described in this paper, though most of the automatic equipment used is covered.

Future Developments

Experience to date with automatic control equipment in telephone offices and with completely automatic plants has been so gratifying that it seems probable the use of such equipment will be extended, particularly to include larger types of offices. Alarms are, of course, provided to secure help in the event that any of the automatic equipment should fail to perform, or unexpected emergencies should develop. The ideal arrangement seems to be a completely automatic plant which will normally function by itself, both during normal times and as far as practicable during emergencies, requiring of the attendant infrequent supervision and routine cleaning, adjustment, and maintenance.

Discussion

For discussion of this paper see page 971.

Telegraph Power Plants

By E. W. GRIFFITH[1]
Non-member

Synopsis.—This paper mentions some of the early methods of providing electrical energy to operate telegraph circuits and contrasts these with a description of the telegraph power plant in the Western Union Telegraph Company's new building in New York City. It also describes the present general methods of distributing power to telegraph circuits and the equipment recently designed for maintaining continuity of power including engine generator sets for standby service.

As the term "power plant" generally conveys to an engineer's mind a station where electric energy is generated, the expression, "telegraph power plants," would have had more significance forty or fifty years ago than it does today, for prior to 1888 electric energy for the operation of telegraph circuits was generated, in all stations, on the premises either by primary batteries or dynamos driven by steam engines. Today, there are only two plants in the United States in the Western Union service where power is generated by a private plant, one being at the Punta Rassa, Fla., cable station and the other at Camp Douglas, Wis. At all other points the Western Union contracts for electric energy from public utility companies and transforms the energy to the potentials and type of current necessary for the operation of a telegraph office by means of transformers, motor-generators, storage battery or rectifiers.

The first installation of a dynamo plant in a telegraph office was placed in service in September, 1880 when three groups of 5 dynamos each, belt driven by three steam engines, were installed by the Western Union at 195 Broadway, New York. A reproduction of a sketch showing this installation as it appeared in the Scientific American of Jan. 31, 1880 is shown herewith. Each machine was wound to deliver 40 amperes at 70 volts. Four potentials were used 70, 140, 210 and 280. The fifth machine served to excite the other four in each group. One group furnished positive potentials, the other negative and the third was a spare for either potential. The higher potentials were obtained by connecting the machines in series and making taps at each machine for the potentials mentioned.

This dynamo plant replaced all of the gravity battery cells then in service for main line operation in that office, totaling almost 15,000 cells. The local circuits were still operated from 4,600 cells. This was the first marked improvement in telegraph power plants. Practically an entire floor of 10,000 sq. ft. was released for other purposes as only about one-tenth of the space occupied by the batteries was necessary for the dynamo plant. The difference in weight of the two plants was 60 tons.

Extensive replacement of primary batteries with dynamo plants was not started until 1888 when the New York dynamos were removed and installed at Pittsburgh, Pa., where another group of machines was also installed for providing 15-volt potential for local circuits. The three groups of dynamos were belt driven by three 110-volt d-c. motors. Power for operating the motors was obtained from the Allegheny County Electric Light Company. This dynamo plant replaced 13,000 gravity cells, and was the first telegraph office in the world where there was not a single chemical cell used. It was also the first telegraph power plant obtaining electric energy from an outside source, and the first station where the power company's 110-volt supply was used directly to operate telegraph circuits.

The removal of the New York machines to Pittsburgh was made after a new plant was furnished at the Western Union office at 195 Broadway. This new

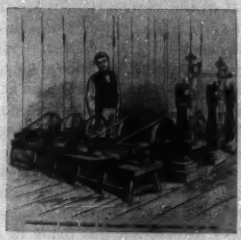

Fig. A—First Telegraph Dynamo Plant, New York City, 1880

plant consisted of three groups of machines larger than the initial machines. Machines for three additional potentials were installed in 1892: 7 volts for locals, 23 volts for loops, and 45 volts for short lines. At the same time three steam engines of 75 hp. belted to three Edison 50-kw. dynamos were installed for lights and power and for operation of a carrier system conveying telegrams to and from various points in the office.

Dynamo plants were installed by the Western Union at Chicago, Albany, Boston, Buffalo, Philadelphia, and several other stations from 1889 to 1894 releasing thousands of cells of primary battery.

The storage battery, although used in England around 1890, was not considered developed to a point to give sufficiently reliable service for operation of telegraph plants, by engineers in the United States, until 1893.

1. Telegraph Power Engr., The Western Union Telegraph Co., New York, N. Y.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

31-62

During that year the first important application of the storage battery in this country was made by the Postal Telegraph Company in its Baltimore office where 430 cells were installed replacing 2,000 gravity batteries. Storage battery was first used by the Western Union in 1893 for local circuits in its office at 21 Cortlandt Street, New York where one 100-ampere hr. cell replaced 30 primary cells The advantage of the storage battery for telegraph operation was soon realized. Only a few main line circuits could be operated from a gravity battery because of its high resistance, whereas any number of circuits could be fed from a battery of storage cells depending upon the capacity of the cells. In view of the steady voltage obtainable with storage battery, its reliability, flexibility, and economy, numerous gravity plants were replaced starting in 1895.

The first large installation was made early in 1895 by the Western Union at Atlanta, Ga. where 700 cells were placed in service releasing 8,000 gravity batteries. Only half of the 700 storage cells were in use at any one time. The battery was charged by two dynamos driven by motors operated from the Electric Light Company's 500-volt d-c. service. During the next two years storage battery plants were installed at most of the larger offices replacing 30,000 primary cells in that period.

The advent of the storage battery for the operation of telegraph circuits brought out numerous charging schemes. In some of the Western Union's branch offices where no charging facilities were available the main line current operating the circuits at such offices was diverted through a single storage cell before connecting to ground and served as a charging current for the battery. This installation, consisting of one 100-ampere hr. storage cell, replaced 30 gravity cells formerly used for local circuits in the office. In the Western Union's office at Allentown, Pa., where a small storage battery installation was made in 1895, 16-cp. carbon lamps were used for a charging resistance and at the same time to light the office.

Notwithstanding the replacements of gravity battery with dynamos and storage cells the number of gravity cells in Western Union service rose from 120,000 in 1877 to 300,000 in 1895. This increase in the use of gravity cells was due in part to the opening of many new small offices where no facilities were available for charging storage batteries or operating dynamos and in part to the addition of new circuits in existing offices where gravity cells were already in service. The peak of the gravity battery installation was reached in 1903 when the Western Union had 528,000 cells of batteries in service. From that time on primary batteries were rapidly replaced by motor-generators or storage cells. At the present time there are only 65,000 cells of gravity battery in the Western Union service, 20,000 of which are in use for main line operation and 45,000 for local service. These cells are principally at isolated railroad stations and in groups of less than 150 cells.

The most modern and probably the largest telegraph power plant in the world was recently placed in service in the Western Union's new building at 60 Hudson Street, New York City. For the operation of telegraph circuits a very steady and reliable source of power is necessary. Every precaution must be taken to keep the lines fed with electric energy and to maintain a steady potential on the telegraph circuits to prevent delays in the handling of telegrams. A power interruption is serious in the telegraph service not only because of message delays for the period of interruption but also because of the time required to reestablish service particularly in those methods of operation requiring manual attention, such as in trunk line multiplex systems. Therefore in the important offices of the Western Union where the service of the power company is subject to interruption, the telegraph plant consists of equipment to maintain continuous service.

Telegrams at the present time are conveyed from one wire to another or from one department to another by means of systems of belt conveyors and pneumatic tubes which systems are operated by electric motors. It is essential that such apparatus also be kept in operation continuously to avoid delay in the handling of telegrams.

In the Western Union's new building in New York facilities are provided to maintain continuous power for telegraph operation under any conditions. Direct-current, 120-240 volts, three-wire power obtained from the New York Edison Company is used for a major portion of the building load and the telegraph load. This d-c. service is obtained over two sets of feeders directly from two separate substations. These feeders follow different routes from the substations and enter the building at opposite sides as a precaution against interruptions due to local disturbance in the streets. A tie to the street network is provided from each entrance room in addition to the direct feeders.

Since a small amount of a-c. power was required for use in the laboratory and in order that the service connection for this a-c. power would be of sufficient size for use also as a reserve power supply, a portion of the building equipment and pneumatic tube compressors was connected to the a-c. service. The use of alternating current was restricted to definite sections of the building so as to avoid duplication of wiring and possible confusion between services. A 300-kw. motor-generator as shown in Fig. 1 was provided to convert this a-c. service to three-wire direct current for emergency power purposes.

The Western Union is greatly indebted to the New York Edison Company and The United Electric Light & Power Company for their cooperation in arranging for the services mentioned. They realized the importance of reliable power service for telegraph operation and provided every facility available to maintain continuity of service. With all of these facilities it would seem practically impossible to have a complete power interruption However, it was considered advisable to provide some source of emergency service independent

of any outside source to guard against catastrophes and explosions in the streets which might destroy both a-c. and d-c. feeders, and also to protect against major troubles in generating stations.

To provide an independent emergency power source a 250-kw. steam turbine generator set as shown in Fig. 2 was installed. This turbine is driven by steam from the regular building boiler plant. A turbine set was se-

Fig. 1—300-Kw. Emergency Motor-Generator Set, New York

lected rather than an internal combustion engine in view of the fact that the boiler plant for building steam supply is operated at 80 lb. pressure and of ample size. Without the necessity of providing a special steam supply considerable economy and saving in space was effected by the use of the turbine set. The turbine selected was of a type which could be started quickly.

When a power failure occurs it is evident that some time will elapse before the emergency a-c. to d-c. motor-generator or turbine set can be started and placed in

Fig. 2—250-Kw. Emergency Steam Turbine Generator Set, New York

service. To avoid any interruption in the operation of telegraph or ticker circuits facilities are provided to maintain continuity of power to such circuits until the emergency units can be substituted for the regular power supply. Two banks of storage battery are provided, a view of which is shown in Fig. 3. Each bank provides 120-240-volt power, the same as the Edison d-c. power and each bank is of sufficient size to carry a load of 400 amperes for a period of 45 minutes with a drop of not over 6 volts. One bank is for ticker service and the other for telegraph service. These batteries are connected to the loads mentioned by means of automatic switches which operate when the voltage of the Edison service falls to 114 volts on either side.

The reserve motor-generator and the turbine generator are intended to carry only the essential loads such as telegraph and ticker circuits, belt conveyors, a portion of the pneumatic tubes, about 15 per cent of the lighting circuits on operating floors, and one elevator.

The main power switchboards located in the basement

Fig. 3—Emergency Storage Battery, New York

are separated into two sections: one as shown in Fig. 4, for controlling building power and general lighting, and one as shown in Fig. 5 for telegraph power including belt conveyors, compressors, a portion of the lights and controls for the emergency units and battery. The telegraph section of the switchboard provides facilities

Fig. 4—Main D-c. Building Power Switchboard, New York

for transfer of all loads connected thereto to regular or emergency services.

Duplicate sets of feeders are provided through separate shafts in opposite sides of the building from the telegraph switchboard to the 11th floor power room, where are located the motor-generators for providing the necessary line and local potentials. A switchboard is

provided in the 11th floor power room for distributing the local and line potentials and the 240-volt d-c. service for belt conveyor power to the various operating floors. A view of this board is shown in Fig. 6. The telegraph and ticker power feeders terminate on the four double-throw switches on the left hand panel.

A signal system is installed to indicate to the engine room attendant which of the four sets of telegraph and ticker feeders from the basement to the 11th floor power

Fig. 5—Telegraph Power Main Switchboard, New York

room, are in use. Specially designed auxiliary contacts are installed on the four feeder switches on the 11th floor switchboard. These contacts control lights in a signaling cabinet placed above the basement switchboard as shown in Fig. 7. A single line diagram of the feeder connections to the 11th floor switches is marked on the face of the cabinet. The completion of the circuit whenever a switch on the 11th floor switchboard is closed is indicated by the illumination of a line on the cabinet representing that switch. It may be seen, that

Fig. 6—Telegraph Power Switchboard, 11th Floor New York

telegraph feeder No. 2 and ticker feeder No. 2 were in use for their respective loads when this picture was taken. Special independent telephone circuits and alarm systems are also provided for communication between these two centers and to give warning of the failure of any motor-generator set or operation of the automatic transfer switches. These automatic switches, which will be described later, are located in the 11th floor power room close to the telegraph load. Two separate feeders run from the batteries through the basement switchboard to the emergency side of the automatic switches and provide against failure of power to the telegraph and ticker service whether such services are being fed from Edison power, the emergency motor-generator or the turbine set.

Distribution of power for the operation of telegraph circuits and belt conveyors from the 11th floor power room is made through separate sets of feeders to each floor in two shafts. From each of these shafts the power is distributed to approximately one-half of each floor. In event of the blowing of a fuse on the distribution panel only half of one floor is affected.

An additional precaution, which is an innovation in so far as the Western Union is concerned, is the use of no-voltage release circuit breakers located in the power shafts on the belt power feeders and telegraph local feeders. Both loads consist of a large number of small motors and much trouble had been experienced in the past as a result of power being restored after a short

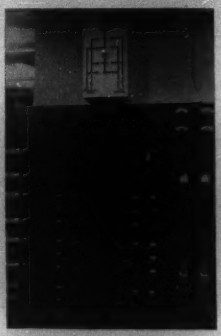

Fig. 7—Signal System on Telegraph Feeders, New York

failure, with these motors stopped but still on the line. The usual result, then, was the blowing of a main fuse or opening of a circuit breaker and a further delay affecting, perhaps, the entire office.

The various telegraph potentials are fed through a distribution cabinet in each shaft. From the shaft cabinet, feeders run to several zone cabinets on each floor. The belt power is fed through two or three circuit breakers in the shafts. Each circuit breaker feeds to a distribution cabinet on the operating room floors. A view of the telegraph distribution cabinet and circuit breaker in a shaft is shown in Fig. 8.

The entire arrangement of distribution is such as to cause a minimum of delay in event of accidental short circuits or power failures.

The maximum demand required in the Western Union's New York office for handling telegrams at present totals 295 kw. divided as follows:

For telegraph circuits..	75 kw.
For ticker circuits.....	50 kw.
For belt conveyors.....	40 kw.
For pneumatic tubes...	130 kw.

The monthly power consumption for these services totals very nearly 150,000 kw-hr.

There are 37 motor-generator sets in service in the 11th floor power room totaling 190 hp. the maximum size of any one set being 10 kw., for the operation of 1,123 automatic and 145 Morse telegraph circuits and about 500 ticker circuits. A view of a group of these motor-generators is shown in Fig. 9.

For the operation of message belt conveyors 15,722

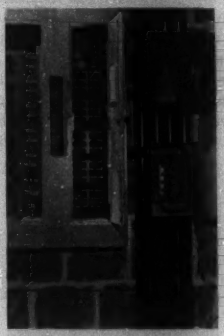

Fig. 8—Typical Installation Cabinets and Circuit Breakers in Shaft, New York

ft. of belt driven by 274 small motors totaling 76 hp. are installed. A section of such equipment with driving motors is shown in Fig. 10. In Fig. 11 is shown the individual table belt motor drives.

There is installed in the building 50,000 ft. of tubing

Fig. 9—Group of Motor-Generators, 11th Floor Power Room, New York

for handling telegrams to 60 branch offices and 48 stations, requiring 356 hp. in blowers and compressors to operate them. Fig. 12 is a view of the 60-hp. compressor units and the high-speed blowers are shown in Fig. 13.

It is now the general practise, with the Western Union, when a telegraph office is moved to a new location or a new repeater station is established, to obtain electric energy from a local power company and to install motor-generators, for operation of its telegraph circuits as the initial cost and maintenance is less than storage batteries and a considerable saving in space is effected. There have been no installations of storage battery plants, except at cable stations and for emergency service, for several years. In some cases, particularly in the south or southwest where thunderstorms are prevalent, a small portion of an existing storage battery plant is used in the new location in conjunction with automatic switches to maintain continuity of service over the short periods of interruptions. There is, however, a number of telegraph stations using

Fig. 10—Belt-Conveyor Driving Motors, New York

storage battery for operation of the circuits. At most of the offices so equipped the storage battery will be replaced with motor-generators when the office is moved or enlarged. Storage batteries have been replaced by motor-generators in all existing plants except cable stations when it is found economical to do so. The Western Union has 120 stations including cable offices

Fig. 11—Direct-Drive Belt Motors, New York

where 33,000 storage cells are installed; the largest at any one point being at Bay Roberts, N. F. cable station where 2,581 cells are in service. The largest size is 400-ampere hour.

The first installation by the Western Union, of storage battery, automatically operated in conjunction with

motor-generators to maintain continuous service, was made at the time a new office was installed in New Orleans in 1923. Considerable trouble had been experienced in the old office due to many short power failures interrupting the service and particularly the circuits emanating from the Cotton Exchange there. At the Exchange in New Orleans cotton orders are sometimes executed with Liverpool, England, in a minute and a half so that the importance of continuity of service can readily be seen.

The installation of such plants was extended to other

Fig. 12—60-Hp. Pneumatic Tube Compressor, New York

large stations where numerous automatic circuits were in use and where power interruptions were frequent. Difficulty was experienced, however, in obtaining an automatic switch which would operate fast enough to maintain synchronism on the automatic circuits. Several types were tried, and it was not until 1927 that a switch was developed by a manufacturer, in conjunction with Western Union engineers, to meet the requirements.

Fig. 13—High-Speed Pneumatic Tube Blowers, New York

In Fig. 14 two of these switches which are in service in New York are shown. These are solenoid type with the blades or moving contacts assembled as one unit on a rocker arm. The current is carried by copper-to-copper contacts but the breaking contacts are copper to carbon. Special relays were designed for operating the switch on a slight drop in potential of the regular power source. A resistance in series with the switch coil is cut in and out by auxiliary contacts on the switch. The resistance is short-circuited when the switch is closed on the bottom or reserve contacts, thereby subjecting the coil to full voltage when the relays are closed. When the switch is closed on the regular side the short circuit is removed thereby reducing the current in the switch coil to just enough to hold it. This reduced current also reduces the lag when the power fails and the switch drops to the reverse side. The two groups of contacts are arranged so that during the operation of the switch there is only one-sixteenth of an inch clearance at the time the regular contacts open and contact is made on the reverse side. Consequently the interval of no current is very small. The change from one source of power to the other is accomplished in less than one-tenth of a second. The switch is arranged so that it can be restored to regular position only by closing the relay contacts manually. The air gap on the relays is

Fig. 14—Automatic Throw-Over Switch, New York

adjusted so that the relays will not close automatically. This was done purposely so as to prevent rapid operation of the switch in event of fluctuations of the regular power source. The switch is kept on the battery until the service is restored and steady.

The switches shown in Fig. 14 are the largest of this type in service. They are 800-ampere capacity. The back contacts of the operating relays are utilized for audible signal when the switch operates.

Storage battery or motor-generators are in service in over 1,200 Western Union offices. Ninety per cent of such installations are motor-generator plants.

Motor-generators for Western Union service are constructed by various manufacturers under special specifications. Very close regulation is necessary not only from no load to full load but at any load as the load of telegraph circuits varies. The maximum temperature rise allowed for field coils and windings is 35 deg. cent. and commutators 40 deg. cent. which insures obtaining liberally rated machines. The manufacturers are also required to specify the minimum guaranteed efficiency.

As practically all telegraph circuits are energized at all times the efficiency of the machines is a factor in determining the value of machines offered by various manufacturers.

In all except very small offices positive and negative polarities are required as most of the telegraph circuits are operated on the duplex system. For each potential

Fig. 15—Group of D-c.—D-c. Motor-Generators for Furnishing One Positive and Negative Polarity, New York

a group of three machines is provided, one for positive, one for negative, and one spare for use of either polarity. Such a group is shown in Fig. 15. These are d-c.—d-c. sets with hand starters. Equalizer connections are provided on the generators to permit paralleling while the load is transferred from one generator to another.

The machines are mounted on benches of angle iron

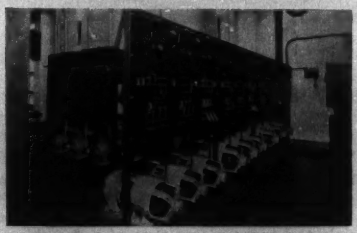

Fig. 16—Group of A-c.—D-c. Machines, Syracuse, N. Y.

construction with ¼-in. sheet metal tops. The motor-generators are mounted on springs to prevent the transmission of any vibration. Ebony asbestos panels are now used exclusively in preference to slate, as trouble was experienced due to conductive veins in slate and breakage in shipment.

A group of nine a-c.—d-c. motor-generators installed in the Syracuse office is shown in Fig. 16. The motors are started by means of double-throw switches placing the machines directly on the line for starting. Otherwise the panels and benches are similar to the d-c.—d-c. type. A group of small motor blowers for operating pneumatic tubes is shown at the left.

Motor-generators of 500 watt size and smaller are generally mounted on what are termed "wall type benches" which are specially designed to conserve space. A view of two such benches installed in the Buffalo office is shown in Fig. 17. They are of angle iron and sheet steel construction and are self-supporting, and can be mounted back to back if necessary. As these benches are designed primarily for use in small offices during the advent of the simplex automatic system, safety type switches are provided. The motor-genera-

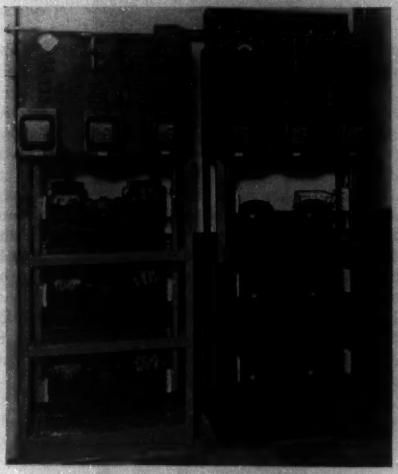

Fig. 17—Wall-Type Motor-Generator Benches

tors shown in the photograph are metallic circuit machines. These sets are also of special design with a double commutator generator. Each machine provides both polarities of 160 volts and is used on an individual metallic circuit. By means of double-throw switches, jacks, and cords and plugs, spare machines can be transferred to any circuit in event of trouble with the regular machine.

When the wall type bench is used in small offices it is equipped with a small panel with simplified apparatus to change machines similar to the larger motor-generator installations.

There are 1,085 stations of the Western Union equipped with a total of 4,230 motor-generators; the largest number at any one station being in the New York office where 37 are installed. The largest size used exclusively for the operation of telegraph circuits

is in Philadelphia where three 20-kw. 110-volt potential machines are in service.

In most of the large offices special motor-generators are installed for synchronizing clocks. These machines are automatically started 20 seconds before each hour and stopped about 20 seconds after the hour. The starting and stopping are controlled by contacts on special clocks through relays.

Fig. 18—Typical Zone Cabinet

Power is also supplied for the operation of time stamps. At each automatic operating position a stamp is installed for the operator to time the telegrams when received. The stamps are operated every minute by master relays.

Specially designed cabinets, termed "zone cabinets," located near the operating equipment, are used to distribute the power to sections of operating apparatus.

Fig. 19—Belt-Power Distribution Cabinet

Several cabinets are located on each floor. A photograph of a typical installation is shown in Fig. 18. This cabinet provides ten feeders of each polarity of 160- and 240-volt potential, twelve for the 110-volt negative, 10 for the 110-volt positive, ten for time stamp circuits, and twelve for grounds. Two copper straps are installed on the inside of the barrier in the upper left hand corner, one being connected to the 110-volt bus in the cabinet through a lamp and the other connected to the ground bus. This arrangement is for testing fuses. In the center a special compartment is provided for storing fuses. A framed chart is installed on the inside of each cover to properly designate the feeders.

Zone cabinets are also provided for belt conveyor motors and are generally mounted on columns similar to the telegraph zone cabinets. Such a cabinet as installed in the New York office is shown in Fig. 19. This cabinet provides for feeding ten 220-volt d-c. circuits and is equipped with a fuse testing device and fuse container. The box at the bottom of the cabinet contains a special starting device for controlling the motor on a "pick-up" belt and the motors on "table belts" feeding into that pick-up belt. It was found desirable to stop all table belts in event the pick-up belt stopped for any reason, otherwise telegrams would accumulate in large numbers on the latter belt. The

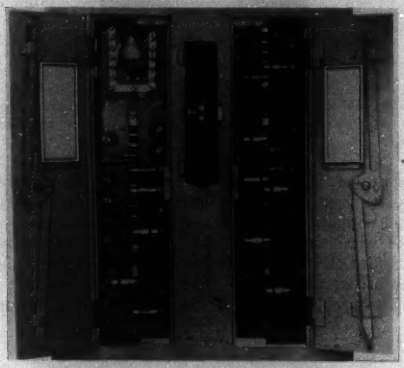

Fig. 20—New Type Zone Cabinet

coil of the starter is connected across the pick-up belt motor and the starter controls a maximum of 12 table-belt motors. This installation at New York was the first of this kind and has proven a great benefit to the service.

In order to reduce the number of styles and sizes of zone cabinets to a minimum and to economize on materials and space a new type of zone cabinet has recently been standardized. A photograph of this cabinet is shown in Fig. 20. The cabinet has been designed to use boxes of three sizes, all of the same width and depth but of three different heights to provide for variable distribution requirements. The fuse clips are mounted directly on the main buses thereby saving two inches in width. Two sizes of panels are used, the larger being twice the height of the smaller. With this arrangement greater flexibility will be effected and at the same time the costs will be reduced since the parts

can be ordered in larger quantities and stocked and assembled as desired.

At all important offices where an interruption to the power source would seriously affect telegraph operation an emergency plant is installed. The first type of engine used for emergency power was a single-cylinder slow-speed gas engine belted to a generator. Several of these single-cylinder sets were installed but most of them have been replaced by multi-cylinder higher speed gasoline engine units. The first multi-cylinder type was installed in 1915. However, the multi-cylinder type was not used exclusively until about 1920. They were 40 to 50 hp. engines running at 600 to 900 r. p. m. and operated mostly on illuminating gas. Considerable trouble was experienced in operating on gas due to the fact that generally during a failure of the electric service an abnormal use of gas occurred which reduced or varied the pressure sufficiently to cause unsteady operation of the engine. Subsequently a gasoline-kerosene type was developed and used. This

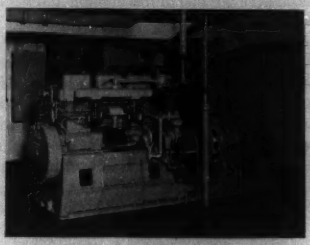

Fig. 21—Emergency Unit, Buffalo, N. Y.

type, although designed for normal operation on kerosene, was started and warmed up on gasoline. With these engines the amount of gasoline was limited to the amount necessary for starting purposes only. Troubles were also reported in connection with such sets due to the rapid collection of carbon and the difficulty in maintaining the duplicate carburetors and fuel supply systems. With one exception all emergency units installed since 1923 have been gasoline engine driven.

There are 114 emergency engine-generator units installed in Western Union offices in the United States. Sixty-seven operate on gasoline, 23 on gasoline and kerosene, 16 on gas, 7 are semi-Diesel and 1 is full Diesel. The smallest unit is 8 hp. and the largest at the present time is 120 hp. although 200-hp. units will be installed in the near future.

The prime requirements for an emergency plant for telegraph service are simplicity of operation, quick starting, capability of handling a load very soon after starting, very close speed regulation on constant load, close regulation from no load to full load and continuous operation once started.

A photograph of a recent installation in the basement of the Rand Building in Buffalo is shown in Fig. 21. This unit consists of an 80-hp., 900-r. p. m., 6-cylinder gasoline engine driving a 40-kw. three-wire 220-volt d-c. generator.

To insure steady operation only engines with four or more cylinders are specified.

All engines of 25 hp. and larger are equipped with two entirely separate ignition systems including separate magnetos with impulse couplings and separate spark plugs. Separate switches are provided so that both sets of ignition systems can be operated simultaneously or either set alone.

An overspeed cutout device is provided to ground the magnetos when the engine reaches a predetermined maximum allowable speed. This device is so arranged that once it operates the magnetos will remain grounded until reset by hand.

Fuel pumps are specified in duplicate and may be either two direct-driven pumps or one direct-driven pump and a hand-operated pump.

A one pint gasoline tank is provided on the engine for gravity feed to the carburetor. This tank is equipped with a gage showing the quantity of fuel also a quick-vent float-air valve. The latter valve will close the vent if the tank is completely filled and is provided to prevent flooding of the room with gasoline in event the main tank is filled beyond its capacity.

Most of the units are not provided with radiators for cooling water. However, each engine is equipped with a water pump for future use in conjunction with a radiator if there appears to be a possibility of water failure. At a few stations where the water supply is unreliable the sets are supplied with radiators initially. At one or two points radiators are provided as a separate unit driven by a motor.

A regulator is specified to automatically open the cooling water supply when the temperature reaches a certain point, to maintain the water at a determined temperature, and shut off the supply when the engine stops.

A control panel is specified for mounting the tachometer, starting switch, ignition switches, oil-pressure gage, thermometer, throttle and choke controls, and a frame for brief operating instructions. In Fig. 21 this panel is shown mounted over the flywheel. From this point the engine can be started or stopped and its complete operation such as speed, oil pressure, and temperature can be observed.

The engines are direct connected to the generators or alternators by means of flexible couplings, and are started with automobile type starters.

The entire assembly of the engine and generator is made on a substantial cast iron base. To lessen the noise and prevent the transmission of vibration to the building the entire sub-base is mounted on springs. All connections such as water inlet and outlet, fuel suction and return lines and exhaust pipe are made with flexible

piping. The spring mounting also lessens the wear of the moving parts of the engine due to the cushioning effect, thereby reducing the maintenance and prolonging the life of the unit.

All moving parts are protected by guards and the exhaust manifold is water jacketed.

A maxim silencer is usually installed in the exhaust line. In the installation at Buffalo it is on the roof at the 20th floor set back.

As considerable condensation occurs in the long run of exhaust pipe a drain cock is placed at the bottom of the

Fig. 22—Emergency Units, Pittsburgh, Pa.

exhaust line in the engine room. This can be seen just to the right of the generator.

In some of the larger offices two smaller emergency units are provided instead of one large unit because it is frequently found to be more economical, it affords greater protection than the single unit installation and the replacement of parts on the smaller engines can be more readily handled by the local force. Two units are installed in Pittsburgh, Pa., a view of which is shown in Fig. 22. These units are similar to the one at Buffalo except the engines drive 220-volt three-phase 60-cycle 50-kva. alternators. At the right can be seen the storage batteries used for starting together with the charging panel. These batteries are furnished in duplicate to insure ample capacity in event of trouble in starting. The batteries are also used in connection with a relay to automatically light a few 12-volt lamps in the engine room when the power fails. Engine units of the type shown can be started very quickly. In some cases after a power failure they have been started and loaded in less than 2 min. However on trial test this has been accomplished in 30 sec.

Where two engines with alternators are installed no attempt is made to parallel them. The various loads are divided and switching facilities provided so that all or any part of the loads can be placed on either unit.

Only one Diesel-engine set is in Western Union service. This is installed at San Francisco and is shown in Fig. 23. It has not been found economical to install Diesel engines instead of gasoline engines for telegraph emergency service as the initial cost of the Diesel engines is much higher and the sets are run so seldom that economy of operation is comparatively a small item. The Diesel set was installed in this case as the storage of gasoline, except in a small quantity, was prohibited. This set is equipped similarly to the gasoline sets; that is, with overspeed cutout device, water regulator, duplicate fuel supply pumps, engine fuel supply tank, control panel, and spring mounting. It was necessary to use two electric starters on this engine as well as to provide compression relief valves for use in starting. This set is a 70-hp. 800-r. p. m. 4-cylinder engine, direct connected to a 40-kw. 3-wire 220-volt d-c. generator.

In a few cases it has been found advisable to install the emergency units above the ground floor because of the danger of high water or floods. One such installation is at New Orleans where two 60-hp. 900-r. p. m. engine-generator sets are installed on the third floor. In this case a motor-driven pump was mounted in a fireproof enclosure a few feet above the ground level for pumping fuel from the underground supply tank to a small reservoir in the base of the engine. This is a rather expensive method of installation and is subject to troubles in operation.

A semi-automatically operated gasoline engine generator set was recently installed in the Western Union office at Shreveport, La., on a trial basis. If proper operation can be obtained it will save considerable time in restoration of service where the unit is several floors from the operating force, as it is in this case.

Such a semi-automatic unit is shown in Fig. 24. In

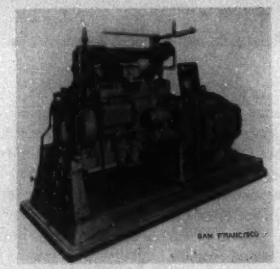

Fig. 23—Diesel Emergency Unit, San Francisco

addition to the standard accessories with which this set is equipped there are also the necessary electrically-operated attachments for starting or stopping the set at a remote station. These consist of a contactor for closing the starting motor circuit, the solenoid-operated choke on the carburetor and the fuel pump control circuits. The set is also equipped with a radiator and fan for cooling purposes in order to make the unit independent of the local water supply. One set of control buttons is mounted on the engine control panel and another or duplicate set is mounted on a power panel in the operating room in another part of the building.

In case of a failure of the regular power an attendant in the operating room may start the emergency unit and transfer the load to it without leaving the room.

In the past five years marked improvement has been made in the design of internal combustion engines prompted by the demand of the automotive and aviation industries for higher powered engines of less weight. This brought about the design of higher speed industrial type engines which proved more reliable and satisfactory in operation. With the economic advantages in the use of higher speed engines for telegraph emergency plants in view, tests were recently made on engines operating at 1,200 r. p. m. which resulted in a decision to make a few trial installations. The first set will be an 80-hp. 4-cylinder engine running at 1,200 r. p. m. and will be installed at the Western Union's office at Jacksonville, Fla. where a saving of about one-third in the initial cost of the plant as well as a saving of space is expected.

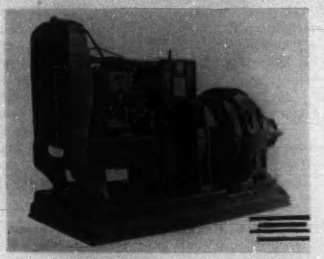

Fig. 24—Semi-Automatic Emergency Unit, Shreveport, La.

Two units similar to the one shown in Fig. 25 consisting of a 200-hp. 8-cylinder 1,200-r. p. m. gasoline-engine direct connected to a 125-kva. 208-volt three-phase 60-cycle alternator will be installed in the new Western Union office at Boston. A considerable saving in space as well as in initial cost will also be made in this case.

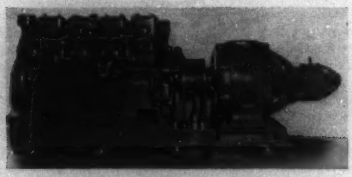

Fig. 25—200-Hp., 1,200-R. P. M. Gasoline Engine-Generator Unit

To provide for the quick restoration of service at offices which may be completely destroyed by fire, flood, earthquake, tornado or hurricane several portable engine-generator sets of 5-kw. capacity each mounted on wheeled trucks together with similar trucks containing motor-generator sets, telegraph switchboards, and operating sets are distributed throughout the country available for quick transportation by rail or truck to any point.

Perhaps there may be some question as to whether the emergency plants described have been of any service. One instance of their use was during a calamity which is remembered by all. When Miami, Fla. was hit by a hurricane in 1926, two small engine units provided power to operate the telegraph plant and provide light for over 72 consecutive hours or until the power was restored.

To provide for the satisfactory operation of new circuits, methods, and apparatus in the most economical and reliable manner development and design of power plant equipment are carried on continually and there is no doubt that the telegraph power plant of 1980 will show as much improvement compared to the present plant as the present day plant compared to 1880.

Discussion

AUTOMATIC POWER PLANTS FOR TELEPHONE OFFICES

(Young and Lunsford)

TELEGRAPH POWER PLANTS

(E. W. Griffith)

J. L. Woodbridge: Possibly a brief mention of certain developments in the design and application of the storage battery for the service described in the first paper may be appropriate.

The function of the storage battery in this field is threefold,—first as a reservoir of energy to insure continuity of service, second as a means for meeting the increasingly exacting requirements for a constant d-c. voltage and third, in many cases, as a means of reducing crosstalk and machine noise which would be disturbing without some type of filter. It has been the constant effort of the battery manufacturer, in cooperation with the engineers of the telephone industry, not only to perfect the storage battery for carrying out these functions but also to simplify its operation and reduce to a minimum the item of attendance and maintenance.

The improvements and refinements in manufacturing methods and processes which have been and are constantly being introduced are of real importance but are not obvious from a superficial examination and can hardly be reviewed in a discussion of this kind. Certain developments in cell construction and operating methods may, however, be of interest.

One of these is the introduction of the sealed glass jar cell, shipped assembled, sealed, and charged. This practise was introduced over 15 years ago for the smaller sizes of cell used in farm-lighting plants and its success has warranted its extension to the larger cells required for telephone installations. One such installation in Denver is shown in Fig. 1, the enclosed cells being on a rack in the foreground and the large batteries in lead-lined wood tanks appearing in the rear.

It is hardly necessary to point out the important advantages of the enclosed cells in respect to convenience of installation and subsequent inspection and maintenance. Reference has been made to the pilot balls or charge indicators which are now provided in these cells. Hollow-glass pilot balls were furnished with the farm-lighting batteries referred to above. Early experience with the wax balls led to some skepticism in regard to the permanence of their indications but improvements which have been introduced by the manufacturer have strengthened confidence in them, provided their limitations are understood.

To avoid misunderstanding on this point, it is well to point out that owing to the tolerances required in practical production methods both in the specific gravity of the balls themselves and in the nominal full charge specific gravity of the electrolyte as well as the variations in the latter with change of level and with time in service, the indications of these charge indicators must be considered as only approximate, but with these limitations in mind they are undoubtedly a useful adjunct. There appears, however, to be a general sentiment to the effect that in view of these limitations two pilot balls in a cell will serve all practical purposes, and probably in most cases the operation is guided by the indications of one of these. A recently-developed three-cell unit in a glass case is shown in Fig. 2, while a two-cell unit of smaller capacity provided with a rubber tray is shown in Fig. 3.

Fig. 1—Battery Room, Denver Telephone Office

Referring to the counter e. m. f. cells as a means for voltage control, a most important and radical improvement has been introduced, in the substitution of nickel electrodes in an alkaline electrolyte for the lead-alloy-acid combination. Such a cell is shown in Fig. 4. Several distinct advantages have been secured. The nickel electrodes are not subject to corrosion in service, being practically permanent in the absence of impurities, provided air is excluded, this latter being accomplished by the layer of oil on the surface of the electrolyte. No capacity is developed in service, permitting the cells to be short-circuited without excessive flow of current through the short-circuiting switches. No sediment is deposited, thus eliminating periodic cleaning. The momentary high voltage sometimes observed when current was first passed through the lead-acid type of cell after a prolonged stand on short circuit does not occur with these nickel-alkaline cells. They also require less space for a given current capacity.

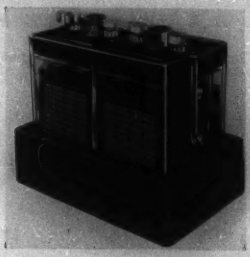

Fig. 3—Two-Cell Battery Unit, with Rubber Tray—2-B1-9, 4 Volts, 40 Ampere-Hours

In respect to operating methods, important improvements have been introduced. The substitution of the single battery in full float operation for the double battery operated on alternating cycles of charge and discharge has been mentioned, as well as the use of the ampere-hour meter for the control of partially worked batteries. One of the limitations of the ampere-hour meter has been referred to in the fact that it requires occasional overhauling on account of the fouling of the mercury.

There is another limitation which should also be kept in mind. The losses in the storage battery are of two kinds. First, that due to the inefficiency of the charging current, a certain portion of the charging energy being necessarily wasted in gassing. In accordance with Faraday's law, that portion of the charging current which causes gassing has no effect in charging the plates. In the case of a battery which is subjected to an appreciable amount of daily discharge, this loss is roughly proportional to the amount of discharge, and can therefore be approximately compensated for by adjusting the ampere-hour meter to run slow by a certain percentage in the charge direction. The other loss in the battery is that due to local action, and is proportional to the elapsed time, varying also with the cell temperature, but bears no relation to the amount of discharge. In the case of a worked battery, this loss is small compared with the charging loss and may be included in the overcharge provided by the adjustment of the ampere-hour meter. In the case of a very

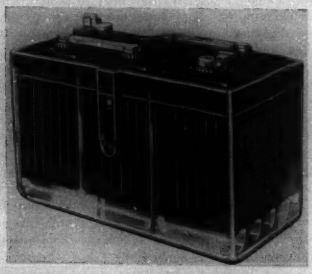

Fig. 2—Three-Cell Battery Unit with Rubber Tray—2-B1-9, 4 Volts, 40 Ampere-Hours

Fig. 4—Nickel-Alkaline Counter Cell—NAK, 2 Volts, 30 Amperes

lightly worked battery, the local action loss, although actually small, may be a very considerable proportion of the total loss, exceeding the limit of adjustment provided in the meter. This condition may be met by supplying to the battery a small constant trickle charge, not subject to control by the ampere-hour meter, but sufficient to compensate for the constant losses.

Much study has recently been given to the control of battery charging by means of a voltage responsive cut-out. In the early history of the storage battery many attempts were made to terminate the charge at a predetermined battery voltage. This method is no more successful now than it ever was, on account of the variation in the full charge voltage of the battery under varying conditions of charging rate, temperature, age of the battery, etc. It has, however, proved quite satisfactory to employ a two-rate charge, the rate being reduced from the higher to the lower rate in response to a battery voltage well below the final value, the lower rate being maintained continuously thereafter and fixed at a value which will not prove injurious. Various modifications of this principle have been successfully applied to meet varying operating conditions.

I. R. Smith: The development of dry disk copper oxide or Rectox rectifiers for the various applications described by Messrs. Young and Lunsford began over four years ago. Since

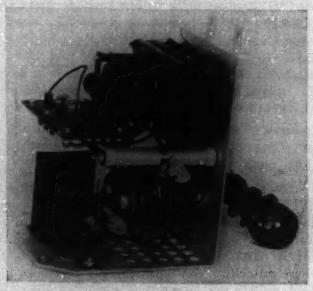

Fig. 5—Rectox Charger for PBX Batteries. Cover Removed

that time I have had the pleasure of cooperating with Mr. Lunsford and his associates of the Bell Telephone Laboratories in the development of a number of different units for these power applications. It may be pertinent to say a few words about these units and their characteristics.

Rectifiers for telephone use differ somewhat from those designed for other applications. In the first place, as pointed out by the authors, reliability is paramount. Operating costs, flexibility, size, and weight are also of importance, but not to the same extent. In the second place, an extensive system of specification and inspection has been built up, based on the long experience of the telephone company in the use of rectifiers. This system, of course, is also applied to the other types of equipment used and I think has a great deal to do with the success of the automatic plant, for it assures interchangeability of parts and duplication of results previously obtained.

For such an application the Rectox rectifier is well suited, as it excels in reliability any other available rectifier, having the ability to run for years with little or no attention, assuming that it has been properly designed and properly applied. Hence, it is well suited for application to telephone work.

The first unit developed was that used for charging 17-20-volt PBX batteries, a view of which is shown in Fig. 5. This unit has a maximum rate of ½ ampere, but it must also be able to charge as low as 0.2 amperes, and of delivering either the maximum or the minimum rate at rated battery voltage and for line voltages varying plus or minus 10 per cent. Furthermore, the rate must be adjustable between the maximum and the minimum in steps of not more than 55 milliamperes each. The resulting unit then is extremely flexible. Adjustment is made by means of transformer taps, for the sake of efficiency, as this rectifier runs continuously. And, as it is used while the battery is furnishing talking current, the normal d-c. wave delivered from the rectifier must be smoothed out somewhat, which is accom-

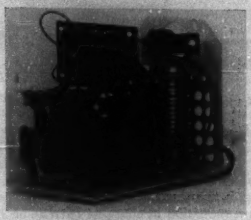

Fig. 6—135-Volt Rectox Charger for Plate Supply Batteries. Cover Removed

plished by means of a small reactor in the d-c. output circuit. Mechanically the unit is designed to be portable, to mount on wall or shelf, and to harmonize in appearance with other equipment in the station.

The next development was that of the 80-milliampere 130-volt unit for charging repeater plate supply batteries, as illustrated in Fig. 6. Mounted in the same case as the PBX charger, this unit is also provided with a wide range of adjustment for flexibility and to assure its being able to handle any application

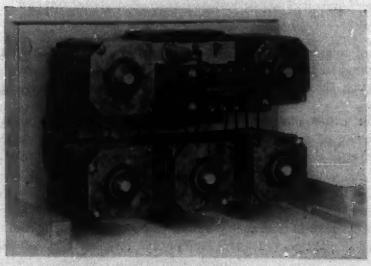

Fig. 7—2¼-Ampere 20-30-Volt Rectox Charger for Filament Supply Batteries

within its range. On lines of secondary importance where outage due to power failure is not serious, this rectifier with a suitable filter is sometimes used to excite the plate of the tube directly, without the battery.

Repeater tube filament supply battery charging called for a somewhat larger Rectox shown in Fig. 7, capable of delivering 2¼ amperes at 20-30 volts. The same provision for wide range of adjustment to meet many conditions has been made. The unit being much larger than the preceding rectifiers was arranged for relay rack mounting.

Several other developments have been carried on, including the magneto power unit, (a combination battery and trickle charger) the superimposed ringing battery charger, and various units used as valves. The latest development is a rectifier which delivers 120 volts and 100 milliamperes with a ripple of less than 3 per cent in the output wave. This rectifier is shown in Fig. 8.

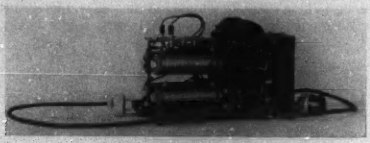

Fig. 8—Power Unit, Case Removed. Rated at 120-Volt 0.1 Ampere D. C.

There are two characteristics of importance. First is the matter of reliability. I think it is of interest to view the life test curve, Fig. 9, showing the average performance of a number of units, with a reduction in charging current of 22 per cent in 3½ years of continuous operation. Those who have had experience with rectifiers know that such operation, absolutely without maintenance or replacements, is really remarkable.

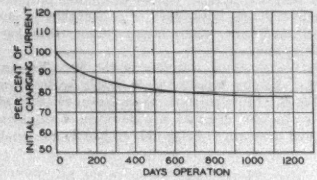

Fig. 9—Rectox Performance on Life Test

On all telephone type rectifiers excess transformer capacity is provided to permit the voltage to be raised when necessary due to aging, thus extending the useful life to some indefinite future date.

The other characteristic is efficiency. Fig. 10 shows the efficiency of a 4-disk bridge-connected rectifier rated at 5 d-c. volts and delivering 0.5 amperes. This illustrates how the efficiency is affected by a change in the operating voltage. In the operating range which would be from 3 to 5 volts, efficiency is quite satisfactory.

K. C. White: It may be interesting to mention that the Pennsylvania Railroad Co. has applied several of the power plants described in this paper to a rather extensive dial telephone system in the Western Pennsylvania and Eastern Ohio area. This system is illustrated schematically in Fig. 11.

There are 9 fully automatic, 7 semi-automatic and 8 manual telephone P.B.X.'s in this system with rather elaborate interconnecting tie lines. Automatic power plants have contributed greatly in making it practicable due to the very much lessened maintenance required resulting in a considerably smaller operating personnel. In addition the better-voltage practically foolproof operation, and longer battery life make for a much higher efficiency than that obtainable in manually-operated plants.

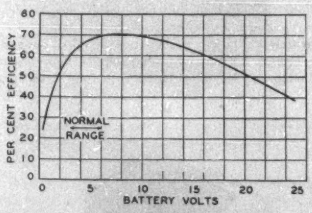

Fig. 10—Efficiency of Four-Disk Rectox Unit at 0.5 Ampere

Some earlier types of automatic power plants of somewhat different characteristics were placed in service in this territory between 1917 and 1927 and formed a basis for the present design which has proven most satisfactory. In fact I can recall no power failure in any of these plants installed since the latter date. This result is particularly gratifying since there is hardly a service where reliability and dependability are more important than that required in the operation of a railroad especially in the central region of the Pennsylvania Railroad Co.

It has been especially pleasing to us to receive numerous favorable comments on the automatic dial telephone P.B.X. system outlined above in which the automatic power plants are contributing their share. These comments have come from superintendents of our operating and traffic departments as well as the rank and file of our yard men.

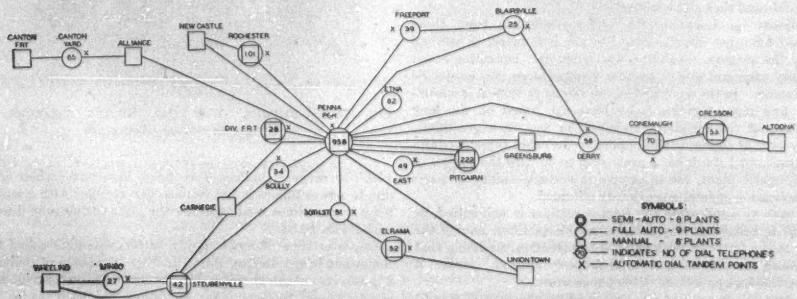

Fig. 11—The Pennsylvania Railroad Central Region

Private branch exchange showing type of power plant and number of dial telephones

G. H. Kendricks: When fire destroyed the office of the Western Union in Pittsburgh in 1885 the Alleghany County Light Company set aside two 500 light d-c. machines for its exclusive use, for it was unable to secure quarters large enough to replace the gravity battery destroyed. It was necessary in order to use this plant to ground the positive or negative polarity of these machines in order to give the proper current to the Company's lines. Up until recently the telegraph service was a single-wire system, being either grounded or "batteried" from the distant end. In order properly to distribute the correct amount of current to the lines, artificial resistance was introduced in the line next to the battery. This was done for various reasons. First, to protect the instruments in case of a short circuit or a nearby ground on the line. At this particular time the 16-candle power 110-volt carbon filament lamp was used. Gravity battery was still maintained for local and sounder service. In order to supply current to the multiplex circuits it was necessary to build up with gravity battery on top of the 110-volt generators.

When the First National Bank Building was rebuilt the company installed the old Siemens-Halske generators used in New York and in addition two Western Electric 15-volt generators for local sounder service. This, it is understood, was the first telegraph office operated exclusively with generators. At this time there were new ways of adding different types of resistance for battery leads. The one in question in the new office was a plaster of Paris cylinder about 15 in. long by 3 in. in diameter and wrapped with No. 36 German silver wire. This was abandoned on account of the space occupied. Then the resistance lamp of various resistances was introduced, which is used at the present time.

With the introduction of the generators it became necessary to provide some contrivance for dividing current for quadruplex operation which was commonly used at that time. In the gravity battery all that was necessary was to tap the battery at the required voltage, but with generators with only one voltage it was necessary to use a rheostat with German silver wire so as to give a proportion from three to one or four to one as the proper amount of current for the polar side of the quad and for the common side of the quad. Quadruplexes have all been discarded since the introduction of high-voltage power plants. The Western Union operating department is now located on the 15th and 16th floors (approx. 25,000 sq. ft.) of the Chamber of Commerce Building and today practically 85 per cent of its service is worked by printer operation. The power plants today are composed of two 25-volt motor-generators, six 110-volt motor-generators, three 160-volt motor-generators, and three 240-volt motor-generators, occupying a space about 6 ft. by 40 ft. (approx. 2,400 sq. ft.) in the main operating room.

The telegraph company has also introduced in the larger cities, pneumatic tubes for carrying messages from the main office to the branches. This is done in Pittsburgh by seven 5-hp. motor-operated blowers to the branch office, and three 1-hp. motor-operated blowers for tubes from one branch office to another in the near vicinity of the larger branch offices.

C. S. Alt: The company with which I am associated has pioneered in the development of emergency engine sets for telephone and telegraph service assisted in a major way by the engineering staff of both systems.

Although our public service supply of power today is of a high order, emergencies do arise which leave us without power, hence the call for an emergency power unit. Primarily such a power unit must of necessity be easily and quickly started and put into service at full load without delay, therefore our development has been carried on with this idea always foremost.

Accomplished accurate speed regulation and governing must be adhered to especially so in the a-c. power plants. Although it has not as yet been necessary in the communication service, a regulation has been accomplished to meet the needs of parallel operation of two or more units.

Freedom from vibration and its attendant results are also a very important item in the operation and development of such units. Extreme caution in balance of all moving parts is necessary. No power demand is so exacting in its requirements in this respect as the telegraph and telephone service, and the years of development bring to one's mind a number of experiences dealing with period and harmonic vibration in buildings peculiarly susceptible to such problems.

The supply and handling of fuel for such a unit is also a problem and the telephone and telegraph company experiences have proven that gasoline as a fuel is most dependable. Gasoline in an office building in the crowded business section of our large cities might seem like playing with a great hazard but experience has shown that this is not the case as the Laboratories of the National Board of Fire Underwriters are continually working hand in hand with us in this field and today a gasoline emergency power unit can be installed in any building with the full approval of this Board. All plants used by the telephone and telegraph service are approved and listed by the Underwriter Laboratories. Simplification and centralization of engine and fuel controls are also very necessary in order that an experienced operator is not required to start the unit. Condensed starting instructions consisting of as few operations as possible are supplied attached to the control panel permitting starting and application of load in a matter of seconds.

Quietness of starting and operation are also needed. Backfiring of an engine in this service must be guarded against. Exhaust silencing is absolutely necessary. The silencer must be located and be of sufficient strength to silence and hold exhaust-pipe explosions and after firing.

Guarding of all moving parts on the unit is necessary from a safety angle. It is surprising to know the number of people who still insist on wiping off, with a piece of waste or rag, a revolving shaft or gear. The remedy is to enclose all these dangerous items.

R. L. Dunlap: Four years ago the first automatic power plant was put into service in this section at the Pitcairn, Pennsylvania Central Office. Since then 72 of these plants have been installed in the Pittsburgh territory. About one third of the number is being used at central offices in the smaller towns and the rest serve private branch exchanges. Among the branch exchanges are department stores, railroads, newspaper offices, insurance companies, banks, a telegraph office, a jail, a court house, private estates and industrial concerns.

Before the automatic plants were developed department stores were a constant source of worry. Whenever the maintenance man read in the newspaper that a large store in the center of town was about to have a special sale, he connected a few more cable pairs to feed battery to the branch exchange in that store. The sale generally ended some time before he remembered to disconnect the pairs, and consequently the battery at the branch received a most thorough overcharging.

It has been found that the automatic power plant at a private branch exchange provides better voltage regulation and longer battery life with less maintenance effort and a saving in power consumption. Another factor has been that cable pairs formerly used for battery charging can now be used for telephone conversations.

At the same time that automatic charging equipment was being developed improvements were made in storage battery design. The closed model of today with its built in charge indicators and large sediment space differs as greatly from the open type of 5 years ago as the 1931 model automobile differs from cars of 1925-6.

R. L. Lunsford: Two of the interesting features of the ringing and signaling equipment, namely the mercury switch interrupters and the inductor alternator, furnish the tones and signals

necessary in the supervision of all telephone calls and some are heard every time a call is made.

The mercury interrupter has been developed because of the fact that the low-speed interrupters make and break substantial currents in circuits having inductance and capacity and it has been found impossible to avoid sparking at the brushes with the well-known segment type rings. This has resulted in pitting of the interrupter segments and has required an undue amount of maintenance.

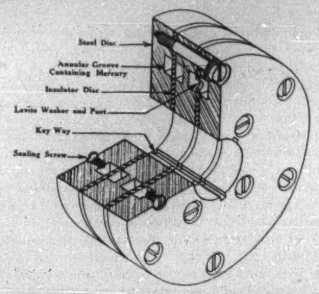

Fig. 12—Mercury Interrupter Unit

The interrupter units (see Fig. 12) though old in theory are believed to be new in the form in which they are used. They consist of either two or three steel disks separated by sheet insulators provided with ports or openings and the whole clamped tightly together. An annular channel is cut in the side of each disk and a small amount of mercury is placed therein. The disks and the interrupter units are so arranged that the channel of one disk is adjacent to the channel of the next disk with the insulator separating the two.

When the interrupter units are partially filled with mercury and rotated on a horizontal axis, contact is made from one ring to the other ring by means of the mercury only when a

Fig. 13—Inductor Alternator for Generating Tones Used in Telephone Power Plants

port of the insulator dips into the pool of mercury. The ports are provided with Lavite washers to withstand the arcing which occurs when the contact is made or broken. The number of ports, their spacing around the insulators, the amount of mercury in the channels, and the speed of rotation determine the timing of the interrupter.

A rather novel feature of these interrupters is the method used in providing an inert gas within them. Several gases were tried out but it finally developed that the simplest and best arrangement was to assemble and hermetically seal them and place them in operation. The sparking inside does the rest and provides atmospheric nitrogen as soon as the oxygen is consumed.

The brush rigging is similar to that used on existing interrupters. Metallic brushes with Baylis brush holders are mounted so that they slide on the rims of the disks to maintain connections with external circuits. The type of brush used requires very little maintenance and is longer wearing than the softer carbon brushes used on the old segment type interrupter.

The tone alternator is the new device mentioned, the development of which has just been brought to a satisfactory conclusion. This produces the "high tones" and "low tones" formerly obtained by segment type interrupters such as shown in Fig. 12 of the paper. On the older type, brushes were used riding on segmental rings to interrupt battery current, thus furnishing current having about 480 and 153 interruptions

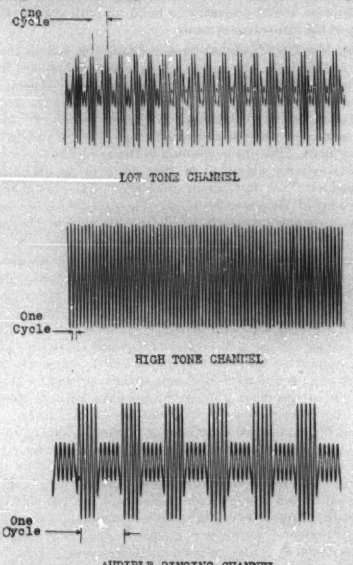

Fig. 14—Tone Alternator—Showing Wave Shapes of Three Channels

per second. Attempts to reduce maintenance on these rings and improve quality of the tone by introducing spark killers made the tone practically disappear.

A multi-channel inductor-type alternator was, therefore, developed as shown in Fig. 13, which has some features not used to our knowledge in commercial applications. The central disk of the rotor generates a continuous high-frequency alternating current for the high-tone channel. When, however, it was attempted to use a low-frequency tone generated in the same manner, it was found that this tone had insufficient energy to go through the repeating coils and other parts of the circuit. This explained the results from spark killers on the earlier interrupters and indicated that the tone was being transmitted by its harmonics rather than its fundamental. Combination rotors were, therefore, designed for the alternator which delivered a few cycles of high-frequency at a relatively higher voltage

alternated with a few more cycles of the same frequency at a reduced voltage. The rotors for these channels are shown in the figure.

Fig. 14 shows oscillographs of the wave shapes obtained with this machine, the high tone being the continuous channel in the center. The low-tone channel shown in the upper curve has the same satisfactory penetration as the old sparking interrupters with high harmonics but it has the same effect upon the ear as a simple sine wave of lower tone. The bottom curve shows a composite wave delivering a still lower effective frequency provided for audible ringing, which is a tone placed upon the circuit of the subscriber being rung and transmitted back to the calling party to notify him that the connection has been established and ringing is in progress. The manner of mounting both the tone alternator and mercury interrupters with their associated ringing machine is shown in Fig. 13 of the paper.

Although these machines are just now being introduced into the telephone system, several trial installations in operation during the past year or more have shown that the tone is more uniform and more satisfactory than anything formerly provided. This result is also obtained over long periods without machine maintenance from the attendant, whereas the older type machines had to be cleaned several times daily in some cases to give satisfactory performance.

Recent Developments in the Operation of
Overseas Radio Telephone Service

By F. A. COWAN[1]
Member, A. I. E. E.

JUST a little more than four years' time has elapsed since the first commercial transatlantic radio telephone service was inaugurated. During this period the scope of overseas service has grown so that today a telephone user in North America may talk with users in Europe, South America, Australia, one point in Africa, and on ships at sea. The establishment of these extensive transoceanic ties has been discussed in a number of papers[2] presented at meetings of the Institute. It is the purpose of this paper to consider some of the more recent developments in the operation of the overseas services.

Those portions of the world which could be reached by telephone from the United States, Canada, Cuba, and Mexico at the end of 1930 are indicated by the shaded areas on the map shown in Fig. 1. This map also shows the radio telephone links which make the overseas connections possible. All of these overseas telephone ties are effected by means of radio operating on short wavelengths in the range between 12 and 100 meters (25,000-3,000 kc.), with the exception of the original circuit between New York and London, which operates at a wavelength of about 5,000 meters (60 kc.). The extensive use of short wavelengths for radio telephone service was occasioned by the particular conditions which were met in the extensions of the overseas services. Such factors as availability of wavelengths, susceptibility to noise in transmission across equatorial regions, available space on ships, character of transmission variations, as well as economic considerations, usually favor the use of short wavelengths. In the case of New York-London circuits, which traverse the North Atlantic region, however, the greater stability of the transmitting medium at long wavelengths together with relatively shorter distances and more favorable locations for the long-wave receiving stations make them somewhat more reliable than short-wave circuits. It has been found, however, that conditions which affect transmission adversely on long-wave circuits rarely occur simultaneously with those factors which affect the short wavelength circuits, so that continuity of service is greatly improved by using the two in conjunction.

Aside from the North Atlantic region, the short wavelength circuits appear to offer the most promising means of extending the overseas service. This is substantiated by the excellent performance of the New York-Buenos Aires, Argentina, circuit which was put in service in the spring of 1930. In view of these considerations, extensions of service now proposed to Bermuda and Hawaii will utilize short wavelength radio.

Since the circuit from New York City to Buenos Aires employs short wavelength radio and is representative of circuits recently established and contemplated, this particular circuit will be considered in some detail. Fig. 2 shows, in schematic form, the layout of the New York-Buenos Aires circuit.

The short-wave transmitting and receiving stations in this country for the South American service, as well as for the European service, are located near Lawrenceville, N. J., and Netcong, N. J., respectively. These transmitting and receiving centers are about 50 miles apart; are about the same distance from New York; and are connected by cable with New York City where the terminals of all of the overseas and ship-to-shore circuits in the Atlantic region are concentrated. In South America, the transmitting and receiving stations are located at Hurlingham and Platanos, Argentina, respectively. Both of these points are within 25 miles of the circuit terminal at Buenos Aires.

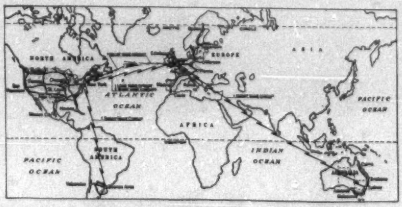

FIG. 1—INTERCONNECTION OF TELEPHONE SYSTEMS OF NORTH AMERICA WITH THOSE OF OTHER CONTINENTS

The equipment and arrangements at Lawrenceville, Netcong, and New York City have been discussed in considerable detail in the papers previously referred to. The equipment used in South America is practically identical with that used in this country. At both transmitting points piezo-electric crystal controlled short-wave transmitters are used. These transmitters employ high-power water-cooled vacuum tubes with an unmodulated power output of between 15 and 20 kw. which, when modulated 100 per cent, corresponds to a

1. Engr. Trans. Long Lines Dept., American Tel. & Tel. Co., New York, N. Y.

2. K. W. Waterson and O. B. Blackwell, A. I. E. E. JOURNAL, April, and May 1928, respectively.

T. G. Miller, A. A. Oswald and Ralph Bown, A. I. E. E. JOURNAL, February, April, and May 1930, respectively.

Lloyd Espenschied and W. Wilson, A. I. E. E. JOURNAL, July 1930.

Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

31-57

peak power output of between 60 and 80 kw. Directive antenna arrays are provided for three separate wavelengths to care for the changing transmission conditions with seasons of the year and hours of the day. The directive characteristics of these arrays increase the power radiated in the desired direction by an amount equivalent to that which would be obtained by increasing the transmitter power from 20 to 50 fold, depending upon the wavelength being used.

At the receiving points, directive antenna arrays are also employed. These arrays, by virtue of their directive characteristics, improve the receiving conditions by strengthening the signal received from the desired direction and reducing the effect of noise or signals coming from other directions. The receiving sets proper are of the double detection type and are equipped with automatic gain controls to compensate for moment-to-moment variations in the strength of the received carrier wave. The complete receiving system is capable of utilizing satisfactorily a signal having a field strength as low as one microvolt per meter when radio noise conditions are satisfactory. This performance has been made possible by the selection of a receiving site which is sufficiently isolated to minimize local interference and by the careful design of the component parts of the receiving system.

At the terminal points voice-operated relay switching equipment is provided which disables the radio path in one direction while speech is traveling in the opposite direction. It also maintains the transmitting side of the circuit in a disabled condition when no speech is being transmitted. This automatic switching simplifies the transmission problem by resolving the two-way circuit into two one-way circuits. The circuits which actuate the switching relays are carefully adjusted in order to compensate for changing transmission conditions. This minimizes interruptions due to false operation of the relays caused by abnormal conditions on either the radio channel or the land line.

Once circuits of the type under consideration have been established, their performance is controlled largely by the condition of the transmitting medium at any particular time. Since the control of the medium is beyond the scope of human endeavor, the operating problem becomes one of maintaining the component parts of the system in good condition and operating the system as a whole to best advantage.

One index of the effectiveness with which radio telephone circuits are operated is the amount of circuit time which is lost. For this reason, this phase of the circuit performance is carefully scrutinized. Fig. 3 gives a comparative analysis of the lost circuit time for one month on the New York-Buenos Aires circuit and one of the New York-London short-wave circuits. It will be noted in each case that the major percentage of lost time is due to atmospheric conditions.

The equipment used in the short-wave radio circuits is so designed and constructed as to minimize the likelihood of trouble. In addition, spare units of equipment have been provided for replacing equipment which would be most subject to trouble or which would require considerable time to repair. These precautions, together with a systematic testing routine, make it possible to operate the overseas circuits with the small amount of lost time, due to equipment troubles, that is indicated by Fig. 3.

The over-all operation of the radio circuits is carried out under the supervision of technical operators located at the circuit terminals. These operators coordinate the activities at their local transmitting and receiving stations with those at the distant terminal. In this country the technical operators receive advice as to what wavelengths are most effective at any time, from a channel observing station which is equipped with special apparatus for maintaining a continuous check on radio transmission conditions. This check on transmission conditions is made by measuring the field strength of foreign stations operating on various wavelengths.

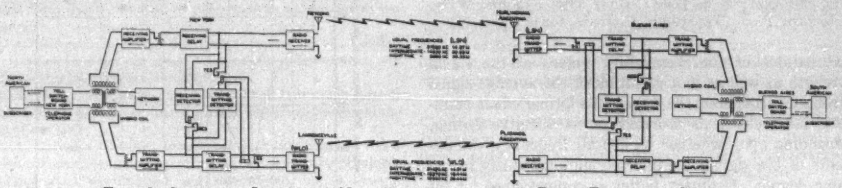

FIG. 2—SCHEMATIC LAYOUT OF NEW YORK-BUENOS AIRES RADIO TELEPHONE CIRCUIT

Very accurate measurements of the frequency of the local radio telephone transmitters and of associated foreign transmitters are also made, periodically, by the channel observing station. In case there is any appreciable deviation from the assigned frequency an investigation is made immediately, the cause determined, and the deviation is corrected.

Fig. 4 shows the results of a series of frequency measurements made during August, 1930, on transmitter WLO, which is used in the South American service. It will be noted from the chart that the frequency deviation was in all cases less than $\pm$ 0.01 per cent whereas the limit set by the Federal Radio Commission allows a deviation of $\pm$0.05 per cent of the carrier frequency. In cases of interference to the telephone channels from

radio stations assigned to neighboring wavelengths, the frequency measurements aid in the correction of the difficulty.

The foregoing describes the steps that are taken to secure the most effective operation of the radio telephone circuits in so far as controllable factors are concerned. As previously stated, the variation of the transmitting medium is at present the major factor limiting the usefulness of short-wave radio channels. These variations in the transmitting medium result in

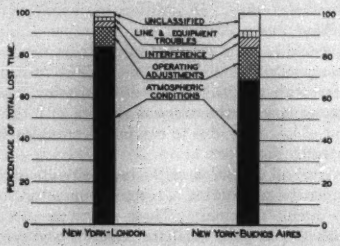

Fig. 3—Analysis of Lost Circuit Time on Short Wave-Length Radio Telephone Circuits for August 1930

fluctuations of the momentary values of the signal strength as well as in a variation of the average signal strength throughout the day. The former effect manifests itself either as ordinary or as selective fading, depending upon whether or not all frequency components of the signals are affected simultaneously to the same degree. Ordinary fading is compensated for by means of the automatic gain control incorporated in the receiver and the only effect which is produced when this type of fading occurs is an apparent variation of

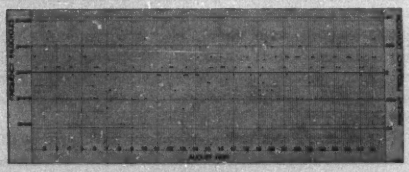

Fig. 4—Frequency Measurements of Transmitter WLO for August 1930 Assigned Frequency 21,420 Kc.

the noise level. Selective fading, however, which usually accompanies ordinary fading to varying degrees, gives rise to distortion effects and may be sufficient at times to render a channel unsuitable for commercial transmission on a given wavelength.

Periods of abnormal activity in the earth's magnetic field, known as "magnetic storms," occur at irregular periods and are accompanied by decreases in the signal strengths at the receiving points to unusable values at times. When these storms occur, transmission on all frequencies is affected to a greater or lesser degree and the time during each 24-hour period when reliable transmission can be obtained is reduced materially.

Fig. 5 shows the average effect of the major magnetic disturbances which occurred during 1930, on the available circuit time of a representative New York-London short-wave radio telephone channel. It should not be construed from this curve that magnetic storms have occurred regularly every two weeks, but rather that when they have occurred the available circuit time, for several days preceding and following the day on which the storm was at its height, has usually been affected as shown.

In view of the adaptability of short-wave telephone systems to transmission over great distances and the fact that rapid progress is being made in improving, simplifying, and reducing the cost of these systems, they are of outstanding importance in the development of world-wide telephonic communication. The brief

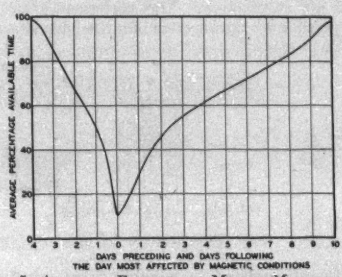

Fig. 5—Average Effect of Major Magnetic Disturbances During 1930 on the Available Circuit Time of a Representative New York-London Short Wave Radio Telephone Channel

period which has elapsed since establishing the first short-wave radio telephone channel has been one of rapid progress in the development and application of short-wave systems and they are, at the present time, generally accepted as a medium for extending the scope of overseas telephone service.

Short-wave radio telephone circuits have been used for the transmission of international broadcast programs, for conducting joint meetings of international conventions, and for telephone interviews. Numerous accounts of examples of the latter type of usage have been published from time to time by American newspapers. One of the most notable examples, perhaps, was recorded by *The New York Times* when a personal interview with General Uriburu was obtained after the recent revolution in Argentina. Services of these types undoubtedly promote mutual understanding between the people of various nations and are helpful in the cementing of international good will.

Discussion

J. P. Scott: This paper presents an analysis of the lost circuit time by causes (Fig. 3) and indicates the number of days that a typical magnetic storm is troublesome (Fig. 5). What is the total per cent lost time due to all causes? How frequently are magnetic storms usually experienced?

F. A. Cowan: As pointed out in the paper the chief cause of lost circuit time is the condition of the transmitting medium. The medium varies considerably from day to day, season to season, and from year to year. The average percentage lost time for one period of several months that I have in mind was about 10 per cent for the New York-London long-wave circuit, about 25 per cent for the short-wave New York-London circuits, and about 5 per cent for the New York-Buenos Aires short-wave circuit. The higher percentage lost time on the New York-London short-wave circuits than on the New York-Buenos Aires circuit is accounted for by the proximity of the transmission path to the north magnetic pole of the earth in the case of transmission between New York and London. In view of the fact that the factors which affect long- and short-wave transmission rarely occur simultaneously, the over-all reliability of the service is materially enhanced by using the two types of facilities in conjunction.

Magnetic storms do not occur with any definite regularity. The number varies from year to year. During 1930 there were about 12 major magnetic storms.

W. C. Hecht: How are the practical operating hours for the several radio circuits affected by differences in time of day at the terminals?

F. A. Cowan: The load distribution curves for the London and Buenos Aires service are not materially different from the load curve for New York-Chicago service.

N. S. Hoff: What steps are involved and how much time is required from the conception of a radio circuit to the establishment of service?

F. A. Cowan: Definite arrangements have to be made with a foreign correspondent, frequency assignments must be obtained, buildings must be designed and constructed, unless already available, and equipment and antenna systems must be engineered, constructed, installed, and tested before commercial service can be established. This will require anywhere from one to two years, depending upon the particular problems involved.

W. L. Shafer: The paper mentions the large gains that have been obtained by the use of directive antenna arrays at the transmitter and receiver. Are there possibilities of further substantial gains by such means? What limits the gain obtainable in practise?

F. A. Cowan: The gain of the antennas is obtained by arranging the elements in such a manner as to direct the energy in the desired direction. The method is analogous to the use of a reflector behind a light. In view of the fact that the transmitting medium is not homogeneous the transmitted waves do not always proceed to the distant point by the shortest straight line paths, but are sometimes reflected from their course. Consequently, if the angle of radiation of the transmitting antenna is made too small the waves might miss the receiving antenna at the distant station. The gain which it is practicable to obtain by the use of directive antenna is limited by this as well as economic considerations.

L. R. Huddler: What over-all length of circuit route is involved when radio circuits are used in tandem with wire extensions at the radio terminals?

F. A. Cowan: One of the longest calls, from the standpoint of distance, was a call from Los Angeles, California to Melbourne, Australia. This call involved a distance of 18,000 or 22,000 miles, depending upon the direction of transmission between London and Sydney. There have been other calls involving only one radio link between the West Coast of this country and the West Coast of South America, which is roughly 10,000 miles. One of the longest calls for the ship-to-shore service was between San Francisco and the *S. S. Belgenland* when the ship was in the vicinity of Japan. This call was established through the Ocean Gate, N. J. ship-to-shore transmitting station and hence the circuit length was about 10,000 miles.

R. T. Griffith: In the answer to the previous question Mr. Cowan states that the transmission between London and Sydney might be in either direction over the great circle route. How is it determined which direction the transmission is in? What arrangements, if any, are made to use the longer great circle route?

F. A. Cowan: The antennas are ordinarily set up to transmit over the shorter great circle route. If it is desired to transmit over the longer great circle route, it is necessary to use a different antenna arrangement.

Temperatures in Electric Power Cables
Under Variable Loading

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Synopsis.—*Considerable need has been felt for accurate methods of solution of the problem of temperature rise under variable loading in order that the maximum use may be made of the large investment in power cables.*

In this paper a rigorous solution of the problem of temperature rise from sheath surface to conductor is attempted, making use of Bessel functions. The heat flow cycle is resolved into harmonics, and each harmonic solved separately for temperature at the conductor. The various harmonics of temperature are then combined in their proper phase relation to obtain the temperature cycle. For purposes of assigning emergency ratings a solution is arrived at for suddenly applied steady loads, making use of the Fourier integral.

The problem is solved rigorously for single-conductor cables and three-conductor cables of shielded (type "H") construction. Modifications of the constants of the cables are described which will allow the theory to be applied to cables of standard belted construction with reasonable accuracy.

The probable errors involved in the assumptions necessary in the solution are discussed. It is believed that knowing the temperature of the air at the sheath surface, the temperature of the conductor can be calculated within 4 or 5 per cent of the correct value if the constants of the cable are known within this accuracy.

The method can also be applied to solution of the temperature rise of the sheath surface provided the constants of the duct bank are known with sufficient accuracy. This temperature can be added to the temperature rise of the conductor above the sheath temperature to obtain the total temperature rise of the conductor above the assumed base temperature.

Introduction

FAIRLY accurate methods are now available for calculating the temperature rise of the conductor above the assumed base temperature when the load is steady. These give results which are as accurate as the known constants of the cable and duct bank.

Until recently no attempt was made to obtain even a moderately rigorous solution for the case of variable loading.

A recent Institute paper[1] describes a point-by-point method of solution for variable loading which gives fairly accurate results. This point-by-point solution is based on the assumption that the temperature of the conductor rises exponentially with time which is only approximately true.

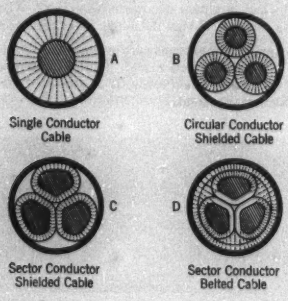

FIG. 1—CROSS-SECTIONS OF CABLES SHOWING DIRECTION OF HEAT FLOW

The theory presented in this paper tends to show that the temperature curve rises faster at first and then slower, say one hour after the load is applied, than the exponential law would allow. Solutions based on the exponential law are thus likely to give lower maximum temperatures and greater time lags than actual for ordinary load cycles.

The present paper solves the problem rigorously for cables with configuration shown in Figs. 1A and 1B. The errors involved in applying it to the configuration shown in Figs. 1C and 1D are discussed in a subsequent paragraph.

Solution for Daily Load Cycle

The general expression for heat flow in the conductor of either a single-conductor or three-conductor cable, assuming constant resistance, circular cross-section of conductor, and uniform current distribution, as derived in Appendix A is as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{S_c I^2 R}{\pi r_1^2} = C_c \rho_c S_c \frac{\partial T}{\partial t} . \qquad (1)$$

The general expression for heat flow in the insulation is the well-known Fourier's equation, and is as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = C_i \rho_i S_i \frac{\partial T}{\partial t} . \qquad (2)$$

where

T = Temperature at any point in the conductor or insulation.
I = Total current in the conductor.
R = Resistance of the conductor, at the frequency of the current, if alternating current, and the average temperature of the conductor.
r = Radial distance measured from the center of the conductor.
r_1 = Radius of the conductor.
C_c = Specific heat of the conductor material.

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1. For references see Bibliography.

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C_i = Specific heat of the insulation material.
ρ_c = Specific gravity of the conductor material.
ρ_i = Specific gravity of the insulation material.
S_c = Heat resistivity of the conductor material.
S_i = Heat resistivity of the insulation material.
t = Time.

When T varies sinusoidally with time, the solution of equation (1) is:

$$\bar{Q}_1 = -\frac{2\,\pi\,q_c\,r_1\,\gamma}{S_c}\left(\bar{T}_1 + j\,\frac{\bar{Q}_0\,S_c}{\pi\,q_c^2\,r_1^2}\right). \tag{3}$$

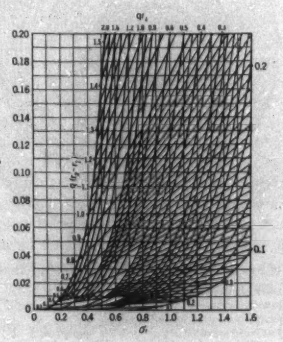

FIG. 2—CHART OF THE HEAT-FLOW CONSTANT: $\sigma = i_2\,k_1 - i_1\,k_2$

$$i_1 = I_0\,\sqrt{j}\,q\,r_1, \quad i_2 = I_0\,\sqrt{j}\,q\,r_2.$$
$$k_1 = K_0\,\sqrt{j}\,q\,r_1, \quad k_2 = K_0\,\sqrt{j}\,q\,r_2.$$

or when $q_c\,r_1$ is small, as in cases met with in solution of power cables:

$$\bar{Q}_1 = \bar{Q}_0 - j\,\frac{\pi\,q_c^2\,r_1^2\,\bar{T}_1}{S_c}. \tag{4}$$

The solution of equation (2) when T varies sinusoidally with time is:

$$\bar{Q}_1 = \frac{2\,\pi}{S_i\,\sigma}\,(q_i\,r_1\,\tau\,\bar{T}_1 - \bar{T}_2), \tag{5}$$

$$\bar{Q}_2 = \frac{2\,\pi}{S_i\,\sigma}\,(\bar{T}_1 - q_i\,r_2\,\psi\,\bar{T}_2). \tag{6}$$

Neglecting the heat storage in the lead sheath, the expression for heat flow in the sheath is for a single-conductor cable:

$$\bar{Q}_2 = \frac{\pi\,D}{K}\,(\bar{T}_2 - \bar{T}_3). \tag{7}$$

and for the three-conductor cable:

$$\bar{Q}_2 = \frac{\pi\,D}{3\,K}\,(\bar{T}_2 - \bar{T}_3). \tag{7a}$$

where

$\bar{T}_1$ = Vector temperature at surface of conductor.
$\bar{T}_2$ = Vector temperature at surface of insulation.
$\bar{T}_3$ = Vector temperature of the air at the surface of the sheath.
$\bar{Q}_0$ = Vector heat flow per conductor, equal to the $I^2\,R$ loss.
$\bar{Q}_1$ = Vector heat flow per conductor at inside surface of the insulation.
$\bar{Q}_2$ = Vector heat flow per conductor, at outside surface of the insulation, or at outside surface of the sheath if sheath losses can be neglected.
r_2 = Outside radius of the insulation.
$q_c = \sqrt{\omega\,C_c\,\rho_c\,S_c}$.
$q_i = \sqrt{\omega\,C_i\,\rho_i\,S_i}$.
D = Diameter of the sheath.
K = Surface heat resistivity of the sheath.

γ, σ, τ, and ψ are complex constants, depending on the constants and the configuration of the cable. They are analogous to the general circuit constants in the solution of a transmission line for voltage and current at the terminals and may be designated the heat flow constants of the cable. Their derivation is given fully in Appendix A. For ease in computation, charts of σ, τ, and ψ have been prepared and are illustrated in Figs. 2 to 4. Curves of the Bessel functions from which these charts were computed are shown in Fig. 5. Tables of these functions are given in Bibliography Nos. 2 and 15.

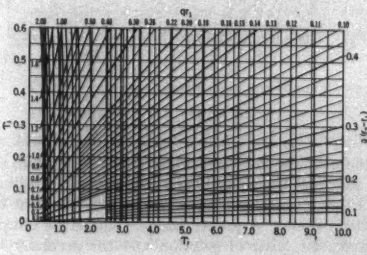

FIG. 3—CHART OF THE HEAT-FLOW CONSTANT: $\tau = i_1'\,k_2 - i_2\,k_1'$

$$i_1' = \sqrt{j}\,I_0'\,\sqrt{j}\,q\,r_1, \quad k_2 = K_0\,\sqrt{j}\,q\,r_2,$$
$$k_1' = \sqrt{j}\,K_0'\,\sqrt{j}\,q\,r_1, \quad i_2 = I_0\,\sqrt{j}\,q\,r_2.$$

Combining equations (4) to (7), the solution for a single-conductor cable is given by the following simultaneous equations:

$$\frac{\bar{Q}_0}{\pi} - j\,\frac{q_c^2\,r_1^2\,\bar{T}_1}{S_c} = \frac{2}{S_i\,\sigma}\,(q_i\,r_1\,\tau\,\bar{T}_1 - \bar{T}_2) \tag{8}$$

$$\frac{D}{K}\,(\bar{T}_2 - \bar{T}_3) = \frac{2}{S_i\,\sigma}\,(\bar{T}_1 - q_i\,r_2\,\psi\,\bar{T}_2) \tag{9}$$

In the solution for a three-conductor cable, equation

(8) remains the same and equation (9) is written as follows:

$$\frac{D}{3\,K}\,(\overline{T}_2 - \overline{T}_3) = \frac{2}{S_i\,\sigma}\,(\overline{T}_1 - q_i\,r_2\,\psi\,\overline{T}_3)\,. \qquad \textbf{(9a)}$$

Where sheath losses are present in appreciable amounts a correction can be made to the values of $\overline{Q}_2$ in equations (7) or (7a), when an accurate solution is desired.

A sample calculation, showing in detail the method employed in the solution of a specific problem is given in Appendix D. Fig. 6 shows the results of this calculation in the form of a curve of temperature against time. Briefly, the method is as follows:

The watts loss in the cable is plotted for the load cycle assumed. This curve is analyzed for its principal harmonics by well-known methods. The solution for temperature is obtained for each harmonic of heat flow and the results combined in their proper phase relation to give the resultant temperature cycle. Usually three or four harmonics will be all that is necessary to

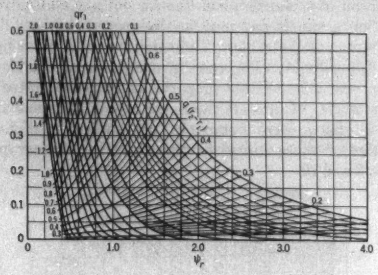

FIG. 4—CHART OF THE HEAT-FLOW CONSTANT: $\psi = i_2'\,k_1 - i_1\,k_2'$

$$i_2' = \sqrt{j}\,I_0'\,\sqrt{j}\,q\,r_3, \quad k_1 = K_0\,\sqrt{j}\,q\,r_1,$$
$$k_2' = \sqrt{j}\,K_0'\,\sqrt{j}\,q\,r_2, \quad i_1 = I_0\,\sqrt{j}\,q\,r_1.$$

solve for unless the rise of watts loss is very rapid, since the effects of the small hourly variations in loading on the cable have very little effect on the resultant temperature cycle. A proper balance should be obtained between the labor of solution and the accuracy with which it is desired to obtain it.

The value of $\overline{T}_3$, the temperature of the air at the outside of the cable, can be obtained in various ways. If the cable is installed in a duct and only approximate solutions are required, this can be assumed constant at the maximum value obtained over a twenty-four hour load cycle. Usually $\overline{T}_3$ will vary from 20 to 30 per cent of the total variation in temperature at the conductor, when the cable is installed in an ordinary duct bank.

If $\overline{T}_3$ is considered constant its value becomes zero in equations (9) and (9a), merely adding to the steady state component of the total temperature at the conductor which is computed by well-known methods. The value of $\overline{T}_1/\overline{Q}_0$ will then be constant in both phase and magnitude for any given cable. Tables of $\overline{T}_1/\overline{Q}_0$ for the principal harmonics may hence be made up for any given cable, as shown in Table I in Appendix D. These may be combined in their proper phase relation to obtain a solution for any load cycle.

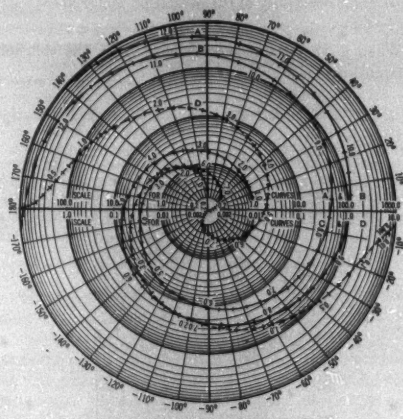

FIG. 5—CURVES OF BESSEL FUNCTIONS OF ZERO ORDER

Curve A—Bessel function of first kind, $I_0\,\sqrt{j}\,x$
Curve B—First derivative of $I_0\,\sqrt{j}\,x$ with respect to x, $\sqrt{j}\,I_0'\,\sqrt{j}\,x$
Curve C—Bessel function of second kind, $K_0\,\sqrt{j}\,x$
Curve D—First derivative of $K_0\,\sqrt{j}\,x$ with respect to x, $\sqrt{j}\,K_0'\,\sqrt{j}\,x$
Examples—$I_0\,\sqrt{j}4.2 = 3.86\,\underline{/146.4^\circ}$; $K_0\,\sqrt{j}\,3.4 = 0.060\,\underline{/-159.0^\circ}$

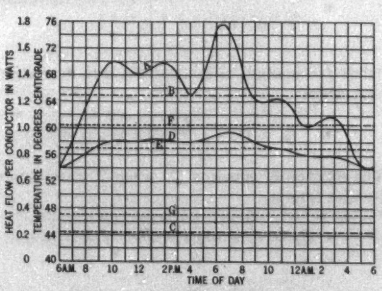

FIG. 6—HEAT FLOW AND TEMPERATURE CURVES FOR THREE-CONDUCTOR, 400,000-CM., 28-KV. CABLE

Curve A—Heat-flow cycle in watts per conductor; $I^2\,R$ loss
Curve B—Average $I^2\,R$ loss per conductor over 24-hour period
Curve C—Dielectric loss per conductor for the average temperature
Curve D—Temperature cycle for load cycle of curve A; temperature at surface of conductors
Curve E—Average temperature at conductor surface
Curve F—Temperature for constant load equal to the maximum
Curve G—Measured maximum temperature of air at sheath surface

If some judgment is used in selection of constants of the duct bank equation (5) may be used for the solution of the air temperature in the duct where $\overline{T}_1$ in the equation represents this temperature and the base temperature is considered zero.

The air temperature then can be expressed as follows:

$$\bar{T}_3 = \frac{\bar{Q}_3 S_e \sigma_e}{\pi q_e D' \tau_e}, \qquad (10)$$

Where

S_e = heat resistivity of duct structure.

$q_e = \sqrt{\omega C_e \rho_e S_e}$

ρ_e = specific gravity of the duct structure.

C_e = specific heat of the duct structure.

D' = inside diameter of duct structure, assuming an equivalent cylindrical structure.

σ_e and τ_e are computed in the same way as σ and τ in equation (5).

Various approximate formulas are given in Appendix C for obtaining the functions σ_e and τ_e which are outside of the range of the charts given in Figs. 2 and 3.

On account of the effect of adjacent cables and the uncertainty of the constants of the duct bank, the best way of obtaining the air temperature at the sheath surface is to actually measure it. Opinions differ as to the proper method of measuring this temperature in the duct outside the lead sheath. It is generally conceded that this temperature is nearly the same throughout the space between the sheath and the duct and can be considered so in the solution for steady heat flow. Since the air has a very low thermal capacity its temperature will follow very closely in phase the variations in the lead sheath. Hence if the maximum value of this air temperature, throughout the load cycle considered, can be obtained it can be added directly to the maximum temperature rise between this air and the conductor to obtain the total rise. The error caused by these temperature components being out of phase will be small since the total variation in the air temperature is only about one quarter of the total.

Maximum reading thermometers inserted between the cable and the duct wall seem to be the best method provided some means are used to prevent the thermometer from being in direct contact with either the sheath or the duct wall.

A thin shield of heat insulating material surrounding the thermometer seems to be the best method of accomplishing this result. The drop in temperature through the shield will be negligible compared to the error obtained if the thermometer is in direct contact with the lead sheath or is enclosed in a metal shield which is in direct contact with the sheath.

Measurements taken in occupied ducts indicate that eight to ten feet in the duct is sufficiently far from the manhole to obtain an accurate reading. In ducts where the temperature was between 50 and 60 deg. cent., the difference in temperature between three and six feet was only three degrees, and from six to ten feet, only about one degree. The air in occupied ducts does not have the freedom of circulation it does in unoccupied ducts so the error due to manhole temperatures is minimized.

If the cable is suspended in air the greatest temperature which the air attains should be used with proper allowance for air movement if there is such. A very slight air movement has a very great effect in lowering the sheath temperature. This case is subject to even greater uncertainty than the case of cables installed in the earth so it is exceedingly difficult to obtain an accurate solution. The best method of attack is to assume a constant maximum for T_3 and compute T_1 from this base temperature.

Solution for a Steady Load Suddenly Applied

The solution for a steady load suddenly applied is as follows:

$$T_0 = \frac{2\,Q_0}{\pi} \int_0^\infty \delta_r \frac{1}{\omega} \sin \omega t \, d\omega . \qquad (11)$$

where

T_0 = The temperature of the conductor at any time t after the steady load is applied.

δ_r = The real part of the expression,

$$\delta = -j \left\{ \frac{\gamma}{\pi q_c r_1} \right\} \times \left\{ \frac{1}{\dfrac{q_i r_1 \tau}{S_i \sigma} - \dfrac{\eta}{S_i \sigma (S_i \sigma D + \eta q_i r_2 \psi)} - \dfrac{q_c r_1 \gamma}{S_c}} \right\}$$

where

$\eta = 2\,K$ for a single-conductor cable in still air,

$= 6\,K$ for a three-conductor cable in still air,

$= 2\,(K + D\,\lambda)$ for single-conductor cables in ducts, and

$= 6\,(K + D\,\lambda)$ for three-conductor cables in ducts.

where

$$\lambda = \frac{N S_e \sigma_e}{q_e D' \tau_e}.$$

N = Number of cables in the duct bank all assumed to be equal and loaded to the same amount.

The expression $\frac{1}{\omega} \delta_r \sin \omega t \, d\omega$ is much too complicated to integrate by formal means, but it may readily be done by graphical methods if some means such as a planimeter is available for obtaining the area under a curve. The value of the function is plotted for different values of t and integrated with respect to ω. Integrating graphically from 0 to ∞ offers no difficulty here since the function converges rapidly as the value of ω is increased. Usually integrating from 0 to 15 will give sufficiently accurate results with t less than three hours, and from 0 to 8 with t greater than three hours. It will seldom be necessary to go beyond six hours in computation of the temperature rise curve unless overload ratings for long periods are desired.

The above method is very powerful in the solution of all transient problems where the impedance function, δ, is very complicated as it is in this case. Where δ

is a simple function the method offers no advantage over other better known methods. The philosophy of the method is described very fully elsewhere[13,14].

Results of a calculation of the temperature rise of the conductor of a 400 m. c. m. shielded cable are shown in Fig. 7. The calculation is carried through in brief form in Appendix D.

A chart showing allowable overloads for various time limits and initial loads, computed from these curves, is shown in Fig. 8. As an example; suppose four

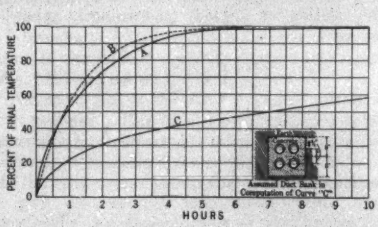

Fig. 7—Temperature Rise Curves; 400,000-Cm. 28-Kv., Shielded Cable

Curve A—Temperature rise; cable suspended in still air
Curve B—Same conditions as in "A", exponential rise assumed
Curve C—Temperature rise: four cables in ducts as shown, with all cables carrying the same load

cables each rated at 100 amperes continuous, are carrying a continuous load of 80 amperes each, a load of 134 amperes each may be applied for two hours without exceeding the temperature limit, with cables of the same characteristics as shown in Fig. 8.

Application to Standard Belted Cable

The well-known equation for obtaining the steady-state temperature rise of the conductor above the outside surface of the insulation is as follows[8,11]:

$$(T_1 - T_2) = \frac{Q_0 S_i G}{2\pi}. \qquad (12)$$

where G is the geometric factor.

For the shielded cable,[12]

$$G = l\,n\,\frac{r_2}{r_1}.$$

Assume that r_2 is the total distance from the center of the conductor to the lead sheath. Obtain G from Simons curves for the cable under consideration[11] and solve for an equivalent S_i. In average cables this equivalent S_i is about double the true value for the insulation under consideration. Use this value of S_i; in equations (8), and (9a).

This approximate solution has been found to give results, within 5 or 10 per cent of the true value as far as the variation in the temperature is concerned. The average temperature corresponding to the zero frequency component of the heat cycle is computed as usual by use of equation (12). Hence for ordinary load cycles the net error of this approximation cannot be over 5 per cent. A rigorous solution for the standard cable would involve much more labor which is useless until the constants of the cable can be obtained with greater accuracy.

Probable Magnitude of Errors

The change in resistance and dielectric losses with change of temperature for the ordinary load cycles met with in practise will not be over 5 or 6 per cent. If it is desired to take this into account a correction may be applied to the watts loss curve corresponding to the estimated temperature at that time.

The value of temperature rise to be assumed in making this correction can be taken as half the ultimate temperature rise produced by the average watts loss over an hourly period. This is based on the fact that in average cables the temperature at the conductor reaches about one-half its ultimate in an hour after a steady load is applied.[1] The error introduced by change in resistance will be less than half of one per cent if this correction is made.

For sector conductors the heat flow will be more concentrated around the regions of smaller radius. This will produce a greater temperature rise in these regions than elsewhere on the conductor. This effect is usually neglected in steady-state solutions, although an approximate solution for sector cables is found in the Bibliography No. 17. The error in neglecting the

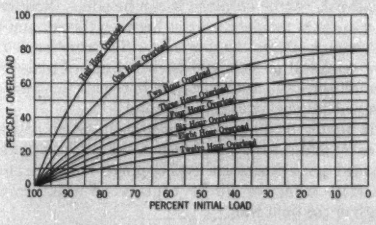

Fig. 8—Overloads, Based on 100 Per Cent Initial Load, which May be Applied without Exceeding the Temperature Rise for 100 Per Cent Initial Continuous Load

Four three-conductor 400,000-cm. 28-kv. shielded cables installed in a four-duct bank; all assumed to be loaded to the same value

effect of the sector shape will not be over 1 or 2 per cent.

Large conductors show considerable proximity effect when three are concentrated in the small area inside the cable sheath. The current tends to crowd to the inside of the cable when three-phase current is flowing. However, under the assumed condition of uniform current distribution the total variation in temperature through the conductor is only one degree for ordinary load cycles. The smallness of this variation is due to the high heat conductivity of the copper. This can be shown by the solution of equation (14) for the temperature at the center of the conductor. Hence if the proper

equivalent a-c. resistance is used for the cable in question, the error in temperature due to unsymmetrical current distribution will not be over 1 per cent. Of course, this assumes all the losses, excluding dielectric losses, are in the conductor. There are undoubtedly some losses in the sheath even in three-conductor cables, especially where large conductors are close to the sheath as they are in shielded cables. Not knowing definitely where all these losses are located, the conservative method would be to assume them all in the conductor.

Neglecting the temperature drop in the copper foil around the conductors in shielded cables introduces about 8 to 9 per cent error in the steady-state solution. A similar error will be introduced in the solution for variable load. This error will be minimized in the total temperature rise if the steady-state component of the temperature cycle is computed taking this drop in the foil into account[12]. This error is partly compensated for by neglecting the heat storage in the lead sheath.

The over-all error in the method is thus not over 5 per cent and is probably much less if the watts loss curve is corrected for temperature. It is doubtful if the value of the insulation heat conductivity is known within 5 per cent, plus or minus, so it is useless to expend the labor necessary in obtaining greater refinement.

Conclusions

Computations based on the theory presented here demonstrate that the temperature curve for any cable and load cycle may be readily calculated without excessive labor with accuracies well within the accuracy of the cable constants. The effects of changes in load cycle from day to day die out in a few hours so the theory can be applied to any day of the week with little effect from the previous day's cycle.

Assumptions are reduced to a minimum, the order of magnitude and direction of the errors are known and hence can be allowed for with any necessary accuracy.

Short-time overload ratings of any duration may be readily applied to correspond to any initial loading with confidence that the calculated temperature rise will not be exceeded.

Acknowledgments

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Appendix A

I—Derivation of Expressions for Heat Flow in the Conductor

The amount of heat entering an annular ring of thickness $d\,r$, and distance r from the center, per unit length per unit time is,

$$-\frac{2\,\pi}{S_c}\frac{\partial}{\partial r}(T\,r) = -\frac{2\,\pi}{S_c}\left(r\frac{\partial T}{\partial r} + T\right).$$

The amount of heat leaving this annular ring is,

$$-\frac{2\,\pi}{S_c}\frac{\partial}{\partial r}\{r\,(T - d\,T)\}\,.$$

where

$$d\,T = -\,d\,r\,\frac{\partial T}{\partial r}\,.$$

This is equal to,

$$-\frac{2\,\pi}{S_c}\left(r\frac{\partial T}{\partial r} + T + d\,r\frac{\partial}{\partial r}\left\{r\frac{\partial T}{\partial r}\right\}\right)$$

or

$$-\frac{2\,\pi}{S_c}\left(r\frac{\partial T}{\partial r} + T + r\,d\,r\frac{\partial^2 T}{\partial r^2} + d\,r\frac{\partial T}{\partial r}\right).$$

Hence the heat stored in the annular ring is,

$$\frac{2\,\pi}{S_c}\left(r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\right)d\,r + 2\,\pi\,r\,i^2\,p\,d\,r$$

$$= 2\,\pi\,r\,\rho_c\,C_c\frac{\partial T}{\partial t}\,d\,r\,.$$

where

i = Current density in the conductor,
p = Resistivity of the conductor.

Hence the expression for heat flow in the conductor is as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{S_c I^2 R}{\pi r_1^2} = C_c \rho_c S_c \frac{\partial T}{\partial t}. \qquad (1)$$

For sinusoidal variation of heat and temperature,

$$\bar{T} = T_m \epsilon^{j\omega t}.$$

Hence

$$\frac{\partial^2 \bar{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{T}}{\partial r} + \frac{\bar{Q}_0 S_c}{\pi r_1^2} = j q_c^2 \bar{T}. \qquad (13)$$

The solution of equation (13) when

$$\frac{Q_0 S_c}{\pi r_1^2} = 0 \text{ is,}$$

$$\bar{T} = \bar{A}\, \bar{I}_0 (q_c r\sqrt{j}) + \bar{B}\, \bar{K}_0 (q_c r \sqrt{j})^9$$

where

$\bar{I}_0 (q_c r\sqrt{j})$ is a Bessel function of first kind and zero order with semi-imaginary argument.

$\bar{K}_0(q_c r\sqrt{j})$ is a Bessel function of second kind and zero order with semi-imaginary argument, and

$\bar{A}$ and $\bar{B}$ are arbitrary constants to be determined by the boundary conditions.

When $r = 0$, $\bar{K}_0 (q_c r\sqrt{j}) = \infty$.

Hence $\bar{B}$ must be equal to zero.

The complete solution of equation (13) is then as follows:

$$\bar{T} = \bar{A}\, \bar{I}_0 (q_c r\sqrt{j}) + j \frac{\bar{Q}_0 S_c}{\pi q_c^2 r_1^2}.$$

Applying the boundary conditions and solving for $\bar{A}$,

$$\bar{T} = \left(\bar{T}_1 + j \frac{S_c \bar{Q}_0}{\pi q_c^2 r_1^2}\right)\left(\frac{\bar{I}_0(q_c r \sqrt{j})}{\bar{I}_0(q_c r_1\sqrt{j})}\right) - j \frac{S_c \bar{Q}_0}{\pi q_c^2 r_1^2} \qquad (14)$$

from which

$$\frac{\partial \bar{T}}{\partial r_1} = q_c\sqrt{j}\left(\bar{T}_1 + j \frac{S_c \bar{Q}_0}{\pi q_c^2 r_1^2}\right)\left(\frac{\bar{I}_0'(q_c r_1\sqrt{j})}{\bar{I}_0(q_c r_1\sqrt{j})}\right).$$

or

$$\bar{Q}_1 = -\frac{2\pi q_c r_1 \gamma}{S_c}\left(\bar{T}_1 + j \frac{\bar{Q}_0 S_c}{\pi q_c^2 r_1^2}\right). \qquad (3)$$

where

$$\gamma = \{\sqrt{j}\, \bar{I}_0' (q_c r_1 \sqrt{j})\} / \{\bar{I}_0 (q_c r_1\sqrt{j})\}.$$

II—Derivation of Expressions for Heat Flow in the Insulation

By a similar method to that shown in Part I, the Fourier equation for heat flow in a cylinder is obtained for the heat flow in the insulation, as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = C_i \rho_i S_i \frac{\partial T}{\partial t}. \qquad (2)$$

The solution for sinusoidal temperature and heat waves is,

$$\bar{T} = \bar{A}\, \bar{I}_0 (q_i r\sqrt{j}) + \bar{B}\, \bar{K}_0 (q_i r\sqrt{j}).$$

Applying the boundary conditions and solving for $\bar{A}$ and $\bar{B}$,

$$\bar{T} = \frac{(\bar{T}_1 k_2 - \bar{T}_2 k_1)\bar{I}_0 (q_i r\sqrt{j}) + (\bar{T}_2 i_1 - \bar{T}_1 i_2)\bar{K}_0 (q_i r\sqrt{j})}{i_2 k_1 - i_1 k_2}$$

where

$$i_1 = \bar{I}_0 (q_i r_2\sqrt{j}).$$
$$i_2 = \bar{I}_0 (q_i r_2\sqrt{j}).$$
$$k_1 = \bar{K}_0 (q_i r_1\sqrt{j}).$$
$$k_2 = \bar{K}_0 (q_i r_2\sqrt{j}).$$

Hence

$$\frac{\partial \bar{T}}{\partial r} = \frac{(\bar{T}_1 k_2 - \bar{T}_2 k_1)(q_i\sqrt{j})\, \bar{I}_0' (q_i r\sqrt{j}) + (\bar{T}_2 i_1 - \bar{T}_1 i_2)(q_i\sqrt{j})\, \bar{K}_0' (q_i r\sqrt{j})}{i_2 k_1 - i_1 k_2}$$

from which

$$\bar{Q}_1 = \frac{2\pi}{S_i}\left(\frac{q_i r_1 \bar{T}_1(k_2 i_1' - i_2 k_1') + q_i r_2 \bar{T}_2(i_1 k_1' - k_1 i_1')}{i_2 k_1 - i_1 k_2}\right)$$

$$\bar{Q}_2 = \frac{2\pi}{S_i}\left(\frac{q_i r_1 \bar{T}_1(k_2 i_2' - i_2 k_2') + q_i r_2 \bar{T}_2(i_1 k_2' - k_1 i_2')}{i_2 k_1 - i_1 k_2}\right)$$

where

$$i_1' = \sqrt{j}\, \bar{I}_0' (q_i r_1\sqrt{j}),$$
$$i_2' = \sqrt{j}\, \bar{I}_0' (q_i r_2\sqrt{j}),$$
$$k_1' = \sqrt{j}\, \bar{K}_0' (q_i r_1\sqrt{j}),$$
$$k_2' = \sqrt{j}\, \bar{K}_0' (q_i r_2\sqrt{j}).$$

It can be shown that,[3]

$$i_1 k_1' - k_1 i_1' = -\frac{1}{q_i r_1} \text{ and,}$$

$$k_2 i_2' - i_2 k_2' = \frac{1}{q_i r_2}.$$

Hence the following two equations represent completely the heat flow and temperature in the insulation:

$$\bar{Q}_1 = \frac{2\pi}{S_i \sigma}(q_i r_1 \tau \bar{T}_1 - \bar{T}_2), \qquad (5)$$

$$\bar{Q}_2 = \frac{2\pi}{S_i \sigma}(\bar{T}_1 - q_i r_2 \psi \bar{T}_2), \qquad (6)$$

where

$$\sigma = i_2 k_1 - i_1 k_2,$$
$$\psi = i_2' k_1 - i_1 k_2',$$
$$\tau = i_1' k_2 - i_2 k_1'.$$

The following notations are equivalent for the Bessel functions used in the above equations:

$$\bar{I}_0 (q r\sqrt{j}) = Ber\, qr + j\, Bei\, qr.$$
$$\sqrt{j}\, \bar{I}' (q r\sqrt{j}) = Ber'\, qr + j\, Bei'\, qr.$$
$$\bar{K}_0 (q r\sqrt{j}) = Ker\, qr + j\, Kei\, qr.$$
$$\sqrt{j}\, \bar{K}_0' (q r\sqrt{j}) = Ker'\, qr + j\, Kei'\, qr.$$

Appendix B

Derivation of an Expression for Temperature Rise with Suddenly Applied Load

For a single-conductor cable suspended in still air at constant temperature, equations (3), (5), and (9) may be combined and an explicit expression obtained for the temperature rise of the conductor above the sheath surface, as follows:

$$\bar{T}_1 = -j\left(\frac{\bar{Q}_0\,\gamma}{\pi\,q_c\,r_1}\right)\times$$

$$\left(\frac{1}{\dfrac{q_i\,r_1\,\tau}{S_i\,\sigma}-\dfrac{2\,K}{S_i\,\sigma(S_i\,\sigma\,D+2\,q_i\,r_2\,\psi\,K)}-\dfrac{q_c\,r_1\,\gamma}{S_c}}\right). \quad (15)$$

For the three-conductor cable in air, equations (3), (5), and (9a) are combined to form the expression,

$$\bar{T}_1 = -j\left(\frac{\bar{Q}_0\,\gamma}{\pi\,q\,r_1}\right)\times$$

$$\left(\frac{1}{\dfrac{q_i\,r_1\,\tau}{S_i\,\sigma}-\dfrac{6\,K}{S_i\,\sigma(S_i\,\sigma\,D+6\,q_i\,r_2\,\psi\,K)}-\dfrac{q_c\,r_1\,\gamma}{S_c}}\right). \quad (16)$$

For single-conductor cables in ducts,

$$\bar{T}_3 = \frac{N\,\bar{Q}_2\,S_e\,\sigma_e}{\pi\,q_e\,D'\,\tau_e}. \quad (10)$$

Substituting for $\bar{T}_3$ in equation (7), the following expression is obtained:

$$\bar{Q}_2 = \frac{\pi\,D\,\bar{T}_3}{(K+D\,\lambda)},$$

where

$$\lambda = \frac{N\,S_e\,\sigma_e}{q_e\,D'\,\tau_e}.$$

Hence for single-conductor cables in ducts,

$$\bar{T}_1 = -j\left(\frac{\bar{Q}_0\,\gamma}{\pi\,q_c\,r_1}\right)\times$$

$$\left(\frac{1}{\dfrac{q_i\,r_1\,\tau}{S_i\,\sigma}-\dfrac{2\,(K+D\,\lambda)}{S_i\,\sigma\,\{S_i\,\sigma\,D+2\,q_i\,r_2\,\psi\,(K+D\,\lambda)\}}-\dfrac{q_e\,r_1\,\gamma}{S_c}}\right) \quad (18)$$

Similarly for three-conductor cables in ducts,

$$\bar{T}_1 = -j\left(\frac{\bar{Q}_0\,\gamma}{\pi\,q_c\,r_1}\right)\times$$

$$\left(\frac{1}{\dfrac{q_i\,r_1\,\tau}{S_i\,\sigma}-\dfrac{6\,(K+D\,\lambda)}{S_i\,\sigma\,\{S_i\,\sigma\,D+6\,q_i\,r_2\,\psi\,(K+D\,\lambda)\}}-\dfrac{q_e\,r_1\,\gamma}{S_c}}\right) \quad (19)$$

It can be shown that.[13,14]

$$T_0 = \frac{2\,Q_0}{\pi}\int_0^\infty \frac{1}{\omega}\,\delta_r \sin\omega\,t\;d\omega.$$

where δ is the impedance function corresponding to sinusoidal variation of heat flow from any frequency from 0 to ∞. Applying this formula to equations (15) to (18), a solution for temperature can be obtained when a rectangular wave of heat of magnitude Q_0 is suddenly applied to each conductor.

Appendix C

Approximate Expressions for Some Heat-Flow Functions

The following expressions are well within 1 per cent of the exact ones when the limiting conditions specified are fulfilled. They can be used where the functions required are outside the range of the charts given in Figs. 2 to 4.

When $q\,r$ is less than 0.3,

$$\bar{I}_0\,q\,r\sqrt{j} = 1-\frac{q^4\,r^4}{64}+j\,\frac{q^2\,r^2}{4}.$$

$$\sqrt{j}\,\bar{I}_0'\,q\,r\sqrt{j} = -\frac{q^3\,r^3}{16}+j\,\frac{q\,r}{2}$$

$$\bar{K}_0\,q\,r\sqrt{j} = (0.116-l\,n\,q\,r)\left(1-\frac{q^4\,r^4}{64}\right)+\frac{\pi\,q^2\,r^2}{16}-\frac{3\,q^4\,r^4}{128}+j\left((0.116-l\,n\,q\,r)\left(\frac{q^2\,r^2}{4}\right)-\frac{\pi}{4}\left(1-\frac{q^4\,r^4}{64}\right)+\frac{q^2\,r^2}{4}\right).$$

$$\sqrt{j}\,\bar{K}_0'\,q\,r\sqrt{j} = -(0.116-l\,n\,q\,r)\left(\frac{q^3\,r^3}{16}\right)-\left(1-\frac{q^4\,r^4}{64}\right)/q\,r+\frac{\pi\,q\,r}{8}-\frac{3\,q^3\,r^3}{32}+j\left((0.116-l\,n\,q\,r)\left(\frac{q\,r}{2}\right)-\frac{q\,r}{4}-\frac{\pi\,q^3\,r^3}{64}+\frac{q\,r}{2}\right).$$

$$\sigma = l\,n\,\frac{r_2}{r_1}+j\left(\frac{q^2\,(r_2^2+r_1^2)}{4}\,l\,n\,\frac{r_2}{r_1}-\frac{q^2(r_2^2-r_1^2)}{4}\right)$$

$$\tau = \frac{1}{q\,r_1}+j\left(\frac{q\,r_1}{2}\,l\,n\,\frac{r_2}{r_1}+\frac{q^2\,r_2^2}{4\,q\,r_1}-\frac{q\,r_1}{4}\right).$$

$$\psi = \frac{1}{q\,r_2}+j\left(\frac{q\,r_2}{2}\,l\,n\,\frac{r_1}{r_2}+\frac{q^2\,r_1^2}{4\,q\,r_2}-\frac{q\,r_2}{4}\right).$$

$$\gamma = j\,q\,r_1/2$$

When $q\,r$ is greater than 2,

$$\bar{I}_0\,q\,r\sqrt{j} = \frac{\epsilon^{\frac{qr}{\sqrt{2}}}}{2\sqrt{\pi\,q\,r}}\Bigg(\left(\sqrt{2}+\frac{1}{8\,q\,r}\right)\cos\left(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\right)$$

$$+\frac{1}{8\,q\,r}\sin\left(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\right)+j\Bigg(-\frac{1}{8\,q\,r}\cos\left(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\right)$$

$$+\left(\sqrt{2}+\frac{1}{8\,q\,r}\right)\sin\left(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\right)\Bigg)\Bigg).$$

$$\sqrt{j}\,\bar{I}_0'\,q\,r\sqrt{j} = \frac{\epsilon^{\frac{qr}{\sqrt{2}}}}{2\sqrt{\pi\,q\,r}}\Bigg(\left(\sqrt{2}-\frac{3}{8\,q\,r}\right)\cos\left(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\right)$$

$$-\frac{3}{8\,q\,r}\sin\left(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\right)+j\Bigg(\frac{3}{8\,q\,r}\cos\left(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\right)$$

$$+\left(\sqrt{2}-\frac{3}{8\,q\,r}\right)\sin\left(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\right)\Bigg)\Bigg).$$

$$\bar{K}_0 q r\sqrt{j} = \frac{1}{2}\sqrt{\frac{\pi}{q\,r}}\,\epsilon^{-\frac{qr}{\sqrt{2}}}\Bigg(\left(\sqrt{2}-\frac{1}{8\,q\,r}\right)\cos\left(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\right)$$

$$+\frac{1}{8\,q\,r}\sin\left(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\right)+j\Bigg(\frac{1}{8\,q\,r}\cos\left(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\right)$$

$$-\left(\sqrt{2}-\frac{1}{8\,q\,r}\right)\sin\left(\frac{q\,r}{\sqrt{2}}+\frac{\pi}{8}\right)\Bigg)\Bigg).$$

$$\sqrt{j}\,\bar{K}_0'\,q\,r\sqrt{j}$$

$$=\frac{1}{2}\sqrt{\frac{\pi}{q\,r}}\,\epsilon^{-\frac{qr}{\sqrt{2}}}\Bigg(-\left(\sqrt{2}+\frac{3}{8\,q\,r}\right)\cos\left(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\right)$$

$$+\frac{3}{8\,q\,r}\sin\left(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\right)+j\Bigg(\frac{3}{8\,q\,r}\cos\left(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\right)$$

$$+\left(\sqrt{2}+\frac{3}{8\,q\,r}\right)\sin\left(\frac{q\,r}{\sqrt{2}}-\frac{\pi}{8}\right)\Bigg)\Bigg).$$

When $\frac{r_2}{r_1}$ is greater than 5,

$$\tau/\sigma = \sqrt{j}\,\bar{K}_0'\,q\,r_1\sqrt{j}\ /\ \bar{K}_0\,q\,r_1\sqrt{j}.$$

$$\psi/\sigma = \sqrt{j}\,\bar{I}_0'\,q\,r_1\sqrt{j}\ /\ \bar{I}_0\,q\,r_1\sqrt{j}.$$

Appendix D

Sample Calculations

I. Calculation of the temperature rise from sheath surface to conductor for a three-conductor 400,000 cir. mils. shielded cable.

The constants of the cable are as follows:

D = 3.1 in. (7.88 cm.).
r_1 = 0.363 in. (0.923 cm.).
r_2 = 0.707 in. (1.797 cm.).
S_e = 0.284 deg. cent. per watt per cm.^{2}
S_i = 650 deg. cent. per watt per cm.^{2}
K = 1,100 deg. cent. per watt per cm.^{2}
ρ_e = 6.9 g. per cm.^{3} (Equivalent for stranded cable).
ρ_i = 1.16 g. per cm.^{3}
C_e = 0.000107 watt-hr. per g.
C_i = 0.000493 watt-hr. per g.
ω = 0.262 radians per hr. for 24 hr. period.

Hence for the fundamental component (24 hr. period),

$$q_i\,r_1 = 0.923\sqrt{1.16\times0.000493\times650\times0.262} = 0.288.$$

$$q_i(r_2-r_1) = 0.874\sqrt{1.16\times0.000493\times650\times0.262} = 0.272.$$

$$q_e\,r_1 = 0.923\sqrt{6.9\times0.000107\times0.284\times0.262} = 0.00685.$$

$$q_e^2\,r_1^2/S_e = 0.00017.$$

Entering the charts shown in Figs. 2 to 4, the following values are obtained for the heat flow constants:

$$\sigma = 0.668 + j\,0.0085 = 0.668\underline{/0.7^\circ}.$$
$$\tau = 3.47 + j\,0.104 = 3.47\underline{/1.7^\circ}.$$
$$\psi = 1.79 + j\,0.085 = 1.79\underline{/2.7^\circ}.$$

Substituting in equation (8) and letting $\bar{Q}_0$ equal 1.0 watt per conductor per foot of cable,

$$0.01043 - j\,0.00017\,\bar{T}_1 = 0.0046\underline{/-0.7^\circ}(1.00\,\bar{T}_1\underline{/1.7^\circ} - \bar{T}_2).$$

Substituting in equation (9a) and letting $\bar{T}_3 = 0$, since the base temperature is to be considered constant,

$$0.00239\,\bar{T}_2 = 0.0046\underline{/-0.7^\circ}\,(\bar{T}_1 - 1.003\,\bar{T}_2\underline{/2.7^\circ}.$$

Solving these equations for $\bar{T}_1$ it is seen that,

$\bar{T}_1/\bar{Q}_0 = 6.43\underline{/-13.9^\circ}$ deg. cent. per watt per conductor per foot of cable.

For the constant or zero frequency component, the following expression is used.[12]

$$Q_0/(T_1 - T_2) =$$

$$61\sqrt{t_1/S_e\,S_i\,r_2}\,G\tanh\phi\sqrt{S_e\,r_2/S_i t_1\,G} + \frac{61\,(\pi-\phi)}{S_i\,G}$$

thermal mhos per foot of cable, where

Q_0 = Watts per conductor per foot of cable.
ϕ = π − Arc of contact of lead sheath and copper shield, expressed in radians, and
t_1 = Thickness of copper shield.

Substituting in this formula,

$(T_1 - T_2)/Q_0$ = 2.45 deg. cent. per watt per conductor per foot.

For the temperature rise in the sheath, under steady load,

$$(T_2 - T_3)/Q_0 = \frac{1100\times3}{3.14\times3.1\times2.54^2\times12} = 4.37 \text{ deg. cent. per watt per conductor per foot.}$$

Hence when T_3 = 0,

$T_1/Q_0 = 6.82$ deg. cent. per watt per conductor per foot.

Table I gives the heat flow constants and the value $\overline{T}_1/\overline{Q}_0$ with the time lag of $\overline{T}_1$ behind $\overline{Q}_0$ for all harmonics up to and including the sixth.

TABLE I

Harmonic	σ	τ	ψ	$q^2_c\, r_1^5/S_c$	$\overline{T}_1/\overline{Q}_0$	Time lag. (hours)
0					6.82	0
1	0.668/0.7°	3.47/ 1.7°	1.79/ 2.7°	0.00017	6.43	0.93
2	0.668/1.4°	2.45/ 3.5°	1.27/ 5.5°	0.00031	5.82	0.87
3	0.668/2.1°	2.01/ 5.2°	1.04/ 8.2°	0.00051	5.17	0.80
4	0.668/2.9°	1.75/ 6.9°	0.91/11.0°	0.00068	4.55	0.74
5	0.668/3.6°	1.56/ 8.6°	0.81/13.8°	0.00085	4.05	0.66
6	0.668/4.4°	1.42/10.4°	0.75/16.6°	0.00002	3.58	0.60

TABLE II

ω	σ	τ	ψ	γ	δ/π
0.226	$0.668 + j\,0.014$	$2.50 + j\,0.145$	$1.28 + j\,0.120$	$j\,0.0032$	63.4 \18.5°
0.900	$0.667 + j\,0.064$	$1.24 + j\,0.288$	$0.63 + j\,0.234$	$j\,0.0064$	38.7 \45.7°
2.023	$0.660 + j\,0.143$	$0.790 + j\,0.435$	$0.375 + j\,0.342$	$j\,0.0090$	21.2 \58.9°
3.61	$0.640 + j\,0.257$	$0.520 + j\,0.565$	$0.220 + j\,0.452$	$j\,0.0127$	14.0 \58.4°
5.67	$0.595 + j\,0.395$	$0.298 + j\,0.705$	$0.080 + j\,0.558$	$j\,0.0159$	10.6 \57.6°
8.12	$0.520 + j\,0.560$	$0.062 + j\,0.815$	$-0.080 + j\,0.635$	$j\,0.0190$	8.48 \57.3°
11.03	$0.380 + j\,0.730$	$-0.195 + j\,0.895$	$-0.295 + j\,0.695$	$j\,0.0222$	6.42 \45.3°

TABLE III

ω	$\sin \omega t$			$\delta_r/\pi\,\omega$	$\frac{Q_0\,\delta_r}{\pi\,\omega} \sin \omega t$		
	$t = 0.4$	$t = 0.8$	$t = 2.0$		$t = 0.4$	$t = 0.8$	$t = 2.0$
0.226	0.0903	0.1800	0.4368	6540	0.788	1.570	3.810
0.900	0.3518	0.6587	0.9744	739	0.346	0.649	0.960
2.023	0.7242	0.9987	−0.7880	133	0.128	0.177	−0.140
3.61	0.9921	0.2453	0.5878	50.1	0.066	0.016	0.039
5.67	0.7660	−0.9848	−0.9397	24.6	0.025	−0.032	−0.031
8.12	−0.1045	0.2079		14.3	−0.002	0.0040	
11.03	−0.9537	−0.9998		10.1	−0.013	0.0077	

Substituting the values given in Table I, for a heat flow represented by the following equation:

$$\begin{aligned} Q_0 = 1.47 &+ 0.300\,(\sin\ \omega t - 3.05 \text{ hr.}) \\ &+ 0.092\,(\sin 2\,\omega t - 0 \text{ hr.}) \\ &+ 0.148\,(\sin 3\,\omega t - 1.39 \text{ hr.}) \\ &+ 0.037\,(\sin 4\,\omega t - 0.75 \text{ hr.}) \\ &+ 0.100\,(\sin 5\,\omega t - 0.60 \text{ hr.}) \\ &+ 0.060\,(\sin 6\,\omega t - 1.50 \text{ hr.}) \end{aligned}$$

the temperature wave is represented as follows:

$$\begin{aligned} T_1 = 10.0 &+ 1.93\,(\sin\ \omega t - 3.98 \text{ hr.}) \\ &+ 0.54\,(\sin 2\,\omega t - 0.87 \text{ hr.}) \\ &+ 0.77\,(\sin 3\,\omega t - 2.19 \text{ hr.}) \\ &+ 0.17\,(\sin 4\,\omega t - 0.01 \text{ hr.}) \\ &+ 0.40\,(\sin 5\,\omega t - 1.26 \text{ hr.}) \\ &+ 0.22\,(\sin 6\,\omega t + 0.90 \text{ hr.}) \end{aligned}$$

Curves of the heat and temperature waves represented by the above two equations are shown in Fig. 6.

II. Calculation of the temperature rise from sheath surface to conductor for a three-conductor 400,000 cir. mils shielded cable, suspended in still air, when a steady load is suddenly applied.

The constants of the cable are the same as those in part I of this Appendix except that S_i is 750 deg. cent. per watt per cm.³ instead of 650.

TABLE IV

T_0/Q_0	$t = 0.2$	$t = 0.4$	$t = 0.8$	$t = 2.0$	$t = 4.0$	$t = \infty$
Deg. cent. per watt	1.44	2.24	3.25	5.15	6.57	7.00
Per cent of final temp.	20.6	32.0	46.5	73.5	93.8	100

The calculation of δ/π is shown in tabular form in Table II.

The calculation of $\frac{Q_0\,\delta_r}{\pi\,\omega} \sin \omega t$ for $t = 0.4$ hr., 0.8 hr., and 2.0 hr. is shown in tabular form in Table III.

The calculation of T_0/Q_0 for $t = 0.2$ hr., 0.4 hr., 2.0 hr., and 4.0 hr. is shown in tabular form in Table IV.

The values in Table IV were obtained by integrating $\int_0^\infty \frac{2\,Q_0\,\delta_r}{\pi\,\omega} \sin \omega t\, d\omega$ for the different values of t. A curve of temperature vs. time, plotted from the above Table IV, is shown in Fig. 7, curve A.

Discussion

F. H. Buller: It is interesting to note from Fig. 7 how much the percentage temperature rise in the ducts is slowed down by the increased heat storage capacity. Actually, however, the rate of rise in degrees is faster for the cables in ducts than in air, as indicated by Fig. 19 of the report of the British Electrical and Allied Industries published in the *I. E. E. Journal* for May 1923.

On page 987 of his paper, Mr. Church states that the heat storage capacity of the lead sheath has been neglected. A simple calculation shows that in a large cable the thermal capacity of the sheath is often equal to 80 per cent of that of the conductor and equal to 25 per cent of that of the cable as a whole; and if the cable is furnished with a reinforced sheath or a heavy armor such as used for buried cables or for exposed bus connection cables, the effective thermal capacity of the "sheath" may be an even larger percentage. One cannot help but feel that Mr. Church's equations should be investigated with a view to discovering what the effect of this omission might be of his results; and that if the effect is considerable, the equation should be revised to include the factor which has been omitted.

Mr. Church gives an integral equation (11) for the thermal transient which will occur if a load is suddenly applied to a cable. He points out that in most cases this integral is difficult to evaluate except by mechanical means. He further mentions that when the impedance function is not too complicated, other methods are available for obtaining the value of the transient. In this connection it might be mentioned that at least two methods are known which give an explicit solution in terms of Bessel functions and exponentials for the case of a steady load suddenly applied to a cable suspended in still air. One of these methods involves the use of the Heaviside operational calculus and has been treated by Mr. Shanklin and the present writer in an article which is shortly to be published in the *General Electric Review*. Briefly, this method consists of setting up an operational equation from differential equations similar to Mr. Church's equation (2) for the boundary conditions which obtain on the cable conductor and sheath and of solving this operational equation by the use of the Heaviside expansion theorem. This method gives a solution which is mathematically more explicit and complete than Mr. Church's equation (11).

This, of course, does not mean that Mr. Church's method is without value; for the more complicated case of a cable in ducts and possibly also of a cable buried in earth, the integral equation offers a practical means of solution which will probably be very useful. Mr. Church deserves credit for drawing attention to this method.

The Heaviside operational equation can also be used to give results analogous to those obtained by Mr. Church for cyclical loads by the use of a very simple substitution. The details of this substitution can be found in standard texts on the operational calculus, notably that written by Dr. Berg. Here, again, the present writer does not desire to detract from the value of Mr. Church's work but merely to add an alternative procedure which may be useful in future investigations along these lines.

E. A. Church: In Fig. 7 of the paper, the cable and earth constants were such that the total final temperature difference between the region in the earth at constant temperature and the conductors, and the final temperature difference between the air adjacent to the sheath and the conductors were in the ratio of 2.74 to one. Assuming a total temperature rise of 75 deg. cent. for the cables in ducts, the rise at the end of two hours would be 23 deg. cent. and for a cable suspended in air, *carrying the same load*, the rise would be 20 deg. cent. This approximates the results given in the Fig. 19 mentioned in Mr. Buller's discussion. If, however, the loading on the cables is adjusted to give the same ultimate temperature rise when suspended in air, the temperature will rise much faster than when the cables are in ducts embedded in the earth.

Although the heat storage capacity of the lead sheath may be in some cases 25 per cent of the total storage capacity of the cable, the actual heat storage in the lead sheath rarely exceeds 10 or 15 per cent on account of the relatively lower temperature of the sheath in comparison with the conductor and insulation adjacent to it. The effect of this is small and as stated on page 987 is partly compensated for in three-conductor cables by neglecting the temperature rise in the copper foil around each conductor, the calculation of which would be very difficult for variable loading.

This heat storage may easily be taken into account if it is felt that, as in the case of armored cables mentioned by Mr. Buller, it is of appreciable amounts. Taking heat storage in the sheath into account introduces two more equations of the form of equations (5) and (6) into the set of simultaneous equations necessary for the solution of the temperature of the cable.

To show that the effect of the heat storage in the sheath is quite small in ordinary cases the following calculation was made, for the same size and type of cable as the one described in Appendix D of the paper:

For the lead sheath:

$$(r_2 - r_1) = 0.125 \text{ in. } (0.317 \text{ cm.})$$
$$r_2 = 1.55 \text{ in. } (3.94 \text{ cm.})$$
$$S = 0.298 \text{ deg. cent. per watt per cm.}^3$$
$$\rho = 11.4 \text{ g. per cm.}^3$$
$$C = 0.0000348 \text{ watt-hr. per g.}$$

Hence by the same method as given in Appendix D,

$$q\,r_1 = 0.0201, \text{ and}$$
$$q\,r_2 = 0.0219, \text{ for the fundamental component of the heat-flow cycle.}$$

Referring to the approximate formulas given in Appendix C it can be seen that with $q\,r$ of the order of magnitude obtained above, very little error will be introduced even in the third or fourth harmonics by assuming that:

$$\sigma = l\,n\,r_2/r_1.$$
$$\tau = l/q\,r_1.$$
$$\psi = l/q\,r_2).$$

These values when substituted in equations (5) and (6) give for the sheath:

$$\bar{Q}_1 = \bar{Q}_2 = \frac{2\,\pi}{S\,l\,n\,r_2/r_1}\,(\bar{T}_1 - \bar{T}_2)$$

which is equivalent to saying that the heat storage in the sheath is negligible.

Proximity Effect in Cable Sheaths

BY HERBERT BRISTOL DWIGHT*

Fellow, A. I. E. E.

Synopsis.—*The electrical characteristics of the lead sheaths of underground cables can be calculated by the methods used for computing proximity effect. Theoretically, any group of round wires and thin tubes can be calculated. Cases which are taken up in this paper are (1) a thin tube and an enclosed wire, (2) the current density, voltage drop, and power loss in the sheath of a three-conductor cable, (3) two single-conductor cables with the sheaths open-circuited, (4) two single-conductor cables with the sheaths short-circuited, and (5) three single-conductor cables in a plane.*

* * * * *

PROXIMITY effect, that is, unequal distribution of alternating current over the cross-section of a conductor caused by current in another conductor, occurs whenever parallel conductors carry alternating current. When the parallel conductors are non-magnetic round wires or infinitely thin round tubes, of any grouping whatever, the effective resistance and reactance can, theoretically, always be calculated. Such a calculation can be carried out for either the loss or the voltage drop. The intensity of the electric field or the magnetic field at any point, due to the irregular current distribution, and the exact amount of shielding, can be computed.

In this paper are given formulas, with numerical examples, for some problems connected with sheaths of underground cables. These are supplementary to the formulas previously published by the writer in the *Electric Journal*.[1]

The problem of the distribution of alternating current in a round wire caused by the proximity of a current in a small filament parallel to the wire, was proposed and solved by Charles Manneback.[2] It seems that this particular problem had not previously been proposed or attacked, but it has proved to be a most useful problem, as it has led to the solution of many others. The result of the above problem is a short algebraical expression for current density, which has the desired characteristic that it gives equal voltage drops at all parts of the conductor section. The current adds up to zero over the section, that is, it is merely a circulating current and does not affect an ammeter connected in series with the conductor. It is not affected by whatever ammeter current there may be and it is not affected by the circulating current caused by another outside filament. These currents are simply added together, as they separately produce equal voltage drops at all parts of the section. The effect of an outside round conductor can be found by integrating the effect of its filaments.

The calculation of the circulating current in an infinitely thin tube caused by a current I in an outside parallel filament has been published by the writer.[3]

The result is

$$i_{(\theta)} = -\frac{I}{\pi c t} \sum_{n=1}^{\infty} \frac{c^n}{s^n} \frac{l^4 + j l^2 n}{l^4 + n^2} \cos n\theta \qquad (1)$$

where

c = mean radius of the tube, in cm.,

t = thickness of the tube, in cm. (considered very small compared with c),

s = distance from the filament to the center of the tube, in cm.,

$$l^2 = \omega \frac{2\pi c t}{\rho_s} \qquad (2)$$

$\omega = 2\pi f$

f = frequency in cycles per second,

ρ_s = resistivity of the tube, in abohms per cm. cube.

This is the same as the last part of (**14**), reference 3, where it is to be noted that the current in the outside filament is $-I$.

A formula for the current density in a very thin tube, caused by a current I in a filament inside the tube, will now be given. If, in reference 3, the filament Y is inside the tube, then instead of equation (**4**) of that article we must use

$$\log n \frac{D}{c} = -\sum_{n=1}^{\infty} \frac{s^n}{n c^n} \cos n\theta \qquad (3)$$

where c is the radius of the tube, in cm. and θ is the angle indicated in Fig. 1, which is to be used in place of Fig. 1 of reference 3.

The calculation will be the same as the derivation of (**14**) in reference 3, except that in every case $\frac{a}{s}$ and $\frac{a^n}{s^n}$ become $\frac{s}{c}$ and $\frac{s^n}{c^n}$. The letter c is used in this paper for the mean radius of the tube or sheath. The current density in the tube due to I at Y, Fig. 1, is

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1. "Losses in Grounded Sheaths of Single-Conductor Cables," by H. B. Dwight, *Electric Journal*, February, 1924, p. 62.

2. "An Integral Equation for Skin Effect in Parallel Conductors," *Journal of Mathematics and Physics*, April, 1922, and *Research Bulletin No. 30*, Massachusetts Institute of Technology.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

3. *Proximity Effect in Wires and Thin Tubes*, by H. B. Dwight, TRANS. A. I. E. E., 1923, p. 850.

$$i_{(\theta)} = -\frac{I}{\pi c t} \sum_{n=1}^{\infty} \frac{s^n}{c^n} \frac{l^4 + j l^2 n}{l^4 + n^2} \cos n \theta \qquad (4)$$

If three-phase currents are carried by conductors Y_1, Y_2, and Y_3 as in Fig. 2, and the sheath is open-circuited,

$$i_{(\theta)} = -\frac{I}{\pi c t} \sum_{n=1}^{\infty} \frac{s^n}{c^n} \left(\frac{l^4 + j l^2 n}{l^4 + n^2} \right)$$

$$\left[\cos n \theta \left(1 - \cos \frac{2 n \pi}{3} \right) + j \sqrt{3} \sin n \theta \sin \frac{2 n \pi}{3} \right] \qquad (5)$$

The reactive drop in the filament at X due to the element of current $i_{(\theta)}\, c\, t\, d\, \theta$ is

$$j\, \omega\, 2\, c\, t\, i_{(\theta)} \log_n \frac{p}{g}\, d\, \theta$$

taking account of flux up to a certain large distance p.

Integrating this expression around the circle and

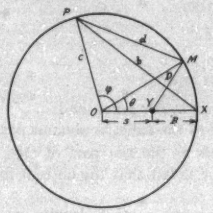

FIG. 1—THIN TUBE AND ENCLOSED FILAMENT

adding the drops due to the three power currents and the resistance drop, the drop in the sheath is found to be

$$j\, \omega\, I \left[\log_n \frac{\left(c + \frac{s}{2} \right)^2 + \frac{3 s^2}{4}}{(c - s)^2} \right.$$

$$+ \sum_{n=1}^{\infty} \frac{2 s^n}{c^n} \left(\frac{-n + j l^2}{n^2 + l^4} \right) \left(1 - \cos \frac{2 n \pi}{3} \right)$$

$$\left. - \sum_{n=1}^{\infty} \frac{2 s^n}{n c^n} \left(\frac{l^4 + j l^2 n}{l^4 + n^2} \right) \left(1 - \cos \frac{2 n \pi}{3} \right) \right] \text{abvolts per cm.} \qquad (6)$$

where I is in abamperes. This formula gives a result equal to zero in a numerical problem. The term $\log_n$ denotes natural logarithm.

The loss in the open-circuited sheath of a three-phase cable is obtained from (5) and is

$$\frac{\text{Loss in open-circuited lead sheath}}{\text{Loss in 3 copper conductors at 0 frequency}}$$

$$= \frac{2 a^2}{c t} \frac{\rho_s}{\rho_c} \sum_{n=1}^{\infty} \frac{s^{2n}}{c^{2n}} \left(\frac{l^4}{l^4 + n^2} \right) \sin^2 \left(\frac{n \pi}{3} \right) \qquad (7)$$

where t is the actual thickness of the lead sheath in cm., ρ_s is the resistivity of lead and ρ_c that of copper, at the temperatures considered, and where a is the radius in cm. of a solid copper wire of the same resistance as the stranded copper conductor. The current is assumed to be uniformly distributed over the sections of the copper conductors. Note that t has here a different definition from that in reference 1. Expression (7) was published by Dr. F. W. Carter, in the *Proceedings of the Cambridge Philos. Soc.*, Vol. XXIII, 1927, p. 905, equation (**19**). It may be mentioned that equation (**16**) of that paper, for the loss in an open-circuited thin tube due to an external filamentary current, is the same result as was published by the writer in paragraph I, Part II, reference 3, in 1923.

Example I. Three-conductor, three-phase cable, 500,000 cir. mils. per conductor, sheath open-circuited, $c = 4.5$ cm., $t = 0.40$ cm., $a = 0.898$ cm., $\rho_s = 25{,}000$, $\rho_c = 2{,}100$ at 75 deg. cent., $f = 60$ cycles, $s = 2.0$ cm., $l^2 = 0.170$.

$$\frac{\text{Loss in open-circuited lead sheath}}{\text{Loss in 3 copper conductors at 0 frequency}}$$

$$= 0.05 = 5 \text{ per cent.}$$

Example II. Three 1,000,000-cir. mil unsheathed single-conductor cables in a 3/16 in. lead sheath or pipe. $c = 4.55$ cm., $t = 0.48$ cm., $a = 1.27$ cm., $\rho_s = 25{,}000$, $\rho_c = 2{,}100$, $f = 60$, $s = 2.31$ cm., $l^2 = 0.207$.

$$\frac{\text{Loss in open-circuited lead sheath}}{\text{Loss in 3 copper conductors at 0 frequency}}$$

$$= 0.15 = 15 \text{ per cent.}$$

Note that lead-sheathed, three-conductor cables as large as in this example have been manufactured.

If the pipe were made of iron instead of lead, the loss would be expected to be considerably greater. See Question No. 2913, *Electric Journal*, April, 1929, page 184, concerning three 1,250,000-cir. mil conductors in an iron pipe, in which the loss in the pipe was neglected.

The e. m. f. induced in the open-circuited sheaths in a single-phase circuit of single-conductor underground cables, assuming that the sheaths are infinitely thin, though of the same total resistance as the actual lead sheaths, and assuming that the main current is uniformly distributed over the cross-sections of the round conductors, is given by the following formula.

Let

$$A_n = \frac{2 c^n}{s^n} \frac{(l^4 + j l^2 n)}{(l^4 + n^2)}$$

$$B_n = \frac{A_n}{2} \sum_{k=1}^{\infty} \frac{c^k}{s^k} A_k \frac{(n + k - 1)!}{(n - 1)!\, k!}$$

$$C_n = \frac{A_n}{2} \sum_{k=1}^{\infty} \frac{c^k}{s^k} B_k \frac{(n+k-1)!}{(n-1)!\,k!}$$

. . . . . .

where n and k denote integers.

$$N_n = A_n + B_n + C_n + \ldots$$

Voltage drop in abvolts per cm. of sheath, for a power current I,

$$= j\,\omega\,2\,I \operatorname{logn} \frac{s}{c} + j\,\omega\,I \operatorname{logn} \frac{s^2+c^2}{s^2}$$

$$- j\,\omega\,I \left(\frac{N_2}{2} - \frac{N_4}{4} + \frac{N_6}{6} - \ldots \right)$$

$$- j\,\omega\,I \left(\frac{c\,N_1}{H} \cos\alpha + \frac{c^2\,N_2}{2\,H^2} \cos 2\,\alpha + \ldots \right.$$

$$\left. + \frac{c^n\,N_n}{n\,H^n} \cos n\,\alpha + \ldots \right)$$

$$- \frac{\omega\,I}{l^2} \left(N_2 - N_4 + N_6 - \ldots \right) \qquad (8)$$

where $H^2 = s^2 + c^2$ and $\cos\alpha = \frac{s}{H}$. The term logn denotes natural logarithm. The drop in all parts of

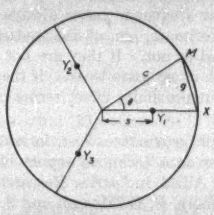

FIG. 2—THREE-PHASE CABLE AND SHEATH

the sheath is the same. This expression was calculated for the upper point of the sheath.

Example III. 2,000,000-cir. mil single-conductor cables, single-phase circuit, 60-cycle, sheaths open-circuited, $c = 2.97$ cm., $t = 0.357$ cm. actual thickness of lead sheath, $s = 10.66$ cm., $\rho_s = 25{,}000$. Absolute value of drop by formula (8), is $\omega\,I\,2.552$. If the first term only of (8) is used, the drop is $\omega\,I\,2.556$. The difference is 0.16 per cent which is practically negligible.

To find the expression for the zero-frequency inductance of the sheath, the terms of (8) which contain ω to the first power only, are required. This will involve A_n only. At low frequency, l will be less than 1.

$$\frac{A_n}{j\,l^2} = \frac{2\,c^n}{s^n} \frac{1}{n + j\,l^2} = \frac{2\,c^n}{n\,s^n} \left(1 + \frac{j\,l^2}{n} \right)^{-1}$$

The part independent of l is $\frac{2\,c^n}{n\,s^n}$.

The part of the last line of (8), which involves ω to the first power only, is

$$- j\,\omega\,2\,I \left(\frac{c^2}{2\,s^2} - \frac{c^4}{4\,s^4} + \frac{c^6}{6\,s^6} - \ldots \right)$$

$$= - j\,\omega\,I \operatorname{logn} \left(1 + \frac{c^2}{s^2} \right)$$

This cancels the second part of (8). The third and fourth parts contain ω to the second and higher powers, and so the zero-frequency inductance of the sheath is given by $2 \operatorname{logn} \frac{s}{c}$, from the first term of (8). The so-called frequency terms, that is, terms involving frequency to higher powers, are seen from Example III to have a negligible effect in a practical case with large conductors. Their effect would be less with smaller conductors.

A series of articles has been published from 1923 to the present time by W. A. Cramp in the *Journal I. E. E.* (England) stating that a term $\operatorname{logn} \frac{s-c}{c}$ should be used in formulas for loss and voltage drop in sheaths at low frequency, in place of the term $\operatorname{logn} \frac{s}{c}$ used by other writers on the subject.

In case of disagreement between formulas of this type for low frequencies, the proximity effect calculation can be used as a criterion, for it gives convergent series at low frequencies. By computing the terms which are being neglected, it can be determined if they are in fact negligible. This can be done both by taking numerical examples, and by finding the formula for practically zero frequency, as has been done in the single-phase problem just taken up. If different assumptions have been made, their effect can usually be determined by separate proximity effect calculations.

It is shown in the preceding paragraphs that the correct low-frequency term is $\operatorname{logn} \frac{s}{c}$. The term $\operatorname{logn} \frac{s-c}{c}$ gives results which differ by a considerable percentage, as is shown in Fig. 3.

Carefully made tests are of value in checking calculations. A set of such tests has been published by W. S. Clark and G. B. Shanklin in the A. I. E. E. TRANS., 1919, p. 917. The results of a number of their measurements on open-circuited sheaths are shown by circles in Fig. 3. They are seen to agree closely with equation (8). Fig. 3 is taken from the thesis of W. M. Swingle of Massachusetts Institute of Technology.

When the measurements given in a set of tests do not plot as a smooth curve, or show discrepancies between

themselves, internal evidence is given that the tests are untrustworthy at least to the extent indicated by the discrepancies. The mutual inductance of non-magnetic conductors depends on their size and position, and is a constant with varying current, the frequency and other conditions reported, remaining constant.

A set of tests on open-circuited sheaths which has been referred to quite often by W. A. Cramp is that by P. Dunsheath, *Journal I. E. E.* (England), Vol. 65, 1927, p. 469. One set of test results on non-magnetic cables at 6-in. centers (Table D, page 493 and Fig. 17) shows 30 per cent change in mutual inductance, while the only change reported is that the current changed from 150 to 300 amperes. This can be seen by dividing each

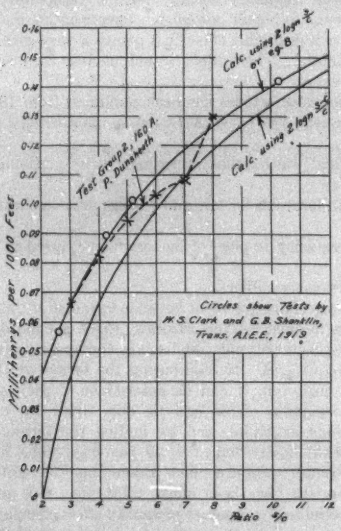

Fig. 3—Mutual Inductance of Cables and Sheaths

value of measured e. m. f. by the corresponding current, and shows a serious discrepancy in this set of measurements. Another similar set of measurements at different spacings (Table A, p. 492 and Fig. 5) is indicated by crosses and a dotted line in Fig. 3. While the height of the complete curve might require changing, owing to incomplete published data, yet the shape of the curve shows that the measurements are not precise enough to give any evidence that would help toward a choice between the two conflicting formulas illustrated. The statement that the dotted line showed that formula (8) was correct at close spacings and that the other formula was more nearly correct at wide spacings is seen to be without justification. These tests were first published in Fig. 5 by Mr. Dunsheath in the form of a curve, and he showed the same departure of the readings from a smooth curve that is shown in Fig. 3 of this paper.

The consideration of the two formulas of Fig. 3 is a good illustration of the usefulness of proximity effect calculations. The precise values of current and voltage in every part can be computed by the proximity effect method. The exact amount of shielding due to the sheath is calculated. Two different solutions are obtained for the cases of open-circuited sheaths and short-circuited sheaths. The open-circuited voltage between the sheaths suddenly becomes zero when a bond is applied and short-circuit current flows, even though the sheaths are separated by a considerable distance. There is no reason why the voltage between the open-circuited sheaths should gradually approach zero as the sheaths come near the position where they touch each other. The open-circuit voltage is not zero just before they touch.

It is satisfactory that the formulas given in the preceding paragraphs are not long. While some proximity effect formulas contain many terms, that is not a reason for giving up all proximity effect calculations. In cable sheath calculations, the higher frequency proximity effect terms are usually of small effect, except for the largest sizes of cables. The initial terms can often be adjusted to be the same as the most convenient and accurate low-frequency formula. It is advisable to have the higher frequency terms always available, so that one will estimate, almost automatically, whether they are needed or not. If they are not needed, they will cause little or no extra work. If they are needed, nothing else but proximity effect terms can take their place.

A number of proximity effect formulas for cable sheaths is given in a technical report of the British Electrical and Allied Industries Research Association by E. B. Wedmore, P. D. Morgan and S. Whitehead.[4] It may be noted that formulas (**118**) and (**119**) of that report are the same as the first terms of the proximity effect formulas published in Section V of reference 3.

Formula (**28**) of reference 1 for the loss in short-circuited, non-magnetic sheaths in a single-phase circuit can be improved by making it more rapidly convergent for wide spacings between the cables. This is done by a slight change in the calculation, making the initial terms the same as in the usual impedance formula. The two formulas give the same numerical results for cases for which they are both convergent.

Using the same notation as described under equation (**1**),

$$P = \frac{-j\,\omega \log n \dfrac{s}{c}}{j\,\omega\, 2 \log n \dfrac{s}{c} + \dfrac{\rho_s}{2\,\pi\, c\, l}} \qquad (9)$$

4. *Journal I. E. E.* (England), Vol. 67, March, 1929, p. 359.

$$F_n = j\,l^2\,(1+2P)\,\frac{c^n}{n\,s^n} \tag{10}$$

$$B = j\,l^2\left[-2\,A\,\text{logn}\,\frac{s}{c} + \sum_{k=1}^{\infty}\frac{c^k}{k\,s^k}\,F_k\right] \tag{11}$$

$$G_n = j\,l^2\left[2\,A\,\frac{c^n}{n\,s^n} - \frac{F_n}{n} + \frac{c^n}{n\,s^n}\sum_{k=1}^{\infty}\frac{(n+k-1)\,!}{(n-1)\,!\,k\,!}\,\frac{c^k}{s^k}\,F_k\right] \tag{12}$$

In the equations for B and G_n, $A = 0$. For C and H_n use the formulas for B and G_n respectively, except change A to B and F to G. Similarly, for D and I_n, change A to C and F to H, and so on.

$$L = P + B + C + \ldots \tag{13}$$

$$M_n = F_n + G_n + H_n + \ldots \tag{14}$$

$$\frac{\text{Loss in sheath}}{\text{Loss in conductor at zero frequency}} = \frac{a^2}{c\,t}\,\frac{\rho_s}{\rho_c}\left[2\,|L|^2 + \sum_{n=1}^{\infty}|M_n|^2\right] \tag{15}$$

where a is the radius of a solid copper wire of the same resistance as the conductor, and where ρ_c is the resistivity of copper, in the same units as ρ_s. $|L|^2$ is the square of the absolute value of L.

For an approximate formula, use

$$\frac{2\,a^2}{c\,t}\,\frac{\rho_s}{\rho_c}\,|P|^2 \tag{16}$$

By using equation (25), reference 1, to calculate the drop in the conductor, the effective impedance of the conductor with short-circuited sheath is found to be

$$R_c + j\,X_c + j\,\omega\,4\,L\,\text{logn}\,\frac{s}{c} - j\,\omega\,2\sum_{n=1}^{\infty}\frac{c^n}{n\,s^n}\,M_n \tag{17}$$

where R_c and X_c are the resistance and reactance of the conductor without sheath. L and M_n are complex quantities. If the numerical value of (17) is found, the real part is the effective resistance, and the unreal part is the effective reactance, of the cable with short-circuited sheath, in a single-phase circuit. The effective resistance will be in agreement with the loss in the conductor plus the loss in the sheath as given by (15).

By putting $L = P$ and $M_n = 0$, the following well-known approximate formulas are obtained:

$$\text{Effective resistance} = R_c + \frac{R_s\,X_m^2}{R_s^2 + X_m^2} \tag{18}$$

$$\text{Effective reactance} = X_c - \frac{X_m^2}{R_s^2 + X_m^2} \tag{19}$$

where X_m is the mutual reactance of the sheath and the conductor, and is given by

$$X_m = 2\,\omega\,\text{logn}\,\frac{s}{c}\ \text{in absolute units.} \tag{20}$$

$$= 2\,\pi\,f\,\frac{140.4}{10^6}\,\log_{10}\frac{s}{c}\ \text{ohms per 1,000 ft. of cable.} \tag{21}$$

See equations (6), (7), (10) and (11) of the article by Donald M. Simmons in the *Electric Journal*, August 1925, p. 366. In numerical calculations, resistance and reactance should be in the same units. The second part of (18) is equivalent to (16). Equation (19) is equivalent to (187) of reference 4.

Example IV. 2,000,000-cir. mil, single-conductor cables, single-phase circuit, 60 cycle, $c = 2.97$ cm., $t = 0.357$ cm. actual thickness of lead sheath, $s = 6.30$ cm. (lead sheaths touching) $a = 1.796$ cm., $\rho_s = 25{,}000$, $\rho_c = 2{,}100$, $f = 60$, $l^2 = 0.1005$.

$$\frac{\text{Loss in sheath}}{\text{Loss in conductor at zero frequency}} = 49\ \text{per cent, by equation (15).}$$

The result by the approximate formula (16) or (18) is 41 per cent. These results agree with those in Example II, reference 1. The rapidity of convergence of (15) of this article is about the same as that of (28), reference 1, for this example in which the cables are very close together. If the cables are on about 6-in. centers, as they would be if in ducts, the convergence of (15) is more rapid than that of (28), reference 1, and its first term is a better approximate formula.

It has recently been pointed out in Holland, England, and the United States that when three single-conductor cables with short-circuited lead sheaths lie in a plane and carry balanced three-phase current, the losses in the two outer sheaths are not the same. This difference is apparent in the zero-frequency expression. To take care of this, a formula for the loss in the second outside sheath should be added to (32) and (34) of reference 1. Using the notation of that article, let

$$P_o = A_{co} + B_{co} + C_{co} + \ldots$$

$$P_n = A_{dn} + B_{dn} + C_{dn} + \ldots \qquad n \neq 0$$

$$\frac{\text{Loss in sheath of second outside cable}}{\text{Loss in conductor at zero frequency}} = \frac{a^2}{c\,t}\left[2\,|P_o|^2 + |P_1|^2 + |P_2|^2 + \ldots + |P_n|^2 + \ldots\right] \tag{22}$$

This gives, for Example V of the above article, a result of 1.80 or 180 per cent to compare with 235 per cent in the sheath of the first outside cable.

The writer wishes to acknowledge valuable suggestions from Mr. R. W. Atkinson, chief electrical engineer of the Standard Underground Cable Company, and Mr. K. W. Miller of the Commonwealth Edison Company, regarding this paper.

In conclusion, various problems in proximity effect and shielding in wires and thin tubes, can be solved, as required. In some cases, it is desired to compute the results to show that certain effects are practically negligible. In other cases, the effects are of engineering importance, and the design of conductors will depend upon the computed values.

Discussion

R. W. Atkinson: I wish to discuss Prof. Dwight's formula (7) relating to sheath losses in three-conductor cables. With the large sizes of cables now frequently used, the sheath losses in three-conductor cables are not negligible and, therefore, the formula developed by Prof. Dwight will be very useful indeed. The formula given is readily available for use and is actually much less imposing than might be assumed from its appearance by the average engineer. In order, however, to improve the convenience of its application, we have transformed it to the following form:

$$P = \frac{24\,\pi^4 f^2 s^2 t\, a^2}{c\,\rho_c\,\rho_s}\left[\frac{1}{1+l^4} + \frac{s^2}{4_c^2}\left(\frac{1}{1+\frac{1}{4}\,l^4}\right) + \ldots\right] \qquad (7')$$

P denotes the ratio of sheath loss to zero frequency copper loss and the rest of the notation is the same as in Prof. Dwight's paper.

The series in the bracket converges so rapidly in all practical cases at commercial frequencies that only the first two terms in the bracket need be taken into account. Moreover, at 60 cycles the term l^4 is nearly equal to $\frac{s^2}{4_c^2}$, the variation being of the order of 2 per cent of the value of P. We may therefore, obtain a fair degree of accuracy by neglecting the bracket entirely at 60 cycles.

By making a simplifying assumption which neglects the effect of the redistribution of flux due to the sheath currents, Mr. Louis Meyerhoff of our company has independently derived a formula for sheath loss. This formula is

$$P = \frac{24\,\pi^4 f^2 s^2 t\, a^2}{c\,\rho_c\,\rho_s}\left[1 + \frac{s^2}{4_c^2} + \ldots\right] \qquad \textbf{(VII)}$$

It will be noted that formulas (VII) and (7′) are identical except for the term l^4 which appears in the bracket of the latter formula. The value of l^4 in practical cables at 60 cycles varies between 0.02 and 0.05, so that the value of sheath loss as determined by (VII) is from 2 to 5 per cent higher than the value as determined by (7). This indicates the degree of error which may be expected if losses of this type are calculated without taking into account the effect of the induced currents on the flux distribution. The calculations are greatly simplified without causing serious error.

If we take out of formula (7) the factor $\pi\, a^2$ and replace it with a symbol representing the cross-sectional area of the conductor, the formula will apply to stranded as well as to solid conductors. We have also found experimentally that it is applicable with reasonable accuracy to sector-shaped conductors as well, provided the current is assumed to be concentrated at the center of the small diameter of the sector.

In order to permit of simpler application of formula (7) all dimensions may be expressed in inches and the conductor area in circular mils. Substituting for f the value of 60, and for ρ_c and ρ_s their values at 20 deg. cent. (1,720 and 22,000, respectively) and neglecting all terms in the bracket, the formula becomes

$$\text{Per cent } P_{60} = \frac{2.3\, s^2\, t\, A}{10^6\, c} \qquad (7'')$$

where

Per cent P_{60} is the sheath loss at 60 cycles expressed in per cent of the copper loss at zero frequency.

s is the distance in inches between the center of each conductor and the center of the cable.

t is the thickness of the lead sheath in inches.

A is the area of each conductor in circular mils.

and c is the mean sheath radius, in inches.

To obtain the loss at any temperature other than 20 deg. cent. the value of per cent P_{60} as obtained by (7″) should be multiplied by the factor

$$\frac{1}{1 + 0.004\,(T_c + T_s - 40)},$$

in which T_c and T_s are the respective conductor and sheath temperatures in degrees centigrade.

Ordinarily, the values of s and c are not readily available, but they can be computed from the following:

For round conductors, the values are accurately:

$$c = 1.078\,d + 2.155\,a + b + \frac{t}{2}$$

$$s = 0.578\,d + 1.155\,a$$

For stranded sector conductors, the values are approximately:

$$c = 0.878\,d' + 2.155\,a + b + \frac{t}{2}$$

$$s = 0.471\,d' + 1.155\,a$$

where

d is the diameter of the conductor in inches,

d' is the diameter of a stranded circular conductor of the same area as the sector conductor under consideration,

a is the conductor insulation thickness in inches,

b is the belt insulation thickness in inches ($b = o$ for type H cables) and c, s and t have the same significance as for formula (7″).

It may be pointed out that the above discussion applies only to three-conductor cables which do not have a magnetic binder.

Another source of losses in three-conductor cables is the skin effect and proximity effect of the conductors. We have measured in our laboratory the losses in stranded sector cable of various sizes from 350,000 cir. mils to 900,000 cir. mils and have found that on the cables tested the value of combined skin effect and proximity effect is approximately equal to 90 per cent of the skin effect calculated by the standard formula for a round single conductor of the same size. By adding this value of skin effect and proximity effect to the sheath loss we obtain the total increase of a-c. loss over d-c. loss. Theoretically, the combined loss will differ somewhat from the simple sum of the two component losses, but the order of difference is smaller than the degree of accuracy with which we are concerned.

The apparent a-c. resistance of a cable will also be affected by the proximity of other cables, the effect depending upon the number, proximity, and physical dimensions of the contiguous cables. Each case must be considered as a special problem. It seems that a fair approximation of this effect can be obtained by making numerical computations on the assumption that the induced currents do not affect the flux distribution. This effect is being studied.

H. B. Dwight: The approximate sheath loss formula given by Mr. Atkinson is a very appropriate one for this problem. It can be used with confidence, as it gives the sheath loss of three-conductor cables up to 1,000,000 cir. mils and more, within a very few per cent.

The test results which he gives are of considerable value. It is to be hoped that further measurements of effective resistance and reactance will be published, for in the cases of stranded conductors and laminated busbars, measurements must be used as well as calculations.

Losses in Transformers for Use With Mercury Arc Rectifiers

BY E. V. DeBLIEUX[1]

Associate, A. I. E. E.

Synopsis.—*This paper developes the mathematical formulas and discusses the assumption involved in the methods for determining the losses of transformers for use with mercury arc rectifiers. The methods discussed have been proposed for the A. I. E. E. Standards.*

From test readings of current taken under service conditions, a rigid calculation of copper losses is made. From this calculation it is shown that the proposed methods give values that are accurate within satisfactory limits.

The paper also presents the advantages of the proposed methods for determining the losses of transformers for use with mercury arc rectifiers.

* * * * *

Part I

A NUMBER of papers and books has been published in the last few years which includes tables and calculations giving the salient features of the various transformer connections which are suitable for use with mercury arc rectifiers.

The calculations set forth in these publications are based on the assumption that the secondary current waves are rectangular in shape.

Similarly, calculations of effective values of current and of losses are based on this rectangular wave shape and on the further assumption that the effective resistance of the winding is the same at all frequencies. Since the reactance of the circuit alters the current wave shape from the assumed form and as the effective resistance is different for the various harmonics which go to make up the current waves, calculations based on the stated assumptions may be very approximate.

It is the purpose of this paper to review these theoretical calculations, to calculate particularly the copper losses of a typical transformer designed for use with power rectifiers and, based on the conclusions obtained from the calculations, to set forth commercial methods of making tests of losses of the commonly used connections.

Current Harmonics

If a rectifier transformer with p secondary phases supplies current to a rectifier and the circuit is assumed to be free of reactance, the current per phase will be as shown for phases 1 and 2 in Fig. 1a. Since there are p phases each phase will carry current for the $\frac{2\pi}{p}$ portion of a cycle, or with the axis as shown, from $-\frac{\pi}{p}$ to $+\frac{\pi}{p}$. The magnitude of the current is designated by the letter J.

With this arrangement, the wave may be analyzed into a series of cosine terms because of its symmetry about the coordinate axis. The amplitude of the nth harmonic is then—

$$a_n = \frac{1}{\pi} \int_{-\frac{\pi}{p}}^{+\frac{\pi}{p}} J \cos n\,\theta\, d\,\theta = \frac{2\,J}{n\,\pi} \sin \frac{n\,\pi}{p} \qquad (1)$$

Solving for the harmonics in the secondary current wave of a six-phase and a three-phase rectifier transformer, the values will be as given in Table I.

Fig. 2c shows one of the commonly used six-phase

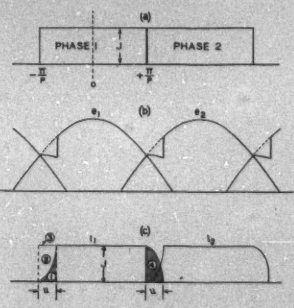

FIG. 1

rectifier connections. Since current flows in each of the (*b*) portions of the winding for $\frac{2\pi}{p}$ or 1/6 of a cycle the values in the $p = 6$ column of Table I will apply to this portion of the winding. The (*c*) portion of the winding carries the current of two successive *b* coils and therefore for ⅓ of a cycle. It is therefore equivalent to a three-phase rectifier and the values in the $p = 3$ column apply to this section of the winding. The current which flows in the primary coils is the resultant of the currents which flow in the *b* and *c* secondary coils which are on the same leg. The even harmonics of the *b* and *c* coils neu-

1. General Trans. Engg. Dept., General Electric Co., Pittsfield, Mass.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

tralize each other because they are, 180 deg. out of phase, of equal magnitude and flow through coils having equal turns. It is therefore the resultant of the odd harmonics that flow in the primary coils.

Effect of Reactance

Due to the reactance of the transformer windings and the supply lines, it requires a definite time for the secondary current to rise to its full value at the start of each period of conduction and again to decrease to zero at the end of the period of conduction.

As a result the current wave is altered from the shape as shown in Fig. 1a and assumes the shape shown by the solid lines of Fig. 1c.

Figs. 1b and 1c show the voltage and current waves of anodes 1 and 2 of the connection shown in Fig. 2c. Anode 1 carries the full d-c. current until the point in the cycle is reached where the voltage of anodes 1 and 2 are equal, at which point anode 2 begins burning.

Beginning at this point two anodes are connected

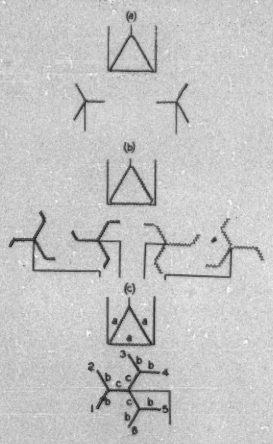

Fig. 2

simultaneously to the cathode and constitute an electrical connection between the connected transformer phases, short-circuiting the difference in voltage between the two phases. This difference in voltage circulates a current through the loop thus formed, which current flows in an opposite direction to the d-c. current carried by phase 1.

This condition continues until the instantaneous value of the circulating current equals the d-c. current carried by phase 1 at which time the resultant current in phase 1 is zero and its arc goes out thus terminating the short circuit and leaving phase 2 carrying the d-c. current.

By this means, commutation of current is effected.

TABLE I

	$p = 6$	$p = 3$
Fundamental	100 %—0.318 I_{DC}	100 %—0.552 I_{DC}
Second harmonic	86.8 %—0.276 I_{DC}	50 %—0.276 I_{DC}
Third	66.8 %—0.212 I_{DC}	0.00
Fourth	43.4 %—0.138 I_{DC}	25 %—0.138 I_{DC}
Fifth	20 %—0.064 I_{DC}	20 %—0.110 I_{DC}
Sixth	0.00	0.00
Seventh	14.3 %—0.045 I_{DC}	14.3 %—0.079 I_{DC}
Average = d-c. comp.	$= \frac{I_{DC}}{p} = 0.167\ I_{DC}$	$0.333\ I_{DC}$
R. m. s. = a-c. and d-c.	$= \frac{I_{DC}}{\sqrt{p}} = 0.408\ I_{DC}$	$0.577\ I_{DC}$
R. m. s. a-c. comp. = I_{DC}	$\sqrt{\frac{1}{p} - \frac{1}{p^2}} = 0.373\ I_{DC}$	$0.471\ I_{DC}$

This period of commutation, *i. e.*, of overlap of current of two phases is denoted by u. (See Fig. 1c.)

Using the instant when anode 2 starts burning as the origin and applying Kirchoff's second law to the closed loop through phases 1 and 2—

$$e_1 = L\frac{d\,i_1}{d\,t} - L\frac{d\,i_2}{d\,t} + e_2 + a_1 - a_2 \qquad (2)$$

in which

e_1 and e_2 are the instantaneous voltages of phase 1 and phase 2 respectively,

a_1 and a_2 are the arc drops of phase 1 and phase 2 respectively,

i_1 and i_2 are the instantaneous currents of phase 1 and phase 2 respectively.

L = the inductance of the circuit between anode and neutral.

Assuming the arc drop of the two burning anodes to be equal the terms a_1 and a_2 cancel out.

Also from Fig. 1c,

$$i_1 + i_2 = J$$

$$e_1 = E\sqrt{2}\cos\left(u\,t + \frac{\pi}{p}\right)$$

$$e_2 = E\sqrt{2}\cos\left(u\,t - \frac{\pi}{p}\right)$$

in which

E = Effective value of transformer secondary line-to-neutral voltage

t = time

$u = 2\,\pi\,f$

Substituting the above values of e_1 and e_2 in (1) and solving simultaneously

$$i_1 = J - \frac{E\sqrt{2}\sin\pi/p}{X}(1 - \cos u\,t) \qquad (2a)$$

in which $X = \omega\,L$

Equating equation (2) to zero at time $t = u$, at which time i_1 = zero, then,

$$\cos u = 1 - \frac{J\,X}{E\sqrt{2}\sin \pi/p} \qquad (3)$$

It should be noted from Fig. 1c that the effect of reactance is to increase the period during which a phase carries current by the angle u without altering the maximum or average value of the current.

As a result it is to be expected that the effective value of current is decreased.

The change in mean square value due to this increase in the period of conduction may be represented by the following equation:

$$\Delta i_1 = \frac{1}{2\pi}\int_0^u i^2\,(1-\cos x)^2\,dx + \frac{1}{2\pi}\int_0^u (J - i\,(1-\cos x)^2\,dx - \frac{1}{2\pi}\int_0^u J^2\,dx \qquad (4)$$

Referring to Fig. 1c, the first term represents the area (1) the second term represents the area (2) which is the same as (4) and the third term represents the area (3) which is enclosed in the dotted lines and is the sum of (1) and (2).

Integrating for i_1

$$\Delta i_1 = \frac{1}{2\pi}\left[\,2\,i^2\left(\frac{3\,u}{2} - 2\sin u + \frac{1}{4}\sin 2\,u\right) - 2\,J\,i\,(u - \sin u)\,\right] \qquad (5)$$

From the figure $i = \dfrac{J}{1 - \cos u}$ at $x = u$

Substituting this value for i in (5) and collecting terms:

$$\Delta i_1 = -\frac{J^2\,(2 + \cos u)\sin u - (1 + 2\cos u)\,u}{2\,\pi\,(1 - \cos u)^2}$$

The minus sign indicates that the change in effective value of the current is a decrease as was anticipated.

The effective or root mean square value of I for flat topped waves is $\dfrac{J}{\sqrt{p}}$ and the mean square value is therefore $\dfrac{J^2}{p}$. Due to commutation, the mean square value therefore becomes

$$I_{eff}^2 = \frac{J}{p^2} - \frac{J^2(2 + \cos u)\sin u - (1 + 2\cos u)\,u}{2\,\pi(1 - \cos u)^2} \qquad (6)$$

which expression is equivalent to and may be reduced to the forms given by various authors.[2]

Equation (6) is based on the assumption that the effective transformer resistance is the same for all the harmonics which make up the current wave for which the expression was derived and that therefore in determining the effective value of current the harmonics may be added in the usual manner to determine the resultant r. m. s. value.

Columns 2, 3, and 4 of Table II show in per cents of the fundamental component the magnitude of each of the harmonics in the various coil currents. The values were obtained when a d-c. current of value J was delivered by a six-phase rectifier, of the connection shown in Fig. 2c, fed from a system whose capacity is large compared with that of the rectifier. The harmonics in the primary and in the anode (b coil) currents were measured. The harmonics in the current of the c coils were determined by proportion from the measured reductions of the corresponding harmonics in the current of the a and b coils.

It will be noted that the actual per cent harmonics in the current are not so large as the theoretical values in Table I and that the ratio between theoretical harmonic current and the test value is greater the higher the order of the harmonic.

This is due to the impedance of the circuit which increases with the order of the harmonic.

Effective Resistance

Based on formulas developed over a period of years and tests of a large number of transformers a formula for eddy loss in transformer windings has been developed, in which the eddy current loss varies approximately in accordance with the following equation:

$$\text{Watts (eddy loss)} = 0.86\,Y\,X^{1.875} \qquad (7)$$

in which

X = frequency and Y represents the sum of the other factors which determine the eddy loss.

It should be noted that in this expression the eddy loss varies in accordance with slightly less than the square of the frequency instead of with the square as given in various other similar formulas, and therefore, calculations based on it will give lower losses than the conventional formulas.[3] For a given winding all of the factors that enter in the Y term of equation (7) are constant, therefore the proportionate resistance to the various harmonics will be as determined from equation (7) and as given in columns 5 and 6 of Table II.

In connection with the loss values determined later in the paper, it should be noted from columns 5 and 6 of Table II that the effective resistance at fundamental

2. "Mercury Arc Rectifiers and Their Circuits," Prince and Vogdes, p. 115.

Archiv. fur Elektrotechnik, Daellenbach and Gerecke, Vol. 14, No. 2, Jan. 15, 1925, pp. 17-246.

"Mercury Arc Power Rectifiers," Marti and Winograd, p. 114.

3. "Magnetic Leakage in Transformers," J. M. Weed, *G. E. Review*, Vol. 15, p. 759, Dec. 1912.

"*Reduction of Eddy Current Losses*, J. M. Lyons, A. I. E. E. Journal, July, 1913.

"Eddy Currents in Large Slot Wound Conductors," A. B. Field, *Proc. A. I. E. E.*, 1905, p. 659.

"Stray Losses in A-C. Armature Windings," Dr. I. Rusch, *Elektrotechnik und Maschinenbau*, Nos. 4 and 5, Jan. 1910.

TABLE II

1 Order of harmonic	2 Test amplitude (peak values) a coil	3 b coil	4 c coil	5 Effective coil resistance a	6 b and c	7 Effective values of I (see equation (8) a coil	8 b coil	9 c coil
D. C.		52.5	60.3	1	1		51.1	58.7
1	100	100	100	1.05	1.12	102.5	105.8	105.8
2		73	42.3		1.44		88.5	50.7
3	22	41.5	0	1.18	1.96	24	58.0	
4		23.5	13.55		2.62		38.0	21.9
5	7.7	7.7	7.7	2.03	3.46	11.0	14.3	14.33
6								
7	5.7	5.7	5.7	2.2	5.6	8.44	13.5	13.5
8		8.0	4.61		6.9		21.0	12.1
9	3.8	7.5		4.07	8.36	7.68	21.4	
10		5.3	3.07		9.99		16.75	9.7
11	2.5	2.5	2.5	5.51	11.8	5.88	8.6	8.6
12								
13	2.2	2.2	2.2	7.18	15.8	5.9	8.75	8.75
14		3.3	1.81		17.6		13.82	7.58
15	1.5	3.0		9.02	18.65	4.5	12.95	
16		2.0	1.17		22.85		9.57	5.6
17	0.9	0.9	0.9	11.21	25.6	3.02	4.55	4.55
18								
19	0.95	0.95	0.95	13.44	30.9	3.46	5.29	5.29
20		1.4	0.78		34		8.16	4.55
21	0.7	1.3		16.08	37.2	2.87	7.93	
22		0.8	0.48		40.75		5.11	3.06
23	0.34	0.34	0.34	18.95	44.2	1.48	2.26	2.26
24								
25	0.43	0.43	0.43	21.86	51.1	2.01	3.08	3.075
R. m. s. a-c. component in per cent of fundamental						75.6	113.5	87.1
Calculated r. m. s. a-c. component (from Table I)						0.816	0.373	0.471
Test r. m. s. a-c. component						0.835	0.361	0.481
D-c. component						0	0.1625	0.324
Calc. r. m. s. (a-c. + d-c.) (From Table I)						0.816	0.408	0.577
Test r. m. s. (a-c. + d-c)						0.835	0.396	0.584
Test r. m. s. in per cent of calc. r. m. s						102.2	97.5	101.2

frequency for the a coils is 5 per cent, and for the b and c coils is 12 per cent, greater than the corresponding d-c. resistance, indicating an eddy and stray loss of 5 per cent for the a coils and 12 per cent for the b and c coils, which are conservative values.

Copper Loss Calculation

From Table II the loss produced by each harmonic may be separately determined.

In order to make the application general, the effective coil resistance to a given harmonic, has been expressed in terms of the resistance to the fundamental as developed from the following equations:

$$\text{Watts loss} = I_h^2 R_h = I_h^2 K R_f = (I_h \sqrt{K})^2 R f = I^2 R_f \quad (8)$$

in which

I_h = Effective value of harmonic current

R_h = Effective coil resistance at harmonic frequency

R_f = Effective coil resistance at fundamental frequency

$$K = \frac{R_h}{R_f}$$

$$I = I_h \sqrt{K}$$

Comparing the percentage values of I' given in columns 8 and 9 of Table II with the corresponding values of Table I expressed on a percentage basis, shows that they are approximately the same, indicating that for each harmonic the effect with respect to the copper losses of the reduction of effective current value due to the reactance of the circuit is approximately balanced by the effect of the increase in effective resistance with the order of the harmonic.

The percentage values of I' will therefore be as shown in columns 7, 8, and 9 of Table II for windings a, b, c respectively.

With this change all harmonics have been expressed on the basis of the same resistance and the r. m. s. value of the series of harmonics which flow in each winding may be determined from the square root of the sum of their squares. This determination gives for the a, b, and c coils respectively, the values shown at the bottom of columns 7, 8 and 9 of Table II.

For the particular transformer used for the test, which is of average design, the losses in the various windings were distributed approximately as shown below when based on the theoretical r. m. s. values of current given in Table I.

Winding	(a)	(b)	(c)
Copper loss	42%	33%	25%

Correcting these percentages for the difference between the theoretical r. m. s. values on which they are based and the test values of Table II, the following percentages are obtained.

Winding	(a)	(b)	(c)
Copper loss	43	32.2	25.3

The sum of the losses of the three windings is 100.5 per cent indicating that the actual losses are 0.5 per

cent greater than the value calculated from the theoretical r. m. s. values of current. Making a similar calculation using the values of Table I gives 133.6 per cent losses.

There are several points in regard to this calculation which should be pointed out. The supply voltage wave contained a number of harmonics of small magnitude which slightly increased the magnitude of the current harmonics and for which no correction was made. The magnitude of these harmonics is given in Table III.

It should be noted from Table II that due to the harmonics in the supply voltage, the 19th and 25th harmonics in the high-voltage winding are larger than

TABLE III

Harmonic	Per cent volts a-c. supply rectifier shut down
Fundamental	100
5	0.50
7	0.44
11	0.11
13	0.06
17	0.32
19	0.06
23	0.015
25	0.020

the 17th and 23rd harmonics respectively, whereas from theoretical calculations, they should be smaller.

Had the tests been made on a connection such as is shown in Fig. 2a or 2b the harmonic currents in the secondary windings would be those for a three-phase circuit, since these connections have multiple three-phase secondaries. The magnitude of the harmonics would therefore approximate those of the *c* coils of the transformer tested. Since the primary harmonic currents are proportional to the secondary currents, the accuracy of the primary winding copper loss determination would approximate that of the test.

The tested connection which has been discussed in detail is therefore representative of the results which would be obtained for any one of the three commonly used connections illustrated.

Conclusion

The transformer copper losses are dependent on the magnitude of the harmonics in the current waves which harmonics vary in magnitude depending on the reactance of the load and of the supply, both of which are variable, as well as on the reactance of the rectifier transformer.

A calculation of losses based on the r. m. s. values of current for flat-topped waves without overlapping (Table I), gives results of the correct magnitude.

Therefore, the proposed A. I. E. E. Standards for Mercury Arc Rectifiers which recommends that the calculation of losses of rectifier transformers be based on flat-topped waves without overlapping, will give values which are accurate within satisfactory limits. Calculation by this method has a number of advantages in addition to eliminating uncertain reactance calculations. These advantages are listed below:

1. It eliminates involved mathematical calculations.
2. It eliminates factors which are design features and known only to the manufacturer.
3. It enables purchaser to check the manufacturer's calculations.
4. It gives safe values of current from which the purchaser may determine the sizes of cables to use in making connections.
5. It reduces cost and shortens the time of delivery since it obviates the necessity of having the manufacturer set up the complete apparatus, duplicating the supply and load reactances and measuring losses by the input-output method. As is well known, the input-output method is subject to considerable inaccuracy due to the fact that it involves the determination of the difference between two large quantities.
6. As will be shown in the third part of the paper, it permits separate and direct measurement of the losses of the transformer by the usual methods employed in testing power transformers.

Part II

It is the purpose of this section of the paper to develop the mathematical basis of the methods of determining the copper losses of power transformers for use with rectifiers which are proposed for the A. I. E. E. Standards for Mercury Arc Rectifiers. Proof will also be developed that the proposed methods give the losses which would be obtained under rectifier operation with flat top secondary current waves without overlapping, it being the conclusion from Part I that the loss thus obtained is equivalent, within satisfactory limits of accuracy, to that obtained with the actual wave forms obtained under service conditions.

The three most commonly used transformer connections are shown in Figs. 2a, 2b, and 2c.

It should be noticed that two of the circuits *a* and *b* have multiple secondaries and that the third *c* has a single interconnected secondary. As a result different tests are proposed for the two types of circuits to obtain the desired results. The various other transformer connections used at times, fall into one or the other of these classes.

Under rectifier operation each anode and the associated transformer winding carries one block of current per cycle as shown in Fig. 1. In order to avoid the saturation of the transformer cores by the direct current carried by the secondaries, an even number of anodes and associated transformer secondary windings, divided into pairs displaced 180 degrees electrically are provided.

Under this condition the primaries carry blocks of current similar to those in the secondaries but only 180 degrees apart, the alternate blocks being of opposite sign.

The heating effect of such a current will be twice that of a current consisting of pulses in one direction only and the r. m. s. value of the primary current will therefore be $\sqrt{2}$ times that of the secondary current when the transformation ratio is unity.

Testing of Transformers with Multiple Secondaries

Considering the connection shown in Fig. 2a:

If I_p is the primary current, I_s the secondary current, and the transformation ratio is unity, then as has been shown in the preceding paragraph $I_p = \sqrt{2}\, I_s$.

If R_p = resistance of one primary coil

and R_s = resistance of one secondary coil

then total copper loss $W_{cu} = 3\, I_p^2 R_p + 6\, I_s^2 R_s$

$$= 3\, I^2_p\, (R_p + R_s) \qquad (9)$$

If only one secondary winding is short-circuited for the copper loss test, then I_p will equal I_s instead of $\sqrt{2}\, I_s$ and the resistance will be R_s.

The total copper loss for this test will therefore be

$$W_{cu}' = 3\, I_p^2 R_p + 3\, I_p^2 R_s = 3\, I_p^2\, (R_p + R_s) \qquad (10)$$

which is identical with equation (9) and therefore gives the correct loss.

Considering the transformer connection shown in Fig. 2b:

For this connection the transformer copper losses under rectifier operation will be

$$W_{cu} = 3\, I_p^2 R_p + 12\, I_s^2 R_s \qquad (11)$$

The various symbols have the same meaning as explained in the preceding paragraph for the connection of Fig. 2a.

It can be shown that for the connection of Fig. 2b.

$$I_s = 0.364\, I_p$$

Substituting this value for I_s in equation (11)

$$W_{cu} = 3\, I_p^2 R_p + 1.59\, I_p^2 R_s \qquad (12)$$

Two tests will be required to determine the copper losses of this connection. For the first test two of the secondary wyes are short-circuited and the losses measured with rated sinusoidal current held in the primary. Assuming a 1:1 ratio of transformation the total copper losses will be:

$$L_1 = 3\, I_p^2 R_p + 1.5\, I_p^2 R_s \qquad (13)$$

For the second test all four secondary wyes are short-circuited and losses measured as in the first test. For the second test the losses will be:

$$L_2 = 3\, I_p^2 R_p + 0.75\, I_p^2 R_s \qquad (14)$$

Solving equations (13) and (14) simultaneously for R_p and R_s

$$R_p = \frac{2\, L_2 - L_1}{3\, I_p^2} \qquad (15)$$

$$R_s = \frac{L_1 - L_2}{0.75\, I_p^2} \qquad (16)$$

Substituting these values of R_p and R_s in equation (12)

$$W_{cu} = 1.12\, L_1 - 0.12\, L_2 \qquad (17)$$

which is an expression for the transformer copper lost in terms of the loss measurements of the two tests connections.

It is essential to successful rectifier operation that the multiple secondary circuits be so placed with respect to the primary that each can operate independently as the full secondary. For this reason short-circuiting only half of the secondary coils does not alter materially the magnitude or distribution of the stray losses. Furthermore, the slight alteration that does occur tends to raise the reactance and therefore the losses. The increase is slight, however, as tests of a number of transformers give values of impedance with one-half of the secondary short-circuited that average only 5 per cent higher than the value with the full secondary shorted-circuited. The maximum increase in reactance obtained in the extreme case was less than 10 per cent.

Testing of Transformers with a Single Set of Secondary Windings

Under rectifier operation assuming flat-topped waves without over-lapping each of the outer b coils (Fig. 2c)

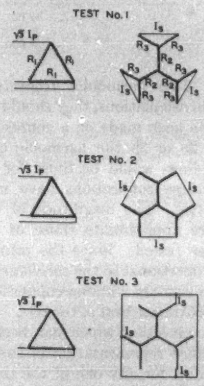

Fig. 3

of the secondary carry the full d-c. for one-sixth of a cycle. Its effective value therefore, is

$$I_b = \frac{I_{DC}}{\sqrt{6}} \qquad (18)$$

Similarly the inner c coils carry the total d-c. for one-third of a cycle and the effective value is

$$I_c = \frac{I_{DC}}{\sqrt{3}} \qquad (19)$$

Assuming a 1:1 ratio of primary to one secondary coil

$$I_a = \frac{2\, I_{DC}}{\sqrt{6}} \qquad (20)$$

The copper losses are therefore

$$W_{cu} = 6\, I_b^2\, R_b + 3\, I_c^2\, R_c + 3\, I_a^2\, R_a \qquad (21)$$

Substituting in (21) for I_a, I_b, and I_c, their values from equations (18), (19), and (20)

$$W_{cu} = (2\, R_a + R_b + R_c)\, I_{DC}^2 \qquad (22)$$

From (20)

$$I_{DC} = \frac{\sqrt{6}\, I_a}{2}$$

Substituting this value for I_{DC} in (22)

$$W_{cu} = (3\, R_a + 1.5\, R_b + 1.5\, R_c)\, I_a^2 \qquad (23)$$

In order to make an independent test of the power transformer and obtain the losses for rectifier operation as given in equation (23) three copper loss tests will be made to obtain three equations and permit solving for the three unknowns—the effective a-c. resistance of the three sections of the windings.

The three test connections are shown in Fig. 3.

Denoting the loss which will be measured in tests (1), (2), and (3) by L_1, L_2, and L_3 respectively, the loss equations will be as follows:

$$L_1 = 3\, I_p^2\, R_1 + 6\, I_s^2\, R_3 = I_p^2 (3\, R_1 + 2\, R_3) \qquad (24)$$

$$L_2 = 3\, I_p^2\, R_1 + 3\, (\sqrt{3}\, I_s)^2\, R_2 + 6\, I_s^2\, R_3 = I_p^2\, (3\, R_1 + 3\, R_2 + 2\, R_3) \qquad (25)$$

$$L_3 = 3\, I_p^2\, R_1 + 3\, I_s^2\, R_2 + 3\, I_s^2\, R_3 = I_p^2\, (3\, R_1 + R_2 + R_3) \qquad (26)$$

Solving equations (24), (25), and (26) simultaneously for R_1, R_2, and R_3

$$L_2 - L_1 = 3\, I_p^2\, R_2 \quad \text{therefore} \quad R_2 = \frac{L_2 - L_1}{3\, I_p^2} \qquad (27)$$

$L_1 - L_3 = I_p^2\, (R_3 - R_2)$ therefore

$$R_3 = \frac{L_1 - L_3}{I_p^2} + R_2 = \frac{2\, L_1 + L_2 - 3\, L_3}{3\, I_p^2} \qquad (28)$$

$$R_1 = \frac{L_1}{3\, I_p^2} - \frac{2\, R_3}{3} = \frac{6\, L_3 - 2\, L_2 - L_1}{9\, I_p^2} \qquad (29)$$

Substituting in (23) the values of R_1, R_2 and R_3, from equations (27), (28) and (29) and collecting terms

$$W_{cu} = \frac{L_1 + 2\, L_2 + 3\, L_3}{6} \qquad (30)$$

which is an expression for the copper loss in terms of the measured copper loss of the three test connections.

Part III

Iron Loss

While the current waves are distorted due to the harmonics discussed in the previous sections of this paper, the voltage wave impressed on the primary, when the rectifier is fed from a system whose capacity is reasonably large, approximates a sine wave.

The core loss tests should therefore be made in the same manner as for ordinary power transformers.

Conclusions

It is therefore possible to test the main transformers of the various rectifier connections independently of the rectifiers, making the tests by the simple methods used for ordinary power transformers. From the readings thus obtained the losses which would be obtained under rectifier operation with flat-topped waves and no overlapping may be determined. This loss as shown in the first part of the paper, approximates within satisfactory limits the loss obtained under actual operating conditions.

Discussion

W. M. Goodhue: Referring to equation (2), the symbol L is used as the inductance of the circuit between anode and neutral and this implies that the inductance of the two anode circuits are equal to each other and are constant at all times. If transformers having only one (two-terminal) secondary coil and one primary coil per "window" of the transformer could be used, the leakage flux paths would have a definite shape and a single value of leakage inductance could be used, provided the exciting current required for the iron is neglected. But with rectifiers, however, this arrangement is impracticable, since the d-c. component in the secondary current cannot be reflected in the primary and the iron would therefore be saturated in a single direction and to an extreme degree. Hence with rectifiers at least two secondary coils (or a three-terminal coil) per window must be used, so that the d-c. components of their m. m. fs. may oppose each other. In Fig. 2c, either coils b_1 and b_2, or coils b_2 and c_1, etc. must all occupy one window. Thus the two coils corresponding to the two anodes which are firing simultaneously must, at least in many parts of the cycle, occupy the same window. Under these conditions the leakage flux paths must be changing *shape* with time as the proportion of current between the two anode currents changes with time, the sum of the two currents at all times during the commutation period being nearly constant. This requires that at least three different leakage flux components (or self and mutual inductances) must be considered instead of one single inductance as in equation (2). A rigorous solution of these conditions would lead to differential equations of at least the third order if all unknowns but one are to be eliminated. Under these conditions it may be shown that the use of several constant self and mutual inductances is sound with the changing shape of the flux path, and with undetermined and peculiar wave forms.

The use of such constant inductances presumes that the leakage flux can be divided into components, each leakage flux being proportional to the m. m. f. producing it, and of constant shape, each m. m. f. being taken *singly*. This holds true in spite of the fact that one m. m. f. when acting alone would saturate the iron. To show that this presumption is correct requires an analysis of the boundary condition of the *unsaturated iron* when *one* m. m. f. *alone* is acting; this analysis may be rigorously made by the use of vector methods. Let A be the *vector* potential of the magnetic field intensity H, that is Curl $A = H$; A, however, may be restricted by making Div $A = O$; Curl H = Curl Curl $A = -\Delta^2 A$, where Δ^2 is the Laplacian operator, so that A is of potential function form with $\frac{1}{4\pi}$ Curl H replacing the density of charge, ρ, in electrostatics; that is,

$$-\Delta^2 A = 4\pi \left[\frac{1}{4\pi} \text{Curl } H \right] = 4\pi\rho.$$

$$A_p = \iiint \frac{\rho}{r_p}\, d\,v = \iiint \frac{\frac{1}{4\,\pi}\,\text{Curl}\,H}{r_p}\, d\,v$$

Where

A_p = vector potential of point p.

$d\,v$ = element of volume.

r_p = length of line drawn from P to $d\,v$.

$\frac{1}{4\,\pi}$ Curl H = current density (of winding).

To take account of the boundary condition $H = O$ at an iron surface with one m. m. f. acting *alone*, set $A = O$ at the iron surface. This gives a *vector* surface charge, σ_1, induced on the iron surface (analogous to electrostatic induced charge). Then the total A due to one m. m. f. acting alone is, with its σ, denoted by A_{p1}

$$A_{p1} = \iiint \frac{\frac{1}{4\,\pi}\,\text{Curl}\,H_1}{r_p}\, d\,v + \iint \frac{\sigma_1}{r_p}\, d\,S$$

Curl H_1 is 0.4 π times the current density (amperes per sq. cm.) in the coil containing m. m. f., F_1 and σ_1 is an induced vector charge due to this same m. m. f., so that A_1 is completely determined. Thus Curl A_1 is determined, giving H_1 the leakage field density due to one m. m. f. *alone* with the boundary condition of unsaturated iron. As σ depends only on the geometry and the current density, H_1 is proportional to the m. m. f., F_1. For similar reasons any one of the three constant self and three mutual leakage inductances may likewise be used, all of them being needed in the differential equations. Propagation and distributed capacitance effects have, however, been neglected; proximity effect on the leakage inductances with certain connections (as conductors in parallel) may occur also, although with conductors in series, it is chiefly the equivalent resistance which is affected.

Referring again to equation (2) the resistance drops have been neglected. *These resistance drops are not negligible during commutation*, because of the small magnitude of the e. m. fs. involved. Also there appear to be errors made in the signs and time origins of e_1 and e_2, for e_1 and e_2 essentially oppose each other at the moment of commutation, considering the closed loop, through phases 1 and 2. Hence e_1 and e_2 should have opposite signs if on the same side of the equation rather than the same sign as given in equation (2).

Furthermore, on page 1005 it is stated that the core loss test is to be made in a manner similar to that used in ordinary power transformers. It should be remembered, however, that the exciting current of ordinary transformers is exceedingly small. Even if a circuit is used which, with proper placing of the coils in the windows of the transformer or transformers avoids extreme saturations due to the full effect of the working load m. m. f., a very slight unbalance due to mechanical and electrical discrepancies in the accuracy of construction and operation of the entire system would produce very considerable saturation. Some method of testing can probably be devised whereby this condition may be similated as for example by circulating a direct current of the correct magnitude in addition to the working alternating exciting current through the windings. The result could then be plotted in some such form as per cent increase in core loss as a function of the per cent unbalance under several operating conditions. To obtain the direct-current component, a small direct-current e. m. f. might be inserted during the core loss test in series between the supply alternator and the transformer, using a direct-current ammeter to indicate the amount of unbalance.

E. V. DeBlieux: It must be recognized that a rigorous solution of the conditions during commutation requires, as pointed out by Mr. Goodhue, that the mutual inductances and the resistance drops be considered. In addition, the fact that the arc drops are not equal at each instant and that there is a ripple in the d-c. current must also be considered.

In order to simplify the calculations incidental to commercial design, these refinements have been eliminated and the simple formula given in equation (2) obtained. This simplification was justified both by test results and the following considerations:

During the interval when current is commutating between two anodes the current in the other anodes which are active, is

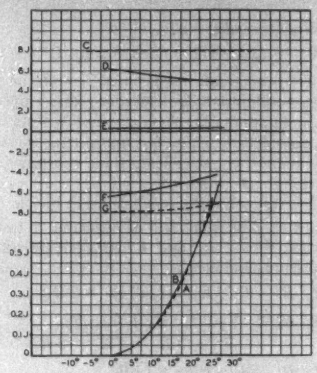

Fig. 1

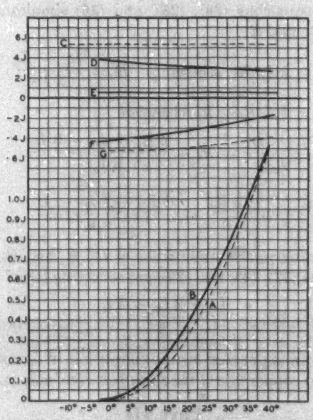

Fig. 2

fairly constant. As a result the mutual effects between the coils active in the commutation and other coils are small. For some connections such as Fig. 2A when current is commutating between two anodes all other secondary coils of the same legs with the two involved in the commutation are idle. Therefore, there are no mutual effects in this case. The same is true of Fig. 2c with the exception that for commutations such as between anodes 2 and 3 there is an inductive effect between coils B_2 and B_3, but as the

design practise is to wind the turns of these two coils side by side for their entire length, the inductive effects are small.

Figs. 1 and 2 show the difference between considering and neglecting resistance drops. Fig. 1 applies to the connection of Fig. 2A of the paper and Fig. 2 to connection Fig. 2c. The calculations were made in accordance with the method given on pages 214 and 216 of the book by Prince and Vodges—"Mercury Arc Rectifiers and Their Circuits." In both figures curve A is the current curve neglecting resistance and curve B the current curve including resistance. The equations from which the various components were calculated are as follows:

$$\text{Curve } C = \frac{\sqrt{2}\, E \sin \dfrac{\pi}{\rho}}{X}$$

$$\text{Curve } D = \left[-\kappa - \sqrt{2}\, E \sin \frac{\pi}{\rho} \sin \left(\omega t_1 - \tan^{-1} \frac{x}{r} \right) \right] \epsilon^{-\frac{V(t - t_1)}{L}}$$

$$\text{Curve } E = \kappa = \frac{J}{4} \text{ for Fig. 2A} = \frac{J}{2} \text{ for Fig. 2c.}$$

$$\text{Curve } F = \frac{\sqrt{2}\, E \sin \dfrac{\pi}{\rho} \sin \left(\omega t - \tan^{-1} \dfrac{x}{r} \right)}{\sqrt{x^2 + r^2}}$$

$$\text{Curve } G = \frac{\sqrt{2}\, E \sin \dfrac{\pi}{\rho} \sin \left(\omega t - \dfrac{\pi}{2} \right)}{X}$$

The calculations are based on the assumption of 3 per cent commutating reactance and 1½ per cent resistance per phase, in order to check the statement in the Treatise of Dallenbach and Gerecke that the resistance may be neglected if it is less than ½ the reactance.

Examination of the current waves and their components shows that while the components are very different under two conditions the resultant waves are almost identical. This is because the error introduced in assuming no resistance and therefore no decrement in the transient accompanying the start of the commutation is well balanced by the error due to neglecting the effect of resistance on the phase and magnitude of the alternating component of the commutating current.

For the connection of Fig. 2c, when resistance is considered, commutation occurred between 2.7 deg. in advance of the time when the open-circuit phase voltages are equal and a point 35 deg. later than this time. If resistance is neglected, commutation is from 0 deg. to 36 deg. Since for normal designs the resistance is from 1/3 to 1/5 of the reactance it is evident that the resistance may safely be neglected.

The effect of variations in arc drop and of the ripple in the d-c. current has been found to be of approximately the same order of magnitude as the effect of resistance and has similarly been neglected.

Test readings of d-c. saturation of power transformers show that the per cent exciting current increases approximately as the per cent saturating current while the core and stray losses increase very roughly as the square of the per cent saturating current for saturations of the order of 10 per cent or less. A saturating current of more than 1 or 2 per cent, therefore, should not be neglected. The records of commercial test which I have available all indicate less than one per cent saturating current.

The Theory of Thermal Breakdown of Solid Dielectrics

BY P. H. MOON*

Member, A. I. E. E.

Synopsis.—*The purpose of the paper is to correlate the work which has been done on thermal breakdown and to put it in a form in which it can be used by the electrical engineer in the calculation of breakdown voltage. Besides a treatment of the Fock theory, the paper includes the derivation of new formulas for breakdown of very thin and very thick samples and for internal temperature rise and current. A startling fact brought out by this analysis is that before breakdown the internal temperature never rises more than about 10 deg. cent. above ambient temperature. The Fock tables of breakdown voltage are extended to cover a wider range of temperature and thickness, and the results are put in several convenient forms for numerical calculation. Experimental data are also presented to verify the theory.*

* * * * *

THE theory of thermal or pyro-electric breakdown of solid dielectrics has been in the process of development since 1922 as a result of the theoretical work of a number of investigators. It is not the purpose of the present paper to present a new theory of thermal breakdown, but rather to put the complete theory in such a form that it can be readily used in the computation of breakdown voltage.

The theory of breakdown of solid dielectrics as a process of thermal instability was first suggested by K. W. Wagner[1] and later modified by Kármán,[4] Rogowski,[2] and Dreyfus,[3] but it was not until 1927 that the high order of mathematical ability of V. Fock[5] gave us the complete solution not only for the resistivity law $\rho = \rho_0 \epsilon^{-\gamma T}$ but also for $\rho = \rho_1 \epsilon^{\beta_1/T}$.

The present discussion follows Fock's original paper with the following changes:

a. The assumptions are explicitly stated.

b. The three tables of results of the Fock theory have been extended to cover *all thicknesses* and a wider range of temperatures.

c. The breakdown voltage has been calculated hitherto by a tedious cut-and-try process using all three tables. In the present paper this has been simplified and the results given in the form of new curves and tables (Appendix C), thus greatly reducing the labor required for breakdown calculations. Expressions are also derived for the special cases of very great and very small thickness.

d. The results are extended to include not only breakdown but also internal temperature rise and current for any condition up to breakdown.

Mathematical Theory

Consider a plate of homogeneous solid dielectric of thickness d and of large area (Fig. 1). The electrodes are of conducting liquid at temperature T_0 or of metal so thin that temperature drop in them is negligible. The assumptions are:

1. All heat flow is in the z-direction. This will be approximately true in practise if the ratio of plate thickness to electrode size is very small.
2. The material is perfectly homogeneous with no weak spots.
3. Only the direct-current case is considered, with the voltage raised so slowly that thermal equilibrium obtains at all voltages below the breakdown value.
4. The dielectric is in a homogeneous electric field.
5. No electrolytic polarization is present. That is, except for the effect of temperature, the resistivity of the insulating plate is independent of z.
6. The thermal conductivity is independent of temperature.

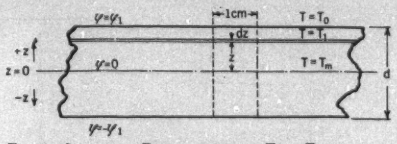

Fig. 1—Sample of Dielectric with Thin Electrodes

*Derivation.** Fock has shown that the equation for thermal equilibrium is

$$k_1 \frac{dT}{dz} + \frac{\varphi}{\rho} \frac{d\varphi}{dz} = 0 \qquad (22a)$$

whose solution is

$$\varphi^2 = 2 k_1 \int_T^{T_m} \rho(T)\, dT \qquad (23)$$

where $\rho(T)$ is the resistivity of the dielectric expressed as a function of temperature. Boundary conditions are introduced, resulting in two simultaneous equations:

$$\left.\begin{aligned} & k_1 \int_{T_1}^{T_m} \frac{dT}{\sqrt{2 k_1 \int_T^{T_m} \rho(T)\, dT}} - \frac{\lambda d}{2 \varphi_1}(T_1 - T_0) = 0 \\ & 2 k_1 \int_{T_1}^{T_m} \rho(T)\, dT - \varphi_1^2 = 0 \end{aligned}\right\} \qquad (27)$$

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1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

*See Appendix A for list of symbols.

Substitution of $\rho(T) = \rho_0 \epsilon^{-\gamma T}$. Equation (27) applies to any form of resistivity law: $\rho = \rho(T)$. For the law, $\rho = \rho_0 \epsilon^{-\gamma T}$, these equations reduce to

$$\left.\begin{aligned} &\frac{2}{\gamma}\epsilon^{\frac{\gamma T_m}{2}} \tan^{-1}\sqrt{\frac{\epsilon^{-\gamma T_1}}{\epsilon^{-\gamma T_m}}-1}-\frac{c}{v}(T_1-T_0)=0 \\ &\epsilon^{-\gamma T_1}-\epsilon^{-\gamma T_m}=\frac{\gamma}{2\,k_1\,\rho_0}\,\varphi_1^2=v^2 \end{aligned}\right\} \quad (31)$$

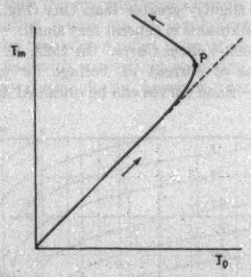

FIG. 2—RELATION BETWEEN MAXIMUM AND AMBIENT TEMPERATURES IN A SAMPLE OF DIELECTRIC

Breakdown occurs at P

where

$v = \sqrt{\dfrac{\gamma}{2\,k_1\,\rho_0}}\,\varphi_1$ which is *directly proportional to applied voltage.*

and

$c = \dfrac{\lambda\, d}{2\,k_1}$ which is *directly proportional to thickness.*

Eliminating the unknown T_1 in (31), we obtain

$$\frac{2\,v}{c}\,\epsilon^{\frac{\gamma T_m}{2}}\tan^{-1}\left(v\,\epsilon^{\frac{\gamma T_m}{2}}\right)-l\,n\,\frac{1}{v^2}\left(\frac{v^2\,\epsilon^{\frac{\gamma T_m}{2}}}{1+v^2\,\epsilon^{\gamma T_m}}\right)+\gamma\,T_0=0 \quad (32)$$

For a given material at a given voltage, the only variables are T_0 and T_m. Thus (32) can be solved for one in terms of the other, giving a curve like that of Fig. 2. Here the maximum temperature (T_m) occurring in the center of the insulating plate is plotted against the ambient temperature (T_0). Fig. 2 shows that if a sample of dielectric be tested by applying a constant voltage and gradually raising the ambient temperature, the maximum temperature T_m will be practically the same as T_0 through a wide range. At sufficiently high values of T_0, however, T_m becomes somewhat higher than T_0 (as shown by the departure of the curve from the straight line). Finally at P thermal instability results and the sample punctures.

The Breakdown Condition. The condition for thermal instability is introduced by differentiating (32) and equating to zero, which gives the particular T_0 and T_m corresponding to the point P. Using the resulting new equation in conjunction with (32), we obtain

$$v = \epsilon^{-\frac{\gamma T_0}{2}}\,X(c)$$

or

$$V_b = 2\sqrt{\frac{2\,k_1\,\rho_0}{\gamma}}\,(10)^{-0.217\,\gamma T_0}\,X(c) \quad (35)$$

where V_b is the breakdown voltage for the sample and $X(c)$ is a function of thickness which is plotted in Fig. 3.

Equation (35) shows that the breakdown voltage is an exponential function of the ambient temperature and is directly proportional to the square root of the resistivity, while Fig. 3 shows the effect of thickness.

Temperature Rise Within the Dielectric. The thermal theory of breakdown concerns itself with the voltage at which instability occurs—the puncture voltage of equation (35). However, the investigator may be interested also in conditions at voltages below V_b and may wish to know the maximum temperature within the dielectric or the shape of the voltage-current curve. The former may be obtained from the relation:

$$\left(\frac{\gamma\,T_0}{2}+2.3026\,l\,g\,v\right)=2.3026\,l\,g\,\frac{\zeta}{\sqrt{1+\zeta^2}}-\frac{\zeta}{c}\tan^{-1}\zeta \quad (36)$$

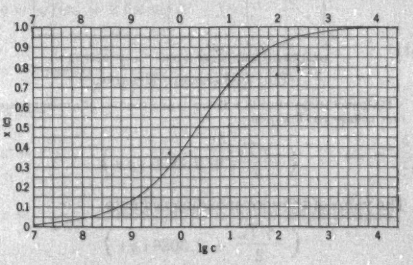

FIG. 3—THE FUNCTION x (c)

used in calculating breakdown on the basis of the resistivity law: $\rho = \rho_0\,\epsilon^{-\gamma\theta}$

c is a parameter which is directly proportional to thickness (See Appendix A)

For any given c, the left side can be plotted as a function of ζ or of $l\,g\,\zeta$. But

$$2.3026\,l\,g\,\zeta=\frac{\gamma\,T_m}{2}+2.3026\,l\,g\,v$$

so

$$\left(\frac{\gamma\,T_0}{2}+2.3026\,l\,g\,v\right)$$

can be plotted against

$$\left(\frac{\gamma\,T_m}{2}+2.3026\,l\,g\,v\right)$$

which gives the set of curves of Fig. 4. These curves allow the calculation of the internal temperature rise for any material of any thickness and at any voltage.

Thus the determination of the internal temperature at the mid-plane of the insulating plate is made from Fig. 4 with very little trouble. The value of c is computed as in breakdown measurements, which determines the particular curve to be used. The given voltage and ambient temperature determine the abscissa:

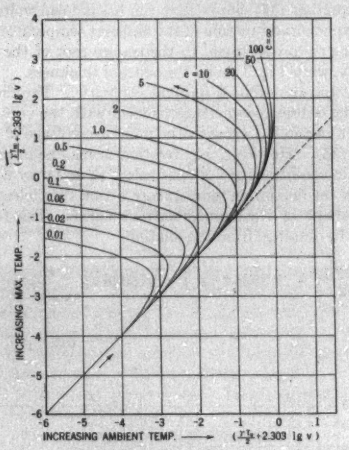

FIG. 4—INTERNAL HEATING

Maximum temperature in a dielectric as a function of ambient temperature, voltage, and thickness

$$\left(\frac{\gamma T_0}{2} + 2.3026\, l g\, v\right).$$

The value of the corresponding ordinate,

$$\left(\frac{\gamma T_m}{2} + 2.3026\, l g\, v\right)$$

is read from the curve and the actual T_m obtained.

These curves show that *over a wide range of voltage, there is no appreciable rise in internal temperature.* Near the breakdown voltage, however, the internal temperature rises rapidly as indicated by the fairly sudden departure of the curves from the linear relationship. But even at the instant of breakdown, the internal temperature rise is not very great. Reference to Fig. 4 shows that throughout a wide range of thickness, the curves at breakdown only rise above the straight line by approximately

$$\frac{\gamma T_m}{2} - \frac{\gamma T_0}{2} = 0.55$$

Thus the maximum internal temperature rise does not exceed about

$$T_m - T_0 = \frac{2\,(0.55)}{\gamma}$$

which, for an ordinary value of $\gamma = 0.1$, becomes

$$T_m - T_0 = \frac{2\,(0.55)}{0.1} = 11^\circ\ \text{C}.$$

This seems surprisingly low in view of the usual conception of a temperature rise of several hundred degrees. For the very thickest samples the temperature rise is seen to be slightly greater than this (Fig. 4), but the effect of thickness is in general very small.

The Current-Voltage Curve. In 1922, K. W. Wagner gave curves of current *vs.* voltage for various solid dielectrics. Such curves can be obtained also from the

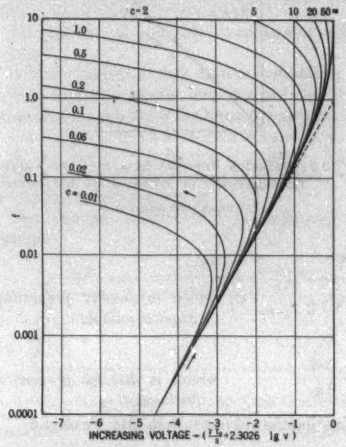

FIG. 5

Values of $f\left[c, \left(\frac{\gamma T_0}{2} + 2.30\, l g\, v\right)\right]$

Used in obtaining current density:

$$I = \frac{2}{v\,d}\sqrt{\frac{2\,k_1}{\gamma\,\rho_0}}\cdot f\left[c, \left(\frac{\gamma T_0}{2} + 2.30\, l g\, v\right)\right]$$

above theory. An equation for current density as a function of v and ζ is easily found:

$$I = \frac{2}{v\,d}\sqrt{\frac{2\,k_1}{\gamma\,\rho_0}}\,\zeta \tan^{-1}\zeta$$

$$= \frac{2}{v\,d}\sqrt{\frac{2\,k_1}{\gamma\,\rho_0}}\cdot f\left[c, \left(\frac{\gamma T_0}{2} + 2.3026\, l g\, v\right)\right] \quad (38)$$

The function $f\left[c, \left(\frac{\gamma T_0}{2} + 2.3026\, l g\, v\right)\right]$ has been plotted in Fig. 5 to facilitate the calculation of current density in dielectrics at any temperature or voltage.

From the known values of c, T_0, and v, the corresponding value of f is found from the curves; and the actual current density is then obtained by multiplying by

$$\frac{2}{v\,d}\sqrt{\frac{2\,k_1}{\gamma\,\rho_0}}.$$

To show the actual shape of the current-voltage curve, a special case has been plotted by the use of Fig. 5 and equation (38). This is shown in Fig. 6. It will be noted that the material obeys Ohm's law to a close approximation nearly up to breakdown. Of course, close to the breakdown point there is an internal temperature rise with a resulting reduction in resistance. Fig. 6 shows very clearly, however, that the ordinary

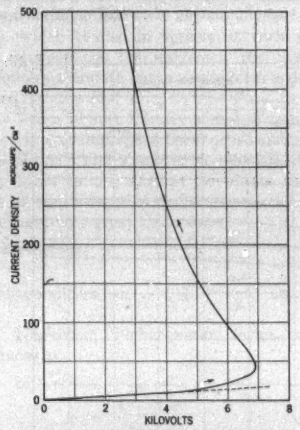

FIG. 6—CURRENT DENSITY IN A SAMPLE OF DIELECTRIC

$\rho_0 = 10^{28}$ ohm-cm.
$\gamma = 0.10$
$T_0 = 400$ deg. K
$k_1 = 0.010$
$\lambda = 0.020$
$d = 1.0$ cm.
$c = 1.0$

departure from Ohm's law obtained on all insulating materials, even at low voltages, cannot be due to internal heating.

Why is it then, that ordinary dielectric resistance tests show such a marked departure from Ohm's law* even at low voltages? It seems likely that this is entirely due to the space-charge effect of the migrating ions (electrolytic polarization). The entire theory of breakdown and internal heating given in this paper is based upon the postulate of *no electrolytic polarization*. Thus it would not be expected that the results here given would agree with resistivity or breakdown tests made in the usual manner. *The resistivity to be used in these calculations must be the unpolarized resistivity and the resulting calculated values will be for the electrolytically unpolarized dielectric.* Recent research at the Massachusetts Institute of Technology with non-polarizing electrodes of molten $Na\,NO_3$ and samples of glass or fused quartz show that Ohm's law is obeyed over a wide range reaching nearly to the breakdown point, a result which is quite in agreement with the above theory. The use of unpolarized resistivity values in the formulas for breakdown voltage also gives predicted puncture voltages which check the experimental values obtained with non-polarizing electrodes.

*See, for instance, H. H. Poole, Bibliography 10.

DERIVATION OF EXPRESSIONS FOR BREAKDOWN VOLTAGE

(Using the law: $\rho = \rho_1\,\epsilon^{\beta_1/T}$)

Equations (27) are valid for any form of resistivity function. We shall now use the law:

$$\rho\,(T) = \rho_1\,\epsilon^{\beta_1/T} \tag{2}$$

This is introduced into (27) and the condition of instability is used, resulting in the three final equations:

$$\frac{1}{x_0} = \frac{1}{x_m + 2\,l\,n\sec\alpha} - \frac{2\,v}{c}\int_0^{\alpha}\frac{\tan\eta\,d\eta}{(x_m+2\,l\,n\sec\eta)^2\sqrt{f(x_m+2\,l\,n\sec\eta)-f(x_m)}}$$

$$\left.\begin{aligned} v &= \sqrt{f(x_m + 2\,l\,n\sec\alpha) - f(x_m)} \\ c &= 2\,v\sec^2\alpha\int_0^{\alpha}\frac{\cos\eta\sin\eta\,d\eta}{\sqrt{f(x_m+2\,l\,n\sec\eta)-f(x_m)}} \end{aligned}\right\} \tag{41c}$$

where

$$x = \frac{\beta_1}{T}, \qquad x_0 = \frac{\beta_1}{T_0}, \qquad x_m = \frac{\beta_1}{T_m}$$

$$c = \frac{\lambda\,d}{2\,k_1}$$ (directly proportional to the thickness d)

$$v = \frac{\varphi_1}{\sqrt{2\,k_1\,\rho_1\,\beta_1}}$$ (directly proportional to the applied voltage) and α is defined by the relation:

$$\cos\alpha = \frac{\epsilon^{x_m/2}}{\epsilon^{x_1/2}}$$

while η is merely a variable of integration.

$$f(x_m) = \int_{-\infty}^{x_m}\frac{\epsilon^x}{x}\,d\,x - \frac{\epsilon^{x_m}}{x_m} = E\,i\,(x_m) - \frac{\epsilon^{x_m}}{x_m}$$

$$= \frac{\epsilon^{x_m}}{x_m^2}\left[1 + \frac{2\,!}{x_m} + \frac{3\,!}{x_m^2} + \ldots\right] \tag{42}$$

There seems to be no way of performing these integrations analytically and thus simplifying the results.

However, the integrals can be evaluated graphically or mechanically for any given values of x_m and α and the calculated values can be tabulated.

Calculation of Breakdown Voltage

(For $\rho = \rho_1 \epsilon^{\beta_1/T}$)

Three methods of calculation will be discussed below: the original Fock method and two modifications devised by the writer to simplify the work. Special cases for very small and for very great thickness will be given and an approximate method for using equation (35) will be discussed.

First Method. Equation (41c) gives three factors: $\frac{1}{x_0}$, v, and c which are functions of x_m and α only. Fock tabulated these factors, making one table for $1/x_0$, one for v and one for c. The range of the tables was $12 \leq x_m \leq 22$ and $0 < \alpha < 60°$. In using the tables in practical work, it was found that the range was often insufficient, and thus further values were computed to extend the tables to all thicknesses ($0 < \alpha < 90°$) and to lower temperatures (x_m up to 42).

The tables of Appendix B are used as follows:

Values of c and of x_0 are first calculated from the known properties of the sample of dielectric.

$$\begin{cases} c = \dfrac{\lambda d}{2 k_1} \\ x_0 = \beta_1/T \end{cases}$$

Table 1 is then entered with the above value of c and the proper value of α is thus determined. This value of α and the known $1/x_0$ are used to fix x_m from Table 2. The values of x_m and α are used in Table 3 to determine the function v which is directly proportional to breakdown voltage.

It will be noted that for small values of α, Table 1 is independent of x_m so that the first step, the determination of α, is simple. In many cases, however, an assumption must first be made as to the value of x_m and several trials may be necessary before both Tables 1 and 2 are satisfied.

Second Method. The three tables of Appendix B give v, c and $1/x_0$ in terms of the parameters x_m and α. In using the tables we eliminate these two parameters and finally obtain v (proportional to breakdown voltage) in terms of c (proportional to thickness) and x_0 (inversely proportional to ambient temperature). Obviously this elimination of x_m and α for each numerical calculation is unnecessary—it should be done once for all in making the table so that $l g v$ can be obtained without the intermediate use of x_m and α.

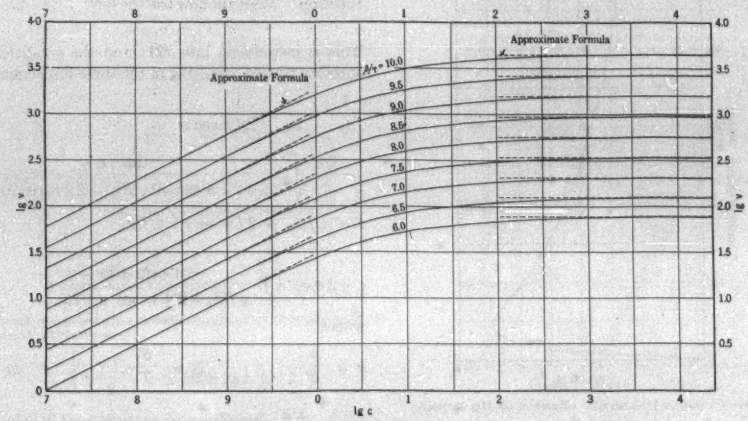

Fig. 7—Breakdown Voltage

calculated according to the thermal theory for the resistivity law:

$\rho = \rho_1 (10)^{\beta/T}$

This has been done in Appendix C, where the three tables have been reduced to one by the elimination of the two intermediate parameters. In making this change it was thought advisable to change the resistivity law from Naperian base to the base ten, since such has been found much more convenient for calculations. Thus the resistivity law

$$\rho = \rho_1 (10)^{\beta/T} \tag{2a}$$

is used, where ρ_1 has the same value as previously and

$$\beta = 0.434295\, \beta_1$$

The table of Appendix C will thus be found much easier to use than the previous tables, especially since

it eliminates most of the troublesome interpolation in tables of double entry. Fortunately the variation of $l\,g\,v$ with β/T is very nearly a linear one, so linear interpolation is permissible even though the tabulated values of β/T are fairly far apart.

The results of the table have been plotted in Fig. 7 to eliminate all interpolation. The curves also give a much better picture of the results than can be obtained from the table. It will be noted that for small values of thickness ($l\,g\,v < 9.0 - 10$, approximately) the $l\,g\,v$-curves are straight lines having the slope of exactly 0.500. This shows that breakdown voltage of thin samples varies as the square root of the thickness, a conclusion previously reached by Rogowski and Kármán.

The curves also show that for great thickness the breakdown voltage becomes almost independent of thickness, which is also in agreement with previous results. Between the two extremes, the breakdown voltage follows a smooth curve whose form is definitely established for the first time by the Fock treatment. That $l\,g\,v$ varies almost linearly with $1/T$ is indicated by the nearly constant space between all curves.

Special Cases. The shape of the curves of Fig. 7 naturally leads to an attempt to obtain analytical solutions of equation (41c) in the two special cases of very small and very great thicknesses.

a. Small Thickness

For this condition, $\sec\alpha \simeq 1 + \frac{\alpha^2}{2}$ and $l\,n\sec^2\alpha \simeq \alpha^2$. Neglecting higher-power terms, equation (41c) reduces to

$$\left.\begin{aligned} v &= \frac{\epsilon^{\frac{x_m}{2}}}{x_m}\,\alpha \\ c &= 2\,\alpha^2 \\ \frac{1}{x_0} &= \frac{1}{x_m}\left[1 - \frac{1}{x_m}\right] \end{aligned}\right\} \qquad (41d)$$

which gives

$$v = \sqrt{\frac{c}{2}}\;\frac{\epsilon^{\frac{x_0}{4}}\,\epsilon^{\frac{1}{2}\sqrt{(x_0/2)^2 - x_0}}}{(x_0/2) + \sqrt{(x_0/2)^2 - x_0}} \qquad (49)$$

or

$$l\,g\,v = \frac{1}{2}\,l\,g\,\frac{c}{2} + 0.4343\,\frac{x_m}{2} - l\,g\,x_m \qquad (49a)$$

where

$$x_0 = \beta_1/T = 2.3026\,\beta/T$$

and x_m is obtained from the last equation of (41d).

It will be noted that the breakdown voltage is proportional to the square root of the thickness as previously enunciated for thin samples. The results calculated by (49a) are shown dotted on the left side of Fig. 7.

A comparison was made of tabulated values (Appendix C) and values calculated from the equations for small thickness:

$$\begin{cases} 1/x_0 = 1/x_m\,[1 - 1/x_m] & (46) \\ l\,g\,v = 0.434295\left(\frac{x_m}{2}\right) + l\,g\,\frac{\alpha}{x_m} & (43a) \end{cases}$$

Calculations were made at $\alpha = 2.5°$ and the results are given in Table I. This shows a good agreement between the two and thus constitutes a check of the above equations.

TABLE I

	$1/x_0$		$l\,g\,v$	
x_m	Formula	Table	Formula	Table
12	0.076389	0.07639	0.16640	0.1666
14	0.066326	0.06632	0.53375	0.5340
16	0.058594	0.05859	0.91004	0.9103
18	0.052469	0.05247	1.29319	1.2934
20	0.047500	0.04750	1.68173	1.6820
22	0.043388	0.043390	2.07463	2.0749
32	0.030273	0.030270	4.08338	4.0840
42	0.023242	0.023239	6.13675	6.1369

b. Great Thickness ($\alpha \to \pi/2$).

Let ν be the small angle given by $\nu = \pi/2 - \alpha$. Expanding in series in terms of ν, we obtain from (41c):

$$v = \frac{\epsilon^{\frac{x_m}{2}}}{(x_m + 2\,l\,n\,1/\nu)}\sqrt{1 + \frac{2\,!}{x_m + 2\,l\,n\,1/\nu} + \ldots} \qquad (51)$$

$$c = 2\,F_1\,(x_m)\,\frac{\epsilon^{\frac{x_m}{2}}\sqrt{1 + \frac{2\,!}{x_m + 2\,l\,n\,1/\nu} + \ldots}}{(x_m + 2\,l\,n\,1/\nu)} \qquad (52)$$

$$\frac{1}{x_0} = \frac{1}{x_m + 2\,l\,n\,1/\nu} \qquad (57)$$

from which

$$v = \frac{\epsilon^{\frac{x_0}{2}}}{x_0}\sqrt{1 + \frac{2\,!}{x_0} + \frac{3\,!}{x_0^2} + \ldots} \qquad (58)$$

or

$$l\,g\,v = 0.217\,x_0 - l\,g\,x_0 + \frac{1}{2}\,l\,g\left(1 + \frac{2\,!}{x_0} + \frac{2\,!}{x_0^2} + \ldots\right) \qquad (58a)$$

which gives the ultimate values given by the dotted lines on the right in Fig. 7.

Third Method. An examination of Fig. 7 shows that all the curves appear to be parallel. If this were strictly true, the results of the two special cases, (43a) and (58a), should be the same except for an additive

constant; that is, the variation of v with temperature should be the same at both extremes of c. To determine how far the curves depart from exact parallelism, values were computed by both equations. A constant was added to the values for small thickness in order to make the two columns identical for $\beta/T = 10.0$.

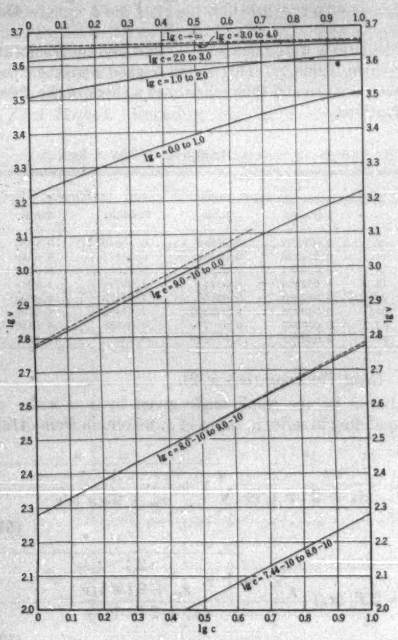

FIG. 8—BREAKDOWN VOLTAGE

calculated according to the thermal theory for $\beta/T = 10.000$

The results are given in Table II. It will be noted that the curves are not exactly parallel, but the difference is less than 1 per cent in all cases and is thus negligible in practical work.

TABLE II—COMPARISON OF FORMULAS FOR SMALL AND GREAT THICKNESSES

β/T	$lg\ v\ (d \to \infty)$	$lg\ v^*(d \to 0)$	Difference
6	1.89778	1.88762	−0.01016
7	2.32417	2.31806	−0.00611
8	2.76157	2.75814	−0.00343
9	3.20701	3.20552	−0.00249
10	3.65863	3.65863	0.00000
12	4.57565	4.57773	+0.00208
15	5.97509	5.97914	+0.00405
20	8.34663	8.35254	+0.00591

*Computed for $d \to 0$, and arbitrary constant added to give same results for both columns at $\beta/T = 10.0$.

This suggests the idea of using only *one* curve of voltage *vs.* thickness (*v. vs. c*) and applying a correction factor to take account of the temperature. The curve $\beta/T = 10.0$ was arbitrarily selected for the standard curve and has been replotted to a larger scale in Fig. 8. For the values of β/T other than 10.0, a correction must be applied to obtain the correct breakdown voltage. In other words, the ($\beta/T = 10.0$) curve of Fig. 8 must be shifted up or down a certain distance depending upon temperature. This correction was computed from equation (**58a**) and is plotted to a large scale in Fig. 9. (Also see Appendix D). The method is seen to be very much more rapid than the previous ones. It requires no interpolation whatsoever and is accurate enough for all ordinary purposes, the assumption of exact parallelism of all the curves introducing less than 1 per cent error in the worst possible case.

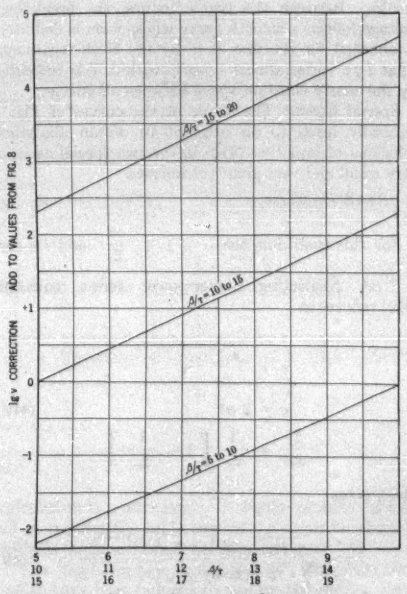

FIG. 9—CORRECTION CURVE

for calculation of breakdown voltage. Corrections to be applied to Fig. 8, to take into account the effect of changes in β/T

THICK ELECTRODES

The previous discussion has been for the case of no temperature drop in the electrodes. The electrodes are assumed to be in intimate contact with the dielectric and of negligible thickness, or they may be in the form of mercury or other molten conductor which is in contact with the specimen and which is assumed to be at

constant temperature. In practise, other arrangements are often used, and it is the purpose of this section to find what modifications must be made in such cases.

Consider first the case of thick plate electrodes in intimate contact with the specimen. The various thermal quantities to be considered are k_1, the thermal conductivity of the dielectric, k_2 the thermal conductivity of the electrodes, and λ_2 the dissipation coefficient for the outer surface of the electrode. The introduction of the extra quantity k_2 merely changes c. The former expression:

$$c = \frac{\lambda\, d}{2\, k_1} \tag{61}$$

changes to

$$c = \frac{\lambda_2\, k_2}{k_2 + \lambda_2 \delta} \cdot \frac{d}{2\, k_1} \tag{64}$$

Having calculated c by (**64**), the rest of the procedure in obtaining V_b is exactly the same as before.

The second possibility is that there is a temperature drop between dielectric and electrode as well as the drop in the sample, in the electrode, and at the outer face of the electrode. This again introduces only a modification in c.

$$c = \frac{\lambda_1\, \lambda_2\, k_2}{(\lambda_1 + \lambda_2)\, k_2 + \delta\, \lambda_1\, \lambda_2} \cdot \frac{d}{2\, k_1} \tag{69}$$

where λ_2 = watts cm.$^{-2}$ deg.$^{-1}$ between sample and electrode. It will be noted that (**69**) reduces to (**64**) of $\lambda_2 \to \infty$ (no temperature drop), while it reduces to (**61**) if $\lambda_2 \to \infty$ and $\delta \to 0$.

Experimental Verification

Introduction. Inge and Walther[13] proved in 1926 that the breakdown voltage of glass varied with temperature as predicted by the thermal theory. The variation with thickness was also checked to some extent, and thus there was considerable reason to believe that the thermal theory was correct. However, the only attempt to check the predicted numerical values was made by Inge and Walther[14] on porcelain in 1927. Flat specimens were clamped between other plates of the same material, and these in turn were held between heavy copper electrodes. The assumption was made that there was *no temperature drop at the various surfaces.* That is, λ was assumed to be infinite. But this is at variance with our experimental results which show that even with the greatest care in using flat surfaces and eliminating air films, λ has a value of about 0.051 watts cm.$^{-2}$ deg.$^{-1}$ between glass and metal surfaces. Using $\lambda \to \infty$, Inge and Walther calculated values of breakdown voltage which were considerably lower than the experimental values as shown in Table III. If our experimental value is used for λ, the discrepancy is still greater so that no check is possible. For instance, the first breakdown voltage in the table falls from 525 volts to 62 volts. This is no reflection on the experimental results of Inge and Walther nor upon the theory. Evidently the use of these comparatively thick samples of not particularly large area does not even approximate the condition of one-dimensional heat flow postulated in the theory.

TABLE III—COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES OF BREAKDOWN VOLTAGE FOR PORCELAIN AT 313° C. (INGE AND WALTHER)[14]

		V_b (volts)	
d	t	calc.	exp.
0.185 cm.	0.31	525	750
0.460	0.31	770	1050
0.900	0.31	1000	1370
0.185	0.91	320	590
0.460	0.91	495	810
0.900	0.91	660	1090

An attempt was made at the Massachusetts Institute of Technology to satisfy the requirements of the theory by using thin hollow spheres of fused quartz.[8] These specimens had a very large area in comparison with the thickness, the radius of curvature was so great that the samples could be considered as flat plates, and the surfaces were in excellent contact with the electrode liquid. Non-polarizing electrodes of sodium amalgam or sodium nitrate were used. Such tests satisfy all conditions imposed in the derivation and should thus give results which can be used to test the quantitative validity of the thermal theory.

In order to calculate breakdown voltage, it was first necessary to obtain data on the thermal conductivity, the surface heat dissipation coefficient, and the unpolarized resistivity of the materials as a function of temperature.

Unpolarized Resistivity. With soda-lime glass used with ordinary electrodes and direct current, the sodium ions under the action of the field drift away from the anode and gradually produce there a poorly-conducting layer deficient in sodium. This well-known fact was discovered by Warburg in 1884. This electrolytic polarization manifests itself in the following ways:

1. The apparent resistivity decreases with an increase in voltage. (The current is not a linear function of voltage.)
2. The apparent resistivity increases with time.
3. It is a function of thickness of sample.
4. After the passage of a considerable quantity of electricity, a change is produced in the structure of the glass near the anode due to the deficit of sodium. This usually gives the glass a milky appearance and the anode surface is apparently etched.

Our tests of unpolarized resistivity eliminated these effects by the use of electrodes of molten $NaNO_3$ or $NaNO_2$, which supplied the glass with Na^+ ions and thus eliminated electrolytic polarization. That polarization was actually prevented is indicated by the absence of its four manifestations:

1. The unpolarized resistivity was found to be *independent of voltage*. This was true, not only for soda-lime glass but also for Pyrex, G-1 lead glass, and fused quartz.

2. No effect of time could be detected. To check this conclusion further, a sample of soda-lime glass was used with molten solder electrodes. The ammeter read 28 milliamperes initially, but the current gradually fell until at the end of two mintues it had reached 4 milliamperes and was still decreasing. Sodium nitrite electrodes were then substituted for the lead, giving a constant current of approximately 28 milliamperes. Since the currents were measured with an ordinary milliammeter, nothing can be said as to changes which may have occurred in the current during the first second. However, it seems doubtful if any marked change with time exists.

3. Samples of soda-lime glass having decidedly different wall thicknesses were tested and gave practically the same resistivity, indicating that the effect of thickness was negligible.

4. Even after use with heavy currents for hours, the samples of all glasses and of fused quartz remained clear and uncolored. No etching was present such as was obtained with mercury electrodes. This indicates that sodium was being supplied from the electrodes at the proper rate.

The above tests appear to show that the method gives the true unpolarized resistivity. It also proves that at high temperatures the conduction in these materials is due, largely if not entirely, to the sodium ion. This is present in all the glasses and exists in the fused quartz as an impurity.

Measurements were made on tubes of several materials at temperatures ranging from about 300 to 380 deg. cent., the results were plotted to a large scale, and the resistivity constants were determined. (Table IV.)

TABLE IV—UNPOLARIZED RESISTIVITY

	ρ_1	β
Soda-lime glass (Kimble Glass Co.)	0.0188	4,320
Pyrex	0.0412	4,750
G-1 Lead Glass (Corning Glass Works)	5.52×10^{-4}	6,100
Fused Quartz (The Thermal Syndicate)	0.0140	7,100

Thermal Conductivity. Thermal conductivity of several substances was measured on flat plates using the generally accepted hot-plate apparatus with guard rings. The results were found to follow the law:

$$k_1 = K\,(10)^{-P/T}$$

and the values of the constants K and P are given in Table V.

Surface Temperature Drops. From these same tests, the following values were obtained for λ:

Glass-copper, $\lambda = 0.051$

Copper-air, $\lambda = 0.0013$

TABLE V—THERMAL CONDUCTIVITY

	K	P
Fused quartz	0.0329	235.5
American window glass	0.1818	499
Pittsburgh plate glass	0.1493	451

Of course, these values depend upon the exact condition of the surfaces and are therefore to be regarded only as rough approximations. However, they give some idea of what values of λ can be expected.

To obtain the temperature drop between glass and molten $NaNO_3$, the breakdown-test apparatus was utilized. A thin glass sphere was filled with the molten salt and was immersed in more of the salt held in a heavily-lagged container. A small heating coil immersed in the salt within the tube furnished a known amount of heat which passed through the sides of the sphere into the surrounding sodium nitrate. A guard ring arrangement prevented most of the heat-flow upward into the tube. Temperatures were measured immediately inside and outside the glass walls by means of thermo-couples, and a small correction was applied for the temperature drop through the thin glass walls. The average of a number of tests made in this way was:

$$\lambda = 0.0403$$

This again is to be regarded as an approximate rather than an exact value.

Comparison of Calculated and Experimental Results. A sample of fused quartz tested at 364.5 deg. cent. had the following characteristics:

$d = 0.0365$ cm.

$\lambda = 0.0403$

$k_1 = 0.0329\,(10)^{-\frac{235.5}{637.6}} = 0.0140$

$\rho_1 = 0.0140$

$\beta = 7{,}100$

$T = 637.6$ deg. K.

$x_0 = 2.3026 \times 7100/637.6 = 25.64$

$x_m = 24.598$

$c = \lambda\, d/2\, k_1 = 0.0403\,(0.0365)/0.0280 = 0.0525$

Because of the small value of c, this is on the straight part of the curve of Fig. 7 and the approximate formula (**49a**) may be used. Thus,

$$\frac{1}{2}\, l\,g\,(c/2) = 9.210 - 10$$

$$l\,g\,v = (9.210 - 10) + 5.3414 - 1.3909 = 3.160$$

or

$$v = 1444$$

The calculated breakdown voltage is thus

$$V_b = 2\,(1444)\,\sqrt{4.60\,(0.0140)\,(0.0140)\,7100} = 7{,}310 \text{ volts}$$

The experimental value was *7,760 volts.*

A complete comparison of the results of thermal breakdown of fused quartz is given in Table VI which shows a reasonable agreement between calculated and experimental values. Each experimental value is the result of a single breakdown test. It is evident that

most of the discrepancies are due to experimental variations rather than to any fundamental difference between theory and experiment.

TABLE VI—COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES OF BREAKDOWN VOLTAGE FOR FUSED QUARTZ

d	c	$l\,g\,\nu$	ν	V_b (volts) (calculated)	V_b (volts) (experimental)
at 364.5° C. Sodium nitrate electrodes					
0.0365 cm.	0.0525	3.160	1,444	7,310	7,760
0.0300	0.0432	3.117	1,310	6,640	7,640
0.0510	0.0733	3.232	1,708	8,650	7,840
0.0105	0.0151	2.889	775	3,920	3,080
0.0076	0.0109	2.822	665	3,360	3,000
0.0096	0.0138	2.869	740	3,740	3,840
0.0285	0.0410	3.106	1,277	6,460	7,360
0.0265	0.0381	3.090	1,230	6,230	5,400
0.0205	0.0294	3.034	1,080	5,470	4,160
at 301.0° C. Sodium amalgam electrodes					
0.0465	0.0738	3.803	6350	30,600	26,500

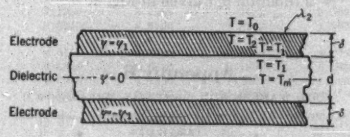

FIG. 10

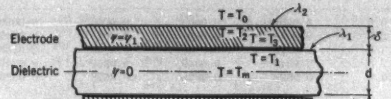

FIG. 11

FIGS. 10 AND 11—SAMPLES OF DIELECTRIC WITH THICK ELECTRODES

LIMITATIONS OF THE THEORY

The thermal breakdown theory as outlined in this paper is applicable to any thickness of any solid insulation operated at any temperature, subject of course to the limitations imposed by the original assumptions. Thus *the results apply only to the d-c. steady-state case for homogeneous materials tested with non-polarizing electrodes whose size is very great compared to the thickness of the sample.*

It should also be emphasized that this paper *treats only of thermal breakdown.* That other mechanisms of breakdown exist is a well-established fact* so that thermal breakdown occurs only for the higher temperatures and greater thicknesses, while other types of failure obtain in thin materials and at sufficiently low temperatures. As pointed out by the writer[9] the thermal theory gives a curve which rises without limit as the temperature is reduced, but the actual curve breaks due to other mechanisms which allow failure at lower voltages. Thus in calculating breakdown voltage, it is first necessary to be sure that the particular conditions of the problem will give a thermal breakdown and not one of another type.

*See, for instance, Bibliography 7 and 8.

In conclusion, the writer wishes to thank Messrs. H. W. Chapman, A. M. Ricciardelli, and G. B. Shaw who did the numerical calculations for the tables of Appendix B, Messrs. V. G. Mukhdjian and J. E. Keely who obtained the experimental values on unpolarized resistivity, and Messrs. V. C. Kauffman and M. A. Jones who furnished the data on thermal properties. Thanks are also due the Thermal Syndicate, Ltd. which kindly furnished the Vitreosil specimens used in the tests.

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Appendix A

LIST OF SYMBOLS

$$c = \frac{\lambda_1 \lambda_2 k_2}{(\lambda_1 + \lambda_2) k_2 + \delta \lambda_1 \lambda_2} \cdot \frac{d}{2 k_1} \quad \text{(general case)} \qquad (69)$$

$$c = \frac{\lambda d}{2 k_1} \quad \text{(thin electrodes)} \qquad (61)$$

d = thickness of dielectric sample (cm.)

$$f(x) = Ei(x) - \frac{\epsilon^x}{x}$$

k_1 = thermal conductivity of the dielectric (watts cm.$^{-1}$ deg.$^{-1}$)

k_2 = thermal conductivity of electrodes (watts cm.$^{-1}$ deg.$^{-1}$)

$l\,g$ = logarithm to the base 10

$l\,n$ = logarithm to the base ϵ

$$u\,(T) = -\,2\,k_1 \int_0^T \rho\,(T)\,d\,T$$

$$u'\,(T) = \frac{d}{d\,t}\,[u\,(T)]$$

$$v = \sqrt{\frac{\gamma}{2\,k_1\,\rho_0}}\,\varphi_1 \text{ for the law } \rho = \rho_0\,\epsilon^{-\gamma T}$$

$$v = \frac{\varphi_1}{\sqrt{2\,k_1\,\rho_1\,\beta_1}} \text{ for the law } \rho = \rho_1\,\epsilon^{\beta_1/T}$$

x = β_1/T

x_0 = β_1/T_0

x_1 = β_1/T_1

x_m = β_1/T_m

z = distance (cm.)

C = a constant

$E\,i\,(x)$ = the exponential integral

$$F_1(x_m) = \int_0^{\pi/2} \frac{\sin\eta\cos\eta\,d\,\eta}{\sqrt{f\,(x_m + 2\,l\,n\sec\eta) - f\,(x_m)}}$$

I = current density (amperes cm.$^{-2}$)

R = resistance (ohms)

T = temperature (deg. abs.)

T_0 = ambient temperature

T_1 = temperature in the dielectric at the surface

T_2 = temperature in the electrode at outer surface

T_3 = temperature in the electrode at inner surface

T_m = maximum temperature in the dielectric

T_b = thread temperature at breakdown

V = volts

V_b = breakdown volts

W = power (watts)

α = a parameter defined by: $\cos\alpha = \dfrac{\epsilon^{\xi_m/2}}{\epsilon^{\xi_1/2}}$

β = a constant of the dielectric defined by $\rho = \rho_1\,(10)^{\beta/T}$

β_1 = a constant of the dielectric defined by $\rho = \rho_1\,\epsilon^{\beta_1/T}$

γ = a constant of the dielectric defined by $\rho = \rho_0\,\epsilon^{-\gamma T}$

δ = thickness of electrode (cm.)

ϵ = 2.71828 . . . . .

ζ = $v\,\epsilon^{\frac{\gamma T_m}{2}}$

η = a variable of integration

λ = watts transmitted across one cm.^{2} of dielectric surface per deg. cent. drop in temperature at that surface

λ_1 = watts cm.$^{-2}$ deg.$^{-1}$ at surface between dielectric and electrode

λ_2 = watts cm.$^{-2}$ deg.$^{-1}$ at outer surface of electrode

ν = $\pi/2 - \alpha$

ϵ = a variable of integration

ρ = $\rho\,(T)$ = resistivity of dielectric (ohm-cm.)

ρ_0 = resistivity at zero temperature

ρ_1 = resistivity at $T \to \infty$

$$\sigma'\,(T) = \frac{d}{d\,t}\,(1/\rho)$$

φ = electric potential at any point (volts)

φ_1 = potential of an electrode (volts)

$X\,(c)$ = a function of c given in Fig. 3.

Appendix B

FOCK TABLES FOR THERMAL BREAKDOWN

(as amplified by Messrs. Chapman, Ricciardelli, and Shaw)

1. TABLE OF c

α	x_m = 12	14	16	18
0.0	0.0000	0.0000	0.0000	0.0000
2.5	0.0038	0.0038	0.0038	0.0038
5.0	0.0153	0.0153	0.0153	0.0153
7.5	0.0349	0.0349	0.0349	0.0349
10.0	0.0628	0.0628	0.0628	0.0628
12.5	0.0998	0.0998	0.0998	0.0998
15.0	0.1467	0.1467	0.1467	0.1468
17.5	0.2048	0.2048	0.2048	0.2049
20.0	0.2755	0.2755	0.2756	0.2757
22.5	0.3606	0.3607	0.3608	0.3610
25.0	0.4626	0.4629	0.4631	0.4634
27.5	0.5844	0.5850	0.5854	0.5857
30.0	0.7300	0.7310	0.7315	0.7321
32.5	0.9048	0.9058	0.9065	0.9074
35.0	1.1142	1.1161	1.1170	1.1180
37.5	1.3670	1.3690	1.3710	1.3730
40.0	1.6740	1.6770	1.6800	1.6830
42.5	2.0490	2.0540	2.0580	2.0630
45.0	2.5120	2.5190	2.5260	2.5310
47.5	3.0900	3.1010	3.1090	3.1160
50.0	3.8180	3.8330	3.8450	3.8560
52.5	4.7500	4.7710	4.7890	4.8040
55.0	5.9600	5.9900	6.0150	6.0370
57.5	7.5580	7.6040	7.6410	7.6710
60.0	9.7170	9.7870	9.8410	9.8860
62.5	12.6862	12.8047	12.8981	12.9609
65.0	16.9473	17.1235	17.2620	17.3592
67.5	23.2430	23.5132	23.7257	23.8502
70.0	32.9272	33.3905	33.7310	33.9840
72.5	48.7957	49.5212	50.0934	50.5279
75.0	76.5458	77.8150	78.8349	79.6214
77.5	129.882	132.415	134.423	135.993
80.0	247.550	253.166	257.644	261.188
82.5	567.371	582.616	594.286	604.673
85.0	1823.98	1883.50	1931.51	1970.41
87.5	13448.3	14011.0	14470.7	14848.8

TABLE OF c (*cont.*)

α	$x_m = 20$	22	32	42
0.0	0.0000	0.0000	0.0000	0.0000
2.5	0.0038	0.0038	0.0038	0.0038
5.0	0.0153	0.0153	0.0153	0.0153
7.5	0.0349	0.0349	0.0349	0.0350
10.0	0.0628	0.0628	0.0628	0.0630
12.5	0.0998	0.0998	0.0999	0.1002
15.0	0.1468	0.1468	0.1469	0.1472
17.5	0.2049	0.2050	0.2051	0.2052
20.0	0.2758	0.2758	0.2759	0.2760
22.5	0.3611	0.3612	0.3615	0.3617
25.0	0.4636	0.4637	0.4643	0.4646
27.5	0.5860	0.5863	0.5869	0.5872
30.0	0.7324	0.7329	0.7341	0.7347
32.5	0.9082	0.9086	0.9106	0.9115
35.0	1.1190	1.1200	1.1418	1.1427
37.5	1.3750	1.3750	1.3794	1.3811
40.0	1.6850	1.6860	1.6917	1.6941
42.5	2.0650	2.0670	2.0746	2.0804
45.0	2.5350	2.5380	2.5485	2.5532
47.5	3.1230	3.1260	3.1552	3.1697
50.0	3.8640	3.8700	3.8912	3.9038
52.5	4.8160	4.8240	4.8541	4.9795
55.0	6.0540	6.0660	6.1098	6.5462
57.5	7.6960	7.7150	7.7984	7.9391
60.0	9.9230	9.9500	10.0429	10.4291
62.5	13.0129	13.0579	13.1962	13.3161
65.0	17.4396	17.5077	18.1118	18.2965
67.5	24.0077	24.1145	24.5407	25.6467
70.0	34.1914	34.3663	34.9942	35.3325
72.5	50.8861	51.1827	54.5219	52.8242
75.0	80.2774	80.8256	82.4739	83.8047
77.5	137.289	138.392	147.701	144.259
80.0	264.120	266.592	274.225	279.874
82.5	612.714	619.551	641.103	651.856
85.0	2002.88	2030.36	2118.65	2186.82
87.5	15167.6	15440.0	16341.3	16948.7

2. TABLE OF l/x_0

α	$x_m = 12$	14	16	18
0.0	0.07639	0.06633	0.05859	0.05247
2.5	0.07639	0.06632	0.05859	0.05247
5.0	0.07638	0.06631	0.05858	0.05246
7.5	0.07635	0.06630	0.05857	0.05245
10.0	0.07633	0.06628	0.05855	0.05243
12.5	0.07629	0.06625	0.05854	0.p5253
15.0	0.07625	0.06623	0.05851	0.05240
17.5	0.07620	0.06619	0.05848	0.05238
20.0	0.07614	0.06614	0.05843	0.05235
22.5	0.07607	0.06608	0.05840	0.05231
25.0	0.07598	0.06602	0.05835	0.05227
27.5	0.07588	0.06595	0.05829	0.05222
30.0	0.07577	0.06586	0.05822	0.05217
32.5	0.07564	0.06576	0.05815	0.05211
35.0	0.07549	0.06565	0.05806	0.05204
37.5	0.07533	0.06552	0.05797	0.05196
40.0	0.07514	0.06538	0.05785	0.05188
42.5	0.07492	0.06522	0.05772	0.05177
45.0	0.07468	0.06504	0.05758	0.05165
47.5	0.07441	0.06484	0.05742	0.05152
50.0	0.07410	0.06459	0.05723	0.05137
52.5	0.07375	0.06432	0.05702	0.05120
55.0	0.07335	0.06401	0.05678	0.05101
57.5	0.07290	0.06368	0.05651	0.05079
60.0	0.07239	0.06330	0.05620	0.05053

2. TABLE OF l/x_0 (*cont.*)

α	$x_m = 12$	14	16	18
62.5	0.07181	0.06285	0.05585	0.05024
65.0	0.07114	0.06234	0.05544	0.04991
67.5	0.07038	0.06174	0.05497	0.04954
70.0	0.06951	0.06106	0.05443	0.04910
72.5	0.06849	0.06027	0.05380	0.04858
75.0	0.06730	0.05934	0.05306	0.04797
77.5	0.06589	0.05824	0.05217	0.04725
80.0	0.06417	0.05689	0.05109	0.04635
82.5	0.06202	0.05519	0.04971	0.04536
85.0	0.05915	0.05290	0.04784	0.04367
87.5	0.05473	0.04933	0.04490	0.04120

TABLE OF l/x_0 (*cont.*)

α	$x_m = 20$	22	32	42
0.0	0.04750	0.04339	0.03027	0.02324
2.5	0.04750	0.04339	0.03027	0.02324
5.0	0.04750	0.04339	0.03027	0.02324
7.5	0.04749	0.04337	0.03027	0.02323
10.0	0.04748	0.04337	0.03026	0.02323
12.5	0.04746	0.04336	0.03026	0.02323
15.0	0.04745	0.04334	0.03025	0.02323
17.5	0.04742	0.04333	0.03024	0.02322
20.0	0.04740	0.04331	0.03023	0.02322
22.5	0.04738	0.04328	0.03022	0.02321
25.0	0.04734	0.04325	0.03021	0.02321
27.5	0.04730	0.04323	0.03020	0.02320
30.0	0.04726	0.04319	0.03017	0.02319
32.5	0.04720	0.04314	0.03015	0.02317
35.0	0.04715	0.04310	0.03012	0.02316
37.5	0.04709	0.04304	0.03010	0.02314
40.0	0.04701	0.04298	0.03007	0.02310
42.5	0.04692	0.04291	0.03004	0.02308
45.0	0.04682	0.04283	0.03000	0.02306
47.5	0.04671	0.04274	0.02996	0.02302
50.0	0.04659	0.04263	0.02990	0.02301
52.5	0.04645	0.04250	0.02984	0.02296
55.0	0.04630	0.04238	0.02978	0.02291
57.5	0.04612	0.04223	0.02970	0.02286
60.0	0.04590	0.04205	0.02961	0.02279
62.5	0.04566	0.04185	0.02952	0.02270
65.0	0.04539	0.04162	0.02937	0.02265
76.5	0.04508	0.04136	0.02927	0.02255
70.0	0.04471	0.04105	0.02911	0.02244
72.5	0.04428	0.04069	0.02893	0.02231
75.0	0.04378	0.04026	0.02871	0.02215
77.5	0.04317	0.03984	0.02845	0.02195
80.0	0.04242	0.03911	0.02812	0.02169
82.5	0.04147	0.03829	0.02769	0.02132
85.0	0.04016	0.03717	0.02710	0.02106
87.5	0.03807	0.03537	0.02613	0.02072

3. TABLE OF $l\,g\,v$

α	$x_m = 12$	14	16	18
2.5	0.1666	0.5340	0.9103	1.2934
5.0	0.4684	0.8358	1.2123	1.5952
7.5	0.6467	1.0131	1.3894	1.7726
10.0	0.7723	1.1397	1.5160	1.8993
12.5	0.8715	1.2390	1.6153	1.9986
15.0	0.9534	1.3209	1.6973	2.0806
17.5	1.0236	1.3912	1.7676	2.1510
20.0	1.0854	1.4530	1.8296	2.2129

3. TABLE OF $l\,g\,v$

α	$x_m = 12$	14	16	18
22.5	1.1409	1.5086	1.8852	2.2687
25.0	1.1916	1.5595	1.9362	2.3196
27.5	1.2386	1.6066	1.9834	2.3668
30.0	1.2826	1.6509	2.0276	2.4112
32.5	1.3244	1.6927	2.0695	2.4532
35.0	1.3644	1.7326	2.1098	2.4935
37.5	1.4028	1.7713	2.1486	2.5324
40.0	1.4402	1.8089	2.1862	2.5702
42.5	1.4770	1.8458	2.2232	2.6074
45.0	1.5131	1.8822	2.2599	2.6442
47.5	1.5491	1.9185	2.2964	2.6809
50.0	1.5852	1.9549	2.3330	2.7176
52.5	1.6217	1.9917	2.3700	2.7549
55.0	1.6588	2.0291	2.4077	2.7928
57.5	1.6968	2.0675	2.4465	2.8318
60.0	1.7362	2.1073	2.4866	2.8722
62.5	1.7766	2.1488	2.5288	2.9146
65.0	1.8202	2.1929	2.5733	2.9594
67.5	1.8668	2.2401	2.6209	3.0074
70.0	1.9174	2.2914	2.6727	3.0597
72.5	1.9732	2.3479	2.7299	3.1174
75.0	2.0363	2.4118	2.7944	3.1825
77.5	2.1090	2.4858	2.8694	3.2582
80.0	2.1967	2.5749	2.9595	3.3492
82.5	2.3083	2.6883	3.0743	3.4652
85.0	2.4643	2.8467	3.2447	3.6271
87.5	2.7306	3.1168	3.5079	3.9029

TABLE OF $l\,g\,v$ (*cont.*)

α	$x_m = 20$	22	32	42
2.5	1.6820	2.0749	4.0840	6.1369
5.0	1.9838	2.3767	4.3854	6.4382
7.5	2.1611	2.5540	4.5628	6.6662
10.0	2.2879	2.6808	4.6897	6.7431
12.5	2.3871	2.7801	4.7890	6.8451
15.0	2.4692	2.8622	4.8711	6.9252
17.5	2.5396	2.9326	4.9417	6.9951
20.0	2.6016	2.9946	5.0037	7.0573
22.5	2.6573	3.0504	5.0597	7.1133
25.0	2.7084	3.1015	5.1108	7.1646
27.5	2.7556	3.1489	5.1556	7.2126
30.0	2.8000	3.1933	5.2029	7.2568
32.5	2.8422	3.2354	5.2453	7.2993
35.0	2.8825	3.2759	5.2859	7.3386
37.5	2.9215	3.3149	5.3252	7.3794
40.0	2.9595	3.3529	5.3635	7.4179
42.5	2.9968	3.3903	5.4011	7.4557
45.0	3.0337	3.4273	5.4384	7.4931
47.5	3.0705	3.4642	5.4756	7.5294
50.0	3.1074	3.5012	5.5130	7.5679
52.5	3.1448	3.5387	5.5509	7.5884
55.0	3.1828	3.5769	5.5896	7.6004
57.5	3.2220	3.6163	5.6294	7.6851
60.0	3.2627	3.6571	5.6708	7.7269
62.5	3.3053	3.6999	5.7143	7.7707
65.0	3.3504	3.7453	5.7700	7.8268
67.5	3.3987	3.7939	5.8098	7.8671
70.0	3.4513	3.8468	5.8637	7.9314
72.5	3.5095	3.9053	5.9511	7.9817
75.0	3.5751	3.9714	5.9907	8.0498
77.5	3.6513	4.0481	6.0691	8.1291
80.0	3.7431	4.1405	6.1635	8.2246
82.5	3.8600	4.2582	6.2838	8.3433
85.0	4.0233	4.4226	6.4519	8.5165
87.5	4.3012	4.7023	6.7377	8.8057

Appendix C

SIMPLIFIED TABLES FOR BREAKDOWN CALCULATIONS

$\beta/T = 6.000$

c	$l\,g\,c$	$l\,g\,v$	v
0.0038	7.7598	0.3006	1.998
0.0153	8.1847	0.6021	4.000
0.0349	8.5428	0.7786	6.006
0.0628	8.7980	0.9045	8.026
0.0998	8.9991	1.0024	10.056
0.1467	9.1664	1.0832	12.112
0.2048	9.3113	1.1518	14.184
0.2755	9.4401	1.2116	16.28
0.3607	9.5571	1.2648	18.40
0.4627	9.6653	1.3129	20.55
0.5846	9.7669	1.3568	22.74
0.7303	9.8635	1.3971	24.95
0.9051	9.9567	1.4347	27.21
1.1150	0.0473	1.4698	29.50
1.3676	0.1361	1.5029	31.84
1.675	0.2240	1.5343	34.22
2.050	0.3118	1.5639	36.63
2.514	0.4004	1.5920	39.08
3.092	0.4902	1.6192	41.61
3.820	0.5821	1.6448	44.14
4.753	0.0770	1.6694	46.71
5.961	0.7753	1.6927	49.28
7.561	0.8786	1.7151	51.89
9.717	0.9875	1.7365	54.51
12.6779	1.1021	1.7566	57.10
16.9220	1.2285	1.7760	58.70
23.1774	1.3641	1.7946	62.32
32.7480	1.5152	1.8125	64.94

$\beta/T = 7.000$

c	$l\,g\,c$	$l\,g\,v$	v
0.0038	7.5798	0.7307	5.379
0.0153	8.1847	1.0323	10.77
0.0349	8.5428	1.2088	16.17
0.0628	8.7980	1.3344	21.60
0.0998	8.9991	1.4328	27.09
0.1467	9.1664	1.5135	32.62
0.2048	9.3113	1.5821	38.20
0.2756	9.4403	1.6416	43.81
0.3608	9.5573	1.6950	49.55
0.4630	9.6656	1.7433	55.37
0.5852	9.7673	1.7872	61.26
0.7312	9.8640	1.8276	67.23
0.9061	9.9572	1.8611	72.62
1.1165	0.0480	1.9003	79.49
1.3696	0.1367	1.9337	85.84
1.678	0.2248	1.9650	92.25
2.056	0.3130	1.9943	99.75
2.522	0.4017	2.0229	105.4
3.104	0.4919	2.0504	112.3
3.837	0.5840	2.0756	119.0
4.776	0.6791	2.1000	125.9
5.996	0.7779	2.1234	132.9
7.612	0.8815	2.1464	140.1
9.796	0.9910	2.1682	147.3
12.814	1.1077	2.1883	154.3
17.129	1.2337	2.2076	161.3
23.502	1.3711	2.2253	168.0
33.330	1.5228	2.2493	177.6
49.348	1.6933	2.2588	181.5
77.348	1.8885	2.2735	187.7
131.08	2.1175	2.2869	193.6
249.06	2.3063	2.2980	198.6
567.3	2.7538	2.3072	202.9
1795.0	3.2541	2.3158	206.9

$\beta/T = 8.000$

c	$lg\,c$	$lg\,v$	v
0.0038	7.5798	1.1707	14.81
0.0153	8.1847	1.4719	29.64
0.0349	8.5428	1.6486	44.53
0.0628	8.7980	1.7739	59.42
0.09 8	8.9991	1.8726	74.58
0.1468	9.1667	1.9531	89.76
0.2049	9.3115	2.0219	105.2
0.2757	9.4404	2.0815	120.7
0.3610	9.5575	2.1348	136.4
0.4633	9.6659	2.1829	152.4
0.5856	9.7676	2.2266	168.5
0.7319	9.8645	2.2673	185.1
0.9071	9.9577	2.3051	201.9
1.1176	0.0484	2.3402	218.9
1.372	0.1374	2.3737	236.4
1.682	0.2258	2.4092	256.6
2.061	0.3141	2.4347	272.1
2.529	0.4029	2.4630	290.4
3.113	0.4932	2.4924	310.7
3.850	0.5855	2.5201	331.2
4.796	0.6809	2.5404	347.1
6.024	0.7799	2.5641	366.5
7.652	0.8838	2.5866	386.0
9.855	0.9937	2.6076	405.1
12.914	1.1101	2.6282	424.8
17.281	1.2376	2.6473	443.9
23.743	1.3755	2.6653	462.7
33.737	1.5281	2.6821	481.0
50.046	1.6994	2.6980	498.9
78.617	1.8955	2.7128	516.2
133.67	2.1250	2.7258	531.9
255.06	2.4066	2.7374	546.3
584.46	2.7668	2.7466	558.0
1869.1	3.2716	2.7542	567.8
13490.0	4.1300	2.7594	574.6

$\beta/T = 9.000$

c	$lg\,c$	$lg\,v$	v
0.0038	7.5798	1.6178	41.48
0.0153	8.1847	1.9194	83.06
0.0349	8.5428	2.0956	124.6
0.0628	8.7980	2.2218	166.7
0.0998	8.9991	2.3195	208.7
0.1468	9.1667	2.4006	251.5
0.2049	9.3115	2.4686	294.2
0.2758	9.4406	2.5288	337.9
0.3611	9.5576	2.5826	382.5
0.4636	9.6661	2.6303	426.9
0.5859	9.7678	2.6740	472.1
0.7323	9.8647	2.7149	518.7
0.9080	9.9581	2.7523	565.3
1.1188	0.0488	2.7878	613.5
1.374	0.1380	2.8213	662.7
1.684	0.2263	2.8524	711.9
2.065	0.3149	2.8818	761.7
2.534	0.4038	2.9098	812.5
3.121	0.4943	2.9369	864.8
3.861	0.5867	2.9629	918.1
4.811	0.6822	2.9876	971.9
6.047	0.7815	3.0117	1027.
7.684	0.8856	3.0342	1082.
9.903	0.9958	3.0547	1134.
12.982	1.1134	3.0749	1188.
17.387	1.2402	3.0941	1242.
23.915	1.3787	3.1128	1297.
34.021	1.5317	3.1295	1347.
50.553	1.7038	3.1346	1395.
79.573	1.9008	3.1586	1441.
135.65	2.1324	3.1724	1487.
259.68	2.4144	3.1834	1525.
597.81	2.7766	3.1930	1560.
1927.2	3.2849	3.2000	1585.
14115.	4.1497	3.2050	1603.

$\beta/T = 10.000$

c	$lg\,c$	$lg\,v$	v
0.0038	7.5798	2.0708	117.7
0.0153	8.1847	2.3720	235.8
0.0349	8.5428	2.5478	353.0
0.0628	8.7980	2.6745	472.6
0.0998	8.9991	2.7728	592.7
0.1468	9.1667	2.8529	712.7
0.2050	9.3118	2.9222	836.0
0.2758	9.4406	2.9821	959.6
0.3612	9.5577	3.0348	1083.
0.4637	9.6662	3.0827	1210.
0.5863	9.7681	3.1279	1343.
0.7329	9.8650	3.1681	1473.
0.9086	9.9584	3.2049	1603.
1.1200	0.0492	3.2407	1741.
1.375	0.1383	3.2738	1878.
1.686	0.2269	3.3055	2021.
2.067	0.3153	3.3353	2164.
2.538	0.4045	3.3636	2310.
3.126	0.4950	3.3907	2459.
3.869	0.5876	3.4160	2606.
4.822	0.6832	3.4397	2752.
6.063	0.7827	3.4644	2913.
7.709	0.8870	3.4872	3070.
9.940	0.9974	3.5077	3219.
13.038	1.1142	3.5278	3371.
17.474	1.2424	3.5472	3525.
24.053	1.3812	3.5657	3679.
34.249	1.5347	3.5821	3820.
50.952	1.7072	3.5973	3956.
80.328	1.9049	3.6117	4090.
137.20	2.1374	3.6238	4205.
263.32	2.4205	3.6353	4318.
608.33	2.7841	3.6450	4416.
1972.5	3.2940	3.6522	4490.
14613.	4.1647	3.6568	4537.

Appendix D

CORRECTION FACTORS TO BE USED WITH APPENDIX C, TABLE FOR $\beta/T = 10.0$

β/T	$lg\,v$ correction
5.0	−2.1710
5.5	−1.9684
6.0	−1.7608
6.5	−1.5493
7.0	−1.3345
7.5	−1.1169
8.0	−0.8971
8.5	−0.6752
9.0	−0.4516
9.5	−0.2265
10.0	0.0000
10.5	+0.2277
11.0	0.4565
11.5	0.6863
12.0	0.9170
12.5	1.1486
13.0	1.3808
13.5	1.6138
14.0	1.8475
14.5	2.0817
15.0	2.3165
15.5	2.5518
16.0	2.7875
16.5	3.0238
17.0	3.2604
17.5	3.4975
18.0	3.7349
18.5	3.9727
19.0	4.2108
19.5	4.4493
20.0	4.6880

The Three-Circuit Transformer

BY C. F. ESTWICK[1]

Member, A. I. E. E.

Synopsis.—*Three electrical circuits which are separate and independent of each other, except that they are magnetically coupled by lines of force which interlink with the conductors in each circuit, can be represented by an equivalent circuit which has been developed from well-known fundamental equations. There are two distinct types of the three-circuit transformer which depend upon the relative directions of the fluxes flowing in the mutual paths in the magnetic circuit. Any arrangement of three coupled circuits will have the characteristics of either one type or the other. It is therefore important to study the magnetic circuits to determine the paths for the flux which interlink with the conductors in each of the electrical circuits. There are stray fluxes which interlink with the conductors in only one circuit and mutual fluxes which interlink with the conductors in two circuits. It may also be that there are mutual fluxes that interlink with the conductors in all three circuits. A problem of each type of transformer has been considered. In Case I the coils in each circuit, or the cores on which they are mounted, are disposed parallel to each other or arranged in the form of a Y in which case it is practically impossible to have a mutual flux thread through all three coils. Case II is where the three coils are wound on one core or arranged on separate cores which are placed so that there is a path for flux which interlink with the conductors in the three circuits.*

WHEN three electrical circuits are magnetically coupled there is a transformer relation between each circuit and each of the other circuits. If the conditions in one circuit remain unchanged the other two circuits may be treated as a two-circuit transformer provided the impedance characteristics of the two circuits are determined in the presence of the third circuit. In a two-circuit transformer, when an alternating potential is applied to the primary circuit and the secondary is left open the voltages in the circuit may be expressed by the equation

$$E = Z_1 I_1 \tag{1}$$

where E, Z_1 and I_1 represent respectively the applied voltage, the total impedance in the primary circuit and the current flowing in the primary. This equation as well as all other equations given in this discussion are in terms of complex quantities. The current, or ampere-turns, in the primary produce fluxes in the magnetic circuit, a portion of which interlink with conductors in the secondary circuit and cause a voltage to be induced in the secondary. As soon as the secondary circuit is closed a current I_2 flows in the secondary and this, in a similar manner, causes an induced voltage in the primary. The conditions in the transformer when the secondary circuit is closed may be expressed by the following equations which include the induced voltages,

$$E + Z_{12} I_2 = Z_1 I_1 \tag{2}$$

$$Z_{12} I_1 = Z_2 I_2 \tag{3}$$

in which Z_{12} represents the mutual impedance between the primary and secondary circuits. The + signs are written before the induced voltages $Z_{12} I_2$ and $Z_{12} I_1$ because the zero phase direction of the current in the secondary, when considered as a rotating vector, is assumed to be in the opposite direction to that of the current in the primary.

Equations (2) and (3) may also be written in the form

$$E = (Z_1 - Z_{12}) I_1 + Z_{12} (I_1 - I_2) \tag{4}$$

$$O = (Z_2 - Z_{12}) I_2 + Z_{12} (I_2 - I_1) \tag{5}$$

The equivalent circuit of the two-circuit transformer has been drawn in the form in which it is usually given as shown in Fig. 1 from the voltage relations in equations (4) and (5).

It can clearly be seen by applying Kirchoff's law to the equivalent circuit that the voltages in the primary and secondary circuit are the same as expressed by the equations (4) and (5).

In the transformer there are two physical circuits coupled by magnetic flux that interlink with the conductors in both circuits whereas in the equivalent circuit there are two circuits, that represent the primary and secondary of the transformer, which are coupled by the

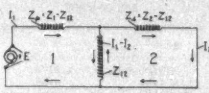

FIG. 1—USUAL ARRANGEMENT OF EQUIVALENT CIRCUIT OF TWO-CIRCUIT TRANSFORMER

impedance Z_{12}. When the secondary circuit is left open the total impedance of the primary is equal to Z_1 and when the primary is left open the total impedance of the secondary will be seen to be Z_2. If the total impedance in the circuits remain the same but the sign of the coupling impedance is changed so that the circuits are coupled by a negative impedance equal to $-Z_{12}$ the current in the primary and secondary will be the same as before except that the direction of the secondary current will be reversed and the current in the coupling impedance will be equal to $I_1 + I_2$ instead of $I_1 - I_2$ as indicated in the diagram of the equivalent circuit in Fig. 2.

It is also possible with the negative impedance coupling to assume the secondary current in the equivalent circuit to flow in the direction as indicated in the diagram in Fig. 1 and assume the direction of the current in the secondary of the transformer to be reversed so that the zero phase direction of the secondary current is taken to be in the same direction as the current in the

1. Elec. Engr., General Railway Signal Co., Rochester, N. Y.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

primary instead of flowing in the opposite direction as stated above.

The equivalent circuit with negative impedance coupling may be used for making calculations but cannot be produced by using simply resistances, reactance coils, and condensers because oft he negative resistance component in the coupling impedance. However a

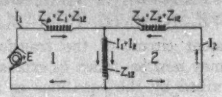

FIG. 2—EQUIVALENT CIRCUIT OF TWO-CIRCUIT TRANSFORMER EMPLOYING NEGATIVE IMPEDANCE COUPLING

negative impedance device is stated to have been used in a form of Wheatstone bridge for measuring the capacities in cables.[2]

There appears to be no advantage to be gained by using the equivalent circuit in Fig. 2 instead of that in Fig. 1. The circuit in Fig. 2 has been given to show how the relative directions of the currents in two coupled circuits can be reversed by the use of negative coupling because this principle will be employed later in some of the equivalent circuits of the three-circuit transformer.

THREE-CIRCUIT TRANSFORMER

Case. I. In the three-circuit transformer the first case to be considered is where the magnetic circuits are

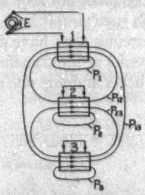

FIG. 3—DIAGRAM DESCRIBING CONDITIONS IN TRANSFORMER IN CASE I (a)

so arranged that it is impossible for mutual fluxes to exist which interlink with the conductors in each of the three circuits. The diagram in Fig. 3 shows a circuit arrangement in which the conductors in each circuit are represented by coils which are placed so that their axes are parallel to each other. The current in the coils is assumed to be flowing in the directions indicated by the arrows and the coils are wound so that the fluxes due to the ampere-turns in each coil flow through the coils from left to right. There are three stray fluxes, that interlink with conductors in only one circuit, which flow in the paths P_1, P_2 and P_3 and six mutual fluxes that flow in paths P_{13}, P_{12} and P_{23} and interlink with the conductors in circuits 1 and 2, 1 and 3 and 2 and 3. Because of the directions of the currents and the arrangement of the circuit the mutual fluxes in each path flow in opposition to each other. There are three distinct transformer relations due to the magnetic coupling between the circuits which are shown schematically in the diagram in Fig. 4.

In circuit 1, in Fig. 3, there is a voltage E from the generator, an induced voltage $Z_{12} I_2$ due to current flowing in circuit 2 and an induced voltage $Z_{13} I_3$ due to current in circuit 3. In circuit 2, there is an induced

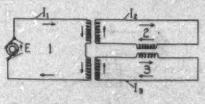

FIG. 4—SCHEMATIC DIAGRAM DESCRIBING TRANSFORMER RELATIONS BETWEEN THREE COUPLED CIRCUITS

voltage $Z_{12} I_1$ due to current in circuit 1 and an induced voltage $Z_{23} I_3$ due to current flowing in circuit 3 and in circuit 3 there are induced voltages $Z_{13} I_1$ and $Z_{23} I_2$ due to current flowing in circuits 1 and 2. The voltage relations in the three circuits may be expressed in the following equations.

$$E + Z_{12} I_2 + Z_{13} I_3 = Z_1 I_1 \quad (6)$$

$$Z_{12} I_1 + Z_{23} I_3 = Z_2 I_2 \quad (7)$$

$$Z_{13} I_1 + Z_{23} I_2 = Z_3 I_3 \quad (8)$$

in which Z_{12}, Z_{13} and Z_{23} represent the mutual impedance due to the flux interlinking with circuits 1 and 2, 1 and 3 and 2 and 3; Z_1, Z_2 and Z_3 are the total impedances in each circuit and I_1, I_2 and I_3 represent respectively the current in each circuit.

Equations (6) (7) and (8) may also be written,

$$E = (Z_1 - Z_{12} - Z_{13}) I_1 + Z_{12} (I_1 - I_2) + Z_{13} (I_1 - I_3) \quad (9)$$

$$O = (Z_2 - Z_{12} - Z_{23}) I_2 + Z_{12} (I_2 - I_1) + Z_{23} (I_2 - I_3) \quad (10)$$

$$O = (Z_3 - Z_{13} - Z_{23}) I_3 + Z_{13} (I_3 - I_1) + Z_{23} (I_3 - I_2) \quad (11)$$

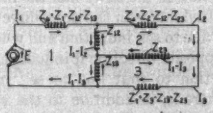

FIG. 5—EQUIVALENT CIRCUIT, CASE I (a)

The equivalent circuit of the three-circuit transformer, in the case under consideration, may be drawn from the voltage relations expressed by the equations as shown in Fig. 5.

In the equations given above and in the equivalent circuit external loads on the transformer have not been considered. Any external loads, or impedances not directly coupled with the adjacent circuits, would be included in the total impedances Z_1, Z_2 or Z_3 in the

2. "Direct Capacity Measurements," by George A. Campbell, *Bell System Technical Journal*, July 1922, pp. 24 and 29.

equations and included in the diagram in the series impedances $Z_p = Z_1 - Z_{12} - Z_{13}$, $Z_s = Z_2 - Z_{12} - Z_{23}$ or $Z_t = Z_3 - Z_{13} - Z_{23}$ in the corresponding circuit.

Case II. When the coils and the magnetic circuit in a three-circuit transformer are so arranged that there is a path for magnetic flux which interlink with all three circuits the condition may be described schematically as shown in the diagram in Fig. 6 in which the coils are represented by heavy black lines and the directions of the current flowing in the coils and the directions of the fluxes are as indicated by the arrows. There are stray fluxes flowing in the paths P_1, P_2, and P_3 and mutual fluxes which interlink with only two circuits in paths

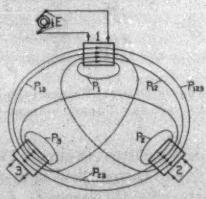

Fig. 6—Diagram Describing Conditions in Transformer in Case II (*h*)

P_{12}, P_{13}, and P_{23} and mutual fluxes that interlink with all three circuits flow in the path P_{123}. The magnetic coupling between circuits 1 and 2 is caused by the fluxes that flow in the paths P_{12} and P_{123} that are due to the ampere-turns in coils 1 and 2. The mutual impedance Z_{12}, in this case, represents the impedance due to all flux interlinkages between circuits 1 and 2. In a similar manner the mutual fluxes which interlink with conductors in circuits 1 and 3 may be seen to flow in the paths P_{13} and P_{123} and the mutual impedance is represented by Z_{13}. The mutual fluxes interlinking with circuits 2 and 3 flow in the paths P_{23} and P_{123} and the mutual impedance due to these fluxes is represented by Z_{23}. When a mutual flux flowing through any coil is in the same direction as the stray flux the induced voltage due to the mutual flux is in the same direction as the induced voltage due to self induction and when the mutual flux is in the opposite direction it induces a voltage in an opposite direction to that due to self induction. In each circuit there is, in addition to the voltage due to self induction, an induced voltage due to interlinkage of flux caused by ampere-turns in each of the other coils.

The voltage equations which include these induced voltages may now be written from an inspection of the diagram in Fig. 6 to describe the relations in the transformer as follows:

$$E + Z_{12} I_2 + Z_{13} I_3 = Z_1 I_1 \tag{12}$$

$$Z_{12} I_1 - Z_{23} I_3 = Z_2 I_2 \tag{13}$$

$$Z_{13} I_1 - Z_{23} I_2 = Z_3 I_3 \tag{14}$$

By comparing these equations with equations (6), (7) and (8) it will be seen that they are of the same form except for the negative signs before the terms $-Z_{23} I_3$ and $-Z_{23} I_2$. If the negative signs are assumed to indicate a negative impedance the equations may be rewritten as in the following equations (15), (16) and (17) which are in the same form as equations (9), (10) and (11).

$$E = (Z_1 - Z_{12} - Z_{13}) I_1 + Z_{12} (I_1 - I_2) + Z_{13} (I_1 - I_3) \tag{15}$$

$$O = (Z_2 - Z_{12} + Z_{23}) I_2 + Z_{12} (I_2 - I_1) + (-Z_{23})(I_2 - I_3) \tag{16}$$

$$O = (Z_3 - Z_{13} + Z_{23}) I_3 + Z_{13} (I_3 - I_1) + (-Z_{23})(I_3 - I_2) \tag{17}$$

The equivalent circuit of the transformer in the present case may now be drawn to express the voltage relations described by the equations as shown in the diagram in Fig. 7.

In deriving the equivalent circuit, circuit 1 has been assumed to be the primary and circuits 2 and 3 two secondary circuits, but circuit 2 might have been selected as the primary, and circuits 1 and 3 the secondary circuits. By following the same line of reasoning the equivalent circuit would then have the same form as that in Fig. 7 but the mutual impedance between circuits 1 and 3 would have the negative sign instead of the impedance between circuits 2 and 3. The mutual impedances would have been Z_{12}, $-Z_{13}$ and Z_{23} and the series impedances $Z_p = Z_1 - Z_{12} + Z_{13}$, $Z_s = Z_2 - Z_{12} - Z_{23}$ and $Z_t = Z_3 + Z_{13} - Z_{23}$. In a similar manner circuit 3 might have been taken as the primary and circuits 1 and 2 as secondary circuits in which case the mutual impedances would be $-Z_{12}$, Z_{13} and Z_{23} and the series impedances $Z_p = Z_1 + Z_{12} - Z_{13}$, $Z_s = Z_2 + Z_{12} - Z_{23}$ and $Z_t = Z_3 - Z_{13} - Z_{23}$. It will be seen in each case that the series impedances are such that the total open circuit impedance in each circuit is Z_1, Z_2 or Z_3. If the three equivalent circuits that have been described are each equivalent circuits of the transformer they must be equivalent to each other and any one may be substituted

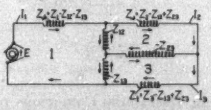

Fig. 7—Equivalent Circuit, Case II (*h*)

in place of another. For instance when a potential is applied to circuit 1 the three equivalent circuits are as shown in the diagrams in Figs. 7, 8, and 9.

The circuit in Fig. 7 represents the condition in the transformer when the zero phase direction of the current in circuit 1 is assumed to be in one direction and the currents in circuits 2 and 3 are in the opposite direction. Fig. 8 represents the condition when the currents in circuits 1 and 3 are assumed to be in the same direction and the current in circuit 2 is in the opposite. The diagram in Fig. 9 indicates that the currents in circuits 1 and 2 are in the same direction and that in circuit 3 is in the opposite direction. There is still another equivalent circuit in which all three of the mutual impedances are

negative impedances which represent the condition when the currents in all three windings are assumed to flow in the same direction.

In the equivalent circuit in Fig. 5, which represents the case where there is no mutual flux which threads all three windings, it will be seen that the mutual impedances are all positive. The signs of any two of the mutual impedances may be changed from positive to negative and the circuit will still be theoretically the

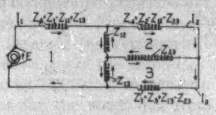

FIG. 8—EQUIVALENT CIRCUIT, CASE II (g)

equivalent of the transformer and equivalent to the circuit in the diagram in Fig. 5, provided corresponding changes are made in regard to the assumption as to the direction of the flow of current in the windings of the transformer.

It can be seen that the mutual impedances in all of the equivalent circuits are connected together in the form of a Y. These impedances may be replaced in any of the circuits by an equivalent Δ combination of impedances which will give a form of circuit from which calculations can be more easily made. The present circuits have a Y combination of impedances within a delta combination and when the proposed substitution is made the arrangement will be a Δ within a Δ combination with the adjacent sides of the two deltas connected in parallel as shown in Fig 14.

TWO-CIRCUIT TRANSFORMER THEORY APPLIED TO THE THREE-CIRCUIT TRANSFORMER

In either of the two cases of the three-circuit transformer it may be that the impedance in one of the

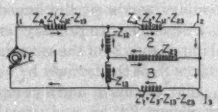

FIG. 9—EQUIVALENT CIRCUIT, CASE II (f)

secondary circuits remains unchanged in which case the other two circuits may be considered as a two-circuit transformer. To describe this condition consider the equivalent circuit in Fig. 5 and assume that the impedance in circuit 3 does not change. The impedances Z_{13}, Z_{23} and $Z_t = Z_3 - Z_{13} - Z_{23}$ are then to be considered as constant quantities. By referring to the diagram it can be seen that these impedances are connected together in the form of a Δ. The Δ arrangement can be replaced in the usual manner by an equivalent Y combination. When this change is made the circuit becomes as shown in the diagram in Fig. 10 in which the impedance $Z_A = Z_{13}\,Z_{23}/Z_3$, $Z_B = Z_{23} - (Z_{13}\,Z_{23} + Z_{23}^2)/Z_3$, $Z_C = Z_{13} - (Z_{13}\,Z_{23} + Z_{13}^2)/Z_3$.

The circuit is clearly an equivalent circuit of a two-circuit transformer in which $Z_{12} + Z_A$ is the mutual impedance and $Z_1 - Z_{12} - Z_{13} + Z_C$ is the series impedance in circuit 1 and $Z_2 - Z_{12} - Z_{23} + Z_B$ is the series impedance in circuit 2. If Z_{12}' represents the mutual impedance and Z_p' and Z_s' the series impedance in circuits 1 and 2 respectively and Z_1' the total impedance in the primary and Z_2' the total impedance in the secondary we have

$$Z_p' = Z_p + Z_C = Z_1 - Z_{12} - \frac{Z_{13}^2 + Z_{13}\,Z_{23}}{Z_3} \tag{18}$$

$$Z_s' = Z_s + Z_B = Z_2 - Z_{12} - \frac{Z_{13}\,Z_{23} + Z_{23}^2}{Z_3} \tag{19}$$

$$Z_{12}' = Z_{12} + Z_A = Z_{12} + \frac{Z_{13}\,Z_{23}}{Z_3} \tag{20}$$

$$Z_1' = Z_1 - \frac{Z_{13}^2}{Z_3} \tag{21}$$

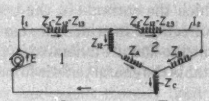

FIG. 10—EQUIVALENT CIRCUIT OF A THREE-CIRCUIT TRANSFORMER REDUCED TO A TWO-CIRCUIT TRANSFORMER

$$Z_2' = Z_2 - \frac{Z_{23}^2}{Z_3} \tag{22}$$

The last three equations are the most important because they fully describe the impedance characteristics of the two-circuit transformer.

In any of the equivalent circuits that have been discussed no matter what the signs are before the mutual impedances when reduced to a two-circuit transformer the total impedance of the primary Z_1' and of the secondary circuit Z_2' are the same as expressed in equations (21) and (22). In a Case I problem, when there is no mutual flux through all three windings, the mutual impedance will be as given by equation (20) or by equation (23).

$$Z_{12}' = -Z_{12} - \frac{Z_{13}\,Z_{23}}{Z_3} \tag{23}$$

and in a Case II problem the mutual impedances will be as given in equations (24) or (25)

$$Z_{12}' = Z_{12} - \frac{Z_{13}\,Z_{23}}{Z_3} \tag{24}$$

$$Z_{12}' = -Z_{12} + \frac{Z_{13}\,Z_{23}}{Z_3} \tag{25}$$

In the following table the circuits a, b, c and d are equivalent to each other and represent the conditions described in the problem in Case I and circuits e, f, g and h are equivalent circuits representing conditions in the problem in Case II. The circuits in either case differ from each other because the equations from which they are derived are based on a different assumption as to the directions in which the currents in the transformer are flowing. The directions in which the currents in each circuit are assumed to flow are indicated in the diagram in Fig. 11. In the table the characteristic impedances of the two-circuit transformer are given in terms of the characteristics of the three-circuit transformer. A set of three fundamental equations may be written in a form which will correspond with any one of the equivalent circuits described in the table. These equations are expressed in terms of the three-circuit characteristic impedances and the currents I_1, I_2, and I_3. In any set of equations the current I_3 may be eliminated algebraically with the result that two equations are obtained which describe the conditions in circuits 1 and 2 and also furnish a proof that the corresponding equivalent circuit shown in Fig. 10 truly represents the conditions in the transformer.

IMPEDANCES IN EQUIVALENT CIRCUITS

Equivalent circuit	Problem	Mutual impedances in three-circuit diagram	Two-circuit diagram Z_1'	Z_2'	Z_{12}'	Equivalent circuit shown
a	Case I	Z_{12}, Z_{13}, Z_{23}	$Z_1 - \frac{Z_{12}^2}{Z_3}$	$Z_2 - \frac{Z_{23}^2}{Z_3}$	$Z_{12} + \frac{Z_{13} Z_{23}}{Z_3}$	Figs. 5 and 10
b	"	$Z_{12}, -Z_{13}, -Z_{23}$	"	"	"	
c	"	$-Z_{12}, Z_{13}, -Z_{23}$	"	"	$-Z_{12} - \frac{Z_{13} Z_{23}}{Z_3}$	
d	"	$-Z_{12}, -Z_{13}, Z_{23}$	"	"	"	
e	Case II	$-Z_{12}, -Z_{13}, -Z_{23}$	"	"	$-Z_{12} + \frac{Z_{13} Z_{23}}{Z_3}$	
f	"	$-Z_{12}, Z_{13}, Z_{23}$	"	"	"	Fig. 9
g	"	$Z_{12}, -Z_{13}, Z_{23}$	"	"	$Z_{12} - \frac{Z_{13} Z_{23}}{Z_3}$	Fig. 8
h	"	$Z_{12}, Z_{13}, -Z_{23}$	"	"	"	Fig. 7

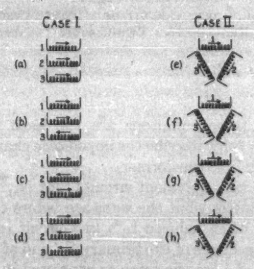

Fig. 11—Showing Relative Directions of the Currents in the Transformer

The two-circuit transformer theory has been applied to describe conditions in circuits 1 and 2 when taken in the presence of circuit 3. In a similar manner equivalent circuits can be drawn that are equivalent to the transformer as regards circuits 1 and 3 in the presence of circuit 2 and there are circuits which are equivalent to circuits 2 and 3 when taken in the presence of circuit 1.

Ratio of Turns in the Windings in the Transformer

Nothing has been said in the present discussion in regard to the ratio of the number of turns in the windings of the transformer and it is not ordinarily necessary to consider the number of turns in making circuit calculations but the equivalent circuit is sometimes used to study conditions that exist within the transformer and for this purpose the equivalent circuit may be reduced to a circuit which represents a transformer that has a 1:1:1 ratio of the number of turns in the windings. If a_2 represents the ratio of turns between the primary and secondary and a_3 the ratio of turns between the primary and tertiary windings, the impedances in the transformer may be reduced to primary terms by multiplying the total impedance Z_2 of the secondary by a_2^2 and the total impedance Z_3 of the tertiary by a_3^2, the mutual impedances Z_{12} and Z_{13} by a_2 and a_3 respectively and the mutual impedance Z_{23} by $a_2 a_3$; furthermore, the voltages in circuits 2 and 3 would vary directly in proportion to the ratio of the number of turns in the windings and the currents would vary inversely as the ratio of the number of turns.

Conditions in the Magnetic Circuit

In the transformer described in Case I, the conditions in the magnetic circuit can be determined from the currents, voltages, and impedances in the electrical circuits. The ampere-turns are determined from the currents flowing and the number of turns in the windings. The permeances of the various paths in the magnetic circuits may be determined from the inductances

in the electrical circuits, because it is well known that the inductances may be written directly in terms of permeances.[3] The flux which interlinks with the conductors in a coil may be determined from the induced voltage they produce, provided it is sufficiently accurate to assume that all lines of force interlink with all convolutions in the coil.

In the particular problem given above in Case I it is possible to determine the permeances of the various paths described in the magnetic circuit shown in Fig. 3. But in more complicated problems in Case I and those in Case II as shown in Fig. 6 it does not appear to be possible to separate the permeance of the path P_{123} from the permeances of the paths P_{12}, P_{13} and P_{23} and it appears to be impossible to separate the permeances of the paths for the stray flux from the permeances of the paths in which there are mutual fluxes.

Discussion of the Equivalent Circuit of the Three-Circuit Transformer

Two very simple problems of the three-circuit transformer have been considered in each of which there is only one coil in each circuit and one path for the mutual flux which interlink with each pair of coils. In practise there may be a number of coils in each circuit and a number of paths for the mutual flux. The fluxes in each path between two circuits, including the path which interlinks with the conductors in all three circuits, cause a component of the total mutual impedance between the circuits. Each component may be represented in the equivalent circuit by a positive or negative component of coupling impedance, depending upon the direction of the fluxes flowing in the mutual path, and the sum of the components determines the sign of the resultant coupling impedance.

It has been shown that there are eight possible combinations of positive or negative coupling impedances in the equivalent circuit, four of which represent the conditions described in the problem in Case I and the other four conditions in the problem in Case II. A proof of this statement can be given in either case of the three-circuit transformer for any condition that may exist as follows:

Consider the equivalent circuits h and g shown in the diagrams in Figs. 7 and 8. The circuit in Fig. 7 represents conditions in the transformer as expressed by the following equations,

$$E_1 + Z_{12} I_2 + Z_{13} I_3 = Z_1 I_1 \tag{26}$$

$$E_2 + Z_{12} I_1 - Z_{23} I_3 = Z_2 I_2 \tag{27}$$

$$E_3 + Z_{13} I_1 - Z_{23} I_2 = Z_3 I_3 \tag{28}$$

and the circuit in Fig. 8 expresses the conditions in the transformer as described by the equations,

$$E_1 + Z_{12} I_2 - Z_{13} I_3 = Z_1 I_1 \tag{29}$$

$$E_2 + Z_{12} I_1 + Z_{23} I_3 = Z_2 I_2 \tag{30}$$

$$E_3 - Z_{13} I_1 + Z_{23} I_2 = Z_3 I_3 \tag{31}$$

In these equations it has been assumed that there may be an a-c. potential applied in each of the three circuits.

By examining the equations it will be seen that the expressions become identical if the sign of the current I_3 and the applied potential E_3 are reversed in either set of equations. Both sets of equations as they are written give the same result in that they accurately describe the magnitude and phase of the currents flowing in the transformer. The difference in the equations is due to the fact that they are based on a different assumption as to the zero phase direction of the current I_3 and the potential E_3 which can be seen by referring to the diagrams in Fig. 11. In the same manner the other two equivalent circuits e and f in Case II can be shown to be equivalent to the circuits in Figs. 7 and 8 and it can also be proven that the four equivalent circuits a, b, c and d in Case I are equivalent to each other and equivalent to the transformer.

A three-circuit transformer when separated from its external loads may have one side of each of the three circuits connected to a common terminal in which case it can be treated as a network having four external terminals. It is well known that such a network cannot be represented theoretically by an equivalent circuit having less than six impedances. The equivalent circuits that have been given cannot be reduced to a circuit having a smaller number of impedances unless it is found to be permissible in some cases to make assumptions that will simplify the problem.

There are equivalent circuits used in the design of power transformers and autotransformers which have been derived by an entirely different method of reasoning by Boyajian[4] and MacLeod[5] on the assumption that the magnetizing currents are negligible or that the mutual impedances are infinite as compared with other impedances in the circuit. Some of these equivalent circuits contain only three impedances and are claimed to give very satisfactory results in solving three-circuit transformer problems that arise in power work.

The present investigation of the theory of the three-circuit transformer was undertaken in connection with a study of the a-c. intermittent train control system of the type installed on the Chicago, Indianapolis and Louisville Railway.

Numerical Example

Consider three coils each mounted on a laminated iron core and arranged as shown in the sketch in Fig. 12 and assume an a-c. potential of 16 volts at a frequency of 360 cycles per second applied to the primary circuit. An ammeter (I_1) and a condenser C_1 are connected in series with the generator in the primary, a load consist-

3. "The Magnetic Circuit," by V. Karapetoff, Fourth Edition, published by McGraw-Hill Book Co., page 9 and Article 58.

4. *Theory of Three-Circuit Transformers*, by A. Boyajian, A. I. E. E. Trans., 1924, p. 508.

"New Theory of Transformers and Autotransformer Circuits," by A. Boyajian, *General Electric Review*, February 1929, p. 110.

5. "New Equivalent Circuits for Autotransformers and Transformers with Tapped Secondaries," by D. R. MacLeod, *General Electric Review*, February 1929, p. 120.

ing of an ammeter (I_2) and condenser C_2 in the secondary and a load connected in the tertiary consisting of the ammeter (I_3). It is required to compute the impedances in the equivalent circuit and the current flowing in each of the coils. The arrangement of the circuit is the same as shown schematically in Fig. 3 and the currents are assumed to flow in the directions indicated by the arrows. The phase relations of the currents in the equivalent circuit have been computed with reference to the phase of the voltage of the generator as standard phase. The phase angles of the currents I_2 and I_3 as determined by calculation are interpreted to mean their phase relative to the zero phase directions shown in Fig. 12 at the instant the voltage of the generator is a maximum in the direction indicated. The condition described is a problem in Case I in which the mutual impedances in the equivalent circuit are all positive impedances and are arranged as shown in the diagram in Fig. 13.

Impedance of Coils.

Primary. $z_1 = 309^\omega$ $r_1 = 13.02^\omega$ $x_1 = 308.7^\omega$ $L_1 = 136.5$ mh. phase angle 87 deg. 35 min., power

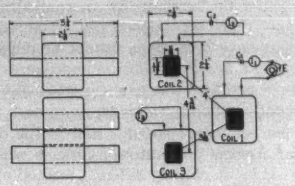

Fig. 12—Arrangement of Three Coupled Circuits Explained in the Numerical Example

factor 0.0421, d-c. resistance 9.15^ω, iron loss resistance 3.87^ω.

Winding. 900 turns, No. 23 A. W. G., s. c. c. copper wire.

Secondary. $z_2 = 233^\omega$ $r_2 = 10.96^\omega$ $x_2 = 232.8^\omega$ $L_2 = 103.0$ mh. phase angle 87 deg. 18 min., power factor 0.0470, d-c. resistance 6.32^ω, iron loss resistance 4.64^ω.

Winding. 800 turns, No. 22 A. W. G., s. c. c. copper wire.

Tertiary. $z_3 = 182^\omega$ $r_3 = 10.8^\omega$ $x_3 = 181.6^\omega$ $L_3 = 80.3$ mh. phase angle 86 deg. 36 min., power factor 0.0594, d-c. resistance 5.2^ω, iron loss resistance 5.6^ω.

Winding. 700 turns, No. 22 A. W. G., s. c. c. copper wire.

Impedance of Loads.

Primary. Ammeter resistance 1.18^ω, condenser $C_1 = 1.633$ μf., $X_{C1} = 270^\omega$.

Secondary. Ammeter resistance 2.37^ω, condenser $C_2 = 2.00$ μf., $X_{C2} = 221^\omega$.

Tertiary. Ammeter resistance 9.6^ω.

Total Impedances.

The total impedance in each circuit consists of the sum of the impedance of the coil and the external load in the circuit. The values are calculated to be as follows,

$$Z_1 = 14.2 + j\ 38.7 = 41.233 \angle 69° 51' \text{ ohms}$$
$$Z_2 = 13.33 + j\ 11.8 = 17.809 \angle 41° 32' \text{ ohms}$$
$$Z_3 = 20.4 + j\ 181.6 = 182.742 \angle 83° 35' \text{ ohms}$$

Mutual Reactances.

The mutual reactances between the circuits have been

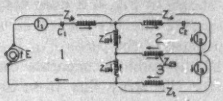

Fig. 13—Equivalent Circuit Described in Numerical Example

determined from tests that were made on the coils when arranged as shown in the sketch in Fig. 12. The results of the tests are:

$$X_{12} = 2\pi f M_{12} = 15.74 \text{ ohms}, \quad M_{12} = 6.96 \text{ mh.}$$
$$X_{13} = 2\pi f M_{13} = 24.6 \text{ ohms}, \quad M_{13} = 10.88 \text{ mh.}$$
$$X_{23} = 2\pi f M_{23} = 7.02 \text{ ohms}, \quad M_{23} = 3.11 \text{ mh.}$$

Mutual Resistances.

The resistive components of the mutual impedances, designated R_{12}, R_{13} and R_{23}, have been separated from the iron loss resistance in the coils in each circuit by assuming that the resistances vary in proportion to the corresponding reactances. The resistance $R_{12} + R_{13}$ has been separated from the iron loss resistance in circuit 1, the resistance $R_{12} + R_{23}$ from the iron loss resistance in circuit 2 and the resistance $R_{13} + R_{23}$ from the iron loss resistance in circuit 3. From the values of these re-

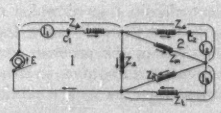

Fig. 14—A Form of Circuit which is Equivalent to the Circuit Shown in Fig. 13

sistances the resistance $R_{12} + R_{13} + R_{23}$ has been obtained. The latter resistance is then separated into the resistances R_{12}, R_{13} and R_{23}, the values of which have been determined to be as follows:

$$R_{12} = 0.322^\omega \qquad R_{13} = 0.502^\omega \qquad R_{23} = 0.143^\omega$$

Mutual Impedances.

The resistive and reactive components of the mutual impedances have been determined and from these values the mutual impedances may be given:

$$Z_{12} = 0.322 + j\ 15.74 = 15.744 \angle 88° 50' \text{ ohms}$$
$$Z_{13} = 0.502 + j\ 24.6 = 24.605 \angle 88° 50' \text{ ohms}$$
$$Z_{23} = 0.143 + j\ 7.02 = 7.021 \angle 88° 50' \text{ ohms}$$

Series Impedances.

The series impedances Z_p, Z_s and Z_t have been determined from the equations:

$$Z_p = Z_1 - Z_{12} - Z_{13}$$
$$Z_s = Z_2 - Z_{12} - Z_{23}$$
$$Z_t = Z_3 - Z_{13} - Z_{23}$$

Numerical values computed from the equations are:

$$Z_p = 13.376 - j\ 1.64 = 13.476 \measuredangle 69^\circ\ 59' \text{ ohms}$$
$$Z_s = 12.865 - j\ 10.96 = 16.901 \measuredangle 40^\circ\ 26' \text{ ohms}$$
$$Z_t = 19.755 + j\,149.98 = 151.375 \angle 82^\circ\ 30' \text{ ohms}$$

Δ Impedances.

The mutual impedances shown in Fig. 13 are connected in the form of a Y and may be replaced by the Δ combination consisting of impedances Z_l, Z_m and Z_n arranged as shown in the circuit in Fig. 14 by employing the well-known equations,

$$Z_l = Z_{12} + Z_{13} + \frac{Z_{12} Z_{13}}{Z_{23}}$$

$$Z_m = Z_{12} + Z_{23} + \frac{Z_{12} Z_{23}}{Z_{13}}$$

$$Z_n = Z_{13} + Z_{23} + \frac{Z_{13} Z_{23}}{Z_{12}}$$

Numerical values computed from the equations are:

$$Z_l = 1.943 + j\ 95.509 = 95.539 \angle 88^\circ\ 50' \text{ ohms}$$
$$Z_m = 0.556 + j\ 27.252 = 27.258 \angle 88^\circ\ 50' \text{ ohms}$$
$$Z_n = 0.869 + j\ 42.591 = 42.601 \angle 88^\circ\ 50' \text{ ohms}$$

The equivalent circuit in Fig. 14 has been reduced to a simpler form of circuit consisting of a series and multiple arrangement of impedances from which the currents in the various branches of the circuit can be easily computed. The required values of current I_1, I_2 and I_3 flowing in the three coils in the circuit have been computed and found to be:

$$I_1 = 0.445 \measuredangle 43^\circ\ 29' \text{ ampere}$$
$$I_2 = 0.421 \angle 4^\circ\ 50' \text{ ampere}$$
$$I_3 = 0.0718 \measuredangle 28^\circ\ 32' \text{ ampere}$$

The values of the currents I_1, I_2 and I_3 when substituted in equations (6), (7) and (8), together with numerical values of the applied voltage and impedances in the transformer, will be found to satisfy the equations. This proves that the results of calculations made from the equivalent circuit are the same as might have been obtained by a solution of the original equations.

Equivalent Circuits of Imperfect Condensers

BY C. L. DAWES* and W. M. GOODHUE†

Member, A. I. E. E. Non-member

Introduction

THE plates of commercial condensers must necessarily be held in mechanical separation by some substance which is a dielectric. Since such substances cannot be perfect dielectrics, the condensers when connected across alternating voltage must absorb power. It is customary to simulate the electrical performance of such condensers, either by an equivalent series circuit consisting of a perfect condenser C_s and a resistance R_s, Fig. 1; or by an equivalent parallel circuit consisting of a perfect condenser C_p in parallel with a resistance R_p, Fig. 2. In the study of dielectric phenomena both methods are employed. Sometimes with the equivalent parallel circuit, a conductance $G_p = 1/R_p$ is used rather than the parallel resistance R_p, since such conductance frequently gives results which are more convenient in form than those given by resistance.

It frequently happens that equivalent series and parallel capacitances and equivalent series and parallel resistances are used indiscriminately without designation as to whether the equivalent series or equivalent parallel circuit is intended. With usual types of condensers this ordinarily does not cause confusion, since the power factor of most condensers is comparatively low. Under these conditions, the equivalent series capacitance C_s is almost equal in magnitude to the equivalent parallel capacitance C_p, so that it is not necessary to make designation; the equivalent parallel resistance R_p is so large in magnitude as compared with the equivalent series resistance R_s that there is little doubt as to which is intended. However, as the study of dielectric and ionization phenomena progresses, it is becoming

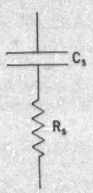

Fig. 1—Equivalent Series Circuit

more common to find that the equivalent series resistance and the equivalent parallel resistance (see Fig. 9) may be of nearly the same order of magnitude; also the equivalent series capacitance may be quite different in magnitude from the equivalent parallel capacitance. (See Fig. 11.) These conditions occur particularly in ionized gases whose power factor is ordinarily high. Therefore, it becomes important not only to discriminate between equivalent series and parallel circuits, but

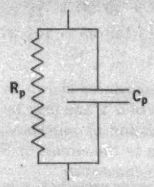

Fig. 2—Equivalent Parallel Circuit

also to investigate the relations existing among their various parameters.

Relationships Among Equivalent-Circuit Parameters

The relationships existing among the parameters of equivalent series and parallel circuits are well known and are as follows:

$$C_p = \frac{C_s}{1 + R_s^2 C_s^2 \omega^2} \tag{1}$$

$$R_p = R_s \frac{1 + R_s^2 C_s^2 \omega^2}{R_s^2 C_s^2 \omega^2} \tag{2}$$

where ω is $2 \pi f$, f being the frequency. The other quantities are given in Figs. 1 and 2.

The power factor of the condenser must be the same whether an equivalent series or an equivalent parallel circuit is considered. In terms of equivalent series-circuit parameters

$$\text{(Power factor)} = \frac{R_s}{\sqrt{R_s^2 + \frac{1}{C_s^2 \omega^2}}} = \frac{R_s C_s \omega}{\sqrt{R_s^2 C_s^2 \omega^2 + 1}} \tag{3}$$

From (1), (2) and (3)

$$R_p = R_s \frac{1}{(\text{P. F.})^2} \tag{4}$$

$$C_p = \frac{C_s (\text{P. F.})^2}{R_s^2 C_s^2 \omega^2} \tag{5}$$

If $R_s C_s \omega = \eta$

Then $$R_p = R_s \frac{1 + \eta^2}{\eta^2} \tag{6}$$

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$$C_p = \frac{C_s}{1 + \eta^2} = \frac{C_s\,(\text{P. F.})^2}{\eta^2} \qquad (7)$$

The power factor in terms of parallel parameters

$$\text{Power factor} = \frac{1}{\sqrt{1 + R_p^2\,C_p^2\,\omega^2}} = \frac{1}{\sqrt{1 + \delta^2}} \qquad (8)$$

where $\delta = R_p C_p\,\omega = \tan\theta = \cot\psi$. θ is the power-factor angle of the condenser; ψ is the angle of defect.

Consider first an impregnated-paper, high-voltage, underground cable having good dielectric characteristics. Fig. 3 shows the power-factor, the power and the capacitance curves taken at 20 deg. cent. of a typical cable of this character. (300,000 cir. mils, 6/32 in. (0.476) cm. wall.) It is to be noted that there is no evidence of gaseous ionization in this cable, since the power-factor and the capacitance characteristics are both linear and horizontal. The equivalent series resistance per foot R_s of this cable is found to be constant

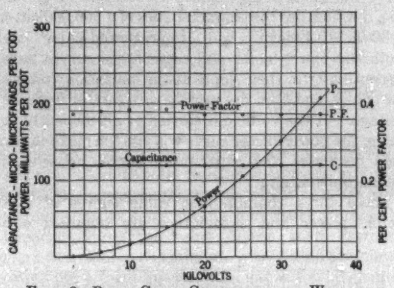

FIG. 3—PAPER CABLE CHARACTERISTICS WITHOUT IONIZATION

300,000 cir. mils, 6/32 in. wall, impregnated paper

at all voltages, up to 40 kv. at least, and is equal to 8.25×10^4 ohms. Likewise, the equivalent series capacitance per foot C_s is found to be practically constant and equal to 119.8 $\mu\mu$f. From equation (2) the equivalent parallel resistance per foot is found to be

$$R_p = 8.25 \times 10^4 \left(\frac{1 + 1.388 \times 10^{-5}}{1.388 \times 10^{-5}} \right)$$

$$= 5.96 \times 10^9 \text{ ohms.}$$

It is to be noted that with this dielectric whose power factor is low, η^2 (equation (6)) is small compared with unity and

$$R_p = \frac{1}{R_s\,C_s^2\,\omega^2} \quad \text{approximately.} \qquad (6a)$$

The equivalent parallel capacitance per foot

$$C_p = \frac{119.8 \times 10^{-12}}{1 + 1.388 \times 10^{-5}} = 119.8\ \mu\mu\text{f.}$$

which is substantially equal to the equivalent series capacitance per foot.

Thus with dielectrics of very low power factor the equivalent parallel resistance is practically equal to the equivalent series resistance multipled by $1/\eta^2$, where $\eta = R_s C_s\,\omega$; the equivalent series and parallel capacitances are practically equal. Hence, with dielectrics of very low power factor, the capacitance as a rule may be given without designating whether equivalent series or equivalent parallel capacitance is meant.

After this same cable (Fig. 3) had undergone 150 hours of accelerated life test, at an average voltage gradient of 225 volts per mil, and under weekly cycles of both voltage-gradient variation (from zero to 225 volts per mil) and temperature variation (from 20 deg. to 65 deg. cent.) both its power factor and capacitance increased.

At 30 kilovolts and 63 deg. cent., the power factor is now 0.1209 and the measured equivalent series capacitance is 147 $\mu\mu$f. per foot. The equivalent series resistance per foot of this cable is now found to be 2.20×10^8 ohms.

The equivalent parallel resistance per foot from equation (2) is now

$$R_p = 2.20 \times 10^8 \left(\frac{1 + 0.01483}{0.01483} \right) = 1.506 \times 10^8$$

ohms, as compared with 5.96×10^9 ohms, its former value. It is to be noted in the first instance, when the power factor of this cable is 0.00373, that η^2 is entirely negligible in comparison with unity; but with the power factor equal to 0.1209, the value of η^2 is just becoming appreciable in comparison with unity, its value being approximately 0.015.

The equivalent parallel capacitance now becomes, from equation (1),

$$C_p = \frac{147.0}{1 + 0.01483} = 145.0\ \mu\mu\text{f.}$$

Likewise the equivalent parallel capacitance now differs from the equivalent series capacitance by 1.5 per cent. The power factor 0.1209 is large for dielectrics, but even so the difference between equivalent series and parallel capacitance is almost negligible.

Hence, with most of the usual dielectrics, the difference between the equivalent series capacitance and the equivalent parallel capacitance is small and generally may be neglected.

COMPOSITE SOLID AND GASEOUS DIELECTRICS

When a dielectric circuit consists of a solid dielectric in series with a film of gas, as a rule it behaves like a simple invariable dielectric up to the ionization voltage of the gas, the only loss being that in the solid dielectric. When the gas becomes ionized, however, its losses become relatively large and its electrical parameters such as capacitance, permittivity, equivalent series and parallel resistance vary with the current density, the functions usually being non-linear. The actual mechanism of the ionization and its physical interpretation

are not simple and need not be considered at this time. Moreover, the authors know of no combination of circuit parameters which can simulate the electrical behavior of such films under all conditions. However, at any single value of current density, the electrical behavior, so far as the fundamental components of the voltage and current waves are concerned, can be simulated either by suitable resistance and capacitance in series, or by suitable resistance and capacitance in parallel. This method is used in this paper.

It has been found experimentally that the power loss

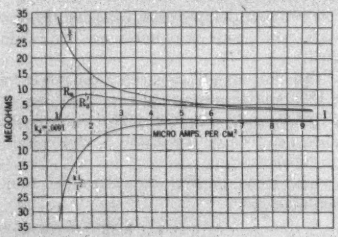

Fig. 4—Equivalent Series Resistance of Glass-Ionized Air Film Model, with 2.17 Mm. Air Gap, at 60 Cycles, 23 Deg. Cent., and 764 Mm. Pressure

in ionized gas films is a simple linear function of the current density.

For example, let the value of current at which ionization begins be I_o. Since the power P increases as a linear function of the current density after ionization begins, its equation becomes

$$P_i = k\,(I - I_o) \tag{9}$$

where k is a constant.

Neglecting harmonics, at room temperature the power in the solid dielectric varies as the current squared and the total power in the composite dielectric becomes

$$P = k_d I^2 + k\,(I - I_o) \tag{10}$$

Since no ionization loss occurs until $I > I_o$, the second term of equation (10) vanishes for all values of $I \leq I_o$. Hence it follows that the first derivative of the function is discontinuous at the point, $I = I_o$.

If R_s is the equivalent series resistance of the composite dielectric,

$$R_s = \frac{P}{I^2} = k_d + \frac{k}{I} - \frac{k\,I_o}{I^2} \tag{11}$$

Equation (11) is composed of three simple functions, a linear function whose graph is parallel to the current axis, a rectangular hyperbola in the first quadrant and an "inverse square" function in the fourth quadrant. Fig. 4 shows this R_s-function and its components for 2.17-mm. film of dry air at 23 deg. cent. a barometer pressure of 76.2 cm. and a frequency of 60 cycles per second. The value of k_d is so small, being only 0.0091, that it cannot be shown with the scale used in Fig. 4.

It is to be noted that R_s rises rapidly to a maximum and then decreases. By differentiating (11) with respect to the current and equating the first differential coefficient to zero, it is found that the maximum value R_s' of R_s occurs when

$$I = 2\,I_o \tag{12}$$

and

$$R_s' = k_d + \frac{k}{4\,I_o} \tag{13}$$

Due to occluded gases, the dielectric of high-voltage cables is a composite one of solid and gaseous dielectric, the gaseous dielectric becoming ionized after the voltage becomes sufficiently large.

The characteristics of high-voltage cables are now generally given by power-factor, power and capacitance curves as in Fig. 3. These three characteristics for a cable having negligible ionization are shown in Fig. 3. When considerable ionization exists in a cable, however, the power-factor, power and capacitance characteristics have the general appearance of the curves given in Fig. 5, which are the characteristics of a 300,000 cir.-mil 6/32-in. (0.476-cm.) wall, impregnated-paper cable. Both the power-factor and capacitance curves have a discontinuity in their slopes at the ionization voltage of the cables, and then rise rapidly with further increase in voltage. If the voltage is carried to a sufficiently high value, the power-factor curve will rise to a maximum and then decrease; the capacitance curve, however, increases indefinitely.

These three characteristic cable curves of power factor, power and capacitance are obtained either

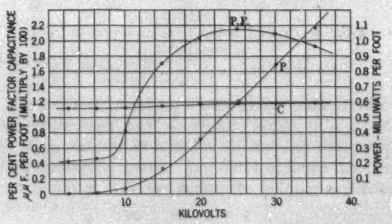

Fig. 5—Paper Cable Characteristics with Ionization

300,000 cir. mils, 6/32 in. wall, impregnated paper

directly or are determined with little computation from data obtained from the methods of bridge measurement now in common use. The bridges which are now commonly used to measure the characteristics of dielectrics at high voltage are the Heaviside type used by the authors and the Schering type.[1] These give directly the equivalent series capacitance. In fact, most types of capacitance bridges give directly the equivalent series capacitance.[2]

Clark and Shanklin,[4] who were among the pioneers in

1. For references see Bibliography.

the study of ionization in high-voltage cables, measured the power taken by cables with a compensated electrodynamometer and in their publications used equivalent parallel resistance as a distinctive cable characteristic. At room temperature, the equivalent parallel resistance of cables is found to remain substantially constant until after ionization begins; then a discontinuity occurs in the slope of the curve, after which the equivalent parallel resistance decreases, at first rapidly and then at a decreasing rate until it reaches a minimum. Fig. 6 shows a typical curve of this character

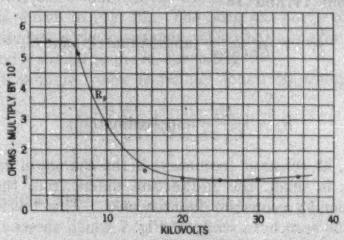

FIG. 6—EQUIVALENT PARALLEL RESISTANCE OF 300,000 CIR. MILS, 6/32 IN. WALL, IMPREGNATED PAPER CABLE

for the cable whose characteristics are given in Fig. 5. This equivalent parallel resistance curve is in distinction to the power-factor curve, which also has a discontinuity in its slope at the ionization voltage, but thereafter shows increasing rather than decreasing values and reaches a maximum, rather than a minimum.

With our present knowledge of ionization phenomena, it is now possible to derive an analytical expression for this equivalent parallel resistance characteristic. The power due to the combined solid-dielectric and ionization power loss is given by

$$P = K E^2 + K_1 (E - E_o) \tag{14}$$

where K is the solid-dielectric power coefficient, K_1 is the ionization-power coefficient and E_o is the intercept of the ionization-power curve with the E-axis. The second term of equation (14) is zero when $E = E_o$. Moreover, in a cable the gas films do not all ionize simultaneously as does a single uniform flat gas film, (Fig. 4). They ionize successively outwards as the voltage increases.[5] Equation (14) therefore, does not apply during this interval of successive ionization, that is between the values $E = E_o$ and $E = E_1$ where E_o is the voltage at which ionization just begins in a cable and E_1 is the voltage at which the ionization power curve becomes a straight line.

The equivalent parallel resistance

$$R_p = \frac{E^2}{P} = \frac{E^2}{K E^2 + K_1 (E - E_o)} \tag{15}$$

Although both the numerator and denominator of (15) are simple algebraic expressions, the equation as a whole does not represent any simple curve or combination of simple curves. This is evident from a study of Fig. 6. However, if the equivalent parallel conductance G_p be used, an equation is obtained which may be analyzed into three terms representing three simple graphic functions. That is,

$$G_p = \frac{P}{E^2} = K + \frac{K_1}{E} - \frac{K_1 E_o}{E^2} \tag{16}$$

The first term K of (16) depends on the properties of the solid dielectric and is a straight line parallel to the voltage axis; the second term is a rectangular hyperbola lying in the first quadrant and the third term is an inverse square function lying in the fourth quadrant. Hence, this function is very similar to (11). Each term of (16) is plotted separately in Fig. 7 for the cable whose characteristics are given in Fig. 6; also the resultant characteristic G_p is plotted, being obtained by combining these three component curves.

In the region of successive ionization, where (16) does not apply, the computed curve is shown dotted. Except in the region of successive ionization, the actual G_p-curve obtained experimentally, and shown solid, coincides with the curve obtained either by combining the component curves or by computation from (16).

It is shown in the Bibliography 5, equation (12) that

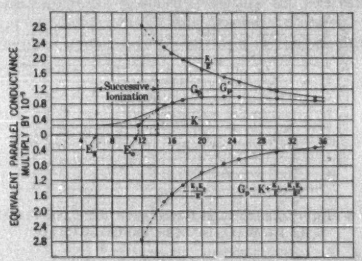

FIG. 7—EQUIVALENT PARALLEL CONDUCTANCE OF 300,000 CIR. MILS, 6/32 IN. WALL, SINGLE-CONDUCTOR CABLE

the power factor of a composite dielectric consisting of a solid dielectric and a gaseous medium is approximately given by $\frac{1}{C\omega}\left(K + \frac{K_1}{E} - \frac{K_1 E_o}{E^2}\right)$ when the power factor is not large. C is either the equivalent series or parallel capacitance, since the two are nearly equal when the power factor is not large; ω is 2π times the frequency.

Hence equation (16) is identical in form with the equation for power factor, differing only by the coefficient $\frac{1}{C\omega}$.

By finding the first differential coefficient of either (15) or (16) with respect to E and equating to zero, the minimum value of R_p and the maximum value of G_p occurs when

$$E = 2\,E_o \tag{17}$$

From, (17) and (16) the maximum value of G_p

$$G_p' = K + \frac{K_1}{4\,E_o} \tag{18}$$

(See Bibliography 6, equation (5).)

This relationship is illustrated in Fig. 7, where E_o equals 11.5 kv. and the maximum value of G_p, G_p', occurs at 23 kv.

For cables having ionization (or other dielectrics having ionization) it also becomes possible to express analytically the equivalent series resistance as a function of voltage. The power

$$P = I^2 R_s = K\,E^2 + K_1\,(E - E_o)$$

Hence

$$R_s = \frac{\{\,K\,E^2 + K_1\,(E - E_o)\,\}\;(R_s^2\,C_s^2\,\omega^2 + 1)}{E^2\,C^2\,\omega^2}$$

If the power factor of the cable or similar dielectric is

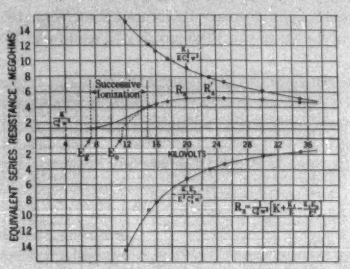

FIG. 8—EQUIVALENT SERIES RESISTANCE OF 300,000 CIR. MILS, 6/32 IN. WALL, SINGLE-CONDUCTOR CABLE

not excessive, $R_s^2\,C_s^2\,\omega^2$ is negligible compared with unity and

$$R_s = \frac{K}{C_s^2\,\omega^2} + \frac{K_1}{E\,C_s^2\,\omega^2} - \frac{K_1\,E_o}{E^2\,C_s^2\,\omega^2} \text{ approximately} \tag{19}$$

If the variation of capacitance with voltage, which in cables is usually small, (see Fig. 5) be neglected, the equivalent-series-resistance characteristic consists of a linear function whose graph is parallel to the E-axis, a rectangular hyperbola in the first quadrant and an inverse square function lying in the fourth quadrant. The equivalent-series-resistance function and its three component curves are shown graphically in Fig. 8 for the same cable whose equivalent parallel resistance and conductance are given in Figs. 6 and 7. This function is similar in character to the equivalent parallel conductance function (equation (16) and Fig. 7) and the power-factor function.[5,6] The maximum value of R_s occurs when $E = 2\,E_o$ and its value is

$$R_s' = \frac{1}{C^2\,\omega^2}\left(K + \frac{K_1}{4\,E_o}\right) \tag{20}$$

It appears, therefore, that many functions relating to ionization phenomena are functions of the character of (11) and (14).

There also follows another interesting relationship when the power factor is not large and the change in capacitance is small.

From (11) and (16)

$$R_s = \frac{P}{I^2} = \frac{P}{E^2\,C^2\,\omega^2} \text{ (approximately) and } G_p = \frac{P}{E^2}$$

Hence

$$R_s = \frac{G_p}{C^2\,\omega^2} \text{ (approximately)}$$

Thus, at constant frequency, if the change in capacitance is small, the R_s-function differs from the G_p-function by a constant quantity only. That this is true may be seen by a study of Fig. 7 which shows the R_s function for the same cable.

EQUIVALENT CIRCUITS OF IONIZED GASES

At any single value of current density and at the fundamental frequency, ionized gaseous media, when considered alone, behave like very leaky condensers. They may therefore be represented by equivalent series and parallel parameters. But it is found that these parameters are quite divergent from those obtained with the usual dielectrics. Furthermore, they are much more difficult to evaluate from measurement data than the dielectrics which have just been considered. For example, when gaseous films become ionized, a relatively large energy loss results and this loss increases as a linear function of the current density, as has already been stated. Also, because of the effects of space charge, the measured capacitance increases with increasing current density and may reach values which are several times greater than the non-ionized value. Hence, the apparent dielectric constant of the ionized gas film will ordinarily have values which are several times the non-ionized value of unity.

These effects cause the power factor to reach values which are much higher than the values ordinarily met in composite solid and ionized dielectrics such as cables, (see Fig. 11). Hence, the approximations which are frequently made with dielectrics, such as assuming that the equivalent series and the equivalent parallel capacitances are substantially equal, are no longer valid.

In Fig. 9 are shown the equivalent series resistance and the equivalent parallel resistance of a 1.53-mm. air film as a function of current density, the frequency being 60 cycles and the pressure 764 mm. The equivalent series resistance obviously is zero until ionization begins. It then rises rapidly to a maximum after which

it decreases, its graph approaching a horizontal line as an asymptote. The analytical function giving the variation of equivalent series resistance R_s with current density is readily derived from (9)

$$I^2 R_s = P = k\,(I - I_o) \tag{22}$$

$$R_s = \frac{k\,(I - I_o)}{I^2} = \frac{k}{I} - \frac{k\,I_o}{I^2} \tag{23}$$

The first term in (23) is the equation of the well-known rectangular hyperbola and the second term is an inverse square function lying in the fourth quadrant. Each component function for the 1.53-mm. air film of Fig. 9 is shown in Fig. 10. The resultant curve R_s is the equivalent series resistance, which has a maximum value and is similar to the R_s-curve, Fig. 9.

The maximum value is readily found by differentiating (23) with respect to I and equating the first differential coefficient to zero. For the maximum value R_s' of R_s,

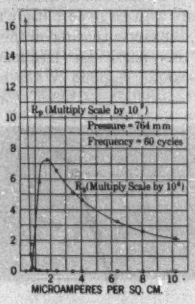

FIG. 9—EQUIVALENT SERIES AND PARALLEL RESISTANCE OF 1.53 MM. IONIZED AIR FILM

$$I = 2\,I_o \tag{24}$$

From (23) and (24)

$$R_s' = \frac{k}{4\,I_o} \tag{25}$$

(See Fig. 23, Bibliography 5 and equation (4) Bibliography 6.)

These relationships are very similar to those for the power-factor, the equivalent parallel conductance, and the equivalent series resistance characteristics of a cable in which ionization exists. (Figs. 5, 7, and 8.)

Fig. 11 shows graphically the equivalent series capacitance, and the equivalent parallel capacitance per cm. cube (or permittivity) of a 1.59-mm. air film as functions of current density. Also the power-factor function is given. It will be noted that the power factor of such a film increases up to nearly 90 per cent, which is a very much higher value than occurs even with composite solid and ionized dielectrics, such as impregnated-paper cables. (See Fig. 5.) Unlike the power-factor curve for cables having ionization (Fig. 5), the power-factor curves which we have so far obtained experimentally for gas films have not shown a maximum, but continue to increase with the current density.

The equivalent series permittivity at the maximum current density of 11.5 microamperes per sq. cm. obtained and shown in Fig. 11 has a value of 24, which illustrates the enormous changes in capacitance which

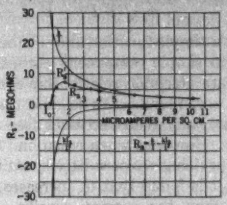

FIG. 10—ANALYSIS OF EQUIVALENT SERIES RESISTANCE FUNCTION OF IONIZED AIR FILM

gas films undergo when ionized. Furthermore, instead of being approximately equal to the equivalent parallel capacitance, the equivalent series capacitance reaches a value at the maximum current density shown in Fig. 11 which is approximately six times the value of the equivalent parallel capacitance. Hence, when analyzing capacitance relations in ionized gas films, it becomes important to specify whether the equivalent series or equivalent parallel capacitance is intended.

It does not appear possible at this time to derive from

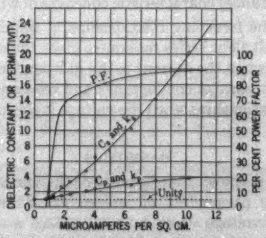

FIG. 11—EQUIVALENT SERIES AND PARALLEL CAPACITANCE OF IONIZED AIR FILM

fundamental considerations any simple equations which give the equivalent series capacitance, and the equivalent parallel capacitance characteristics shown in Fig. 11.

For example, with the equivalent series circuit

$$I = \frac{E}{\sqrt{R_s^2 + \dfrac{1}{(C_s\,\omega)^2}}} = \frac{E\,C_s\,\omega}{\sqrt{R_s^2 C_s^2\,\omega^2 + 1}} \tag{26}$$

$$C_s = \frac{1}{\omega}\,\frac{I}{\sqrt{E^2 - (I\,R_s)^2}} \tag{27}$$

From (22), $P = I^2 R_s = k (I - I_o)$; substituting for R_s in (27)

$$C_s = \frac{1}{\omega} \frac{I}{\sqrt{E^2 - \frac{k^2}{I^2} (I - I_o)^2}}$$

$$= \frac{I^2}{\omega \sqrt{(E I)^2 - k^2 (I - I_o)^2}} \qquad (28)$$

This is not a simple function and it cannot be readily separated into simple component curves. Moreover, no approximate algebraic relationship even between E and I has yet been found. We do know that E increases to a maximum, and then decreases (see Fig. 10, Bibliography 6). However, when equation (28) is plotted as in Fig. 11 (which gives permittivity rather than total capacitance) it is found that C_s increases at a slightly greater rate than the current density. A graphical analysis shows the equation of this curve to be

$$k_s = A + B (I - I_o)^{1.19} \qquad (29)$$

where k_s is the equivalent series permittivity and A and B are constants. Specifically for Fig. 11.

$$k_s = 1 + 1.28 (I - 0.8)^{1.19} \qquad (30)$$

Also, the equivalent parallel permittivity k_p is plotted as a function of current density in Fig. 11. Again the explicit function for this curve is far from being simple and it does not appear that the curve can be analyzed into simple fundamental curves. Also the equivalent series permittivity (or capacitance) characteristic concaves upwards: the equivalent parallel permittivity (or capacitance) characteristic concaves downwards.

From the foregoing relationships, it appears that as a rule equivalent series resistance gives the simple relationships when the current is the independent variable; equivalent parallel conductance and resistance give the simple relationships when the voltage is the independent variable.

Harmonics

It is well known that because of the variable character of the parameters of ionized gas films, a simple harmonic voltage wave cannot produce a simple harmonic current wave, and a simple harmonic current wave in such gas films cannot produce a simple harmonic voltage wave across the gas films. In practise, flat ionized gas films as a rule can exist only under the conditions of "restricted ionization." That is, the volt-ampere characteristics of the ionized gas film alone are highly unstable, and without a stabilizing impedance in series the cumulative reactions would cause a power arc to develop almost instantly. Solid dielectric in series with the gas film acts to stabilize the current and voltage across the film in the same manner as ballast stabilizes a power arc. If a simple harmonic voltage wave is impressed across the ionized gas film and solid dielectric in series, the current wave cannot be a simple harmonic wave because it is determined in part by the variable impedance of the ionized gas film. However, if the impedance of the solid dielectric is large in comparison with that of the ionized gas film, the current wave under these conditions may be nearly a sine wave. Under these conditions, the voltage across the ionized gas film cannot be a sine wave because of the variable character of the gas-film impedance.

Oscillograms show that with short air-gaps and comparatively low voltage, the current wave contains harmonics ranging from the lowest harmonics, well up through the audio-frequency range; at the higher voltages, and also with air films of greater thickness, the current wave contains harmonics from the lowest to probably well beyond the audio-frequency range. When such waves are studied with the usual Duddell string-type oscillograph vibrator, it is found that the harmonics near the resonant frequency of the vibrator cause it to vibrate with large amplitude at its own resonant frequency, thus accentuating on the oscillogram the harmonics of this particular frequency. Moreover, such vibrators cannot respond to the high frequencies

Fig. 12—Oscillogram of Ionization Current for Air Gap 1.59 Mm., Voltage 5,500, Frequency 60 Cycles

that some waves must contain. Hence, such oscillograms may not always give a true representation of the wave; they do, however, give useful information, in that they show changes in the wave shapes and in the lower frequency harmonics that ionization produces in such current waves under different conditions. As an example, the oscillogram* in Fig. 12 shows the voltage and current waves for a 1.59-mm. film of dry air, at 5,500 volts r. m. s. and at a frequency of 60 cycles per second. The voltage wave is not that across the gas film itself, but across the gas film and a slab of glass in series. However, the glass has a very high dielectric constant and very low power loss so that its impedance is small compared with that of the gas film. The voltage at which this oscillogram was taken is known to be well above the ionization voltage. It will be noted that the current increases almost sinusoidally with voltage until the voltage is sufficiently high to start ionization. At the instant of initial ionization, high-frequency harmonics occur in the current wave. These are im-

*A vacuum-tube amplifier of the type developed by S. K. Waldorf was used in obtaining this oscillogram. See Bibliography 7.

mediately followed by a sudden increase or large peak in the current wave.

This oscillogram shows that the harmonics of lower frequencies have the greatest effect on the wave distortion; also the wave has a considerable number of harmonics of high order. For more accurate reproduction of such waves, a cathode-ray type of oscillograph should be used. However, it is difficult to obtain sufficient current sensitivity with this type of oscillograph. Owing to the difficulty of making suitable potential connection across the gas film itself, oscillograms of this potential, so far as is known, have not as yet been obtained. (A complete series of oscillograms for different conditions of ionization will be published later.)

Definitions of Dielectrics with Ionization

The presence of the foregoing harmonics makes ambiguous many of the terms and definitions which are applicable to dielectrics that are practically invariable or in which the ionization is so small that it has but little influence on the wave shape. In view of the rapidly increasing study of gaseous ionization, it has become almost necessary to agree on some definitions of the parameters of ionized gas both when alone and when in combination with solid dielectrics.

One method is to base definitions on r. m. s. values. For example, if the voltage E and current I are measured or computed on a r. m. s. basis the equivalent series resistance $R_s = P/I^2$, where P is the power in watts; and the capacitance $C_s = \frac{1}{\omega} \frac{I}{\sqrt{E^2 - (I R_s)^2}}$.

These parameters might then be called the r. m. s. equivalent series resistance; the r. m. s. equivalent series capacitance; etc.

On the other hand, if the characteristics of the gas film are measured in a bridge having invariable parameters in three of its arms such as a standard air condenser and resistances, and a harmonic voltage is applied across the bridge, the bridge can be balanced with a detector, which responds only to the fundamental frequency. A vibration galvanometer is an example of such a detector. The parameters determined under such conditions might well be termed fundamental equivalent series resistance; fundamental equivalent series capacitance; etc.

It is conceivable that definitions depending on parameters other than the foregoing may also be evolved. The authors believe, however, in view of the magnitude and large number of harmonics which are present in the current wave, particularly, and the complications which these harmonics introduce, that for the present at least the more simple definitions of the parameters of ionized gas films based on fundamental frequencies should be used.

Conclusions

1. The electrical characteristics of imperfect condensers at any one frequency may be represented by resistance and capacitance in series, and by resistance and capacitance in parallel; definite algebraic relationships exist among the parameters of such equivalent series and parallel circuit.

2. Even with dielectrics having relatively high power factor, the equivalent series and parallel capacitances are substantially equal.

3. With composite dielectrics consisting of an invariable dielectric in series with ionized gas films, the equivalent series resistance characteristic and the equivalent parallel conductance characteristic are very similar to the power-factor characteristic provided the total change in capacitance is not large.

4. Ionized gas films have such high losses and high power factor that approximations which are applicable to ordinary dielectrics are not applicable to them.

5. With increasing knowledge of the laws of ionization, it is becoming possible to express series and parallel parameters of ionized gas films as explicit functions of known quantities.

6. The usual dielectric definitions are not applicable to ionized gas films, and to dielectrics containing such films; some appropriate definitions should be formulated; and it is recommended that these definitions be based on the first harmonic components of the voltage and current waves.

The authors are indebted to Professor H. E. Clifford, Dean of the Harvard Engineering School, for his helpful suggestions in the preparation of this paper.

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Discussion

M. G. Malti: Prof. Dawes and Mr. Goodhue have pointed out some very valuable and pertinent facts relative to the representation of imperfect condensers by equivalent circuits. Their treatment is complete and straightforward when, and if, a sine voltage, impressed on an imperfect condenser, produces a sine current. But the variation of the characteristics of an

imperfect condenser with impressed voltage is non-linear. Hence with a sine impressed voltage a non-sine current (Fig. 12 of the paper) is obtained. Moreover the distortion suffered by the current is a function of the nature of the dielectric used, its temperature, its humidity, the magnitude and wave form of the impressed e. m. f. and other factors. Under these conditions it becomes difficult to conceive of a series or a parallel circuit, consisting of a variable resistance and capacitance, which would simulate the behavior of an imperfect condenser. Thus such a circuit can not possibly give the e. m. f. and current oscillograms shown in Fig. 12 of the paper.

The writer is of the opinion that equivalent circuits should be altogether abandoned and the facts studied directly from the oscillograms. Thus Fig. 12 gives the e. m. f. and current curves of an imperfect condenser. The instantaneous power supplied to the condenser may be easily obtained by multiplying corresponding ordinates of e and i and plotting the products to a convenient scale. The average power dissipated in the condenser is the average ordinate of the instantaneous power curve. Here we have a means of determining the e. m. f. current, and power relations of an imperfect condenser. No mention is made of an equivalent series or parallel circuit because, physically, there is no such a thing. The imperfect condenser is an entity which behaves uniquely and cannot be imitated by an equivalent circuit.

C. L. Dawes: The statement of Prof. Malti that the variation of the characteristics of imperfect condensers is non-linear, in a strict sense, is undoubtedly true. That is, the absorption loss does not vary as the voltage squared and the conductance may vary with the voltage gradient. There are, however, many imperfect condensers in which the total loss as measured by precision methods is found to vary sensibly as the voltage squared. The dielectrics of such condensers must then have characteristics which are practically linear and they will not therefore produce any appreciable distortion in the current wave when a sine wave of voltage is impressed across them. For all practical purposes then it becomes possible to simulate such condensers either by an equivalent series circuit or by an equivalent parallel circuit. For example, we have found that cable paper when impregnated with the higher grades of cable-impregnating compounds appears to have characteristics at room temperature which are practically linear, provided all occluded gases are substantially removed.

It is true, however, as Prof. Malti points out, that it would be difficult or perhaps impossible to devise either an equivalent series circuit or an equivalent parallel circuit that would simulate under all conditions dielectrics having non-linear characteristics. The authors were well aware of this fact as is shown in the statement at the top of page 1032 inwhich they state that they "know of no combination of circuit parameters which can simulate the electrical behavior of such (gas) films under all conditions." However, in spite of the fact that such equivalent circuits cannot under all conditions simulate dielectrics having non-linear characteristics they frequently are very useful for purposes of analysis and I cannot agree with Prof. Malti that they should be abandoned entirely. For example a knowledge of the power loss in such dielectrics often is very important as with high-voltage cables. If the impressed voltage wave is sinusoidal the power loss is given by $E^2 G_p$ where E is the r. m. s. value of the impressed voltage and G_p is the equivalent parallel conductance determined for the fundamental component of current. G_p will vary with E as is shown for example in Fig. 7, but the loss at any particular voltage may be readily determined by using the value of G_p corresponding to that voltage. Likewise if the current wave is sinusoidal, the power loss is given by $I^2 R_s$ where I is the r. m. s. value of the current and R_s is the equivalent series resistance to the fundamental.

To depend entirely on oscillograms for the determination of the electrical properties of imperfect condensers, as suggested by Prof. Malti, would, I believe, be not only inconvenient but would frequently be unsatisfactory as regards accuracy. For example, before the oscillogram shown in Fig. 12 could be obtained it was necessary to construct and adjust a 7-tube amplifier, as well as to devise and test out an elaborate shielding system. The taking, developing, printing, and calibrating of the oscillogram was in itself a relatively laborious procedure. To obtain the power by multiplying together the instantaneous values of E and I in itself is also laborious, and is also subject to errors in calibration and in the fact that the width of line is relatively large as compared with the amplitude. Hence the value of power obtained by this method would undoubtedly be less precise than the value obtained by the use of an equivalent circuit.

There are, however, special problems such for example as occur when both voltage and current waves are non-sinusoidal where the use of such oscillograms are necessary if accurate analyses are to be made. We are confronted with several of this type of problem at the present time.

As a matter of fact from an analytical point of view, non-linear dielectric characteristics are not different from non-linear magnetic characteristics. The analysis of dielectric phenomena is however more recent and is therefore less understood. For convenience in solving practical problems we have become accustomed to represent the circuit linked by magnetic material, even though it has non-linear characteristics, by equivalent series and parallel circuits. A common example is the equivalent parallel circuit so frequently used to represent the no-load conditions of transformers and induction motors. Rigorous analysis would require the use of oscillograms of voltage and current, as suggested by Prof. Malti for the dielectric circuit. As a matter of practical convenience, however, we actually use either an equivalent series or parallel circuit under all conditions of operation, although such do not, strictly speaking, simulate the actual circuit under all conditions. This same practical necessity, I believe, will necessitate in many instances the use of equivalent circuits to simulate the behavior of dielectric circuits, even although such circuits do not simulate rigorously the actual circuit.

Measurement of Noise in Electrical Machinery

BY BENJ. F. BAILEY[1]

Fellow, A. I. E. E.

WHEN a new and useful machine is first introduced, nobody thinks much about possible refinements. Thus when electric motors were first used, reliability was the essential quality. Soon afterward price became of importance and still later efficiency was considered. Now at the present time attention is being turned to noise and in the not too distant future, appearance will become of distinct importance.

When the public has had some means of expressing its opinion quietness has always been demanded. In the case of the automobile this demand has led to the development of a machine that is unbelievably quiet especially to those who drove cars twenty years ago. On the other hand, it is hard to see why the public has not protested long ago against the needless noise of street cars and trains. Little progress has been made in this direction in thirty years and it is only now that the competition of quieter vehicles is forcing those responsible to take some action.

Where an electric motor is used to drive a washing machine or a mangle, noise is not so important, but if a motor is used to drive an electric refrigerator the case is quite different. The motor is usually located in close proximity to the living quarters and is in operation much of the time and at any time of night or day. Obviously such a motor must be quiet.

Need of Methods of Measurement

The point has now been reached where the need of adequate means for measuring noise is felt. At present the intensity of a noise is stated about as accurately as the Irishman's reply who when asked how large a certain object was said it was about as large as a lump of coal. Two distinct types of measuring devices are needed. One should, if possible, give with a single reading an indication of the intensity of a sound; much as an ammeter gives at once the current in a circuit. The other should make possible the analysis of a sound into its components in much the same manner that the components of a current can be analyzed by means of an oscillograph. The first type will be useful to the manufacturers and users of machinery in determining quickly and easily whether or not a machine comes within certain prescribed limits of quietness. The second type will be useful to the designer of machinery to enable him to determine and eliminate the sources of noise.

The inspection of parts by ear for undue noise is old in industry. Thus in the manufacture of gears, an inspector runs the different gears and accepts or rejects them in accordance with his judgment. An average gear may be provided which the inspector may use as a standard of comparison. Such an inspection while better than nothing is very imperfect. Probably no two people hear things alike and the sensitivity of the ear differs from day to day.

One gear manufacturer to test the accuracy of sound inspection submitted 100 gears to one of his inspectors. Of these 95 were passed and 5 rejected. Another inspector who knew nothing of the first inspection passed 93 and rejected 7. Only one or two of the gears were rejected by both inspectors. A few days later the same gears were again submitted to the first inspector who did not know that he had previously inspected them. This time he rejected 11. Of the 11, only one had been included in the first five. The other ten he had previously passed.

The accuracy of a direct comparison method may be greatly improved if a number of observers is used. A prominent laboratory was recently called upon to rate a considerable number of motors in regard to noise. These motors were all of the same horsepower and speed. For comparison, five motors were selected and arbitrarily given ratings from 1 to 5, No. 1 being an exceedingly quiet motor, No. 5 a very noisy one while No. 3 was supposed to represent about the average. Six observers compared the motors with these standards. Occasionally all the ratings would agree, particularly in the case of very quiet motors, but in other instances the same motor would be rated all the way from 1 to 3 by different observers. The average of the six ratings was probably fairly reliable.

In somewhat similar tests conducted by a prominent manufacturer a standard of noise was provided as well as the motor under test. The observer placed himself between the two motors and varied his position until they sounded equally noisy. This method is similar to that used in the photometry of lamps.

Difficulties of the Problem

When an attempt is made to develop an instrument which will give an objective indication of the degree of noise from a given source serious difficulties are encountered. If it were merely a question of measuring the physical intensity of the sound the difficulties would not be so great. Unfortunately the effect upon the human ear is not directly proportional to the physical intensity of the sound. A sound of very high pitch, for example, cannot be heard at all and the same

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is true of sounds of very low pitch. Moreover the range which the ear can hear varies with different persons and at different times. The problem is further complicated by the fact that what is really desired is to find out how disagreeable a certain noise is. Some sounds, such as the scratching of a slate pencil upon a slate, while not very loud are exceedingly disagreeable. It is hard to see how an instrument can be made which will take all these things into account.

Reflections

Another difficulty arises from the fact that in an ordinary room we hear not only the sound itself but many reflections of the sound. A sound indoors at a certain distance may easily produce ten times the effect of the same sound at the same distance in open air. Moreover in any ordinary room there will be certain points at which the sound will be focused and will consequently be much louder than at other parts of the room or perhaps at the object emitting the sound. When trying to measure sound by using a microphone as a pick-up it has frequently been observed that the sound three feet from the motor under test may be much louder than at a distance of one foot from the motor. Of course one obvious remedy is to make the test in a sound proof room. This while possible in the laboratory would introduce serious difficulties if an attempt was made to make sound measurements in the course of routine production.

Another method that has been used with some success is to provide a very large rotating vane. This tends to break up reflections and prevent them concentrating at any given point.

Measurement of Vibration

One interesting suggestion is that the noise itself should not measured but the vibration which produces it. If a transformer, for example, produces noise it is because certain parts of the transformer are vibrating. If it is possible to measure the vibration it will give a reasonably accurate indication of the intensity of the sound. This method has actually been applied to the routine testing of roller bearings. In this case the problem is much simplified in that the objects to be tested differ if at all only in size. It would not be so easy to apply this to the case of electric motors, for example, where much of the sound comes from the rotating parts.

Analysis into Components

As previously indicated we can analyze the sound into its various components and determine the intensity of each of these components. One method consists in using a microphone to pick up the sounds and generate a feeble electric current. This is amplified by vacuum tubes and then passed through a variable filter, so that only one component at a time passes beyond the filter. This component can then be measured by a galvanometer or similar indicating device. This method, while obviously slow, gives complete information in regard to the sound produced. In order to determine its effect upon the ear it would be necessary to assign to each frequency an arbitrary importance depending upon the effect of the particular frequency upon the average human ear. It would not appear that such a method could be made sufficiently quick and simple so that it could be used in routine production. It is, however, obviously of great value to the designer, in helping him to reduce noise.

Organizations Interested

Several manufacturers of various types of machinery have become greatly interested in methods for measuring accurately and quickly the noises produced by their product. In the past five years approximately $250,000 has been spent upon such projects at the University of Michigan alone and methods have been devised for measuring the noise produced in ball and roller bearings, gears, electric carpet sweepers, and similar devices. Instruments have been constructed and are used in routine production in several factories.

Certain national societies, among them the National Electrical Manufacturers Association and the American Society of Mechanical Engineers, have become interested in the problem of noise and have appointed committees to consider the subject. The National Electrical Manufacturers Association's committee was instructed, first, to find or devise a method of measuring the noise produced by electrical machinery, secondly, when such a device had been found to formulate standards of allowable noise in electrical machinery. Some of the difficulties in the first part of this problem have already been indicated. When we consider the enormous variation in horsepower, speed, frequency, number of phases, etc. of electric motors alone it will be apparent that the difficulties of the second part of the assignment are no less.

Conclusion

It is very evident that the American people as well as the manufacturers of electrical machinery are becoming "noise conscious." Great progress has been made in the past few years in devising means of measuring noise as well as improving machinery so that it will be less noisy. Although the difficulties of the problem are great there is reason to hope that some device will in the near future be perfected to give an immediate indication of the effect of any given noise upon the human ear. After all what is desired is to eliminate noise. But with such an instrument available progress will be much more rapid. A definite measure of noise in place of the present very imperfect estimate based entirely upon personal judgment will then have been achieved.

Discussion

For discussion of this paper see page 1071.

Indicating Meter for Measurement and Analysis of Noise

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Non-member Non-member Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—This paper describes a visual indicating meter for the measurement of noise and other sounds. Its design is based on the known characteristics of sound and hearing, which are summarized. Particular attention has been paid to the response of the meter to sounds of short duration The aim has been to make the meter both simple in operation and portable. An attachment for the frequency analysis of noise is under development. Several fields of use of the meter and analyzer are indicated.

FOR the measurement of acoustic noises of various kinds, the advantages of a visual indicating meter are generally recognized. This paper describes such a meter which has recently been developed in the Bell System. As this meter measures noise and sounds in general, it has been called a "sound meter." In designing this meter the aim has been to have its indications in close accord with the present information as to the response of the ear, in so far as is practicable in a simple portable device. In this regard, particular attention has been paid to the response of the meter to sounds of short duration which frequently are an important part of noise. There is under development also an attachment for this sound meter to permit the analysis of sound both on a single-frequency and a band-frequency basis.

To indicate the requirements for a meter of this type this paper discusses the characteristics of noise and hearing, and the attributes which it is desirable that a meter for noise measurement should have in order that its indications be correlatable with the effects of noise.

What a Meter for Noise Should Measure

Every noise problem arises from the fact that noise is objectionable, *i. e.*, produces certain undesirable effects upon people (such as interference with hearing, annoyance, effect on working efficiency, effect on health). It might appear desirable, therefore, to construct a meter which would measure these effects directly. Most of them, however, are dependent upon the psychology of the individual and require for their evaluation a large amount of further investigation. Furthermore, a meter for measuring one effect probably would have different characteristics from a meter for measuring some other effect, leading to a diversity of meters whose readings would not be comparable.

On the other hand, a meter giving a purely objective description of the noise, *i. e.*, measuring the physical characteristics of acoustic waves, would also be undesirable, as the measurements would bear an unknown relation to the noise as people would hear it, and, therefore, to the effects produced by the noise.

There remains as a practical solution of the problem of noise measurement, a meter having as far as practicable the attributes of hearing. This is a logical solution, since when measuring noise with an indicating meter, the ear is replaced by the meter. Since reactions to noise are dependent upon hearing, the readings of a meter based upon hearing should be correlatable with noise effects.

The sound meter described in this paper is of this type. It aims to fill a need now felt in many diverse kinds of noise problems for a rather simple meter which can be made generally available and can be used with some facility to obtain a large variety of noise data. With preliminary models of such a meter, much valuable information has already been obtained, some of which is summarized herein. Such data should be of assistance in laboratory studies of the various reactions of people to noise.[1] As laboratory data on the effects of given noises on human beings become available, approximate relations between sound meter readings and the amounts of effect of various types can be obtained.

As the basis for the design of the sound meter, loudness has been employed, since loudness is the most elemental and universally appreciated attribute of noise. Since the loudness of a sound depends on its intensity (sound power passing through unit area), frequency, duration, and complexity of wave shape, the effects of each of these upon loudness must be considered in determining the requirements for such a meter.

Essentials of the Sound Meter

The sound meter discussed in this paper consists of a sound pick-up for converting acoustic into electrical energy; a five-stage vacuum tube amplifier; a calibrated gain control; an electrical network for weighting energies at different frequencies in a manner similar to that in which they are weighted for loudness by the ear; a full-wave rectifier and a visual indicating meter having a time of response similar to that of the ear; and suitable means for making an over-all acoustic calibration of the

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1. For references see Bibliography.

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instrument. The calibrated gain control makes it possible to measure sound levels over a range of 90 db. (corresponding to a range of 1,000,000,000:1 in input powers) on a d-c. meter having an effective range of only 15 db. The sound meter proper comprises two units: the amplifier unit, and a battery box which also contains the calibrating equipment and has space for carrying the sound pick-up. Fig. 1 shows a schematic diagram of the complete measuring set while Figs. 2

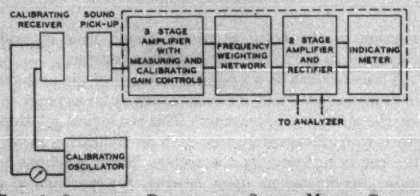

Fig. 1—Schematic Diagram of Sound Meter Circuit

and 3 show the external appearance of an experimental model of the meter and battery box.

The sound meter is calibrated in terms of a 1,000-cycle pressure of 0.001 dyne per sq. cm. in a free progressive wave. This value is called "reference sound level" and has been chosen because it is a convenient reference point and for a 1,000-cycle tone is near the threshold of audibility for an average observer in a completely quiet place.* The level of a sound is taken as the level of a 1,000-cycle tone which gives a meter reading equal to that given by the sound in question,

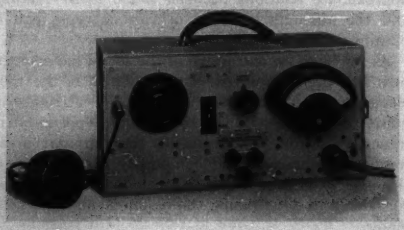

Fig. 2—Model of Sound Meter

and is expressed in decibels above this reference point. The advantage of this reference point is that it is definite and reproducible, and does not depend on personal equation. It can be related to the threshold of audibility, as accurately as the latter is known.

Characteristics of Noise

The characteristics of noise must be studied since they determine the range over which the characteristics of hearing must be considered and impose very severe requirements on measuring apparatus. Noises vary widely in intensity, frequency, duration, and wave shape.

Noise covers the entire audible range of intensities. It varies from the barely audible ticking of a watch to the screaming of a steam whistle or the detonation of heavy artillery which may actually damage the ear of a nearby observer. Fig. 4** illustrates the levels of some commonly encountered noises.[2,3] The available data are not detailed enough to determine accurately the range of noise levels to be expected in specific types of location where particular kinds of machinery may be installed.

As to the frequency range covered by noise, building vibrations sometimes are of such low frequency that the listener is in doubt whether he can hear a sound or only feel vibration; on the other hand, some noises

Fig. 3—Model of Sound Meter Battery Box

involve very high frequencies, as, for instance, the scraping of a knife on glass or the jingling of keys. Most noises are composed of a large number of frequencies.

The duration characteristics of noises vary widely. A noise may be steady, like the hum of rotating machinery; it may vary in definite cycles, as the noise of an oscillating fan; it may increase to a maximum and from there on decrease till it disappears, as the roar of a passing train or an airplane. A great deal of noise consists of one or more isolated peaks, like the banging of a door, hammer blows, etc.; or of a series of peaks with or without steady noise. Fig. 5 gives a diagram of noise measured in a restaurant. This figure indicates the magnitude and frequency of occurrence of the clearly

*Reference sound level is between about 5 and 10 db. above the 1,000-cycle threshold of audibility. A more accurate relation is being determined.

**The "zero noise level" of this figure was determined as the threshold of audibility for certain common types of noise.

discernible peaks which stood out above the general noise level, and the causes of these peaks. In noise produced by a great many sources at different distances, as for instance, street noise, generally the nearby noise peaks stand out, whereas those farther away tend to merge with the general noise.

Some noises have very peaked wave shapes. For common noises such as ordinary room or street noise, the ratio of the maximum instantaneous pressure to the r. m. s. pressure over an interval of a few tenths of a second probably varies from 5 to 10. For noises consisting of hammer blows, the banging of doors, etc., this ratio may be as high as 30 or 40.

OUTDOOR NOISES IN NEW YORK CITY		db ABOVE ZERO NOISE LEVEL	OTHER NOISES
DIST. FROM SOURCE IN FEET	SOURCE OR DESCRIPTION OF NOISE		SOURCE OR DESCRIPTION OF NOISE
		100	
35	RIVETER		SUBWAY — LOCAL STATION WITH EXPRESS PASSING (1)
15-20	ELEVATED ELECTRIC TRAIN ON OPEN STRUCTURE	90	
15-75	VERY HEAVY STREET TRAFFIC WITH ELEVATED LINE	80	SUBWAY CAR AT FREE-RUNNING SPEED, INTERIOR OF CAR (2)
15-80	MOTOR TRUCK	70	
15-75 15-50	BUSY STREET TRAFFIC PASSENGER AUTOMOBILE		AVERAGE OF 18 FACTORY TELEPHONE LOCATIONS (1)
15-300	RATHER QUIET RESIDENTIAL STREET, AFTERNOON	60	LARGE HALL WITH AUDIENCE TALKING (2)
		50	AVERAGE BUSINESS LOCATION (1)
15-300 15-500	MINIMUM NOISE LEVELS IN ENTIRE CITY DAYTIME: 10-20 MINUTE AVERAGE; IN MID-CITY NIGHT: 5 SECOND AVERAGE	40	AUDIENCE NOISE DURING PERFORMANCE IN LARGE AUDITORIUM (2)
		30	AVERAGE RESIDENCE LOCATION (1)
		20	
		10	OBTAINABLE IN SOUND PROOFED ROOMS (2)

ABOVE DATA FROM REPORT BY NEW YORK CITY NOISE ABATEMENT COMMISSION

(1) DATA OBTAINED AT TELEPHONE LOCATIONS BY JOINT SUBCOMMITTEE ON DEVELOPMENT AND RESEARCH, NATIONAL ELECTRIC LIGHT ASSOCIATION AND BELL TELEPHONE SYSTEM.
(2) DATA OBTAINED BY ELECTRICAL RESEARCH PRODUCTS, INC.

FIG. 4—LEVELS OF NOISES

CHARACTERISTICS OF HEARING AFFECTING NOISE PERCEPTION

The exact relations between the physical properties of a sound and its perception include psychological factors which are difficult to measure.[4] It is not the object of this paper to discuss these relations in detail, as a great deal has been published on this subject. Some of this work is referred to herein. The principal question of interest in the design of the sound meter is how the sensation of loudness is related to the physical properties of a sound, such as intensity, frequency, duration, and complexity of wave shape. These relations are indicated in the following.

The relationship between the loudness of a sound and its intensity and frequency is quite complicated. To produce a given loudness, low frequency tones require more intensity than those of higher frequency (500 to 5,000 cycles); and the loudness of low frequency tones increases more rapidly with increasing intensity than does that of the higher frequencies Curves showing the relation between the loudness of pure tones and their frequency and intensity are given in the Bibliography.[5-8]

The loudness of tones of very short duration, less than a few tenths of a second, is a function of the time during which the tone persists. The available information on this subject is rather meager. Fig. 6 indicates the relation between loudness and duration for short tones,[7] and indicates that the ear fully appreciates the loudness of a tone persisting for two-tenths of a second. More work along these lines, however, will be necessary, especially to determine how the relationship applies to complex sounds.

The relation between loudness and complexity of wave shape of sounds is extremely involved[8] and is by no means fully understood. It appears that for very low levels, the contribution from different frequency bands to the total loudness of a complex sound is approximately proportional to the squares of the pressures in these bands as compared to threshold pressure. At high levels, the loudness of a complex sound seems to be materially affected by the non-linear characteristic of the ear, and the relations become much more complex.

FUNDAMENTAL DESIGN

Consideration will now be given to the way in which the sound meter meets the requirements imposed on it by the characteristics of noise and hearing.

Intensity Range. While an extremely wide range would be necessary to cover all possible types of noise,

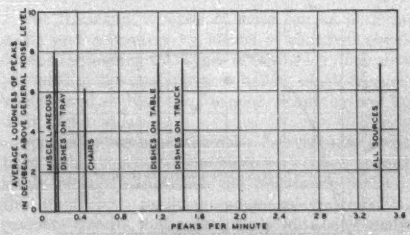

FIG. 5—AVERAGE NOISE PEAKS IN A RESTAURANT CLASSIFIED ACCORDING TO SOURCES

it is generally unnecessary for practical purposes to measure accurately extremely low or high noise levels. The sound meter has been provided with a range from about 10 to about 100 db. above reference sound level, which should be satisfactory in a very large majority of cases. It is direct reading in decibels above reference sound level.

Frequency Weighting. From the data given in the Bibliography,[5] the curves of Fig. 7 have been plotted to show how the shape of frequency-weighting curves

based on loudness would change with variations in the level of the sound being measured. On this account, consideration has been given to the provision of an adjustable frequency weighting in the sound meter. For general use, such refinement does not seem necessary, since it would appreciably change the results of only a very small proportion of the measurements. It will be seen from Fig. 7 that only at low frequencies are there large divergences between the different curves. Some experimental data are available on the differences in meter readings obtained with weightings corresponding to the 30- and 60-db. curves.

Source of noise	Difference in readings
Electric drill	0 db.
Electric fan, 1 ft. from pick-up	4 db.
Several telephone ringer bells	not greater than 1 db.

From consideration of the above and in view of the complications which a variable frequency weighting would entail, it seems desirable to employ a single weighting based on a loudness curve at a medium level. The sound meter weighting is based on the 40-db. level in the range 60-4,000 cycles, where experimental data are available. Outside this range its characteristic does not differ greatly from that of the threshold of audibility.

Duration Characteristics. The response of the meter to sounds of short duration approximately simulates that of the ear, as can be readily demonstrated by watching the movements of the meter needle and listening to the variation of the noise being measured. In such a test it will be observed that the visual and aural impressions are approximately synchronized.

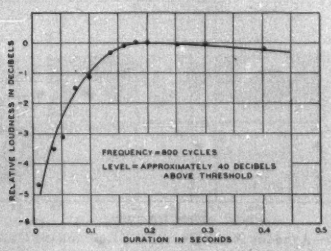

FIG. 6—RELATIVE LOUDNESS OF TONES OF SHORT DURATION

Data taken from article by V. Békésy in *Physikaliache Zeitschrift* XXX, 1929, p. 118

The output meter used gives a full deflection on pulses of noise lasting about 0.2 second or more. It does not overshoot more than about 0.5 db. With these characteristics there is no difficulty in reading noise peaks to within ± 1 db. The measurement of non-recurring rapidly varying noises has been greatly facilitated by the use of an output meter having a long and easily readable db. scale.

The following test results illustrate how the readings of a meter are affected by its dynamic characteristics. Certain peaked noises were measured by the sound meter and by another meter identical except for its dynamic characteristics, which were as follows: when a pure tone was applied, the second meter took about 5 seconds to come within 0.5 db. of its ultimate steady-state value. The difference in maximum readings on

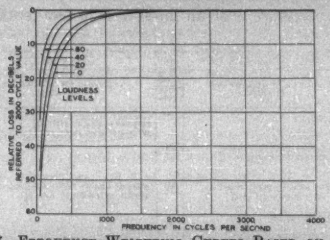

FIG. 7—FREQUENCY WEIGHTING CURVES BASED ON EQUAL LOUDNESS AT VARIOUS LOUDNESS LEVELS

Data taken from article by B. A. Kingsbury, "A Direct Comparison of the Loudness of Pure Tones," *Phys. Rev.*, April 1927, p. 588

these noises was as follows, the slower meter reading lower.

Type of noise	Approximate difference in maximum readings
Short blast from automobile horn	25 db.
Sharp blow on large metal can	30 db.
Single short piano note	15–30 db.
Hammer blow on metal plate	25 db.
Cough	20 db.

Response to Complex Waves. Taking account of present knowledge as to the way in which the ear combines components of a complex sound, practical considerations have been controlling in the design of the meter. The rule of combination employed by the meter is known and simple, and is not a poorer approximation to the performance of the ear, as far as this is known, than other practicable rules. The rectifier employed is of the full-wave type, and approximately follows a "square law;" that is, the meter readings are approximately the same when a sinusoidal voltage is impressed on the rectifier as when there is impressed a steady complex voltage having the same r. m. s. value. With a square law device, the meter indication is a function merely of the average impressed power, and not of wave shape. With devices following other simple laws, the meter indication, in general, is a function of the impressed wave shape, even for waves differing only in the phases of their components. Since phase relations would be changed in the sound meter circuit, any dependence on phase would be arbitrary. With rectifiers not of the full-wave type, the meter indi-

cation may depend on which way the connection between pick-up and amplifier happens to be poled.

Practical Design

In addition to the above, a number of practical requirements has been met in the design of the sound meter.

Portability. The general usefulness of the sound meter depends to a large extent upon its portability, as in most cases of noise measurement it is necessary to take the meter to the location of the noise. In the interests of portability, a light and efficient type of amplifier has been designed, and the battery supply has been reduced to the minimum consistent with reasonable life. The sound meter proper consists of two units, which together are portable by one man.

From the standpoint of both portability and cost, the analyzing equipment was built as a separate attachment.

Linearity. In designing a sound meter to cover a wide range of levels, and especially with an amplifier using a frequency-weighting network having a high loss at low frequencies, special precautions are necessary to reduce non-linearity to a minimum. Non-linearity in the sense used here means the failure of the meter to read correctly differences in sound level; it may be due either to feed-back from a high level part of the circuit to a low level part, or to overloading in circuit elements for which the transmission loss varies with level. It is by no means sufficient for the meter to measure single frequencies without overloading since many noises have very peaked wave shapes.

The sound meter amplifier has been designed so that non-linearity is negligible. It can handle without overloading pure tones as low as 20 cycles, and complex voltage waves having ordinary peak factors. Since measurements are seldom made near the upper end of the meter scale, there is for most noises a working factor of safety.

Shielding. Since sound meters may be employed to measure noise levels in the presence of strong electromagnetic or electrostatic fields, they should be sufficiently shielded to make the effects of stray fields negligible. This is particularly important when an analyzer attachment is used.

The sound meter is practically completely shielded against electrostatic fields. It was necessary to compromise between portability and complete protection from stray electromagnetic fields. The electromagnetic shielding is sufficient for ordinary purposes, and there are several features which compensate for the fact that it is not complete. In most locations where intense magnetic fields are encountered, the noise levels are so high that no material error should be introduced even by a moderate effect from such stray fields. Also, since the sound meter is readily movable, in many cases the effects of electromagnetic fields can be made negligible by properly orienting or locating the meter.

Method of Calibration. The primary absolute calibration of the sound meter was made by placing the sound pick-up in a 1,000-cycle free progressive wave at a point at which the pressure was determined from Rayleigh disk measurements.[9] This wave was essentially plane in so far as its effect on the pick-up is concerned. By means of the calibrating control on the meter amplifier, the gain was then adjusted to make the meter read the level of the 1,000-cycle wave in decibels above reference sound level. The meter reading for given acoustic pressures at other frequencies can be found from the frequency characteristic of the sound meter.

Great care has been taken to make the sound meter amplifier as stable as possible. In addition, in order to verify in the field that the meter is reading correctly, a means of making an over-all acoustic calibration has been included. The apparatus for this secondary calibration consists of a very stable type of telephone receiver, a small 2,000-cycle oscillator for energizing the receiver with a known current, and a means of coupling the receiver to the sound pick-up in order to produce a definite acoustic pressure on the pick-up diaphram. The frequency of 2,000 cycles was chosen since at this point the sensitivity of the sound meter is greatest, and the frequency characteristic is practically flat in this region. When the battery voltages are within their normal limits, calibration once a day is sufficient. The process of calibration is a very simple one, requiring only a few seconds.

Individual sound meters will have the same electrical characteristics within very close limits.

Analyzer Attachment

The frequency analysis of noise can be performed easily and accurately by means of electrical circuits. Means for making two types of frequency analysis are provided in the analyzer attachment, single-frequency analysis and analysis into frequency bands several hundred cycles wide.

For continuous-spectrum noises or other noises which have their frequency components very closely spaced, a single-frequency analysis is not practicable because of the insufficient resolving power of any available resonant circuits, and consequently wide-band analysis must be employed. These noises are those which either do not contain predominant single-frequency components or else are a mixture of noises from so many sources that the individual frequency components from any one source have lost their significance. Street noise, office noise from typewriters and computing machines, and noise from a large number of factory machines are examples.

In the development of the analyzer attachment, much assistance has been obtained from the work done by others on the development of harmonic analyzers, especially those for use in studies of inductive interference on telephone circuits.[10] In recent work con-

siderable effort has been exerted to design an instrument which is truly portable, and at the same time is very simple to operate.

The selectivity, sensitivity, and modulation requirements in an analyzer depend upon the types of noise to be analyzed and the purposes of the analysis. These requirements have been considered together since small components of a noise cannot be measured if other components, including undesired modulation products, affect the indication of the analyzer appreciably. When the analyzer is arranged for single-frequency analysis, the response curve in the immediate neighborhood of the tuned frequency will be sufficiently flat so that small variations in the frequency of the noise component being measured will not cause large variations in the reading of the indicating meter; this curve, however, will not be flat over so great a range that the accurate location of low-frequency components will be difficult or the discrimination against adjacent components suffer.

Uses

The field of usefulness of the sound meter is as yet only partially developed, as it is applicable in the solution of a great many acoustic and noise problems. Before considering the uses it is desirable to have in mind certain limitations.

Limitations. A sound meter cannot discriminate between noise and desired sounds, nor can it pick out one noise from a combination of noises unless this noise differs radically from the others. In case it is desired to measure a single noise whose source is known, sometimes all of the other noises may be stopped temporarily. For example, in order to be able to measure the noise of a certain machine in a factory room, it may be necessary to stop all other machines in this room, and also to exclude street noise and noise from other parts of the building. If the noise background is 20 db. or more below the noise from the source under investigation, it may be neglected. In other cases, a correction can sometimes be made by measuring the background separately.

Secondly, the amount and character of the noise due to a given source are affected by the acoustic properties of its surroundings. The principal acoustic properties which must be considered are the volumes of rooms in which measurements are made, the total acoustic absorption present, and the characteristics of this absorption over the audible spectrum. Allowance for these must be made when applying the results of noise measurements made in a particular location, such as a test booth, to predict the amount of noise from the same source in some other location.[11,12]

Another limitation is the effect of the location of the pick-up. Small changes in the distance from the source to the pick-up, particularly when the distances themselves are short, may result in large changes in the measured noise level. Certain types of noise are more directive than others; this must be taken into account in determining the best location for the sound pick-up.

General Noise Measurements. Many general noise measurements may be made with the meter without the analyzer attachment. Meters have been employed* in telephone, municipal, and industrial surveys to determine points where noise conditions are most severe, and have yielded valuable data on the relative contributions of various sources to the total noise. Information of this type is essential in preparing any well-founded engineering plan for the reduction of noise.

The effect of noise on the efficiency of workers is a matter of growing public concern. Architects and builders are being called upon to plan their structures so that they will be quiet. The meter can be used to aid in ascertaining desired levels of noise in various cases, and to determine when these levels have been met. It can also be used to measure the noise produced by a machine, which will be of value in forecasting the noise conditions which will obtain in the building containing the machine and in related locations. Meter measurements may be made in residential neighborhoods prior to and after installation of power plants, substations, etc., where it is desirable to know the effect, if any, on the general noise level in the neighborhood caused by such installations.

As indicated in the selection of the name "sound meter," the meter can be used to measure sounds other than noise. In the recording and broadcasting of speech and music, valuable data may be obtained by the use of this instrument. Optimum sound levels may be determined for speech and music in various situations; studies may be made of the sound distribution throughout a studio; and meter determination of balance of orchestras appears possible, permitting standardization of methods with consequent saving in time.

Analysis. The analyzer may be used to obtain more detailed information about noises than can be obtained from over-all measurements. With this information it should be possible to make better estimates of particular effects. For example, analysis will generally be required to determine whether an undesired sound such as machinery noise will be covered up by the general noise, or to determine whether a desired sound, *e. g.*, a signal, will be audible through this noise.

In cases of noise reduction by the use of sound insulating or sound absorbing materials, determination of the frequency composition of the noise will permit a choice of materials which will be most effective at the most important noise frequencies.

The arrangement for single frequency analysis opens opportunities for the study of noise in detail. This feature is important in the case of machinery noise. A determination of the components existing in the noise produced by a machine may yield, by comparison with the mechanical features of the machine, accurate data as to the contribution to the total noise of each individual source in the machine. Such data have been

applied in the redesign of machines to reduce noise, and in many cases have permitted of carefully planned designs involving the minimum of changes to secure given results.

The control of uniformity of manufactured products by means of acoustic measurement offers a wide potential field of application of the sound meter. It is anticipated that the results in this field will be most valuable when careful single frequency analysis is made of the particular machine or process under study by means of special adaptations of the meter constructed for the particular problem.

Measurement of Electrical Quantities. The sound meter is to be provided with a suitable electrical input, so that it can also be used to measure electrical levels (or to analyze electrical quantities) in cases where the frequency weighting and other features provided are suitable.

Conclusion

This paper has outlined the general requirements for an indicating meter for measuring noise and has described a meter meeting these requirements which has been developed. In the design of this meter, great care has been exercised to have it both simple and portable. It is believed that this new meter will provide a valuable tool in studying a great many noise and sound problems.

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Discussion

For discussion of this paper see page 1071.

The Measurement of Machinery Noise

BY H. B. MARVIN*

Associate, A. I. E. E.

Synopsis.—The audio noise meter described in this paper is a portable instrument for measuring and analyzing noise and vibration of machinery. The analyzer obtains its high selectivity through the use of a mechanical "band pass filter" of four stages. Some test results and a discussion of the measurement problem are given.

* * * * *

Introduction

THE purpose of this paper is to describe a new instrument for measuring the loudness and frequency composition of noise and to present some typical test results. The description is preceded by a brief discussion of some aspects of the problem.

Noise is a comprehensive term which includes almost all sounds except speech and music. The usual noises created by machinery are recurrent and approximately steady as contrasted with the random character of street noise. They are caused by vibration of the machine as a whole or in parts. The vibrating parts radiate sound waves which reach the ear directly and also set up vibration waves which travel through the supports to floors, etc., which radiate sound waves. The noise perceived is usually a mixture of the direct and indirect radiation.

In order to control noise, an understanding of the causes and mode of transmission is necessary. It is in this field that measuring instruments are especially helpful. Information may now be obtained with instruments of the strength of the source of noise, and of its magnitude and quality in space. Coincidence of noise frequency with rotational frequency of machines and with vibration frequency due to magnetic forces may be established and serve as a basis for design changes. The effects of noise on the nervous systems of human beings are also important but satisfactory methods of measurement have not yet been developed.

Units and Methods

It is desirable to measure vibration amplitudes of the machine, the loudness of the noise, and the frequency and loudness of each component. The measurements should include vibration at various points on the machine and its support, and loudness at various points in space around it. The response of the instrument should correspond with the loudness heard.

The sensation of loudness corresponds more or less to brightness, heaviness, coldness, sweetness, etc. The method of measurement of all these sensations has long been a problem. The natural unit of sensation is the smallest perceptible increment, but this kind of unit is difficult to establish and reproduce. This is because the relation between sensation and exciting stimulus (measured in physical units) is not a simple one. It is approximately a logarithmic one over part of the range. Accordingly a logarithmic scale in which loudness is expressed as the number of decibels[1] above threshold of audibility is here used. This choice of threshold or zero loudness as a reference point is chiefly because results are then comparable with published data on room and street noise.[2]

Dependence of the sensation of loudness on frequency for various intensities has been shown by B. A. Kingsbury[3] as a series of contours of equal loudness. The contour which at 700 cycles is 30 decibels above threshold of audibility of the average ear has been used as the basis of the response-frequency characteristic of the noise meter for total loudness. In the analysis of noise, frequency-weighting by the instrument is not necessary, since only one frequency is present at a time. The loudness of each component is measured separately and referred to the equal loudness contours. This method gives greatest sensitivity.

Much information can be obtained by analysis of noise which is not obtained from total noise tests. In the case of machinery noise, high selectivity or ability to separate adjacent harmonics is essential because the noise contains harmonics of rotational frequency which is often 60 cycles or less. This requirement led to the adoption of a mechanical type of filter which offers the possibilities of a high degree of selectivity with small size and weight.

The portable audio noise meter to be described is capable of making the above measurements. The method involves conversion of sound into electric current by a microphone, amplification, detection by a sensitive a-c. milliammeter, and calibration by comparison with sounds of known loudness.

Description of Audio Noise Meter

The complete analyzing noise meter is contained in three cases as shown in Fig. 1. One case contains the measuring amplifier and has space to hold the sound

*General Electric Co., Schenectady, N. Y.

1. *Decibel—the Name for the Transmission Unit*, W. H. Martin, Journal A. I. E. E., Vol. 48, p. 223, March 1929.

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3. "A Direct Comparison of the Loudness of Pure Tones," *Physical Review*, 2nd Series, Vol. 29, p. 588, 1927.

pick-up and tripod. Another contains batteries. The third contains the analyzer. This arrangement was chosen so that the meter may be used with or without the analyzer. Fig. 4 illustrates the circuits.

The sound pick-up consists of a condenser microphone

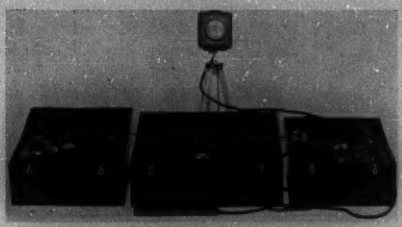

Fig. 1—Audio Noise Meter Set Up for Analysis

Fig. 2—Audio Noise Meter—Amplifier Unit

Fig. 3—Audio Noise Meter—Analyzer Unit

and a three-stage amplifier enclosed in a separate metal case, shown mounted on a tripod in Fig. 1 and packed for carrying in Fig. 2. Power for the amplifier is obtained from the battery unit.

The amplifier unit consists of a four-stage amplifier with calibrated output meter of the rectifier-milliammeter type, a calibrated attenuator, and an ear compensation network. Scale and dial are marked in decibels. Fig. 6 shows response of the noise meter for constant loudness tones of different frequencies. Tests have shown the readings to be closely proportional to loudness over the range of the instrument.

The analyzer shown in Fig. 3 may be connected between sound pick-up and amplifier to obtain harmonic analysis of noises. This analyzer has been designed to have high selectivity to obtain complete separation of

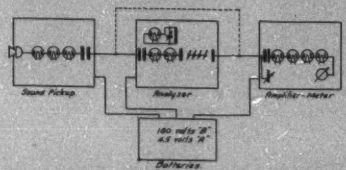

Fig. 4—Schematic Diagram of Audio Noise Meter

harmonics, yet sufficiently broad at resonance so that small variations of speed or frequency do not seriously affect operation. The results are accomplished by an electro-mechanical filter as shown schematically in Fig. 5. The complex current to be analyzed is passed through windings on the poles of the driving magnet and a sinusoidal current from a variable frequency vacuum tube oscillator is passed through an exciting winding. The resultant flux in the air gap produces a torque on an armature or vibrator equal in frequency to

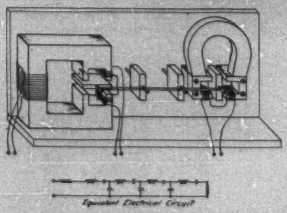

Fig. 5—Mechanical Filter Used in Analyzer

the difference between the complex wave and the analyzing sine wave and proportional to the product of their amplitudes. The moment of inertia of the armature and the moment of compliance of its supporting stem are designed to give angular vibration resonance at 6,000 cycles. Four vibrators are tuned to the same frequency and coupled in tandem by wires under tension. A permanent magnet and associated pole windings are mounted in such relation to the fourth vibrator that its motion induces in the windings a voltage proportional to the velocity of angular vibration. Mass and compliance of the vibrators and tension of the

coupling wires are adjusted so that the combination has characteristics similar to an electrical "band pass filter."

In the equivalent circuit of Fig. 5, inductance represents moment of inertia, series capacitance represents moment of compliance of vibrator stem, shunt capacitance represents longitudinal compliance of coupling

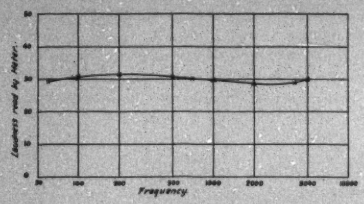

Fig. 6—Response of Audio Noise Meter to Tones of 30 Loudness

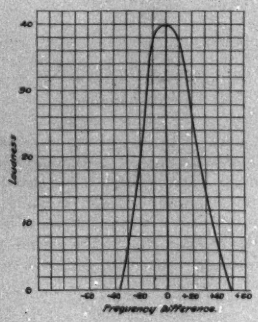

Fig. 7

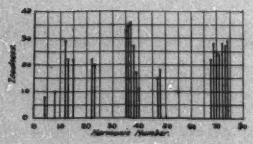

Fig. 8—Noise Analysis of Small Synchronous Motor, ¼ Hp., 1,800 R. P. M., 60 Cycle, Three-Phase, 220 Volts, No Load

The fundamental of the harmonic series is rotational frequency, 30 cycles per sec.

wires, and resistance represents electromagnetic damping. Fig. 7 shows the measured selectivity of the filter. Since forty loudness units correspond to 100 fold in current change, it is seen that the current drops to 1 per cent of its resonant value with a change of less than 50 cycles in frequency, while five-cycle changes have negligible effect.

In operation the frequency of the oscillator is varied slowly from 6,000 to 11,000 cycles. Whenever the difference between this and the frequency of a harmonic in the complex wave equals 6,000, the analyzer responds. The frequency range is therefore 5,000 cycles maximum. The minimum is at present 30 cycles. This heterodyne method of analysis has an important property; namely, a selectivity which is the same for all harmonics.

The combination of analyzer and amplifier units has

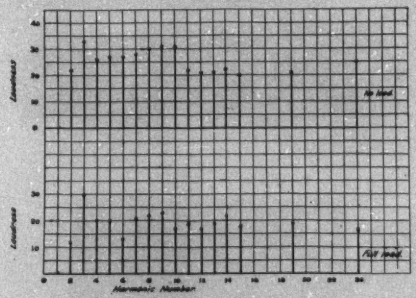

Fig. 9—Noise Analysis of 300-Watt Transformer. Fundamental of the Series is 120 Cycles Per Sec.

a broad field of usefulness. It may be used directly on electric circuits of suitable voltage for harmonic analysis where phase angles are not needed and is particularly useful on higher harmonics where analysis from oscillograms is difficult. In addition to sound analysis, it may be used with attachments for the analysis of mechanical vibrations.

In the measurement of vibration, a device, shown in Fig. 10, is held against the machine and connected to

Fig. 10—Vibration Pick-Up

the amplifier. The vibration generates a voltage proportional to its velocity. This vibration pick-up is substituted for the microphone so that both total vibration and harmonic analysis measurements are possible. Since the noise is more often proportional to velocity than to amplitude of vibration, this measurement gives an idea of the strength of the noise source and may be used to supplement or substitute for sound measurement.

Results

The uses of an audio noise meter are manifold. For some work, a total noise test is sufficient as for example comparing noise levels in factories, test booths, etc., and acceptance testing of machinery. If tests are made with the idea of changing design, harmonic analysis is most useful. Fig. 8 shows an analysis of noise from a small synchronous motor, in which the noise arises principally from higher harmonics of rotational frequency which is 30 cycles. Fundamental, second and third harmonics are below threshold while harmonics as high as the 74th produce appreciable noise. In synchronous apparatus nearly all noises are multiples of rotational frequency but in induction motors and other variable speed machines there are usually two series, one referred to power supply frequency and one to rotational frequency.

Analysis of hum from a 325-watt 60-cycle 110-:1100-volt transformer is shown in Fig. 9. Core clamps were loosened and the applied voltage was raised 20 per cent above normal in order to increase the amount and complexity of the noise. Here again the harmonics fall in one series having a fundamental of 120 cycles per sec. All harmonics up to the 15th are present in a measurable amount but the fundamental is inaudible. This would probably not be surmised in a listening test owing to imagination, which would supply a fundamental in order to be logical. The effect of loading the transformer is to decrease the total noise, but the analysis shows that some harmonics notably second, sixth, and tenth are weakened much more than the average.

Calibration

Laboratory calibration is made by comparison with primary sound standards on sine-wave tones of variable loudness and frequency. The instrument contains voltmeters for checking battery condition but in addition a convenient scheme is used for an over-all one point check which can be made in the field. This is accomplished by plugging a telephone receiver used with the noise meter into the output circuit and placing the receiver against the microphone. The attenuator having been set at a predetermined point, the amplification is adjusted by an auxiliary control until the noise meter is on the verge of self-oscillation. Since the oscillation is due to acoustic feed-back, the amplifier is again stable when the receiver is removed and is adjusted to a standard sensitivity.

Conclusions

Although knowledge of the sense of hearing is incomplete, it is felt that the instrument described serves a very useful purpose in studying machinery noise, the manner of its generation and transmission, its composition and magnitude.

A new analyzer having a high degree of selectivity and convenience of operation has been developed and combined with sound and vibration pick-up devices so that quantitative analyses can be made on electrical, mechanical, and acoustical waves.

Acknowledgment is made of the able work of Messrs. M. S. Mead and T. M. Berry in the development of the noise meter and analyzer, and of C. D. Greentree in the development of the vibration pick-up.

Discussion

For discussion of this paper see page 1071.

Induction Regulator Noise

BY J. P. FOLTZ* and W. F. SHIRK*

Associate, A. I. E. E. Non-member

IN recent years designers of machines have been confronted with the necessity of making them operate with a minimum of noise. This has been brought about by a general rebellion of the public against unnecessary noises. The Noise Commission in New York City is both an effect and a cause to bring about further action on the subject. For many years there has been some agitation along this line, but lately it has become an important factor affecting the sale of the machines.

At the same time the design of quiet machinery has been made more difficult due to a tendency to reduce the size and weight of machines. This makes it necessary to work electrical and magnetic materials harder, figure stresses higher and allow greater deflections. With the improvement of electrical and magnetic properties of materials it is possible to build machines smaller from an electrical point of view. This produces a weaker structure mechanically which will vibrate more since the forces involved have not been changed or may even have been increased under certain conditions. It is necessary then for the engineer to look to improvements in mechanical design which will overcome these difficulties that are being aggravated by the trend in electrical design.

One of the most important prerequisites to the study of noise in induction regulators is the establishment of a method whereby quantitative measurements of noise can be made. Surroundings have a very large affect on the values of sound intensity obtained by sound measurement for any particular machine, so for consistent results all sound measurements would have to be made under identical conditions, as regards not only background noises, but also surfaces in the vicinity. Induction regulators, however, are usually too heavy to handle readily without heavy cranes and it is therefore quite inconvenient to move them to a place suitable for making sound measurements.

In order to eliminate many of these difficulties it has been found desirable to measure surface vibration instead of sound produced. That the surface vibration bears a quantitative relation to the sound produced can be seen from the fact that in regulators there are no fans or other moving parts which might act on the air directly to produce a sound; therefore, the sound must all be radiated from some vibrating surface. By following this idea all the need for special surroundings is eliminated and direct comparison of machines is possible on the test floor during working hours and with a minimum expenditure of time. This principle also makes it possible to make representative measurements on the various elements of the machine tracing the vibration to its source and determining the effect of different conditions on the source.

The noise produced by induction regulators and transformers is quite simple in character as compared with noise produced by rotating machinery. The noise, exclusive of clicking of relays during the period of voltage adjustment is radiated directly from the tank walls, cooling tubes, and boxes housing the control panels which fasten directly on to the side of the regulator, and has a fundamental frequency double that of the supply voltage. There may be harmonics of the fundamental present depending on the supply voltage, but they are usually not large where the voltage supply is reasonably pure.

Experience gained on small single-phase regulators in the past has indicated that invariably noise could be reduced by a well-balanced gap and small clearances in the bearings. More recent experience, however, gained with larger machines, has shown that there are several types of vibration, especially in larger regulators, which no amount of accuracy as to fits and centering will cure.

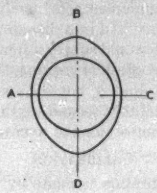

FIG. 1—DEFORMATION OF STATOR INTO AN ELLIPTICAL SHAPE BY THE FORCES OF THE FIELD—SHOWN EXAGGERATED

According to the source, regulator vibration may be divided under three headings. In its elementary form an induction regulator consists of a stator of hollow cylindrical shape built of steel punching rings stacked to a considerable length and assembled in a frame, a rotor separated from the stator by a small air gap and held concentric by means of bearings fastened to the stator at either end. This whole assembly is immersed in a tank of oil for cooling and insulation. Both stator and rotor are slotted to provide space for the coils. The coils on the rotor are usually the primary and are connected across the supply line.

The force due to the flux across the gap is radial and is balanced. Its only effect then is to deform the stator into an ellipse, in the two-pole regulator, with minor axis corresponding to maximum of flux wave. This

1. Both of the Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

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deformation is shown much exaggerated in Fig. 1. In the case of a single-phase machine the position of the deformation does not move but its amplitude changes with time since the field is alternating. Since the parts are not polarized, both halves of the cycle produce the same deflection in the same direction and the resulting vibration has a frequency double that of the line. In the case of a three-phase machine the value of the field does not change but the axis or maximum point moves around the gap at synchronous speed. In this case the

FIG. 2—VIBRATION OF STATOR UNDER RADIAL FORCES OF FIELD—SHOWN EXAGGERATED

The drawing is not quite accurate since the end portions vibrate some but the increased rigidity due to the end frames prevents the ends vibrating as much as the center

deformation does not change magnitude but rotates in space with the field. The result of this deformation into an elliptical shape is a vibration of the stator in a radial direction. This may be termed "hoop vibration" to distinguish it from the other types to be mentioned later. This vibration is transmitted directly through the oil to the tank. Fig. 2 gives an exaggerated view of this motion.

If the rotor is out of center or eccentric with regard to the stator (as shown exaggerated in Fig. 3) the forces

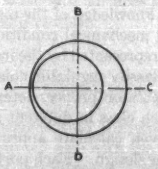

FIG. 3—EXAGGERATED SKETCH SHOWING ROTOR OUT OF CENTER IN STATOR

on either side of the rotor are not balanced and there is a force tending to vibrate the rotor and stator as beams supported at the ends. This vibration is transmitted to the tank partly through the top and partly through the oil directly from the stator.

A third type of vibration, evident in some single-phase regulators, is torsional. Torsional vibrations are carried to the cover through the worm and segment. While this type does not contribute much to the noise directly, if of considerable magnitude, it is liable to cause impact between worm and segment with a resultant rattle that is quite objectionable.

These types of vibration are the same frequency but it is necessary to distinguish between them when the question of source or cause of vibration arises.

The more important type of vibration in large three-phase two-pole regulators is produced by the elliptical deformation of the stator under the action of the field. In the case of four-pole machines the field acts at four points with the result that the effective rigidity of the stator is increased and the deflection decreased. Most machines, however, are of the two-pole type, since they are more economical of floor space. Then this type of vibration becomes the most important in larger machines. In small machines the stator rigidity is high enough to limit this vibration to a point where the other types become of importance.

In designing regulators it is desirable to calculate the probable amount of vibration that will be produced in the finished machine. It is possible to do this approximately by the usual theoretical formula provided certain assumptions and approximations are made.

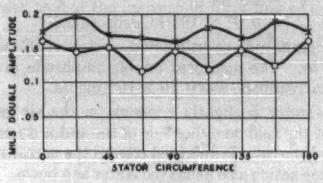

FIG. 4—RELATIVE MOTION BETWEEN CORE AND FRAME

X—Vibration amplitude on stator
O—Vibration amplitude on frame
Punching bars at 0-45-90-135-180 degrees

In the case of hoop vibration the annular rigidity of the stator is the factor controlling the amplitude, so the amplitude of vibration is the same as the static deflection under the load produced by the force of the field. In the case of a three-phase regulator the field is fairly well distributed and it may be considered as having a sine wave of distribution. Then for a two-pole regulator the deflection can be calculated approximately by the following expression:

$$= \frac{q\,R_1^3\,R_2}{18\,E\,I}$$*

q = maximum magnetic force—lb. per inch of circumference.

R_1 = radius to neutral axis of ring.

R_2 = radius at air gap.

In computing I it is necessary to correct for the fact that the stator is built from punchings. In this example I is calculated as 75 per cent of I for a solid ring of the dimensions of the stator exclusive of teeth.

*"Etude Meneanique Et Uswage Des Machines Electriquet," by H. De Pistoye, p. 560.

Again the I of the frame should be included but the frame and punching are not rigidly connected so the two moments of inertia will be added. Fig. 4 gives comparative readings on frame and core of a large regulator operating at rated voltage and frequency. This shows the flexibility between frame and core. When computing q_0 it is necessary to correct for the effect of slotting. A discussion of this problem is given in the appendix.

$$q = 0.014\, B_m^2\, L\, F$$

L = length of core

F = factor to correct for slotting

For a representative three-phase two-pole regulator the following values have been computed using a value of F for the punchings 75 per cent of the I of a solid ring.

R_1 = 13 in.

R_2 = 9.6 in.

I punchings 114 in.^{4}

I frame 4 in.^{4}

I total 118 in.^{4}

q = 210 lb.

= 0.07×10^{-3} in. deflection.

Double amplitude as calculated $= 2 \times 0.07 = 0.14 \times 10^{-3}$ in. The measured double amplitude (average of several readings) was 0.19×10^{-3} inches.

If the rotor is slightly eccentric in the air gap the forces of the field on either side of the stator do not balance and the rotor is deflected toward the narrower gap. This same action also bends the stator as a beam. When the field changes this deflection takes the form of a vibration having a frequency double that of the supply voltage. This vibration adds to the hoop vibration on one side and subtracts on the other in the case of a three-phase, two-pole machine. The vibration of the rotor is carried to the covers and thence to the outer tank surface. If the rotor was very eccentric in the gap this vibration might become bad but it is quite possible to keep the eccentricity low enough in practise on large regulators to keep this type of vibration within limits.

Torsional vibrations of the rotor, due to considerable flexibility in the shaft and worm are controlled by the mass reaction of the rotor. This type of vibration is of more importance in small regulators, or large regulators on 25-cycle service. The frequency of the torsional vibrations is double the frequency of the line as with all the other types and the amount of frequency reaching the outer surfaces is usually not large so ordinarily it does not directly contribute much to the noise. However, in the worm and segment there is a system which has a variable spring constant so if this torsional vibration becomes large, higher frequencies are introduced due to impact between worm and segment. These lie in the range of greater ear sensitivity and are very objectionable. The introduction of flexibility between rotor and segment or between worm and cover eliminates this difficulty preventing the vibration reaching the cover and preventing impact between worm and segment.

While the vibration has its origin in the stator core it must pass through the oil and move the tank wall before it can be radiated as sound. The oil is nearly incompressible and serves to carry the vibration to the tank. The tank however, does not follow the core. It appears to have a number of locally resonant regions on the surface whose positions are not at all definitely determined. These do not necessarily locate themselves symmetrically on two similar machines or even on the two sides of one machine. They even quite frequently shift position without any apparent reason so that it is seldom possible to check amplitude measurements made on a particular point on the tank. As a result, in order to compare machines, it is necessary to take a number of readings in both a vertical and horizontal plane and obtain an average.

The presence of cooling tubes further complicates the action of the tank wall. Tubes are liable to be resonant in themselves. This should be guarded against not only because of the noise caused, but because due to the large amplitudes that occur at resonance it is likely to fatigue the weld where the tube enters the tank resulting in a leak. In these tubes one may find anything from nearly rigid support to nearly perfect flexibility at end welds. In order to be safe then it is necessary to work outside the region between these two extremes. This resonance region can be computed by consideration of weight of tube full of oil, inertia of cross section and length. It is then the problem of the uniformly loaded beam. Where the tubes are supported upon one another or on the tank at any place other than the end, the problem becomes indeterminate analytically.

With information gained from vibration measurements and some knowledge of the magnetic, electrical, and fundamental mechanical conditions in the general type of structure represented by the induction regulator, it is possible to classify regulator noise according to its source in the regulator proper, determine the relative importance of each class in different sizes and types of regulators and even calculate approximately what to expect from a new design. Each portion of the regulator may be approximated by some mechanical system whose characteristics can be determined. Then checking with actual measurements on finished machines it is possible to correct any erroneous mathematical assumptions as well as obtained correction factors which will bring the calculated values closer to the actual values obtained by measurement and make it possible to calculate accurately performance of new designs as regards noise.

The authors wish to acknowledge their indebtedness to Mr. J. P. DenHartog for theoretical work in connection with stator hoop deflection and to Mr. L. G. Tubbs for suggestions relating to the experimental tests.

Appendix

In calculating the force between stator and rotor in an induction regulator it is necessary to take into consideration the effect of the slotting of both stator and rotor. The effect of the slots is to increase the reluctance of the gap. The flux does not decrease, however, since the voltage induced by it in the primary must balance the supply voltage. The result is an increase of energy in the gap or an increase of force between the stator and rotor. The exact amount of this force will vary with the relative position of teeth on the stator and rotor. By use of the fundamental equation of potential magnetic energy and Carter's coefficients for calculating reluctance of a gap between toothed surfaces, it is possible to obtain a factor representing the average over the pole face of the increase of force due to slotting or due to concentration of flux in the gap over a tooth.

If W represents the potential magnetic energy in the field between any two magnetic bodies separated by an air gap and r is the reluctance of the gap:

$$W = k\,r\,\phi^2$$

where k is a constant depending on the units used.

Suppose the gap is decreased a differential amount $d\,g$; this will change the reluctance by $d\,r$, then the change in energy

$$d\,w = k\,\phi^2\,d\,r \tag{1}$$

Also if f is the force between the two bodies

$$d\,w = -f\,d\,g \tag{2}$$

From (1) and (2)

$$f = -k\,\phi\,\frac{2\,d\,r}{d\,g} \tag{3}$$

The sign indicates that when the body moves in the direction of the force the field does work and hence loses energy.

In the case of two smooth surfaces separated by an air gap where the gap is uniform

$$r = C\,g$$

and

$$\frac{d\,r}{d\,g} = C \tag{4}$$

When the surfaces are slotted, the reluctance will vary not only with the gap but also with the dimensions and number of slots. If the increase in reluctance due to slotting is taken care of by means of Carter's coefficients then

$$r = C\,v$$

where v equals Carter's coefficient times the actual air gap. Then

$$\frac{d\,r}{d\,g} = C\,\frac{d\,v}{d\,g} \tag{5}$$

when $\frac{d\,v}{d\,g}$ approaches unity we have the condition of smooth surfaces.

Then if f_1 and f_2 represent the force between the two surfaces with and without slots respectively

$$f_1 = -k\,\phi^2\,C \qquad \text{from (3) and (4)}$$

$$f_2 = -k\,\phi^2\,C\,\frac{d\,v}{d\,g} \qquad \text{from (3) and (5)}$$

The change in force

$$-f_2 f_1 = -k\,\phi^2\,C\left(\frac{d\,v}{d\,g} - 1\right)$$

Using Carter's coefficients to determine v

$$v = \frac{g^2 + \frac{b}{5}}{g + \frac{b}{5} - \frac{b^2}{5\,c}}$$

$$\frac{d\,v}{d\,g} = \frac{2g\left(\frac{g}{2} + \frac{b}{2} - \frac{b^2}{5\,c}\right) + \frac{b}{5}\left(\frac{b}{5} - \frac{b^2}{5\,c}\right)}{\left(g + \frac{b}{5} - \frac{b^2}{5\,c}\right)^2} = v$$

where b and c represent slot dimensions as shown in Fig. 5.

In order to find the increase of force due to slotting one member, it is necessary to evaluate $\frac{d\,v}{d\,g}$ for both

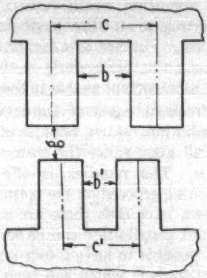

Fig. 5

stator and rotor. Then according to the use of Carter's coefficient, the factor by which the force calculated for uniform gap must be multiplied is $\frac{d\,v}{d\,g}$. A similar factor exists to correct for slotting the other member, so the total correction factor

$$f = v_s\,v_r$$

where v_r and v_s represent values for rotor and stator respectively. The average total force then between the rotor and stator, where both are slotted, is the force calculated for smooth surfaces having a similar gap and flux density, multiplied by F.

Discussion

For discussion of this paper see page 1071.

Magnetic Noise in Synchronous Machines

BY QUENTIN GRAHAM*, STERLING BECKWITH*, and FRANK H. MILLIKEN*

Associate, A. I. E. E. Non-member Non-member

Synopsis.—Magnetic noise in synchronous machines is shown to be of two principal kinds. One is a vibration in the tooth frequency range which usually occurs whether load is present or not; the other has twice line frequency and is dependent on load current. The vibrating part of the structure is usually the stator but may be the rotor and in nearly all cases of troublesome noise mechanical resonance is present. Some measurements of vibration on noisy machines are given.

WHILE magnetic noise in synchronous machines, particularly of the low-speed class, has often been a source of annoyance both to designers and operators, the literature on the general subject of noise is notably free from any discussion of the specific causes of noise in this class of machines. It is proposed in this paper to point out some of these causes and to show the extent to which noise may be controlled by the designer.

When the noise problem in synchronous machines was first investigated several years ago there was a great deal of speculation concerning the particular part of the machine that produced the principal vibration. Suspicion was cast on the stator teeth, on the overhanging tips of the poles, and on the poles as a whole. It finally became evident, however, that one of the principal sources of trouble was the vibration of the stator as a unit. Any condition which caused the stator to change from a circle to an ellipse periodically was likely to produce noise. Further investigation has revealed other sources of noise, particularly in the rotor, which may be almost as important as that in the stator.

The most frequent cases of stator vibration arise from flux distributions which, because of unequal magnetic pull on all axes, cause the stator to assume an elliptical shape. This requires, in effect, a two-pole flux distribution superposed on the normal distribution. It will be shown later that there are several ways in which such a flux distribution can be set up. In addition, it is also possible to have a four-pole distribution (or any larger number) which will tend to distort the stator at four (or more) points. Since the stator structure is much more rigid with respect to forces of this kind than to the two-pole force, they are not such frequent sources of trouble.

Rotor vibration takes more forms than stator vibration. One possibility is for the rotor rim between arms of the spider to move in and out, either in phase or 180 degrees out of phase with the segment between the succeeding spider arms. Another possibility is for the poles to vibrate circumferentially. A third mode of vibration occurs when parts of the spider rim vibrate axially. This last type of motion resembles that occurring in gears.

Under certain conditions it is possible to have magnetic forces tending to pull the rotor out of center. This requires a distribution of force having only one wavelength in the circumference of the machine. It might be called, by analogy, a one-pole distribution and is similar to what occurs when the air gap is unequal. Such forces are found in induction motors[1] due to combinations of rotor and stator harmonics, but are seldom encountered in synchronous machines. They would occur if odd numbers of slots were used, but since the number of slots is almost invariably an even number, especially in large machines, this type of vibration rarely occurs and will not be discussed here in detail.

Consideration of Periodic Forces

Two important classes of noise or vibration with reference to frequency have been noted and these will be discussed under separate headings. The first is tooth frequency noise, usually occurring at no-load and independent of armature current, but sometimes proportional to or increasing with armature current. Although called tooth frequency, it may be slightly greater or less than tooth frequency, due to the rotation of one member. The second type of noise is not present at no-load, but depends upon the armature current for its existence. The frequency is pole frequency, that is, twice the line frequency, and is thus much lower in tone than the other type of noise. It is, in fact, usually spoken of as a vibration and while still in the audible range, it is more often felt than heard.

It is realized that noises of other frequencies may exist but as they are relatively unimportant they will not be considered here.

Tooth-Frequency Forces. In order to explain the cause of tooth-frequency noise, it is convenient to consider the change in flux of a single pole as it moves over one stator tooth pitch. Clearly there is some point as in (*a*) Fig. 1, where the flux is a maximum and some point as in (*b*) Fig. 1, where it is a minimum. Thus the flux in each pole rises and falls slightly each time a tooth pitch is traversed and the magnetic pull between that pole and the stator goes through a corresponding cycle. If the number of slots per pole is an

*Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

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1. *Quiet Induction Motors*, L. E. Hildebrand, A. I. E. E. Trans., 1930.

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integer, each pole on the rotor is passing through the same cycle at the same time. That is, the variation in magnetic pull is in time phase in all poles. If the rotor is centrally located the result is simply a variation in the radial pull which is the same across each axis.

Suppose now that the number of slots per pole is not an integer. Then the magnetic pull of all poles is not

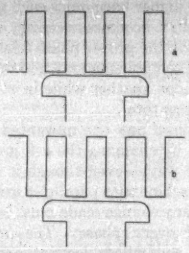

Fig. 1—Position of Maximum and Minimum Permeance

a. Position of maximum permeance
b. Position of minimum permeance

in time phase and it becomes necessary to examine the relative position of slots and poles in more detail.

The simplest case is that in which the number of slots per pole consists of an integer plus a fraction having one in the numerator, such as 5 1/7. It is evident that if No. 1 pole has maximum flux at a given instant, No. 2 pole, being 1/7 tooth pitch from the same relative slot position, will have slightly less flux. Each succeeding

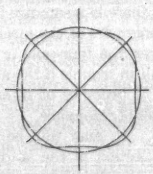

Fig. 2—Distortion of Stator Due to Four-Pole Type of Pull

pole will have a little less flux than the preceding one until the minimum is reached, after which there will be a gradual increase. If the machine under consideration has fourteen poles there will be two areas diametrically opposite having high flux density and two areas on an axis at 90 degrees to these having low density. The result is a higher magnetic pull along one axis and a tendency to give the stator an elliptical shape. In the case of a 28-pole machine having the same number of slots per pole there would be four high density areas and four low density areas giving a distortion as shown in Fig. 2.

It now remains to see what happens to the magnetic pull or stator distortion as the field member rotates. Using the example above, it may readily be seen that when the rotor has moved 1/7 tooth pitch the point of maximum density has shifted from Pole No. 1 to Pole No. 2. When a tooth pitch has been traversed, the point of high density has moved seven pole pitches *as measured on the rotor*. With reference to the stator the distorting force has moved seven pole pitches plus one tooth pitch.

Thus it may be seen that a given point on the stator goes through a complete cycle from maximum pull through minimum pull and back to maximum, while the rotor moves slightly less than one tooth pitch. It is

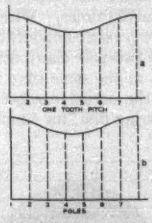

Fig. 3—Variation of Pull on One Pole

a. Variation of pull on one pole as it moves over one tooth pitch
b. Variation of pull-per-pole over a group of d poles (Case 1)

shown in Appendix 1 that for the general case the frequency of the deformation is: tooth frequency + twice line frequency [slots per pole − nearest integer].

It is also shown as Case 1 in Appendix 1 that in order to have a gradual distribution of density or pull over a group of adjacent poles, certain fractional numbers of slots must be used. With certain other fractional

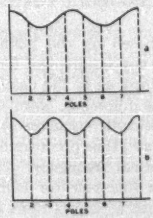

Fig. 4—Variation of Pull on One Pole (Cases 2 and 3)

a. Variation of pull-per-pole over a group of d poles (Case 2)
b. Variation of pull-per-pole over a group of d poles (Case 3)

numbers it is shown that the poles having high density at a given instant are so alternated with poles of low density that the frame deformation is of a different form. The curves of magnetic pull plotted against pole position in these cases have two or more maximum points as shown in Fig. 4. To continue the example of a machine having 14 poles, the predominant component

of the pull now has the effect of a rotor of more than two poles. Due to the greater rigidity of the frame, a pull of short wavelength is usually unimportant. There may be a small component of pull which has a two-pole distribution, but its magnitude is so much less than the Case 1 distribution shown in Fig. 3b that there is seldom any trouble from noise. As a result, it is possible to classify fractional slot machines into those that are usually free from serious no-load noise (Case 2, Case 3, etc.) and those that are quite susceptible to it (Case 1).

Pole-Frequency Forces. It has previously been pointed out[2] that the armature m. m. f. waves of machines having fractional slot windings contain components of low harmonic order. With certain slot arrangements there may be a component wave of m. m. f., and flux, having a two-pole distribution. Other components may exist at the same time which are also of fewer poles than the actual number of poles on the machine. Considering first, for simplicity, a two-pole

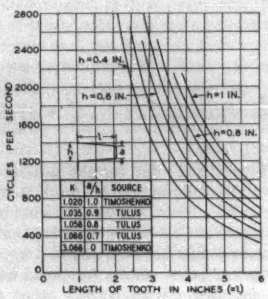

FIG. 5—RESONANT FREQUENCY OF STATOR TEETH FOR TANGENTIAL VIBRATION

distribution superposed on the main distribution of flux, we again have the requirement for elliptical distortion of the stator just as in the case described under no-load noise. The difference, however, is that in this case the two-pole field travels at the synchronous speed of a two-pole rotor and thus produces a vibration of the stator at twice normal frequency. Similarly, a subharmonic having a four-pole (or higher) distribution may distort the frame at four or more points and cause vibration. The frequency will be twice normal in all cases since the rate at which poles of the superposed distribution pass a given point on the stator is the same regardless of the number of poles.

While it is convenient to start from the assumption of a single two-pole component of armature m. m. f. in describing the effect of a two-pole pull, this simple case does not complete the story. It is shown in Appendix 2 and illustrated in Fig. 10 that when there exist any two components of flux differing in harmonic order by two, there will be a resulting modulated wave which gives the same effect as a simple two-pole distribution.[3] When the order of magnitudes is examined it is found that these modulated waves are really the important ones.

Since it is possible to have a two-pole pull due to a single sub-harmonic and at the same time to have combinations of two higher harmonics each giving an effective two-pole pull, it becomes necessary to combine the two or more sets in the proper phase relation. In some cases the several components may add in such a way as to partly cancel one another while in other cases they add to form a higher total.

The elimination of any one unwanted harmonic can be accomplished by changing the coil grouping[4] in two relatively simple ways—even though it is complicated somewhat in a machine with paralleled armature circuits by the fact that any change made must be duplicated in each parallel of every phase. The first method is merely to cut out coils whose electrical position coincides with the vector position of the harmonic to be eliminated. The second way is to shift coils from one phase group to the next adjacent group in such a way that the troublesome harmonic is decreased without reducing the terminal voltage of the machine appreciably.

MECHANICAL ASPECTS

The existence of various forces tending to distort the machine has been established. It has been pointed out that the most serious cases of trouble arise from the two-pole type of pull, since forces distributed in this way produce the greatest deflection. Experience, however, seems to show that nearly all cases of objectionable noise* occur when there is resonance between the magnetic forces and some part of the mechanical structure. For this reason even the four-pole or higher force distribution may result in trouble if the necessary resonant condition exists. It becomes of interest then to consider the possibilities of resonance.

Stator Vibration. It may not be entirely obvious that a rotating deflection of the stator can be affected by resonance. It can be shown however, that the rotating pull or deflection may be considered as the result of two stationary pulsating pulls which are 90 degrees apart in space and time. Thus resonance to a traveling elliptical deflection is quite possible. In practise, one of these components is usually damped considerably by the frame supports.

Tests made on a number of machines have shown that resistance to a distortion in the plane of the laminations is usually dependent entirely upon the frame or clamping plates and not upon the laminations. The

2. *The M. M. F. Wave of Polyphase Windings*, Graham, A. I. E. E. TRANS., 1927, Vol. XLVI, p. 19.

3. P. L. Alger and W. V. Lyon, discussion of *The M. M. F. Wave of Polyphase Windings*, Graham, A. I. E. E. TRANS., 1927, Vol. XLVI, p. 29.

*We are well aware of the difficulties of defining an objectionable noise. The term is necessarily relative and is used here to mean a noise considerably worse than that of average commercial machines for industrial plants or power stations.

laminations become so much dead weight attached to the frame but add nothing to its stiffness. The following formula developed by Timoshenko[4] for simple rings has been applied to complete stators in a number of cases and has given useful results, as shown by Table I and by the comparison between calculated and test results in Figs. 7 and 8.

TABLE I—NATURAL FREQUENCIES OF STATORS OF TYPICAL SYNCHRONOUS MOTORS FOR COMPRESSOR DRIVE

Frame size suitable for	Natural frequency of stator				
	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$
100 hp. 360 r. p. m.	190	540	1,030	1,670	2,450
150 hp. 277 r. p. m.	127	360	690	1,120	1,640
200 hp. 200 r. p. m.	85	240	460	750	1,100
400 hp. 200 r. p. m.	70	200	370	620	900
600 hp. 150 r. p. m.	54	150	200	470	690

$$f = \frac{1}{2\pi r^2}\sqrt{\frac{g E I i^2 (1 - i^2)^2}{\gamma A (1 + i^2)}} \qquad (1)$$

where E = Modulus of elasticity.
γA = Weight of ring per circumferential inch.
r = Mean radius of ring.
I = Moment of inertia of frame (punchings considered as dead weight).
i = Wavelengths per circumference.
g = Acceleration of gravity.

Rotor Vibration. While it has been shown that stator vibration is of consequence only when there is a force tending toward elliptical distortion of the frame, rotor vibration may also be set up by a periodic variation in the magnitude of the normal radial gap force, such as that occurring in integral slot machines.

Rotor resonance is affected so much by the rotor arms that it is not susceptible to such accurate calculation as the stator. Two formulas,[5] however, have been found useful in giving the probable upper and lower bounds of resonant frequency for radial vibration of the rim. One, for a section of rim pivoted at both ends, is:

$$f = \frac{C_3}{2\pi r^2}\sqrt{\frac{g E I}{\gamma A}} \qquad (2)$$

Where g, E, I, r, and γA are the same as in formula (1), and α is the angle subtended by the rim segment.

$\alpha = 0 - 20° - 40° - 60° - 80° - 100°$

$C_3 = \infty - 321 - 78.5 - 33.6 - 17.8 - 10.65$

The other, for a section of rim fixed at both ends, is:

$$f = \frac{C_1}{2\pi r^2}\sqrt{\frac{g E I}{\gamma A}} \qquad (3)$$

Where g, E, I, r, and γA are the same as in formula (1), and α is the angle subtended by the rim segment.

$\alpha = 20° - 40° - 60° - 80° - 100°$

$C_1 = 504 - 124 - 53.8 - 29.2 - 17.9$

A formula for the resonant frequency of gears with different modes of rim vibration is also available,[6] and has been found useful, but requires so much empirical correction when applied to plate spiders that it will not be given or discussed here.

Stator Tooth Vibration. Tangential vibration of the stator teeth is often thought of as the simplest and most probable form of mechanical vibration causing tooth-frequency noise. If it be agreed that resonance is necessary for appreciable tooth motion, then Fig. 5 shows[7] that almost all machines are out of the range

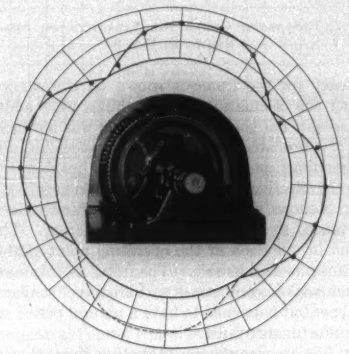

FIG. 6—MAGNITUDE OF VIBRATION AT VARIOUS POINTS AROUND THE STATOR

where tooth resonant frequency approaches tooth frequency. The assumption that resonance is essential is justified by past experience, and appears reasonable from a consideration of the stiffness of the teeth.

Test Results. Figs. 6 to 9 show the results of several vibration tests of machines having pole frequency vibration. The measurements were made by amplifying the resistance variations in a phonograph pick-up the needle of which followed the motion of the frame. The accuracy of this method is indicated by the consistency of the test points.

Fig. 6 shows the variation in the amplitude of the vibration at different points around the circumference. (The nodes could be located easily by feeling the frame.) The picture of the machine in the center of the figure is

4. "Vibration Problems in Engineering," S. Timoshenko, p. 297, Van Nostrand Co.

5. "Vibration of Frames of Electric Machines," Den J. P. Hartog, *ASME*, presented May, 1927.

6. "Natural Frequency of Gears," R. E. Peterson, *ASME*, presented December, 1929.

7. Resonant Frequency for Wedges and for Rectangular Teeth from S. Timoshenko, Vibration Problems in Engineering, pages 234 and 267. Factors for tapered teeth derived by E. A. Tulus of Westinghouse Elec. & Mfg. Co.

correctly oriented with the vibration curve. Measurements were made on the back of the punchings half way between the ends of the core although the results would have been the same had they been made at any point on the same axial line.

Fig. 7 shows the change of vibration amplitude as the supply frequency is varied. The calculated resonant

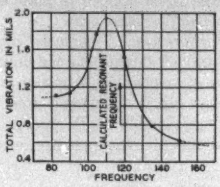

Fig. 7—Resonance Curve for Vibration of the Type Shown in Fig. 6

frequency for the frame is shown on the curve. In making the calculation the punchings were added as dead weight but the armature copper was not included since it is probable that the slot clearance, flexibility of end windings and damping effect of insulation may cause the effective mass to be zero or even negative.

Fig. 8 shows similar tests on a machine which was later braced to add stiffness to the structure. The reduction in vibration at the operating point (120-cycle vibration) is considerable. The frame of this machine was not weak in comparison with frames of similar sizes and types but happened to have a natural period which gave unfortunate results.

Fig. 9 is an exceedingly interesting curve showing a

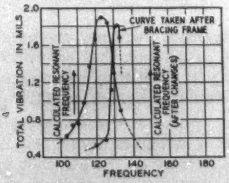

Fig. 8—Resonance Curve for Vibration of the Type Shown in Fig. 6

dip in the center which was found to be due to end-bell resonance at that point. The violent oscillation of the rather light end-bells absorbed sufficient energy to decrease the frame vibration at what otherwise would have been its region of greatest amplitude. Since calculation of the resonant point of a complex structure like an end-bell is practically impossible and since its likelihood of setting up objectionable overtones is great it illustrates the difficulties that face the designer in trying to predict his results.

Vibration of a stator in the tooth-frequency range has not been measured accurately although it has been observed and definitely identified. One machine exhibiting this type of vibration had eight distinct nodes (the four-pole type of pull). The number of slots per pole was such as to give a vibrating frequency of 635 cycles while the calculated resonant frequency of the frame for this mode of vibration was 645 cycles. The same frame was later tested with four-node (two-pole) vibration and a resonant frequency of 115 cycles was found. Since there is definite ratio between the resonant frequencies for the four-node and the eight-node type of vibration as shown by equation (1) it can be concluded that the resonant frequency for the eight-node case was actually 623 cycles. This value

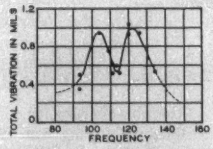

Fig. 9—Resonance Curve for Vibration of the Type Shown in Fig. 6

compared with the frequency of 635 cycles for the disturbing force indicates that a high state of resonance existed.

Appendix 1

Let the magnetic pull on each pole as it moves through one tooth pitch be represented by the curve of Fig. 3a. In the case of fractional slot windings each pole passes through the same cycle but not in time phase with all other poles. Let the curve, Fig. 3a, be divided in d parts where d is the denominator of the fraction expressing the slots per pole. Then at a given instant the pull of each pole can be taken from the intersection of the curve with one of the vertical lines. The particular point on the curve applying to any given pole is found from a consideration of the fraction expressing the slots per pole.

$$\text{Let, slots per pole} = \text{any integer} \pm \frac{n}{d}$$

There are several cases to be considered depending on the value of n.

Case 1; $n = \pm 1$. If $n = +1$ it is easily seen that adjacent poles will have values of pull taken from adjacent points on the curve in Fig. 3a. That is, between the center of pole No. 1 and pole No. 2 there is an integral number of tooth pitches plus $1/d$ tooth pitch. Thus the pull of pole No. 2 at any instant is the same as the pull of pole No. 1 will be after it has moved $1/d$ tooth pitch. A curve showing the pull of a group of d poles is given in Fig. 3b.

It can be seen that a similar condition would exist if $n = -1$.

Case 2; $n = \pm 2$. When $n = +2$ adjacent poles will have values of pull taken from every second division of the curve in Fig. 3a, since each pole is an integral number of tooth pitches plus $2/d$ tooth pitch from the pole next to it. The curve of pull for a group of d poles in this case takes the form shown in Fig. 4a. A similar curve is obtained if $n = -2$.

Case 3; $n = \pm 3$. *Case 4*; $n = \pm 4$, etc. Curves are obtained similar to Fig. 4 with three, four, etc., waves per group of d poles.

It is possible to have vibration in any of these cases, but experience indicates that it is usually the machines having Case 1 distribution that produce troublesome noise. Thus machines having a number of slots per pole consisting of an integer plus a fraction having numerator equal to plus or minus one are most likely to be noisy.

In Fig. 4b a smooth curve has been drawn through a very few points so as to give a curve which exactly repeats. This simplification of representing a few points by a single harmonic curve, even if the curve passes through all the points, is not quite exact. Furthermore, the assumption that the pull of each pole may be concentrated on its centerline is not strictly true. For these reasons the analysis must be taken as an approximate one which is convenient for illustration and which, fortunately, is close enough to the facts to give practical working rules. It may be possible, however, in Cases 2, 3, 4, etc. to have a small component of force distributed in space as in Case 1 (but traveling at a different speed) and, if a resonant condition exists, to have troublesome vibration. No such cases, however, have been observed.

Frequency of Vibration. For Case 1 distribution it is evident that the wave of force moves one pole pitch on the rotor while the rotor itself moves $1/d$ tooth pitch. When $n = +1$ the wave of force moves against the direction of rotation and when $n = -1$ it moves with rotation. Since the wavelength of the force wave is d pole pitches it is evident that the frequency of the force with reference to the rotor is $2fs$ where f is line frequency and s is slots per pole. That is, it is tooth frequency. During one cycle with respect to the rotor the wave is carried forward or backward one tooth pitch by the motion of the rotor. Since there are $s\,d$ teeth per wavelength the frequency of the vibration with respect to the stator is increased or decreased by $1/s\,d$ times the rotor frequency. Or, stator frequency equals $2fs \pm \dfrac{2f}{d}$.

For Cases 2, 3, etc., where there are n wavelengths in $s\,d$ teeth, stator frequency equals

$$2fs \pm \frac{2fn}{d}.$$

This can be expressed as:

Tooth frequency + twice line frequency [slots per pole − nearest integer] (4)

Appendix 2

Since any given harmonic of flux will not have the same effect in producing pull when it exists by itself as when it is superposed on other harmonics, as illustrated graphically in Fig. 10, it is necessary to consider all harmonics together and take cross-products into account in obtaining pull from flux. In the following analysis, however, only the harmonics cutting the stator with 60-cycle frequency will be considered, as the others can contribute nothing to the 60-cycle vibration phenomena observed. Thus the harmonics

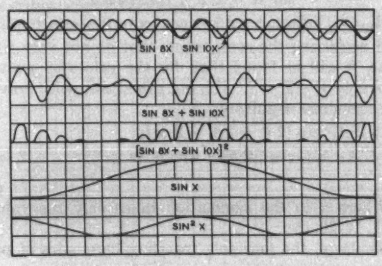

Fig. 10—Combination of Harmonics

left to consider will be only the fundamental of field flux and harmonics due to the armature magnetomotive force.

The general Fourier equation for flux is:

$$\Phi = \sum_n A_n \sin\left(n\,(\theta + \beta_n) \pm \omega t\right) \qquad (5)$$

where n = order of harmonic ($n = 1$ for two pole harmonic)

θ = physical degrees

β_n = phase position of nth harmonic (physical degrees)

$\omega = 2\pi f$

t = time

As indicated[8] in Table II, the numerical summation is

TABLE II—POSSIBLE SUB-SYNCHRONOUS HARMONICS

(For three-phase machines with an even number of slots and 60-degree phase belts)

1. For machines with $10 + 12K$ poles,	
	Possible harmonics are..... +1, −5, +7, −11, +13, −17, +19, etc.
2. For machines with $14 + 12K$ poles,	
	Possible harmonics are..... −1, +5, −7, +11, −13, +17, −19, etc.
3. For machines with $8 + 12K$ poles,	
	Possible harmonics are..... +2, −4, +8, −10, +14, −16, +20, etc.
4. For machines with $4 + 12K$ poles,	
	Possible harmonics are..... −2, +4, −8, +10, −14, +16, −20, etc.

K = any positive integer, including zero.

Negative harmonics rotate rotorwise.

Positive harmonics rotate counter-rotorwise.

Machines with 6k poles are omitted since no two-pole pull exists in these cases.

8. Table II was derived from formulas developed in *The M. M. F. Wave of Polyphase Windings*, Graham, A. I. E. E. Trans., 1927, Vol. XLVI, p. 19.

considerably simplified by the fact that all harmonics do not exist in any one machine.

Substituting the values $n = +1, -5, +7, -11, +13, -17, \ldots$ etc. (where + or − refer to direction of rotation) and squaring:

$$\Phi^2 = \text{constant} \times \text{pull} = \sum_n A_n^2 \sin^2 (n(\theta + \beta_n) \pm \omega t)$$
$$+ \sum_n [2 A_n \sin(n(\theta+\beta_n) \pm \omega t) \sum_{n=1}^{n=\infty} A_{n+1} \sin\{(n+1)(\theta + \beta_{n+1}) \pm \omega t\}] \quad (6)$$

Where the second summation of the second term is performed for every value of n in the first summation (*i. e.*, it is a doubly infinite series).

Using the relations:

$$\sin^2 \alpha = 1/2 - 1/2 \cos 2\alpha$$

and, $$2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

and neglecting the components that do not vary with time, it will be found that, for the values of n given above $(+1, -5, +7, -11$, etc.) the "two-pole" traveling harmonics of pull are:

$$- 1/2 A_1^2 \cos(2\theta + 2\beta_1 - 2\omega t)$$
$$+ \sum_n A_n A_{n+2} \cos\{2\theta + n\beta_n + (n+2)\beta_{n+2} - 2\omega t\} \quad (7)$$

Where n in the summation takes only the values 5, 11, 17, 23, etc.

Similarly, the four-pole traveling harmonics of pull are:

$$\sum_n A_n A_{n+4} \cos(4\theta - n\beta_n + (n+4)\beta_{n+4} + 2\omega t) \quad (8)$$

Where n takes only the values 1, 7, 13, 19, etc.

Harmonics producing 6, 8, 10, etc., pole pulls can be obtained by the same process.

For a machine in which the third or fourth series of harmonics is indicated in Table II, the process can be carried through in the same manner, and, for the two-pole pull, gives the following:

$$\sum_n A_n A_{n+2} \cos(2\theta - n\beta_n + (n+2)\beta_{n+2} - 2\omega t) \quad (9)$$

where $n = 2, 8, 14, 20 \ldots$ etc., in the summation.

For four-pole pull the components are:

$$- 1/2 A_2 \cos(4\theta + 4\beta_2 + 2\omega t]$$
$$+ \sum_n A_n A_{n+4} \cos[4\theta - n\beta_n + (n+4)\beta_{n+4} + 2\omega t] \quad (10)$$

Where $n = 4, 10, 16, 22$, etc., in the summation.

The above analysis applies to machines with an even number of slots. A machine with an odd number of slots, can be considered as half of a machine with an even number of slots, and the results determined and interpreted accordingly.

Discussion

For discussion of this paper see page 1071.

Elastic Supports for Isolating Rotating Machinery

BY E. H. HULL* and W. C. STEWART*

Non-member Associate, A. I. E. E.

IN spite of the increasing refinement in the design of electrical machinery, the combination of high rotative speeds, smaller machine parts, and less massive buildings, makes the problem of noise constantly more acute. In general, two types of noise originate from electrical machinery; direct air noise, and noise transmitted through the machine foundations to the framework of the building in which the apparatus is installed.[1] Difficulties of the latter type may often be remedied by a properly designed elastic support between the machine and building.

Elastic supports can be made to isolate from two types of disturbance; shocks of any frequency typified by the automobile spring suspension, or from vibration of one or several known and constant frequencies. The supports described in this paper fall in the second class. The function of these supports is to *isolate*, not *absorb*, vibration.

The effect of such a support may be demonstrated with the apparatus shown in Fig. 1. The mass M,

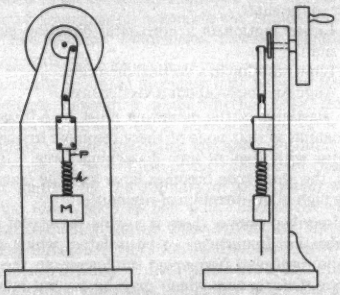

FIG. 1—DEMONSTRATION APPARATUS

constrained to move in a vertical direction, is supported through a spring, k, from the piston, P, which moves through a constant amplitude in approximately simple harmonic motion. M is elastically supported. At a very low speed the weight moves through the same amplitude and in phase with the driving member; with increasing speed the weight amplitude becomes larger until a maximum is reached at the critical speed of the system. Here the phase between the amplitude of the weight and the impressed force shifts through 180 deg., and at progressively higher speeds the weight amplitude becomes smaller, until, at a high speed, the weight remains practically stationary. Substantially the same phenomena is shown by a shaft while passing through its critical speed. The curve marked $b = 0$ in Fig. 2 shows the relation between amplitude (ordinate) and speed (abscissa). From this curve it can be seen that the elastic support is not useful until a speed well above the critical speed is reached.

Fig. 3 represents the general case in which a machine

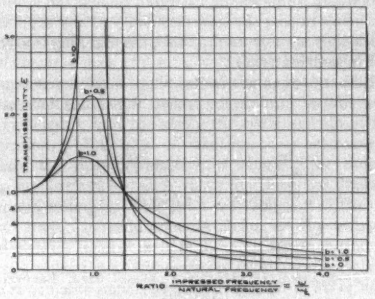

FIG. 2—TRANSMISSIBILITY CURVES

of mass M is supported by a spring of stiffness k, having a damping constant b. A sinusoidal force of maximum value F and angular velocity ω, is impressed on the mass. A certain proportion of F is transmitted to the base through the support and appears as a sinusoidal force of maximum value f. Call the ratio of the force transmitted to the force impressed the transmissibility ϵ. Appendix I gives in equation (4)

$$\epsilon = \frac{f}{F} = \frac{\sqrt{b^2\,\omega^2 + k^2}}{\sqrt{b^2\,\omega^2 + (k - M\,\omega^2)^2}}$$

When $b = 0$ this reduces to

$$\epsilon = \frac{1}{\left(\frac{\omega}{\omega_c}\right)^2 - 1} \qquad (5)$$

where $\omega_c = \sqrt{\frac{k}{M}}$, the critical speed of the system.

As before the curve marked $b = 0$ in Fig. 2 gives the values of transmissibility, ϵ, plotted against a ratio of

*General Electric Co., Schenectady, N. Y.

1. For references see end of paper.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

impressed frequency, ω, to the natural frequency of the system, ω_n. When the damping, that is, friction in the support, is increased, more force is transmitted to the foundation. Curves marked $b = 0.5$ and $b = 1.0$ show this effect plainly.

In our model (Fig. 1) the disturbing medium is an impressed amplitude on the spring, whereas in the actual case the disturbance is a force impressed on the mass. It can be shown, however, that the same expression for transmissibility is obtained in each case.

Several materials are available for elastic support. Metal springs may usually be designed to fit any given installation, but they require considerable space, and the ratio of stiffnesses in the various directions cannot be easily varied. Metal has an obvious advantage in its resistance to deterioration of all sorts.

Some materials such as cork, felt, and gelatinous substances, show a gradually decreasing thickness under load which solidifies the material, raising the stiffness, and hence raising the transmissibility of any support in which they may be used. Also, the damping is usually high in such material.

Rubber compounds containing a large percentage of pure rubber form a convenient substance for use in

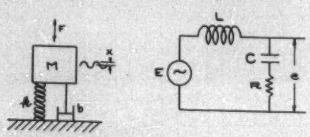

Fig. 3—Elastically Mounted Mass with Electrical Analogy

supports. Small volumes only are necessary, the natural life is fairly long, and may be extended with antioxidants. Oil slowly dissolves rubber, but protection can be arranged in most cases. Once its elastic properties are understood supports may be designed for nearly any combination of stiffness. Damping in rubber is not sufficiently high to cause increased transmissibility.

Damping in the support itself is detrimental since it must be designed to operate where damping raises the transmissibility. On the other hand damping in the foundation under the elastic support is beneficial. In the case of a lightly constructed building this damping would decrease resonance effects in the floors and walls, and decrease the amplitude at any frequency.

The noise and vibration of machines which may be transmitted through the foundation, and which may be isolated by an elastic mounting, fall into three general classes: unbalance, mechanical brush and bearing noise, and magnetic noise. Vibration due to unbalance occurs at rotational frequency which usually is near 1,200 r. p. m. or 20 cycles per second. These frequencies are too low to cause noise at the amplitudes ordinarily encountered, but may cause excessive motion of the foundations occasionally resulting in rattles.

Bearing noise is negligible in journal bearings. Ball bearings, however, cause considerable direct air noise and in some cases are the source of noise which is transmitted to the foundations. Brush noise transmitted to the foundations is not serious in the case of slip-ring motors.

Magnetic forces in the machine are the greatest source of noise. In a-c. machines the fundamental noise frequency is twice the applied electrical frequency or 120 cycles for 60-cycle machines. Harmonics of this fundamental frequency and other high frequencies also occur, but of relatively less intensity.

Elastic mountings used to isolate the above noises and vibrations usually do not constitute a simple system. In general, six natural frequencies of the machine on its mounting occur. These frequencies have been calculated by the expressions given in Appendix II and checked by observations. The modes of vibration of the machine at these frequencies are approximately as follows:

1. Angular motion about a horizontal axis parallel to the shaft.
2. Linear motion in a horizontal direction perpendicular to the shaft.
3. Angular motion about a horizontal axis perpendicular to the shaft.
4. Linear motion in a horizontal direction parallel to the shaft.
5. Linear motion in a vertical direction.
6. Angular motion about a vertical axis.

The elasticity of the mounting must be adjusted to the machine so that none of these resonant frequencies coincides with that of any disturbing force. If this occurs, the amplitude becomes large and the transmissibility high in the direction of resonance.

As described above, there is a wide variety of noise and vibration frequencies to be isolated, which range from the rotational frequency, the lowest, to the high magnetic disturbances which may be several hundred cycles per second. By inspection of the formula for the transmissibility, it is seen that if a mounting is designed to isolate a certain frequency, the higher frequencies will be isolated more effectively. For this reason the high frequencies, unless their amplitudes are greater than ordinarily encountered, do not present any difficulty.

The foundation upon which the machine is mounted affects the mounting design. Noise problems at the present concern installations in hotels, apartment houses, schools, and office buildings, where the foundation is usually a concrete floor. This floor is most sensitive to forces transmitted from the motor in a vertical direction which tend to bend the floor slab. For this reason the transmitted force normal to the floor should be given particular attention.

Damping, as has been described above, is beneficial in the foundation as it reduces the vibration amplitude of the floor which is the source of sound radiation. This is particularly true for high frequencies as they are damped more than the low frequencies.

Since the high frequencies are isolated best by the mounting and are also damped most by the foundation, there usually remain only two frequencies which are troublesome. These are the rotational frequency and

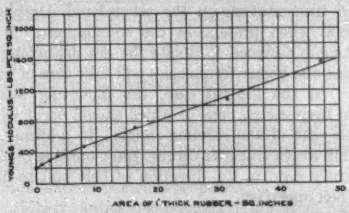

Fig. 4—Variation of Young's Modulus with Area

the fundamental magnetic frequency. In estimating the transmissibility of a proposed mounting, the lowest disturbing frequency should be used as the impressed frequency in the calculation. The vertical resonant frequency of the mounting should be used as this determines the transmission normal to the foundation. The other five resonant frequencies, however, should also be comparatively low.

Elastically mounted machines may be separated into two classes depending upon whether they transmit torque to apparatus on a separate base or are self contained. Motor-generator sets are an example of the latter class. The stiffness of the support for these sets is not limited by the stability of the system against external forces such as belt pull. Hence the vertical natural frequency can be made sufficiently low to isolate the unbalance vibration of the machine, which is the lowest frequency of disturbance present. For these low frequency vibrations, which are below the audible range, a ratio of $\frac{\omega}{\omega_e}$ of 2 or 3 is sufficient.

A number of motor-generator sets has recently been mounted on rubber pads with success. In specifying rubber pads for a mounting of this kind account must be taken of the variable elastic properties of the material. Rubber is practically incompressible if entirely confined. When the material is compressed, the deflection is taken up by a change in shape rather than a change of volume. If the change in shape is restrained, the material becomes stiffer. For this reason Young's modulus in compression increases rapidly with an increase of the area of a rubber pad with respect to its thickness. If the area is very large and the rubber thin, the result is almost total confinement of the rubber and a correspondingly high stiffness. Fig. 4 shows a curve of Young's modulus in compression for 1 in. thick soft rubber as a function of the area.

The modulus of rubber also increases considerably with the deflection. For a given sample of rubber, the stiffness increases with load on the rubber. These variations of stiffness of rubber may be combined for the purpose of specifying the size of pad to be used under machines of various weights. Fig. 5 shows a useful arrangement of the data. This curve gives the area of a pad of rubber to be used for various loads to give a constant value of the vertical natural frequency of the system. This natural frequency is the criterion of transmissibility of the mounting. The curves given here are for 10 and 14 cycles. Similar curves can be drawn for other frequencies.

Four pads of suitable proportion to give the motor-generator set the proper vertical natural frequency are chosen from the above curve. These pads are slipped under the corners of the base between flat surfaces.

In the more general motor applications the mounting is of the first class as it must withstand belt pull. The mounting must have sufficient elasticity in the direction of the disturbing force to give satisfactory isolation, yet be sufficiently stiff to satisfy the requirements of belt operation. Mountings of this type have been used to isolate the torque pulsation of single-phase motors. The support can be made elastic in a torsional direction and stiff radially. This is an easy solution of the conflict between the requirements of elasticity for isolation, and rigidity against belt pull. The result is that the torque pulsation at 120 cycles is satisfactorily isolated. There remains some 120-cycle vibration in other directions which occurs both in single-phase and polyphase motors.

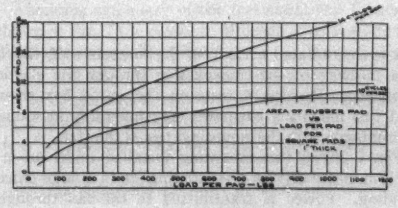

Fig. 5—Area of Rubber Pad Required for Various Loads and Frequency

The application of a mounting to polyphase motors is more difficult. The magnetic vibration in this type of machine may be in any direction, therefore the stiffness must be low in all directions in order to isolate the vibration. On the other hand, the mounting must be sufficiently stiff to withstand belt pull which usually makes it impossible to isolate the low frequency unbalance vibration. This vibration can be cured at the source by an accurate running balance.

Fig. 6 is an illustration of a mounting as worked out for a polyphase induction motor having a speed range from 50 to 100 per cent normal speed. To isolate the 120-cycle magnetic frequency the ratio of the impressed to the natural frequency should be about 8, or for 120-cycle vibration the natural frequency should be 15 cycles. This is in the usual range of rotational frequency and resonance may occur. The six natural frequencies of the mounting mentioned above were adjusted, by varying the size and disposition of the rubber, to be either above or below the range of running speeds.

This mounting has been compared with all of the usual types of mountings on a specially prepared sound box. This box consists of an elastically mounted concrete slab upon which the motor and mounting to be tested are placed. The slab serves as a roof to a box large enough for an observer to enter comfortably. The box sides and door are made of sound proof material which prevents the transmission of extraneous sounds. The only sound which can be heard inside the

Fig. 6—Installation Under Motor Driving Ventilating Fan

box is the radiated noise from the concrete roof, which is a measure of the transmission through the elastic mounting. Results of tests on the sound box show that this mounting compares very favorably in sound transmission with the mountings tested thus far.

Fig. 6 shows this mounting installed under an induction motor driving a ventilating fan in a large office building. Power is transmitted to the fan through a V-belt drive. The belt guard was not fastened to the motor as is usually the case for this would form a rigid contact with the foundations and transmit noise. Flexible leads were also used to prevent vibration being transmitted to the control panel and thence to the foundation. The steel plate underneath the elastic mounting is not necessary for its operation but is used in this particular case to make connection with the foundation bolts which were not located properly for this mounting. No noise transmitted from the motor could be detected in the floor. The motor itself was stable throughout the speed range, vibrating through a maximum amplitude of 2 mils at a low frequency, due to forces transmitted through the belt from the fan. This slight motion is not objectionable.

To summarize; elastic supports are useful only when the ratio $\frac{\omega}{\omega_c}$ is greater than 1.41. A ratio of 2 or 3 is sufficient for unbalance vibration, while a ratio of 7 or 8 is necessary for noise isolation. Damping in the elastic member of a support is undesirable since it causes increased transmission; on the other hand damping is beneficial when present in the foundations. If the mounting is designed to isolate the low disturbing frequencies, the higher frequencies will take care of themselves. In the general case, the mounting has six natural frequencies, none of which should occur within ten per cent of the running range. In addition, for machines which transmit torque, the mounting must be sufficiently rigid in the direction of belt pull to allow stable belt operation.

In conclusion we wish to express our gratitude to Mr. A. L. Kimball of this laboratory for the ideas and assistance which he contributed while this work was being carried out.

Appendix I

Derivation of the Expression for Transmissibility of an Elastic Support

Problems in mechanical sinusoidal motion may be solved, for the steady-state conditions, by setting up the analogous electrical circuit, solving that by the use of complex quantities, and then substituting the mechanical terms for their electrical analogies.[2] Table I shows these equivalents. This process involves the difficulty of transforming to the electrical system and back again, as well as the objectionable fact that the result appears in terms of velocity (current) rather than amplitude (charge).

Certain changes allow the problem to be solved directly in mechanical units as shown below.

$$i = \frac{E}{Z} \text{ or } v = \frac{F}{Z} = \omega x \tag{1}$$

Dividing by ω gives

$$x = \frac{F}{\omega Z} \tag{2}$$

Phase angles computed from (2) will be the complements of the proper phase angles. Multiplying ωZ by j to remedy this difficulty gives[3]

$$x = \frac{F}{j \omega Z} = \frac{F}{Z_m} \tag{3}$$

where Z_m = mechanical impedance, or amplitude impedance, which can be formed from the following components just as electrical impedance is set up from ω, L, C, and R.

TABLE I—ELECTRO-MECHANICAL EQUIVALENTS

Mechanical	Electrical
M...Mass.....	L...Inductance
v....Velocity.....	i...current
x....Amplitude...	q...charge
b....Friction coef..	R...Resistance
$\frac{1}{k}$...$\frac{1}{\text{Stiffness}}$...	C...Capacity
Z....Impedance...	Z...Impedance
F....Force.....	E...Voltage
Z_m..Mechanical impedance.	$j\ \omega\ Z$

Viscous friction (proportional to velocity) is represented by $j\ \omega\ b$.

Mass reaction is represented by $-\ M\ \omega^2$.

Spring reaction is represented by k.

Two general rules based on amplitude may be given for adding the mechanical components of impedance, just as two rules are given, based on current, for obtaining impedance in electrical problems.

1. When various components of a system have the same amplitude impressed upon them, their impedances are added directly to obtain the total impedance.

2. If it is possible for two or more parts of a system to divide an impressed amplitude between them, depending on their impedances, then the reciprocal of the total impedance is the sum of the reciprocals of the individual impedances.

The analogy with the series and parallel cases in electrical theory is obvious.

Referring to Fig. 3 the transmissibility[4] of the mounting, ϵ, equals $\frac{f}{F}$ where F is the maximum value of the impressed force and f is the maximum value of the alternating force on the foundation. In this case the mass, spring, and damping elements have the same amplitude impressed upon each, hence they are in series and their impedances add.

From (3) $$x = \frac{F}{j\,b\ \omega + k - M\ \omega^2}$$

Also $f = x\ (j\,b\ \omega + k)$

Solving for F and taking the ratio gives

$$\epsilon = \frac{f}{F} = \frac{j\,b\ \omega + k}{j\,b\ \omega + k - M\ \omega^2}$$

Taking the absolute magnitude yields

$$\epsilon = \frac{\sqrt{b^2\ \omega^2 + k^2}}{\sqrt{b^2\ \omega^2 + (k - M\ \omega^2)^2}} \tag{4}$$

When $b = 0$

$$\epsilon = \frac{k}{k - M\ \omega^2} \text{ or since } \omega_c^2 = \frac{k}{M}$$

$$\epsilon = \frac{1}{1 - \left(\frac{\omega}{\omega_c}\right)^2}$$

This expression becomes negative at values of $\omega > \omega_c$, indicating that F and f are opposite in phase. Since we are not interested in phase relations this sign may be reversed, giving

$$\epsilon = \frac{1}{\left(\frac{\omega}{\omega_c}\right)^2 - 1} \tag{5}$$

Appendix II

Resonant Speeds of an Elastic Mounting

The six resonant speeds of a machine resting on elastic feet may be calculated using the symbols given below.

k_x = Stiffness of the support in the X direction
k_y = Stiffness of the support in the Y direction
k_z = Stiffness of the support in the Z direction
I_x = Moment of inertia about the X-axis
I_y = Moment of inertia about the Y-axis
I_z = Moment of inertia about the Z-axis
M = Mass of the machine
x = Linear amplitude of vibration of the center of gravity along the X-axis
ϕ = Angular amplitude of vibration about the Z-axis
ω = Angular velocity of the impressed vibration
T = Maximum value of a sinusoidal applied torque.

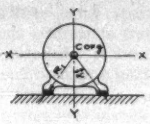

FIG. 7

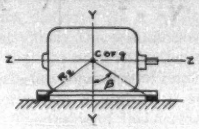

FIG. 8

Assuming symmetry of the feet about the Y-axis as shown in Fig. 7 the equation of linear motion along the X-axis for zero applied force is

$$(M\ \omega^2 - k_x)\ x + \phi\ R_1\ k_x \cos\alpha = 0 \tag{1}$$

The corresponding equation of angular motion about the Z-axis for applied torque $= T$

$$[I_z\ \omega^2 - R_1^2(k_x \cos^2\alpha + k_y \sin^2\alpha)]\phi + x\ R_1\ k_x \cos\alpha = -\ T \tag{2}$$

Solving equations (1) and (2) simultaneously for ϕ

$$\phi = \frac{-\ T\ (M\ \omega^2 - k_x)}{(M\ \omega^2 - k_x)\ [I_z\ \omega^2 - R_1^2\ (k_x \cos^2\alpha + k_y \sin^2\alpha\ )\ ] - R_1^2\ k_x^2 \cos^2\alpha} \tag{3}$$

Resonance occurs when the amplitude of vibration, ϕ, is infinite. Accordingly, setting the denominator of (3) equal to zero and solving for ω, we get two different numerical values, which are

$$\omega_1 = \sqrt{\frac{\omega_a^2 + \omega_b^2}{2} + \left[\left(\frac{\omega_a^2 + \omega_b^2}{2}\right)^2 - \frac{\omega_a^2\ \omega_b^2}{1 + \frac{k_x}{k_y}\cot^2\alpha}\right]^{\frac{1}{2}}}$$

and

$$\omega_2 = \sqrt{\frac{\omega_a^2 + \omega_b^2}{2} - \left[\left(\frac{\omega_a^2 + \omega_b^2}{2}\right)^2 - \frac{\omega_a^2\,\omega_b^2}{1 + \frac{k_x}{k_y}\cot^2\alpha}\right]^{\frac{1}{2}}}$$

where $\omega_a^2 = \dfrac{k_x}{M}$

$$\omega_b^2 = \frac{R_1^2\,(k_x \cos^2\alpha + k_y \sin^2\alpha)}{I_z}$$

ω_a represents the natural frequency of this system if the machine is constrained to move as a unit in the X-direction and is not allowed to oscillate torsionally about the Z-axis. ω_b is the natural frequency which would occur if the machine were allowed to oscillate about the Z-axis but prevented from moving linearly along the X-axis.

In a similar manner, two resonant frenquencies may be found in the Y-Z plane (see Fig. 8). They are

$$\omega_3 = \sqrt{\frac{\omega_c^2 + \omega_d^2}{2} + \left[\left(\frac{\omega_c^2 + \omega_d^2}{2}\right)^2 - \frac{\omega_c^2\,\omega_d^2}{1 + \frac{k_z}{k_y}\cot^2\beta}\right]^{\frac{1}{2}}}$$

and

$$\omega_4 = \sqrt{\frac{\omega_c^2 + \omega_d^2}{2} - \left[\left(\frac{\omega_c^2 + \omega_d^2}{2}\right)^2 - \frac{\omega_c^2 + \omega_d^3}{1 + \frac{k_z}{k_y}\cot^2\beta}\right]^{\frac{1}{2}}}$$

where

$$\omega_c^2 = \frac{k_z}{M}$$

$$\omega_d^2 = \frac{R_2^2\,(k_z \cos^2\beta + k_y \sin^2\beta)}{I_x}$$

There remain two resonances, one for linear motion along the Y-axis, ω_5, and another for angular motion about the Y-axis, ω_6.

These are,

$$\omega_5 = \sqrt{\frac{k_y}{M}}$$

$$\omega_6 = \sqrt{\frac{k_z\,R_1^2 \sin^2\beta + k_z\,R_2^2 \sin^2\beta}{I_y}}$$

References

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Discussion

For discussion of this paper see page 1071.

Noise Mitigation in Substations

BY E. A. BISHOP[1]

Associate, A. I. E. E.

IT is the intention of this paper to deal with the noise encountered in several types of substations and the methods employed to reduce or eliminate the noise. The following classes of substations will be considered in the order named: 230-volt direct current located in a downtown district; 600-volt direct current for traction service; 1,500-volt direct current for electrified suburban service and a-c. remote-controlled distribution substation.

Since the problem of noise due to converters is common to three of the types of substations mentioned it may be well to discuss it first. The noise emanating from converters is due to the following causes: d-c. commutation, windage, magnetic noises, a-c. brushes on slip rings, and vibration of the building housing the equipment. Commutation noise has two elements, (1) caused by brushes riding on the commutator bars, and is of a mild scraping nature, and (2) caused by the commutator slots passing under the brushes. With a commutator in good condition, the latter noise is not troublesome, but high mica or commutator bars causes a chattering noise which is rather objectionable. Windage depending upon the speed of the machine can be anything from a gentle murmur to a very high pitched and penetrating whistle. Magnetic noises are usually so low pitched that they are drowned out by the other noises present. Brushes on the slip rings contribute a rasping noise.

Vibration in substation structures is rather unusual and occurs chiefly if the unit, for any reason, runs open phase. When it occurs it is very pronounced, not necessarily adjacent to the unit but at points very remote from the unit. For example, in a substation located in the sub-basement of an office building, the vibration due to open-phase operation has been very pronounced thirteen floors above, yet the operator in charge was not aware of anything wrong. The noise encountered in any unit is in direct relation to the speed and frequency of the unit. For example, the slow-speed 25-cycle units were very seldom the cause of any serious complaints but with the advent of the higher speed 60-cycle units the complaints began to multiply very rapidly.

The first measure employed to reduce the noise coming from the 60-cycle units was to have the manufacturers provide as a part of the unit a semi-enclosure consisting of a sheet metal housing, with wire glass doors and windows, enveloping the d-c. end of the converter. Whenever any attempt is made to enclose a converter, a secondary problem of ventilation is immediately manifest. In fact, the two problems go hand in hand right through the entire consideration. Therefore, when considering noise mitigation it is necessary to consider ventilation. In the case of the semi-enclosures, the ventilation of the converters was accomplished by leaving an opening in the front end of the housing approximately the same size as the opening inside the commutator. Cool air was then drawn into the armature through the commutator, passed through the ventilating slots of the armature core and windings and discharged through a chimney on top of the enclosure. The chimney in some cases was connected by means of a duct to a point outside the substation. In others it simply discharged in the converter room. The semi-enclosures were first applied to lighting converters and made quite a reduction in noise. They were not quite satisfactory from a standpoint of either ventilation or accessibility for maintenance work around the commutator.

Three 600-volt railway converters with semi-enclosures were ordered, but when they were installed the enclosures were omitted as the hazard due to commutator flashover was great and it was not deemed advisable to take the risk. One of the units was installed in a new substation in an apartment district. As a result of serious complaints from neighbors the semi-enclosure was installed. It was felt that the noise reduction was not sufficient so a total enclosure consisting of one-half inch thick transite supported on a structural steel framework was installed. The inside of this enclosure was lined with one inch thick asbestos hair felt. A baffled chimney on top of the housing permitted the air to be discharged and deadened a large part of the noise.

From a noise reduction view point the total enclosure was rather successful since it eliminated the complaints, but the problem of heating was quite serious. Fortunately the layout was such that a blower to force air through the housing could be installed with slight expense. A new problem then presented itself. When flashovers occurred small particles of hot copper or carbon were discharged into hair felt lining. This material did not burn readily but would continue to smolder due to the draft of air, making it necessary to use a fire extinguisher. This was overcome by again lining inside the hair felt with one-eighth inch porous asbestos paper. The thin asbestos could not be applied to the baffles in the chimney so the baffles were omitted. Although the relatively poorer noise absorbing quality of the thin asbestos slightly decreased the over-all noise reduction, the net result was satisfactory.

The general outcome of the above was the development of the total enclosure. It was designed along two

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general lines by the two principal manufacturers. The first consisted simply of a cubical shaped housing completely enveloping the entire converter, the dimensions being such that the sides were a few inches clear of the machine bedplate at all points. A partition at the main field yoke divided the housing into two sections. For ventilation the air is admitted at the d-c. side of the unit through openings in the foundation. The air flows up past the commutator and d-c. brush rigging, into main and booster armatures, past the a-c. rings and brushes, and is discharged through a chimney located in the top on the a-c. side. A duct connects the chimney to the outside air. A blower is required either on the intake or discharge side to circulate the air.

The second design, while completely enveloping the converter conforms to the general shape of the unit, that is rounded off at the top. The method of ventilation is identical with the first design. The advantages of the total enclosure are: (1) better noise reduction, (2) better ventilation of the equipment, and (3) sufficient space available for maintenance and minor repairs. The disadvantages are: (1) complete disassembly for major repairs and (2) added cost due to elaborate ventilation facilities which are required.

In a recent installation the problem of noise being communicated to adjacent parts of a large downtown building had to be solved. In this case the situation was very critical. The least amount of noise or vibration would be very annoying to a large theater located immediately over the substation space. The following measures were taken to provide a quiet installation. The converters and transformers were supported on a cork insulated foundation to isolate vibration from the remainder of the structure. The blowers and motors were likewise mounted on cork foundations. The connection to the air exhaust ducts was made with a canvas joint. The main d-c. terminal connections were made using flexible laminated copper strips. To further eliminate the possibility of vibration being communicated along the main copper connections and in turn through the supports to the building steel, several flexible joints were made in these long runs. The converters of course, were completely housed. The results of the installation have been very satisfactory in every way. The installation has been in service two years and so far no complaints have been received.

In another recent installation, on company owned property, where cork insulated foundations were found to be impracticable, a one-inch space was left between the substation and the adjacent building. This space was filled with ground cork with a suitable binder. The principal purpose of this filler was not for its insulation value, as the air space would serve this purpose very well, but rather to prevent an accumulation of solid material such as brick bats, which would form a solid means of communicating vibration to the adjacent building.

At the present time, there is under consideration the problem of noise mitigation in 1,500-volt traction substations. The units installed consist of two 750-volt machines having their commutators connected in series. They have a speed of 600 r. p. m. and give rise to an extremely high-pitched penetrating noise. The noise consists principally of two elements, one due to commutators and the other to windage. The air emerging from the armature radial ventilating slots impinges on the field pole tips and is alternately cut off and allowed to flow freely as it passes the field poles. The resultant whistle is responsible for an objectionable portion of the noise. The manufacturer has designed a new form of pole tip which has its edges gradually rounded off thus eliminating the sudden cutoff of air. He claims that a very appreciable reduction in noise can be obtained by substituting the new pole tips. There is no authentic data on the amount of noise reduction obtainable by use of the new pole tips. This company has at present in operation slow-speed 230-volt converters equipped with both types of pole pieces and comparative tests are under way to determine the relative merits of the two types. Unfortunately these tests will not be concluded in time to include the results in this paper.

The alternate scheme for mitigating the noise is to totally enclose the two units with the resultant hazards of 1,500-volt flashovers and the cost of providing ventilating facilities in substations not readily adaptable to this type of ventilation. Other methods, such as noise absorbing walls surrounding the converters, have been tried but they have not proved successful.

Before leaving converting equipment it should be stated that from a noise mitigating standpoint, the mercury arc rectifier is the most satisfactory device so far developed. Its operation is silent except the hum in the transformer and the noise of the rotary vacuum pump. The latter device has been much improved since the original installations and is now satisfactory.

In a-c. distribution substations, the character of the noise produced is entirely different, consisting chiefly of 120-cycle hum inherent in transformers and induction regulators. The latter also produces a noise due to operation of the regulator motor control equipment. The noise in a-c. substations while not nearly as intense as in d-c. substations is considerably above the noise level of their neighborhoods especially in residence districts. In one substation, this noise was found to be transmitted through building steel to the outside walls and thence through to an adjoining apartment house. This difficulty was overcome by placing cork under the transformers and regulators, and replacing the rigid terminal connections with flexible ones.

Transformers installed outdoors for a remote-controlled substation have also been the cause of serious complaints. The transformers are, as a rule, installed in an open court adjacent to the building which houses

the switching equipment and regulators. One end and side of the court usually consist of brick walls with small openings for ventilation. The third side is a wire mesh screen containing the gate for admitting apparatus. The noise is confined to the court by placing sound absorbing felt over the wire mesh screen and by constructing a roof consisting of sheet metal arc-shaped sections having the under side covered with sound absorbing felt. These sections are arranged with respect to each other so as to baffle the sound waves and at the same time permit free passage of air.

In summarizing the situation at least as far as rotating equipment is concerned very little has been accomplished toward reducing the noise inherent in the machines. Practically all efforts up to the present time have been toward confining the noise to the equipment and involved with it is the serious problem of ventilation.

Discussion

MEASUREMENT OF NOISE IN ELECTRICAL MACHINERY
(Bailey)

INDICATING METER FOR MEASUREMENT AND ANALYSIS OF NOISE
(Castner, Dietze, Stanton, and Tucker)

MEASUREMENT OF MACHINERY NOISE
(Marvin)

INDUCTION REGULATOR NOISE
(Foltz and Shirk)

MAGNETIC NOISE IN SYNCHRONOUS MACHINES
(Graham, Beckwith, and Milliken)

ELASTIC SUPPORTS FOR ISOLATING ROTATING MACHINERY
(Hull and Stewart)

NOISE MITIGATION IN SUBSTATIONS
(Bishop)

R. E. Pumphrey: What is generally accepted to be one of the principal sources of magnetic noise in synchronous machines, namely, the periodic variation in magnetic pull as the poles pass the stator teeth, has been described by Messrs. Graham, Beckwith, and Milliken in a very clear manner. While it has been known for some time that such a variation in radial pull existed, the methods of calculating the extent of this variation have been laborious and unsatisfactory. Some three or four years ago in collaboration with Mr. H. D. Taylor we carried out a series of investigations to determine by actual test the magnitude of the forces involved, their relation to various design details of the pole face such as width, radius of pole arc, amortisseur winding slots, etc. The testing equipment consisted essentially of a section of stator core so supported that the force exerted on it by a single pair of poles could be accurately measured. Means were provided for preserving the proper air gap and rotating the poles past the stator teeth, recording readings of the pull for various positions. Some 90 odd combinations were tested and have provided the basis for an approximate quantitative analysis of the results to be expected from a given design.

Fig. 1 shows some of the results of these tests plotted in per cent pulsation against pole face width per stator slot pitch. Maximum pull always occurred with the pole center directly opposite either a stator tooth or slot. Pulsation was arbitrarily taken as the difference in pull between these two positions, and per cent pulsation was defined as this difference divided by the average. The effect of rounding the pole face, that is, of increasing the ratio of maximum to minimum gap is clearly shown.

E. J. Abbott: The availability of noise meters as portable and convenient as those described should greatly encourage the use of sound measurements of machinery noises. The desirability of having such apparatus can be especially appreciated by those who have transported bulky laboratory analyzers and amplifiers for field measurements.

Experience at the University of Michigan indicates that one or two weighting curves is hardly sufficient for the range of sounds likely to be encountered. We have measured noises from electric refrigerators in the neighborhood of 25 db. above the 1,000-cycle threshold, and also noises from large reduction-gear units which were from 90 to 100 db. above the 1,000-cycle threshold. Enormously different weighting circuits are required in such cases.

Even with convenient and accurate apparatus for measuring the magnitude of sound at a given point in space, the problem of rating machines for noise in terms of meter measurements is far from being solved. One would naturally assume that good comparative measurements on a group of similar machines could be obtained by placing the microphone at the same

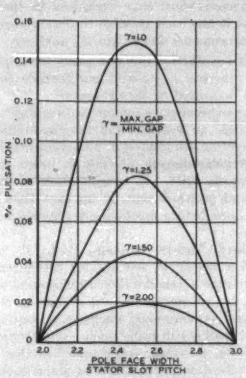

Fig. 1. Per Cent of Radial Magnetic Pull in Per Cent Average Pull for Various Pole Face Widths and Radii

relative position for each machine. Experience shows that this usually is not the case. The discrepancies encountered in such cases are usually very much greater than the difference between the loudest and quietest machines in the group.

In case of the large reduction-gear units mentioned above, readings were made at 10 stations at corresponding positions on each side of eight identically constructed units. The value obtained by averaging the ten readings on each machine agreed perfectly with aural observations, but this was by no means true for the individual stations. For example, machine 2, which was the next to the loudest on the average, was the quietest of the group at microphone stations 1 and 10, and the loudest at stations 5 to 9. Machine 1, which was the quietest of the group, measured next to the loudest at station 10, and the quietest at several other stations. Machine 3, which was the fourth in loudness, measured the quietest of the group at stations 9 and 10, and next to loudest at stations 2 and 3. At station 6, machine 4, the noisiest of the group, measured almost the same as machines 1, 11, and 12 which were the quietest of the group. These results are by no means unusual, but rather the ordinary experience.

The causes of these large variations in sound from point to point and from instant to instant lie not only in the familiar acoustic standing wave patterns due to sound reflection from neighboring objects, which are troublesome enough; but also in the nature of the source itself, which is usually even more important. Machinery does not seem to vibrate as a whole at audio frequencies, but breaks up into a large number of small areas vibrating with different phases and magnitudes. Each of these radiates its own set of sound waves, the largest of which by no means come from the region nearest the original source of the noise. Insignificant variations in design, materials, construction, speed, load or other causes may entirely shift these patterns on the machine without greatly changing their average value. Hence the inconsistent readings obtained with a given microphone position on different machines.

The difficulties listed above can usually be overcome if time and conditions permit the averaging of a large number of readings. At the University we have had considerable success in obtaining such averages rapidly by installing the machine to be measured in a very reverberant room. A large moving metallic reflector serves to shift the sound energy continually, and by moving the microphone and using a slow-acting indicator meter to average-out short time variations in the intensity at the microphone it has been possible to obtain very satisfactory total noise measurements and frequency analyses.

Obviously this method is not applicable for impulsive noises, and in many practical cases it is not feasible to remove the machine to be measured to the laboratory. The frequency of the noise from most machines cannot be "warbled" several times a second to obtain an average, and the possibilities of moving the microphone often are limited if measurements must be made close to an irregular-shaped machine in order to reduce the effect of extraneous noise. In consideration of these practical aspects of sound measurement it appears that considerable remains to be done before machine noise measurement is on a production basis.

H. D. Taylor: Messrs. Graham, Beckwith, and Milliken have described a theory to account for tooth-frequency noise which is very similar to one developed by some of my associates and confirmed by several years' experience in designing and testing low-speed synchronous machines. Continual progress has been made—and is still being made—on this problem. But, generally speaking, it seems well established that the noise is usually due to vibration of the stator at about tooth frequency, and that the vibration is caused by pulsation of the radial magnetic pull of the field poles as they revolve past the stator slots and is independent of stator load current. Our experience confirms the authors' statement to the effect that the worst cases of noise are likely to be associated with pole and slot combinations giving a nearly but not quite integral number of slots per pole.

With respect to frequency of the noise, our analysis indicated that it should be a multiple of twice the line voltage frequency,—$2fx$ (integer nearest to the number of slots per pole); and this rule was confirmed by a number of observations. The authors' rule gives slightly different results.

Supplementing the authors' illustration of the production of tooth-frequency vibration and noise by pulsation of the magnetic pull of the poles in moving over one tooth-pitch (Fig. 3), I should like to call attention to the magnitude of the forces involved. At ordinary magnetic densities, the average radial pull is about 25 lb. per square inch of pole face area,—which amounts to several hundred pounds per pole even for quite small pole pieces. Furthermore, for such design proportions as are illustrated in the authors' Fig. 1, the amount of pulsation may be as high as 20 per cent—that is, 10 per cent above and below the average. Considering the fact that vibration amplitudes of only a few tenths of a mil at the frequencies in question will produce large volumes of noise, it is not surprising that such forces have been making themselves heard.

In order to avoid noise from this source, there would seem to be two general courses of action: (1) to make the stator construction rigid enough to eliminate these almost microscopic deformations; or (2) to eliminate the fluctuations in magnetic pull by suitable control of the design details. Our experience has indicated that the latter course is likely to be more successful. We have made tests which show conclusively that by proper proportioning, pole pieces of almost any dimensions can be designed for practically zero pulsation; also, that the noise usually disappears when such poles are substituted in noisy machines of the type described by the authors.

We have observed a number of cases of other types of tooth-frequency noise in synchronous machines,—notably:

(1) Cases involving poles having little or no pulsation, but with a decided tendency to have the flux, and pull, swing from side to side of the pole piece,—the number of slots per pole being an integral number plus one-half.

(2) Cases intermediate between these and the type described by the authors, in which the flux-swinging action and the pulsation effect are cumulative.

(3) One case of tooth-frequency noise which was almost entirely dependent on load current in the stator, and which seemed to be associated with the use of a very nearly but not quite integral number of stator slots per pole *per phase*.

The first and second types responded to the same kind of treatment as I have described for simple pulsation; the load noise was eliminated by changing the number of stator slots.

C. E. Kilbourne: The paper by Messrs. Graham, Beckwith, and Milliken states that the stator is the chief source of the noise generating vibrations, but that the rotor may also contribute to the trouble. In a number of observations made on machines having several types of rotors, cast laminated steel plate, circular disk centers with welded rims, and keyed rims with structural centers, I have never found a case where the noise could be traced directly to the rotor. An analysis of the noise frequency and the stator vibration frequency has shown them to coincide and a stethoscopic analysis of the stator itself has always, with the exception of localized resonant areas, yielded an increase in the volume of noise as the stator core was approached. I should like to ask the authors if in their experience they have found cases where the rotors did vibrate and how they assured themselves that it was the rotors. The authors give some equations for the frequencies of rotor vibration and state that approximate resonant frequencies can be determined. I should like to ask if the approximations so obtained are close enough for actual use.

The paper shows that the effect of the traveling waves of force on the stator is to produce an elliptical distortion of the frame. It states Timoshenko's equation of such vibration as applied to a thin ring and uses the results to determine the natural frequency of the stator. This application is justifiable as a first approximation and gives good results for a small number of nodes; but as the number of nodes and consequently frequency increases, some of the assumptions made in the deduction of the equations, lead to results containing considerable error.

Timoshenko's equation is based on the assumption that the cross-sectional dimensions of the ring are small in comparison with the radius of its center line. A more general equation for the vibration of a homogeneous ring has been obtained by Mr. A. L. Ruiz as part of an investigation on stator vibration now in progress. This equation is limited only by the following assumptions.

1. A stator frame may be treated as a thick circular ring of uniform cross section.

2. The cross section is made up of punchings, air, and structural parts.

3. The neutral axis of the section passes through the center of gravity.

4. The energy of deformation of each cross section is negligible.

5. There are no damping or frictional losses.

6. There are suspended masses at a distance from the center of gravity which move with the section, adding to its kinetic energy, but not adding to its stiffness.

The first two assumptions are necessary to justify the application of the equation to a frame having feet. Experimental results have shown that these approximations do not introduce much error if the frame is of the type usually used in low-speed machinery. The next three assumptions seem justifiable from the physics of the problem. The last assumption is in accordance with the conclusions drawn by the authors of the paper, namely that the laminations add to the weight of the section but do not add to its stiffness. This statement must be modified as the frequency increases. At low frequencies, where the wavelength is long, it is practically true, but as the frequencies become higher, and the wavelengths shorter, the laminations do add stiffness to the frame.

The method of attack is to set up expressions for the potential and kinetic energies of deformation of a circular ring. The general expressions for these are:

$$V = a \int_0^{2\pi} \left(\frac{M_B^2}{2EI} + \frac{S^2}{2AE} - \frac{M_B S}{AEa} + \frac{Q^2}{2\mu} \right) d\theta \qquad (1)$$

which may be reduced to

$$V = a \int_0^{2\pi} \left\{ \frac{EI}{2a^4} \left(u + \frac{\partial \psi}{\partial \theta} \right)^2 + \frac{EAe^2}{2} - \frac{EIe}{a^3} \left(u + \frac{\partial \psi}{\partial \theta} \right) + \frac{\mu}{2a^2} \left(\frac{\partial u}{\partial \theta} - \psi \right)^2 \right\} d\theta \qquad (2)$$

and

$$T = a \int_0^{2\pi} \left\{ \frac{\rho}{2} \left[\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right] - \frac{M}{a} \left(\frac{\partial w}{\partial t} \right) \left(\frac{\partial \psi}{\partial t} + \frac{\partial w}{\partial t} \right) + \frac{J}{2a^2} \left(\frac{\partial \psi}{\partial t} + \frac{\partial w}{\partial t} \right)^2 \right\} d\theta \qquad (3)$$

Hamilton's Principle, namely

$$\delta \int_{t_1}^{t_2} (T - V)\, dt = 0 \qquad (4)$$

is then applied and the resulting expression is solved for a particular solution involving ω, which is 2π times the vibration frequency, and n which is the number of nodes in the circumference. The general expression is:

$$\left\{ \rho a \omega^2 (n^2 + 1) - \left(2M - \frac{J}{A} \right) \omega^2 - n^2 \left(\frac{EI}{a^3} + \frac{\mu}{a} n^2 \right) \right\} \left\{ - \frac{EI}{a^3} n^2 + \frac{J}{a} \omega^2 - \frac{\mu}{a} \right\} \left\{ - \rho a \omega^2 + \left(2 \frac{EI}{a^3} + \frac{\mu}{a} n^2 \right) + \left(\frac{EI}{a^3} + \frac{EA}{a} \right) \right\} + 2n^2 \left\{ \left(M - \frac{J}{a} \right) \omega^2 + \left(\frac{EI}{a^3} + \frac{\mu}{a} \right) n^2 \right\} \left\{ 2 \frac{EI}{a^3} + \frac{\mu}{a} \right\} \left\{ - \rho \omega^2 a + \left(2 \frac{EI}{a^3} + \frac{\mu}{a} n^2 \right) \right\} + n^2 \left\{ - \rho a \omega^2 + \left(2 \frac{EI}{a^3} + \frac{\mu}{a} n^2 \right) \right\}^2 \left\{ - \frac{EI}{a^3} n^2 + \frac{J}{a} \omega^2 - \frac{\mu}{a} \right\} - \left\{ \left(M - \frac{J}{a} \right) \omega^2 + \left(\frac{EI}{a^3} + \frac{\mu}{a} \right) n^2 \right\}^2 \left\{ - \rho a \omega^2 + \left(2 \frac{EI}{a^3} + \frac{\mu}{a} n^2 \right) + \left(\frac{EI}{a^3} + \frac{EA}{a} \right) \right\} + n^2 \left\{ 2 \frac{EI}{a^3} + \frac{\mu}{a} \right\}^2 \left\{ \rho a \omega^2 (n^2 + 1) - \left(2M - \frac{J}{a} \right) \omega^2 - n^2 \left(\frac{EI}{a^3} + \frac{\mu}{a} n^2 \right) \right\} = 0 \qquad (5)$$

This expression is unwieldy to handle though perfectly possible of solution. Numerical substitution for a given value of n will yield a cubic in ^{2}, the smallest positive root of which is the desired result.

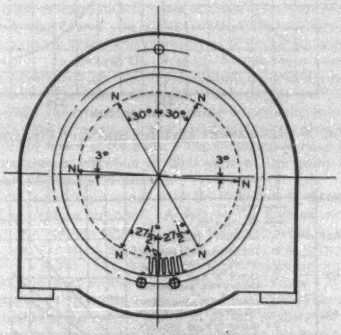

FIG. 2—NODE LOCATION FOR SIX-NODE VIBRATION, FORCE APPLIED AT A

A number of approximations which approach Timoshenko's assumption and eventually reduce to it, considerably simplify the labor of calculation. If we treat the motion as being inextensional letting $e = 0$, the result is:

$$\left\{ \frac{M^2 a}{\mu} - \frac{Ja}{\mu} (n^2 + 1) \right\} \omega^4 + \left\{ \frac{\rho EI}{\mu a} n^2 (n^2 + 1) + \frac{J}{a} (n^2 - 1)^2 + 2M (n^2 - 1) + \rho a (n^2 + 1) \right\} \omega^2 - \frac{EIn^2}{a^3} (n^2 - 1)^2 = 0 \qquad (6)$$

If in addition shearing deformation is neglected, there is:

$$\left\{ \frac{J}{a} (n^2 - 1)^2 + 2M (n^2 - 1) + \rho a (n^2 + 1) \right\} \omega^2 - \frac{EIn^2}{a^3} (n^2 - 1)^2 = 0 \qquad (7)$$

If the dead weights are also neglected there is:

$$\left\{ \frac{J}{a} (n^2 - 1)^2 + \rho\, a\, (n^2 + 1) \right\} \omega^2 - \frac{E\, I\, n^2}{a^3} (n^2 - 1)^2 = 0 \tag{8}$$

And finally if rotary inertia is neglected the solution becomes Timoshenko's equation:

$$\rho\, a\, (n^2 + 1)\, \omega^2 - \frac{E\, I\, n^2}{a^3} (n^2 - 1)^2 = 0 \tag{9}$$

Details of the solution are not given here as it is hoped that when the investigation is complete there will be sufficient useful information to make publication of the whole advisable.

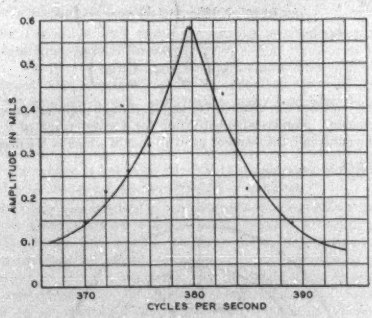

FIG. 3—RESONANCE CURVE FOR SIX-NODE VIBRATION

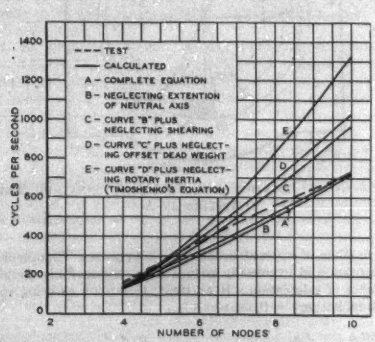

FIG. 4—COMPARISON OF TEST AND CALCULATED NATURAL FREQUENCIES

In our study of the phenomena a number of commercial stator frames complete but without coils were set up in a laboratory and vibrated by means of a magnet excited from a variable frequency source. Applied frequency was varied from 50 to nearly 1,000 cycles and whenever a resonant point was detected a detailed study of the frame was made. With resonant frequencies applied a stethoscopic examination of the punchings, side plates, and wrapper plate showed that the frame was vibrating as a whole, and the nodes were thus located more accurately than they could have been by hand. Amplitude-frequency curves taken at these points showed the nature and range of the resonant condition. In all cases the lower numbers of nodes were located as if the frame were a true ring, thereby justifying the first two assumptions. As the frequency was increased the effect of the feet became more pronounced and a shifting of the nodes from symmetrical positions resulted.

Results obtained on a frame used for about 200-hp., 300 r. p. m. motors are shown in a number of figures. Fig. 2 shows the location of nodes for six-node vibrations. The results of amplitude frequency measurements for the 4- and 6-node vibrations are shown in Table I and the latter plotted in Fig. 3. The general shapes of these curves check well with similar results obtained by the authors of the paper. A plot of resonant frequencies against number of nodes was made and compared with the theoretical results obtained from equations (5), (6), (7), (8) and (9). This is shown in Fig. 4. It will be noted that as the approximation approaches the true equation, the error decreases. Therefore, it is logical to expect that the use of equation (9) will generally lead to considerable error.

TABLE I—RESONANCE CURVES

Four-node frequency cycles/sec.	Vibration amplitude corrected to constant force $\frac{1}{1000}$ Inch	Six-node frequency cycles	Vibration amplitude corrected to constant force $\frac{1}{1000}$ Inch
156	0.046	370	0.145
158	0.167	372	0.216
160	0.890	374	0.260
161	1.388	376	0.320
162	4.33	377.5	0.430
163	1.48	379.5	0.580
165	0.980	382.8	0.434
166	0.553	385	0.215
170	0.206	389	0.142
174	0.117		
Maximum force	33.0 lb.	Maximum force	68.0 lb.

In conclusion, I wish to express my belief that frame resonance is an important contributor to synchronous machine noise, and that the authors have made a valuable addition to our knowledge of the noise problem.

NOMENCLATURE

a = radius of neutral axis of ring.
θ = angular coordinate of any point P on the neutral axis.
u = normal displacement of p, positive inwards.
w = tangential displacement of P.
$\frac{\psi}{a}$ = slope of deformed curve when the frame is in vibration (shear neglected).
e = extension of the neutral axis.
t = time.
ρ = mass of ring and added weight per unit length of arc of neutral axis.
I = area moment of inertia of ring section.
J = mass moment of inertia of unit cross section of ring and added mass, about the center of gravity of the ring section.
M = moment of mass per unit length of added mass about center of gravity.
T = total kinetic energy of vibration.
V = total potential energy of deformation.
E = Youngs modulus.
G = modulus of rigidity.
A = cross-sectional area of ring section.
k' = shear factor for cross section.
μ = $k' A G$
M_B = bending moment at P.
Q = shear at P.
S = tension along neutral axis at P.

J. A. Jackson: It seems to me that we are not going to completely solve our noise troubles in buildings until there is a much closer cooperation between the electrical industry and the architects and consulting engineers. There has been a strong tendency within the last few years to reduce the cost of buildings, the result being to make them much lighter and with much thinner walls, floors, and ceilings. Little attention seems to be paid to the fact that this makes them vibrate more readily and act as sounding boards. Therefore, the problem of preventing objectionable noises in inhabited space is a more difficult one. This fact does not seem to be fully recognized by many of those in the architectural and building engineering professions, with the result that when a noise complaint does arise the machinery manufacturer is asked and expected to bear all expense for curing the trouble. I think it is generally agreed that rotating electrical machinery and, for that matter, most other rotating machinery cannot be made absolutely noiseless without incurring a prohibitive cost and being unduly large and, that all manufacturers are now doing a good job in making their equipment as quiet as is commercially possible.

It is the duty of the architects and engineers to so design and acoustically treat their buildings so that such noises as do remain in the electricl machinery will not become objectionable. Electrical engineers should do all they can in the necessary cooperative and educational work with the architects and building engineers to bring about this much desired condition.

P. E. Stevens: Vibration and noise have always existed in power plants, but with the adoption of the lighter forms of switchboard manufactured of steel, these have come to have a more than unpleasant significance. We have had in one case a false operation of a relay which dumped a load of some 2,000 kw. Another instance may be cited in which the false operation of the relay dumped a load of 120,000 kw. The false operation of the relay was caused by vibration in the panel of the switchboard. The vibration in the building was not of such a magnitude as to cause any complaint. It was just the ordinary vibration which is common in the modern power plant. Drastic steps were taken to remedy these conditions. In one case the relays were moved to an entirely different switchboard. In another case, they were set up first to twice the gap and later to four times the gap at which they were designed to operate.

The false operation of relays is not the only difficulty which attends vibration in a switchboard. The graph drawing instrument and the integrating instruments suffer inaccuracies in their measurement and require frequent attention to keep them in operating condition. In some cases jewel bearings have been injured and broken. Our organization felt that this was a matter which called for a rather thorough investigation, the purpose of which should be to determine the source of vibration and the mode of propagation from the source to the switchboard; and if possible the magnitude of vibration under which instruments would operate successfully. In order to make the study as broad as possible we invited and secured the cooperation of Sargent and Lundy of Chicago. A group of five men was organized into a surveying committee. In ten stations visited measurements were taken of vibration of noise; tests of frequencies; and a very searching examination for minute vibrations which are too small to register on the usual vibrometers. In one case, records were also taken with a vibrograph. There were probably over five thousand observations, which have yet to be assembled in proper form for determining the parallels and divergencies in the various indications.

Definite measurements indicate that noise is accountable in certain cases for at least a part of the vibrations in switchboards and sometimes in much heavier structures. It is, therefore, necessary that a measuring instrument which is to be fully useful in surveying the effects of noise in a power plant, should not only measure the effect upon the human ear or the auditory effect, but should also be prepared to measure its probable vibration effect on the physical elements of the plant.

It is a well-known fact that a resonant condition will greatly magnify any impressed vibration. In the study of one of our plants we found three conditions in our switchboards; when the board was first delivered the blank panels had a vibration frequency of approximately 26 cycles per second. This is below the frequency of the vibrations impressed by the machine which operated at 1,800 r.p.m. or 30 cycles per second, but is too close to this frequency for a satisfactory quieting of the vibrations. The board was found to vibrate violently in certain places but no measurements were made. The manufacturer suggested the addition of certain stiffeners. These were applied and this condition changed the vibration period of the panels to somewhat over 40 per second. This is sufficiently above the frequency of the machine so that a rather small vibration might have been expected except for the fact that many of the panels are loaded with equipment which tends to lower their frequency. This is the purpose of the panel, that is, to support equipment. Further study of the natural frequency of the panel revealed the fact that the relays which had suffered most from the vibration were almost exactly the correct weight to bring the frequency down from somewhat over 40 to 30 per second, or in perfect resonance with the frequency of the impressed vibration from the turbine.

The relative magnitude of some of these vibrations is shown in the following figures. In one of the plants the turbo generator had a maximum transverse vibration of 1.4 mills (thousandths of an inch). In general the floor of the operating room had smaller vibrations than this, or a maximum transverse vibration of one mill. The floor immediately around the switchboard and on which the switchboard rested had a maximum transverse vibration, that is to say, a vibration normal to the plane of the panels of the board of 0.4 mills. The switchboard relay panels had a maximum vibration of 20.0 mills at the point before mentioned as being in resonance.

Trouble is also experienced with the vibration of the modern type of steel office furniture. A desk on the floor of the room in which the vertical vibration is 0.4 mills was found to have a vibration at the point where the chief operator was expected to write of 4.5 mills. This was rather a poor place on which to write but the effect on the telephone was even worse. It broke up the speaker's voice into a series of beats which were absolutely unintelligible to the listener at the other end of the wire. It is suspected that some of this vibration might be due to air borne waves or noise. The human ear does not appreciate air borne waves of low frequency although many can hear distinct sound vibrations as low as 28 per second. There is no reason to believe that because these low-frequency atmospheric vibrations are not heard that they may not be present and may not influence structures which may be in resonance with their frequency. At one station measurements were made on the switchboard in the operating room, following which the doors between the operating room and the turbine room were opened. An increase in the sound readings taken with a Western Electric 3-A audiometer was 30 per cent. The increase in the amplitude of vibrations of the panels was approximately 20 per cent.

Noise conditions present a practical problem calling for immediate and perhaps drastic steps to overcome the conditions which have arisen from the step-by-step reduction in weights of mechanical equipment and structures. The manufacturers are not being accused of skimping their work but the reduction of weights is a natural evolution brought about by the necessity for less weight per kilowatt if machines of the vast magnitude of today are to be manufactured.

Designers of power plants, engineers, and designers of machinery and equipment should assemble into an organized body which will attack this problem and correlate the point of view of the two interests involved.

E. H. Hull and W. C. Stewart: Mr. P. E. Stevens in his discussion describes the excessive vibration of a switchboard panel due to forces transmitted to it both through the air and through the foundations. Such vibration can be limited by the proper use of an elastic mounting. Fig. 2 of the paper shows the transmissibility ϵ as a function of the ratio of the disturbing frequency to the natural frequency of a mounting. ϵ may be either the ratio of the force transmitted to the foundation to the disturbing force or the ratio of the amplitude of an elastically mounted body to that of a vibrating foundation.*

The latter case, of course, applies to the mounting of a switchboard panel to isolate it from foundation vibration. The switchboard, as originally installed, had a natural frequency of 26 cycles, which presumably was due to elasticity in the mounting of the panel. This gives a frequency ratio of 1.15. Referring to curve marked $b = 0.5$ in Fig. 2 of the paper, this ratio gives $\epsilon = 1.8$. The panel was then stiffened until the natural frequency reached 40 cycles. This gives a frequency ratio of 0.75 and again $\epsilon = 1.8$. Therefore, the motion of the panel would be exactly the same after stiffening as it was originally. Further stiffening would reduce the panel amplitude slightly until for an infinitely rigid mounting $\epsilon = 1.0$ or the panel has the same amplitude as the floor.

Thus no amount of stiffening could reduce the panel amplitude below that of the foundation. On the other hand, by decreasing the mounting stiffness and lowering the natural frequency to 10 cycles, the frequency ratio would be 3.0 and ϵ would be reduced to 0.24. This means that the panel amplitude would be about ¼ of that of the foundation.

If the panel vibration is due to air borne forces alone, a very rigid mounting would be desirable, providing that resonance occurred somewhat above the disturbing frequency, as the motion of the panel would be zero for infinite stiffness. In the case of large air forces, where the foundation vibration is negligible, relays and instruments may be supported elastically from the panel, as the air force on them would be small due to the small area. If still better isolation is desired, the instruments could be elastically supported within cabinets fastened to the panel. The elastic support would isolate the vibration of the board from the instruments and the cabinet would protect them from air forces.

*See "Influence of Damping in the Elastic Mounting of Vibrating Machines," by E. H. Hull presented at the Applied Mechanics Meeting of the A. S. M. E., Purdue University, June 16, 1931

Sterling Beckwith: The rule for frequency of the stator vibrations given by Mr. H. D. Taylor is the same as the rule developed in the paper, but has been further simplified. This simplified rule is: vibration frequency = twice line frequency [integer nearest to the number of slots per pole].

Fig. 1 of the discussion by Mr. R. E. Pumphrey is very valuable, although one must not regard it as indicating a "cure-all" for any type of machine, as the ratio of maximum gap to minimum gap, and the number of slots per pole usually cannot be varied to reduce per cent pulsation without undesirably affecting other machine characteristics such as load loss, pole-face loss, machine size, core loss, telephone interference factor, etc.

Noise due to resonant vibration of parts of the rotor has been identified only by "circumstantial evidence" as it is exceedingly difficult to measure actual vibration of a rotating structure when vibration amplitudes are only a few mils or less. The circumstantial evidence may be listed as follows:

1. Changing spider construction on a line of machines built of the same parts has been found to affect probability of certain types of noise in the machines.
2. Six slots per pole has been found to cure tooth frequency noise on a certain size machine with a six-arm spider, which machine is noisy with four and a half slots per pole; whereas, on a similar, but slightly smaller machine with a four-arm spider, six slots per pole was noisy and four and a half slots per pole was quiet. The formulas for rotor resonance given in the paper indicated that the above results could have been anticipated.
3. Where tooth-frequency noise has been due to the stator, the vibration has been easily detected, so that when similar machines are noisy without any type of stator vibration of a comparable order of magnitude, the rotor must be the source of the trouble.

The work of A. L. Ruiz in obtaining an exact mathematical solution for resonance frequency of different modes of stator vibration is very valuable. The graphical summary in Fig. 4 shows at a glance both the relative error in the simple formula and the accuracy of the complete formula. It is hoped that the possibility of publishing the details of Mr. Ruiz' work and the results of the complete investigation of which this work is a part will become a reality in the near future.

Lightning

Characteristics of Lightning—Induced Voltages—Direct Strokes—Coordination—Transmission Line Design

BY F. W. PEEK, Jr.*

Fellow, A. I. E. E.

I. Introduction

It is the purpose of this paper to show how the various characteristics of lightning can be obtained from measurements on lines, and how the results can be applied in design. *Oscillograms of traveling waves give a remarkably complete and easily read story of the life and characteristics of the lightning which produces them.* For example, such oscillograms give data from which the height, the time of discharge, the current and the voltage of the clouds producing them can be estimated. Induced voltages on any given transmission line can be found from these data. Direct stroke and induced voltages can be determined from direct-stroke currents, etc. Many measurements have been made during the past five years on the lines of the Pennsylvania Power & Light Company, the American Gas & Electric Company and elsewhere.

In making the analytical study, curves are plotted showing how voltage, time, and current are numerically interrelated over a wide range, for comparison with measured values from which the actual limits must be finally determined. Lightning measurements are very difficult; many are known to be accurate, while others are still uncertain. It is not possible, for instance, to fix the range of direct-stroke current until further measurements are available. Although some numerical values will probably be modified and their relative importance changed, as the reliability of field measurements is improved, the methods of analysis will remain unchanged and valuable in interpreting and utilizing data. This study should also point the way for future experiments. To avoid confusion, the derivation of the mathematical equations has been confined to the appendix or references in the Bibliography. Some actual designs are made for purpose of illustration.

II. Voltages on Lines and Methods of Controlling Them

Induced Voltages. Since voltages on lines are caused by induction or direct strokes, it is of practical importance to be able to estimate the numerical value of these voltages for different tower designs. When voltages occur by induction they are of a polarity opposite to that of the cloud and can be calculated by the following formula[1,2,3]

$$V = g\,h\,\alpha \qquad (1)$$

Where V is the voltage above ground, g is the gradient in volts per foot established by fairly good measurements to be about 100,000. α is a factor depending upon the time of cloud discharge and distribution of bound charge and is always less than unity. Values of α calculated from the rate of cloud discharge and used in former calculations are given in Fig. 1. The exponential rate of cloud discharge used in these curves probably more nearly represents actual conditions than any other. α is the factor for the induced voltage during the forming period at the origin, while α' is for the resulting free traveling wave.[4] In order to use these curves to determine α, and thus the induced voltages possible or probable, it is necessary to have

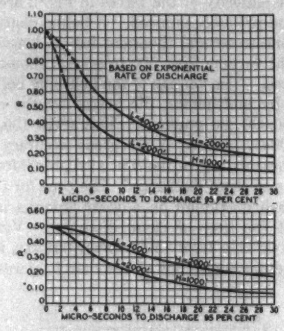

Fig. 1—α for Induced Voltages (Exponential Discharge)

measured values of the time of the cloud discharge. As already mentioned, cathode-ray oscillograms of traveling waves of induced lightning voltages taken reasonably near the source of the disturbance, and not badly distorted by sparkover, losses or reflections give practically the complete life history and characteristics of the stroke. In fact, the time from zero to crest is a measure of the length of bound charge, while the time from crest to zero on the falling wave is the time of cloud discharge. The equivalent distance for one-half the time to crest also approximates the height of the cloud.

There is a large number of such oscillograms with voltage and polarity available. Fig. 2 (inset) is a typical wave shape for a large group of these oscillograms. From this oscillogram the time of discharge is 11 microseconds. The length of the bound charge is 4 micro-

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1. For references see Bibliography.

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seconds or 4,000 ft., while the height of the cloud is

$$\frac{4{,}000}{2} = 2{,}000 \text{ ft.}$$

When the waves become distorted the front is not of value in obtaining information on the cloud height or length of bound charge since the values are too high. The lengthened tail would indicate a longer time than actually obtained. However, an approximation of the time of discharge is twice the time from crest to half voltage on the tail. Only the first loop is of interest for this purpose. A wave often persists for a number of loops due to reflections, etc. The complete wave is important from the standpoint of internal stresses in apparatus. The total length of a direct-stroke wave, if not completely chopped by an insulator sparkover, would indicate the time of discharge.

Measurements of a large number of oscillograms of lightning voltages obtained in Pennsylvania and Ohio are given in Table I and plotted in Fig. 2. This curve includes all voltage oscillograms not causing line sparkover. The majority of the waves in Fig. 2 are of (+) polarity and probably induced. A bound charge varying from one to several thousand feet in length and a cloud height varying from 500 to several thousand feet are indicated. The voltage of the lightning bolt can be estimated by multiplying the height by the cloud in feet by 100,000 the gradient. This gives 150,000,000 for a 1,500-ft. cloud. While great accuracy is not claimed, these data give the best available information on the time of cloud discharge. The time for the voltages to reach maximum on a short antenna is also a measure of the time of cloud discharge. Very little data are available from this source, but ten readings give a cloud discharge time of less than 5 microseconds.

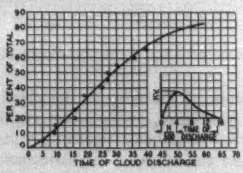

Fig. 2—Measured Time of Cloud Discharge

All 1929 Pennsylvania and Ohio oscillograms equal to or over 100 kv. and not coincident with trip-out

It is important to establish the time of discharge accurately by field tests.

Using these measured values of time, values of α varying from 0.1 to more than 0.4 are indicated from Figs. 1 and 2. These values of α substituted in equation (1) give induced voltages as high as 2,000,000 for storms over the usual line. See Figs. 2 and 6. Current corresponding to α can be found in Fig. 3.

The voltage of the free traveling wave is lower than the voltage during the forming period by the ratio of $\frac{\alpha'}{\alpha}$. Traveling waves are also rapidly reduced by attenuation. The voltages in Table I, measured at a considerable distance from the disturbance are therefore much lower than at the source. Since the object of Fig. 2 was to determine the time of cloud discharge, low-voltage values not causing sparkovers and as free as possible from distortion were purposely selected.

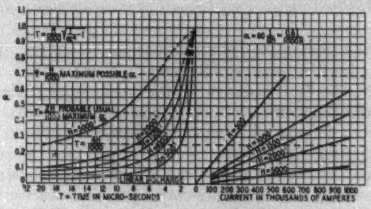

Fig. 3—Variation of α with Lightning Currents and Time of Cloud Discharge for Different Cloud Heights

(Based upon point cloud and linear rate of cloud discharge)

When the time and manner of the discharge is fixed the current is also fixed. In this study two types of discharge, exponential and linear, approximately representing respectively the maximum and minimum current range, are used. An example is of interest. Assume a cloud 1,000 ft. high and a 10 microsecond time of discharge. For exponential discharge, α from Fig. 1 is 0.28. The corresponding induced voltage on a line 50 ft. high is 1,400 kv. See Fig. 6. The current corresponding to α from Fig. 3 is about 400,000 amperes. *The current requirements are thus quite large and confirmation by actual current measurements is necessary before final conclusions are drawn.* In this connection, the fusing effects of high currents for short times shown in Fig. 14 are of interest. See Section III. For a 10 microsecond linear discharge, α from Fig. 3 is 0.10, and the corresponding current value is 170,000 amperes. For a given current, the induced voltages for both types of discharge are the same but the time for the linear discharge is one-third that of the exponential.

The range of α can be arrived at by starting from direct-stroke current measurements instead of time. In fact, certain direct stroke and induced voltage relations are interdependent and one cannot be changed without consideration of the other. An instrument which records only when the current is considerably above the maximum value possible by induced voltages, was developed to differentiate between induced and direct-stroke currents in towers. As the readings are proportional to the current in the tower, the use of the instrument has been extended to current measurements.[12,14] While readings as high as several hundred thousand amperes are indicated, further calibration is

TABLE I—MEASURED TIME OF CLOUD DISCHARGE

American Gas & Electric Company—1929
Penn. Power & Light Company—1929

Oscillogram number	Measured time of cloud discharge μ sec.	Voltage	Polarity	Approximate per cent: Waves over 50 μ sec.	Approximate per cent: All waves	α (From Fig. 1)
(1) Waves not Coincident with Trip-out (See Fig. 2)						
901282 Penn	5	220	−	8	5	0.70
901241 "	9	120	+	15	10	0.50
901292B "	9.5	110	+	23	15	0.48
457A Ohio	16	245	+	31	20	0.32
901179 Penn	16	160	+	38	25	0.32
901252B "	18	100	+	46	30	0.28
901283 "	19	208	+	54	35	0.27
901183 "	25	180	+	61	40	0.21
901178 "	27	245	−	69	45	0.20
460 Ohio	27	108	+	77	50	0.20
901189A Penn	30	180	+	85	55	0.19
462 Ohio	36	190	+	92	60	0.17
901187 Penn	40	260	+	100	65	0.15
901252A Penn	Over 50	105	+		70	
901189B "	"	140	−		75	
901194 "	"	810	−		80	
901198 "	"	150	−		85	
901226 "	"	105	−		90	
901230C "	"	120	−		95	
901295 "	"	127	−		100	
(2) Waves Coincident with Trip-out						
901190 Penn	11	280	−	10	8	0.43
901193 "	15	740	+	20	17	0.33
901188 "	16.5	530	+	30	25	0.30
901285 "	18	630	+	40	33	0.28
901231 "	20	330	+	50	42	0.25
901235 "	26	260	+	60	50	0.21
901197 "	27.5	190	+	70	58	0.20
901200 "	30	260	+	80	67	0.19
901240 "	32	300	+	90	75	0.18
901275 "	37.5	500	+	100	83	0.17
901201B "	Over 50	740	−		92	
901222 "	"	155	−		100	
(3) Waves Possibly Coincident with Trip-out						
901214 Penn	2	135	−	12.5	7	
901191B "	5	250	+	25	14	0.70
901249 "	12	390	+	37.5	21	0.40
901191A "	21	600	−	50	29	0.25
901199 "	21.5	180	−	62.5	36	0.24
901186 "	22	330	+	75	43	0.24
901203 "	30	260	+	87.5	50	0.19
901251A "	49	390	+	100	57	0.13
901185 "	Over 50	360	+		64	
901251B "	"	100	+		71	
901202 "	"	100	−		79	
901229B "	"	330	−		86	
901254A "	"	220	−		93	
901272B "	"	600	−		100	

necessary. This instrument has shown the polarity of most direct strokes to be (−). One measurement of a direct-stroke current from a different source and made in an entirely different manner indicates 160,000 amperes and about 30 microseconds time of discharge.[11] For a 1,000 meter cloud discharge, α is 0.10 from Fig. 1. The current corresponding to $\alpha = 0.10$ and $H = 1{,}000$ (from Fig. 3) is 170,000 amperes, a remarkably good check. However, further data are necessary before definite conclusions can be drawn.

Values of α, based upon linear discharge, for various currents and cloud heights are given in Fig. 3. The corresponding values of time for cloud discharge are also given in the curve on the left hand side of the figure. Values of current for exponential discharge can be obtained by finding α for the given time from Fig. 1 and reading the amperes from the right hand curve in Fig. 3. It is of interest that a value of α of 0.7 obtains when the time of cloud discharge is equal to the time for a wave to travel from cloud to ground. This should be the maximum possible value of α. The actual maximum may be 0.45 or that corresponding to the time required for the wave to travel from cloud to ground and return. *The extent that this range of values occurs in practise can only be determined from observations.* A further examination of the curves shows that very low values of α can be obtained for special cases. In an example frequently cited as a proof that only very low induced voltages are possible, the special case of 200,000 amperes and a 5,000 ft. cloud height was taken.[5] This gives a value of α (from Fig. 3) of 0.024 or a voltage of 120,000 on a 50-ft. line. The calculations were really made as in equation (1). The assumption of a 5,000-ft. cloud is largely responsible for the low values. The time of cloud dis-

charge for this particular case is 200 microseconds. The wide range of possibilities illustrated in Table II shows that it is unsafe to generalize from a special case.

TABLE II—VARIATION OF INDUCED VOLTAGES WITH DIFFERENT ASSUMPTIONS AS TO CLOUD HEIGHT AND LIGHTNING CURRENT

Cloud height feet	Lightning current amperes (assumed)	α (calculated)	Induced voltages kv. (calc.)	Approximate time of discharge μ sec. (calc.)
LINEAR DISCHARGE				
Line 50 ft. High (No Ground Wires)				
500	600,000	0.70	3,600	0.5
1,000	600,000	0.36	1,800	3.
2,000	600,000	0.18	900	11.
500	200,000	0.24	1,200	2.
1,000	300,000	0.18	900	6.
2,000	300,000	0.09	450	22.
*5,000	200,000	0.024	120	200.
Line 80 ft. high (No Ground Wires)				
500	600,000	0.70	5,760	0.5
1,000	600,000	0.36	2,880	3.
2,000	600,000	0.18	1,440	11.
1,000	300,000	0.18	1,440	6.
*5,000	200,000	0.024	190	200.
EXPONENTIAL DISCHARGE				
Line 50 ft. High				
500	200,000	0.25	1,250	6.
1,000	410,000	0.25	1,250	11.
2,000	840,000	0.25	1,250	21.
Line 80 ft. High				
500	200,000	0.25	2,000	6.
1,000	410,000	0.25	2,000	11.
2,000	840,000	0.25	2,000	21.
500	90,000	.125	1,000	12.
1,000	195,000	.125	1,000	22.
2,000	400,000	.125	1,000	45.

$$T = \frac{H}{1000}\sqrt{\frac{1}{\alpha^2} - 1} \qquad \alpha = \frac{0.6\,I}{1000\,H} = \frac{60\,I}{g\,H}$$

H = cloud height in feet.
I = current in amperes.
g = volts per ft.—100,000.
*Special case frequently cited (See Bibliography No. 5).

Examples of induced voltage values based upon assumed currents are given in Table II for a 50-ft. and 80-ft. conductor height (from Figs. 1 and 3).

Equation (1) becomes

$$V = g\,h\,\alpha\,\omega \tag{2}$$

when the ground wire protective ratio factor ω is added. Since

$$g = 100{,}000$$
$$V = 100{,}000\,h\,\alpha\,\omega$$

V = lightning voltage
h = average conductor height in feet (in dry, sandy, or rocky country with high resistance grounds the effective h may be much higher than indicated by the height of line)
α = usually less than 0.5
ω = 0.5 or less, depending upon ground wire arrangement.

The polarity of the induced voltage is opposite to that of the cloud.

A large variation of α and induced voltages is possible for the values of current and cloud height assumed. The lightning sparkover for a 230-kv. line with 14 suspension insulator units and the ½/5 wave is about 1,950 kv.; the corresponding voltage for the 1½/40 wave is about 1,300 kv. The effects of the ½/5 laboratory wave correspond to steep or short lightning waves; and the effects of the 1½/40 laboratory wave to the less abrupt and longer lightning waves. It is believed that the 1½/40 waves apply more generally to the lower voltages. Table II shows that under certain assumptions induced voltages caused by nearby strokes to ground could exceed the line insulation strength and cause sparkover of high-voltage lines. It also shows that under other assumptions nearby strokes could occur without causing sparkover on such lines. This

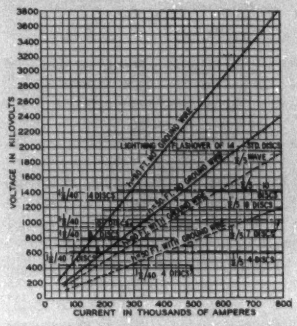

FIG. 4—VOLTAGES INDUCED ON LINES 50 FT. AND 80 FT. HIGH FOR VARIOUS LIGHTNING CURRENTS FROM A 1,000-FT. CLOUD

seems to account for conflicting reports or observations on the magnitude of induced voltages caused by nearby lightning strokes. With ground wires the above voltages could be reduced to one-half or less.

Table II and Fig. 4 indicate that a line insulated for 230 kv. (14 to 16 units), and without ground wires, could have occasional sparkovers by induction; that it is unsafe to assume induced voltages negligible until more data are available; that the addition of ground wires should eliminate sparkovers caused by induction; that higher lines or lines of the same height but with less insulation should have sparkover unless special ground wire arrangements are made.

A ground resistance of less than 50 ohms is desirable from the standpoint of induced voltages. For this resistance the ground wire efficiency is reduced in the order of 25 per cent. It is indicated later that resistances as low as 5 ohms might be desirable from the standpoint of direct strokes.

Fig. 4 shows the induced voltages on lines 50 ft. and 80 ft. high corresponding to various assumed lightning

currents from a cloud 1,000 ft. above ground. The sparkover voltages for different line insulation for the ½/5 and the 1½/40 waves are also indicated.

Fig. 5 shows the number of insulators necessary to resist induced lightning voltages calculated for various currents from a cloud 1,000 ft. high. These curves show that the proportion of sparkovers due to induced voltages would be expected to increase rapidly as the line insulation is reduced below 14 units when the insulation strength is based upon the ½/5 wave. The beneficial effect of the ground wire is also shown.

In Fig. 6 induced voltages are plotted for different values of time of cloud discharge. Fig. 6 is of especial interest because by comparing with Fig. 2 the possible voltages due to measured values of time of cloud discharge can be obtained. Based upon the measured time of discharge, a cloud 2,000 ft. high and a line 50 ft. high, 10 per cent of the discharges could induce a voltage over 2,500 kv.; 20 per cent over 1,600 kv.; 40 per cent over 1,150 kv.; and 60 per cent over 800 kv. The data given in Table I were, in most cases, for waves originating a number of miles from the oscillograph. Due to distortion, the actual time is shorter than measured. Most of the waves in Fig. 2 were (+) and probably induced. The voltages at the source were probably over twice the measured values. A similar study was made of the front of waves.[10] The data are

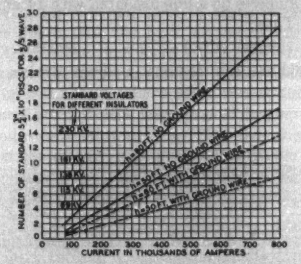

FIG. 5—NUMBER OF STANDARD 5¾-IN. BY 10-IN. SUSPENSION INSULATORS

Necessary to resist induced lightning voltages caused by various currents from a 1,000-ft. cloud

given in Table IV. An average cloud height of 1,500 ft. is indicated, with a range of 500 to 5,000 ft.

It can be concluded that:

1. Low lines are desirable.
2. Ground wires are desirable.
3. As the insulation is reduced or the conductor height increased, induced voltage sparkover becomes increasingly important.
4. Induced voltage troubles can be eliminated by ground wires.
5. Measurements made from cathode-ray oscillograms give data for estimating the induced voltage on any line. Further measurements are necessary to establish numerical values definitely.

DIRECT STROKES

Chance of Being Struck. The chance of a line being struck increases with increasing height of line. On a flat plane without projections, a tower may be hit when the distance from the projection of the storm center is less than four to ten times its height. This factor is based on the assumption that the nearest object will be hit and has been called the *direct hit ratio, r.*[3,6] Tests

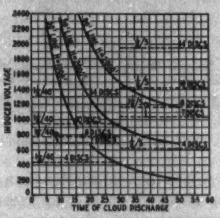

FIG. 6—VOLTAGES INDUCED ON LINES 50 FT. AND 80 FT. HIGH FOR DIFFERENT VALUES OF TIME OF CLOUD DISCHARGE

made on models in the laboratory check this ratio. The higher the cloud compared to the line, the greater this factor. The ratio r for different cloud-tower ratios is given in Table III.

TABLE III—DIRECT-HIT RATIO

$\frac{\text{Cloud height}}{\text{Tower height}} = \frac{H}{y}$	Direct-hit ratio r
50	10
40	9
30	7.5
20	6
10	4

Danger zone when storm center is over a strip $2\,r\,y$ ft. wide with tower as center.

The direct-hit ratio r is approximate and corresponds to the (−) cloud. The danger zone is less with the (+) cloud. This seems to be due to the fact that corona streamers from a (+) point (tower) are of much greater length, and thus more directive than from a (−) point (tower). For a (+) cloud the direct-hit ratio of four corresponds to an H/y ratio of about twenty. For laboratory tests see Bibliography reference 3, p. 289.

An example of the use of Table III follows:

With a ground wire or tower 50 ft. high and a cloud 1,000 ft. high

$$\frac{H}{y} = 20 \qquad \text{and } r = 6$$

A direct stroke may go to the ground or to a 50-ft. tower when the projection of the storm center comes within 300 ft. of the tower. In this case, a direct hit may take place to the tower when it is above a zone 600 ft. wide with the line as center. For a 100-ft. tower and a 1,000-ft. cloud this strip would be 800 ft. wide.

$$\frac{H}{y} = \frac{1{,}000}{100} = 10 \qquad r = 4$$

$$4 \times 100 \times 2 = 800 = \text{width of danger zone}$$

The hazard is thus greater for the taller tower.

When there are projections on the plane the chance of a given object being hit is very greatly reduced as shown in Fig. 7. Fig. 7a shows that a lightning stroke may begin to hit the tower when x and y become equal. Passing to Fig. 7b, a tree half the height of the tower and half way between it and p would take the strokes since

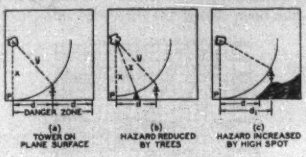

FIG. 7—LIGHTNING HAZARDS

the distance z would then be shorter than the distances y and x. The direct-hit ratio is approximate since it also depends upon polarity.[3] See note on Table III.

It is thus apparent that trees of even moderate height, within several tower lengths of the line, very greatly reduce the danger zone and the chance of a direct stroke to the line. Furthermore, many lines are brought through a right-of-way with trees practically equal to the line height. The hazard of direct strokes to such lines should be very small. It is a good reason for a low tower since the hazard is reduced to a much greater extent than would be indicated by the direct reduction in height. This follows because the chance of being

TABLE IV—PERCENTAGE OF LIGHTNING WAVES OF VARIOUS FRONTS MEASURED ON TRANSMISSION LINES

Per cent of waves	Measured wave front (μ sec.)	Estimated front at origin (μ sec.)	Cloud height	Impulse ratio for breakdown[10] on rising front of waves (Time from Column No. 3) Insulators	Gap
20	1	0.5	500	2.2	2.8
40	2	1.0	1,000	1.9	2.2
50	3	1.5	1,500	1.7	2.0
60	4	2.0	2,000	1.6	1.9
80	7	3.5	3,500	1.5	1.6

hit depends largely upon the relative heights of the line and secondary objects. It should be pointed out that a tree can direct a lightning stroke or "attract" lightning as readily as a metal tower because the charging takes place with small currents over a relatively long time. It also follows that the chance of any particular tower being struck does not depend upon its footing resistance. The resulting voltage on the insulators following a hit does, however, depend almost directly on this resistance.

Fig. 7c illustrates a line on a high point. If storms pass over such a point the hazard may be great. A consideration of the hazard of a line being struck emphasizes the importance of a tower relatively low compared to trees, etc.

Line Voltages Due to Direct Hits. Usually the tower is more likely to be struck than the line. When a tower is struck the resulting voltage from line to tower or ground wire, or across the insulator string, is

$$V_s = K I R \tag{3}$$

Where K is a constant depending upon the tower and ground wire configuration, I is the current in the tower, which will be less than the current in the stroke by the current in the ground wire, and R is the tower footing resistance; 0.80 may be taken as an average value of K for purpose of calculation.[7]

This formula shows the importance of keeping the tower footing resistance very low if immunity from direct-stroke sparkovers is to be hoped for. It was shown above that a line with ground wires and 14 to 16 insulator units should be free from sparkovers due to induced voltages. It is now of interest to test such a line from the standpoint of direct strokes to the tower.

Assume $I = 100{,}000$ amperes in the tower

$K = 0.8$ and $R = 40$ ohms

then $V = 100{,}000 \times 40 \times 0.8 = 3{,}200{,}000$ volts

The resistance R measured by a megger or at low voltage will be somewhat greater than the high-voltage lightning value. A heavy direct stroke with a 40-ohm tower footing resistance would thus cause a sparkover. Furthermore, it would be necessary to keep the footing resistance to about 20 ohms to prevent sparkover at 100,000 amperes on a line insulated with 14 units. See Fig. 8. Thus:

$V = 100{,}000 \times 20 \times 0.8 = 1{,}600{,}000$ volts

If 400,000 amperes obtains in direct strokes a 5-ohm tower footing resistance would cause 1,600 kv. Since a given current is associated with a given time of dis-

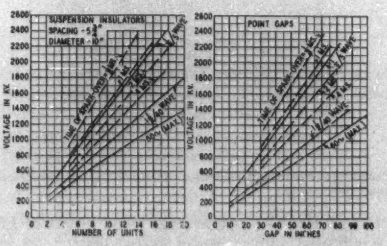

FIG. 8—LIGHTNING AND 60-CYCLE SPARKOVER OF INSULATOR AND POINT GAP

charge, it is possible to estimate the current and thus possible direct-stroke voltages from the oscillograms discussed above.

It is important to establish the actual range of direct-stroke currents by field measurements. For high currents a low resistance is necessary to make a line immune from outages due to direct hits at the tower.

The maximum insulator voltage at the tower struck is limited by the footing resistance but the tail of the

wave is rapidly reduced by reflections on the ground wire from the towers on either side. This reduction begins when the wave on the ground wire has traveled to the other tower and back. For a 1,000-ft. span the time is $\frac{2 \times \text{span}}{1{,}000} = 2$ microseconds; for a 500-ft. span it is 1 microsecond. The effect is that of applying a shorter wave across the insulators for the shorter span. The sparkover voltage is greater for the shorter waves. This is one advantage of the short span. Curves for

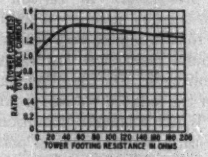

Fig. 9—Calculated Relation of Sum of Tower Currents to Total Current in Lightning Bolt for Tower with Different Footing Resistances

(For meter measuring maximum instantaneous crest currents)

sparkover in ½, 1, 2, and 4 microseconds are plotted in Fig. 8. The 1 and 2 microsecond waves correspond to the waves for 500- and 1,000-ft. spans respectively. It will be noted that the ½/5 laboratory curve falls between these values and should thus apply to direct-stroke sparkovers on the usual line. As already noted, when a tower is hit the total current of the stroke does not go directly to ground; part goes to ground through the ground wire to other towers. The division depends largely on the footing resistances. When the footing resistance is low, the total current of the stroke may be obtained approximately by adding the currents of the various towers. (See Fig. 9.)

It might appear that in certain cases a direct stroke could be steep enough to produce sufficient voltage between line and tower to cause sparkover even with zero footing resistance. This does not appear probable on a high-voltage line and the usual tower. On a 50-ft. tower the reflected wave would begin to reduce the voltage in about 0.1 microsecond. A front increasing at from 10,000,000 to 20,000,000 volts per microsecond would be necessary.

Location of Ground Wire to Shield Conductors from Initial Hits. The ground wire or wires should be at a sufficient distance above the line wires to "attract" the lightning and thus take the initial hit. The shielding effect of a ground wire depends upon its vertical height relative to the line wire, and to the horizontal distance between the projections of the two wires. If this horizontal distance is X and the vertical height of the line wire is h, the ground wire should have a vertical height greater than y.

y can be obtained from Fig. 10 or calculated from the equation

$$\frac{X}{H} = \sqrt{2\left(\frac{y}{H}\right) - \left(\frac{y}{H}\right)^2} - \sqrt{2\left(\frac{h}{H}\right) - \left(\frac{h}{H}\right)^2}$$

which gives y and h for equal hits. y should be at least 10 per cent greater than the calculated value for protection of h. Fig. 10 curves are for a cloud height $H = 1{,}000$ ft. While this position of the ground wire may shield the line wire from the initial hit, it does not follow that a side flash or sparkover will not take place from the struck ground wire to the line if certain other conditions are not fulfilled. This is so if the hit takes place at some distance from the tower and the effect is maximum for a hit at the center of the span.

Direct-Hit Wire—Protection from Side Flashes. The only difference between the effect of a direct hit at the tower and one somewhere on the ground wire is that caused by the time taken by the wave to travel from the point hit to ground and for the reflected wave to travel back. This is so because the point struck does not "know" there is a ground until the reflected wave returns. Since the reflected wave is of opposite polarity it begins to reduce the voltage at this point upon its arrival. If the front of the direct-stroke voltage is not steep enough to reach a dangerous voltage before the wave returns, sparkover will not occur between the ground wire and the line if the ground resistance is low enough. The time required for the return wave is one microsecond for every thousand feet of complete travel. The maximum distance to the ground would occur for a hit in the center of the span. There are several ways

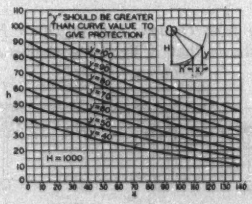

Fig. 10—Location of Ground Wire to Take Initial Hit

by which this factor can be taken care of, as for example:

a. Very high direct-hit wires.

b. Short spans or short distance between ground wire grounds.

c. A moderately high direct-hit wire and a moderate length of span.

d. By separate towers or lines to take direct hits.

With the high direct-hit wire the separation between ground wire and the line wire requires a longer time for the voltage to reach the sparkover value. Because of its height it is undesirable from the standpoint of "attracting" direct hits and not being very effective as a ground wire. With long spans 1,000 ft. or more and

abrupt direct strokes the separation between this wire and the line or ground wire at the center of the span should be 25 ft. to 50 ft. or more. Methods of stringing may permit greater clearance at the center of the span. These distances for different lengths of span can be obtained from Fig. 11. The *minimum distance required* on the assumption of 10,000,000 volts reaching the line is given at the intersection of the sparkover curve and the curve of voltage between ground wire and line wire.

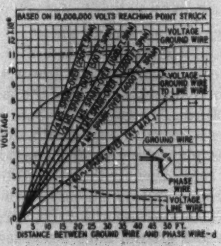

FIG. 11—MINIMUM CLEARANCE NECESSARY TO PREVENT SPARKOVER FROM GROUND WIRE TO LINE FOR A DIRECT HIT TO GROUND WIRE IN CENTER OF SPAN

The sparkover curve is based upon the point gap sparkover with voltage rising to breakdown value in the time for the wave to travel to the tower and back.[10-8] Usually it may not be necessary to design for a hit at the center of the span since the chances of such a hit are small. Further experience with direct-hit wires of varying heights on sections of operating lines is desirable.

Immunity from Outages Due to Direct Hits. This discussion shows that if lightning strikes the tower,

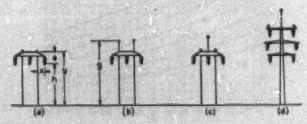

FIG. 12—TOWERS WITH INDUCED VOLTAGE AND DIRECT-STROKE PROTECTION

immunity from sparkover can be approached only with very low tower footing resistance and 230-kv. insulation (14 to 16 units). Direct-stroke wires in addition to the ground wires may be necessary. A line with comparatively short spans (500 ft.) and a direct-hit wire sufficiently high to take the initial strokes from the ground wires, as well as the line wire, may generally be more practicable than excessively high direct-hit wires. (See Fig. 12.) This same effect could be obtained on a long span line by placing a ground wire or direct-stroke wire sufficiently high to take the initial discharges on what is, in effect, a short span line. This could be accomplished by permitting the main towers to support the direct-stroke wire and using the intermediate towers for grounding purposes only.

Low-Footing Resistance. Low-footing resistance can usually be obtained by means of buried wires connected to the tower legs. These wires may be either extended out radially from the tower or extended directly from the towers along the line. Radial wires of moderate length should usually be more effective. These have been called counterpoise wires, a term having a somewhat different meaning in radio. However, the method used will be governed largely by cost.

Protection Against Direct Strokes When Towers Have High-Footing Resistance. In certain parts of the country where there is little or no soil and the towers are located in rock it may be very difficult or impossible to reduce ground resistances low enough to prevent direct-stroke outages. On the other hand, resistances low enough for the induced voltage ground wire may be practicable. When economic conditions permit, this can be taken care of by extending parallel lines at a distance x on either side of the sections of the transmis-

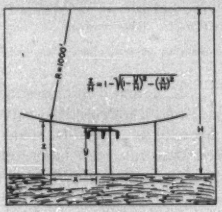

FIG. 13—SPECIAL TYPE OF CONSTRUCTION FOR PROTECTION AGAINST INDUCED VOLTAGES AND DIRECT STROKES

sion line. The object of these lines is to "attract" the stroke away from the transmission line and thus prevent it from being hit. For ideal protection a cloud directly overhead should be nearer the direct-stroke line than the main line as shown in Fig. 13. The minimum height of these lines Z, can be calculated from the formula

$$\frac{Z}{H} = 1 - \sqrt{\left(1 - \frac{y}{H}\right)^2 - \frac{x}{H}^2}$$

This line need not have low-footing resistance and the conductor need not be any larger than mechanical conditions require. This form of protection would in many ways seem ideal.

Another solution, possibly not economical, would be a very low line with a net fence and net top. Towers placed to take all of the direct hits could also be used for this purpose.[6]

Dischargers or Diverters. It is possible to prevent outages by some form of discharge device or diverter at the tower permitting the lightning to discharge freely to ground but preventing the power current from following. The requirements for such a device are much more severe for direct strokes than for induced voltages.

In the case of direct strokes the resistance must be very low. The fused grading shield is operating successfully in practise. Other devices have also been successful but they are still in the developmental stage. One weakness of such a device is that when located at the tower it would not necessarily prevent a power arc between line and ground wire due to a lightning discharge at the center of the span.

III. Operating Experience

The Ground Wire. There is now overwhelming operating evidence of the value of the ground wire in reducing outages, operating reports generally indicating a reduction varying from two to one, to ten to one after the installation of ground wires. Probably better results will be obtained with more attention to location and grounds. Measurements of the protective ratio of ground wires on full size outdoor antennas give good agreement with the values in Table V obtained from laboratory measurements on models. These ratios are lower than those obtained mathematically where corona is neglected.

The value of the ground wire for direct-stroke protection has also been indicated. This applies to the ground wire as usually located, as well as the high direct-stroke wire, both of which have been experimented with on the lines of the Pennsylvania Power & Light Company. On one bad section of this line the outages were 6, 9, and 14 respectively in the years 1926, 1927, and 1928. During 1929 the ground resistances on this section were reduced from 50 to 150 ohms to values of the order of one ohm and the usual ground wire was unchanged. There were no outages on the section in 1929 and 1930. "Diverter" wires or high-ground wires erected about 50 ft. over the line wires on another section were struck without outages.[13]

Direct Strokes vs. Induced Voltages as a Cause of Outages. It is very difficult to decide definitely whether an outage is caused by induced voltages or direct strokes even when lines are equipped with instruments for that purpose. There is very good evidence that 70 per cent of the outages on the 220-kv. (14 to 16 insulator units) lines of the Pennsylvania Power & Light Company were caused by direct hits. A result of this order might be expected from the analysis above, particularly from the results in Figs. 2 and 6.

A very interesting analysis of outages on the 132-kv. lines of the American Gas & Electric Company has been made by Mr. Sporn.[9] Table VII in this paper shows that 8 outages out of 12 occurred where the footing resistance was less than 5 ohms. In the remaining 4 the resistances varied from 18 to 33 ohms. Similar data are given for other lines. These data would at first appear to be at variance with the conclusion that low-ground resistance is important, because more flashovers occurred on the towers of low-ground resistance than on those of high resistance. A possible explanation is as follows:

TABLE V—EFFECT OF GROUND WIRES ON LIGHTNING INDUCED VOLTAGES
(From Tests on Models)

Arrangement	Protective ratio	
.	1	0.50
o o o	2	0.44
1 2 3	3	0.50
. .	1	0.40
o o o	2	0.34
	3	0.40
. . .	1	0.34
o o o	2	0.28
	3	0.34
.	1	0.28
. o o o .	2	0.25
	3	0.28
.	1	0.43
o o	2	0.50
o o	3	0.62
o o		
. .	1	0.34
o o	2	0.39
o o	3	0.48
o o		
.	1	0.40
o o	2	0.48
o o	3	0.36
o o		
.	1	0.35
o o	2	0.35
o . o	3	0.43
o o		
. . .	1	0.25
o o .	2	0.30
o o	3	0.37
o o		

o indicates conductor
. indicates ground wire

$$\text{Protective ratio} = \frac{\text{Voltage with ground wire}}{\text{Voltage without ground wire}}$$

Average values for various relative conductor and ground wire distances that have been used in practise.
Includes corona effect.

PROTECTIVE RATIOS
ω
Showing Effect of Varying Distance between Line Wire and Ground Wire
h = 50 ft.

x	Values of y					
	60 ft.	70 ft.	80 ft.	90 ft.	100 ft.	125 ft.
0 ft.	0.41	0.48	0.53	0.58	0.63	0.73
10 ft.	0.45	0.50	0.55	0.59	0.64	0.74
20 ft.	0.51	0.53	0.57	0.61	0.65	0.75

h = 80 ft.

x	Values of y					
	60 ft.	70 ft.	80 ft.	90 ft.	100 ft.	125 ft.
0 ft.			0	0.39	0.43	0.52
10 ft.			0.39	0.41	0.45	0.54
20 ft.			0.44	0.45	0.47	0.56

For a single ground wire.
x = horizontal distance between projections of ground wire and phase wire.
y = vertical height of ground wire.
h = vertical height of line wire.

For the lower ground resistance values sparkover by direct strokes would not be expected, while for the higher values direct-stroke outages would be expected from application of equation (3). On the other hand, most of the footing resistance values are low enough so as not to affect materially the efficiency of the ground wire for induced voltages. Approximately equal effects would be expected from induced voltages as recorded in the tables. Thus 100 per cent of the outages could be accounted for by induced voltages and 33 per cent by direct hits at the tower. Therefore the cause of outages might be 66 to 100 per cent by induction or up to 33 per cent by direct hits and the remainder by induction. The percentage of direct-hit outages might be increased somewhat by side flashes from hits out on the span.

Lightning has been observed to strike near lines without causing induced voltages high enough to flashover insulation; it has also been observed to strike at a distance with simultaneous splintering on wood poles or insulator flashovers on steel towers. Because of the short duration of the phenomena, exactly what happens is frequently uncertain. However, as pointed out above, induced voltages would be expected to vary widely

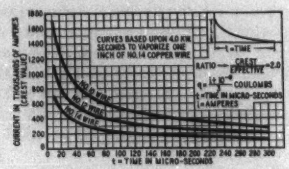

FIG. 14—CURRENT REQUIRED TO VAPORIZE WIRE FOR VARIOUS TIME LENGTHS OF CURRENT APPLICATION

with the character of the discharge. The reported observations are, therefore, not contradictory.

On July 16, 1930 a radio antenna of No. 12 copper wire was struck at Pittsfield, Massachusetts. One hundred feet of the wire was completely vaporized by the lightning current. The middle curve in Fig. 14 gives the calculated current necessary to vaporize a No. 12 copper wire for varying time.* It is of interest that this current is 800,000 amperes at 20 microseconds; 510,000 amperes at 50 microseconds and 112,000 amperes at 1,000 microseconds. No. 14 rubber-covered copper wire used on telephone circuits and house wiring is also frequently fused.

In order to give the various factors the proper weight in tower design it is important to obtain more data on how frequently the conditions necessary to cause high induced voltages occur during storms and also on the hazard of a line being struck.

*These curves are based upon 4.0 kw. seconds to vaporize 1 inch of No. 14 copper wire.

IV. TOWER DESIGN

Several different tower designs will now be considered for the purpose of illustrating the application of the above principles.

230-Kv. Line Conductors in Horizontal Plane

Assume that a single-circuit steel tower will be used with conductors in a horizontal plane and limited to an average height of 50 ft.

With two ground wires the general arrangement would be as in Fig. 12a. Take $\alpha = 0.45$ as the probable maximum value, then

Induced Voltages

$$V = g\,h\,\alpha = 100 \times 50 \times 0.45 = 2{,}250 \qquad (1)$$

or with ground wires when $\omega = 0.5$

$$V = g\,h\,\alpha\,\omega = 1{,}125 \qquad (2)$$

The footing resistance should be less than 50 ohms and preferably not over 25 from the standpoint of induction. From Fig. 8, eight insulator units would be sufficient with the ½/5 wave or twelve on the 1½/40 wave. Thus induced voltages are readily taken care of if ground wires are used.

Direct-Hit Voltages $V = K\,I\,R = 0.8\,I\,R$ (3)

Take $I = 200{,}000$

then for $R = 25$ ohms

$V = 0.8 \times 200{,}000 \times 25 = 4{,}000{,}000$ volts $= 4{,}000$ kv.

This is a high voltage for which to insulate. Reduce the footing resistance, then

$$R = 12 \text{ ohms}$$
$$V = 1{,}920 \text{ kv.}$$

This requires 14 units on the ½/5 wave which seems reasonable for most direct strokes (See Fig. 8). However, a more conservative arrangement, until more information is available on the common direct-stroke currents, would be 5-ohm resistance and 14 to 16 units. It will be noted that up to the present the induced and direct-stroke requirements are the same except that the direct stroke necessitates a lower ground resistance. The clearance from the conductor to tower can be obtained from the point gap curve in Fig. 8.

Shielding the Line Wires from Direct Hits. It is now necessary to determine if the ground wires are high enough to shield the conductor from the initial hit. Referring to Fig. 10, for $X = 10$ ft. and $h = 55$ ft. at the tower, y must be greater than 60. Insulator lengths and other considerations have fixed y at 70 ft. The shielding condition is more than met without impairing the efficiency of the ground wires.

Direct Hits to Ground Wires at Center of Span. To provide against direct hits from the ground wire to the line at the center of the span consideration of a further condition is necessary.

If the span is 500 ft. the requirement is a minimum clearance of 20 feet. See Fig. 11. A somewhat greater clearance at the center of the span can probably be provided for without greatly reducing the ground wire

efficiency. Check the clearance decided upon with the protective ratio in Table V to see if the value is greater than 0.5 as used above.

With a 1,500-ft. span the requirement is greater than 29 ft. and the ground wire is reduced in efficiency.

From the calculation of the voltage due to direct strokes with a 5-ohm ground it is seen that 14 to 16 units are necessary. It would then appear that 230-kv. insulation is necessary where protection from direct strokes is desired. The tower in Fig. 12a seems to meet the requirements for 230-kv. insulation both from the usual induced voltage and direct-stroke standpoints, with a possible small factor of safety for mid-span hits on the longer span. It meets the induced voltage conditions for 138-kv. insulation but not all of the direct-stroke conditions. A one-ohm tower resistance would be desirable. *Operating experience is desirable with direct-hit wires of different heights on sections of lines exposed to lightning.* Good results have been obtained with the usual ground wire with low-footing resistance as noted above.

If longer spans are used or greater safety factor for mid-span hits is desired, the height of the ground wire may be increased as in Fig. 12b with 16-unit insulation. This could be done without too great reduction in the induced voltage efficiency.

For lines of lower voltage or at lower insulation a combination of direct-stroke wires can be used as in Fig. 12c. Figs. 12b and 12c have the disadvantage of increasing the hazard of being hit. For high insulation, the ground wires in Fig. 12c could be omitted, leaving only the single direct-stroke wire properly located to take direct hits.

Conductors in Vertical Plane.

A tower with a vertical conductor arrangement increases the difficulty of designing a line highly resistant to lightning. See Fig. 12d.

Induced Voltages. $V = h\,g\,\alpha\,\omega = 100\,h\,\alpha\,\omega$

$$= 100 \times 80 \times 0.45 \times 0.5 = 1{,}800 \text{ kv.}$$

would require 14 to 16 insulator units with ground wires for the ½/5 wave. See Fig. 8.

Direct-Hit Voltages. With 12-ohm footing resistance —200,000 amperes

$$V = K\,I\,R = 0.8 \times 200 \times 12 = 1{,}920$$

Requires 14 to 16 units for the ½/5 wave.

Obtain clearance to tower from gap curves (Fig. 8).

Shield Wire. The 15-ft. clearance between line and ground wires should be sufficient for shielding.

Direct Hits to Ground Wires at Center of Span. If desired, a direct hit wire can be provided as shown by dotted line.

With this tower the number of direct hits should be greater than for tower 12A.

Short Spans Important

Since the height of the direct hit wire must be increased with increasing span length, short spans between ground wire grounds are important. As already noted, short spans also increase the effectiveness of the insulation.

Special Method of Protection Against Direct Hits.

Where low-ground resistances are not possible, or are difficult to obtain, the arrangement in Fig. 13 could be used. In fact, if economically feasible it would be ideal for any condition since it takes the hit away from the tower. The usual tower is used with the ground wires that are necessary for induced voltages. The ground resistance of these wires need not be particularly low. At either side, at a distance from the line about equal to the height of the ground wire, shielding wires are erected and grounded at each pole. No care need be taken in obtaining low-ground resistance. The wire need not be larger than mechanical conditions demand. The object of these wires is to take the direct hits and thus prevent the line from being struck.

With the ground wire height $y = 60$ and the distance between ground wire and shielding wire $x = 60$, the minimum height of the shield wire is $z = 62$ from

$$\frac{Z}{H} = 1 - \sqrt{1 - \left(\frac{y}{H}\right)^2 - \left(\frac{x}{H}\right)^2}$$

The same effect is obtained by trees along the right-of-way of many lines. Z should be at least 10 per cent greater than the calculated value. Direct-stroke towers could also be used.

The towers shown in Figs. 12 and 13 are not intended for definite designs but merely for the purpose of illustration.

The above principles can be used in the design of lines with wood pole towers. With such towers advantage should be taken of the wood to increase the insulator sparkover. The lightning strength of wood has already been given elsewhere.[2,3]

V. Wave Shapes—Coordination

From the above analysis it appears that the steep wave-front effects, causing insulator sparkover on the more highly insulated or higher voltage lines, are approximated by the ½/5 wave. This applies particularly to direct strokes to the ground wire or to tower, or severe induced voltages. The ½/5 curve should then apply in determining the lightning strength under such conditions. When a direct stroke goes to a line wire, high sparkover values such as are caused by the ½/5 wave will occur but the resulting wave on the line is likely to be very long.

Lower induced voltages may have very long waves. While voltages of such waves will not usually reach sufficiently high values to cause sparkover on the highly insulated lines, they may be the predominant cause of trouble on the lower voltage lines, especially on distribution circuits. The 1½/40 wave approximates the effects of this type of lightning voltage as the lower extreme. Both waves should be considered in the design of the lower voltage lines.

The coordination of line insulation with apparatus

insulation has been so successful in practise that it will probably soon be in general use. In adjusting the sparkover values of the various insulations and gaps for purposes of coordination it is important to have the relative values hold for any possible lightning wave. Usually, if the coordination adjustment is made with the ½/5 wave, the same relative sparkover positions of the various insulators will remain the same for any wave. However, it may sometimes be desirable to know the relative voltages for a long wave as well as a short one. All conditions are well covered by tests with the ½/5 and the 1½/40 waves. A wave falling between the two and giving an impulse ratio of about 1.5 might also be useful. Further information on lightning strength for different waves can be found elsewhere.[10]

VI. Conclusions

1. The cloud height, the charge, the time of discharge, and the current and voltage of the bolt can be estimated from lightning oscillograms on transmission lines.

2. An independent check on oscillographic measurements can be obtained from direct-stroke current measurements.

3. Transmission line voltages due to induction and direct strokes can be calculated from the above information.

4. In some instances more accurate field measurements are necessary before final conclusions can be reached.

5. Line sparkover can be caused by either induced voltages or direct hits. As line insulation is increased, the direct hit becomes of increasing importance as the cause of sparkover.

6. Induced voltages can be reduced to safe values on high-voltage lines by ground wires. An important factor in direct-stroke protection is low tower footing resistance.

7. The general requirements for a safe line from the standpoint of direct hits or induced voltages are the same. They are; low lines, short spans, low tower footing resistance, and ground wires. However, direct strokes require lower footing resistance and higher ground wires.

8. The lightning strength of insulators for induced voltages and direct strokes can be estimated.

9. A lightning proof line seems feasible. Special designs are discussed.

10. Methods of tower design are illustrated.

11. More accurate field measurements are desirable to determine the numerical range of voltage, current, etc.

The valuable assistance of Mr. L. V. Bewley in developing formulas, making calculations and preparing the appendix is acknowledged.

Mathematical Appendix

Induced Strokes

$$e = \frac{(\omega)}{2}\int_0^t \left\{ f[x+v(t-\tau)] + f[x-v(t-\tau)] \right\} \frac{\partial F(\tau)}{\partial \tau}\, d\tau \qquad (1)$$

$f(x)$ = Distribution of voltage due to instantaneous release of bound charge.

$F(t)$ = Law of cloud discharge

$$= \begin{cases} \dfrac{t}{T} & \text{linear law} \\ (1 - \epsilon^{-\frac{t}{3T}}) & \text{exponential law} \end{cases}$$

T = Time of cloud discharge

$$(\omega) = \frac{1}{h_r} \begin{vmatrix} z_{11} & \cdots & z_{m1}\, h_1 \\ \cdots & \cdots & \cdots \\ z_{1m} & \cdots & z_{mm}\, h_m \\ z_{1r} & \cdots & z_{mr}\, h_r \\ z_{11} & \cdots & z_{m1} \\ \cdots & \cdots & \cdots \\ z_{1m} & \cdots & z_{mm} \end{vmatrix}$$

= theoretical protective ratio for line wire r due to m ground wires

h = Heights of the conductors

$$z_{xx} = 60 \log_\epsilon \left(\frac{2h}{\rho} \right) = \text{self surge impedance}$$

$$z_{xy} = 60 \log_\epsilon \left(\frac{a}{b} \right) = \text{mutual surge impedance}$$

$$e' = \frac{(\omega)}{2}\int_0^t f[x+v(t-\tau)] \frac{\partial F(\tau)}{\partial \tau}\, d\tau \qquad (2)$$

= voltage of the free traveling wave

$$V = \alpha\, g\, h(\omega), \therefore = \frac{V}{g\, h(\omega)} \qquad (3)$$

$$V' = \alpha' g h(\omega), \therefore = \frac{V'}{G\, h(\omega)} \qquad (4)$$

$$I = Q \frac{\partial F(t)}{\partial t} = \text{Cloud discharge current} \qquad (5)$$

Q = Initial charge of cloud.

By (5) and (1)

$$e = \frac{(\omega)}{2Q}\int_0^t \{ f[x+v(t-\tau)] + f[x-v(t-\tau)] \}\, I(\tau)\, d\tau \qquad (6)$$

Front of free wave $\simeq L \simeq 2H$

Total length of free wave $\simeq (L + T)$

$$L = \frac{\int_{-\infty}^{+\infty} f(x)\,.\,dx}{f(0)} = \text{length of equivalent rectangular bound charge} \qquad (7)$$

Direct Strokes

Stroke at tower	Voltages	Stroke at mid-span
$\frac{2 R z_{11} E}{(z_{11} + 2 z_o) R + z_{11} z_o}$	$V_{g\cdot w.}$ (tower)	$\left(\frac{2 R}{2 R + z_{11}}\right)\left(\frac{2 z_{11}}{z_{11} + 2 z_o}\right) E$
"	$V_{g\cdot w.}$ (mid-span)	$\left(\frac{2 z_{11}}{z_{11} + 2 z_o}\right) E$
$\frac{2 R z_{1r} E}{(z_{11} + 2 z_o) R + z_{11} z_o}$	V_{line} (tower)	$\left(\frac{2 R}{(2 R + z_{11}}\right)\left(\frac{z_{1r}}{z_{11} + 2 z_o}\right) E$
"	V_{line} (mid-span)	$\left(\frac{z_{1r}}{z_{11} + 2 z_o}\right) E$
$\frac{2 R (z_{11} - z_{1r}) E}{(z_{11} + 2 z_o) R + z_{11} z_o}$	$V_{insul.}$ (tower)	$\left(\frac{2 R}{2 R + z_{11}}\right)\left(\frac{z_{11} - z_{1r}}{z_{11} + 2 z_o}\right) E$
"	$V_{insul.}$ (mid-span)	$\left(\frac{z_{11} - z_{1r}}{z_{11} + 2 z_o}\right) E$

z_{11} = Equivalent self surge impedance of all ground wires.
z_{1r} = Equivalent mutual surge impedance of all ground wires to any line wire r.
z_o = Surge impedance of lightning bolt.
R = Tower footing resistance.
E = Voltage of incident wave from lightning stroke.

In terms of the current in the tower the voltages are (for a strike at the tower)

$$V_{g\cdot w} = V_{tower} = R I$$

$$V_{line} = R I \left(\frac{z_{1r}}{z_{11}}\right)$$

$$V_{insul.} = R I \left(\frac{z_{11} - z_{1r}}{z_{11}}\right)$$

Ordinarily $z_{1r}/z_{11} = 0.20$ so that $V_{insul.} = 0.80\, R\, I$

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Discussion

For discussion of this paper see page 1146.

Lightning Discharges and Line Protective Measures

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and

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Synopsis.—*This paper describes the mechanics of the development of a lightning stroke in a cloud and the formation of a surge on a line. It shows that the energy of a stroke can vary between wide limits so that the actual surge on the line may be of practically any magnitude.*

Fundamental types of high-tension line construction are analyzed to show the performance that would be obtained with them.

Experience obtained on the 220-kv. lines of the Public Service Electric and Gas Company is offered to substantiate the belief that induced surges are of relatively small importance and that the direct stroke is the criterion for good design.

The effects of line construction on the character of surges impressed on the terminal substation apparatus is discussed to indicate the degree of protection that would be required.

An understanding of the formation of lightning surges on transmission lines is essential to the study of the problem of protecting lines and substations against these surges. In the study of lightning protection, the magnitude and wave form of the surges formed on the line are of fundamental importance; attenuation also becomes important when considering the surges to which a substation may be subjected during any thunderstorm.

Complete data on the limiting value of lightning stroke potentials are not available, but from known data the approximate values of potentials that may be encountered may be estimated. The highest surge yet recorded on any transmission line occurred during the past season on a 110-kv. wood pole line in Arkansas. This surge was formed on the line as the result of a stroke of lightning which splintered the pole from top to bottom; a side flash from the transmission line jumped 25 ft. of air to a neighboring telephone line. The cathode-ray oscillograph, located four miles from the point at which the lightning struck the line, gave an oscillogram of a surge which reached its maximum of 5,000 kv. in one microsecond. From known data it has been estimated that, at the point struck, the potential due to the lightning stroke must have reached a value of approximately 15,000 kv. before flashover took place from line to ground over the wood pole. From laboratory data on creosoted wood poles and needle gaps it is estimated that the rate of rise of the potential due to the lightning stroke before flashover took place must have been between 10,000 kv. and 20,000 kv. per microsecond.

Mechanism of Lightning Strokes

To understand the factors that have a bearing on the limitation of the potential of the lightning stroke when it hits a line or, what amounts to the same thing, the maximum limit of the lightning stroke current, one must consider the nature of the thundercloud itself and the manner in which electric energy is stored in it. The thundercloud consists of small particles of water-vapor each carrying a minute charge of electricity. If a free negative charge is present it attaches itself to a particle of vapor and thereby loses mobility. A thundercloud retains its charge by reason of the fact that the particles of water-vapor repel each other and, since the size of the particles is maintained constant, remain suspended in the air by the action of upward components of the air currents.

Since there are very few free ions in the cloud atmosphere and since the mobility of the charged moisture particles is very small, the only way such a volume can be discharged is by ionization. This process in a cloud atmosphere may be extremely slow. During the formation of the lightning streamer very little current is drawn from the cloud, and therefore the volume of cloud atmosphere required to be ionized is small. As the head of the streamer progresses by ionizing the air space in front of it, a space charge of requisite value must be built up and maintained. This ionization of air requires energy and time. Since the energy must be obtained from the cloud itself, the rate at which the point of the streamer will travel will be governed by the rate at which cloud ionization can release energy.

Even if it were assumed that ionization streamers inside the cloud atmosphere could form with the velocity of light, it would require 10 microseconds to discharge completely a cloud volume one mile in diameter. However, the speed of formation of streamers is much slower than the velocity of light, probably being not more than one-twentieth of that value as determined from time lag measurements of sphere-gap breakdown. To discharge completely a spherical volume one mile in diameter would therefore take at least 105 microseconds. If a reflection from the ground over the lightning channel is considered, 5 microseconds should be added to the above values.

Some streamers have been noted which never reach ground as the charge in the cloud is not sufficient to maintain and propagate the streamer. It may therefore be inferred that lightning strokes to earth or to a

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structure will vary their severity over a wide range depending upon the amount of energy the cloud is capable of discharging through the lightning channel.

The energy in the streamer itself, until it reaches ground, a transmission line or any structure, is mainly potential energy because the motion of the charges in the channel is relatively slow as compared to the velocity of light and because the currents required to produce ionization are relatively small. When the earth or a structure is reached by the streamer its potential energy is instantly changed to kinetic energy and a current wave moving at the velocity of light flows into the earth or the structure. Negative reflections pass up the lightning channel, increasing the discharge current in the channel as they proceed, until they reach the cloud where they increase the gradient in the cloud atmosphere, causing the ionization streamer to penetrate further into the cloud volume and thus tapping more of the cloud energy to supply current to the channel. As the volume of ionization extends within the body of the cloud, the rate of discharge gradually decreases until the point is reached where the available energy is insufficient to stabilize the channel from cloud to earth and the current ceases to flow.

Formation of Line Surges Due to Lightning Strokes

Formation of line surges due to lightning strokes should be analyzed in accordance with the type of transmission construction used of which there are two general classes. In the first general class are included the wood pole lines and steel tower lines not equipped with ground wires. The second consists of steel tower lines equipped with ground wires. Three cases occur in this class; first, where inadequate ground wire protection is provided and the lightning stroke is allowed to strike the conductors; second, where the design of ground wires is adequate to prevent the contact of the stroke with any conductor but the conductor insulation is inadequate to prevent flashover either at the tower or mid-span; and third, where all strokes are terminated on the ground wires and the insulation of the conductors is adequate to prevent flashover.

Considering first surges occurring on transmission lines unprotected by ground wires, when lightning strikes a wood pole line unprovided with a spill-over gap at that point, the potential attained by the surge before the wood structure breaks down may be very high. Such a lightning stroke usually involves all conductors before the failure of the wood pole unless the clearances between wires are extremely large. When the structure flashes over, the potential is reduced quickly to a value which depends upon the surge impedance of the lines and of the wood pole and ground, considering the wood pole as a conductor to earth. The surge impedance of the wood pole and earth may range between 10 and 200 ohms. Assume a value of 50 ohms and a lightning stroke that will have a potential of such a value that, when it terminates on a conductor of the same surge impedance as the lightning channel, it will deliver a surge of 15,000 kv. crest and will rise at a rate of 15,000 kv. per microsecond. If the surge impedance of a lightning channel is 200 ohms the calculations for potential on the line are as follows:

$$\frac{V_a}{V_o} = \frac{2\,Y_o}{Y_o + 2\,Y_a + Y_c} \qquad (1)$$

Potential at point of stroke $= V_a$
Dynamic lightning potential $= V_o$
Surge admittance of lightning channel $= Y_o$
Surge admittance of conductors involved $= Y_a$
Surge admittance of pole and ground $= Y_c$
$Y_o = 0.005$ mhos
$Y_a = 0.002$ mhos for single conductor
$= 0.004$ mhos for three conductors
$Y_c = 0.02$ mhos

Voltage reduction factor for stroke to three conductors:

$$\frac{V_a}{V_o} = \frac{0.01}{0.005 + 0.008} = 0.77$$

Voltage reduction factor for stroke to three conductors and ground:

$$\frac{V_a}{V_o} = \frac{0.01}{0.005 + 0.008 + 0.02} = 0.303$$

Steepness of surge on the line ($0.77 \times 15{,}000$ kv.) is 11,550 kv. per microsecond.

If the line under consideration has physical characteristics similar to the Arkansas line,[6] flashover may be assumed to take place when the potential rises to 10,000 kv. or in 0.87 microseconds.

Potential of the conductor immediately after the pole flashes over will be

$$\frac{10{,}000 \times 0.303}{0.77} = 3{,}940 \text{ kv.}$$

This potential at the end of one microsecond will be

$$15{,}000 \times 0.303 = 4{,}550 \text{ kv.}$$

Fig. 1 shows the approximate shape and value of the surge that will be impressed on the line due to the lightning stroke and the flashover of the wood pole. When the surge reaches the next structure, flashover may take place there also but at a greater time lag, reducing the tail of the surge to a much lower value. Surges on the line resulting from lightning strokes of lower values are also shown in Fig. 2. It should be noticed that the lower the potential the longer will be the unchopped portion of the surge. This always will be true if the lightning stroke has plenty of energy and therefore, the values represented by the longer surges shown in the curves are the most severe types of waves due to lightning which may be encountered in a line of this type.

In the case of steel towers without ground wires the surge potential of the conductor after flashover of the

6. For references see Bibliography.

insulator will be that of the tower. One insulator string only may be flashed over if the surge impedance of the tower is low, since the surge traveling along the line that is struck induces a potential of the same sign on the other conductors thereby reducing the difference of potential between them and the tower. In horizontal

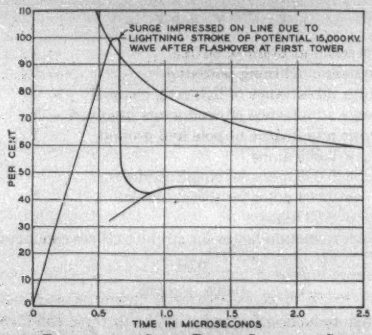

FIG. 1—FORMATION OF SURGE DUE TO LIGHTNING STROKE ON WOOD POLE LINE

construction, if an outside wire is struck, the other outside wire has the next higher probability of flashing over and if it should flash over the middle wire will generally be well protected from flashover. The likelihood of another flashover taking place at adjacent towers is

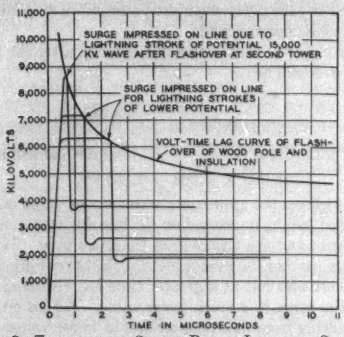

FIG. 2—FORMATION OF SURGES DUE TO LIGHTNING STROKE ON WOOD POLE LINE

dependent upon the intensity of the stroke and the impedance of the tower where flashover first occurred.

Considering the case of a steel tower line protected by ground wires, the primary function of the ground wire is to intercept the stroke of lightning and conduct the current to a tower through which it dissipates itself into the earth. To accomplish this the correct relationships of ground and line wires for both horizontal and vertical construction are shown in Figs. 3 and 4. For the horizontal configuration, the ground wire should be so placed that a line drawn through it and the outside conductor will form an angle with the vertical which is no greater than 20 deg. With angles greater than this, side flashes to the outside conductors are very likely. For the vertical configuration, the ground wires on the double circuit towers should be placed vertically above

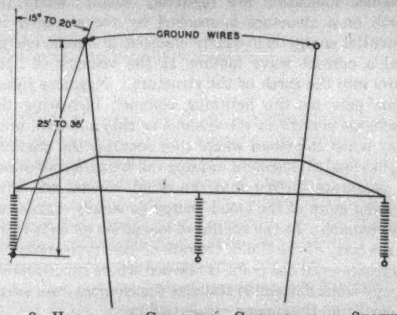

FIG. 3—HORIZONTAL CONDUCTOR CONFIGURATION SHOWING GROUND WIRE ARRANGEMENT

the outermost conductors. With both types of construction the spacing between the ground wire and conductor should be such that a stroke to the ground wire at mid-span will not cause a flashover between the ground wire and the conductor. Theory and limited practical experience indicate that this spacing for 1,000-ft. spans should be approximately 40 ft.

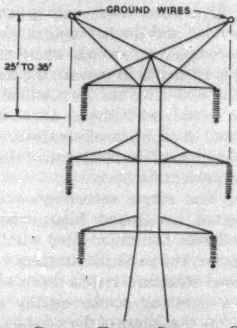

FIG. 4—DOUBLE-CIRCUIT TOWER SHOWING GROUND WIRE ARRANGEMENT

To evaluate the surge that is impressed on a line inadequately protected with ground wires and sufficiently insulated against tower potentials only, first consider the potential set up on the conductor when struck in mid-span before flashover takes place. The value of this potential V_a, may be obtained in the same manner as

described under the discussion of lines not equipped with ground wire. The insulator will flash over on the front of the wave at some value which is less than V_a, and the potential will be reduced to a value computed by formula (I) with the added admittance of the ground wires included. After flashover of the insulator, part of the surge is propagated over the ground wires and is reflected from the adjacent towers reducing the potential of the tower and conductor involved in the flash-

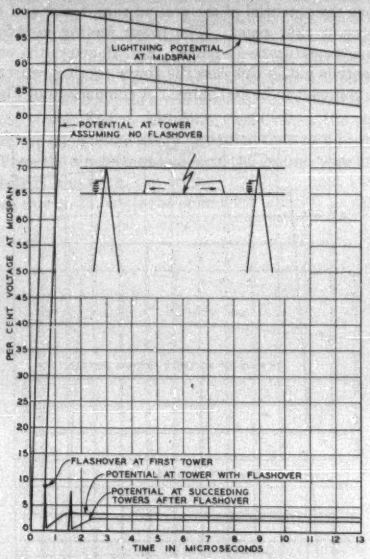

Fig. 5—Potential at Tower Due to Lightning Stroke on Line Conductor at Mid-Span

over. Meanwhile the rest of the surge is carried away by the conductor and tower. An analysis of the form indicated above was made but with the stroke terminating on the conductor in the middle of the span. The results follow, being presented in simple numerical values and illustrative curves.

Assume a 15,000-kv. wave to be impressed upon the conductor in the middle of the span (Fig. 5). The ground wire will at the same time assume a similar potential wave with a magnitude determined by the coupling which in this case is 0.295. The surge will travel along the line until it strikes the tower. The front part of the wave which is below flashover will travel along the line, but with a slight reduction. The current drawn by the ground wire through the tower reduces the potential of the ground wire and allows a charge to form between the ground wire and the conductor. This reduces the potential of the surge traveling beyond the tower to 89 per cent of its original value which is not reduced at succeeding towers except by attenuation.

Assuming that flashover takes place when the voltage rises to 1,000 kv. so that the potential of the tower, ground wires and conductor now becomes the same, the rest of the wave will divide as follows:

A part will travel beyond the tower on the conductor, a part will travel in both directions on the ground wire, a part will travel over the tower into ground, and the remainder will be reflected over its original path. Thus the magnitude of the surge traveling over the ground wire is the same as that on the conductor following the flashover. At the succeeding tower, however, the potential of the ground wires is reduced to a negligible value. This change, through coupling with the conductor reduces the conductor potential for the section of the wave following flashover, to 63 per cent of its original value. Thus if the potential of the tower where flashover occurred rises to 159 per cent of the flashover

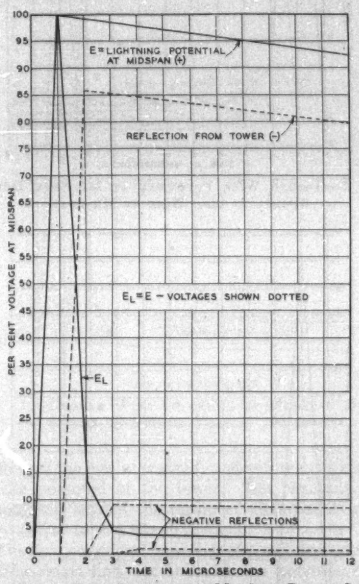

Fig. 6—Line Wire Potential at Mid-Span Due to Stroke on Line Wire at Mid-Span

value of the insulator string, the insulators will flash over at only one of the succeeding towers.

Flashover at the first towers will set up reflections over the conductor and cause waves to travel back and forth over the ground wires. The potentials developed at various points are given in the following curves. Fig. 6 shows the potential (E_L) of the conductor at mid-span. This wave is a resultant of the lightning

potential impressed by the lightning stroke and the subsequent reflections from the towers. The reflections shown with dotted lines are of opposite polarity, thus subtracting from the impressed wave. The ground

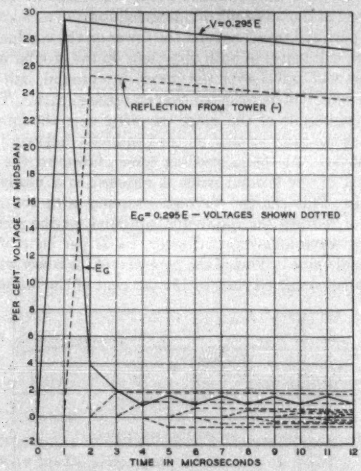

FIG. 7—GROUND WIRE POTENTIAL AT MID-SPAN DUE TO STROKE ON LINE WIRE AT MID-SPAN

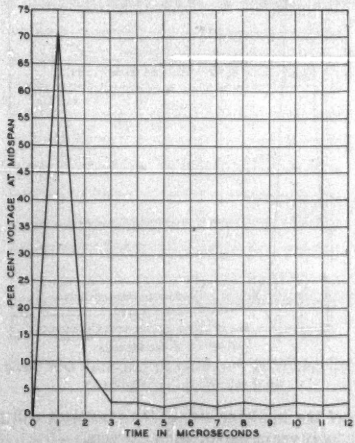

FIG. 8—VOLTAGE BETWEEN LINE WIRE AND GROUND WIRE DUE TO STROKE ON LINE WIRE AT MID-SPAN

wire potential at mid-span due to the surge on the conductor is as illustrated in Fig. 7. The first part of the voltage wave on the ground wire is generated by virtue of its coupling with the conductor. Traveling waves modify the latter part of the wave as shown by dotted lines. Fig. 8 shows the voltage between the ground wire and conductor at mid-span and is derived by subtracting E_G from E_L. It is quite obvious that high voltages exist at mid-span requiring a large separation between ground wire and line conductor to prevent flashover at that point.

The potential to which the tower will rise when flashover takes place is shown in Fig. 9. Incidentally this is the potential of the ground wire, tower and conductor at that point. The tower footing impedance was assumed to be 10 ohms. Fig. 10 shows the tower potentials but in this case the tower footing resistance was assumed to be 200 ohms. A comparison of Figs. 9

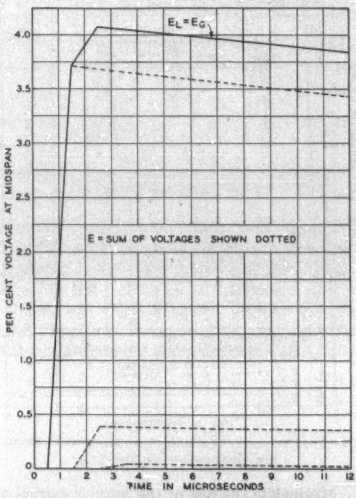

FIG. 9—POTENTIAL AT TOWER OF 10 OHMS FOOTING RESISTANCE DUE TO STROKE ON LINE WIRE AT MID-SPAN

and 10 illustrates the value of low tower footing resistance.

The case of a line properly protected with ground wires but with insufficient insulation is to be considered next. Inadequate insulation may be considered to exist at either of two points, at the tower or at mid-span. When the lightning terminates on the ground wires at mid-span the ground wires will be raised to a potential which may be computed by formula (1). This surge will travel to the adjacent towers where part of it will be absorbed by the tower, a part will continue on the ground wire, and a reflection will take place. With inadequate insulation at the tower, flashovers may take place at the adjacent towers on one or more conductors so that towers, ground wires, and affected conductors will assume the same potential. After

flashover occurs, reflections on the ground wires similar to those occurring in the case where lightning strikes conductors inadequately protected by ground wires, will take place from these towers. When the flashover takes place at mid-span from the ground wire to one of the conductors, the surge will travel over the ground wire and conductor to adjacent towers. A part of the surge on the ground wire will be absorbed by the tower, a part will continue on the ground wire and a reflection will take place at the tower as before. As in the case in which the conductor inadequately protected by ground wires is struck, the potential on the conductor will be of sufficient magnitude to cause flashover at adjacent towers. Because of the shorter arc path at the tower the 60-cycle arc is more apt to be maintained at the tower than in mid-span, flashover at the tower therefore being more easily identified than at mid-span.

When the stroke terminates on the ground wire in the middle of the span, the voltage on the ground wire is given in Fig. 11, while the conductor potential is shown

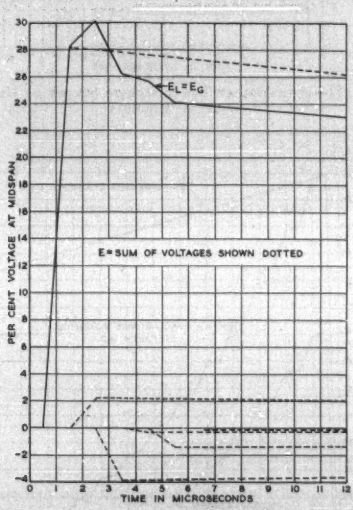

Fig. 10—Potential at Tower of 200 Ohms Footing Resistance Due to Stroke on Line Wire at Mid-Span

in Fig. 12. The difference in potential between ground wire and conductor is shown in Fig. 13. The tower potential will be of the nature shown in Fig. 14. The induced portion of the voltage on the conductor will vanish beyond the tower structures where the ground wire potentials become negligible. A flashover at the first tower impresses upon the conductor a traveling wave of the nature shown in Fig. 14. This wave will be reduced to only 63 per cent of the tower voltage beyond the afflicted section, and as such, subject to attenuation, will continue along the conductor until it finally enters a substation.

Should the spacing between the ground wire and conductor be insufficient, a flashover will take place at mid-span impressing upon the conductor the same potential as the ground wire. A voltage wave of the nature shown in Fig. 11 will be transmitted by the conductor. This potential will exceed the potential of the tower by an amount dependent on the magnitude of the reflection of the surge on the ground wire, will

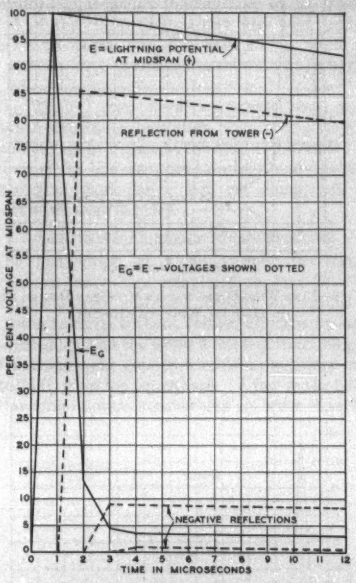

Fig. 11—Ground Wire Potential at Mid-Span Due to Stroke on Ground Wire at Mid-Span

flash over the insulator string, and will then be reduced to the potential of the tower. The resultant wave propagated beyond the region of disturbance will be very similar to that shown in Fig. 5.

The final consideration is that of a well-protected and well-insulated line. In this case the potential of the stroke to a tower, or to the ground wire in mid-span is quickly reduced by reflection from the adjacent towers and becomes of very small magnitude three spans distant from the point hit. The potential assumed by the tower when struck is shown in Fig. 15, while the potential of the adjacent towers is shown in Fig. 16. In these figures the voltage across the insulator string is shown also. Fig. 14 indicates that the potential of the towers with 10 ohms ground resistance is 4 per cent of the voltage at mid-span, the point struck.

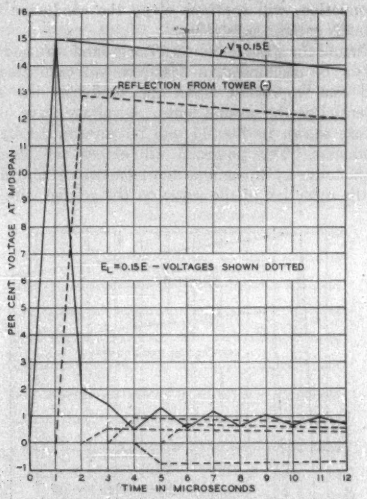

FIG. 12—LINE WIRE POTENTIAL AT MID-SPAN DUE TO STROKE ON GROUND WIRE AT MID-SPAN

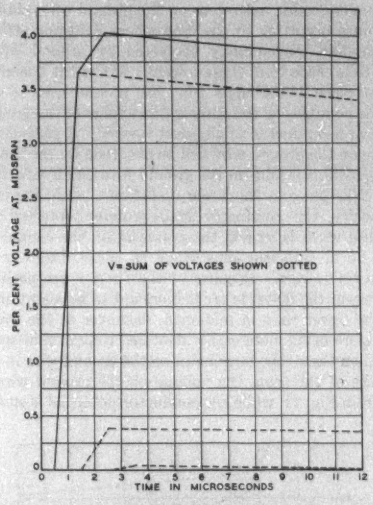

FIG. 14—POTENTIAL AT TOWER DUE TO STROKE ON GROUND WIRE AT MID-SPAN

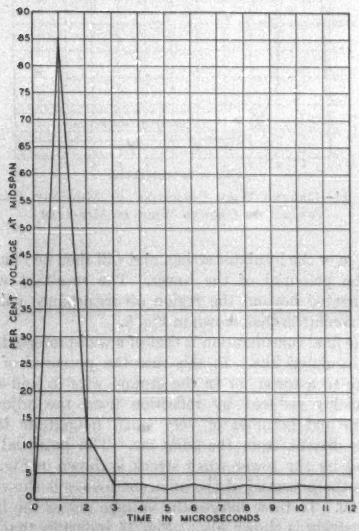

FIG. 13—VOLTAGE BETWEEN LINE WIRE AND GROUND WIRE DUE TO STROKE ON GROUND WIRE AT MID-SPAN

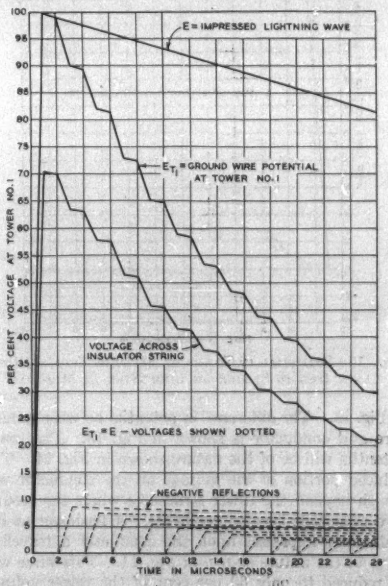

FIG. 15—POTENTIALS AT TOWER 1 DUE TO STROKE ON TOWER 1

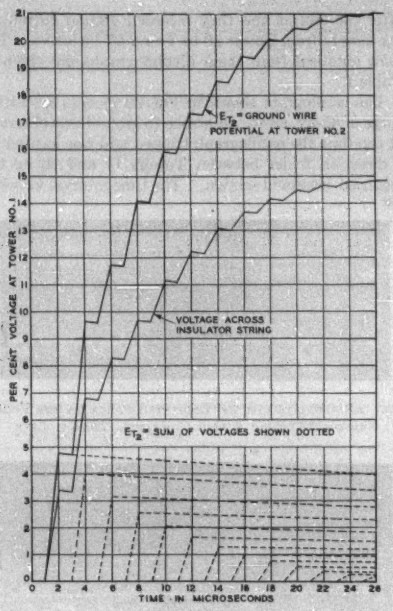

FIG. 16—POTENTIALS AT TOWER 2 DUE TO STROKE ON TOWER 1

SUMMARY OF 1930 LIGHTNING INVESTIGATIONS

The results herein reported were obtained on the Public Service Electric & Gas Company's 220-kv. lines during the lightning season of 1930.[1] During these investigations a portable cathode-ray oscillograph was located at the Roseland Switching Station, Roseland, New Jersey. The 220-kv. lines in use consisted of a six-mile section of the Roseland-Plymouth Meeting line and a six-mile section of the Siegfried-Roseland line. Surge tests with a portable million-volt lightning generator supplemented the lightning work. Phases 1 and 3 of the Siegfried-Roseland line were jumpered together at Tower 34. The method by which these line sections were connected is shown schematically in Fig. 17.

The 220-kv. lines were constructed with 16-unit standard suspension insulators, with arcing rings, and 18-unit strings in tension at the dead-end towers. Both sections of the line were dead during the investigation, construction work being not yet completed. Two ground wires were in place.

LIGHTNING CURRENT MEASUREMENTS

The installation of klydonographs for recording currents in towers was completed on June 26. These instruments were installed directly in parallel with the tower footings without voltage dividers, by locating the instruments on the tower a few feet from the ground and running light copper wire to an auxiliary ground 50 to 75 ft. from the tower base. A few weeks later the klydonographs were reconnected in order to utilize a

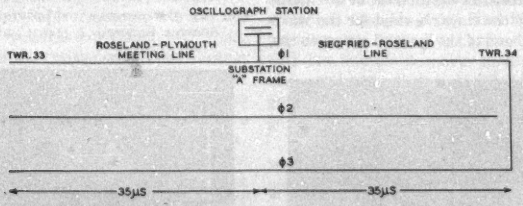

FIG. 17—SCHEMATIC DIAGRAM SHOWING METHOD USED FOR SURGE INVESTIGATIONS ON THE PUBLIC SERVICE ELECTRIC AND GAS COMPANY'S 220-KV. LINE

If the lightning voltage at mid-span is assumed to be 15,000 kv. the tower potential will be 600 kv. If the effect of the inductance of the tower is taken into consideration, this voltage may rise during the first microsecond to 1,200 kv. while the voltage across the insulator string would be 840 kv., a value insufficient to cause flashover on a 16-unit insulator string such as is used on 220-kv. transmission lines.

It has been pointed out in previous paragraphs that if no flashover takes place at mid-span or at the towers, the potentials induced by coupling on the conductors will become nil beyond the region of the disturbance. Thus no surge will be propagated over the line to terminal apparatus and the line will continue to operate unaffected by the lightning stroke.

resistance voltage divider. The voltage divider consisted of a high-resistance wire running from the tower to the auxiliary ground, the terminals of the klydonograph being connected across a small section of this resistance wire. This is shown schematically in Fig. 18. By considering the tower ground resistances, ratios were chosen to record 75,000 amperes ground current in the towers for positive surges. For negative figures this value was considerably greater.

Fig. 19 is an oscillogram taken during the course of a general storm over both sections of the line. The surge is positive and reaches a crest value of 30 kv. A second wave occurs at an interval of 55 microseconds. The second wave, which is a reflection of the first, shows that the surge probably originated on the Roseland-

Plymouth Meeting section about 7,000 ft. from the station. No klydonographs had been placed on this section of the line because the 220-kv. towers were overtopped throughout the entire distance by a parallel line of 132-kv. towers.

Fig. 20 is an oscillogram of a surge obtained during a storm on July 1 and shows a positive surge of 35-kv. crest value. A heavy stroke to ground was observed just east of Tower 1 on the Siegfried-Roseland section

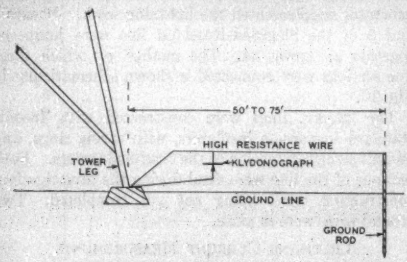

Fig. 18—Schematic Diagram of a Klydonograph Installation

of the line at the time the instrument tripped. Thunder and lightning seemed almost simultaneous, the interval being not greater than one second. Observers placed the stroke within 1,000 ft. from the station and not more than 300 ft. from the line. A second wave is noted on the oscillogram after an interval of 65 microseconds, approximately the time required for the wave to reflect from the open end of the line and return to the

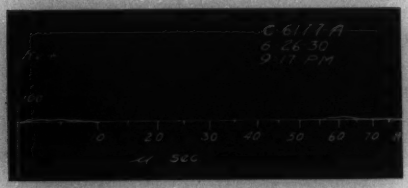

Fig. 19—Oscillogram of Lightning Surge on the Public Service Electric & Gas Company's 220-Kv. Line

station. Further confirmation of this theory is obtained by comparing the time of the second wave in this oscillogram with that of the oscillogram shown in Fig. 21 taken on the same time scale. Data from observations indicate that this surge was induced and originated close to the station.

Fig. 21 is an oscillogram taken during the same storm and shows a positive surge of 65-kv. crest. Analysis of the oscillogram indicates that the disturbance occurred on the Siegfried-Roseland section near Tower 23. The klydonograph films were collected shortly after the storms and they recorded ground current in the towers from Tower 20 to Tower 27 inclusive. The data obtained from these klydonographs are given in Table I.

The oscillogram shown in Fig. 22 records a positive surge of 85-kv. crest. A stroke to ground was observed at the time the oscillograph tripped and was located by a direction finder between Towers 11 and 20 on the Siegfried-Roseland section. The time interval between

Fig. 20—Oscillogram of Lightning Surge on the Public Service Electric & Gas Company's 220-Kv. Line

Fig. 21—Oscillogram of Lightning Surge on the Public Service Electric & Gas Company's 220-Kv. Line

Fig. 22—Oscillogram of Lightning Surge on the Public Service Electric & Gas Company's 220-Kv. Line

the lightning and thunder was about 15 seconds. In the oscillogram a second wave follows the first after an interval of 38 microseconds. By the aid of information given by the second and third reflections the disturbance was determined to have taken place about 14 microseconds from Roseland. Klydonograph films indicated that currents were present in Towers 15 and 16. Data from these films are given in Table II.

Another group of current records was obtained on Towers 4 to 9 inclusive. The surge resulting on the conductors was evidently so low in this instance that

TABLE I

Tower No.	Polarity	Radius of fig. (mm)	Instrument kv.	Instrument ratio	Meggered tower ground resistance	Tower amperes approximate
20	Pos.	10.0	7.2	1/1	1.2	6,000
21	Instrument broken			1/1	1.8	
22	Pos.	9.5	6.8	1/1	2.1	3,200
23	Pos.	3.5	3.4	1/1	2.4	1,400
24	Pos.	11.5	8.5	1/1	2.1	5,500
25	No. fig				2.7	
26	No. fig				5.0	
27	Pos.	13.0	9.7	1/1	20.0	500

TABLE II

Tower No.	Polarity	Radius of fig. (mm.)	Instrument kv.	Instrument ratio	Meggered tower ground resistance	Tower amperes approximate
14	No record		3 (max.)	1/7	5.1	4,000 (max.)
15	Pos.	8.0	5.8	1/7	4.2	9,700
16	Pos.	9.8	7.1	1/5	2.4	14,800
17	No record		3 (max.)	1/6	3.5	5,100 (max.)

TABLE III

Tower No.	Polarity	Radius of fig. (mm.)	Instrument kv.	Instrument ratio	Meggered tower ground resistance	Tower amperes approximate
4	Pos.	3.5	3.3	1/1	2.2	1,500
5	Pos.	12.0	8.8	1/1	2.5	3,500
6	Pos.	12.7	9.5	1/1	6.8	1,400
7	Pos.	Possible flashover	20.0(?)	1/1	7.5	2,700(?)
8	Pos.	13.0	9.7	1/1	13.1	750
9	Pos.	5.0	4.0	1/1	3.2	1,200

it did not cause operation of the cathode-ray oscillograph. The tabulation of these current records is given in Table III.

On the basis of the induced surge theory, the currents indicated by the klydonograph measurements are not in agreement with the voltages correlated with these values and measured on the transmission line. Potentials of two or three million volts would have to appear on both the conductor and the ground wire if these currents were set up by induced voltages. The recorded voltages on the conductor were extremely low, showing quite conclusively that lightning struck the ground wire. The current disturbances on the tower and ground wire system were limited to a distance of approximately six spans due to low tower footing resistances. The potentials induced on the conductor arising from surges on the ground wires become nil beyond the region of the disturbance. Thus no surge on the conductor can be propagated beyond this region unless flashover takes place.

The voltages recorded on the oscillograph for these cases are probably due to the electrostatic disturbances resulting from the ionization of air during the formation of the lightning stroke. This ionization of the air draws energy from the transmission line, creating a positive impulse of short duration and fairly low magnitude.

To summarize, these data show that the voltages on the conductors with a stroke very close to the line or terminating on the tower or ground wire will be extremely low unless flashover takes place.

Effect of Line Construction on Potentials Impressed on Substations

The potentials appearing in a substation will be discussed with relation to the various forms of line construction: first, wood pole lines highly insulated against impulses but having no ground wires; second, lines with low impulse flashover such as steel towers having no ground wires; third, steel structures with inadequate ground wire protection; and fourth, a tower line having good ground wire protection and adequate insulation.

With wood pole structures using high insulation, and without ground wires, a stroke terminating on the conductor will impress upon it a surge of exceedingly high magnitude. This surge may travel for many miles and still be of a dangerously high potential. Such a surge entering a substation will be impressed across the substation insulation. Because of the lower insulation against impulses generally used in substations, flashover will occur within the substation causing a trip-out. To prevent such trip-outs or flashovers, exceptionally good lightning protection is required.

In the case of lines having low impulse insulation the conductor struck will flash over at the adjacent towers and the surge that is propagated over the conductor will be of a chopped nature. The tail of this wave, as previously described, will have a magnitude depending upon the tower footing resistance at the point where flashover took place. However, the wave, which will be triangular in form, will be of short duration and will attenuate very rapidly. With surges of this nature

entering into the substation, the stresses on the substation insulator will be fairly moderate and less lightning protection will be required to safeguard the substation equipment.

This analysis can be applied to the case of tower lines where inadequate ground wire protection is used, as the nature of the surge propagated over the conductor is very similar to the preceding case with the exception that in this case the tail of the wave will be of much shorter duration.

The final case to be considered is that in which adequate insulation and good ground wire protection are used on the steel tower line. It has been shown that under these conditions only surges of very low magnitude will be propagated over the transmission line and thus substations will not be subjected to dangerous surges. In the event of a stroke very close to the substation, thus involving the substation in the disturbed region, the voltages on the conductor may be moderately high. However, the station lightning arresters will limit the voltages appearing on the conductors to a value well below the substation insulation. Under this condition, the current carried by the lightning arrester is fairly low being only that required to charge up the conductor to about the potential of the substation ground.

Summary

1. The charge in a thundercloud, being held by particles of moisture in suspension which are insulated from each other by the intervening air, can form a lightning stroke only if the air between these particles becomes conducting.

2. Energy is transferred from one section of a cloud to another through streamers. Laboratory experiments show their rate of development to be on the order of one-twentieth the velocity of light.

3. With the finite rates at which energy can be collected, a cloud volume one mile in diameter will require approximately 100 microseconds to discharge.

4. Flashover at mid-span will be followed by a flashover at the adjacent tower. Because of the shorter arc path at the tower, the 60-cycle arc is more apt to be maintained at the tower than in mid-span.

5. With the high rates of rise of potential measured on wires struck by lightning, large spacing between ground wires and conductors is required at mid-span to prevent flashover at that point.

6. The voltage to which a tower will rise when either it or the ground wire is struck is a function of the tower footing resistance.

7. When the tower or ground wire is struck, the voltage difference before flashover between the tower or ground wire and the conductor is equal to the potential of the tower or ground wire less the potential assumed by the conductor through mutual coupling with the ground wire.

8. When the ground wire or tower is struck and flashover takes place, a surge having the same potential as the tower is propagated over the conductor.

9. If a tower or ground wire is struck and no flashover results, severe electrical disturbance is limited to a few structures, and any surge propagated over the conductor will be of extremely low magnitude.

10. During one year's investigation on lines equipped with ground wires, no flashovers have occurred where tower footing resistances are below five ohms.

11. The ground wire protection adjacent to the station should be strengthened as much as possible to eliminate flashover and direct strokes to the conductor, thus preventing strokes close to the station from impressing dangerous surges upon the station equipment.

12. On well-protected lines, reduced insulation for structures adjacent to substations is not advisable as for strokes terminating at these points, the hazard of flashover is increased with the consequent greater possibility of transmitting a surge into the substation.

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2. "Lightning Investigations," by C. L. Fortescue, *Electric Journal*, February, March, April, May, August, and November, 1930.

3. *Rationalization of Station Insulating Structures With Respect to Insulation of Transmission Lines*, by C. L. Fortescue, Trans. A. I. E. E., October 1930, p. 1450.

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7. *Impulse Insulation Characteristics of Wood Pole Lines*, by H. L. Melvin, Trans. A. I. E. E., January 1930.

8. *An Experimental Lightning Protector for Insulators*, by J. J. Torok, A. I. E. E. Winter Convention, January 1931.

Discussion

For discussion of this paper see page 1146.

Lightning Investigation
on the 220-Kv. System of the Pennsylvania Power & Light Company (1930)

BY EDGAR BELL* and A. L. PRICE†
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—This paper describes the 1930 results (fifth consecutive year) of a lightning investigation conducted on 220-kv. lines located in eastern Pennsylvania and where lightning storms are prevalent. During 1930 the investigation was expanded; two new devices, the surge (insulator assembly flashover) indicator and the lightning (storm) severity meter being successfully applied.

Unique and comprehensive data were secured on the magnitudes and wave shapes of natural lightning surges both at and remote from the point of origin on the transmission line; numbers of phases faulted during trip-outs; numbers, magnitudes and effects of direct lightning strokes; some data on the relative importance of induced strokes; results of measurements of atmospheric voltage gradients; and the indicated effect of overhead ground wires and tower footing resistances.

General

THE extensive investigation of lightning on the Pennsylvania Power and Light Company's 220-kv. system during 1930 is a continuation of research begun in 1926. Data obtained prior to 1930 have been presented in papers[1,2] before the A. I. E. E. in 1928 and 1930.

The field study included measurements of magnitude and wave shape of lightning surge voltages on 220-kv. transmission lines, location and current magnitudes of direct lightning strokes, atmospheric electric field intensities, induced voltage gradients, lightning storm severity, insulator flashovers (including flashovers which left no visible burns or marks on insulator assemblies), and careful and systematic collection of weather and operating data.

The results of the 1930 investigation have furnished a key to the mechanism of lightning influence on high-voltage (220-kv.) transmission lines, and the problems of evaluating the effects of overhead ground wires, tower footing resistance, and direct and induced strokes are in process of solution.

Line Data

The 220-kv. system of the Pennsylvania Power & Light Company consists of the Wallenpaupack Tap, the Siegfried-Delaware River section of the Siegfried-Roseland line, and the greater portion of the Plymouth-Siegfried line. The sections of line between Wallenpaupack and Siegfried (formerly known as the Wallenpaupack-Siegfried line) are now called the Wallenpaupack Tap-Siegfried Roseland line. Practically all measurements were made on this line, and unless otherwise specified, references in this paper will be to this line.

General characteristics of the above mentioned lines including a description of the so-called "counterpoise" installed along a 2½-mile section of line on which flashed insulators had been very numerous have been described[2] in a previous paper. These lines, together with present and future interconnections, are shown in Fig. 1.

During 1930 two radical changes were made to sections of the Wallenpaupack Tap. The tower footing resistances of all towers equipped with overhead ground wires (exclusive of the "counterpoise" section) were reduced by connection of tower footing grounding cables to the tower footings. Each cable consisted of a 50-ft. length of 00 stranded copper cable trenched in the ground to a depth of about a foot, and connected to one corner of the tower. Four cables extending radially

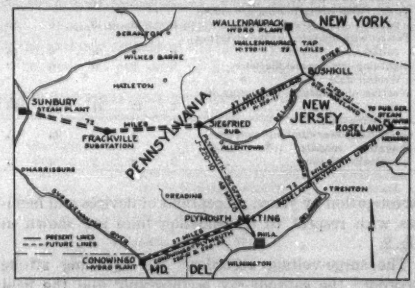

FIG. 1—PENNSYLVANIA-NEW JERSEY 220-KV. INTERCONNECTION

outward were used at each of the 80 towers. The immediate effect of these tower footing grounding cables was to reduce tower footing resistances to about one-half their former values.

A 3½-mile section of this line outside the ground wire section, and which was particularly subject to insulator flashovers, was equipped with lightning stroke diverting cables,[3] or overhead ground wires of unusual design.

Facilties and Measuring Devices

The investigation was conducted and facilities were made available cooperatively by the Pennsylvania Power and Light Company and the General Electric

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†Penn. Power & Light Co., Hazleton, Pa., formerly of General Elec. Co., Schenectady, N. Y.

1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

Company, the Electric Bond and Share Company acting as consultant for the former. The Public Service Electric and Gas Company of New Jersey and the Philadelphia Electric Company also cooperated by supplying lightning storm and weather data from their respective operating territories, the latter company also furnishing operating data from its 220-kv. lines which are interconnected with those of the Pennsylvania Power and Light Company. All five companies participated in analysis of data.

Measuring devices and facilities used are listed in Table I which also indicates trends in scope of the investigation by years. Locations of devices and facilities with respect to transmission lines are shown in Fig. 2.

TABLE I—MEASURING DEVICES AND FACILITIES

	Number of instruments or facilities				
	1926	1927	1928	1929	1930
A—Instruments used on 220-kv. lines					
Surge-voltage recorders	9	26	46	28	21
Lightning-stroke recorders	0	0	0	284	314
Surge (flashover) indicators	0	0	0	0	1,041
Cathode-ray oscillographs	0	0	1	1	2
Magnetic oscillograph elements or high-speed graphic ammeters	2	2	2	5	12 to 22
B—Other instruments (not coupled to 220-kv. lines)					
Surge-voltage recorders	0	0	6	12	13
Cathode-ray oscillographs	0	0	0	1	0
Field-intensity recorders	0	0	0	1	3
Rate of change of field recorder	0	0	0	1	0
Storm severity meters	0	0	0	0	4
C—Special installations					
Antennas	0	0	5	10	10
Towers with counterpoise	0	0	0	14	14
Towers with tower footing grounding cables	0	0	0	0	80
Towers with lightning stroke diverting cables	0	0	0	0	18
Lightning arrester installations	0	0	0	0	2
D—Weather data					
Weather observations (operating stations)	7	26	27	142	138
E—Operating data					
Overhead (tower climbing) patrols	1	1	7	23	5
Operating records, from 220-kv. operating stations	2	2	2	3	3

The surge-voltage recorder,[4] the lightning stroke recorder,[5] the cathode-ray oscillograph[6] and the field intensity recorder[5] have been described in other papers before the Institute.

The surge (insulator assembly flashover) indicator[7] is a small device applied to each insulator assembly of a tower, and designed to cause a target, visible to a ground patrolman, to show upon occurrence of flashover of that assembly. This instrument was developed during the winter of 1929-30 by the General Engineering Laboratory of the General Electric Company. As used during 1930 towers having one (or more) targets showing were climbed, all insulator assemblies carefully inspected for marks of flashover and a new indicator link installed. Broken pieces of the old link were saved, and inspected by experienced engineers. At the end of the season a tower climbing inspection was made of the entire Wallenpaupack Tap-Siegfried Roseland line for the purpose of checking the performance of the surge indicators and to secure the best possible insulator assembly flashover data. Fig. 3 shows surge indicator installations on a typical steel tower.

The lightning (storm) severity meter[7] is a small device of simple construction, consisting of a roll film box camera containing a special glow tube. This tube is connected between a short vertical antenna (30 ft. long) and ground. Collapse of electric charges collecting on the antenna during lightning storms causes the tube to glow, and record a spot on the film. The intensity or degree of darkness of this spot is an integrated function of the number of charges on the antenna and their magnitudes. As used during 1930, films were advanced by hand once a week to a new position, and all records secured were on a weekly basis. Installations were made in open fields as free as possible from trees, buildings, and electric power lines.

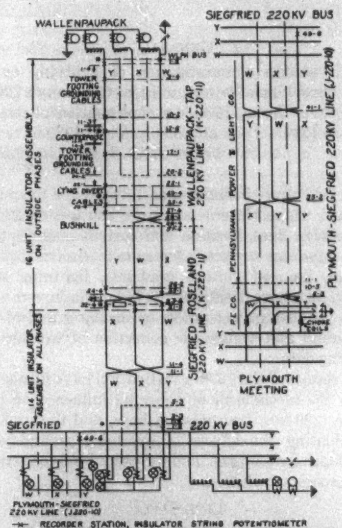


FIG. 2—DIAGRAM OF WALLENPAUPACK TAP-SIEGFRIED ROSELAND AND PLYMOUTH-SIEGFRIED LINES

Showing locations of recording devices and facilities

The lightning severity meter was developed by the General Engineering Laboratory of the General Electric Company.

During 1930 the Wallenpaupack Lightning Laboratory was abandoned, and a new one established near

tower 23-4 of the Siegfried Roseland line. Fig. 4 is an illustration of the new laboratory and its site. At this location, known as Cherry Valley, the line is not equipped with overhead ground wires, and flashed insulators had been particularly numerous. During the

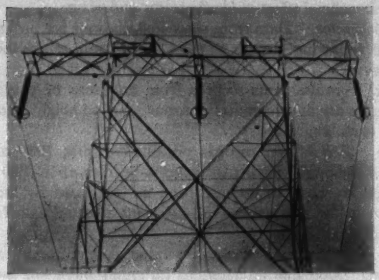


FIG. 3—SURGE-INDICATOR INSTALLATION ON TYPICAL 220-KV. STEEL TOWER

1930 season a direct stroke of lightning contacted the line conductor at not more than 500 ft. from where the cathode-ray oscillographs were coupled. The oscillogram of this voltage surge is described later.

Two cathode-ray oscillographs were employed, both being coupled to the same line conductor. One instrument, called the "fast sweep" had a time axis of 50 microseconds duration; the other or "slow sweep" oscillograph had a time axis of 2,000 microseconds duration. The latter instrument was particularly useful in determining the approximate locations of the line faults associated with oscillograph records. This result was possible because of voltage or current reflections from faults, line terminals, and line junctions.

A transmission line voltage indicator located at the laboratory and controlled by the electrostatic field from the line conductors was utilized for exact and positive time correlation of oscillograms with line trip-outs.

Auxiliary oscillograph circuits[2] were substantially as used in 1929. Sensitivity of the trip circuit of the fast

FIG. 4—CHERRY VALLEY LIGHTNING LABORATORY

Tower *SR* 23-4 immediately behind laboratory

sweep oscillograph was increased by changes in circuit constants and by exposure of trip gaps to ultra-violet light. The slow-sweep oscillograph circuit was initiated by the cathode voltage of the fast-sweep oscillograph.

For over 90 per cent of line faults associated with

TABLE II—TRIP-OUT DATA—1930—WALLENPAUPACK TAP-SIEGFRIED ROSELAND LINE

Trip-out number	Phases faulted	Insulator assembly flashovers: No. of towers involved	Insulator assembly flashovers: No. of assemblies involved	Measured direct lightning strokes	Overhead ground wires installed	Location of fault (tower and phases where flashover occurred)	Cathode-ray oscillogram obtained
1	X, W	2	4	Instruments	No	SR16-2X, W; 16-3X, W	
2	W			not	?	?	
3	Y, X, W	2	4	installed	No	SR19-4X; 20-1Y, X, W	
4	W				?	?	
5	W			"	?	?	
6	W			"	?	?	
7	Y, W*			"	?	?	Yes
8	X	2	2	"	No	SR11-4X; 11-5X	Yes
9	Y			?	?	?	
10	Y			?	?	?	Yes
11	X, W	1	2	Yes	No	WT 22-4X, W	Yes
12	X	2	2	Yes	No	SR34-1X; 34-2X	Yes
(13)	W	1	1		Yes	Plymouth-Siegfried Line	No
14	Y	1	1	No	Yes	WT15-5Y	
15	W			?	?	?	Yes
16	W	2	2	Yes	No	SR11-1W; 11-2W	Yes
17	Y	2	2	Yes	No	SR31-2Y; 31-3Y	
18	W*			?	?	?	Yes
19	Y	2	2	Yes	No	SR8-4Y; 9-1Y	Yes
(20)	W	1	1		Yes	Plymouth-Siegfried Line	Yes
21	W	4	4	Yes	No	SR16-3W; 16-4W; 16-5W; 17-1W	
22†	X	2	2	No	No	SR23-3X; 23-4X	Yes
23	Y	2	2	Yes	No	SR24-5Y; 25-1Y	Yes
24	X	2	2	No	No	SR14-5X; 15-1X	Yes
25	Y	2	2	Yes	No	SR9-5Y; 10-1Y	Yes
26	W	3	3	Yes	No	SR13-5W; 13-6W; 14-1W	Yes

*No. of phases faulted not known with certainty. Possibly one other phase involved.
†Trip-out due to stroke near lightning laboratory. See oscillogram 902379.
Total number of lightning trip-outs 24 (excluding numbers 13 and 20).

trip-outs the number of phases involved was successfully determined by means of high-speed graphic recording ammeters at Wallenpaupack and magnetic oscillographs at Siegfried.

Results

Line Trip-outs. Of the 24 lightning trip-outs during 1930, the location of the fault, the number of phases and insulator assemblies involved, and whether or not a direct stroke exceeding approximately 50,000 amperes occurred, were successfully determined in fifteen cases. In the remaining nine cases, complete data are either lacking or correlation is uncertain. Table II presents summarized data for each trip-out.

The Plymouth-Siegfried line tripped twice, each case being due to a single-phase fault.

Insulator Assembly Flashovers. Table III summarizes 1930 insulator assembly flashover data with respect to overhead ground wires. Of the 65 line insulator assemblies flashed on this line during 1930, 71 per cent showed glaze burns characteristic of dynamic arcs. Of 38 surge (flashover) indicator operations 29 (76 per cent) were associated with marked insulator assemblies. Possibly 24 per cent of the flashovers during 1930, therefore, left no discernible marks on the insulators. As in previous years, most flashovers occurred on phase conductors occupying the outer positions in the towers. The numbers are 36, 6, and 23 respectively for the west, middle, and east conductors.

TABLE III—SUMMARIZED INSULATOR FLASHOVER DATA 1930

Line Name	Under ground wires		No ground wires		Total for line	
	No.	No./mi.	No.	No./mi.	No.	No./mi.
Wallenpaupack Tap	5*	0.26	8	0.94	13†	0.46
Siegfried-Roseland	3	0.67	49	1.51	52	1.41
Plymouth-Siegfried	2	0.04			2	0.04

*Two of these flashovers occurred before installation of tower footing grounding cables.

†Excludes four flashovers at Wallenpaupack protective gaps.

Table IV shows that the great majority of insulator assembly flashovers involve only one phase (32 groups out of 36) but that in 15 of these 32 cases more than one assembly was involved (single-phase flashover on two or more adjacent towers).

TABLE IV—GROUPING OF FLASHED INSULATOR ASSEMBLIES

Number of phases involved in group	Number of assemblies composing group (1)	(2)	(3)	(4)	Total
1	17	12	2	1	32
2		1	0	2	3
3			0	1	1
Total	17	13	2	4	36

Fig. 5 shows the flashed insulator assemblies by line miles for 1930 and for the five-year period 1926 to 1930 inclusive.

Lightning Strokes. All towers of the Wallenpaupack Tap-Siegfried Roseland line were equipped with lightning stroke recorders. These devices were serviced after each trip-out or group of trip-outs. Table V lists detailed data.

Table VI summarizes important results, and indicates the beneficial effect of overhead ground wires.

Antennas. Data from these installations are meager. The following results appear to be well established, however.

a. Simultaneous records from antennas of varying heights are approximately proportional to height above the earth's surface.

b. The fifteen records obtained during 1929 and 1930 although varying in magnitude from about 300 to 2,700 kv. have in no case been accompanied by more than a low voltage on the adjacent transmission line conductors.

c. Voltages of flashover value occurring on the transmission line adjacent to antennas have not been accompanied by records on the latter.

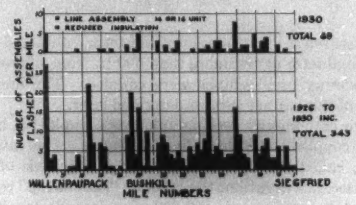


Fig. 5—Insulator Assembly Flashovers

Wallenpaupack Tap-Siegfried Roseland Line

d. A 60-ft. high-grounded antenna erected over a 50-ft. high antenna indicates a protective ratio of about 0.5 to 0.6.

Cathode-Ray Oscillograms

During 1930 oscillograms of 22 lightning surges which exceeded 100 kv. at the point of measurement were obtained. These have been classified into three groups, as follows:

Group I. Surges accompanied by line trip-out caused by a fault involving the conductor phase to which the oscillographs were coupled (phase X).

Group II. Surges accompanied by line trip-out caused by a fault not involving the conductor phase to which the oscillographs were coupled.

Group III. Surges unaccompanied by line trip-out and line fault (nature and location of origin unknown).

Table VII presents pertinent data of these 22 oscillograms.

Fig. 6 shows a typical pair of oscillograms of Group I. Both oscillograms are records of the same surge. The latter record (902388) began at about − 400 kv. and is about one microsecond later in starting than the corresponding fast-sweep oscillogram or 902387.

These records were obtained coincident with trip-out

TABLE V—LIGHTNING STROKES RECORDED ON WALLENPAUPACK TAP-SIEGFRIED ROSELAND LINE

Stroke No.	Structure at which record was obtained	Magnitude of structure current amperes¶	Polarity of structure current	Overhead ground wires	Structure footing resistance ohms	Insulator flashovers	Conductor position	No. of phases affected	Correlating trip-outs
(A) *Lightning strokes which caused trip-outs* (This list includes one doubtful case, No. 2)									
1	WT22-4	80,000	Osc Neg	No	91	WT22-4W WT22-4X	E M	2	11
2	SR17-4	110,000	Neg	No	23	None discovered	?	?	Probably none
3	SR34-1 SR34-2	90,000 110,000	Neg Neg	No No	33 37	SR34-1X SR34-2X	E E	1	12
4	SR36-2	110,000	Neg	No	10 13	SR36-2Y SR36-1Y	W W	1	10
5	SR8-1 SR8-2	100,000 90,000 100,000 60,000	Neg Neg	No No No	34 23 28	SR8-1W SR8-2W SR8-3W	W W W	1	15 or 18
6	SR11-1 SR11-2	90,000 100,000	Neg Neg	No No	27 17	SR11-1W SR11-2W	W M	1	16
7	SR16-5 SR17-1	80,000 Less than 60,000	Neg Neg	No No No No	85 43 30 22	SR16-5W SR17-1W SR16-3W SR16-4W	E E E E	1	21
8	SR31-3	110,000	Neg	No No	34 12	SR31-3Y SR31-2Y	W W	1	17
*	SR23-3 SR23-4	None recorded		No No	51 23	SR23-3X SR23-4X	W W	1	22
9 †	SR24-5 SR25-1	100,000 100,000	Neg Neg	No No	32 16	SR24-5Y SR25-1Y	W W	1	23
10	SR9-5 SR10-1	150,000 120,000	Neg Neg	No No	34 15	SR9-5Y SR10-1Y	E E	1	25
11	SR13-6 SR14-1	110,000 100,000	Neg Neg	No No No	100 21 42	SR13-6W SR14-1W SR13-5W	E E E	1	26

Stroke No.	Structure at which record was obtained	Magnitude of structure current amperes¶	Polarity of structure current	Overhead ground wires	Structure footing resistance ohms
(B) *Lightning strokes which did not cause trip-outs or flashovers*					
12	SR4-4	90,000	Neg	Yes	20
13	WT3-2	60,000	Neg	Yes	27
14	WT6-4 WT6-5	130,000 120,000 60,000	Neg Neg	Yes Yes	23 47
15	WT16-3	260,000 220,000	Neg	Yes	79
16	WT17-4 WT17-5	70,000 160,000	Neg Neg	Yes Yes	52 18
17 ‡	WT22-1 (Guy) WT22-2 (Guy)	Less than 60,000 by L. S. R. Approx. 120,000 by S. V. R. in shunt Less than 40,000 by L. S. R.	Neg Neg	Diverting Cables Diverting Cables	32 Approx. 30
18	WT23-1 (Tower) WT23-1 (Guy) WT22-4 (Guy)	60,000 112,000 Less than 40,000	Neg Osc Neg Osc Neg	Diverting Cables Diverting Cables Diverting Cables	Approx. 30 Approx. 30

*Surge-voltage recorders coupled to all three-phase conductors at tower SR23-4 indicated that the maximum possible voltages on phases Y and W (22½ and 45 ft. from phase X which flashed over) were −1,620 kv. and − 710 kv. respectively. On phase X conductor a voltage in excess of − 2,400 kv. was recorded. Also see oscillogram 902379.

†A surge-voltage recorder coupled to phase X conductor at tower SR25-1 (45 ft. from phase Y which flashed over) indicated that the maximum possible voltages on phase X were −1,190 kv. and + 1,280 kv.

‡A surge-voltage recorder coupled to Y phase conductor at tower WT22-1 indicated + 2,400 kv. coincident with this stroke. No insulator flashover or trip-out occurred.

¶According to present calibrations, subject to revision.

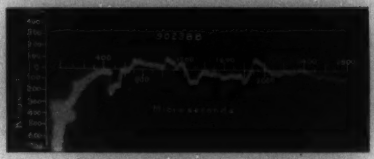
TABLE VI—SUMMARIZED LIGHTNING STROKE RECORDER DATA WALLENPAUPACK TAP-SIEGFRIED ROSELAND LINE

No. of towers in line	314
No. of towers equipped with recorders	314
No. of current records obtained	35
Probable No. of strokes causing these records	18
Maximum current recorded, amperes	260,000
Minimum current recorded, amperes	40,000
No. of negative strokes	16
No. of oscillatory strokes (negative predominating)	2
No. of recorded strokes which definitely caused trip-outs (no ground wires in any case)	10
No. of recorded strokes which did not cause trip-outs (ground wires in every case)	7
No. of recorded strokes which may have caused trip-outs	1

24. A single-phase fault developed on phase X at towers SR 14-5 and 15-1, about 8.7 miles south of the lightning laboratory. Flashed insulators were found on phase X at these towers, and the reflections recorded on oscillogram 902388 show that the location of the fault was approximately 9 miles south of the laboratory.

Similarly Fig. 7 shows a typical pair of oscillograms of Group II., 902365 and 902366 are the short and long duration oscillograms, respectively. These records were obtained coincident with trip-out 16. A single-phase fault developed on phase W at towers SR 11-1 and 11-2, about 12.5 miles south of the lightning laboratory. Flashed insulators were found on phase W at these towers, and the reflections recorded on oscillogram 902366 show that the location of the fault was about 13 miles south of the lightning laboratory. In this case,

A



B

FIG. 6—OSCILLOGRAMS OF LIGHTNING SURGE—TYPICAL GROUP I SURGE

Cathode-ray oscillograms 902387 and 902388, obtained July 24, 1930 at 1:50.5 p. m. coincident with trip-out No. 24. Ammeters at Wallenpaupack and magnetic oscillographs at Siegfried indicated a fault on phase X. Surge indicator targets showed on phase X at towers 14-5 and 15-1 (Siegfried-Roseland line) and the insulators were found flashed. Reflections on the slow-sweep oscillogram indicate origin of the surge as about 9 miles south of the laboratory or at about this location

TABLE VII—CATHODE-RAY OSCILLOGRAMS OBTAINED AT CHERRY VALLEY, PA. LIGHTNING LABORATORY

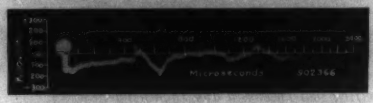
Oscillogram numbers		Corresponding trip-out number	Phases faulted	Maximum recorded kv.		Nature of wave	Polarity characteristics		Approx. distance from oscillograph in miles‡	Time in microseconds to reach				
													Zero of first loop	
Fast sweep 50 μ sec.	Slow sweep 2,000 μ sec.			Fast sweep	Slow sweep		First loop	Second loop		75% of max. (front)	Max. voltage	50% of max. (tail)	Fast sweep	Slow sweep
Group I—Fault on phase X (to which oscillographs were coupled)														
902315	902316	8	X	−500	−480	Uni	Neg.		12	S	0.8	2.0	6.5	42. +	140
902329	902330	11	X, W	+100	−525	Osc	Pos.	Neg.	20	N	0.1	0.1	2.0	2.5	
902338	902339	12	X	−380	−375	Uni	Neg.		10	N	1.5	15.	50.	50. +	250
902379	No S. S.	22	X	−2760		Uni	Neg.		0.1 or less	?	?	?	5.9	
902387	902388	24	X	−750	−725	Uni	Neg.		9	S	8.0	11.0	31.	53. +	460 +
Group II—Fault on phase Y or W or both, but not on phase X														
902306A	902308A	7	Y, W*	+380	−320	Osc	Pos.	Neg.	18	N	1.5	2.5	11.0	13.5	
902325	902326	10	Y	+140	+162	Osc	Pos.	Neg.	11	N	0.1	0.1	3.0	6.0	
902344	No S. S.	See Note†	W	+150		Uni	Pos.		100	S	1.0	5.0	30.	47. +	
902363	902364	15	W	+200	+155	Uni	Pos.		14	S	4.5	6.0	9.0	13.0	
902365	902366	16	W	+100	−175	Osc	Pos.	Neg.	13	S	5.0	6.0	6.5	8.0	
No F. S.	902367	18	W*		−175	Osc	Pos.	Neg.	15	S					35
No F. S.	902368	19	Y		−150	Osc	Pos.	Neg.	15	S					25
No F. S.	902370	20†	W		+285	Osc	Pos.	Neg.	60	S					40
902383	902384	23	Y	−250	−300	Uni	Neg.		1	N	0.3	0.3	1.0	2.0	
902409	902410	25	Y	+150	−200	Osc	Pos.	Neg.	13	S	2.0	2.5	4.2	10.0	
902411	902412	26	W	+310	+300	Osc	Pos.	Neg.	10	S	1.0	2.0	7.0	9.0	
Group III—No corresponding trip-out (location of fault, if any, unknown)														
902307B	902308B	None	Unknown	?	−230	Osc	Neg.	Pos.	3.5	N?	?	?	?	5.0	
902307C	902308C	"	"	+120	−430	Osc	Pos.	Neg.	42.5	N?	2.5	3.5	3.8	4.0	
902313	902314	"	"	+180	−450	Osc	Pos.	Neg.	?	S	0.1	0.1	9.0	12.	
902335	902336	"	"	+230	+240	Uni	Pos.		Near lab.	?	1.5	2.0	6.0	50. +	400
902337	No S. S.	"	"	+350		Uni	Pos.				2.0	3.5	25.	50. +	
No F. S.	902369	"	"		−150	Osc	Neg.	Pos.				40.			170

*No. of phases faulted not known with certainty. Possibly one other phase involved.
†Trip-out of interconnected 220-kv. line. No fault on Wlpk. Tap.-Sieg. Rose. Line.
‡Judging from reflections on slow-sweep oscillograms. N (North) and S (South).

lightning-stroke recorder records were obtained at both towers, indicating that a direct negative stroke occurred to phase W conductor between towers 11-1 and 11-2, and that the magnitude of current was sufficient to produce records at both towers.

Fig. 8 showing oscillogram 902379 is a Group I record, unusual in that the direct stroke causing the line fault and trip-out occurred not more than 500 ft. away from the lightning laboratory. No lightning-stroke recorder records were obtained, (minimum sensitivity being about 60,000 amperes). The fault was confined to phase X on towers SR 23-3 and 23-4, and caused trip-out 22. The record shows that the rise of voltage was rather gradual for the first 3.5 microseconds, but quite rapid during the next 2 microseconds or until insulator flashover occurred on the crest of the wave at over − 2,760 kv. After flashover the voltage collapsed

A



B

Fig. 7—Oscillograms of Lightning Surge—Typical Group II Surge

Cathode-ray oscillograms 902365 and 902366 obtained July 9 at 5:27.3 p. m. coincident with trip-out No. 16. Short-circuit 60-cycle currents indicated a fault on phase W. Surge-indicator targets showed on phase W at towers 11-1 and 11-2 (Siegfried-Roseland line). Insulators were found flashed and lightning stroke recorders indicated at these structures, currents of 90,000 and 100,000 amperes, respectively. Reflections on the slow-sweep oscillogram indicate origin of the surge as about 13 miles south of the laboratory or at about this location

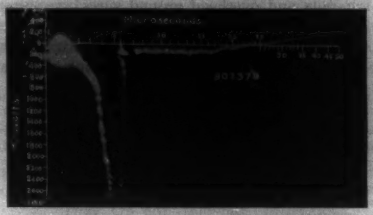
very quickly in about 0.2 microsecond to zero, later increasing to about − 150 kv. and gradually diminishing to zero again at 23 microseconds.

The wave front of this surge resembles that of the voltage across a capacitor when being charged through an inductance from a constant voltage source.

Based on wave-shape characteristics before reflections occur, the records of Group I appear, in general, to be unidirectional and negative. Those of Group II appear to be oscillatory, first loop positive, second loop (longer time duration) negative. Composite wave shapes based on these records are shown in Fig. 9.

Most of the records in these two groups have been associated with specific groups of insulator flashovers, and a number was also accompanied by measured direct strokes.

As shown in Table VII the times to reach crest voltage ranged from less than 0.1 to 15 microseconds: Composite wave shapes representative of Groups I and II are summarized in Table VIII.



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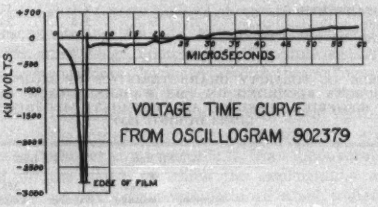

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Fig. 8—Oscillogram of Lightning Surge Voltage Measured at Point of Origin

Cathode-ray oscillogram 902379 obtained at the Cherry Valley lightning laboratory—July 24, 1930 at 1:41 p. m. and coincident with trip-out No. 22. Flashover occurred across phase X insulator assembly at tower 23-4, a point about 125 ft. south of where the oscillographs were coupled. The flashover was checked by (1) visual observation, (2) surge indicator operation, and (3) flash marks on insulators. Flashover on phase X at tower 23-3 (adjacent tower) was also indicated. This record begins at − 100 kv., increases to − 720 kv. in 3.5 microseconds and is off scale (− 2,760 kv.) in 4.8 microseconds. Record reaches zero due to the accompanying insulator flashover in 5.9 microseconds

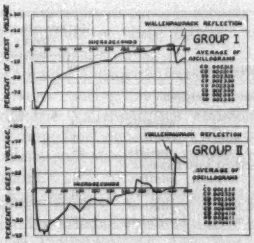

Fig. 9—Composite Lightning Surge Wave Shapes

Surge-Voltage Recorders

As in previous years surge-voltage recorders located at rather widely separated intervals along the line yielded records of all magnitudes up to about 2,700 kv. (breakdown value of line insulation) and of bothpositive

and negative polarities. Polarity characteristics and magnitudes of surge voltages measured during 1930 are indicated in Table IX.

Data for five consecutive years classified with respect to trip-outs and overhead ground wires are presented in Table X. It appears that surge-voltage recorder records are largely dependent upon the distance between the originating disturbance and the point of measurement, whether or not overhead ground wires are present, and whether or not the measurements are made on the same phase as that involved in the original disturbance. In the limited number of cases in which correlation can be made with a reasonable degree of confidence, high negative voltages have been associated with flashed insulators on the phase to which the surge-voltage recorder is coupled, either at the same tower or a very few spans away. The high positive voltages do not always correlate with the insulator flashovers. When they do, the flashover occurred at the surge-voltage recorder tower but not on the conductor phase coupled to the recorder. Table X shows that all voltages of high magnitude which are not coincident with line trip-outs, are positive in polarity and are measured on sections of line protected by ground wires.

TABLE VIII—SUMMARIZED CATHODE-RAY OSCILLOGRAM CONSTANTS

Time in microseconds to reach	First loop	Second loop
Group I		
75 per cent crest	5.0	
Crest	11.0	
50 per cent crest on wave tail	80.	
Zero	420.	
Group II		
75 per cent crest	2.1	17
Crest	2.5	25
50 per cent crest on wave tail	5.0	92
Zero	8.0	335

TABLE IX—POLARITY CHARACTERISTICS OF LIGHTNING SURGES RECORDED ON THE WALLENPAUPACK TAP-SIEGFRIED ROSELAND AND PLYMOUTH-SIEGFRIED LINES DURING 1930

	WT-SR line		PS line	
	Number	Max. voltage	Number	Max. voltage
Unidirectional				
Positive	1	450	0	
Negative	0		8	550
Oscillatory				
Highest crest value positive	6	2400	0	
Highest crest value negative	21	2640-3080	0	
Positive and negative crest values equal	1	tr	2	tr
Total	29		10	

TABLE X—NUMBER, LOCATION AND POLARITY OF VOLTAGE SURGES EXCEEDING VARIOUS TIMES NORMAL, AS MEASURED BY SURGE-VOLTAGE RECORDERS OVER THE FIVE-YEAR PERIOD 1926 TO 1930, INCLUSIVE

Location and polarity	No. exceeding times normal (180-kv.) shown		
	(1)	(5)	(10)
Surges coincident with trip-outs			
On open line ... positive	8	8	6
On open line ... negative	29	21	8
Under ground wire ... positive	4	2	1
Under ground wire ... negative	26	11	2
Surges not coincident with trip-outs			
On open line ... positive	5	1	0
On open line ... negative	6	2	0
Under ground wire ... positive	16	8	7
Under ground wire ... negative	5	0	0

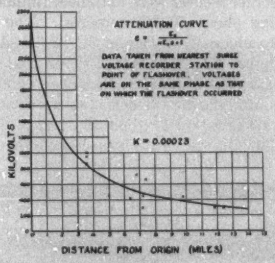

Fig. 10—Attenuation of Lightning Surge Voltages

Fig. 10 shows the magnitude of surge voltages at various distances from their source. All of these voltages were measured on sections of line unprotected by ground wire and on a phase which flashed over at the point of surge origin.

Electric Field Intensity and Lightning Severity

Electric-field intensity recorders installed at three locations from two to three miles away from the Cherry Valley lightning laboratory recorded voltage gradients up to − 37 kv. per ft. and + 35 kv. per ft. During any storm period, voltages at a given location are usually preponderantly of one polarity, which may be either positive or negative.

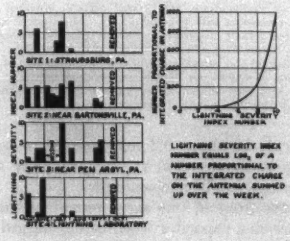

Fig. 11—Lightning (Storm) Severity Meter Data

Lightning severity meters were installed at each of the three sites mentioned above, and also near the lightning laboratory. Fig. 11 shows graphically the records obtained from these four locations. The lightning severity index number is proportional to the

logarithm to the base 2 of the charge on the severity meter antenna summed up for a week and is an arbitrary rather than an exact index of lightning storm severity.

Records from both types of instruments indicate that lightning at one location does not create sufficient field intensity to cause records on similar recorders at points 3 to 4½ miles away.

Overhead Ground Wires and Tower-Footing Resistance

The most reliable data available for evaluating the effect of overhead ground wires are those of flashed insulators. Data from 1929 and 1930 are shown in Fig. 12. It will be remembered that only about 37 per cent of the line is equipped with overhead ground wires.

Although many flashed insulator data were secured prior to 1929, these are not wholly reliable, with respect to time of occurrence and total numbers per season.

Fig. 12 shows the effect of overhead ground wires and tower footing resistance. The effect of the former (bearing in mind that these data are applicable only to

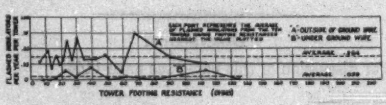

FIG. 12—INDICATED EFFECT OF TOWER FOOTING RESISTANCES ON INSULATOR ASSEMBLY FLASHOVERS (WITH AND WITHOUT OVERHEAD GROUND WIRES)

the construction used on this line) is to reduce the number of flashovers to about 15 per cent of those which would otherwise occur. The effect of tower footing resistance, at least above 17 ohms, seems to be negligible.

SUMMARY AND CONCLUSIONS

1. At least a large proportion of the lightning trip-outs of the Pennsylvania Power & Light Company's 220-kv. transmission lines are caused by direct strokes to line structures or wires.

2. Voltages induced in the power conductors by the collapse of cloud fields, coincident with lightning discharges to other objects, usually do not appear to be of sufficient magnitude to endanger service over circuits as highly insulated as these lines. For antenna voltages ranging up to 2,500 kv., the highest recorded corresponding transmission line voltage was 350 kv. Usually, no measurable induced voltages existed on the transmission line at times when appreciable voltages were impressed on the antennas.

3. With the insulation and clearances employed on these lines most lightning strokes contact only one object (ground wire, power conductor, or tower top) without forking to other objects (power conductors) in the same vicinity, and usually do not give rise to sufficient potential difference between the contacted object and surrounding non-contacted objects to cause local side-flashing.

4. Lightning strokes apparently vary through a considerable range of current intensity. Structure currents interpreted as ranging from approximately 40,000 amperes to approximately 260,000 amperes have been recorded.

5. Most lightning strokes are of negative polarity (earth positive and cloud negative) as indicated by measured structure currents. A smaller proportion of structure currents appear to be oscillatory with negative polarity predominating.

6. Positive-surge voltages between conductor and tower are usually indicated at locations where a ground wire, or a conductor not coupled to the surge-voltage recorder, is contacted by a direct stroke of negative polarity. They are not, therefore, indications of induced surges due to cloud fields.

7. The wave form of lightning surge voltages as recorded at the Cherry Valley Laboratory at the time of transmission line trip-outs, appears to be dependent, among other things, upon whether the surge originated and created fault on the phase to which the oscillographs were coupled, or on either or both of the other two phases. Coincident with the reported direct lightning stroke in the immediate vicinity of the laboratory which caused flashover at a tower 125 ft. from the laboratory and on the phase to which the oscillographs were coupled, an oscillogram was obtained which indicated that the line voltage rose to something in excess of − 2,760 kv. before flashover occurred. An average voltage change of 1,540 kv. per microsecond between the range of − 750 kv. and − 2,760 kv. was recorded.

8. The location of surge origin can be determined approximately from reflections shown on the oscillogram from the slow-sweep (2,000 microsecond time scale) oscillograph. The nature and time of these reflections supported by the records of the recording ammeters at Wallenpaupack and magnetic oscillographs at Siegfried, and the indications of the surge (flashover) indicators enable identification of fault locations in many cases, even when several trip-outs occur in a single day.

9. Most trip-outs of these 220-kv. lines are occasioned by single-phase faults.

10. The number of insulator assemblies involved in a given disturbance causing line trip-out seems, from the 1930 data for the Wallenpaupack Tap-Siegfried Roseland line, to range from one to four, with two (both on the same phase and at adjacent towers) as a rather usual condition.

11. A protective ratio of the order of 0.5 to 0.6 is indicated for the ground wire installed over the 50-ft. antenna near Wallenpaupack Tap Tower 1-3. The voltages recorded on the voltage measuring antennas appear to be approximately proportional to antenna height above ground. These voltages as recorded display no pronounced trend toward any given polarity characteristic.

12. The absence, during the past two summers, of insulator flashovers in the High Knob section of the

Wallenpaupack Tap leads to the conclusion that the buried "counterpoise," installed there in the spring of 1929, increases the effectiveness of the overhead ground wires. During the years 1926 to 1928 insulator assembly flashovers in this section of line were numerous.

13. Conventional overhead ground wires, as now applied, and especially when grounded through towers of comparatively low-footing resistance, seem to provide protection against a majority of the direct strokes encountered. Of the twenty-four lightning trip-outs of the Wallenpaupack Tap-Siegfried Roseland line (37 per cent of which is equipped with conventional overhead ground wires), one (or possibly four) resulted from faults which developed in the ground wire sections. The Plymouth-Siegfried line (which is equipped with conventional overhead ground wires throughout) tripped twice.

ACKNOWLEDGMENTS

Acknowledgment is made to Messrs. J. R. Fisher, G. J. Gross, H. B. Newsom, H. M. Poller, N. Rohats, J. E. Treweek, A. F. Werner and others, including the personnel of the Pennsylvania Power and Light Company and the General Electric Company for field work, analysis, and correlation of data, and to Messrs. C. M. Foust, C. A. Jordan, E. S. Lee, W. W. Lewis, W. L. Lloyd, Jr., F. W. Peek, Jr., H. S. Phelps, H. K. Sels, A. E. Silver, J. A. Snyder and N. Stahl for general direction of this investigation.

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Discussion

For discussion of this paper see page 1146.

1930 Lightning Investigations

On the Transmission System of the American Gas and Electric Company

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Fellow, A. I. E. E. Member, A. I. E. E.

Introduction

THIS paper covers two field investigations carried on in 1930 on the lightning problem. One of these was made on the 132-kv. transmission system of The Ohio Power Company and was really a continuation of the investigation carried on in 1929 and previously described.[1] The Philo-Canton Transmission line and the system of which it is a part have both been recently and fully described.[2] The second investigation was made on the 132-kv. transmission line of the Atlantic City Electric Company between Deepwater, New Jersey, and Atlantic City, New Jersey. A map showing the location of the two termini and the route of this line is shown in Fig. 1. The entire double circuit line is 63.5 miles long, 57 miles being steel tower 132-kv. construction with a 397,500 A. C. S. R. conductor, and the balance, between Pleasantville and Atlantic City, being wood-pole 66-kv. construction. The steel tower portion, except for a short distance from Deepwater is insulated with ten and twelve 4¾-in. disks, using two ground wires in a vertical plane through the center of the tower, and plain, approximately 15 in. by 30 in. rings, both top and bottom. The entire line was operated throughout the investigation period at 66 kv.

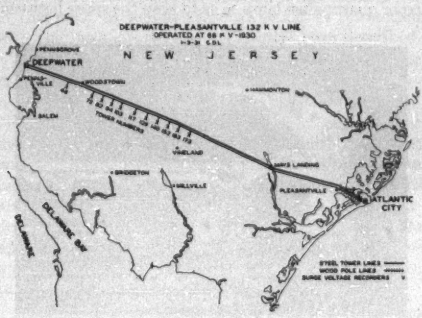

FIG. 1—ROUTE OF DEEPWATER-ATLANTIC CITY LINE OF ATLANTIC CITY ELECTRIC COMPANY

*American Gas and Electric Company, New York.

†General Electric Company, Pittsfield, Mass.

1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29–May 2, 1931.

This paper is, of necessity, limited to as brief a summary of the salient data as possible; if at times the resulting impression is somewhat hazy, it should not in fairness be ascribed to a similar condition in the data.

Instruments Employed and Their Location

The instruments used in these investigations were the surge-voltage recorder, the cathode-ray oscillograph, the direct-stroke recorder and the flashover indicator. The first three instruments have been described elsewhere.[3,4,1] Surge-voltage recorders were installed at every tower indicated in Fig. 1, on the Deepwater-Atlantic City line; on the Philo-Canton line instruments were placed at towers 200, 190, 181, 172, 164, 159, 156, 152, 148, 143, 134, 129, 123, 118, 113, thus giving a greater continuous stretch of line over which to obtain data than was available in 1929. Surge-voltage recorders were also installed on 132-kv. lightning arresters at Zanesville* and Newcomerstown.† The cathode-ray oscillograph station at Newcomerstown was operated throughout the entire lightning year; no oscillograph was in service on the Deepwater line.

Direct, or lightning, stroke recorders of the same type and connected in the same manner as previously described[1] were installed on both the Philo-Canton and Deepwater-Atlantic City lines. On the first line instruments were installed on towers 72 to 200 inclusive; on the second line 200 instruments were installed, one on each tower, from towers 20 to 219 inclusive.

The flashover indicator, or surge indicator, is an instrument used for the first time this year. When properly connected the instrument gives a visual indication that an insulator string has flashed over as a result of the passing or flowing across it to ground of a lightning surge current following the breakdown of its insulation by a lightning or surge voltage. A more detailed description of this instrument has been given elsewhere. A view showing such an instrument installed is shown in Fig. 2. A total of 200 of these was in service on the Philo-Canton line. On the Deepwater-Atlantic City line 300 of these instruments were installed on towers 41 to 140 inclusive.

These instruments were connected across a portion of the steel tower arm. Laboratory tests were made on the instrument before it was installed in the field. These consisted of surge voltages of known magnitude and wave shape applied to the line end of the insulator

*Thyrite.

†Oxide Film.

31-79

assembly by means of an artificial lightning generator. The tests were made at a voltage of 450 kv., and the insulator flashover current was limited by resistance to a value of approximately 2,500 amperes; this was less than the minimum value of lightning current that would be carried by the tower arms when the line string flashed over. Using this value of flashover current, considerable time and effort were spent in finding the

FIG. 2—FLASHOVER- OR SURGE-INDICATOR INSTALLATION ON PHILO-CANTON LINE

proper lead length, location and instrument position to give the best flashover indicator performance without at the same time offering hazard to normal line operation. With the setup finally adopted 100 per cent operation was obtained in the laboratory.

The 1930 circuit arrangement of the cathode-ray oscillograph is shown in Fig. 3. This circuit is a modi-

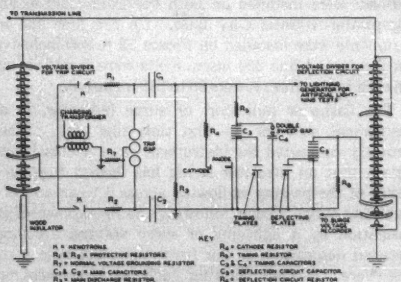

FIG. 3—1930 CIRCUIT ARRANGEMENT OF CATHODE-RAY OSCILLOGRAPH AT NEWCOMERSTOWN

fication of the 1929 circuit. Its principal changes consist of the following:

1. A change in the method of sweeping the cathode beam across the time axis. This method consisted of the use of a rather small capacity C_2 for first giving a rapid sweep of beam across the time axis, thus making it possible to obtain the front of a wave on a fairly long time scale. After the voltage across condenser C_2 has reached a value sufficiently high the gap in series with condenser C_4 (whose value of capacity is approximately from 20 to 40 times that of C_2) breaks down, the voltage during the process of breakdown being for an instant reduced to zero and a new sweep rate established on a much slower basis owing to the larger capacity of C_4. It was thus possible to obtain the tail of a long wave on the more compressed scale. An oscillogram of a long tail wave showing the double scale obtained by the double sweep gap is shown in Fig. 4. It will be noted that this represents a wave having reached a crest of 170 kv. in 9 microseconds and attenuating to 50 per cent crest in 30 microseconds; the total length of the wave is 160 microseconds.

2. Changes were made in the initiating of the trip gap to eliminate previously found erratic behavior. It had been found, for example, that under certain conditions the gap would be initiated apparently by air currents without any increase in voltage. To eliminate this difficulty the gap was entirely enclosed. Further, it was subjected continuously to the action of a mercury-vapor quartz-glass lamp so as to keep the space between

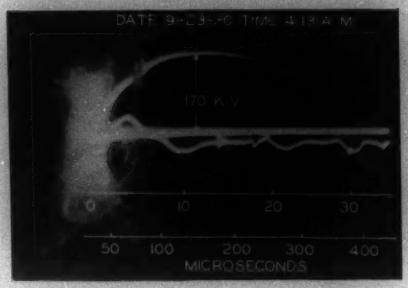

FIG. 4—OSCILLOGRAM OF LONG TAIL WAVE SHOWING DOUBLE SCALE OBTAINED BY DOUBLE SWEEP

the gap in a highly ionized state and thus reduce any initial time lag owing to the necessity for ionization.

This is believed to be the first installation of a single cathode-ray oscillograph in the field to do the work of two. Heretofore, two oscillographs have been required to get oscillograms of a single lightning wave at two different sweeps. The method described above permits these two oscillograms to be taken with the same oscillograph at a saving in equipment and maintenance. Positive correlation is obtained.

SCOPE OF INVESTIGATION

In undertaking the investigation it was hoped to obtain data on the following:

Magnitude of Lightning Voltage. The installation of surge-voltage recorders on both the Philo-Canton and the Deepwater-Atlantic City lines was expected to yield further information on this point. On the Philo-Canton line approximately 22 miles were quite

thoroughly covered by 15 instruments as previously outlined. These instruments were of further value for furnishing correlating data in connection with the information yielded by the direct lightning stroke recorders. On the Deepwater-Atlantic City line approximately 23 miles of line were covered by 11 surge-voltage recorders located as indicated in Fig. 1; at each one of these points, too, direct-stroke recorders were installed so that the surge-voltage recorders were available for furnishing correlating data in connection with any records obtained from such direct-stroke recorders. It was also expected that the data obtained by means of the surge-voltage recorders would furnish an approximate measure as to the intensity and severity of lightning during this year's investigation as compared with other years.

Characteristics of Lightning Voltages. It was expected to obtain further oscillograms of natural waves through the cathode-ray oscillograph installed at Newcomerstown. The reason for the location of an oscillograph at this point as well as physical details in connection with the installation have previously[1] been described.

Measurement of Current in Lightning Strokes. Last year an instrument was placed upon the circuit, designed to differentiate between induced voltages and direct strokes. This was accomplished by adjusting the instrument so that it would not record until the current was well above the maximum possible current due to induction. Since the instrument appeared to record approximate current its use was extended to that purpose. While it is realized that the current readings are approximate and subject to correction as calibrations are changed, it appears worth while to include them in this report.

Relation between Direct and Induced Strokes and between Direct Strokes and Flashover. It was expected that the large number of lightning-stroke recorders installed on both the Philo-Canton and Deepwater-Atlantic City lines, combined with the surge-voltage recorders, would yield data on the relationship between direct and induced strokes. The installation of flashover indicators, it was hoped, would also result in disclosing information as to the relationship between flashovers at towers and direct strokes of lightning terminating at the same towers.

Lightning Arrester Performance. In discussing the present status of rationalization of transmission system insulation[6] it was pointed out how important a *knowledge* of lightning arrester performance is in any attempted solution of the problem and further, how small an amount of actual data were available on that phase of the problem. It was hoped, that the surge-voltage recorders installed at Zanesville and Newcomerstown, would yield some very definite data on that point. The surge recorder, for current measurement, in each case was connected across a resistance which was in series with a lightning arrester at the grounded end. By placing the resistor inside the surge-recorder housing, disturbance from stray fields was reduced to a minimum and the connecting leads were shortened from a few feet to a few inches.

Analysis of Data Obtained

Surge-Voltage Data. No records were obtained from the surge-voltage recorders on the Deepwater-Atlantic City line. However, it should be pointed out, the surge-voltage recorders were not installed until all lightning trip-outs on the line had taken place.

On the Philo-Canton line 107 surges were obtained, divided as follows:

Lightning	35
Lightning and switching	3
Switching	64
Unknown	1
Testing	4
Total	107

In Table I there is shown a classification of these 107 surges according to five groupings, namely, positive, negative, predominantly positive, predominantly nega-

TABLE I—PREDOMINATING NATURE OF SURGES
(Number Recorded)

Cause of surge	Positive		Negative		Oscillatory	Total
	Pure	Predom.	Pure	Predom.		
Lightning	23	7	3	0	2	35
Lightning and switching	0	0	0	1	2	3
Switching—energizing	0	0	0	0	1	1
Switching—deenergizing	0	0	1	0	0	1
Switching—mixed energizing and deenergizing	14	5	14	3	26	62
Unknown	0	1	0	0	0	1
Testing	4	0	0	0	0	4
						107

tive, and oscillatory. It will be seen that of the lightning surges 74.3 per cent were unidirectional, 20 per cent were predominantly unidirectional and 5.7 per cent were oscillatory; further, 65 per cent were purely positive, 20 per cent predominantly positive, 9 per cent purely negative, 0 per cent predominantly negative and 6 per cent oscillatory.

In the light of the observed data that the polarity of direct strokes as recorded by the lightning-stroke recorder was in every case negative, it is of particular interest that only 9 per cent were either negative or predominantly negative. It suggests very definitely that 91 per cent of these surges were of an induced origin.

In Table II is shown the voltage amplitude in times of normal of the highest recorder surge in each polarity classification. It will be noted that the highest surge voltage recorder record was 6½ times normal. Only

TABLE II—PREDOMINATING NATURE OF SURGES WITH MAXIMUM RECORDED VOLTAGES
(Times Normal to Ground)

Cause of surge	Positive		Negative		Oscillatory	
	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.
Lightning	5.6	0	0	3.1	6.4	6.4
Lightning and switching			2.6	4.0	6.5	Trace
Switching—energizing circuit No. 1					3.5	3.5
Switching—deenergizing circuit No. 1			0	2.3		
Switching—mixed energizing and deenergizing circuit No. 1					2.6	2.3
Unknown	2.7	0				

two line trip-outs occurred during the investigation indicating that the lightning storm severity was unusually light and accounted to a large extent for the absence of high-voltage surge measurements.

Cathode-Ray Oscillograph Data. Sixty-two lightning

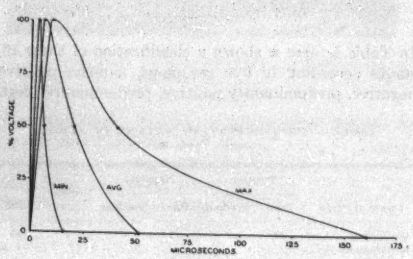

FIG. 5—THREE WAVE SHAPES REPRESENTING ENVELOPES OF MINIMUM, AVERAGE, AND MAXIMUM TIME ELEMENTS TO CREST, 50 PER CENT VALUE ON TAIL AND ZERO VOLTAGE OF 29 OSCILLOGRAMS 50 KV. CREST OR ABOVE

surges were recorded by the cathode-ray oscillograph. Of these 61 or 98.5 per cent were of positive polarity. The time to crest value of these surges varied from 2 to 13 microseconds; to half crest value on the tail from 5 to 85 microseconds and the time to zero voltage from 9 to 160 microseconds. The lowest crest value recorded was 20 kv. and the highest 180 kv.

In Fig. 5 there has been shown graphically three wave shapes which represent minimum, average, and maximum time elements to crest, 50 per cent value on the tail, and zero voltage. These curves are based on a selected group of 29 oscillograms recording 50 kv. on crest or above. It is particularly striking that the highest crest value recorded was only 1.67 times normal (180 kv.); this wave was a 9 by 16 microsecond wave.

Direct-Stroke Data. In Table III there are shown the lightning stroke currents obtained by direct addition of measured tower currents for 13 strokes recorded on the Philo-Canton line and for 12 strokes recorded on the Deepwater-Atlantic City line. A typical distribution of current on each side of a tower struck by lightning is shown in Fig. 6. This represents case No. 16, in Table III, in which tower 72 on the Atlantic City line was struck. It will be noticed that the lightning current traveled a distance of approximately 3,000 ft. in one direction, covering three spans, and approximately 2,000 ft. in the other direction, covering approximately two spans. The distribution of current on each side of

TABLE III—SUMMATION OF INDICATED TOWER CURRENTS

Reference	Date—1930	Tower No.	Lightning stroke recorder readings*
1	6–12 to 6–19	128	40,000
2	6–11 to 6–17	176	540,000
3	6–13 to 6–20	87	40,000
4	6–19 to 6–27	126	470,000
5	6–24 to 7–1	196	475,000
6	7–8 to 7–15	181	60,000
7	7–11 to 7–18	99	155,000
8	8–26 to 9–9	194	110,000
9	8–27 to 9–10	156	355,000
10	9–11 to 9–25	122	155,000
11	9–11 to 9–25	131	130,000
12	9–9 to 9–23	173	515,000
13	9–25 to 10–2	113	515,000
14	6–12 to 7–8	50	410,000
15	6–16 to 7–8	61	70,000
16	6–16 to 7–7	72	670,000
17	6–16 to 7–7	89	90,000
18	7–8 to 7–25	49	60,000
19	7–4 to 7–16	143	200,000
20	7–4 to 7–15	180	120,000
21	7–4 to 7–14	192	600,000
22	7–2 to 7–13	202	90,000
23	7–4 to 7–15	164	130,000
24	7–4 to 7–15	168	230,000
25	7–4 to 7–15	175	170,000

From 1 to 13 Philo-Canton.
From 14 to 25 Deepwater-Atlantic City.
*Approximate amperes—subject to correction.

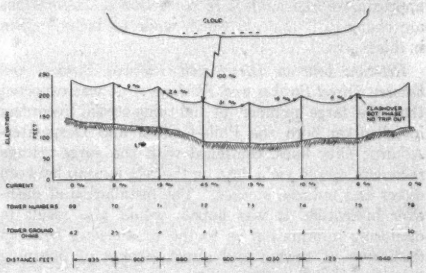

FIG. 6—TYPICAL CURRENT DISTRIBUTION ON EACH SIDE OF A TOWER STRUCK BY LIGHTNING

the towers struck for all 25 cases recorded is shown in Table IV.

An examination of Tables III and IV shows first, several cases where the approximate currents measured in individual towers may be expressed in terms of hundred thousands of amperes; second, if currents in adjacent towers due, apparently, to the same stroke are directly added, much larger currents are indicated. It is realized, of course, that the time-phase relation of the currents existing in the various parts of the circuit

TABLE IV—DIRECT-STROKE RECORDS APPROXIMATE KILOAMPERES—SUBJECT TO CORRECTION

Ref.	Adjacent towers				Tower struck	Adjacent towers				S. V. R. readings kv. at feet	
1					40					0	900
2		50	20	90	300	80				+600	6,110
3					40					0	31,680
4				85	310	75				+300	3,710
5				85	290	100				+360	7,270
6					60					+280	0
7					100	55				+360	103,000
8				55	55					+690	6,610
9				85	270					0	0
10				70	85					+280	716
11				65	65					−520	8,870
12		50	70	260		70	65			+260	0
13				90	300	75	50			+260	0
14				60	200*	60	40	50			
15					70						
16		60	100		300	100	70	40*		No surge-voltage recorders as yet installed when these direct strokes took place.	
17					90*						
18					60						
19					200†						
20					120						
21	60	90		70	280	100					
22					90†						
23				60	70†						
24					160	70					
25				70	100						

* = Flashover of insulator string (total 3 cases).
† = Flashover of insulator string and trip-out (total 3 cases).
1 to 13 = Philo-Canton.
14 to 25 = Deepwater-Atlantic City.

are not definitely known and that an accurate analysis of traveling wave phenomena involved might give values of current less than the totals indicated in Table III. On the other hand, preliminary analysis has indicated that a rigidly correct addition of currents would increase the value obtained in the tower directly struck to at least 50 per cent of the total obtained by adding directly all the currents in the other towers. The results indicate that lightning strokes of several hundred thousand amperes do occur.

It is significant that in the case of the Philo-Canton line there was only one automatic line trip-out where there existed a possible correlation with a direct-stroke record. On September 23, 1930 line No. 1 tripped automatically at Newcomerstown and Philo and line No. 2 at Canton due to lightning. During this period (September 9 to 25) three lightning strokes were indicated by lightning-stroke recorders (cases 10, 11, and 12 in Tables III and IV) showing a possible correlation with the single trip-out. The remainder of the lightning-stroke records could not be correlated with any known trip-outs, although flashovers may possibly have occurred. On the Deepwater-Atlantic City line there is a correlation between line trip-out and line insulator assembly flashover in three of the twelve cases. These are reference Nos. 19, 22, and 23 in Table III. In all the other nine cases no correlation with any known trip-outs could be obtained, although inspection of the line completed on December 2nd indicated that insulator flashover without line trip-out had apparently occurred in three other cases. Twenty-five per cent of the direct strokes to the Deepwater-Atlantic City line therefore produced a trip-out with the possibility that another 25 per cent produced flashover not followed by trip-out.

Flashover (Surge) Indicator Records and Other Data Bearing on Question of Direct vs. Induced Strokes. The flashover or surge indicators on the Deepwater line were not placed in service until late in the lightning season and after all line trip-outs due to lightning had taken place.

No insulator assembly flashovers were indicated by the flashover indicators on the Philo-Canton line during the period of the investigation. The three principal reasons for this are: first, the fact that the instruments were installed rather late in the lightning season, second, they were not placed on every insulator assembly, and third, lightning conditions were not as severe as in the past years. Records of damage to the line insulators were rather few and in every case were at towers where there were no flashover indicators installed.

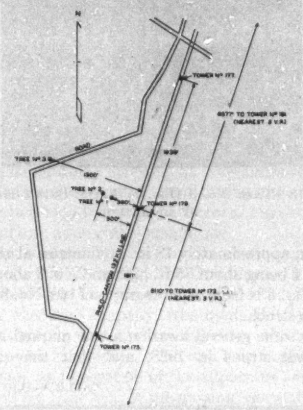

Fig. 7—Location of Trees Struck by Lightning in Vicinity of Tower No. 176 on Philo-Canton Line

It will be seen from the above that little was contributed to the knowledge of the relationship between direct and induced strokes by the flashover or surge indicators. Other data, however, were obtained which shed some light on this question. During the lightning season two trees were struck by lightning in the vicinity of tower 176. The location of these trees with respect to the line is shown in Fig. 7. Both trees Nos. 2 and 3 were struck apparently at the time of a storm which occurred on September 23. The nearest surge-voltage recorder (located at tower No. 172 more than a mile away) indicated a highly damped voltage surge of − 4.8 and + 2.4 times normal at 4.13 A. M. on September 23. It has not been found possible, however, to correlate these lightning strokes to trees Nos. 2 and 3 with any trip-out. They are apparently cases of direct strokes 500 ft. and 1,300 ft. from the line which, although possibly producing a flashover, produced no line trip-out or high-voltage record on any instrument connected to the line (the nearest instrument, of course, being more than

a mile away). Both trees had the bark split and torn off around the periphery of the tree, the lightning apparently striking the top of the tree and extending down to the ground in four or five parallel paths. In each path the bark torn off was 2 to 3 in. wide. The trees

FIG. 8—TREE NO. 3 (FIG. 7) AFTER BEING STRUCK BY LIGHTNING

were oak, approximately 18 in. in diameter at the base, tree No. 2 being about 75 ft. high and No. 3 about 50 ft. high. Fig. 8 is from a photograph of tree No. 3 after it had been struck.

In the same general location a tree marked No. 1 in Fig. 7 was struck in 1928, and that time a surge recorder was located at tower No. 176 only 360 ft. away. This instrument recorded at that time maximum voltage 2,100 kv. positive; trip-out occurred. It seems evident from the 1928 record that a direct stroke of lightning striking tree No. 1 had caused an induced voltage on the line of 2,100 kv., flashover, and trip-out. The 1930 record indicates that one or more direct strokes of lightning striking trees Nos. 2 and 3 resulted in relatively low, if any, induced voltages on the line, no outages, and therefore just the opposite of the 1928 experience.

Lightning Arrester Data. Surges obtained on lightning arresters showing both current discharges and voltages are given in Table V.

The arresters at Newcomerstown are three legged oxide film type, 400 cells per leg. The series gaps on these arresters were set at 3.5 in. throughout the entire lightning season. Thyrite arresters are installed at Zanesville. Referring to Table V it will be seen that the values of voltages obtained across lightning arresters are at or below 300 kv. in all cases except on surge No. 14 B where a voltage of 500 kv. was recorded. A surge-voltage registration equivalent to 300 kv. is obtained under conditions where the reliability and accuracy of the registration is comparatively poor; the same thing holds true for currents of 100 amperes or less. Under these conditions no great reliability can be placed in the precision of measurements obtained. The best that can be gathered from the data is to state that it seems fairly certain that neither of the arresters at Newcomerstown or Zanesville was called upon to discharge surges of high voltage or current of any appreciable magnitude.

From the summary of the data obtained it will be seen that while everything was set for the gathering of a

TABLE V—LIGHTNING ARRESTER SURGE DATA

Surge No.	Location			Amperes		Kilovolts		Cause of surge
	Station	Circuit	Phase	+	−	+	−	
5	Zans. sub.	2	2	N	tr	N	N	Switching
6	Zans. sub.	2	2	N	115	N	N	Switching
7	Zans. sub.	1	1	N	tr	(2)	(2)	Switching
7	Zans. sub.	1	3	N	130	N	300	Switching
8	News. sub.	bus	2	100	N	(2)	(2)	Lightning
14	Zans. sub.	1	3	(1)	110	(2)	(2)	Switching
14 B	Zans. sub.	2	2	340	N	N	N	Lightning
14 B	Zans. sub.	1	1	270	N	510	230	Lightning
15	News. sub.	bus	2	110	N	280	N	Lightning
16	News. sub.	bus	2	(1)	tr	(2)	(2)	Lightning
17	News. sub.	bus	2	(1)	tr	(2)	(2)	Lightning
38	Zans. sub.	2	1	N	100	N	tr	Switching
38	Zans. sub.	2	2	N	tr	N	N	Switching
41	Zans. sub.	1	3	110	(1)	300	370	Switching
48	Zans. sub.	2	2	N	tr	N	N	Switching
49	Zans. sub.	2	2	N	tr	N	N	Switching
61	Zans. sub.	1	3	tr	N	(2)	(2)	Switching
62	Zans. sub.	1	1	105	(1)	230	230	Switching
64	News. sub.	bus	1	90	(1)	230	300	Lightning
76	Zans. sub.	1	1	110	(1)	280	tr	Switching
82	News. sub.	bus	2	(1)	120	(2)	(2)	Lightning
93	Zans. sub.	2	2	210	N	250	N	Lightning
95	Zans. sub.	1	1	N	tr	(2)	(2)	Switching
95	Zans. sub.	1	2	N	105	tr	N	Switching
95	Zans. sub.	1	3	N	160	(2)	(2)	Switching

tr = trace.
(1) 70 amperes or less.
(2) 225 kv. or less.
(N) No record above normal line voltage.

rather extensive set of data, the amount of actual information obtained was not as great as had been expected. Nevertheless some very important new data were obtained and confirmation obtained of other previously gathered observations. The results obtained, it is believed, fully justify the conclusions below.

Conclusions

1. The highest lightning voltage obtained by surge-voltage recorder was 6.5 times normal, positive; the highest switching surge recorded was 3.5 times normal, oscillatory and highly damped; the highest cathode-ray oscillograph voltage recorded was 1.67 times normal, positive. The low severity of lightning storms during the year accounts to a great extent for the absence of high-voltage surge measurements.

2. Practically all lightning voltage surges on the transmission line of appreciable magnitude are unidirectional. The investigation disclosed:

74.3 per cent wholly unidirectional
20.0 per cent predominantly unidirectional
5.7 per cent oscillatory but highly damped.

The oscillatory figures may be due to superposed surges, which the timing of the surge recorder is unable to separate.

3. Positive lightning surges on the conductors predominate. The surge records show:

23 pure positive
23 predominantly positive
3 pure negative
0 predominantly negative
2 oscillatory

Out of 62 cathode-ray oscillograms, only one negative surge was recorded; the balance being positive.

4. From 29 oscillograms of natural lightning voltages above 50 kv. obtained on the line, wave fronts ranging from 2 to 9 microseconds were recorded. Tail time values to 50 per cent crest ranged from 6 to 40 microseconds; and total length to zero from 10 to 160 microseconds.

It should be noted in this connection that these waves were not measured at the point of origin where steeper fronts and possibly shorter tails may have existed.

5. The lightning-stroke recorders indicate that direct strokes are negative in polarity.

6. Direct lightning strokes to the overhead ground system of the transmission line take place frequently, 13 having been recorded on the Philo-Canton line and 12 on the Deepwater-Atlantic City line. Considering the fact that out of these 25 cases only 4 trip-outs resulted, it appears that the induced surge voltages on the conductors under these conditions are in the great majority (84 per cent) of cases insufficient in amplitude to cause line trip-out.

7. Direct strokes to the tower or ground wire also occurred without apparent flashover of the insulator assembly. Out of 13 direct-stroke records on the Philo-Canton line 8 were obtained under conditions where flashover indicator operation might have taken place with top or bottom insulator assembly flashover. Not a single case of such operation was recorded.

8. The study of direct-stroke records shows frequent cases where currents of the order of several hundred thousand amperes or more are indicated in a single lightning stroke.

9. Direct lightning strokes to the overhead ground system produced discharge currents through as many as 6 adjacent towers of a transmission line.

10. Repeated discharges (possibly up to 4 or 5) in the lightning stroke are indicated by the presence of superimposed Lichtenberg figures on several of the lightning-stroke recorder films. Another possibility, however, is that these are individual strokes some short time apart.

11. The presence of a large number of surges of positive polarity indicates that many induced surge voltages are present on the line conductors. In at least one known case a tree was struck approximately 360 ft. away from the line, inducing a positive voltage of 2,100 kv. on a conductor at a tower that distance away from the tree; this caused a flashover of the insulator string and a trip-out of the line.

12. The lightning arresters at Newcomerstown and Zanesville were not called upon during the period of active data taking to discharge surges of high voltage or current of any appreciable magnitude.

Acknowledgment is due to the executives of the General Electric Company, The Ohio Power Company, the Atlantic City Electric Company and the American Gas and Electric Company, who made this investigation possible; to the various members of the above companies for their cooperation and help in the design, installation, and operation of the apparatus and equipment, and for their help in supervising, correlating, and working up the data; in particular to Messrs. H. L. Rorden, J. C. Dowell, S. D. Day, G. D. Lippert, E. W. Whitmer and S. S. Smith, who actually carried out the work and were in charge of the various tests and field investigations.

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Discussion

For discussion of this paper see page 1146.

Lightning Investigation

On the Appalachian Electric Power Company's Transmission System

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Associate, A. I. E. E. Associate, A. I. E. E.

INTRODUCTION

Plan of Investigation. Upon the organization of the Lightning Subcommittee of the Power Transmission and Distribution Committee of the A. I. E. E., in the fall of 1926, it was decided that a comprehensive investigation should be started, in an effort to obtain some definite information regarding lightning and its effects, which were causing havoc with electric power service and which, at that time, were relatively unknown. For this purpose, the American Gas and Electric Company placed its 132-kv. system at the disposal of the Committee, and in the following spring an investigation was started using the klydonograph, at that time the only instrument available for the purpose. This investigation was conducted cooperatively by the American Gas and Electric Company and the two principal electrical manufacturers, and most of the equipment was pooled for the purpose.

The following year, 1928, the activity was considerably expanded, and the efforts of the different participating groups were concentrated on two different parts of the system. The tests in which the Westinghouse Electric & Manufacturing Company cooperated with the American Gas and Electric Company, the results of which are being reported here, were performed on the Southern System, or that of the Appalachian Electric Power Company. Since its initiation the work has been continued up to the present, and use was made of all additional equipment, such as the cathode-ray oscillograph, and the application of the klydonograph to the recording of occurrence and current in direct strokes, as these applications became available.

During the four years in which this investigation has been conducted, it has been paralleled by similar work in other localities. A number of papers and articles has appeared, and these, of course, have been influenced by all the information currently existing. Also, all the results of this investigation have been continuously at the disposal of the Lightning and Insulator Subcommittee. However, a wealth of data has been accumulated and it is felt that a complete analysis and summarization of these data would constitute a worthy contribution on the subject. Not only does it, because of its volume, lend weight of confirmation by smoothing out the curves of previous papers, but several records of special interest have been obtained which throw light on various phenomena not completely understood. Furthermore, because of different settings of apparatus, previous curves have been extended.

OBJECTIVE

The information needed in connection with lightning was listed at the beginning of the investigation and these items have, in general, remained the objectives throughout the work. On many points the information is now fairly definite while on others it is still uncertain. The factors to be determined may be grouped under three headings, as follows:

1. *Nature of Lightning, that is:*
   a. Its magnitude, voltage, and current.
   b. Its polarity, unidirectional or oscillatory.
   c. Its wave shape, front, duration, and tail.
   d. Its effect on transmission lines, direct or induced.
2. *Behavior of Surges on Systems, that is:*
   a. Attenuation.
   b. Reflections at junctions and ends.
   c. Ability to flashover line structures.
3. *Influence of System Characteristics, such as:*
   a. Configuration of conductors, including height.
   b. Right-of-way terrain.
   c. Ground wires.
   d. Protective apparatus, lightning arresters, gaps, etc.
   e. Condition of system neutral ground.
   f. Tower and footing impedance.

SCOPE OF INVESTIGATION

System Investigated. The parts of the transmission system,[1] on which the investigation was concentrated comprised three sections of line and seven substations. The sections of line involved the first 9 miles out of Turner on the Turner-Logan line, the first 26 miles out of Glen Lyn on the Glen Lyn-Roanoke line, and the entire 24 miles of the Turner-Cabin Creek line. The substations where the investigation was carried on were Turner, South Point, Cabin Creek, Saltville, Kingsport, Switchback and Glen Lyn.

The transmission lines are of two-circuit steel tower construction, with conductors in vertical configuration and in reverse phase order on each side of the tower.

One ground wire is employed, being located at the

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†Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

1. For references see Bibliography.

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center peak of the tower except on the Turner-Logan line. On this line no ground wire was in service in 1927 and 1928, the first two years of the investigation, but the conventional one ground wire was added late in 1928, so that on this line field data have been collected on the line both with and without ground wires.

Grading shields were in service on all lines where tests were made, these shields consisting of a 15 in. by 30 in. oblong ring at the line side and either a 30-in. horn or oblong strap ring at the ground end of the string.

FIG. 1—FIELD INSTALLATION OF KLYDONOGRAPHS, TOWER 5

Turner-Logan line showing potentiometers and ground wire connection

The line insulation is nominally ten 5⅛-in. disks in suspension strings, and two strings of twelve 5⅛-in. insulators each at dead ends.

The line wires are 336,400 cir. mils. A. C. S. R. on the Turner-Logan and Turner-Cabin Creek lines, and 397,500 cir. mils A. C. S. R. on the Glen Lyn-Roanoke line. The ground wire is A. C. S. R., being 177,000 cir. mils, 159,000 cir. mils, and 397,500 cir. mils, respectively, on the three lines.

The nature of the terrain over which the transmission lines run is very rough and mountainous, ranging in elevation from 600 to 3,000 feet. The soil is mostly shale or a mixture of shale and clay, and even solid rock is encountered at some locations, where blasting is necessary in placing the tower footings. As might be expected, the tower footing resistances of these lines are relatively high, in a number of instances exceeding 250 ohms. The elevation, footing resistance, and locations where flashovers were found are shown in Figs. 18, A, B, and C.

Instruments Used in the Investigation. In studying lightning phenomena in the field three instruments have been used, namely, the klydonograph, the cathode-ray oscillograph, and the direct-stroke recorder.

The klydonograph[2] used was of the three or four electrode type and was coupled to the line by pipe type potentiometers having a ratio of about 65 to 1. A typical field installation of the klydonograph at a tower, as used in this investigation, is shown in Fig. 1. A close-up view of the potentiometers, together with the circuit arrangement as used in investigating lightning arrester operation, is shown in Figs. 9A and B.

Wherever klydonograph installations were made at towers in the field the instrument ground was connected to a separate ground system consisting of the six potentiometer metal anchors and one or more 8 ft. ground rods, all located at least 10 ft. from the base of the tower. This was done in an attempt to measure the maximum voltage from line to ground and not the line to tower potential. Actually, this may not have been completely achieved in some cases as the separate

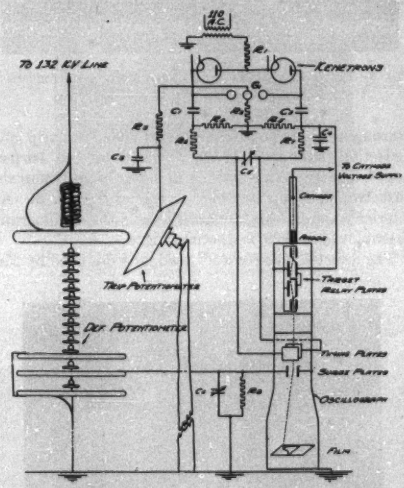


FIG. 2—SCHEMATIC WIRING DIAGRAM OF CATHODE-RAY OSCILLOGRAPH AND POTENTIOMETERS AT TOWER 20, TURNER-LOGAN LINE

grounds were only of the order of 25 ft. distant from the tower legs. At installations in the substations, the instrument ground was made to the station ground bus which had a resistance of only a fraction of an ohm.

The cathode-ray oscillograph used was of the cold cathode type, using the Norinder relay, which allows of continuously exciting the cathode, thus insuring that the electronic beam is ready for recording the transient at the instant the transient arrives. The schematic diagram of the oscillograph wiring and the coupling to the 132-kv. line is shown in Fig. 2. When the surge

to be measured arrives at the oscillograph, the excess potential on the trip potentiometer causes the gap G 1 to break down imposing a voltage from the condenser C 2 on the oscillograph relay plates. This allows the beam to focus on the film. The charging of condenser C 5 sweeps the beam across the film, providing the timing circuit; and the main surge from the deflection potentiometer excites the surge plates causing the beam to reproduce the voltage magnitude of the surge.

The field installation of the lightning laboratory

Fig. 3—Field Installation of Laboratory and Potentiometers at Tower 20, Turner-Logan Line

housing the oscillograph, together with the trip and deflection potentiometers at Tower 20 of the Turner-Logan line where the oscillograph was in operation the past two lightning seasons, is shown in Fig. 3. An interior view of the laboratory with the oscillograph as employed in 1929, is shown in Fig. 4.

The direct-stroke recorder used, and designed by Mr.

Fig. 4—Interior of Lightning Laboratory, Tower 20, Turner-Logan Line, Showing Oscillograph Auxiliary Equipment

J. J. Torok of the Westinghouse Company, is essentially a small klydonograph with timing accessories omitted. The instrument consists of a regular telephone receiver shell having metal electrodes extending from both ends. The telephone diaphram is replaced with a photographic film on a micarta disk which rests on a metal plate. The disassembled instrument is shown in Fig. 5 and the field installation in Fig. 6. The potentiometer method of using this direct-stroke recorder at a tower, where the resistance wire is in parallel with the tower footing resistance, is shown in Fig. 7. The point at which the direct-stroke recorder was tapped to the resistance wire

Fig. 5—Disassembly View of Direct-Stroke Recorder

Fig. 6—Field Installation of Direct-Stroke Recorder at the Base of Tower

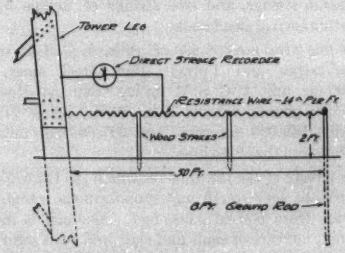


Fig. 7—Schematic Wiring Diagram of Direct-Stroke Recorder

was so chosen that the instrument would not flash over with 100,000 amperes in the tower, and yet would be able to record currents as low as 10,000 amperes.

Location of Instruments. The locations of klydonographs, cathode-ray oscillograph and direct-stroke recorders are shown for all line installations in Fig. 8.

In addition, klydonographs were also installed at the South Point, Switchback, Saltville, and Kingsport 132-kv. substations. The direct-stroke recorders were placed at each tower on the Turner-Logan line for the first 36 towers out from the Turner substation.

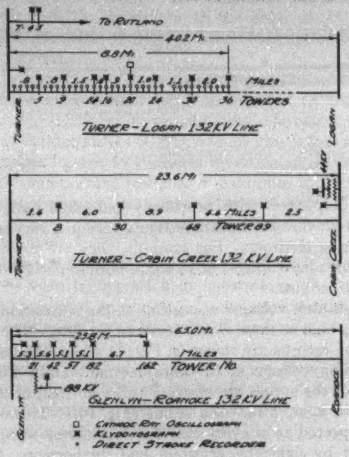


Fig. 8—Location of Lightning Measuring Instruments on the Line During Four Years' Investigation

During lightning seasons the cathode-ray oscillograph was in service in 1929 and 1930, the direct-stroke recorders in 1930, and the klydonographs in 1927 to 1930, inclusive. The locations of klydonographs during these four years are shown in Table I.

TABLE I—KLYDONOGRAPHS IN SERVICE

Location	1927	1928	1929	1930
Glenlyn–Roanoke Line		5	5	
Turner–Cabin Creek Line		4	4	
Turner–Logan Line	2	3	8	8
Turner–Rutland Line	1	1		
Cabin Creek substation		2(1)	2(1)	2(1)
Glenlyn substation	3(2)	3(2)	2(3)	2(3)
Kingsport substation		2(4)	2(4)	2(4)
Saltville substation		3(5)	3(5)	3(5)
South Point substation			1	1
Switchback substation		2(3)	2(3)	2(3)
Turner substation	1	1	1	1
Total	7	26	30	21

(1) One instrument on 44-kv. bus.
(2) One instrument on 13.2-kv. transformer tertiary.
One instrument on 88-kv. auto-transformer.
(3) One instrument on 88-kv. auto-transformer.
(4) One instrument on 132-kv. lightning arrester.
(5) One instrument on 88-kv. auto-transformer.
One instrument on 132-kv. lightning arrester.

In studying lightning arrester performance, the klydonograph was used to measure both the voltage on the line and the discharge currents in the arrester. Zircon resistors were placed in the legs of the arrester and in the ground connection common to the three arrester legs. The resistances used were approximately thirty and ten ohms, respectively. Zircon has the property of having a constant resistance under impulse conditions, and, as proved by laboratory tests after one year's operation, appears to be free from aging after repeated use.

The schematic wiring diagram of the klydonograph

Fig. 9a—Potentiometer Installation at Kingsport for Measuring Voltage at Lightning Arrester

Fig. 9b—Klydonograph Installation for Measuring Lightning Arrester Discharge Currents

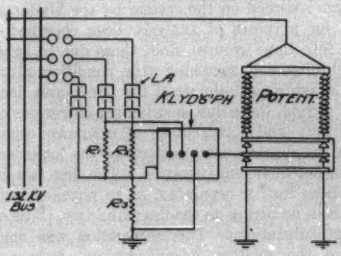


Fig. 9c—Schematic Wiring Diagram of Klydonograph Installation on Lightning Arrester

as coupled to the circuit for the lightning arrester tests is shown in Fig. 9c, where one instrument measures two arrester leg currents, the total current and one phase potential. This instrument was backed up by a separate klydonograph on the bus measuring all phase voltages. This separate klydonograph was located about 150 circuit ft. away from the arrester connections, due to space limitations at the station.

During the winters of 1927 and 1928, klydonographs were kept in operation at substations, where they could

be conveniently serviced, to observe if any troublesome voltages existed on the system under normal operation, at times when lightning was not prevalent.

Analysis of Data

During the four years' investigation there were recorded some 900 voltage surges yielding over 3,000 Lichtenberg figures, 21 cathode-ray oscillograms of

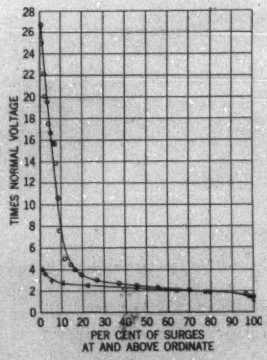


Fig. 10—Voltage Surges, 1927, 1928, 1929, 1930. Transmission Records

oo = 143 surges—lightning plus lightning and switching
xx = 98 surges—switching

lightning voltages on the line, and two direct-hit records.

Klydonograph Records on Line and Substations. The voltages recorded on the system by the klydonograph have, for purposes of analysis, been classified as to cause into three groups: first, those due to lightning, or lightning and switching (the condition obtaining when a lightning flashover of the line causes line trip-out); second, those due to switching operations on the line itself; and third, those of unknown origin, or caused by disturbances on parts of the lower voltage system not under investigation. The first two groups have been further separated, as to physical location; one, those occurring on the line; and two, those occurring at substations. This segregation was made to observe the relative magnitude and frequency of transient voltages on the line itself, and on the substation apparatus separately.

The record of transient voltages on the transmission line is given in Fig. 10, where it is shown that switching surges reach a maximum of four times normal (normal being taken as the crest voltage of the 60-cycle voltage from line wire to ground—108 kv. for the 132-kv. lines). The lightning voltages reach a maximum value of 26 times normal; and 10 per cent of the surges were above seven times normal.

To determine the relative frequency of these high lightning voltages on lines in the territory investigated, the total lightning year miles for the four years of the investigation were computed (using total lengths of Turner-Cabin Creek and Glen Lyn-Roanoke and one and one-half miles beyond the last klydonograph on the Turner-Logan Line) and by using Fig. 10 the following Table II was obtained:

TABLE II—LIGHTNING SURGES PER 100 MILES OF LINE PER YEAR

Times normal voltage	Crest kv.	No. of surges
7	757	10.9
10	1,060	8.3
15	1,620	4.6
20	2,160	3.5
25	2,700	1.0

By comparison with previously published data, it is apparent that the lines investigated are in a very severe lightning territory. For example, they are subject to 4.6 lightning voltages of 15 times normal per 100 miles of line per year.

Lightning voltages appearing at the substations are not as high as those which occur on the line. The substation records are shown in Fig. 11 where the highest lightning voltage is 12 times normal. Only 4.5 per cent of the surges are above 7 times normal, or 9.2 in number against 14.3 on the line (Fig. 10). This would be expected as in every case the substations were protected by lightning arresters and furthermore, there

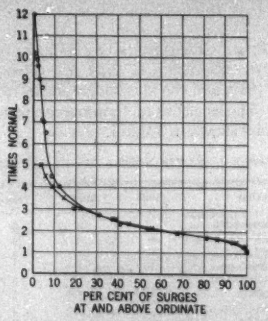


Fig. 11—Voltage Surges, 1927, 1928, 1929, 1930. Substation Records

o = 205 lightning plus lightning and switching
x = 170 switching

are in general divided paths at substations into which an incoming surge can be dissipated. Totaling the substation lightning years (22 for the four-year period) it appears that a station is subjected in a ten-year period, on an average basis, to five surges above seven times normal (757 kv.) to two surges above ten times normal (1,060 kv.), and to one-half surge of twelve times normal (1,300 kv.).

In addition to the surges indicated on the curve for

switching surges, five records were obtained which indicated from 6.2 to 9.0 times normal voltage. These were all recorded at Turner on the middle phase potentiometer where close physical spacing had raised the ratio. Therefore, these values were discounted in the curve, as it is believed that they were actually not greater than the maximum, in the order of six times normal, as measured on other parts of the system and in other investigations.

The magnitude of switching surges at the substation is of the same general order as observed on the line, being slightly higher, however, as would be expected.

The voltage surges of unknown origin, or due to disturbances foreign to the lines investigated, were relatively unimportant reaching a maximum of 3.8 times normal, except in two cases where 4.5 and 4.0 times normal were recorded at the end of the South Point 132-kv. line.

During the two winters (1927-1928) when klydonographs were in operation at substations, the voltage surges recorded were very few reaching a maximum of 3.5 times normal, and occurring at the time of switching operations.

Effect of the Ground Wire—Klydonograph Records. The beneficial effects of the use of a ground wire are clearly indicated by the records obtained on the Turner-Logan line, as shown in Table III.

TABLE III—LIGHTNING VOLTAGES ABOVE 700 KV. RECORDED ON TURNER-LOGAN 132-KV. LINE WIRES

1927–1928 No ground wire	1929–1930 One ground wire
− 700	−2,240
−1,700	−2,400
−1,800	
+1,900	
2,100	
−2,900	

While the numerical order of the highest voltages observed are the same, as might be expected due to the line insulation limiting this voltage, there are three times as many surges of a magnitude which might be expected to give trouble on the line or station apparatus when no ground wire exists. It should also be pointed out that in the two years of operation with a ground wire about twice the extent of line was investigated with the klydonograph as was the case during the previous two years of operation without a ground wire which further accentuates the indicated advantage of the ground wire.

Lightning Arrester Records—Klydonograph. Lightning voltages at the two substations where investigation was made on lightning arresters are given in Table IV. In only two cases was a discharge current recorded in the arrester, one of these where the maximum voltage on any phase was −970 kv., and the other where the voltage was − 1,300 kv. The corresponding discharge currents were − 300 and − 500 amperes. In another instance, at Saltville, a voltage of + 1,100 kv. was recorded without a current discharge measured on the arrester. The minimum current in the arrester which could be measured was about 80 amperes, which, with the 30-ohm Zircon resistor used, produces 2,400 volts on the klydonograph, which is near the low limit of recording.

The arresters used are of such a rating that they are expected to begin discharge at a voltage of about 700 kv. Some of the records listed in Table IV indicate voltages considerably above 700 kv., both with and without discharges.

It should however be pointed out that three factors were present which might account for voltage records higher than those which actually existed:

1. The voltages were recorded some 150 circuit ft. from the terminals of the arrester.
2. Tests have shown that stray fields which exist in the vicinity of substation structures have, under certain conditions, a marked influence on the klydonograph records.
3. No flashover or apparatus failure occurred in the substation at the time.

Ground Resistance and Line Flashover. An attempt to relate line flashover to tower footing resistance proved futile. Reference to Figs. 18A, B, and C indicates some tendency for flashovers to concentrate near the points of high elevation on the line, a fact which correlates with the reasonable expectation that lightning will strike to the most exposed points in the storm path; and these points are those of highest elevation.

Wave Fronts of Lightning Surges—Klydonograph Records. The klydonograph records of 21 lightning

TABLE IV—132-KV. LIGHTNING ARRESTER PERFORMANCE 1929–1930

Location	Line voltage–kv.			Arrester current–amperes			
	Phase 1	Phase 2	Phase 3	Phase 1	Phase 2	Phase 3	Total
Kingsport S. S.	− 890	± 240	−970	*	±270	−340 +280	+ 625
Kingsport S. S.	+ 288	+ 524	+322	*	None	None	None
Kingsport S. S.	+ 288	+ 465	+322	*	None	None	None
Kingsport S. S.	−1000	−1,300	−900 +270	*	−500	−650	−1180
Kingsport S. S.	+ 325	− 512	+247	None	None	None	*
Saltville S. S.	+ 640	+1,100	+825	None	None	None	*

*Not measured.

surges of 600 kv. and above obtained during the four years were analyzed to determine the wave fronts. The analysis is given in Table IX. It is not contended that this method of determining wave fronts is very accurate, but it is believed the record may show the predominating characteristics of the wave fronts. Fast fronts of one microsecond or less were observed in nine of the 21 surges, and wave fronts of the order of five microseconds were observed for 11 surges. Only

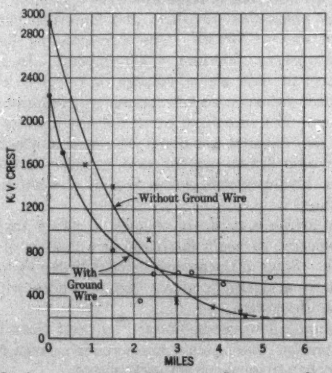


FIG. 12—ATTENUATION OF NEGATIVE LIGHTNING SURGES ON LINE, WITH AND WITHOUT GROUND WIRE

x—No. 233—line trip-out
o—No. 424—line trip-out

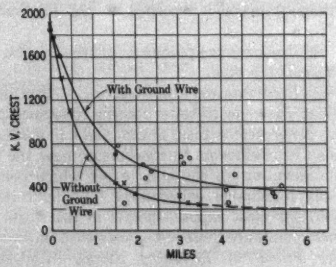


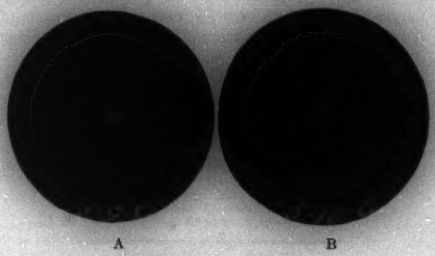
FIG. 13—ATTENUATION OF POSITIVE LIGHTNING SURGES ON LINE, WITH AND WITHOUT GROUND WIRE

x—No. 169—no line trip-out
o—No. 424—line trip-out

one slow front surge was observed, this being a positive surge of 685 kv. No positive surge with a fast front (one microsecond or less) was found.

Since the records analyzed were of surges above 600 kv., and the 60-cycle crest flashover of the line is of the same order, it is believed these surges were recorded near the point where the surge originated. The conclusion is therefore drawn that at the point of inception some lightning voltage surges have fast fronts, in the order of one microsecond or less, the corresponding rates of voltage rise being as high as 2,500 kv. per microsecond.

Voltages on Top, Middle, and Bottom Conductors. At the start of the investigation it was expected to find some relation between the voltages observed on the top, middle, and bottom conductors on a transmission line. That no fixed relation exists on the line under service lightning conditions is apparent from Tables VIII, VI-A and VI-B. An average of all voltages in Table VI-A and VI-B (the highest being taken where the surge was oscillatory) shows 1,165 kv. on the top, 1,060 kv. on the middle, and 965 kv. on the bottom conductor. These are 100 per cent, 91 per cent, and 82.6 per cent in terms of relative voltage. There are individual cases, however, where the voltages on the bottom and middle conductors are equal and considerably higher than on the top conductor.



FIGS. 14A, 14B—DIRECT-STROKE RECORDER RECORDS AT TOWERS 9 AND 10

The conclusion drawn from these data is that over a period of time the top conductor will be subjected to an average voltage of 10 per cent higher than the middle or bottom conductor, and the middle conductor will be subjected to a voltage 10 per cent higher than the bottom conductor. Individual surges may give values just the reverse of this.

In the event of induced strokes the magnitudes of the voltages on the various conductors in vertical configuration would be expected to be almost equal with a ground wire since the amount of protection afforded by a ground wire against induced strokes practically equalizes the difference in the heights above ground. Referring to Table VIII, the first 13 surges were recorded before the ground wire was installed and the last four after the ground wire was installed. These records show that no particular change in the distribution of voltages occurred as a result of the ground wire. Actually the differences to be expected would not be easily detected with the klydonograph. In general the differences between the voltages on the three conductors for the lower valued surges were not great, which might be

taken either as an indication of induced strokes or a direct stroke to the ground wire which did not flash over. In the event of direct strokes to tower or ground wire, which cause flashover, a random distribution of voltages would be expected depending upon the number of conductors which flashed over. If a flashover occurred on only one conductor it would be just as likely to do so on the bottom conductor since the coupling between the various conductors and ground wire would be least

FIG. 15—KLYDONOGRAPH RECORD AT TOWER 9 AT THE TIME RECORDS IN FIGS. 14A AND 14B WERE RECORDED

on the bottom conductor, due to its greater distance of separation, and therefore the potential difference between this conductor and the tower would be greatest. In case of a stroke to a conductor it might be expected to take place on the top conductor most often, as it is most exposed, and with no ground wires the top conductor should be subjected to the highest surges. The records show that in a large percentage of the higher valued surges there was a wide variation between the voltages on the different conductors.

TABLE V—SURGE CHARACTERISTICS OF NATURAL LIGHTNING
1929–1930

No.	Polarity	Crest kv.	Wave characteristics Front (1)	Tail (2)	Length (3)	Lightning
1(5)	+	185	2.0	6.5	10.5	Moderate
2(5)	+	180	5.0	15.0	35.0	Slight
3(5)	−	160	2.5	8.0	15.0	Moderate
4	−	70	2.0	5.0	7.0	Severe
5	−	75		Indefinite		Moderate
6	−	75	1.0	5.0	10.0	Moderate
7	+	75		Indefinite		Severe
8	−	100		Indefinite		Slight
9	+	100	15.0	55.0+	70.0+	Slight
10	+	110	2.0	80.0+	80.0+	Moderate
11	−	120		Indefinite		Slight
12	+	125	2.0	15.0	25.0	Severe
13	+	125	2.0	20.0	50.0	Severe
14	+	140	2.0	25.0	50.0	Severe
15	+	150	2.0	30.0	55.0	Slight
16	+	175	12.0	35.0	40.0	Severe
17	+	230	2.0	52.0	60.0	Severe
18	+	240	2.0	45.0	50.0	Severe
19	−	250	35.0	80.0+	80.0+	Slight
20	+	350(4)	2.0	8.0	80.0	Severe
21	+	1,000(4)	2.0	3.0	4.0	Severe

(1) Microseconds to crest.
(2) Microseconds to 50 per cent crest on tail.
(3) Microseconds to first zero.
(4) Line trip-out.
(5) 1929 records

Cathode-Ray Oscillograms. During the two years the oscillograph was in operation, 21 oscillograms were obtained of natural lightning. The characteristics of these surges are shown in Table V. Excluding waves below 125 kv. in magnitude, also No. 20 and No. 21, due to line flashovers occurring at the time, this table may be summarized as follows:

NATURAL LIGHTNING VOLTAGE CHARACTERISTICS
MICROSECONDS

	Front	to 50% tail	Length
Minimum	2.0	6.5	10.5
Maximum	35.0	80.0+	80.0+
Average	6.2	30.1	43.1

The oscillograms of lightning voltage obtained at Tower 20 on the top phase of the Turner-Logan circuit under test when line trip-outs occurred are shown in Figs. 16 and 17. Fig. 16 shows the largest surge recorded, which was of the order of 1,000 kv. It is oscillatory in nature, having a period of the order of ten microseconds, with higher frequency oscillations superimposed. This surge was accompanied by a flashover. Although it was not possible to fit these oscillations to readily apparent circuit constants, it is possible for reflection phenomena to be quite complicated, and the oscillations are believed to be due to reflections resulting from the line flashover rather than to oscillations in the

FIG. 16—OSCILLOGRAM OF LIGHTNING SURGE AT TOWER 20 AT THE TIME FIGS. 14A AND 14B AND FIG. 15 WERE RECORDED

lightning stroke itself. The superimposed high frequency oscillations are probably due to reflections of the wave between tower top and ground.

The klydonograph record at Tower 20 correlating with this surge indicated voltages of + 685 and − 625 kv. on the top phase, + 685 and − 625 kv. on the middle phase, and + 610 kv. on the bottom phase. Taking into consideration the 60-cycle potentials which influence klydonograph records, but were eliminated in the oscillograph circuit, the voltages from the cathode-

ray oscillograph record (+ 600 and − 1,000 kv.) check reasonably well the klydonograph data.

The correlating klydonograph record at Tower 9, Fig. 15, which is 3.07 miles from Tower 20, is interesting in indicating negative impulse voltages with a very fast front on the ground wire and top conductor, with a slower front negative on the middle wire, and no appreciable negative on the bottom conductor, and also high positive voltages on all line conductors.

The polarity analysis of the surges recorded by the oscillograph shows 13 positives, 7 negatives, and one oscillatory.

Records of Special Interest

a. Arcing Grounds. All of the voltages recorded by the klydonograph were either unidirectional or highly damped oscillations with the exception of one record which showed a sustained oscillation similar to those

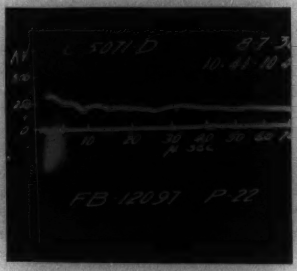


Fig. 17—Lightning Surge Recorded at Tower 20

recorded on isolated neutral systems. Upon investigation it was found that at the time this record was obtained, the system connections were such that the line to which the klydonograph was connected was operating momentarily ungrounded. The voltages recorded at the time were 315 kv. (3 times normal) on two phases, and 215 kv. (2 times normal) on the other phase on which the arc was later located.

b. Bushing Flashover-Klydonograph Record. Klydonographs registered a potential of 860 kv. across the high- and low-tension bushings of a transformer during a lightning storm at Glen Lyn in 1927, which checked with the air line distance at which breakdown would be expected to take place. The occurrence was attributed to a direct stroke of lightning to the 88-kv. system near a point where the 88-kv. and 132-kv. systems crossed. It is believed a negative direct stroke to the 88-kv. line transmitted a voltage to the transformer which was registered as − 650 kv. on the 88 kv. bushing; and the induced voltage on the 132-kv. line entered the station at the same time where it was recorded at + 210 kv., giving a total voltage of 860 kv.

c. Lightning Voltages and Trip-Outs. Surges recorded above 600 kv., which caused line trip-outs are listed in Table VI-A; those above 600 kv., which did not cause line trip-outs are listed in Table VI-B. As would be expected, many of the surges in Table VI-A are considerably above the 60-cycle crest flashover of the line insulation (about 600 kv. crest), indicating that some flashovers occurred on the front of the wave. The voltages listed which are near the 60-cycle crest flashover of the line are in some cases, at least, attenuated values of surges originating at some distance from the point of measurement.

TABLE VI-A—LIGHTNING SURGES ABOVE 600 KV. ON 132-KV. LINES RECORDED ON LINE WIRE AND CAUSING LINE TRIP-OUT

Ref.	Kv. Top	Kv. Mid.	Kv. Bot.
8	+ 635	+ 575	+ 600
13	− 1800	− 1500	+ 650
		+ 980	− 620
14	− 1080	− 1020	− 580
			+ 375
15	− 735	− 625	− 390
			+ 190
16	− 2900	− 2900	−1,600
			+ 680
23	− 890	± 240	− 970
28	+ 236	− 1010	−1,010
29	−2,500	± 300	± 300
36	+1,850	+1,610	+1,780
	−2,240	−1,710	
37	+ 320	+ 470	+ 750
	− 525	− 308	−1,470
44	+ 705	+ 256	+ 770
	− 820		− 525
46	+ 610	+ 550	+ 480
	− 350	− 615	
47	+ 685	+ 685	+ 610
	− 625	− 625	

TABLE VI-B—LIGHTNING VOLTAGES ABOVE 600 KV. NOT CAUSING LINE TRIP-OUT ON 132-KV. SYSTEM

Ref.	Top φ	Mid. φ	Bot. φ	Grd. wire	Where recorded
2(1)	− 710	− 212	− 238		Substa.
10	− 700	± 300	+ 300		Substa.
12	+1400	+1900	+1100		Line
20	±1800	±1290	(2)	−25	Line
22	+1710	+1710	+1710	+30	Line
24	−1800	−2400	−2400	−30	Line
40	+ 640	−1100	+ 825		Substa.
41(1)	± 345		± 640		Substa.
42	− 805	− 745	− 925		Substa.
19(3)	−1190	−1695	−1695	−35	Line

Notes: (1) On 88-kv. system.
(2) Not recorded.
(3) Line not energized

A factor which might account, in some cases, for recorded surge voltages considerably above line flashover voltages without a corresponding trip-out is that when a direct hit to the tower occurs, the potential of the tower is altered with respect to ground due to the surge impedance of the tower and tower footing resistance. The voltage across the insulator strings, being the difference between the potential of the line conductor and tower top, will not, therefore, be the same as the potential of the conductor to ground.

Applying this consideration to Table VI-B fails, however, to explain an absence of line trip-out for all cases listed, as flashover in all probability occurred for the higher valued records such as surges, references 12, 20 and 24.

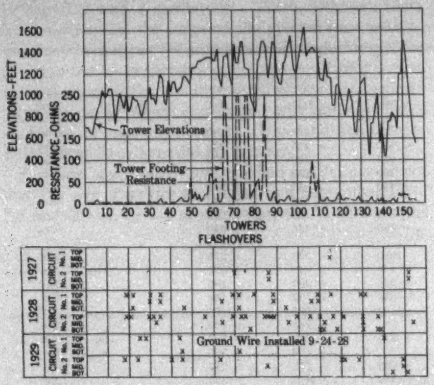


Fig. 18a—Line Flashover Record of Turner-Logan Line

of natural lightning waves on the line gave a limited amount of data. These data were obtained on the Turner-Logan line under two conditions, first, before the ground wire was installed; and second, after it was installed. Figs. 12 and 13 show the attenuation of the crest values of the lightning voltages over about five miles of line.

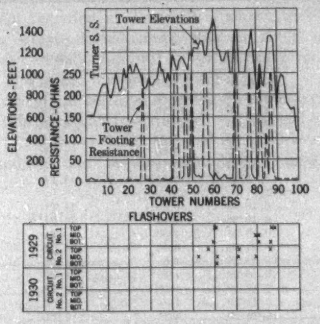


Fig. 18c—Line Flashover Record of Turner-Cabin Creek Line

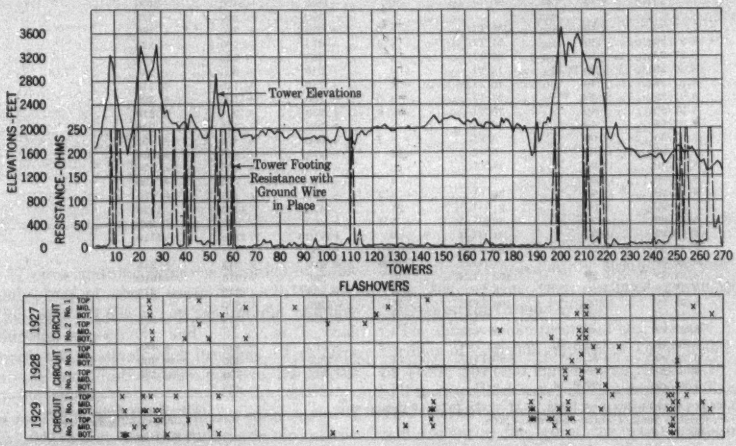


Fig. 18b—Line Flashover Record of Glenlyn-Roanoke Line

An important fact brought out by these data is that voltage surges well above the flashover value of the line occur without causing line outages and the corollary follows that lightning impulse flashover on insulators is not necessarily accompanied by power follow-up.

d. Attenuation of Lightning Surges—Klydonograph Records. The attenuation study with the klydonograph

Two characteristics of attenuation are brought out by these data, so far as the accuracy of the klydonograph in this type of analysis permits. One is that the attenuation is greater for positive lightning voltage surges than for negative surges; and the other is that the attenuation is greater when there is no ground wire. It shows definitely that the attenuation for high-voltage

surges is very rapid, the maximum crest values in the order of 2,000 kv., being reduced to one-half value within about a mile. Considering the influence which the wave steepness and length has on attenuation and the indefiniteness of these values of the waves here recorded, these curves agree quite closely with those obtained in artificial surge investigations. Where the line was not equipped with a ground wire, and the surge was negative (surge 233), the attenuation appears less rapid at the start. The data from one surge only, however, are too meager to justify great confidence in the particular shape of the curve.

The data obtained on attenuation of natural lightning voltage surges show that klydonographs must be placed at frequency intervals along the line (one mile or so apart) if any quantity of information is to be obtained. It is also clear from the curves in Figs. 12 and 13 that the klydonograph data can only record the general order of attenuation and are accurate enough to indicate the trend, but not to study the effects produced by the presence of ground wires, positive and negative surges, etc., where the relative voltages to be measured may vary some 20 per cent or less.

TABLE VII—SURGES ON GROUND WIRE

Ref.	Date	Time	Tower location	Surge kv. Ground wire	Top	Mid.	Bot.	Cause	Line T. O.
1	7/16/27	6:27 P. M.	4 TR	+ 2.3		± 285	± 240	Ltn.	No
2	8/ 1/27	10:20 "	4 TR	+ 9.6	‡ 520	± 285	± 230	"	Yes
3	8/ 1/27	10:20 "	4 TR	+ 9.0	+ 285	+ 220	+ 270	"	Yes
4	8/ 1/27	10:31 "	4 TR	+ 5.0	+ 190	+ 230		"	No
5	8/ 6/27	3:30 "	4 TR	− 4.7	+ 180	+ 255	+ 210	"	No
6	8/ 8/27	5:30 "	4 TR	+14.8	+ 340	+ 515	+ 240	"	No
7	9/18/27	10:52 "	4 TR	+20.0	± 585	± 285	+ 285	"	Yes
8	6/ 9/28	2:55 "	14 TL	+ 4.0*	+ 435	+ 435	+ 340	"	Yes
							− 490		
9	6/16/28	5:30 "	14 TL	− 6.2*	−1080	− 920	+ 270	"	Yes
10	6/19/28	9:30 "	14 TL	+ 3.3*	± 180	+ 340	+ 230	"	No
11	6/20/28	3:50 "	14 TL	+ 3.8*	+ 270	+ 300	+ 180	"	No
12	8/21/28	6:10 "	5 TR	+ 6.8	+ 230	+ 160	+ 160	"	No
†13	8/24/28	1:00 "	14 TL	−35 *	−1190	−1695	−1695	"	No
14	6/18/29	1:10 "	5 TL	+20		+ 245	+ 190	"	Yes
15	6/18/29	1:10 "	9 TL	+10	+ 245	+ 250	+ 200	"	Yes
16	6/22/29	4:35 "	5 TL	+ 7.5		+ 250	+ 300	"	No
17	6/22/29	4:35 "	9 TL	+ 5.7	+ 260	+ 275	+ 215	"	No
18	7/ 1/29	4:30 "	20 TL	−20	+ 250	+ 340	+ 250	"	No
19	7/ 1/29	4:30 "	30 TL	+ 3.0	+ 230	+ 250	+ 230	"	No
20	7/ 1/29	4:30 "	36 TL	+ 2.7				"	No
21	7/ 1/29	4:50 "	36 TL	+ 3.0		+ 295		"	Yes
22	7/ 9/29	4:15 "	30 TL	−30	−1800	−2400	−2400	"	No
					− 360	− 360	− 220		
23	7/ 9/29	4:15 "	36 TL	+ 3.0	+ 250	+ 270	+ 180	"	No
24	8/27/29	7:30 "	24 TL	+ 4.5	+ 275	+ 190	+ 225	"	Yes
25	8/27/29	7:30 "	30 TL	+ 4.	+ 255	+ 250	+ 185	"	Yes
26	8/27/29	7:40 P. M.	14 TL	+ 4.	+ 240	+ 300	+ 250	Ltn.	Yes
27	8/27/29	7:40 "	16 TL	± 4	+ 110	+ 110	+ 110	"	Yes
28	8/27/29	7:40 "	24 TL	+ 4.0	+ 250	+ 190	+ 225	"	Yes
29	8/27/29	7:40 "	30 TL	+ 3.6	+ 230	+ 225	+ 210	"	Yes
30	8/27/29	7:40 "	36 TL	+ 4.5	+ 255	+ 355	+ 240	"	Yes
31	8/ 7/30	11:00 "	5 TL	+ 6.2	±1030	± 486	+ 352	"	Yes
32	8/ 7/30	11:00 "	9 TL	−35	±2175	±1665	+1920	"	Yes
33	8/ 7/30	11:00 "	14 TL	±10	± 985	+ 265	± 770	"	Yes
34	8/ 7/30	11:00 "	20 TL	± 4	+ 600	± 775	+ 520	"	Yes
35	8/ 7/30	11:00 "	30 TL	+ 6.2	+ 250	+ 375	+ 270	"	Yes
36	5/30/29	4:00 "	21 GL-R	25	1420	1420	1420	"	Yes
37	5/31/29	3:30 "	82 GL-R	± 5.4	+ 490	+ 245	+ 290	"	Yes
38	6/18/29	10:30 "	21 GLR	+30	+1710	+1710	+1710	"	No
39	7/10/29	5:00 "	82 GLR	−35	+ 255	+ 325		"	No
40	8/22/29	6:20 "	42 GLR	+14.8	+ 220	+ 215	+ 215	"	Yes
41	8/22/29	6:20 "	162 GLR	−30	± 236	±1010	±1010	"	Yes
42	8/21/28	4:40 "	8 TCC	+ 9	+ 265	+ 315	+ 215	"	No
43	8/29/28	5:00 "	168 TCC	−25	±1290	±1800		"	No
44	6/22/29	4:35 "	8 TCC	+ 7.3	+ 250	+ 350	+ 295	"	No
45	6/22/29	8:30 "	89 TCC	+ 3.7				"	No
46	8/23/29	4:55 A. M.	168 TCC	− 3.3	± 525	± 525	± 485	"	Yes
47	8/23/29	4:55 "	89 TCC	+15	−2520	± 300	± 300	"	Yes
48	9/ 7/29	12:30 "	30 TCC	+ 8.2	+ 275	+ 300	+ 230	"	No
49	9/24/30	5:20 P. M.	9 TL	−12.1	+ 450	+ 455		"	No

†Line not alive at this time, being out of service to install ground wire.
*Measured at peak of tower—no ground wire.
‡Horizontal tower.

TR—Turner-Rutland Line.
TL—Turner-Logan Line.
GLR—Glenlyn-Roanoke Line.
TCC—Turner-Cabin Creek Line

e. Surges on the Ground Wire. Voltages recorded on the ground wire by the fourth electrode of the klydonograph with the corresponding voltages recorded on the line conductors are listed in Table VII. It is not believed these records are accurate as to voltage magni-

tude, as the klydonograph which was located near the ground was connected to the ground wire by a vertical wire about 100 ft. long extending up the side of the tower. However, they are interesting in three respects. First, they are recorded only at times of lightning storms. Second, of the 47 surges, 33 are positive, 10 negative, and 4 oscillatory. Third, although there is a tendency for the surge on the ground wire to have the same polarity as the surges on the conductors, there is a number of exceptions where the conductor surges are oscillatory with a unidirectional surge on the ground wire, and again where the ground wire and conductor surges are of opposite polarity. This condition obtains both when a line did or did not trip out.

f. Pure Lightning Surges. When the Turner-Logan line was out of service in August and September 1928, for installation of the ground wire, the klydonographs recorded 17 pure lightning surges as recorded in Table VIII. Only one of these, recording − 1,190 kv., − 1,695 kv., and − 1,695 kv. on top, middle, and bottom conductors, respectively, and − 35 kv. at the peak of the tower, was relatively high in value. This surge was negative; and all others were positive, the highest positive being 385 kv.

Except for the high negative voltage surge recorded, it is remarkable to note that the voltages on top, middle, and bottom conductors are nearly equal on all three conductors, there being no tendency for higher voltages to be recorded on the top phase. The average for all 17 surges shows 17 per cent higher voltage on the middle conductor than on the top, and 9 per cent higher on the bottom conductor.

Assuming, since they were of relatively low magnitude, that the positive surges might have been caused by induced lightning strokes, it appears that the induced voltages are approximately equal in magnitude on all three phases when in vertical configuration, whether the ground wire is installed or not.

g. Direct-Stroke Recorder Records. Two records were obtained on direct-stroke recorders, both being associated with one storm, and believed to be due to one lightning stroke on the Turner-Logan line. Simultaneous records were obtained on the klydonographs between Turner substation and Tower 36. The klydonogram recorded at Tower 9 is shown in Fig. 15. Also, a simultaneous record was obtained on the cathode-ray oscillograph at Tower 20, three miles distant. This record is reproduced in Fig. 16A.

The maximum voltages recorded on the line conductors of this circuit at Tower 9 were − 2,200 and + 1,850 kv., − 1,700 and + 1,610 kv., and + 1,780 kv. on the top, middle, and bottom conductors, respectively. The voltage recorded on the ground wire was − 34 kv. This low voltage was discussed previously under "Surges on the Ground Wire."

The direct-stroke records, Figs. 14A and B, indicate currents of − 46,000 amperes in Tower 9 and − 97,000 in Tower 10. No current was recorded in Tower 8, and the recorders were not in operation at Towers 7 and 11. The circuit tripped out three times within twenty minutes during this storm, the relay targets of the line under test indicating on the middle and bottom conductor in the first case; on the top conductor in the second case; and on the top and bottom conductors in the third case. The only flashover which could be located on this section of the line, by an inspection after the storm, was on the top phase at Tower 8 of the circuit under test. It should be pointed out that the above currents were those measured in the tower footing. Since the current in a lightning stroke on hitting a tower

TABLE VIII—PURE LIGHTNING SURGES ON TURNER-LOGAN LINE
8-1-28 to 9-24-28
(Ground wire being installed)

	Ref.	Date	Time	Location	Surge kv. Top	Surge kv. Mid.	Surge kv. Bot.	Cause
GROUND WIRE Not Installed	1	8- 1-28	7:20 P. M.	9	+ 240	+ 220	+ 200	Lightning
	2	8- 1-28	7:20 "	14	+ 200	+ 220	+ 200	"
	3	8- 1-28	7:20 "	20	+ 150	+ 200		"
	4	8- 1-28	7:40 "	9	+ 300	+ 300	+ 270	"
	5	8- 1-28	7:40 "	14	+ 220	+ 270	+ 240	"
	6	8- 1-28	8:00 "	9	+ 270	+ 240	+ 220	"
	7	8- 1-28	8:00 "	14	+ 180	+ 200	+ 200	"
	8	8- 4-28	12:0 "	9	+ 240	+ 240	+ 215	"
	9	8- 4-28	12:20 "	14	+ 255	+ 265	+ 240	"
	10	8- 4-28	12:15 "	20	+ 200	+ 220	+ 200	"
	11	8- 4-28	5:45 "	9	+ 180	+ 180	+ 180	"
	12	8- 4-28	5:45 "	14	+ 155	+ 205	+ 155	"
	13	8- 4-28	5:45 "	20	+ 110	+ 110	+ 110	"
GROUND WIRE Installed	14	8-21-28	5:10 "	9	+ 385	+ 300	+ 255	"
	15	8-21-28	4:45 "	14	+ 225	+ 265	+ 245	"
	16	8-21-28	4:45 "	20	+ 215	+ 215	+ 200	"
	17	8-24-28	1:00 "	14	−1,190	−1 695	−1,695	"
Average					277	314	302	
Per cent					100	113	109	

TABLE IX—WAVE-FRONT DETERMINATION OF LIGHTNING SURGES FROM KLYDONOGRAPH RECORDS. 21 SELECTED SURGES OF 600 KV. UP

Ref.	Location	Polarity and microsecond fronts			
		Top	Mid.	Bot.	Gr. wire
2	Glenlyn 88 kv	−Fast*			
4	Glenlyn 88 kv	−Fast*			
8	Tower 4-T-R	+Slow	+Slow	+Slow	
10	Turner bus	+5	+Slow	+Slow	
12	Tower 9-T-L	+5	+5	+5	
			−5		
13	Tower 20-T-L	−Fast	+Slow	+Slow	
14	Tower 14-T-L	−5	−5	±5	−5
15	Tower 9-T-L	−5	−5	±5	
16	Tower 20-T-L	−Fast*	−Fast	±5	
19	Tower 14-T-L	−Fast*	−Fast	−Fast	−Fast
20	Tower 68-T-CC	−Fast*	−5		−Fast
23	Kingsport 132 kv	−5		−5	
24	Tower 30-T-L	−Fast*	−Fast	−Fast	−Fast
26	Kingsport 132 kv	−Fast*	−Fast	−Fast	
28	Tower 162-GLR	−5	−Fast	−Fast	−Fast
36	Tower 9-T-L	±Fast*	±5	+5	−Fast
37	Tower 5-T-L	±5	±5	±5	±5
38	Switchback 132 kv	+5	+5	+5	
39	Glenlyn 88 kv	+5	+5	+5	
40	Saltville 132 kv	+5 +	+5 +	+5 +	
42	Switchback	−5	−5	−5	
	Of 21 surges	9 Fast*	5 Fast*	4 Fast*	5 Fast*
		11-5 μ sec.	11-5 μ sec.	11-5 μ sec.	2-5 μ sec.
		1 Slow	3 Slow	3 Slow	

*Less than one microsecond.

divides into multiple paths consisting of the tower, the ground wire in both directions, and such line conductors as become involved, and since the stroke current is further modified by the reflected wave from the ground when it reaches the tower top, the maximum current in the stroke probably is not the same as the ground current. Calculations for typical cases indicate that the latter is of the order of 120 per cent of the maximum stroke current.

Conclusions

1. In general the conclusions to be drawn from these tests are similar to those in previously published publications but contain quite a number of additional points of considerable interest.

2. The lightning voltages recorded on the line reached magnitudes of 2,500 kv. Ten per cent of the total recorded were above 750 kv. Reduced to a basis of surges per 100 miles of line per year there were nine surges above 1,060 kv.

3. At substations lightning voltages reached a magnitude of 1,200 kv. Reduced to a yearly basis there were 0.2 surges per substation per year above 1,060 kv. Comparing this with the above paragraph it seems that a substation is equivalent to 2.2 miles of line in being subjected to lightning surges of this magnitude.

4. Lightning surges of the order of 2,500 kv. have been found on a line without trip-out, which, with the insulation values used, indicates clearly impulse flashover of the line without power follow-up.

5. Switching surges as high as six times normal voltage from line to neutral at substations and four times normal on the line were recorded, which confirms past data.

6. Except at times of lightning occurrence impulse voltages on these transmission lines throughout the year are of minor importance, not exceeding 3.5 times normal.

7. Attenuation of lightning surges is rapid for high voltages, attenuating from 2,000 kv. to 1,000 kv. in one mile. Keeping in mind the limited accuracy of the klydonograph the records indicate that the attenuation of lightning surges is more rapid with positive than with negative voltages and also it is more rapid where the line is not equipped with a ground wire.

8. Lightning arresters must be placed close to the equipment they are intended to protect. Voltages some 60 to 80 per cent higher than the value to which the arrester should hold the voltage were recorded some 150 ft. distant from the arrester.

9. The beneficial effect of the ground wire in reducing the lightning voltages is indicated by the fact that six surges above 700 kv. were recorded during a period with no ground wire and only two above 700 kv. during a similar period when one ground wire is used.

10. The maximum current recorded in a tower struck by lightning was in the order of 100,000 amperes which calculations indicate to be of the order of 120 per cent of that in the lightning stroke. The records indicate a current of 50,000 amperes in one adjacent tower and calculations indicate that the voltage may have been sufficient to flash over the line conductors on a total of five towers. Calculations also indicate that the voltage of the tower top struck was of the order of 4,000 kv. The voltage measured at the oscillograph station three and one-half miles distant was one million kv.

11. The characteristics of natural lightning on transmission line investigated with the cathode-ray oscillograph show minimum and average fronts of 2.0 and 6.2 microsecond fronts; 6.5 and 30.1 microseconds to 50 per cent on the tail; and 10.5 and 43.1 microseconds total lengths. A maximum front of 35.0 and total length of 80.0 microseconds were observed. None of these surges so far as is known was recorded near the point of origin. The majority of the waves were positive but the highest recorded was negative.

12. An arcing ground producing oscillatory voltages of three times normal to ground was observed in one case at a time when the system was operating ungrounded. This shows a type of voltage that is dangerous; but it was not as high as theoretical analysis shows as the maximum.

13. Wave fronts of natural lightning recorded by the klydonograph apparently near the point of origin have indicated that negative surges have wave fronts of one microsecond or less. Positive lightning surge records indicated that the fastest fronts were in the order of five microseconds. This indicates that nega-

tive lightning voltage surges initially appear on transmission lines with steeper fronts than positive waves.

14. The relative lightning voltages on top, middle, and bottom conductors of a vertical configuration circuit for voltages over 600 kv. (approximately 60-cycle crest flashover) appear as a matter of chance. Cases were observed (Tables VI-A and B) where the highest voltage appears on top, middle, or bottom conductor and also on combinations of any two conductors. In general there was less variation between the voltages on different conductors in the case of lower voltage surges. It seems reasonable to assume that where a surge produced approximately equal voltages on the three conductors, and where no line trip-out occurred, that the surge was caused by an induced stroke and that where the relative voltages on the three conductors vary widely the surge was due to a direct stroke.

15. The records of high voltages on the line at times of line trip-out (Table VI-A) indicate 60 per cent of these are predominantly negative and 40 per cent positive. From a study of flashover locations and tower footing resistance no relation could be found between tower footing resistance and line flashover on this line.

16. Surge voltages were observed on the ground wire only at times of lightning storms. Of these records 70 per cent were positive, 21.5 per cent negative and 8.5 per cent oscillatory. The oscillatory records are believed to be due to reflections on the line rather than oscillations in the lightning stroke.

17. Lightning voltage on a deenergized line (Table VIII) indicates positive voltages of approximately equal magnitude on all three phases regardless of whether the ground wire is installed or not. The approximate equal and low magnitude of the voltages on the three conductors seems to indicate that these were caused by induced strokes. It should also be pointed out that all of these voltages were positive. No high or negative surges were recorded during the period when the ground wire was not installed.

ACKNOWLEDGMENTS

The assistance of the participating groups of this cooperative investigation is hereby acknowledged, and especially the help of the Appalachian Electric Power Company on whose physical property the tests were made. In particular, the authors wish to express their appreciation for the assistance rendered by Messrs. R. C. Slack and F. D. Brook of the Appalachian Electric Power Company, Messrs. R. Sparks and W. R. Ellis of the Westinghouse Electric & Manufacturing Company, and Mr. G. D. Lippert of the American Gas and Electric Company, in carrying on field work and analyzing the records.

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1. *1929 Lightning Experience on the 132-Kv. Transmission Lines of the American Gas and Electric Company*, Fig. 1, by Philip Sporn, A. I. E. E. TRANS., June 1931, p. 574.
2. J. F. Peters, *Electrical World*, April 19, 1924.

Discussion

For discussion of this paper see page 1146.

Experimental Studies in the Propagation of Lightning Surges on Transmission Lines

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Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*A discussion is given of cathode-ray oscillograms showing the attenuation and distortion of artificial lightning surges on a power transmission line, the voltages induced by such surges in parallel wires and the effect on tower footing impedance of earth wires or counterpoises.*

* * * * *

INTRODUCTION

THE study of the laws of propagation of electromagnetic waves along parallel conductors has become of increasing interest to the power engineer, since it was realized that these waves play an important part in lightning disturbances on power transmission systems.

Electromagnetic wave propagation has received considerable theoretical treatment at the hands of the telephone engineer[1] and exact formulas for attenuation and distortion have been derived under certain simplifying conditions. The conditions existing in lightning surges however, differ in several important respects from those encountered in electrical communication in that the circuit parameters (inductance, resistance, capacitance, and leakance) cannot be treated as constants, but must be considered as variable. As mathematical analysis consequently becomes very complicated, the experimental approach to the problem seems particularly desirable.

The present paper presents some of the results of experimental work which has been carried out during the last two summers on the S.19 line of the Consumers Power Company between Croton Dam and Grand Rapids, Michigan. The work was done by the Research Section of the Lightning Arrester Engineering Department, General Electric Company, Pittsfield, Mass., and the Production and Transmission Departments of the Consumers Power Company of Michigan.

Previous papers have reported on different aspects of this work and described the equipment used.[7,9,10] High-voltage surges were applied to the transmission line by means of an impulse generator generating one and one-half million volts on open circuit. The surges were measured at any desired point on the line by means of a portable cathode-ray oscillograph which was connected to the line through a resistance potentiometer.

I. ATTENUATION AND DISTORTION

Choice of Wave Shapes. It had been shown by sphere-gap tests[7] that the rate of attenuation of a surge was affected by its voltage, polarity, wave-shape, and its distribution over one or more conductors. Accordingly, certain test waves were chosen to bring out these features in particular.

A selection of the oscillograms obtained is shown in Fig. 1. For more ready comparison, these have been replotted to a uniform time and voltage scale in Fig. 2.

Corona. The first thing which strikes one in examining these oscillograms is the clearly defined point on the front of the wave at which the effect of corona becomes evident on the high-voltage surges; it is also evident that the wave is attenuated much more rapidly above corona voltage than below it. (The calculated value of critical corona voltage is 85 kv. for a surge on one conductor, 125 kv. for a surge on three conductors in parallel.)

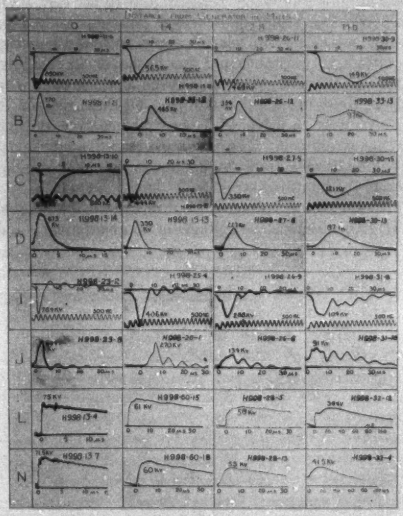

FIG. 1—SELECTED ATTENUATION OSCILLOGRAMS

Resistance. It can also be seen however, that there

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1. For references see Bibliography.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29–May 2, 1931.

are other influences besides corona affecting the surge. This is most clearly brought out by the oscillograms of low-voltage surges in which the voltage is too low for corona to play any appreciable part (there will always be a little corona on sharp points of insulator fittings, etc.).

On these surges we see the type of change to be expected from series resistance in the line. A physical picture of this type of attenuation and distortion can be obtained in the following way:

Consider a rectangular wave of finite length. This is

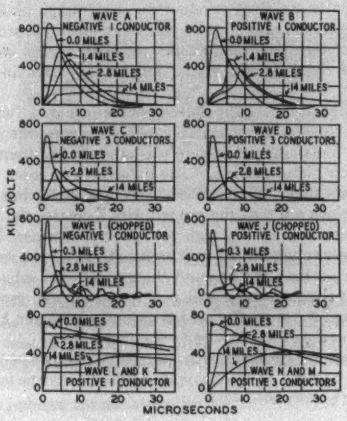

FIG. 2—REPLOT OF OSCILLOGRAMS TO SHOW CHANGE OF WAVE SHAPE

associated with a fixed amount of electric charge on the line. Assuming no leakance in the line, the amount of this charge remains constant although its distribution on the line may change. Then imagine that the resistance instead of being distributed is lumped at regular intervals. At every point at which the surge strikes such a lumped resistance, one part of the wave will be transmitted as a rectangular wave, while the remaining part will be reflected. The front of the wave will therefore undergo a continuous attenuation. The reflected portion will however, again be reflected on its way back so that ultimately it contributes an increase of voltage in the tail of the forward-going wave. If the resistance is distributed instead of lumped, there will not be separate waves but the distortion of a single wave shown in Fig. 3a. The very front of the wave will suffer the greatest attenuation; succeeding parts will be attenuated less and less and the whole wave will be lengthened in such a way that although the voltage of the wave is decreased, the total charge still remains constant. Such a change may be noticed on the oscillograms of the low-voltage waves.

It will also be noticed that the front of the low-voltage waves becomes less and less steep as the surge proceeds along the line. A constant series resistance in the line leaves the perpendicular wave front unchanged; but a constant resistance is not physically possible with a perpendicular front on account of skin-effect which would make the resistance infinite for such a front. This effect makes the resistance higher, the steeper the slope of the wave, and so has the effect of decreasing the slope of a very steep wave. The effect on wave shape as calculated theoretically[3] is shown in Fig. 3b. These effects are all observable in the oscillograms of waves L and N (Fig. 1).

A striking fact is that for a surge on three conductors the slope of front decreases more rapidly than for a surge on one conductor. A possible explanation of this is that the higher concentration of current in the earth in the case of three conductors increases the effective ground resistance. Another cause may be found in the mutual effect between conductors. This effect will be discussed more fully in Section II of this paper.

Distortion. Returning to the case of the high-voltage surges, it will be seen that in these also the maximum voltage is decreased while the whole wave is increased in length. Frequent attempts have been made to calculate the effect of corona by assuming an equivalent leakance, suggested by the fact that the power loss for steady alternating voltages is approximately proportional to the square of the voltage. It is possible however, to get a physical picture of the effect of leakance in the same way as was done for series resistance, and it will be found that attenuation in this case is accompanied by a distortion which decreases the length of the wave. (In the so-called distortionless line of telephone theory, resistance and leakance balance

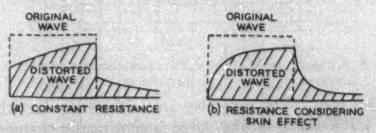

FIG. 3—ATTENUATION AND DISTORTION PRODUCED BY RESISTANCE (THEORETICAL)

each other in such a way that the distortion produced by one is exactly equal and opposite to the distortion produced by the other.) This is not the effect of corona observed experimentally. Closer consideration will show that the conception of corona loss as a leakance is erroneous, since the corona does not, except at very high voltages near sparkover, establish a conducting path by means of which charges may leak off the line. Usually the area surrounding a conductor becomes conducting only within an envelope of limited radius. As the voltage rises above corona voltage, charges enter this envelope thereby decreasing the voltage on the

front of the wave. The charge which enters this envelope will remain there as long as the voltage of the conductor is high; when it falls the charge will return to the conductor, thus holding up the voltage on the tail of the wave. In this exchange of charge between the conductor and the corona envelope *energy* is lost, but no *charge*.[11]

The process has some similarity with the discharge of one condenser into another through a resistance, the difference being that on the transmission line the quantities are distributed instead of lumped and the magnitude of these quantities depends upon the voltage.

Attenuation. In Fig. 4 are shown curves of maximum voltage plotted against distance of travel of selected surges. These curves may be called attenuation curves, but in fact they involve both attenuation and distortion unless we define attenuation in a very special way. The difficulty arises from the fact that the wave shape of the surge is continually changing, and as we have seen, the rate of decrease of maximum voltage depends on the wave shape.

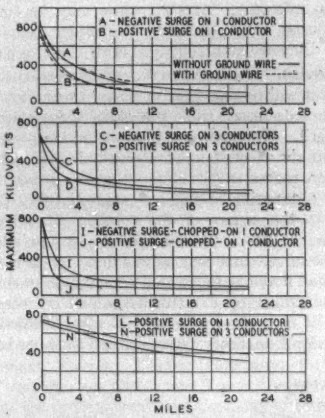

Fig. 4—Change of Maximum Voltage for Selected Cases

It is however, possible to get a great deal of information from the curves in Fig. 4. They bring out very clearly the fact that positive surges are attenuated (and distorted) more rapidly than negative surges by corona. They show also that short waves are attenuated more rapidly than long waves (compare *e. g.*, waves *I* and *J* with waves *A* and *B*).

Surge on Three Conductors. A comparison of the rate of attenuation of a surge on three conductors with that of a surge on one conductor is not immediately possible because the wave shape of surge *C*, for example, is not the same as that of surge *A*. It is much shorter and would therefore be expected to attenuate more rapidly than *A*, other things being equal. A better comparison is obtained by considering surges *C* and *I* which are of approximately the same length at a voltage of about 600 kv. Considering the "attenuation" curves of Fig. 4 for these waves, it is seen that surge *C* attenuates more slowly than surge *I*; the same is true of surges *D* and J. Thus we are led to the conclusion that if the voltage and wave shapes are the same, a surge on three conductors will attenuate more slowly than the corresponding surge on one conductor. This is in accord with the fact that, if the same voltage is produced by a charge on a single conductor as by charge distributed on three conductors, the voltage gradient will be greater around the single conductor than around each of the three conductors; therefore corona would appear more readily in the former case than in the latter.

Ground Wires. The dotted curves in Fig. 4 (waves *A* and *B*) show the attenuation of these waves when one of the remaining conductors was grounded at frequent intervals along the line. The differences between this case and that without ground wire are slight, but the indications are that the ground wire increases the rate of attenuation at very high voltages but decreases it at the lower voltages. At very high voltages corona would appear on both the line and the ground wire, while at lower voltages corona will not appear on the ground wire, and the only effect of the ground wire would be to decrease the effect of ground resistance.

This is in agreement with the results obtained in 1929 where, because of the low voltages used, only the latter effect was noticed.

II. Surge Voltages Induced in Parallel Conductors

The theory of induced surges in parallel conductors was very early developed by K. W. Wagner[4] for the case of the non-dissipative line. Wagner's formulation has been used by many authors, principally in the more recent theory of ground wires.[5] Briefly, this theory leads to a set of simultaneous linear equations expressing the surge voltage on any conductor in terms of the surge currents on all the conductors. (The usual convention is observed of regarding all voltages as being composed of two traveling waves proceeding in opposite directions.) The coefficients in these equations are known as the self and mutual surge impedances of the conductors.

Because of the linear relationships in these equations an induced surge will have exactly the same wave shape as the inducing surge. An isolated conductor will have a potential due to its position in the field but no current along the conductor.

During the summer of 1929 sphere-gap measurements of induced voltages on an isolated conductor were made and it was found that the ratio of induced to inducing voltage increased as the surge proceeded along the line. Since there was considerable attenuation, it was natural

to expect that the theory of the non-dissipative line would not hold accurately.

Further light is thrown on this phenomenon by the oscillograms now available. A selection from these is shown in Fig. 5. In these oscillograms it will be seen that the wave shape of the induced surge is appreciably

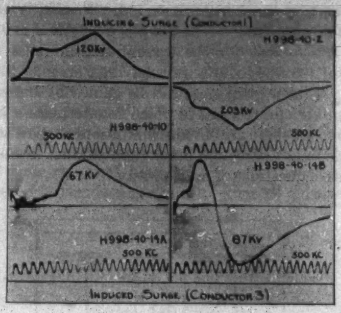

FIG. 5—OSCILLOGRAMS OF INDUCED VOLTAGE AT TOWER 93

different from that of the inducing surge. For both the positive and the negative wave the voltage of the induced surge is lower at the beginning and higher later on than that due to the field of the inducing surge; in the case of the negative surge the induced voltage is actually of opposite polarity at the beginning. It seems probable that the difference between positive and negative induced surges is due to the fact that the electric field is more affected by the corona envelope for the positive than for the negative surge.

That this peculiarity in the induced wave is not produced only when corona is present is shown by the fact that it also exists on low-voltage surges; Fig. 6 shows such a case. It resembles more nearly the negative high-voltage case than the positive, which is in accord with the fact that for the same voltage the negative corona envelope is smaller than the positive.

In Fig. 7 is shown the induced voltage at a point one

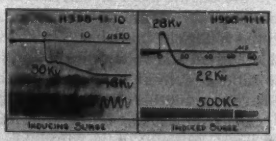

FIG. 6—INDUCTION ON FREE CONDUCTOR AT LOW VOLTAGE

mile from the impulse generator end of the line. Only the beginnings of a divergence from the wave shape of the inducing surge are here seen. The cumulative effect of this divergence is brought out very clearly in Fig. 8. It appears as though there is a continuous separation of charge taking place within the induced surge (for in no other way than by a separation can charge of a definite polarity appear on an isolated conductor). Charge of the opposite polarity to that of the main surge collects in the front of the induced surge, thus decreasing the resultant voltage there, while charge of the same polarity increases the voltage in the latter portion of the induced surge.

It is not easy to see the mechanism by which this separation takes place, but it seems that it must be in

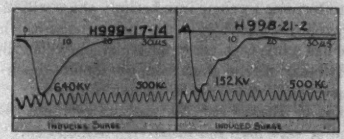

FIG. 7—OSCILLOGRAMS OF INDUCTION AT TOWER 10

some way connected with the process of attenuation and distortion of the main wave. Not only is the self surge impedance no longer a constant for wave propagation on a dissipative line, but also the mutual surge impedances have become more complicated operators.

It is now easy to see how an induced voltage of this nature will affect the propagation of a surge on three

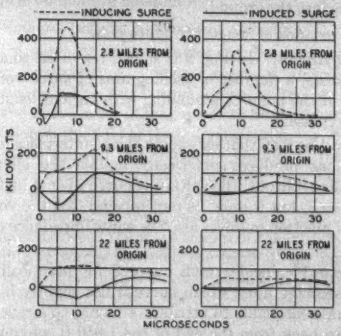

FIG. 8—REPLOT OF SELECTED OSCILLOGRAMS SHOWING INDUCTION ON FREE CONDUCTOR

conductors so as to produce that flattening of the front which was noticed in the foregoing section of this paper (Fig. 2, waves *C*, D and *N*).

III. COUNTERPOISE

General. The use of counterpoise has been suggested to improve grounding conditions in high resistance soil, especially in connection with ground wires.

The tests to be described in this section were conducted to determine the effectiveness of counterpoise and the relative merits of different arrangements.

An effort was also made to obtain a clearer insight into the manner of action of the counterpoise.

Considering for a moment the action of the counterpoise from the standpoint of theory it is evident that there will be two factors tending to reduce voltage at the tower footing; namely, the capacitance of the counterpoise to ground (in which true ground potential will occur at an appreciable depth below ground level) and a conductance effect due to the leakage of charge from the conductor into the soil. On surge voltages however, time will be necessary for the whole length of the counterpoise to come into action; there will in fact be a traveling wave effect on the counterpoise which will behave like a line with very high leakance; this includes the possibility of reflections from the remote (open) end making themselves felt at the tower footing end.

Method of Tests. For the purpose of these tests a location of very high soil resistivity was chosen. At a tower whose footing resistance measured (50 cycles) approximately 400 ohms, various arrangements of counterpoise were connected and a surge was discharged directly from the line into this grounding system. At a distance of 40 ft. from the tower leg a probe was driven to provide a reference ground and the voltage of the point at which the line was grounded was measured relative to this reference ground. In some tests the line was grounded through a known resistance; the voltage across the resistance was used to measure the current flowing into the ground.

A. *Variation of Length of Counterpoise*

Test Results. In Fig. 9 is shown replotted to a uniform scale, a number of oscillograms of surge voltages taken at the point of grounding with various lengths of counterpoise.

It will be seen that extending the counterpoise in two directions produces practically double the effect of extending it in one (the voltage of curve 1 is not half that of curve 2, but it must be remembered that the voltage at this junction involves also the surge impedance of the line).

Curve 3 coincides with curve 2 for practically six microseconds, after which curve 3 lies slightly higher. It seems reasonable to attribute this rise to a reflection arriving back from the end of the 1,050 ft. of counterpoise. A similar rise is evident in curve 4 but beginning much earlier; namely, about three microseconds from the start. This is in accord with the supposition that these rises are caused by reflections, since the times are approximately in the ratio of the respective lengths. (It is also interesting to note that this gives a velocity of propagation of approximately 375 ft. per microsecond in the counterpoise.) In curve 5 is noticed for the first time an increase in maximum voltage, because now the reflection has arrived back before crest voltage. There is also evidence of two reflections in this case.

In deciding how much advantage is to be gained by increasing the length of the counterpoise, it is clear that for this particular case no great advantage is gained by extending the counterpoise further than 1,000 ft; but this already involves two spans so that if we are dealing with a *region* of high resistance it involves going the whole distance from one tower to the next. How much shorter than 1,000 ft. the counterpoise can be made without losing a great deal of its effect will depend on the length of surge to be expected, particularly on the length of wave front; for example, a length of 500 ft. is quite satisfactory for the waves used on these tests, but would not be so for a wave having a six microsecond front. That a great deal can be gained by increasing the number of directions in which counterpoise is laid is evident, but there are obvious limitations to the distance possible in any direction other than parallel to the line because of questions of right-of-way.

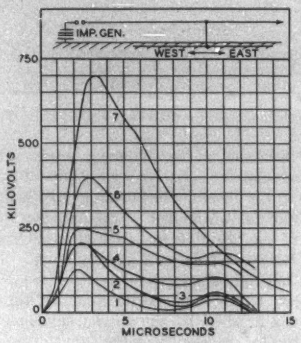

Fig. 9—Effect of Varying Length of Counterpoise

1. Counterpoise 1,527 ft. west and 921 ft. east
2. Counterpoise 1,527 ft. west
3. Counterpoise 1,056 ft. west
4. Counterpoise 529 ft. west
5. Counterpoise 225 ft. west
6. Tower footing alone
7. Line not grounded

B. *Current and Voltage at the End of Counterpoise*

In Fig. 10 are shown plots of current entering the ground (counterpoise) at the point of grounding, together with the voltage at this point. It is immediately apparent that current and voltage are not proportional; in fact, the ratio of current to voltage increases with time. We have here another case of non-constant surge impedance, chiefly on account of the high leakance of the counterpoise.

If a constant voltage be impressed at the end of a line with leakance, it is clear that the current at this end will increase with time although the voltage remains constant. This is due to the effect of the leakance at distant points being reflected back to the source at successive points of time; if the length of the line were infinite the current at the end of the line would ulti-

mately become infinite (provided the resistance of the line is zero). On the other hand, if the voltage at the end is not held constant but is supplied through a resistance (as in the case of grounding a rectangular wave of surge voltage on a line by a counterpoise, where the surge impedance of the line enters as a series resistance) the voltage will decrease as the current increases towards a constant value. Consequently a counterpoise becomes more and more effective as time goes on, provided the counterpoise is long enough. This is of importance in considering the effectiveness of counterpoise on different wave fronts. It is clear that for a counterpoise of unlimited length, the smallest effect on maximum voltage would occur on surges of shortest wave front. For a surge of rectangular front, leakance would have no effect initially, the entire effect of the counterpoise then being equal to that of a resistance $= \sqrt{\frac{L}{C}}$. Limitations of time allowed of no further tests being made to check this, but this conclusion from the voltage-current relationship is plain.

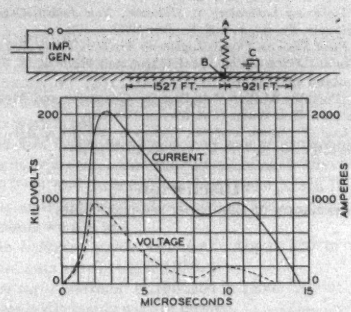

Fig. 10—Current and Voltage at Point of Grounding of Counterpoise

This last fact is important in view of the increasing weight of evidence that direct strokes are a major factor in producing outages. In this case flashover would be produced by the tower rising to a sufficiently high potential above that of the line and the object of the counterpoise would be to keep the tower footing impedance low enough to prevent this from happening. It is reasonable to believe that a direct stroke on a tower would produce a voltage wave of very steep front on the tower approximating the condition of rectangular front discussed above.

Summary

Attenuation and Distortion

1. Lightning surges undergo considerable changes both in crest voltage and in wave shape in traveling along transmission lines.

2. Attenuation is more rapid above corona voltage than below.

3. Positive surges are attenuated more rapidly by corona than negative surges. This fact will influence the interpretation of lightning measurements made by surge recorders and other instruments inasmuch as surges of equal magnitude but opposite polarity at the point of origin will not be equal at the point of measurement.

4. On both positive and negative waves the attenuation is accompanied by a distortion which lengthens the wave.

5. The rate of voltage rise (or fall) decreases as the surge proceeds along the line.

6. Short waves attenuate more rapidly than long waves. This and (5) means that a lightning surge chopped by insulator flashover becomes relatively harmless after traveling one or two miles, since it will be of low crest voltage and will also have lost all abrupt changes in magnitude.

7. There is an appreciable attenuation of low-voltage (below corona) surges, probably mostly due to ground resistance.

8. A surge on three conductors in parallel is attenuated less rapidly by corona than a surge of the same voltage and wave shape on one conductor.

9. The front of a surge is slowed down more rapidly when the surge is on three conductors in parallel than when it is on a single conductor.

Surge Voltages Induced on Parallel Conductors

10. A surge on one conductor induces on a parallel isolated conductor a voltage which is initially of approximately the same wave shape as the inducing surge and of the order of magnitude to be expected from its position in the field of the inducing surge.

11. The ratio of the maximum voltage of induced surge to the maximum voltage of inducing surge increases as the surge proceeds along the line.

12. As the surge progresses there is a cumulative divergence between the wave shapes of the induced and the inducing surges in which the front of the induced surge tends to acquire a charge opposite in polarity to that of the inducing surge, while the latter portion gains a charge of the same polarity; it is this gain in the latter portion of the surge which accounts for the increased ratio noted in 11, while the tendency for the front to assume opposite polarity produces the flattening of front noted in (9) above.

Counterpoise

13. The impedance offered to surge voltages by tower footings is materially reduced by the addition of a counterpoise.

14. A counterpoise behaves like a line with very high leakance.

15. Depending on the length of the counterpoise, reflections from the remote end will be noticeable in the voltage at the end connected to the tower.

16. The first and all successive reflections will be attenuated to a negligible value if the counterpoise extends beyond a certain distance.

17. The ratio of current to voltage at the tower end of the counterpoise increases as the effect of the leakance of distant portions of the counterpoise is reflected back to this end.

18. An increase in the number of directions in which counterpoise is laid, increases the "surge admittance" of the counterpoise practically in proportion (this will be true as long as the mutual effect between the different conductor lengths is negligible).

19. The effectiveness of the counterpoise depends, among other things, on the length of front, as well as on the total length of the wave. The effectiveness decreases with increasing slope of front and for the extreme case of a perpendicular front is a minimum being equal to the surge impedance of the counterpoise with no leakance.

ACKNOWLEDGMENTS

The authors wish to express their thanks to all those who contributed in collecting the results embodied in this paper. In particular they wish to mention Messrs. K. B. McEachron and J. G. Hemstreet, who initiated this series of tests, Messrs. E. J. Wade, W. J. Rudge and E. J. Shimek (of the General Electric Company) and Messrs. C. C. Tanner, F. D. Blackwell, and D. H. Geiger (of the Consumers Power Company) who took active part in all the tests, and all the operators and men of the Consumers Power Company whose cooperation was essential to the success of the tests.

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Discussion

For discussion of this paper see page 1146.

Lightning Investigation on Transmission Lines—II

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Synopsis.—The results of a five years' investigation of the effects of lightning on transmission systems are reviewed. It is concluded that direct strokes play a prominent part in causing flashover. Induced strokes are also important.

Conventional overhead ground wires with low tower footing resistance take care of direct strokes, as well as induced strokes, and prevent flashover. The mechanism of the lightning stroke is not yet understood, but the indications are that polarity effects are highly important and must be given full consideration.

* * * * *

IN previous papers[1,2,3] the authors have summarized the results obtained for four years (1926, 1927, 1928 and 1929) in a surge voltage investigation conducted by the General Electric Company in cooperation with various power companies.

In 1929 and 1930 progress in this investigation was greatly accelerated. This was partly due to concentration of the work on a few systems and largely due to the liberal use of several new instruments especially developed by the General Electric Company for this investigation.

Instruments Used in the Investigation

The *surge-voltage recorder* of the two electrode type.[4] Approximately 250 of these instruments were in use.

The *cathode-ray oscillograph*.[5] Six of these instruments were in use.

The *field intensity recorder* and *rate of change of field intensity recorder*.[3] One instrument of each kind was in use in 1929, and three field intensity recorders in 1930.

The *severity meter*[6] introduced in 1930. Eleven instruments were in use.

The *lightning stroke recorder*.[3] About 800 of these instruments were in use in 1930.

The *surge indicator*[7] introduced in 1930. About 1,500 of these instruments were in service.

Systems on Which Investigation was Conducted

In 1929 and 1930 the work was concentrated mostly on the Pennsylvania Power & Light Company's Wallenpaupack-Siegfried line, the Ohio Power Company's Philo-Canton line, and the Atlantic City Electric Company's Deepwater-Pleasantville line (the last mentioned in 1930 only). In addition, an investigation was conducted with artificial lightning on a 45-mile, 70-kv. line of the Consumers Power Company in Michigan.

See companion papers for details of the investigation on these systems.[8,9,10]

The main change on the Wallenpaupack-Siegfried line in the 1930 set-up as compared with the 1929 set-up was the removal of the lightning laboratory from tower 1-3 near Wallenpaupack to tower 23-4 of the Siegfried-Roseland section (at Cherry Valley).

In this laboratory two cathode-ray oscillographs were installed, one operating with a 50-microsecond time axis and the other with a 2,000-microsecond time axis. Both oscillographs were connected to phase X of the transmission line.

In 1929, a "counterpoise" was installed parallel to the line conductors and trenched in the ground, connecting the bases of the towers from tower 11-4 to tower 14-1 of the Wallenpaupack tap section of the Wallenpaupack-Siegfried line. In 1930, four tower footing grounding cables, each 50 ft. long, extending out in four directions from the bases of the towers and trenched in the ground, were installed at each tower from tower 1-4 to tower 11-3 and from tower 14-2 to tower 20-2 of the Wallenpaupack tap section of the Wallenpaupack-Siegfried line.

Two "lightning diverter wires,"[11] 180,000 cir. mils A. C. S. R. extend over the line conductors from tower 22-1 to tower 25-4 (18 consecutive towers) of the Wallenpaupack tap section of the Wallenpaupack-Siegfried line. These were installed in June 1930.

Each tower of the Wallenpaupack-Siegfried line was equipped with a lightning stroke recorder and each insulator string with a surge indicator.

Results of Investigation

1. Crest Value of Surges due to Lightning and Switching.

Lightning surges up to 3,000 kv. in magnitude have been measured on the Wallenpaupack-Siegfried line, or 16.7 times normal crest value line-to-neutral voltage. This is approximately 58 kv. per ft. of height of conductor. On the Ohio system lightning surges up to 1,640 kv. or 15.2 times normal have been measured. It appears that the limit for these values is set by the line insulation, and that this limit increases as the line insulation is increased.

Fig. 1 gives a curve of lightning voltage measured by surge-voltage recorders (times normal line-to-neutral voltage) against percentage of time occurring. The data are based on a study of 678 surges due to lightning, occurring on 14 systems during the years 1926 to 1930 inclusive. Fig. 1 also shows a curve of switching surges measured by a surge-voltage recorder, based on 724 surges occurring during the years 1927 to 1930 inclusive on

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1. For references see end of paper.

Presented at the North Eastern District Meeting of the A. I. E. E., Rochester, N. Y., April 29-May 2, 1931.

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practically the same systems from which the lightning data were taken.

2. Flashovers and Trip-outs Caused by Lightning.

The development and installation of the surge indicator and the use of the high-speed magnetic oscillograph at the line terminals, greatly assisted in the study of insulator flashover and trip-outs on the Pennsylvania system. By means of the data from these instruments supplemented by data from the slow-sweep (2,000 microseconds time axis) cathode-ray oscillograph and the lightning-stroke recorders, it was possible in 1930, for the first time, to locate the point of breakdown with reasonable certainty for about two-thirds of the lightning trip-outs.

On the 220-kv. lines in Pennsylvania for the five-year period, 1926 to 1930 inclusive, the Wallenpaupack-Siegfried line had 324 flashovers of insulator assemblies. In addition, there were 19 relief gap operations at Wallenpaupack and Siegfried. For the three-year period, 1928 to 1930 inclusive, the Plymouth-Siegfried line had six flashovers, the Conowingo-Plymouth line

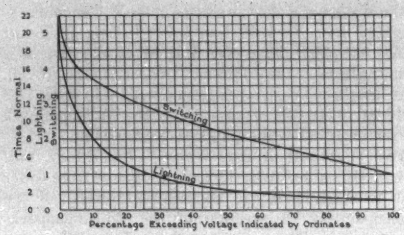

Fig. 1—Curves of Voltage Against Percentage of Time Occurring, for Lightning and Switching

Based on data from 14 systems for years 1926 to 1930 inclusive

No. 1 three, and the Conowingo-Plymouth line No. 3, four flashovers.

Each outside conductor on the Wallenpaupack-Siegfried line suffered two to three times as many flashovers as the middle conductor. The majority of flashovers involved only one phase and one or two assemblies, but in one case as many as three phases and four assemblies were involved.

During the five-year period, 1926 to 1930 inclusive, the Wallenpaupack-Siegfried line tripped out 98 times. During the three-year period, 1928 to 1930 inclusive, the Plymouth-Siegfried line tripped out five times, the Conowingo-Plymouth line No. 1, four times, and the Conowingo-Plymouth line No. 3, five times.

On the Wallenpaupack-Siegfried line in about 80 per cent of the cases only one phase was involved in a trip-out.

3. Direct-Stroke Records.

In 1930, after June 1, 18 records were obtained on the Wallenpaupack-Siegfried line with lightning stroke recorders. These have been interpreted as indicating direct strokes.

The list is divided into two groups, the first group of 10 or 11 are associated with trip-outs and the second group of 8 or 7 are not associated with trip-outs. The strokes associated with trip-outs took place on the section of line not equipped with overhead ground wires, while 7 strokes not associated with trip-outs took place on the section of line equipped with overhead ground wires or diverter wires.

Of the 24 trip-outs experienced on the Wallenpaupack-Siegfried line in 1930, 8 took place before the lightning recorders were in operation. Of the remaining 16 trip-outs, 10 or possibly 11 were interpreted to be associated with direct strokes. In four cases, no lightning stroke records were obtained and the remaining one or two cases were indefinite.

On the Ohio Power Company's 132-kv. Philo-Canton line, 130 lightning stroke recorders were installed in the vicinity of Newcomerstown. Many interesting records were obtained, involving in some cases as many as five adjacent towers. On the Atlantic City Electric Company's system, 200 recorders were installed. Some records involved as many as six towers.

On the Philadelphia Electric Company's 220-kv. and 66-kv. systems about 65 recorders were installed and some interesting data obtained.

In a high percentage of the cases the direct strokes recorded on the Ohio Power, Atlantic City, and Philadelphia Electric Companies' systems were not accompanied by trip-outs. All of these lines were equipped with overhead ground wires.

4. Cathode-Ray Oscillograms.

During 1928, one cathode-ray oscillogram was obtained on the Wallenpaupack-Siegfried line; during 1929, 95 oscillograms and during 1930, 23 oscillograms. These are of a wide variety of shapes and durations. The origin of some of the surges was near and some far from the oscillograph.

In 1930, there seemed to be a tendency for the waves to have a certain general shape, if the phase on which the oscillograph was installed was involved in the fault, while another distinct shape of wave was recorded if the phase on which the oscillograph was connected was not involved in the fault.

One oscillogram was obtained of a direct stroke, which apparently hit the conductor to which the oscillograph was connected, on the span adjacent to the laboratory. About 3,000 kv. negative was recorded on this conductor, both by cathode-ray oscillograph and surge-voltage recorder. On the other two conductors not over 1,620 kv. negative and 720 kv. negative, respectively, were recorded by a surge-voltage recorder, and on the 50-ft. antenna adjacent to the transmission line no voltage was recorded.

5. Storm Severity

The average storm severity on an arbitrary scale as

measured by the storm severity meter at four stations in Pennsylvania and one station at Schenectady for 20 weeks, May to September, varied from one to two. At two stations in Colorado the average storm severity for eight weeks in July, August, and September, was 9.3 at one station and over 10 at the other.

6. Overhead Ground Wires

On the section of the Wallenpaupeck-Siegfried line equipped with overhead ground wires and counterpoise, tower WT-11-4 to tower WT-14-1 (14 towers in all), the flashed insulator assemblies for the years 1926 to 1930 were as follows (the counterpoise was in service in the last two years only):

TABLE I

Year	Flashed insulator assemblies
1926 (No overhead ground wires)	6
1927 (Ground wires installed)	9
1928	14
1929 (Counterpoise installed)	0
1930	0

The Overhead Systems Committee of the Great Lakes Division, N. E. L. A., has gathered data for several years on the performance of transmission lines in its district, and has made a special study of the effect of overhead ground wires.

The miles of line under observation equipped with overhead ground wires have increased from 1,400 miles in 1927 to 2,660 miles in 1929. The miles without overhead ground wires have decreased in the same period from 759 to 673 miles. The interruptions per 100 miles of line due to lightning on the lines with overhead ground wires have increased from 5.8 in 1927 to 7.0 in 1929. On the lines not equipped with overhead ground wires interruptions per 100 miles have increased from 12.2 in 1927 to 20.9 in 1929. The indications from this study are that from two to three times as many interruptions occur on the lines without overhead ground wires as on the lines with overhead ground wires.

The American Gas & Electric Co. operates a very extensive 132-kv. network in Ohio, Indiana, West Virginia and Virginia (approximately 1,800 miles).[13] This network is divided into 24 lines, all but two of which are equipped with overhead ground wires. On the lines equipped with overhead ground wires, the outages (1928) averaged 11.7 per 100 miles. On the lines not equipped with overhead ground wires the outages for the same year averaged 38 per 100 miles.

The Public Service Elec. & Gas Co. of New Jersey operates an extensive system of 13.2, 26.4, 66, 132 and 220-kv. lines. The lines 66 kv. and 132 kv. are mainly on double-circuit steel towers with conductors vertically arranged and two ground wires over the outermost (middle) conductors. The ground wires are about 10 ft. above top conductor. The lines pass through farm, meadow, and swamp lands, and the towers have a naturally low-footing resistance. Whereas the total interruptions on this system in 1930 were 231, only two of these took place on the 66-kv. system and four on the 132-kv. system. The remainder took place on the 13.2- and 26.4-kv. systems. This territory is subjected to a normal amount of lightning.

The Philadelphia Electric Co. has a number of 66-kv. double-circuit steel tower lines, with two overhead ground wires placed vertically above the outermost (middle) conductors. These lines in general are subjected to a limited number of interruptions, but one line in particular, that from Philadelphia to Chester, a distance of 14 miles, has operated 14 years without an interruption due to lightning. The distinguishing feature of this line is that throughout the entire length the line follows a double-track railroad, with the tower structures located a short distance from the nearest track rail. In addition the line parallels a river throughout practically its entire length, the distance between the line and river varying greatly. The tower-footing resistances on the line are unusually low and uniform (of the order of 4 ohms). This line is in territory which is subjected to about the same number of storms per year as the Pennsylvania Power & Light Co. system and the storms are apparently of comparable severity as measured by the severity meters.

7. Direct and Induced Strokes.

It is quite important to know how relatively frequent are direct and induced strokes, the magnitudes of potentials placed on the power conductors by such strokes and the effect of overhead ground wires and other protective measures against such strokes. Unfortunately, reliable data on this subject are still largely missing. Some of the meager data that are available are examined.

In a recent paper[12] appear the statements given in the following three paragraphs:

> Observations by skilled engineers show conclusively that lightning strokes within ½ to 2 miles from the line resulted in surges having values not over the line voltage.
>
> In May, 1930, a photograph was taken of a lightning stroke which hit the earth 1,600 ft. (estimated by sound) from a cathode-ray oscillograph station at Roseland, New Jersey. This stroke had hit the earth within 400 feet of the line to which the measuring instruments were connected. Neither the klydonographs connected to this line nor the cathode-ray oscillograph indicated any substantial voltage, although these instruments were so calibrated that they would indicate any surge above the line voltage, which in this case was 200 kv.
>
> In another case, a 33-kv. line in Arkansas was struck by lightning and the poles split from top to bottom. At right angles to this line was another line to which recording apparatus was connected. The instruments gave no record of any surge, although they were located only 700 ft. from the pole which was struck.

In 1929, at the Wallenpaupack laboratory an attempt was made to correlate the potential gradient, as measured with field intensity recorder, with distance

of the stroke from the laboratory. The distance was determined roughly by an observer in the cupola of the laboratory, with the assistance of a stop watch and a device for sighting on the stroke. Of 253 strokes over 15 kv. per meter recorded, it was possible to correlate in this manner only 81 strokes or 32 per cent of those occurring. The other strokes were too uncertain in correlation or else no satisfactory distance measurements were obtained. Fig. 19 of the authors' previous paper[3] shows the results of this attempt at correlation. It will be noticed that the correlation is very rough, but that there is a tendency for a curve showing greater field intensity with shorter distance from the laboratory. Field intensities as high as 280 kv. per meter (85 kv. per ft.) were recorded in this investigation.

From the standpoint of gradient alone, therefore, it appears reasonable to assume that induced voltages may be of such amplitude as to cause insulator flashover. It is appreciated, however, that induced voltage on a line conductor is a function of rate of change of field gradient as well as absolute field gradient. Information concerning rate of change of field gradient is not sufficiently complete to permit a conclusion regarding the amplitude of induced voltage.

In 1928, on the Philo-Canton line of the Ohio Power Co. a tree designated as tree No. 1, located 360 ft. from the transmission line at tower 176 was struck by lightning. A surge-voltage recorder located on the bottom conductor at tower 176 recorded a voltage of 2,100 kv. positive. The surge was accompanied by a line trip-out. The correlation between the transmission line record and the direct stroke at the tree is not certain.

In 1930 two trees in this same vicinity were struck, apparently during a storm which occurred on Sept. 23. Tree No. 2 was 500 ft. from the transmission line and tree No. 3, 1,300 ft. from the line. There was no surge-voltage recorder at tower 176 this year,the nearest recorder being at tower 172 (approximately one mile away). There were no trip-outs on the line or surges which correlated with these strokes recorded on instruments connected to the line.

On the Louisiana Power & Light Company's 110-kv. system, in 1929, lightning strokes occurred to two trees adjacent to the right-of-way of one of the lines. Tree No. 1 was approximately 550 ft. from the line and opposite one of the highly insulated wood pole *H* frame structures which had no guys, pole grounds, or bonding. The next structure south was similar. The second structure south was a guyed structure with a bonding wire along the cross arm, a grounding wire down each pole, except for a 13½-ft. gap in each grounding wire, and 18 ft. wood guy insulators in each guy. This structure was approximately 1,000 ft. from the tree that was struck. No evidence was left of flashover on the unguyed structures, but the guyed structure flashed over the 13½-ft. wood pole gap at the time of the lightning stroke.

At another time, tree No. 2 which was approximately 480 ft. from the line and 670 ft. from the same guyed structure was struck, and two of the 18-ft. wood guy insulators were flashed over with no indications of flashover on any of the unguyed structures.

On the Consumers Power Company's 140-kv. 30-cycle system in Michigan, a direct stroke hit the top of tower 4334. The line was single-circuit on steel towers with one overhead ground wire, conductors vertically arranged. Power arcs occurred from all three conductors to the tower. Potentials of 2,800 kv. positive were recorded on two phases at tower 4334. On the adjacent tower north (tower 4333) a positive potential of 1,375 kv. was recorded. On other towers (4323 and 4304) located one and three miles respectively, north, negative potentials of 1,480 and 900 kv. were recorded. On towers (4366 and 4385) located one and three miles respectively, south, negative potentials of 1,840 and 1,300 kv. were recorded.

The following explanation has been advanced for this occurrence: A direct negative stroke to the tower, resulted in a flow of current down the tower and out in both directions on the overhead ground wire. The flow of current down the tower encountered the tower footing resistance (55 ohms) and raised the tower to a potential above the conductors (conductors positive with respect to the tower). The conductors flashed to the tower, and the conductors became negatively charged, which accounts for the negative potentials at the towers one and three miles north and south of tower 4334. The positive potential on the adjacent tower 4333 may have been caused by the flow of current out on the ground wire and down the tower, raising the tower to a potential above ground and conductor.

On the right-of-way of the Wallenpaupack-Siegfried line of the Pennsylvania Power & Light Co. there were erected at five points highly insulated antenna wires about 300 ft. long and at different heights above the ground, varying from 12½ ft. to 100 ft. The antenna wires were approximately 75 to 100 ft. from the nearest line conductor.

A voltage (presumably induced) of 2,700 to 3,150 kv. positive was measured on the 100-ft. antenna at Wallenpaupeck, with a corresponding voltage of 350 kv. positive on the transmission line. In another case, a voltage of 2,700 kv. negative was measured on the 50-ft. antenna at High Knob with no registration on the transmission line. In no cases were high voltages measured on the transmission line which correlated with antenna registrations.

On June 26, 1930, the diverter wire was apparently subjected to a direct stroke at tower WT-22-1. At the same time a voltage of 2,400 kv. positive was measured on the conductor on *Y* phase.

In the past, numerous high-voltage potentials of positive polarity have been recorded on transmission lines, although high-voltage surges of negative characteristics were apparently in the majority. There is a tendency to attribute the high-voltage negative surges

to direct strokes. The high-voltage positive surges may be due to induced strokes, or they may be due to direct strokes to towers or ground wires, which cause sufficient current to flow down the tower to give an appreciable difference of potential between the tower and the conductors. In this case the conductor would have a positive polarity with respect to the tower.

Apparently, there is evidence to the effect that direct strokes play an important part in transmission flashovers and outage. There is a great deal of evidence that induced strokes are also highly important. There is insufficient evidence at the present time to evaluate more exactly these two types of strokes. It may eventually be found that one type of stroke is more important than the other on lines of one voltage class and insulation, and that in another voltage class the other type of stroke becomes more prominent.

Discussion of Results

1. Mechanism of Lightning Stroke. There is, of course, as yet no adequate understanding of the lightning stroke even from a qualitative standpoint. Furthermore it is improbable that from existing data an indisputable explanation of the mechanism of the stroke can be built. However it appears to the authors that there are suggestions in the results already obtained which should be given careful consideration.

Measurements of cloud field potentials[2,14,15] have established beyond question that fields of both polarities are present. It has been shown that the field polarity at a point on the ground may change from positive to negative or from negative to positive during the same storm. Many storms have been shown to have a predominant polarity characteristic but in almost all cases both positive and negative fields are present to some degree.

Studies of polarity effects in gas discharges suggest that no satisfactory understanding of the mechanism of the lightning stroke will be reached until due consideration is given polarity effects.[16,17] Laboratory experimental work on the breakdown of gases between dissimilar electrodes[18] reveals that the shape of the positive electrode is of greatest importance in determining voltage breakdown. Lichtenberg figure studies[19] have supplemented this information and in addition have contributed valuable experimental proof that in most cases the propagation of the steamer tip away from the positive electrode is the essential aspect of the breakdown mechanism.

Laboratory studies[20,21] of the "protective zone" effect of the vertical lightning rod have shown that polarity is the determining factor. With the lightning rod polarity positive and the cloud negative a very high degree of protection is afforded. With the lightning rod negative relative to the cloud some protection is still obtained but the effectiveness of the rod is greatly reduced.

As previously stated all lightning-stroke recorder records of direct strokes to towers indicate that the top of the tower is negative with respect to the bottom and from this it appears reasonable to infer that these are cases of negative clouds and positive ground charges. With the tower structure positive with respect to the cloud, breakdown proceeds from the structure to the negative cloud. Numerous strokes terminate on the grounded structure under this polarity condition. Under conditions of tower structure negative with respect to the cloud the stroke would proceed from the cloud and the probability of the tower structure being struck would not be great. While this conception appears to reverse our picture of the cloud-to-ground stroke nevertheless it is consistent in that it is in accord with laboratory and field experience.

The direction of current in the lightning stroke has also been given considerable study.[22] As the breakdown streamer propagates rapidly away from the positive electrode electrons flow toward the positive electrode. The lightning stroke currents therefore classified in accordance with our practical definition always flow from the positive to the negative polarity.

2. Ground Wires. The theory regarding the particular mechanism of operation of the ground wire in case of induced charges is generally agreed upon. Ground wires are presumed to protect in this case because they reduce the induced charge on the line conductor and increase the capacitance of the conductor to ground.

However, in the case of direct strokes it appears that a somewhat different viewpoint is necessary. The fact of the elevation of the line structure above the ground plane causes many strokes to terminate on the line structure.

When the cloud polarity is negative and the ground positive all strokes within a distance of the order of ten times the height of the line on either side will strike the line. This, of course, is the condition obtained when there are no other objects projecting above the ground plane of the line within the distance mentioned.

When the cloud polarity is positive and the ground negative the projecting transmission line still functions to attract many strokes. However, in this case the number terminating on the structure will be smaller, the line attracting all discharges within a distance of 2 to 3 times the height of the line on either side instead of 10 times. This explains the great number of negative strokes indicated during the lightning investigations.

The overhead grounded structure (towers and ground wires) serves as the ground terminal for the cloud-to-ground stroke, forming a shield for the line conductors. The customary ground wire attached to the tower structure serves to extend the shielding effect provided by the towers to the space between towers. In addition the ground wire also lowers the impedance between tower top and ground. Current resulting from a direct stroke at the tower top may pass out over the ground wires as well as down the tower structure. The conventional ground wire therefore functions in two ways to afford protection to line conductors; first to provide

over the line conductors a grounded structure which can serve as a terminal for direct strokes, and second to reduce impedance to ground in case of direct stroke to the grounded structures.

The so-called diverting wires, one form of which is being experimented with in an actual installation, are ground wires which may or may not be insulated from the tower structure and in general are located at a greater distance above the line conductors than the conventional ground wire.

Diverting wires supported on the tower structures by metallic tower extensions can have no advantage over the conventional ground wire other than a possible increased effectiveness in providing a terminal for direct strokes due to greater height and more advantageous position over the line conductors.

Diverting wires located above the line conductors but insulated from the tower structure and provided with independent grounds of low resistance have the advantage that lightning currents are not conducted through the tower structure to ground. As will be briefly discussed later this advantage is a significant one.

Data obtained in the field investigations and extensive operational experience testify to the effectiveness of the conventional ground wires. The number of direct strokes recorded compared to the number of trip-outs on a line with overhead ground wires indicates that only in rare cases will a line conductor be struck when steel towers and conventional ground wires are used. Knowledge of the commanding influence of the polarity effects described above permits at least a partial understanding of these results.

3. Insulator Assembly Flashover. Conditions under which the line insulation may flash over as a result of direct strokes appear to be three in number. First, of course, a direct stroke to a line conductor will provide sufficient voltage for insulation breakdown. Second, a direct stroke to the tower will cause insulator flashover by raising the potential of the tower structure with reference to the conductor. The voltage drop across tower and footing impedance is responsible for this flashover voltage. Third, a direct stroke to the ground wire between towers may result in such potentials on the ground wire that flashover to the line conductor takes place.

Direct strokes to line conductors would appear to be of rare occurrence where steel towers and overhead ground wires are used. Insulator flashover with this construction would therefore appear to result in most cases through tower potentials due to high tower footing resistance. Under this condition the tower structure would almost invariably be raised to a negative potential with respect to the conductor.

Information gathered by the surge indicators on insulator assembly flashover shows that in the greatest number of cases a single assembly flashes over. Next to this condition in frequency of occurrence is the case where two adjacent assemblies on the same phase flash over. Data on flashover of insulator units is still too meager to permit more than speculation on the influence of the ground wire, or a conductor connected to the tower by insulator flashover, in extending or limiting the total number of assemblies flashed for one stroke. However, it appears that where flashover occurs from the tower to a conductor the spread of flashover to other insulator assemblies on the same tower may be prevented by the resulting reduced impedance between tower and ground. Also insulator assemblies on adjacent towers on the same phase may flash over as the surge proceeds along the conductor from the tower struck and where the first flashover occurred.

In natural lightning measurements and also in artificial lightning measurements in field and laboratory, it has been shown that insulator flashover is not necessarily followed by line trip-outs.[10] One key to an explanation of this is found in the fact that flashover must occur at a time when the power voltage to ground and the current flowing through the conductor both have sufficient amplitude and proper polarity relations to establish a permanent arc for the power current to follow. Experience has shown that in many cases a permanent arc is not established because these conditions are not satisfied.

4. Wave Shape. Oscillographic work on transmission lines under natural lightning conditions has shown that wave shapes to be expected at a specific point on the line range in time to crest from about one microsecond to several hundred microseconds and that the times on the wave tail extend over a wider range than times to crest. The lower voltage surges are of longer time duration.

Wave shapes measured at or near the origin of the surge have fronts which bring the conductor potential up to insulator flashover in the order of a microsecond. These high amplitude waves of steep front usually cause flashover and are therefore reduced very quickly to low potential values. It would appear that a voltage wave rising to a high value but still somewhat under insulator assembly flashover would be of longer time to crest and have a wave tail value of some 10 to 100 microseconds.

Much has been learned concerning attenuation or diminution of voltage amplitude of the surge as it travels along the conductor. For general purposes a formula which has been derived from natural lightning measurements[8] is of sufficient accuracy for practical calculation. This formula is as follows:

$$e = \frac{e_0}{K\,s\,e_0 + 1} \text{ and } A = -k\,e^2$$

where

e_0 = initial surge voltage at the point where the surge originated

K = a proportionality factor found empirically

S = distance in miles from the origin of the surge
e = voltage at distance s
A = attenuation in kilovolts per mile at any point where the line voltage is e

The factor k has been shown to vary between 0.0001 and 0.0007 depending upon the particular line under consideration. This formula agrees with results of attenuation studies where artificial lightning was used on an actual transmission.

Detailed studies have proved that attenuation is dependent upon polarity and wave shape. Positive surges decrease more rapidly in amplitude than negative surges and chopped waves of short time element attenuate more rapidly than longer and unchopped waves.

Change in wave shape above corona voltage has been shown to consist of a flattening of the wave front and a lengthening of the wave tail.[10] It has been shown that in effect this may be thought of as resulting from a lower velocity of propagation for the higher voltage portions of the traveling charge. Such a viewpoint is undoubtedly helpful but it must not be accepted as an explanation of the mechanism of surge propagation without a closer examination than has as yet been given. The influence of corona and polarity effects which have been shown to be great in connection with surge propagation are probably determining factors. Conditions of relative energy loss on the front, crest, and tail of the wave are little understood. The shift of energy relations within the wave may account for the wave changes which have been demonstrated.

In connection with attenuation studies of surge propagation by means of surge-voltage recorders distributed along a line conductor it has been noticed that very frequently positive surges of high amplitude are recorded at one station with very little or no voltage measurements at adjacent stations. These voltages have been explained in two ways.

The positive conductor voltage has been attributed to bound charge on the line from a negative cloud which charge is released when the stroke occurs. In other words, it is an induced voltage.

Another explanation which has been offered is that these voltage registrations are the result of ground surges accompanying the negative stroke to the tower or some object near the line. Under this condition the instrument would record a positive voltage between conductor and ground. In view of the preponderance of negative direct strokes obtained by field measurements and the agreement of these data with laboratory studies of discharge mechanism this explanation appears to justify close consideration.

Conclusions

The following conclusions are based on the data presented or referred to in this paper.

1. Many trip-outs of high-voltage lines are caused by direct lightning strokes to the line structure. This conclusion is based on approximately 16 records of direct strokes correlating with line trip-outs.

2. Many trip-outs on sections of transmission line not equipped with overhead ground wires are due to direct strokes terminating on the line conductor.

3. Direct strokes terminating on the overhead ground structure may cause flashover of the line insulation. This appears to be due to the voltage drop across the tower footing resistance.

4. Direct strokes to the transmission line structure may occur without a line trip-out. Records of approximately 37 lightning strokes which did not result in trip-outs bear out this conclusion.

5. Overhead ground wires and steel towers serve as a terminal for lightning strokes, thereby protecting the line conductor and in many cases preventing insulator flashover.

6. Insulator flashover may occur without sufficient power current flowing through the arc to result in a line trip-out.

7. Conventional overhead ground wires have proved effective in intercepting direct strokes and preventing line trip-out.

8. Low tower-footing resistance functions to improve protection when a direct stroke terminates on the overhead ground structure.

9. Where overhead ground wires are used, currents resulting from direct-lightning strokes flow to earth in many cases through several adjacent towers. Records have been obtained involving as many as six tower structures.

10. Lightning strokes of negative polarity terminating on the transmission line, far outnumber strokes of positive polarity. In fact, no strokes of positive polarity have been recorded.

11. Strokes terminating on objects projecting from the ground near the transmission line, produce high voltage on the conductors and in some cases are known to have caused flashover of the line insulation.

12. In a great majority of cases of flashover of line insulation, only one or two insulator assemblies are involved. When two are involved, they are usually on the same phase and adjacent towers.

13. At the origin of the disturbance the rate of voltage rise across the line insulation is of such order as to reach flashover value in a very short time, perhaps of the order of one microsecond.

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Discussion

LIGHTNING
(Peek)

LIGHTNING DISCHARGES AND LINE PROTECTIVE MEASURES
(Fortescue and Conwell)

LIGHTNING INVESTIGATION
(Bell and Price)

1930 LIGHTNING INVESTIGATIONS
(Sporn and Lloyd, Jr.)

LIGHTNING INVESTIGATION
(Gross and Cox)

EXPERIMENTAL STUDIES IN THE PROPAGATION OF LIGHTNING SURGES ON TRANSMISSION LINES
(Brune and Eaton)

LIGHTNING INVESTIGATION ON TRANSMISSION LINES—II
(Lewis and Foust)

J. H. Cox: Messrs. Lewis and Foust have discussed the question as to the proportion of surges caused by direct and induced strokes. This is a very important question since it has a direct bearing on the methods used in protection. Eighteen records of direct stroke measurements are listed. Those recorded on the unprotected part of the line were accompanied by trip-outs and those recorded on the ground wire protected part of the line were, in at least the majority of cases, not accompanied by trip-out. These data are a gratifying indication of the benefits derived from the use of a ground wire.

The authors further state, "First, of course, a direct stroke to a line conductor will provide sufficient voltage for insulation breakdown." I do not believe that present data justify this statement and feel it entirely possible for a direct stroke to be insufficiently severe to cause a flashover on a high-voltage line. Furthermore, the fact that all direct-stroke records obtained on the unprotected part of the line were accompanied by flashover does not indicate that no direct strokes occurred which were not recorded. With the methods of measurement used a direct stroke to a line conductor without an insulator flashover would not yield any direct-stroke record. It may be likely that a direct stroke would be to the tower but not at all certain.

Another factor brought out is that all the direct-stroke records indicated negative strokes and in conclusion 10 the statement is made that this indicates that negative direct strokes far outnumber the positive. It is entirely possible that positive strokes, even when direct to towers, do not cause currents sufficiently heavy to yield a record with the instrument settings used. So far as the data show, it still seems likely that the range of severity of direct strokes may be sufficiently great so that some do not cause flashover, even on unprotected lines, and also that some cause flashover on lines with the best protection now available.

In connection with the records obtained by Messrs. Lewis and Foust on antenna, I wish to point out that antenna measurements, unless the antenna is connected to ground through a resistance equal to the antenna surge impedance, which would be of the order of 500 ohms, indicate the cloud field gradient but do not yield a measure of the potential which would be induced on the line. This fact is demonstrated by the experience of the authors as stated by them: "In no cases were high voltages measured on the transmission line which correlated with antenna registrations." The antenna was close to the line. Another fact which indicates that induced voltages may be low, for at least certain kinds of strokes nearby, is the case mentioned of a direct stroke to the transmission line with no recordable induced voltage on the antenna, and in this case the antenna being connected to ground only through a high resistance the conditions were most favorable for a high induced voltage.

Regarding the strokes to trees adjacent to transmission lines as mentioned in the paper, these data may be misleading. For instance, a stroke to a nearby tree without a line surge, a direct stroke or a trip-out record indicates definitely that the induced voltage was below the required value. However, a stroke to a nearby tree with a correlated record of one kind or another does not definitely indicate that the record was caused by the particular stroke which hit the tree, unless the correlation was complete and an actual observation was made which tied the two points of evidence together, since it is possible that the record on the line and the injury to the tree were caused by two different strokes of the same storm. The greatest care should be taken to take into account possibilities of this nature which completely modify the interpretation.

The use of a buried cable, called a counterpoise by the authors, was suggested some time ago as a method of reducing tower footing resistance. The results of at least one year's experience on one line, as contained in Table I of the paper, which shows a perfect record, are very gratifying.

The authors state that the lightning stroke always starts from the electrode of positive polarity. Simpson suggested this in 1926 but more recent data indicate this to be incorrect. Torok's work at Trafford on arrested discharges indicate that a discharge streamer always started from the electrode of high potential, that is the electrode of smallest effective dimensions, whether such electrode is positive or negative.

I am unable to understand the discussion under "Wave Shape" of the short distance of surge propagation. If the surge is induced and consists of the release of a bound charge, this bound charge is actual and must be dissipated in some manner. Therefore, it should perform as a simple traveling

wave and should not dissipate in two or three towers. The second explanation is somewhat obscure but it seems rather likely that the extent of the ground surge due to the lightning current is not great. I feel that the correct explanation is contained in the Fortescue and Conwell paper. As pointed out there, in the case of a stroke to a ground wire without a flashover the potential on the line conductor is an induced part of the traveling wave and involves no actual charge. Thus, when the surge on the ground wire is dissipated, which it will be in a very few towers, the potential on the line conductor will disappear simultaneously.

Fig. 1—Potential at Tower Struck

J. J. Torok: The calculations made in the R. N. Conwell and C. L. Fortescue paper were based upon the accepted theories of traveling waves. In setting up the equations it was assumed that the tower length was such that it could be neglected in these calculations. The authors of the paper have realized that in a general treatise it would be desirable to include the effects of the tower in these reflections, consequently, at their request the calculations were revised so as to include the effects of the tower.

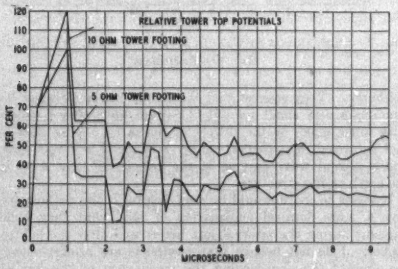

Fig. 2—Effect of Tower Footing Resistance on Potential at Tower Top

The surge impedance of the tower was determined to range from 75 to 150 ohms, the average being approximately 100 ohms. The assumptions in these calculations were that a tower to which two ground wires are attached was struck. Under this condition a train of traveling waves would be set up on the ground wires and on the tower. The waves on the ground wires will be reflected between adjacent towers while the waves on the tower will travel up and down the tower. In these calculations it was assumed that the nature of the wave traveling over the lightning channel was as follows: it rose to crest value in one microsecond and then dropped to half value in 50 microseconds.

The results of these calculations are shown in the following figures. Fig. 1 shows the potentials of the tower at both the top and bottom. A is the potential at the top and B the potential at the bottom. High over-shooting of the potential at the top of the tower during the first microsecond which is shown by curve A is due chiefly to the impedance of the tower itself. The tower having physical length will have, therefore, during the period of time that the current in the lightning stroke is increasing, a considerable potential across it. However, as soon

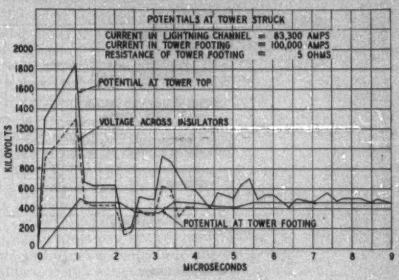

Fig. 3—Tower Potentials

as the wave ceases to increase in magnitude, the potential across the tower will drop to nearly the same potential as that of the tower footing.

The effect of the change in tower footing resistance is shown in Fig. 2. The tower top potentials have been shown with two conditions: (1) with the tower having a 5-ohm footing resistance, and (2) with a 10-ohm footing resistance. From this we see that by raising the resistance from 5 to 10 ohms the tower top potential has been increased only 20 per cent. These curves have been replotted on a voltage scale and are shown in Fig. 3. Three curves have been plotted: (1) the potential and the tower

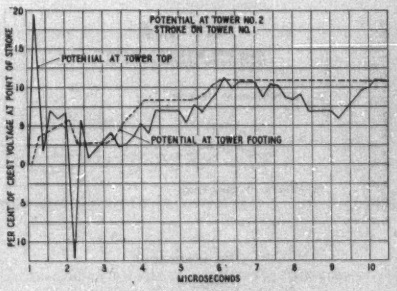

Fig. 4—Potential at Second Tower

top, (2) the potential at the footing of the tower, and (3) the potential across the insulator string. To obtain the potential across the insulator string a 30 per cent coupling was assumed between the ground wire and the conductors, thus the potential across the insulator string is approximately 70 per cent of the potential at the tower top. The maximum current in the lightning channel is assumed to be 83,000 amperes, the maximum current in the tower footing is 100,000 amperes, and the resistance of the tower footing is 5 ohms. The potential at the

adjacent tower has been found to be very much lower, having a volt-time characteristic as shown in Fig. 4. The highest voltage appearing at this tower is some 19 per cent of the potential at the first tower.

The relative currents in the first three towers are shown in Fig. 5. The highest current, of course, appears on the tower on which the stroke terminated.

In order to check these calculations the set up was made in the field in which tests were made on a non-energized line. A portable surge generator and cathode-ray oscillograph were

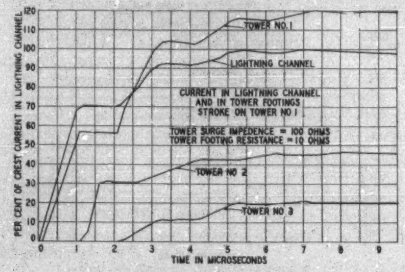

Fig. 5—Relative Currents

utilized in this test. A surge was impressed upon one conductor and allowed to travel over the line, finally reaching a point at which the conductor is grounded through a tower. The wave appearing at this tower had a three microsecond front and dropped to half value in 10 microseconds. This wave is not nearly as severe as that assumed in the calculations and due to experimental limitations a more severe wave could not be obtained. The wave traveling beyond this point was measured by cathode-ray oscillograms of the nature shown in Fig. 6. Inasmuch as the wave has a duration of only 10 microseconds it is to be expected that it could not maintain the potential of

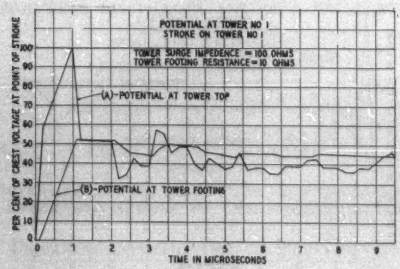

Fig. 6

the tower top to a value shown in the calculated waves, however, the first part of the wave is very similar to that obtained through calculations. The comparison of the two waves is shown in Fig. 7.

The analysis and accompanying curves as presented in this paper are of considerable importance, as they give for the first time a clear insight into the happenings on a transmission line at the time it is struck. From them the types of waves that are to be considered under different conditions can be ascertained. For example, a tower having low tower footing impedance (of the order of 5 ohms) when struck will stress the insulators with a wave having a high potential for one or two microseconds and then abruptly falling to a low value where it remains for a relatively long period of time. However, should the impedance of the tower footing be high (of the order of 50 ohms or more) the potential will rise to a high value and remain there for a considerable time. In the former case only the short time characteristics of insulators need be considered. In the latter, however, because of the flat top characteristics, the long as well as the short time characteristics must be considered. It is information of this type which will greatly facilitate the establishment of standard methods of adjudging insulation.

K. B. McEachron: In discussing the mechanism of lightning strokes Messrs. Fortescue and Conwell discuss the part played by water vapor in the production of sufficiently high charges to cause electrical discharge as in a thunder storm. I believe that it is generally understood that clouds consist of condensed water vapor in the form of actual drops and that these drops of water carry charges which are increased as the result of the action of the wind until finally an electrical discharge occurs.

The authors advance a partial theory of the mechanism of lightning strokes, indicating that the speed of formation is probably not greater than one-twentieth that of light and thus considerable time is involved in the production of the lightning

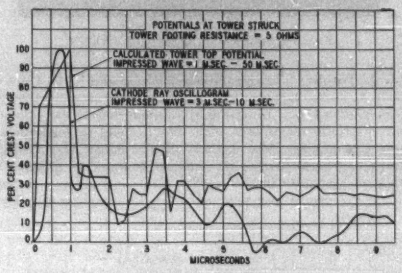

Fig. 7—Comparison of Calculated Potential and Cathode-Ray Oscillogram

discharge. They suggest that the energy in the streamer itself contains a very small amount of current until after contacting with some object on the surface of the earth. After such contact is formed the potential energy is changed to kinetic energy and a current wave moving at the velocity of light flows into the earth or the structure.

It seems to me it would be very helpful if the authors could give a more detailed explanation of just what they believe happens during the interval just before and after the lightning strikes. I do not believe that physicists are in agreement at the present time concerning the mechanism of the lightning discharge. Two different theories have been worked out in much greater detail than that advanced by the authors, one being developed by Simpson and the other by Dorsey. Simpson believes that the discharge channel is formed from the positive end, while Dorsey believes it is formed from the negative end. Dorsey in his "dart" theory suggests a mass of electrons and ions which move with high velocity at the front of the stroke following an irregular path which is determined more or less from each point of travel of the discharge.

It is appreciated that the calculations made by the authors in the first part of their paper are based on several assumptions for which little exact experimental data exist. For instance, it is assumed that the surge impedance of the lightning discharge path is 200 ohms and yet it is assumed that the continuation of this same discharge path on to ground after con-

tacting with the transmission line may be between 10 and 200 ohms. If the discharge path of the lightning can be considered as a surge impedance it undoubtedly is a variable quantity becoming smaller as we approach the surface of the earth. It seems to me that in making this calculation the authors assigned values to unknown quantities about which less is known than the unknown which is calculated, particularly as a cathode-ray oscillogram was taken giving some idea at least of the potential which might occur at the time of the direct stroke.

Based upon a considerable experience gained from the study of the operation of several transmission lines under lightning conditions the separations given between the line wires and the ground wires shown in Figs. 3 and 4 of the paper seem unnecessarily large, as many lines similar to that shown in Fig. 4 with much smaller clearances between line and ground wires with low tower footing resistance have had an excellent operating record. It is interesting to note that if these large spacings are necessary on extra high-voltage lines, equally large ground wire spacing is necessary on lower voltage lines if the spans are of equal length.

In connection with the study on the Siegfried-Roseland line the authors apparently conclude that high voltages cannot be obtained on a highly insulated transmission line except from direct strokes. This conclusion is based on the observation that direct strokes occurred close to the transmission line and yet voltages less than 100 kv. were observed on the transmission line. However, this does not mean that other direct strokes nearby might not cause high voltages on such a line. As an illustration of this situation reference should be made to the paper by Messrs. Sporn and Lloyd in which apparently two different lightning strokes did not give rise to induced voltage, while the third direct stroke apparently caused a recorded voltage at 2,100 kv. on the transmission line. These direct strokes occurred between 300 and 1,300 ft. from the transmission line. It may be that because of the variability of lightning discharges some do cause much higher induced voltages than others so that the conclusion to be drawn is that most direct strokes or at least many strokes are not responsible for high induced voltages.

Messrs. Lewis and Foust in discussing the mechanism of the lightning stroke point out the probability of discharges proceeding from the tower to the cloud when the cloud is negative. While this agrees with experiments made between a point and a plane in the laboratory, assuming the point to be positive, yet not all of the observations in the field confirm this point of view. If the clouds are usually negative for severe direct strokes one would expect it would be a common occurrence to witness long streamers from towers just before the flashover occurs. It is true that such streamers have been observed in a few cases, but apparently they are not a frequent occurrence. In the laboratory test it has been noted that the streamers extend away from the positive electrode and this led Simpson (*Proceedings of Royal Society A*, 1926 Vol. III, p. 56) to comment on the fact that only three out of more than two hundred photographs of direct strokes of lightning showed streamers upward, indicating that the earth was positive.

If long streamers do proceed upward from transmission line conductors and towers frequently some flashovers of transmission line insulation might be accounted for in this manner. This idea was originally suggested to me by Mr. H. P. Seelye as he had seen certain evidence of flashovers which led him to suggest the possibility of intense ionization on towers during lightning storms, which might cause flashovers even though no lightning discharge took place, which either struck the line or gave rise to induced effects.

Further study should be given to the determination of the manner of propagation of discharges in order that definite evidence may be obtained from which an adequate theory may be built up.

L. V. Bewley: There are seven main factors which characterize the magnitude and shape of induced voltage waves due to a lightning discharge.

They are:

Law (and time) of cloud discharge.
Distribution of bound charge.
Current in the stroke.
Charge on the cloud (amount and distribution).
Height of the cloud.
Field gradient.
Average height of the line.

Heretofore investigators have assigned values to some particular group of these variables, without respect to the effect of such arbitrary specifications on the remaining variables, and have arrived at unduly divergent conclusions as to the magnitude of induced voltages. However, as Mr. Peek's paper has so clearly shown, the above variables are not entirely independent of each other, but are definitely related, so that values cannot be arbitarily assigned to any one of them which violate the known confines of any of the others. He has made an outstanding contribution to our knowledge of the magnitude of induced strokes by tying all of these factors down with a single set of curves, Fig. 6 of his paper. By means of these curves, the influences of the time of cloud discharge, of the height of the cloud, and of the current in the stroke, can be readily estimated. Since the curves are based on a maximum field gradient of 100 kv. per ft. (an established value apparently agreed upon by all of the authorities on the subject), they are in conformity with that assumption.

It may be of interest to demonstrate the application of the equations in the Appendix of Mr. Peek's paper, to a specific case.

Cloud: A point charge (or uniformly charged sphere) Q at a height H.

Law of Cloud Discharge: Linear, in time T.

Ground Wires: None, therefore $\omega = 1.0$.

This particular example has previously been worked out by Mr. Fortescue.[1] It is included here merely to illustrate the general equations of which it is a special case. The vertical component of the field gradient near the surface of the earth due to a point charge (Q) at a height (H), and its image ($-Q$) at a depth ($-H$) below the surface, is

$$g(x) = \frac{2\,Q\,H}{(H^2 + x^2)^{3/2}}$$

and therefore the potential on the line at height h for an instantaneous cloud discharge is

$$f(x) = h \,.\, g(x) = \frac{2\,Q\,h\,H}{(H^2 + x^2)^{3/2}}$$

If the cloud discharge is linear, then

$$F(t) = \begin{cases} t/T & \text{for } t \leq T \\ O & \text{for } t > T \end{cases}$$

and the current in the stroke therefore is

$$I = Q\,\frac{\partial F(t)}{\partial t} = \begin{cases} Q/T & \text{for } \leq t\ T \\ O & \text{for } t > T \end{cases}$$

Then by equation (6) in the Appendix of Mr. Peek's paper

$$e = \frac{h\,I}{H\,v}\left\{\frac{x + v\,t}{\sqrt{H^2 + (x + v\,t)^2}} - \frac{x - v\,t}{\sqrt{H^2 + (x - v\,t)^2}}\right\} \quad \text{for } t \leq T$$

where v is the velocity of propagation of waves on overhead lines, and is approximately 1,000 ft. per microsecond. This potential distribution is seen to consist of a pair of waves traveling in opposite directions. They superimpose for a maximum, at $x = O$ and $t = T$, of

1. Discussion, C. L. Fortescue, A. I. E. E. Trans. Vol. 49, p. 1503, October, 1930.

$$V = \frac{2}{v} \frac{h}{H} \frac{I}{\sqrt{1 + (H/v\,T)^2}}$$

and this increases with the time of discharge T, as the current is assumed to be constant. For the time of cloud discharge in microseconds large (say 3 times) compared with the height of the cloud in thousands of feet, and converting to practical units,

$$V \cong 60 \frac{h}{H} I \quad \textit{volts}$$

where I is in amperes.

By equation (3) the reduction factor is

$$\alpha = \frac{V}{g\,h} = \frac{60\,I}{g\,H} \frac{1}{\sqrt{1 + (H/v\,t)^2}} \cong \frac{60\,I}{g\,H}$$

The equivalent rectangular bound charge, by equation (7), is

$$L = 2\,H = 2\,(\textit{cloud height}) = (\textit{front of traveling wave})$$

The fact that the front of the free traveling wave is practically equal to the length of the equivalent rectangular bound charge was established by calculating the wave shapes for a variety of different distributions of bound charge and noticing that the relationship holds when the time of cloud discharge in microseconds is long compared with the length of the bound charge in thousands of feet—a condition that is true in most cases.

There are only a few simple cases which can be calculated directly by the analytic formulas, due to the difficulty of performing the prescribed operations. However, equation (1) can be expressed as the limit of a summation, and therefrom solutions obtained by tabular step-by-step and graphical methods.[2]

Theoretically, from an oscillogram of an induced lightning surge which has not suffered attenuation or distortion, the following information is obtainable:

1. (2) (*Height of cloud*)
   = (*length of bound charge*) = (*front of wave*)
2. (*Time of discharge*)
   = [(*total length of wave*) − (*front of wave*)]
3. *Law of cloud discharge* from the solution of the integral equation

$$e'(t) = \tfrac{1}{2} \int_0^t f\,[x + v\,(t - \tau)]\, \frac{\partial F(\tau)}{\partial \tau}\, d\tau$$

The function $f(x)$ may be taken as that due to a point charge, or even as a rectangle, without appreciable error; because the shape of $e'(t)$ is not sensitive to changes in $f(x)$. If an analytic solution cannot be obtained, then trial functions $F(t)$ may be tried until one is found which approximately satisfies the relationship. Since the form of $e'(t)$ for many representative functions $F(t)$ have been worked out and tabulated, a fair idea of $F(t)$ for a given $e'(t)$ is not hard to arrive at, within the permissible limits of error.

Since $F(t)$ must be such a function that it reaches a value of unity at $t = T$ (the completion of cloud discharge) and since the crest of $e(t)$ is given, the maximum value of $f(x)$ is also determined by this integral equation. Then

4. The *maximum gradient* is $g(x) = \dfrac{2\,f(x)}{h}$

5. The *reduction factors* are

$$\alpha = \frac{e_{max}}{h \cdot g(o)} = \frac{e_{max}}{h \cdot g(o)}$$

$$\alpha' = \frac{e'_{max}}{h \cdot g(o)} = \frac{e'_{max}}{2\,f(o)}$$

6. The *charge on the cloud* (assuming a spherical cloud uniformly charged) is

$$Q = \frac{H^2\,g(o)}{2}$$

7. The *current in the lightning stroke* is

$$I = Q \frac{\partial F(t)}{\partial t} = \frac{H^2\,g}{2} \frac{\partial F(t)}{\partial t}$$

8. The cloud potential (assuming a spherical cloud of radius R) is

$$E = \frac{Q}{R} = \frac{H^2\,g}{2\,R}$$

It may be challenged that a sphere is not a true representation of a charged cloud. That is true, but by fortunate circumstances the shape of the cloud (or of the bound charge) does not seem to have much effect on the shape of the traveling waves, and consequently the assumption of a spherical cloud is as good as any for numerical accuracy, and has the distinct advantage of being relatively easy to handle mathematically.

As a result of their studies Messrs. Brune and Eaton have concluded that the counterpoise is in effect a very leaky short transmission line connected to the tower and open-circuited at its other end. This conclusion is evidenced by the highly attenuated and badly distorted reflections shown on the oscillograms. It is surmised, however, that the major benefit to be derived from the

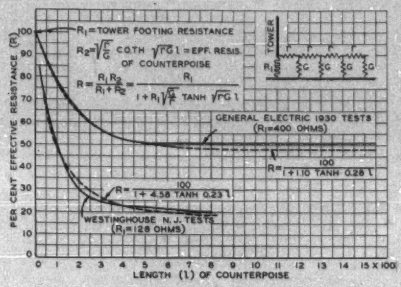

FIG. 8—EFFECT OF COUNTERPOISE

use of a counterpoise, is merely that of the reduction in true grounding resistance which it provides; and that the surge impedance and reflections are quite subsidiary effects. In support of this notion there have been plotted in Fig. 8 the field tests of two different investigations on the effects of varying the lengths of counterpoises, and the corresponding calculated curves based on assuming the counterpoise to be a simple distributed series resistance r and conductance to ground g. The constants used in the calculated curves were chosen to agree with the experimental results, so that the agreement between the test and calculated curves is of importance only in showing that they presumably follow the same law—in other words a distributed r and g is sufficient to account for the observed data within the limits of the experimental error in this type of field study. The equations are:

$$e = E \frac{\cosh \sqrt{r\,g}\,x}{\cosh \sqrt{r\,g}\,l} = \text{voltage at } x$$

$$i = E \sqrt{\frac{g}{r}} \frac{\sinh \sqrt{r\,g}\,x}{\cosh \sqrt{r\,g}\,l} = \text{current at } x$$

$$R_x = \frac{e}{i} = \sqrt{\frac{r}{g}} \coth \sqrt{r\,g}\,x = \text{effective resistance at } x$$

2. *Traveling Waves Due to Lightning*, L. V. Bewley, A. I. E. E., TRANS. Vol. 48, p. 1050, July 1929.

l = total length
x = distance from open end
r = resistance per unit length
g = conductance per unit length

The net resistance at the tower is the tower footing resistance R_T in parallel with the effective resistance of the counterpoise R_l, or

$$R = \frac{R_T R_l}{R_T + R_l} = \frac{R_T}{l + R_T / R_l} = \frac{R_T}{l + R_T \sqrt{g/r} \tanh \sqrt{r g}\, l}$$

which is the equation plotted in Fig. 8. In this equation it must be remembered that R_T and (l/g) under transient conditions are only about 75 per cent of the measured d-c. resistance; and that r is much greater than the measured d-c. resistance due to skin effect. Also the conductance g is difficult to measure, and probably varies over a wide range with the condition of the soil (moisture content), so that the practical utility of such an equation is very doubtful. Its chief merit lies in showing a reasonable correspondence with the observed experimental data, and therefore establishing to that extent the reason for supposing that a counterpoise is principally merely a low resistance to ground. But if that is so, then a given length of wire is most efficiently employed as a counterpoise by using it as a number of short lengths in parallel radiating out from the tower, rather than as a single length. And on this basis also, there is no particular virtue to the direction of the counterpoise—it might just as well be run perpendicular to the transmission line as parallel to it.

F. E. Andrews: The experience on the 132-kv. lines of the Public Service Company of Northern Illinois is of interest relative to the question of protection afforded by overhead ground wires. These lines are in general of double-circuit steel tower construction, using a single ground wire only over the one circuit installed on the single-circuit portions of these lines, and using two ground wires, one over each circuit, on the double-circuit sections. The single-circuit lines with one ground wire are all built on double-circuit towers, one side being left vacant for the future circuit.

The following tabulation summarizes this experience:

TWO GROUND WIRES

Year	No. interruptions	No. circuit miles	Trip-outs/100 miles due to lightning
1926	0	47.57	0.00
1929	1	158.06	0.63
1930	*3	158.06	1.90
Interruptions per 100 circuit miles per year over entire period			1.10
Interruptions per 100 tower-line miles per year over entire period			2.20

*Two of these simultaneous flashovers, third flashover from 132-kv. line to 33-kv. line on same right-of-way.

ONE GROUND WIRE

Year	No. interruptions	No. circuit miles	Trip-outs/100 miles due to lightning
1925	0	9.10	0.00
1926	0	9.10	0.00
1927	3	84.93	3.54
1928	3	112.25	2.67
1929	2	102.27	1.96
1930	9	132.50	6.80
Interruptions per 100 miles per year over entire period			3.78

ALL LINES SINCE GROUND WIRES WERE INSTALLED

Year	No. interruptions	No. circuit miles	Trip-outs per 100 miles due to lightning
1925	0	9.10	0.00
1926	0	9.10	0.00
1927	3	84.93	3.54
1928	3	159.82	1.88
1929	3	260.33	1.15
1930	12	290.65	4.13
Interruptions per 100 miles per year over entire period			2.58

It will be noted that with the lines protected by two ground wires, interruptions per 100 circuit miles per year average 1.1, whereas with the lines protected by one ground wire only the average is 3.78 interruptions per 100 miles per year. If it is considered on the double-circuit lines that the exposure to lightning is proportional to tower line miles rather than circuit miles, the interruptions per 100 tower-line miles per year with two ground wires would be 2.2. Either of the two figures for the lines equipped with two ground wires indicates a material improvement in performance in comparison with the lines equipped with one ground wire only.

Another interesting instance showing the protection of the ground wires can be cited on our Chicago Heights—Joliet line which had a four-mile section of ground wire removed during 1930 in the vicinity of the Frankfort lightning oscillograph station. During 1927, 1928, and 1929 there had been no trip-outs due to lightning in this section protected by the ground wire although during the two years previous to this, *i. e.*, in 1925 and 1926 during which there was no ground wire installed, there had been a number of lightning interruptions through this section. During 1930, with the ground wire removed, there were three lightning trip-outs in this section.

C. L. Fortescue: The first part of Mr. Peek's paper which seems to me to be a plea for the induced stroke theory is written in a convincing manner. If it were not for the fact that I have what I believe to be good physical grounds for disagreeing with him I might even be convinced myself. The basis on which the conclusion was arrived at by the group of engineers working on the lightning investigation, with which I have been associated, was that induced surges were probably not a factor in producing outages due to lightning on high-voltage lines. Last summer I made a mathematical analysis of the magnitude of induced voltages that would appear on a line due to a stroke to earth nearby and the value I obtained for a cloud discharging at the rate of 200,000 amperes was of the order of 120 kv. Mr. Peek says in his paper that the low value which I obtained was due to my assumption that the cloud was 5,000 feet high and he states, though I do not clearly follow his reasoning, that if a lower height had been chosen the induced voltage obtained would be much higher. If I understand his reasoning, it is based on the fact that with 20 coulombs the field due to the cloud at the lower height would be very much more intense. However, the 20 coulombs and the height chosen will give a field of 50 kv. per foot at the earth's surface which is a fairly intense field and with lower heights the same field gradient will require correspondingly lower charges.

It would seem reasonable that the formation of the lightning streamer would be associated with some minimum value of field intensity and therefore the clouds at the lower height would be apt to discharge at a lower energy content and therefore produce lower induced surges than clouds of higher altitude. This seems to be in accord with field experience, in that in the parts of the country where the ceiling is low the thunderstorms are more frequent but apparently not so violent as in those parts where the ceiling is high.

As to the physical conditions which I feel absolutely prevent the cloud from discharging at the rates required to produce a high induced voltage, the thundercloud itself is bipolar, that is to say it has a positively and a negatively charged portion. This has been determined by actual observation and is an experimental fact. The very nature of the thundercloud requires that its atmosphere shall be a good insulator. In fact, it is probable that the cloud atmosphere is a better insulator than atmospheric air at the earth's surface on account of the absence of free electrons. This atmosphere of the thundercloud consists of discrete particles of water vapor charged with electricity. There are several theories accounting for the presence of these charged particles and the bipolar nature of the thundercloud. That of Dr. Simpson of the British Meteorological Department, is perhaps the best known and the one that is generally accepted. According to this theory the large drops of water are broken into spray by the wind and in doing so the spray is negatively charged and carried upward by the wind and in descending forms the negative portion of the cloud, the first portion of the cloud consisting of large water drops becomes positively charged. The two portions of the cloud atmosphere are kept separated by the wind due to a difference in their mobility. However, if free electrons were present in large quantities these would not be affected by the wind but only by the electric field in the cloud, as a consequence of which they would discharge the two portions of the cloud as fast as they become charged. It is a fact that the particles of water vapor have the faculty of capturing the stray electrons that make possible the formation of the thundercloud.

The particles of water vapor have very little mobility and consequently when the lightning channel is formed between cloud and earth they cannot by their motion discharge the cloud. The only way that the charge of the cloud can be carried to earth is by ionization of the cloud atmosphere itself and this is a slow process requiring a high gradient, and the velocity of discharge instead of being of the order of the velocity of light will probably be of the order of one-twentieth this velocity.

I cannot follow Mr. Peek's reasoning regarding the relation between the time from zero to crest of the wave and the height of the thundercloud. I can see no relation whatever myself. For example, in the case which I considered last summer with an infinite line and the cloud discharging at a uniform rate, the analysis shows that the induced voltage will continue to increase until the discharge ceases and then it will decrease logarithmically. Here, where I took a uniform rate of discharge of the cloud of 200,000 amperes, the total quantity discharged being 20 coulombs, the time from zero to crest would be 100 microseconds. It also seems to me that the assumption that the undistorted positive surge, where no outage had taken place on the line, is induced, is not tenable, for in probably one-half the cases where a direct stroke takes place to the line no outage occurs even though the insulator flashes over.

Regarding the time of discharge of surges due to direct strokes, as I have pointed out many times, a cloud discharge may or may not reach earth depending upon whether the energy given up by the cloud is sufficient to support the streamer in its path towards earth; consequently, the lightning strokes to a line may be weak and of short duration or intense and of long duration. I think it may be stated quite definitely that those of short duration have very little potential while those of long duration have a high potential. To support Mr. Peek's theory of induced voltage this condition would have to be reversed, that is to say, intense strokes would have to be of short duration and the weak strokes would have to be of long duration. These, in brief, are the physical reasons why I do not believe that it is possible to get a very high voltage induced in a transmission wire due to a stroke of lightning to the earth that does not hit the wire.

Referring to the paper by Lewis and Foust, I would like to point out that one is not justified in assuming that a stroke which reaches the earth near a transmission line will not also make contact with the line itself. In fact it is highly probable that even though the main part of the stroke hits the earth a streamer may strike the line or that a second stroke following in general the path of the first may make electrical contact between cloud and line in either of which cases, of course, the surge resulting cannot be considered as an induced surge. Furthermore on transmission lines well protected with ground wires when the ground wire is struck the conditions are ideal for producing a high-induced surge were such a surge possible, since there is a bound charge on the line wires which is released when the lightning stroke is discharged to earth through the ground wires and towers. However, we have no record of any such high-induced surges despite the fact that our cathode-ray oscillographs have been connected to lines which were known to be struck by lightning.

In the paper by Mr. Lloyd the means by which he arrived at the 670,000 amperes of lightning current, shown in this table, is not permissible. I understand that the currents in his six towers were added directly. Since these maxima occur at different times and a considerable time after the maximum crest of the lightning stroke they cannot be added directly. As far as I can see, if the measurements are correct the current that can be assigned to the lightning stroke is not over 300,000 amperes.

F. W. Peek, Jr.: Contrary to Mr. Fortescue's impressions, the first part of my paper is not a plea for induced voltages. I have shown how induced voltages are affected by such factors as height of cloud, time of discharge, current, height of line, gradient, etc. Then, by inserting *measured values* of current, time, etc., I have shown the possible range of induced voltage values on lines. It is quite evident that much higher induced voltages result from certain types of lightning discharges than from others. Some discharges near a line would be expected to cause high voltages while others would not. The relative occurrence of these types and therefore the frequency of high-induced voltages is being determined statistically by cathode-ray measurements. High-induced voltages can occur. How often are the conditions such that they do occur?

A similar analysis has been made in my paper of direct strokes and methods of determining their effect developed.

In general it can be said that as the operating voltage, or rather the line insulation is increased, the direct stroke becomes an increasing cause of outage. For the low-voltage line the induced voltage is the principal cause of trouble. It cannot be said that there is any definite dividing voltage at which one or the other is the more important. There are too many varying factors involved for different locations. *For high-voltage lines the design must be made with full consideration of direct strokes.* Fortunately, direct-stroke remedies are also good induced voltage remedies. I have given a number of examples in my paper.

Mr. Bewley in his discussion has brought out some very important facts. One of these that I also wish to emphasize is that the different variables such as time, voltage, current, are definitely related and that erroneous conclusions will be reached if arbitrary values are assigned to one group without reference to the other. This has been largely done in the past by some investigators.

Perhaps the most interesting fact brought out in my paper is that complete information regarding a lightning stroke including height of cloud, voltage, etc., is given by the oscillogram of the traveling wave on a transmission line. Such a wave is in fact an illustrated autobiography of the stroke.

Mr. Fortescue has asked how a cloud lower than 5,000 ft., which he assumed in his example cited in my paper, could cause a higher induced voltage. Mr. Fortescue will find that the equation which he used in calculating his example can be reduced to the following form:

$$V \simeq 60 \frac{h}{H} I \text{ volts}$$

Where h = height of line in feet
H = height of cloud in feet
I = current in amperes

With the equation in this form it can be readily seen that if the cloud height H is reduced with a given constant discharge current the voltage is increased. For example, with a cloud 5,000 ft. high the induced voltage at 200,000 amperes discharge is 120,000 volts on a 50-ft. line wire; with a cloud 1,000-ft. high the induced voltage at 200,000 amperes discharge is 600,000 volts on a 50-ft. line wire. Examples of this are given in Table II of my paper.

Mr. Fortescue speculates on the time that it takes a cloud to discharge. I have avoided this speculation and have considered values of time measured by the cathode-ray oscillograph. The time of cloud discharge should be no longer than the total time recorded by the oscillograph. (See Fig. 2 of my paper.) Mr. Fortescue states that his 5,000-ft. cloud discharging at a velocity of 1/20 of that of light would require 100 microseconds to complete its discharge. A great majority of the cathode-ray oscillograms of natural lightning are much less than 100 microseconds and some of the more severe ones are of the order of 10 microseconds.

In support of his belief that no relationship holds between the front of the wave and the height of the cloud Mr. Fortescue states that the final crest is not reached until after 100 microseconds. It is quite true that the final crest is not reached until 100 microseconds have elapsed in his example. However, if the calculated curve is plotted it will be seen that the voltage reaches 90 per cent of its final value at the end of 10 microseconds and it takes 90 microseconds to reach the remaining 10 per cent. *Practically speaking, therefore, the crest* of the wave is reached in 10 microsec. Thus,

$$2\,H = 10 \text{ microsec.}$$

or

$$H = 5 \text{ microsec.} = 5{,}000 \text{ ft.}$$

In other words, the height of the cloud is determined from the time from zero to approximate crest of the wave.

If the bound charge had been rectangular the front of the wave would have been exactly the length of the bound charge. For other distributions it is very nearly so.

The energy necessary to complete a lightning discharge is less for short streamers than for long ones. It follows, therefore, that although the energy of a low cloud may be less than for a high cloud a discharge may nevertheless take place.

Mr. Fortescue has questioned Mr. Lloyd's method of arriving at the total current in the lightning stroke by adding the currents measured in the adjacent towers. I have analyzed this method and illustrate the results in Fig. 9 of my paper. This analysis shows that with moderately low tower footing resistances approximately correct values of total current are obtained by adding the values measured in the several separate towers. Correction may be made from Fig. 9.

C. L. Fortescue: I pointed out in my discussion of Mr. Peek's paper that the nature of the thunder cloud does not permit of very quick discharge of its energy, and, therefore, the induced surges are not of high enough magnitude to affect high-voltage transmission lines. It is quite possible, however, that for the lower voltage lines, those below 66,000 volts, the surges produced by induction may be a factor in causing outages.

In answer to Mr. McEachron's questions I think that I have made it plain in my discussion why I believe it impossible for a thunder cloud to discharge at a high rate. However, I have mentioned the speed of propagation of the lightning channel between ground and earth as being of the order of one-twentieth the velocity of light. My basis for this estimated velocity is the speed of propagation of streamers between two spheres. In tests with suppressed charges this velocity was estimated to be of the order of one-twentieth the velocity of light. This, however, is merely an approximate figure. It is probable that the speed of propagation is not as high as this. If I remember correctly, Dr. Toepler estimates it even lower than this. I think his figure, based on analysis, gave it as of the order of one-thousandth the velocity of light.

With regard to the nature of the lightning discharge during the progress of the streamer, the space charge on the outside of the streamer is progressively fed by the actual current in the channel which is comparatively small. However, the air in the core of the channel is probably highly conducting through thermionic disassociation of the air molecules. On account of the fact that current in this core is small most of the energy in the channel before it reaches the earth is potential energy in the form of space charge. However, when the streamer reaches a transmission line, this potential energy is at once changed into kinetic energy which results in a surge propagated in the transmission line away from the channel and a receding surge propagated in the channel changing the potential energy into kinetic energy, or, in other words, producing current flow in the channel. When this receding wave reaches the cloud, it at once sets up a high gradient in the cloud causing the release of more of the cloud's energy. This gradient is, of course, propagated into the cloud producing ionization and even streamer formations.

I do not agree with Dorsey's dart theory nor do I believe with Simpson that the streamer is initiated at the positive end. I believe that the streamer is set up by conditions at the surface of the cloud which produces high enough gradient to cause ionization and that this condition will persist so long as the cloud is able to furnish sufficient energy to keep the streamer progressing. This means that at the end of the streamer space charge must be formed high enough to cause progressive ionization. The streamer, therefore, progresses as fast as it can with the energy available, the speed being controlled by the rate at which ionization can take place.

After the channel has been established between the cloud and earth, it then forms a conducting path and the propagation of the energy from the cloud to the earth along this channel is determined by the law of propagation of surges along conductors. However, as Mr. McEachron remarks, surge impedance along the channel is not uniform but will depend upon the height above earth, but the only value that we are concerned with in estimating the effect on transmission lines is the value at the point of the stroke which I have estimated to be 200 ohms. This figure may be too high or too low, but it is convenient to use as a basis for estimating the surge potentials that may rise on transmission lines due to lightning. Of course, just at the instant before the streamer strikes the transmission line, its actual potential will be double that assigned to the kinetic surge, which conforms to the law of surge propagation. I agree with Mr. McEachron that research should be carried out to determine the proper value to give to this surge impedance and also the mechanism of the streamer formation should be given considerable study, but we cannot wait for the results of these investigations in designing transmission lines.

Mr. McEachron also remarks that the values given for spacing between ground wires and lines in the middle of the span seem to him excessive. This may be true and there are lines with maximum spacings of the order of 15 to 17 ft. which are giving a good record in certain parts of the country. Some recent laboratory work indicates that a lower spacing will be satisfactory, perhaps a maximum of from 30 to 35 ft. This lower spacing results from the fact that the coupling between ground wire and line wires is much higher than that obtained from the physical dimensions of the wires, on account of the large diameter due to corona formation on the ground wires at the instant of stroke. In short, I am in accord with most of what Mr. McEachron says and agree with him that we have a lot more to

learn about lightning though we have made very marked progress in the last few years. However, I feel that it is better to err on the safe side and use somewhat larger clearances than necessary than to use too small clearances and regret it later on.

In his discussion of the induced stroke versus direct stroke theory, Mr. McEachron assumes that when the earth or a tree is struck in the neighborhood of a transmission line that the high voltage appearing on a transmission line as a result of the stroke is necessarily induced. I wish to suggest that when a high resistant object is struck the potential of the streamer may have a value of 40,000,000 volts. Under these conditions it is quite possible for a side streamer to also hit the transmission line. The lightning stroke as we see it is made up of a succession of discharges, some of which might hit the tree and others of which might be diverted to the transmission line. The conclusions that outages on high-voltage transmission lines are caused by direct strokes and not by induced surges are not only based on observations of the past few years but also on analysis. In order to produce potentials of the order of several million volts, it will be necessary to have discharge current in the lightning channel of the order of several million amperes. Such measurements as have been taken in the past show no indication of discharge currents of such large order of magnitude.

W. W. Lewis and C. M. Foust: Referring to Mr. Cox's discussion, the antennas adjacent to the Pennsylvania line are connected to ground through a few megohms resistance. They are intended to give a measure of the cloud field potential at the height of the antennas and not to measure the potential which would be induced on the line.

In the next to the last paragraph, Mr. Cox states that Mr. Torok's work indicated that a discharge streamer always started from the electrode of high potential (gradient), that is, the electrode of smallest effective dimensions, whether such electrode is positive or negative. This is equivalent to stating that polarity has no importance in connection with breakdown between electrodes of dissimilar shape.

As pointed out in our paper, a considerable amount of published data, showing the importance of polarity effects on such breakdown is available. Attention is especially called to Reference 18. The authors have presented considerable evidence which demonstrates the vital importance of polarity in the mechanism of breakdown in an article, entitled, "Direct Strokes to Transmission Lines," *G. E. Review*, August 1931.

The final paragraph of Mr. Cox's discussion refers to a suggested explanation for the high-positive conductor potentials recorded at one station with very little or no voltage at adjacent stations. We agree with Mr. Cox that the apparent rapid attenuation indicates that these are not induced surges of the commonly accepted kind. The second explanation the authors feel to be more reasonable. There are considerable data showing that the tower structure and adjacent ground are elevated to high potentials when the direct-stroke currents pass through the tower structure. The conductor tends to remain at ground potential. The measuring instrument connected between the conductor and the bottom portion of the tower measures a potential between the highly-charged tower structure and the conductor. The instrument, of course, indicates a conductor polarity opposite to that of the tower itself. The line conductor record of positive polarity on this basis indicates a direct stroke to the tower structure of negative polarity.

Mr. McEachron refers to Simpson's classification of streamers branching away from and toward the cloud. Further consideration of this classification might indicate somewhat different conclusions. The large number of unbranched discharges would appear to the authors to be logically classified as strokes between negatively charged clouds and positively charged earth. The upward streamer branching is very likely entirely concealed from the observer, as it takes place in the cloud itself. The preponderance of branched discharges found among collections of photographs is, undoubtedly, influenced by a desire to retain the more spectacular pictures.

The field distribution between cloud and transmission line tower is such that any extension of breakdown streamers from the tower toward the cloud would result in an increasing gradient at the streamer tip and, thus, a rapidly increasing streamer length. Long streamers proceeding from the tower top and terminating in space would not be common on this basis.

An Electric Analog of Friction

For Solution of Mechanical Systems Such as the Torsional-Vibration Damper

BY H. H. SKILLING[1]

Associate, A. I. E. E.

Synopsis.—When the mathematical solution of a problem is so difficult as to be impracticable it sometimes is possible to obtain the desired answer from the behavior of an analogous electric circuit. This article presents an analog of "coulomb-friction," and demonstrates the application of this analog to the previously unsolved problem of the torsional-vibration damper.

SOLUTION of the equation of motion for the torsional-vibration damper represents one of the mathematical problems which is extremely difficult to solve. Almost endless complications are involved on account of the peculiar combination of vibratory motion and friction which the problem contains. An approximate solution can be arrived at by introducing several assumptions, but this is not entirely satisfactory; in the more intricate cases, approximate solutions are of no value at all. In order to predict the effectiveness of new designs, mechanical models sometimes are built and tested. Difficulty often is experienced, however, in the construction of these models, and in addition, the results obtained have been found to be somewhat uncertain.

In this article is proposed a method of solution wherein electric analogs are set up for the various types of mechanical motion involved. An electric circuit can then be set up with constants analogous to the mechanical characteristics of the apparatus being studied. The principal advantage in treating the problem in this manner is in dealing with a system easily put together and altered, and with quantities easily metered. Although the analogous method in general is not new, difficulty has been experienced previously in representing mechanical friction electrically. In the present investigation this obstacle has been overcome by the use of gaseous conduction tubes.

The torsional-vibration damper is in effect a flywheel that fits loosely on a shaft. A common application is on the crank-shaft of a Diesel engine, where there is danger of torsional stress in the shaft becoming extremely large because the natural frequency of oscillation of the shaft may be the same as the firing frequency of the cylinders. The damper flywheel is placed on the shaft at the point where torsional vibration is greatest; it is not keyed fast but turns with the shaft because of the friction between the two. When rapid torsional vibration of the shaft exists, the friction force is not great enough to accelerate the flywheel as rapidly as the shaft; hence the shaft slips within the flywheel and in so doing dissipates sufficient energy to prevent resonant oscillation of the shaft. In actual design, the damper although simple is not so elementary as this account has indicated. A detailed description may be found in a paper by Den Hartog and Ormondroyd (*A. S. M. E. Trans.*, Vol. APM-52-13, p. 133) which also treats its action by an approximate mathematical solution.

With ordinary dry friction—so-called "coulomb friction"—which is involved in the torsional-vibration damper, the frictional force depends upon the nature of the rubbing surfaces; also upon the force holding them together, but not upon velocity. An electric analog for this application must therefore produce a constant voltage drop, variable at will but independent of the amount of current flowing.

TABLE I—MECHANICAL-ELECTRICAL ANALOGIES

Mechanical quantity	Electrical quantity
Torque	Voltage
Velocity (angular)	Current
Displacement (angular)	Charge
Moment of inertia	Inductance
Elastance (torsional)	Elastance
Mechanical hysteresis	Resistance

A mechanical system which is the practical equivalent of a Diesel engine may be seen in Fig. 1. The main flywheel is shown at A; moment of inertia of the pistons, cranks, and other moving parts is represented by another flywheel B; the damper flywheel which fits loosely on the shaft is shown at C. Input torque from the cylinders, which may be considered as a steady torque with a superposed alternating torque, enters the system at the pistons. Any load on the engine will be applied to the left of the main flywheel.

The electrical analogy to this mechanical arrangement is shown also in Fig. 1 where, with the exception of friction between the shaft and the damper flywheel, the electrical quantities used are as shown in Table I. Analogous to the shaft is condenser C; analogous to the piston flywheel B is inductance L. These cooperate to form the oscillating system, since the main flywheel is so large that for the present it can be considered infinite and therefore impassable by any vibrations. The damper flywheel C is represented by the inductance L, but because of its looseness on the shaft a shunt must be

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connected around the inductance to make the analogy complete. In this shunt is the apparatus which, to be analogous to friction of the flywheel on the shaft, must prevent current from flowing until a certain voltage is reached, and then permit current to pass without any further increase of voltage.

The peculiar property of constant voltage drop is possessed by a mercury arc; it is also characteristic of glow discharge under special conditions. For this

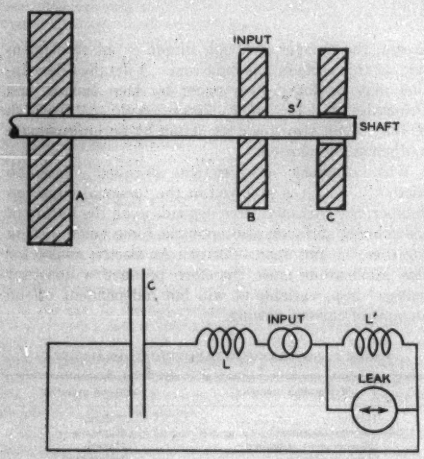

FIG. 1—(ABOVE) MECHANICAL SYSTEM SCHEMATICALLY REPRESENTING THE DIESEL ENGINE; (BELOW) THE ELECTRIC ANALOG OF THIS MECHANICAL SYSTEM

investigation raytheon gaseous conduction tubes were employed. Two tubes were used, one to pass current in each direction; the voltage drop in the tubes as they were connected in this case was 85 volts independent of current. In series with each tube was a source of potential which either added to or subtracted from the 85-volt tube drop depending upon other conditions. For the input to the circuit an alternator was used the speed and voltage of which could be varied over an extremely wide range; its resistance, together with that of the rest of the system, was small. For mechanical hysteresis losses in the elastic parts, and for any viscous friction that may exist, resistance is sufficiently analogous to permit its use.

The validity of the complete analogy may be seen from the differential equations of the two systems which are adapted from Den Hartog. For the mechanical system,

$$I \frac{d^2 \alpha}{d t^2} + I' \frac{d^2 \alpha}{d t^2} + k \alpha = M \sin \omega t$$

when the damper flywheel is not slipping, and

$$I \frac{d^2 \alpha}{d t^2} + k \alpha \pm T = M \sin \omega t$$

when this flywheel is slipping.

I = moment of inertia of system being damped
I' = moment of inertia of damper flywheel
k = elastance of shaft
α = flywheel displacement from neutral
$M \sin \omega t$ = input torque as a function of time

The analogous electrical equations, when the shunt is not working and when it is, are respectively,

$$L \frac{d^2 q}{d t^2} + L' \frac{d^2 q}{d t^2} + \frac{1}{C} q = e \sin \omega t$$

$$L \frac{d^2 q}{d t^2} + \frac{1}{C} q \pm v = e \sin \omega t$$

All terms are well known except v, the voltage drop in the leak, which is added or subtracted depending upon the direction of the current. The analogy of corresponding terms, displacement, and charge, for example, may be seen to agree with Table I.

The purpose of making a detailed study of damper performance, whether by mathematics, model, or analogy, is to determine whether a particular damper

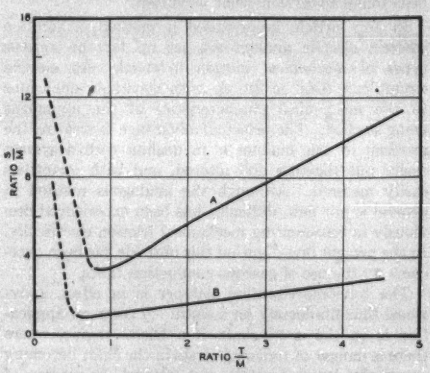

FIG. 2—PERFORMANCE CURVES FOR THE TORSIONAL-VIBRATION DAMPER AS DETERMINED BY ELECTRICAL ANALOGY. (A) WITH RIGID DAMPER SHAFT; (B) WITH ELASTIC DAMPER SHAFT

is so proportioned as to prevent dangerous stresses in the shaft. Momentary stresses in a shaft are exceedingly difficult to measure, but the analogous voltages across the condenser in the electric circuit may be read quite easily on a crest-reading voltmeter; thus for any given combination of quantities and dimensions an indication of maximum stress is obtained immediately by simply reading that meter.

The crest voltage of condenser C (Fig. 1) is analogous

to the maximum torque in the shaft and will be designated by S; analogous to the torque required to slip the shaft inside the damper flywheel, is the voltage drop in the leak which will be denoted by T. Results of a study by such analogy may be plotted showing S as a function of T, where curves similar to those of Fig. 2 will be obtained. If the value of S at the lowest point of the curve represents a safe stress, the damper is large enough to be useful; the value of T corresponding to this minimum value of S gives the optimum adjustment for tightness of the damper flywheel.

To avoid changing back and forth from mechanical to electrical units, it is well to avoid absolute values of quantities. Relative values need not be so transformed. Let M be the amplitude of the alternating input torque; instead of plotting S as a function of T, S/M may be plotted as a function of T/M. This scheme involves neither volts nor foot-pounds but only ratios, the ratio of voltage to voltage being of course numerically equal to the analogous ratio of torque to torque. The scale

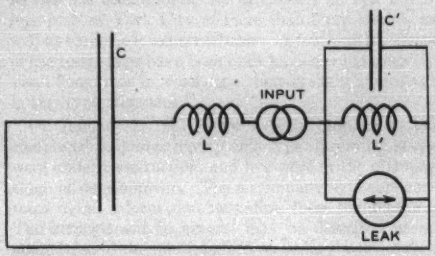

Fig. 3—Electric Analog of the Torsional-Vibration Damper with Elastic Damper Shaft. Compare with Fig. 1

of the electrical model is equally unrestricted; only the proper ratios between parts need be specified. The frequency of the input voltage is of course the frequency causing the most severe stress; in the simple mechanical system the crucial stress will occur when the frequency of firing is the same as the natural frequency of the vibrating system.

As a typical example, assume a damper with moment of inertia of the damper flywheel equal to the moment of inertia of the system being damped. In the analog any convenient capacitance and inductance may be used to represent C and L (Fig. 1), but they must be resonant at some frequency attainable by the alternator used for *input*. The inductance L' equal to L is then added, and the vacuum-tube leak is connected across L'. The input voltage is adjusted to any convenient value, and the voltage drop in the leak circuit made comparatively large (for instance, 15 volts input and 85 volts drop in the leak). Now, as the frequency of the input voltage is varied, the crest voltage across condenser C will rise to a maximum and then decline. The maximum value of this voltage (in this particular case about 208 volts) is analogous to the maximum torque in the engine shaft. Dividing this by the maximum value of the input voltage, (about 21.2 volts) and the result S/M is about 9.8; divide the 85-volt drop also by 21.2 and the quotient, which is 4, is the value of T/M. By varying the leak and input voltages, enough points can be determined to give curve A of Fig. 2. This curve gives the whole story of the operation for this particular damper. Moreover, it gives the entire story of operation for all dampers having the same moment of inertia as has the system being damped. In present practise, however, a damper has from one-fifth to one-tenth the amount of inertia of the system being damped, and S/M is proportionately larger.

So far, the damper has been considered as being applied to the simplest possible vibrating system. This simple system may be solved mathematically, although with some difficulty if approximations are avoided. Accordingly, this case was used as a general check on the method, and the results were compared with Den Hartog's computed values; agreement was so close as to be within the limits of experimental error.

With the more complicated systems, however, solution by analogy is of far greater value. When the elastance of that part of the shaft labelled S' (Fig. 1) is considered, a mathematical solution is out of the question; in the analog a condenser of the proper size is simply shunted around L', and the solution is as simple as before. The study of this circuit disclosed an interesting and important relationship that previously was not recognized. As it is a typical analog, it will be used as a further example of the application of the method.

The circuit used with C' representing the elastance of the damper-shaft is shown in Fig. 3. The elastance of condenser C' must bear the same proportion to condenser C as elastance of the damper-shaft does to the main shaft. Now as the input frequency is varied, the voltage measured across C no longer has one maximum, but three. The reason for this is apparent from the diagram, since there are now two resonant parts of the circuit, consisting of (1) C and L and (2) C' and L'. At some definite frequency the reactance through C and L in series is zero; at two additional frequencies the reactance of the complete circuit is zero. However, if the natural frequency of C' and L' is the same as that for C and L, when the reactance through C and L is zero, the reactance through C' and L' is infinite; thus the circuit current under these conditions is small and one maximum eliminated. The other two maxima, one falling at a higher and one at a lower frequency, must be prevented from becoming serious by the action of the leak.

Experiments with this second analog showed that the damper is much more effective with an elastic

damper-shaft than it is with a rigid shaft as in the first case. When the damper flywheel has one-fifth the moment of inertia of the system being damped, and when the damper-shaft is one-fifth as stiff as the main shaft, the analog method gives results as shown in curve *B*, Fig. 2. To make due allowance for the light weight of the damper, all ordinates have been multiplied by the ratio of moments of inertia (one-fifth); this makes the curves comparable to those of Den Hartog and Ormondroyd if a factor of $1/\pi$ is introduced into the vertical scale.

Since the primary purpose of this article is to point out the merits of the electrical means of solution, no particular attention is given to the mechanical merits of this new design of damper. Certain relationships were discovered by study of the analog, however, which seem worthy of mention. The "elastic damper-shaft" is not in the nature of a spring, for a shaft one-fifth to one-tenth as rigid as the main shaft of a Diesel engine is needed. The torque on the damper-shaft never can exceed T, the slip torque of the damper, and the shaft may be designed accordingly. The smaller the damper, the more effective will be a proper damper-shaft in improving its performance.

In conclusion, the principal advantages to be gained by the use of the electric analog are that (1) it shows the extent to which an approximate mathematical solution may be trusted, and (2) it offers a new method of approach to designers of dampers and other frictional devices. It is particularly useful in the study of new designs and at present there appears no limit to the possible applications of the method.

Vertical Networks in Metropolitan Office Buildings

BY A. H. KEHOE[1] and BASSETT JONES[2]

Fellow, A. I. E. E. Fellow, A. I. E. E.

Adoption in New York City

A TYPE of a-c. distribution known as vertical network service has been adopted for the supply of twenty-two large buildings in the Borough of Manhattan, New York City, from the distribution system of The United Electric Light and Power Company. This kind of installation was first offered by companies comprising the New York Edison System in December 1928, and at that time, the arrangement was accepted by the owners of the Irving Trust Company building at 1 Wall Street as suitable for the electric supply to that building. Similar arrangements were made almost simultaneously with several other buildings which at that time either were being planned or were under construction. Since then, this type of service has been adopted for practically all new buildings in New York City of more than forty stories, as well as for some lower structures. A few high buildings of the tower type have been built having comparatively small floor areas in which case there is slight advantage in this type of service.

On January 1, 1931, there were in Manhattan ten completed buildings having this type of service, seven were under construction, and five were in the planning stage of development. The accompanying Table contains detail information regarding these installations. The arrangement in general may be described as an adaptation of the multiple-feed secondary a-c. network to the supply of tall or large buildings. Fig. 1 shows typical examples of street and building network supply systems.

High-Tension Supply Above the Street Level

Alternating-current distribution for densely-loaded urban areas in recent years has been made multiple-feed network in many localities. The first installation of this type in New York City was completed in 1922. This arrangement gives an economical and reliable system which satisfies the electrical distribution requirements of most urban districts. In many cities networks are of the three-phase, four-wire type, supplied at 120/208 volts. The distance from the distribution transformers over which the energy at such voltages can be economically transmitted is definitely limited if both a reasonable amount of copper for distribution and satisfactory regulation are to obtain.

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Presented at the Middle Eastern District Meeting of the A. I. E. E., Pittsburgh, Pa., March 11-13, 1931.

In a number of cities, there has been a decided tendency in recent years to increase the height of buildings; this has resulted in an increase of total floor area many times the ground area, with a corresponding increase in the electrical density of distribution. While in these large buildings the electrical-load density per sq. ft. of the floor area has increased somewhat, the percentage increase, of course, is much less than the increase in load density based on ground area.

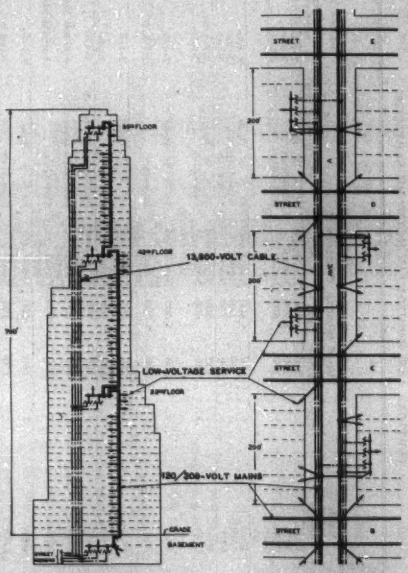

Fig. 1

While this load can be considered as a dense load at the street level, yet the area may be so located that it is cheaper in over-all costs to supply it from distribution transformer capacity in the upper floors of the building than to distribute the energy at 120/208 volts from the street distribution system on which the building faces. Thus it might be considered that the utility has been provided with the equivalent of a large addition to its territory to be supplied. This will be readily under-

Date: 1-1-31

VERTICAL AND HORIZONTAL NETWORK SERVICES

Name and type of building	Stories	Gross floor area (sq. ft.)	Total feeders supplying service 13.8 kv.	Total feeders supplying service 3 kv.	Vault location (floor)	No. of banks	Size of banks (kva.)	Initial capacity in vault (kva.)	Character of inter-vault ties	Terminal points of inter-vault ties	Sizes of inter-vault ties	Load and estimated demand	Status of installation
Chrysler	77	1,160,000	5	3	Base	4	300	1,200	Non-fused	St. N. W.-Base	7-4/0	Light =2,460 kw.	Complete
	&				30th	4	300	1,200	None	Base-30th	None	Power =3,378 hp.	Complete
(Office)	Tower				60th	4	150	600	None	30th-60th	None	Demand =2,500 kw.	Complete
					74th								Held pending flood-light development
Bank of Manhattan	72	800,000	5		Base	5	300	1,500	Non-fused	St. N. W.-Base	7-4/0	Light =3,241 kw.	Complete
	&				15th	5	150	750	None	Base-15th	None	Power =3,765 hp.	Complete
(Bank & Office)	Tower				65th	5	150	750	None	15th-65th	None	Demand =2,000 kw.	Complete
Daily News	36	723,500	4		Base	4	150	600	Non-fused	St. N. W.-Base	2(7-4/0)	Light =1,115 kw.	Complete
					25th	4	150	600	None	Base-25th	None	Power =1,292 hp.	Complete
												Demand = 900 kw.	
	9	314,090	5		Base	5	300	1,500	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light = 639 kw.	Complete
(Office and Printing Plant)												Power =2,782 hp.	
					2nd	4	300	1,200	Fused Non-tapped	Base-2nd	2(7-4/0)	Demand =1,500 kw.	Complete
Downtown Club	42	520,154	4		Base	4	150	600	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light =1,043 kw.	Complete
												Power =1,117 hp.	
(Club House)					19th	4	150	600	None	Base-19th	None	Demand = 805 kw.	Complete
Irving Trust Co.	50	500,000	4		Base	4	150	600	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light =1,379 kw.	Complete
												Power =4,047 hp.	
					21st	3	300	900	Fused Tapped	Base-21st	4(4-300 MCM V.C.)	Demand =1,500 kw.	Complete
(Bank & Office)					37th	3	300	900	Fused Tapped	21st-37th	4(4-300 MCM V.C.)		Complete
Bardiam Plaza	40	460,000	3		Base	3	150	450	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light = 742 kw.	Complete
												Power =1,120 hp.	
(Hotel)					37th	3	150	450	None	Base-37th	None	Demand = 645 kw.	Complete
Salmon Towers	60	649,403	4		Base	3	300	900	Non-fused	St. N. W.-Base	7-4/0	Light = 974 kw.	Complete
												Power =1,928 hp.	
					22nd	2	150	300	Fused Tapped	Base-22nd	2(4-300 MCM V.C.) 1(4-400 MCM V.C.)	Demand = 940 kw.	Complete
					42nd	2	150	300	Fused Tapped	22nd-42nd	3(4-250 MCM V.C.)		Complete
(Office)					59th	3	150	300	Fused Tapped	42nd-59th	3(4-3/0 V.C.)		Complete
	38	460,000	4		Base	3	150	450	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light =1,170 kw.	Complete
												Power =1,208 hp.	
					19th	2	150	300	Fused Tapped	Base-19th	3(4-500 MCM)	Demand = 750 kw.	Complete
(Office)					38th	3	150	450	Fused Tapped	19th-38th	3(4-500 MCM)		Complete
Lefcourt	43	703,000	4		Base	2	150	300	Non-fused to St. N. W.	St. N. W.-Base	7-4/0	Light =1,214 kw.	Complete
												Power = 770 hp.	
					22nd	2	300	600	Fused Tapped	Base-22nd	2(11-4/0)	Demand = 522 kw.	One bank cut in
(Apt. Hotel)					42nd	2	150	300	Fused Tapped	22nd-42nd	2(8-4/0)		Complete
	30	330,000	4		1st	1	300	300	Non-fused to St. N. W.	St. N. W.-1st	7-4/0	Light = 650 kw.	Complete
												Power =1,090 hp.	
(Office)					21st	3	300	900	Fused Non-tapped	1st-21st	4-600 MCM	Demand = 630 kw.	Under construction
Farmer's Loan & Trust Co.	56	863,821	4		Base	4	300	1,200	Non-fused to St. N. W.	St. N. W.-Base	7-350 MCM	Light =1,296 kw.	Complete
												Power =4,201 hp.	
					21st	4	300	1,200	Fused Non-tapped	Base-21st	4-500 MCM	Demand =1,780 kw.	Under construction
(Bank & Office)					54th	3	300	900	Fused Non-tapped	31st-54th	4-500 MCM		Under construction
	37	800,000	3		Base	2	450	900	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Light =1,322 kw.	One bank cut in
												Power =1,430 hp.	
(Office & Loft)					36th	3	300	900	Fused Non-tapped	Base-36th	2(3-800 MCM) 2(1-400 MCM)	Demand =1,110 kw.	Two banks cut in
Continental	44	335,545	4		Base	1	450	450	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Light = 503 kw.	Complete
												Power =1,138 hp.	
(Office)					43rd	3	300	900	Fused Non-tapped	Base-43rd	4-500 MCM	Demand = 583 kw.	Two banks cut in
Nelson Towers	48	477,720	3		Base	3	300	900	Non-fused to St. N. W.	St. N. W.-Base	1(7-4/0) 1(7-2/0)	Light = 715 kw.	Complete
												Power =1,420 hp.	
(Office & Loft)					48th	3	300	900	Fused Non-tapped	Base-48th	4-500 MCM	Demand = 715 kw.	Complete

Date: 1-1-31

VERTICAL AND HORIZONTAL NETWORK SERVICES—*Continued*

Name and type of building	Stories	Gross floor area (sq. ft.)	Total feeders supplying service 13.8 kv.	Total feeders supplying service 3 kv.	Vault location (floor)	No. of banks	Size of banks (kva.)	Initial capacity in vault (kva.)	Character of inter-vault ties	Terminal points of inter-vault ties	Sizes of inter-vault ties	Load and estimated demand	Status of installation
R. C. A.	50	462,288	3		Base	3	150	450	Non-fused to St. N. W.	St. N. W.-Base	7-350 MCM	Light = 693 kw. Power = 1,577 hp. Demand = 727 kw.	Two Banks cut in
					22nd	3	150	450	Fused Non-tapped	Base-22nd	4-500 MCM		Under construction
(Bank & Office)					49th	3	150	450	Fused Non-tapped	22nd-49th	4-500 MCM		Under construction
Empire State	85 & Tower	2,754,180	5		Base	5	600	3,000	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Light = 4,431 kw. Power = 9,593 hp. Demand = 5,031 kw.	4 Banks cut in
					41st	4	600	2,400	Fused Non-tapped	Base-41st	4-500 MCM		Complete
(Office)					84th	4	600	2,400	Fused Non-tapped	41st-84th	4-500 MCM		Complete
Stone & Webster	Initial 24 Ultimate 54	Initial: 350,212 Ultimate: 501,012	Initial 3 Ultimate 4		Base	1	450	450	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Initial: Light = 525 kw. Power = 914 hp. Demand = 407 kw.	Under construction
					24th	2	300	600	Non-fused Non-tapped	Base-24th	2(7-4/0)		Under construction
					52nd	1	800	300	Non-fused Non-tapped	24th-52nd	3(7-4/0)	Ultimate: Light = 752 kw. Power = 1,270 hp. Demand = 637 kw.	Planning
(Office)													
River House	27	481,000	3		Base	1	450	450	Non-fused to St. N. W.	St. N. W.-Base	2(7-350 MCM)	Light = 633 kw. Power = 523 hp.	Planning
(Club & Apartment)					15th	2	300	600	Non-fused Non-tapped	Base-15th	2(7-4/0)	Demand = 371 kw.	Planning
City Service	63	1,046,000	4		Base	3	450	1,350	Non-fused to St. N. W.	Cedar St. N. W. Base Pine St. N. W. Base	2(7-350 MCM) 2(7-350 MCM)	Light = 1,345 kw. Power = 4,725 hp. Demand = 1,963 kw.	Planning
					18th	3	300	900	Fused Tapped	Base-18th	2 Tubular Busses		Planning
					31st	3	300	900	Fused Tapped	18th-31st	1 Tubular Bus		Planning
(Office)					61st	3	300	900	Fused Tapped	31st-61st	1 Tubular Bus		Planning
Port of N. Y. Authority Inland Stat. No. 1	15	Offices & Stores, 8th & 9th Aves. = 273,966 Loft Mfg. = 997,767 Warehouse = 997,767 Total = 2,269,500			Base (8th Av.)	0	0	Sec. Ser. Pt.	Non-fused to St. N. W.	St. N. W.-Sec. Ser. Pt.	4(3-500 MCM) (1-4/0)	Light = 2,258 kw. Power = 7,775 hp. Demand = 4,147 kw.	Planning
(Offices & Stores) & Loft & (Warehouse)					Roof (8th Av.)				Non-fused Non-tapped	Sec. Ser. Pt. Roof (8th Av.)			Planning
					Base (center)				Non-fused to St. N. W.	St. N. W. Base (center)	4(3-500 MCM) (1-4/0)		Planning
					Roof (center)				Non-fused Non-tapped	Roof (c)-Base (c) Roof (c)-Roof (8th) Roof (c)-Roof (9th)			Planning
					Base (9th Av.)	0	0	Sec. Ser. Pt.	Non-fused to St. N. W.	St. N. W.-Sec. Ser. Pt.	4(3-500 MCM) (1-4/0)		Planning
					Roof (9th Av.)				Non-fused Non-tapped	Sec. Ser. Pt. Roof (9th Ave.)			Planning
Metropolitan Life	(Unit No. 1) 29	1,012,564			Base (4th Av.)				Non-fused to St. N. W.	St. N. W.-Base (4th)	4(3-500 MCM) (1-4/0)	Light = 1,519 kw. Power = 9,455 hp. Misc. = 35 kw. Demand = 3,475 kw.	Planning
					15th (Part 1)				Non-fused Non-tapped	Base-15th (Part 1)			Planning
	(Unit No. 2) 84	2,144,165			Base (Mad. Ave.)				Non-fused to St. N. W.	St. N. W.-Base (Mad.) Base (4th)-Base (Mad.)			Planning; Unit No. 2 Will be postponed approx. 4 years
					15th (Part 2)				Non-fused Non-tapped	Base (Mad.)-15th (P2) 15th (P1)-15th (P2)			" "
					45th				Non-fused Non-tapped	15th (P2)-45th			" "
(Office)					80th				Non-fused Non-tapped	45th-80th			" "
N. Y. C. R. R. St. John's Park Freight Terminal	12	3,529,000			3rd (Clarkson St.)				Non-fused to St. N. W. Non-fused Non-tapped	St. N. W.-3rd (Clar.) 3rd (NC)-3rd (Clar.)	4(3-500MCM:1-4/0)	Light = 1,484 kw. Power = 5,666 hp. Demand = 2,035 kw.	Planning
					3rd (N. Cent.)				Non-fused to St. N. W. Non-fused Non-tapped	St. N. W.-3rd (N. C.) 3rd (S. C.)-3rd (N. C.)	4(3-500 MCM; 1-4/0)		Planning
					3rd (So. Cent.)				Non-fused to St. N. W. Non-fused Non-tapped	St. N. W.-3rd (S. C.) 3rd (Sp.)-3rd (S. C.)	4(3-500 MCM; 1-4/0)		Planning
					3rd (Spring St.)				Non-fused to St. N. W. Non-fused Non-tapped	St. N. W.-3rd (Sp.)	4(3-500 MCM-1-4/0)		Planning
(Freight Terminal)													

stood when it is realized that building heights of from 500 to 1,000 ft. above the street are becoming commonplace and such buildings require large banks of elevators, the motors of which are located in the upper stories of the building. Some of these elevators have motors ranging in size from 100 to 150 hp. for the higher buildings, with short period demands several times these values.

With such dense loads at an appreciable distance from the street level, it is found to be definitely economical in over-all costs to distribute with multiple high-voltage supplies feeding a low-voltage network throughout the building. This system is identical in principle with the street network system.

Early Considerations, Economic Aspects

The introduction of this type of service in New York City with transformer vaults on the upper floors of buildings did not mark the first use of transformers above the street level in this territory. A few installations had previously been made on the radial supply system with transformers located as high as the sixth floor.

The tapped riser type of building wiring was first considered in 1920 when an economic study was made of a 50 story building for the New York Telephone Co. at 140 West Street. This study contemplated the installation of supply transformers on the floors of the building where the elevator machine rooms were to be located and a low tension tie between transformer rooms was to be tapped to supply load on each floor. A considerable saving was estimated for this system of wiring over the old scheme wherein all building feeders originate at the house switchboard located in the basement, but certain restrictions made it impossible to make use of this type of installation at that time.

In April 1928, comparative estimates of building wiring costs for the Irving Trust Company Building, 1 Wall Street showed a saving of $35,000, for the vertical network system as compared with the system in which the service transformers are all located in the basement. However, some modifications in plans were made at the request of the local inspection authority and all of the above estimated saving was not realized.

In the Empire State Building, the network system has been applied to the individual floors. Four-wire 120/208-volt feeders from one or more centers of distribution, where they are fused, feed into a four-wire main circling the floor. Unfused branch circuits are tapped off this main to feed one or more bays.

With the radial system of building wiring, enough copper must be run from the house switchboard to each load group (say three floors, for lighting, an elevator bank, or other motor group) to carry the maximum load of that group. With the network system using a tie bus between transformer banks, the advantage of load diversity is realized in reducing the amount of copper required. The utilization factor can be worked out by the known probability factors for the case (see "Power Calculations for Elevators by the Method of Probabilities," *G. E. Review*, October-November 1930). This method applies not only to elevators but is applicable also to variable loads of any kind for which an "operating factor" can be determined. The fifteen-minute demand (r. m. s. value of the load over this period) on the usual elevator bank is about 35 per cent of the connected load. With the old radial system of wiring, a separate feeder or duplicate feeders must be run to each such bank having at least the load capacity of the fifteen-minute demand of each bank.

Ownership of Service Installations and Supply Feeders

Since a distribution installation in a building cannot be considered to have the permanency of a street system, it being subject to the whims of the building owners, and since it cannot be utilized for any service other than for that particular building, there is naturally a distinct limitation to the amount of investment that the utility company can make for such an installation. On the other hand, the building owner realizes a definitite saving in building wiring if the vertical distribution network is used instead of distributing throughout the building from street services. It is thus reasonable to have a different division of expense for vertical distribution than that which would be used in the case of general distribution throughout a city.

The division used in New York is to have the utility company supply, own, and maintain the high-voltage cables and the distribution transformers with their associated network equipment. These are placed in position by, and at the expense of, the building owners in ducts and vaults supplied as part of the building installation. All low-voltage distribution conductors and ducts are owned and installed by the building owners except the 120/208-volt a-c. network ties from the street mains to the first distribution point within the building. While the building owners install the transformers and high-voltage cables, all connections are made by the utility company. This division makes it possible for the utility company to recover all its equipment in case the building is abandoned, and does not materially change its costs from those that would be incurred in supplying the building direct from the street system.

The cost of the wiring installation in the case of high buildings is considerably reduced. Certain expense, of course, is incurred in using what might otherwise be usable in building space for vaults for distribution transformers but in most cases space yielding little of value can be used for this purpose.

Low-Voltage Tie Connections Between Transformer Vaults

If all conditions of supply for vertical distribution were the same as those found in the common network system, the installation would be unusual only from

the standpoint of establishing the division of ownership and maintenance between the building owner and the utility. However, as the types of installations made to date will indicate, there are numerous conditions which cause these services to differ. The vertical networks used so far can be classified according to the arrangement of the secondary connections within the building; such as isolated networks without connecting ties, networks with untapped ties, and networks with

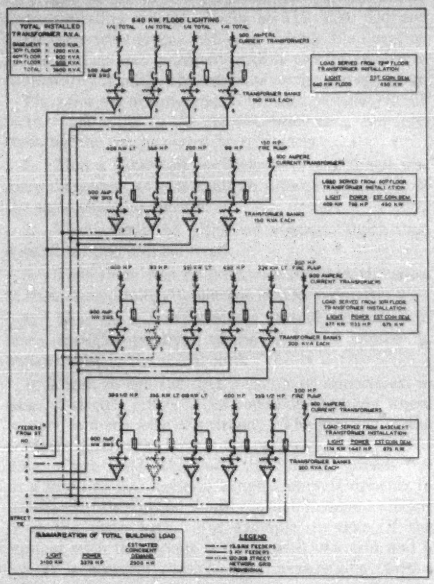

FIG. 2

tapped ties. (Figs. 2 and 3.) In the isolated or spot network, which also has been called a junior network, the energy used at any one point in the building is supplied solely from a plurality of step-down transformers at a single location as there are no secondary ties to any other network. The network with untapped low-voltage ties is similar, except in this case the ties do connect to other sources and there does not necessarily have to be a multiplicity of step-down transformers at the location as the ties form a network with other sources. However, in this case, service has to be supplied from the one location over special utilization feeders in the same manner as with a spot network. In the case of the network with tapped ties, the ties are used to distribute load to the several floors between transformer locations. Each of these types of installation has been used to meet conditions which existed at the time they were planned. None of the tapped ties has been installed without fuses and there is little incentive for the owner to install the secondary ties in conduit buried in concrete when he installs fuses which make it unnecessary to surround the conduit with concrete. Recent installations are being made with untapped and unfused ties installed in conduit surrounded by two or more inches of concrete; and in such cases, the building service is supplied locally from each step-down location.

Future Considerations

One of the advantages obtained with vertical network service is the ability to increase the electrical capacity in the building in the future without expensive reconstruction of the original wiring. The ability to double, or triple the supply capacity, if needs be, results from the fact that each of the several high-voltage cables supplying the building is required for mechanical

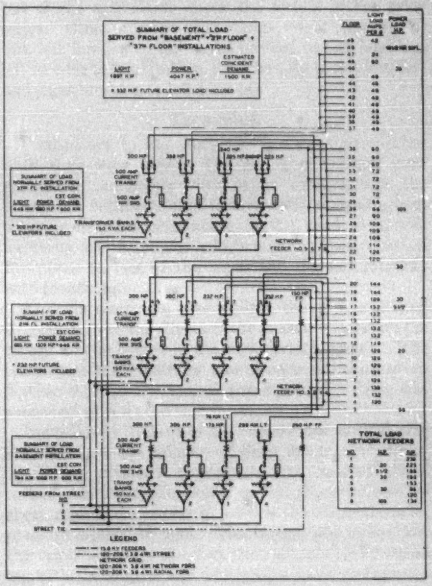

FIG. 3

reasons to have considerably more carrying capacity than is needed to serve any of the buildings contemplated up to the present time. Of course, the installation of additional distribution transformers would be required either at existing vaults or at new locations nearer to the utilization point than the present vaults; and, in addition, there would be required the necessary utilization wiring to deliver the new energy to the places in the building where it would be used. Such

a procedure is much simpler and less expensive than would be required if the increase were to be supplied from the street network.

All vertical high-voltage feeder installations planned to date have used a dry type of three-conductor shielded cable with an outside wrapping of steel wires with which the cable is held up at the top of its run. It is possible, and obviously desirable, that future developments should make it feasible to use standard types of high-voltage cables for this purpose.

Considering the operating experience in New York City covering a period of about two years, the vertical network installations being made, are regarded to be more than adequate for the service requirements now foreseen. Improvements in methods and in equipment which will simplify the wiring installation and reduce its cost may be expected in the future as inspection authorities, architects, and supply company engineers become better acquainted with the special requirements of this type of installation.

Effects of Electric Shock—II

BY W. B. KOUWENHOVEN* and O. R. LANGWORTHY†

Member, A. I. E. E. Non-member

Introduction

In a paper[1] published in January 1930 the authors discussed the results of a series of experiments on electric shock in which the electrodes were applied to the head and tail of animals. In those tests rats were used as the experimental animal and continuous and alternating voltage shocks at 110, 220, 500 and 1,000 volts were given. At each voltage the duration of the contact was increased until it was impossible to resuscitate the subject by means of artificial respiration.

The prior set of experiments brought to light certain facts which apply to the present series and, therefore, these will be recapitulated here, namely:

1. That a 1,000-volt continuous-current circuit was more dangerous to rats than an alternating circuit of the same effective voltage.

2. That a large rat withstood a greater shock than a small rat and still survived.

3. That there was no difference between the sexes in their susceptibility to electric shock.

4. That the a-c. experiments were characterized in many cases by paralysis of the hind legs, caused by hemorrhages in the spinal cord.

It is well recognized that a factor of importance in cases of electric injury is the points on the body where contact is made with the circuit. The location of the contacts determines largely the pathway of the current flow. Once inside the body the current spreads out in a more or less fusiform shape, but it is difficult to believe that the current density is high in organs distant from the main path. For example, in man, if the contact with the circuit is made with one arm and a leg, it is scarcely possible that considerable current will pass through the brain.

With the electrodes on the head and tail of an animal, the brain lies directly in the current path. In the previous study it was found that the brain appeared to be injured readily by the passage of an electric current. After the death of the rats Dr. Langworthy[2] found evidences of macroscopic and microscopic hemorrhages in the brain and also of considerable damage to nerve cells in many of the rats. In legal electrocution one electrode is placed upon the skull.

In industrial accidents, however, the current rarely passes through the head. In the 479 cases discussed by Maclachlan[3] the path of the current was commonly from the hands to the feet. The present series of experiments was undertaken to study the influence of the current path upon the effects produced by an electric shock. Are injuries to brain cells produced when the current does not pass through the head? Rats again were chosen as animals for study, both because of our knowledge of their behavior and because a rat seldom, if ever, dies of damage to heart after an electric injury, but rather from respiratory failure.

In the present series, 102 rats were shocked with a 1,000-volt circuit, half with alternating current and half with direct current. In all cases the duration of the contact was two seconds. The position of the electrodes was varied in these experiments. A few tests were made with the electrodes on the head and tail of the animals to check the results of the prior series. Other current paths were, right fore-leg to tail, left fore-leg to tail, right fore-leg to left hind-leg, left fore-leg to left hind-leg, right fore-leg to left fore-leg, and right hind-leg to left hind-leg.

Technique

Preparatory to the shock the rats were given ether until they were quiet. The points of contact were moistened thoroughly with saline solution and the electrodes, voltmeter clips filed smooth, were attached. When connection was made to the head, a special electrode was employed which made contact only on the upper surface of the skull, and did not interfere with breathing.

After the electrodes were in place, the rat was allowed to recover from the effects of the anaesthetic before the closure of the switch in the high-tension circuit. The circuit was held closed for two seconds and the current flowing through the animal was recorded. Time was measured with a stop watch.

Immediately following the shock, the animal was removed from the circuit. If there was no evidence of breathing, artificial respiration was applied. This consisted of the application of digital pressure upon the chest, reproducing as far as possible the Schaefer prone-pressure method in man. The tongue was pulled out, the vagus nerves in the neck were stimulated by stroking with the fingers, and efforts were made to keep the respiratory tract free from mucus.

Observations

Following the a-c. injuries, the chest of the animal was fully expanded. This seemed to be an important aid to the resumption of spontaneous breathing or to adequate artificial respiration. On the other hand, after contact with the d-c. circuit the chest was collapsed and it was difficult to get air into the lungs. The a-c. circuit produced a very strong contraction of muscula-

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1. For references see Bibliography.

ture of the body and this in the cases of males was accompanied almost invariably by an emission. Clonic movements of the muscles of the leg were observed in many of the a-c. injuries. They were probably produced by irritation of the nerve cells due to asphyxiation. Bleeding of the conjunctiva occurred in many cases.

The results are arranged in groups in accordance with the location of the electrodes upon the surface of the body. The data for the principal groups are stated in tabular form. The tables give the animal number, sex, weight in grams, the type of circuit, the current, whether or not artificial respiration was required, and the condition of the animal following the injury.

Electrodes on Head and Tail

Six rats in the present series and ten in the prior series were subjected to a two-second contact with a 1,000-volt a-c. circuit with the electrodes on the head and tail. As may be seen from Table I, artificial respiration was applied in 88 per cent of the cases, and five, or 31 per cent, recovered. The balance died or were paralyzed by the shock. Three of the five that recovered were normal. After breathing was established it was rapid, shallow, and somewhat irregular for several hours following the injury. Later it became normal.

Only one of the six rats in this series was paralyzed as against five of the ten in the prior series. In fact in this present group paralysis was less common and fewer hemorrhages in the spinal cord were found than in the tests of a year ago. No explanation for this discrepancy can be given.

In the tests previously published no rat survived a contact of one second or over with a d-c. circuit of 1,000 volts. Therefore, in this series only two animals were given a two-second shock at this voltage. Neither of them survived, although their hearts beat strongly for several minutes following the contact. Autopsy showed only few abnormalities. The microscopic examination of the brains of the rats of the prior series revealed, however, that the current had caused nerve-cell injuries incompatible with life.

The observations in the present group check those of the prior experiments where the electrodes were applied to the head and tail of the animal. It is clearly evident that contact with a 1,000-volt d-c. circuit is much more deadly to a rat than contact with an a-c. circuit of the same effective voltage. In this series, artificial respiration was applied to all of the animals tested with continuous current, and 100 per cent of them died. In the a-c. tests, however, artificial respiration was needed in only 88 per cent of the cases; 31 per cent of the animals died; 38 per cent were paralyzed; and 31 per cent recovered from the injury.

Electrodes on Right Fore-Leg and Tail

Twelve rats were given a two-second shock with 1,000 volts alternating current the electrodes being on the right fore-leg and tail. Five of these rats breathed as soon as the circuit was opened, and artificial respiration was required in the other seven cases. Death occurred either immediately or within a few minutes in 42 per cent of the cases, while three rats were paralyzed. One of the paralyzed rats died four hours after the injury, another nine hours, but the third survived until killed preparatory to autopsy.

This group was of particular interest in emphasizing the fact that an animal was not necessarily on the road to recovery when breathing was first started. This phenomena is mentioned often in records of human electric shock. Breathing was started in three of the rats, Nos. 1, 2, and 8 of Table II, only to cease after a few minutes. A second application of artificial respiration saved only one. Several of the rats in this group were hyper-irritable to all sensory stimuli for some minutes following the injury. Bleeding from the eyes was a common occurrence. The paralyzed rat that

TABLE I—TWO-SECOND SHOCK—1,000 VOLTS ELECTRODES—HEAD AND TAIL

Rat No.	Circuit	Sex	Wt. Gms.	Current Milliamp.	Artificial Respiration Required	Result	Remarks
1	A. C.	M	240	700	Yes	Recovered	Breathed rapidly
2	A. C.	F	206	840	Yes	Recovered	Breathed rapidly
3	A. C.	M	155	800	Yes	Died	Never breathed
4	A. C.	M	140	720	Yes	Died	Never breathed
5	A. C.	M	130	750	Yes	Died	Few breaths
6	A. C.	M	238	740	Yes	Paralyzed	
7	A. C.				No	Paralyzed	Prior series[1]
8	A. C.	F	230		Yes	Paralyzed	Prior series[1]
9	A. C.			650	Yes	Paralyzed	Prior series[1]
10	A. C.	F	190	800	Yes	Recovered	Prior series[1]
11	A. C.	F	230	830	No	Paralyzed	Prior series[1]
12	A. C.	F	240	900	Yes	Recovered	Prior series[1]
13	A. C.	M	200	950	Yes	Died	Prior series[1]
14	A. C.	F	270	980	Yes	Paralyzed	Prior series[1]
15	A. C.	F	180	1,000	Yes	Died	Prior series[1]
16	A. C.	F	210	1,000	Yes	Recovered	Prior series[1]
17	D. C.	F	162	750	Yes	Died	Never breathed
18	D. C.	F	131	700	Yes	Died	Never breathed
19	D. C.	M		730	Yes	Died	Prior series[1]

TABLE II—TWO-SECOND SHOCK—1,000 VOLTS ELECTRODES—RIGHT FORE-LEG AND TAIL

Rat No.	Circuit	Sex	Wt. Gms.	Current Milliamp.	Artificial Respiration Required	Result	Remarks
1	A. C.	M	130	740	Yes	Died	Few breaths
2	A. C.	F	180	750	Yes	Recovered	
3	A. C.	F	170	830	Yes	Paralyzed	Died 9 hr. later
4	A. C.	F	130	750	Yes	Died	No breaths
5	A. C.		132	760	No	Recovered	Hyper-irritable
6	A. C.	M	237	840	No	Recovered	Breathed at once
7	A. C.	M	228	800	No	Paralyzed	Hyper-sensitive
8	A. C.	M	137	680	Yes	Died	Started breathing, died
9	A. C.	M	266	700	Yes	Died	Cyanosis
10	A. C.	M	139	600	No	Paralyzed	Died 4 hr. later
11	A. C.	F	208	720	Yes	Died	Died 5 min. after
12	A. C.	M	134	720	No	Recovered	Breathed at once
13	D. C.	F	80	600	No	Recovered	Breathed at once
14	D. C.	M	124	950	No	Recovered	Breathed at once
15	D. C.	M	58	500	No	Recovered	Breathed at once
16	D. C.	M	40	950	Yes	Died	Started breathing, stopped
17	D. C.			1,000	No	Paralyzed	Breathed at once
18	D. C.	F	156	850	No	Paralyzed	Breathed at once
19	D. C.	F	110	700	No	Paralyzed	Breathed at once
20	D. C.	F	125	900	Yes	Paralyzed	
21	D. C.	F	68	500	Yes	Died	Few gasps
22	D. C.	F	201	950	Yes	Died	Never breathed
23	D. C.	M	145	840	Yes	Recovered	Breathed almost at once
24	D. C.	F	135	880	Yes	Recovered	Breathed almost at once

survived had a hemorrhage in the spinal cord and there was no chance of its permanent recovery.

With the alternating current flowing through the animals from right fore-leg to tail, four of the rats, 33 per cent, recovered; 42 per cent died; and 25 per cent were paralyzed. The injury was slightly less severe than when the brain formed part of the current path as in the previous series.

An equal number of rats was tested on the continuous-current circuit with the electrodes on the right fore-leg and tail. In this test the brain did not lie directly in the current path and the results were vastly different from those of Table I where one contact was made with the top of the skull. Here six of the rats breathed spontaneously when the circuit was opened, whereas none were saved in the first series even after long applications of artificial respiration. There was only one rat in this group of twelve that did not attempt to breath after the injury.

Artificial respiration was applied in 50 per cent of the d-c. cases; five rats or 42 per cent recovered; 25 per cent died; and 33 per cent were paralyzed. These figures do not, however, tell the whole story. The burns on the d-c. circuit were much more severe than those observed with the alternating. Because of these burns, it was not possible to keep any of the animals that survived alive for a period longer than three days following the d-c. shock. On the other hand, any rat that survived an a-c. shock without paralysis appeared quite normal twenty-four hours later. They ate, drank, and were active. It is also interesting to note that in this d-c. group there were two cases, No. 16 and 21—Table II, where breathing was started, only to stop again. In both of these instances a second application of artificial respiration was unsuccessful. From these results it is clearly evident that in cases where the continuous current path does not include the brain there is at least a fifty-fifty chance of resuscitating the animal.

Electrodes on Left Fore-Leg and Tail

Twenty-four rats were tested with the electrodes on the left fore-leg and tail, half with alternating and half with continuous current. The results are given in Table III. Under alternating current artificial respiration was applied in 75 per cent of the experiments; 58 per cent recovered, 17 per cent were paralyzed, and 25 per cent died. With direct current, artificial respiration was used in only 58 per cent of the cases; 42 per cent recovered, 33 per cent were paralyzed, and 25 per cent died.

The results of this series also emphasized the fact that breathing will not necessarily continue after having been established either spontaneously or by artificial respiration. The reactions and behavior of the rats in this group were closely identical with those where the current path was from right fore-leg to tail. It was, however, not quite as dangerous to life. The d-c. results here also confirm the importance of eliminating the brain from the current path if any hope of the recovery of the animal is to be entertained.

Electrodes on Both Fore-Legs

Eight rats were tested with the electrodes on the fore-legs, thus involving the heart and lungs directly in the path of the current flow. Four were shocked with alternating, and four with continuous current. Their average weight was 144 grams, and the average current was 1.24 amperes.

Although artificial respiration was tried in every case, no animal survived. This is not strange when one considers the current that flowed for two seconds. The chest was warm to the touch when the circuit was

opened. In only one case, however, was there no heart beat following the shock. It is remarkable that there was any heart beat at all after this injury, and that two of the rats attempted to breathe following the a-c. contact and two following the d-c. contact. There is no possibility of survival with this current path.

Electrodes on Both Hind-Legs

Fourteen rats were tested at 1,000 volts with electrodes attached to the hind legs, seven on an a-c. circuit and seven on a d-c. circuit. Their average weight was 127 grams and the average current that flowed for two seconds was 1.17 amperes.

Only one rat died as a result of the two-second contact with the a-c. circuit and two from that of the d-c. circuit, when the circuit path involved principally the caudal region of the body. The rat that died after a-c. contact had started breathing spontaneously as soon as the circuit was opened. He was considered as recovered and had been placed back in the cage; ten minutes later he was found dead, probably from neglect.

Artificial respiration was not applied to any of the seven rats that were injured with alternating current. All breathed spontaneously and 86 per cent recovered completely. They were all very active and lively within a few minutes following the shock. One, or 14 per cent, died. The authors believe that his life could have been saved had artificial respiration been applied at the proper time. The a-c. shocks were accompanied by an emission in the case of the males.

Following the d-c. shock, five of the animals breathed at once, artificial respiration was applied in but two cases, 29 per cent. These two animals died after a few attempts at breathing. The other five, 71 per cent, recovered and in a few minutes became very active.

This series demonstrates clearly the fact that when none of the vital organs lie directly in the path of the current the electrical injury usually is small.

Electrodes on Fore-Leg and Hind-Leg

The effects produced by the flow of current from the right fore-leg to the left hind-leg were studied in eight rats; from the left fore-leg to the left hind-leg in eight other rats; and in a third group of eight the electrodes were on the left fore-leg and the right hind-leg. Half of these were subjected to the a-c. circuit and the remainder to the continuous-current circuit. The results are given in Table IV.

Not a single rat was permanently saved after contact with the d-c. circuit. One, No. 24, was resuscitated only to die two and a half hours later. In several cases no heart action could be detected and, at most, many of the animals gave only a few gasps. There was greater success in saving those that had been injured with the alternating circuit. In five cases breathing was established by a long application of artificial respiration, one of these rats, however, was paralyzed. They all, including the paralyzed one, became quite active soon after the shock.

The results may be summarized for the current path from fore-leg to hind-leg as follows:

Artificial respiration was applied in every instance, both for the a-c. and continuous-circuits. With the alternating current, 33 per cent of the animals recovered, eight per cent were paralyzed and 59 per cent died. With continuous current 100 per cent of the animals died.

Normally one would expect that there would be little, if any, difference between the effects produced by a current flowing from either fore-leg to tail or by one flowing from either fore-leg to either hind-leg.

TABLE III—TWO-SECOND SHOCK—1,000 VOLTS ELECTRODES—LEFT FORE-LEG AND TAIL

Rat No.	Circuit	Sex	Wt. Gms.	Current Milliamp.	Artificial Respiration Required	Result	Remarks
1	A. C.	M	149	750	Yes	Recovered	
2	A. C.	M	195	900	Yes	Recovered	
3	A. C.	M	155	700	Yes	Died	No breaths
4	A. C.	M	197	730	Yes	Died	No breaths
5	A. C.	M	114	680	Yes	Paralyzed	Fluid from mouth
6	A. C.	M	121	720	No	Recovered	Breathed at once
7	A. C.	M	214	770	No	Recovered	Breathed at once
8	A. C.	M	258	860	No	Died	Died 1 hr. later
9	A. C.	M	104	800	Yes	Recovered	
10	A. C.	M	131	720	Yes	Paralyzed	Died 4 hrs. later
11	A. C.	M	124	800	Yes	Recovered	
12	A. C.	M	115	620	Yes	Recovered	
13	D. C.	F	144	950	Yes	Died	Few gasps
14	D. C.	F	120	750	No	Paralyzed	Breathed at once
15	D. C.	F	175	1,050	No	Paralyzed	Irregular breathing
16	D. C.	F	115	750	No	Paralyzed	Irregular breathing
17	D. C.	F	154	800	No	Paralyzed	Irregular breathing
18	D. C.	F	120	900	Yes	Recovered	
19	D. C.	F	110	700	No	Recovered	Breathed at once
20	D. C.	M	131	850	Yes	Recovered	Died 35 hr. later
21	D. C.	F	135	760	Yes	Recovered	Rapid breathing
23	D. C.	M	160	840	Yes	Recovered	Shallow breathing
23	D. C.	F	95		Yes	Died	Died 20 min. after
24	D. C.	F	85	840	Yes	Died	Never breathed

TABLE IV—TWO-SECOND SHOCK—1,000 VOLTS ELECTRODES—FORE-LEG TO HIND-LEG

Rat No.	Circuit	Sex	Wt. Gms.	Current Milliamp.	Artificial Respiration Required	Result	Remarks
			RIGHT FORE-LEG—LEFT HIND-LEG				
1	A. C.	M	240	1,140	Yes	Died	Started breathing, stopped
2	A. C.	M	145	1,100	Yes	Died	No breaths
3	A. C.	M	143	1,060	Yes	Paralyzed	Active soon
4	A. C.	M	114	1,140	Yes	Recovered	Active soon
5	D. C.	F	122	1,450	Yes	Died	Few gasps
6	D. C.	F	69	1,700	Yes	Died	No heart beat
7	D. C.	F	264	1,700	Yes	Died	No heart beat
8	D. C.	M	127	1,200	Yes	Died	Heart beat
			LEFT FORE-LEG—LEFT HIND-LEG				
9	A. C.	M	54	920	Yes	Died	Fluid from nose
10	A. C.	M	155	1,060	Yes	Died	Fluid from nose
11	A. C.	F	130	680	Yes	Recovered	Clonic movements
12	A. C.	M	125	940	Yes	Died	Few gasps
13	D. C.	M	223	1,300	Yes	Died	Few breaths
14	D. C.	F	135	1,700	Yes	Died	Few breaths
15	D. C.	F	60	1,300	Yes	Died	No heart
16	D. C.	F	105	1,250	Yes	Died	Few gasps
			LEFT FORE-LEG—RIGHT HIND-LEG				
17	A. C.	F	130	1,060	Yes	Died	Never breathed
18	A. C.	M	150	1,110	Yes	Died	Never breathed
19	A. C.	F	102	820	Yes	Recovered	Clonic movements
20	A. C.	M	120	1,000	Yes	Recovered	Clonic movements
21	D. C.	M	95	840	Yes	Died	Never breathed
22	D. C.	F	95	820	Yes	Died	Few gasps
23	D. C.	M	100	1,500	Yes	Died	Fluid from mouth
24	D. C.	F	110	930	Yes	Died	Died 2½ hr. later

Yet in the first group with the current path from fore-leg to tail, 44 per cent of the rats recovered; in the second group, where one electrode was attached to a hind-leg instead of the tail, only 16 per cent were saved. Wherein, then, lies the deadliness of this current path from fore-leg to hind-leg? The authors believe that the high death rate found here is caused by the low resistance path that the animal offers to the current flow when its tail is eliminated from the circuit. The average resistance of the current paths from fore-legs to hind-legs for the 24 rats was less than 70 per cent of that found for the 48 rats (Tables II and III) where the current paths were from either fore-leg to tail. The higher resulting current is believed responsible for the deaths of the animals. A second argument supporting this conclusion is the almost complete absence of serious injury to the rats when the current passed from one hind-leg to the other, even though the current values were high.

Conclusion

These experiments confirm Urquhart's[4] conclusion that when an electric current passes through the brain, a temporary physiological block was produced in the respiratory center, and that spontaneous breathing ceased for a time. He stated that if no serious injury to the heart occurred, adequate artificial respiration might give time for the center in the medulla to recover and for normal breathing to begin. Maclachlan[3] reports in his field notes a case of recovery from shock following eight hours of continuous application of the Schaefer prone-pressure method, which also confirms Urquhart's findings. Urquhart,[4] in a second series of experiments, demonstrated that a temporary block could be produced in the spinal cord or in a nerve trunk by the passage of an electric current.

Langworthy[2] found, by his microscopic examination of sections of the brains of the rats injured by high-voltage electric currents, that in many cases the damage to the cells was such as to be incompatible with life. Microscopic study of the central nervous systems of all of the rats in this present series will be made.

Results clearly demonstrate the presence of a temporary respiratory block in many of the rats. The location of the block, whether in the brain stem or in the vagus nerves, depends upon the current path. Resuscitation is possible provided the damage done does not require a recovery time too long to permit life to be supported in the interval.

The effects produced by the various current paths through the animals are summed up in Table V. From this table and from the results presented, the following conclusions may be drawn:

1. A 1,000-volt continuous-current circuit is more dangerous to rats than an alternating circuit of the same effective voltage, irrespective of the current path.
2. The chances of recovery are best when the brain does not lie directly in the pathway of the current.
3. The injury produced by the current path from left fore-leg to tail is less severe than that caused by the same current flowing from right fore-leg to tail.
4. When the main pathway of the current did not

TABLE V—CONTACT WITH 1,000-VOLT CIRCUIT FOR TWO SECONDS

Circuit	Position of Electrodes		Number Shocked	Per Cent Requiring Artificial Respiration	Per Cent Immediate Recovery	Per Cent Paralyzed	Per Cent Immediate Death
Alternating	Head	Tail	16	88	31	38	31
Continuous			3	100	0	0	100
Alternating	Right fore-leg	Tail	12	58	33	25	42
Continuous			12	50	42	33	25
Alternating	Left fore-leg	Tail	12	75	58	17	25
Continuous			12	58	42	33	25
Alternating	Either fore-leg	Tail	24	67	46	21	33
Continuous			24	54	42	33	25
Alternating	Right fore-leg	Left fore-leg	4	100	0	0	100
Continuous			4	100	0	0	100
Alternating	Right hind-leg	Left hind-leg	7	0	86	0	14
Continuous			7	29	71	0	29
Alternating	Either fore-leg	Either hind-leg	12	100	33	8	59
Continuous			12	100	0	0	100

include the brain, spinal cord or nerves required for respiration, most of the animals breathed at once and were active a few minutes after the injury.

5. The presence of a respiratory block has been demonstrated, and the fact that when natural breathing is first established the animal is not necessarily on the road to complete recovery.

6. The respiratory block is less severe, if not absent, when the current traverses the posterior portion of the body.

7. Burning of the tissues with continuous current is much more severe than occurs for the same effective alternating current.

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Communication in the Western Division of the Niagara Hudson System

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EXTENSIVE interconnection of electric generating and transmitting systems has brought the problem of rapid, dependable communication to the fore. Present day systems cover such large areas that the communication lines have become almost as important as the power lines themselves.

Men operating these systems must have some means of immediate communication with several points on the system. More than that, the system of communication must be highly reliable and as safe from the destructive forces of nature as it can be made, because it is usually most needed when power system operation is upset by storms.

The area covered by the Western division of the Niagara Hudson System and the location of the various generating and switching points are given below:

The Niagara Hudson System spans New York State from the Pennsylvania border, near Jamestown, to the Massachusetts border, near Albany, with a branch of the lines extending northward to the International boundary between New York and Quebec. The western division of this system includes all that part west of Lyons. This area is shown graphically on the copy of the system map in Fig. 1.

Main generating stations of the western division are located at Niagara Falls and Buffalo, but power is also available from plants on the Adirondack streams and in the Mohawk and Hudson river valleys. Through interconnections, power may also be secured from plants on the Canadian side of the Niagara River and from an adjoining system in Pennsylvania.

Main switching stations of the western division are located at Buffalo, Niagara Falls, Gardenville, Lockport, Mortimer, Jamestown and Olean. These points are clearly designated on the system map.

In the economical operation of a large system such as this, which includes both steam and hydroelectric plants, rapid and dependable communication between the division load supervisor's office in Buffalo and the various generating and switching points, as well as between those points, is vital. Recognizing this fact, the system now in use was devised and is believed to fulfill all requirements.

Prior to the organization into one system of the various companies comprising the western division, each company arranged for its own communication facilities. In some cases this involved the building of telephone circuits by the power company, in some cases the leasing of circuits from the telephone company and in some cases merely the use of telephone toll service. It was only natural that different companies should have different ideas on the subject.

Following the grouping of the companies into the Niagara Hudson System, it immediately became apparent that communication facilities should be rearranged and revised to meet the needs of the coordinated system. Greater flexibility was needed. Engineers of the telephone company were called in to make a survey and a report. Upon the completion of this survey, they submitted a report covering the complete communication needs of the western division. This report is divided into three sections: inter-office communication in the Electric Building at Buffalo,

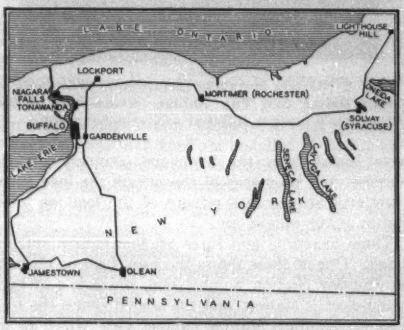

FIG. 1

communication between substations, extensions in the Service Building and lines to Huntley Station. These are all extensions from the private branch exchange and comprise more than 700 miles of extensions; interdepartment communication at Huntley Station, lines connecting Terminal Houses A, B, C and D in Buffalo, and lines to Niagara Falls and the Tonawandas; communication channels to main switching and generating stations and other strategic points of the system throughout western New York State.

Part I will be discussed later in this paper, as it is concerned mostly with inter-office and Buffalo city communication.

Parts II and III were considered and fully discussed by the various operating men concerned. Following this discussion and the adoption of various modifications

1. Elec. Engr., Buffalo, Niagara & Eastern Power Corp., Buffalo, N. Y.

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proposed by the operating staff, the telephone company was authorized to provide the facilities outlined in the plans.

Remote Metering

The load supervisor's office had been moved to the Electric Building in Buffalo. This made it necessary to centralize operating telephone lines at that point and provide remote metering installations from the generating plants at Niagara Falls and Buffalo. Lines

Fig. 2—Remote Metering Installation in Load Supervisor's Office and Two 20-Line, Single-Sided Special Dispatcher's Turrets in Office of System Operator

were leased from the telephone company for this purpose, and by means of the meters the load supervisor gets an accurate picture of the load on either station at any time of day.

Three recording and three graphic meters are provided. One of these shows the total load in kilowatts on Huntley Station number one, which is a 25-cycle steam-electric station, and the second shows the total load on Huntley Station number two, which is the 60-cycle steam station. The third is connected to Harper Station, the switching unit at Niagara Falls, and shows the load on units 19 and 20 in Schoellkopf hydroelectric station. These are the regulating generators at the station, and the meter reading, in connection with data supplied to the supervisor, makes the total load on the station available at a glance.

Huntley Station is about five miles from the supervisor's office and Schoellkopf Station is about 20 miles away. The meters are operated by three separate circuits leased from the telephone company. These are shown in Fig. 2.

Full Talk Circuits

Three vital points of the system are tied together from a communication standpoint by lines leased from the telephone company on a 24-hour a day basis. While these lines are built and maintained by the telephone company as part of its regular toll facilities, the power company has as full use of them as it would have of privately owned lines . . . . and without the responsibility of maintenance. In the event of trouble on these lines, the telephone company is usually in a position to make available other lines for reaching the same points.

The points thus tied to the division load supervisor's office are the Buffalo steam plant, the system operator's office at the Niagara Falls hydro plant and the Lockport switching station. Through Lockport, the load supervisor can also talk to Mortimer, Lyons, Solvay, and Syracuse. Using the automatic equipment in the Buffalo office, he can also reach Gardenville (just outside the city), Jamestown, and Olean.

Lines were also leased from the telephone company to connect the four main switching points of the western division directly with each other. Niagara Falls, Lockport, Gardenville, and the Buffalo steam station are so tied in that any one can call directly and talk to the other three.

These four points have district load supervisors, and by means of the leased circuit a two to four party conversation may be held directly by the men. The load

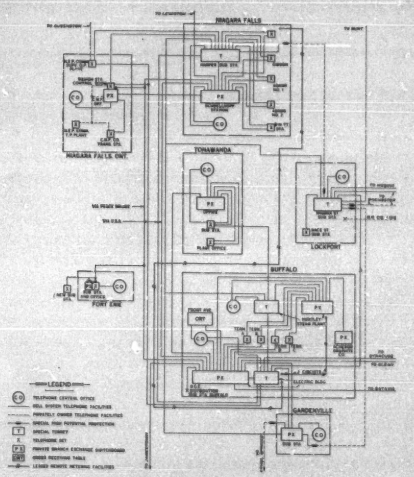

Fig. 3A—Communication Facilities, Buffalo, Tonawanda, and Niagara Falls

Buffalo, Niagara and Eastern Power Company

supervisor for the western division gives his orders to the district supervisors over circuits reserved for that purpose entirely, and the line connecting the four points directly is reserved for the use of the four district supervisors in direct intercommunication.

The arrangement of communication circuits at Buffalo is shown in Fig. 3A.

Lines entering the division load supervisor's office in Buffalo, as well as those entering each switching

station are brought to a special dispatchers' turret. One telephone instrument connects the operators to the entire communication network, the turret being so simplified that the operator needs only to throw a key to call the station he wants.

Turrets are of three capacities, 10 line, 20 line, and 40 lines, and are provided in both single and double face to meet various requirements. The turret in the division supervisor's office at Buffalo is a 20-line single-face installation.

Through these turrets, communication lines of various types can be tied together. Dial central office lines, dial extension lines, common battery extension lines, magneto-battery central office lines, magneto-battery leased lines and magneto-battery privately owned lines can be interconnected in the turrets. For example, the load supervisor in Buffalo may call Syracuse on a leased line, be switched through the turret and talk to Lighthouse Hill using the privately owned line from Syracuse to that point. This eliminates the delay and error danger involved in relaying the message. The turrets provide that only one call like this may be switched through at a time, however. This limitation was purposely provided in order to avoid making the load supervisor a telephone operator.

Another advantage of the turret is its speed. All lines end in keys rather than plugs, and it is faster and easier to switch the key than to pick up a plug and make the connection.

While the communication system was being planned, several instances were discovered where operating needs could not justify the leasing of lines on a 24-hour a day basis. Such lines were desirable from an operating standpoint, but the operating department alone did not feel justified in incurring the necessary expense.

It was found, however, that if these circuits could be reserved for the operating department at certain hours and open to other departments the rest of the time, the expenditure was well worth while. These circuits were accordingly leased, with a resulting saving in long distance toll calls and closer contact between district offices and the main office.

Interconnection of Private Lines

By what has gone before, it might be assumed that private lines owned by the various companies before the consolidation, were entirely discarded, but this was not the case. Where private lines were available and in good condition, protective equipment was leased from the telephone company, in accordance with the power company's agreement, in order that private lines might be connected to leased lines to provide intercommunication when desirable.

Power company privately owned lines usually follow the transmission line right-of-way, and where these lines interconnect with telephone company circuits there is danger of damage to equipment and possible personal injuries due to the building up of voltage on telephone lines through induction or direct contact between communication and power transmission wires. This hazard has been eliminated through the installation of protective equipment leased from the telephone company.

Where the interconnection of communication lines is in a station building, horn gap arresters located in a sheltered spot on the outside of the building are installed on the lines. These are set at 0.05 inches, to operate at about 5,000 volts. The lines then pass through disconnecting switches, at which point fuses are provided and thence to a second arrester. This second arrester is set for as low a voltage as is consistent with that normally induced on the private line. The ground of the arresters is connected to the station ground bus. The lines then pass through a transformer for further protection, since it acts as an insulating barrier. From the secondary side of this transformer the lines pass through a standard telephone protector to the point of connection with the telephone company's conductors.

Where privately owned lines connect with telephone circuits at points distant from buildings, a similar installation is made on poles. One pole is set between the last high-tension line pole and the first telephone company pole for the installation of protective equipment.

In order to take care of excessive ground potentials where telephone equipment is installed in power and larger switching stations, two methods are used; insulating transformer, and distant ground.

The distant ground method is suitable in the case of smaller stations and where the telephone instruments can be placed so that persons using them are insulated from the ground potential of the station. Telephone equipment and wiring are adequately insulated from all grounds and grounded structures that are subject to excessive potential variations, and the equipment is connected to a ground a sufficient distance away to be free from the influence of power station ground potential variations.

With the insulating transformer method it is not necessary that the telephone instruments be placed in insulated locations in the power station. Where grounding is required, the telephone equipment is grounded to the power station ground, and the outgoing lines pass through an insulating transformer before they connect with telephone company circuits.

All equipment on the power station side of the transformer is grounded to the station ground; all equipment on the telephone company side of the transformer, within range of the influence of power station ground potential variations, is carefully insulated from that influence. The insulating transformer is the barrier between the power station ground potential variations and the telephone company's system.

Carrier Current

Carrier current is used in addition to the leased lines and privately owned lines. Briefly, in the event that not everyone is familiar with this means of communication, it is a system which puts carrier current for voice transmission at very high frequency on the power lines, the frequency of this carrier current being in the radio frequency range.

Sets are installed at Niagara Falls, Buffalo, Gardenville, Olean, Jamestown, Mortimer, Solvay, and Lighthouse Hill. These sets are single frequency duplex, providing two-way communication between stations and in general are rated at 50 watts.

Each telephone instrument on this system is provided with a dial for selective ringing. Various stations are assigned dial numbers, as on the conventional dial telephone system, and any one station may call any other by dialing it directly. Where leased lines are available, the carrier-current system is not used to any great extent under normal conditions because of the better quality of voice transmission on the leased lines. However, each set is tested thoroughly every day to make sure that it is in operating condition in the event of an emergency.

In situations where leased lines are not available, such as between Solvay and Lighthouse Hill, the carrier-current equipment is much more valuable. There is also an arrangement whereby the carrier-current equipment at Lighthouse Hill, which is on the edge of the Adirondacks, can be used for communicating with the generating stations in the St. Lawrence district. Its operation under these conditions is entirely satisfactory. Fig. 3B shows the power system communication plan.

Practically all the lines leased from the telephone company are in cables, reducing to a minimum the probability of failure during storms. In addition, where two circuits are leased between the same points, the circuits follow entirely different routes. This also helps to minimize the possibility of storm damage.

In this connection there is an important consideration in regard to power company private lines. Usually these lines are on transmission line rights-of-way in order to avoid the expense of securing separate rights-of-way. Communication is most needed during periods of destructive storm, and private communication lines often go out when transmission lines fail. The leased circuits usually follow different routes from the transmission lines, providing an additional factor of safety.

Prior to the establishment of the present communication network, a particularly disastrous sleet storm, swept across practically the entire territory. Many of the stations were cut off at the beginning of the storm and the private lines were severely crippled. Of course the first job was to restore the power lines.

Two interesting occurrences during this period are worthy of mention. Extensive use was made of telephone company toll circuits and one call from Albany to Buffalo was routed entirely around the storm area via Montreal and Chicago. Short-wave radio was also used. Amateur stations in Buffalo and Syracuse, as well as at a number of intermediate points, broadcast messages which were relayed by local telephone to our switching stations. At times during the storm period the carrier-current system was the only means of communication between load centers.

Communicaton for the operating department was

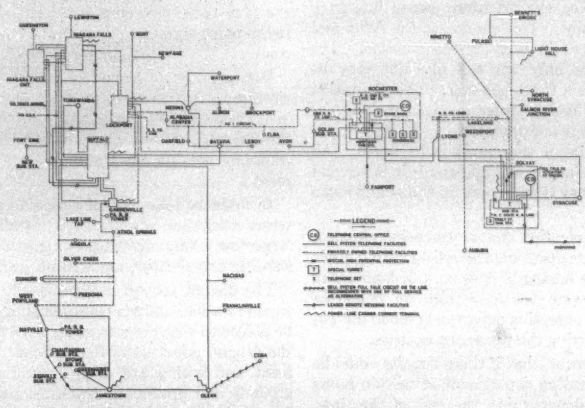

Fig. 3B—System Communication Plan

Buffalo, Niagara and Eastern Power Company

carefully planned following an exhaustive study of the needs and daily traffic. As it is now laid out, the load supervisor has at least two different channels of communication with every major switching and generating station on the system, as shown by the Buffalo district diagram in Fig. 4. The communication system now installed has met every practical test so far satisfactorily.

Interdepartment Communication

With the establishment of western division headquarters in the Electric Building, population of the building doubled in less than a year. Prior to 1930, three separate switchboards were maintained for the three separate companies, with resulting confusion.

Employees of one company calling the employees of another first signaled their own operator and asked for

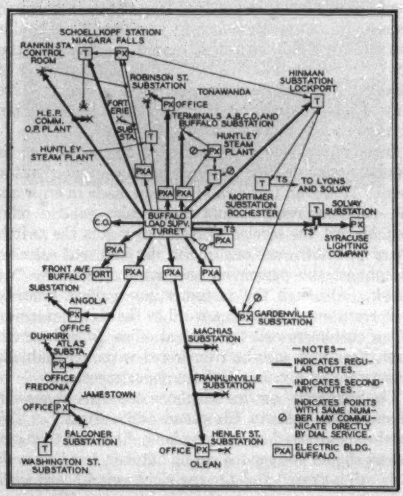

Fig. 4—Buffalo Load Dispatcher's Circuit Diagram

the other company. When the second operator answered, they asked for their party. With the rearrangement of departments by floors, irrespective of companies, employees working side by side were often on two different switchboards. There was also a great deal of congestion on the tie lines between company switchboards, and a large staff of operators was required to handle the traffic. Customers were also confused and there was a considerable number of calls intended for one company received on the switchboard of one of the others.

This situation was somewhat relieved by bringing all the boards together, but the traffic problem, particularly on interdepartment calls, was still present.

The problem was solved by establishing a dial private branch exchange leased from the telephone company, for interdepartment communication. Calls between departments are now dialed directly and are not handled by operators. Calls to the telephone company's central office are also dialed through directly and do not pass through the manual switchboard. Operators are now concerned only with incoming calls to the general offices; customer and all service calls being handled by a special method which is explained later.

The company's general switchboard, following the adoption of the dial system, was cut from five positions to three positions, only two of which are used in normal operation. The staff of operators was reduced from nine to five, despite the fact that the number of telephones served by the private branch exchange has increased from 189 to 325, with a corresponding increase in telephone traffic.

The extra position on the manual switchboard is not needed in normal operation. But in peak traffic, such as that occasioned by service interruptions, it is possible to put an extra operator at this position to help in handling customer calls. While trouble calls usually go to an entirely separate order table receiving apparatus, as described later, customers have a habit of calling the general office when the customers' service department is busy. This occasionally occurs during an unusual service interruption. This extra position helps to relieve the peak caused when customers in trouble call the general offices instead of the customers' service telephone department.

In some of the smaller offices of the western division, dial equipment has been installed to give better service to customers. In time of trouble, single position boards in these offices have been swamped by customers calling in, and the operating staff, trying to call out in order to clear the trouble, has been handicapped.

With dial equipment, the operating men can dial out on a separate group of outgoing trunks and the operator can devote her entire time to answering incoming trouble calls until the trouble has been cleared. The results have been quicker restoration of service and better treatment of customers.

Where the dial system is in use, one telephone instrument is provided for both inside and outside calls. On an inside call the number of the extension desired is dialed directly. On an outside call, 7 is dialed and when the second dial tone is heard, indicating that the instrument is connected with the telephone company's exchange, the number wanted is dialed.

Since the installation of the dial system, time consumed in completing both inside and outside calls has been cut in half. Employees have become thoroughly familiar with the dial telephone and find it faster, more accurate, and generally more satisfactory. The new system has also ended the irritating habit of having a telephone operator put in a call and keep the person called waiting on the line until the caller is ready to talk. It has been found that people have acquired the more

courteous habit of being ready to talk when the person they call answers.

Incoming Calls from Customers

Public relations dynamite lurks in every telephone call. Executives who ponder upon the public relations problem would do well to listen in on some of the incoming calls from customers and learn at first hand how employees deal with customers over the telephone.

Under the method previously used by the company, customer calls came to the general switchboard. The customer made his request to the telephone operator who knew by experience what department was wanted. Usually the operators were all busy with other calls, and this delayed answering incoming calls. Customers were not always sure what department they wanted. It was a common occurrence for a customer to ask for the purchasing department, and upon being connected, say that he wanted to purchase an appliance.

Calls to have service cut in or cut out and similar requests were routed to a commercial table. Here, after the customer had repeated his request, the message was taken and relayed to the department handling the work.

Customers who reported trouble were connected with an order table in the distribution department, operated by men, and while they handled their work well, the telephone contact with customers was not always as pleasing as it might have been.

Entirely aside from the cumbersomeness of this arrangement, there were several undesirable features from the standpoint of public relations:

1. The customer was forced to make his request two or three times to different people, explaining his trouble to each one.
2. The company caused the customer loss of time and patience.
3. Often customers hung up before the proper department was located, with a resulting loss of both good will and business to the company.
4. No record was made of how any given case was handled and whether or not the customer was satisfied; no one, with the possible exception of one department head (whose department had happened to handle the particular case) knew what service had been given.
5. There was no follow up to make sure that the customer was satisfied.

Following the survey and report by the telephone company, a department was established known as the customers' service telephone department. This is a centralized point where all customer telephone calls regarding connections and disconnections of electric service, interrupted service, bill complaints, and requests for information are received.

Calls are received over a special group of 18 central office trunk lines which terminate in what is known as an order receiving table. This is a telephone switchboard in the form of a long table, with places for seven persons on each side facing each other. Green lights in the center of the table indicate a call instead of the ordinary telephone bell. The board is so multipled that the same call may be taken at any one of three positions, and a white light flashes when the call is answered.

Each service representative is equipped with headset receivers and a chest transmitter, leaving both hands free. From all positions at the order table, direct lines run to the meter and trouble departments and the order table files, making it possible to get information quickly from these departments while holding the customer on the line. In addition, 10 tie lines connect the order table with the dial switching equipment of the private branch exchange, making it possible for the service representatives to pass the call along to any of the 325 extensions from the company's regular switchboard in the event that the customer wants to talk further to some particular person. In this event, the service representative gets the company employee while holding the customer call, explains what is wanted and thus obviates the necessity of the customer's repeating his request. Through these tie lines, service representatives may also get information from any one of the 325 extensions on the company's regular exchange.

Incoming trunks to the order table pass through the regular exchange, but give no signal to the operators there. However, if the call has been made in error and the customer really wants the regular general office switchboard, the service representative gets the switchboard operator over one of the tie lines and asks her to connect the person on the trunk number to Mr. Blank. Although the customer has made a mistake, he is transferred without a word to the right extension. When customers call the general office switchboard in error, they may also be transferred without trouble to the customers' service telephone department.

The customers' service telephone department answers all customer calls to Cleveland 5260 directly. This number is so listed in the telephone directory that customers should not get the wrong number, but if they do their call is passed along to the right switchboard without any inconvenience to the calling party. The young ladies who comprise this department were carefully selected and put through a course of training extending over a six weeks period before the department was put into operation. During this time, they were in all the various service departments, getting a background regarding the work done so that they might handle customers' requests intelligently.

The staff of the department includes fourteen attendants and a supervisor. The department is in operation 24 hours a day, every day in the year. The usual load on the department is about 1,200 calls a day, but during times of general trouble over a section of this city, calls rise to a peak of 3,000 or more. When the lines are normal, attendants who are not needed "double" as typists and filing clerks, but are ready to take up their duties as attendants the moment a peak impends.

When desired information can be obtained in a few moments, the attendant "holds" the customer on the line and provides it; when considerable time is needed, the customer's number is taken and the information is telephoned back when it is secured.

By having the customer calls come into this centralized point instead of directly to the private branch exchange switchboard, the necessity of the customer being cross-examined by the private branch exchange operator to find out what the customer's desire might be is obviated. After this, the call was referred to the department that might be involved, where again the customer was requestioned and if he had not been connected to the right department, transferred to another telephone and again questioned, much to the irritation of the customer. The customer's telephone service bureau not only receives the call, but completes the entire customer contact without transferring the call to another person or to another department. This arrangement not only provides for speedier handling of customer calls, which is essential to giving real customer service, but allows all calls of this nature to be handled by a group of people who can specialize in this type of work. The equipment is shown in Fig. 5.

Fig. 5—Order Table—Buffalo

Teletype Machines

Calls to have service cut in or out, bill or service complaints, and trouble calls are handled by the department directly. The customer provides the information and the operator follows it through to completion. For this purpose, five teletype machines are provided, affording direct communication with the meter and distribution departments in the Service Building, two miles away. The teletype installation is shown in Fig. 6.

A telephone typewriter instrument is an electrically operated machine which has a keyboard and other mechanical parts similar to a regular typewriter machine but with this difference; that the sending machine can be at one location and a similar receiving machine can be at a distant location either in the same building, the same city or two widely separated cities. The machines are so arranged that whatever is typed on the sending machine, such as an order, a letter or other information, is instantaneously transmitted by electrical impulses over wires to the receiving machine. These impulses operate the keyboard of the receiving machine in such a manner that it types the message at the receiving point exactly as it was typed at the sending point. Through the medium of five of these machines in the customers' telephone service department, all orders and reports of trouble are immediately sent to the Service Building for prompt attention.

All service requests are kept in duplicate in a pending file. This file is inspected every morning to make sure that all promises of the day before have been kept. In this way, the customers' service telephone department provides a constant check on the service rendered by other departments.

There has also been installed a monitor observing set which taps every company telephone line. By means of this, samples are taken of telephone conversations carried on by employees and these are used as a basis for training employees in the correct use of the telephone.

Fig. 6—Five No. 15 Sending and Receiving Telephone-Typewriter Instruments

Three equipped with special platens for positive drive and remote control, two equipped with standard platens—one with remote control

This work has been so arranged that there has been no charge of "spying." Each department knows in advance when it is to be observed over the monitor. Samples are taken verbatim by a stenographer at various periods of the day for several days. These are discussed later at a meeting of employees of the department observed and means of improving the different conversations are pointed out.

The vital importance of telephone contacts with customers has been realized and for that reason steps

have been taken to see that their impressions of the company gained over the telephone, are favorable.

It is believed that the system of communication which has been set up in the western division of the Niagara Hudson system is practical, efficient, dependable, and up-to-date. The telephone company deserves a great deal of credit for the way in which it handled the complete survey of our system and for the cooperation it has given us in working out our communication problem.

Communication on the Arkansas-Louisiana-Mississippi Interconnected System

BY E. C. STEWART[1]

Non-member

Synopsis.—*This paper describes the communication facilities in use on the Arkansas, Louisiana and Mississippi Power & Light Companies' interconnected system, and describes the operating performance of the communication equipment.*

Carrier-current telephone equipment is used to a large extent for relatively long distance communication, the longest distance being 459 miles. The reliability of this means of communication under unusual storm conditions is discussed. The experience resulting from over six years use of carrier-current telephony for load dispatching purposes is described.

Introduction

THE Arkansas, Louisiana and Mississippi Power and Light Companies are operating in these three states an interconnected transmission system comprised of a total of approximately 1,800 miles of 110-kv. lines. Supervision of the operation of this system is centered in the dispatcher's office at Woodward Switching Station near Pine Bluff, Arkansas. Operation on the southern part of the system is handled from the dispatcher's office located at Jackson, Mississippi. Other systems which operate in parallel with this are the New Orleans Public Service, Memphis Power and Light, Southwestern Gas and Electric, and the Arkansas-Missouri Power Companies. Each of these has its own dispatching force which handles inter-company operating matters with the Pine Bluff dispatcher.

It is the purpose of this paper to describe the method of communication used in the operation of this system, which is largely by means of carrier-current telephony.

History

Prior to 1925, while this system was yet small, all communication was carried on by means of the toll service of the telephone company and considerable use is still made of this service, some for dispatching, but the larger part for business matters.

Upon the completion of Remmel Dam near Malvern, Arkansas in the latter part of 1924 and connection of this into the system over a 66-kv. line to Pine Bluff and a 33-kv. line to Little Rock, need of fast and reliable communication was anticipated. In considering means of obtaining such service, it was natural to favor the shortest route possible. Such a route was covered, except for a matter of about eight miles, by a privately owned and operated telephone system. This, however, was only a single-circuit line and in rather a poor state of repair. It would thus have been necessary for the power company to string a circuit of its own after satisfactory arrangements had been made, and to replace a large number of poles in order to obtain a strong and reliable line. The cost of this was such that it was felt to be too high when the reliability of such a line was compared to the heavier construction employed on the power line.

It was thus that carrier current came up for consideration, and the result was the purchase of two Westinghouse carrier-current sets. Operation of this equipment was begun the first of 1925, and the original set is still in operation at Remmel Dam, although the set at Pine Bluff was replaced by another early in 1926, because

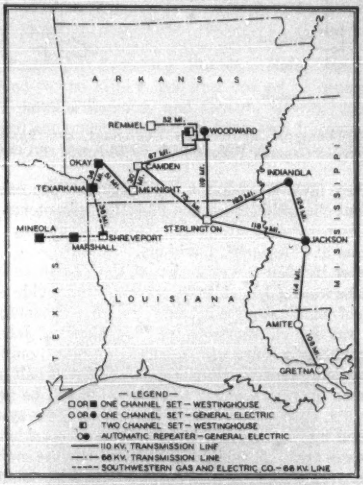

FIG. 1—LOCATION OF CARRIER-CURRENT SETS

of the creation of the Woodward Switching Station about two miles west of the original location of the Pine Bluff carrier-current equipment.

Results of this first installation were so satisfactory that the use of carrier current was decided upon for the Sterlington Station and connecting lines when completed early in 1926, and accordingly five more carrier-current sets were purchased. This gave carrier-current connection from Pine Bluff to Remmel Dam,

1. Arkansas Power & Light Co., Pine Bluff, Ark.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

El Dorado and Crossett in Arkansas; Sterlington in Louisiana, and Vicksburg and Jackson in Mississippi. It is not deemed necessary here to go into the details of how some of these sets have been moved from place to place as system changes caused certain substations to lose importance, which made it more desirable to have the carrier-current facilities at other points.

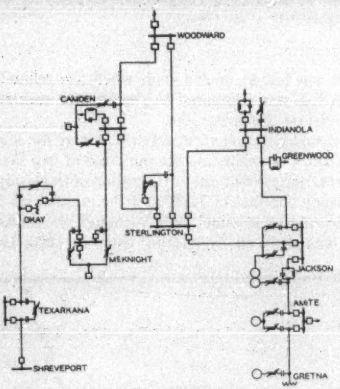

Fig. 2—Location of By-Pass Equipment and Traps to Confine Carrier Frequencies to Definite Channels

Early in 1930, operation was begun on the carrier-current channel between the Pine Bluff dispatcher and the Southwestern Gas and Electric Company dispatcher at Shreveport, Louisiana.

Later in 1930 the extension of the system to New Orleans was made and it was necessary to provide some form of communication in that direction. Communication was desired from both the Pine Bluff and Jackson dispatchers to New Orleans and the Amite, Louisiana substation, a main switching point about half way between Jackson and New Orleans. It will be noted from Fig. 1 (which shows only the portion of the system covered by carrier-current channels) that considerable distances are involved in such a connection, the shortest route over the transmission lines from Pine Bluff to Jackson being 237 miles and from Jackson to Gretna substation (which is just across the river from New Orleans) 222 miles, or a total distance from Pine Bluff to New Orleans of 459 miles. Such distances are most economically covered according to our experience by carrier-current equipment, supplemented in the few cases necessary by the use of the toll service of the telephone company. Without going into detail, the purchase of new equipment was decided upon, and a total of five type KCA-1 sets and one type KCA-100 set was made. The KCA-1 equipments were installed as follows: one at Pine Bluff, Arkansas, one at Indianola, Mississippi, two at Jackson, Mississippi (arranged as an automatic repeater) and one at Gretna, (New Orleans) Louisiana. The KCA-100 equipment was installed at Amite, Louisiana.

The installation of all of the above sets created a new channel and permitted the move of the Westinghouse equipments from Indianola and Jackson to other points on the system. One is to be installed at Carpenter Dam, now nearing completion, which is located at the headwaters of the lake created by Remmel Dam, and the other will be installed at some other point, the proposed location at present not being fixed.

Preparation of Channels

Fig. 2 indicates what has been done in the preparation of the carrier channels over this system by means of by-passing equipment and frequency blocking on unused portions of the system where necessary. It will be noted that dependence is placed between Pine Bluff and Sterlington, and from Sterlington to Indianola or Jackson upon at least one side of the transmission line loops being intact, and it has been our experience that it is in extremely rare cases that the communication channel is broken between Pine Bluff and the aforementioned points.

The location of carrier-current equipment on the lines of the Southwestern Gas and Electric Company,

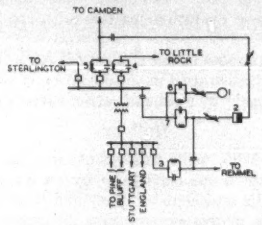

Fig. 3—Schematic Diagram of Method of Coupling, Tuning and Frequency Blocking in Use at Central Dispatching Point at Woodward Switching Station of the Arkansas Power & Light Company Near Pine Bluff, Arkansas

Legend: 1. General Electric carrier set.
2. Westinghouse carrier set, operating on two channels.
3. Resonant chokes for termination of Remmel channel.
4 Resonant chokes to prevent loss of energy from General Electric set.
5. Resonant chokes for termination of Camden circuit at frequencies used in communication with Shreveport.
6. Trap circuit to exclude Westinghouse carrier from General Electric set.
7. Trap circuit to exclude General Electric carrier from Westinghouse set.

namely, at Okay, Texarkana, Shreveport, Marshall and Mineola have been indicated because of the relation they bear to the operation of the carrier equipment on the Arkansas-Louisiana-Mississippi system. These sets operate on a separate frequency channel and it has not been found necessary to date for either party to resort to frequency blocking between the two systems. No

attempt will be made here to describe this company's installation other than the portion involved in communication between the Pine Bluff and Shreveport dispatchers. This is accomplished on a separate frequency channel by the use of a second receiver at

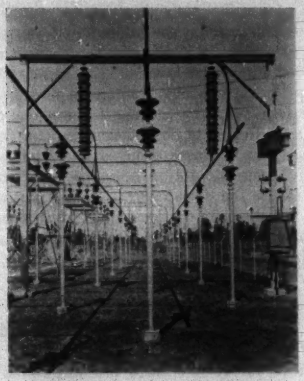

Fig. 4—Woodward 110-Kv. Bus Coupling

Shreveport and at Pine Bluff, both transmitters being arranged to deliver the proper frequency when calls are made between these points. The frequencies in use are 65 and 84 kilocycles. The by-passing circuits shown in Fig. 2 at Camden, McKnight, Okay, and

Fig. 5—Westinghouse Carrier-Current Equipment at Woodward

Texarkana are for preserving this communication channel.

Coupling

With the exception of the Remmel Dam set, which uses antennas, all coupling is by means of capacitors. There are only two kinds in use on the interconnected system: the Westinghouse suspension units and the General Electric cable type. No failures have occurred on either of these during the past two years, which covers the period of our experience with capacitors.

Fig. 3 shows a somewhat unusual application adopted

Fig. 6—General Electric Carrier-Current Equipment at Woodward

at Woodward Switching Station. Here, we have coupled to the 110-kv. bus over one set of capacitors both the Westinghouse and General Electric carrier-current equipments for the channels used in communication on the interconnected system. The General

Fig. 7—Cable-Type Coupling Capacitors and Carrier-Current Traps at Jackson

Electric equipment is operated at 70 kilocycles and the Westinghouse at 40 and 55 kilocycles. Operation of this arrangement has proven satisfactory.

Protection

There have been two occasions at Remmel Dam, when the 66-kv. wires came in contact with the antenna

wires, for observing how well the protective system functioned. Neither case resulted in damage to anything except the protective equipment itself. On one of these occasions the Remmel operator was talking on the carrier current at the time of the trouble and experienced no physical inconvenience whatsoever.

Operation

The carrier-current installation on the interconnected system, supplemented by the use of toll service and one or two leased or specially arranged circuits, is primarily for use by the dispatcher, but considerable business traffic is handled by the dispatching force outside of any rush period by means of messages, which are relayed by the dispatcher to a station operator, and thence to the party addressed.

A check of the number of times the Westinghouse transmitter has been on at Pine Bluff for the past year shows that the total of placed and received calls averages over 3,400 per month. A similar check on the General Electric set at Pine Bluff over the last five months shows that it has averaged close to 1,400 operations per month. A record was kept over three months of the total time of use and indicates that one and one-quarter minutes per conversation is representative of the average communication.

In the operation of the carrier-current equipment on the interconnected system, communication between outlying stations has not been allowed since it has been of little advantage in the system operation for any of these outlying stations to communicate with any other point than the Pine Bluff dispatcher. Pine Bluff has therefore been made a "central" and is called by the mere taking down of the telephone. It is necessary though, for Pine Bluff to dial the various stations to call them, but to save time and provide a rapid check on the selective ringing system, the successful call of a station is evidenced by the automatic starting up of the distant transmitter which has been called.

To date, carrier current has not been provided along the lines for the use of patrol or maintenance men. Reports of routine patrolling are not turned in by telephone unless some condition found warrants immediate attention, in which case the nearest available form of communication is used. Emergency patrolling to locate line breaks has been made during the last three or four years by means of aeroplane, weather permitting. Report of observations is made in a good many cases by messages dropped in pasteboard containers to some substation operator who relays the information to the dispatcher. Exact structure numbers are given, to fix the location, through the use of line maps specially prepared for this purpose and distinguishing markers on top of key structures, which are readily visible from the plane.

Reliability

There is a number of instances on record which convince us that carrier current affords very reliable communication. A few cases are mentioned briefly below.

In April 1927, during flood, carrier current was the sole means of communication at Remmel Dam for a couple of days.

The latter part of April and the first part of May 1927, during the great flood, carrier current was the only form of communication for several days to the Sterlington Station.

Several times, during storms, when other means of communication were not available uninterrupted service was maintained over the carrier current.

Communication Facilities of a Metropolitan Power System

BY P. B. JUHNKE[1]

Member, A. I. E. E.

Synopsis.—*A description of the communication needs of a power company serving a large metropolitan area, including detailed information with respect to load dispatching problems and use of remote metering facilities as an effective aid in conducting load dispatching operations.*

ELECTRIC light and power, in all its metropolitan perfection, is not entirely independent of several other agencies of public service. Not the least of these is communication, which makes the light and power system the living organism which it really is.

Because the electric power system is everywhere, so far flung in supply centers and activities of various kinds, it depends more upon communication in the discharge of its obligations than most other forms of business. For this reason an examination of the communication facilities of the Commonwealth Edison Company is of interest.

Fig. 1 shows a map of Chicago, with all fixed plant locations of the Commonwealth Edison Company. This fixed plant includes not only generating stations and substations, but also service buildings and yards, garages, etc. These many locations are served by about 19 telephone switchboards, with a total of about 2,600 telephone stations. The main board with seven sub-boards in the Edison Building is indicated by the double star. The sub-boards outside of the Edison Building are indicated by a single star.

Load Dispatching

Of special interest is the employment of communication in the technical direction and supervision over the operation of the system. The technical group especially entrusted with this function is the load dispatcher's office, located in the main office building at 72 West Adams Street, and in direct private wire communication with all generating and distribution stations and most substations.

The load dispatcher's office, shown in Fig. 2, is composed of three sections, two dealing with the routine of load dispatching and one with the performance of the system. They are designated as:

1. Load dispatcher, section No. 1
2. Load dispatcher, section No. 2
3. Load control section

The first two are charged with the responsibilities which have always characterized this work. Responsibility for supply of power to all principal distribution centers, direction of switching in connection with work to be done, responsibility for safe working conditions on service elements removed for work, responsibility for correct procedure for putting into or removing from service any system elements. Each one of these two sections has directing charge over approximately three

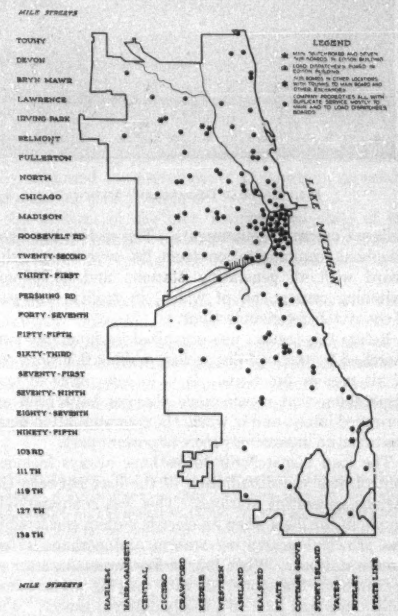

FIG. 1

generating stations and a total of approximately 175 substations and transformer vaults. It has direct communication with all generating stations and most substations. With important load centers it has a number of direct telephone wires from its own telephone switchboard.

The load control section is of relatively recent

1. Chief Load Dispatcher, Commonwealth Edison Co., Chicago, Ill.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

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origin, and is charged with supervision over the supply of power from the different generating centers selected to contribute to same. It further has supervision over the connecting tie line elements between the stations, mostly of 66,000- and 132,000-volt order. This supervision not only embraces the generating stations and interconnections within the city limits, but also the interconnections with and the supply from the generating stations located outside of the city and having responsibility of contributing to the Commonwealth

Fig. 2—Load Dispatcher's Office

Edison Company requirements. This section has direct telephone communication from its own two-position board with all generating stations and 66,000-volt switching centers, all of which, in reality, are part of the system interconnections.

Before this section was organized its functions were exercised by the load dispatcher, making this work one of his regular functions. It is in this phase of load dispatching that considerable changes have been experienced lately, and in which the communication plant has taken an increasingly more important part.

The load dispatcher's duties have always included control over the distribution of the load between the various generating stations. This was comparatively simple in the days when an electric system consisted of one principal generating station and perhaps a few reserve stations. When four or five generating stations appeared on the scene, all with varying performance ability, the task took on a vastly different aspect. The load has to be divided between all of them in a manner taking account of their relative performance efficiency.

The correct principle of operation was recognized from the beginning, *viz.*, to put the largest amount of load on the most efficient stations, and lesser quantities on those of successively poorer efficiency. Progressive development was, therefore, possible only in regard to methods employed in conforming to the aforesaid principle. A series of definite steps in the development of the methods now in force are distinctly discernible.

Earliest methods, when a new station came on the scene, provided for that station, usually the more efficient one, to carry all the load possible, predicated upon its capacity, the outgoing lines and the size of its interconnecting ties. This method was intelligent, but sometimes made the required performance of the older station rather difficult. When a third station of still greater efficiency appeared, the situation became more involved and compelled the load dispatcher to pay more attention to the instantaneous total load and to apportion it between the three as the economies of the situation dictated. This was done from hour to hour and worked well, but was very laborious because it required almost continuous readings to distribute the load properly.

From this developed the load totalizer, which collected the loads of individual stations in the load dispatcher's office. In the collection of these data, telephone lines were employed for transmission of the power readings. This was a considerable step forward, but still required continuous direction on the part of the load dispatcher, based on the totalization. It was followed by another development, known as the load totalizing scheme with reverse indication. In this scheme the totalization effected in the load dispatcher's office is sent back to each of the principal generating

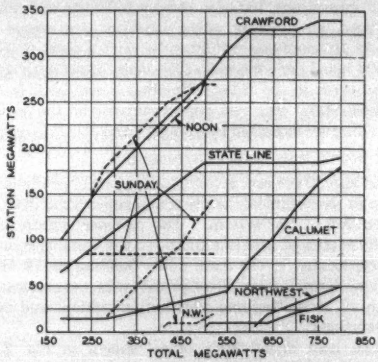

Fig. 3—Commonwealth Edison Co. 60-Cycle Load Schedule

stations, and on this totalization, with indication to each station, is based the scheme of operation. Each generating station is given a load chart showing every station's individual responsibility toward the system for all the different load values encountered in the course of a day or longer, as is shown in Fig. 3. In the preparation of this chart full and complete account is taken of the relative efficiency of different stations, and the distribution of load between the different stations is planned with a view to leaving the bulk of the load on the base load stations, and progressively less load quantities on the less efficient ones.

It is recognized that even inefficient stations, that run mostly for peak purposes, must be given a certain load considerably ahead of the time they are desired to carry the peak. It is also recognized that sometimes less efficient stations must be kept running in certain locations in the interest of adequate system or district protection. All these requirements can well be indicated on the load schedule and this fact makes the scheme practically automatic, as each operator at every moment can see his own share of the total load and conform thereto. The load control function simply has to concern itself with seeing that the schedules are adhered to and that the proper departures are made when and if changing system conditions so dictate.

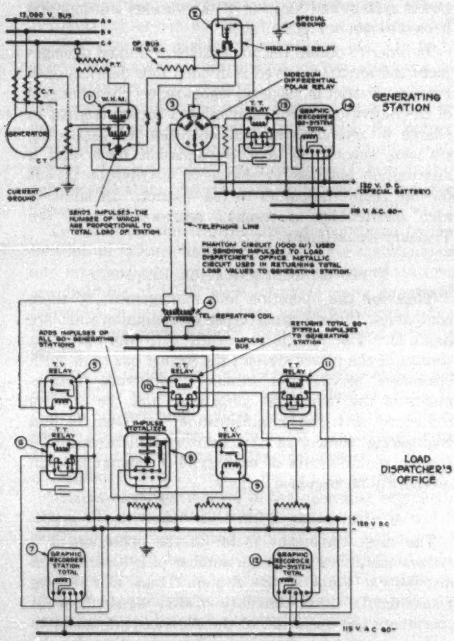

FIG. 4—SCHEMATIC DIAGRAM OF REMOTE METERING SYSTEM

REMOTE METERING SYSTEM

The information required for proper load control is obtained largely from the remote metering systems, details of which appeared in an earlier article by the author.* Briefly the arrangement consists of utilizing received alternating current impulses for operating the controlling coils of an indicating graphic recorder.

IMPROVEMENTS IN THE REMOTE METERING SYSTEM

Instead of balancing the effect of the impulses by mechanical means, some of the recorders have recently been changed to operate on a wheatstone bridge arrangement. The coils of the meter act as the galvanometer, in which there is no current as long as the impulses are constant. As the impulses vary the circuit becomes unbalanced and current flows through the meter coils, tending to close a contact, causing the operating motor to run in a direction to indicate more or less load, depending on the direction of current through the meter coils or galvanometer. The operating motor also moves a sliding contact on a resistance coil, tending to restore the balance of the wheatstone bridge circuit.

With this new meter circuit arrangement it is not necessary to have all the meter elements installed on the load dispatcher's board. This has made it possible to install several of these recorders in a relatively small space, all of which can be read by the men at the load control switchboard. The charts are all controlled by the same synchronous motor, and therefore all record for the correct time, so that there is no question about comparing the chart for any specific time.

In this metering scheme as installed in the Commonwealth Edison Company metallic telephone circuits are used exclusively, but the fact that the impulses can now be transmitted by the carrier-current system has materially enhanced the usefulness of the system, especially for metering distant stations.

On account of the high maintenance costs of the mechanical totalizers, research work is being conducted in an effort to replace these totalizers and relays with vacuum tubes.

This installation no doubt makes possible the most intelligent operation of large interconnected systems. The task of directing is continuous and practically automatic, and the economies gained in plant operation of a system of moderate geographic extent more than warrant the cost of the equipment and the incidental telephone line rental.

Discussion

F. R. Innes: The function of communication is the transmission of information so that operation may be controlled. Information may be expressed in speech, intelligible to the human mind or it may be expressed in some other manner to which mechanical and electrical devices are receptive. To the latter class belongs remote metering.

In systems where generation stations are separated and must, for over-all economy, be operated with the most favorable division of load between them, the remote metering installation is a valuable tool. It is quite possible and it does happen on systems where load control is not closely watched that load swings at one station or another due entirely to transient conditions, may be very large, much larger than is good for the operation of the system.

A remote metering system, quickly responsive to changing load conditions, bringing to one point and to one man a complete picture of the system load, makes possible the reduction of differences from the ideal conditions of load division between stations and results in a much more stable system operation. It is possible, with such close control, to keep the regulating station secure in its function and to prevent the usurpation of its job by another station that may for any reason be more responsive to changes in system load.

*"Budget-Plan Load Dispatching," by P. B. Juhnke and L. N. Moore, *Electrical World*, Vol. 89, pp. 1317-1320.

Cooperative Study of Power System Communication

Results in Improved Facilities

BY C. A. BOOKER[1] and M. E. CLARK[2]
Non-member Associate-member

NEW England Power Association operates a large group of interconnected wholesale and retail electric companies in a highly developed industrial area. Energy is generated at eleven major hydro stations on the Connecticut and Deerfield Rivers, and

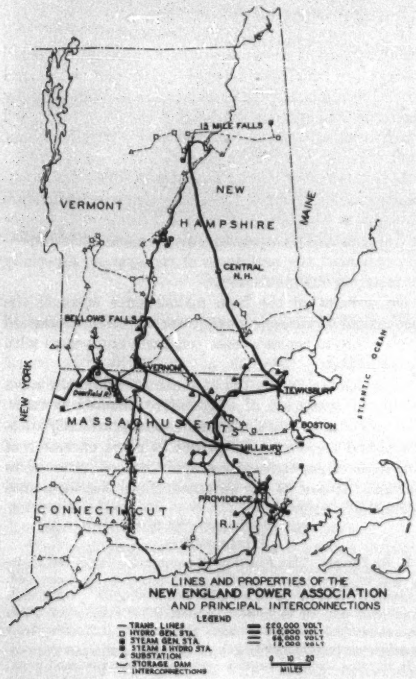

FIG. 1

transmitted on four 66-kv., four 110-kv., and two 220-kv. circuits to the load centers in central and eastern Massachusetts and Rhode Island, where interconnection is made with several large steam generating plants by other high-voltage lines.

To assure maximum economy in operation, a centralized control over all producing and distributing units is necessary. This control is exercised by the operating engineer through the Dispatching Department, which has its headquarters at Millbury. The extent of the power system and location of the various transmission lines is shown in Fig. 1.

In order to obtain the advantages of unified management the territory served is divided into districts each having a district manager to whom report the managers of the individual retail companies, a superintendent in charge of maintenance and operation of generating stations, substations and transmission lines, and a distribution engineer in charge of distribution layout for the retail companies in his district. In addition each district has a resident representative of the Treasury Department.

Types of Communication Required

Thus for the operation and management of these companies, three distinct types of communication are required. The first is concerned with the operating control of the power system; the second has to do with customers' service and ordinary commercial requirements of the individual companies; and the third is concerned with the administration, or executive and engineering control of the territorial districts. The specific requirements of each type of communication service will be discussed in detail.

Operating Control

The most important factor in the operation of a system comprised of a large number of generating and distributing units is the System Load Dispatching Department. It is the duty of this department to coordinate the operation of the various parts so as to achieve maximum economy. This objective can be gained only by very prompt and reliable communication from department headquarters at Millbury to each individual unit.

An analysis of the actual functioning of the department discloses a considerable number of closely related methods through which the coordination and control is exercised.

1. Complete centralized control over production and load distribution.
2. Control over every piece of apparatus available for service.
3. Supervision of system tagging and safety work,

1. New England Power Association, Boston, Mass.
2. New England Tel. & Tel. Co., Boston, Mass.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

both during ordinary maintenance and emergency repair work.

4. Forecast and scheduling of coming operation about 15 to 24 hours in advance.

5. Arrangements for interchange of power with other interconnected systems.

In order to plan intelligently the operation of the system, the dispatcher must have available complete data on existing conditions. These data are secured by hourly reports from the various stations which transmit by telephone the necessary information, of which the following items are examples: production, sales, voltage, water elevation, machines in operation and others available for service, condenser operation, precipitation reading, gate and flashboard operations, etc. In fact, everything having a bearing on system operation is included in the hourly reports.

Reports are also received in regard to apparatus or line failures and arrangements made with the proper departments for the necessary repairs. During such repairs, elaborate safety precautions are taken to prevent energization of equipment on which work is being done.

To facilitate the transmitting of reports as to the location of failures as well as to expedite maintenance and return of equipment to service, telephone stations are desirable at fairly close intervals along important transmission lines. It has been found to be good practise to have such stations located at about six-mile intervals.

Recently it has been found advantageous to delegate dispatching authority over certain limited areas to district dispatching offices. These district offices exercise the first three functions of dispatching under the general supervision of the chief system dispatcher at Millbury, who in turn is responsible to the superintendent of power control in the operating engineer's office. This results in a unified organization in which methods are standardized and by which policies and procedures may be speedily dispatched without misunderstanding. It is evident that this method of operation requires instant, dependable, and continuous communication between all points involved.

Customers' Service

The individual retail companies usually require a private branch exchange system to provide a flexible communication service for receiving customers' and other outside calls, as well as to handle interdepartmental calls. These switchboards must be supplied with ample facilities including outside lines and private branch extensions to handle all calls without delay. In the larger communities 24-hour service is essential, which necessitates constant attendance of one or more telephone operators or the use of a full dial P. B. X.

The telephone extensions to the Customer's Service Department are so arranged that all calls involving service of anything from flat irons to radios or electric refrigerators are received at one place. This is in order to enable such calls to receive prompt and sympathetic treatment without irritating transfers from one department to another.

Administration

As a network of communication lines converging at Millbury is necessary for the operating control of the power system, just so a similar network converging at Boston is convenient for the administration of the commercial and engineering aspects of the business.

The provision of a relatively few lines provides such a network between the executive offices and the territorial

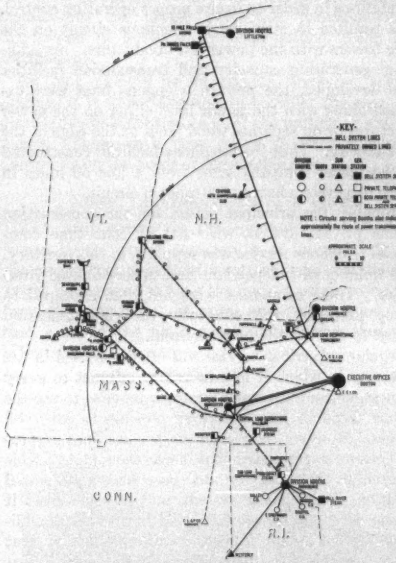

Fig. 2—Principal Communication Circuits of New England Power Association

offices by which the business is greatly expedited. Also, by interconnection made at the territorial office switchboards, economical and speedy communication is available with every part of the system. Thus when designing the telephone system for administrative purposes, flexibility is kept uppermost in mind, it being particularly necessary to consider carefully the effect on the whole network when any addition or change is being planned.

Existing Communication Arrangements

The extent and character of existing arrangements for supplying the above requirements are shown in Fig. 2.

On a map of this reduced size it is of course not possible to indicate the location of all P. B. X.'s of which there are twenty altogether.

History of Private Telephone System

Before entering upon a discussion of the methods by which the above requirements have been fulfilled and coordinated, a brief historical review of the beginning and development of the private telephone system will provide some necessary background.

At the time of the construction of the first transmission line in 1908 from Vernon to Clinton, it was believed by the designers that instant communication was necessary between the two termini, as well as with intervening stations in order to insure proper operating control. This resulted in stringing a telephone circuit on the same towers with the power line conductors. As additional generating capacity and transmission facilities were developed, the private telephone lines were extended along with the power lines either on the power structures or on separate wood poles at the edge of the right-of-way. Thus the telephone facilities constructed by the power company grew from a line 50 miles in length to a system having 600 miles of circuit.

Probably the principal reason for the construction of this private system was that at that time commercial telephone service was supplied, in this territory, by a number of independent and uncoordinated companies. These companies were not then equipped to supply promptly and efficiently the communication needed to operate the power system.

Regular subscriber service was of course used in the same way in which it is a necessary adjunct to every business, although it was a common practise to use the private dispatch lines wherever possible to save toll charges. However, traffic on the private lines eventually became so congested that it was thought advisable to prohibit their use for any purpose except actual operation of the power system, and in some cases it became necessary to provide relief in the form of additional service. Careful consideration indicated that service suitable for this need could be secured most economically by the use of regular commercial telephone service.

When the executive and engineering offices were concentrated at Boston and affiliations with distributing companies increased, the great increase in messages radiating from the Boston office justified the use of a leased network for handling the administrative business.

By 1928 the number of messages handled by the private telephone system at the central dispatching office at Millbury had grown to over 600 per day in times of normal operation or about twice that number during periods of trouble. It thus became necessary to make further improvements in the communication arrangements, which now handle about three times the above number of calls daily.

More knowledge of the fundamental requirements for each type of service was needed as well as a definite picture of the desirable arrangement and concentration of such services. It was also recognized that both privately owned lines and regular commercial telephone service were involved, and that an intelligent study of the situation required a thorough knowledge of modern developments in the communication field, as well as intimate familiarity with the operation of the power system.

At a meeting of Power company executives with those of the New England Telephone and Telegraph Company, this need was expressed with the result that a cooperative study was entered upon to determine how to obtain improved communication service from existing facilities, as well as to plan for coordinated extensions.

Joint Study

This study considered all parts of the system and its objectives were as follows:

1. The determination of existing communication facilities, methods of operation and service usage.
2. The determination of present and future communication requirements.
3. The analysis of these requirements to determine what improvements or changes should be made, and what additions were required.

A large part of the initial survey has been completed, and it has become apparent that the changing nature and growth of requirements merit continued and sustained study on a joint basis. The survey has already resulted in improvements of which the following are typical.

Modification or rearrangement of private branch exchange systems serving local areas, districts, or divisions has provided a more adequate and efficient handling of the needs of the business. In some instances, it has been possible to effect substantial economies in the cost of service coincident with the rearrangement. Somewhat allied with this problem has also been the conversion from regular toll service to leased line service in all cases where this is justified. Leased service is adopted when the cost of toll calls between two areas increases to the extent that the leased service becomes particularly attractive, due consideration being given to such factors, as accessibility and usage, as well as cost.

Load Dispatching Turrets

Probably the most interesting of the special station and switchboard equipment developed are the load dispatching turrets shown in Figs. 3 and 4. The desirability of these turrets was indicated in the joint study, and they were designed and furnished by the Telephone company as a part of its service. These turrets are conveniently placed at the dispatcher's desk and serve to terminate and interconnect communication circuits

associated with the private branch exchange system. They have been so designed as to meet the following outstanding requirements of the load dispatchers.

It was desired to provide access to all lines and yet minimize the actual work of telephone operating which is required of the dispatcher. This was accomplished by locating the turrets conveniently, by using switch

Fig. 3

keys instead of cords and plugs, and by providing means for the station P. B. X. operator to handle all calls not intended for the dispatcher. At the same time the load dispatchers must have full control of all the lines at times of system disturbance or at night when the regular switchboard was unattended. This requirement was met by arranging for permanent patching of all lines from the P. B. X. to the turrets whenever desired.

Visual as well as audible indication of incoming calls and their status is important because of its many advantages. For example, the visual indications of incoming calls appear on the turret as fast as the calls come in, and remain until answered. Thus the amount of traffic waiting to be handled is at all times apparent, and the number of dispatchers on duty can be adjusted depending upon its volume. In this connection a visual indication of a busy line is also provided at all positions so that, when a call is answered by any dispatcher, the others are notified to that effect and take care of the next unanswered call.

These various conveniences are accomplished while all but two telephone instruments are eliminated from the dispatcher's desk, one for the private line and one for the Bell system lines.

Transmission, Protection, and Signaling

A vital part of every study in the review of existing facilities and in the planning of new ones has been that of transmission, that is, the grade of "talk" obtained. It is not unusual to connect two or more lines for communication within the private network. In such cases a line which is designed to give satisfactory transmission between its terminating P. B. X's may be entirely inadequate when connected to another line to reach a more distant P. B. X. or station. It is, therefore, important to determine beforehand what interconnections are desirable in order to meet such situations in the design of the circuits.

Of equal importance is the matter of protection from abnormal electrical voltages which may be impressed upon the communication circuits by lightning, induction from power lines, or ground potential conditions. Existing installations have been inspected to make sure that protective equipment was properly installed and suited to the specific conditions. On new or proposed installations the proper protection arrangement is determined in advance by observations, tests, and calculations.

On long circuits or on circuits involving several bridges, it is important also to consider the signaling arrangements best fitted to meet the particular requirements. These have been found to vary somewhat depending on the needs of the specific case.

Noise Treatment—Private Lines

The joint survey of the communication facilities owned by the power company is of interest. It was recognized that consistent treatment and maintenance would be required to keep the lines in proper condition from the standpoint of noise and transmission, and that some of the lines were much noisier than was believed to be necessary.

Fig. 4

A systematic attack on this problem was made, beginning with a complete field inspection and survey. As a result of this survey, all necessary tree trimming, replacement of broken insulators, etc., were accomplished as well as the preparation of a complete exposure and transposition diagram of all the lines in the system. At the same time measurements were made of the noise,

both metallic and to ground, which was present on all of the lines and on the various parts of these lines. From the exposure diagrams obvious transposition unbalances were noted and corrected. The noise survey provided the basic data from which an intelligent attack could be made on testing and locating the sources which caused the noise.

The next step was to design a combination of equipment which would accurately measure insulation resistance, series unbalances, etc., so that bad joints, leaky insulators, and other contributing causes could be located without sacrificing adequate protection to the testers from the induced voltages present on the telephone lines. The work of cleaning up the lines with the use of this testing equipment is finished. Although all the data are not yet assembled, it can be stated that this testing and clean up work has resulted in marked improvement of the noise conditions on the lines and it is planned to continue the use of this testing equipment as a regular part of the maintenance work.

Fifteen Mile Falls Patrol Service

When, in 1929, it was decided to proceed with the construction of the Fifteen Mile Falls power development with its 220-kv. transmission system, authorization was secured on the basis of constructing the usual wood-pole telephone line parallel to the power lines. The effect of induction on the communication system was carefully considered by the power company and discussed with the telephone engineers on the joint study basis.

This discussion resulted in the telephone company's suggesting as an alternative proposition that this service be leased from it. A preliminary plan of the details of obtaining leased service at the various patrol stations was made. A thorough consideration and discussion showed that the desired service could advantageously be obtained in this way. This resulted in a decision to enter into the proposed arrangement on a trial basis.

The prime influence in arriving at this decision was the ability of the telephone company to furnish one or more duplicate paths of communication in case of trouble with the regular path. Such duplication is available on the existing private system by virtue of its layout in loop feeds but was unobtainable on the Fifteen Mile Falls line except at very considerable expense. Another factor considered is that within a few years the telephone company plans to have sleet proof construction throughout the length of the line. A third factor was the lower annual cost.

The layout of this communication circuit is indicated in the upper right hand portion of Fig. 2. It is designed to serve precisely the same function for load dispatching and for patrol service along the transmission line as do the privately owned lines paralleling the other transmission lines.

The features of design and the maintenance arrangements which insure the desired continuity of communication service may be of interest. The leased circuit is routed over regular toll lines from Tewksbury to Fifteen Mile Falls with a permanently bridged tap to the central New Hampshire substation. Access to the 23 patrol booths on the transmission line is given by lateral spur lines extending from central offices through which the main line passes. The average distance from the main line to the patrol booths is five miles and the average spacing between successive booths is five miles. This line is under the complete control of the power company and the services of regular telephone operators are not required, except when exchange service is desired from a patrolman's booth.

An interesting equipment feature is the arrangement whereby the patrol station spurs are normally disconnected from the main circuit, but which permits them to be connected to the main circuit at will by the operation of a switch at the patrol booth. With this arrangement, shown in Fig. 5, the closing of the switch at any patrol station causes a relay at the central office to connect the patrol loop to the main circuit. The switch

Where Line is Bridged at a Magneto Office

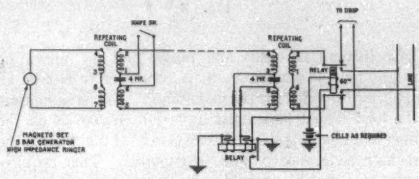

Fig. 5—Patrolman's Line Circuit

Special for New England Power Company Tewksbury-Barnet Line

is normally left open so that the bridging losses of the patrol station spurs may be avoided.

Provision is also made to enable the patrol station to signal the operator in case exchange or toll service is desired. This desirable feature is accomplished by connecting the back contacts of the bridging relay to a drop in the central office. It provides greater continuity of service, for, should the relay fail to operate, a patrol station will automatically signal an operator when the patrolman rings on the line. Connection can then be made either to the leased lines or to the toll or exchange network of the telephone company.

Adequate transmission is obtained on this circuit by the use of vacuum tube repeaters at three intermediate central offices. These repeaters also make it possible to carry on satisfactory conversations between Millbury and Littleton when the respective tie lines are connected.

Field tests indicated the necessity for special high-voltage protective equipment at the Tewksbury substation and the Fifteen Mile Falls generating station, and also at the central New Hampshire substation to prevent interruptions of telephone service due to rise

of station ground potential at times of power line fault. These tests indicated that the ground potential in case of power line fault will rise above the corresponding central office ground enough to operate standard substation and central office protection and put the telephones out of service. The power line is *Y* grounded at Tewksbury and Fifteen Mile Falls and the station ground at these points is therefore susceptible to such voltages in case of any fault throughout the entire length of the 220-kv. line.

The protection provided consists essentially in isolating the telephone plant in the vicinity of the power station from the rest of the telephone plant by using insulating transformers in all telephone lines on the load feeding the power station. Transformers insulated for 25,000 volts between the windings are used. They are protected from lightning by 3,000-volt lightning arresters where the line adjacent to them is open wire. These transformers are mounted on *H* structures in the aerial telephone plant feeding the stations.

Recognizing the important part played by communication in the operation of the power system the maintenance practises which have been placed in effect are designed to ensure the highest degree of dependability and continuity of this service.

Among the points covered in the maintenance routine, the following have been emphasized as of primary importance:

1. One man has been designated, to whom trouble on any part of this circuit is reported, either by the power or telephone company employees. This man coordinates and administers all maintenance or repair work on this circuit.

2. At times of trouble in the outside plant, he arranges to have it cleared in the shortest possible time.

3. A cabinet of patching jacks has been installed in every central office which was not already equipped with a toll testboard or toll patching jacks. Both the spur lines and the main line are terminated in these jacks. Regular toll circuits have been designated for use at times of circuit failure on the leased line and also appear in these patching boards so as to facilitate the setting up of emergency service.

4. Instructions are set up with test men and with various central offices to patch with these circuits at time of trouble and to re-route the entire base circuit if necessary to maintain continuity of service.

5. Arrangement is made for routine tests and for manual connection of spur lines at offices not equipped for test to the main circuit so that tests can be made from distant test boards.

Conclusions

Although the communication requirements of the New England Power Association have presented some unusual problems, joint consideration of these requirements has resulted in a satisfactory communication system. The provision of adequate communication is essential to the operation of the power company, and in order that this may be accomplished economically and with minimum duplication of effort, as the requirements change in the future, it is planned to continue the joint survey work.

Reestablishing Excitation of a Loaded Alternator in Parallel with Others

BY D. D. HIGGINS[1] and E. WILD[2]
Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The paper describes operating tests with large turbo generators under load and gives a résumé of answers to a questionnaire, bearing on the phenomena accompanying the interruption of the direct-current excitation supply, followed by reclosing after a short interval.

The object of the investigation was to determine, if possible, a reasonably safe operating procedure that could be followed in reestablishing normal conditions on a large turbo generator whose excitation had been interrupted while carrying load.

The authors conclude that the reexcitation of a generator without withdrawing it from the bus is a justifiable and safe operating procedure and the one that will restore all conditions to normal in the shortest time.

These conclusions have been verified by twelve prearranged tests, five "experiences" in the local company and seventeen "experiences" by other companies. In all of these cases the field switches have been reclosed without trouble of any kind.

* * * * *

ONE of the early questions which arose in the parallel operation of alternating-current generators was, "what will happen if the field circuit is opened while a generator is carrying load and in parallel with other generators?" This proposition is somewhat academic and no doubt has been solved to the complete satisfaction of students and others using laboratory set-ups or small generators whose behavior, in so far as customers' service was concerned, was of no serious consequence.

Operators of large power stations and interconnected systems have disposed of the question in various ways as evidenced by replies to a questionnaire. It was found that there was a lack of conclusive data bearing on the behavior of turbo alternators when operated with an open field and, in general, it was the impression that something very serious might result if the field was reestablished before disconnecting the generator from the bus. The usual practise was therefore to take the unit off the system, reexcite it and resynchronize.

While the foregoing was the general feeling, there was no evidence to support it; in fact, the evidence produced was negative. As the size of individual units increased and large, single machines constituted a considerable percentage of a station's capacity, and an interruption on one of them seriously affected the stability and uniform operation of the system, this old question came to the front for serious consideration.

With the advent of multiple or group units consisting of two or more generators, the time required to restore a unit to the system after once being disconnected from the bus, when not excited, would probably be not much less than an hour. This is because the two or more machines comprising the unit will retard at different rates and consequently will be so far out of synchronism that when the excitation of the various generators is reestablished, they will not pull into step. It therefore becomes necessary to allow the machines to come to rest, close the field circuits and accelerate in the usual way. Such an interruption on units of 50,000-kw. to 150,000-kw. capacity would obviously be most serious at the time of heavy load.

In order to establish a reasonably safe operating procedure for such emergencies and one that would reestablish normal conditions just as soon as possible, the authors have collected data on experiences and statements from various sources and have determined, by tests, the behavior of large units under this abnormal condition.

Replies from 10 operating companies and one manufacturer brought information about 16 cases in which field switches were reclosed without cutting the generator off the bus. No one reported any disturbance on the reclosing of the field. The authors have rather complete information on 18 other cases wherein the field circuits were reclosed without any difficulty.

Opinions of Operating Companies

Seven stated they had promptly reclosed the field switch or would not object to it being done.

Three stated they would not reclose the switch but would cut the generator off the bus.

One manufacturer of large units recommended the field switch be reclosed.

These statements concerned machines of from 8,000 kw. to 65,000 kw., both steam and hydro.

It is the practise in unattended hydroelectric plants to have the generators thrown onto the bus unexcited when about up to synchronous speed, after which the field is applied and the unit pulls into step.

The replies from operating companies probably represent the accumulated experience over a great many years and while probably not complete, no single case is cited wherein trouble resulted from reclosing the field circuit. It seems reasonable to conclude that where the general feeling is in opposition to such a practise, that had there been anything to support such a stand reference to it would have been made.

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It would seem from some of the replies to the questionnaire that the behavior of unexcited generators is not very well understood, because several references are made to the anticipated danger of "pole slippage," "out-of-step," and "out-of-phase" conditions. In practically all cases of record referred to in answering the questionnaire and in 18 cases of which the authors have first hand information, the generators continued to carry load, sometimes they did not drop any, sometimes as much as 30 per cent but in all cases they carried some. This indicates that the generators did not drop below synchronous speed but probably went above speed and that pole slippage is of no moment. In the tests on the 60,000-kw. generator there is positive evidence that the rotors drifted ahead of synchronous speed and by different amounts. This was probably true in the other cases. It is therefore reasonable to believe that pole slippage as associated with possible complications incident to the opening of the field circuit is not harmful.

With the reestablishment of excitation the rotor poles build up slowly and as they do, the speed of the rotor changes slowly and the rotor and stator poles lock into step with properly related polarities.

The question of subnormal excitation of a generator has a bearing on this subject because of experience which indicates what the effect on a generator is when it slips a pole due to reversal of exciter polarity resulting from weak exciter field, etc. The point that should be borne in mind is, that the exciter should never be operated below the knee of its saturation curve. If it is, it may be overpowered by the induced voltage in the rotor circuit when the field switch is reclosed, causing the rotor poles to reverse as fast as they try to lock in with the stator poles. The harmful effect of such an occurrence on a large system is moderate as measured by bus voltage disturbance and apparently negligible in so far as harm to the generator. Aside from the remote possibility of accumulated dust and dirt being ignited by sparks from poor electrical joints in the steel structure of the rotor, no damage would be anticipated. No doubt, currents of high magnitude exist in the steel of the rotor at the time when the rotor speed differs from that of the armature poles.

Cases of this kind of disturbance and on units of 35,000 kw. have resulted in a 10 to 12 per cent dip in bus voltage and aside from a groan of the unit, did not interfere with parallel operation in the system.

As sub-normal excitation may allow pole slipping, this point was taken into consideration in the tests described below and normal setting of all rheostats maintained.

Tests

Tests have been made on the restoration of the excitation of alternators of 30,000-kw., and 60,000-kw. capacity. Preliminary tests, four in number, made on the 30,000-kw. machine were more or less of a trial nature and served to show what the instrument set-up should be for the test on the 60,000-kw. multiple unit. Oscillograph records were not made but the observed indications of current, voltage, etc., were recorded. This unit was rated 35,000 kva., 85 per cent power factor, 12,000 volt, 1,200 r. p. m., 60 cycle; tandem compound, having 3.06 ohms synchronous impedance; no external reactance; operating in parallel with the Chicago system. The following readings were taken:

	Bus volts	Machine amperes	Machine kw.	Power factor
Before test	12,400	1,000	15,000	0.72 lag.
Field open	11,200	2,000–2,600	14,000–15,000	Off scale lead
After test	12,400	900	15,000	0.78 lag.

The field switch was left open for approximately one and one-half minutes on this test. The change in current and power factor after the field was reclosed was due to increase in excitation in other connected stations and on other generators in the same station while the field switch was open.

There was no violent fluctuation of current or voltage while the switch was open, the change from 1,000 amperes to 2,000 amperes was at a slow and approximately uniform rate covering about two seconds. The change from 2,000 to 2,600 amperes was irregular and without any definite frequency though observers agreed that it did seem as though average values might have a definite period.

A second test with the same set-up of values resulted in the same conditions. This was followed with a test when the unit was loaded to 21,000 kw. Results were about the same except the bus voltage dipped to 11,000—a 1,300-volt drop. After this test it was observed that the field discharge resistance was being heated. A voltmeter was then connected across the discharge resistance and a fourth test made which showed, by calculation, that the induced current in the field circuit was 100-120 amperes and fluctuating and reported "not to have dropped to zero at any time." This report is doubted.

Starting with a bus pressure of 12,400 volts, five seconds elapsed before the pressure dropped to 11,200 volts, going down at a slow and uniform rate. After the field switch was reclosed it took about one second for the exciter to build up the current to full value but took five seconds after that for the bus pressure to reach full value again.

The unit layout and instrument set up for the tests on the 60,000-kw. machine are shown in the accompanying diagrams.

This unit is of 60,000-kw. capacity, cross compound, 60 cycles, 12,000 volts. The high-pressure generator runs at 1,800 r. p. m., is of 17,000-kw. capacity and has a synchronous impedance of 5.8 ohms. The low-pressure generator is of 43,000-kw. capacity, 1,200

r. p. m., and has 1.92-ohms synchronous impedance. The external reactors between the unit and the bus are 827 kva., 3,500 amperes and have 3 per cent reactance on 12,000 volts.

During two preliminary tests the unit was given a load of 20,000 kw. at 85 per cent power factor. The switch was opened which cut off the excitation of both fields. The switch was left open for approximately three minutes, during which time the hand of the synchroscope made 16 revolutions, showing that the field

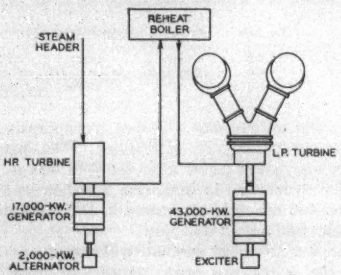

Fig. 1—Crawford Ave. Station

Plan of two-element 60,000-kw. turbo-generator unit No. 2. Commonwealth Edison Co. Feb. 28, 1930.

of at least the high-pressure generator was slipping ahead of the system slowly. The speed of the high-pressure unit was compared with the system by means of the synchroscope which was connected between the house alternator on the high-pressure unit shaft and the 2,300-volt bus which was alive from the main 12,000-volt bus.

The reclosing of the field switch occurred about the time the synchroscope showed the high-pressure rotor to be 180 deg. out of phase. That is, the mechanical time-phase relation of the poles of the rotor was 180 deg. away from the normal position. There was a total absence of surging or violent fluctuations; the excess current on both generators simply subsided, the synchroscope needle stopped at the "in" phase position and the voltage came back to normal.

Final Oscillograph Tests

A second set of tests was run on the 60,000-kw. unit during which oscillograph records were made, but owing to the length of the films (24 ft.) only the important sections are given in the accompanying figures and these curves are faithful reproductions of the oscillograph record.

The field switch was opened three times in all. On the first test, the machine had 30,000-kw. load at unity power factor. A short, fast film was run to determine the proper scale on the oscillograph and the time relation between various waves.

On the second test the machine had the same load of 30,000 kw. at unity power factor but the field switch was kept open over 30 seconds while a 24-ft. oscillogram was taken. On the third test the machine had 50,000 kw. at 80 per cent power factor when the switch was opened. A 24-ft. film was again run for about 30 seconds.

The general performance of the machine in each test was similar but the amplitude of the current and voltage oscillations was much greater for the higher load. A plot has, therefore, been made for the latter case only, and root-mean-square values of current and voltage, instantaneous values of field current and speed have been shown.

A plan of the principal steam connections and another of the electrical connections used in the test, are also given.

Opening the Field

When the field switch was opened the field current immediately dropped to less than half value. Then for a period of four seconds the fields gradually became demagnetized. The station bus voltage dropped as the fields became demagnetized and the armatures gradually drew heavy magnetizing currents from the system.

From the end of the fourth second till the field switch was reclosed, generally transient conditions existed.

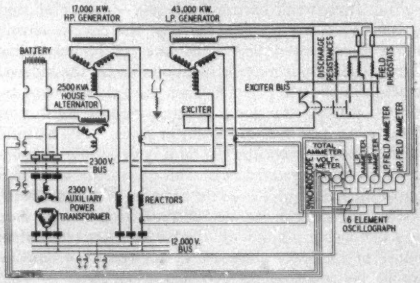

Fig. 2—Crawford Ave. Station

Two-element 60,000-kw. turbo-generator unit No. 2 instrument connections for open field test. Commonwealth Edison Co.

Both machines ran considerably above synchronous speed and slipped past their regular pole positions many times. The speed of the high-pressure turbine was very erratic. Violent governor action was observed. The low-pressure turbine, not having a speed regulating governor, ran at a more uniform speed.

The steadiness in the speed of the low-pressure turbine can probably be accounted for by the fact that the reheat boiler served as a stabilizer for the rapidly pulsating steam that was passing through the high-pressure turbine. Since the heat in the reheat boiler

was constant, it would momentarily give off more reheat to a smaller amount of steam at a lower pressure and less reheat to a larger amount at a higher pressure.

As the magnetic coupling between the field and armature poles was removed, the energy produced by the turbine being constant, the machine accelerated above synchronous speed. The field winding then cut lines of force and had induced in it a current which had a frequency proportional to the slip or the difference between armature and field speed. Its general characteristics were those of an induction generator.

As the turbine speeded up, the governor, set for a given load at synchronous speed, reduced the amount of steam into the turbine and the machine slowed down. When the machine approached synchronous speed, the governor opened up again. This cycle repeated itself very rapidly at high loads, much less rapidly at light loads.

When a field pole came under an armature pole it had induced in its winding a current which set up a magnetic pole of opposite polarity to that of the armature pole. Then as the field passed over to the next armature pole, the field pole became demagnetized and then remagnetized in the reverse direction. These reversing field currents are shown in Fig. 3 where they are plotted as instantaneous values.

The armature currents which are plotted as root-mean-square values, are seen to have a maximum and minimum value for each reversal of the field current. Ordinarily an induction generator would have constant armature current for a given speed. The peak values found on these tests can be explained by the fact that the field does not have a uniformly distributed winding such as would be found in the rotor of an induction machine. When the center of the field pole is not directly under the center of the armature pole, the field windings are not symmetrical with respect to the armature pole and it takes a larger armature current to induce the same field current than when the poles are centered.

The station bus voltage was reduced about 1,500 volts. This reduction was greatest when the armature currents were a maximum and was least when the armature currents were a minimum. The maximum armature currents checked very closely with predetermined values which were 2,100 and 6,000 amperes respectively for the high- and low-pressure generators.

There is some question that if the field switch had been closed when the induced field current was a maximum in the reverse direction, the exciter (shunt wound) might have become demagnetized. In the worst case found here, the low-pressure field was found reversed 550 amperes and passed through a discharge resistance of 0.56 ohms so that the voltage which would have been thrown across the exciter would be 308 volts, only momentarily. This might have been enough to demagnetize an exciter set at 275 volts. It would also have been necessary for the high-pressure field

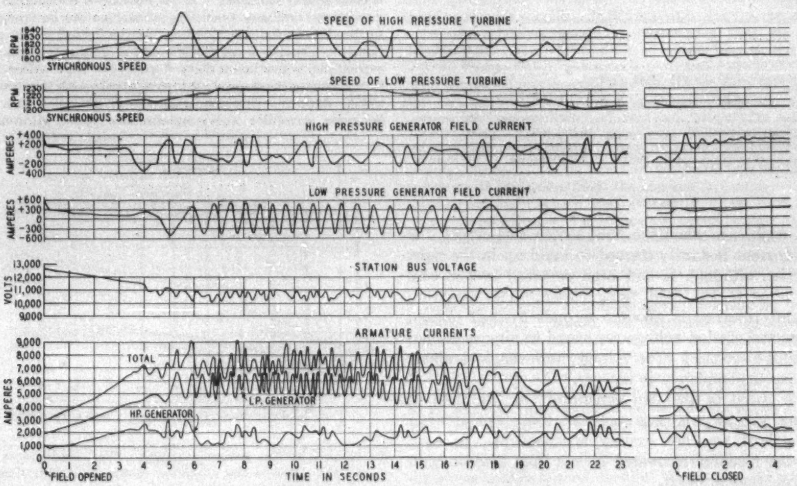

Fig. 3—Generator Opened-Field Test

Unit 2—Crawford Ave. Station. Feb. 28, 1930. 60,000-kw. cross-compound turbine. Machine carrying 50,000 kw. at 80 per cent power factor when field switch was opened.

current to be negligible or also reversed at the same time, since the net current in the common exciter is the instantaneous sum of the high- and low-pressure field currents.

It is thought that the time required to weaken and reverse the exciter would be greater than the duration of the induced field current at its maximum negative value. Therefore, the exciter would probably not be reversed.

The belief that excessive currents might be induced in the field winding which would prove injurious was not borne out by the observed values of current.

The tabulation which follows gives a comparison between the action on the two loads:

Load	30,000 kw	50,000 kw.
High-pressure speed (max.)	1,855 r. p. m.	1,872 r. p. m.
Low-pressure speed (max.)	1,220 r. p. m.	1,233 r. p. m.
High-pressure field current (max. positive)	120 amperes	410 amperes
Low-pressure field current (max. positive)	180 amperes	550 amperes
Bus voltage (minimum)	10,500 volts	10,000 volts
High-pressure armature current (max.)	1,510 amperes	3,000 amperes
Low-pressure armature current (max.)	3,340 amperes	6,700 amperes

Closing the Field

In each case when the field switch was closed, the field current instantly started to build up in the right direction although it had been reversed in one case just prior to the field being closed. The armature currents returned in about 5 seconds to their normal values and station voltage recovered to almost normal in about 5 seconds. The voltage, however, took about 20 seconds completely to recover.

The speed of the two generators, at the time of closing the switch, was not far from synchronism although it was not exactly synchronous. In each case the machine recovered in less than 2 seconds to its synchronous speed.

In the 30,000-kw. load test, the unit dropped a maximum of 30 per cent load and in the 50,000-kw. test, 40 per cent.

Conclusions

It is recognized that different types of generators behave in different ways when the excitation is cut off. Old units having low-resistance field windings and operating with 125-volt excitation are more erratic than later machines employing 250-volt excitation probably due to lower energy loss in the rotor. If the unit is carrying less than ¾ load, there is no danger in reclosing the field switch. If the unit is carrying more than ¾ load it will be prudent to lower on the governor before reclosing the field switch.

If the bus voltage has dropped to 60 per cent of normal it will be prudent to cut off the generator to prevent the possibility of other generators dropping out of step, but such a reduction in voltage due to an open field is unlikely on large systems.

Owing to its reactance, it is impossible for the rotor-field to snap up to full strength and correspondingly owing to a weaker magnetic coupling it is impossible for the rotor to change speed instantly, therefore, there can be no shock such as paralleling a machine out of phase.

Discussion

L. P. Shildneck: The proper procedure in case of accidental loss of excitation while under load has long been a moot question among operating companies. A critical study of this problem shows that no harm should result to the machine from re-closing the field switch. In fact, it reveals that the excitation should be re-applied as soon as possible to prevent damage to the rotor.

If viewed from the synchronization standpoint, this problem is considerably simplified. If the heating of the rotor is disregarded, any ordinary synchronous machine can be brought up to speed as an induction motor. At the instant before excitation, the machine is on the line at full voltage, with a stator current and power factor determined by the slip. After excitation, the rotor must speed up to synchronize with the stator flux wave. A turbine generator upon loss of excitation runs as an induction generator above synchronous speed at some slip

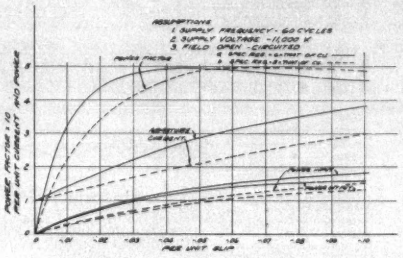

Fig. 1—Induction Generator Characteristics

determined by the governor setting. This condition is very nearly the same as the induction motor condition, since the machine is on the line at full voltage, with a stator current and power factor determined by the slip speed. Thus synchronization under this condition is actually easier than in the former case of an induction motor, for the rotor slows down to synchronous speed.

The induction generator characteristics of a solid turbine generator can be approximated by assuming that the rotor is a solid steel cylinder, with no slots or field copper. The rotor currents are mainly concentrated in a shell of about one-half inch depth on the outside periphery of the rotor. Curves of stator current, power factor, and power against slip are shown in Fig. 1, for various values of resistivity of rotor steel. These characteristics, together with the governor characteristics, shown in Fig. 2, then determine the operation of the set.

Referring to Fig. 3 of the paper, in the interval between 8 and 11 sec. after removal of excitation, the slip frequency, as observed from the low-pressure generator field current, is 1.83 cycles per sec., or the slip is − 0.03. According to the calculated induction generator characteristic curves, the stator current at 3 per cent slip is about twice normal, or nearly 5,000 amperes. This is seen to check the lower envelope of the L.P. generator armature current curve Fig. 3. The lower envelope constitutes a check point for the following reasons. There is one point in the cycle of operation of the rotor which justifies the assumption

that the rotor was a solid cylinder with no slots or field winding. Consider now the actual rotor with a stator flux field revolving slowly around it. At one instant, the field coils are inductively related to the stator flux, and currents are induced in them. Ninety degrees later, the field coils are non-inductively related to the stator flux, and no current is induced in them. At the first instant, then, there are two parallel paths for rotor current to flow, one in the steel, and one in the copper. The impedance is then low. At the later instant, the field coils are "out of the circuit"; there is but one path, and that is in the steel. The impedance is then high. At this instant, current is flowing down one pole face and back along the adjacent one. Since the impedance is high, the stator current will be low. Thus the lower envelope of the armature current constitutes a check point for the calculations.

With the set carrying 50,000 kw. before loss of excitation, it is seen that the governor begins to throttle the steam as the speed rises above the system speed. Thus at 2 per cent overspeed the governor is admitting sufficient steam to carry approximately 30,000 kw. But at 2 per cent slip, the output of the generators is about half load, or 30,000 kw. Thus the set would finally settle down and carry a continuous load of 30,000 kw. However, the power factor is but 35 per cent, consequently the armature current is high, about 150 per cent of normal. While this condition for the stator is harmful, it will be seen that the rotor is in a worse plight.

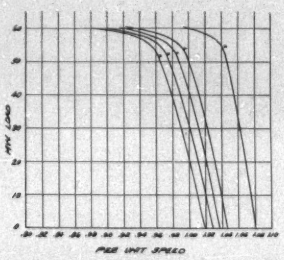

Fig. 2—Governor Characteristics for Various Governor Settings

An inspection of the test curves for armature current of the low-pressure unit reveals an interesting comparison between the values of currents flowing in the steel and in the copper portions of the rotor. These curves are but the effective current curves, or 0.707 times the envelope of the 60-cycle stator current. Notice that this effective stator current reaches a peak value of 6,500 amperes whenever the field coils are inductively related to the stator flux waves, and has a value of 5,000 amperes, whenever the field coils are non-inductively related (at 3 per cent slip). Thus the m.m.f. contributed by body or steel currents is proportional to 5,000, while the m.m.f. contributed by the copper currents is proportional to 6,500–5,000 or 1,500. The steel current m.m.f. is thus 3.33 times the m.m.f. set up by currents in the field copper (at 3 per cent slip). Herein lies the real danger in operating as an induction generator or motor for any length of time. The extremely high body currents concentrated in the outer shell of the steel rotor cause an intense heating of the rotor surface. Considerable damage may be done due to burning of the rotor wedges, especially at the ends of the rotor body. The extent of the danger is readily apparent from a consideration of the fact that at 2 per cent slip, the rotor losses are 2 per cent of the output of the machine. Normal full-load rotor loss is about ¼ per cent. Taking into account the fact that at 2 per cent slip, the load is but one-half normal, it is seen that the rotor loss is about 4 times normal when operating at 2 per cent slip. Furthermore, Fig. 2 shows that the slip is greatly increased when the machine loses excitation at full load. The sustained values will be: slip 4.5 per cent, load 85 per cent. Therefore, the rotor losses will be 0.85 x 4.5 per cent or 3.8 per cent as compared to a normal rotor loss of ¼ per cent. In other words, the rotor loss under this condition will be *15 times normal*. Moreover, this loss is not uniformly distributed over the periphery of the rotor surface; it is greatest at the ends of the machine where the current has to pass across the wedges. If heated sufficiently, these wedges may lose their physical strength to an extent which would allow them to be pushed out of the slots by the centrifugal force of the coils.

These are the dangers attending operation as an induction generator or as an induction motor. For these reasons, it is certainly inadvisable to operate any longer than necessary at other than synchronous speed. Excitation should be reestablished as soon as possible. If this cannot be done in very short order, the machine should be disconnected from the line immediately.

L. L. Perry: This discussion is particularly applicable to a triple compound unit comprised by one high-pressure generating unit and two low-pressure generating units. With this type of unit we have an installation wherein each low-pressure member can, when the high-pressure member is down, operate up to rated capacity by admitting high-pressure steam to the low-pressure turbines. In the event of accidental opening of the high-pressure generator field, the operator is confronted with two alternatives:

(1) He may at once open the main oil circuit breaker, cutting off the high-pressure generator from the bus and thereby lose that amount of generator capacity for many minutes. Meanwhile he can, in a few seconds, obtain the rated capacity of the two low-pressure generators by permitting high-pressure steam to enter the low-pressure turbines through the reducing valves provided therefor. (Indeed, this admission of high-pressure steam may be made automatic whenever the kilowatt load on the high-pressure turbine falls below a predetermined value.) In the first interim period the momentary loss to the system may be easily 90 to 100 per cent of the load carried by the total unit; 60 per cent may be restored in a few seconds, but it may require some minutes to restore the balance of 40 per cent of the load originally carried by the unit; or

(2) He may allow the high-pressure generator to operate on the system until he restores the field. In this case the momentary loss to the system may not exceed 50 to 60 per cent of the load carried by the total unit, and the restoration of the load previously carried may be made in much quicker time than the first interim period in the first alternative. Furthermore the loss of time necessary in the alternative to synchronize does not occur.

The facts presented in the paper permit the station electrical engineer to issue orders to the watch engineer that may easily prevent a serious disturbance to service.

L. E. Frost: There have been three occurrences at the Hudson Avenue Station which are of particular interest in connection with the paper by Messrs. Higgins and Wild. All of these involved the same machine, an 80,000-kw. cross-compound unit with generators of equal capacity on the two shafts.

The scheme of field current supply for this unit is shown in Fig. 3. The fields of the two generators are controlled through a duplex rheostat. The supply to both fields is derived jointly from a shaft exciter on the high-pressure unit, and from an excitation bus. Each of these sources of supply is controlled by an automatic air circuit breaker and reverse current relay.

In a period of one year, there were three occasions when this unit became unstable and pulled out of step while carrying loads ranging from 60,000 to 75,000 kw. Several plausible explana-

tions have been advanced as to what started the disturbance in each case, but the proof has not been conclusive and my description of the sequence of events represents an opinion which is held by many who are familiar with the case, but which is not unanimous.

Whether the original disturbance was at the steam or at the electrical end of the unit, it seems clear that the unit pulled out of step and speeded up until the governor brought the steam input down to meet the output as an induction generator, at an unsteady speed corresponding (in its average) to a 61-cycle

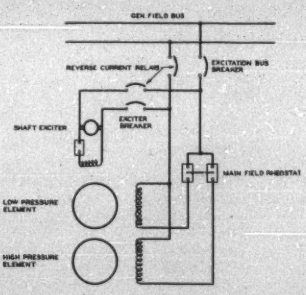

Fig. 3—Typical Generator Field Circuit Connections

synchronous frequency. In each case the circuit breaker between the excitation bus and this unit opened automatically, and it appears that this was due to transient reversals of field current back into the excitation battery as the poles slipped past synchronous positions. As the unit had been drawing about a fourth of its excitation from the bus, the tripping of this breaker aggravated the disturbance by materially weakening the field supply. In each case the generator remained connected to the high-tension bus and in a minute or so pulled back into step. During this minute, the output of the machine fluctuated in fairly regular swings from 20,000 kw. up to 65,000 kw. or more at a slip frequency of about one cycle per second. At the same time the system voltage underwent swings of the same frequency but fortunately it did not reach serious values.

At the first occurrence, which was in December, 1929, the controls of the machine were not touched by the operators until it had come back into synchronism of its own accord. At the second and third occurrences, which were in June and September of 1930, the operators increased the field current in order to force the unit back into synchronism. This step proved very effective. It was reported that as the field current was brought up the amplitude of load swings increased somewhat at first but that the unit quickly pulled into step and settled down to normal operation.

These cases are, of course, different from the situation presented by Messrs. Higgins and Wild, which was to apply field to a unit whose field current supply had been interrupted entirely; however, these experiences confirm in my mind the opinion that there is no inherent hazard in restoring normal field strength to an alternator which is out of step but still running somewhere near synchronous frequency.

D. D. Higgins: The thought the authors had in mind was not "what will happen if the field switch is opened," but rather "what will happen when it is closed." We know there is no reason for operating a generator unexcited and that there are very good reasons for not operating it so. At least there is one case in which the end wedges in the slots became overheated and were deformed by centrifugal force after some time. The rates of heating of various parts were carefully considered before the tests were made.

Inspection after the tests showed no visible defects had resulted during the tests lasting from one-half minute to three minutes.

The opening of a field switch is distinctly something the operating engineers do not want to happen, but which, nevertheless, does happen from time to time, possibly not oftener than once in five years.

These tests have given us information upon which we can confidently proceed to reduce immediately a case of major trouble to a simple operation and restore normal conditions in a minimum time.

Calculation of Synchronous Machine Constants—

Reactances and Time Constants Affecting Transient Characteristics

BY L. A. KILGORE*

Associate, A. I. E. E.

Synopsis.—*Recent advances in the theory of synchronous machines have introduced a large number of new constants. The method of symmetrical components requires sequence reactances, and an accurate theory of transients requires transient and subtransient reactances and time constants. Most of the published discussion on the constants has been concerned with the application, rather than the calculation of values.*

In this paper, all of the most significant constants are calculated, except the subtransient time constant. A general method of calculation is discussed in which the reactances are accurately resolved into components. Comparisons of test and calculated values are given. The formulas for salient-pole machines and turbine generators are given in Appendixes A and B.

The principle of superposition is applied to resolve accurately the reactances into components which can be readily calculated. The induced currents in the field and additional damping circuits are accounted for simply by applying the constant interlinkage theorem.

I. Introduction

UNTIL recently, the theory of synchronous machines has required relatively few constants. A single value of reactance, (usually called armature leakage) was used to calculate the initial short-circuit currents, and standard decrement curves used to determine the decay. Recent advances in theory have introduced a large number of new constants.

The method of symmetrical components[9] applied to single-phase, short-circuit calculations requires *negative-sequence reactance* (x_2) and *zero-sequence reactance* (x_o), as well as the positive-sequence reactances, which depend on the condition of application of current.

An accurate theory of transients[6] recognizes a subtransient (rapidly decaying component due to currents induced in additional damping circuits), as well as a transient component of symmetrical armature current. This requires several (positive-sequence) reactances, the *subtransient reactance* (x_d''), the *transient reactance* (x_d') and a *synchronous reactance* (x_d). The rate of decay of the transient components of current is determined by time constants. The time constant of the asymmetrical component is termed the *armature time constant* (T_a). The time constant of the rapidly decaying component of symmetrical current is the *subtransient time constant* (T_d''), and the time constant for the main transient component of current is the *transient time constant* (T_d').

As a reference, it is desirable to use the time constant of the field, which is called the *open-circuit transient time constant* (T_{do}').

A most complete solution[6,7] may require resolution of currents into a direct-axis component (reacting directly on the main field axis), and quadrature-axis component (reacting on the axis between poles). Where the constants are quite different in the two axes, this method requires *quadrature-axis constants:* (x_q, x_q'' *and in some cases* x_q', T_q' *and* T_q''), as well as the corresponding direct-axis constants (x_d, x_d'', x_d', T_d', and T_d''), described above.

These new methods consist essentially in resolving the actual currents and voltages into their essential components, and defining a sufficient number of constants to determine the initial and final values and the rates of decay. These constants have all been defined in other papers;*[1,6,9] however, since the general method of defining the reactances is vital to the method of calculation, it is well to state it here. The reactance of a synchronous machine is the ratio of fundamental reactive armature voltage produced by the given component of armature current applied to the value of that component of current at rated frequency, and with the rotor running at synchronous speed.

These theories have been developed on the basis of no saturation, or at least assuming fixed permeability, but in practical calculations the effects of saturation cannot be neglected. These effects are most readily dealt with in most cases by modifying the constants. The effects of saturation on each of the constants is discussed in the companion paper[1]. It is shown that *for most practical purposes, only one value of each constant is necessary for a machine.* Except for the synchronous reactances and open-circuit time constant, the final values calculated will be the "saturated values," corresponding to the currents obtained on sudden short-circuit from full voltage.

Most of the published discussion of these constants has been concerned with the application, rather than the calculation of the values. However, all designers

*Power Engineer, Westinghouse Elec. and Mfg. Co., East Pittsburgh, Pa.

9. For references see Bibliography.

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*Proposed definitions of these constants are included in a report of the Transmission and Distribution Committee of the A. I. E. E. This report, which was prepared by Prof. G. Dahl, will be presented at the Winter Convention, New York, N. Y., Jan. 25-29, 1932.

of large synchronous machines have had to meet this problem of calculating the constants, and those who use them are interested in how well they can be calculated.

A recent paper[5] treats armature leakage reactance quite thoroughly, but does not cover the other constants. Another[4] describes an accurate method of treating synchronous motor constants by constructing a complete equivalent circuit. In this paper, formulas are developed for calculating the following constants: x_d, x_q, x_d', x_d'', x_q'', x_2, x_o, T_{do}', T_d' and T_a. This includes all of the more important constants, except the subtransient time constant ($T_a''T_d''$), which can be estimated from the test values given in the companion paper.[1] The quadrature transient reactance (x_q') is not included, since it is quite commonly assumed equal to x_d', in short-circuit calculations.

II. General Method of Calculation

To calculate any of the reactances as defined, it is only necessary to determine the flux produced by the component of current applied in the given manner, and to calculate the resulting armature voltage. The resultant field of flux may be complicated, but by applying the principle of superposition,[6] the resultant fields can be resolved into components which can be more readily calculated.

Components of Reactance. It is convenient to divide all the positive-sequence reactances into two components, one of which is the armature leakage (x_l) defined in a definite manner. The synchronous reactance is then the sum of the armature leakage and the reactance of armature reaction: $x_d = x_l + x_{ad}$ for the direct-axis, and $x_q = x_l + x_{aq}$ for the quadrature-axis. The reactance of armature reaction (x_{ad}) is defined as the reactance due to the space fundamental of air gap flux produced by the armature current acting alone.

The unsaturated transient reactance (x_{du}') is calculated as the sum of the armature leakage and an "effective field leakage" (x_F'): (x_F') is not the true leakage of the field with respect to the armature, but is a definite component of the total leakage of the armature with respect to the field. The saturated value of the transient reactance (x_d') is calculated by multiplying the unsaturated value (x_{du}') by an empirical constant (F_{st}), determined from test in the companion paper[1] as 0.88.

The subtransient reactance is calculated as the sum of the armature leakage and the "effective" damper winding leakage: $x_d'' = x_l + x'_{Dd}$, and $x_q'' = x_l + x_{Dq}'$. x_{Dd}' and x_{Dq}' are components of the total leakage of the armature with respect to the damper winding determined by assuming no net change of interlinkage with any of the damping circuits.

Armature Leakage Reactance Defined. Several recent writers[8] have treated armature leakage as an arbitrary component of the synchronous reactance. The definition might be stated as follows: The armature leakage reactance is the reactance due to the difference between the total flux produced by the armature current acting alone, and the space fundamental of the "flux in the air gap." To make this definition as useful as possible, the flux in the air gap will be understood to include the fringing flux at the ends which enters the rotor, since this is very nearly the component of the end leakage field which is mutual with the rotor circuits. For turbine generators with magnetic retaining rings, the end zone flux entering the retaining ring surface should be included in the end leakage, and not in the air gap flux.

Defined in this way, the armature leakage can be shown to be very accurately a component of all the positive-sequence reactances. This may be seen from the fact that the fundamental flux in the air gap is determined almost entirely by the space fundamental of the m. m. f., and that the remaining component consists only in slot and end leakage, and harmonic fluxes in the gap which are not appreciably affected by the condition of application of current. The effects of saturation on the armature leakage are generally negligible, except for the increased end leakage due to magnetic retaining rings in turbine generators.

Form of the Results and Formulas. The most useful forms for expressing the reactance coefficients are per cent and per unit, on a machine kva. base. In this paper, per cent notation is used, since this is the form most generally adopted when specifying the values of reactances. The time constants are given in seconds.

The reactance formulas must, of course, involve either directly or indirectly the armature turns, the frequency, and certain dimensions of the machine. The actual form of the equation makes considerable difference in the usefulness. Some writers have used the conception of effective permeance per unit length of slot, and others the permeance per phase belt per unit of length. In extending these conceptions to the calculation of all the reactances, it was found desirable to change the conception to a permeance per pole per inch of length, since the flux across the gap is most conveniently visualized as a flux per pole.

All the component reactances may be calculated as a product of a reactance factor (X), and a specific permeance (λ) for the given component, multiplied by flux distribution coefficients when necessary.

The specific permeance (λ) is defined as the effective flux per pole per inch of core length produced by unit ampere turns per pole.

The reactance factor (X) used here is a group of terms common to all the reactance formulas, but it is chosen in such a way as to have a definite physical significance. It is the per cent reactance for unit specific permeance, or the per cent of normal voltage induced by a fundamental flux per pole per inch numerically equal to the fundamental armature ampere turns at rated current.

The reactance factor can be written in several ways.

In terms of the turns, frequency and rated phase voltage and current, this factor can be shown to be:

$$X = 100\%\left(\frac{I_{ph}}{E_{ph}}\right) f\, l\, m\, p \left[q\left(\frac{n_s}{C}\right) k_p\, k_d\right]^2 10^{-8} \qquad (1)$$

The factor can be expressed in terms of the ratio of effective ampere conductors per inch ($A\,k_p$) to the fundamental air gap density (B_1):

$$X = 100\%\left(\frac{A\,k_p\,k_d}{\sqrt{2}\,B_1}\right) = 100\%\left(\frac{k_d}{\sqrt{2}\,C_1}\right)\left(\frac{A\,k_p}{B_g}\right) \qquad (2)$$

This is very useful if the effective ampere conductors ($A\,k_p$) and air gap density (B_g) are thought of as fundamental quantities in design, the ampere conductors per inch being a measure of current loading, and the air gap density a measure of the magnetic loading of the machine.

The chief advantages of this method are that it expresses the reactances as products of two factors, a specific permeance which involves only the proportions of the machine, and a reactance factor which is a simple function of the fundamental quantities of design. This makes it possible to see at a glance the effect of changing any of the proportions or factors. Also, it simplifies the work of calculation.

Calculation of Time Constants. The open-circuit time constant of the field may be calculated as the self inductance of the field (in henrys) divided by the field resistance in ohms:

$$T_{do}' = \frac{L_f}{R_f}.$$

The short-circuit transient time constant is calculated as:

$$T_d' = \left(\frac{x_d'}{x_d}\right) T_{do}'.$$

It is shown[1] that using the saturated value of x_d' in this relation gives a good approximation to the effective saturated value of T_d'.

The armature time constant (T_a) is the time constant of the asymmetrical component of armature current which is due to the flux trapped by the armature winding at the instant of short circuit. The effective reactance to the average asymmetrical current is the same as for negative-sequence current (x_2), and the resistance is the d-c. armature resistance (r_a) in per cent. Hence,

$$T_a = \frac{x_2}{2\,\pi\, f\, r_a}.$$

III. Comparison of Test and Calculated Values

The formulas of Appendixes A and B have been checked by the numerous tests described in the companion paper.[1] Tables showing the tests and calculated values are given below. Most of the test values were obtained from special sudden short-circuit or locked tests, as indicated in the tables. The synchronous reactance (x_d) can be obtained from the commercial tests, hence a great many more checks on the synchronous constants have been obtained than those listed below. Also certain other special tests[1] were made to determine the effects of saturation, but will not be included here.

Table I gives the results of sudden short-circuit tests on salient-pole machines. Table II shows the results of locked tests obtained on other salient-pole machines with dampers, and Table III shows the results of similar tests on machines without dampers. Table IV shows the results obtained from sudden short-circuit tests on turbine generators.

TABLE I—CONSTANTS OF SALIENT-POLE MACHINES; COMPARISON OF TEST AND CALCULATED VALUES

Data from Three-Phase Short-Circuit Tests

All x's Are Per Cent Values All T's Are in Seconds

										% error	
Kva.		5,000	100	750	30,000	20,000	15,000	331	240		
Kind		Gen.	Gen.	No damper H. W. G.	Cond.	Cond.	Cond	Motor	Motor	Max.	Avg
Frequency		60	60	25	60	60	60	60	60		
Volts		7,600	2,300	6,600	5,000	11,000	11,500	440	2,200		
R. p. m.		900	1,200	187.5	720	720	900	1,200	277		
x_d	Test	90	125	120	150	160	165	160	105	6.8	3.6
	Calc.	90	130	115	160	170	170	160	110		
x_d'	Test	22	24	41	37	45	34	23	42	12.5	6.0
	Calc.	21.5	27	39	34	40	33	23	40		
x_d''	Test	13	16.5		19	27	21	16	32	11.0	8.0
	Calc	14	15.0	40	21	30	19.5	15	30		
x_2	Test	13	17.0	62	18	26	20	16	33	8.0	5.0
	Calc	13.5	16.0	57	19	28	19	16	33		
x_0	Test	6.3	3.30		7.20		11.0			33.0	18.0
	Calc	5.5	2.20	15.0	8.40	13.5	11.0	9.3	18.5		
T_a	Test	0.095		0.180	0.130	0.210	0.180	0.033	0.048	38	16.0
	Calc.	0.100		0.200	0.180	0.250	0.160	0.030	0.040		
T_d'	Test	0.90	0.22		2.80		1.85			11.0	7.3
	Calc	1.00	0.24	0.100	2.70	2.60	1.75		0.270		

TABLE II—CONSTANTS OF SALIENT-POLE MACHINES; COMPARISON OF TEST AND CALCULATED VALUES
Data from Locked Tests
All x's Are Per Cent Values
Machines With Dampers

Kva.		7,000	3,000	30,000	7,500	7,500	1,000	30,000	30,000	22,500	15,000	5,000	1,250	% error	
Kind		W.W.G.	W.W.G.	Cond.	Cond.	Cond.	Cond.	Motor	Cond.	Cond.	Cond.	Cond.	Cond.		
Frequency		60	50	60	60	60	60	50	60	60	60	60	60	Max.	Avg.
Volts		6,600	6,600	13,600	4,800	4,000	2,300	11,500	5,000	12,500	11,950	12,470	480		
R. p. m.		450	750	720	720	900	900	600	720	300	900	900	1,200		
x_d	test	97	160	170	175	145	160	78	170	120	155	170	190	6.3	2.0
	calc.	96	160	170	180	150	160	80	170	120	165	175	200		
x_d''	test	26	16.5	23	24	21	18	13	20	29	20	27	23	7.5	4.6
	calc.	25	17.0	24	23	20	18.5	13	21	27	18.5	25	22		
x_2	test	25	16	20	23	19	16	12	18	28	19	27	22	10.0	3.7
	calc.	23	16	22	22	19	17	12	18.5	27	18.5	25	22		
x_0	test	14.5	3.60	11.0	12.0	3.0		3.0	7.2	15.5	3.1	4.4		31.7	12.5
	calc.	13.5	3.45	11.5	13.0	2.45	6.55	3.95	8.2	18.0	2.8	3.9	8.0		

TABLE III—CONSTANTS OF SALIENT-POLE MACHINES; COMPARISON OF TEST AND CALCULATED VALUES
Data from Locked Tests
Machine Without Dampers

Kva		13,750	7,000	4,250	2,330	30,000	187	730	% error	
Kind		W.W.G	W.W.G	W.W.G	W.W.G	Gen.	Gen.	Motor		
Frequency		60	50	60	50	60	60	60	Max.	Avg.
Volts		6,600	6,600	2,300	6,600	11,500	240	2,300		
R. p. m.		90	94	600	125	600	200	1,200		
x_d	test	110	115	135	115	115	94	130	4.2	1.8
	calc.	110	120	135	115	120	94	135		
x_{du}''	test	43	44	37	45	33	41	30	6.0	2.5
	calc.	44	43	38	44	35	42	30		
x_2	test	58	61	62	65	58	55	55	10.3	4.5
	calc.	57	60	60	60	52	57	53		
x_0	test	25	24		24	9.3			10.4	5.7
	calc.	25.5	23.5		21.5	8.5				

TABLE IV—CONSTANTS OF TURBINE GENERATORS; COMPARISON OF TEST AND CALCULATED VALUES
Data from Three-Phase Short-Circuit Tests
All x's Are Per Cent Values All T's Are in Seconds

Kva		7,500	8,575	9,375	12,500	18,750	43,750	68,750	75,000	% error	
Frequency		60	60	60	60	60	60	60	60		
Volts		6,600	4,150	6,600	11,000	13,800	13,800	13,200	11,600	Max.	Avg.
R. p. m.		3,600	3,600	3,600	3,600	3,600	1,800	1,800	1,800		
x_d	test	120	96	110	100	115	130	97	110	2.0	0.6
	calc.	120	97	109	98	116	130	97.5	110		
x_d'	test	16.5	12	14	13.5	14	25	21	24	10.8	4.1
	calc.	16.0	12	13	13.0	15.5	26	21	23		
x_d''	test	9.2	8.4	7.5	8.8	8.5	17	12	14	14.4	7.4
	calc.	9.6	7.2	8.1	7.9	8.1	16	13	14.5		
x_0	test	1.80	1.60	1.60		3.1	5.1	5.9	10.0	18.7	7.1
	calc.	1.95	1.50	1.30		2.7	5.0	5.9	9.8		
T_a	test	0.043	0.11	0.085	0.11	0.13	0.28	0.25	0.26	53.3	17.4
	calc.	0.066	0.11	0.086	0.105	0.12	0.23	0.31	0.34		
T_d'	test	0.56	0.44	0.49	0.54	0.59	1.00	1.60	1.60	16.4	8.0
	calc.	0.48	0.415	0.41	0.55	0.57	1.10	1.70	1.70		

It will be seen that the agreement between test and calculated values is quite good, except that in a few cases where the value of the zero-sequence reactance is quite small the per cent error is fairly large. Also, the calculated value of armature time constant in a few cases is considerably higher than the test value. This may possibly be due to external resistance in the armature circuit or breaker contacts, for the armature resistance is so low that a very little external resistance will appreciably affect the time constant.

IV. Conclusions

A general method has been given for calculating the constants of synchronous machines affecting the tran-

sient characteristics. In Appendixes A and B formulas have been given for all the more important constants, except the subtransient time constant. Comparison of test and calculated values shows that they give sufficiently good results over a wide range of types and sizes.

The formulas are based on a physical conception of the actual currents and flux fields involved. An attempt has been made to obtain simple formulas for design calculations, and still retain all the most significant factors, so that the formulas will cover a wide range of types and sizes of machines. The formulas are definitely limited to the usual two-layer type of winding. Only laminated-pole salient-pole machines, and solid rotor turbine generators are considered. The methods used can readily be extended to cover other special cases.

The general method of resolving the reactances into components has advantages both in calculating and in visualizing the flux fields involved. The method of expressing the component reactances as a product of a reactance factor, and specific permeances and flux distribution coefficients for the several components, has been found very useful.

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Appendix A—Formulas for Salient-Pole Machines

I. ARMATURE LEAKAGE REACTANCE (x_l)

Slot Leakage. Complete formulas have been given[8] for calculating slot leakage, derived by assuming the flux to go straight across the slot. With good approximation, Mr. Alger showed that the "slot constant" (effective permeance per cm. of slot) could be calculated as

$$k_s \left(\frac{h_2}{b_s} + \frac{h_1}{3\,b_s} \right),$$

and k_x could be determined as

$$\frac{1}{4} \left(\frac{3\,y}{m\,q} + 1 \right)$$

for three-phase, and as $\left(\frac{y}{m\,q} \right)$ for two-phase.

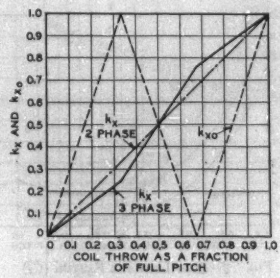

FIG. 1—REDUCTION OF SLOT REACTANCE DUE TO CHORDING

k_x = factor for positive- and negative-sequence reactance
k_{xo} = factor for zero-sequence reactance

These results expressed as a specific permeance give:

$$\lambda_s = C_s \frac{20}{m\,q} \left[\frac{h_2}{b_s} + \frac{h_1}{3\,b_s} \right],$$

where

$$C_s = \frac{k_x}{k_p^2\,k_d^2}.$$

See curves of Figs. 1 and 2.

Tooth Tip and Zig-Zag Leakage. In the paper referred to above, formulas were given for belt and zig-zag leakage, due to flux produced across the gap by the harmonics of armature m. m. f., but these did not include any increase in leakage due to flux going from tooth to tooth. In the following development, the principle of superposition is used to establish an accurate basis for calculating all of these components.

The total resultant field of flux, (within the core length) produced by the armature current may be accurately resolved into components produced by two

imaginary sets of currents which, when superposed, give only the real current. The first component is due to a set of currents flowing on the armature surface, each uniformly distributed over a slot pitch and equal to the current in the slot; the second component is that due to a set of currents flowing in the conductors and returning uniformly distributed over the slot pitch. The m. m. f. due to the first set of imaginary currents may be analyzed as a fundamental and harmonics. The fundamental of this m. m. f. produces the fundamental flux; and the harmonics produce the belt leakage, which is due to the concentration in a finite number of phases. The second set of currents produces the slot, tooth tip and zig-zag leakage fields. The $\frac{b_t}{\tau_s}$ portion of the current of one slot, which returns over a half tooth width on each side, produces a field as shown in Fig. 3.

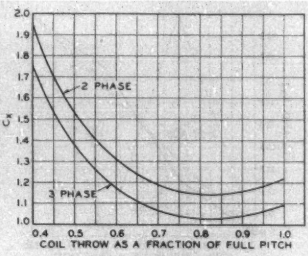

FIG. 2—SLOT LEAKAGE FACTOR (C_x)

A number of such plots shows that the effective permeance per cm. of slot (not included in the slot leakage) may be approximated as

$$\left[\frac{b_t^2}{8\,\tau_s\,g} + 0.35\,\frac{b_t}{\tau_s}\right].$$

Using an average value of $\frac{b_t}{\tau_s} = 0.56$, it reduces to

$$\left(0.2 + 0.07\,\frac{b_t}{g}\right).$$

This includes the tooth tip and the so-called zig-zag leakage.

It is convenient to calculate a specific permeance λ_i for the slot tooth tip and zig-zag leakage combined:

$$\lambda_i = C_x\,\frac{20}{m\,q}\left[\frac{h_2}{b_s} + \frac{h_1}{3\,b_s} + 0.2 + .07\,\frac{b_t}{g}\right] \qquad \textbf{(1a)}$$

Belt Leakage. As defined, the belt leakage is due to the harmonics of armature m. m. f., determined by uniform current distribution. For a full pitch winding, where the slots per pole per phase (q) are integral and large, the reactance calculated by summing up the harmonics[8] can be shown to be 0.002 x_{ad} for three-phase, and 0.015 x_{ad} for two-phase. (This was derived assuming a uniform gap and no damping.) For a smaller number of slots, these formulas are not very accurate, but they are sufficiently good since the value is small. The case of fractional slots per pole is difficult if a complete solution is desired. However, the unbalance in m. m. f. resulting from fractional slots is kept to a minimum, and the additional reactance may be neglected in normal machines.

Expressing these results as a specific permeance, and assuming that the two-phase leakage varies with

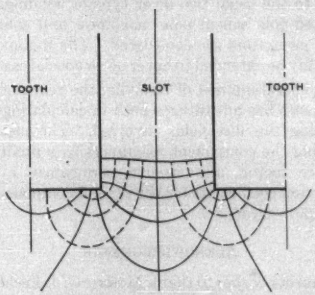

FIG. 3A—TOOTH TIP LEAKAGE FIELD DUE TO DIFFERENTIAL CURRENT—LARGE AIR GAP

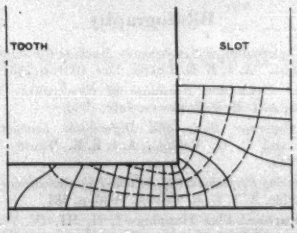

FIG. 3B—TOOTH TIP LEAKAGE FIELD DUE TO DIFFERENTIAL CURRENT—SMALL AIR GAP

chording in the same way as that part which is due to the third harmonic:

$\lambda_B = 0$ (negligible) for three-phase;

$$\lambda_B = 0.095\,\frac{d}{p\,g'}\left[\frac{\sin\frac{3\,y\,\pi}{m\,q\,2}}{k_p}\right]^2 \text{for two-phase} \qquad \textbf{(2a)}$$

End Winding Leakage. The end leakage formula presented here is essentially the same as that developed by Mr. B. G. Lamme. Professor Alfred Still has recently published a paper[2] in which he develops a formula on the same basis.

The method referred to consists in treating the end winding leakage as a revolving field in air. The pres-

ence of the iron is neglected in the derivation and, if necessary, an empirical correction is applied to the result. Both Messrs. Lamme and Still assumed that the flux went from pole to pole without spreading out axially. Further, in order to integrate the flux, each assumed that the flux paths were arcs of circles.

This method of analysis can be carried one step further by using the exact solution[8] for a sinusoidal m. m. f. distribution in air, assuming a two-dimensional field. The exact solution gives the very simple result that the field is sinusoidally distributed, and the flux per pole per inch is 3.19 times the fundamental ampere turns.

The pitch of the winding for the straight coil extension (l_{e2}) is the same as for the gap, but it drops off to zero at the end of the coil. An effective length 1/2 (l_{e1}) will be used as in Lamme's formula.

The end winding field is not truly a two-dimensional field as assumed, but spreads out axially. This effect was investigated further by making approximate plots of the three-dimensional field in the several views. The increased leakage due to local flux closing about the individual conductors was calculated by resolving the field into components in a manner similar to that described for the tooth tip leakage. It was found that these factors, although small, are not negligible, and may be approximately accounted for by a constant factor 1.25.

Expressed as a specific permeance, the end winding leakage is (1.25) . (3.19) times the ratio of effective end winding extension on both ends, to the core length

$$\lambda_e = 4/l\,(2\,l_{e2} + l_{e1}) \qquad \textbf{(3a)}$$

The armature leakage reactance is calculated as the product of the reactance factor (X) determined by equation (**1**) or (**2**), and the specific permeances, equations (**1a**), (**2a**), and (**3a**).

$$x_l = X(\lambda_l + \lambda_e + \lambda_B) \qquad \textbf{(4a)}$$

II. Reactances of Armature Reaction (x_{ad}, x_{aq}) and Synchronous Reactances (x_d, x_q)

The specific permeance of the air gap (λ_a) is most conveniently determined on the assumption that the gap is uniform, and the actual flux distribution may be calculated by the use of flux distribution coefficients given below. The effective gap (g') is determined by multiplying the actual gap (g) by Carter's coefficients for stator slots and vents, and by a reduction factor for end fringing flux.

$$\lambda_a = 3.19\left(\frac{2\,d}{p\,g'}\right) \qquad \textbf{(5a)}$$

The flux distribution coefficients used here are defined as follows: *C_1 is the ratio of the fundamental to the actual maximum value of the field form.* (The field form is the wave of flux density due to the field only.) *The "pole constant" (C_p) is the ratio of average to maximum of the field form. The ratio of equivalent field to armature ampere turns (C_m) is the ratio of fundamental air gap flux produced by the fundamental of armature m. m. f. to that produced by the field for the same maximum m. m. f. C_{d1} is the ratio of the fundamental air gap flux produced by the direct-axis armature current to that which would be produced with a uniform gap equal to the effective gap over the pole center. C_{q1} is the corresponding coefficient for the quadrature-axis.*

All of these coefficients may be determined by accurate flux maps, but it is often more convenient to use formulas, except for C_p and C_1. These two constants are determined directly or indirectly in almost any design procedure, by laying out the field form either from a flux map or other accurate method.

A formula for C_m may be derived assuming the flux to go straight across a uniform gap over the pole face. The formula is much more accurate than might appear

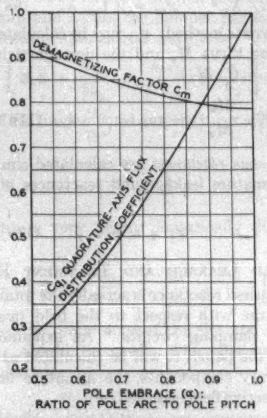

Fig. 4—Flux Distribution Coefficients for Salient-Pole Machines C_m and C_{q1}

from the assumptions stated, since the fringing and increase in gap at the pole tip affect both quantities in the ratio about equally. This formula was checked by accurate flux plots on a wide range of pole shapes from 4 to 88 poles.

$$C_m = \left(\frac{\alpha\,\pi + 4\sin\alpha\,\pi}{4\sin\alpha\,\dfrac{\pi}{2}}\right) \qquad \textbf{(6a)}$$

Where α is the pole embrace. See curve, Fig. 4.

From the definition of C_m, it may be seen that $C_m = C_{d1}/C_1$. Having determined C_m and C_1 as described above,

$$C_{d1} = C_m\,C_1 \qquad \textbf{(7a)}$$

An approximate formula for C_{q1} may be derived on the

assumption that the flux goes straight across a uniform gap over the pole face.

$$C_{q1} = \left[\alpha - \frac{\sin \alpha \pi}{\pi} \right];$$

making some allowance for end fringing and an average increase in gap at the pole tips, a modified formula was obtained which checked sufficiently well with the series of flux plots mentioned.

$$C_{q1} = \left[\frac{4\ \alpha + 1}{5} - \frac{\sin \alpha \pi}{\pi} \right] \qquad \textbf{(8a)}$$

The flux distribution coefficients which have been designated here as C_1, C_{d1} and C_{q1} can be calculated for a particular pole shape from the curves given in a paper[10] by R. W. Wieseman.

Reactances of armature reaction may be calculated as:

$$x_{ad} = X\, C_{d1}\, \lambda_a \qquad x_{aq} = X\, C_{q1}\, \lambda_a \qquad \textbf{(9a)} \text{ and } \textbf{(10a)}$$

As an alternate method: x_{ad} may be calculated from the gap ampere turns M_g and the demagnetizing ampere turns ($M_m = C_m M_A$).

$$x_{ad} = 100\%\ \frac{M_m}{M_g} \qquad x_{aq} = \frac{C_{q1}}{C_{d1}}\, x_{ad} \quad \textbf{(11a)} \text{ and } \textbf{(12a)}$$

Synchronous reactances are calculated simply as the sum of armature leakage and reactances of armature reaction:

$$x_d = x_l + x_{ad} \qquad x_q = x_l + x_{aq} \qquad \textbf{(13a)} \text{ and } \textbf{(14a)}$$

III. Field Leakage and Transient Reactance

The transient reactance is actually the total leakage of the armature with respect to the field (assuming no additional damping circuits). As explained in Section II of the paper, it will be calculated as the sum of two arbitrary components, the armature leakage (x_l) and an effective field leakage (x_F').

The specific permeance for field leakage (λ_F) will be the side and end leakage flux per pole per inch for unit field ampere turns. The side leakage flux per inch may be determined accurately by a flux map for the field only, similar to those shown in Fig. 1 and Fig. 50 in a series of articles[5] by J. F. Calvert. Each element of flux is multiplied by the fractional part of the turns linked.

It is often more convenient to use an approximate formula. The following formula was derived assuming that the flux goes straight across the interpolar space (at right angles to the center line between poles), except near the armature surface where the flux is assumed to go straight across half way from the pole head to the center line between poles, and then to spread out so that the upper boundary intersects the armature surface and the center line between poles at 45 deg., as it should theoretically. A further correction was made to allow for the shortening of the path of part of the flux which terminates on the under side of the pole tips.

The *side leakage* expressed as a specific permeance based on the above assumptions is:

$$\lambda_{F_s} = (3.19)\ \frac{4}{3} \left[\frac{3\ (h_h + g - .055\ \tau_r)}{(\tau_r - b_h)} + \frac{h_{f1} + 3\ h_{f2} + 0.1\ \tau_r \left(1 - \frac{10}{p}\right)}{\frac{\pi}{p}\ (d_r - 2\ h_h - 0.4\ h_{f1}) - b_p} \right] \qquad \textbf{(15a)}$$

This may appear rather long, but it will be noted that it is expressed directly in the main dimensions of the pole, and does not require a pole layout. This formula was found to check well with the results obtained by flux maps on a wide range of pole shapes for machines from 4 to 88 poles.

To determine the end leakage field, a number of two-dimensional flux plots was made in two views, which show the flux which fringes from pole to pole and the flux which closes about the field coil ends. The resultant field is obviously not two dimensional, but an approximate formula based on these plots should be sufficiently accurate.

$$\lambda_{F_e} = 3.19 \left[\frac{4(l_h - l) + 2\ h_{f1} + 0.5\ b_p}{l} \right] \qquad \textbf{(16a)}$$

The specific permeance for the effective pole leakage is:

$$\lambda_F = \lambda_{F_s} + \lambda_{F_e} \qquad \textbf{(17a)}$$

The effective field leakage reactance (x_F'), which added to armature leakage (x_l) gives the transient reactance, may be calculated as follows: With unit fundamental armature ampere turns suddenly applied on the direct-axis, an initial field current (I_f) will be induced of such value that the net interlinkage with the field produced by both the armature and field currents is zero. The interlinkage (per inch) produced by the armature current is

$$\left(\frac{\pi}{4}\ C_1\ \lambda_a \right) N_p,$$

and by the field current

$$N_p\, I_f \left(\frac{\pi}{2}\ C_p\ \lambda_a + \lambda_F \right) N_p;$$

hence

$$N_p\, I_f = \left[\frac{\frac{\pi}{4}\ C_1\ \lambda_a}{\frac{\pi}{2}\ C_p\ \lambda_a + \lambda_F} \right].$$

The net fundamental air gap flux (per inch) produced by both armature and field is then

$$\left[C_{d1}\,\lambda_a - \frac{\frac{\pi}{4}\,C_1^2\,\lambda_a^2}{\frac{\pi}{2}\,C_p\,\lambda_a + \lambda_F} \right].$$

The effective field leakage is:

$$x_F' = X\left[C_{d1}\,\lambda_a - \frac{\frac{\pi}{4}\,C_1^2\,\lambda_a^2}{\frac{\pi}{2}\,C_p\,\lambda_a + \lambda_F} \right]$$

$$= x_{ad}\left[1 - \frac{\frac{\pi}{4}\,\frac{C_1}{C_m}}{\frac{\pi}{2}\,C_p + \frac{\lambda_F}{\lambda_a}} \right] \qquad \textbf{(18a)}$$

The unsaturated value of transient reactance is:

$$x_{du}' = x_l + x_F' \qquad \textbf{(19a)}$$

The saturated transient reactance (x_d') is determined by multiplying by an empirical factor (F_{st}). The test results described in the companion paper[1] indicate that this factor may be taken as a constant $F_{st} = 0.88$, with sufficiently good results on normal machines.

$$x_d' = F_{st}\,.\,x_{du}' \qquad \textbf{(20a)}$$

IV. Damper Winding Leakage (x_{Dd}', x_{Dq}') and Subtransient Reactances (x_d'', x_q'')

The initial flux set up by suddenly applied direct-axis armature current is limited by the damper winding. The flux maps of this field were constructed on the assumption that there is no net change of interlinkage with any of the damper winding circuits. The field is quite complicated, but it was found that the fundamental flux could be calculated with fair approximation from the permeances of the main parts of the flux paths.

Approximately

$$\left[\cos\frac{(n_b - 1)\,\tau_b}{\tau_r}\,\frac{\pi}{2} \right]$$

part of the armature m. m. f. acts across the field, leakage permeance (λ_F) in series with the permeance of the end damper bar (λ_b) combined in parallel with the permeance of the gap over the pole tip (λ_{pt}).

$$\lambda_b = 6.38\left(0.5 + \frac{h_{b2}}{b_{b2}} + \frac{h_{b1}}{3\,b_{b1}}\right) \qquad \textbf{(21a)}$$

Where 0.5 is an approximation to the tooth tip permeance per cm. of damper slot. For round bars, use 0.62 instead of $\frac{h_{b1}}{3\,b_{b1}}$.

Using an average increase in gap at the pole tip:

$$\lambda_{pt} = 6.38\left[\frac{b_b - \tau_b\,(n_b - 1)}{3\,g'} \right] \qquad \textbf{(22a)}$$

Although the flux is not sinusoidally distributed under these conditions, the distribution is such that the total flux is a good approximation to the fundamental. Combining these permeances and multiplying by the fractional part of the ampere turns acting, the resultant formula for the specific permeance is:

$$\lambda_{Dd} = \left[\cos\frac{(n_b - 1)\,\tau_b}{\tau_r}\,\frac{\pi}{2} \right] \cdot \left[\frac{(\lambda_b + \lambda_{pt})\,.\,\lambda_F}{\lambda_b + \lambda_{pt} + \lambda_F} \right] \qquad \textbf{(23a)}$$

$$x'_{Dd} = X\,\lambda_{Dd} \qquad \textbf{(24a)}$$

The direct-axis subtransient reactance (x_d'') of a machine with a damper winding is calculated as the sum of the components (**4a**) and (**24a**):

$$x_d'' = x_l + x_{Dd}' \qquad \textbf{(25a)}$$

For a machine without dampers:

$$x_d'' = x_d' \qquad \textbf{(26a)}$$

The quadrature-axis damper leakage is determined on the same base. In this case, the flux goes tangentially down the gap and across the damper bar slots. The effective permeance per cm. for one slot is

$$\left[0.5 + \frac{h_{b2}}{b_{b2}} + \frac{h_{b1}}{3\,b_{b1}} + \frac{g}{\tau_b} \right].$$

The part of the fundamental ampere turns acting across one slot is

$$\left(\pi\,\frac{\tau_b}{\tau_r} \right),$$

and the flux is very nearly sinusoidally distributed; hence,

$$\lambda_{Dq} = \frac{20\,\tau_b}{\tau_r}\left[0.5 + \frac{h_{b1}}{3\,b_{b1}} + \frac{h_{b2}}{b_{b2}} + \frac{g}{\tau_b} \right] \qquad \textbf{(27a)}$$

In this case, the dimensions of the slot and bar in the pole center should be used. The effective damper winding leakage is:

$$x_{Dq}' = X\,\lambda_{Dq} \qquad \textbf{(28a)}$$

The quadrature-axis subtransient reactance (x_q'') for machines with dampers is calculated as the sum of the components (**4a**) and (**28a**):

$$x_q'' = x_l + x_{Dq}' \qquad \textbf{(29a)}$$

For machines without dampers (**12a**) or (**14a**):

$$x_q'' = x_q \qquad \textbf{(30a)}$$

V. Negative Sequence (x_2)

As explained in Section II, the negative sequence reactance may be calculated as the average of the subtransient reactances.

$$x_2 = \frac{1}{2}(x_d'' + x_q'') \qquad \textbf{(31a)}$$

VI. Zero-Sequence Reactance (x_0)

Since the zero-sequence currents are all in phase, the fundamental m. m. fs. of the three phases cancel out, and there is no appreciable fundamental flux across the

gap. Hence, it consists in a slot and belt leakage, with a small belt leakage in the end windings. Mr. Alger[8] has derived a factor for the effect of chording on zero-sequence slot reactance. This factor will be designated here as k_{zo}. See curves, Fig. 1. The specific permeance for slot and tooth tip leakage can be shown to be:

$$\lambda_{io} = \left[\lambda_i \left(\frac{k_{zo}}{k_z} \right) + \left(\frac{20}{m\, q\, k_p^2\, k_d^2} \right) \left(\frac{h_1 + 2\, h_3}{12\, b_s} \right) \right] \tag{32a}$$

For machines with a damper winding, the belt leakage flux over the pole face is restricted in the same manner as the quadrature subtransient reactance flux. Neglecting the reluctance of the flux path across the gap, the specific permeance for a full pitch winding would be λ_{Dq}, equation (**27a**). Since this leakage varies with the throw in very nearly the same manner as the slot leakage, it may be written:

$$\lambda_{Bo} = \left(\frac{k_{zo}}{k_z} \right) \lambda_{Dq} \tag{33a}$$

For machines with no dampers, the effective permeance per cm. of phase belt would be $\frac{\tau}{36\, g'}$, if the winding was full pitch and the gap was uniform. Expressed as a specific permeance, this would be

$$\frac{20}{m\, k_d^2} \left(\frac{\tau}{36\, g'} \right) \cong 0.1\, \lambda_a,$$

but it should be reduced by about 0.7 for an average pole embrace. To take into account chording, it may be multiplied by $\left(\frac{k_{zo}}{k_p^2} \right)$. Hence, for machines with no damper winding:

$$\lambda_{Bo} = \frac{k_{zo}}{k_p^2} (.07\, \lambda_a) \tag{34a}$$

In the end windings, the fundamental m. m. fs. cancel, but the local flux about the phase belt is equivalent to about 0.2 of the positive sequence end winding leakage. The zero-sequence reactance will be calculated as the sum of the slot, belt, and end leakage components:

$$x_o = X(\lambda_{io} + \lambda_{Bo} + 0.2\, \lambda_e) \tag{35a}$$

VII. Time Constants (T_{do}'), (T_d'), (T_a)

The self inductance of the field in henrys is

$$L_f = N_p^2\, p\, l\, 10^{-8} \left(\frac{\pi}{2}\, C_p\, \lambda_a + \lambda_F \right).$$

As explained in Section II, the open-circuit time constant may be calculated as a ratio of L_f to the resistance of the field (R_F); the resistance (R_f) should be the value in ohms at 75 deg. cent.

$$T_{do}' = \frac{L_f}{R_f} \tag{36a}$$

The unsaturated value of the (short-circuit) transient time constant can be shown to be exactly

$$\left(\frac{x_{du}'}{x_d} \right) T_{do}'.$$

Tests[1] show that a good approximation to the saturated value may be had by assuming the transient time constant to be reduced by the same factor as the transient reactance; hence:

$$T_d' = \frac{x_d'}{x_d}\, T_{do}' \tag{37a}$$

The armature time constant may be calculated as explained in Section II:

$$T_a = \left(\frac{x_2}{2\, \pi\, f\, r_a} \right) \tag{38a}$$

Where r_a is the d-c. armature resistance in per cent at 75 deg. cent.

Appendix B—Formulas for Turbine Generators

Due to the differences in construction, many of the formulas for turbine generators are different from those for salient-pole machines, but most of the conceptions involved are the same. Hence, only the differences in the formulas will be discussed.

I. Armature Leakage (x_l)

The reactance of armature reaction of turbine generators is calculated by assuming the flux to go straight across the gap. Therefore, it is necessary to include the fundamental flux going tangentially in the gap as a part of the armature leakage. This component is most conveniently accounted for by adding $\frac{g}{2\, \tau_s}$ to the permeance per cm. of slot. The specific permeance for the embedded portion is then:

$$\lambda_i = C_x \frac{20}{m\, q} \left[\frac{h_2}{b_s} + \frac{h_1}{3\, b_s} + 0.2 + \frac{g}{2\, \tau_s} \right] \tag{39a}$$

The end winding leakage for machines with non-magnetic field coil retaining rings is very similar to that for salient-pole machines, but an additional empirical factor of 1.2 is used for the increased axial divergence. The formula may be further simplified by using average proportions:

$$\lambda_e' = \frac{13\, d}{p\, l} \tag{40a}$$

For machines with magnetic retaining rings, the leakage under steady state conditions is increased by the presence of the ring. Under normal conditions, the greatest effect of saturation is in the tapered section next to the rotor body, and a rough approximation to the resultant field may be had by assuming that the effect is equivalent to cutting out this tapered section.

Numerous two dimensional flux plots constructed on this basis indicate that the increased leakage may be accounted for by adding $\left(\frac{13\,d}{p\,l}\right)$ to the specific permeance. Hence, for machines with magnetic retaining rings, it is necessary to use a different specific permeance for the end winding leakage for steady state conditions:

$$\lambda_e = \lambda_e' + \frac{13\,d}{p\,l} \qquad \textbf{(41a)}$$

Under initial sudden short-circuit conditions, this additional leakage path is completely saturated, and the increased leakage will be neglected.

The total leakage reactance for transient and subtransient conditions is:

$$x_l' = X\,[\lambda_i + \lambda_B + \lambda_e'] \qquad \textbf{(42a)}$$

For machines with non-magnetic retaining rings, the leakage reactance for steady state conditions is the same as (x_l') given above, but for a machine with magnetic retaining rings:

$$x_l = X\,[\lambda_i + \lambda_B + \lambda_e] \qquad \textbf{(43a)}$$

II. Reactances of Armature Reaction (x_{ad} and x_{aq}), and Synchronous Reactances (x_d and x_q)

The flux distribution coefficients may be defined in the same way as for salient-pole machines, but a factor (C_f) is required in place of the average of the field form (C_p). *C_f is the ratio of the interlinkage with the field to that which would be produced with a uniform gap and a concentrated field winding.*

The constants will be calculated assuming the flux to go straight across the air gap. On this basis:

$$C_1 = \frac{8}{\pi^2 K_{rs}}\left(\frac{\sin\gamma\frac{\pi}{2}}{\gamma}\right) + \frac{4}{\pi}\left(\frac{K_{rs}-1}{K_{rs}}\right)\cos\gamma\frac{\pi}{2} \qquad \textbf{(44a)}$$

See curves, Fig. 5.

The exact equation for C_m is long, but if the increase in gap due to slotting is neglected, a very good approximation is obtained.

$$C_m = \frac{\pi^2}{8}\left(\frac{\gamma}{\sin\gamma\frac{\pi}{2}}\right) \qquad \textbf{(45a)}$$

See curve, Fig. 5. The coefficient C_{d1} is exactly

$$C_{d1} = \left[1 - \frac{K_{rs}-1}{K_{rs}}\left\{\gamma - \frac{\sin\gamma\pi}{\pi}\right\}\right],$$

but it may also be calculated as:

$$C_{d1} = C_m\,C_1 \qquad \textbf{(46a)}$$

The quadrature-axis coefficient

$$C_{q1} = \left[1 - \frac{K_{rs}-1}{K_{rs}}\left\{\gamma + \frac{\sin\gamma\pi}{\pi}\right\}\right] \qquad \textbf{(47a)}$$

The factor C_f may be calculated as:

$$C_F = \left[(1-\gamma) + \frac{\gamma}{3\,K_{rs}}\right] \qquad \textbf{(48a)}$$

The reactances of armature reaction are calculated by the same formulas as for salient-pole machines: (**9a**) and (**10a**) or (**11a** and **12a**); using the coefficients calculated by (**44a**), (**45a**), (**46a**), and (**47a**) given above. The synchronous reactances are given by equations (**13a**) and (**14a**)

III. Field Leakage and Transient Reactance (x_d') Field Leakage Specific Permeance (λ_F)

The rotor slot leakage permeance per cm. is similar to the stator, but expressed as a specific permeance for the field it is:

$$\lambda_{F_s} = \frac{4\,(3.19)\,P}{Q_r}\left[\frac{h_{r2}}{b_r} + \frac{h_{r1}}{3\,b_r} + 0.2 + \frac{g}{2\,\tau_{rs}}\right] \qquad \textbf{(49a)}$$

The end leakage was calculated by determining the

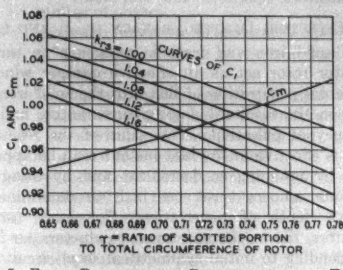

Fig. 5—Flux Distribution Coefficients for Turbine Generators

flux per pole per inch closing about the coil ends. The retaining ring saturation was accounted for by assuming the effect equivalent to cutting out the tapered section of the ring, as discussed for (λ_e). An approximation based on these assumptions and using average proportions was found to be:

$$\lambda_{F_e} = \frac{35\,d}{p^2\,l} \qquad \textbf{(50a)}$$

The specific permeance for field leakage is:

$$\lambda_F = (\lambda_{F_s} + \lambda_{F_e}) \qquad \textbf{(51a)}$$

The unsaturated transient reactance (x_{du}') can be derived in the same manner as for salient-pole machines, except that C_f should be used instead of C_p. However, for any normal distribution of field turns, a simpler solution may be obtained by neglecting the harmonics of the turns distribution of the field. Such harmonics produce entirely negligible mutual interlinkage, and might be treated as additional field leakage, but for

normal machines this is also negligible. All the flux across the gap may then be assumed mutual flux, and if the mutual reactance were very high, the fundamental ampere turns of the field would equal that of the armature. The actual field ampere turns would be C_m times the fundamental of the armature, and the fundamental flux across the gap required to produce zero net interlinkage is $\frac{4}{\pi} C_m$ times the actual field leakage. Hence, the true field leakage with respect to the armature, expressed as an armature reactance, is:

$$x_F = X \frac{4}{\pi} C_m^2 \lambda_F \qquad \textbf{(52a)}$$

The effective field leakage is the true field leakage (x_F) added in parallel with the mutual reactance, which on the above assumptions is x_{ad}.

$$x_F' = x_F \left(\frac{x_{ad}}{x_{ad} + x_F} \right) \qquad \textbf{(53a)}$$

The transient reactance is determined as for salient-pole machines, equations (**19a**) and (**20a**).

IV. Effective Damper Leakage (x_{Dd}') and (x_{Dq}')

The solid iron and metallic wedges of a turbine generator rotor act as an effective damper winding. The actual penetration of flux into the solid iron depends on the resultant ampere turns acting and the frequency of pulsation. The value of the reactance which is effective in the first instant may be approximated by assuming no decrement and by using the maximum ampere turns per inch to determine an effective depth of penetration. For most turbine generators, the maximum ampere conductors per inch, corresponding to initial symmetrical short-circuit current, is around 16,000.

The direct-axis damper leakage is determined by the depth of flux penetration in the rotor teeth and wedges under sudden short-circuit conditions. The ampere conductors per inch over the center of this section pulsate between zero and twice the value, due to the symmetrical current. The effective permeability at the surface, for these maximum ampere turns, is about 2. Assuming this constant throughout, the formula given by Steinmetz for flux penetration in deep solid parts can be applied. For 60 cycles, the flux penetrates to about 0.6 inch, which is equivalent to a path in air of depth $\delta_d = 1.2$ inches at 60 cycles; (for 25 cycles, rated frequency $\delta_d = 1.8$). The direct-axis leakage may be still further restricted by the slot. The flux across the slot will be assumed to penetrate to the bottom of the wedge. Half of the leakage flux going tangentially in the gap was included in the armature leakage. Expressing these relations as a specific permeance:

$$\lambda_{Dd} = \frac{3.19\ p}{d} [g + \delta_d + h_{r2}] \qquad \textbf{(54a)}$$

The quadrature-axis effective damper leakage depends mainly on the flux penetration in the solid pole center. The magnitude and frequency of the pulsation of ampere turns per inch acting over the pole center vary with conditions of short circuit. For a line-to-line fault, a double frequency pulsation of ampere turns is produced by the symmetrical current and a rated frequency pulsation due to asymmetrical current. The resulting maximum ampere conductors per inch are a little more than half the value used above, but at double frequency the penetration is essentially the same ($\delta_q = 1.2$) for 60 cycles (and $\delta_q = 1.8$ for 25 cycles). For a short circuit from load, where quadrature-axis flux is involved, the resultant ampere conductors per inch are about the same as given above for the direct-axis, and the effective penetration is again δ_q. Expressing these relations as a specific permeance:

$$\lambda_{Dq} = \frac{3.19}{d} [g + 2\,\delta_q] \qquad \textbf{(55a)}$$

The effective damper leakage reactances (x_{Dd}') *and* (x_{Dq}') are calculated as for salient-pole machines, equations (**24a**) and (**28a**).

V. Subtransient and Negative-Sequence Reactances

The subtransient reactances x_d'' and x_q'' may be calculated as for salient-pole machines with dampers (**25a**) and (**29a**). The negative-sequence is determined as the average of x_d'' and x_q'', equation (**31a**). It will be seen that there is very little difference between x_d'' and x_q'', and the tests described in the companion paper[1] show that they do actually become very nearly equal at high currents. For most practical purposes, it should be necessary to calculate and test only x_d''.

VI. Zero-Sequence Reactance

For turbine generators, the zero-sequence belt leakage may be approximated by using the direct-axis damper leakage specific permeance (λ_{Dd}) reduced by the ratio $\frac{k_{zo}}{k_z}$ for chording.

$$\lambda_{Bo} = \frac{k_{zo}}{k_z} \lambda_{Dd} \qquad \textbf{(56a)}$$

The remainder of the formula is the same as for salient-pole machines (**32a** and **34a**):

$$x_o = X (\lambda_{io} + \lambda_{Bo} + 0.2\ \lambda_s') \qquad \textbf{(57a)}$$

VII. Time Constants T_{do}', T_d', and T_d

The self inductance of the field in henrys is:

$$L_f = N_p^2\, p\, l\, 10^{-8} \left[C_f \left(3.19 \frac{\tau}{g'} \right) + \lambda_F \right] \qquad \textbf{(58a)}$$

The open-circuit time constant will be calculated, neglecting damping currents in the solid iron, hence is the same as equation (**36a**).

The short-circuit time constant T_d' and armature time constant (T_a) are calculated as for salient-pole machines (**37a**) and (**38a**).

Symbols

NOTE: The symbols for the characteristic reactances and time constants are given in the introduction, Section I. The symbols for the components of reactance are given in Section II. The symbols for the specific permeances and the flux distribution coefficient are given as they are defined and calculated in Appendixes A and B.

For symbols representing machine dimensions, see also Fig. 6.

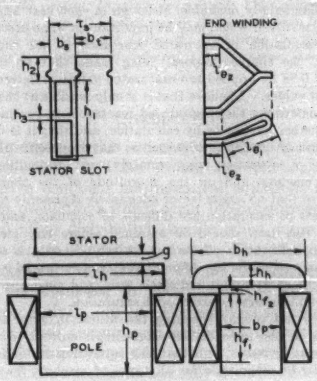

FIG. 6—DETAILS

A = ampere conductors per inch (amperes per slot ÷ slot pitch)
b_{b1} = width of damper bar
b_{b2} = width of slot above damper bar
b_h = width of pole head
b_p = width of pole
b_{rs} = width of rotor slot
b_s = width of stator slot
b_t = width of stator tooth
B_g = air gap density over pole center (at rated voltage)
B_1 = maximum fundamental density at rated voltage
C = number of parallels in the stator winding
d = inside diameter of armature (armature bore)
d_r = rotor diameter
E_{ph} = rated phase voltage (phase winding voltage)
f = rated frequency
F_{st} = ratio of saturated to unsaturated transient reactance (an empirical constant used here as 0.88)
g = actual (single) air gap in pole center
g' = effective air gap, including increase in gap due to stator slots and vents and end fringing
h_{b1} = depth of damper bar
h_{b2} = depth of slot above the damper bar
h_{f1} = depth of field coil
h_{f2} = distance between field coil and pole head
h_{r1} = depth of conductors in the rotor slot
h_{r2} = depth of rotor slot above the conductors
h_1 = depth of conductors in stator slot
h_2 = depth of stator slot above the conductors
h_3 = distance between top and bottom coil sides
I_{ph} = rated phase winding current
k_d = distribution factor for stator winding
k_p = chord factor of the stator winding
k_x = reduction of slot reactance due to chording
k_{xo} = reduction of zero sequence slot reactance due to chording
k_{rs} = Carter's coefficient for rotor slots
l = core length (including vent ducts)
l_b = length of damper bar
l_{e1} = extension of bent section of end winding
l_{e2} = length of straight section of end winding
l_h = length of pole head (including magnetic coil supports)
L_f = total self inductance of the field
m = number of phases
M_A = maximum fundamental armature ampere turns
M_g = air gap ampere turns at no-load and rated voltage
M_m = "demagnetizing ampere turns;" field ampere turns required to balance armature reaction at rated current (zero power factor)
n_b = number of damper bars per pole
n_s = number of conductors per slot
N_p = number of field turns per pole
p = number of poles
P_s = permeance per cm. of slot
q = slots per phase per pole
Q_r = total rotor slots
r_a = d-c. resistance of armature in per cent
R_f = field resistance in ohms at 75 deg. cent.
X = reactance factor, per cent reactance for unit specific permeance. See Section II
y = coil throw (number of slots spanned)
α = "pole embrace" ratio of pole arc to pole pitch
γ = ratio of slotted portion to total circumference of a turbine generator rotor
δ_d = depth of an equivalent path in air for the pulsating flux in the rotor teeth
δ_q = depth of an equivalent path in air for the pulsating flux in a solid pole center
λ = "specific permeance:" effective flux per pole per inch of length for unit ampere turns. See Section II
τ = pole pitch on stator diameter
τ_b = pitch of damper bars
τ_r = pole pitch on rotor diameter
τ_{rs} = rotor slot pitch
τ_s = stator slot pitch

Discussion

M. L. Henderson: Mr. Kilgore's paper is unquestionably an important contribution to the science of synchronous machine design. He has given formulas for the calculation of a larger number of the principal design constants than have heretofore been collected in a single paper. Many of these formulas have been derived on a more fundamental basis than before, while some are more approximate than previous expressions.

The end-winding leakage is accounted for by a method slightly different from that used by Mr. Alger, but comparative results for average machines indicate but little difference in the final results. In fact if the effective end-winding extension $(2\,l_{e2} + l_{e1})$ is assumed equal to the peripheral span of the coil Mr. Kilgore's formula gives almost exactly the same end-winding leakage reactance as does Mr. Alger's expression. The expression (**3a**) for the end-winding leakage permeance, as printed, should evidently be divided by the core length, l. Formula (**40a**) for the end-winding leakage permeance of turbine generators results from (**3a**) provided that the effective end winding extension, including both ends, is taken as 86 per cent of the pole pitch. Considering the range of pitch and the relative importance of end-winding reactance in turbine generators, it seems better to retain the actual dimensions in this formula. For an actual two-pole machine on which these formulas were tried out, (**40a**) gave an end-winding leakage permeance of 6.3, while the expression derived from (**3a**) gave a value of 4.7, including the empirical factor of 1.2.

Mr. Kilgore states that his air-gap flux distribution coefficients, C_1, C_{d1} and C_m can be calculated for salient-pole machines from curves given in Mr. Wieseman's paper on *Graphical Determination of Magnetic Fields*, TRANS. A. I. E. E., Vol. 46, p. 141, but does not give the relation of his coefficients to those given by Mr. Wieseman. The relations are as follows:

by Kilgore		by Wieseman
C_1	=	A_1
C_{d1}	=	A_{d1}
C_{q1}	=	A_{q1}
C_p	=	$\dfrac{2\,A_1\,K_a}{\pi}$
C_m	=	K_a, as defined by Mr. Alger

Some unpublished work of Prof. A. A. Bennett, since the publication of Mr. Wieseman's paper, has shown that in agreement with Mr. Kilgore, K_a (or C_m) is dependent only upon the ratio of pole arc to pole pitch, but he obtains values 1 to 2 per cent higher than Mr. Kilgore's. However, we believe, from Prof. Bennett's work, that the quadrature coefficient, C_{q1}, depends upon the ratios of maximum to minimum gap and minimum gap to pole pitch as well as upon the ratio of pole arc to pole pitch. For example C_{q1} for a 70 per cent pole arc varies from 0.43 to 0.55 with reasonable variations in the other ratios, while Mr. Kilgore's curve gives 0.50.

From the standpoint of those who attempt to use and compare the various formulas for synchronous machine constants it is to be hoped that the nomenclature for machine dimensions, number of phases, number of slots, etc., will soon reach something like a steady state. Regardless of the merits of the individual systems, a great deal of the user's time would be saved if it were not necessary to literally translate one set of formulas into another nomenclature to compare them with the results of another writer.

P. L. Alger: The most novel elements of Mr. Kilgore's paper are the inclusion of the tooth tip leakage fringing as an additional term in the slot reactance (equation (**1a**)), the use of flux distribution coefficients independent of the rounding of the pole (Fig. 4), and the formulas for the damper winding leakage. All of these proposals seem to be in the right direction, and fundamentally desirable, although in each case approximations are involved which may be important in some cases.

The calculation of saturated reactance is not considered, however, the simple proposal being made that an empirical factor of 0.88 be applied to unsaturated values to determine the saturated values. I believe that it is only feasible at the present time to determine the unsaturated reactances in routine design work, whether by test or by calculation, and that it is desirable to establish saturation factors such as that proposed. However, the effect of saturation must certainly vary, depending on the type of machine and on the magnitude of the short-circuit current, so that no single factor adequately represents the facts. The effects of saturation are difficult to calculate, and a large body of test data should be obtained before any method of calculating saturation effects can be established. A complete theory must reconcile the different values of saturated reactance obtained by short circuit and static tests, and must also explain the variations in time constant with saturation.

I believe that, roughly speaking, the saturation factor decreases uniformly as the ratio of the short-circuit current to the full load current increases, and that this saturation factor is of the same order of magnitude for all types of machine for a given value of per unit short-circuit current. This effect can be roughly represented by taking the saturated value of the transient reactance as a definite fixed amount, say 0.05 in per unit value, less than the unsaturated reactance value. Thus, a machine having an unsaturated transient reactance of 40 per cent might have a saturated value of 35 per cent, while one having an unsaturated reactance of 15 per cent might have a saturated reactance of 10 per cent.

I think that as a temporary measure some such basis as this should be agreed upon for the determination of the saturated reactance values, rather than taking any arbitrary constant ratio for all cases, and I certainly believe that the factor of 0.88 proposed by Mr. Kilgore, is too high for the cases of low reactance machines such as turbine alternators.

Transient Oscillations in Distributed Circuits
With Special Reference to Transformer Windings

BY L. V. BEWLEY*
Associate, A. I. E. E.

Synopsis.—*This paper is essentially a mathematical study of transient voltages in transformer and other distributed windings, and is primarily concerned with showing the effects of wave shape, circuit constants and neutral impedances on the voltage distributions, and how these distributions may be controlled or changed. In the appendix there is derived the general differential equation for a circuit consisting of distributed self and mutual inductance, series and shunt capacitance, series resistance, and conductance along the stack and to ground. This equation is solved for the following conditions: (1) initial and final conditions with generalized impedance in the neutral; (2) grounded neutral; (3) isolated neutral for zero losses; and (4) capacitance in the neutral when the losses and mutual inductance are neglected. Equations are given corresponding to a number of different applied waves, showing the effects of wavelength, wave front, damped and sustained oscillations, and typical lightning waves, and these effects are illustrated by curves and oscillograms. Equations are also given for the potential difference between any two points on the winding and for the voltage gradients. Tables are included showing the influence of the various circuit constants on the amplitude and frequency of oscillation, linear velocity and surge impedance of harmonic waves, and the type of propagation. These effects are illustrated by curves of the potential distribution at different instants of time. Methods of controlling the transient so as to prevent or alleviate abnormal voltage distributions are discussed at length. An equivalent circuit is developed which serves as an approximation for a transformer as far as external reactions are concerned.*

* * * * *

I. Introduction

THE purpose of this paper is to continue the study of oscillations in transformers and particularly to derive complete mathematical equations for the transient oscillations which occur in transformer windings and other distributed circuits when subjected to incident waves of arbitrary shape. The effects of the losses, neutral impedances, circuit parameters, and shields are discussed in detail, and the appropriate equations derived. While the analysis is necessarily based on ideal circuits and subject to the restrictions of certain simplifying assumptions; nevertheless the theoretical considerations not only describe the general characteristics of the transient, but give closer quantitative agreement with tests than might be expected. The principal discrepancies are due to the assumptions (made in the interests of mathematical simplicity) concerning the series capacitance paths and the mutual inductance. In order to avoid a complicated partial differential equation with variable coefficients it was necessary to adhere to the assumptions of a linearly graded mutual inductance.[3] Moreover, it was expedient to ignore all circuit discontinuities, such as the lumping of the winding into pancake coils, breaks for taps, interleaving, etc. These circuit discontinuities give rise to localized oscillations which are superimposed on the principal oscillations, and while these do not add materially to the maximum potentials to ground, yet they may greatly increase the turn-to-turn stresses.

The author's indebtedness to the papers in the bibliography, which he has consulted very freely, will be obvious to the informed reader on this subject.

*Power Trans. Engg. Dept., General Electric Co., Pittsfield, Mass.

3. For references see Bibliography.

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II. The General Circuit

The Circuit Constants. The transient in a distributed winding, such as that of a transformer, is characterized by a complicated network of circuit constants. These include the self and mutual inductances; the capacitances from turn to turn, to tank, to the core and between coils; the dielectric loss constants, and the series resistance of the winding as modified by skin effect.

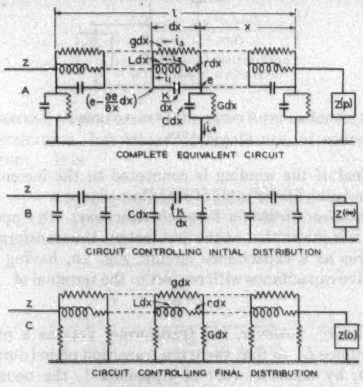

FIG. 1—IDEAL COMPLETE CIRCUIT OF A TRANSFORMER

For purposes of analysis the circuit may be simplified to that shown in Fig. 1A. The capacitances from the winding to the core and to the tank have been combined as a single capacitance C per unit length to ground. The coil-to-coil capacitance along the stack is K per unit length. The total flux linkages at any point of the winding consists of those due to the mutual flux of the

entire winding, and the partial interlinkages of the leakage flux. The effective inductance L, which includes these partial interlinkages is not, therefore, a pure self-inductance. It is defined, on the assumption of a linearly graded mutual flux, in Appendix I.

Since the transient is of high frequency, the skin effect in the conductors probably represents the principal copper loss. This transient skin effect is too complicated to express analytically, but since it is known that the losses do not have much influence on the first cycle of the fundamental oscillation, it is sufficient to represent the copper losses by a series resistance r. There remain the dielectric losses caused by the difference of potential between winding and ground, and from point to point along the stack. These are accounted for by the conductances G and g respectively. In practise, the neutral of a transformer may be directly grounded, isolated, or connected to ground through some sort of impedance, such as a resistance, reactor, or impedor. This grounding impedance is represented in Fig. 1 as $Z(p)$ where the $p = \partial/\partial t$ implies that the impedance can be any general combination of resistances, inductances, capacitances, etc. The other

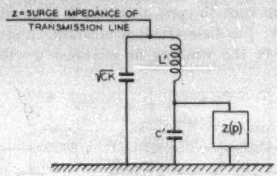

Fig. 2—Equivalent Circuit of Transformer to Traveling Waves

terminal of the winding is connected to the incoming transmission line or cable of surge impedance z.

Equivalent Circuit for Terminal Reactions. In Appendix I it is shown that at the first instant the transformer behaves as a capacitance circuit, Fig. 1B, having an effective capacitance with respect to the terminal of

$$\sqrt{C\,K}$$

Eventually, however, the transformer acts as a pure inductance L', so that there is a transition period during which, by equation (63) of Appendix I, the neutral voltage of an isolated neutral transformer oscillates with a fundamental frequency[3]

$$f = \frac{\pi}{4\sqrt{L\,(4\,C + \pi^2\,K)}}$$

and rises to practically double the terminal voltage. If the neutral is grounded, there are, of course, no oscillations at the neutral. These considerations suggest the circuit shown in Fig. 2 as the approximate terminal equivalent circuit of a transformer, where the capacitance C' is chosen so that for an isolated neutral the neutral voltage has a period of oscillation equal to f above. That is

$$f = \frac{1}{2\,\pi\,\sqrt{L'\,C'}} = \frac{\pi}{4\,\sqrt{L\,(4\,C + \pi^2\,K)}}$$

$$\therefore\ C' = \frac{4\,L\,(4\,C + \pi^2\,K)}{L'\,\pi^4}$$

where L is the effective inductance defined following equation (8) in Appendix I, and L' is the short-circuited inductance of the transformer. It will be observed that the circuit satisfies the following conditions:

1. At the first instant the circuit behaves as a capacitance $\sqrt{C\,K}$ to ground.
2. For a grounded neutral the neutral voltage is zero.
3. For an isolated neutral the neutral voltage oscillates at the fundamental natural frequency of the transformer, and approaches an upper limit of double the terminal voltage.
4. Resistance in the neutral decreases the period of oscillation and limits the rise of the neutral voltage to a value less than twice the terminal voltage.
5. The circuit finally behaves as an inductance L'.

This circuit has been verified by tests results, and is instrumental in tremendous simplifications in the analysis of transformer oscillations. While the introduction of such approximations is never justifiable from a mathematical point of view, yet they are appropriate for engineering purposes. Still further simplifications can usually be made. The surge impedances of the harmonic waves comprising the internal oscillations in the windings are many times the surge impedance of the transmission line, and because of this great disparity in value these harmonic waves are reflected from the line terminal practically as from a grounded end. Conversely, the incident wave on the transmission line is practically completely reflected, so that the terminal voltage is very nearly twice the traveling wave voltage. For this reason the surge impedance of the transmission line does not come into the picture, except for very long waves; and it is often sufficient to compute the internal oscillations of the transformer on the assumption that twice the incident wave is directly applied to the terminals. When greater accuracy is required the equivalent circuit of Fig. 2 can be used to determine the terminal voltage, and then this voltage used as the impressed voltage for calculating the internal oscillations. Ordinarily the capacitance $\sqrt{C\,K}$ may be ignored, unless there is an inductance in series with the transformer.

The Initial Distribution. When a wave with an abrupt front reaches the terminal of a transformer, the rate of change of potential in that first instant is infinite, and therefore the currents flowing through the capacitance paths of the winding are also infinite. But the current in the inductance is zero, and the currents in

the resistance elements are all finite. Therefore the initial distribution depends only on the capacitance elements of the circuit, Fig. 1B, and the nature of the neutral impedance. In Appendix I it is shown that the distribution is of the form

$$e = \frac{A \cosh \alpha x + \sinh \alpha x}{A \cosh \alpha + \sinh \alpha}$$

where

$\alpha = \sqrt{C/K}$
$A = [p\, Z\,(p) \sqrt{C\,K}]$ as $p \to \infty$
$Z\,(p)$ = neutral impedance
K = series capacitance
C = shunt capacitance
x = fraction of winding from neutral end.

In a normal transformer α is of the order of 5 to 30. In a non-resonating transformer it is nearly zero.

A few representative combinations for A are given in the following table:

Neutral impedance	$Z\,(p)$	A
Directly grounded	0	0
Isolated neutral	∞	∞
Resistance	R_o	∞
Inductance	$p\,L_o$	∞
Capacitance	$1/(p\,C_o)$	$\sqrt{C\,K}/C_o$
Inductance and capacitance in series	$p\,L_o + 1/p\,C_o$	∞
Inductance and capacitance in parallel	$p\,L_o/(1 + p^2\,L_o\,C_o)$	$\sqrt{C\,K}/C_o$
Resistance and capacitance in parallel	$R_o/1 + p\,R_o\,C_o)$	$\sqrt{C\,K}/C_o$
Resistance and inductance in parallel	$R_o\,L_o\,p/(R_o + p\,L_o)$	∞

In all cases for which $A = \infty$ the initial distribution is given by

$$e = \frac{\cosh\ \alpha\ x}{\cosh\ \alpha}$$

It is evident that only an uninterrupted capacitance from the neutral to ground, or a directly grounded neutral can change the initial distribution from that corresponding to an isolated neutral.

In Fig. 3 is a set of curves showing the initial distribution corresponding to a perpendicular wave front.[3] Notice that for values of α in the neighborhood of $\alpha = 10$ the initial distribution is practically independent of the neutral impedance (as characterized by A). These curves also clearly show that the distribution becomes more nearly linear as α decreases, that is, as the ratio $\sqrt{C/K}$ decreases. In other words, the distortion in the initial distribution is caused by the capacitance to ground C, and can be improved either by decreasing C or by increasing the series capacitance K. This possibility is of primary importance in connection with electrostatic shielding, and is fully discussed in Part V of this paper.

The Final Distribution. After the transient oscillations caused by the application of an infinite rectangular wave at the line terminal have died out (theoretically at $t = \infty$), there remains a residual distribution. The capacitance elements then act as open circuits and the inductance elements as short circuits, and the distribution is entirely determined by the resistance elements, Fig. 1C. The damping out of the oscillations is due not only to the internal losses (skin effect, dielectric and iron losses), but also to the junction losses at the terminals where energy is lost by the refraction of waves to the line and to the neutral impedance. In Appendix I it is shown that the final distribution is given by an

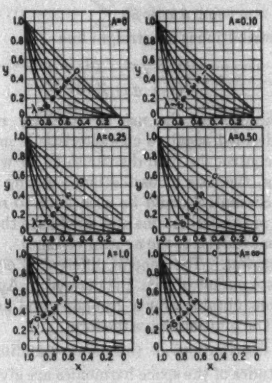

FIG. 3—INITIAL AND FINAL DISTRIBUTION FACTOR

$$\frac{e}{E} = \frac{A \cosh \lambda\, x + \sinh \lambda\, x}{A \cosh \lambda + \sinh \lambda}$$

$\lambda = \alpha$ for initial distribution
$\lambda = \beta$ for final distribution

equation having exactly the same form as for the initial distribution, but whose constants are, of course, different. It is

$$e = \frac{A \cosh \beta\, x + \sinh \beta\, x}{A \cosh \beta + \sinh \beta}$$

where

$$\beta = \sqrt{\frac{r\,G}{1 + g\,r}}$$

$$A = Z\,(o) \sqrt{\frac{G}{r}\,(1 + g\,r)}$$

r = series resistance
g = conductance along winding
G = conductance to ground
$Z\,(0) = Z\,(p)$ when $p = 0$

In all cases for which $Z\,(o) = 0$, and this includes all combinations in which there is a pure inductance between the neutral and ground, the distribution is

$$e = \frac{\sinh\ \beta\ x}{\sinh \beta}$$

In all cases for which $Z\,(0) = \infty$, and this includes a

capacitance in series as part of the neutral impedance, the distribution is

$$e = \frac{\cosh \beta x}{\cosh \beta}$$

The distributions are given for a range in the values of β and A in Fig. 3. In an actual transformer, the value of β is so small that

$$\cosh \beta x \cong 1$$
$$\sinh \beta x \cong \beta x$$

and the distribution is the straight line

$$e = \frac{(1 + g r)\, Z(0) + r x}{(1 + g r)\, Z(0) + r}$$

Theoretically, by the use of auxiliary resistances having a decided negative characteristic it may be desirable to control the final distribution—or what is the same thing, the axis of oscillations—during the transient. Such control could be used both to reduce the amplitude of the oscillations and accelerate their decay, or to prevent the rise of the neutral voltage.

The Transient. The general differential equation of the complete circuit of Fig. 1 is derived in Appendix I, and solutions obtained for a grounded and an isolated neutral. The solutions have the form

e = (final distribution) + (harmonic standing waves).

The amplitudes of the space harmonics are given by the Fourier analysis of the initial distribution with respect to the final distribution, and are therefore smaller, the more nearly these two distributions are alike. The two solutions, when the losses are neglected, are discussed in detail under Parts III and IV of this paper. The solution, including the losses, for the grounded neutral case, is of the form

$$e = E \frac{\sinh \beta x}{\sinh \beta} + E \sum_{1}^{\infty} \frac{2\,(-1)^s\, s\, \pi\, (\alpha^2 - \beta^2)}{(\beta^2 + s^2\pi^2)\;\;(\alpha^2 + s^2\pi^2)}\, \epsilon^{-\gamma_s t} \cos \omega_s t \sin s \pi x \tag{45}$$

where

$$\alpha = \sqrt{\frac{C}{K}} \text{ and } \beta = \sqrt{\frac{r G}{1 + g r}}$$

If $r = g = G = 0$, then $\beta = 0$ and $\gamma_s = 0$ and the solution reduces to that given in Part III. The effect of the losses is four-fold:

1. Oscillations are damped out by exponential decrement factors.
2. Decreased amplitudes of oscillations.
3. Decreased frequencies of oscillation.
4. Non-linear final distribution.

It is evident that all oscillations vanish if $\alpha = \beta$. But this is the condition that the initial and final distributions are alike. Therefore the criterion for no oscillations is that the initial and final distribution shall be the same, and it is immaterial whether the adjustment be made on either distribution, or on both.

A general solution for any impedance $Z(p)$ in the neutral is impossible, nor can explicit solutions be obtained for the most simple specific cases, but must depend upon finding the roots of transcendental equations. For this reason pure mathematical analysis ceases to be profitable. However, good approximate results may be obtained by using the equivalent circuit of Fig. 2 and applying the principle of superposition. This method is discussed in detail in Part V.

III. Grounded Neutral

In this section there are considered the circuits shown in Fig. 4. The wavy line coil indicates that the flux is due to pure self-inductance, whereas the looped coil indicates that mutual inductance is also present between

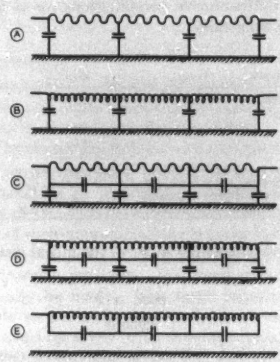

Fig. 4—Circuits Used in Calculations

points along the winding. In the interests of simplicity the losses are neglected. Solutions are obtained for the different wave shapes shown in Fig. 6, and the effect of wavelength, wave front, and frequency of applied voltage are evaluated.

The potential e at point x of any of these five circuits, measuring from the grounded terminal, can be expressed, as shown in Appendix I, by the identical equations (for an infinite rectangular applied wave)

$$e = xE + E \sum_{1}^{\infty} A_s \sin s \pi x \cos \omega_s t \tag{1a}$$

$$= xE + E \sum_{1}^{\infty} \frac{A_s}{2} \left[\sin (s \pi x + \omega_s t) + \sin (s \pi x - \omega_s t)\right] \tag{1b}$$

$$= E \sum_{1}^{\infty} \left[A_s \cos (s \pi - \omega_s t) - \frac{2}{s \pi}\right] \cos s \pi \,.\, \sin s \pi x \tag{1c}$$

These three equations represent the same phenomenon from three different points of view:

Equation	Point of view
1 a	(Fixed distribution) + (harmonic standing waves)
1 b	(Fixed distribution) + (pairs of harmonic traveling waves)
1 c	Simple traveling waves in the case of circuit Fig. 4A.

The amplitude A_s, angular velocity ω_s, linear velocity v_s, surge impedance z_s, initial and final distributions, and type of propagation for each of the four circuits under consideration are given in the following table.

Circuit	(L, C)	(M, C)	(L, C, K)	(M, C, K)
A_s	$\frac{2 \cos s\pi}{s\pi}$	$\frac{2 \cos s\pi}{s\pi}$	$\frac{2\, c^3 \cos s\pi}{s\pi\,(s^2\pi^2 + \alpha^2)}$	$\frac{2\, \alpha^2 \cos s\pi}{s\pi\,(s^2\pi^2 + \alpha^2)}$
ω_s	$\frac{s\pi}{\sqrt{L\,C}}$	$\frac{s^3\pi^2}{\sqrt{M\,C}}$	$\frac{s\pi}{\sqrt{L\,(K\,s^2\pi^2 + C)}}$	$\frac{s^2\pi^2}{\sqrt{M\,(K\,s^2\pi^2 + C)}}$
v_s	$\frac{1}{\sqrt{L\,C}}$	$\frac{s\pi}{\sqrt{M\,C}}$	$\frac{1}{\sqrt{L\,(K\,s^2\pi^2 + C)}}$	$\frac{s\pi}{\sqrt{M\,(K\,s^2\pi^2 + C)}}$
z_s	$\sqrt{\frac{L}{C}}$	$\frac{1}{s\pi}\sqrt{\frac{M}{C}}$	$\sqrt{\frac{L}{C}}\sqrt{\frac{s^2\pi^2}{\alpha^2} + 1}$	$\sqrt{\frac{M}{C}}\sqrt{\frac{1}{\alpha^2} + \frac{1}{s^2\pi^2}}$
$e_{t=0}$	0	0	$\frac{\sinh \alpha x}{\sinh \alpha}$	$\frac{\sinh \alpha x}{\sinh \alpha}$
$e_{t=\infty}$	x	x	x	x
Type of propagation	As a rigid distribution	High harmonics ahead	High harmonics behind	High harmonics ahead

These conditions are illustrated in Fig. 5 for the first four circuits of Fig. 4, when $L = M = 5$, $C = 10^{-9}$, and $K = 10^{-11}$.

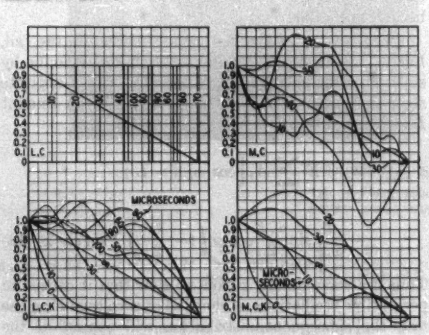

Fig. 5—Internal Distributions

Incident Waves of Arbitrary Shape. The foregoing equations and curves describe the response of the grounded-neutral circuits in Fig. 4 to an infinite rectangular applied wave. If the solution for this case be called $\varphi(t)$ and if the solution corresponding to any applied wave $E(t)$ is required it can be found by means of Duhamel's theorem as

$$e = E(t)\,\varphi(0) + \int_0^t E(\tau)\,\frac{\partial}{\partial t}\,\varphi(t-\tau)\,d\tau \qquad (2)$$

where, from equation (1),

$$\varphi(t) = x + \sum_1^\infty A_s \sin s\pi x \cos \omega_s t \qquad (3)$$

Finite Rectangular Wave. Applying (2) to the finite rectangular wave of length L, Fig. 6B, there is

$$\left.\begin{aligned} e_{(t<L)} &= x\,E + E\sum_1^\infty A_s \sin s\pi x \cos \omega_s t \\ e_{(t>L)} &= E\sum_1^\infty A_s \sin s\pi x[\cos \omega_s t - \cos \omega_s (t-L)] \\ &= -E\sum_1^\infty A_s \sin s\pi x\left(2\sin\frac{\omega_s L}{2}\right)\sin \omega_s\left(t - \frac{L}{2}\right) \end{aligned}\right\} \qquad (4)$$

Thus the solution is the same as for the infinite rectangular wave up until the time $t = L$. But for $t > L$ the axis of oscillations has shifted from the $x\,E$ line to the zero axis, and the amplitudes of the harmonics are multiplied by the reduction factor

$$2 \sin \frac{\omega_s L}{2} \qquad (5)$$

Any particular harmonic is then a maximum for

$$\frac{\omega_s L}{2} = (2n - 1)\,\frac{\pi}{2}$$

where n is an integer. Therefore the wavelength L necessary to make any harmonic s a maximum is

$$L = \frac{(2n-1)\pi}{\omega_s} = \frac{(2n-1)\pi}{2\pi f_s} = \frac{2n-1}{2f_s} = (2n-1)\frac{T_s}{2} \quad (6)$$

Thus the wavelength must be an odd multiple of the natural half period of oscillation $T_s/2$. In this case the amplitude of oscillation for that particular harmonic is twice as great for $t > L$ as for $t < L$, but since the axis of oscillations has shifted to the zero axis the voltage to ground does not necessarily increase at any

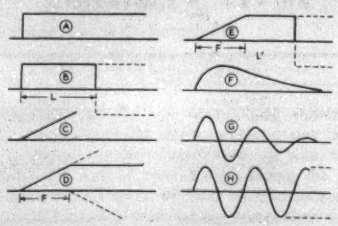

FIG. 6—WAVE SHAPES USED IN CALCULATIONS

point along the winding. Considering any harmonic alone there is for $t < L$

$$e' = E\,(x + A_s \sin s\,\pi\,x \cos \omega_s t) \quad (7)$$

and this is a maximum when $\cos \omega_s t = \cos s\,\pi$ and for

$$\frac{\partial e'}{\partial x} = 0 = E\,(1 + A_s\, s\,\pi \cos s\,\pi\,x) \quad (8)$$

$$\therefore x = \frac{1}{s\,\pi} \cos^{-1} \frac{(-1)}{s\,\pi\,A_s} \quad (9)$$

$$e'_{max} = E\left[\frac{1}{s\,\pi} \cos^{-1} \frac{(-1)}{s\,\pi\,A_s} + A_s \sqrt{1 - \left(\frac{1}{s\,\pi\,A_s}\right)^2}\right]$$

$$= \frac{E}{s\,\pi}\left[\cos^{-1} \frac{(-1)}{s\,\pi\,A_s} + \sqrt{s^2\,\pi^2\,A_s^2 - 1}\right] \quad (10)$$

In a grounded-neutral winding A_s is greatest for the fundamental, but cannot exceed $2/\pi$, so that the maximum to which fundamental can raise the voltage is

$$e'_{max} = \frac{E}{\pi}\left[\cos^{-1}\frac{(-1)}{2} + \sqrt{3}\right] = 1.218\,E \text{ for } t < L \quad (11)$$

as compared with

$$e''_{max} = 2\,A_s\,E = \frac{4}{\pi}\,E = 1.272\,E \text{ for } t > L \quad (12)$$

These relative values are, of course, altered by the contributions of the other harmonics, and A_s is actually always less than $2/\pi$.

Effect of a Linear Front. The solution for the infinite linear wave, Fig. 6C,

$$E\,(t) = a\,t \quad (13)$$

by Duhamel's theorem is

$$e = a\,t\,x + \sum_1^\infty \frac{a}{\omega_s} A_s \sin s\,\pi\,x \sin \omega_s t \quad (14)$$

Herefrom, the solution for a wave with a linear front of length F and an infinite rectangular tail is found by superimposing two such solutions of opposite sign and displaced by an amount $t = F$, Fig. 6D. It is

$$\left.\begin{aligned} e_{t<F} &= x\frac{E}{F}\,t + E\sum_1^\infty A_s \frac{1}{F\,\omega_s} \sin s\,\pi\,x \sin \omega_s t \\ e_{t>F} &= x\,E - E\sum_1^\infty A_s \\ &\left[\frac{2}{F\,\omega_s} \sin \frac{F\,\omega_s}{2}\right] \sin \omega_s \left(t - \frac{F}{2}\right) \cos s\,\pi\,x \end{aligned}\right\} \quad (15)$$

For $t > F$ the amplitude of any harmonic is proportional to

$$y_s = \frac{2}{F\,\omega_s} \sin \frac{F\,\omega_s}{2} \quad (16)$$

This reduction factor is plotted in Fig. 8. It has a maximum of unity occuring at $F = 0$, and then rapidly decreases in magnitude similar to a damped oscillation.

$$y_s = 0 \text{ for } \frac{F\,\omega_s}{2} = n\,\pi \quad \text{or} \quad F = \frac{2\,n\,\pi}{\omega_s} = \frac{n}{f_s} \quad (17)$$

Thus if the wave front is a multiple of the natural period of oscillation of a particular harmonic, then that harmonic vanishes. If the front is sufficiently long so that the fundamental vanishes, then the harmonics also become negligible.

When a wave of this nature is chopped on the tail, Fig. 6E, the subsequent voltage distribution is found by superimposing an infinite rectangular wave of opposite sign, and displaced back from the origin by an amount L'. The solution then is

$$e_{t>L'} = -\,E\sum_1^\infty A_s \left[\cos \omega_s\,(t - L') - \frac{2}{\omega_s F} \sin \frac{\omega_s F}{2} \cos \omega_s \left(t - \frac{F}{2}\right)\right] \sin s\,\pi\,x \quad (18)$$

Thus if a wave having a front so slow that it causes no perceptible oscillations, is chopped at a voltage E on the tail, the subsequent distribution will consist of the oscillations corresponding to an infinite rectangular wave of $(-E)$ potential, but these oscillations will take place about the zero axis instead of the $(-x\,E)$ axis.

Typical Lightning Wave. The characteristic shape of a natural lightning wave—disregarding the minor irregularities—is shown in Fig. 6F, and is represented by

$$E\,(t) = E\,(\epsilon^{-at} - \epsilon^{-bt}) \quad (19)$$

Then by Duhamel's Theorem there is

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})\,x + E\sum_{1}^{\infty} A_s \sin s\,\pi\,x \left[\frac{a^2\,\epsilon^{-at}}{a^2+\omega_s^2} - \frac{b^2\,\epsilon^{-bt}}{b^2+\omega_s^2} + \frac{\omega_s}{\sqrt{a^2+\omega_s^2}}\cos\left(\omega_s t + \tan^{-1}\frac{a}{\omega_s}\right) - \frac{\omega_s}{\sqrt{b^2+\omega_s^2}}\cos\left(\omega_s t + \tan^{-1}\frac{b}{\omega_s}\right)\right] \quad (20)$$

A wave with an infinite tail is obtained by making $a = 0$. Then the above equation reduces to

$$e = E\,\epsilon^{-at}\,x + E\sum_{1}^{\infty} A_s \sin s\,\pi\,x \left[\frac{a^2\,\epsilon^{-at}}{a^2+\omega_s^2} + \frac{\omega_s}{\sqrt{a^2+\omega_s^2}}\cos\left(\omega_s t + \tan^{-1}\frac{a}{\omega_s}\right)\right] \quad (22)$$

Here again, by putting $a = 0$ the solution reduces to that for an infinite rectangular wave.

Damped Oscillations. The importance of this case justifies a separate paper.[3]

Potential Difference Between Points. The potential

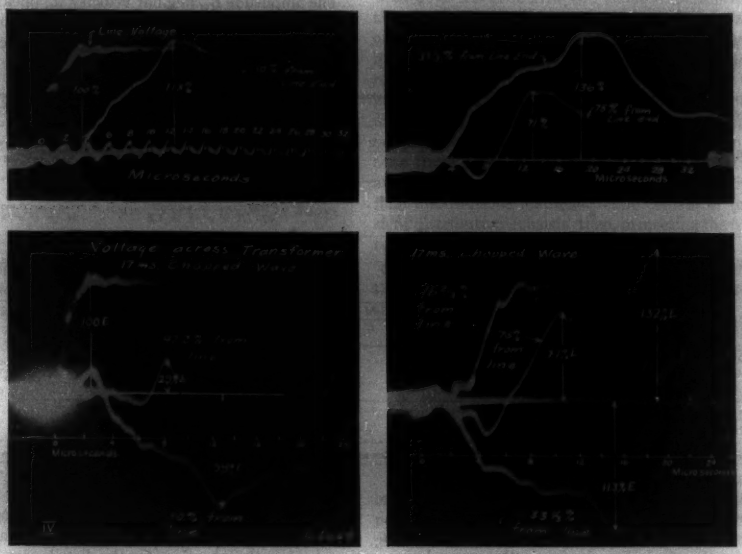

FIG. 7—EFFECT OF CHOPPING WAVE ON THE TAIL

Top, very long applied wave
Bottom, wave chopped at 17 microseconds

$$e_{a-b} = E(1-\epsilon^{-bt})\,x + E\sum_{1}^{\infty} A_s \sin s\,\pi\,x \left[-\frac{b^2\,\epsilon^{-bt}}{b^2+\omega_s^2} + \frac{b}{\sqrt{b^2+\omega_s^2}}\cos\left(\omega_s t + \tan^{-1}\frac{\omega_s}{b}\right)\right] \quad (21)$$

Thus the oscillations are partially proportional to the factor b determining the length of the front. When $b = \infty$ the front is perpendicular, and the solution reduces to that of the infinite rectangular wave.

A wave with a perpendicular front and an exponential tail is given by putting $b = \infty$ in (19). Then

difference between any two points x_1 and x_2 of the winding, for an infinite rectangular wave is

$$e_1 - e_2 = (x_1 - x_2)\,E + E\sum_{1}^{\infty} A_s\,(\sin s\,\pi\,x_1 - \sin s\,\pi\,x_2)\cos\omega_s t$$

$$= (x_1 - x_2)\,E + E\sum_{1}^{\infty} A_s\,2\sin\frac{s\,\pi}{2}(x_1-x_2)\cos\frac{s\,\pi}{2}(x_1+x_2)\cos\omega_s t \quad (23)$$

Consequently the harmonic voltage between any two points vanishes if

$$\left.\begin{aligned} s\,(x_1 + x_2) &= (2\,n - 1) = \text{an odd integer} \\ s\,(x_1 - x_2) &= 2\,m \qquad\quad = \text{an even integer} \end{aligned}\right\} \tag{24}$$

For example, if $x_1 = 2/3$ and $x_2 = 1/3$, then all odd harmonics and all even harmonics which are multiples of six, vanish. Thus the only harmonic voltages between these two points are the 2nd, 4th, 8th, 10th, 14th, etc. The harmonic voltages which can exist between two points of the winding, as fraction of full value, are best pictured by charts. For instance, at $x_1 = 3/4$ and $x_2 = 1/4$ the 1st, 3rd, 5th, etc., cancel out; the 4th, 8th, 12th, etc., are zero; and the 2nd, 6th, 10th, etc., have doubled differences. More generally, for points equal distances from the ends $x_2 = (1 - x_1)$ and

$$\left.\begin{aligned} s\,(x_1 + x_2) &= s = 2\,n - 1 \\ s\,(x_1 - x_2) &= s\,(2\,x_1 - 1) = 2\,m \end{aligned}\right\} \tag{25}$$

$$= E - E \sum_1^\infty \frac{B_s}{2} \left[\cos\left(\frac{s\,\pi\,x}{2} - \Omega_s\,t \right) + \cos\left(\frac{s\,\pi\,x}{2} + \Omega_s\,t \right) \right] \tag{1b}$$

$$= -\,E \sum_1^\infty \left[B_s \sin\left(\frac{s\,\pi}{2} - \Omega_s\,t \right) - \frac{4}{s\,\pi} \right] \sin \frac{s\,\pi}{2} \cos \frac{s\,\pi\,x}{2} \tag{1c}$$

The amplitude B_s, angular velocity Ω_s, linear velocity V_s, surge impedance Z_s, initial and final distributions, and type of propagation for each of the four circuits of Fig. 4 are given in the following table:

Circuit	(L, C)	(M, C)	(L, C, K)	(M, C, K)
B_s	$\dfrac{4 \sin s\,\pi/2}{s\,\pi}$	$\dfrac{4 \sin s\,\pi/2}{s\,\pi}$	$\dfrac{16\,\alpha^2 \sin s\,\pi/2}{s\,\pi\,(s^2\,\pi^2 + 4\,\alpha^2)}$	$\dfrac{16\,\alpha^2 \sin s\,\pi/2}{s\,\pi\,(s^2\,\pi^2 + 4\,\alpha^2)}$
Ω_s	$\dfrac{s\,\pi/2}{\sqrt{L\,C}}$	$\dfrac{s^2\,\pi^2/4}{\sqrt{M\,C}}$	$\dfrac{s\,\pi/2}{\sqrt{L\,(K\,s^2\,\pi^2/4 + C)}}$	$\dfrac{s^2\,\pi^2/4}{\sqrt{M\,(K\,s^2\,\pi^2/4 + C)}}$
V_s	$\dfrac{1}{\sqrt{L\,C}}$	$\dfrac{s\,\pi/2}{\sqrt{M\,C}}$	$\dfrac{1}{\sqrt{L\,(K\,s^2\,\pi^2/4 + C)}}$	$\dfrac{s\,\pi/2}{\sqrt{M\,(K\,s^2\,\pi^2/4 + C)}}$
Z_s	$\sqrt{\dfrac{L}{C}}$	$\dfrac{2}{s\,\pi}\sqrt{\dfrac{M}{C}}$	$\sqrt{\dfrac{L}{C}\left(\dfrac{s^2\,\pi^2}{4\,\alpha^2} + 1\right)}$	$\sqrt{\dfrac{M}{C}\left(\dfrac{1}{\alpha^2} + \dfrac{4}{s^2\,\pi^2}\right)}$
e $t = 0$	0	0	$E\,\dfrac{\cosh \alpha\,x}{\cosh \alpha}$	$E\,\dfrac{\cosh \alpha\,x}{\cosh \alpha}$
e $t = \infty$	E	E	E	E
Type of propagation	As a rigid distribution	High harmonics ahead	High harmonics behind	High harmonics ahead

Thus there are no odd harmonics in the potential difference between points equidistant from the ends.

Gradients. The gradient, or stress along the stack per unit length of winding, is simply the space derivative $(\partial\,e/\partial\,x)$ of the corresponding voltage equation. Therefore, since x does not appear in the foregoing equations except as (x) or in $(\sin s\,\pi\,x)$, the gradients are given by replacing in the above equations

$$\left.\begin{aligned} &x \text{ with } 1 \\ &\sin s\,\pi\,x \text{ with } s\,\pi \cos s\,\pi\,x \end{aligned}\right\} \tag{26}$$

IV. Isolated Neutral

When the neutrals of the circuits shown in Fig. 4 are isolated, and the applied wave is infinite rectangular, the solution is

$$e = E - E \sum_1^\infty B_s \,.\, \cos \frac{s\,\pi\,x}{2} \,.\, \cos \Omega_s\,t \tag{1a}$$

The behavior of an isolated neutral winding to waves of arbitrary shape can be determined by an application of Duhamel's Theorem, as in Part III. In fact, since time t enters the equations for the distribution in either the grounded or isolated neutral case, merely as a cosine function, and since Duhamel's Theorem affects only the time function, the equations for the isolated-neutral case can be written immediately by inspection from the corresponding grounded-neutral equations for waves of arbitrary shape. The same conclusions and curves, concerning the effect of wave shape, apply to either case.

V. Neutral Impedance

When a generalized impedance $Z\,(p)$ is inserted in the neutral of the transformer, the mathematical difficulties involved in attempting a solution become unsurmountable from an engineering point of view. An approximate method of solution which yields fair results is as follows:

1. Solve the equivalent circuit, Fig. 2, for the neutral voltage e_n.

2. Find the transient distribution due to the applied terminal voltage, on the assumption that the neutral is grounded.

3. Find the transient distribution due to the neutral voltage e_n on the assumption that the line terminal is grounded, using the grounded neutral equations and Duhamel's Theorem.

4. The resultant distribution is then given by adding the distributions found under (2) and (3).

The application of the above method will be illustrated for the case of a capacitance C_s in the neutral and for an infinite rectangular applied wave. In order to simplify the problem as much as possible, the capacitance $\sqrt{CK}$ will be neglected. Then the circuit consists of the surge impedance of the transmission line in series with L' and $C_n = C' C_s/(C' + C_s)$. By the theory of traveling waves the neutral voltage is

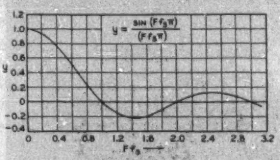

FIG. 8—HARMONIC REDUCTION FACTOR FOR WAVE FRONT

$$e_n = \frac{2/p\,C_n}{\dfrac{1}{p\,C_n} + p\,L' + z}\,E = 2\,E\,[1 - B\,\epsilon^{-at}\sin(b\,t + \theta)\,] \tag{1}$$

where

$$a = \frac{z}{2\,L'},\quad b = \sqrt{\frac{1}{L'\,C_n} - a^2},\quad B = \frac{1}{\sqrt{1 - a^2\,L'\,C_n}},$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

It will be sufficient to take the terminal voltage as $2\,E$. The solution corresponding to $2\,E$ and grounded neutral is

$$e_1 = 2\,E\,x + 2\,E\sum_1^\infty A_s \sin s\,\pi\,x\,.\,\cos\omega_s\,t \tag{1}$$

Now considering the line end grounded, and the potential e_n applied at the neutral, the solution is, by Duhamel's theorem,

$$e_2 = (1 - x)\,e_n + 2\,E\sum_1^\infty A_s \sin s\,\pi\,(1 - x)\,\Big\{\cos\omega_s\,t$$

$$- B\,\epsilon^{-at}\sin(b\,t + \theta)$$

$$+ B\,\frac{\omega_s^2\,\epsilon^{-at}}{(a^2 + b^2 + \omega_s^2)^2 - 4\,b^2\,\omega_s^2}\,[2\,a\,b\cos(b\,t + \theta)$$

$$+ (b^2 - a^2 - \omega_s^2)\sin(b\,t + \theta)\,]$$

$$- B\,\frac{\omega_s}{2}\left[\frac{a\cos(\omega_s\,t + \theta) + (\omega_s - b)\sin(\omega_s\,t + \theta)}{a^2 + (b - \omega_s)^2}\right.$$

$$\left.\left. - \frac{a\cos(\omega_s\,t - \theta) + (b + \omega_s)\sin(\omega_s\,t + \theta)}{a^2 + (b + \omega_s)^2}\right]\right\} \tag{2}$$

Then by the principle of superposition, the actual resultant distribution is the sum of the distributions due to the terminal and neutral voltages, or

$$e = e_1 + e_2 \tag{3}$$

VI. CONTROL OF TRANSIENT DISTRIBUTION

In both the grounded and isolated neutral windings the transient distribution consists of an infinite series of space harmonics, the amplitudes of which are given by the Fourier analysis of the initial distribution with respect to the final distribution. It should not be inferred, however, that this is true for any generalized impedance in the neutral. When a neutral impedance is used there will be, in general, other transient terms than those given by the Fourier series, and such terms are not necessarily eliminated by making the initial and final distributions identical. The control of the transient distribution consists, then, in meeting the following two conditions:

A. Maintaining the neutral voltage constant during the abnormal voltage transient, for all applied wave shapes.

B. Elimination of the internal transient oscillations

1. By making the initial and final distributions identical for an infinite rectangular wave, or
2. By increasing the losses during the transient to such values that all oscillations are at least critically damped, or
3. By modification of the applied wave shape.

Maintaining Constant Voltage at the Neutral. The only connection which will result in a perfectly constant neutral voltage under all conditions is the directly grounded neutral. However, if an isolated neutral or specified neutral impedance (such as a resistance or reactor) is necessary under normal operating conditions, then in order to simulate effectively the grounded-neutral condition during the transient, there are three means available:

1. A capacitance to ground with a charging period very long in comparison with the fundamental natural period of oscillation of the transformer, and with the length of the applied wave.
2. An oscillatory circuit with a period very long in comparison with the fundamental natural period of the transformer, and with the applied wave.
3. A resistance with an exaggerated negative characteristic which will effectively limit the rise of the neutral voltage.

Any of these or their combinations may be used provided that the proper functioning of the normal neutral

impedance under the conditions for which it was intended is not interfered with.

However, while such impedances can be designed to hold the neutral voltage below a specified limit for any required length of time, yet the potential of the neutral rises by a slow transient.[4] This transient is approximately governed by the neutral impedance $Z(p)$, the transformer inductance L, and the surge impedance of the transmission line z. In accordance with the theory of traveling waves, the voltage at the transformer terminal, and the neutral voltage, corresponding to a traveling incident wave e on the transmission line, are respectively:

$$e_1 = \frac{2\,[Z(p) + p\,L]}{Z(p) + p\,L + z}\,e(t)$$

$$e_n = \frac{2\,Z(p)}{Z(p) + p\,L + z}\,e(t)$$

Case I. $Z(p) = 1/p\,C_o$

For a capacitor in the neutral, and an infinite rectangular wave, there is

$$e_1 = 2\,E\left[1 - \frac{2\,\alpha}{\omega}\,\epsilon^{-\alpha t}\sin\omega\,t\right]$$

$$e_n = 2\,E\left[1 - \frac{\omega_o}{\omega}\,\epsilon^{-\alpha t}\sin\left(\omega\,t + \tan^{-1}\frac{\omega}{\alpha}\right)\right]$$

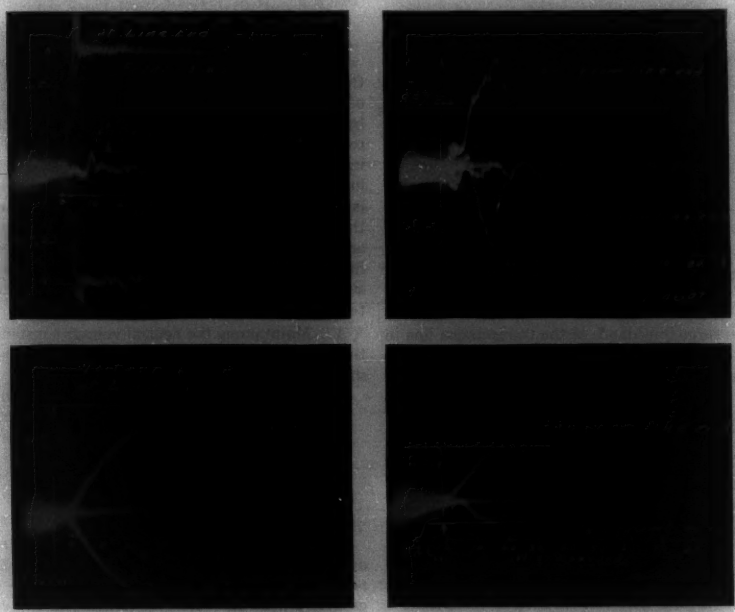

Fig. 9—Effect of Wave Front

Top, applied wave of 3-microsecond front
Bottom, applied wave of 50-microsecond front

where

$$\alpha = z/2\,L$$
$$\omega_o^2 = 1/L\,C_o$$
$$\omega^2 = \omega_o^2 - \alpha^2$$

Thus the neutral voltage oscillates with a damped amplitude of practically $2\,E$ about the $2\,E$ line as an axis; so that the neutral voltage may rise to nearly four times the voltage of the incident wave for infinite rectangular wave excitation. Superimposed hereon are the internal high frequency oscillations, and if the fun-

damental natural frequency of the winding is in the neighborhood of that of the neutral, voltages of the order of three times the terminal voltage will be experienced at points in the winding. Of course, if the applied wave happens to be oscillatory with a near resonant frequency, then exceedingly high voltages will be built up and destruction of the apparatus may occur. For these reasons a capacitor by itself is not a sufficient guarantee that the neutral voltage will be controlled for all applied wave shapes and conditions.

Case IV. The Impedor

Another method of controlling the neutral voltage under transient conditions is by means of the impedor.[4] It consists, essentially of three elements: (1) the normal frequency neutral impedance, (2) a capacitor which is effective in holding down the neutral voltage for most lightning waves, and (3) a resistor having a decided negative characteristic in series with a gap, to take care of extremely long lightning waves, or arcing ground and switching surges. As an example of how the effects of such an impedor may be calculated by a tabular step-by-step method of successive approximations, let:

L_o = inductance of the normal-frequency neutral reactor
C_o = capacitance of the impedor
z = surge impedance of transmission line
e = incident wave
e_n = neutral voltage
i = total current in the neutral
i_L = current in the neutral reactor
i_C = current in the neutral capacitor
i_R = current in the neutral resistor

Then by traveling wave theory

$$2e - zi = L\frac{di}{dt} + e_n$$

$$L_o\frac{di_L}{dt} = e_n$$

$$C_o\frac{de_n}{dt} = i_C$$

$i_R = f(e_n)$ from the negative resistance characteristic.

Substituting increments for differentials and rearranging

$$\Delta i_L = \frac{e_n}{L_o}\Delta t$$

$$i_L = \sum \Delta i_L$$

$$\Delta i = \left(\frac{2e - zi - e_n}{L}\right)\Delta t$$

$$i = \sum \Delta i$$

$$i_C = (i - i_R - i_L)$$

$$\Delta e_n = \frac{i_C}{C}\Delta t$$

In the above equations the average values of the variables over the interval Δt are to be used. Hereby a step-by-step method of successive approximations may be followed as shown in the following table. Such a table is filled in lightly with pencil, and each figure arrived at is readjusted by trial until all of the conditions specified by the above equations are fulfilled to the required degree of accuracy.

Δt	t	$2e$	e_n	Δe_n	i_C	i_R	i_L	Δi_L	Δi	i	Zi
	0	2000	0		0	0	0			0	0
10				33				1	199		
	10	2000	33		198	0.1	1			199	60
10				91				3	166		
	20	2000	124		344	17	4			365	110
10				116				6	168		
	30	2000	240		343	170	10			523	158
10				83				9	154		
	40	2000	323		162	495	19			677	203

Elimination of Oscillations by Shielding. If the neutral is maintained at substantially zero potential during the transient, then the axis of oscillations is

$$e_{t=\infty} = E\frac{\sinh \beta x}{\sinh \beta}$$

where

$$\beta = \sqrt{\frac{rG}{1 + rg}}$$

r = series resistance in the winding
g = shunted conductance along the stack
G = conductance to ground

However, in a normal transformer β is so small that

$$e_{t=\infty} = E\frac{\sinh \beta x}{\sinh \beta} \cong E\frac{\beta x}{\beta} = Ex$$

The initial distribution is

$$e_{t=0} = E\frac{\sinh \alpha x}{\sinh \alpha}$$

where

$\alpha = \sqrt{C/K}$
C = capacitance to ground
K = series capacitance along stack

Evidently the initial and final distributions are identical if

$$\alpha = \beta \quad \text{or} \quad \frac{C}{K} = \frac{r\,G}{1 + r\,g}$$

and all transient oscillations vanish. It is then simply a question as to which of the two it is most feasible to adjust. In practise it has been found much better to modify the initial distribution to conform to the final distribution than the reverse, because the initial distribution causes an excessive concentration of stress near the line end. There are three ways in which this may be accomplished:

Case I.

The conventional method of electrostatic shielding as employed on non-resonating transformers is shown in Fig. 10B. The auxiliary capacitances C_1', C_2', C_3', etc., are designed so that they supply the charging current of the capacitances C to ground, thereby relieving the series capacitances K of that current. At any point x

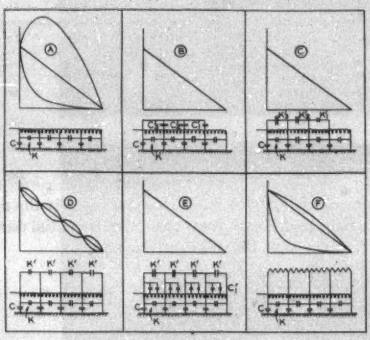

Fig. 10—Methods of Shielding

the potential of the winding above ground for a linear distribution is $x\,E$, and the voltage between this point and the line terminal is $(1 - x)\,E$. Therefore in order that C_x' may supply the charging current to C at point x there must be

$$(1 - x)\,E\,C_x' = x\,E\,C$$

$$\therefore C_x' = \frac{x}{1 - x}\,C$$

Consequently, the same current flows through the entire series capacitance path, K, and the distribution of voltage along the stack is linear. In effect, the ground capacitance C has been canceled or nullified by the shielding capacitance C', and therefore $\alpha = \sqrt{C/K} = 0$ and

$$e = E\,\frac{\sinh \alpha\,x}{\sinh \alpha} \cong E\,\frac{\alpha\,x}{\alpha} = E\,x$$

Case II.

Fig. 10C illustrates the method of shielding by means of auxiliary series capacitances K_1', K_2', K_3', etc. Suppose that the winding has been divided into n equal sections, and let the junctions be numbered $0, 1, 2, \ldots n$ from the neutral. Then for a linear distribution of voltage the conditions at junction x are

$$e = \frac{E}{n} = \text{voltage per section}$$

$$e_x = E\,\frac{x}{n} = \text{potential to ground}$$

$$E - e_x = \left(1 - \frac{x}{n}\right) E = \text{potential to the line terminal}$$

By Kirchhoff's law

$$\begin{aligned} C\,e_x &= C\,e\,x = (K + K_x)\,e - (K + K'_{x-1})\,e \\ K_x' &= x\,C + K'_{x-1} \\ &= x\,C + (x - 1)\,C + K'_{x-2} \\ &= x\,C + (x - 1)\,C + (x - 2)\,C + K'_{x-3} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &= [x + (x - 1) + (x - 2) + \ldots\ldots + 1]\,C + 0 \\ &= \frac{x\,(x + 1)}{2}\,C \end{aligned}$$

Case III.

Increase K, by adding auxiliary series capacitances K' as shown in Fig. 10D to such an extent, that

$$\alpha = \sqrt{\frac{C}{K}} \to 0$$

$$E\,\frac{\sinh \alpha\,x}{\sinh \alpha} \cong E\,\frac{\alpha\,x}{\alpha} = E\,x$$

The capacitance of the K' units can be made quite sufficient to give a practically linear initial distribution of the connected points, but since the sections of the winding between points are not equalized, local oscillations take place as shown in Fig. 10D. If it is desirable to suppress these local oscillations it may be effectively done by combining the schemes shown in Fig. 10B and Fig. 10D as indicated in Fig. 10E. Here the auxiliary capacitances $(C_1', C_2', \ldots)$ have been added to the circuit. There is a group for each section of the winding fed from the higher potential end of the section. The design is similar to that of Case I. Let points x_1 and x_2 be adjacent points on the winding maintained at the appropriate potentials $e_1 = x_1\,E$ and $e_2 = x_2\,E$ by the large auxiliary series capacitors K'. Let x be a point intermediate between x_1 and x_2 at which it is required to connect a capacitor C_x' such that the point x will be at potential $x\,E$. Then the voltage across C_x' is $(x_2 - x)\,E$, and therefore in order that C_x' may supply the charg-

ing current of the capacitance C of the winding to ground there must be

$$(x_2 - x)\, E\, C_x' = x\, E\, C$$

$$C_x' = \frac{x}{x_2 - x}\, C$$

It will be noticed in this scheme that the C_x' capacitances need not be insulated from the winding by more than the potential difference across one section of the winding, thus permitting a very economical type of shield to be used. However, the local oscillations in the individual sections are not dangerous, and ordinarily would not be compensated.

Elimination of Oscillations by Losses. In the general solution given in Appendix I it was shown that the effect of losses is threefold:

1. All harmonics suffer a decrement
2. The periods of oscillation are increased
3. Certain oscillations degenerate to exponential decay.

By increasing the losses sufficiently it is possible to prohibit all oscillations. The initial distribution then diffuses into the final distribution without over-shooting, and dangerous abnormal voltages are avoided. It may not be practicable to absolutely prohibit all oscillations, and a small amount of over-shooting may occur as shown in Fig. 10F. Perhaps the most simple method of controlling the distribution by losses is through the use of a tied-in shunt resistor. However, this shunt resistor must have such a low resistance to be effective against high-voltage surges, that its losses and heating under normal conditions become excessive, unless the material has a decided negative characteristic. Thyrite has been used to excellent advantage as a shunt resistor for current limiting reactors.[8]

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Appendix I

The General Differential Equation. Referring to Fig. 1, the circuit constants per unit length of winding are:

L = effective inductance including the mutual flux.
$M(x, y)$ = mutual inductance between elements at x and y.
C = shunt capacitance to ground.
K = series capacitance along the winding.
G = shunt conductance to ground.
g = shunt inductance along the winding.
r = series resistance.
n = turns.

The variables involved at any point of the winding are:

e = potential to ground.
i_1 = current in series capacitance K.
i_2 = current in the inductance L.
i_3 = current in the shunt conductance g.
i_4 = current to ground through G.
ϕ = total flux linkages at a point.
B = flux density.
t = time.
p = $\partial/\partial t$ = partial derivative with respect to time.
x, y = points along the winding, measured from the neutral end.
l = length of the winding.
$(m\,l\,t)$ = mean length of turn.
h = length of the leakage path.

The fundamental relationships are:

$$i_1 = K \frac{\partial^2 e}{\partial x\, \partial t} \tag{1}$$

$$i_3 = g \frac{\partial e}{\partial x} \tag{2}$$

$$i_4 = \left(G + C \frac{\partial}{\partial t}\right) e = \frac{\partial}{\partial x}(i_1 + i_2 + i_3) \tag{3}$$

$$\frac{\partial e}{\partial x} = r\, i_2 + \frac{n}{10^8} \frac{\partial \varphi}{\partial t} \tag{4a}$$

$$= r\, i_2 + \frac{\partial}{\partial t} \int_0^l M(x, y)\, i_2(y)\, .\, d\, y \tag{4b}$$

$$= r\, i_2 + \frac{\partial}{\partial t} \left\{ L'\, i_2(x) + \int_0^l M(x, y)\, [i_2(y) - i_2(x)]\, d\, y \right\} \tag{4c}$$

where

$$L' = \int_0^l M(x, y)\, d\, y = \text{self inductance}$$

$$\varphi = \varphi_m + \varphi_l = \varphi_m - (m\, l\, t) \int_0^x B_1\, d\, y + (m\, l\, t) \int_x^l B_2\, d\, y \tag{5}$$

where

φ_m = flux mutual to the entire winding.
φ_l = flux due to partial interlinkages.
B_1 = flux density for $0 < y < x$
B_2 = flux density for $x < y < l$

Assume, as simple approximation, that the flux density is distributed as a linear function

$$B_1 = \frac{-\,.4\,\pi\, n}{h} \int_x^l i_2\,(y)\, d\, y \tag{6}$$

$$B_2 = \frac{+\,.4\,\pi\, n}{h} \int_0^x i_2\,(y)\, d\, y \tag{7}$$

From (4a), (5), (6) and (7) there is

$$\frac{\partial^4 e}{\partial x^4} = r\, \frac{\partial^3 i_2}{\partial x^3} - \frac{.8\,\pi\, n^2 (m l t)}{h\, 10^8}\, \frac{\partial^2 i_2}{\partial x\, \partial t}$$

$$= r\, \frac{\partial^3 i_2}{\partial x^3} - \frac{L}{l^3}\, \frac{\partial^2 i_2}{\partial x\, \partial t} \tag{8}$$

where

$$L = \frac{.8\,\pi\, n^2\, l^3\, (m\, l\, t)}{h\, 10^8} = \text{effective inductance}$$

By (1), (2), and (3) there is

$$\frac{L}{l^3}\, \frac{\partial^2 i_2}{\partial x\, \partial t} = \frac{L}{l^3} \left(G + C\, \frac{\partial}{\partial t} \right) \frac{\partial e}{\partial t}$$

$$- \frac{L}{l^3}\, \frac{\partial^2 i_1}{\partial x\, \partial t} - \frac{L}{l^3}\, \frac{\partial^2 i_3}{\partial x\, \partial t} = \frac{L}{l^3}\, G\, \frac{\partial e}{\partial t}$$

$$+ \frac{L}{l^3}\, C\, \frac{\partial^2 e}{\partial t^2} - \frac{L}{l^3}\, K\, \frac{\partial^4 e}{\partial x^2\, \partial t^2} - g\, \frac{L}{l^3}\, \frac{\partial^3 e}{\partial x^2\, \partial t} \tag{9}$$

and

$$r\, \frac{\partial^3 i_2}{\partial x^3} = r\, G\, \frac{\partial^2 e}{\partial x^2} + r\, C\, \frac{\partial^3 e}{\partial x^2\, \partial t} - r\, K\, \frac{\partial^5 e}{\partial x^4\, \partial t}$$

$$- g\, r\, \frac{\partial^4 e}{\partial x^4} \tag{10}$$

Substituting (9) and (10) in (8) there results

$$r\, K\, \frac{\partial^5 e}{\partial x^4\, \partial t} + (1 + g\, r)\, \frac{\partial^4 e}{\partial x^4} - \frac{L}{l^3}\, K\, \frac{\partial^4 e}{\partial x^2\, \partial t^2}$$

$$- \left(r\, C + g\, \frac{L}{l^3} \right) \frac{\partial^3 e}{\partial x^2\, \partial t} - r\, G\, \frac{\partial^2 e}{\partial x^2} + \frac{L}{l^3}\, C\, \frac{\partial^3 e}{\partial t^3}$$

$$+ \frac{L}{l^3}\, G\, \frac{\partial e}{\partial t} = 0 \tag{11}$$

If the losses can be neglected equation (11) reduces to

$$\frac{\partial^4 e}{\partial x^4} - \frac{L\, K}{l^3}\, \frac{\partial^4 e}{\partial x^2\, \partial t^2} + \frac{L}{l^3}\, C\, \frac{\partial^2 e}{\partial t^2} = 0 \tag{12}$$

Hereafter it will be convenient to take $l = 1$.

The total current is, from (3)

$$(i_1 + i_2 + i_3) = \left(G + C\, \frac{\partial}{\partial t} \right) \int e\, d\, x \tag{13}$$

The solution to these equations must satisfy

a. The differential equation
b. The terminal conditions at $x = 0$ and $x = l$
c. The initial distribution at $t = 0$
d. The final distribution at $t = \infty$

If the solution corresponding to a constant sustained potential suddenly applied at $x = l$ can be found, then the solution for any other applied terminal voltage is given by Duhamel's Theorem. The usual procedure in solving a partial differential equation is to assume the form of the solution and try it by direct substitution in the differential equation and the boundary conditions. Each tentative trial usually suggests the necessary changes and adjustments in order to meet the complete specifications. Therefore, in order to choose the proper solution from among the infinite number of functions which will satisfy the differential equations, it is necessary to first investigate the boundary conditions.

The Initial Distribution. When an infinite rectangular wave is applied at the terminal of the winding, the currents in the capacitances at the first instant are infinite, since the time rate of change of voltage is infinite; whereas the current in the inductive winding is zero, and in the resistances the currents are all finite. Therefore the initial distribution of potential depends only upon the capacitances, Fig. 1B, and can be determined by solving the differential equation for the capacitances alone. Consequently, considering only the capacitances of the circuit, the combination of equations (1) and (3) gives ($G = 0$).

$$\frac{\partial^2 e}{\partial x^2} - \frac{C}{K}\, e = \frac{\partial^2 e}{\partial x^2} - \alpha^2\, e = 0 \tag{14}$$

where

$$\alpha = \sqrt{C/K}$$

This equation also follows from the general differential equation (11) upon dividing through by $p^2 = \partial^2/\partial t^2$ and putting $p = \infty$, according to the procedure in operational calculus.

The solution to (14) is:

$$e = A\, \epsilon^{\alpha x} + B\, \epsilon^{-\alpha x} \tag{15}$$

and from (1) the corresponding current is

$$i = K\,p\,\frac{\partial e}{\partial x} = K\,p\,\alpha\,(A\,\epsilon^{\alpha x} - B\,\epsilon^{-\alpha x}) \tag{16}$$

where $p \to \infty$, and the initial rush of current is therefore infinite.

Suppose that the winding is grounded at $x = 0$ through a generalized impedance $Z\,(p)$, and that the voltage applied at $x = 1$ is E. Then

$$\left.\begin{aligned} &\text{at } x = 1,\ e = E = A\,\epsilon^{\alpha} + B\,\epsilon^{-\alpha} \\ &\text{at } x = 0,\ e = Z\,(p)i = Z(p)p\sqrt{C\,K}(A-B) = A+B \end{aligned}\right\} \tag{17}$$

Herefrom the integration constants are

$$\left.\begin{aligned} A &= \frac{1}{2}\,\frac{[Z\,(p)\,p\,\sqrt{C\,K} + 1]\,E}{Z\,(p)\,p\,\sqrt{C\,K}\cosh\alpha + \sinh\alpha} \\ B &= \frac{1}{2}\,\frac{[Z\,(p)\,p\,\sqrt{C\,K} - 1]\,E}{Z\,(p)\,p\,\sqrt{C\,K}\cosh\alpha + \sinh\alpha} \end{aligned}\right\} \tag{18}$$

and the initial distribution therefore is

$$e = E\,\frac{Z\,(p)\,p\,\sqrt{C\,K}\cosh\alpha\,x + \sinh\alpha\,x}{Z\,(p)\,p\,\sqrt{C\,K}\cosh\alpha + \sinh\alpha}\Bigg]_{p\,=\,\infty} \tag{19}$$

For a grounded neutral $Z\,(p) = 0$ and

$$e = \frac{\sinh\alpha\,x}{\sinh\alpha}\,E \tag{20}$$

For an isolated neutral, $Z\,(p) = \infty$ and

$$e = \frac{\cosh\alpha\,x}{\cosh\alpha}\,E \tag{21}$$

For a capacitance C_o in the neutral $Z\,(p) = 1/p\,C_o$ and

$$e = \frac{\sqrt{C\,K}\cosh\alpha\,x + C_o\sinh\alpha\,x}{\sqrt{C\,K}\cosh\alpha + C_o\sinh\alpha}\,E$$

For an inductance L_o in the neutral $Z\,(p) = p\,L_o$ and

$$e = \frac{L\,p^2\,\sqrt{C\,K}\cosh\alpha\,x + \sinh\alpha\,x}{L\,p^2\,\sqrt{C\,K}\cosh\alpha + \sinh\alpha}\Bigg]_{p\,=\,\infty} E$$

$$= \frac{\cosh\alpha\,x}{\cosh\alpha}\,E \tag{23}$$

Equation (19) may be expressed as a half-range sine series

$$e = 2\sum_{1}^{\infty}\sin s\,\pi\,x\int_{0}^{1} e\sin s\,\pi\,x\,d\,x$$

$$= E$$

$$\sum_{1}^{\infty}\left\{\frac{p\,Z(p)\,\sqrt{CK}(1-\cos s\,\pi\cosh\alpha) - \cos s\,\pi\sinh\alpha}{p\,Z(p)\,\sqrt{C\,K}\cosh\alpha + \sinh\alpha}\right\}$$

$$\frac{2\,s\,\pi}{\alpha^2 + s^2\,\pi^2}\sin s\,\pi\,x \tag{24}$$

The Final Distribution. After the transient incident to the application of an infinite rectangular wave at the line terminals has died out (theoretically at $t = \infty$) the residual distribution is direct current. The capacitance elements then act as open circuits and the inductance elements as short circuits. The distribution is then entirely determined by the resistance elements, Fig. 1c. The differential equation for this circuit from (2), (3), and (4) is

$$\frac{\partial^2 e}{\partial x^2} - R\,G\,e = \frac{\partial^2 e}{\partial x^2} - \beta^2\,e = 0 \tag{25}$$

where

$$R = \frac{r}{1 + g\,r} = \text{resistance of } r \text{ and } g \text{ in parallel}$$

and

$$\beta^2 = R\,G$$

This equation also follows from the general differential equation (11) upon substituting $p = \partial/\partial\,t = 0$ according to the procedure in operational calculus.

The solution to (25) is

$$e = A\,\epsilon^{\beta x} + B\,\epsilon^{-\beta x} \tag{26}$$

and from (2) and (4) the corresponding current is

$$i = \frac{1}{R}\,\frac{\partial\,e}{\partial\,x} = \sqrt{\frac{G}{R}}\,(A\,\epsilon^{\beta x} - B\,\epsilon^{-\beta x}) \tag{27}$$

The terminal conditions are

$$\left.\begin{aligned} &\text{at } x = 1,\ e = E = A\,\epsilon^{\beta} + B\,\epsilon^{-\beta} \\ &\text{at } x = 0,\ e = Z(p)i = Z(p)\sqrt{\frac{G}{R}}(A-B) = A+B \end{aligned}\right\} \tag{29}$$

Herefrom the integration constants are

$$\left.\begin{aligned} A &= \frac{1}{2}\,\frac{[Z\,(p)\,\sqrt{G/R} + 1]\,E}{Z\,(p)\,\sqrt{G/R}\cosh\beta + \sinh\beta} \\ B &= \frac{1}{2}\,\frac{[Z\,(p)\,\sqrt{G/R} - 1]\,E}{Z\,(p)\,\sqrt{G/R}\cosh\beta + \sinh\beta} \end{aligned}\right\} \tag{30}$$

and the final distribution therefore is

$$e = E\,\frac{Z\,(p)\,\sqrt{G/R}\cosh\beta\,x + \sinh\beta\,x}{Z\,(p)\,\sqrt{G/R}\cosh\beta + \sinh\beta}\Bigg]_{p\,\to\,0} \tag{31}$$

For a grounded neutral or an inductance in the neutral $Z(p) = 0$ and

$$e = E\,\frac{\sinh\beta\,x}{\sinh\beta} \tag{32}$$

For an isolated neutral or a capacitance in the neutral $Z\,(p) = \infty$ and

$$e = E\,\frac{\cosh\beta\,x}{\cosh\beta} \tag{33}$$

For a resistance R_o in the neutral, $Z\,(p) = R_o$ and

$$e = E\,\frac{R_o\,\sqrt{G/R}\cosh\beta\,x + \sinh\beta\,x}{R_o\,\sqrt{G/R}\cosh\beta + \sinh\beta} \tag{34}$$

In a normal machine β is a very small quantity, so

that approximately, using only the first terms in the cosh and sinh series,

$$e = E\,\frac{Z\,(p)\,\sqrt{G/R} + \sqrt{R\,G}\;x}{Z\,(p)\,\sqrt{G/R} + \sqrt{R\,G}} = E\,\frac{Z\,(p) + R\,x}{Z\,(p) + R} \qquad (35)$$

Solution for Grounded Neutral

As a tentative solution to equation (11) assume

$$e = E\,\frac{\sinh \beta\, x}{\sinh \beta} + \sum A\,\epsilon^{ax + bt} \qquad (36)$$

The first term represents the axis of oscillations as given by equation (32) in which

$$\beta = \sqrt{\frac{r\,G}{1 + g\,r}}$$

and the summation term is the usual trial solution for a differential equation of this type. Substituting (36) in (11) there results

$$r\,K\,a^4\,b + (1 + g\,r)\,a^4 - L\,K\,a^2\,b^2 - (r\,C + g\,L)\,a^2\,b - r\,G\,a^2 + L\,C\,b^2 + L\,G\,b = 0 \qquad (37)$$

Solving this equation for b there is

$$b = -\,\frac{r\,K\,a^4 - (r\,C + g\,L)\,a^2 + L\,G}{2\,L\,(C - K\,a^2)}$$

$$\pm\,\frac{\sqrt{(r\,K\,a^4 - r\,C\,a^2 - g\,L\,a^2 + L\,G)^2 - 4\,L(C - K\,a^2)\,(a^4 + g\,r\,a^4 - r\,G\,a^2)}}{2\,L\,(C - K\,a^2)}$$

$$= -\,\gamma \pm j\,\omega \qquad (38)$$

Write $\lambda^2 = -\,a^2$ or $a = \pm\,j\,\lambda$ so that (36) takes the form

$$e = E\,\frac{\sinh \beta\, x}{\sinh \beta} + \sum \epsilon^{-\gamma t}\,[A \sin \lambda\, x\,.\,\cos \omega\, t + B \sin \lambda\, x\,.\,\sin \omega\, t + C \cos \lambda\, x\,.\,\cos \omega\, t + D \cos \lambda\, x\,.\,\sin \omega\, t] \qquad (39)$$

Comparing this equation at $t = 0$ with equation (24) for the initial distribution it is evident that $C = 0$ and

$$\lambda = s\,\pi \qquad (40)$$

and the corresponding values of γ and ω are determined from (39). When $x = 1$, equation (40) gives

$$e = E + \sum \epsilon^{-\gamma t}\,(C \cos \omega\, t + D \sin \omega\, t) \cos s\,\pi \qquad (41)$$

and since $e = E$ at $x = 1$ it follows that $C = D = 0$. Therefore,

$$e = E\,\frac{\sinh \beta\, x}{\sinh \beta} + \sum_{s=1}^{\infty} \epsilon^{-\gamma_s t}\,(A_s \cos \omega_s\, t + B_s \sin \omega_s\, t) \sin s\,\pi\, x \qquad (42)$$

The A_s coefficients are determined by a comparison with equation (20). Putting $t = 0$ in (42), equating to (25) and expressing the *sinh* as a half-range *sine* series there is

$$E \sum_1^{\infty} \frac{-\,2\,s\,\pi}{\beta^2 + \pi^2\,s^2} \cos s\,\pi \sin s\,\pi\, x + \sum_1^{\infty} A_s \sin s\,\pi\, x = E \sum_1^{\infty} \frac{-\,2\,s\,\pi}{\alpha^2 + s^2\,\pi^2} \cos s\,\pi\,.\,\sin s\,\pi\, x \qquad (43)$$

Therefore

$$A_s = \frac{2\,s\,\pi\,(\cos s\,\pi)\;(\alpha^2 - \beta^2)}{(\beta^2 + s^2\,\pi^2)\;(\alpha^2 + s^2\,\pi^2)}\,E \qquad (44)$$

The coefficients B_s are zero, as is evident from the condition that the current i_2 through the inductance is zero for $t = 0$. Then the differential equation and the initial, final, and terminal conditions are all satisfied. Therefore the solution is

$$e = E\,\frac{\sinh \beta\, x}{\sinh \beta} + \mathrm{E} \sum_1^{\infty} \frac{2\,(-1)^s\,s\,\pi\,.\,(\alpha^2 - \beta^2)}{(\beta^2 + s^2\,\pi^2)\;(\alpha^2 + s^2\,\pi^2)}\,\epsilon^{-\gamma_s t} \cos \omega_s\, t \sin s\,\pi\, x \qquad (45)$$

If the losses are zero ($r = g = G = 0$), then $\beta = 0$ and equation (38) reduces to

$$b = \pm\,\frac{\sqrt{-4\;L(C - K\;a^2)a^4}}{2\,L(C - K\,a^2)} = \pm\,j\,\frac{\lambda^2}{\sqrt{L(C + K\,\lambda^2)}}$$

$$= \pm\,j\,\frac{s^2\,\pi^2}{\sqrt{L(C + K\,s^2\,\pi^2)}} = \pm\,j\,\omega_s \qquad (46)$$

and the solution for zero losses therefore is

$$e = E\,x + E \sum_1^{\infty} \frac{2 \cos s\,\pi}{s\,\pi} \left(\frac{\alpha^2}{\alpha^2 + s^2\,\pi^2}\right) \sin s\,\pi\, x \cos \frac{s^2\,\pi^2\,t}{\sqrt{L(C + K\,s^2\,\pi^2)}} \qquad (47)$$

This equation cannot be obtained by a direct application of the Heaviside expansion theorem.

Solution for Isolated Neutral

As mentioned in the last section, the Heaviside expansion theorem breaks down when applied to the solution of the differential equation given in this Appendix as equation (12). The author wrote to Dr. E. J. Berg of Union College for an explanation of the failure of the theorem. In subsequent discussions between Dr. Berg and his associates, Dr. J. J. Smith pointed out that the Heaviside expansion theorem depends upon an expansion in terms of partial fractions, and that in the particular problem, functions are encountered which do not lend themselves to such an expansion. Professor S. J. Haefner then modified the procedure so that the solution could be obtained by operational methods closely allied to the Heaviside expansion theorem, but in certain respects more extended in scope. The following operational solution of equation (12) for the case of an

isolated neutral is by Haefner's method, and is given here pending a more detailed and rigorous treatment by him.

Rewriting equation (**12**) symbolically

$$(D^4 - L\,K\,p^2\,D^2 + L\,C\,p^2)\,e = 0 \tag{48}$$

where $D = \partial/\partial x$ and $p = \partial/\partial t$.

The operational solution to (**48**) then is

$$e = A \sinh \lambda x + B \cosh \lambda x + A' \sinh \lambda' x + B' \cosh \lambda' x$$

where the roots of (**48**) with respect to D are

$$\left.\begin{aligned} \lambda &= \sqrt{\frac{L\,K\,p^2}{2}\left(1 - \sqrt{1 - \frac{4\,C}{L\,K^2\,p^2}}\right)} \\ \lambda' &= \sqrt{\frac{L\,K\,p^2}{2}\left(1 + \sqrt{1 - \frac{4\,C}{L\,K^2\,p^2}}\right)} \end{aligned}\right\} \tag{49}$$

The current is

$$i = C \int p\,e\,d\,x = C\,p\left[\frac{A \cosh \lambda x + B \sinh \lambda x}{\lambda} + \frac{A' \cosh \lambda' x + B' \sinh \lambda' x}{\lambda'}\right] \tag{50}$$

For an isolated neutral

$$\left.\begin{aligned} i &= 0 \text{ at } x = 0 \\ e &= E \text{ at } x = 1 \end{aligned}\right\} \tag{51}$$

Therefore the terminal conditions are satisfied by

$$\left.\begin{aligned} A &= A' = B' = 0 \\ B &= \frac{E}{\cosh \lambda} \end{aligned}\right\} \tag{52}$$

and the solution is

$$e = E\,\frac{\cosh \lambda x}{\cosh \lambda} \tag{53}$$

Now the partial fraction theorem gives

$$\frac{Y(\lambda)}{Z(\lambda)} = \sum_{s=1}^{\infty} \frac{Y(\lambda_s)}{Z'(\lambda_s)}\,\frac{1}{(\lambda - \lambda_s)} = \frac{Y(0)}{Z(0)} + \sum_{1}^{\infty} \frac{Y(\lambda_s)}{\lambda_s\,Z'(\lambda_s)}\left(\frac{\lambda}{\lambda - \lambda_s}\right) \tag{54}$$

where λ_s are the roots of $Z(\lambda) = 0$. Applying (**54**) to (**53**) there is

$$Z(\lambda) = \cosh \lambda = \prod_{s=1}^{\infty}\left[1 + \frac{4\,\lambda^2}{(2\,s-1)^2\,\pi^2}\right] \tag{55}$$

$$\therefore \lambda_s = \pm j\,(2\,s - 1)\,\frac{\pi}{2} \tag{56}$$

$$Z'(\lambda_s) = \left.\frac{d\,Z(\lambda)}{d\,\lambda}\right]_{\lambda = \lambda_s} = \sinh(\lambda_s) = \pm j \sin(2\,s - 1)\,\frac{\pi}{2} \tag{57}$$

$$Y(\lambda_s) = E \cosh \lambda_s x = E \cos(2\,s - 1)\,\frac{\pi\,x}{2} \tag{58}$$

Therefore, using both forms of (**54**) there is

$$e = -2\,E\sum_{1}^{\infty}(-1)^s \cos(2\,s-1)\,\frac{\pi\,x}{2}\left(\frac{(2\,s-1)\,\pi/2}{\lambda^2 + (2\,s-1)^2\,\frac{\pi^2}{4}}\right)$$

$$= E + 2\,E\sum_{1}^{\infty}(-1)^s\,\frac{\cos(2\,s-1)\,\pi\,x/2}{(2\,s-1)\,\pi/2}\left(\frac{\lambda^2}{\lambda^2 + (2\,s-1)^2\,\frac{\pi^2}{4}}\right) \tag{59}$$

from which the half-range cosine series for E is easily obtained.

$$E = -2\,E\sum_{1}^{\infty}(-1)^s\,\frac{\cos(2\,s-1)\,\pi\,x/2}{(2\,s-1)\,\pi/2} \tag{60}$$

Substituting (**59**) in (**48**) and making use of (**60**)

$$\lambda^2 = \frac{L\,C\,p^2}{(2\,s-1)^2\,\frac{\pi^2}{4} + L\,K\,p^2} \tag{61}$$

so that (**59**) becomes

$$e = E + 2\,E\sum_{1}^{\infty}(-1)^s\,\frac{\cos(2\,s-1)\,\pi\,x/2}{(2\,s-1)\,\pi/2}\left(\frac{C}{C + K\,(2\,s-1)^2\,\pi^2/4}\right)\left(\frac{p^2}{p^2 + \omega_s^2}\right)$$

$$= E + 2\,E\sum_{1}^{\infty}(-1)^s\,\frac{\cos(2\,s-1)\,\pi\,x/2}{(2\,s-1)\,\pi/2}\,\frac{\alpha^2}{\alpha^2 + (2\,s-1)^2\,\pi^2/4}\cos \omega_s t \tag{62}$$

where

$$\omega_s = \frac{(2\,s-1)^2\,\pi^2/4}{\sqrt{L\,C + L\,K\,(2\,s-1)^2\,\pi^2/4}} \tag{63}$$

Appendix II

Negligible Mutual Inductance and No Losses

The fundamental circuit equations are

$$i_1 = K\,\frac{\partial^2 e}{\partial x\,\partial t} \tag{1}$$

$$i_4 = C\,\frac{\partial e}{\partial t} = \frac{\partial}{\partial x}(i_1 + i_2) \tag{2}$$

$$\frac{\partial e}{\partial x} = L\,\frac{\partial i_2}{\partial t} \tag{3}$$

From these three equations the differential equation is

$$K L \frac{\partial^4 e}{\partial x^2 \partial t^2} + \frac{\partial^2 e}{\partial x^2} - L C \frac{\partial^2 e}{\partial t^2} = 0 \qquad (4)$$

or substituting $p = \partial / \partial t$ and rewriting in symbolic notation

$$\frac{\partial^2 e}{\partial x^2} - \frac{L C p^2}{K L p^2 + 1} e = 0 \qquad (5)$$

The solution to (5) with respect to x is

$$e = A \epsilon^{\sigma x} + B \epsilon^{-\sigma x} \qquad (6)$$

where

$$\sigma = \sqrt{\frac{L C p^2}{K L p^2 + 1}} \qquad (7)$$

From (2) and (6) the total current in the series path is

$$i = (i_1 + i_2) = \int C \frac{\partial e}{\partial t} d x = \frac{C p}{\sigma} (A \epsilon^{\sigma x} - B \epsilon^{-\sigma x}) \qquad (8)$$

The terminal conditions are

$$\left.\begin{aligned} e &= E \text{ at } x = 1 \\ Z(p)\, i &= e \text{ at } x = 0 \end{aligned}\right\} \qquad (9)$$

From (6), (8) and (9) the integration constants are

$$\left.\begin{aligned} A &= \frac{C p Z(p) + \sigma}{C p Z(p) \cosh \sigma + \sigma \sinh \sigma} \frac{E}{2} \\ B &= \frac{p Z(p) - \sigma}{C p Z(p) \cosh \sigma + \sigma \sinh \sigma} \frac{E}{2} \end{aligned}\right\} \qquad (10)$$

and substituting (10) in (6) there is

$$e = \frac{C p Z(p) \cosh \sigma x + \sigma \sinh \sigma x}{C p Z(p) \cosh \sigma + \sigma \sinh \sigma} E = \frac{Y(p)}{H(p)} \qquad (11)$$

This operational equation may be solved by the Heaviside expansion theorem

$$e = \frac{Y(p)}{H(p)} = \frac{Y(o)}{H(o)} + \sum \frac{Y(p_s)\, \epsilon^{p_s t}}{p_s H'(p_s)} \qquad (12)$$

where the summation is to include all of the roots of $H(p) = 0$

Applying (12) to (11) there is

$$Y(p) = C p Z(p) \cosh \sigma x + \sigma \sinh \sigma x \qquad (13)$$

$$H(p) = C p Z(p) \cosh \sigma + \sigma \sinh \sigma \qquad (14)$$

$$\frac{Y(o)}{H(o)} = \frac{Z(o) + p L x}{Z(o) + p L} \Big/_{p=0} \qquad (15)$$

The roots of (14) are given by the transcendental equation

$$\tanh \sigma = - \frac{C p Z(p)}{\sigma} \qquad (16)$$

and from (7)

$$p = \pm \sqrt{\frac{\sigma^2}{L (C - \sigma^2 k)}} \qquad (17)$$

In the general case, the roots of (16) must be found by methods of approximation.

$$p \frac{d H(p)}{d p} \Big/_{p=p_s} = \left(\sinh \sigma + \frac{\sigma}{\cosh \sigma}\right) \sigma \left(1 - \frac{\sigma^2}{\alpha^2}\right)$$

$$+ p_s \cosh \sigma C \frac{d}{d p} [p Z(p)]_{p=p_s} \qquad (18)$$

This is as far as the generalization can be carried.

If the neutral impedance is a capacitance C_o then

$$Z(p) = \frac{1}{p C_o} \qquad (19)$$

and the last term of (18) vanishes. Now substituting $\sigma = j \zeta$ equation (16) becomes

$$\frac{C_o}{C} \zeta = \cot \zeta = \frac{1}{\zeta} - \frac{\zeta}{3} - \frac{\zeta^3}{45}$$

$$- \frac{2}{945} \zeta^5 - \frac{\zeta^7}{4725} - \ldots \qquad (20)$$

or

$$\zeta^8 + 10 \zeta^6 + 105 \zeta^4 + \left(1575 - 4725 \frac{C_o}{C}\right) \zeta^2 - 4725 \cong 0$$

from which the most important roots of (16) may be determined by approximations. Suppose that this has been done for a specific ratio C_o/C. Then by (17)

$$p = \pm \frac{j \zeta}{\sqrt{L (C + \zeta^2 K)}} = \pm j \omega \qquad (21)$$

Substituting (13), (15), (18) and (21) in the expansion formula (12) there results,

$$e = E - 2 E \sum \frac{C \cos \zeta x - C_o \zeta \sin \zeta x}{C_o \left(\sin \zeta + \frac{\zeta}{\cos \zeta}\right) \zeta \left(1 + \frac{\zeta^2}{\alpha^2}\right)} \cos \omega t \qquad (22)$$

where the roots are given by equation (20).

For a grounded neutral, $Z(p) = 0$ and (16) gives

$$\sinh \sigma = \sigma \prod_1^\infty \left(1 + \frac{\sigma^2}{s^2 \pi^2}\right) = 0 \qquad (23)$$

from which

$$\sigma = \pm j \zeta = \pm j s \pi \qquad (24)$$

and the solution is

$$e = x E + E \sum_1^\infty \frac{\cos s \pi}{s \pi}$$

$$\frac{2 \alpha^2}{\alpha^2 + s^2 \pi^2} \sin s \pi x \cos \frac{s \pi t}{\sqrt{L (C + s^2 \pi^2 K)}} \qquad (25)$$

For an isolated neutral, $Z(p) = \infty$, and (16) gives

$$\cosh \sigma = \prod_{1}^{\infty} \left[1 + \frac{4\,\sigma^2}{\pi^2\,(2\,s-1)^2} \right] = 0 \qquad (26)$$

from which

$$\sigma = \pm j\,\zeta = \pm j\,\frac{(2\,s-1)\,\pi}{2} \qquad (27)$$

and the solution is

$$e = E + E \sum_{1}^{\infty} \frac{\sin s\,\pi/2}{s\,\pi} \frac{16\,\alpha^2}{s^2\,\pi^2 + 4\,\alpha^2} \cos \frac{s\,\pi\,x}{2} \cos \frac{s\,\pi\,t}{\sqrt{L\,(4\,C + K\,s^2\,\pi^2)}} \qquad (28)$$

when $K = 0$ and $\alpha = \sqrt{C/K} \to \infty$, this equation reduces to the equation of an open end transmission line, or

$$e = E + 4\,E \sum_{1}^{\infty} \frac{\sin s\,\pi/2}{s\,\pi} \cos \frac{s\,\pi\,x}{2} \cos \frac{s\,\pi\,t}{2\sqrt{L\,C}} \qquad (29)$$

The Heaviside expansion formula may be used for solving equation (11), whereas the method fails when applied to the differential equation of Appendix I (which includes mutual inductance). For this reason it is feasible to use the less general equations of this Appendix II when investigating the effect of neutral impedances. The suggestion is offered as an alternative to the method described under Part V of the paper.

Field Transients in Magnetic Systems

A Study of Field Transients in Iron Cores Partially Laminated and Partially Solid

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Synopsis.—The complete mathematical solution of applying suddenly a d-c. voltage to an electromagnetic circuit composed of an electric coil with a partly laminated and partly solid iron core containing an air-gap is presented in this paper. Starting from Maxwell's field equations for the eddy currents and the magnetic field in the iron core, the solution of the time conditions is reached with the aid of Heaviside's operational calculus. The result is expressed by means of a time constant of the magnetic field, composed of the time constant of the electric circuit and a fraction of a new-defined time constant of the solid iron part due to the eddy currents in that part. The importance of this result is illustrated by its application to a numerical example and to the effectiveness of quick response excitation in the case of large synchronous generators. Finally some other important applications are discussed.

I. Introduction

When switching an electric circuit on or off without an iron core or with an iron core of constant permeability and infinitely thin laminations, the current and the field (the magnetic flux may be designated shortly by the word "field," while the vectors of the magnetic field H and B may be called magnetic field strength and magnetic induction, respectively) have the same form of time variation,[1] *i.e.*, the same "time constant," by which the exponential curve can be easily characterized. In the case of a solid iron core, eddy currents arise when any change of the exciting current occurs, due to the law of induction. This case was generally treated[2,3,4] as a special problem of the decay of a magnetic field when switching off the circuit rather than considering the true condition of the interlinkage of the eddy currents with the exciting current. In this case of the special problem the time constant of the decreasing magnetic field may be defined.[5]

A further investigation of this subject lead to the solution of a periodically alternating field in a magnetic circuit with solid iron core.[6] This treatment also neglected the interlinkage of the eddy currents and the exciting current. Later the effect of the eddy currents was replaced by a secondary electric circuit.[7] In this case the solution was obtained as in the treatment of coupled electric circuits. This method of solution is given to avoid the use of the field equations, but the results with respect to the magnetic time constant are not sufficiently accurate.

Having the solutions of the decay of a magnetic field and that of an electric current separately and without interconnection, the problem is, how to combine the time constants to get the real time constant of the magnetic field in cases where finite changes take place and the interlinkage of the eddy currents with the exciting current cannot be neglected. It might be supposed that the arithmetic addition of the time constants would be a satisfactory approximation; but this paper proposes to show that this method of treatment leads to large error. This cannot be readily proven by experimental procedure, because there are so many influencing factors.

This matter is very important in certain practical applications of electrical engineering. The need of a general mathematical solution is, therefore, necessary. Probably the mathematical difficulties have precluded the solution heretofore. But in Heaviside's operational calculus[8,9] a very convenient and useful method is made available to extend the field of solutions.

II. Mathematical Solution of Switching on an Electric Circuit Having a Partly Laminated and Partly Solid Iron Core Containing an Air-Gap

1. Mathematical Statement by Maxwell's Field Equations. To obtain a mathematical solution of the proposed problem, it is not necessary to consider the complicated magnetic structure of an actual generator but rather assume the relatively simple case of a torroidal iron core containing an air-gap of length δ. The iron core itself consists of a solid part of length l, and a laminated part of length l_l as shown in Fig. 1 and Fig. 1A. The excitation is given by an electric circuit connected to a d-c. voltage source.

There are three circuits to be considered, two electric circuits, the exciting coil and the eddy current circuit in the solid core, both interlinked by the magnetic circuit. When switching on the d-c. voltage, it is assumed that no eddy currents will occur in the laminated part due to the thinness of the laminations. But in the solid iron part eddy currents will appear which will oppose the increase of the magnetic field.

The mathematical statements for the switching phenomenon will be given in the so-called practical units, which means that the magnetic field strength is measured in amperes per centimeter, the electric field

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1. For references see Bibliography.

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strength in volts per centimeter, the absolute permeability constant Π in farads per centimeter,[10] and the electric conductivity Λ in mhos per centimeter. For convenience it is assumed that Π is a constant quantity not only with reference to the cross-section of the solid iron part but also with respect to time. The influence of temperature on the eddy currents as well as on the electric current will be neglected.

It would be possible to start from Maxwell's field equations directly in the differential form. But it is more convenient and desirable to establish the equations in the integral form.[11,12] The electric current

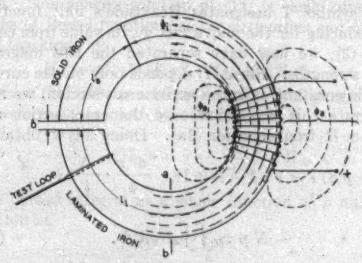

FIG. 1—THE TORROIDAL IRON CORE WITH SOLID AND LAMINATED PART AND WITH AIR GAP

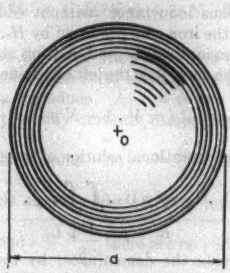

FIG. 1A—SHOWING THE LAMINATION OF THE LAMINATED IRON CORE

Section through a-b

through the coil produces the magnetic field not only through the iron core but also in the surrounding air. So the total magnetic flux Φ can be subdivided, for convenience, into Φ_i, the flux in the iron core, as it would be measured by any loop situated as shown in Fig. 1, and into the leakage flux Φ_a, which can be assumed proportional to the current. Under these assumptions we get for the electric circuit, summing up the voltage drops and equalizing them to the impressed voltage V, the following differential equation:

$$R i + S \frac{d i}{d t} + N \cdot \frac{d \Phi_i}{d t} = V, \qquad (1)$$

in which S means the leakage inductance, and N designates the number of turns of the exciting coil.

The flux in the iron core is composed of two components. The first component, the flux density or magnetic induction $B_1 = \Pi_0 H_1$, with the air-gap field strength H_1, is produced by the exciting current. The second component, the flux density or magnetic induction $B_2 = \Pi_0 H_2$, with the air-gap field strength H_2, arises from the eddy currents in the solid part of the iron core. The total flux in the iron core is given by the integral over the cross-section

$$\Phi_i = \Pi_0 \iint_A (H_1 + H_2) \cdot d A \qquad (2)$$

The field strength H_1 is connected with the electric current by the closed line integral

$$\oint H_1 \, d l = N \cdot i \, . \qquad (3)$$

Since H_1 designates the field strength in the air-gap in amperes per centimeter, we get the part of the integral given by the air-gap as δH_1, whereas the part of the integral due to the iron path is given by $\frac{l - \delta}{\mu} \cdot H_1$ where μ designates the relative permeability of the iron. Introducing these expressions in (3) we get, abbreviating

$$\left(\delta + \frac{l - \delta}{\mu} \right) \cdot H_1 = l' \cdot H_1 = N \cdot i, \qquad (3a)$$

in which l' is the equivalent length of the iron core and air-gap. The field strength produced by the eddy currents and its interlinkage with these currents is expressed by a similar equation in which the left hand term gives the difference of the two line integrals shown in Fig. 2, and in which the right hand term gives the enclosed amount of eddy currents. If the density of these eddy currents is designated by G, the line integral equation becomes,

$$l' \cdot H_2 (r) - l' \cdot \left[H_2 (r) + \frac{\partial H_2}{\partial r} \cdot d r \right]$$

$$= \iint_A G \cdot d A = l_s \cdot G \cdot d r \, . \qquad (4)$$

From this we get the differential equation

$$\frac{\partial H_2}{\partial r} = - \frac{l_s}{l'} \cdot G = - \frac{l_s}{l'} \cdot \Lambda E, \qquad (4a)$$

in which E = the electric field strength due to the eddy currents according to Ohm's Law. The eddy currents are produced by the change of magnetic field in the solid iron part. The law of induction applied to the solid iron part gives (refer to Fig. 2A)

$$\oint E \cdot d s = - \frac{\partial}{\partial t} \iint B \cdot d A \qquad (5)$$

Since we have no variation of the quantities E and B with the angle φ, we may then write

$$2\,r\,\pi\,E = -\frac{\partial}{\partial t}\left(2\,\pi\,.\int_{r=0}^{r} r\,B\,d\,r\right).$$

By differentiating the above equation with respect to r we get, remembering $B = \Pi_0\,(H_1 + H_2)$,

$$\frac{\partial}{\partial r}\,(r\,E) = -\,r\,\Pi_0\,.\,\frac{\partial}{\partial t}\,(H_1 + H_2)\,. \qquad (5a)$$

The differential equations (1), (2), (3a), (4a) and (5a) give the mathematical statement of the entire problem.

2. *Introduction of Heaviside's Notation; the Differential Equation for the Magnetic Field Strength Due to the Eddy Currents.* The differential equation for the magnetic field strength due to the eddy currents is derived from the combination of the above five general equations into one equation containing only the function H_2. In doing this, the most convenient method is the application of Heaviside's notation by which the time variation may temporarily be eliminated, leaving only the radius r as the variable.

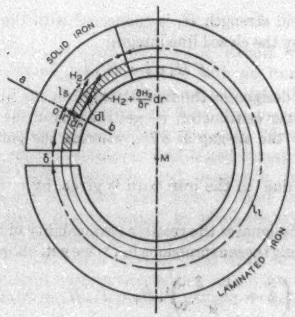

FIG. 2—REPRESENTING THE LINE INTEGRAL OF THE MAGNETIC FIELD STRENGTH

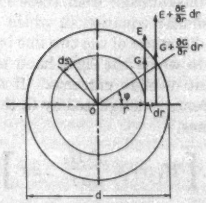

FIG. 2A—SHOWING VIEW A-B FROM FIG. 2 AND THE CYLINDRICAL COORDINATE SYSTEM

Proceeding in the elimination from the above equations to obtain the final differential equation, let us combine equations (4a) and (5a) to give,

$$\left[\frac{\partial^2 H_2}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial H_2}{\partial r}\right] = \frac{l_s}{l'}\,.\,\Lambda\,\Pi_0\,.\,\frac{\partial}{\partial t}\,(H_1 + H_2) \qquad (6)$$

The partial derivatives with respect to time may be replaced by the total derivatives, because all circuits are relatively at rest.

Introducing Heaviside's notation of his operational calculus,[8,9] we may replace the total differentiations with respect to time by the symbol p. Beginning with equation (1), we write it in the form

$$(R + p\,S)\,.\,i = V\,.\,1 - N\,p\,\Phi_i\,. \qquad (7)$$

The symbol 1 designates Heaviside's unit function. Introducing for the above flux Φ_i, its value from equation (2), we now have separated the two magnetic field strengths. Since H_1 depends only on the current and is constant over the entire cross-section, we may integrate this expression over the cross-section and replace H_1 from equation (3a). Doing this we obtain

$$(R + p\,S)\,i = V\,.\,1 - N\,p\,\Pi_0\cdot\frac{d^2\,\pi}{4}\cdot\frac{N}{l'}\,i$$

$$-\,N\,p\,\pi_0\iint_A H_2\,d\,A\,. \qquad (7b)$$

The product of the magnetic flux due to H_1 and the number of turns is identical with $L_0\,i$, where L_0 stands for the fictitious inductance (without eddy currents) of the flux in the iron core produced by H_1. Introducing this expression in (7b) and shifting all terms proportional to the current to the left hand side, we get

$$(R + p\,S + p\,L_0)\,i = V\,.\,1 - N\,p\,\Pi_0\iint H_2\,d\,A$$

from which the operational solution of the current is

$$i = \frac{V\,.\,1 - N\,p\,\Pi_0\,.\iint H_2\,.\,d\,A}{R + p\,S + p\,L_0} \qquad (8)$$

We may indicate the denominator by the symbol P as the transient impedance of the electric circuit:

$$P = R + p\,(L_0 + S). \qquad (9)$$

Hence,

$$i = \frac{V}{P}\,1 - \frac{N\,\Pi_0}{P}\cdot p\iint_A H_2\,d\,A\,. \qquad (10)$$

In order to eliminate H_1 of equation (6) substitute i of equation (10) in equation (3a). Writing this in operational form gives the final differential equation for the magnetic field strength due to the eddy currents, as

$$\left[\frac{d^2 H_2}{d\,r^2} + \frac{1}{r}\cdot\frac{d\,H_2}{d\,r} - \frac{l_s}{l'}\cdot\Lambda_0\,\Pi.\,p\,H_2\right]$$

$$= \frac{l_s}{l'^2}\cdot\frac{\Lambda\,\Pi_0\,p\,N}{P}\cdot\left[V\,.\,1 - 2\,r\,N\,\Pi_0\,.\,p\int_{r=0}^{d/2} H_2\,d\,r\right] \qquad (11)$$

3. The Operational Solution for the Magnetic Field and the Electric Current. The left hand side of equation (**11**) is a differential equation in r leading to the special solution for the magnetic field strength H_2. The right hand side is merely a function of the time, not dependent on r. Therefore, the total solution of (**11**) may be given as the sum of the solution of the so-called homogeneous differential equation:

$$\frac{d^2 H_2}{d r^2} + \frac{1}{r}\frac{d H_2}{d r} - \frac{l_s}{l'} \cdot \Lambda \Pi_0 p H_2 = 0 \qquad (12)$$

and a partial integral of (**11**) as any arbitrary function of p. The general solution of (**12**) is easily given in terms of Bessel functions of the first order and of the first two kinds[13,14]

$$H_2' = C \,.\, J_0 (K r) + D \,.\, N_0 (K r) \,, \qquad (13)$$

in which C and D are arbitrary constants of integration. Introducing this solution in (**12**) we get

$$K = \sqrt{-\frac{l_s}{l'} \,.\, \Lambda \Pi_0 p} = j \sqrt{\frac{l_s}{l'} \,.\, \Lambda \Pi_0} \,.\, \sqrt{p}. \qquad (14)$$

The partial integral as a single function of p may be written as,

$$H_2'' = F (p) \,. \qquad (15)$$

The composition of (**13**) and (**15**) gives as the total solution

$$H_2 = H_2' + H_2'' = C J_0 (K r) + D N_0 (K r) + F (p). \qquad (16)$$

To determine the integration constants C and D and the arbitrary function $F(p)$ we have first two boundary conditions to introduce. At $r = 0$, the electric field strength of the eddy currents disappears, because we have a circular current. At $r = \frac{d}{2}$, the magnetic field strength of the eddy currents must disappear, because beyond $r = \frac{d}{2}$ there are no eddy currents.

Introducing equation (**16**) into (**4a**) gives

$$E = -\frac{l'}{l_s} \cdot \frac{1}{\Lambda} \cdot \frac{d H_2}{d r}$$

$$= -\frac{l'}{l_s} \cdot \frac{1}{\Lambda} [-C K r J_1 (K r) - D K r N_1 (K r)]$$

From the first boundary condition putting r equal to zero, $J_1 (o) = 0$ and $N_1 (o) = \infty$. Therefore, D must equal zero,

$$D = 0 \qquad (17)$$

Observing (**17**) we get by the second boundary condition

$$H_2 \Big|_{r=\frac{d}{2}} = C \,.\, J_0 \left(K \frac{d}{2} \right) + F (p) = 0$$

and from this

$$C = -\frac{F (p)}{J_0 \left(K \frac{d}{2} \right)}, \qquad (18)$$

so that we now have from equation (**16**),

$$H_2 = F (p) \,.\, \left[1 - \frac{J_0 (K r)}{J_0 \left(K \frac{d}{2} \right)} \right]. \qquad (19)$$

The solution (**19**) entered in (**11**) must satisfy this fundamental equation. The required integral $\int_{r=0}^{d/2} H_2 r \, d r$ in (**11**) may now be solved with the aid of (**19**), giving

$$\int_{r=0}^{d/2} H_2 r \, d r$$

$$= F (p) \cdot \frac{d^2}{8} \cdot \left[1 - 2 \frac{J_1 \left(K \frac{d}{2} \right)}{\left(K \frac{d}{2} \right) J_0 \left(K \frac{d}{2} \right)} \right]. \qquad (20)$$

Introducing now (**19**) and (**20**) in (**11**), gives

$$F (p) \left\{ \frac{d^2 \pi}{4} \cdot \Pi_0 \,.\, p N \left[1 - 2 \frac{J_1 \left(K \frac{d}{2} \right)}{\left(K \frac{d}{2} \right) J_0 \left(K \frac{d}{2} \right)} \right] - \frac{l'}{N} P \right\} = V . 1$$

and, therefore, the arbitrary function $F(p)$ by the operational expression becomes

$$F (p) = \frac{N}{l'} \cdot \frac{V}{p L_0 \left[1 - \frac{2 J_1 \left(K \frac{d}{2} \right)}{\left(K \frac{d}{2} \right) J_0 \left(K \frac{d}{2} \right)} \right] - P} \cdot 1 \qquad (21)$$

Replacing (**21**) in (**19**) and simplifying gives the final operational solution for the magnetic field strength in the form

$$H_2 = \frac{N}{l'} \,.$$

$$\frac{-K\frac{d}{2}\cdot\left[J_0\left(K\frac{d}{2}\right)-J_0(K\,r)\right]}{\left(K\frac{d}{2}\right)J_0\left(K\frac{d}{2}\right)(R+p\,S)+2\,p\,L_0\,J_1\left(K\frac{d}{2}\right)}\cdot V1. \tag{22}$$

The operational expression for the current given by (8) may now be changed using (20) and replacing $F(p)$ by (21) into

$$i=\frac{\left(K\frac{d}{2}\right)\cdot J_0\left(K\frac{d}{2}\right)}{\left(K\frac{d}{2}\right)J_0\left(K\frac{d}{2}\right)\cdot(R+p\,S)+2\,p\,L_0\,J_1\left(K\frac{d}{2}\right)}\cdot V1 \tag{23}$$

The total magnetic flux in the iron core as given by equation (2) now becomes

$$\Phi_i=\frac{L_0}{N}\cdot\frac{2\,J_1\left(K\frac{d}{2}\right)}{\left(K\frac{d}{2}\right)J_0\left(K\frac{d}{2}\right)\cdot(R+p\,S)+2\,p\,L_0\,J_1\left(K\frac{d}{2}\right)}\cdot V1 \tag{24}$$

4. Conversion of the Operational Solutions into Real Time Functions. Heaviside's unit function is known as the complex integral[9,3]

$$1=\frac{1}{2\,\pi\,j}\int_{-j\infty}^{+j\infty}\frac{\epsilon^{pt}}{p}\,d\,p, \tag{25}$$

where p means any variable in the complex plane. This time function is to be applied to the magnetic field strength given by equation (22). Introducing (22) in (25) gives the rather complicated complex integral

$$H_2=-\frac{N}{l'}\cdot\frac{V}{2\,\pi\,j}\cdot\int_{-j\infty}^{+j\infty}\frac{\left(K\frac{d}{2}\right)\cdot\left[J_0\left(K\frac{d}{2}\right)-J_0(K\,r)\right]}{\left(K\frac{d}{2}\right)\cdot J_0\left(K\frac{d}{2}\right)(R+p\,S)+2\,p\,L_0\,J_1\left(K\frac{d}{2}\right)}\cdot\frac{\epsilon^{pt}}{p}\,d\,p. \tag{26}$$

The evaluation of this integral depends on the integrand. We may prove that in this case the integrand is a rational function of p. Since the Bessel functions are generally given by the power series.[13,14]

$$\left.\begin{aligned}J_0(x)&=1-\frac{(x/2)^2}{1\,!^2}+\frac{(x/2)^4}{2\,!^2}-\frac{(x/2)^6}{3\,!^2}+-\ldots\\ J_1(x)&=\frac{x}{2}\cdot\left\{1-\frac{(x/2)^2}{1\,!\,2\,!}\right.\\ &\left.\quad+\frac{(x/2)^4}{2\,!\,3\,!}-\frac{(x/2)^6}{3\,!\,4\,!}+-\ldots\right\}\end{aligned}\right\} \tag{27}$$

in which x, the argument, is identical with $\left(K\frac{d}{2}\right)$ in (26), it may be seen from that equation that $\sqrt{p}$ cancels throughout the fraction and that, therefore, the fraction is a rational function in terms of p.

In the case where the integrand of a complex integral is a rational function of the variable, it is permissible to replace the integral by the sum of the residues. This theorem in connection with the time function is known as Heaviside's expansion theorem.[9] So that, in other words, we may apply to the complex operational solutions, the general expansion theorem. It may be emphasized that this holds only in the case of rational functions in p.

For convenience replace the fraction in (26) by the symbolical expression

$$H_2=-\frac{N}{l'}\cdot\frac{V}{2\,\pi\,j}\int_{-j\infty}^{+j\infty}\frac{Y_2(p)}{Z(p)}\cdot\frac{\epsilon^{pt}}{p}\,d\,p$$

and readily obtain a solution in time functions as

$$H_2=-\frac{N}{l'}\cdot V\cdot\left\{\frac{Y_2(o)}{Z(o)}+\sum_{\nu=1}^{\infty}\left[\frac{Y_2(p)}{p\,\frac{d}{d\,p}\,Z(p)}\right]_{p=p_\nu}\cdot\epsilon^{p_\nu t}\right\} \tag{28}$$

whereby

$$Y_2(p)=\left(K\frac{d}{2}\right)\cdot\left[J_0\left(K\frac{d}{2}\right)-J_0(K\,r)\right] \tag{29}$$

$$Z(p)=\left(K\frac{d}{2}\right)J_0\left(K\frac{d}{2}\right)\cdot(R+p\,S)+2\,J_1\left(K\frac{d}{2}\right)\cdot p\,L_0\cdot \tag{30}$$

Now the problem is to find the roots of the denominator function $Z(p)$.

In the case of the current we have again to prove that the integrand in (23) is a rational function in p. This also may be proved by means of equations (27). For convenience we introduce for the numerator function in (23) the symbolical expression

$$Y_i(p) = \left(K\frac{d}{2}\right) \cdot J_0\left(K\frac{d}{2}\right) \cdot \tag{31}$$

The development according to the expansion theorem gives

$$i = V\left\{\frac{Y_i(o)}{Z(o)} \cdot + \sum_{\nu=1}^{\infty}\left[\frac{Y_i(p)}{p\frac{d}{dp}Z(p)}\right]_{p=p_\nu} \cdot \epsilon^{p_\nu t}\right\} \cdot \tag{32}$$

The total field flux is given by equation (24). Denoting the numerator function by

$$Y_\Phi(p) = 2J_1\left(K\frac{d}{2}\right), \tag{33}$$

we can prove also in this case the integrand to be a rational function in p. In terms of the expansion theorem the expression for the total field flux becomes,

$$\Phi_i = \frac{L_0 V}{N} \cdot \left\{\frac{Y_\Phi(o)}{Z(o)} + \sum_{\nu=1}^{\infty}\left[\frac{Y_\Phi(p)}{p\frac{d}{dp}Z(p)}\right]_{p=p_\nu} \epsilon^{p_\nu t}\right\} \cdot \tag{34}$$

5. *General Discussion of the Results.* Now it becomes necessary to establish and check the time conditions of the problem. The eddy currents arise when switching on the electric circuit and they disappear under steady state conditions, because we have direct current. The steady state term is given in equation (28) by $\frac{Y_2(o)}{Z(o)}$ and must become zero. From the definition of K by equation (14), it is evident that $p = 0$ if $K = 0$. Introducing the latter values in the equations (29) and (30) gives after canceling $\left(K\frac{d}{2}\right)$

$$\frac{Y_2(o)}{Z(o)} = \frac{o}{R} = 0 \tag{35}$$

The steady state terms of the magnetic field strength produced by the eddy currents are therefore zero.

The steady state value of the electric current i is given, since we have d-c. voltage, by

$$i_{t=\infty} = \frac{V}{R} \cdot \tag{36}$$

Since after canceling $\left(K\frac{d}{2}\right)$ equations (31) and (30) give

$$\frac{Y_i(o)}{Z(o)} = \frac{1}{R},$$

from which (36) checks as a result of (32).

Finally it is necessary to check the steady state term of the magnetic flux. Introducing $K = 0$ in (33) and (30), the steady state member in (34) becomes

$$\frac{Y_\Phi(o)}{Z(o)} = \frac{1}{R}, \quad \Phi_{t=\infty} = \frac{L_0}{N} \cdot \frac{V}{R}, \tag{37}$$

which value is found to be correct.

The dimension of p_ν must be 1/sec. because $p_\nu t$ as the exponent of the exponential function has the dimension 1. By observing these values we can determine the dimension of K. The argument of the Bessel function $\left(K\frac{d}{2}\right)$ must have the dimension 1. Since K is given by equation (14) we get for the root term the condition,

$$D\left(K\frac{d}{2}\right) = 1 = D\left(j\sqrt{\frac{l_s}{l'} \cdot \Lambda\,\Pi_0\frac{d^2}{4}} \cdot \sqrt{p}\right) \cdot$$

This means that the expression under the first root sign has the dimension of time which we may designate as θ and call the *magnetic time constant for the solid iron part.*

$$\theta = \frac{l_s}{l'} \cdot \Lambda\,\Pi_0\frac{d^2}{4} \text{ sec.} \tag{38}$$

This time constant will be of great importance in the following considerations. Its physical meaning is similar to the *electric time constant* given by

$$T_1 = \frac{L_1}{R}, \tag{39}$$

where L_1 designates the total inductance of the electric current from the relation

$$L_1 = L_0 + S. \tag{40}$$

As mentioned before, S indicates the leakage inductance and L_0 the inductance of the magnetic flux in the iron core only. Therefore the electric time constant may be decomposed into

$$T_1 = \frac{L_0}{R} + \frac{S}{R} = T_0 + T_s. \tag{41}$$

T_0 may be called the *fictitious magnetic time constant,* holding in that case where the current and its field have the same shape, no leakage being present.

The roots p_ν must have negative and real values, therefore, for convenience we may define a new quantity by

$$\theta p_\nu = -q_\nu^2. \tag{42}$$

The argument of the Bessel functions takes the real values

$$j\sqrt{\theta p_\nu} = +q_\nu = K\frac{d}{2} \tag{43}$$

It is useful to change the expressions for the magnetic field and the current into functions of this new variable q. First introduce q in the numerator functions getting

$$\left.\begin{aligned} Y_2(q) &= +q\,.\left[J_0(q) - J_0\left(q\,\frac{2\,r}{d}\right)\right] \\ Y_i(q) &= +q\,.\,J_0(q) \\ Y_\Phi(q) &= +2\,.\,J_1(q) \end{aligned}\right\} \quad (44)$$

The general denominator function $Z(p)$ may be changed into

$$Z(q) = q\,.\,J_0(q)\left(R - \frac{q^2}{\theta}\,S\right) - 2\,J_1(q)\cdot\frac{q^2}{\theta}\,L_0\,.$$

Taking $q\,\dfrac{L_0}{\theta}$ as a constant factor gives

$$Z(q) = q\,\frac{L_0}{\theta}\cdot\left[J_0(q)\cdot\frac{\theta}{T_0} - q^2\cdot\frac{S}{L_0}\cdot J_0(q) - 2\,q\,J_1(q)\right]. \quad (45)$$

Here we may introduce for the factor

$$\frac{S}{L_0} = \sigma\,. \quad (46)$$

The symbol σ is called the "leakage coefficient" as it is used in the design of electrical machinery and transformers. It is the ratio of leakage flux to useful flux. Rearranging (45) gives the final expression

$$Z(q) = q\,\frac{L_0}{\theta}\left[J_0(q)\left(\frac{\theta}{T_0} - q^2\,\sigma\right) - J_1(q)\,.\,2\,q\right]. \quad (47)$$

Putting $Z(q)$, as given in equation (47), equal to zero we have the *transcendental equation for the roots*

$$\frac{J_0(q)}{J_1(q)} = \frac{2\,q}{\dfrac{\theta}{T_0} - \sigma\,q^2} \quad (48)$$

The solution of this equation (48) will be given in the next part. But we can see already the importance of the so-called *relative time constant* $\dfrac{\theta}{T_0}$.

It is also necessary to transform $p\,\dfrac{d}{d\,p}\,Z(p)$ into a function of q. Introducing (42) and (43) in the derivative of the general function $Z(q)$ (30) and with some simplification we obtain

$$\left[p\,\frac{d}{d\,p}\,Z(p)\right] = \frac{1}{2}\,q\,\frac{L_0}{\theta}\cdot\left\{J_0(q)\cdot\left[\frac{\theta}{T_0} - q^2\,(3\,\sigma + 2)\right] - q\,J_1(q)\left[\frac{\theta}{T_0} - (q^2\,\sigma - 2)\right]\right\}. \quad (49)$$

The final expressions for the magnetic field strength H_2, the current i and the total flux in the iron core Φ_i are now to be found as

$$H_2 = -2\,\frac{\theta}{T_0}\cdot\frac{N}{l'}\cdot\frac{V}{R}\cdot\sum_{\nu=1}^{\infty}\left[\frac{J_0(q) - J_0\left(q\,\dfrac{2\,r}{d}\right)}{J_0(q)\left[\dfrac{\theta}{T_0} - q^2\,(3\,\sigma + 2)\right] - q\,J_1(q)\left[\dfrac{\theta}{T_0} - (q^2\,\sigma - 2)\right]}\right]_{q=q_\nu}\epsilon^{-q_\nu^2\,\frac{t}{\theta}} \quad (50)$$

$$i = \frac{V}{R}\cdot\left\{1 + 2\,\frac{\theta}{T_0}\cdot\sum_{\nu=1}^{\infty}\left[\frac{J_0(q)}{J_0(q)\left[\dfrac{\theta}{T_0} - q^2\,(3\,\sigma + 2)\right] - q\,J_1(q)\,.\left[\dfrac{\theta}{T_0} - (q^2\,\sigma - 2)\right]}\right]_{q=q_\nu}\epsilon^{-q_\nu^2\,\frac{t}{\theta}}\right\} \quad (51)$$

$$\Phi_i = \frac{V\,T_0}{N}\cdot\left\{1 + 4\,\frac{\theta}{T_0}\cdot\sum_{\nu=1}^{\infty}\left[\frac{\dfrac{1}{q}\cdot J_1(q)}{J_0(q)\left[\dfrac{\theta}{T_0} - q^2\,(3\,\sigma + 2)\right] - q\,J_1(q)\left[\dfrac{\theta}{T_0} - (q^2\,\sigma - 2)\right]}\right]_{q=q_\nu}\epsilon^{-q_\nu^2\,\frac{t}{\theta}}\right\} \quad (52)$$

III. Practical Use of the Theory and Discussion

1. Determination of the Roots of $Z(q)$. Before applying the above equations to special problems it becomes necessary to determine the roots of $Z(q)$. The fundamental equation for the roots is given by

$$\frac{J_0(q)}{J_1(q)} = \frac{2q}{\dfrac{\theta}{T_0} - \sigma q^2}. \tag{48}$$

The unknown quantities in this expression are σ and $\dfrac{\theta}{T_0}$. The leakage coefficient σ may be assumed to have the value 0.25 as is commonly found in electrical machinery. Introducing this numerical value the roots q_ν become functions only in $\dfrac{\theta}{T_0}$, which will appear as a determining parameter.

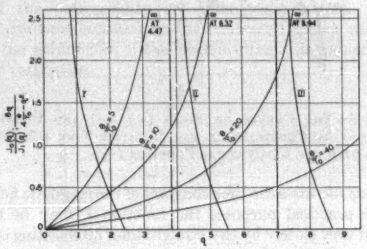

Fig. 3—The Determination of the Roots of $Z(q)$

The solution of the highly transcendental equation (48) can be given either graphically or by approximate calculation. To obtain the results more precisely we shall combine these two methods. To obtain a graphical solution as a first approximation plot curves for the left hand side and the right hand side of equation (48), and locate the intersections of these two families of curves. In Fig. 3 is shown the family of curves $\dfrac{J_0(q)}{J_1(q)}$, the left hand member of (48) which are similar to the function $\tan q$ having a variable frequency. The curves of the right hand side have involved the parameter $\dfrac{\theta}{T_0}$ for which the numerical values 5, 10, 20, and 40 have been arbitrarily chosen. From Fig. 3 the abscissas of the intersecting points may be taken as a first approximation and their numerical values may be checked by calculation of the ordinates. Table I shows such calculated results. For more detailed explanations see the mathematical Appendix 1.

TABLE I—THE FIRST 6 ROOTS OF $Z(q)$

$\theta/T_0 =$	5	10	20	40
$q_1 =$	1.645	1.955	2.168	2.285
$q_2 =$	3.964	4.400	4.900	5.20
$q_3 =$	6.60	6.885	7.435	8.05
$q_4 =$	9.45	9.594	9.94	10.65
$q_5 =$	12.38	12.44	12.66	13.19
$q_6 =$	15.42	15.47	15.56	15.77

2. The Curves of the Magnetic Field and of the Current. The final expressions for the current and for the total flux in the iron core are given by (51) and (52). They may be written symbolically:

$$\frac{i}{i_\infty} = 1 + \sum_{\nu=1}^{\infty} I_\nu \cdot \epsilon^{-\left(q_\nu^2 \frac{T_0}{\theta}\right) \cdot \frac{t}{T_0}}, \tag{54}$$

$$\frac{\Phi_i}{\Phi_{i\infty}} = 1 + \sum_{\nu=1}^{\infty} F_\nu \cdot \epsilon^{-\left(q_\nu^2 \frac{T_0}{\theta}\right) \cdot \frac{t}{T_0}}, \tag{55}$$

in which the steady state values of current and field have been designated by i_∞ and $\Phi_{i\infty}$, respectively, and the exponents have been changed by the introduction of T_0 to maintain the form of the parameter $\dfrac{\theta}{T_0}$ as follows:

$$-\frac{q_\nu^2}{\theta} t = -\left(q_\nu^2 \cdot \frac{T_0}{\theta} \right) \cdot \frac{t}{T_0}. \tag{53}$$

The time scale is changed into a relative time scale $\dfrac{t}{T_0}$, as seen from (53). The expressions (54) and (55) in the given form can be readily compared with the solution in the case where no eddy currents are present:

$$\frac{i}{i_\infty} = 1 - \epsilon^{-\frac{t}{T_1}}.$$

The single term in this last expression has to be replaced by the sum in (54) and (55). From Table II and III the values of the harmonic amplitudes I_ν and F_ν may be taken as functions of $\dfrac{\theta}{T_0}$. The amplitudes of higher order become positive according to the fact that the total sum must be equal to minus one.

To get the magnetic field and the current as time functions, the amplitudes as shown in Tables II and III are to be multiplied by the exponential time-functions. The results may be seen in Fig. 5. For detailed explanation see the mathematical Appendix 2. As $\dfrac{\theta}{T_0}$ increases, the curves of the magnetic field have decreasing values for each abscissa, considering distinct values of the relative time scale $\dfrac{t}{T_0}$. The time

TABLE II—THE FIRST SIX HARMONIC AMPLITUDES I_ν OF THE CURRENT

θ/T_0 =	5	10	20	40
I_1	−0.619	−0.395	−0.218	−0.116
I_2	−0.268	−0.324	−0.244	−0.132
I_3	−0.079	−0.169	−0.252	−0.177
I_4	−0.027	−0.063	−0.155	−0.232
I_5	−0.010	−0.025	−0.065	−0.189
I_6		−0.011	−0.028	−0.092
Total	−1.003	−0.987	−0.962	−0.938

TABLE III—THE FIRST THREE HARMONIC AMPLITUDES F_ν OF THE MAGNETIC FIELD

	5	10	20	40
F_1	−1.000	−0.931	−0.85	−0.790
F_2		−0.087	−0.15	−0.158
F_3				−0.066
Total	−1.000	−1.018	−1.00	−1.014

constant of this family of curves must be a function of the ratio $\frac{\theta}{T_0}$. The curves of the current show a high rate of increase at low values of the time scale but they tend toward definite constants at very high values of the time scale.

3. The Time Constant of the Magnetic Field. From Table III it will be noted that the first harmonic of the magnetic field has very high values compared with the harmonics of higher orders. Therefore, the value of

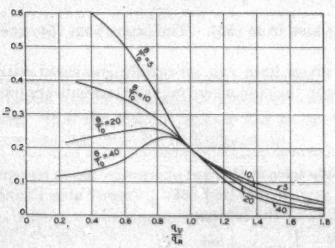

FIG. 4—THE HARMONIC AMPLITUDES OF THE CURRENT AS FUNCTIONS OF THE ROOTS

the time function is determined generally by the time constant of the first harmonic. This time constant is given by

$$T_\Phi = T_0 \cdot \left(\frac{\theta}{T_0} \Big/ q_1^2 \right) \qquad (56)$$

and for convenience this expression may be changed into

$$T_\Phi = F \cdot T_1 = \left(\frac{T_0}{T_1} \cdot \frac{\theta}{T_0} \Big/ q_1^2 \right) \cdot T_1 \qquad (57)$$

The ratio $\frac{T_0}{T_1}$ equals $\frac{1}{1+\sigma}$ as may be seen from equation (7). The factor F is now a function of $\frac{\theta}{T_1}$. The calculation gives the curve in Fig. 6 which shows its asymptotic character. The straight line to which this curve approaches is given by the first root of the Bessel function of zero order, which value[13] is 2.4048. At $\frac{\theta}{T_1} = 0$ the curve of F approaches unity which means that if the solid iron part of the magnetic circuit is considered to disappear the time constant of the magnetic field will become identical with the time constant of the exciting current. The *presence of eddy*

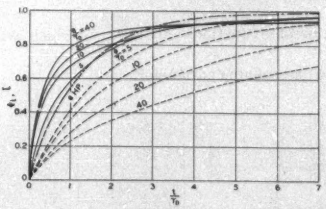

FIG. 5—THE CURVES OF FIELD (- - - -) AND CURRENT (———) AS FUNCTIONS OF THE RELATIVE TIME t/T_0 COMPARED WITH THE EXPONENTIAL FUNCTION (— - —)

currents increases the time constant of the magnetic field.

For practical purposes the function F may be replaced very closely by two straight lines intersecting one another at $\frac{\theta}{T_1} = 5$. The best approximation is given by

$$F = K_1 + K_2 \frac{\theta}{T_1}, \quad K_1 = 1 \quad , K_2 = 0.12 \text{ if } \frac{\theta}{T_1} \leqq 5,$$

$$K_1 = 0.75, K_2 = 0.175 \text{ if } \frac{\theta}{T_1} \geqq 5. \qquad (58)$$

Introducing these values in (57) the time constant of the magnetic field takes the very simple form

$$T_\Phi = T_1 + 0.12\,\theta \qquad \text{if } \frac{\theta}{T_1} \leqq 5,$$

$$T_\Phi = 0.75\,T_1 + 0.175\,\theta \text{ if } \frac{\theta}{T_1} \geqq 5. \qquad (59)$$

The total time constant T_Φ is composed of T_1, the time constant of the electric circuit, and a fraction of θ, the time constant of the solid iron part. There is *no addition* of these time constants but only a composition of their fractional values. This is one of *the most important results* in this paper.

The definition for θ is given by equation (38). This

relation may be simplified if Π_0 is replaced by its numerical value $4\,\pi\,.\,10^{-9}$ and writing in place of $\frac{\pi\,d^2}{4}$, the area A of the cross-section of the iron core. Although the derivation is based on the assumption of circular cross-section, the results may be used in all cases where the cross-section is approximately circular. Introducing this statement, θ becomes

$$\theta = 0.4\,\gamma \cdot A\,\frac{\lambda_s}{\lambda_\varphi + \frac{1}{\mu}} \cdot 10^{-4}\ \text{sec.} \qquad (60)$$

The electric time constant was given as

$$T_1 = \frac{L_1}{R} = \frac{(1+\sigma)\,L_0}{R}\,. \qquad (61)$$

It is easy to see that θ depends upon the dimensions of the solid iron part, that is, the area of its cross-section and its length. In the same manner the conductivity and the permeability are influencing factors. The electric time constant on the contrary depends merely on the parameters of the electric circuit, as inductance and resistance.

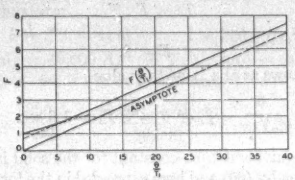

FIG. 6—THE TIME CONSTANT OF THE MAGNETIC FIELD $T_\Phi = F\,T_1$

4. The Equivalent Self-Inductance for the Eddy Currents. The magnetic time constant as given by (57) may be written in the form of the electric time constant

$$T_\Phi = \frac{L_\Phi}{R}\,,$$

in which L_Φ indicates any fictitious self-inductance with respect to the field. Taking from (57) the expression T_Φ and replacing T_1 by (61) we get the identity

$$L_\Phi = F \cdot L_1\,.$$

Subtracting the total inductance, the remainder must be produced by the eddy currents. It may be called the *"equivalent self-inductance" for the eddy currents.* and the expression for it becomes,

$$L_0 = (F - 1) \cdot L_1\,. \qquad (62)$$

The physical meaning of (62) is that of an additional self-inductance, whenever magnetic circuits, containing a solid iron part, are considered with respect to a switching phenomenon. It may be emphasized that this increase of self-inductance holds only for the magnetic field. The electric current, however, shows, as may be seen from Fig. 5, a large decrease of the time constant, but the shape of the current curve cannot be expressed as a simple exponential function. The assumption of a representative time constant has no physical sense in the case of the current.

5. Numerical Example. To get a conception of the numerical value of the physical quantities, involved in this problem, a simple numerical example may be considered. The magnetic circuit may be given by a torroidal iron core as assumed in Section 3 and shown in Fig. 1. The lengths of the different parts of the iron path may be

$$l = 250\ \text{cm.},\quad \delta = 2.5\ \text{cm.},\quad l_s = 100\ \text{cm.}$$

and the diameter of the cross-section be $d = 30$ cm. The material of the solid iron part as well as of the laminated iron part may be cast steel with the material constants

$$\gamma = 8\ \text{Mho}\ m/m\,m^2,\quad \mu = 1000.$$

The electric circuit is characterized by the inductance $L_0 = 12.77\ H$, the leakage coefficent $\sigma = 0.25$ and the resistance $R = 5.72$ ohm.

The magnetic time constant of the solid iron part is, according to (60),

$$\theta = 0.4 \cdot 8 \cdot \frac{30^2\,\pi}{4} \cdot \frac{0.4}{0.01 + 0.001}\,10^{-4} = 8.18\ \text{sec.}$$

The electric time constant is, according to (61),

$$T_1 = \frac{12.77. + .25}{5.72} = 2.8\ \text{sec.}$$

The ratio $\frac{\theta}{T_1}$ is 2.93, which being less than 5 dictates the use of the first values in equation (59). The total magnetic time constant becomes, therefore

$$T_\Phi = 2.8 + 0.12\,.\,8.18 = 3.80\ \text{sec.}$$

The electric time (current) must be increased by 36 per cent to give the magnetic time constant. Arithmetic addition would have given nearly three times this value. Consequently the importance of having an accurate formula for the magnetic time constant due to eddy current effects is made clear.

The variation of the time constant T_Φ with the length of the solid iron part is shown in Fig. 7. It is assumed there that the electric time constant T_1 has two different values, one for each of two different exciting coils without change of the iron core. From these two curves the influence of θ relative to the electric time constant is made clear. In the case $T_1 = 2.8$ sec., the range of T_Φ is from 1 to 2 but in the case $T_1 = 1$ sec., the range of T_Φ becomes 1 to 4.33.

To lower the magnetic time constant in the case of solid iron in any magnetic circuit, it is necessary to change the dimensions of the solid iron part *as well as* to change the parameters of the electric circuit. It depends on the ratio of the electric to the magnetic

time constant which change will be more effective. This can be decided by calculation according to formulas (**60**) and (**61**).

IV. Application to Switching Problems in Synchronous and D-C. Machines

1. Quick Response Excitation of Large A-C. Generators. In order to improve the system stability under short-circuit conditions of short duration it has been found advantageous to increase suddenly the excitation power input.[15-16] A measure of the effectiveness of this method is thought to be the rate of change of voltage at the terminals of the main exciter machine in volts per second. The rate of change in the magnetic field of the generator is obviously much less than the rate of change in the quick response excitation voltage. The greater the rate of change of the exciting current the greater the rate of change of the eddy currents in the solid iron part of the magnetic circuit of the generator.

The theory treated in this paper gives a solution in

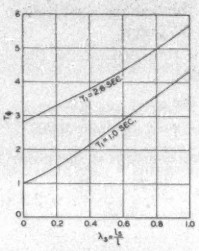

Fig. 7—The Magnetic Time Constant as Function of the Relative Length of Solid Iron Core in a Special Case

the case of a d-c. voltage suddenly impressed upon an electromagnetic system, such as a synchronous machine. In the synchronous machine as in the torroidal core, there is a magnetic circuit partly laminated and partly solid. The high rate of increase in the excitation voltage may with close approximation be replaced by a suddenly impressed d-c. voltage, which value is that of the maximum exciter voltage. In the case of an alternator the stator winding introduces a difference in the theory thus far considered. Further investigation on this subject may take into account the two electric circuits that is, the field and stator circuits. For first approximation only the effect of the impressed excitation voltage will be considered. The opposing induced eddy currents produce a time lag of the magnetic field, the time constant of which is given by equation (**59**). This time constant must be made a minimum in order to have the best effect in producing a quick response excitation. Even the highest increase of voltage in the exciting circuit cannot be successful with respect to the system stability when the time constant of the magnetic circuit is too large. The effect lies in the rate of change of the magnetic field rather than in the rate of change of the exciting voltage. Equation (**59**) shows the most economic way to increase the effectiveness of the super-excitation of an alternator without using larger exciters. On the other hand it is known that the synchronizing torque of an alternator is decreased when all parts of the magnetic circuit are laminated. The asynchronous eddy currents due to the oscillations of the pole system with respect to the system voltage vector increase the synchronizing power.

It is seen, therefore, that there are two opposing conditions in regard to the proportions of laminated and solid iron parts, and the best proportion may be stated in terms of the amount of solid iron to be used in an alternator to restrict the magnetic time constant to a given limit. The time range in which the effect of super-excitation shall have become established is about 0.2 to 0.6 seconds according to the kind of short circuit. The magnetic time constant T_Φ must be at least within this range, so it may be taken as an average value of 0.4 seconds. The application to other values is easily accomplished. It may be assumed that the electric time constant T_1 is one-half of the total magnetic time constant, which means the very low value of 0.2 second. Then from (**57**)

$$T_\Phi = F \,.\, T_1 \,,$$

in which

$$F = 2.$$

Fig. 6 shows as abscissas the value

$$\frac{\theta}{T_1} = 8, \text{ or } \theta = 8\, T_1 = 1.6 \text{ sec.}$$

The equivalent time constant for the solid iron part, θ, is given by (**60**) and here expressed in the form

$$\theta = 0.4\, \gamma \frac{A \,.\, l_s}{\delta + \dfrac{l}{\mu}} \cdot 10^{-4} \text{ sec.} \qquad (63)$$

In the case of the alternator it is useful to introduce $A\, l_s$ as the volume of the solid iron part. When different parts with different dimensions are in series the total sum is to be taken. This is permissible only in accordance with the proportionality of θ to the volume, as expressed in (**63**) and furthermore with the proportionality of T_Φ to θ, as shown in equation (**59**). If there were several air-gaps, also the sum of their lengths must be entered in (**63**).

The relative permeability μ, as a physical quantity, may be usefully replaced by an expression in ampere-turns, in order to simplify the application in electrical engineering mathematics. Equation (**3a**) which stated that the line integral of the magnetic field strength H_1 was identical with the sum of the ampere-turns for the different paths, gives the definition for μ as,

$$\delta\, H_1 + \frac{l - \delta}{\mu}\, H_1 = N\, i$$

$$A\, T \text{ (air-gap)} + A\, T \text{ (iron)} = A\, T \text{ (total)}.$$

So it may be written

$$\frac{l}{\mu} = l\,\frac{N\,i - \delta\,H_1}{(l-\delta)\,H_1}$$

$$= \frac{l\,\delta}{l-\delta}\cdot\frac{A\,T\,(\text{total}) - A\,T\,(\text{air-gap})}{A\,T\,(\text{air-gap})}\cdot \quad (\mathbf{64})$$

In this definition all the quantities are well known both in the theory and in the design of electrical machines. Finally the conductivity γ may be replaced by the numerical value for cast steel, because in alternators cast steel is generally used for the solid iron part, $\gamma = 8$ Mho $m/m\,m^2$. From (**63**) it follows, that large values of the air-gap δ, and large values of $\frac{l}{\mu}$, the length of a fictitious air-gap equivalent to the iron path, which means low values of permeability for whatever kinds of iron are used, are both desirable. Similarly low values of conductivity of the iron and a small volume of solid iron are advantageous.

To continue the numerical example, the magnetic circuit in the case of an alternator is formed by two poles, the connecting yoke, two air-gaps, and the path through the stator.

Let $\delta = 2{\cdot}2 = 4$ cm., $l = 350$ cm.,
$A\,T$ (total) $= 40{,}000\,A$, $A\,T$ (air-gap) $= 30{,}000\,A$.
According to (**64**) it will be found that

$$\frac{l}{\mu} = \frac{350{\cdot}4}{346}\cdot\frac{40{,}000 - 30{,}000}{30{,}000} = 1.35 \text{ cm.}$$

and therefore

$$\theta = 0.4\cdot 8\cdot\frac{\Sigma\,(F\,.\,l_s)}{4 + 1.35}\cdot 10^{-4} = 0.6.\ \Sigma\,(F\,l_s)\,.\,10^{-4}\text{ sec.}$$

whereby the summation is taken over the entire solid iron parts in the magnetic circuit. Substituting in this expression the numerical value 1.6 seconds, the condition is found for the volume of the solid iron as

$$\Sigma\,(F\cdot l_s) \leq \frac{1.6}{0.6}\cdot 10^4 = 2.67\,.\,10^4\text{ cm.}^3$$

This is the criterion for the amount of solid iron in the magnetic circuit to maintain the total magnetic time constant below or about 0.4 seconds.

In each case such a criterion may be easily derived. But a general expression for it cannot be given because of the transcendental character of the factor F. To use the approximation (**59**), first $\frac{\theta}{T_1}$ must be known, but the value of this ratio can be determined only at the end of the computation, so, therefore, it might be necessary to repeat the entire procedure. Only the following condition of the total volume of solid iron, as dependent on the magnetic time constant θ, may be generalized in the form

$$\Sigma\,(F\cdot l_s) \leq \frac{\delta + \dfrac{l}{\mu}}{3.2}\cdot 10^4\cdot\theta_c\text{ cm.}^3 \quad (\mathbf{65})$$

where θ_c means the critical value of θ in the special case as considered above.

Generally it is noticed that a continuous increase of voltage instead of a suddenly impressed voltage, as assumed in this paper, would increase somewhat the final magnetic time constant. But this influence is neither so important nor large that it should change the theory developed in this paper.

The effect of saturation could be introduced by composing the final solution of a sum of partial solutions each holding for a range of the magnetizing curve in which it may be considered as an approximate straight line. The suggestion of a standard magnetizing curve[17,18] might be of great value in applying this method of getting a general solution. But this method would be very laborious. Therefore, the results given in this paper may be taken as a first and apparently very important approximation to the final solution.

2. *Other Fields of Application.* A very important field of application may be found in all magneto-electric systems where regulation is influenced by the magnetic inertia.

The large d-c. motors in reversing rolling mills have a limitation imposed upon the succession of their different processes by the magnetic time constant of a solid iron yoke. In critical cases it is of importance to know with more precision the real magnetic time constant of a solid iron part as a member in the magnetic circuit. It is especially important in cases where the speed of production should be increased to know the possibilities of existing motors and to know what changes should be made when the time constant is too large.

One of the most important fields of application, however, is in the computation of the short-circuit currents in large generators. Such calculations are highly inaccurate as evidenced by the permissible international tolerances of $\pm$ 30 per cent in the peak value.[19] Such values are most unsatisfactory when compared with the relatively high precision of other electrical calculations, and it is hoped that refined methods in short-circuit calculations may be prepared in an additional paper using as a basis the method suggested in this treatise.

ACKNOWLEDGMENT

The author wishes to acknowledge with appreciation the aid of Professor Robin Beach who so kindly improved the English presentation.

Mathematical Appendix

1. To obtain the curves in Fig. 3 showing the ratio $\frac{J_0\,(q)}{J_1\,(q)}$ the tables of E. Jahnke-F. Emde[13] and Gray and Mathews[14] are used. For simplification the values of the roots of the Bessel functions $J_0\,(q) = 0$ and $J_1\,(q)$

$= 0$ are marked by making the ratio $\frac{J_0(q)}{J_1(q)}$ equal to zero and infinity, respectively. So we get the intersections with the axis of abscissas and the asymptotic straight lines. The slope of the curves in their intersections with the axis of abscissas is given by

$$\frac{d}{dq}\left(\frac{J_0(q)}{J_1(q)}\right)=\frac{-J_1^2(q)-J_0^2(q)+\frac{1}{q}\,J_1(q)\,J_0(q)}{J_1^2(q)},$$

and introducing the value $J_0(q) = 0$ we get the simple numerical value

$$\frac{d}{dq}\left(\frac{J_0(q)}{J_1(q)}\right)\Big/_{J_0=0} = -1.$$

2. To prove the calculation of the harmonic amplitudes of the field and of the current, we plot these amplitudes against the roots of the denominator function $Z(q)$. For simplification a so-called "resonance-root" of this function may be introduced. Placing the denominator of the right hand member in equation (**48**) with $\sigma = 0.25$ equal to zero, we obtain the definition

$$4\frac{\theta}{T_0} = q_R^2, \quad q_R = 2\sqrt{\frac{\theta}{T_0}}.$$

Taking instead of the roots q_r themselves, the ratio of the roots to this resonance root q_R as abscissas, we get the curves in Fig. 4 in the case of the current. The value for the fictitious harmonic amplitude at $q = q_R$ may be taken from (**51**), as after introducing $J_1 = 0$ in order to make $\frac{J_0(q)}{J_1(q)} = \infty$,

$$I(q_R) = 2\frac{\theta}{T_0}\cdot\frac{1}{\frac{\theta}{T_0} - 275\,q_R^2}.$$

Introducing q_R according to the definition, this expression becomes

$$I(q_R) = -0.2.$$

All curves must, therefore, intersect the axis of ordinates at a value -0.2. This is seen to be true from an inspection of Fig. 4.

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Discussion

H. Poritsky: The author is to be congratulated upon his successful treatment of an important subject for which no adequate solution has thus far been offered. The example chosen, while highly idealizing the actual structure of a generator, yet illustrates all the essential features of the problem, and the treatment by means of Heaviside's Operational Calculus, leaves nothing to be desired.

The reviewer wonders, however, whether an equally illustrative yet simpler example could not even further simplify the

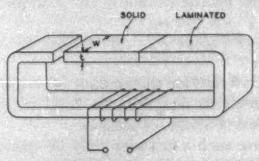

FIG. 1

problem. Thus, if in place of an iron core a yoke is used such as shown in Fig. 1, with width w considerably larger than the thickness t, we obtain a case resulting in a *one-dimensional* problem in the heat equation. The Bessel functions then would be eliminated in favor of hyperbolic or circular functions.

Again, the case of a long, solid, cylindrical rotor, placed in a laminated stator, has been briefly examined by the reviewer. This case allows direct treatment along similar lines, with no added complexity, yet approximates even better to the actual case of a turbo alternator.

Interconnection Services

Their Classification and Evaluation

BY ALEX. E. BAUHAN*

Member, A. I. E. E.

Synopsis.—*The services rendered by an interconnection between two or more electric power systems may be classified as:*

1. Emergency service—by which assistance is given by one system to the other in event of breakdown of equipment or other abnormal conditions.

2. Load diversity—arising from the non-coincidence of the times of peak loads in the interconnected systems.

3. Firm power—transactions involving a definite advance commitment by one company to sell power and energy to the other for use in lieu of installed capacity.

4. Storage power—by which off-peak energy from one system is converted into peak capacity in the other through the medium of storage.

5. Intra-company use—referring to the use of interconnecting facilities for the internal purposes of one of the parties.

6. Economy flow—by which generation in one system is replaced by more economical generation in the other system; and

7. Unintentional flow.

The foregoing services may be evaluated, as far as capital cost is concerned, by determining the aggregate saving in generating capacity and multiplying by the estimated cost per kilowatt of generating capacity and then comparing the result with the capital cost of the interconnection.

The more important annual-cost comparison is made by:

1. Determining the difference between the fixed charges on the avoided generating station investment and the fixed charges, including operation and maintenance, on the interconnecting facilities.

2. Determining the saving in fixed element of production expense arising from the avoided capacity.

3. Determining the change in the variable element of production expense due to interconnection.

THE word *interconnection* as used in this paper refers to the physical tying together of two or more independently owned or managed electric systems for the purpose of realizing financial benefits or improvement of service by the interchange of power and energy. The subject of interconnection may be considered under the following divisions:

1. Classification of Services
2. Evaluation, Economics
3. Contract
4. Design and Construction
5. Operation

of which this paper deals only with the first two.

I—Classification of Services

The determination of the economic merits of an interconnection, its evaluation, is much facilitated by an orderly and logical classification of the services which that interconnection will render. Recognition of the various classes of service is not only helpful but is quite essential in the preparation and application of the interconnection contract, as each class of service requires different provisions and rates of payment. Such classification is helpful in rationalizing design,—for instance in determining whether the services to be rendered by the interconnection warrant a double circuit or only a single circuit. The various services are of course the substance of operation.

What is offered here is not intended to be an enumeration of the benefits rendered by interconnection but rather a delineation of types of transactions which take place over interconnections. It is an attempt at earmarking or labelling the various amounts of power or blocks of energy in the light of what they do, of what service they perform, as necessitated principally by evaluation and contractual expression. The classification is submitted with the thought that it may form the basis of discussion and trial which will eventually bring out a generally acceptable classification and nomenclature. It recognizes the following seven main service divisions:

1. Emergency service
2. Load diversity
3. Firm power
4. Storage power
5. Intra-company use
6. Economy flow
7. Unintentional flow

1. *Emergency Service*

By emergency service is meant the assistance which one of the interconnected companies renders to the other on account of breakdown of equipment, unusual load demands, or other abnormal conditions resulting in need for power in excess of the capacity available from the normal power sources of the company in difficulty.

For the purposes of evaluation, and possibly for contractual reasons, it may be necessary to distinguish between *system emergency service* and *transmission emergency service*. The former has to do, in a well-consolidated system, with conditions of load and generating capacity in the whole system. It is associated with the more substantial ties between important centers of the systems. Sometimes when this service makes possible a definite recognizable reduction in the

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amount of reserve generating capacity it is referred to as *reserve diversity*. Transmission emergency service on the other hand is concerned with small interconnections between the fringes of neighboring systems. Such service reinforces the normal transmission, which perhaps consists of only a single circuit, to a particular outlying part of one or both systems. Of course each type of emergency service brings with it something of the other and in some interconnections both are of importance.

Another class of emergency service arises from transactions which involve the transmission of emergency service from or to a third party or between a third and fourth party through the interconnection of the two parties under consideration. This classification need be recognized only if such flow be accorded contractual treatment different than that given to emergency transactions between the two parties immediately at interest.

The energy associated with emergency service is commonly looked upon as being simply that necessi-

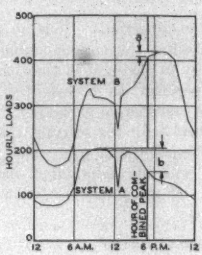

FIG. 1—DAILY DIVERSITY = $a + b$

tated by the emergency, the flow being limited to the extent and duration of the need for outside assistance.

2. *Load Diversity*

Load diversity refers to those transactions which result in a saving of generating capacity by reason of the non-coincidence of the times of the peak loads of the interconnected companies.

For instance, if a system having a morning peak and a relatively light load in the late afternoon is interconnected with a system having its peak at five o'clock in the evening and considerably less load in the morning, a saving in required generating capacity results. This saving in generating capacity, which is spoken of as the diversity, is equal to the sum of the individual peak loads minus the peak load of the combined systems. If the diversity is found to be say 40,000 kw. and is equally divided between the two companies, each at the time of its peak could receive 20,000 kw. from the other and thus realize a saving of installed generating capacity of that amount. The power and energy blocks flowing over the interconnecting lines as a result of this transaction are spoken of as diversity power and diversity energy.

It is not necessary that the load-diversity energy be considered to flow equally in opposite directions, as in the foregoing case. As a matter of agreement the flow can be divided in any other way. In event one of the companies is not in a position to curtail its installed generating capacity by reason of already having a surplus, all the capacity saving might be taken by the other company. This means that all of the diversity power would flow in one direction and instead of being

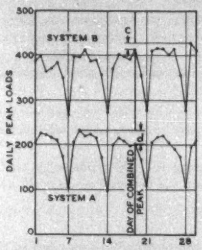

FIG. 2—MONTHLY DIVERSITY = $c + d$

returned in kind, could be adjusted by a money payment. Such a transaction and the flow resulting might, however, be considered to be equally divided and equally exchanged load diversity, with a one-way firm-power sale in the amount of one-half of the diversity saving superimposed on it.

Daily, Monthly, and Yearly Diversities. As an aid in determining diversity and later in laying out operating schedules for its realization it is sometimes advisable

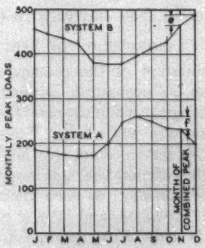

FIG. 3—YEARLY DIVERSITY = $e + f$

to subdivide diversity into daily diversity, monthly diversity, and yearly diversity.

Daily load diversity is the diversity which occurs within any particular day. Monthly diversity is the diversity as among the daily peak loads of any particular month. Yearly diversity is the diversity as among the monthly peak loads in any particular year. Figs. 1, 2, and 3 illustrate respectively these three classes of diversity.

Note that in these illustrations the diversity is

determined as being the sum of the differences for each system of its peak load and its load at the time of combined peak. This is somewhat different than the common expression for diversity, *viz.*, "the sum of the individual system peaks minus the peak of the combined systems;" but it is the mathematical equivalent. The form used in the illustrations will be found far more convenient in thinking of and working with diversity problems, particularly in cases where the combined peak occurs simultaneously with the peak of one of the systems. When this is the case, and it usually is when one of the systems is appreciably larger than the other, the diversity is determined independently of the system whose peak is simultaneous with the combined peak and is equal simply to the difference between the other's peak load and its load at the time of the combined peak.

Installed-Capacity versus Operated-Capacity Reductions. Our interest in load diversity arises mainly in the reduction in installed generating capacity which it makes possible. This reduction of installed generating capacity is equal to the daily diversity of the combined-peak day plus the monthly diversity of the combined-peak month plus the yearly diversity. There may in the non-peak periods of the year be monthly and daily diversities which effect no saving in the installed generating capacity. These can, however, be taken advantage of to reduce the amount of capacity operated. The parties to an interconnection may therefore wish to distinguish between load-diversity service reducing installed capacity and that reducing operating capacity. Diversity which only reduces the operating capacity is of relatively small importance. In fact, it is better not to consider this service as being diversity. It really comes under the category of "economy flow,"—discussed in detail later.

Scheduled Diversity and Call Diversity. Another classification of load diversity, one which is called for only by contractual and operating convenience, and then only in highly refined operations, is that which divides load diversity into *scheduled diversity* and *call diversity.* Scheduled diversity is that diversity which is predictable as to amount and time of its occurrence and call diversity that which is predictable as to amount but not as to time of its occurrence. The character of call diversity may be discerned from Fig. 2 illustrating monthly diversity. It is evident that the day-to-day load variations make it impossible to predict the day of combined peak or the day of either system's peak. There is, however, adequate expectancy that the two systems will not have their individual peaks on the same day, *i. e.*, that there will be monthly diversity. But in order that this portion of the total diversity may result in capacity savings, and inasmuch as the necessary power interchanges cannot be scheduled in advance, arrangements must be made for a previously determined amount of diversity-power flow which can be called for on short notice by the party experiencing a heavy or peak day. On another day when the other party approaches its estimated peak load it in turn calls for this diversity flow. Hence the expression *call diversity.*

To be of value, the amount of both scheduled diversity and call diversity must be established two or two and one-half years in advance. Both are firm entitlements and firm obligations,—scheduled diversity during periods which may be scheduled in advance, and call diversity at such times as either company's loads are such as to justify calling for it.

Usually the call diversity approximates the monthly diversity; and the scheduled diversity is likely to be the yearly and daily diversity. However, to the extent that the monthly diversity has a predictable time characteristic, such as might arise from a regular downward or upward trend of the daily peak loads, it can be considered as scheduled diversity. Similarly the yearly and daily diversities may have uncertain time characteristics and would therefore contribute to call diversity.

3. *Firm Power*

Firm power arises when one company, instead of installing necessary generating capacity in its own system, chooses to buy power from the other system. It involves a sale contract, usually made two or three years in advance, with a firm obligation on the part of the seller to hold capacity available for the use of the buyer in lieu of installed capacity.

Short-Term and Long-Term Firm Power. Firm power may result from either short-term contracts running for only two or three years or long-term contracts extending ten, fifteen, or more years.

Under the heading of short-term firm-power contracts would be those arrangements sometimes entered into under the name of staggered construction, where one company instead of building a new power house will, for a year or two, buy its additional requirements from a neighboring company which has surplus capacity. In a later year the situation may be reversed, the first company building its generating station and selling to the other company, whose own needs by this time have more than used up its own available capacity.

Sometimes several neighboring companies will form a pool of their capacity, and stations will be developed, as dictated by economy, in the system of one company or the other for the benefit of the entire pool,—witness the power pool in and around Chicago. In such cases firm power is really flowing from one system to the other for short terms, the flows shifting back and forth from year to year depending on how the loads and capacities of the systems develop.

An arrangement by which a company obligates itself to take power for an extended time such as 10, 15, or 20 years, where it is distinctly not a temporary expedient, is classified as a long-term firm-power transaction.

Peak Power and Base Power. Firm-power provisions

are sometimes spoken of as being *peak power*, or *base power*.

Peak power refers to power which is either available for only a few hours a day, or which carries an energy rate which is high in comparison with the incremental costs per kilowatt-hour of the poorer stations in the receiving system. Therefore it is used for peak purposes only. If the energy price of the purchased power is very high, and if there is no load-factor guarantee in the contract, it might even result in no energy ordinarily being used. This would be the purchase of a capacity entitlement for use only when other equipment having lower incremental costs per kilowatt-hour is not available.

Base power is power which carries an energy rate which is low in comparison with the incremental costs per kilowatt-hour of the better stations in the receiving system and therefore it is used in the base of the load curve at high-load factor. Where the energy rate of the firm power is neither distinctly high nor low in reference to the incremental energy costs of the various generating stations in the receiving system, the purchased power takes an intermediate place in the receiver's load curve, as determined by economies.

Quasi-Firm Power. Ordinarily, the expression firm power implies that the selling company obligates itself to maintain the spare capacity necessary to make the power *firm*, and therefore relieves the buying company from the necessity of providing any spare capacity for the purchased power. Sometimes, however, power is sold with the understanding that its delivery is conditional on the availability of certain special equipment. Company *A* may for instance install a generator in one of its stations and sell part or all of the output of that generator to company *B*, with the understanding that power will not be delivered if that generator breaks down. In this case, as far as *B* is concerned, *A's* generator resembles any of *B's* own generators which is liable to break down at any time and *B* must therefore provide the spare capacity for the purchase. It may be best to refer to such a transaction as being quasi-firm power, although as far as the interconnection is concerned it has all of the characteristics of firm power when it is flowing and does not warrant a separate classification.

4. *Storage Power*

The term storage power has been applied to transactions involving the conversion of energy generated off peak in one system into peak capacity in another system through the medium of storage. The particular storage medium used is immaterial, although in America at least and in this discussion we are practically concerned only with the medium of hydraulic potential energy.

Two classes of storage power may be recognized; that associated with pumped storage and that associated with improved utilization of stored stream flow.

Pumped Storage. In the case of pumped storage, water is actually pumped into the storage pond during off-peak hours, using relatively low-cost energy, and later that water is used for the generation of energy on peak when it has capacity value.

Improved Utilization of Storage. In the other and more general class of storage power, energy is not put into storage; instead there is a substitution for energy which would otherwise be taken out of the available storage. The available stored energy is thus conserved for utilization at a higher rate of discharge and at a time when capacity value will be created.

The present discussion is concerned only with those cases where two companies are involved and where the energy to be stored or used in substitution is transmitted over an interconnection. This energy may be generated by steam power or it may be energy from another hydro-development on the same or another stream.

The classification of storage power is mainly concerned with the creation of capacity by the conversion

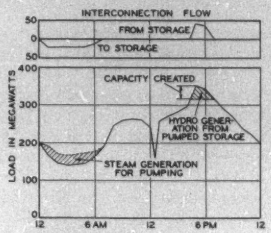

FIG. 4—PUMPED STORAGE

of energy having relatively low value. This capacity may be sent to the cooperating company, retained by the hydro company, or divided between them, depending upon where it may be used to best advantage. There may be instances, however, where energy of relatively low value is converted through the medium of storage into energy of higher value, but the savings to be realized will be small in proportion to those obtained where capacity value is created.

The power and energy quantities involved may be more readily observed in the accompanying diagrams. The quantities involved under the pumped-storage scheme are shown in Fig. 4 for the case where the capacity created is returned to the company supplying the off-peak energy. The common case of better utilization of natural storage is represented by Fig. 5, in which it is assumed that the capacity created is returned to the steam company. Fig. 6 represents the special case where two companies having similar peak characteristics cooperate to obtain the greatest capacity value from available hydro energy. The scheme depends upon the fact that the combined peak of the two companies

is sharper than that of the hydro company alone and therefore that the available hydro energy will have a greater capacity value on the combined peak. In operation the non-hydro company must generate energy in addition to its own load immediately before and immediately after its peak. This requires careful load dispatching and well-conceived contractual relations.

5. *Intra-Company Use*

Intra-company use refers to use of the interconnecting facilities by one of the companies in place of other facilities which it would have had to provide for its own purposes. Interconnecting facilities which may be thus used include transmission lines and structures, substation equipment, tertiary windings in interconnection transformers which take the place of separate two-winding transformers, and rights-of-way.

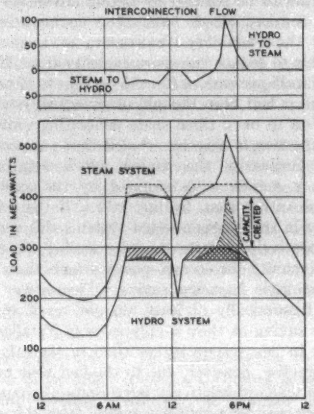

FIG. 5—IMPROVED UTILIZATION OF STORAGE

Systems having dissimilar peak characteristics
———Load - - - - - - -Generation

In some of these transactions energy for intra-company use is commingled with interconnection flows and in such cases contractual arrangements are necessary to provide for their separation and to place limits upon the intra-company use so that interconnection flows are not unduly hampered. In other cases it is necessary simply to provide for division of the resulting benefits in the manner intended by the parties and there is no necessity for recognizing the classification of intra-company use so far as energy flows are concerned.

An example of intra-company use appears in Fig. 7 where the interconnection lines between the generating sources A and B of two companies are tapped so that they may be used by company A in transmitting energy from A to M, one of its outlying load centers, instead of A's reinforcing its own direct transmission between A and M.

6. *Economy Flow*

Economy flow occurs when energy used by one system and which could have been generated by it, is actually generated by another system because it is more economical to do so. For instance, if A has idle capacity capable of operating at an incremental cost

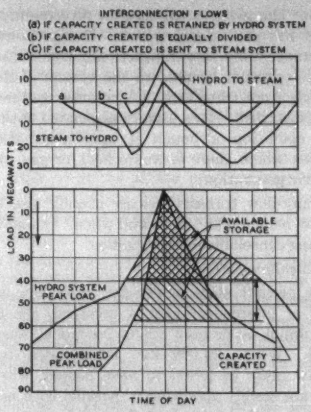

FIG. 6—IMPROVED UTILIZATION OF STORAGE

Systems having similar peak characteristics

of 3 mills per kilowatt-hour at a time when B is running generating capacity having an incremental cost of 4 mills per kilowatt-hour, obviously interconnection permits substitution of the less expensive generation, and the resulting transaction is called economy flow.

In order to determine the cost and value of power and

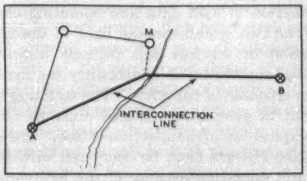

FIG. 7—INTRA-COMPANY USE

Generating stations
Substations

energy thus transferred it may sometimes be desirable to distinguish that part of the transaction which results in a change in standby or fixed element of operating costs—such as might come about by its shutting down generators or boilers in the receiving system. It is suggested that the term *economy power* might be applied to this part of the transaction to distinguish it from that part in which the saving is proportional to the amount

of energy transferred, which might be called *economy energy*. In some cases, as when a standby station is shut down by another company's agreeing to supply the standby service ordinarily obtained from this station, the power phase of the transaction is dominant, and the energy flows of secondary or negligible importance.

Custom has given a subdivision of economy flow a name of its own—*dump power*. Dump power is a particular economy-flow transaction in which the energy involved could not be used in the system of its origin and which if not transferred to the other system would have been wasted. It is usually associated with hydro-electric operation although it may arise also from operations where there is waste heat or gas available for electric generation.

Economy-flow transactions are usually also characterized by being on an "if, as and when" available basis or by being cancelable on short notice.

7. *Unintentional Flow*

In the present state of the art, especially where a network of interconnections is involved, the flow of energy cannot be or may not be controlled exactly as intended. Where 20,000 kw. is intended to flow, 22,000 kw. may be found to have passed; where two systems are merely operating in parallel with no interchange intended, inadvertent flows may occur. Such energy quantities cannot be entirely disregarded since they cost money to generate and they save money for the receiving company. It is therefore convenient to recognize unintentional flow in the inter-company relations.

II—Evaluation

Before decision can be reached as to whether or not a proposed interconnection should be installed, it is necessary to estimate and present its economies.

Examination of load data and operating characteristics of the two systems in the light of the foregoing classification of services will perhaps indicate that interconnection will make unnecessary the installation of certain amounts of capacity. The resulting avoided investment in generating stations must be compared with the capital cost of the interconnection. The avoided annual fixed charges must be compared with the fixed charges and operating expense of the interconnection. The change in the annual generating station production expense brought about by the various services must be determined. The comparisons mentioned must be made ordinarily not for only one year, but for an extended period, probably not less than five years nor more than ten years, for the reason that wide variations may occur in the amount of the savings from year to year.

If the result of these comparisons is a net saving the decision would probably be favorable to making the interconnection. If the net saving is small or slightly negative, it does not follow that the interconnection should not be made. Consideration of the intangible and indeterminate benefits may warrant favorable decision. Among such indeterminate benefits may be mentioned catastrophe assistance, lessened hazard from under-estimation of loads, possibility of taking chances on the non-availability or non-installation of equipment which could not be taken without interconnection, and opportunity for future but at present unforeseen benefits.

The following is intended to indicate the general procedure and the mode of attack to be used in evaluating a proposed interconnection.

1. *Generating Capacity Savings*

From the load data and operating characteristics of the systems it will be determined that certain capacity savings are to be realized by taking advantage of the various services which have been enumerated. For instance, the availability of emergency service may make it possible to reduce the spare capacity from 20 or 25 per cent to 15 per cent, or from 15 per cent to 10 per cent; or, where it had been thought necessary without interconnection to have three spare generating units, with interconnection it may be decided that two units will suffice. Ordinarily the saving which results from emergency service is determined by the exercise of judgment rather than by any rule. If there is load diversity in the interconnected systems this results in a comparatively definite capacity saving. Similarly, the opportunity for storage-power transactions yields a determinable capacity saving. Firm-power transactions theoretically at least, do not result in a net capacity saving as they simply involve installation of capacity in one system rather than in the other. In actual practise, however, due to the fact that installations are made in relatively large units of commercial size, there may be a capacity saving from this source.

In fact, if the problem is one involving only two or three years it is safer to determine the actual capacity installation program with and without interconnection, rather than to use the capacity savings theoretically indicated. However, if the benefits of the interconnection accrue over a long period of years, during which it is expected that the business will continue to grow, a study based on commercial unit sizes is not of any superior value. The error arising from not considering commercial unit sizes is present in both the with-interconnection and without-interconnection programs and in the comparison the errors tend to drop out. In any event they become unimportant over a period of years. Further, the impossibility of predicting specific installation programs with any adequate degree of accuracy for more than a very few years in advance, favors the use of the theoretically indicated amounts of capacity saving.

In arriving at the capacity savings in any detailed study it is necessary to consider conditions month by

month as the theoretical capacity saving may vary cyclically within each year.

Another caution to be exercised in arriving at the avoided capacity is to consider the effect which the requirements of scheduled outage of units for maintenance will have on the possible savings. This effect will be of importance only if the seasonal variation of the combined load is small.

2. Capital Cost Savings

After the amount of capacity saved has been ascertained, the resultant capital cost saving must be determined. This may be accomplished by applying appropriate unit costs to the kilowatts of avoided capacity or if a more concrete study is being made, by setting up the particular program of station installations "without interconnection" together with its capital costs, then doing the same for "with interconnection," and finally obtaining the difference in capital costs.

Of course any capital savings which result from intra-company use of the interconnecting facilities, or from firm-power transactions, must be taken into account.

It remains then only to compare the total capital cost savings with the capital cost of the interconnection.

Rarely is a determination of capital cost savings sufficient *per se* to indicate the economic merits of an interconnection. Nevertheless, capital cost determinations must be made as the first step in computing the annual costs, and they are in any event of importance to the executives who are concerned with financing the project.

3. Annual Fixed Charges

In the evaluation of an interconnection the determination of its effect upon fixed charges is necessary since any avoided investment in facilities naturally results in a reduction of fixed charges and on the other hand fixed charges on the interconnecting facilities are incurred.

The term *fixed charges* is ordinarily meant to include the annual costs of money, insurance, taxes, retirement expense and general expense, and in the case of facilities other than generating stations, it is sometimes convenient to include operation and maintenance expenses. Fixed charges are usually determined and expressed as a percentage of capital cost.

The necessity for careful determination of fixed charges warrants some discussion of the fixed-charge percentage to be used in each case.

Money Cost. The annual cost of money represents the compensation which must be given to capital in order that it be attracted to the industry for the purposes under consideration. In the absence of abnormal conditions, it is likely to be the prevailing allowable return on public utility property. This is neither the cost of borrowed money nor the yield on equity funds, but rather the weighted-average cost of all funds. There may, however, be cases where the cost of the next increment of financing should be used. If this happens to be by bonds, the money cost includes not only the coupon yield, but also the amortization of debt discount and expense.

Insurance. Insurance includes the cost of fire, tornado, earthquake, public liability, boiler, machinery, fly-wheel, workmen's compensation and other insurance. Insurance should also include the cost of self-insurance. Where the company's operations are large or have continued over an extended period of time the cost of self-insurance can be estimated from the actual losses incurred for which insurance premiums might otherwise have been paid.

Taxes. Taxes differ in each state and municipality and in order to arrive at the proper percentage it is necessary to understand the manner in which the taxes are levied. In many cases it is necessary to determine only the relation of the total state and municipal taxes paid by the company to its fixed capital; in other cases the tax rate in a particular locality should be applied to facilities in that locality. In still other cases, where taxes are levied on the gross receipts of a company and would not be directly affected by any interconnection considerations, they may be ignored. In all cases, however, income taxes should be included. Because of the manner of income tax determination the relation between borrowed funds and equity funds must be known and the prevailing tax rate applied to only the taxable income. For example, with the present U. S. federal income tax of 12 per cent, if the money cost is 8 per cent and 60 per cent of the capital funds are borrowed at a cost of 5 per cent, the taxable income will be 5.682 per cent and the income tax 0.682 per cent of capital.

Retirement Expense. Retirement expense should be determined on the sinking-fund basis using the rate of interest currently earned on securities suitable for investment in trust funds. The retirement rate to be used will depend upon the rate of interest and the economic life of the equipment under consideration, which varies for different classes of equipment. If the rate of interest is 5 per cent per annum, the retirement rate for a steam generating station having an economic life of 18 or 19 years will be 3.25 per cent or 3.50 per cent; for a hydroelectric development having, due to its inherently simple and sturdy construction and high efficiency, an economic life of 50 years, only 0.5 per cent; for a substation having an economic life of 22½ years, 2.5 per cent; and for a transmission line having an economic life of 25 years, 2 per cent.

General Expense. General expense includes administrative salaries, general office expenses, communication, law expenses, relief and welfare work, and other items not included elsewhere. Ordinarily general expense will amount to 1 per cent or less, of fixed capital, but frequently it may be considered that general ex-expense will be unaffected by interconnection.

Usually fixed charges on interconnecting facilities, exclusive of their operation and maintenance, will be slightly less than on steam stations, so that for the same capital investment there is some saving on this score. Naturally no generally applicable fixed charge percentages can be cited. Fixed charges must be individually determined for each case. However, to indicate the order of the fixed charges it may be said that for steam generating stations they will frequently be about 12 or 13 per cent and for hydroelectric developments perhaps 9 or 10 per cent. For steel-tower transmission lines fixed charges are affected somewhat by the proportion of the investment in right-of-way but they may run from 12 to 14 per cent with operation and maintenance expenses included. Substations, likewise including operation and maintenance expenses, will be somewhat higher, usually between 14 and 17 per cent.

4. Annual Production-Expense Savings

After the saving in fixed charges has been determined it remains to evaluate the effect of interconnection on production expense. This requires, first, determination of station load schedules with and without interconnection and, second, application of the individual station production-expense formulas to the end that the changes in the fixed and variable elements of production expense be determined.

Station Load Schedules. The output of each station and the character of its load for most economic conditions with and without interconnection can be obtained by resorting to integrated duration curves or duration curves, for the loads of the individual systems on the one hand and for the combined system on the other, on an annual, seasonal, or monthly basis, according to the degree of precision desired. Integrated duration curves with percentage scales are helpful where much work is to be done. (See Hutchinson, A. I. E. E. TRANSACTIONS, 1914.) For more precise studies loading schedules may be obtained from ordinary load curves of typical weekdays, Saturdays and Sundays. Due allowance must be made for the outage of generating units for normal and abnormal maintenance. It must be remembered that the interconnection transactions will be limited by transmission losses, the capacity of interstation and interconnection transmission, the necessities of service protection in local areas, minimum steaming conditions, and the minor imperfections of operation and of contracts.

Behavior of Generating Station Costs. Before the production expense savings due to interconnection can be determined it is necessary to know for each generating station in the interconnected systems, the behavior of production expense as a function of station output. For interconnection studies the two-part annual production expense formula has been found to be sufficiently accurate and most suitable. Such a two-part formula arises from the experience that within a limited range of annual capacity factors the production expense is made up of a fixed element independent of annual output and a variable element directly proportional to the number of kilowatt-hours generated in the period. The fixed element includes a large part of the items of superintendence, supervision, labor, supplies, and maintenance, together with (usually) a small part of fuel expense. The variable component consists principally of fuel costs, although part of the maintenance and small amounts of other items may also be found to come in this class. Expressed in mills per kilowatt-hour it represents the average cost throughout the year of generating an additional unit of output, and is therefore commonly termed the "incremental cost."

Usually a single formula covering a limited capacity-factor range suffices for each station. If, however, the station is to be examined under widely differing capacity factors a two-part formula for each of several capacity-factor ranges may be necessary. If the station is to operate at a peak load predominately less than its capacity, as may be the case with the poorest stations, corrections may be necessary principally in

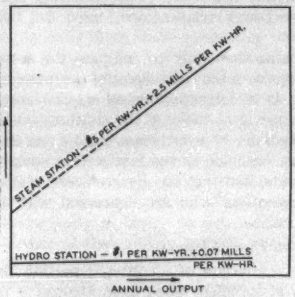

FIG. 8—ANNUAL PRODUCTION EXPENSE CHARACTERISTICS FOR STEAM AND HYDRO STATIONS

the fixed element of the two-part formula, or a formula introducing a peak-preparedness term as developed by Funk and Ralston (Philadelphia Electric Company) must be used.

Fig. 8 is representative of the two-part formula for a steam station having a fixed element of about $5.00 per kilowatt of installed capacity and a variable element of about 2½ mills per kilowatt-hour. For comparison a hydro-station formula with a fixed element of about $1.00 per kilowatt of installed capacity and an almost negligible variable element is shown.

A bibliography of the scant but authoritative literature on this subject is attached.

Fixed Element of Production Expense. Obviously the avoidance of an installation of generating capacity brings about a reduction in the fixed element of production expense. Approximately speaking, and frequently with sufficient accuracy in the case of a long-term study, the saving in fixed-element expense is the number of kilowatts of avoided capacity resulting from the inter-

connection transactions multiplied by a figure, possibly between $3.00 and $5.00, representing the fixed element of production expense per kilowatt of avoided capacity. Its effect is similar to avoided fixed charges.

For a more precise determination in which actual installation programs and station loadings have been predicted, it is advisable to tabulate and add the appropriate fixed elements for all generating stations under the without-interconnection condition and then under the with-interconnection condition. The difference between the two will be the fixed-element saving. In such precise study it may be recognized that, aside from fixed-element saving on avoided capacity, there is sometimes an additional saving of fixed costs in the marginal stations due to the interconnection's encouraging curtailment of *running* spare capacity.

Variable Element of Production Expense. The change in variable element of production expense involves only the multiplication of the energy quantities determined for each generating station without and with interconnection, by the corresponding increment cost per kilowatt-hour from the two-part formula and obtaining the difference between the two plans of operation.

Frequently the net effect of interconnection on this item is not a saving, but a decided loss. The explanation is, of course, that the interconnection forestalls installation of modern, highly efficient equipment, and increases the duty on older plants having higher variable costs.

5. *Net Savings*

Net savings resulting from the interconnection are obtained from the combination of net fixed-charge savings and net production-expense savings, which may well be presented by means of a tabulation showing the amount of these savings for each of the years studied. If comparisons are to be made between different plans of interconnection, these net annual savings over the period of study should be accumulated and due account taken of interest on the accumulations.

No attempt has been made herein to show how savings may be divided between the interconnecting parties, since this is a matter for agreement between them. Nevertheless it is a factor which may affect some of the items of evaluation, and certainly requires careful recognition of the equities for each particular class of interconnection service in the agreements relating to the project.

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Discussion

For discussion of this paper see page 1280.

Interconnection—New England District

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Historical and Physical Data

THE geographical characteristics and the early method of development of the New England region make it a natural field for interconnection. The total area covered is small compared with that of other interconnected districts, but industrial growth has been intensive and many strong local companies have been developed. There are tidewater locations for steam plants affording ample condensing water and low carriage charges for fuel, and there are inland rivers with sufficient runoff, fall and control possibilities to make water power developments of various types attractive. It is natural that a close network of tie lines has been built.

The standpoint of those companies which now form the New England Power Association, together with that of the systems directly interconnected with the Association, has been taken for the sake of brevity and in the belief that their experiences are typical of the territory. The experience of the companies now forming the Association is somewhat unique in that many of them were interconnected when operating under separate financial control. A direct comparison of the economies of both methods of operation is therefore possible. The companies operating in the state of Maine have been generally interconnected presumably for the same purposes that have existed in the remainder of New England, but they have been isolated as a group by legislative restrictions upon the flow of power across the boundaries of the state.

The New England Power System started principally as a hydroelectric system with plants on the Connecticut and Deerfield Rivers located in the northwestern part of Massachusetts and southern Vermont. Lines were built easterly through major cities and towns terminating near Worcester. Steam reserve was obtained from the local companies to whom power was sold, and also from some large industrial purchasers. Later, transmission was extended into Rhode Island, connecting with Woonsocket, Pawtucket, Providence, and Fall River. The first major interconnection (of about 30,000-kva. capacity) was made in 1913 at an existing crossing with the lines of the Turners Falls Power & Electric Company. The Turners Falls Power & Electric Company had considerable surplus hydro output at times which the New England Power system was able to absorb, and in addition was occasionally able to sell steam auxiliary. This tie, in view of the small amount expended, has probably shown a greater return per dollar invested than any of those subsequently made. It later provided a path in the reverse direction to market large quantities of surplus steam capacity, and still later some firm power has been sold through the tie.

The South Street steam plant in Providence was next connected with 20,000 kva. of cable capacity. This tie furnished a source of steam supplied largely from spare capacity, and also a market for surplus hydro power. After a few years an additional 50,000-kva. high-tension tie was made here and an elaborate agreement was drawn with the Narragansett Electric Company of Providence under which firm and surplus steam power was purchased particularly to cover large industrial contracts in that general territory.

The next major connection was with the Montville steam plant of the Eastern Connecticut Power Company (now the Eastern Division of the Connecticut Light & Power Company). This arrangement involved about 60 miles of line, which was partly in the New England Power System territory and partly in the Eastern Connecticut territory, and served, especially in the case of the Eastern Connecticut Company, as a source of power to customers along its length. The extra expense due to the larger capacity built into the line for power transfer and also some charges on account of additional boilers to make available the capacity of a spare unit at Montville were carried by the New England Company. As part of this arrangement, it was possible for the New England Power System to purchase at any time when the unit was available, a reasonably firm quantity of steam generation up to 10,000 kw. This connection was made during the period of extraordinary growth of 1919 and 1920 when costs were high and development of hydro would not have been economically sound.

A similar arrangement was made with the Edison Electric Illuminating Company of Boston at approximately the same time, making available to the New England Power System surplus capacity in the "L" Street station of this company, to the extent of 15,000 kva., as well as a market for surplus hydro power. The charges on Edison's expense were carried for a number of years by New England. Afterward this line developed into a customers' line for both parties, as well as remaining an interconnection.

In 1924 negotiations were carried on with the then Adirondack Power and Light Company for a connection between systems. The New York State Company at that period had primarily an unregulated hydroelectric system. The New England Power System had grown to approximately equal parts of hydro and steam, and the hydro was largely regulated so that it could absorb up to 50,000 kw. of surplus hydro from New York State. The construction of this tie established conditions

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Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

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whereby, for the first time, three systems were involved in power transfers because much of this water generated power found its way through the New England Power System to the Edison company at Boston. Each company carried the expense of transmission in its own territory with a slight adjustment for inequality of length.

An additional interconnection was made in 1927 with the Edison company at 110,000 volts which connected its new Edgar plant with the system center at Millbury. Both parties covered the expense of the line in their own territories. With this line it was possible to

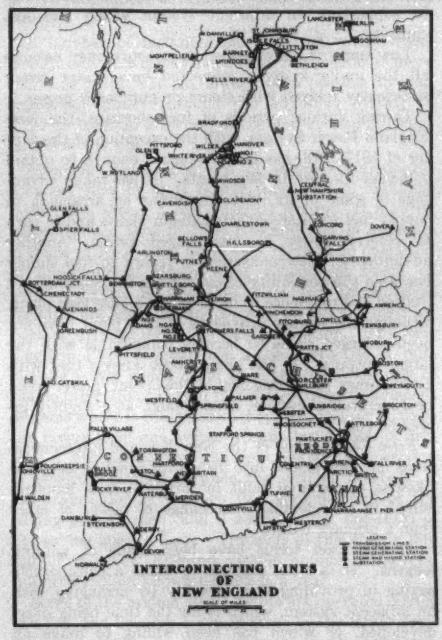

FIG. 1

transfer up to 50,000 kw. of low cost steam from the Edgar station to the New England Power System. No contract was made.

In 1929 prior to the development of Fifteen Mile Falls, and prior to the sale of a large block of primary power to the Edison company on the north side of their territory, it became a matter of mutual saving to backup the northern territory of the Edison Company through the New England Power System from the connection already made between Millbury and the Edgar station on the south side of the territory. New England advanced its own construction schedule one year and built, prior to the necessity on account of Fifteen Mile Falls power, a connection from Millbury to the north side of the Edison territory.

In addition to the interconnections between the New England Company and adjacent systems, there are at least two major interconnection arrangements in operation; that of the Connecticut Valley Exchange Board, and the Montaup Electric Company.

The Turners Falls Power & Electric Company, taking its power mainly from a large hydro plant in northern Massachusetts, was early connected with the local electric company at Springfield, Massachusetts, which depended primarily on steam power. In 1925, the Connecticut Valley Exchange started operation involving the Hartford Company with the two mentioned above, and in the following year included power transfer with the Connecticut Light & Power Company in the southwestern part of Connecticut. The Central Hudson Company was interconnected in 1927. These companies, without common control, arranged to operate their power facilities as a group, at first under general orders from a small organization formed for the purpose, and later by actual dispatching.

The pool was charged the cost of generation in excess of local requirements by each company furnishing power to it. Each company was charged for power received from the pool an amount equal to the cost that company would have incurred in producing the same amount in its own plants. The expense consisted of management costs and some fixed charges for interconnection. The New England Power System is physically connected through the Turners Falls Company, and has in late years exchanged power with the pool as a unit.

The Montaup Electric Company, which is controlled by the local companies at Pawtucket, Woonsocket, Brockton, and Fall River, started operation in 1925. A steam station and transmission lines connecting the participating companies were built. Capital was furnished by the local companies in proportion to their capacity requirements, and the operating expense was paid for by sales at cost to the local companies. The New England Power System is connected to and exchanges power with the system as a unit.

The reasons for the construction of these interconnections are: (1) The absorption of surplus hydro power from systems generating, largely, from unregulated hydro sources: (2) The absorption of large quantities of surplus steam with resultant good load factor on relatively high-grade steam plants of adjacent systems: (3) Arrangements for the purchase of surplus steam capacity in various stations which resulted in a certain amount of firm capacity: (4) The sale or purchase of firm power.

Benefits Derived from Interconnection

Peak Diversity. Companies which are not under the same financial control have not taken advantage of peak

diversity by virtue of interconnections to save capacity installation. Each company has assumed responsibility for its own primary load. A study of this subject indicates that between the major interconnected groups in New England there is approximately 5 per cent diversity, amounting to say 50,000 kw. A worthwhile safety factor is gained though by each of the interconnected companies, and it has become practise to take advantage of this by continually tying together, even though an exchange of kilowatt-hours is not particularly desirable at the time and may even result in some slight losses.

On the other hand, there is an appreciable diversity saving between the commonly owned and controlled properties such as Lowell, Lawrence, Worcester, Providence, and the original wholesale company, known as the New England Power Company which is of the order of 7 per cent, or at the present time about 30,000 kw. This diversity exists on account of peaks occurring on different days, rather than at different hours of the same day.

A large diversity of peaks in the same day might well be looked for on a system spread into two time belts but no dependable diversity of this sort appears from a study of the New England systems since they all operated on eastern standard time except during summer months when heavy peaks are not to be expected.

Reserve Diversity. No particular advantage is taken of reserve diversity. The systems usually carry reserve equal at least to the capacity made unavailable by the loss of their largest units. The New England Power System carries a percentage of operating spare, generally 10 per cent or more, which is greater than the effect of losing its largest unit. Probably some of the smaller systems connected do take some advantage of reserve diversity, but generally the gain is considered as a matter of additional safety to service. The effect of a loss of unit, or of a major connection supplying power, is greatly reduced by spreading the effect over the two or three remaining systems connected. In general, it has been demonstrated that the loss of 25,000 to 50,000 kw. on the system results in less than 1/10 of a cycle frequency drop.

Emergency Interchange. Conditions necessitating emergency interchange may be brought about by loss of stations, low water, or fuel shortage. Parallel operation of systems at all comparable in size is a desirable procedure and the interconnected companies have been desirous of keeping ties closed as much as possible. All companies have been saved from embarrassing situations by this practise. In our case emergency interchange has been most useful on occasions of a system pull-out, when, during an attempt to transfer for economic reasons too large blocks of power over lines of limited capacity, some upset has caused these lines to trip and in this case, the end of the system lacking capacity has been partly supported by the interconnection. One case out of many which could be cited may be of interest. The New York Power & Light System some time ago requested an unusual amount of power to assist them in sending power west to replace the loss of a machine in the Buffalo-Niagara district. The Boston Edison Company was readily able to supply this deficiency for the time required and each system between relayed the power. The best of relations obtain in this matter and we know of no cases where any special charges have resulted on account of this sort of service. In the early days when standards of service were not as exacting as they are today, occasionally when requested one system lowered its frequency in order to phase with and assist in supporting an adjacent system which had lost capacity.

Low water is not classified as an emergency because all the major systems connected have water power sufficiently relayed with steam or purchased power.

During various periods of fuel shortage, the New England Power System, with the exception of the winter of 1917-1918, has been fortunate enough to obtain sufficient fuel for its own plants. It has taken advantage of some of its interconnections by arranging to supply fuel to interconnected steam plants and in turn to receive power at prearranged prices. Another advantage of interconnection in regard to fuel supply has been the possibility of converting the South Street Station at Providence to more or less a base load plant, although the storage provisions at that plant have not been enlarged so as to become comparable with the larger outputs. It has always been known that with a temporary fuel stringency at that point, power could be generated elsewhere, even though at a slightly increased cost.

Firm Sale. Some of the New England interconnections have developed into means for primary sale, and likewise connections built for primary sales have resulted in considerable interchange of power of other than a primary nature.

The contract terms have been made of varying lengths. It is often possible to make a considerable saving in providing firm capacity by arranging with an adjacent system, which is for the time being overdeveloped, or which has been willing to make an apparent over-development because of joint economies. An outstanding example is the development of Fifteen Mile Falls lower plant at a time when the need of additional capacity on the New England Power System coincided with that of the Edison System at Boston. This made possible the development of a large and economical block of power and insured its rapid absorption with a consequent saving to both parties.

Likewise, within the New England Power System, power contracts have been made between the wholesale companies and many of the local retail companies, which have resulted in a conservation of natural resources and joint benefits by the selling of water power, or tide water steam generated power to the smaller

companies. The smaller companies gain by the ability to make commitments for only one or two years in advance, rather than make relatively large station extensions which would require many years to absorb.

Intra-Company Use. It has been possible to use the transmission facilities in a number of cases to supply a customer of an adjacent system at a point remote from their lines. For a number of years the New York Power & Light Company furnished a customer in the neighborhood of Lake Champlain through our system and that of the Central Vermont Public Service Company until, as has recently become the case, it has proved economical to construct lines to serve the load directly.

A number of instances has occurred where interconnections between systems have been made at more than one point. It has always been possible to pass emergency power through the adjoining system in order to back-up a section of the first system adjacent to one of the ties.

Economy Flow. The use of interconnections for economy flow or as it is sometimes called, exchange power, has shown a real financial return. It may be generally said that in New England and in adjacent systems in New York State that there is no operation of uneconomical steam when economical steam or surplus hydro is available, provided transmission facilities are adequate and there is no especial need for backing up local service by local generation. The New England Power System has made a practise of purchasing power which could be generated at less cost in plants other than its own furnishing power to adjacent systems where it could generate for less cost, or furnishing power from one system to another across its own lines. Any saving resulting from this sort of operation is generally divided between the systems involved; if the transaction is between two systems, half of the saving goes to each; and if among three, one-third of the saving goes to each. Transfers of this sort in and out of the New England System amount to several hundred million kilowatt-hours in a year.

Some of the system steam plants are operated during the day and evening hours in some of the larger communities where tie lines are not wholly adequate. These plants operate usually at approximately one-half of the local load and with standby capacity just sufficient so that the entire load can be carried in case of any sudden emergency. Other than this so-called "service steam" the generation is distributed strictly on an economy basis. Service steam operation will disappear when system ties become sufficiently reinforced to provide the necessary reliability.

Contract Relations

The New England Power System was primarily benefited in the earlier connections by the ability to purchase auxiliary power and it was customary to make a guarantee to purchase a block of kilowatt-hours for a number of years. This guarantee of taking power was backed by a forfeit, in case of failure to take power, sufficient to cover the expense involved by the interconnection. Later, the character of the use of the lines shifted to that of a straight interchange basis with the facilities becoming more useful for serving customers in the territory through which they ran, and forfeits became less necessary.

The Providence contract gave the New England Power System the right to purchase the surplus capacity of the station when available and firm capacity to the extent of a certain minimum reservation. New England paid for the reservation a predetermined percentage charge on the book value of the plant and for kilowatt-hours at a slight margin over the average operating cost. The Providence Company agreed to purchase surplus at a margin of saving.

The connection to New York State and the later connection with the Edison company were put in with no advance agreements for interchange of power because it was apparent to both companies that the expense of the ties would be covered by the business over them, based on each company's knowledge of each system and the generating facilities that existed.

In the early days, prices were usually agreed upon from time to time, to be effective for certain periods and for certain classes of power. These often remained constant for periods of a year or more. If either company felt that the price was unjust, the matter was brought up, discussed and adjusted. This method was generally applied to comparison of cost of surplus hydro to steam generation, and is still used for steam and hydro exchange with smaller companies where the benefits would not warrant the bookkeeping involved for a refined method.

More recently the major systems interchange has been made between steam plants and it has become necessary to go thoroughly into the increment and decrement cost of power. A definite man has been designated on each system and he furnishes necessary cost data to the dispatching departments interested. Representatives of the various interested parties get together at the start of these arrangements and from time to time during their operation to insure fairness and acquaint themselves with the methods employed. All methods have not yet become identical, but are sufficiently close so that each company feels satisfied with the intent of each of the other companies. This procedure has resulted in a really remarkable amount of business between the dispatching departments, entirely based on these estimated increment and decrement costs. The New England Power System's primary business amounts to approximately one and one-half billion kilowatt-hours per year, and exchange of this type from and to the system now amounts to around 300 million kilowatt-hours. This sort of transaction, of course, is only practicable where there is a reasonable difference in the costs of power generation.

The extreme differentials, for example, are represented by high-pressure stations on tide water, as against older and inland steam stations, and make a spread of several mills. No interchange is attempted with a differential of less than 2/10 of a mill, in view of the uncertainty in estimating costs accurately.

Plans for interchange are usually made daily in advance, so that definite schedules can be arranged late in the afternoon for the following day. The New England Power System dispatcher is in touch with the dispatchers of the adjacent systems and determines the quantities and values of power which can be furnished or absorbed. These, compared with his own sources and costs, readily determine the economical set-up for the day and schedules are arranged. As stated above, where power is exchanged between two companies the saving that results is split equally, the line losses coming into the calculations as part of the cost where they are of sufficient importance.

When it is impossible for the New England Power System to participate in exchange to advantage with two adjacent systems and there is advantage for the adjacent systems to interchange power, arrangement is made on what is known as a three-way basis, in which it is assumed that the transmission company is a necessity to the transaction; therefore the profits are split among the three equally.

There is, of course, some difficulty in all these transactions in attempting to make the power flow exactly as intended and it is not unknown to have it reverse for short periods. In periods when companies are tied together and there is no intent to exchange power, some exchange may occur in an unprofitable direction. All these losses are deducted from current gains, and no effort is made to return the kilowatt-hours which may be received in this inadvertent manner.

The complicated procedure of estimating saving which involves backing down machines, shutting down machines, banking boilers, saving of additional boilers on the line, added or saved maintenance, together with complications of metering where a line is used for both primary and exchange power simultaneously, have forced the dispatching departments to become bookkeepers and accountants in order to turn over adequate data to the billing departments of their respective companies.

Generally there is no guarantee of services to be rendered in other than contracts for firm power delivery. It is, however, understood that each company will assist the other in an emergency, and if one company has arranged to furnish another company with power, and unforeseen conditions make it undesirable for it to do so, it will nevertheless continue to furnish the schedule planned until the other company can make arrangements to carry the load.

Contracts have been desirable when a definite saving can be made by pooling capacities but kilowatt-hour interchange or "economy flow" has been quite effectively taken care of by the operating personnel without contracts. Certain officials in each company are given authority to complete arrangements binding for a sufficient length of time to accomplish the transfers desirable. These can always be stopped if they do not appear to be working in a fair and profitable way. Fortunately, relations between companies have been pleasant and without such good will it would be impossible to carry on interchange either with or without contracts.

Disadvantages and Difficulties

We do not believe that any disadvantages have come about in any interconnection in New England that are not offset entirely by benefits derived. The major difficulty is probably in being unable to exchange power in exactly predetermined amounts with a number of other systems at the same time. The New England Power System has done a great deal of business with the Connecticut Valley Power Exchange as a unit, which in turn has frequently been interchanging power with the Central Hudson Gas and Electric Company, which is tied nearly all the time with the New York Power & Light on the north, and Metropolitan Edison and others on the south. It is believed that in order to reduce the difficulty of departure from schedule to a minimum all systems should attempt to run with rather exact speed control at 60 cycles. By allowing one system to assume responsibility for this speed control, and the other systems each to look out for the exchange over the connections, we have been able to control interchange reasonably well.

Several of the major systems are giving time service and the combined problem of correcting time and maintaining 60 cycles is a difficult one for unavoidable things shift the systems away from exact time, and the necessary correction sometimes upsets tie line control. This has been minimized by arranging for the dispatcher to know daily the time error on the systems adjacent and to adjust his time at a very slow rate to normal, when he can do so by agreement with the adjacent systems, or to adjust when separated from them.

It is not uncommon for systems all the way from Boston west to Niagara Falls, and south to Philadelphia, to operate in parallel and it is therefore necessary for word to be passed from dispatcher to dispatcher concerning the shifting of tie line loads and the taking over and changing of the point of master speed control. The large distances involved make communication of utmost importance and require the best efforts of all the companies involved, so that there may be no undue delay in passing essential information from one end of the interconnected systems to the other.

It is inevitable that there will be cases when because of special conditions the interchange schedules have not been adhered to strictly and one company or the other feels that it has been put to some disadvantage.

It has been found that informal meetings between one or more operating men representing each system involved, immediately following major occurrences of this sort, have had a highly satisfactory result in smoothing out and preventing recurrences of such difficulties. At these meetings, usually in New York City, which are called together by a temporary chairman, the various representatives present all the available data in the way of charts and records, showing exactly what did take place during the time when difficulty was experienced. It has usually been easy to trace the trouble to its source. It has been possible to come to some agreement in every case, or to outline a policy which will largely eliminate future troubles of the same nature. The utmost frankness has always prevailed and the helpful spirit of the various company representatives has made progress possible.

We have had for 10 years an association known as the "System Operators of New England" which has usually held four meetings a year and these meetings have enjoyed the heartiest popularity. The meetings are intended to bring the dispatchers of the different systems together personally, but many other officials and those most intimately responsible for the day-to-day operation of adjoining plants and systems have been interested enough to attend.

While the lack of a common financial control is not necessarily a difficulty and is certainly not a disadvantage to interconnection, it should be pointed out that the common control of interconnected units will generally work toward additional economies over straight interconnection. Effective peak diversity, better coordination of system planning, benefit of direct action, and other kindred betterments accrue. Interconnection establishes gains in operating economy and improvement in service reliability, and common financial control of interconnected companies carry these advantages a step further.

Construction Program Based on Joint Planning

The New England Power System has had the good fortune to have had many alternatives in planning additional sources of power. There have been possible water power developments of various types as well as a variety of methods of extending steam generating facilities. The development at Fifteen Mile Falls was of such a size that it could not be justified without various interconnections to assist in its absorption. Its character is such as to make it a desirable adjunct to any system depending entirely on steam and this fact made it possible for us to sell a large block to the Edison company of Boston.

The New England Power System load has grown to a size in which the capacity and power requirements for growth should not be studied as a whole but divided into three major classes: (1) base load; (2) 2,000-hr. power or day load; (3) peak and spare capacity. These various classes of load grow about equally on a percentage basis but the short hour type of load grows far faster on a quantitative basis. Additonal requirements of base steam of the best grade grow slowly and if this type of plant is increased too far in advance of requirements it crowds into short hour use expensive plants designed for better load factor. For the second class, day load or 2,000-hr. power it si apparent that plant capacity should be secured for a lesesr first cost than for class one. A few hydro possibilities are peculiarly suited to this service. If the hydro plant site affords sufficient storage, permits the power house to be close to the dam and does not restrict its size, the increment first cost of unusual capacity may be favorable. These conditions existed at the Fifteen Mile Falls development. The water available at this plant, though sufficient to operate at full capacity but 2,000 hours per year, can be used when it is most valuable and involves a negligible operating cost for additional capacity.

The third requirement of the systems is for a capacity suitable for meeting annual peaks and carrying day to day spare through the year and during the peak period. The necessity of this sort of facility has been well illustrated by the development of the Rocky River Pumping Plant of the Connecticut Light & Power Company. The Fifteen Mile Falls plant is also well adapted to supply this type of power. While the older and rather uneconomical steam plants are also suited to this type of service, the cost of being ready to serve for peaks and spare is higher than that of suitable hydro plants. The Fifteen Mile Falls type of plant will confine their use for this purpose to a very small part of the year.

Interconnection, therefore, has made possible a large development of a character of power which was decidedly the type needed on the New England System which also fitted well into the Edison system but which would not have been feasible of development without a very large or interconnected system. It has made the entire output of this development more immediately useful, and put off the construction of further base load steam capacity, which will in turn be desirable to construct in large blocks and which will be more economically absorbed later, in the combined New England systems.

Features of Design

Frequency Control. In the early days of interconnection no special efforts were made to control frequency beyond those which were in use on individual systems before they were tied together. It was found desirable, however, in the interests of maintenance of desired interchange schedules to hold the combined system speeds within closer limits than were found practicable by manual and natural governor control whether hydraulic or steam.

Considerable developmental work was done on the New England Power System as far back as 1927 in

conjunction with various manufacturers of automatic equipment, and it is believed notable success has been obtained. Special apparatus has been installed in three hydroelectric plants in order that it may be possible for a great many hours of the year to have at least one of the plants in position to handle the major burden of speed control.

On any system of hydroelectric plants of varying characteristics, it is undesirable to place the control of speed constantly in the same plant. A run-of-the-river type of plant should be operated wide open during flood periods while during the lower water periods it may be used to carry the swings of load and the characteristics of plants using stored water are directly opposite for they are normally shut down during flood periods and used heavily during dry periods. Thus, by having automatic apparatus in key plants of various types, it is insured that one of them will almost always be available for speed regulation without departing materially from its normal economic schedule.

It has been found that a frequency deviation from normal of not more than $\pm 1/20$ of a cycle approaches the best possible conditions for load interchange. Regulation as close as this on a scattered group of small hand controlled plants would place a burden on the operators that would be almost impossible, and even the best governors are unable to maintain the speed within such close limits. Automatic frequency control has also been found of considerable help in maintaining accurate time as recorded on the large number of synchronous clocks in the New England area.

While considerable work has been already done, it is thought we must equip more plants with automatic apparatus, so that the burden may be as widely distributed as possible, with a view to less wear and consequent maintenance in any one place, and less operation at a lowered efficiency, which necessarily follows when the load of any given units is constantly changing over rather wide limits.

Transmission

The increasing interconnection of transmission lines together with the increasing use and reliance upon electric power is necessitating always greater reliability and freedom from disturbances. Not only is it necessary to isolate disturbances closely and quickly but it is also necessary to avoid them in so far as it is economically possible. A comprehensive interconnected system will serve customers to whom a voltage surge is as disastrous even as an interruption of short duration.

Lightning disturbances present our most formidable problem and this problem is one of the most important before the industry today. It is not always possible to greatly benefit the performance of lines already in existence although in one case it was possible to add insulation and ground wires to an existing line so that it is now experiencing only about 40 per cent the lightning disturbances that it had previously. Considerable advances have been made that are particularly applicable to new construction. We are attempting to design lines with unusually short spacing of supporting structures so as to get a low average height above ground and we provide these lines with overhead ground wires where we are justified economically. Horizontal construction with single-circuit towers and two overhead ground wires are being used generally on all major lines. The importance of low footing resistance has been recognized but so far it has not been possible to arrive at any economical solution of the naturally high resistance conditions that pertain generally throughout New England. Steel grillages for tower footings have been used, however, for a number of years and these undoubtedly are of some benefit.

Sleet is another formidable foe to the successful operation of transmission lines in New England. Sleet destruction is caused at comparatively rare intervals, but the ensuing interruptions are of a major consequence on account of the inherent length of time necessary to repair the damage. The sleet storm of 1921 was more severe than any storm of established record in New England. This storm proved the inadequacy of off-set vertical lines and established the necessity of providing tower strength to insure against tower failure on important lines. We are attempting to design major lines to operate through the 1921 conditions with not more than broken conductors. Considerable use has been made of sleet thawing on lines that can be taken out of service.

Circuit Breakers

It is perhaps of fundamental importance to avoid disturbance in so far as possible, however, the rapid removal of faults, once they occur, is a very important consideration and it is only recently that circuit breakers even approaching adequate speed have been secured. The Fifteen Mile Falls development was equipped with 220-kv. circuit breakers that will have an over-all relay and breaker time of the order of eight cycles (0.013 seconds) and it is hoped to secure similar characteristics in breakers to be purchased in the future for the lower voltage developments. Tests on the 220-kv. breakers, some of which were taken under actual operating conditions, showed the removal of short circuits in quite a spectacular fashion. Only very small voltage disturbances were caused and a remarkable lack of damage resulted at the point of application of the short circuit. Considerable increase in synchronizing power during short-circuit conditions between the Fifteen Mile Falls plant and the remainder of the system was also secured. It is our observation that the disturbances causing damage to circuit breakers have been very materially lessened in the past several years and it is thought that one of the largest factors has been the design of breakers of sufficient mechanical strength to be able to withstand gas ex-

plosions. However, considerable progress can be made in bettering the mechanical reliability of oil circuit breakers. It is imperative that oil circuit breakers be designed so that the possibility of their failing to open or to close be entirely avoided.

Synchronous Condensers

Our experience indicates that the economical operation of a high-tension interconnected system necessitates such power-factor correction as to assure the operation of the high-tension lines at practically unity power factor. Interconnected lines must also operate at somewhat less drop in potential to avoid voltage variation when the point of feed is shifted on the system. There is approximately 180,000 kva. of synchronous condenser capacity in operation on the New England Power System.

Transformation

There appears to be little possible gain in attempting to improve the reliability or the efficiency of modern high-voltage transformers. We have come to look upon this piece of equipment as being as safe against disturbances as any other type of equipment in service.

There has been no occasion on the New England system to make use of load ratio control for shifting load nor balancing power between parallel circuits. There have, of course, been many occasions when it has been difficult to get the proper distribution of power, yet it has not been felt that any situation would justify the expense of load ratio control. The accurate regulation of system frequency together with the wide use of synchronous condensers has obviated to a considerable degree the need of phase shifting equipment. Existing conditions can easily change or new conditions arise so that we shall be forced into this expense.

Tap Shifting

Extensive use has been made for many years of arrangements whereby transformer taps can be changed readily and rapidly when the transformers are de-energized. Several of the intersystem ties have been made through transformations consisting of at least duplicate capacity equipped with ratio adjusters and it has been practical to change transformer taps one at a time during changes of load conditions. This method of operation has been so successful that the New England companies were among the first to standardize upon external tap shifting devices for practically all important transformers.

Discussion

For discussion of this paper see page 1280.

The Pennsylvania-Ohio-West Virginia Interconnection

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Synopsis.—This paper describes a specific interconnected system wherein ten independent operating companies of western Pennsylvania, eastern Ohio, West Virginia, and Maryland operate normally in parallel to secure the manifold advantages of the interconnection of their facilities. Some of the benefits derived from the interconnection of these ten companies and the principles and methods followed in actual operation including very brief descriptions of physical plant are treated. The feasibility of coordinating the operations of an interconnected system through a working operating committee functioning in part through the joint dispatching organization of two of the companies without sacrificing independence of operation, is discussed.

Introduction

MODERN power system management fully recognizes the importance of the linking of power sources, transmission facilities and load areas, whether these interconnections are ties between power systems independently owned and operated or connections between power sources and loads of a single system and ownership. Many papers on this subject describe the fundamental plan of power supply and the operating problems of interconnections of systems largely under one management, treating of a basic load area, a fundamental plan of power supply, and a developed method of operation.

The purpose of this paper is to describe the plan of interconnection and operation of contiguous power systems, independently serving a vast industrial region. Each system is under separate control, both financial and operating, with all the diverse standards and ideas of technical, commercial, and accounting nature. Each system has its own tale of romantic growth and progress from the small local generating plant to the highly developed load area with its network of power lines served by modern power stations.

It was inevitable that as each system grew, a mutual interest should develop between neighboring utilities in the planning of power supply extensions into the intermediate territory. It was proper that these neighboring independent companies should cooperate to realize any possible benefits to be derived through interconnection. It was natural that a common understanding of the every day problems of parallel operation should follow, this consensus of opinion being expressed by the Working Committee of operating men from each of the ten individual systems.

Territory Served

Character of Territory. The territory served by the ten operating companies constituting this interconnected system has an approximate total area of 101,000 sq. mi. and a population of about 9,000,000 including 300 cities and towns. The metropolitan areas of Cleveland and Pittsburgh each have a population in excess of 1,250,000. Other cities of over 60,000 people are Akron, Toledo, Canton and Youngstown, Ohio; Erie, Altoona, and Johnstown, Pa.; Charleston and Huntington, W. Va. Industrial towns average from 5,000 to 25,000 people. A large part of the remainder of the population resides in 3,200 communities classed as rural,—less than 2,500 people. The character of territory served by these companies illustrates the recent trend toward decentralization of industry and population.

Nature of the Load. An estimate of the peak load of this group for 1930, occurring in December between 9 and 11 a. m. is 1,750,000 kw. The annual load factor is about 70 per cent.

Although 85 per cent of the consumers are domestic, 1,400,000, industry constitutes the bulk of the load. The industries served consist principally of bituminous coal mines and coking plants; sheet steel mills; electric steel melting and ore reduction furnaces; rubber goods and tire factories; glassware and plate glass plants; cement plants; refractory, brick, tile and porcelain works; and countless machine shops, foundries and small plants manufacturing mine, mill, railroad, and electrical equipment.

Electric railway systems under the same holding company management with the power companies which serve them, operate in Pittsburgh and Allegheny County, Pennsylvania, throughout the coal regions of southwestern Pennsylvania and northern West Virginia, and in Akron, Canton, and Massillon, Ohio.

Classification of Services—Contractual Relations

The contractual purposes of making interconnections according to the important benefits to be derived are represented on this combined system in their every essential; reliability of service, capital investment economies, operating economies, and the stabilization of frequency, voltage or reactive kilovolt ampere conditions and transmission systems. Typical examples are given of contractual relationships, the inception of which was due to one or more of these purposes.

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TABLE I—DATA ON GENERATOR RATING AND OUTPUT OF 10 INTERCONNECTED COMPANIES

System	Generator rating Dec. 31, 1929				Added capacity 1930 steam plants		Generator rating Dec. 31, 1930	Total 1929 kw.-hr. output
	Steam plants		Hydro plants					
	No.	Kw. rating	No.	Kw. rating	No.	Kw. rating		
Appalachian Electric Power Co	8	302,540	5	36,900		5,000	344,440	1,484,761,000
Associated Gas & Elec. System (West. Penna. Group)	8	89,175	2	48,200			137,375	491,037,034
Cleveland Electric Illuminating Co	4	447,000			1	150,000	597,000	1,447,193,594
Duquesne Light Co	3	384,000			1	60,000	444,000	1,573,445,327
Ohio Edison Co	4	279,000	1	2,175			281,175	1,018,759,180
Ohio Power Co	15	395,345	4	2,850			398,195	1,544,252,000
Ohio Public Service Co	8	130,100					130,100	611,482,217
Penn Central Light & Power Co	4	76,100	3	4,420			80,520	237,817,248
Toledo Edison Co	4	160,200	2	6,800			167,000	514,989,108
West Penn Electric Co	26	457,374	9	55,595			512,969	1,938,955,818
Total of all companies	84	2,720,834	26	156,940	2	215,000	3,092,774	10,862,692,526

Frontier Exchange. Neighboring companies first contacted on the frontiers of their respective systems and interconnected for the mutual protection of those districts having only one source of power. Such interconnection of transmission lines made it possible, without suspending the customers' service, to take a section of line out of service for repairs, especially at week-ends, thus eliminating the hazard of working on live lines and insuring the highest degree of maintenance. These lines were usually small in capacity and did not permit of continuous parallel operation. However, in 1916 the Duquesne Light Company of Pittsburgh and the West Penn Power Company of Western Pennsylvania, operating in territory immediately adjacent to each other, remained in parallel for 100 hours continuously over two connections of light capacity 22-kv. transmission lines. Although it was believed at this time that the extreme watchfulness essential to parallel operation did not warrant the continuance of parallel operation, nevertheless it was proven this early in the transmission history of these two systems that they could be paralleled for emergency purposes in case of failure of generating units or for the transfer of suitable blocks of load until the disabled system could regain its generating capacity.

Investment and Operating Economies. During the war period, 1916 and 1918, an exchange energy contract between the Duquesne Light Company and the West Penn Power Company necessitated the paralleling of these systems twice each day to transfer blocks of load. The peak load of the Duquesne system was greater in the evening than in the morning and the reverse condition prevailed on the West Penn System so that each company had a surplus of generating capacity at the time when the aggregate demand of the consumers of the other company was the greatest.

Beginning with June 1916 and continuing to the present day, energy has been exchanged to supply those fringe areas of each system, which are at a considerable distance from their respective generating stations, to the manifest advantage of both parties and of the service.

Dove-tailing Construction Programs. This early contract of 1916 also provided for the development of the respective systems of the Duquesne and the West Penn with such proper harmonious cooperation in the construction and operation of additional generating stations and additional substations that in the interests of both companies and of the public, avoidable expenditures would be prevented and the cost of supplying energy would be minimized. From the initial sale of 10,000 kw. by the Duquesne Light Company to the West Penn Power Company, 1916-1918, in which period the West Penn System was constructing its Springdale power station, there has been a total of four firm energy short-term contracts between these systems for the purpose of dove-tailing the steps in power station building programs. One of these contracts terminated August 31, 1930 shortly before the placing into operation of the James H. Reed power station of the Duquesne Light Company. In this manner on this interconnected system, economic waste that goes with building generating stations too far in advance of the individual system load has been eliminated by staggering construction between independent companies.

Combined Financing—Power Stations. An example of this type of interconnection is the Windsor power station jointly owned by the Ohio Power Company of the American Gas & Electric System and the West Penn Power Company. After considerable study in 1915, a site was located on the Ohio River, about 9 miles above Wheeling, W. Va., for a station to meet the increasing needs of the Ohio Power Company, as further expansion of its Canton plant was impossible due to lack of water, and to give a firm source of power to the West Penn Power Company in its western area, which was being fed by a long low-voltage transmission line from its Connellsville plant with considerable voltage and power losses. The joint Windsor station was then built in 1917 which in effect produced all the major benefits of the interconnection of generating facilities as well as many other operating and investment economies. This was the first time any arrange-

ment of this kind had been made in the United States. It is also interesting that the Windsor station was the first station of this magnitude to be built at the mouth of coal mines and securing coal for fuel without the use of river or rail for transportation.

Although the contract for joint operation of the Windsor station provided that one company should operate the power station and the other company operate the coal mine and that these operations should alternate each year, so satisfactory have been the results of the joint operation that the Ohio Power Company has operated the power station since the beginning and through the increases of station capacity to its present rating of 185,000 kva. and the West Penn Power Company has operated the coal mine. Each company is particularly well fitted in its relation to Windsor to continue to perform that part of the work which it initiated and no request has ever been made by either company for a change in the authority of the respective operations.

Combined Financing—Transmission Lines. The interconnection between the Associated Gas & Electric System and the West Penn Power Company was caused by the desire of each company at about the same time to build high-voltage transmission lines, connecting its main properties to outlying properties served by old power stations. The Associated Company desired to build a transmission line between the Piney hydro station on its main system to the Warren station and the West Penn Power Company desired to build a transmission line from Kittanning on its main system to the Ridgway station. By collaborating, the West Penn Power Company built a line from Kittanning to the Piney hydro station and each company built sections of line between Piney, Ridgway, and Warren within its own territory, thus completing in 1926 an interconnection of all the desired properties and making possible the joint use of the same transmission facilities. Five very successful years of perfect cooperation have been a result of this unique arrangement.

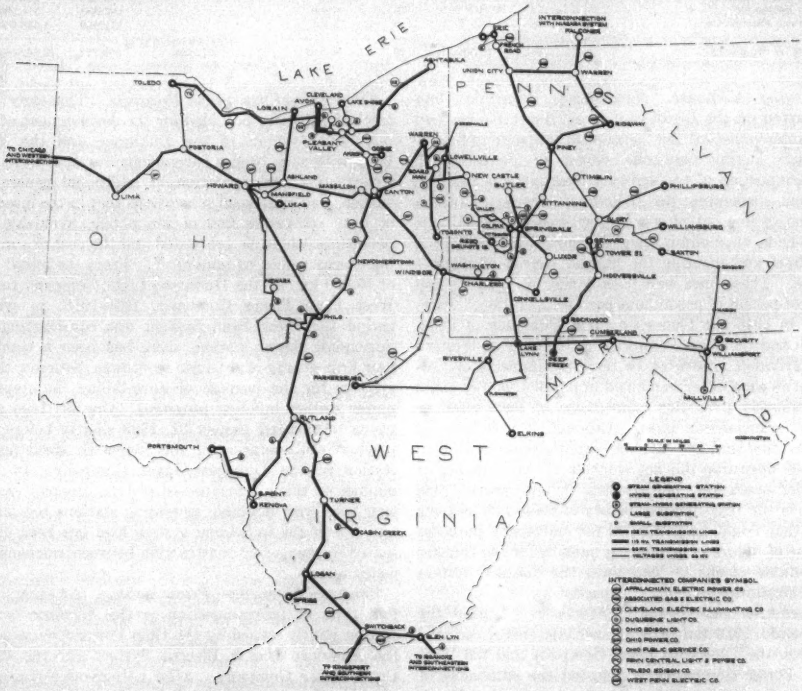

Fig. 1—The Transmission Backbone of the Pennsylvania-Ohio-West Virginia Interconnection

Reduction in Spare Generating Capacity. Because of the strong transmission backbone throughout the interconnection, the whole system is in itself the best example of how planning can be done and operation

accomplished safely with a much lower percentage in spare generating capacity than without such an interconnection. On this interconnection the emergency reserve capacities are spread over the various transmission systems that are interconnected, so that any one system will be called upon to furnish what, in many cases, represents only the overload capacities of the machines used in usual operation; all of which results in less capital invested in the individual properties and consequently a lower fixed charge to be absorbed by the output of each plant. In addition, therefore, to the more specific purposes of contractual relationship, this attractive advantage has weighed heavily in the consideration of interconnection made finally by several systems of the interconnected group.

Protection of Service. Of all the possibilities of interconnection, that of mutual assistance when assistance is necessary is commercially the most important. The ability to call on the resources of other power companies in addition to those owned, gives a feeling of great assurance to the man who is responsible for the continuity of service. Such an interconnection for continuous parallel operation was made when the 66-kv. Roxbury tie was built in 1926 between the Penn Central Light and Power Company and the Potomac Edison Company of the West Penn System. This interconnection has proven to be helpful in several instances of emergency at the Saxton or Williamsport power stations and on a number of occasions to serve consumers of the terminal substations or on the interconnecting line when a section of it was in trouble.

Classification of Services—Operating Benefits

Reserve Generating Capacity—Practise. The study of the problem of reduction in spare generating capacity on this interconnected transmission system has resolved itself into two phases. One is the utilization of the operating or spinning reserve capacity of the generating units and the other is the coordination of the operation of the reserve capacity between these various companies so as to carry the required spinning reserve in the most economical manner. The resultant diversity by combining spinning reserve capacities makes possible a decrease in the total reserve capacity in service without the loss of reliability, and the proper proportioning of the spinning reserve capacity between the component parts of the system decreases materially the amount carried on any one system. On this interconnected system it is believed that the spinning reserve carried by any one company should be 10 per cent of the load carried by a steam station and 5 per cent of the load carried by a hydro station. When any one company generates more than 20 per cent of its load on any one steam unit, it should carry a spinning reserve equivalent to 50 per cent of the load carried on this unit, as its proportion of the spinning reserve.

Each of the two large metropolitan systems, namely; the Cleveland Electric Illuminating Company and the Duquesne Light Company, serving respectively Cleveland and Pittsburgh and their adjacent territories, carries its own reserve which is equal to the largest unit on its system. These urban systems follow the practise of holding this reserve only in the overload and less efficient capacities of the generating equipment actually in operation. The tie lines from these two companies to the main interconnected system might not carry all the load placed upon them without damaging the cable, transforming, and regulating equipment in the event of the loss of the largest unit with insufficient reserve.

It is essential that member companies participating in the benefits of such a pool of spinning reserve capacity should be able to draw their requirements from the pool whenever an emergency arises on their system. It is therefore, necessary to have a means of determining what reserve actually exists in the pool to insure the existence of an adequate amount of reserve at all times. On this system Windsor, being the central point of the interconnected system, acts under supervision of the Working Committee as a clearing house for all data on existing reserve capacity. Each company relays to the Windsor load dispatcher, not later than 3:00 p. m. of each day its expected conditions with respect to load, capacity in operation, load to be carried by the largest unit and spinning reserve for the following day. A further report of these conditions is made not later than 6:00 a. m. the following day. Each company makes a frequent check of these conditions throughout the day to ascertain if its spinning reserve capacity is equal to or in excess of the requirements specified. The Windsor load dispatcher is notified at once of any condition which arises to change the status of the reserve from the expected condition previously reported.

It is also essential that the specified spinning reserve be maintained by each company, either by placing in operation idle capacity or by securing reserve capacity elsewhere. Should a condition arise which causes one company to draw on the pooled reserve or to reduce its proportion of reserve due to loss of capacity or excess load, the company notifies the Windsor dispatcher and makes arrangements to increase its proportion of the spinning reserve to that required as soon as possible. In no case is the spinning reserve deficient for more than two hours. The companies concerned in the reserve are notified at once of any condition which causes a deficiency of the spinning reserve. If an accumulation of circumstances causes the required spinning reserve to be decreased by more than 25 per cent, the Windsor dispatcher at once has a check made of all possible reserve and if the actual spinning reserve is not equal to at least 7½ per cent of the then total system peak, he requests that additional capacity be placed in service as soon as possible at the most convenient location.

All ten systems of the interconnected group furnish

the Windsor dispatcher, not later than Friday of each week, information on their expected operating conditions for the following week. Any system desiring to make use of available spare capacity can find out from the Windsor dispatcher where this reserve may be secured. All negotiations for supplying spinning reserve capacity in this manner are made between the individual companies as a two-party agreement. Such an arrangement naturally leads to a schedule between the various systems for dove-tailing maintenance of large units and other contemplated work which may temporarily change operating conditions.

Perhaps the most outstanding two-party verbal reserve capacity agreement ever entered into in this country was made in September 1930, between the Duquesne Light Company and West Penn Power Company. The occasion for this unique arrangement to supply spinning reserve capacity arose when the navigable waters of the Monongahela River below Fairmont, W. Va. and as far down river as Brownsville, Pa., became so depleted by unprecedented and continual drought that the pools of this section were scarcely navigable. The government authorities were faced with the necessity of withdrawing what water remained in the upper pools to maintain navigation in the lower pools between Brownsville and Pittsburgh. Such an act would have affected the water supplies of a very large population in West Virginia and would have caused the cessation of mining operations by many mines loading to the river with all the attendant idleness of labor and tie-up of machinery. The losses of labor and the stand-by charges on equipment alone were estimated to be $1,000,000 per month for the mines affected. To relieve this urgency the West Penn Power Company offered all of its impounded waters in the Lake Lynn hydro development at no cost to the Federal government. However, in order that the West Penn Power Company might be able to fulfil its power contracts in case of emergency in the loss of large generating capacity, it became necessary to arrange for spinning reserve capacity to replace Lake Lynn hydro station. The Duquesne Light Company offered its help, thus making possible the gift of all water in Lake Lynn to the government and the use of 80 per cent before the drought was broken. Such energy as the Duquesne might furnish the West Penn, should such a contingency occur, was to be returned by the West Penn at a time when the Lake Lynn hydro station could again generate energy. The agreement for this stand-by reserve capacity of the Duquesne Light Company to the West Penn Power Company was a verbal one and although it was never used in this exigency the reserve capacity was continually available as was the means of transmitting it to the West Penn Power Company.

Fig. 2—View of Switchboard Room and Load Dispatcher's Headquarters at Windsor Station

Reserve Generating Capacity—Performance. Troubles are encountered on this interconnected system the same as they are on any other system for apparatus, no matter how perfect, and plants, no matter how well designed, break down. Interconnection, however, has wrought wonders so that the public served is often unaware that trouble has ever occurred. According to the law of probabilities it would seem on an interconnected system that has 56 generators of 19,000 kw. capacity and over, that the probability of losing more than one generator at the same time would be more than a coincidence. Except those troubles due to instability, such cases are quite rare.

In November, 1927, unit No. 1 at the Philo station of the Ohio Power Company, 40,000 kw., developed internal trouble and was taken out of service for inspection and repairs.[2] Two weeks later, and before unit No. 1 was replaced on the line, unit No. 2 of the same capacity developed internal trouble also and was removed from service, thus completely shutting down the Philo station and reducing the capacity of the Ohio Power Company System by 80,000 kw. It was not possible to get unit No. 1 back in service. The accident to unit No. 2 was on a Saturday, when the load was light, so that the system did not feel the loss. But all day Monday and until noon Tuesday the interconnections, each adding their quota of help to the 30,000 kw. above normal supplied by the Cleveland Electric Illuminating Company, carried the interconnected system load of the Ohio Power Company without the slightest disturbance or hitch until one of the Philo units was returned to service.

The West Penn Power Company had a case of trouble in July 1927 at its Springdale station when unit No. 2 developed blading trouble and had to be removed from service. The switchboard operator pulled unit No. 1 off the line at the same time, as this turbine was hunting and he believed it also was in trouble. Springdale station capacity was thereby reduced 40,000 kw.,

2. For references see Bibliography.

5,000 kw. of which could be picked up on the remaining units in this station and the interconnected system picked up the remaining 35,000 kw. of this load. It took one hour for the West Penn System to bring capacity into service at its various plants during which period the interconnected system served all the load perfectly.

On February 11, 1931 at 8:06 p. m. one transformer, 16,667 kva., in Canton No. 1 bank failed at Windsor,—the joint station of the Ohio Power Company and the West Penn Power Company. Differential relays cleared transformer bank No. 1 from service. There are normally two banks operating in parallel, banks No. 1 and No. 3, and at the time of the failure of bank No. 1 each bank was carrying 45,000 kw. to the west. When bank No. 1 tripped the sudden overload on bank No. 3 tripped it out on overload relays thereby separating the eastern and western systems at Windsor. At Windsor 90,000 kw. to the west was dropped almost instantaneously and was absorbed by Philo, Cleveland and other western and southern stations, with a loss of frequency of only 0.17 cycle and no service disturbance. The frequency of Windsor and the eastern stations increased 0.35 cycle with no service disturbance. The eastern and western systems were paralleled at 8:09 or three minutes after the time of the initial trouble.

Load Diversity. When individual systems, now a part of this group, began to be interconnected some of the advantages that were obtained previously in the tying together of small communities within the same system were no longer obtainable. One of the things that disappeared, and very quickly, was the saving due to diversity. A typical case can be taken of the western part of the system on which with a system peak of 490,000 kw. the diversity between the four individual systems was only 15,000 kw. or 3 per cent.[2] In general, studies of other groups have given similar results.

As a large portion of the industrial load is coal mining, in which industry the working day is practically over at 4:00 p. m. it would seem that the factor of diversity could be used to advantage with respect to the lighting load of a metropolitan area, such as Pittsburgh. This has been somewhat true in the past, especially in December, but in the last two years the evening peak of the transmission groups, who are essentially industrial serving systems, has approached within 8 per cent of the morning peak. The mere elimination of normal expected hydro capacity in the evening due to low water conditions such as has prevailed since November 1929, creates an evening peak for the steam stations.

The industrial load is noticeably much more diversified within the last two years, no doubt due to the use by many industries of off-peak power,—4:00 p. m. to 7:00 a. m.,—especially electric furnaces and municipal water pumping stations. The economic desire of many manufacturers to operate at least a part of their plants during the entire 24-hour period coupled with the lower cost of off-peak energy in regard to demand rates for a highly fluctuating load is also probably responsible for this condition.

This interconnected group operates entirely on eastern standard time excepting Pittsburgh and adjacent towns which operate on daylight saving time during the summer and which departure from standard is scarcely noticeable to the group as a whole. The Western and Chicago Systems, operating on central time, affect the frequency regulating station at Windsor to give it two noon-hour variations each week day.

Stability

In the early history of parallel operation of generators and power plants on this interconnected system, instability was accepted as one of the many problems resulting from the attempt to keep the component parts of the system in parallel with weak transmission lines. With the installation of the 132-kv. transmission backbone and with more attention in the selection of proper equipment characteristics in the terminals of this system, instability ceased to be such a serious problem.

Windsor. As Windsor is the center of all interconnected operation so is this station the center of the stability problem. The eastern and western systems are here paralleled on the 11-kv. generating bus to which are connected the six generators and one synchronous condenser, each with a capacity of 30,000 kva. Between generators are reactances of 5 per cent., and as far as possible loads are taken off the bus sections to agree with generator outputs. The eastern system is principally connected to this bus through three banks of transformers totaling 86,000 kva. and the western system solely through two banks of transformers totaling 100,000 kva. All banks are fixed ratio and transform to the 132-kv. system with about 10 per cent reactance. An emergency tie between the 132-kv. systems is provided.

The generators at Windsor are of the large air gap type which requires a heavy no-load magnetization. Consequently a low maximum voltage is indicated with full excitation and no-load. These machines are not much affected by load changes, have a high short-circuit ratio and are inherently stable. The voltage regulators at Windsor are of the Tirrill type, one per exciter which is direct connected to its generator. This arrangement of exciters is satisfactory from the point of view of stability, since steam units do not overspeed to any great extent. The voltage regulators operate in parallel through the usual compensating coils which force a correct sharing of the reactive kva. Automatic frequency control operates normally upon three units.

Throughout its history the Windsor station has stood like a rock against the many momentary surges transmitted to it from the east or the west. It has successfully handled many rapid changes of load in the order

of 50,000 to 90,000 kw. The problem of stability on each system is mainly of staying in synchronism with Windsor.

Eastern System. To the east of Windsor station there are the Springdale steam and the Lake Lynn hydro stations of the West Penn System joined together at the Charleroi transmission substation which is distant 41 miles from Windsor, 31 from Springdale and 29 from Lake Lynn. From the Springdale station radiate the connections to the Duquesne Light Company-Colfax, Reed and Brunot Island stations,—and to the Associated Gas and Electric System—Seward, Erie, Piney and Deep Creek stations. From the Lake Lynn station radiate the West Virginia connections—Rivesville and Parkersburg stations, and the Western Maryland connections—Williamsport and Cumberland stations, both of the West Penn System, and from Williamsport the connection to the Penn Central Light and Power Company, Saxton station. The synchronizing power of the lines between Windsor and Springdale and between Windsor and Lake Lynn is 500,000 kw. which is ample to hold these stations together.

Fig. 3a—Front View of Automatic Frequency Control Panel at Windsor Station with Recording Charts of Frequency, Time, and Loads of Tie Lines to Eastern and Western Connections—All Located Adjacent to Switchboard Operator's Desk

The Lake Lynn hydro station and the Springdale steam station generate at 11.2 kv. and each step-up through transformers of about 10 per cent reactance to a 132-kv. bus. The generating units at both stations have very high no-load full excitation voltages—17.5 kv. and consequently a low short-circuit ratio.

There is an unusual situation with respect to the Piney tie between the West Penn System and the Associated Gas and Electric System which is made through three 5,000-kva. auto-transformers 132-110 kv. with dead-grounded neutral. Thus either system may receive ground fault disturbances from the other system. A similar installation is made at Cumberland where auto-transformers 132-66 kv. are installed. So far no difficulty has been encountered from these connections.

The connection between the Springdale and Colfax stations is made through a transformer bank consisting of three 12,000-kva. single-phase units connected delta-delta 132-66 kv. and an auto-transformer bank, three-phase core type 36,000 kva. 66 kv. Y, 72.6 kv. Y.[3] This auto-transformer is equipped with load ratio control making it possible to obtain 10 per cent buck or 10 per cent boost in voltage without removing the transformer from service. Over a wide voltage range, these systems thus have a means of regulating the wattless current flowing between them.

The most vulnerable point of this eastern system is near the Springdale station where there is a short-circuit capacity of 1,200,000 kva. Calculations indicate that a three-phase short-circuit fault must be isolated within 16 cycles or the units are likely to go out of step. On March 21, 1929 at 4:15 p. m. there occurred a case of dynamic generator instability when a fault occurred on both of the two 132-kv. lines toward Luxor substation at a point three miles from Springdale. During the fault the two 25,000-kva. generators tripped out on the steam end upon overspeed and one of the 35,300-kva. generators was removed from service by the operators, as it was thought to be surging. Only one 35,300-kva. generator was left in operation and 96,000 kw. of load was dropped at Springdale. Of this amount Windsor picked up 55,000 kw. The duration of the fault was three minutes and the 25,000-volt buses at the adjacent transmission substations were reduced to about one-half voltage. However, at Lake Lynn and Rivesville

Fig. 3b—Rear View of Automatic Frequency Control Panel at Windsor Station Showing Spring Mounting to Eliminate Vibration

stations there was only a momentary 20 per cent drop in voltage.

As a result of the above case of instability due to ground faults, short circuit and line clearing, with attendant changes in loads and load power factors, the excess voltage protective devices of the voltage regulators at Springdale were removed from service. The wisdom of this procedure was borne out when a more severe short circuit occurred on December 23, 1929 near the Springdale 132-kv. bus and no generators tripped out of service.

The power factor at Springdale is high, it being difficult to average lower than 95 per cent on the five generators, which have low short-circuit ratio characteristics. In several instances one of the generators has fallen out of step due to the operators placing a leading power factor upon it during load shifting. No interruption or disturbance to service was involved in any case.

Improving the governor action is a problem upon which the operating engineers of the Duquesne Light Company have done much experimental work including load dumping tests.[4] Modifications in the apparatus have resulted in a much faster and more accurate following of the speed changes, allowing the governors to hold the turbines below the limit of the auto-stop setting. The governors of this system are, therefore, probably more responsive to sudden load changes than those of any other operating group on this interconnected system. Nevertheless tests have proven that the period of the governor oscillation and the load oscillation on the generators do not have a time phase that would sustain a surge and that the governors are sensitive enough to hold the turbine from overspeeding and tend to check the speed of the turbine during a pole slip.

Western System. To the west of Windsor Station there are the Philo Station of the Ohio Power Company, Toronto Station of the Ohio Edison Company and the three stations; Avon, Lake Shore, and Ashtabula of the Cleveland Electric Illuminating Company. These are joined together at the Canton transmission substation of the Ohio Power Company, this being distant 55 miles from Windsor, 73 miles from Philo, 102 miles from Toronto and 28 miles from the Pleasant Valley substation, which in turn joins the three generating stations of the Cleveland Electric Illuminating Company. From the Philo Station radiate toward the south the connections to the Appalachian Electric Power Company, Cabin Creek, Kenova and Glen Lynn Stations, which in turn interconnect with the Carolina Power and Light Company at the Virginia-North Carolina state line near Danville, Virginia. Also from the Philo Station toward the west radiate the connections to the Howard, Lima and Fort Wayne, Indiana transmission substations, the Twin Branch steam station and the Chicago interconnected systems.

The Philo and Toronto steam stations generate at 11 kv. and each steps up through transformers of about 9 per cent reactance to a 132-kv. bus. Recent oil circuit breaker tests at Philo developed a short-circuit capacity of 1,735,000 kva. on the 132-kv. system at Philo.[6]

From the Canton Substation extends a line through the Alliance Substation of the Ohio Public Service Company and the Boardman Substation of the Ohio Edison Company to the Valley Substation of the Duquesne Light Company. Also from the Canton Substation extends the connection to the Windsor Station, to the Charleroi Substation and Springdale steam station of the West Penn, to Colfax steam station and Valley Substation of the Duquesne Light Company. These two lines form a 250-mile high-tension transmission ring involving five power companies and a total generator capacity of nearly 1,250,000 kw. Occasional operating with this ring closed on itself has demonstrated the inflexibility of interchange control and therefore, the loop is left open at the Valley Substation. A measured angle exists at 19.6 degrees which corresponds to a calculated angle of 17 degrees described in an A. I. E. E. paper.[5]

Comparatively few actual cases of static instability have been reported on this interconnected system. An A. I. E. E. paper of January 1931[6] describes a case which occurred on the Appalachian Electric Power Company's system at Kingsport, Tennessee, where a comparatively weak interconnection is made with the Carolina Power and Light Company. A disturbance occurred on the Carolina system resulting in a burden on the interconnection in excess of the static stability limit of the lines involved with the result that the two systems pulled entirely apart and the voltage at Kingsport dropped practically to zero in approximately two seconds as recorded by high-speed instruments. Subsequent tests using instantaneous relays showed a power swing to 80,000 kw. which checked with approximate calculations of the stability limit for these interconnections.

Reactive Component Flow Control and Voltage

The regulation of the flow of reactive power and the distribution among generating units is not a problem of this interconnected system due to the high power factor maintained on lines and generating stations. This is partially due to the character of the load with a large amount of synchronous equipment but in the main due to the synchronous condenser capacity located at the transmission substations. The West Penn Power Company has 63,500 kva. in eleven synchronous condensers located at ten transmission substations. The Ohio Power Company has 85,000 kva. in synchronous condenser capacity located at its various transmission substations. If the capacity of the spare generator at Windsor, operating as a synchronous condenser, giving about 80 per cent of its rating, or 24,000 kva. leading is added, there is a total capacity of synchronous condenser equipment alone of 172,500 kva. in these two systems not including the generating units at hydro

stations which may be operating with low gate opening.

The voltage of the generating units at the principal power stations is held practically flat, there being not more than a 3 per cent variation between light load and heavy load. By means of the synchronous condensers the voltage at the transmission substations is held to less than 2 per cent variation.

Power Flow Control and Frequency

Close regulation of the load among the interconnected systems of this group and among the individual power stations of each system has been a most difficult problem. As each system joined the interconnected group it would have to scrap, in many instances, much of the operating practise of load and frequency regulation it had built up over a long period of operating experience. The Working Committee of operating executives has held many meetings, in which these problems have been analyzed by all and a common basis for action determined. These meetings have been followed by continual educational work among the load dispatchers and switchboard operators until satisfactory operation was accomplished. From the very beginning all operating men were in accord that the interconnection must work and that it was their job to make it work. The sacrifice of personal opinions and private practises and the desire to do whatever was best for the common good have resulted in notable progress being made.

It was recognized early in the history of this interconnected system that in the problem of plant loading or tie line control and in fulfilling the essential terms of the many contracts through which each system interconnected with its neighbors, that frequency control was a matter of vital consequence. As early as 1924 the operating executives agreed, after there had been much confusion and continual bickering between the operators on shifts, that the frequency as recorded at Windsor Station whether it was correct or not, would be the standard system frequency. Windsor station was then equipped with duplicate sets of the latest type of frequency recorder and the usual frequency or time clocks.

Those companies which were interconnected with more than one other company and at different points, were greatly limited in their ability to maintain correct loading on the various interconnecting tie lines. While it was not difficult for a company to maintain the load on its tie lines between its stations, or between its system and the central frequency control regulating station, it was difficult for those companies interconnected to it and still farther away from the center of frequency control to regulate the amount of power flowing in the interconnecting tie line.

For instance, one company would regulate the flow of power from its station by opening and closing the turbine governors, which would tend to change the system frequency.[7] Such a change being made at the first station without the knowledge of the frequency regulating station would result in a variation of the frequency before it was caught. The frequency adjusting station would then observe the change in frequency and pick up or drop load to bring the frequency back to normal. These variations would cause the load to change on other turbines having sensitive governor adjustments all over the interconnected system, and numerous adjustments would have to be made before the proper divisions of load were restored. High frequency often resulted, especially during light load periods. On the other hand, during periods of very heavy load or when the steam pressure became low at one station, the operators would shake off load to relieve the overloaded condition of their station and periods of low frequency would occur. It was usually 9:00 a. m. before the system settled down to the morning load which steady condition would last until 11:45 a. m.

This kind of operation was very unsatisfactory and often created peaks between the different companies, which were difficult to explain and to adjust when the billing was being discussed. It was believed by the working committee that if the frequency could be held at 60 cycles, with but a very slight deviation—less than 1/20 cycle,—the outlying companies would need only to adjust the load on their interconnecting tie lines, the next company in turn adjust its tie line load and so on to the last tie line connecting to the central frequency regulating station. In this manner the tie line load equilibrium would not be disturbed.

Windsor Station, being the center of all other operating features, was assigned the task of holding a flat frequency with six 30,000-kw. generators in the early part of 1929. As no automatic frequency control system had been installed, one operator on each shift was assigned practically the sole duty of holding a steady frequency by adjustment of the six governors. This was a radical departure from all previous practise for heretofore load had been picked up or dropped at Windsor by simply adjusting the governors as long as the frequency was not unduly altered. Now Windsor operators were to be more watchful over frequency and more agile in adjusting it than the other operators of the system had been and were to concentrate not on loading the station but on maintaining a close frequency range, whereas all other operators were to concentrate on maintaining a close tie line load adjustment and forget frequency.

This new scheme of operation worked successfully from the start. Troubles with tie line regulation seemed to disappear like magic. Whereas it had been the usual practise to adjust tie lines three or four times an hour, now three or four times a morning or afternoon or evening suffices.

In May 1929 an exhibition sample of an automatic frequency control device was temporarily installed at Windsor, thus relieving the shift operator of his onerous duties. This equipment had an over-all adjustment of two cycles plus or minus 60. Permanent equipment in duplicate was then ordered and installed early in 1930

with an adjustment range of 0.25 cycle plus or minus 60. Although this equipment was installed to control all six generators, it was found in normal operation that three or four generators would adequately hold the system frequency within 1/40 of a cycle plus or minus 60.

After a year's study of frequency regulation by one station only namely, Windsor, during which period several other large interconnected systems were added and the control of their frequency also taken over by Windsor, it was found that the only real difficulty of this scheme of regulation occurred at the rapidly load shifting periods of the day, as 7:00 a. m., noon hour, and 6:00 p. m. At these periods it was not unusual for the load on the Windsor station to shift from 60,000 to 90,000 kw. in 8 or 10 minutes. Although the boiler room operating forces at Windsor Station handled these radical changes in load in a most splendid manner, rarely ever popping the boilers or losing steam pressure, it was believed by the working committee that the installation of several other frequency control sets on this interconnected system would materially alleviate the conditions at Windsor. At the time this paper was written (March 1931) automatic frequency control equipment was being installed by two manufacturers for trial at the Windsor, Philo, and Springdale stations. The methods of operating under these new conditions have not yet been determined and positive results cannot be foretold. Preliminary tests would indicate that the objective had been achieved.

With very close frequency adjustment it has been possible to maintain electric time with a maximum error of not more than four seconds fast to four seconds slow in each month and with an average error when checking twice each day with Arlington of considerably less than one second. An inconvenience in operation due to this close time keeping operation results when the systems are separated for repair work on the tie lines or from fault reasons, for when the systems are paralleled the clocks on each system must correspond to the second or the connection must be broken again to adjust the frequency and consequently the time until the clocks do correspond.

Communication and Telemetering

In recent years considerable attention has been directed toward those situations which have arisen in the communication field as the result of the rapid growth in power output, the increased transmission line mileage, and more particularly the extensive interconnections. The requirements brought about by the vastly improved services rendered by the power utilities have necessitated the seeking of more adequate communication facilities. The problems in this special field require every facility of communication to the end that the most economical and serviceable systems may be made available. One answer to this problem now being used by several power companies of the interconnected system is the unification of private telephone services with the Bell Telephone System. Agreements have been entered into which provide for the termination of both privately owned circuits and leased circuits with suitable protection in the same private branch exchange switchboard where such a connection affords the most desirable arrangement.

The unified service plan makes possible circuits of proper transmission characteristics including the necessary freedom from noise. On power system communication lines the noise difficulty has been combated by using voice currents which tend to over-ride the induced currents to such an extent as to be intelligible. There is a limit, however, as to what can be accomplished along the line of high powered transmitters without the use of costly vacuum tube amplifiers. Transmitters using large currents have short life and high maintenance costs. By utilizing the lines of the telephone company for the long jumps and the lines of the power company for radial short distance communication service, a circuit is obtained which is not noisy and which is highly efficient.

Another method employed on this interconnected system for providing essential communication between power stations, load centers, and dispatchers is the carrier current telephone. The use of this system of communication has reached the highest stage of the art on the American Gas & Electric Company System, of which the Ohio Power Company and Appalachian Electric Power Company are members.[8] In July 1921 the Ohio Power Company conducted tests on the Windsor-Canton 132-kv. line which was the first successful communication over high-tension power lines of such a high voltage in the history of the art. The unique use of a common frequency by two independent operating companies for intercommunication has been carried out in territory around Canton, Ohio. A frequency of 90 kc. is used as a common frequency between the systems of the Ohio Power Company and that of the Ohio Public Service Company. The Ohio Power Company stations at Canton and Shelby are able to communicate with the Ohio Public Service Company station at Massillon over the common intersystem channel between these three stations.

One of the special services, to which reference should be made and part of the work of communication of a large interconnected system is that of distance metering. Such a scheme is in use between the Windsor Power station, Charleroi Load Dispatching headquarters and the Lake Lynn Hydro Station of the West Penn Power Company, a total distance of 75 miles, utilizing the telephone wires on the 132-kv. tower line structures to transmit the total load of the Windsor Power Station to the West Penn System to the load dispatchers and the hydro station.[9] Another use made of telemetering is to transmit the load taken by the Ridgway division of the West Penn Power Company to the Piney hydro station of the Associated Gas & Electric System, which enables the latter system to

control correctly the flow in the interconnection between these companies and which interchange is involved with the use of the joint use facilities.

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Discussion

For discussion of this paper see page 1280.

Interconnection Development and Operation

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Synopsis.—*The possibilities of interconnection can only be realized when the system is effectively developed and operated.*

The interconnected system functions as a unit but the companies owning the different parts have diverse interests which must be sufficiently coordinated both in development and operation to make the benefits of the interconnection available to all the participants. There is question as to how much coordination is sufficient.

The differences which exist between an interconnected system and a similar system under one ownership result either from the larger size of the interconnected system or from the variations in interests of the ownerships involved.

Interconnection development usually tends to reduce generation costs while increasing transmission costs and hazards. Possibly there are some limits to the size of the interconnected system and the degree of coordination desirable between ownerships. Closed loop operation involving several companies requires attention.

Interconnections may be operated by centralized direction or by cooperative methods between adjoining system operators. The unit characteristics of the system require some coordination of the group as a whole. Operation should be simplified as far as possible with adequate operating facilities.

* * * * *

INTERCONNECTION is only one of the many factors contributing to the accomplishment of the purpose of the electric utility industry. The industry endeavors to provide the customer an electric supply, adequate in quantity, quality, and reliability for his requirements, at the place and time he wants it and at the minimum cost commensurate with a sufficiently encouraging return to obtain the men and money needed in the service. As far as practicable, natural resources are conserved and the development of the electric supply system is harmonized with public irrigation, navigation, flood control, and water supply projects. Justified interconnections contribute their bit only when effectively developed and operated.

Mr. Bauhan's paper on Interconnection Services defines interconnection, delineates and classifies its uses and indicates the procedure in evaluating a proposed interconnection. This paper attempts to outline methods used and problems encountered in interconnected system development and operation.

Synchronized operation of adjoining electric utilities has become such an established fact that no other type of interconnection will be considered here.

Distinguishing Characteristic of Interconnection

The synchronized electric supply system functions as a unit, all the component parts responding to variations in the demands made upon it. This inherent unit characteristic is naturally recognized under the single interest of one ownership.

The interconnected system is a synchronized system, but the parts of it are owned by different companies, each of which must look out for its own interests. The unit characteristic must, therefore, be seen through a screen of diverse interests, and naturally is not as well defined nor as readily recognized as under one ownership.

The distinguishing characteristic of interconnection is coordination of the efforts of a group of independent ownerships in the development and operation of a unified electric system.

How much coordination is required? Some believe all the benefits of interconnection can only be obtained by developing and operating the system just as though it were all owned by one company; that the trend in that direction is inevitable; and that the individual company's interests need not be submerged. Others believe all the practical benefits can be obtained with much less coordination; that continuing interconnection over greater and greater areas so complicates complete coordinating arrangements as to make them economically unjustified; and that greater results will be obtained by maintaining stronger company individuality.

It is generally agreed that there must be at least sufficient coordination to care for equities in established contracts and to avoid penalizing any of the participating companies. Any additional coordination economically justifiable from the point of view of all the interests involved is equally acceptable.

Development of Interconnected Systems

There are two outstanding questions in interconnection development. How far is the enlargement of the interconnected system justified? Shall power flow in closed loops be controlled by means of special equipment or shall the power be allowed to flow as it would under one ownership and the equities of each company be cared for by net billing of the algebraic sum of the power flow on all of its connections into the loop?

Other engineering problems involved in the development of the interconnected system are essentially those of a similar system under one ownership. However, since few independently owned systems develop to the size of the moderate interconnected system, it may be appropriate to outline some of the problems and possibilities which have arisen.

Generation. Reduced capital and operating costs of generation usually result from interconnection. The development problem is so to take advantage of all of these possibilities as to achieve minimum total cost

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with maximum reliability in the total generating capacity of the system. Economically balanced distribution of capacity among the various types of steam stations and hydro stations, properly correlated for the absorption of all available by-product power from irrigation, navigation, flood control, water supply, and industrial developments is an intricate problem new in the breadth of its scope and possibilities. Heavy transmission for interconnection purposes at times justifies development of generating station sites otherwise unavailable due to transmission costs.

The trend in steam generating stations has been toward the largest unit which will economically fit into the system with practically no reserve provided in the individual stations. However, there are inviting possibilities in considering the transmission network as the bus and strategically locating large economical units in relatively small stations close to the load.

The flatter generating station load curve resulting from all types of diversity offers opportunities for the use of more economical vapor cycles not flexible enough to meet the demands of smaller systems. The station however, must be sufficiently flexible to meet the responsibilities imposed upon it by the variations in the hydro and by-product power.

The large peaks and proportionately large steam capacities of interconnected systems justify water power developments which could not be absorbed by smaller systems. The load characteristics of the system combined with the diversities in the flow of different streams and the increased quantities of low cost power available at off peak periods appreciably affect the justification and design of pumped storage and all other types of hydro stations. The design is further affected by the possibilities of using the generators as synchronous condensers and automatic stand-by capacity.

Transmission. Interconnection usually increases both capital and operating costs of transmission by increasing the quantity of transmission, the capacity of the units and the reliability requirements. This increase as a rule is partially offset but rarely equalled by the decrease produced in the internal transmission costs of the individual companies.

The transmission development problem is to discover means of reducing costs while providing an adequate reliable transmission network from the generator to the distribution system. Possibly these means of reducing cost do not exist in available equipment but can only be found in some entirely new development such as high-voltage direct-current transmission.

Interconnection tends to increase the transmission hazards to which each company is subjected by increasing the exposure due to the increased mileage of transmission lines tied together, by increasing the number of equipment units subject to inherent failures, and by increasing the current demands on equipment. The problem in development is to minimize the number of faults resulting from these hazards and the effect of the faults on the service.

Units of transmission and generation are going to fail even with faults from hazards reduced and adequate localization accomplished. At times such failures are going to separate sections of the system. The design therefore, must permit prompt resynchronizing. When the section separated from the system does not have adequate capacity to carry its load the set-up should be sufficiently flexible to drop the required quantity of load quickly.

The mechanical behavior at the point of support of the large conductors on high capacity long distance transmission lines is not well enough understood. Some scattered studies and tests are being carried on but a broad coordinated scientific attack is needed first to establish what is actually taking place at the point of support of these conductors, and second to devise methods of support to handle satisfactorily the actual conditions.

Much has been done to analyze and control short-circuit current, stability, voltage regulation, and power flow in closed loops, but with the increasing size of synchronized systems the problems are becoming both more important and more difficult of solution. Closed loops are developing rapidly and in some cases are hundreds of miles in circumference, involving many ownerships. Some general coordination of development is essential if the ownerships involved in these loops are to continue to get the benefits which justified their individual interconnections.

The development of the synchronized transmission system would be greatly simplified, the cost reduced, and operation improved by standardization of transmission voltages and phase relations.

System Operating Facilities. Planning, engineering, and operating groups cooperatively develop the system operating facilities necessary to indicate at all times the performance of the system and the adjustments required. As nearly as may be, controlling information on system performance is transmitted directly from the point at which it is observed to the point where compensating adjustment must be made. A fundamental communication and control plan should be developed, economically utilizing as required one or more of the following types of equipment: telemeters, teletypes, telecords, pneumatic tubes, open-wire telephone, cable telephone, carrier-current telephone, radio, remote control, supervisory control, automatic frequency control, automatic load regulation, and dispatching boards.

Experience seems to indicate that the communication and control system is rarely planned with the breadth of scope, the unity of purpose, and the economical utilization of available elements which the importance of operation justifies. Consolidation and correlation of guiding information are expedited by totalizing metering, and by system diagram boards both auto-

matically and manually controlled which portray what is occurring, establish the status of the elements in the system, and indicate necessary adjustments.

An ideally regulated interconnected system would economically and promptly distribute load variations over all generating equipment in operation. Instantaneous consolidation and centralization of complete load information seems at present economically impractical. Something other than load therefore, must be used as the guiding factor. The automatic distribution of the necessary load control impulses to all generating stations to cause them to share the variations in load is possible but has not been used. It seems probable however, that refined frequency indication offers a guide to the variations required of generating equipment.

A standard frequency of great accuracy can be made available at all generating stations. Since the actual interconnected system frequency will be the same at all points, a measure of the divergence of this actual from the standard frequency offers a means of instantaneously and simultaneously adjusting generation at all stations to meet load variations. The problem has not been solved however, and a complete analysis is needed of the functions and the relations of steam turbine and hydraulic turbine governors, over-speed stops, automatic frequency and load regulators, and all other factors.

Metering. The metering equipment used in billing interconnection power requires more rational treatment than it has received in the past. Complicated, expensive, and yet inadequate metering has frequently resulted from attempts to predict definitely and to provide exactly for all theoretical equities. Actual operation has usually indicated a dearth of operating meters and an excess of billing meters.

Flexible transmission busing and switching arrangements almost invariably force the metering equipment onto the high-tension circuit. Equipment at these high voltages is a relatively new development about which much additional information is necessary. Both the voltage and the newness of the development make the equipment expensive and apparatus should be developed to accomplish the desired result at reduced cost.

Interconnection Operation

Objective of Operation. The objective of electric supply system operation is to produce most economically the most reliable service possible with the system provided while efficiently accomplishing maintenance. The coordination of the performance of all of the operating and maintenance personnel and operating equipment on the system to accomplish this objective requires planning and execution. The planning necessitates a forecast of the load demands and maintenance requirements and the establishment of a schedule of the activities to meet these demands and requirements economically. The execution of the plan requires coordination of load demands, maintenance requirements, personnel, and equipment to meet the schedule with such adjustments as are necessary in actual performance.

Interconnected system operation differs from system operation under single ownership only in the method by which the objective is accomplished, and in the necessity for properly taking care of the equities resulting from differences in ownership.

Since little has been written on the planning and execution of system operation it may be appropriate to review briefly the manner in which this is usually handled on a system under one ownership realizing that centralized interconnection operation follows practically the same procedure.

Planning Operation. Centralized system operation is the universal practise under one ownership.

Predictions are usually made of the monthly average and peak load demands and the maintenance requirements as far in advance as necessary. A guiding annual schedule for major maintenance is then prepared. It is assumed that arrangements have been made for adequate installed capacity or for purchased capacity to care for the maximum annual demand. It is recognized that daily maintenance requirements and variations from annual predictions on load demands must be taken care of by more detailed estimates and schedules prepared monthly and weekly and adjusted daily and even hourly.

Weekly forecasts usually prepared on Thursday or Friday for the following seven days are based upon a detailed study of load requirements and up-to-date information on daily maintenance work. The shape of the load curve at periods of rise and fall and at peak and valley periods of less than one hour duration must be known. The height as well as the contour of the system load curve must be established as accurately as possible. This total system load and maintenance information must then be broken down into load areas.

With this predicted load demand and maintenance requirement information, generating equipment is provided to protect against hazards and provide continuous service not only for the system as a whole but for each of the load areas. Knowledge of the characteristics of operating response of all of the equipment on the system to varying types of normal and emergency conditions is required.

Generators, frequency changers, and synchronous condensers and the loads on them must be scheduled to produce the lowest total production cost while utilizing all of the hydro and by-product power available up to the point where the cost of this power delivered exceeds the cost of other power. Knowledge of by-product and dump hydro power which is going to be available and of the factors entering into the cost of power delivered at any given point on the system is necessary. Cost of maintenance work at different

periods must be balanced against the increased operating cost it necessitates.

Available hydro must be predicted based upon the factors entering into the run of stream hydro, pumped, and natural storage hydro, and hydro available from irrigation, navigation, flood control, and water supply developments. Information is necessary on probable weather conditions, ground water, snow on ground, rainfall over water sheds, long time predictions of rainfall and all of the other factors that enter into regulation of water power. Where there is more than one ownership involved in a water power development the utilization should be coordinated.

Complete and accurate starting and stopping costs, no load running costs and instantaneous incremental fuel and maintenance costs must be available on all fuel stations. It is important here to stress the difference between the incremental cost referred to for estimating purposes in Mr. Bauhan's paper and the instantaneous incremental cost necessary in scheduling operation. Cost of labor and supplies in generating stations are so nearly fixed that except in special cases where stations can be shut down for long periods they are not usually included. Methods of arriving at incremental costs and using them to get most economical system loading have been described in the technical press.

Costs at the generating station when considered in scheduling load distribution must be properly compensated for transmission losses.

The loading schedules arrived at in this manner must be continuously studied in relation to the corresponding actual results and the necessary adjustments and improvements made. Operation planning has not always received the attention it deserves. Methods of load forecasting require improvement for greater accuracy. Additional knowledge is needed on practically all of the factors entering into prediction of available hydro especially where large storage and complete regulation of stream flow is possible. Thorough coordination is needed with industrial and public developments having by-product power available.

Methods of establishing instantaneous fuel increment costs on running machines, starting and stopping costs, and no load running costs are well developed but much better basic test information is needed on many systems. The maintenance element of instantaneous incremental cost is influenced by many factors which are not capable of correct quantitative analysis as a function of load. Some studies have indicated that the ratio of fuel plus maintenance increment costs to fuel increment costs alone may vary under different assumptions from 1.05 to 1.45.

No exact method exists for conveniently determining the effect of transmission losses under operating conditions on the cost of the power delivered.

The method of developing the most economical load schedule from all these data needs considerable analysis and study.

Planning on Interconnected System. Planning operation on the interconnected system where centralized direction has been established follows practically the procedure outlined, but each ownership is responsible for providing all the required information from its system in such form that it may be easily coordinated into an interconnected system picture. Due to the size and expanse of the system, the problem becomes correspondingly complicated. For example, how much operating reserve does such a large system need, how shall this reserve be apportioned between the ownerships and how shall those ownerships not carrying their proportionate share compensate the companies who are sustaining them?

When centralized direction has not been established each company carries through its own planning as outlined and then checks with adjoining companies. Knowing the costs of the blocks of generation above and below that sufficient to carry its own load each company determines with its neighbor where load and reserve can be placed most economically.

Executing the Operating Plan. Execution of the operating plan under one ownership is invariably directed by a central position variously called system operator, load dispatcher, power director, and so on. System operator will be the name applied to the position directing the operation of the individual company, and local system operator will be the name applied to the sub-centers on the individual companies. The local system operator takes coordinating direction from the system operator.

The system operator keeps in touch with the demands made upon the system and the performance of the personnel and equipment in meeting those demands. His course is charted by the schedule of operation which has been prepared. He relates the demands as they develop to the predicted demands and from his thorough knowledge of all of the elements on the system, adjusts the performance of equipment and personnel most economically and effectively to compensate for deviations of the actual from the schedule demand.

The details of directing are so numerous it is essential to develop a method of procedure which delegates detailed observation of indications and responsibility for compensating adjustments as far down the line as possible, superimposing direction only where coordination is necessary between operating points. Every switchman, substation operator, and generating station operator is usually allowed to do all that he can without coordinating direction from the local system operator who in turn handles all he possibly can in his area without coordinating direction from the system operator. Methods of procedure and standard practises are established through written operating instructions so as to reduce to a minimum the amount of direction necessary

both in emergency and in normal operation. Complete and thorough training of all the operating personnel is essential to be sure that everyone can be counted upon to respond promptly as expected and thus get harmonized action over large areas though individuals in the performance do not know what other individuals are simultaneously doing.

When such a system is well established with well trained personnel, lightning storms and similar emergencies are satisfactorily handled without any one person being acquainted with all that is taking place. In fact it often becomes necessary to spend some little time on assembled records in order to determine what actually has happened and whether or not the equipment and personnel have functioned properly.

The operating personnel must have a broad knowledge of surrounding system conditions to accomplish this harmonized performance. Such continuous information on system conditions as will assist in performing the function of a particular operating point must get to that point. Changes in generation to compensate for differences in actual and scheduled load conditions require a convenient method for the system operator to determine the most economical adjustment. Methods of correlating and automatically acting upon information are being developed and will undoubtedly be more widely applied.

Local system operators and system operators must be increasingly able, well trained, experienced men, considerate and diplomatic, but with the broad knowledge and judgment necessary to make quick accurate decisions and translate them into action through direction over the telephone. The performance of the system in fulfilling its objective finally depends upon the quality of system operation. The importance of these men and their perfomance has not always been sufficiently appreciated and there is a shortage of well trained, well educated, able men.

Executing the Operating Plan on Interconnected System. An interconnected system operator coordinates the performance of the individual companies where centralized direction is provided on the interconnection. System operators do not report to the interconnected system operator any more than a switchboard operator reports to a local system operator. Only such coordinating directions are given and received in each case as are required to produce dependable, economical, unified operation. In neither case is there any need for loss of individual initiative or responsibility. The function of the interconnected system operator is in some cases simply assigned to some system operator by mutual consent of the companies involved.

Most existing interconnections have no centralized direction. The system operators of adjoining companies coordinate the performance of their companies and by such action progressing through the interconnected group produce unified operation. Many companies have handled their own operation in exactly the same manner between the switchboard operators at two or three generating stations before system operators were justified. The effectiveness of the decentralized method is increased by occasional meetings of those directing the operation of the individual companies. Operating problems and practises are reviewed and mutual agreement reached on methods of procedure.

In a closed loop interconnection without phase shifting transformers or other special equipment it is impossible to direct the power flow. Adjustments can only be made for the individual company's conditions by so regulating that company's generation as to produce the net power flow desired. No satisfactory method of accounting for equities in interconnected loop operation through several companies has yet been developed.

Little attention is paid to the power flow on tie lines in a system under one ownership until there is danger of reaching some physical limitation on the circuit. Power flow on tie lines between different ownerships in interconnection however, becomes one of the bases on which money is exchanged. All the different classes of power referred to in Mr. Bauhan's paper have different values and even the same type or classification will have different values at different times. It becomes part of the execution of the operating plan in interconnection therefore, to sort out, classify, and account for the different classes of power flowing. Unless care is exercised in establishing the principles not only in operation but in the contract, this work is likely to become excessively complicated.

So-called incidental or uncontrollable power will flow across the interconnection as long as the properties are synchronized even though no power flow is scheduled. It is necessary, therefore, to have an incidental rate which will encourage complete utilization of the instantaneous diversity without penalizing anyone.

An encouraging emergency rate which will induce everyone to rely upon the interconnection reserve, but which will at the same time adequately compensate those properties producing the power in an emergency, is necessary. The rate used for a sustained emergency must compensate the property supplying the capacity and yet not too severely penalize the company using the capacity unless the necessity for this use has arisen from negligence in maintenance, poor planning in development, or willful leaning to prevent investment in capacity.

Summary

The possibilities of interconnection can be realized only when the system is effectively developed and operated.

The interconnected system is an electric supply unit which functions like a similar system under one ownership. Such differences as exist result either from the larger size of the interconnected system or from the

multiplicity of interests resulting from the number of ownerships involved.

Possibly there are some limits to the size of the interconnected system and the degree of coordination desirable between ownerships. Sufficient coordination to satisfactorily operate closed loops involving several companies is becoming essential. Ample facilities should be provided to simplify the operation as much as possible.

Is not interconnection much like democratic government in which the companies are the individuals? The total benefits to be shared are in proportion to the total participation and the regulation imposed on the individual participants should be the minimum necessary to make the mutually attainable benefits proportionately available to all.

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Discussion

INTERCONNECTION SERVICES

(BAUHAN)

INTERCONNECTION—NEW ENGLAND DISTRICT

(DILLARD AND BELL)

PENNSYLVANIA-OHIO-WEST VIRGINIA INTERCONNECTION

(FITCH)

INTERCONNECTION DEVELOPMENT AND OPERATION

(KEENAN)

H. K. Sels: Mr. Bauhan has presented very logical and lucid definitions of interconnection and the various services to be rendered by it. As a member of a group working on definitions under the direction of the Joint Interconnections Subcommittee I should like to see these definitions accepted as a standard. One engineer refers to load diversity where another calls it peak diversity and economy flow called dump power. If these terms could be standardized it would prevent possible confusion and misunderstandings. It would serve to fix the relative importance of the various services in engineering design as undoubtedly each class of service will justify different amounts of spare capacity and equipment. For example, economy flow might not justify any spare capacity or equipment where undoubtedly firm power could justify sufficient spare equipment to insure a high continuity of service. In contractual relations accepted definitions would provide useful terms in writing contracts which would have a better legal status.

To amplify Mr. Keenan's example of a multifamily house compared to interconnection and the cooperation necessary, it would be possible for a group of families to have things in common such as billiard rooms and ballrooms. So too with interconnections it is possible to have common plants built economically at the most desirable location to obtain maximum operating economies. In this way individual plants would become more or less independent of the load and the problem then becomes how to operate the total generation plant the most efficiently. So we have a concentration and saving in generation compared to a lesser addition of transmission and distribution. While systems will continue to grow due to consolidations interconnection has and will continue to increase materially the size of systems.

Attention may be called to the added importance of load and frequency control and the importance of investigations of lightning in transmission lines, development of standard test waves, and coordination of insulation with interconnection. Incorporating in our standards the results of some of the investigations will facilitate the development of interconnected systems in a standardized and economical manner.

H. G. Harvey: Mr. Bauhan's paper deals primarily with the interchange of power or kilowatts between operating companies. However, the phraseology of his paper throughout indicates that he had the interchange of reactive kva., or voltage control very clearly in mind although he did not specifically mention it. It might be pointed out that with a few very obvious modifications, the main service divisions which he outlined can be applied to reactive kva. as well as to true power flow or kilowatts.

There is a subdivision of his first division "Emergency Service" which Mr. Bauhan does not bring out in his paper, probably because it does not affect contracts. However, since it is a potential source of difficulty it might be well to point it out.

What is ordinarily thought of as emergency service, lasts for an appreciable length of time, anywhere from 15 minutes to a number of days or even weeks. However, there is another class of emergency service, of a more transitory nature, which lasts from something like one-half a second to one or two minutes. This occurs, for example, when a distant tie line or generator comes off the line due to a switching operation and a surge or bump is thereby set up in the system. Under such conditions, the presence of the interconnection may mean the difference between dropping load and riding through successfully. The value of this service naturally cannot be measured by the power taken over the interconnection during the very limited time necessary for the readjustment of the load to a normal condition. In instances where a company with a relatively low standard of service is interconnected with a company with relatively high standard of service, the result of the interconnection is that the first company gets a higher grade of service, with fewer momentary interruptions, while the second company may have a lower grade of service with more voltage disturbances and the like as a result of the interconnection. As far as I know, no means have as yet been devised to adjust equitably for this condition. I have, however, known of cases where this condition has been a source of friction and difficulties among the load dispatchers and that part of the operating personnel charged with the control of the actual physical interconnection.

M. M. Samuels: Engineers are prone to forget the fact that the answer to the majority of engineering investigations should not be of an engineering but of an economic character, because in the final analysis, the answers must be submitted to executives in simple form, and executives are satisfied to let the engineers go through with elaborate engineering investigations, but they are not satisfied with engineering answers; they must have economic answers. Mr. Bauhan correctly stresses the economic side. Yet he does not altogether eliminate intermediate engineering features from his answers.

Since fixed charges make up the principal part in the cost of a kilowatt-hour, it is important that an attempt be made to come to a definite understanding as to what items enter into fixed charges. Steam plant fixed charges are variously given between 10 and 18 per cent, substation fixed charges are found given as low as 8 per cent and as high as 17 per cent. This is not caused by the fact that one plant actually shows a lower fixed charge than another, but that one considers an item a fixed charge, whereas the other considers the same item an operating charge. Furthermore Mr. Bauhan speaks of the "fixed element of production expense" and goes so far as to say correctly that the effect of its avoidance is "similar to avoided fixed charges." This method is universal, but why? If it is a fixed charge, and everyone, including Mr. Bauhan, seems to agree that it is, why not call it a fixed charge and include it as an item in the total fixed charges?

The evaluation as described in the second chapter of the paper, with its subdivision, is very logical and systematic, but it is of an intermediate nature and not the final analysis to be presented to executives. Under the heading "Capital Cost Savings," Mr. Bauhan states: "and they are in any event of importance to the executives who are concerned with financing the project." In this remark lies the key to the incompleteness of Mr. Bauhan's second chapter. He considers the engineering features as of primary importance and the purely financial and economic features of interest to executives as of secondary importance, whereas the reverse is the case. It is no doubt for this reason that he discusses production in such detail. The following are suggestions in systematic form of what is of interest to executives, and what should finally be presented to them for *each year* of *each plan after* the evaluation feature studies suggested by Mr. Bauhan have been completed.

1. Total kilowatt-hours delivered to load centers, whether generated or purchased, for each year, each plan, each system.
2. Total cost per kilowatt-hour at load center for each year, each plan, each system.
3. Over-all plant factor of all generating stations for each year, each plan, each system.
4. Capital to be invested each year, each plan, each system.
5. Degree of reliability for each year, each plan, each system.

Total Kilowatt-Hour Delivered to Load Center. The executives are not interested so much in the kilowatt-hour generated or purchased, but more in the kilowatt-hour delivered to load center for distribution. As a matter of fact, they are really interested only in the cost per kilowatt-hour delivered to customers. It is quite fair and sufficient for the range of Mr. Bauhan's paper to estimate energy delivered to load center, because the distribution will be the same whether there is interconnection or not.

Over-All Plant Factor. It would seem offhand that the plant factor would be of very little interest after capital investment and cost per kilowatt-hour are known. But it is of interest as an indication of plant utilization, and if the plant factor is low, some explanation is often called for.

Capital Investment. The cost of energy is not always the only determining factor for the adoption of a plan. Sometimes it may be advisable to have a higher energy cost if it can be done at a lower investment. Market conditions, uncertainty as to future load, and other features may make high investments inadvisable even if they appear to promise economies.

Degree of Reliability. Absolute reliability can only be had with plants that are shut down. With plants and lines in operation, reliability is a matter of degree, and a decision must be made in each case as to the degree of reliability that can be obtained for a definite price. It would be well to discuss this matter further and eventually establish standards for degrees of reliability. The following ratings are suggested:

A. Each load center has sufficient local generating capacity for emergency and also at least one incoming outside source.

B. Same as A, except that some load centers have no outside source of supply.

C. Some load centers have insufficient generating capacity for emergency, but at least two incoming sources from outside, each at least a double circuit.

D. Same as C, except single circuit.

E. One source, double circuit.

F. One source, single circuit.

The estimated degree of reliability should be given for each important load center, for each plan, each year, each system.

In conclusion, I want to mention that at present it is very difficult to make any rational prediction as to future load. President Hoover estimates a growth of 25,000,000 hp. in generating capacity for the next 20 years. Translated into kilowatt-hours, this figure would seem extremely conservative, and yet

the President is generally sure of his figures. Perhaps he had in mind that more kilowatt-hours can be gotten out of existing equipment. The growth of per caput consumption and the resulting per caput generation are likewise quite doubtful. Just now it is difficult to use past statistics to estimate future requirements. This uncertainty should lead to more interconnections, but there are cases in which the same uncertainty will prevent interconnection.

L. L. Perry: The operation of widely interconnected systems, whether under one ownership or under several ownerships, has already demonstrated, as so many executives and operators well know, the improvements in power supply that are obtained in several particulars, such as

1. Lower fixed charges.
2. Better operating economy.
3. More uniform frequency.
4. Lessened voltage fluctuations.
5. More reliable service during major accidents incident to power station and transmission operation.

As Messrs. Dillard and Bell point out, executive staffs and operation staffs are undergoing an evolution. Operational supervision of interconnected systems is being delegated to one central group. This will enable each interconnected section to reap the benefits of interconnection such as those enumerated above without sacrificing appreciably any important assets of separate operation.

There are several lines along which rapid developments are to be expected. Little is said in these papers about actual experience with simultaneous automatic frequency control at several power stations in an interconnected system, although as mentioned in Mr. Fitch's paper experimental equipment is now in place for this and experiments are in hand. Widely different ideas exist as to what such automatic frequency control equipment does and should do, but I am confident great progress will be made in reducing instantaneous fluctuations on regulating stations and main tie lines, at the same time getting improved frequency.

Few transformer phase control equipments are yet in service. Yet, as overhead and underground circuits get loaded to capacity, economics will dictate their installation in some instances, particularly where it is desirable to close loops and still have control of power flow. Where buses at strategic points are tied together through circuits of different voltages, satisfactory operation has been attained, both in the east and in the west, in several instances without transformer phase control, but in other instances studies show that it will be very difficult to make the higher voltage line properly share the load with the lower voltage line unless transformer phase control equipment is installed. In instances where parallel circuits of the same voltage follow routes differing greatly in length, sometimes load ratio control will produce sufficiently equal division of current, even though most of the kilowatts take one route and most of the reactive kilovolt-amperes take the other with the result that the cost of phase control equipment is not justified.

H. R. Kurth: The pioneer efforts of New England companies in solidifying the several systems through the medium of interconnection has been amply justified by the savings in operating expense, and the mutual assistance rendered during periods of stress. Illustrative of this foresight is the 110,000-volt connection between Edgar Station and Millbury which was originally erected without contract but over which large quantities of secondary power have been transferred.

During dry seasons sales of surplus steam power by the Edison Electric Illuminating Company of Boston have totaled sixteen million kilowatt-hours in a single month and this spring purchases of dump hydro power exceeded thirty-six million kilowatt-hours during April alone. At some periods of the year the normal lighting peak characteristic load curve of Edgar Station has been converted to nearly a flat load, with greatly improved load factor, by the sale of surplus steam power during the lighter load period and purchase of firm power over the peak. During other times the purchase of dump hydro power causes a lowered load factor, but the over-all system economy is simultaneously greatly improved due to the low net cost of the surplus hydro power. Interconnections of ample capacity have permitted operation for maximum over-all economy of the several systems without loss of their corporate identities.

There remains however between some sections of the interconnected systems certain tie lines whose inadequate capacity causes frequent trip-outs with resultant upsets in load distribution, frequency, and voltage, the effect of which upon our customers may not receive sufficient recognition. Those industries employing continuous processes, dependent on a steady flow of energy at constant frequency and voltage are at times seriously affected by the heavy fluctuations which accompany the tie line interruptions.

The stability of tie line operation is directly determined by the degree of accuracy with which each system regulates frequency at its own stations. It should not be a normal function of interconnected companies to maintain frequency by changing tie line load because of laxity on the part of another system in changing generation in step with its own load.

The existence of long transmission line loops through several systems permits maximum protection to the several systems with the loop closed, but undesirable power flow around the loop frequently causes them to be operated open, with less stable operation of tie lines. The introduction of phase angle shifting devices into the loops seems inevitable for most satisfactory operation.

The high grade of service normally rendered by the metropolitan type of system utilizing underground transmission methods perhaps makes its customers particularly susceptible to disturbances transmitted from another system through interconnections. The more severe of these disturbances, occurring during lightning storms, sometimes force separation of the interconnected systems in the interest of reducing the area affected.

The occasional very severe disturbances due to loss of a large amount of interconnected system capacity has several times caused a load pick-up on the Edison system of 70,000 kilowatts without undue distress, thereby indicating the adequacy of the policy of protecting the largest single unit, be it generator or tie line. In the process of restoration to normal frequency and voltage the stations previously designated to pick up such load shifts may be dilatory in functioning, forcing other systems to assume the burden of load pick-up.

These difficulties encountered in interconnection operation are frequently accentuated by inadequate communication and metering facilities. The Edison load dispatcher has direct telephone lines to the system operators of the New England Power and Eastern Massachusetts Electric systems.

Past experience indicates that the success of interconnection operation depends primarily on the cooperation existing between the several system operators, and we are pleased to report that the spirit of cooperation is paramount in New England.

S. S. Seyfert: The analysis of the services and economics through interconnection made by Mr. Bauhan is most logical and complete. Messrs. Dillard and Bell point out the fact that these services and economics are considerably augmented under unified ownership or control. Mr. Fitch, in his paper, makes the point that mutual understanding and good will among companies may produce the same advantages in kind as common ownership. In either of these two cases there is presented to the public a condition that suggests a "power trust."

The public has now long accepted the idea of the power company as an economically desirable monopoly within its territorial limits. It has also accepted the almost complete integration of the wire communication system as working toward the greatest

good of all. The numerous interconnections already effected or in the making have created a new situation in the power field which immediately suggests or even calls for a fuller degree of integration than now obtains. It would seem that if the public could be educated as to the advantages and possibilities of interconnection and integration, and if the facts engineers are fully aware of could be expressed in their own terms and divorced from our technical language, it could be brought to stand behind and support our integrated power systems financially and otherwise as it has done for other worthy projects in the past.

A. H. Sweetnam: The Edison Electric Illuminating Company of Boston is the largest single customer of the New England Power Association and as such has been keenly interested in the development of the system of the New England Power Association, and in its interconnection activities with other New England systems.

The first attempt at interconnection with the Boston system was initiated in an effort to utilize the facilities of both systems in providing capacity needed in the minimum time and with the least investment. This connection, looked upon in the light of later developments, was a very limited attempt to apply the principles of interconnection.

The capacity of the connection was approximately 15,000 kva., and is made at 66 kv., with lower voltage transmission through the thickly settled portions of the city of Boston. The line is on wooden poles and transformers are without ready means of tap changing, which has somewhat handicapped both parties in the use of the connection to maximum advantage, as the power factor at which the line operates is not under control. The relay system initially was not adequate, and the capacity of the tie is so small in comparison with the capacity of the systems interconnected, that it is often unstable at times of system disturbance.

In recent years the operation of this line has been improved by modernizing the relay system, and by the installation of synchronous condenser capacity at the approximate midpoint of the line. The capacity of the connection has also been increased to 22,500 kva. by adding water cooling connections to the transformers.

The next interconnection, made a few years later, was of very much larger capacity, and utilized modern methods in the control of reactive energy. This connection was made between the Edison Company's new Edgar steam station and the largest dispatching center of the New England system at Millbury. It consists of a double circuit 110-kv. steel tower line with a nominal capacity of 100,000 kva. Transformers at the Edgar Station are now of 75,000-kva. capacity and are equipped for tap changing under load. There are two distributing substations of the Edison Company on this line, one of 30,000-kva. capacity, and one of 5,000 kva. This line was built, as has been mentioned in the paper, without formal contract between the parties, but with the knowledge that once in service it would serve for mutual support and provide opportunities for power interchange on a profitable basis. Experience has indicated the wisdom of this construction, as instances are numerous of the benefits of mutual support and the accounts of both companies show substantial amounts saved due to interchange operations.

The next interconnection was accomplished, not on the basis of interchange, but on the purchase of prime power from the New England system in lieu of the construction at that time of a new steam station by the Edison Company on the waterfront to the north of the city. This prime power is taken at the Woburn Station in the northern section of the Company's territory. By agreement energy can be sent from the Edgar Station on the south to the New England Power Association's system at Millbury, and a like amount drawn from the Association lines at Tewksbury and delivered to the Woburn Substation on the north. This receiving point on the north will be connected later with the proposed steam station just mentioned, when the additional distribution facilities and steam capacity are required.

One very marked saving accomplished by the agreement permitting power transfer from Edgar Station via New England Power Association's lines to the Woburn Station, was the postponing for the time being of a program which was then being actively considered by the company calling for the construction of a 110-kv. double circuit tower line from a point on the line between Edgar and Millbury to the present receiving point on the north at Woburn. This line would have accomplished the same transfer of power as is now possible via the lines of the New England Company. Thus duplication in the construction of some 30 to 40 miles of line was avoided.

At the Woburn Substation transformers are equipped with load ratio control and synchronous condensers are provided. When the connection is made to the new generating station, which will in turn be connected to existing stations, there will be need for phase angle control equipment.

It will be noted that the Edison Company has equipped itself at all important transformer banks involved in these interconnections with facilities for tap changing under load, and with synchronous condensers, and is contemplating phase angle control equipment. It is felt that in the not distant future the necessity for more accurate control of kilowatts and kilovolt-amperes and voltages will necessitate the more general use of this class of equipment, and that the New England Company has been exceptionally fortunate in being able to operate to the satisfaction of so many interconnected parties, without having as yet to resort to this refinement and consequent expense.

The interconnection activities with the New England Company have indicated very plainly the advantage and necessity for the closest cooperation in the development of the relaying system as applied to the interconnecting lines.

Frequency troubles were numerous in the early days, but these have now disappeared entirely as a result of the application of automatic frequency control equipment. Interconnection brings increasing voltage disturbances, but these undoubtedly will be gradually overcome by improved switching and relaying equipment and by more adequate shielding of lines.

We wish to lay particular emphasis on one point mentioned by Messrs. Dillard and Bell, that of the importance to the successful operation of any interconnection of a mutual spirit of cooperation and good will regardless of formal contracts.

Alex. E. Bauhan: Mr. Sels has intimated that the general adoption of a nomenclature for interconnection services would improve the legal status of interconnection contracts. This is very true. In the absence of a nomenclature generally accepted by the industry it is necessary for the various services to be specified and defined in considerable detail in the interconnection contracts. Several paragraphs may be required to give the same meaning as a few words from an accepted nomenclature. A nomenclature generally accepted by the industry will be given due recognition by the courts. This is, therefore, one good reason why it might be advisable to develop an official nomenclature for interconnection services.

Mr. Harvey points out that one of the services rendered by interconnection is the amelioration of voltage fluctuations due to minor disturbances and that the benefits resulting therefrom are difficult to evaluate. It is of course quite true that this service exists and under the present classification it is undoubtedly a manifestation of emergency service. Because of its difficulty of measurement and evaluation it must be considered as one of the several intangible benefits which arise from interconnection.

In calling attention to the reactive component service which might be rendered by an interconnection, Mr. Harvey touches on a subject which was considered in the preparation of the paper but left out in the interest of simplicity and brevity. The reactive component service rendered by an interconnection can be

classified very much the same as the energy service. All of the main classifications which have been offered, with the exception of that designated as *storage power*, are applicable to reactive component transactions. It may be mentioned that my own company has been engaged in reactive component economy-flow transactions with one of its neighboring companies—reactive component from synchronous condensers being supplied by one of the companies in replacement of the more expensive supply of reactive component from a turbine generator in the system of the other company.

A classification such as presented in the paper, as Mr. Samuels remarks, is not the result of a brief examination of the subject, but rather the result of several years of development and experience as well as of an analysis of what services are being undertaken and contracted for in the industry generally.

Mr. Samuels is of the opinion that fixed element of production expense can be combined with fixed charges and considered as a single item in any evaluation procedure. It is true that the fixed element of production expense behaves very similarly to fixed charges and I see no objection to their being combined for report purposes. However, they arise from entirely distinct accounts and must in any event be separately determined in anything but the most superficial study.

Mr. Samuels also gives a word of caution regarding the uncertainty of future loads and installation plans. We engineers are prone to set up certain basic assumptions in our studies involving loads and development programs and then after performing various cleancut mathematical operations, arrive at a conclusion which we regard as definite and infallible. As Mr. Samuels states, the estimation of future loads is fraught with uncertainty and notwithstanding the fact that development programs are frequently laid out for ten or more years in advance it is doubtful that such programs can be dependably predicted more than three years in advance. The results of evaluation calculations must therefore be used with good judgment.

Professor Seyfert believes that the contents of the papers of this session should be made available generally to the public in non-technical language to show what the much-maligned power utilities are doing by interconnections and mergers to reduce the cost of electric service. In this connection I am prompted to venture an opinion as to the effectiveness with which these interconnection savings are realized under independent operation and under merged operation. My guess would be that ordinarily something in the order of 50 per cent of the possible savings are realized in interconnection between companies. With the application of an unusual degree of skill in the discovery of these savings and their realization by well conceived contractual relationships possibly an upper limit of 80 per cent of the savings can be realized. With merger and consolidation of the system operations substantially all of the savings are realized.

Mr. Sibley states that in his own systems the savings from economy flow are relatively small, being of the order of 5 per cent of the total production expense. This is the usual experience, although in some few cases the economy-flow saving is the main justification for the interconnection and is sizeable. In the 220,000-volt interconnection which my own company has entered into with companies in eastern Pennsylvania the main justification for the interconnection arose from load diversity. Since it has been in partial operation we have discovered, however, that the capacity savings arising from emergency service are still more important than those arising from load diversity. The savings arising from economy flow, in this case, though appreciable, are relatively unimportant, appearing to be about 5 or 10 per cent of the total. Those savings which result in the avoidance of capacity installation are generally far more important than the relatively small production-expense savings arising from economy flow. Nevertheless, small or large, a saving is a saving and should not be overlooked, especially since economy-flow savings are so easily realizable.

I. M. Moultrop: The paper by Messrs. Dillard and Bell gives much valuable information on the subject of natural interconnection in the New England District, and emphasizes many of its advantages.

One point not touched on is that of frequency control. In the past many people have felt that there was serious difficulty in connecting hydro with steam plant, because the frequency control of the hydro plant was not up to the same quality as that of the steam plant. This was undoubtedly true a while back, but need not be true today. The engineers of the Boston Edison Company can speak from actual experience on this subject; and they wish to go on record that when the hydro plant is operated with frequency control equal to that of our steam plant, the frequency is kept uniformly to a high standard. By means of improved apparatus and more care in operating, the hydro company is matching us in the quality of its frequency control and possibly under certain conditions of load the hydro plant is a little better than the steam plant. No one need to hesitate in the future about tying steam plant and hydroelectric plant together so far as the matter of frequency control is concerned.

Another point that is very essential to good service over an interconnection is load dispatching. The load dispatchers of the two systems must be in close contact with one another. It is not necessary for the load dispatching of both systems to be under one control, and it is probably better for each system to have its own load dispatcher, but they must cooperate and work in harmony with one another. One thing that will materially help this is the load dispatchers conferences that are held at intervals where the interested people of both systems get together, discuss their mutual and particular problems and exchange ideas. All these things are very helpful, but what is equally helpful is establishing the personal relationship between the two groups of load dispatchers. When two persons are sufficiently acquainted to call each other by their first names and meet at frequent intervals to discuss their work, they come to understand each other and each acquires a respect for the other's ability and his problems, thereby eliminating much friction and misunderstanding. This is working out very well indeed with the New England Power System and the Boston Edison Company.

Communication Services of Electric Utilities Underlying Considerations

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Synopsis.—The paper outlines the communication requirements of electric utilities and suggests effective methods for meeting those requirements. Operating, customer service, and administrative activities are considered separately. However, the advantages of a unified communication system for all activities are outlined.

The relative merits of cable, open wire, power line carrier and radio are reviewed. The use of communication channels for telemetering and supervisory control purposes is discussed.

The desirability of basic plans for establishing and operating communication systems for electric utilities is discussed in detail.

THE elements of an electric utility must be effectively coordinated in operation to provide satisfactory electric service to the customer. A properly designed and constructed electric system, adequate tools and able personnel are essential, but until all of these elements are coordinated and operated as an electric supply unit, no service is rendered and no revenue earned.

Effective coordination of the activities of an electric utility necessitates the establishment of communication channels for the continuous flow of information into organization centers and the prompt transmission of orders and suggestions from these centers to the points where action is taken. This paper outlines the communication requirements of an electric utility and suggests effective methods for meeting those requirements. Particular attention is given to voice communication, as it seems to be especially adapted to the requirements of the electric utility. This is evidenced by the fact that about 85 per cent of the electric utility's communication expense is for the transmission of the spoken word. The use of the word "communication" throughout the paper assumes that the path used may be cable or open wire, power line carrier current or radio, either privately owned or leased.

The work of an electric utility may be considered as divided into three classes of activities:

1. Operating activities, including dispatching, generating and substation operation, emergency line patrolling and repairs.

2. Customer service activities, including new installations, disconnects, moves, service complaints, and comments, and

3. Administrative activities, including administrative sales and business functions as differentiated from the service functions.

Operating Activities

Fig. 1 shows a typical organization for the handling of activities classed as "operation," and indicates the communication channels usually available for the necessary flow of information between controlling centers and the points where action originates. A complete picture of happenings on the system at any

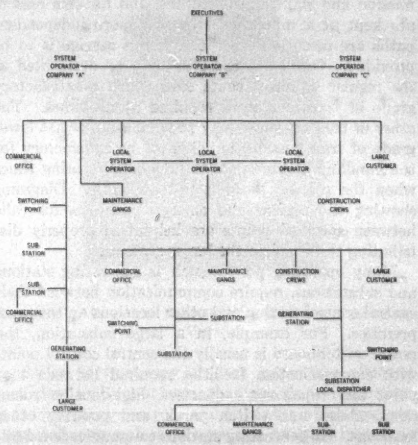

FIG. 1—TYPICAL ARRANGEMENT OF COMMUNICATION CHANNELS FOR OPERATING ACTIVITIES OF ELECTRIC UTILITIES

one moment is rarely available immediately because operation is simplified and speeded up by allowing each operating center to proceed on its own responsibility until the need for coordination with other operating points becomes essential. When this need arises, direction must be introduced from a dispatching point which may be the directing center of the generating or substation, a local dispatching office, a local system

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operating office or the operating office for the entire system.

The communication facilities for operating activities need to be:

1. Easy to operate and so arranged that communication service can be established quickly.
2. Available at times when generating and distributing facilities are in trouble and, therefore, as far as possible, not subject to the same hazards as power facilities.
3. Of such quality as to avoid misunderstandings and permit conversations to be carried on with ease.
4. Safe for use but not so protected as to be unavailable for service during electric service difficulties, and
5. Of sufficient capacity so that under the worst emergency conditions no serious delays will be experienced in restoring electric service.

Heavy or important traffic requires the establishment of direct channels. Relatively light or unimportant traffic may be handled over routes established by connecting two or more direct channels together. Switched connections of this latter type tend to create transmission and signaling difficulties, and for this reason, are kept at a minimum. Two or more independent paths are essential where continuous service is to be provided. One of these paths may be designated as the regular or direct route over which a satisfactory grade of transmission is required at all times. The other or back-up route may provide a somewhat lower grade of transmission, but it must be satisfactory for the handling of essential communication during times when the regular route is not available. Diagrams showing the regular and emergency routes for calls between specified points are helpful in properly distributing the traffic in the larger systems.

Many operating points, such as generating stations and substations, require communication between their central control station and other locations on the same premises. For example, in a large substation, the power switchboard is usually the central control point, with communication facilities required between that point and repairmen's quarters, high-tension room, pump house, and switch yards, and possibly other locations. In generating stations communication facilities are required from the central point to locations where coal and ashes are handled, to the boilers, turbines, pumps, switch yards, etc.

The equipment provided at generating or substations to terminate communication channels may be either manual or dial. Dial equipment will generally be desirable in those cases where most of the traffic is confined to intercommunication between telephone stations on the same premises and complicated tie lines or long extensions do not have an important bearing.

At points where small private branch exchange switchboards are installed, it may be difficult to establish convenient arrangements for answering calls at these switchboards at night. In some of these cases night connections have been established through the P. B. X. and in other cases it has been arranged for the communication utility to answer these calls at its central office and properly route them just as would a private branch exchange switchboard operator.

Dispatchers require communication channels to practically all operating points on the system, and, in certain cases, to locations along the transmission lines, to the various departments and officials of the company, to large power customers, and to system operators of interconnected utilities.

Various equipment arrangements have been provided at dispatching points for the termination of communication channels. The cordless type of equipment, where answering, ringing, and holding features are obtained by merely operating a key, is usually considered the simplest and most convenient arrangement. This type of equipment, illustrated in Fig. 2, appears suitable for use where not more than 50 channels are to be terminated or where the need for simultaneous through connections does not exceed five. Where more than 50 channels or more than five through connections are

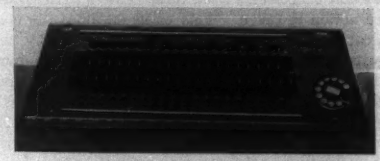

Fig. 2—Typical Cordless Turret for Termination of Communication Channels

involved, it is necessary to use the familiar cord and plug type arrangements. With either type of equipment, it is usually necessary to limit the height of the turret in order that the dispatcher may have an unobscured view of the power system diagram board. This requirement is met by special assemblies whereby the height of the cord and plug turret may be reduced to about six inches, or by the use of slant-faced cordless turrets, or by sinking the equipment in the top of the dispatcher's desk.

Convenient arrangements are desirable for transmitting routine information such as meter readings, switching orders, etc., required in connection with load dispatching work. Teletypewriters, which provide a typed record, are used to some extent for such work. This apparatus provides a means whereby the dispatcher may deal simultaneously with generating plants and substations. Objections have been raised to teletypewriter service on the basis that the employees using it must acquire some degree of facility in operating a keyboard. This objection should not prove serious, however, and there is some probability

that the teletypewriter will provide a valuable service to supplement the telephone in power dispatching work.

Some electric utilities have considered it advisable to install devices on the private dispatching lines for recording the telephone conversations of load dispatchers. The wax records used are kept for reference purposes until there is no likelihood that need will arise for transcribing the conversations. Reports as to the results obtained are favorable. Improvement is noted in the work of the load dispatchers, and from the standpoint of management, such arrangements are valuable in that responsibility can usually be definitely fixed in case the need arises for reviewing past operations.

Little progress has been made toward standardizing equipment arrangements for load dispatchers. Considerable study of existing installations and analysis of results obtained will be necessary before definite conclusions can be drawn as to the arrangements which are most desirable.

Disturbances occurring on a power system may make it necessary to reach patrolmen and line foremen to perform switching operations and emergency repair work. When a power line has failed it is the usual practise to patrol simultaneously small sections of the line to determine the location of the trouble. After the trouble has been localized and repair work started, need may develop for communication between the repair crews and the dispatching point. Communication facilities necessary in this connection have been provided in several different ways. Patrolmen have been furnished with portable telephone equipment suitable for connection to open-wire communication circuits at any point along the power line right-of-way. Other companies have considered the portable equipment too cumbersome and have established permanent communication stations along the transmission lines. These permanent stations have been connected in most cases to lines along the right-of-way, in a few cases to spur lines connected to trunk lines off the right-of-way, and occasionally to antennas used in connection with power line carrier-current channels.

Detailed reviews of representative reports of patrolmen indicate that the need for separate communication circuits for this purpose may be over-emphasized. However, much depends upon whether the territory traversed by the transmission line is thinly or thickly settled and upon the relative importance of maintaining uninterrupted power service over a particular route. Apparently, each situation requires individual study and engineering to determine its particular requirements.

The value of telemetering, remote indicating, and supervisory control services in effecting centralized control of a power system is rapidly gaining wide recognition. Where long distances are involved, apparatus operated by direct current over separate channels or by means of impulses sent over narrow frequency channels appears to be the most economical. For shorter hauls, the cost of conductors is usually not controlling, and simple terminal apparatus using separate wires for each metering or control service seems to be the most economical and desirable. Since large investments in power equipment are normally involved, and manual assistance cannot be obtained quickly at unattended stations, the conductors used must give reliable service.

The present methods of regulating frequency are not considered satisfactory in some of the larger electric systems. Arrangements are desired which will permit the sharing of frequency control by several generating plants or possibly, in the ultimate, provide for load distribution to stations on an automatic basis in direct proportion to the relative costs of generation. Apparatus has been installed and tests are under way to determine the possibility of obtaining independent

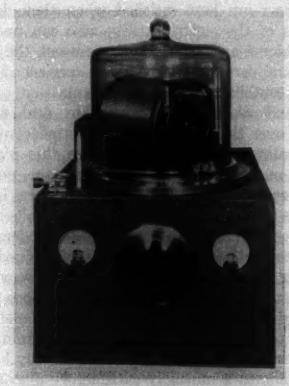

Fig. 3—One Complete 100,000-Cycle Standard Frequency Unit

regulation of frequency at each generating plant to such close limits that more than one station can share in the frequency control. Another plan which has been suggested contemplates the use of a constant frequency supply which might be furnished over communication channels and used to synchronize frequency controlling devices in generating plants. A quartz crystal oscillator circuit used in conjunction with vacuum tube circuits for control would probably provide the most stable and conveniently controlled source of constant frequency. Fig. 3 shows such a standard, now available in laboratory form, which has a frequency stability of approximately one part in ten million or about three seconds in one year. This apparatus together with suitable telephone conductors or channels should provide a satisfactory means of synchronization for power frequency control purposes, should the need develop.

Customer Service Activities

The second principal class of electric utility activity is customer service. Electric utility operation is devoted to the furnishing of a grade of service to the customer which he will consider entirely satisfactory. A vital factor in supplying such service is the provision of suitable means whereby the customer may easily and quickly convey to the utility his impressions and reactions in regard to his electric service. A large majority of the contacts with customers is made by telephone, and electric utilities have generally recognized the need for providing arrangements to insure that the customer calling by telephone will be handled as intelligently and courteously as if the call were made in person. In fact, greater importance is normally attached to telephone contacts because of the emergencies generally associated with such calls from the customer's point of view.

Customers are best satisfied when they deal with employees of the electric utility who can understand their problems and satisfactorily present the utility's attitude and service objectives. Experience indicates that the establishment of an organization especially trained and assigned exclusively to deal with the customer over the telephone is a most effective means of personalizing the service. Where this plan is in effect, all customer calls are routed to this personnel. Male attendants are considered necessary by some companies due to the character of the calls received and the feeling that men have a better understanding of technicalities. However, groups of trained female attendants have been assigned to this work with considerable success in a number of organizations recently established. Their generally pleasing method of handling telephone contacts tends to eliminate any feeling of irritation which the customers may have and usually leads to a mutually satisfactory disposition of the calls. Under this plan of centralized handling of customer calls, direct observations can be made to determine which practises are most effective in building up good will among the customers. Any desirable changes in practise can be put into effect quickly on a uniform basis.

Where centralized service bureaus are established, telephone lines are necessary between the bureau and the various departments in the company so that the attendants may readily obtain any information which may be necessary in transacting business with customers. The work of the bureau is simplified as much as possible by centralizing the file of customers' records, by short-cut methods of handling customers' orders, and by the installation of convenient arrangements of telephone facilities.

It is, of course, important that any service arrangements or commitments made over the telephone be carried out to the letter. The establishment of ideal communication arrangements and courteous treatment of customers calling by telephone will prove to be of little value if the actual service furnished and action taken with respect to complaints fall short of the customers' requirements.

Administrative Activities

Communication services are required by the electric utility for the handling of what may be broadly considered as administrative activities which include executive, legal, engineering, sales, and accounting matters together with certain features of the operating work where speed of action is not the governing factor. The electric utility's territory is divided into divisions and districts, with offices in central locations responsible for the conduct of administrative affairs over considerable areas. Communication service is required to keep these offices in touch with field forces and headquarters, and to maintain the necessary contact with other business concerns and individuals.

The type and volume of administrative activity handled by telephone generally necessitates extensive circuit and equipment arrangements. These communication facilities usually are provided by the telephone companies and it is important that the electric utilities make their objectives and requirements clear to the telephone companies so that the arrangements provided will accomplish the desired results.

The requirements for communication channels for administrative traffic are no different than those for similar traffic in other lines of business. Where the volume is sufficiently heavy, channels are leased for full time use. Arrangements may also be made with the telephone company for short period talking channels to be set up at certain specified hours of the day. Where the traffic is light, the public message channels of the telephone company usually provide satisfactory service.

Where quick transmission of letters, orders, accounting matters or other administrative affairs is necessary and a written record is desired, teletypewriter service has been found particularly helpful.

In those cases where it is necessary to reach employees who do not have a fixed post of duty, code-calling systems are often desirable. These may be installed in connection with private branch exchange systems or separately, as conditions may justify.

It is usually possible to install in the private branch exchange switchboard circuit and equipment arrangements for holding conferences by telephone.

Convenient switching plans are available which enable the secretary at a master station to answer and hold one or a number of controlled stations. The controlled station may have access to the master station line if desired. Switching plans are also available which allow an employee at one desk to answer calls on other lines without leaving his desk.

Sufficient attention has not generally been given to the location of private branch exchange switchboards. Such features as proper light, heat, ventilation, and

quietness, which are essential to efficient operation, are frequently neglected. The private branch exchange operator is often loaded with a multiplicity of duties having no connection with her regular telephone operating work and the communication service naturally suffers. Highly satisfactory results are obtained in those cases where the operator's duties are restricted to work directly in connection with the operation of the switchboard.

Methods of Providing Communication Channels

Cable and Open Wire

Most of the communication channels used by electric utilities are obtained by means of cable and open-wire conductors. Cable is relatively free from trouble during storms and is considered a desirable type of facility. Open-wire channels are normally considered less desirable from the standpoint of continuity of service. However, in specific cases sufficient open-wire facilities are often available over the regular or alternate routes to ensure the desired reliability of communication. When necessary, extra guying, short spacing of poles and similar measures are employed to strengthen open-wire lines.

Open wires have been strung on the structures carrying power circuits or on separate poles along the power line rights-of-way. It has been the experience that open-wire circuits thus situated are normally subject to a large amount of interference, and disturbances occurring on the power wires may put the telephone circuits out of service completely at the time when they are most urgently required. In other cases the failure of telephone conductors has put the power lines out of service. As a result of these experiences and the improvement in other types of communication service available for the purpose, there is a growing disinclination to rely on open-wire telephone facilities routed along the same right-of-way with the power facilities over which service is to be protected.

Power Line Carrier Current

Communication channels are also provided for the use of electric utilities by means of power line carrier-current telephone apparatus. In a number of instances during storm breaks, power line carrier has provided the only means of communication available, and it is generally considered a reliable form of service. There appears to be a definite, but somewhat limited, application for this type of communication channel over long hauls where it is necessary to reach generating plants and substations in remote and sparsely settled locations. The cost of providing suitable cable or open-wire telephone facilities in such situations may be prohibitive.

Probably the chief difficulty experienced with carrier is the variation in quality and volume of transmission. The power line is not usually a simple circuit, but rather, is a part of an extensive high-voltage network and any switching or circuit changes in the network alter its communication characteristics. Choke coils have been used in branch circuits to assist in keeping the carrier current along the main channel and improved transmission has been obtained in some instances by this means. Difficulties have also been experienced in operating power line carrier successfully over long two-wire extensions or interconnecting it with two-wire circuits. These difficulties have in many instances had the effect of isolating the carrier channels from all other communication facilities. This arrangement does not appear to be flexible nor economical.

Opinions as to the value of carrier current differ and further experience is necessary in order to arrive at generally satisfactory conclusions as to the extent to which carrier current logically may be employed in an electric utility's communication system.

Radio

A few companies have installed radio telegraph apparatus with the view of providing back-up service for the regular communication facilities. However, the necessity for employing licensed operators and the regulations limiting normal operating time to a ten-minute test period each day have not only made new installations unattractive, but have resulted in the abandonment of some of the original installations. No use is being made of point-to-point radio telephone channels due to lack of wavelength assignments and the high cost of securing the necessary reliability of communication. Considering the limitations placed on the use of radio channels and the other means of communication now available for use by electric utilities, it would seem that there is little field for radio in their communication systems.

Consideration of System Communication Requirements

Basic communication planning is necessary in order to coordinate individual services and facilitate the growth of the electric utility. It is, of course, desirable that maximum use be made of all communication facilities, that a satisfactory grade of communication service be available for instantaneous use at the principal operating points and that future needs be cared for slightly in advance of actual demand and without replacement of any large part of the existing communication facilities. These objectives cannot be met economically without establishing a basic communication plan for the electric supply system as a whole. It may be helpful, therefore, to consider somewhat in detail the information required and analytical methods which may be used in developing such basic plans.

Development of Basic Communication Plan

In formulating a basic plan, it is highly desirable that close contacts be established for the interchange of ideas and information and for coordination of action by those concerned with the development, operation,

and maintenance of the electric utility's communication service. All departments of the electric utility are involved, together with the companies supplying communication services and facilities.

It is desirable that the basic communication plans adopted by electric utilities be developed along consistent lines. The extensive interchange of power between electric utilities and the frequent consolidation of operating properties is emphasizing the need for some degree of uniformity in the communication arrangements adopted by the various operating units in the industry. Each electric utility should determine its requirements for communication in the light of the overall objectives of the electric industry and should estimate for its particular territory what will be the probable effect of proposed changes in the territory served, in organization and in power operating practises upon its specific communication needs.

In conducting a detailed analysis of communication requirements a field inspection is made of representative sections of line and items of equipment in the existing communication system. Observations are made of the traffic passing over the private network. The volume and character of calls made over the telephone company's public message circuits is carefully studied. The average daily flow and hourly distribution of all telephone traffic is separated into incoming, outgoing, and intercommunicating classifications. Important calls are separated from those of a relatively unimportant nature. The average holding time is determined for the trunk lines to the telephone company's central offices and the tie lines between private branch exchanges. Busy conditions on individual station lines are noted. Particular attention is devoted to the volume and character of traffic during times of emergency. Methods of operating are carefully scrutinized. Studies are made of private branch exchange operating loads, including not only the operator's regularly assigned telephone duties, but also other special assignments such as secretarial or office work. Information with respect to the volume, character, and cost of communications handled by means other than the telephone is obtained.

Unification of Communication Services

Basic plans for the communication services required by an electric utility may be developed either to provide for a unified system for all activities or separate communication facilities for particular activities.

Traffic loads and operating contingencies can usually be cared for with a greater margin of safety and with less expense where all telephone services and facilities are unified and operated as one system. In some companies, however, the needs of the operating department have appeared to be of sufficient importance and the traffic so heavy in volume that a communication network has been provided for the exclusive use of that department. In other territories where this has not seemed practicable, convenient switching arrangements have been installed which allow the operating department to seize quickly and hold any desired circuits for an indefinite period of time. These arrangements permit the exclusive use of circuits by the operating personnel whenever, in their opinion, such use is necessary and would seem to provide in effect the equivalent of an exclusive communication network.

In a number of instances communication studies have indicated that the best results could be secured through connection of the electric utility's privately owned communication plant to the facilities furnished by the telephone company. Mutually satisfactory interconnection arrangements have been agreed upon in such cases. Provision is made thereby for the complete interconnection of privately owned and leased communication facilities, thus enabling the electric utility to eliminate all useless duplication of circuits and equipment, obtain a broader use of privately owned plant, and retire investment in such plant in an orderly fashion.

In some companies the responsibility for adequate and satisfactory communication service throughout the entire system has been centralized in one department. This department gives specialized, continuous attention to the needs of the utility for communication. Where such a plan is in effect, it is important that the department be fully conversant with the general overall objectives of the company and have sufficient authority to initiate such action as may be necessary to provide communication service which will be of the greatest assistance in meeting those objectives.

Establishment of Basic Plan

The data obtained by analyzing the communication service required in connection with the various activities of an electric utility, as outlined in previous sections of this paper, provide the material for the preparation of a basic communication plan.

The establishment of such a plan requires the exercise of judgment to evaluate the relative importance of the various activities of the utility and the extent to which available communication services and facilities can be used practicably to expedite such activities. The value of the plan adopted will depend upon the quality of judgment exercised. It is extremely important, therefore, that the best available knowledge and experience in the power and communication field be enlisted in establishing a basic plan.

The establishment of a basic plan and its specific application will necessitate periodical reviews of existing communication arrangements to insure that the plan is being followed. Modification of the plan itself may also be necessary from time to time on account of new developments, changes in operating practises, etc.

Application of Basic Plan

The specific form and detailed application of a basic communication plan for an electric utility is dependent upon local conditions and is therefore outside the

scope of this paper. Information in regard to the specific communication facilities used by electric utilities in meeting their requirements has been given in a number of important papers recently presented before the Institute. The contents of these papers are briefly summarized in the attached bibliography. Also, further information regarding the communication arrangements in a number of interesting and important cases involving widely different operating requirements is given in the other papers of this symposium.

The growth of the electric utility industry is extremely rapid. The activities of the operating companies are continually broadening in scope as the areas served increase in size and the customers' demands become heavier and more exacting. It is clear that adequate, convenient communication service cannot be obtained by the electric utility economically without a plan based on a thorough knowledge of requirements and the availability of various communication service and facilities to meet those requirements. It is important that the activities of the electric utility be constantly studied from a communication standpoint to determine how communication can be used to greater advantage, and to anticipate requirements so that the communication service necessary will be available on the date required. Consideration along these lines will assist to a marked degree in effecting continual improvement in customer service.

Bibliography

Communication Systems for Electric Utilities

The Communication System of the Southern California Edison Company, Ltd., by Roy B. Ashbrook and Fred B. Doolittle, A. I. E. E. Journal, November 1930.

The Southern California Edison Company's communication system is described. The development and construction of the "serjdetour" arrester is discussed. The communication building, which is the center of the telephone system, is described together with the method of handling traffic through this point. A brief description of the equipment used for making initial tests on new lines and maintenance of existing facilities is given.

Experience with Carrier-Current Communication on a High-Tension Interconnected Transmission System, by Philip Sporn and Ray H. Wolford, A. I. E. E. Trans., Volume 49, p. 288.

A description of carrier-current communication over a 132-kv. network having an extent of 2,500 linear miles is given. The necessity for zoning and inter-zoning is brought out. Data are cited as to cost, maintenance, reliability, traffic, and safety. The fundamentals of power line carrier-current communication over a transmission network are summarized and the need for certain development work outlined.

The Communication System of the Conowingo Development, by W. B. Beals and E. B. Tuttle, A. I. E. E. Trans., Volume 47, p. 909.

The communication system installed to serve Conowingo, (Md.) generating plant and its associated transmission line is described. The important features to be considered in designing a telephone system for an electric utility are pointed out. The paper also discusses the communication facilities provided for patrolling purposes along the transmission line, the connection of the private branch exchange to the general telephone system and the special electrical protection necessary on the long lines leaving the power house

Carrier Current and Supervisory Control on Alabama Power Company's System, by W. I. Woodcock and E. W. Robinson, A. I. E. E. Trans., Volume 48, p. 214.

The experience of the Alabama Power Company with power line carrier-current telephony and supervisory control is outlined. The early development of these systems in the Alabama Power Company's territory is discussed as well as systems later installed. Extracts are given from the general specifications for the equipment used.

Power Line Carrier-Current Apparatus

Power Line Carrier Telephony, by L. F. Fuller and W. A. Tolson, A. I. E. E. Trans., Volume 48, p. 102.

This paper gives a brief outline of the electric utility's requirements for power line carrier-current communication, the problems involved in furnishing such communication and briefly describes the different types of equipment available.

Problems in Power Line Carrier Telephony and Recent Developments to Meet Them, by W. V. Wolfe and J. D. Sarros, A. I. E. E. Trans., Volume 48, p. 107.

The problem of applying high frequency communication to power systems is outlined in this paper. The effects of branch lines and switches in the power system network and sources of noise in the carrier-current channel from operation of the power circuits are discussed. Methods of obtaining simplex and duplex operation of carrier systems as well as carrier and side-band suppression are outlined briefly. A brief description is included of a single frequency, single side-band, carrier suppressed type of power line carrier-current communication system.

Transmission of High Frequency Currents for Communication Over Existing Power Networks, by C. A. Boddie and R. C. Curtis, A. I. E. E. Trans., Volume 48, p. 227.

The fact that carrier-current communication presents certain difficulties not encountered in other types of communication channels is brought out in this paper, which describes the use of tuned choke coils to isolate the power line carrier-current communication channels from the remainder of the power system. It is stated that this gives in effect a simple communication circuit between communicating points.

Telemetering and Supervisory Control

Telemetering, by C. H. Linder, C. E. Stewart, H. B. Rex and A. S. Fitzgerald, A. I. E. E. Trans., Volume 48, p. 766.

Operating requirements and channels available for the transmission of meter readings are discussed. The field of application for telemetering is surveyed and discussed in its relation to supervisory control. The installation of a specific type of telemetering is described.

Development of a Two-Wire Supervisory Control System with Remote Metering, by R. J. Wensley and W. M. Donovon, A. I. E. E. Trans., Volume 49, p. 1339.

The conditions leading up to the development of a two-wire supervisory control system are described. The paper enumerates the outstanding requirements of supervisory control systems in general and indicates the advantages of a two-wire system. The operation of the two-wire system is briefly described.

An Electron Tube Telemetering System, by A. S. Fitzgerald, A. I. E. E. Trans., Volume 49, p. 1321.

The paper describes a varying frequency telemetering system which employs an electron tube, beat frequency oscillator. A method of furnishing a number of telemetering indications over a single conducting channel by the use of automatic telephone type selectors at transmitting and receiving stations, is described.

A thyratron circuit is described, by means of which a number of varying frequency telemetering signals may be totalized.

Centralized Control of System Operation, by James T. Lawson, A. I. E. E. Trans., Volume 49, p. 1344.

Centralized control of system operation of the Public Service Electric & Gas Company of New Jersey is described. Detailed descriptions are included of the power and telephone apparatus installed and the methods used to provide indications of the operation of electrical apparatus. The results obtained by means of centralized control are described.

Discussion

Howard S. Phelps: With the present tendencies toward system expansion, either through ownership control or operating agreements between companies in adjacent areas, there are two items that should be given particular attention when making fundamental studies of communication services of electric

utilities which it seems to me the authors of this paper have not particularly stressed.

The first of these is the quality of voice transmission that will be provided for on that part of the system devoted to the use of the system load dispatchers. This is essential to insure as far as possible against misunderstanding of operating instructions. For this service the quality of transmission should undoubtedly be at least the equal of that for high-grade commercial telephone communication.

The second item is that factor in voice communication generally spoken of as "transmission margin" which means the amount by which a given volume of voice transmission, irrespective of quality, can be decreased to a point where transmission is just intelligible. Undoubtedly, in general, it would be good engineering to provide for considerable transmission margin in order to assure that satisfactory voice communication is available as the system may expand or in the event it is necessary to reroute communication circuits following storm or other trouble that may occur on the communication lines.

Any company that is seriously considering the installation of a new communication system, or revamping its present system, certainly should give detailed attention to the two factors in voice communication to which reference has just been made. An ill-advised attempt to save money through the installation of fair quality transmitting and receiving sets, or the installation of the minimum size of wire in the communication system, or both, might easily impose a very serious burden on the future communication services to be rendered.

International Standard of Electromotive Force

and Its Low-Temperature-Coefficient Form

BY MARION EPPLEY*

Member, A. I. E. E.

"THE electromagnetic unit e. m. f. exists between two points when one erg of work is done by one electromagnetic unit of current flowing for one second between the two points."[1]

"Unit current is the current which, flowing through a circle of one centimeter radius in air, exerts a force of 2 π dynes on a unit magnetic pole at the center of the circle."[2]

"The c. g. s. electrostatic unit difference of potential exists between two points when one erg of work is done in the movement of a c. g. s. electrostatic unit of charge from the one point to the other point."[3]

The above definitions are fundamental but it is not simple to give being to the physical conditions for which they call.

Even the international volt ("the electrical pressure which when steadily applied to a conductor the resistance of which is one international ohm will produce a current of one international ampere")[4] is defined actually in terms of the silver voltameter and the mercury ohm. This combination does not lend itself to routine measurements, even though the mercury ohm can have substituted for it a wire resistance-standard. Moreover, the voltameter measures the total quantity of electricity (the number of coulombs) that pass through it. This total, divided by the time of passage in seconds, gives the *average* current in amperes, so that only steady electromotive forces can be determined by the voltameter with the ohm, unless the accuracy of the results be vitiated by this integrating effect. Hence, to have determinations of electromotive force in the sense of the above definition give definite meaning with respect to voltage, it is essential to have an outside source of constant potential difference by means of which an unchanging drop in the voltameter circuit can be maintained. Up to the present time the most precise and the most convenient method of producing a steady potential difference with the required precision is by means of a galvanic cell of the proper characteristics. When an element with a constant electromotive force has been used once so to maintain the voltameter circuit with a steady drop across the standard ohm, equal to the e. m. f. of the cell, this last obviously has been calibrated in terms of the international volt, and its independent use as a standard immediately is possible.

Elements so used are called "standard cells." They must be constant in electromotive force, or at the worst their electromotive forces should change slowly and regularly with time. Their temperature-coefficient should be such that a variation of $\pm$ 0.01 deg. cent. in the neighborhood of 25 deg. cent. causes a change in e. m. f. of less than one microvolt, and they should exhibit no effects due to temperature-hysteresis greater than this for a like temperature change. They should be reproducible as to e. m. f., but this is a matter of convenience only, since calibration with the voltameter and the ohm could be practised upon every cell used as a primary standard.

The container of a standard cell should be of such a design that concentration effects at the electrode faces are eliminated. That is, a more concentrated, heavier solution must not be able to flow, under the action of gravity, to one electrode and not to the other. Also the path between the electrodes should be as long as is consistent with low internal resistance, since the two electrode systems, by the very nature of things, are not in equilibrium with each other. For this reason, the diffusion from one to the other of ions that interact is to be hindered as much as possible. These two conditions are fulfilled by the "H-tube" due to Rayleigh which is in general use. Other forms have been proposed, notably a pair of concentric cylinders,[5] the inner cylinder, and the annular space between the inner cylinder and the outer one holding the two electrodes. This receptacle offers certain difficulties in glass-blowing, and its main advantage, uniformity of temperature of the two electrodes, can be secured more readily by other means.

From the standpoint of handling there are two types of cell, the portable and the non-portable. Portability is secured by some form of septum inserted in each leg of the cell vessel to hold the solid constituents in place. This can be a perforated cork disk covered with silk or linen, a porous stopper of porcelain wrapped with some sort of fiber, or any similar mechanical device. Non-portable cells have no septa, and must be held in a vertical position to prevent mixing of their contents.

Laboratory standards certified to 0.01 per cent should be portable; primary standards, reproducible to five to ten microvolts, should be non-portable, since septa introduce another factor in the system of primary standards. This is not to be desired. (The specific values above apply to the cell in general use today.)

From the physical-chemical standpoint (which involves behavior) there are also two types, the saturated or normal cell, and the unsaturated cell. The saturated or normal cell has an excess of a salt of the metal of each

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1. For references see end of paper.

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electrode. In practise, the two salts have the same acid radicle. The solid of the salt of the less electropositive metal is in the vicinity of its electrode only. It should not be too soluble, or diffusion to the opposite pole will be too rapid. The solid of the salt of the more electropositive metal can be in the vicinity of both electrodes. The unsaturated cell has no solid phase of the salt of the more electropositive metal, its electrolyte being saturated with this at some temperature well below the lowest probable room temperature. Owing to the excess of the salts of both metals in the normal cell, the electrolyte becomes saturated with these, when maintained for a proper length of time at any temperature within the useful range of the cell, to the concentration corresponding to the particular temperature. Since the e. m. f. is a function of these concentrations, saturated cells are reproducible in so far as their e. m. f. depends upon concentration. Because of the difficulty of bringing once and for all a solution to a single predetermined degree of saturation, unsaturated cells are not reproducible, strictly speaking.

Unsaturated cells have a lower temperature-coefficient than saturated cells. Therefore, laboratory standards should be of the unsaturated type, exposed as they are to changes or room temperature. Primary standards, in which reproducibility is so convenient as to be an essential, should be saturated cells. They must be maintained at constant temperature in a thermostat. Laboratory or secondary standards are calibrated in terms of these primary standards.

A number of systems has been proposed for the purpose under discussion, and some of these made use of in default of something better, but all except one are of historic interest only. This one, the combination upon which are based the primary and the secondary standards of the present period is the following: mercury—mercurous sulfate—aqueous solution of cadmium sulfate—cadmium amalgam.

It was proposed by Dr. Edward Weston, and its unsaturated type patented by him in 1893. To Dr. Weston belongs great credit for the long step forward made by his introduction of a standard of electromotive force with a low-temperature-coefficient (negligible for all practical purposes) and in addition free from the mechanical instabilities of the Clark and Clark-Carhart cells which were its predecessors.[6]

In his honor, the two types of this cell are known as the normal Weston cell or the saturated Weston cell, and as the unsaturated Weston cell or simply the Weston cell. The word cadmium is quite as often used as the word Weston and instead of it, cadmium cell and Weston cell being synonymous as applied to the composition of the *system*, but Weston cell requiring no further qualification to denote it refers to an *unsaturated* element.

Cadmium sulfate ($CdSO_4 . 8/3H_2O$) is a well defined salt. It can be secured readily as clear crystals by spontaneous crystallization. Hulett[7] states that it does not seem to be isomorphous with any known salt. However, an unfinished investigation in this laboratory, has shown even clear, once re-crystallized crystals of high-grade commercial salt, after twelve washings, to have contained appreciable quantities of manganese. Also, crystals can be prepared which contain a considerable proportion of mercuric mercury. Iron too appears to be present in clear crystals in a closely-bound state at times. The mercuric mercury is no doubt present in such form as to produce a mixed-crystal rather than a double salt, though the final proof of this is still to be made. The same is no doubt true of manganese and iron. The author has found no mention in the literature of manganese as an impurity in cadmium sulfate other than a statement made by him in 1926.[8] The impurities usually listed[9] as probable are zinc, lead, ferrous, and ferric iron and occasionally nickel. A rather wide review of the literature plus chemical checks in this laboratory make it appear reasonably certain that too much faith has been placed by investigators in the effectiveness of the simple methods used to purify the cadmium sulfate used by them in standard cells, and that manganese, which the author has found in a wide range of cadmium sulfates tested, may have been present in most cases. Also iron.

Qualitative experiments made by us as far back as 1924[10] showed manganese to cause higher initial electromotive forces when present as an impurity in cells than were found when its concentration was reduced to very low limits. These higher electromotive forces did not remain constant.

Through the courtesy of the Bureau of Standards a sample of the cadmium sulfate prepared by Wolff and Waters[11] in 1906 has been received recently for investigation. This salt, with which, it is understood, the fundamental standards of the United States were set up, gives a negative test for manganese. The excellence of these standards has been proven by time.

While the case against manganese by no means has been proved, it would be safest, until it has been found to have no injurious effects, to specify its elimination in primary standards, and its reduction to low limits in other cells.

A low manganese content is secured with no great difficulty. Its elimination requires no small degree of care. A method, soon to be described, has been developed by the author, but it is so far from convenient for the preparation of large quantities of the salt, that a simpler one is being sought.

The preparation of mercury that gives the desired results presents no difficulty. It is washed in a fine spray in dilute nitric acid,[12] and is distilled in a partial vacuum. The tendency to "bump" is prevented by passing a slow current of air into the mercury through a capillary. This also serves to oxidize the base metals. The noble metals remain behind in the residue.[13]

Cadmium can be purchased in the metallic form in a satisfactory state of purity. However, only one such

source seems to be available for a routine supply. Cadmium sulfate can be used as a source of cadmium as will be mentioned in the next paragraph.

As has already been indicated, leaving out of consideration the solid cadmium sulfate, one pole of the Weston system consists of mercury, covered with mercurous sulfate, and the other of cadmium amalgam. The cadmium amalgam to give a smooth temperature-electromotive force curve must consist of two phases, a solid phase which is a solid solution of cadmium in mercury and a liquid phase which is a saturated solution of the solid in mercury.[14] Within the temperatures between 15 deg. cent. and 40 deg. cent. amalgams of about 8 to 13 per cent[15] cadmium are satisfactory. There seems to be cause of irregularity at 15 deg. cent. (about) which may interfere with accurate measurements below this point. The amalgam can be made by dissolving the correct weight of cadmium in mercury, or by electrolyzing the cadmium from an appropriate weight of cadmium sulfate into mercury.

Mercurous sulfate plays an important role in the Weston cell, so important in fact that it is customary to charge all irregularities in cells to it. The salt is sensitive to many conditions, one of which, at least, makes it useful as a "depolarizer." This is its ready reducibility, which prevents cadmium ion from entering the mercury electrode. It is also sensitive to light, and should be protected from this, since it causes a decomposition which is injurious to the constancy of cells. It also hydrolyses[16] in neutral or very slightly acid cadmium sulfate solution, hence the electrolyte should be somewhat acid. The hydrolysed salt is a very pale green,[17] not yellow as sometimes stated in the older literature. The salt, when exposed to oxygen can form a yellow basic mercuric salt of low solubility.[18] This basic mercuric sulfate seems to be reduced to the normal mercurous salt in the presence of sulfuric acid in cadmium sulfate solution, and mercury.[19] This basic mercuric salt forms only as a coating[20] on the grains of the normal mercurous sulfate and protects them against further oxidation, it would appear, and in a slightly acid solution of cadmium sulfate is dissolved and reduced by the mercury of the cell. The solubility in acid solutions of cadmium sulfate has been determined to be considerable by experiments completed last year and as yet unpublished.

The mercury limb of a cell has been rotated by Hulett,[21] and the e. m. f. determined against an amalgam electrode which could be connected with it when desired. Upon rotation the e. m. f. rose to about 1.02 volt, but fell upon standing still. The author has repeated Hulett's experiments, in a slightly different form, with practically complete agreement with his results. Wolff and Waters[22] rotated the cathode of a cadmium cell, but found no variation of e. m. f. greater than 0.01 per cent. It has not been possible to reconcile the results of these investigators as yet, but one thing may be said with confidence, namely that at the mercury face the system is in equilibrium. It is at this face that the potential difference due to the mercury electrode has its being.

Mercurous sulfate can be made by a number of methods,[23] but that prepared by electrolysis is undoubtedly the best. The gray electrolytic salt, deriving its color from small globules of mercury gives better results than a white salt.

Since current should not be drawn from a cell, cells should be used only in a "null-point" or compensation method. Some form of potentiometer provides this. It is usual to compare the entire electromotive force of the cell used as standard against the entire e. m. f. of the cell the electromotive force of which is to be determined. In this method the drop in the potentiometer acts as a transfer standard. It is essential that this drop be kept constant, so that it is well to lag the storage cell with heat-insulating material to prevent changes in room temperature from causing fluctuations in the measuring current, or drop. Accumulators are best kept on a stand, above the floor, to avoid drafts. With this precaution, and that of shifting from standard to unknown several times with readjustment of the measuring current if necessary and the attainment of a new balance with the unknown, results can be obtained with a potentiometer of proper design that will be reproducible from day to day within six microvolts. This method has the convenience of being direct reading.

But the above method has the disadvantage, perhaps more theoretical than real, of not comparing the standard and the unknown at the same instant. For measurements beyond criticism, therefore, it is best to oppose the unknown and the standard, reading the residual e. m. f. on a potentiometer or by some other suitable means. This reduces the effect of fluctuations in the measuring current to a minimum and makes use of only one or two dials of the potentiometer with reduction in the number of corrections to be applied. It is not direct reading, however, and the sign of the residual must be watched carefully.

It is necessary of course that the measuring current be allowed to flow for a long enough time before measurements are made for it to have come to a steady state, whatever method be used. Careful attention should also be given to shielding the installation against stray leakage currents.[24]

In any system, provision should be made to eliminate the effect of thermoelectric forces. A very excellent method has been proposed by the Bureau of Standards. It is an adaptation, for standard cell comparisons, of the assemblage of Lindeck and Rothe,[25] with new and useful features.

It is made clear by the diagram herewith. (Fig. 1.) This assemblage consists of a standard resistance of appropriate value, R_s. This is in series on the one hand with a reversing switch, an ammeter A, a battery and a rheostat, indicated as circuit I. In an opposed branch

circuit there are a standard cell S, the unknown electromotive force X differing but slightly from that of the standard cell, and a galvanometer G. Means are provided for disconnecting the standard cell and the unknown and replacing them with a neutral resistance R_c about equal to their combined resistances. There is also a resistance, R_t, in this circuit. This entire branch is marked circuit II.

Across the resistance R_t there are connected through a pole-changing switch an ammeter (for convenience), a battery, and a rheostat. This is marked circuit III.

The purpose of this assemblage is to permit the difference of electromotive force which is being compensated to be thrown out of the circuit at the same time that the current in the side branch I is interrupted. In order to keep the galvanometer damping the same, the resistance R_c is substituted for that of the standard and the unknown. Then by means of the branch circuit III the parasitic thermal electromotive forces that may exist in circuit II are compensated by the battery and rheostat in side circuit III. Then the resistance R_c is thrown out, and the standard cell and the unknown electromotive force "bucking" it, opposed to the side circuit I, are thrown into action and a balance obtained. In this way it is possible to get a difference reading between S and X with parasitic electromotive forces eliminated.

The diagram with values inserted as shown in

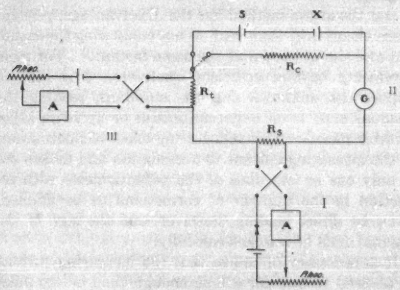

FIG. 1—OPPOSITION METHOD

As suggested by Bureau of Standards

(Fig. 2) is the form in which the principle is used in this laboratory. It gives the desired flexibility. No explanation of the details is necessary.

The London International Electrical Congress[26] in 1908 adopted specifications for the mercury ohm; the silver voltameter, and the Weston normal cell, fixing a provisional value of 1.0184 international volts at 20 deg. cent. for this latter.

The value of 1.0184 was chosen because it was the mean of the figure 1.01843 obtained at the National Physical Laboratory in 1906-7, and the figure 1.01834 obtained at the Reichsanstalt in 1907-8. Both of these values were based on an electrochemical equivalent of silver equaling 1.11800 mg. per coulomb.

The specifications for the mercury ohm and the Weston cell were fairly complete, but those for the silver voltameter were inadequate, hence the above value for the cell was adopted as provisional only, until more precise values should have been determined by further experiments.[27]

For the encouragement of such experiments, and to promote the cooperation of the different countries throughout the world in maintaining a single uniform system of electrical units, the London Conference es-

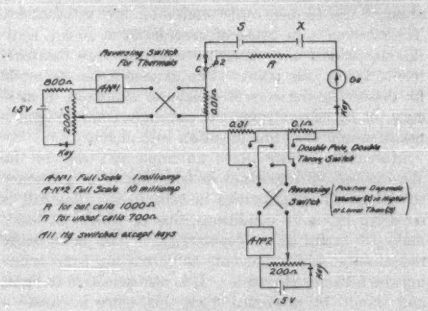

FIG. 2—OPPOSITION METHOD

Modified form, The Eppley Laboratory

tablished an International Committee on Electrical Units and Standards.

On March 1, 1909 the acknowledged figures for the value of the normal Weston cell at 20 deg. cent., obtained by means of the silver voltameter, using 1.11800 mg. per coulombs as the electrochemical equivalent of silver were as follows:

At Nat. Phys. Lab., using filter paper.....1.01843 volts at 20 deg. cent.
At Phys.-Tech. Reichs., using silk and porous cups.....1.01834 volts at 20 deg. cent.
At Bur. of Stands., using porous cups.....1.01837 volts at 20 deg. cent.
At Bur. of Stands., using filter paper.....1.01863 volts at 20 deg. cent.
At Lab. Cen. d'Eleo., using filter paper.....1.01879 volts at 20 deg. cent.

In view of the wide spread in these values, delegates from the several national standardizing laboratories met in Washington on April 4, 1910. This body was known as the International Technical Committee, and was brought into being through the initiative of Dr. S. W. Stratton, Director of the Bureau of Standards. One-quarter of the traveling expenses of the delegates was defrayed by the American Institute of Electrical Engineers.

The delegates[28] made determinations with the silver voltameter, and set up and compared normal Weston cells; forty-eight cells were made by them in Washington, using four different preparations of mercurous sulfate. In addition 15 cells were brought over and submitted from the PTR, 34 from the NPL, 15 from the LCE and 7 cells were chosen as representative of the standards of the Bureau of Standards in addition to forty new cells submitted by it.

The agreement of the cells submitted is given in the next table:

Cells		Mean of group minus general mean; difference in 10^{-6}	Average deviation of single cells from mean of group
No.	Laboratory		
15	PTR	0	±10
34	NPL	−2	± 9
15	LCE	+9	±25
40	NBS	−9	± 6

The group means for the new cells set up in Washington by the delegates [Twelve cells each. Four groups of three cells, the groups being made with one of the preparations of mercurous sulfate made by each delegate according to the method of his own national laboratory] are listed in the following table:

	Set up by				Mean omitting LCE
Hg_2SO_4	NBS	LCE	NPL	PTR	
NBS	−17	+147	−29	−34	−27
LCE	+19	+62	+18	+15	+17
NPL	−35	+142	−12	−17	−21
PTR	+11	+121	+3	+19	+11
Means	−6	+118	−5	−4	−5

The LCE cells were set up under conditions, due to interruptions that made their proper agreement a practical impossibility.[29]

As a result of the work done by it in Washington The Technical Committee decided "to recommend to the International Committee for Electrical Units and Standards the following value for the e. m. f. of the Weston normal cell:

$E = 1.0183$ international volts at 20 deg. cent."[30]

Upon submission of a proposal to do so and acceptance by it of the proposal, the International Committee requested the governments involved to adopt officially the new value for the Weston normal cell on January 1, 1911.

In the case of the Bureau of Standards no legislation was necessary to make the new value effective, so the Bureau adopted it in advance of legislative action, on the first of January 1911, carrying it out one more decimal place to 1.01830.

It also accepted the temperature coefficient formula of the Weston normal cell of the London Conference, namely,

$$E_t = E_{20} - 0.0000406\ (t - 20) - 0.00000095\ (t - 20)^2 + 0.00000001\ (t - 20)^3$$

The values for the normal Weston cell in use previous to 1911 are of interest. They are

In the United States	1.0189	international volts at 25 deg.
equivalent to	1.019126	" " " 20 "
In Germany	1.0186	" " " 20 "
In Great Britain	1.0184	" " " 20 "

(Great Britain, until January 1, 1909, when the value 1.0184 adopted provisionally by the London Conference was adopted provisionally by the NPL, used a value slightly larger than that of the United States.)[31]

It should be noted that the electromotive force of a normal cell depends upon temperature not only as concerns the system's correspondence to the equilibrium conditions at a temperature, but also to the actual value of the particular temperature on a definite scale. The first condition is a function of the closeness with which the temperature is controlled; the second is met by determining the temperature and its fluctuations with the platinum resistance thermometer by which the International Temperature Scale is defined.[32] A tenth of a degree error in temperature causes about 5 microvolts error in e. m. f. at 25 deg. The error caused by fluctuations through a temperature range of one-tenth degree may cause an error twice as great as this in some cells, due to hysteresis.

The cause of hysteresis is not thoroughly understood. It has been explained as due to a shift in the mercurous-mercuric ion equilibrium with temperature,[33] and also to supersaturation of the viscous cadmium sulfate solution with mercurous sulfate.[34] It is less when the temperature is raised than when the temperature is lowered. Acid in the electrolyte reduces hysteresis to a low value. So does finely divided mercury in the mercurous sulfate. It probably cannot be explained by a single, simple cause. For some reason it is greater in portable cells than in non-portable, and greatest in neutral, normal, portable cells with their solid cadmium sulfate.

At the Bureau of Standards normal cells are maintained to within 0.01 deg. cent. or better in electrically-heated oil baths. These baths are not lagged with heat-insulating material, as it is not wished to have the bath temperature rise over night sufficiently to "cook" the cells in case of mal-functioning of the regulating device. A cooling system for the room as a whole is provided for use in hot weather. This is a much better plan than that of endeavoring to regulate the baths by a constant abstraction of heat from them by a cooling coil in the oil when room temperature is higher than the bath temperature. Temperatures are determined by means of platinum resistance thermometers and the Mueller temperature bridge. (The main coils of the bridge are at constant temperature in one of the thermostats, and only the very small resistance made

up of low-resistance coils, necessary to balance the slight changes of thermometer resistance, are at room temperature.)

Comparisons of electromotive force are made by the difference method with the old type of Feussner potentiometer constructed by Otto Wolff. This instrument has been calibrated with great care. It is understood that an arrangement to compensate for parasitic thermals has been installed or is about to be installed for use with the Wolff potentiometer though no thermals are evident. All wiring is installed with especial regard to the elimination of thermals, and "shielding" is carefully attended to.

In this laboratory, the installation is essentially the same as that at the Bureau except that the baths, two in number are heavily lagged, and no part of the metal of the bath in contact with the heated oil projects into the room, except the stirrer shaft. One bath is nearly twice the size of those in the Bureau and holds one hundred cells. A special type of potentiometer reading to ten microvolts, and an improved Feussner potentiometer, both direct reading, are used for comparing the entire e. m. f. of the unknown with that of the standard. Accumulators furnishing the measuring current are lagged against temperature changes and are off the floor. Difference measurements can be made by the assemblage described above and by a White potentiometer reading to microvolts. Temperatures are measured by the platinum resistance thermometer and a modified type of Mueller bridge, oil immersed with temperature control. No cooling system for the room has been installed as yet, as the room is not expected to heat up in summer, due to its position and on the basis of experience with smaller baths maintained in it in the past. The present baths have not been in operation throughout a summer. Regulation of temperature is within 0.01 deg. cent.

There can be no greater authority upon the maintenance of the volt than G. W. Vinal, physicist, charged with this important duty at the Bureau of Standards. There is spread below verbatim an extract from an article by him upon this subject.[35]

Maintenance of the Standard

Between 1910 and 1912 our basis of reference was the group of 36 cells, left at the Bureau after the departure of the delegates. It became evident, however, that many of our cells, which had shown consistent values on the old basis of reference, were gradually increasing in value on this new basis as represented by the 36 cells which the technical committee left at the Bureau. All the evidence pointed to the fact that the mean of these 36 cells was decreasing, and it was then decided that a group of 19 selected cells, which had shown a satisfactory degree of constancy before 1910, and which had shown nearly equal increases in value in terms of the new basis of reference, had actually remained constant. These cells were then selected as a new reference group.

Some substitutions have been made in this group between 1912 and 1919, but no change has been made since 1919. The primary reference group (Group I) at the present time consists of 20 cells, 15 of which were made 22 years ago by F. A. Wolff and C. E. Waters. These cells have shown a remarkable degree of constancy, and no correction for any yearly decrease in voltage is made. Table I shows values of the reference cell, which is one cell of the group in daily use during the last four years. The maximum variation is only a few microvolts.

To safeguard the unit and at the same time provide further information as to the constancy of these cells, in 1925 two secondary reference groups of 10 cells each (Groups II and III) were selected from the Bureau's stock of seasoned cells. One of these groups is kept in the safe in the south building of the Bureau with the national prototype kilogram and meter, and the other secondary group is kept in the safe in the electrical building. The secondary groups are compared with the primary group about once in six months. The values are given in Table II. The maximum variation in each group is four and six microvolts respectively.

TABLE I—VALUE OF THE REFERENCE CELL IN TERMS OF THE PRIMARY REFERENCE GROUP

Date	Difference in microvolts from mean of group
Oct. 3, 1924	−6.9
July 15, 1925	−6.9
Nov. 17, 1925	−7.6
May 1, 1926	−7.8
Nov. 4, 1926	−6.5
Apr. 21, 1927	−8.1
Nov. 5, 1927	−5.8
June 5, 1928	−5.7

TABLE II—SUPPLEMENTARY REFERENCE GROUPS IN TERMS OF PRIMARY REFERENCE GROUPS

Date	Difference in microvolts from mean of Group I	
	Group II	Group III
Jan. 8, 1925	−1.9	−16.3
Feb. 16, 1925	−3.1	−17.3
May 26, 1925		−15.9
Nov. 18, 1925	−5.4	−20.8*
May 17, 1926	−1.7	
June 1, 1926		−22.9
Dec. 18, 1926		−24.0
Jan. 11, 1927	−3.0	
July 22, 1927	†	−26.8
April 17, 1928	−2.8	−25.9

*Three cells were changed in Group III prior to this measurement.
†One cell in Group II was found to be broken and it was replaced at this time.

International Comparisons

At the British National Laboratory the standard for voltage is maintained in quite a different way. Groups of new cells are set up annually, and the basis of reference is the average of the cells for the preceding three years.

Recent comparisons of the Bureau's basis of reference with that of the British Laboratory have indicated that the difference between our value for the volt and theirs is about one part in 100,000. Confirmation of this has been received through a report from the Central Chamber of Weights and Measures at Leningrad, indicating a difference between the above standards of seven millionths of a volt.

The value for the volt which was presumably the same in the various countries on January 1, 1911, has not remained the same in all countries, however, as shown by Fig. 3, which is based partly upon a report of the Chamber of Weights and Measures at Leningrad, and partly upon the exchange of cells between the Bureau of Standards and National Laboratories of other countries. The fact that the value for the volt as defined by the Weston cell in several countries differs so widely cannot be interpreted, however, as meaning that the cell is an unreliable

standard, since these values involve not only errors in the cell, but the errors in the standards of resistance and the measurement of current. This lack of uniformity is not surprising in view of the fact that no adequate international specifications for either the standard cell or the voltameter have been adopted.

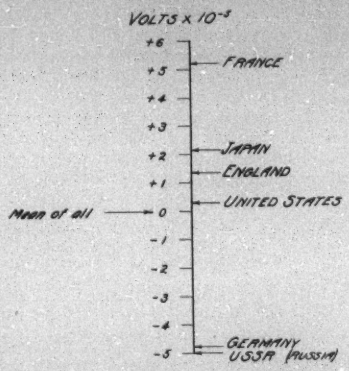

FIG. 3—RELATIVE VALUE OF THE VOLT AT THE PRESENT TIME IN SIX COUNTRIES

Vinal, *Trans. Am. Electrochemical Society*, 54, 251 (1928) Fig. 1

Five determinations with the Smith form of voltameter reported in the above article, gave a mean for the value of the normal Weston cell of 1.018307. Six determinations with the Richards or porous cup form of voltameter gave a mean of 1.018218. Even this last value, which is lower than any obtained during the investigation of 1908, and therefore probably put in error by some interfering factor, is low by 0.01 per cent only.[36]

The cells just discussed are the normal higher temperature-coefficient type. The unsaturated type or Weston cell now becomes of interest. As said before this is not strictly reproducible, but its voltage can be made to fall within narrow limits, and the cell can be standardized by comparison with a normal cell since its e. m. f. is to all intents constant.[37]

Vosburgh and Eppley found in a series of forty-five Weston cells that the electromotive force increased in an approximately linear manner as the cadmium sulfate concentration decreased. The e. m. f. ran from 1.018270 volts for 43.22 per cent $CdSO_4$ to 1.02095 volts for 41.84 per cent $CdSO_4$.

The electromotive forces of these cells were linear functions of the temperature between 15 deg. and 35 deg. cent., but were not linear functions between 35 deg. and 45 deg. cent.

The temperature coefficient between 15 deg. cent. and 35 deg. cent. was a function of the electromotive force. It decreased from 0.0000028 volts per degree for cells with an electromotive force of 1.01827 volts to − 0.000013 volts per degree for cells with an electromotive force of 1.0210 volts, being zero for a cell with an electromotive force of 1.01873 volts.

These cells contained approximately 0.0115 moles of sulfuric acid per liter. The cell with zero temperature coefficient would be saturated at 4 deg. cent., if the effect of sulfuric acid on the solubility of cadmium sulfate is the same at lower temperatures as between 15 and 20 deg.

Three groups of portable cells, each group having a different form of construction, have just been investigated for temperature coefficient and hysteresis in this laboratory. They were maintained until the cells were constant to within 10 microvolts (about), except at 45 deg. and 50 deg., at each of the temperatures shown below, and their e. m. f. determined each day. The mean of the readings on the last two days was taken as the e. m. f. at equilibrium, but any reading for a particular temperature could have been taken as such without error greater than 0.01 per cent. Temperature control was within ± 0.02 deg. cent.

The temperatures and the temperature coefficients for the corresponding intervals, together with the acidities of the electrolytes of the cells with respect to sulfuric acid are given in the table herewith.

Temperature interval	Temperature coefficient microvolts per 1 deg. cent.		
	1st group def. 0.04 N	2nd group pre. 0.04 N	3rd group pyr. 0.03 N
15°.04–20°.00	−3.70	−2.86	−4.59
20°.00–25°.08	−2.80	−2.85	−3.09
25°.08–30°.04	−4.03	−2.72	−2.72
30°.04–35°.02	−5.64	−4.67	−7.38
35°.02–40°.11	−1.37	− .74	−1.37
40°.11–45°.38	−1.78	+9.27	−1.53
45°.38–50°.04	+1.86	−8.45	+3.54

After measurements were completed on this rising temperature cycle, a cycle was run through with the same temperatures in the reverse order. This cycle is not yet completed for 20 deg. and 15 deg. However, as far as it has been carried, the results are in agreement with the first cycle to within 0.008 per cent in the worst case. Equilibrium was slower, as has been so often noted, on the downward cycle.

The three groups after having been at equilibrium at 30 deg. on the downward cycle were suddenly cooled to 25 deg. They were measured at this new temperature seventy minutes on the average after having been cooled to it. Cooling from 30 to 25 deg. required about 20 minutes. The average of the e. m. f. of two of the groups was within 0.02 per cent of the average of their values at 25 deg. taken before the test began. The average of the other group was within 0.027 per cent of its corresponding value.

Unsaturated cells show a downward drift with time.

In this connection the Bureau[38] of Standards has the following to say of the Weston cells in its laboratories in 1910. "The electromotive force of these cells generally decreases slightly with age. To illustrate, the Bureau of Standards has 15 of these cells in use in its laboratories which have on the average decreased only one ten-thousandth of a volt in the last four years, the change in the various cells being from nothing to three ten-thousandths of a volt." That is, the average in per cent was very nearly 0.01 per cent for four years or 0.003 per cent per annum, and the maximum 0.008 per cent per annum, very closely. These values are in close agreement with the values for this decrease found in this laboratory, 0.003 per cent to 0.004 per cent per annum.[39] This slow decrease can be well explained by the diffusion of mercurous ion to the amalgam face and its discharge there with cadmium going into solution and increasing the cadmium concentration of the electrolyte. The drop due to this cause should cease when the e. m. f. of a saturated cell is reached. Another more rapid drop is often observed in unreliable cells, and is beyond doubt

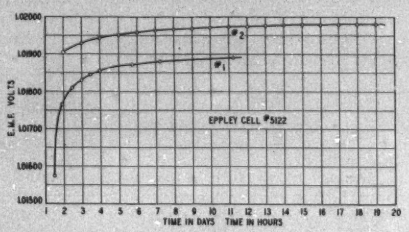

FIG. 4—CURVES SHOWING TIME REQUIRED AND VALUES DURING THIS TIME FOR AN UNSATURATED CADMIUM CELL TO RECUPERATE AFTER IT WAS SHORT-CIRCUITED FOR 30 MIN.

Curve No. 1 voltage vs hr. 1st day
Curve No. 2 voltage vs days up to 19 days
Total time to recuperate 36 days
Value before short circuit 1.019875 volts
Value after recuperation 1.019800 volts

Difference 0.000075 volts
Aug. 19, 1920

caused by hydrolysis. This does not stop at the value of the saturated cell. Such cells in practically every case—certainly in every case observed—have shown large hysteresis effects. They should be discarded. Hence, for an accuracy of 0.01 per cent unsaturated cells should be recertified every two years, and in addition, when in use they should be maintained within 5 deg. cent. of the temperature for which the certified value is given.

A cell short-circuited for one-half hour took eleven hours to recover to within 0.07 per cent of its former value, and 36 days to come back to within 0.007 per cent of its previous value. (Fig. 4.)

Three cells were short-circuited for ten minutes. At the end of seven hours they had recovered to within 0.02 per cent of their original values. A short-circuited cell should be regarded with suspicion, however, as the original equilibrium has been upset permanently because of the addition of cadmium to the electrolyte, if for no other cause.

In another experiment, three unsaturated cells (Weston type) with an internal diameter of 25 mm. were each connected across a resistance of such value as to cause 5 microamperes, 50 microamperes and 100 microamperes to flow in each cell respectively. The internal resistance of these cells was approximately 80 ohms. The initial e. m. f., the closed circuit e. m. f. after 6 minutes of current flow and the open circuit e. m. f. immediately after opening the circuit are given below.

	Microamperes		
	5	50	100
E. m. f. before closing circuit	1.01890	1.01886	1.01884
E. m. f. circuit closed 6 min	1.01851	1.01481	1.01073
E. m. f. immediately on opening	1.01890	1.01882	1.01880

The same thing was done with three cells of 13.5 mm. internal diameter. The internal resistance of these cells was approximately 350 ohms. The results were as follows:

	Microamperes		
	5	50	100
E. m. f. before closing circuit	1.01890	1.01882	1.01884
E. m. f. circuit closed 6 min	1.01698	1.00386	0.98994
E. m. f. immediately on opening	1.01883	1.01877	1.01874

The above indicates that not more than 0.7 microamperes per square millimeter of cross-section (in these cells, electrode area) can be taken for 6 minutes from a cell if a precision of 0.01 per cent is to be obtained.

It often has been stated that "not more than 0.0001 ampere should be taken from a cell." The results of the above experiment emphasize that as a matter of safety in measuring against a standard cell the closing key should be only just tapped and lengthy closing of the potentiometer circuit avoided.

Cells should not be exposed to the radiation from an electric light bulb. A 25-watt bulb 15 inches away and to one side of unmounted cells caused an increase in e. m. f. when the mercurous sulfate (positive) limb was exposed and the amalgam limb covered. The opposite arrangement caused a lowering of the e. m. f. Such exposure for five minutes brought about differences from the equilibrium value of the e. m. f. of from 100 to 200 microvolts (0.01 to 0.02 per cent). Even mounted cells have been known to show divergences of

these magnitudes, or greater, when unequally exposed to the heating effects of lamps or to the disturbing effects of draughts of hot or cold air. The same can be said of heating by the hand in carrying from set-up to set-up.

Ten cells were packed in "dry ice." Two were proved to have been frozen solid. All recovered to within 0.1 per cent of their former values. This test is not conclusive but indicates that for precision work cells should not be frozen. This involves no hardship since the cryohydric point of cadmium sulfate is about −16 deg. cent.

It is surprising how many attempts are made to determine the e. m. f. of a standard cell by connecting it directly to a voltmeter. This method not only gives very poor accuracy, for reasons it is not necessary to go into, but injures the cell, for reasons of this same class.

From the foregoing discussion it would seem logical to conclude that the system of the cadmium standard cell is a sensitive one. This is no doubt true. It might also be concluded that the system cannot be set up so as to be stable. This is very far from true. Over fifteen years work upon the cadmium cell have given the writer ever-increasing confidence in its reliability. It is by no means foolproof, but then very few precision devices are foolproof. Ceaseless care must be given to its manufacture, and a thorough knowledge gained of its chemistry to insure the maximum of constancy and agreement. But there are few electrical measurements —if any, indeed—where so high a precision can be obtained with so little trouble as in comparisons of electromotive force based directly upon the standard cell.

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Discussion

For discussion of this paper see page 1324.

Design of Potentiometers

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Member, A. I. E. E.

Synopsis.—This paper discusses potentiometer design under the following headings:

I. Early history. II. Factors affecting voltage range. III. Factors affecting potentiometer resistance. IV. Low-voltage potentiometers. V. Analysis of potentiometer errors. VI. Self-checking features of potentiometers. VII. Galvanometer considerations. VIII. Comments on volt-boxes. IX. Deflection potentiometers. X. The potentiometer in industry.

No attempt has been made to give the complete history of potentiometer development, as the paper is not intended to be primarily historical.

Neither has any attempt been made to include all of the circuits which have been used in high-resistance potentiometers and in the earlier forms of low-resistance potentiometers, as this would require too much time and space. Material of this kind has been limited to the minimum amount necessary to establish the proper background for the discussion of the subject.

* * * * *

Introduction

THE paper is limited to the discussion of d-c., laboratory type potentiometers. It would be difficult to include a discussion of a-c. potentiometers without taking up numerous specific applications, because the a-c. potentiometer is to a large extent a special purpose instrument. The principal difference between the laboratory type of d-c. potentiometers included in this discussion and the industrial plant types of potentiometer is in the modifications necessary for the adaptation to particular applications. A general discussion of specific applications of the potentiometer in industry is beyond the scope of this paper and those interested in that subject are referred to a recent paper[1] in the *Journal of the Franklin Institute.*

I. Early History

A significant point in the early history of the potentiometer is that the circuit was developed for comparing the electromotive forces produced in primary cells; and then as now it was desired to measure the true e. m. f. of each cell, that is, the open circuit voltage. To accomplish this purpose, Johann Christian Poggendorf, a German philosopher, about the middle of the nineteenth century devised a "compensation" method which is shown in Fig. 1 and in which we have the relation

$$e = E\frac{r}{R+r}$$

where e is the open circuit voltage of cell X and E is the closed circuit voltage of cell S. Poggendorf's circuit was modified by the Frenchman DuBois Reymond[2] and further modified by Latimer Clark[3] in England about 1874 to give the general form of circuit now used in many potentiometers. (Fig. 2.) Here

$$e_2 = e_1\frac{r_2}{r_1}$$

where e_1 and e_2 are the open-circuit voltages of S and X, respectively, and r_1 and r_2 are the values of r to produce a balance with e_1 and e_2, respectively.

II. Factors Affecting Voltage Range

In all of the early work, the open-circuit voltage of a primary cell was involved. Today, in all precise potentiometers, the open-circuit voltage of a primary cell is used for standardizing the potentiometer. This is the primary reason why most of the precise potentiometers have a range from zero to about 1.5 volts. There are, however, important exceptions to this.

In general, if the e. m. f. of the cell used for standardizing the potentiometer is approximately the same as the e. m. f. or potential difference to be measured, the measurement approaches a purely substitution method and the errors in the potentiometer itself become negligible. For instance, if in some research problem, requiring the utmost accuracy of measurement, it were desired to measure the relation between current and some other condition; then if the resistance of the current shunt were chosen to give, for the desired current, a potential drop of about 1.018 volt (the approximate value of a Weston standard cell), any error in the potentiometer itself could cause no measurable error in the determination of the current value.

Some potentiometers are designed so that ten times the normal current may be used, in which case the standardizing of the potentiometer current may be done with a series of ten standard cells. In this way, a range up to 10 or 15 volts may be provided without placing any greater dependence on the accuracy of the potentiometer resistances than is done in the potentiometers having a range up to about 1.5 volt. If such a potentiometer has also a low range, having one-tenth the value of the high range, then using ten times the normal current and only one standard cell, the current may be standardized on the low range, which thus becomes the 1.5-volt range and the nominal 1.5-volt range becomes a 15-volt range. However, if the standardizing is done in this way, the accuracy of the final measurement involves the accuracy of the range changing coils.

Potentiometer range changing devices are discussed

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1. For references see end of paper.

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in a paper by Behr[4], and a recent potentiometer of the type shown in Fig. 2, but having three ranges is shown in Fig. 3.

The use of ten standard cells provides a somewhat more reliable reference e. m. f. than is available with a single standard cell. Practically the range cannot be increased indefinitely in this way because of difficulties

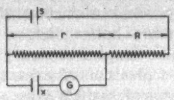

Fig. 1—Poggendorf Circuit

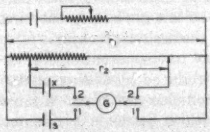

Fig. 2—Modified Latimer Clark Circuit

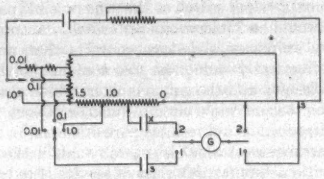

Fig. 3—Modern Three-Range Potentiometer with Separate Standard Cell Dial (2)

due to leakage which would arise if a much higher voltage were applied directly to the potentiometer and for higher voltages a "volt box" is necessary.

For low-range potentiometers, having a maximum range of the order of 0.1 or 0.01 volt, full dependence must be placed on the accuracy of the potentiometer resistances as no standard of voltage is available which will permit making a direct substitution measurement. Such low-range potentiometers are essential for the precise measurement of temperature with thermocouples.

III. Factors Affecting Potentiometer Resistance

The total resistance of the potentiometer circuit is an important consideration in the design. Basically, the potentiometer is a voltage-measuring instrument, but the galvanometer, which is recognized as the most satisfactory device for indicating when the potentiometer is balanced or unbalanced, is basically a current detecting device. For any particular voltage unbalance in the potentiometer, the resulting galvanometer deflection varies with the current in the galvanometer circuit and this of course varies inversely with the total resistance in the galvanometer circuit. Accordingly, a higher resistance in the potentiometer reduces the sensitivity. This is true even though the galvanometer characteristics are correctly chosen for the high-resistance circuit. This point is discussed more fully below.

For the same voltage range, the current through the potentiometer resistances varies inversely with the potentiometer resistance; for example, if a 1.5-volt range potentiometer has a resistance of 75 ohms, the current will be 0.02 ampere and if the potentiometer resistance is 15,000 ohms, the current will be 0.0001 ampere. As the impressed voltage is the same in either case, the leakage current around the potentiometer resistances will be the same, but a leakage current of a

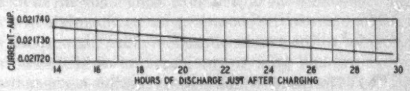

Fig. 4—Current Change in Lead Storage Cell 60-Ampere-Hour Cell Discharging through 100 Ohms

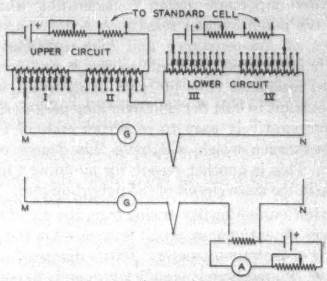

Fig. 5—(Upper) White Potentiometer Circuit (Lower) Addition of Lindeck and Rothe Circuit

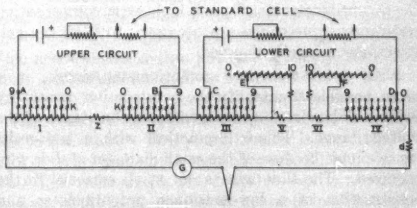

Fig. 6—Two-Range 5-Dial Low Resistance Potentiometer

particular value will cause 200 times the error in the 15,000-ohm potentiometer that it will in the 75-ohm potentiometer. This is a serious disadvantage in working with the very small potentiometer current which results if a very high resistance is used in the potentiometer circuit.

An advantage sometimes claimed for the smaller

potentiometer current, but which is more apparent than real, is the smaller drain on the working battery. In this connection, it should be remembered that storage batteries having capacities upwards of 50 ampere-hours are now readily available at very low cost and the effect of a drain of 0.02 ampere on such a battery is very small. (Fig. 4.) Unless the battery is lagged or thermostated, temperature changes are likely to be of more importance than current drain in causing unsteadiness of the potentiometer current and the temperature effect is the same for a drain of either 0.0001 ampere or 0.02 ampere on a battery of 50 ampere-hour capacity. The temperature coefficient of the voltage[5] of a lead storage cell is 0.0004 volt per deg. cent. Probably the objection to using a current of 0.01 or 0.02 ampere in the potentiometer had its origin in the days prior to the advent of the low-price, high-capacity storage-cell and prior to the invention of M. E. Leeds and E. F. Northrup[6] which made it possible to check the potentiometer current instantly without changing the potentiometer setting.

Another important design consideration which involves the potentiometer resistance is the comparative stability of low-resistance and high-resistance coils. While it is extremely difficult to design and construct unsealed resistance coils of 1,000 ohms which will remain constant to 0.01 per cent over long periods of time, it is comparatively easy to construct coils of 1 to 10 ohms resistance which will have this degree of constancy. This is another reason for avoiding a high resistance in the main circuit of the potentiometer.

An additional difficulty arising from the use of a high-resistance potentiometer circuit is involved in the proper damping of the galvanometer. This is discussed more in detail below, and here it is sufficient merely to point out that the resistance introduced into the galvanometer circuit by an ordinary potentiometer varies considerably as the potentiometer setting is changed and if the potentiometer resistance is high, extra compensating resistance adjustments are necessary to keep the galvanometer properly damped.

With a low-resistance potentiometer circuit, it is often possible to simplify the construction greatly by using a slide wire to cover as much as one-tenth of the voltage range. This is impractical with a high-resistance circuit because of the small diameter of slide wire required. The slide wire is not at all essential to the construction of a low-resistance potentiometer and some low-resistance potentiometers are constructed with dial decades throughout. Potentiometers of this latter design are discussed in detail below.

IV. Low-Voltage Potentiometers

For making precise temperature measurements with thermocouples, accurate low-range potentiometers are required. A copper constantan thermocouple produces a voltage of only about 40 microvolts per deg. cent. and a precision of 0.001 deg. cent. is not an uncommon requirement. This would mean a precision of 0.04 microvolt if only one thermocouple were used, and it is sometimes desirable or necessary to use only a single thermocouple. A practical case would include the following considerations. The "cold junction" or reference junction of the thermocouple would be maintained at 0 deg. cent. because of the accuracy with which the icepoint can be determined and the "hot junction" or measuring junction probably would be at about 30 deg. cent. in a constant temperature bath. Better control can be obtained if the bath temperature is a little above room temperature. If a precision of a few thousandths of a deg. cent. were required, say 0.003 deg. cent., this would mean a precision of 0.01 per cent and 0.12 microvolt. It will be seen at once that a total parasitic e. m. f. of a few microvolts anywhere in the circuit would prevent obtaining the desired precision.

Referring now to the potentiometer circuit, parasitic e. m. fs. are introduced by sliding contacts whether of the dial switch or slide wire type. A fair value to assign to these parasitic e. m. fs. is 1 microvolt per switch contact and 2 or 3 microvolts for a well designed slide-wire contact. Dial switches frequently are designed with 2 contacts per switch so that the possible parasitic e. m. f. would be 2 microvolts per switch. Accordingly, ordinary switch or slide-wire contacts that must be moved frequently cannot be tolerated in the thermocouple circuit; and the galvanometer, which also must be in the thermocouple circuit, must be of such design as to introduce no appreciable parasitic e. m. f.

The arrangement which is generally used in the design of potentiometers for this class of service is to transfer those sliding contacts which must be moved frequently, to the potentiometer battery circuit or to some other circuit in which the contact e. m. f. does not combine directly with the measured e. m. f. (See Fig. 6.) For such service, the potentiometer resistance must be low (not much in excess of 100 ohms) because of the sensitivity required. In a low-resistance circuit even the high-grade slide wire contacts which serve admirably in the galvanometer circuit of a potentiometer of the ordinary range have too much variation in resistance (0.01 to 1.0 ohm) to permit their use in a circuit which must remain constant in resistance while the potentiometer adjustments are being made.

It must not be assumed that by avoiding sliding contacts directly in the thermocouple circuit, the problem is entirely solved. Parasitic e. m. f. must be avoided in the galvanometer itself (see below), but even then, there may remain a small and comparatively constant parasitic e. m. f. which must be taken into account in work of extreme precision by using a false zero-setting of the galvanometer.

The best null potentiometer for this work, therefore, would appear to be a low-resistance instrument with dial decades for all settings. A further refinement is to keep the galvanometer circuit resistance constant for all potentiometer settings. This permits interpreting

deflections of the galvanometer to give readings in between steps of the lowest dial and increases the speed of operation because good galvanometer damping is maintained. It also avoids disturbing the "false zero" of the galvanometer if this expedient is used.

Probably the simplest, complete potentiometer circuit of this general form is that devised by Walter P. White[7] and shown in Fig. 5. Here it will be seen that the sliding contacts of the two dials which are used for measuring temperature changes are in the battery circuit. The contacts of the two dials which are moved only very infrequently are shown in the thermocouple circuit. They could be placed in the battery circuit very readily, but there is little to choose between the two arrangements because constancy of battery circuit resistance in the two higher voltage dials is just as important as absence of parasitic e. m. f., the important consideration being that these switches are seldom, if ever, moved, once the initial setting is made.

It will be noted that the White circuit permits the use of four dials, values lying between steps on the fourth dial being determined from the deflection of the galvanometer which operates in a constant resistance circuit. The arrangement is really that of two 2-dial potentiometers with the thermocouple circuits connected in series. Two separate balances must be made against the standard cell, one for each potentiometer, but this is not a serious disadvantage because when the first or second dial is in circuit, the accuracy of the current adjustment through the third and fourth dials need not be high; accordingly, the current in this circuit needs adjustment only very infrequently.

The White potentiometer is ordinarily made with a single range, either 10,000 or 100,000 microvolts. The galvanometer circuit resistances are 40 ohms and 300 ohms, respectively. Instruments of both ranges may be equipped with a second set of dials to provide a double potentiometer. In the double potentiometer, two values of potential differing widely from each other may be measured without disturbing the dial settings. This arrangement is an adaptation of the independent standard cell balancing dial[6] previously mentioned.

The equivalent of an additional dial on the White potentiometer may be provided by connecting the device of Lindeck and Rothe[8] in the thermocouple circuit. (See Fig. 5.)

A two-circuit potentiometer suggested by the author for this class of work is shown in Fig. 6. Here the two thermocouple circuits, instead of being connected to give the sum of the two potentials, are connected to give the difference of the two potentials. This arrangement avoids the requirement of reducing both potentials to zero to obtain a zero reading on the potentiometer and therefore makes possible the inclusion of one or two shunted decades[9] to give 5 or 6 dials instead of the 4 dials provided in the White potentiometer. Only 5 dials would be used at one time, the extra dial being a convenient means for making the range of the potentiometer either 10,000 microvolts or 100,000 microvolts.* It will be apparent that numerous variations of this general arrangement are possible. The potentiometer of Fig. 6 has a practically constant resistance of about 11 ohms in the galvanometer circuit. This general circuit requires two batteries for supplying the potentiometer and, consequently, two adjustments against the standard cell just as in the White potentiometer.

Low-voltage potentiometers have been devised which utilize the same "difference" principle as is used in the potentiometer of Fig. 6, but which use only one battery and require only one balance against the standard cell. These are commonly called "split-circuit" potentiometers and are shown in Fig. 7 and Fig. 8. The potentiometer of Fig. 7 was proposed by Diesselhorst[10] and involves some elements proposed by Hausrath, White, Waidner and Wolff.[11] The one shown in Fig. 8 uses a development of the circuit suggested by Wenner.[12] The former has a normal range of 10,000 microvolts and a practically constant resistance in the galvanometer circuit of about 14 ohms. The latter has two ranges of 100,000 and 10,000 microvolts and has a practically constant resistance in the galvanometer circuit also of about 14 ohms.

There are at present no published papers giving a complete description of the modified Wenner potentiometer of Fig. 8, but such a paper is in preparation and will shortly be published.

V. Analysis of Potentiometer Errors

Reference has been made to the stability of high-resistance and low-resistance coils and to resistance variation and parasitic e. m. f. at switch and slide wire contacts. The following analysis should help to make clear the significance of these factors.

Reference to Fig. 2 will show that when both potential contacts are set on zero, they are connected to the identical point in the current circuit. Accordingly, there can be no potential difference between them no matter what may be the value of the potentiometer current. In other words, the potentiometer has a true zero point. This arrangement makes possible a simple low-resistance potentiometer and, as indicated above, low resistance in the potentiometer is much to be desired.

Fig. 9 shows the well-known potentiometer circuit due to Feussner.[13] This potentiometer is usually made with a very high total resistance of (20,000 ohms). This high resistance is not desirable, but is necessary because of the large number of contacts in the circuit. It is important to note that the error in microvolts, due to residual resistance in a circuit of this kind, is a constant one and not a proportional one; in other

*The use of all six dials at one time, to measure absolute voltage values, would imply a higher accuracy in the main potentiometer coils and in the value of the standard cell than is justifiable. All six dials may be used at one time for measuring small changes in voltage such as might be encountered in calorimeter measurements in which the temperature changes are observed by means of thermocouples.

words, it produces a false zero. The error is equal to the residual resistance multiplied by the potentiometer current. In the Feussner design, a very high resistance is used to keep the potentiometer current small (0.0001 ampere) so that the error due to the residual resistance is usually negligible in the class of potentiometers in which the true Feussner circuit is used. If a potentiometer were made using a circuit similar to the Feussner circuit, but which required a potentiometer current of 0.01 ampere, it would be evident that the constant error due to residual resistance in such a potentiometer would be 100 times as large as the error in the Feussner instrument for the same residual resistance.

Referring now to the potentiometer circuit shown in

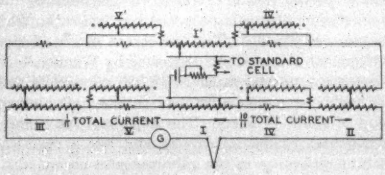

Fig. 7—Diesselhorst Potentiometer Circuit

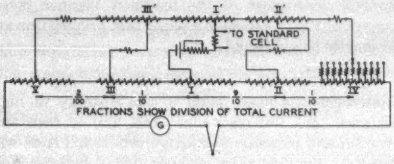

Fig. 8—Wenner Potentiometer Circuit

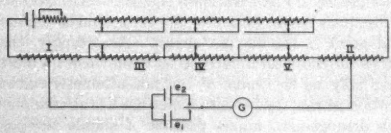

Fig. 9—Feussner Potentiometer Circuit

Fig. 6, the contacts A, B, C and D are all in the battery circuits where the voltage is about 2 volts and the resistance in the upper battery circuit is 200 ohms and in the lower battery circuit 2,000 ohms. In view of the comparatively high voltage of the battery circuit, the error due to parasitic e. m. f. at the contacts is, of course, negligible. The microvolt error, due to variation of resistance at the contacts, is proportional, not constant, and will never cause an error in per cent greater than $\frac{\Delta r}{R} \times 100$, where Δr is the variation in contact resistance and R is the battery circuit resistance. Assuming 0.0002-ohm variation in resistance, for a 200-ohm battery circuit, this would be 0.0001 per cent, and for a 2,000-ohm battery circuit, this would be 0.00001 per cent.

Contacts E and F in Fig. 6 produce negligible errors from either variation in contact resistance or parasitic e. m. f. The circuit in which these contacts are placed is such as to make the effective variation due to resistance entirely negligible and to make the effective parasitic e. m. f. less than 1 per cent of the actual contact e. m. f. (See Appendix I.) The manipulation of contacts E and F causes a negligible variation in the current in the lower potentiometer circuit. If an arrangement of this kind were used in the upper potentiometer circuit where the total resistance is lower, compensating coils would be necessary to correct for the variation in potentiometer current.

The additional errors which need consideration in the potentiometer proper are the errors in the resistance coils. These fall into three groups: (1) the main potentiometer coils; (2) the compensating coils, and (3) the coils bridged by the standard cell in standardizing the potentiometer current. Frequently, coils are placed in circuit in such a way that they belong in more than one of these groups. Here it is desired merely to say that if coils not much above 100 ohms are used, there is no great difficulty in making them and adjusting them so that the error will not exceed 0.01 per cent over long periods.

A summation of errors due to the above causes for the 100,000 and 10,000 microvolt ranges of the potentiometer, shown in Fig. 6, is given in Appendix II. These are maximum errors, the probable errors being somewhat less. It will be seen that for both ranges, the maximum error is less than the value of one step on the last dial.

VI. Self-Checking Features of Potentiometers

The measurements for which potentiometers are used are usually those on which great dependence is to be placed and, accordingly, the user should never be in doubt about the accuracy of his potentiometer. Even with the best design, workmanship and materials, a potentiometer may be made inaccurate by abuse or use by a careless operator, and, to guard against errors from such causes, the potentiometer design, if possible, should be such as to permit convenient checking by the user. Fortunately, the nature of the ordinary potentiometer circuit is such as to permit checking to a high degree of accuracy with instruments which are of a much lower order of accuracy. However, to facilitate the work of checking, the potentiometer circuit should be kept as simple as possible.

Usually, there are provided low-resistance plug blocks, all alike, for making connections to each of the main potentiometer coils so that their values may be compared for equality using a wheatstone bridge. Since it is necessary to measure only the difference between values which in a good instrument are alike

within 0.01 per cent, errors as large as 10 per cent in the bridge used for checking would cause an inaccuracy of only 0.001 per cent in the result. After the equality of the main resistors has been determined in this way, the calibration of the slide wire is checked against the main resistors; see Fig. 10. If the comparative values of the main resistors are known to 0.01 per cent, the uniformity of the slide wire may be determined with this same precision.

The final step is to determine the ratio between the main potentiometer resistors and the resistance across which the standard cell is connected when standardizing the potentiometer current. By making the standard cell balance across the identical resistors that comprise the main potentiometer resistance, it is obvious that

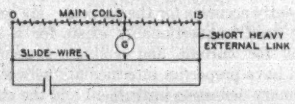

Fig. 10—Checking Potentiometer Slide-Wire Against Main Coils

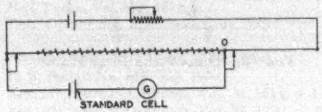

Fig. 11—Checking Standard Cell Adjustment Against Main Slide Wire

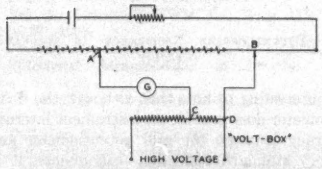

Fig. 12—"Volt-Box" Connections

the ratio must be exactly unity. For convenience, it is desirable to make a small part of the resistance, across which the standard cell is balanced, a separate adjustable unit. This small adjustable resistance may be checked against the main slide wire as shown in Fig. 11. With the main slide wire on zero and the standard cell dial at its maximum setting the battery current is adjusted to produce a balance against the standard cell. The standard cell dial is then changed in steps and the resistance change for each step is found by adjusting to a balance with the main slide wire.

VII. Galvanometer Considerations

The improvement in sensitivity resulting from low resistance in the potentiometer circuit has been previously mentioned. A common example will be found in the measurement of current in the standardizing laboratory. Here, if a comparison is made on the basis of using galvanometers of the same general type and period, it is found that generally the energy consumption in the current shunts is much less when working with a low-resistance potentiometer. The lower energy consumption reduces the cost of the shunts and avoids the inconvenience of providing oil and water cooling. If the comparison is made on the basis of equal energy consumption in the current shunts, then the low-resistance potentiometer has the advantage that a more sturdy and convenient and less expensive galvanometer can be used. For instance, if a 300-ampere current shunt is made with a resistance of 0.001 ohm, it will produce a voltage drop of 0.3-volt at full load. A precision of measurement of 0.01 per cent would require a sensitivity of 30 microvolts. With a potentiometer having a total resistance of 100 ohms, the galvanometer circuit resistance due to the potentiometer for a setting of 0.3-volt is 12.75 ohms. With a potentiometer having a total resistance of 20,000 ohms, the corresponding resistance is 2,550 ohms. A convenient galvanometer suitable for use in the low-resistance circuit would have a resistance of 20 ohms, a current sensitivity of 0.25 microampere per millimeter, a voltage sensitivity of 25 microvolts per millimeter with an external critical damping resistance of 80 ohms and a period of 3 seconds. The difference between 12.75 ohms and 80 ohms would be made up with a resistance coil of about 67 ohms. Reading to the nearest millimeter would give the desired precision.

If this same galvanometer were used with the high-resistance potentiometer, the voltage sensitivity with 2,550 ohms in circuit would be 643 microvolts per millimeter. This would require reading to about 0.05 millimeter, which is impractical, and, in addition, the galvanometer would be badly underdamped. If proper damping is taken into consideration, the required sensitivity of 30 microvolts is impossible to obtain with the design of galvanometer under consideration. Accordingly, it is necessary to use a more sensitive and more costly form of galvanometer or materially to increase the resistance of the current shunt with a consequent increase in energy loss and cost.

Proper damping of the galvanometer is essential to rapid working with the potentiometer and the damping of modern galvanometers, designed to have short periods and high sensitivities, is much more affected by the external circuit resistance than were the older galvanometers having long periods and much lower sensitivities. Consequently, it is very desirable that the circuit resistance external to the galvanometer should be approximately constant. In a 20,000-ohm potentiometer, the probable resistance variation in the galvanometer circuit, as the dials are manipulated, is in excess of 4,000 ohms, while in a 100-ohm potentiometer, the corresponding resistance variation is approximately

20 ohms. Accordingly, when working with a very high-resistance potentiometer, it is impossible to keep the galvanometer properly damped unless some auxiliary compensating resistance is adjusted each time a substantial change is made in the potentiometer setting. (See Appendix III for calculations.)

When working with low-range potentiometers, such as those shown in Figs. 5 to 8, it is not sufficient merely to avoid parasitic e. m. fs. in the potentiometer proper because the galvanometer itself may be the seat of a parasitic e. m. f. amounting to 5 or 10 microvolts under ordinary working conditions. By making the entire galvanometer circuit of copper, the parasitic e. m. f. is reduced to a small fraction of a microvolt.[14]

VIII. Comments on Volt-Boxes

Usually, for extending the potentiometer range above 1 or 2 volts, a voltage divider known as a "volt-box" is used. The connections are shown in Fig. 12. It is desirable to make the total resistance of a volt-box reasonably high, several hundred ohms per volt, first, to avoid lead and contact errors in the high-voltage circuit, and, second, to reduce the energy dissipation in the coils. A much higher resistance is undesirable because the higher resistance coils are likely to be less stable and because the higher resistance in the galvanometer circuit reduces the sensitivity. Some publications[15,16] state that a low-resistance volt-box should be used with a low-resistance potentiometer and that a high-resistance volt-box should be used with a high-resistance potentiometer. This is incorrect. In considering lead and contact errors or energy dissipation in the coils, the potentiometer resistance is not involved. The consideration of sensitivity does involve the potentiometer resistance, but if the maximum resistance introduced into the galvanometer circuit *A-B* (Fig. 12) by a low-resistance potentiometer is 25 ohms and by a high-resistance potentiometer is 5,000 ohms, it is obvious that the galvanometer circuit resistance *C-D* in the volt-box may be 4,975 ohms higher with the low-resistance potentiometer to give the same sensitivity.

IX. Deflection Potentiometers

In the deflection potentiometer, the principal part of the potential being measured is balanced against a known potential just as is done in an ordinary null potentiometer and the remaining small portion of the unknown potential is read from the deflection of a calibrated galvanometer. While the general idea was suggested by Stansfield[17] as early as 1898 and was used by Hoffman and Rothe[18] in 1905, the complete development of the idea as applied to precise laboratory potentiometers is due to H. B. Brooks of the Bureau of Standards. The principles underlying the design of deflection potentiometers have been so well presented in the publications[19] of the Bureau of Standards that they need not be discussed here. It will suffice to say that the fundamental requirement in the design of a deflection potentiometer is that for all connections and adjustments of the potentiometer, there must be no appreciable change in the galvanometer circuit resistance.

Instead of going into the design of the deflection potentiometer, it is desired to state the advantages of the instrument in one important class of measurements. Aside from its use in photometric measurements, the most important application of the deflection potentiometer is in checking large numbers of portable indicating instruments. Discussing this latter problem, Brooks, in his original publication[19] on the deflection potentiometer points out that null potentiometers are the most accurate for the purpose, but require too much time to use and that long scale "laboratory standard" deflecting instruments, while faster in operation, are not sufficiently accurate for the purpose. He concludes that "the need unquestionably exists for a type of instrument for current and voltage measurements which will have properties intermediate between those of the ordinary deflection instrument and the elaborate potentiometer."

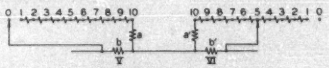

Fig. 13—Dials V and VI of Fig. 6

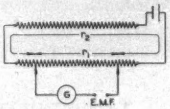

Fig. 14—Potentiometer Resistance in Galvanometer Circuit

It is interesting to note that although the deflection potentiometer does provide an instrument intermediate in accuracy between the null potentiometer and the laboratory standard deflection instruments, it is not intermediate in speed of operation. It is faster than either of the other two types of instrument. There are several reasons why the deflection potentiometer is more rapid in operation than the laboratory standard deflection instrument. In the deflection potentiometer, the deflecting pointer is shorter and lighter and this results in a shorter period; it is unnecessary to take the time to estimate tenths of a division because reading to the nearest division is all that is required and finally, because in the deflection potentiometer, the position of the pointer does not change very much and need not be observed more closely than one division, it is unnecessary for the operator to change his position frequently in order to remain in line with the pointer. This latter precaution is essential in reading laboratory standard deflection instruments to avoid parallax errors.

X. The Potentiometer in Industry

Mention has been previously made of the use of the potentiometer in industrial plants. It may be of interest to point out that in recent years the use of potentiometers in industrial plants has increased to such an extent that a fair estimate indicates that nine-tenths of potentiometers put into use each year go into industrial plant applications and only one-tenth go into laboratory service. This is not due to any falling off in the laboratory uses of potentiometers. The potentiometer is more generally used in laboratories than ever. It is due entirely to the very rapid increase in industrial plant uses of the instrument, in the form of portable indicating potentiometers and automatic recording and controlling potentiometers.

Acknowledgment

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Appendix I

Calculation of Errors in Waidner-Wolff Shunt-Decade

In the circuit where these shunted decades are used (see Fig. 6), the e. m. f. opposes the principal e. m. f. in the potentiometer circuit. Accordingly, a decrease in the e. m. f. in this circuit will cause a net increase in the potentiometer e. m. f. Therefore, with a constant current, the effective resistance should decrease for increasing dial readings. This decrease of resistance is obtained by shunting the coils b and b', the shunting resistance decreasing in value as the dial readings increase. The coil values are given in the following table:

TABLE I—COIL VALUES (OHMS) DIALS V AND VI OF FIG. 6

	Dial V Shunted coil (b) $= \sqrt{1.26} = 1.1225$ ohm		Dial VI Shunted coil (b') $= \sqrt{0.126} = 0.3550$ ohm	
Dial position	Total shunting resistance (R_s)	Individual coil resistance	Total shunting resistance (R_s)	Individual coil resistance
10.....	$126.0 - \sqrt{1.26}$	$124.8775 - a$	$126.0 - \sqrt{0.126}$	$125.645 = a'$
9.....	140.0 − "	14.0	140.0 − "	14.0
8.....	157.5 − "	17.5	etc.	17.5
7.....	180.0 − "	22.5		22.5
6.....	210.0 − "	30.0		30.0
5.....	252.0 − "	42.0		42.0
4.....	315.0 − "	63.0		63.0
3.....	420.0 − "	105.0		105.0
2.....	630.0 − "	210.0		210.0
1.....	1260.0 − "	630.0		630.0
0.....	Infinity.....	Infinity...		...Infinity

Note: A similar selection of coil values for shunt-decades was described by E. F. Mueller,[20] who derived values for the shunted resistance b and b' to make most of the individual coil resistances simple values and identical in both decades.

In addition to errors in the resistance coils, the possible sources of error in such shunt-decades are the resistance and the parasitic e. m. f. at the contact. Most of the resistance coils are low in value so that, with good construction, the error due to lack of stability is negligible. Lack of stability in any of the few higher resistance coils needs to be considered.

The error due to contact resistance will be equal to the error introduced in the joint resistance of the shunted and shunting coils, and will have its maximum value when the shunting resistance has its minimum value. The joint resistance is

$$R_j = \frac{r\,(R_s + \Delta R)}{r + R_s + \Delta R}$$

where

R_j = joint resistance of shunted and shunting coils
r = shunted resistance (b or b')
R_s = shunting resistance (minimum value a or a')
ΔR = variation in contact resistance

The error in R_j due to contact resistance ΔR, expressed in per cent is:

$$\text{per cent error} = \frac{r\,.\,\Delta R}{R_s\,(R_s + \Delta R + r)} \times 100$$

Taking approximate values for the coils of dial V and assuming a contact resistance of 0.0002 ohm, this gives,

$$\text{per cent error} = \frac{0.02}{125\,(125 + 0.0002 + 1)} = 0.00000125$$

Expressed another way and for the particular resistance values used, the change in the joint resistance R_j for a contact resistance ΔR is less than $0.0001\,\Delta R$.

The joint resistance in dial V is approximately 1 ohm and changes precisely 0.001 ohm for each step of the dial. The joint resistance in dial VI is approximately one-third of an ohm and changes precisely 0.0001 ohm for each step of the dial.

The error due to contact e. m. f. will be the e. m. f. produced in the galvanometer circuit by the contact when all other potentials in the circuit are assumed to be zero. Any effect in or because of the battery circuit is too small to be considered. The following notation is used:

e = $i\,r$ = e. m. f. produced in galvanometer circuit by contact e. m. f.

r = resistance of shunted coil (b or b')

i = current produced in r by contact e. m. f.

ϵ = contact e. m. f.

I = total current produced by ϵ

R_s = shunting resistance

R_g = resistance in galvanometer circuit exclusive of shunted resistance of r

R_i = joint resistance in series with ϵ

Then:

$$I = \frac{\epsilon}{R_i} = \frac{\epsilon}{R_s + \dfrac{r\,R_g}{r + R_g}}$$

$$= I \,.\, \frac{\dfrac{r\,R_g}{r + R_g}}{r}$$

$$= \epsilon \,.\, \frac{\dfrac{r\,R_g}{r + R_g}}{r\left(R_s + \dfrac{r\,R_g}{r + R_g}\right)}$$

$$e = i\,r = \epsilon \,.\, \frac{r\,R_g}{r\,(R_s + R_g) + R_s\,R_g}$$

Again taking approximate values for dial V, taking R_g as 40 ohms (Appendix 2) and assuming a contact e. m. f. of 1 microvolt, this gives

$$e = \frac{40}{125 + 40 + 5000} = 0.008 \text{ microvolt}$$

Expressed another way and for the particular coil values used

$$e = 0.008\,\epsilon$$

Referring now to stability of resistance coils, there are only two coils in the decade which are much above 100 ohms. These have values of 210 ohms and 630 ohms. There is no observable difference in stability between a coil of 210 ohms and one of about 100 ohms, both being able to retain their values well within 0.01 per cent. By reference to the above equation showing the minute error caused by a contact resistance, ΔR, of 0.0002 ohm, it will be seen that a contact resistance of 16 ohms would cause an error of only 0.1 per cent. An error in the 630-ohm coil of 16 ohms would cause a hundredth of this error because of the value of R_s when the 630-ohm coil is in circuit. An error of 16 ohms would correspond to a change of over 2 per cent in the value of the 630-ohm coil and any such large change in a 630-ohm coil which was not defective would be impossible.

Accordingly, the only errors in the shunt-decade which need be taken into account in a summation of errors are the error in the value of the shunted resistance r (b or b') and the error introduced by the contact e. m. f. ϵ. The value of r in dial V is approximately one ohm and therefore can be depended upon to better than 0.01 per cent. The error caused by the contact e. m. f. ϵ is less than 1 per cent of the value of ϵ. Similar calculations using the coil values for dial VI show a negligible error for variation in contact resistance and a value for e of 0.003 microvolt.

Appendix II

Calculation and Summation of Errors Low-Range, Five-Dial Potentiometer—Fig. 6

This instrument is referred to as a five-dial potentiometer although six dials are used. The reason for this is that only five dials are used at one time, the sixth providing a convenient means for changing the range. When dials I to V are used, the range is 100,000 microvolts and when dials II to VI are used, the range is 10,001 microvolts.

The current in the upper circuit is 0.01 ampere and in the lower circuit 0.001 ampere for either range. Using 2-volt batteries, the battery circuit resistance in the upper circuit is 200 ohms and in the lower circuit is 2,000 ohms. The actual galvanometer circuit resistance is approximately 11 ohms, but an auxiliary coil d is included to bring the resistance up to 40 ohms, which is the external critical damping resistance value of a good galvanometer of proper sensitivity and period.

The coil values for dials V and VI are given in Appendix I. The principal potentiometer coils for the remaining dials are given below.

Dial I.....nine 1.0-ohm coils
Dial II.....nine 0.1-ohm coils
Dial III....nine 0.1-ohm coils
Dial IV.....nine 0.01-ohm coils

It will be seen at once that there are no sliding contacts directly in the galvanometer or thermocouple circuit. The contact e. m. f. produced in the galvanometer circuit by dials V and VI is given in Appendix I. The contacts of dials I, II, III and IV are in the battery circuits. As any parasitic e. m. f. at the contacts of

these four dials is effective only in changing the battery currents, the author has found it convenient to convert these parasitic e. m. fs. into equivalent resistances. Assuming a maximum parasitic e. m. f. of 1 microvolt per contact, the equivalent resistance is 0.0001 ohm per contact in the 200-ohm, 2-volt battery circuit and 0.001 ohm in the 2,000-ohm, 2-volt battery circuit. Contact resistance changes in these four dials are assumed to be 0.0002 ohm per contact. The final assumption is that the compensating coils k are adjusted to cause no error greater than 0.01 per cent in dials I and II and no error greater than 0.1 per cent in dials III and IV.

It should be noted that any change in battery current in the upper circuit affects dials I and II and any change in battery current in the lower circuit affects dials III, IV, V and VI.

The coil z in the upper circuit is of such value as to produce a potential drop in that circuit just equal to the maximum drop in the lower circuit. This occurs when all dials of the lower circuit are set on zero and is 2467.5 microvolts. Dials I and II add 99,000 microvolts so that the maximum potential drop in the upper circuit is 101,467.5 microvolts.

For small resistance changes, the maximum per cent error in current in either circuit, due to operation of the dials, is the same as the maximum per cent variation in equivalent resistance and is obtained as follows:

$$\text{Variation in current} = \frac{\Delta R}{R} \cdot 100 \text{ per cent}$$

where R is the battery circuit resistance and ΔR is the maximum variation in equivalent resistance due to operation of all the dials in the circuit.

Computation of ΔR for Upper Circuit

Contact resistance change	2×0.0002 ohm	= 0.0004 ohm
Equivalent resistance change (contact e. m. f.)	2×0.0001 "	= 0.0002 "
Compensating coil error Dial I	9×0.0001	= 0.0009 "
Compensating coil error Dial II	0.9×0.0001	= 0.00009 "
ΔR = maximum equivalent resistance variation		= 0.00159 ohm

Computation of ΔR for Lower Circuit

Contact resistance change	2×0.0002 ohm	= 0.0004 ohm
Equivalent resistance change (contact e. m. f.)	2×0.001 "	= 0.0020 "
Compensating coil error Dial III	0.9×0.001	= 0.0009 "
Compensating coil error Dial IV	0.09×0.001	= 0.00009 "
Resistance change due to Dial V	10×0.001 ohm	= 0.0100 "
Resistance change due to Dial VI	10×0.0001 "	= 0.0010 "
ΔR = maximum equivalent resistance variation		= 0.01439 ohm

Microvolt Error in Upper Circuit due to Resistance Changes

$R = 200$ ohms

$$\frac{\Delta R}{R} \cdot 100 = 0.5\,\Delta R = 0.0008 \text{ per cent}$$

Maximum microvolt error $= 101{,}467.5 \times 0.000008$
$= 0.81$ microvolt

Microvolt Error in Lower Circuit due to Resistance Changes

$R = 2{,}000$ ohms

$$\frac{\Delta R}{R} \cdot 100 = 0.05\,\Delta R = 0.00072 \text{ per cent}$$

Maximum microvolt error $= 2467.5 \times 0.0000072$
$= 0.018$ microvolt

SUMMATION OF ERRORS DUE TO DIAL OPERATION 100,000-MICROVOLT RANGE

Error due to resistance changes, upper circuit	0.807	microvolt
Error due to resistance changes, lower circuit	0.018	"
e. m. f. produced in galvanometer circuit by contact of dial V	0.008*	"
e. m. f. produced in galvanometer circuit by contact of dial VI	0.003*	"
Total microvolt error	0.836	"

*See Appendix I.

This summation includes slight errors due to operation of dial VI, which, as stated before, would be used on the 100,000 microvolt range, only when observing small changes of voltage such as encountered in calorimetry.

To obtain the total error for the 10,000 microvolt range, the error due to dial I should be omitted, as dial I would be set on zero and would not be operated.

Microvolt Error in Upper Circuit—10,000 microvolt range

Contact resistance change	0.0002	ohm
Equivalent resistance change (contact e. m. f.)	0.0001	"
Compensating coil error Dial II 0.9×0.0001 =	0.00009	"
ΔR = maximum equivalent resistance variation =	0.00039	ohm

$R = 200$ ohms

$$\frac{\Delta R}{R} \cdot 100 = 0.5\,\Delta R = 0.0002 \text{ per cent}$$

Microvolt error $= 11{,}467.5 \times 0.000002$
$= 0.023$ microvolt

SUMMATION OF ERRORS DUE TO DIAL OPERATION 10,000-MICROVOLT RANGE

Errors due to resistance changes, upper circuit	0.023	microvolt
Errors due to resistance changes, lower circuit	0.018	"
e. m. f. produced in galvanometer circuit by Dial V	0.008*	"
e. m. f. produced in galvanometer circuit by Dial VI	0.003*	"
Total microvolt error	0.052	"

*See Appendix I.

When using the 100,000-microvolt range, one step of the lowest dial (dial V) is 1 microvolt and when using the 10,000-microvolt range, one step of the lowest dial (dial VI) is 0.1 microvolt. Accordingly, the maximum error due to switch operations is less than one step on the low dial for either range and of course the probable error is even less.

It should be noted that these summations of errors do not include the usual errors due to the residual inaccuracy in the main potentiometer coils. An inaccuracy of 0.01 per cent in these coils would cause a maximum microvolt error equal to one step on the fourth dial in use for either range. Facilities must be available for determining the values of the main coils to 0.001 per cent if the main coil resistance error is not to exceed the error due to switch operations.

Appendix III

Variation in Galvanometer Circuit Resistance of Potentiometers

If,

R = total resistance in battery circuit of potentiometer

R_g = effective potentiometer resistance in galvanometer circuit

r_1 = potentiometer resistance directly in galvanometer circuit

r_2 = resistance shunting r_1

Then,

$$R = r_1 + r_2$$

$$R_g = \frac{r_1 r_2}{r_1 + r_2} = \frac{R r_1 - r_1^2}{R}$$

$$\frac{d R_g}{d r_1} = \frac{R - 2 r_1}{R}$$

R_g is maximum when $\frac{R - 2 r_1}{R} = 0$ or when $R = 2 r_1$

$$\therefore R_g \text{ (max)} = \frac{R}{4}$$

From this it will be seen that the usual high-resistance potentiometer, having a total battery circuit resistance of 20,000 ohms, introduces a variable resistance of 5,000 ohms into the galvanometer circuit and that the usual low-resistance potentiometer, having a total battery circuit resistance of 100 ohms, introduces a variable resistance of only 25 ohms into the galvanometer circuit.

Discussion

For discussion of this paper see page 1324.

Electrical Units and Their Application

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Introduction

THE importance of measurements in any art or science is great; and progress is not made without measurement. Progress and improvement are made possible to the extent that those interested in and working along the line of developing and improving measuring methods and instruments are able to anticipate the needs of those interested in the development and operation of systems, devices, and apparatus.

Electrical units are based on or definitely related to units of length, mass, and time. In the ordinary sense these units are not electrical but because so much depends on these fundamental units; we must be interested primarily in how definite and permanent the fundamental units are and how precisely length, mass, and time may be determined by reference to existing standards; also how accurately copies of units may be made and the degree of precision attainable in producing for practical use multiples or submultiples of the fundamental unit chosen.

Probably the so-called c. g. s. system of units is most familiar. In this system the centimeter is taken as the unit of length; the gram as the unit of mass, and the second as the unit of time. This system has been quite generally accepted for many years. For some purposes it may be a little old fashioned but in general it is likely to endure for a long time. It may be better to say that l, m, and t are likely to be required as fundamental units in the future as they have been in the past. At this point it would be permissible to define l, m, and t but this will not be attempted. So many things have occurred within the last few years to upset older conceptions of what anything really is, that satisfactory definitions are not written down offhand. However, all those interested in electrical science and art have an idea of what these quantities mean, at least sufficiently for the present purpose.

Whatever is done, we may always retain the c. g. s. system, or something equivalent to it, and therefore be tied permanently to the fundamental units of length, mass, and time which have given evidence of being so reliable and permanent. Many determinations leading to the absolute value of the volt, ampere, and ohm have been made and these efforts have attained a large degree of success. However, the procedure by any methods that have been employed requires a very long time and infinite knowledge and pains and even then the precision arrived at is not all that could be desired. Easier and more satisfactory methods it is hoped may be discovered.

Before making any statements that may be useful for comparison, I would like to emphasize a point; *viz.*, that only very general statements will be made and comparisons will be based on groupings, each one roughly 10 times or 1/10 of the magnitude of the one next to it. A precision of 1 to a possible 9 parts in 100,000,000 will be called 10^{-8} and so on down to 10^{-2}, which will give a sufficient range. There will then be seven classes 10^{-8}, 10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2} into one or another of which all things to be considered will be placed. It may well happen that some may feel that certain estimates are not correct even when referred to such wide limits.

We are usually quite satisfied in ordinary work with accuracy of standard and precision of comparison considerably below the best obtainable, but workers in the field of primary reference standards must of necessity be able to make determinations 10 to 100 times more precise than the precision demanded in practise so that the practical art has something to grow up to and its possibilities are not limited. Sometimes unusual practical requirements arise suddenly that can only be met with precision far greater than any previously employed.

It is in this region that national laboratories and some independent workers find a very useful field and in this field the national Bureau of Standards stands second to none. These groups should all receive every encouragement to go forward for it is not known when the advancing art will need much greater precision than has heretofore been attained.

C. G. S. Units

Mass standards are permanent and can be compared to a precision of one or at least a very few parts in 100,000,000 and standards of length about the same. These, therefore, lie definitely in the 10^{-8} class. Time, one of the fundamental elements in many things, can hardly be regarded as in the same class as length and mass.

There are 86,400 seconds in a day. Therefore a minimum departure from a uniform rate of about 0.001 second a day is required in order to have a clock included in the 10^{-8} class. This is better than has been attained over an extended period of time. Time is therefore in the 10^{-7} class. It even has a tendency to slip into the 10^{-6} class.

Roughly 500 years ago the first instruments that could really be called clocks were brought out and they were quite definitely in the 10^{-2} class, probably the lower end of this band; in 300 years they were brought into the 10^{-6} class; in about 150 years more into the 10^{-7} class. During the last few years the clocks

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of Shortt and of Schuler, each quite different from the other, have given unmistakable signs of being able to qualify for the 10^{-8} class.

This shows remarkable progress. Twice within the last say 40 years the error in time keepers has been divided by 10.

This brief consideration of time is not really a digression for within the last few years the demand for accurate measurement of time on which frequency or wavelength measurements are based came almost too soon. This is a definite example of where we found ourselves too close for comfort to the precision maintainable in primary reference standards.

But the picture is not without its brighter side. The crystal oscillator, which in a certain way caused the trouble by requiring accurate adjustment and timing or which, to take the other point of view, had great value because it was capable of being given this fine adjustment, has been found to be a most reliable timing device itself. First checked with clocks it now bids fair to turn about and enter the higher time keeping class itself. Comparisons between crystal oscillators can now be placed in the 10^{-8} class and timing of crystals in the 10^{-7} class.

As a general statement it is fairly certain that a perfectly free pendulum is apt to be more certain in its performance than the motions of the planet on which we live but on the other hand observation of the action of crystal oscillators, treated with the same care and respect as precision clocks, seems to make it also fairly certain that no fault as to permanence or precision of performance is chargeable to the crystal itself. Sufficiently long periods of continuous operation have not been observed so that it would be perhaps unwise to be too sanguine of the final result, but we may have a time measurer that is more perfect than any pendulum can ever be because the oscillating crystal is not affected by gravity which may change some in any locality and because it is probably much more easily protected against shocks than a pendulum. On the other hand it is at present rather afflicted by complicated auxiliary apparatus which while it functions well leaves many chances for failure. Soon the motions of the earth may be checked by clocks or their equivalent instead of employing the present process of determining the clock rates from the relative motions of the earth and the stars. Irregularities in these motions may be shown by the clocks.

But why say so much about length, mass, and time? It is because it is of interest to know just about where we stand on these fundamental standards and the conclusion is that they are in a fairly satisfactory position; time far in the rear but rapidly advancing to a position of equality with the other two. I therefore feel that length, mass, and time are good things to tie to and start from. The difficulties seem to be in the steps necessary to realize electrical standards from length, mass, and time. Improvements may come and it is to be hoped that they will soon arrive so that the precision assignable to the fundamental units, length, mass, and time, may be more nearly realized in the electrical units that are used for fundamental reference.

Electrical Units

We are more directly interested in the ohm, the volt, and the ampere or the absolute units of which these are derived multiples. As will be seen later there is a very wide gap between the precision assignable to length, mass, and even time, and that assignable to the absolute values of the practical electrical units, for a very good reason. Fundamental length and mass standards are in a sense arbitrary. This does not give concern because the meter need not be exactly 1-10,000,000 of the polar quadrant of the earth and the gram may not have exactly the mass, of 1 cu. cm. of water at maximum density provided the established standards remain permanent. There is reason to believe that they are permanent within satisfactory limits. On the other hand in considering electrical units their relation to length, mass, and time must be known. They should not be arbitrary if agreement with mechanical units and others which are derived from length, mass, and time also is to be maintained. The electrical standards must therefore be kept from change and must also be brought into a definite and predetermined relation to the fundamental units; so that conversion factors are as simple as they can be. The necessary powers, roots, and $4\pi s$ are quite sufficient without the addition of odd values differing only a small amount from multiples of 10. From the way in which the art has developed the ohm and the volt or definite multiples of either are more readily available and transportable from place to place than is the ampere.

I believe that the ohm is now defined as the resistance at 0 deg. cent. of approximately 106.30 cm. of mercury, 1 sq. cm. section. This may be certain to five significant figures or 10^{-4}. It can hardly be said to be dependably known to six figures, 10^{-5}. However, work is constantly being done to better this situation. Copies and comparisons of resistances in general may be made to about 10^{-6} and the permanence of the primary reference standards is good, within about the same limits.

Therefore, it may be said that the ohm may be considered in a safe situation, but more certainty as to the absolute value would be desirable. I should like to see this unit known and its permanency and absolute value unquestioned to 10^{-6}. This seems quite possible of attainment. It may be that those working in this field will say that they have already arrived; if so, good. At any rate, gratifying progress has been made. Looking back to the first ohm of the British Association Committee it was about 1½ per cent smaller than the present ohm, and it has taken nearly

50 years to eliminate the greater part of this error, although most of this change was in one important step. Precision and permanence have just about kept out of the way of progress in the practical field.

The volt had rather a bad start for some time. The preference was to derive this from the ampere and the ohm. With much time and care galvanometers were constructed that depended for their accuracy on measurements of length and of the magnetic field of the earth. Again, we were tied up with a property of the earth as we are now with time, but unfortunately we cannot transmit amperes around the countries by telegraphic signal or by radio, and although the earth's magnetic field is capable of very precise determination, the ampere or multiples when realized could not very well be carried home and used. There was nothing to carry the ampere in or with. Instruments using no permanent magnet and which did not depend on the earth's magnetic field were made—electrodynamometers. Some were very good, but not always dependable or precise within the required limits.

Certain voltaic cells were known all along, but up to the advent of the Clark cell no competitor of the galvanometer or electrodynamometer appeared. With the advent of the Clark cell something was available which was a very great advance indeed. It could be reproduced in various localities with considerable precision. However, it had a considerable temperature error and it was very difficult to keep the containers sealed, also terminals cracked, necessitating the making up of new cells.

Enormous improvement was subsequently made by Edward Weston when his Standard cell was given to the world. And it is this cell in one form or another that is now used together with copies of the ohm and multiples as the basis of most electrical measurements.

How good are our volts and how permanent? I should say very nearly in the same class as the ohms, 10^{-4} class, and as a result the ampere in terms of these two standards may come perhaps in the same class. This refers to absolute values, the comparisons are probably possible to 10^{-6}.

A class of instruments for the determination of or the realization of ohms, volts, amperes, and watts, using standard cells and carefully compared resistances, are capable of giving multiples of the units throughout a wide range. I refer to the so-called bridges and potentiometers. By means of these instruments values within any reasonable range may be given and with precision well up into the 10^{-5} class. The instruments are convenient to use and including the portable or standard cells with which they are usually used are permanent and reliable. When it is considered that they are usually employed in the calibration of portable and working instruments the precision of which they are capable is for most purposes unnecessarily high. There is a still further class represented by the deflection potentiometer which is perhaps in the middle part of the 10^{-4} class.

These instruments may be used as reference standards and also for direct observations. In fact for continuous currents there is the opportunity at any time to secure definite and accurate results beyond, in most cases, the practical requirements.

Transfer to Alternating Current

However, many measurements must be made where alternating current is the medium and here in special cases the instrument commonly used employs some sort of a reflecting device for observing with some precision the indications of the moving part. These instruments may be placed in the border line between the 10^{-3} and 10^{-4} class. They do not fill the requirements with complete success. Considering what is available for the intermediate class of work of which so much must be done, *i. e.*, between the 10^{-5} class and the 10^{-3} class, time and patience are surely taxed to get what is required. Agreement on this point may not be complete and I do not offer this comment as a criticism of anything that has been done. On the other hand I feel that we need something here that we do not have at the present time and the nearer it can be to a portable instrument the better. It should be quite definitely in the 10^{-4} class for volts, amperes, and watts. For other general measurements we have the well-known portable instruments capable of a precision of say 1 part in 500 or perhaps better if advantage can be taken of a group of readings; perhaps not as good as this if small scale readings are employed.

Something of the general class of the deflection potentiometer conveniently usable on alternating current would be very helpful. There is also to consider the step which has been taken from direct current to alternating current and which rests on the assumption that a given instrument will give the same indication on alternating current as it does on direct current for the same r. m. s. values. I am not sure that this step rests on a perfectly secure basis. By this I do not mean that reasonable security cannot be assured by applying suitable corrections but many times much is taken for granted.

Higher frequencies than 60 cycles must sometimes be reckoned with but we will not yet speak of this point. However, low power factor is a serious problem.

In using portable direct-reading instruments or instruments of the intermediate class on alternating current or on alternating-current systems it is sometimes difficult to realize the precision of which the instruments are capable because of slow pulsations in the supply source and because all moving systems have not the same time of swing. In fact, in passing into alternating-current work we are of necessity compelled to work on rotating machinery and this brings in more problems. Some success has been attained by recording photographically or by other means the position of the

pointers of a group of instruments but this is not always satisfactory. It is usually best to record enough readings to overcome the effects of the fluctuations. Matters can be greatly improved in a well equipped laboratory by using very refined regulators on the supply lines. This is a great help but fills only a small part of the general need.

Many times we do not get all the precision out of portable instruments of which they are capable because we are apt to consider them as having no permanence of their own. It is common practise to compare portable instruments with a potentiometer or some other arrangement, say 10 to 100 times as precise, but not always anywhere near it, and reject all values found by previous comparisons. Almost any instrument is good enough to regard a previous comparison as having some weight. It is only a question of how much; it should not be neglected altogether.

In dealing with large values of alternating-current amperes, volts, and watts one usually cannot use the resistances so precisely comparable with reference standards. In this field it has become customary to employ instrument transformers. These transformers serve the purpose also of protecting the observer from contact with the circuit voltage. Instrument transformers are capable of quite precise work. They may be placed, when carefully checked and properly used, in the lower part of the 10^{-4} class.

Within comparatively recent years the electrical measurement art has required expansion into the high frequency field and also into the field of very low power-factor measurements which brings in a new set of requirements and in which very many curious effects must be guarded against. The recording of wave fronts which are expressed in microseconds together with a very great variety of problems which have come up in connection with sound and communication systems using wires and also the radio art have been for the most part met with a fair degree of satisfaction. This general group cannot be referred to in detail. A tremendous range of voltage current, power, and frequency must be covered, up to millions of volts and thousands of amperes and also millions of cycles per second and correspondingly small values.

Great increases in sensitivity and decreases in the amount of power required to make measurements have been made possible by the use of vacuum tubes and a large expansion in this field may be looked for. Permanence and reliability, if they could be increased, would avoid the necessity of using these methods in most cases for comparisons and placing no great reliance on permanence.

Capacitors or condensers are very useful devices as they are able to give out a definite quantity of electricity when charged with a voltage which can be determined very precisely from standard cells and suitable multiplying arrangements. It has not been my personal experience, however, that condensers are permanent and reliable to the same extent that resistors and standard cells are and improvements along this line would be desirable. For example, in magnetic measurements it would be much simpler to work from a reference standard condenser than to use mutual inductances determined by some of the well-known methods of comparison or by direct reference to length standards.

Materials for resistance standards have been greatly improved and it may seem that there is little need for further improvement. The accuracy with which comparisons can be made is perhaps ten times greater than the precision or permanence of resistance units assembled in practical apparatus without taking undue precautions which need not be taken if the temperature effects were even less than they are now.

The development of an increased class of instruments in the intermediate class (10^{-4}) is hoped for. The requirement is for something which will stand in the alternating-current field, at normal and if possible high frequencies and low power factor, in the same place that the deflection potentiometer, resistances, and standard cells stand to direct-current applications.

Very little advance among practical workers has been made along the line of combining and weighing of results and in the application of well-known and simple probability relations. It is possible to do much along this line and to develop a few simple relations which will cover most cases and which will be of enormous advantage in use. In fact, experience can develop an intuitive sense in this regard and it is surprising what can be done with comparatively crude apparatus if the results obtained with a number of determinations are made use of and sufficient care is taken to control the conditions under which the observations are made.

Summary

It is not the intention that the conclusion which should be drawn from this brief review of the situation is that conditions are unsatisfactory. Of course, the point of view might be taken that we have not made satisfactory progress in the last fifty years and that the best reference standards are not at all in the same class with available standards of length, mass, and time.

Ordinary portable instruments seem very crude in comparison with portable and every day instruments for determining mass, length, and time. On the other other hand I think we should not regard the situation at all in this light. Length, mass, and time standards have had at least ten times as long a period of development. The one that has advanced the most is time. This advance has been about 100 fold in the past 40 years.

In the electrical field, reference standards in relation to the fundamental units, and the precision of comparison with reference standards have been increased in the same period by about the same amount. In the port-

able instrument field and in indicating instruments generally, I feel that errors have been divided by about 10. We can, therefore, review the situation with some degree of satisfaction. We have by comparison equalled the rate of progress in other branches but there is still much to be done and much to be learned by occasional excursions into other lines a little outside of the electrical field.

Most important of all we can feel assured that the interest in going forward is undiminished and that during the next few years further and important advances will be made.

Discussion

For discussion of this paper see page 1324.

The Unit of Electrical Resistance;

Past History and Impending Change*

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Synopsis.—*In 1923 an amendment to the International Convention of 1875 relating to weights and measures gave the International Committee of Weights and Measures authority to deal with the electrical units. This committee was thereby confronted with two major problems concerning the magnitudes of the units and the best procedure for their legalization and maintenance. It set up an Advisory Committee of experts to advise it on all such questions. At its second meeting, in 1930, the Advisory Committee adopted a resolution to the effect that further comparisons of the manganin wire standards of resistance with mercury ohms are unnecessary.*

The mercury ohm has so long been the legalized material standard of resistance that it is regarded by many workers as a permanent institution, subject only to such slight revision of the length and mass of the mercury column as might be shown to be necessary as the technique of absolute measurements becomes more accurate. The present paper first gives a background by outlining the entire history of the unit of electrical resistance. It then gives briefly the reasons why the mercury ohm is obsolescent, as follows: the technical difficulties attending its use are very great; determinations with it are useless unless a relatively large number is made with great pains by highly skilled observers; some of the factors entering the result (waviness of the bore of the tube, uncertainty regarding the terminal resistance) remain insoluble to the present; the mercury ohm was devised in the day of the individual worker, separated from his fellows by great difficulties of communication, and hence obliged to prepare his own standards from specifications; now great national standardizing laboratories issue the most accurate standards available, backed by legal authority. The mercury ohm has always been a necessary, expensive evil. It has held its place because it could be reproduced more precisely than the agreement between the results of absolute determinations of the ohm. The technique of absolute determinations has improved until now it may fairly be said to be at least on a par with the mercury ohm as to reproducibility. The laborious and time-consuming repetitions of mercury-ohm determinations may therefore cease, and the high-grade technical effort they would require may be more profitably spent in improving the technique of absolute measurements of resistance.

IN 1923 an amendment to the international convention of 1875 relating to weights and measures was proclaimed, containing the following provision:

> "After the (international) committee (of weights and measures) shall have proceeded with the work of coordinating the measures relative to electric units and when the general conference (of weights and measures) shall have so decided by a unanimous vote, the (international) bureau (of weights and measures) will have charge of the establishment and keeping of the standards of the electric units and their test copies and also of comparing with those standards, the national or other standards of precision."

The International Committee of Weights and Measures, having thus been given legal authority to deal with electric units, was confronted with two major problems; first, whether to maintain the accepted values of the units as nearly as possible, or to revise them to accord with the fundamental c. g. s. system; second, whether to maintain the mercury ohm and silver voltameter as legalized material standards to represent the units, or to legalize the absolute units themselves. In the latter case it would become the duty of the national standardizing laboratories, having the necessary equipment and personnel, to carry out such absolute measurements as might be necessary to fix the values of their working standards. This work would be done in cooperation with the International Committee. Whatever course this committee might follow, there was an evident need for the prosecution of intensive researches to provide the technical basis for final decisions on the type of standards and on the units to be adopted for international use.

In a paper before the summer convention of the A. I. E. E. in 1927, Mr. E. C. Crittenden discussed the status of the international electric units, particularly as maintained at the Bureau of Standards, and pointed out the urgent need for further investigations of the international standards and of the experimental methods for realizing the absolute units upon which they are based. The important questions raised in this paper were referred to the Institute's Committee on Instruments and Measurements for consideration. The subcommittee appointed to study the matter prepared resolutions urging the Bureau of Standards and foreign national standardizing laboratories to undertake, as soon as possible, the additional researches needed before suitable legislation to reduce the discrepancies between the international units and the absolute units can be enacted; and to carry out the further researches that will eventually make possible the legalization of the absolute units in place of the international units. In 1928 these resolutions were approved by the Standards Committee, adopted by the Board of Directors of the Institute, and transmitted to the appropriate committees of the United States Senate and House of Representatives, the Bureau of Standards and other national standardizing laboratories, and to the International Committee of Weights and Measures at Sèvres.

*Approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

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Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

To assure itself the necessary expert counsel in regard to the important electrical questions before it, the International Committee in 1927 had set up as a subsidiary an Advisory Committee on Electricity, with a membership limited to 10. Eight members were appointed, including: one representative each from the national laboratories of Germany, Great Britain, Japan, Soviet Republics, and the United States; one from the Laboratoire Central d'Electricité at Paris; M. Guillaume, Director of the International Bureau of Weights and Measures; and Professor L. Lombardi of the Royal School of Engineers at Rome. At its first meeting in Paris in 1928 the Advisory Committee passed resolutions which were adopted by the International Committee a year later. These resolutions stressed the importance of unifying the systems of electrical measurements on a basis free from all arbitrary characteristics, by the adoption of the absolute system of units derived from the c. g. s. system; and expressed the wish that to this end researches to fix with exactness the ratios between the absolute units and the international units then in use be pursued in suitably equipped laboratories.

The second meeting of the Advisory Committee on Electricity was held in 1930 at Paris. Those who have shared the feeling that legalized material standards, such as the mercury ohm, are permanent institutions, possibly subject to slight changes in specifications, but not to be superseded by legalized abstract units, will perhaps be surprised by the following resolution, the first of two adopted at this session:

> 1. "With regard to the unit of resistance, the ohm, considering that methods of determining the absolute ohm are sufficiently advanced and that the agreement between the measurements of the coils (secondary standards) of the different (national) laboratories remains within the limits of precision of the measurements, it is not necessary at present to undertake further comparisons of the resistance coils with mercury ohms."

In order to bring out the full significance of this seemingly radical proposal, and the sound reasons for the ultimate abandonment of the mercury ohm as a legalized material standard, it will be of interest to review the history of the ohm.

After the announcement, in 1827, by Ohm of his famous "law," electrical workers began to feel the need for a unit of electrical resistance. Nature had been kinder in the matter of standards of electromotive force, and the early workers could use the electromotive force of a Daniell cell or a gravity cell as a unit. Lenz, in 1838, suggested as a standard for the unit of resistance "one foot of No. 11 copper wire;" Wheatstone, in 1843, "one foot of copper wire weighing 100 grains." Buff, in 1847, proposed to use a German-silver wire 1.5 mm. diameter and 75 cm. long. Jacobi, in 1848, had a certain piece of copper wire as a material standard for his unit, and sent copies of it to other workers. The telegraph engineers were not idle. In England they introduced a unit which was the resistance of an English mile of No. 16 copper wire; in France, the resistance of a kilometer of iron wire 4 mm. in diameter was used; in Germany, that of a German mile of No. 8 iron wire. This does not exhaust the list of arbitrary standards of resistance.

All of these proposals were faulty. Simple metal wires were difficult to obtain in sufficient purity; also, their resistivity is affected by internal stresses and the degree of annealing. The alloy wires were not of definite and exactly reproducible composition.

The use of mercury as an ideal metal for the purpose seems to have been first suggested by Marie-Davy, but Werner Siemens, in 1860, made the first practical proposal concerning it, namely, that the unit of resistance should be "The resistance at 0 deg. cent. of a prism of mercury 1 meter long and 1 square millimeter in cross section." He demonstrated that he could reproduce a unit of resistance by means of mercury in glass tubes with an error of less than 5 parts in 10,000. The unit he proposed came into use under the name "Siemens Einheit" (Siemens unit), denoted in the older literature by the abbreviation "S. E."

In 1861, at the suggestion of Sir Wm. Thomson (later Lord Kelvin) the British Association for the Advancement of Science appointed a Committee on Standards of Electrical Resistance. The original committee of six members included Prof. Wheatstone and Dr. Matthiessen. Other outstanding men added during the first eight years included Maxwell, Carey Foster, Latimer Clark, and Joule. Other names which appear later are those of Ayrton, Perry, Rayleigh, Oliver Lodge, Muirhead, Preece, Fleming, Glazebrook, J. J. Thomson, J. V. Jones, Silvanus P. Thompson, Mather, and F. E. Smith.

The "B. A. Committee" recognized that under the conditions then existing it was necessary to have material standards of resistance, but it wanted them to refer ultimately to an absolute basis. Thomson proposed as the basis the foot-grain-second system; Weber and Gauss, the millimeter-milligram-second system. The Committee finally adopted the centimeter-gram-second system. Because of the high precision (1 in 100,000) with which two nearly equal resistances could be compared, and the great scattering (about 10 per cent) between the results of absolute measurements of resistance by various observers, the Committee adopted as one of its cardinal doctrines the idea that a material standard should be developed, to have a value as nearly as possible equal to that of an ideal absolute unit; that this standard, once made, was to be regarded as a prototype standard, never to be changed even though succeeding absolute measurements might show it to be appreciably in error.

Three members of the Committee made absolute measurements of resistance at King's College, in 1863

and 1864, by rotating a short-circuited coil of copper wire, about a vertical diameter and with a measured velocity, and observing the resulting deflection of a magnetic needle at the center of the coil. From the known angular velocity, the deflection of the magnet, and the linear measurements of the coil, the resistance of the coil in absolute units was computed. On the basis of this work, the Committee constructed standard coils of platinum-silver wire, and began to issue copies of the "B. A. Unit." The Committee was dissolved in 1870 without having established a unit for either electromotive force or current.

In 1878 the investigations of Rowland at Baltimore showed that the B. A. unit was more than 1 per cent smaller than the value (10^9 c. g. s. electromagnetic units) which the B. A. Committee had intended to give it. This result was verified soon afterward by other investigators.

In 1880 the B. A. Committee was reappointed. In its reports during the next few years it was brought out that as the B. A. unit was known to be over 1 per cent in error, the question of whether to maintain this unit indefinitely or to define the ohm as 10^9 c. g. s. units might well be reconsidered. In the meantime a number of mercury standards of resistance had been set up in accordance with the proposal of Werner Siemens. The agreement between these was satisfactory from the standpoint of that time, and the results of absolute determinations had consequently been referred, directly or indirectly, to the Siemens Unit.

In 1881, at the invitation of the French Government, an International Congress of Electricians met at Paris for the purpose of establishing definitions of the electrical units in a form suitable for enactment into legislation. This Congress adopted resolutions proposing: that the ohm and the volt preserve their previous definitions, namely, 10^9 and 10^8 c. g. s. electromagnetic units, respectively; that the ohm be represented by a column of mercury 1 square millimeter in cross section, at a temperature of 0 deg. cent.; and that the ampere be defined in terms of the ohm and the volt. The coulomb and the farad also were defined. The Congress proposed that an international commission be charged with the determination, by new experiments, of the *length* of the mercury column to have a resistance of 1 ohm; that is, 10^9 c. g. s. electromagnetic units of resistance. This commission, the International Conference for the Determination of the Electrical Units, at its first meeting in Paris in 1882, decided that definite action should be deferred until further absolute measurements could be made.

In 1883, coils sent in to be tested by the B. A. Committee were certified in B. A. units, with an alternative figure in "R. ohms" (real ohms) on the basis of 1 B. A. unit equal to 0.9867 R. ohms. In 1883 the British Association granted funds to its Committee on Electrical Standards for the purchase of "Standards of resistance in terms of the (real) ohm." The purchase of these standards was deferred to await whatever action might be taken at the second meeting of the International Conference for the Determination of the Electrical Units. At this second meeting, held in 1884 at Paris, this Conference adopted 106 centimeters as the length of the mercury column, in spite of the fact that the best absolute determinations indicated that the correct length was very close to 106.3 centimeters. This compromise with the facts was made because it was considered advisable for a provisional value to be correct to the last figure given. The unit of resistance so defined was given the misleading name "legal ohm," but although it was used to some extent, it was never legalized.

The B. A. Committee indirectly defined the relation between the B. A. unit and the "legal ohm" by stating that the resistance at 0 deg. cent. of a column of mercury 100 cm. long and 1 sq. mm. cross section was equal to 0.9540 B. A. units. Thereafter, coils sent to the B. A. Committee's laboratory for test were reported in legal ohms, with a supplementary statement of the value of the legal ohm in B. A. units.

The B. A. Committee's reports of 1888 to 1892 reveal increasing doubts and difficulties because of variations in the relative values of their wire standards. In the attempt to explain some of these changes some improbable assumptions were made; for example, that in some way the nameplates of two of the coils had been interchanged, and that previous observers had used wrong values of temperature coefficient. (These difficulties, in the light of today, are explicable on the basis of the bad practise then followed, namely, the embedding of the windings in a mass of solid paraffin, which material has at least two serious drawbacks; an extremely high expansivity and a very low thermal conductivity.) These difficulties effectually negatived the Committee's original plan for a wire standard never to be altered, and established the fact that some independent and reliable method for checking the constancy of the wire standards was urgently required.

In 1892 the B. A. Committee announced that it had discontinued issuing standards in terms of the legal ohm in favor of the ohm defined as the resistance of a column of mercury 106.3 cm. long.

The 1893 report of the B. A. Committee contained an appendix by Prof. J. V. Jones describing his Lorenz apparatus for the absolute measurement of resistance and giving the results of some measurements made with it. He states that "Such an apparatus placed in the National Laboratory might with advantage be kept in constant use, not only for the calibration of low resistances, but also as embodying in concrete form a proper ultimate standard of electrical resistance." In an appendix to the 1894 report Prof. Jones communicates the result of a determination of the ohm in absolute measure with his Lorenz apparatus. He obtained 106.32 cm. as the length of a mercury column

1 sq. mm. in cross section having a resistance of one "true ohm."

An international electrical congress met in Chicago during the Exposition of 1893, with representatives of ten governments in attendance. It adopted resolutions defining eight electrical units. To distinguish them from their predecessors, they were called the *international* ohm, international ampere, etc. The international ohm was "*based upon* the ohm equal to 10^9 units of resistance in the c. g. s. system of electromagnetic units, and is *represented by* the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14.4521 grams in mass, of a constant cross-sectional area and of the length of 106.3 cm."

Because of certain defects and inconsistencies in the resolutions of the Chicago Congress, the six national governments which afterwards legislated on the matter defined the electrical units in different ways. One of the outstanding defects of the Chicago resolutions was that the ohm, ampere, and volt, which are physically related by Ohm's law, were all defined in terms of material standards.

The inconsistencies of the resolutions of the Chicago Congress were not recognized when the U. S. legislation (Act approved July 12, 1894) was enacted. They were recognized in the German law of June 1, 1898, which defines the ohm in terms of a column of mercury, the ampere in terms of a rate of deposition of silver, and the volt in terms of the ohm and the ampere. At the call of the German Reichsanstalt, an international conference was held in Berlin in 1905, at which this logical procedure was recognized as the correct one. The Berlin conference also favored replacing the Clark cell by the Weston normal cell.

In 1908 an international conference on electrical units and standards was held at London, with delegates from 21 countries in attendance. At this meeting resolutions were adopted which for the first time clearly differentiated between the fundamental (absolute) electrical units and a system of units representing the fundamental units, and sufficiently near them for practical use and to serve as a basis for legislation. The London Conference recommended the adoption as this second system, of the international ohm defined in terms of a mercury column, the international ampere defined in terms of a time-rate of deposition of silver, and the international volt and international watt defined in terms of the international ohm and international ampere. The Conference provisionally adopted the value 1.0184 international volts as the electromotive force at 20 deg. cent. of the Weston normal cell.

In 1910 an intercomparison of Weston normal cells and of silver voltameters was carried out at the Bureau of Standards by members of that institution together with one delegate each from the national standardizing laboratories of England and Germany and one from the Laboratoire Central d'Electricité of Paris. As a result of this work it was agreed to define the electromotive force of the Weston normal cell at 20 deg. cent. as 1.0183 international volts. This value was officially recognized by the Bureau of Standards on January 1, 1911, and is still in force. The international ohm was taken to be the mean of the values realized by the use of mercury ohms at the national laboratories of England and of Germany, as represented by manganin coils brought by the representatives of these institutions.

Mercury standards of resistance had been constructed by F. E. Smith at the English National Physical Laboratory during the years 1902 to 1907. A preliminary calibration of 70 glass tubes gave the data on which 11 tubes of the greatest possible uniformity of bore were selected. This matter of uniformity of bore is very important. The legal specifications assume a column of mercury of unvarying cross section, which cannot be realized. In practise, after selecting the most uniform tubes, a prolonged experimental study of the irregularities of bore is followed by a mathematical analysis to determine the "caliber correction," which is the factor by which the resistance of the column of mercury of variable cross section is greater than the resistance of an ideal column of mercury having the same length and mass, and an absolutely uniform cross section. It is desirable that this factor be as nearly unity as possible. The calibration factor of Smith's tubes exceeded unity by from 8 to 200 parts per million.

The elaborate technique of cleaning, calibrating, and cutting off the tubes, and measuring their length; of determining the mass of contained mercury; and of comparing the resistance of the tubes with the existing wire standards, is too laborious and involved to be outlined here. To be worth doing at all, this technique must be carried out by scientific men of great manipulative ability, with the observance of extreme precautions against errors in various parts of the measurement. The amount of time required is very great; so much so that in spite of the legal standing of the "mercury ohm" only a relatively small number of outstanding determinations have been made during the past thirty years.

Smith's final result was that the unit which had been in use at the National Physical Laboratory was 85 parts per million lower than the international ohm as realized by his mercury standards, and 19 parts per million lower than the international ohm as maintained at the German Reichsanstalt.

Smith's account of his work is notable for its frankness and freedom from bias. He seems to be the only worker in this field to appreciate and to discuss a source of error not even mentioned by the rest. The measured "length of the tube" which has been so carefully determined with all possible accuracy is actually the length between the end faces of a hollow glass body of approximately cylindrical form. The length which ought to be measured is that of the axis of the *wavy*

column of mercury which fills this hollow cylinder, which length must be greater than the measured distance between the ends of the tube. Even today there is no known method for determining the true length of the mercury column. Smith carried out a calculation which showed that a waviness of the axis of the column, so slight that if the axis were drawn as a line it could not be distinguished by eye from a straight line, would introduce an error of 36 parts per million.

Seven mercury-ohm investigations have been carried out at the German Reichsanstalt. Of these, that of Lindeck, reported in 1891, must be regarded as preliminary only, because of the lack of various precautions not appreciated at that time. The work begun by Kriechgauer, completed and described by Jaeger in 1895, was much more thorough in nature, but included only two tubes carried to completion, and three others begun. The two completed tubes (No. XI and No. XIV) were not of very uniform bore; their caliber factors, before cutting to length, were respectively 1.002980 and 1.001731. By choosing the most favorable locations for cutting off the surplus end portions, the caliber factors were reduced to 1.001878 and 1.000341 respectively. Four 1-ohm manganin standards, employed in this work, have played a very important role in the maintenance of the unit of resistance at the Reichsanstalt. Their relative values have been accurately determined at frequent intervals, for over 30 years, and all the mercury standards since made at the Reichsanstalt have been compared with them. From a study of the relative values of these four manganin coils the Reichsanstalt has felt justified in assuming that their *mean* value may be taken to be constant over a period of years between mercury-ohm investigations. In fact, this mean value has continued to be taken as the basis on which resistance standards have been certified by the Reichsanstalt, even though succeeding mercury-ohm investigations have indicated an average increase in the unit (as maintained by the mean of the four coils) of about 1 part per million per year. This procedure has been considered warranted by the resulting convenient continuity in the records.

Other mercury-ohm determinations at the Reichsanstalt, which cannot be discussed here in detail, are listed in the following table:

TABLE I

Investigation No.	Observers	Date of publication
3	Jaeger and Kahle	1900
4	Jaeger and Diesselhorst	1904
5	Jaeger and Diesselhorst	1905
6	Jaeger and von Steinwehr	1913
7	von Steinwehr and Schulze	1927

The work of von Steinwehr and Schulze, presumably the last investigation of this kind that will be made at the Reichsanstalt, revealed the fact that some of the older tubes had changed their linear dimensions appreciably. For example, tubes XI and XIV, originally agreeing to 7 parts per million, had drifted in 12 years until they differed by 70 parts per million. Furthermore, the agreement between these old tubes and five newer ones was not very satisfactory. It is impossible to make an accurate check of the caliber correction factor of a tube after the end portions have been cut off. The dimensional changes can be confirmed by remeasuring the length of the tube and redetermining the mass of mercury required to fill it.

Four mercury-ohm tubes were constructed by Benoit in 1884. The length of the tubes appears to have been measured with great care, but the account of the electrical technique is too meager to permit an evaluation of the accuracy of the final results. In 1908 Benoit made ten tubes, but his account is not sufficiently detailed to allow a comparison between his results and those of the national laboratories.

In 1911-12 four mercury standards were made at the Bureau of Standards by Wolff, Shoemaker, and Briggs. The technique in general was similar to that of the Reichsanstalt. The 1-ohm manganin coils with which the tubes were compared were of the sealed, oil-filled type developed by Rosa. The results of the work showed that the international ohm as defined by the mean of the four tubes was 26 parts per million smaller than the international ohm as maintained by the Bureau's wire standards, 30 parts per million smaller than the unit as defined by the German tubes, and 20 parts per million smaller than the unit defined by the English tubes.

Five mercury-tube standards were made at the National Physical Laboratory for the Electrotechnical Laboratory of Japan, and five additional standards were made at the latter institution. The work was described in a report by Obata in 1914. The results showed a good agreement between the individual tubes, and between the mean of the five tubes made in the Japanese laboratory and the mean of the five made in England.

The principal advantages of the mercury-ohm standard are as follows: The conducting material is free from internal molecular stresses, and is purifiable to a high degree. Because it can be reproduced from specifications, the mercury-ohm standard serves to record the unit of resistance of any given period, within a few parts in 100,000, for future reference. The standard is simple in principle of construction, but its numerous drawbacks have caused it to be much neglected for many years, even in countries which have legalized it.

The disadvantages of the mercury-ohm standard are as follows: The conducting material has a high temperature coefficient of resistance, which in conjunction with the high thermal resistivity of glass makes it difficult to measure the temperature of the mercury with sufficient exactness, and limits the allowable test current through the mercury column to a small value.

It is now known that mercury is not a simple substance but a mixture of isotopes. (This is not a serious objection from the practical standpoint, because the particular mixture of isotopes of mercury found in nature seems to be very closely the same everywhere.) The mercury is liable to slight contamination from the metal electrodes immersed in it. The glass tubes cannot be made straight and of uniform bore, and the error arising from waviness of the bore, though of known sign, is of undeterminable magnitude. The terminal bulbs which are necessary to make connection to the ends of the mercury column introduce an uncertainty because no method has yet been found for definitely calculating the terminal resistance with sufficient accuracy. This fact makes it necessary to adopt an empirically chosen size of end bulb. The wires which lead the current into the terminal bulbs must be small so as to minimize the conduction of heat to the mercury; they are therefore of relatively high resistance, which is a drawback in some methods of effecting the electrical comparisons.

It is of interest to summarize some of the other major reasons for the original adoption of the mercury ohm and its present obsolescence. It was originally proposed at a time when electrical workers were much isolated. The existing means of communication were slow and imperfect, and hence each worker felt the need of preparing his own standards. Although the underlying absolute electrical units were recognized from the beginning of the B. A. Committee's work, the crudities of the apparatus and technique of absolute measurements resulted in such a scattering of the values obtained by means of them, as to constitute an imperative reason for a physical standard of resistance reproducible from specifications.

The situation has altered greatly since the beginning of the present century. The technique of the mercury ohm was almost fully developed at the start, having been originated and perfected by the thermometer maker, the physicist, and the chemist. In spite of the vast amount of high-class technical labor which has been expended on the mercury ohm, it seems physically impossible to push its limit of reproducibility beyond that attained thirty years ago, namely, a few parts in 100,000. On the other hand, the technique of absolute measurements has continuously improved until now it can fairly be said to be at least on a par with the mercury ohm as to reproducibility. The day of the isolated worker, setting up his own standards, has passed, and great national standardizing laboratories now not only establish standards with an accuracy beyond the resources of most individual workers, but have a legal status which gives the stamp of authority to their certified values. These national laboratories keep in touch with one another by the exchange of standards, thus securing a measure of international uniformity.

The technique of absolute measurements can undoubtedly be improved. But even if it could do no more than maintain its present parity (as to reproducibility) with the mercury ohm, the latter, always a necessary evil, has no further reason for existence.

It is beyond the scope of this article to give more than the briefest reference to absolute measurements. The spinning coil of the B. A. Committee has already been mentioned. The Lorenz apparatus is an elementary dynamo in which a non-magnetic metallic disk is rotated within the magnetic field of two solenoids wound on non-magnetic cores and traversed by a steady direct current from an external source. The apparatus thus constitutes a sort of homopolar generator in which an electromotive force is induced radially in the rotating disk. At a certain definite speed this induced electromotive force will just balance the $I\,R$ drop between the potential terminals of a four-terminal resistance standard through which passes the steady current circulating in the solenoids. From the accurately determined speed of rotation and the mechanical measurements of the dimensions of the disk and of the field coils, the resistance of the standard between its potential terminals can be determined in terms of the c. g. s. absolute electromagnetic unit. The most outstanding Lorenz apparatus thus far built is the one at the British National Physical Laboratory.

An absolute apparatus, simpler in some respects, is used at the German Reichsanstalt and at the Bureau of Standards. It employs single-layer solenoids wound upon accurately machined cylindrical forms of non-magnetic material, such as marble. The self inductance of such a coil, in terms of the fundamental units, can be computed with high accuracy from measurements of its dimensions. The self inductance can also be measured in terms of the international ohm and the second by means of alternating-current methods, using currents of which the frequency is accurately known. From the two results one may derive the ratio of the international henry to the absolute henry, equal to 10^9 c. g. s. units of inductance. It may be shown that this ratio (which is 1.00052, plus or minus a few units in the last place) is also the ratio of the international ohm to the absolute ohm. The results obtained by this a-c. method are in close agreement with those obtained with the Lorenz apparatus. These two methods do not exhaust the list of possible means for the realization of the absolute ohm.

It would be premature to attempt to set even an approximate date for the establishment of the absolute electrical units on a legalized basis, but there are urgent reasons for reaching this goal as soon as possible. The rapid development of electrical technology in recent years has imposed new and exacting demands for accuracy of measurement and permanency of standards. Nowhere is this demand more impressive than in some phases of the radio art, where (for example) frequency measurements are carried out and standards of frequency have been developed to an accuracy (a few

parts in ten million) so great as to threaten to expose the failings of even the most highly developed time-keepers used in observatories. The world-wide sweep of radio makes *international* uniformity of frequency standards an imperative necessity. Electrical standards of various kinds, embodying the errors known to exist in the present units, are being made and distributed more widely than ever. The longer the legalization of the absolute units is deferred, the more costly and troublesome will be the necessary readjustment of the standards used in engineering and science when such legalization is ultimately accomplished.

Discussion

INTERNATIONAL STANDARD OF ELECTROMOTIVE FORCE

(EPPLEY)

DESIGN OF POTENTIOMETERS

(STEIN)

ELECTRICAL UNITS AND THEIR APPLICATION

(ROBINSON)

THE UNIT OF ELECTRICAL RESISTANCE

(BROOKS)

H. B. Brooks: Dr. Robinson's paper gives a valuable general survey of the electrical units and of the fundamental units and standards upon which the electrical units and standards are based. I am especially pleased to note his conviction that the electrical units should not be arbitrary if agreement with the fundamental units is to be maintained. This is in harmony with one of the resolutions of the Advisory Committee on Electricity at its first meeting in 1930, namely, that it is desirable to establish the electrical units on a basis free from all arbitrary character. This thought is not as recent as it may seem. At the International Conference on Electrical Units and Standards in London in 1908, the chairman (Lord Rayleigh) characterized the mercury ohm as a fifth wheel to the coach, and expressed the hope that in time it might be eliminated from the definition of the ohm. Dr. F. E. Smith, who did outstanding work on the mercury ohm years ago at the British National Physical Laboratory, expressed a firm conviction later that the time was ripe for the adoption of the absolute ohm in place of the arbitrary mercury ohm.

Dr. Silsbee and I were much interested to note Dr. Robinson's requirement for "something which will stand in the alternating-current field, . . . in the same place that the deflection potentiometer, resistance, and standard cells stand to direct-current applications." We have felt this need very clearly and Dr. Silsbee has had under development for some time a type of instrument which we hope may meet this demand. We have named the type "composite-coil instruments" although they operate directly on the electrodynamic principle. They can be wound as wattmeters, ammeters or voltmeters, and have a reading error equivalent to that of a pointer instrument having about 1,250 scale divisions. The unavoidable cost of this precision lies in the complication of supplying an auxiliary direct current and standard cell, and in somewhat less convenience in reading. A description of this type of instrument will be published shortly.

Dr. Eppley's paper is a very inclusive and carefully prepared treatment of the standard cell. Standard cells as everyday tools have become so customary that we seldom realize either the vast amount of research work which has been spent on them, or the distressing situation of the art of electrical measurements if the standard cell or an equivalent device should cease to be available.

My comments refer to the use of standard cells and to means for comparing them. I am very partial to the opposition method, in which one actually measures only the small difference between e.m.f. of the cell under test and that of the reference cell. Relative values to one part in a million are easily obtained in this way, provided one takes extraordinary pains to eliminate parasitic e.m.f., and yet the actual measuring device needs to have only moderate accuracy. I have under design a special potentiometer for standard-cell comparisons which is based on the opposition method but has novel features which avoid the two small inconveniences of the method, mentioned by Dr. Eppley. The observer need not determine whether the e.m.f. of the cell under test is higher or lower than that of the reference cell. The usual addition or subtraction is not required; the value of e.m.f. of the cell under test may be read off directly to the nearest microvolt, with interpolation to 0.1 microvolt. An exact balance can always be obtained, within the limit set by the sensitivity of the galvanometer used. Methods in which the microvolts figure (even tens of microvolts, in one prominent laboratory) is obtained by interpolation of two opposite galvanometer deflections are (in my opinion) objectionable in cases where the highest accuracy is desired. In such cases, the passing of even very small currents through the cell should be avoided as much as possible.

In the electrical instrument laboratory of the Bureau of Standards unsaturated cells for ordinary purposes are kept in heavy copper cases covered with heat-insulating material. These two expedients tend to prevent two undesirable thermal conditions; namely, inequality of temperature of the two limbs of the cell, and abrupt changes of the temperature of the cell as a whole.

I should like to recommend to users of unsaturated standard cells that they occasionally measure and record the internal resistance of their cells. Occasionally, gas bubbles may develop in the amalgam limb, and as the bubble grows in size the internal resistance of the cell increases. In time, the internal resistance may become practically infinite, and the cell useless. I have reconditioned such cells by carefully passing the amalgam limb back and forth through the flame of a bunsen burner. The heating of the entrapped gas drives it up in minute bubbles past the stopper which serves as a septum. Such cells have had their value of e.m.f. disturbed only temporarily by this process, and are useful after a reasonable period of rest.

To determine the internal resistance of a standard cell, it may be connected in opposition to a similar reference cell and the small difference of e.m.f. measured. If a resistance of one megohm (strictly, 1.018 megohm) is now connected across the cell under test, a new balance must be made because the potential difference at the cell terminals is lowered by the flow of one microampere through its internal resistance. Then the observed drop of e.m.f., in microvolts, is numerically equal to the internal resistance of the cell in ohms.

Mr. Stein's discussion of the factors affecting potentiometer design brings out the great importance of having the "internal resistance" of the potentiometer suitable for the constants of the galvanometer appropriate for the given measurement range. Formerly, this matter was not appreciated, but at present it is beginning to receive attention. However, in considering the matter of adapting the internal resistance of the potentiometer to that of the galvanometer, one should not feel obliged to obey too literally the old tradition which requires these two values of resistance to be as nearly equal as possible. It has been found independently by A. Schuster and myself[1] that the resistance of the galvanometer may differ very greatly from that of the rest of the circuit without the loss of a proportionate amount of working sensitivity. For example, if the resistance of the galvanometer coil is 100 ohms, the greatest "working sensitivity" will be had if the resistance of the potentiometer (including

1. A. Schuster, *Philosophical Magazine*, 39, 1895; p. 175; H. B. Brooks Bureau of Standards *Journal of Research*, 4, 1930, p. 297.

resistance of galvanometer suspensions and that of any other inert parts of the galvanometer) is also 100 ohms. However, the same galvanometer may be rewound with wire of the same space factor to give a coil of from 20 ohms to 500 ohms resistance, with a loss of working sensitivity nowhere in excess of 26 per cent of the optimum sensitivity.

In discussing the relative stability of low resistance and high resistance coils, it should be remembered that this matter, as regards the potentiometer, is very different from the same question as related to most other apparatus composed of resistance coils. Absolute stability in the coils of a potentiometer is unnecessary; all that is required is that the various coils of any given potentiometer shall maintain their relative values to a suitably high degree.

Mr. Stein speaks of taking account of parasitic e.m.f. by using a false zero-setting of the galvanometer. There are cases, such as in the comparison of standard cells, where this method is not reliable. A much better plan is to use a compensating device consisting of a copper-wound slide rheostat with its winding in the galvanometer circuit. A flashlight cell or similar small cell sends a current of about 0.1 milliampere through a resistor of about 15,000 ohms to a tap from the center of the copper winding. The slider of the rheostat completes the circuit to the cell. This device may be cheaply made and provides the best known way of taking care of parasitic e.m.f.

It is pleasing to see Mr. Stein call attention to the possibility of parasitic e.m.f. in the galvanometer, of as much as 5 or 10 microvolts. Some workers in the field of precise measurements are very reluctant to believe this. Under a very unfavorable combination of poor design and exposed location of galvanometer, winter weather, and heating by circulated warm air, I once observed a parasitic e.m.f. in the galvanometer of 40 microvolts.

While the relative speed of working with the deflection potentiometer does not constitute a striking advantage for the very occasional test of an instrument, it becomes important in the production of instruments, or in the checking of large numbers by users. The greatest advantage which the deflection potentiometer possesses, as compared with the laboratory standard deflection instrument, is that on the average about 98 per cent of its reading is based on the e.m.f. of the standard cell. An operating advantage of the deflection potentiometer, of some importance, is that it is difficult to bend the pointer of its galvanometer, and easy to replace the galvanometer if it should be damaged. The bending of the pointer of a laboratory standard instrument is a much more serious matter, and causes a relatively long delay in getting the instrument back into service.

H. W. Bearce: An important step in the direction of increasing accuracy of length measurement was that taken in 1927 by the International Committee of Weights and Measures by which a supplementary definition of the meter was set up in terms of light waves. The provisional equivalent was established as 1 meter equals 1553164.13 wavelengths of the red radiation from cadmium under specified standard conditions. This relation already has found extensive use in a wide variety of interferometric apparatus.

L. T. Robinson: It is pleasing to note that as far as the ohm is concerned Dr. Brooks is in accord with the idea that the primary reference standard should be the absolute determination rather than any arbitrary standard. I did not know of the results of the work of the Advisory Committee on Electricity nor of Lord Rayleigh's earlier statement. These statements and other references by Dr. Brooks indicate that the fact is quite generally appreciated that the adoption of the absolute ohm in place of the arbitrary mercury ohm is near at hand.

The degree of precision that has been attained by the Bureau of Standards in the absolute determination of the ohm is most gratifying and indicates that the ideals set down in my short paper are possible. I hope that some equally satisfactory arrangement may be made with regard to the volt.

The figures given in my paper were arrived at only by considering the needs and to obtain accuracies comparable with those of the fundamental standards. Any misgivings that I may have had have been dispelled by the papers and discussions at this session.

The instrument that Dr. Brooks and Dr. Silsbee have under development to fill the gap in the alternating current field is very interesting. If application of the first type produced is limited by some complications I am sure that once the need is fully recognized and the precision demanded is met any minor difficulties and complications will soon disappear.

In Dr. Eppley's carefully prepared and instructive paper the fact is also quite clearly brought out that the permanence of standard cells is much more satisfactory than their reproducibility in absolute values. If the absolute value of the ohm and the volt is adopted and if such satisfactory progress is made with the volt as has been reported by Dr. Brooks with reference to the ohm, the requirement for working reference standards becomes only that they be permanent and transportable. The absolute values relating to any particular copy may be accurately determined and affixed.

Several minor comments occurred to me during the reading of Mr. Stein's paper. Referring specifically to page 1304 "parasitic e.m.fs." the figures given in the paper seem high. In the dial type potentiometer with which I am most familiar these parasitic e.m.fs. do not exceed 2 microvolts and are usually less than one microvolt total for all the dials.

Mr. Stein has spoken of high resistance and low resistance potentiometers and of the advantages of the low resistance type as far as the galvanometer is concerned. Perhaps any failure to agree with the author on all his conclusions may be that most users have associated "low resistance" in their minds with instruments having slide wires.

A dial potentiometer having about 100 ohms per volt seems best and this is quite satisfactory with commercial galvanometers.

Mr. Stein shows in Appendix 3 that within reasonable limits the galvanometer requirements for potentiometers is about the same whatever the resistance.

The best performance in the higher resistance galvanometers is limited by the very large number of turns required and the consequent small space factor of the winding. On the other hand the low resistance type is limited in the very low resistance range by the resistance of the suspensions becoming such a large part of the total galvanometer circuit.

I think all the well appreciated advantages of the 20,000-ohm potentiometer can be and in fact have been realized in the dial type of about 200 ohms.

H. L. Curtis: Dr. Brooks has shown the weaknesses of the mercury ohm and indicated the intention of the International Committee to adopt the absolute ohm as the unit of electrical resistance. He has asked me to summarize the information on the absolute ohm that probably will be available to the International Committee at its next meeting in 1933.

There are only two published results which are now of importance in deciding on the difference between the present international ohm and the absolute ohm. One of these researches was completed in 1913 at the National Physical Laboratory (F. E. Smith: *Phil. Trans. Roy. Soc.*, 214, p. 27, 1913). The experimental work for the second was completed in 1914 at the Physikalisch-Technische Reichsanstalt (Grüneisen and Giebe, *Ann. der Physik*, 368, p. 179, 1920) but was not published until 1920.

There is now in progress a determination by a second independent method at the National Physical Laboratory and two entirely independent methods are being worked on at the Bureau of Standards. The Bureau of Standards is also concerned with a third method, which is being done partly at the Bureau and partly at the Bell Telephone Laboratories, Inc., of New York City.

The result obtained by the National Physical Laboratory was:

One international ohm (NPL) = 1.00052 ± 2 absolute ohms

The result obtained by the Physikalisch-Technische Reichsanstalt was:

One international ohm (PTR) = 1.00051 ± 3 absolute ohms

However, there was a comparison of standards between the Physikalisch-Technische Reichsanstalt and the National Physical Laboratory in 1914 which shows that:

One international ohm (PTR) = 1.00003 international ohm (NPL) or a difference of three parts in 100,000. Hence

One international ohm (PTR) = 1.00051 ± 3 absolute ohms (PTR)
= 1.00055 ± 2 absolute ohms (NPL)

The difference between the two absolute determinations is four parts in 100,000, which can be accounted for by the experimental errors in the determinations.

Although the Bureau of Standards has not published any detailed description of its determinations, a preliminary result was announced by Curtis, Moon and Sparks at a meeting of the National Academy of Sciences in April, 1930. This result was:

One international ohm (B.S.) = 1.00053 absolute ohms

Although this result agrees very satisfactorily with those quoted above, which were obtained 15 years earlier in the national laboratories of England and Germany, it should be emphasized that the agreement may be partly accidental, because it is not possible to determine, with an accuracy of one part in 100,000, the relations between the units of resistance now used by these laboratories and those used by them 15 years ago.

Based on present information, a resistance unit can be established which will differ from the absolute ohm certainly by less than one part in 10,000, and probably by only a few parts in 100,000. Before the 1933 meeting of the International Committee, additional results may become available. It is to be hoped that sufficient data will be at hand so that some definite action can be taken at that time.

W. B. Kouwenhoven: The use of potentiometers and standardizing equipment by utilities companies is becoming common, and today many companies have one or more Weston Standard cells. They note that the e. m. f. of these cells varies from day to day. These variations are caused by unequal temperature changes, as pointed out by Mr. Eppley. Most of these company laboratories are not in a position to justify the expense of installing equipment to maintain their cells at a uniform temperature. I have found that a simple inexpensive and fairly satisfactory solution of the problem may be obtained by protecting the cells with heat insulating material. A two-inch layer of cork gives good results.

G. Thompson: Mr. Stein's very able paper on potentiometers draws merited attention to a measuring instrument which, with the wheatstone bridge, ranks among the most precise measuring devices of any field of science. It is capable of making measurements with a precision better than one part in 10,000 using the most ordinary precautions, and with an amount of care equal to that given to precise instruments in other fields, will yield results reliable to one part in 100,000, comparable to the precision obtained with a good chronometer, theodolite or precision surveyor's tape. Designed in the early days to make measurements of voltage of the order of one volt, the potentiometer is now being employed more and more for the precise measurement of voltages of the order of 0.1 and 0.01 part of a volt. For this latter class of measurements potentiometers having a resistance as high as 10,000 ohms per volt become unsuitable.

It would be a mistake, however, to draw the conclusion from Mr. Stein's paper that the high resistance potentiometer no longer has its place in voltage measurements or even that it is inherently less accurate and reliable than the low resistance types. When all the theoretical considerations have been advanced and weighed, the results obtained in actual practical operation over a long period of time remain the deciding factor. In our company we have enjoyed daily experience, excepting Sundays, with both types of potentiometer for something over 20 years. During that time the low resistance potentiometers have given excellent service; they are used with portable galvanometers as though they were indicating instruments. Yet all of them have shown gradual changes in the resistance of the slide wire due to wear from the sliding contact near the ends of the wire, necessitating finally a return to the maker for readjustment. On the other hand, the high resistance potentiometers have never been repaired or returned to the maker, while the initial accuracy has been retained. A check of one of the high resistance potentiometers made 18 years ago showed errors which, in readings involving the use of the first dial, amounted in no case to as much as 0.005 per cent. A recent check of this same potentiometer shows that this accuracy of reading has been maintained and, furthermore, that the resistances of the 1,000-ohm coils and of the 100-ohm coils differ from those of 18 years ago by less than 0.007 per cent. It would appear, therefore, that 1,000-ohm coils may be adjusted with at least the same precision and will remain at least as permanent, as coils of lower resistance.

One of the inherent inaccuracies of the low resistance potentiometer with single dial and slide wire was due to the fact that the standard cell balance was obtained using the part of the potentiometer between the 5th and 15th coils of the main dial, whereas the unknown voltage was measured on the part beginning from the other end of the dial. This objection has been met in the arrangement used in the recent edition of this potentiometer, shown in Fig. 3 of Mr. Stein's paper. Likewise, the current flowing through the potentiometer set for the 0.1 range could not be checked against the standard cell; it was necessary to return to the 1.0 range whenever the current was to be checked. This objection also has been overcome as shown in Fig. 3. In the new arrangement the reliability of readings obtained with the 0.1 and 0.01 ranges depends, of course, upon the maintained accuracy of the shunting resistances and of the additional resistances which must be introduced for the standard cell balance. In this respect, however, the new arrangement is in no way more susceptible to error than other multiple-range potentiometers or its predecessor.

The choice of potentiometer to be used depends on the type of measurements to be made. For rapid measurements in work of an investigational character we find the single dial and slide wire type of potentiometer to be more suitable; the accuracy expected is of the order of 0.05 or 0.02 per cent. For calibration work and for precision measurements the high resistance Feussner type has given excellent service.

It has been found convenient to use dry cells as the source of current for both types of potentiometer. Mr. Stein's curve of Fig. 4 shows the slow rate at which the current changes between the 14th and 30th hours after charging the storage cell which is the source of current, but the early part of the first 14 hours, while gas is still escaping, is not so satisfactory. It has been our experience that an appreciable time must elapse after the cell has been charged before the current steadies down to the point where rechecking of the current needs to be made only at long intervals. Dry cells may be used with the low resistance type of potentiometer where the current required is 0.02 ampere since the effect of changes in room temperature takes place slowly, the discharge curve is practically horizontal for long periods of time, and rechecking of the battery current can be made with rapidity. The battery circuit is opened as soon as use of the potentiometer has been completed, and only a few minutes are required when the circuit again is closed until the current attains a stable value. With the high resistance potentiometer the draft of current is so small that the dry cells are left permanently connected in circuit. In this way it is necessary to recheck current only at

intervals of an hour or so, and the cells need to be replaced at intervals of several months. The dry cells represent a saving both in testing time and in battery expense.

Mr. Stein makes frequent reference to an apparent disadvantage in speed of galvanometer operation with the high resistance potentiometer as contrasted with the low resistance type. Reference to the catalog of his company shows that a galvanometer of a satisfactory type, having a period of six seconds, can be obtained with a 10-ohm coil or with a 500-ohm coil. In the case of the former, the published sensitivity is 0.1 microvolt per millimeter measured at the galvanometer terminals, while the recommended external damping resistance is 50 ohms. As he shows in Appendix III, the usual low resistance potentiometer introduces a resistance of only 25 ohms into the galvanometer circuit so that this particular galvanometer would be somewhat overdamped when used with such a potentiometer. On the other hand, the galvanometer with the 500-ohm coil has a recommended external damping resistance of 10,000 ohms; in consequence, the high resistance potentiometer mentioned by Mr. Stein as introducing a resistance of 5,000 ohms in the galvanometer circuit would likewise give the same degree of overdamping. The published volt sensitivity of this galvanometer, however, is 0.25 microvolt measured at the galvanometer terminals. There is no choice therefore between the two methods as to speed of operation and the sensitivity of both galvanometers is adequate where the voltage to be measured is of the order of at least 50,000 microvolts. This agrees with our general experience. As a matter of fact, work is retarded more by having the galvanometer slightly underdamped than when it is appreciably overdamped. A certain amount of time is required to adjust the potentiometer dials to a new setting and with a galvanometer having a period of the order of six seconds no time is lost if the galvanometer is somewhat overdamped.

Mr. Stein's theoretical contentions concerning volt boxes are correct. Practically, however, it is not feasible to use a volt box with as little as 10 ohms per volt for the measurement of voltages of the order of 100 volts, on account of the high power loss with resultant uncertainties of resistances in the volt box. If 20 ohms per volt be permissible then a suitable galvanometer, used with a low resistance potentiometer having a total battery circuit resistance of 100 ohms, is reduced in sensitivity to one-half the value obtained when the volt box is not used. On the other hand, the high resistance type of volt box does not ordinarily have more than 1,000 ohms per volt, so that when a potentiometer is used in which the total battery circuit resistance is 20,000 ohms, the use of the high resistance volt box reduces the galvanometer sensitivity by only one-sixth of the value enjoyed when the potentiometer was used without a volt box. Perhaps a more correct statement of the situation would be, therefore, that a high resistance volt box with consequent avoidance of heat errors can be used with a high resistance potentiometer without the loss in sensitivity that takes place when a volt box is used with a low resistance potentiometer.

There is no question that low resistance potentiometers are essential in modern industry. It is doubtful, however, if they can take the place of the high resistance potentiometer as an instrument of precision. In order to avoid the use of a slide wire which is subject to wear and sometimes to difficulties of operation, the desirable 4 and 5 dials of fixed resistance units must be secured by means of shunt circuits of some sort, often with numerous compensating units. These call for coils of odd values of ohms which can hardly be measured with the precision possible with decimal value coils although the accuracy of the potentiometer is equally dependent upon the adjusted and maintained accuracy of these coils.

The suitability of low resistance potentiometers for the measurement of thermocouple e.m.f. needs little argument. Viewed as a practical consideration, however, it is doubtful if a temperature difference as small as 0.001 cent. should be measured with a single thermocouple and a low resistance potentiometer, when an accurate determination of such a temperature difference with a resistance thermometer can be carried out so much more comfortably.

Marion Eppley: I think Mr. Stein has overestimated the difficulty of making 1,000-ohm coils that are constant to 0.01 per cent and has underestimated the difficulty of making lower valued coils of the same constancy. To illustrate this point, I have known "sealed" one ohm coils, *i. e.*, coils so enclosed as to be protected against humidity changes, action of oil, etc., to vary by more than 0.02 per cent and, on the other hand, instruments which I have checked, having many 1,000-ohm coils, have remained constant to 0.01 per cent, even though the coils were of the ordinary unenclosed type. With resistances higher in value than 1,000 ohms it is a different story.

I have used both high resistance and low resistance potentiometers and the advantages and disadvantages of both types seem to be very evenly balanced.

I am glad that the points having to do with galvanometer damping, and especially that example dealing with the reading of shunts on page 1307, second column, have been made more clear by the author's verbal explanation. It should be stressed constantly that galvanometer characteristics must be suited to the circuit and that a galvanometer giving the desired results with a low resistance network will not do the same in a network of high resistance. However, largely due to Mr. Stein's efforts, galvanometers of individual characteristics suitable to a wide range of circuits are no longer special instruments, but are stock patterns. Whether the circuit be of high resistance or of low resistance, galvanometer damping cannot be ignored. It is only fair to say that galvanometers with high critical damping resistances and adequate sensitivity are apt to have slightly longer periods than those of low critical damping resistance.

One point with reference to galvanometer damping has not been touched upon. This is the difficulty in thermocouple potentiometers of having the galvanometer critically damped when in the low resistance potentiometer circuit proper and also when across the standard cell coil which must be in the nature of things of a much higher resistance than that of the potentiometer circuit proper. Either a shunt on the galvanometer, if it has a critical damping resistance suited to the thermocouple circuit, or series resistance, if the critical damping resistance is suited to the standard cell coil, must be provided for use in one or the other circuit. It is preferable to have the critical damping resistance a little higher than the effective resistance of the potentiometer circuit plus that of the thermocouple, making up the difference with a resistance free from parasitic e.m.fs. Then a shunt on the galvanometer can be used when connected across the standard cell coil without too greatly lowering the galvanometer sensitivity.

On page 1307 at the end of the first paragraph in speaking of the calibration of the slide wire potentiometer, it is said that "If the comparative values of the main resistors are known to 0.01 per cent, the uniformity of the slide wire may be determined with this same precision." It should be noted that the uniformity of the slide wire is checked only at the 13 points given by this set-up. This check cannot tell what is between the points tested. It should be emphasized, however, that in practise slide wires can be made extremely uniform. On the other hand, the dial instrument can be checked at every point.

On page 1305, last paragraph, notice is taken of the "zero drop" in the Feussner potentiometer. This is constant if the switches are properly made. As stated, it is not proportional. In two German instruments of the highest grade which were examined, the zero drop in each instrument had an average value of 0.8 microvolts with wide variations as the switches were shifted off zero and back again. It has been reduced to 0.5 microvolt in certain other instruments with which I am familiar and is practically constant. It is thus less than 0.1 of one stud on the last dial. It would appear from an investigation of this circuit

which I have just made that the zero drop can be reduced to approximately 1/3 of 0.5 microvolts. If this is so, it will be possible to reduce the resistance of the circuit to 1,000 ohms per volt for use where such a reduction is advantageous. Such a reduction would cause the zero drop to rise to say 2 microvolts as indicated by Mr. Stein, but still this would cause an error of only 0.2 of one stud on the last dial.

The White potentiometer is made in two ranges—"10,000 microvolts" and "100,000 microvolts." This rating is somewhat ambiguous, for in either case the instrument has only four dials. The precision based upon a full scale reading is the same in either instrument, *i. e.*, 0.1 of the last dial, gained by galvanometer deflection. Stated as it is, it might seem that the 100,000 microvolt instrument would read to six significant places and, therefore, have the greater precision. This is not so. Both instruments cover only five significant places and one starts one place higher and ends one place higher than the other.

N. E. Bonn: It is very gratifying to note that the potentiometer is at present attracting considerable attention in the technical press. Only two months ago Eppley and Gray[2] published a paper describing a new American edition of the Feussner potentiometer.

While the present paper touches upon a large number of topics pertaining to potentiometer design, two features are given especial prominence. One of these is a discussion of the superiority of the low resistance potentiometer over instruments having a high internal resistance; the other is the new modification of the White potentiometer shown in Fig. 6.

The advantages of the low resistance potentiometer in affording greater sensitivity are of real importance only in measuring e.m.fs. of less than 0.1 volt. For higher voltages or in cases where the source has a high internal resistance, these advantages largely disappear, and the choice of an instrument must be based on other and more important considerations.

In the section under "Galvanometer Considerations" a case is cited involving the measurement of 0.3 volt. It is pointed out that a particular low resistance galvanometer, when used in conjunction with a certain low resistance potentiometer, exhibits good sensitivity and damping, but if the *same galvanometer* were used with a high resistance potentiometer the results would not be satisfactory. This statement is unquestionably correct, but is by no means a valid argument against the high resistance potentiometer. No one familiar with the fundamentals of electrical measurements will even attempt to use a galvanometer of such low resistance with a high resistance potentiometer. The argument could easily be reversed by assuming the availability of a suitable high resistance galvanometer, and in such an event greater sensitivity can be obtained with the high resistance potentiometer than would be the case if a low resistance potentiometer were used. As a matter of fact, one does not select a potentiometer to suit a given galvanometer, but vice versa. The cost, period, and convenience of a galvanometer are in no way dependent upon its internal resistance and critical damping. Simple and convenient galvanometers are available that are admirably suited to high resistance circuits.

In calculating the maximum errors that may arise from the manipulation of dial switches, Mr. Stein makes the assumption that compensation will always remain accurate to 0.01 per cent. In other words, a nine-ohm coil initially adjusted to equality with a group of nine one-ohm coils may be relied upon to maintain that equality to within 0.01 per cent over extended periods of time. Such an assumption does not appear to be warranted in the light of every day experience. Even granting Mr. Stein's own contention that low resistance coils can maintain their adjustment to $\pm$ 0.01 per cent, it may well happen that the changes in the coils will be of opposite sign, particularly since the nine-ohm coil must be geometrically different from the one-ohm coils. Under such conditions the maximum error from compensation must be assumed to be 0.02 per cent. If this is done, the total error from dial manipulation exceeds one step of the fifth dial for the higher range, Mr. Stein's statement to the contrary notwithstanding.

Another important consideration is the question of zero shift. In discussing the Feussner potentiometer, Mr. Stein duly points out that this instrument has an inherent false zero, yet he fails to mention the fact that the same is true of his favorite arrangement of Fig. 6. When all dials are set on zero, there are present in this circuit two presumably equal and opposite electromotive forces. One of these is due to coil Z of the upper potentiometer carrying a current of 0.01 ampere; the other is present in the coils of the lower potentiometer carrying a current of 0.001 ampere. Now assume that the resistance of coil Z will have increased to the extent of only 0.01 per cent in the course of time and that the resistance of the lower potentiometer coils will at the same time have decreased by the same relative amount. Since each of the two opposing e.m.fs. amounts to 2467.5 microvolts, the zero shift from this source alone may amount to 0.5 microvolts, and this equals five full steps on the last dial for the low range. The situation is still further aggravated if one takes into consideration that the standard cell coils of the two potentiometers may also change in opposite directions, one being of the order of 100 ohms, while the other has a value exceeding 1,000 ohms. In such an eventuality (and this is entirely within the realm of probability) the zero shift error may well exceed one full step of the fourth dial (dial V) of the low range.

In view of such relatively large possible errors, there would appear no justification for the use of two batteries and six dial switches. Better results may be obtained with a single battery and five dial switches, particularly with the Wenner circuit shown in Fig. 6.

Marion Eppley: Dr. Brooks has described his potentiometer in detail and I am of the opinion that when this is made available it will mark a great stride forward not only in the accuracy of measuring electromotive forces, but also in ease and speed.

I am heartily in accord with the method of maintaining unsaturated cells mentioned by Dr. Brooks.

Bubbles, if they form, can be removed not only by heating the amalgam side as described by Dr. Brooks, but also by tapping the amalgam limb. This is so much a knack that I hesitate to try to describe it. Cells treated by either method are quite uninjured, but should be given a period of rest.

I. Melville Stein: Referring to Dr. Robinson's comments, the value of parasitic e.m.f. given for dial switches is intended to be a safe maximum value, as are also the values given for well-designed slidewire contacts. Usually the parasitic e.m.fs. will be lower, but probably it is not safe to assume lower values. In this connection, it should be borne in mind that usually there is only a single slidewire contact in the circuit, whereas the usual number of switch contacts varies from five to ten or more. I agree fully with Dr. Robinson's comments referring to high- and low-resistance potentiometers. However, I cannot agree with his comments pertaining to galvanometer requirements. This will be clear from my later remarks in answer to other discussions of the paper.

I agree fully with Dr. Robinson that the well appreciated advantages of the 20,000-ohm potentiometer can be realized in dial type potentiometers of much lower resistance, but in designing for lower resistance, numerous precautions must be taken, as I have tried to indicate in the paper.

Referring to Dr. Brooks' discussion, I should like to add the following comments: While it is true that relative stability is the most important point in the resistance coils of a potentiometer, this consideration does not help the situation appreciably if the design requires coils of widely different values, some of them being as high as 1,000 or 10,000 ohms. This is particularly important when the effect of atmospheric humidity on relatively high resistance coils of substantially different values is considered.

2. Review of Sci. Inst., New Series, Vol. 2, No. 4.

In referring to the use of a false zero setting of the galvanometer, I did not intend to place emphasis on any particular means for obtaining a false zero. I quite agree with Dr. Brooks that electrical means for establishing the false zero are frequently more convenient than mechanical means.

Referring to Mr. Thompson's remarks, I had no intention of indicating that a high-resistance potentiometer is useless. What I attempted to point out is that the high-resistance potentiometer is always susceptible to more sources of error than a well-designed low-resistance potentiometer, and, for certain classes of measurements, the high-resistance potentiometer cannot be used at all. On the other hand, the low-resistance potentiometer is equally as good or better for making measurements of the class that can be made with a high-resistance potentiometer.

I believe that Mr. Thompson has entirely misunderstood some of my statements referring to low-resistance potentiometers. He seems to have assumed that all low-resistance potentiometers are of the slidewire type, and this is distinctly not the case, as will be seen from the illustrations and statements in the paper. However, his remarks bearing on the relative performance of high-resistance potentiometers and low-resistance potentiometers of the slidewire type deserve some comment. He makes it clear that the changes he has observed in slidewire potentiometers are due to wear on the slidewire. In a properly designed slidewire potentiometer, such wear occurs only when the instructions to clean and lubricate the slidewire, occasionally, are not followed. Repeated tests, involving as many as 500,000 movements of the contact along the slidewire, show that, within the accuracy of reading the slidewire setting, there is no change of resistance of the slidewire if the wire is reasonably clean and lubricated. Mr. Thompson's remarks referring to the constancy of high-resistance potentiometers are not clear for several reasons. No information is given pertaining to the atmospheric humidity at the time the measurements were made. Also no information is given as to the number of years that the coils had aged before the first measurements were made; in other words, the stability during the first two or three years after the potentiometer has been built is at least as important as the stability from, say, the 3rd to the 21st year.

Mr. Thompson refers to an inherent inaccuracy in a particular form of low-resistance potentiometer, due to the point on the potentiometer resistance at which the standard cell connection is made. He then states that the inaccuracy has been eliminated in a more recent design of the same instrument. Obviously, therefore, if there were any error due to this cause, it certainly would not be an inherent one. This improvement in the later design is theoretical rather than real because all of the potentiometer coils are low in value and are identical in construction.

Referring to Mr. Thompson's comments on the constancy of potentiometer batteries, it is well known that a storage battery should not be used for precise measurements with a potentiometer immediately after the battery is taken off charge. If, however, the user takes the battery off charge and connects it to the potentiometer when he leaves the laboratory, say, at 6 o'clock in the evening, the first 14 hours of discharge will have occurred by 8 o'clock in the morning when he returns to the laboratory. A 60-ampere-hour storage cell, which can be purchased for about $6.00, when used with a potentiometer drawing 0.02 ampere, will require charging only once every two or three weeks to keep it in good condition for the most precise work with a potentiometer.

Referring to Mr. Thompson's remarks on galvanometers, there is no difficulty at all in obtaining a very satisfactory galvanometer for use with a high-resistance potentiometer in making measurements at relatively high values of e.m.f. On the other hand, when it is necessary to measure very low values of e.m.f., there are no galvanometers of any make or type available that would permit the use of a high-resistance potentiometer. It is in the intermediate range of e.m.f. where the high-resistance potentiometer may still be used, but is handicapped by lack of sensitivity, that its use involves working with a galvanometer having characteristics poorer than those of a galvanometer that could be used with a low-resistance potentiometer for making the same measurements. To make clear the difficulties involved in proper damping of the galvanometer when working with a high-resistance potentiometer, it is necessary only to consider specifically the performance of the galvanometer which Mr. Thompson has mentioned as being satisfactory for use with the high-resistance potentiometer. It is logical to assume that this galvanometer has sufficient sensitivity for the measurements that are to be made with the high-resistance potentiometer and that it has this sensitivity with its proper external damping resistance in series. As Mr. Thompson has indicated, the proper external damping resistance is 10,000 ohms. With the high-resistance potentiometer, 5,000 of this 10,000 ohms is in the potentiometer, and by manipulation of the potentiometer dials, may be reduced to almost zero, thus producing a very large change in the effective damping resistance. If a low-resistance potentiometer were used, the 10,000-ohm damping resistance could be a 10,000-ohm fixed resistance coil in series with the galvanometer and the damping resistance would be altered by not more than 12.5 ohms in 10,000 due to manipulation of the potentiometer; in other words, when working with the high-resistance potentiometer, the damping resistance may vary as much as 5,000 ohms in 10,000, whereas when working with the low-resistance potentiometer, the damping resistance will not vary any more than 12.5 ohms in 10,000. It should be obvious from these figures that the damping problem, when working with the low-resistance potentiometer, is very much simplified; in this particular instance, practically eliminated.

I do not believe that Mr. Thompson will find many who will agree with him that it is better to work with a galvanometer slightly overdamped rather than slightly underdamped. It has been my own experience, and the experience of all those with whom I have discussed the matter, that it is more convenient to work with a galvanometer slightly underdamped.

Referring to volt boxes, I agree with Mr. Thompson that it is not practical to use a very low resistance in the volt box, and I indicated in the paper that it was desirable to make the total resistance of the volt box several hundred ohms per volt. The point which I attempted to make, and which, apparently, Mr. Thompson has misunderstood, is that *for the same sensitivity*, when working with a low-resistance potentiometer, it is possible to use *higher* resistance in the volt box than may be used when working with a high-resistance potentiometer. Mr. Thompson's argument in connection with the reduction in sensitivity caused by the insertion of the volt box is equivalent to saying that if the sensitivity is already very poor because of the high resistance in the potentiometer, it cannot be made much worse because of the resistance added by the volt box.

Referring to Mr. Thompson's comments on the relative accuracies of high- and low-resistance potentiometers, I think it best to refer to Appendix II, which gives concrete values for the errors to be expected in low-resistance potentiometers of good design. As to the relative merits of resistance thermometers and thermocouples for measuring temperature, it is impossible to construct a satisfactory resistance thermometer which would be small enough to make some of the temperature measurements which can be made with single junction thermocouples.

Referring to Dr. Eppley's remarks, I should say first that the statements in the paper pertaining to the relative stability of high-resistance and low-resistance coils are not estimates, as Dr. Eppley has indicated, but are based on actual observations on over 1,000 coils measured in connection with research work on the stability of resistance coils and on careful measurements

on many thousands of resistance coils in regular production. However, to further support the statement made, I quote below from Circular No. 21 of the Bureau of Standards:

> A few words may be said regarding the relative advantages of high- and low-resistance potentiometers. The chief advantage of the low resistance is in sensitivity, which increases as the resistance is reduced. In the low resistance instrument, the effect of poor insulation and possible leakage between parts is less important than in the case of high resistance. The change of the resistances with time and their variation with atmospheric humidity are also less in the case of low resistances.

I must disclaim most of the credit which Dr. Eppley very generously accords me in connection with galvanometer design. If any credit is due, it should go to the organization, of which I am but a small part.

Referring to Mr. Bonn's comments, I agree with him that in measurements that do not require high sensitivity, the high-resistance potentiometer may be used, but at some sacrifice in accuracy, as I have already indicated.

In commenting on the part of the paper having to do with "Galvanometer Considerations," Mr. Bonn bases his remarks upon a misinterpretation of what is stated in the paper. He apparently has confined his attention to certain introductory remarks, and has ignored the main point, which is that, even if the particular form of galvanometer were rewound to better adapt it to working in a high-resistance circuit and provided proper damping were taken into consideration, the required sensitivity would be impossible to obtain. Mr. Bonn's remarks relative to reversing the argument, by assuming the availability of a suitable high-resistance galvanometer, are clearly inaccurate. If the galvanometer has sufficient sensitivity for use with a high-resistance potentiometer, it will necessarily have sufficient sensitivity for use with a low-resistance potentiometer, and, in addition, the damping characteristics will be superior when used with the low-resistance potentiometer, as has been indicated above in responding to Mr. Thompson's discussion. Mr. Bonn is also in error in stating that one does not select a potentiometer to suit a given galvanometer, but rather does the reverse. It is true that in many cases where the designs of potentiometers and galvanometers have been carefully worked out for a particular purpose and the conditions are such as to permit some latitude in the choice of the galvanometer, the user may select the potentiometer first and the galvanometer afterward. However, in measurements that require either very high sensitivity or very high speed in making measurements, first attention must be given to the availability of a suitable galvanometer, and having available the proper galvanometer, a potentiometer must be chosen so as to retain, not destroy, the essential characteristics of the galvanometer. This point has been emphasized by Dr. Brooks in his discussion of the paper. I must disagree also with Mr. Bonn's further comments relative to cost, period, and convenience of galvanometers. Space is not available to make a complete answer, but most of what I should want to say in response to these comments will be found in the paper and in the discussion above.

Referring to the potentiometer shown in Fig. 6 of the paper, the assumptions which Mr. Bonn makes in discussing the error resulting from inaccuracy in the compensating coils are his own and do not agree with the statement in the paper which is as follows: "The final assumption is that the compensating coils k are adjusted to cause no error greater than 0.01 per cent in dials I and II and no error greater than 0.1 per cent in dials III and IV." This assumption, as stated in the paper, is justified by the low value of the coils in question, combined with proper design for use in a very precise potentiometer. Every day experience, even if correctly interpreted, can hardly be taken as a guide in considering the performance of an instrument designed especially for extremely precise work, as few can claim every day experience with instruments of that character. There is no justification for doubling the error indicated in the paper, as Mr. Bonn has attempted to do.

The false zero, even if it existed in the potentiometer shown in Fig. 6 of the paper, would not be an inherent one, as Mr. Bonn has indicated. The assumptions which Mr. Bonn makes relative to resistance changes, which would cause a false zero, are unwarranted, but, even if they were proper assumptions, his conclusions would still be wrong. As I have indicated in the paper and as Dr. Brooks has clearly brought out in the discussion, when using a potentiometer to make precise measurements of very low values of e.m.f., it is usually necessary to work with a false zero in order to compensate for parasitic e.m.fs. in the circuit. The false zero adjustment would take care of any error resulting from minute changes in the resistance of the coils in question. It would be ridiculous to assume that the resistance coil errors were less stable than the parasitic e.m.f. in the circuit, resulting from extraneous causes. In any precise low-voltage potentiometer, the stability of the standard cell coils needs unusual attention. In the potentiometer circuit shown in Fig. 6 of the paper, the standard cell coil in the "upper" potentiometer needs to be more stable than the one in the "lower" potentiometer because relatively low accuracy is required in the "lower" potentiometer circuit. Accordingly, it is appropriate to use a higher value for the standard cell resistance coil in the "lower" potentiometer circuit. However, should any question arise about the stability of either standard cell coil, it would be a simple matter, because only two coils are involved, to make these coils of the sealed type as developed at the Bureau of Standards.

I must disclaim any favoritism toward the potentiometer circuit shown in Fig. 6 of the paper. It was described in more detail than the others because it had not been described before. The Wenner circuit, which has not been described before, is to be described in detail in a separate paper which is to be published at an early date. This was stated in the paper.

Determination of Synchronous Machine Constants by Test

Reactances, Resistances, and Time Constants

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Associate, A. I. E. E.

Synopsis.—*In recent years there have been many advancements made in the art of analysis of three-phase synchronous machine operation. Most treatments to date have very properly stressed the theoretical point of view, the practical point of view receiving less attention due in part to the complications involved, particularly those due to saturation effects. In this paper the practical standpoint is stressed and discussed on the basis of observations made from many tests; an important feature of these tests is that saturation effects under short circuit were studied by applying short circuits to machines operating at rated voltage.*

Due to serious saturation effects, the so-called constants of synchronous machines may have different values under different conditions. What are considered to be the most important of these values are discussed, and by defining certain saturated and other values, an attempt is made to provide a definite basis for the specification of machine constants.

Part I reviews certain basic concepts and discusses practical considerations of test methods. Part II describes methods of determining the more important constants by test. Part III illustrates the application of these methods to actual power machinery, and presents tabulated test results as well as a table of typical constants. The Appendixes are devoted to certain additional tests and conclusions, and to development of formulas.

* * * * *

INTRODUCTION

THE importance of the analysis of synchronous machine operation is well evidenced by the considerable literature on this subject, representing definite steps in the advancement of the art. Briefly, the major steps in this advancement are as follows: The two-reaction theory was introduced by A. Blondel,[1] and the method of symmetrical components by C. L. Fortescue.[2] An extension of Blondel's two-reaction theory for transient conditions was recognized and adopted by C. F. Wagner and R. D. Evans,[3] for stability studies; this has been developed and amplified by R. E. Doherty and C. A. Nickle and their associates.[4,5]

The novelty of the method of analysis of synchronous machine operation under various conditions lies in the fact that the machine is characterized by a considerable number of constants—reactances, time constants, and resistances. At present there is no accepted set of definitions† which standardize the meanings of these constants. It is the purpose of this paper to discuss three-phase synchronous machine constants from the practical standpoint, to describe methods for their determination from tests, and to describe the application of these methods in the form of extensive tests made on those types of machines found in a power system. The calculation of machine constants by design formulas is treated in a companion paper by L. A. Kilgore.[6]

A major step in the development of machine theory has been the recognition and analysis of the effects of additional rotor circuits such as damper bars (amortisseurs), solid iron paths, etc. Certain machines are characterized by certain additional rotor circuits; further, the more important synchronous machines are those found in three-phase power systems. Accordingly, the machines discussed are those three-phase synchronous machines falling under the following convenient classification.

1. Turbine generators
   This refers to the common non-salient pole turbine generator with solid-iron rotor.
2. Salient-pole machines with dampers
   This refers to the various salient-pole machines equipped with damper windings (amortisseurs, etc., or their equivalent) and includes synchronous condensers and motors.
3. Salient-pole machines without dampers
   This refers to salient-pole laminated-rotor machines which have no rotor circuits other than the field winding. If a machine of this general type has additional rotor circuits such as short-circuiting pole collars, rivets, etc., it should be classified either under (2) or (3) according as the effect of these circuits is significant or insignificant.

Part I of this paper discusses practical considerations involved, and reviews certain basic concepts. Part II describes test methods by which machine constants may be determined. Part III illustrates the application of these methods to actual power machinery—such

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1. For numbered references, see Bibliography.

†A subcommittee on "Definitions of Terms used in Stability Studies," under the chairmanship of Prof. G. Dahl, has in preparation a report which gives definitions as well as descriptions of test methods of synchronous machine constants. Because of space limitations, precise definitions are omitted here; however, Prof. Dahl's committee report will be presented at the Winter Convention, N. Y., January 25-29, 1932.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

as turbine generators, synchronous condensers, etc.

I—Fundamental Concepts and Practical Considerations

Phase Sequence Currents in a Three-Phase Synchronous Machine

The analysis of synchronous machines operating under balanced and unbalanced conditions may be considered as based on two fundamentals, the method of symmetrical components, and Blondel's two-reaction theory.

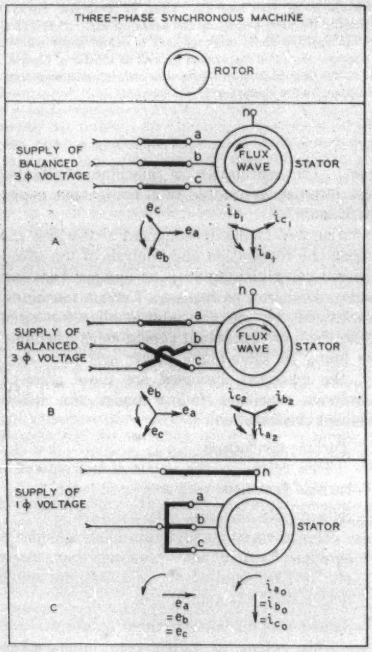

Fig. 1

A—Positive-Sequence Current
B—Negative-Sequence Current
C—Zero-Sequence Current

The application of the three sequence currents (rated-frequency) to a wye-connected machine is illustrated in Fig. 1. The set of three equal stator currents producing a flux wave which rotates at synchronous speed *with* the rotor is thereby identified as positive phase sequence (or simply positive sequence), designated I_{a1}, I_{b1}, and I_{c1} in Fig. 1A; the subscript (1) implies, conventionally, positive sequence. That set producing a flux wave rotating at synchronous speed in a direction opposite to that of the rotor is thereby identified as negative-sequence current, designated I_{a2}, I_{c2}, and I_{b2} in Fig. 1B, the subscript (2) implying negative sequence. Zero-sequence current produces no net fundamental air-gap flux because the three currents are in time phase and the three phases are spaced 120 elec. deg. apart; being in time phase (and equal) identifies them as zero-sequence currents, designated I_{a0}, I_{b0}, and I_{c0}, in Fig. 1C. Positive-, negative-, and zero-sequence currents may exist not only as fundamental-frequency currents as above, but also as harmonics; for example, under balanced conditions all triple-harmonic currents are zero sequence. Unless otherwise qualified in this paper, rated frequency quantities are implied.

Positive- and negative-sequence currents may, as far as their effect on the rotor is concerned, be replaced by a set of permanent magnets concentric with the rotor, alternately north and south poles, and equal in number to the rotor poles. The equivalent effect on the rotor of negative-sequence current is obtained by rotating the magnets at synchronous speed against the rotor direction. The equivalent effect of positive-sequence current is obtained by rotating the magnets at synchronous speed *with* the rotor; the equivalent of a *change in magnitude* of positive-sequence current is to move the magnets radially toward or away from the rotor, keeping their angular velocity constant.

Blondel's two-reaction theory deals mainly with positive-sequence current. The flux wave produced by positive-sequence current is stationary with respect to the rotor, and if the relative position of flux wave and rotor is such that a maximum amount of this flux links the field winding, tending to magnetize or demagnetize the rotor in the axis of the field poles, then that current is a direct-axis current and is affected by a direct-axis (positive-sequence) reactance. Positive-sequence current which tends to magnetize or demagnetize the rotor in the interpolar axis, so there are no net flux interlinkages with the field winding, is a quadrature axis current and is affected by a quadrature-axis (positive-sequence) reactance.

Another classification of positive-sequence currents, important for transient conditions, is according to whether armature current is suddenly or continuously applied. Neglect saturation for the moment and consider the three-phase short circuit from no load of a salient-pole machine whose only rotor circuit is the field winding. In terms of the equivalent magnets, the short circuit is equivalent to suddenly moving the magnets radially toward the rotor; this induces direct current in the field. This phenomenon may be explained in terms of the theorem of constant flux interlinkages.[17,7,8,9] If the total field current were known, the actual armature current could be calculated as the ratio of (the no-load voltage corresponding to that field current), to (synchronous reactance). It is more convenient to account for the increase in rotor excita-

tion by a fictitious reactance such that the short-circuit current is the voltage previous to short circuit (at no load) divided by this fictitious reactance. For a machine whose only rotor circuit is the field winding, this is the transient reactance;[3] the presence of dampers causes greater short-circuit current, and the fictitious reactance is called the subtransient reactance, while transient reactance retains the same significance as before—*i. e.*, it may be considered as that reactance which *would* determine the current if the only rotor circuit were the field winding.

Machine Constants as Affected by Saturation and Other Practical Considerations

Of the factors affecting machine constants, saturation is one whose effect is quite serious and which must be accounted for definitely. For example, the subtransient reactance of a turbine generator may be 14.5 per cent for a dead short circuit (no external impedance) and 16.5 per cent when external impedance causes lower current and less saturation. Saturation effects are different for different constants and types of machines. In other words, machine constants are not truly constant. It is obviously impractical to use a different value of a constant for each condition of operation. However, there is one value of each constant which appears to be generally more important than any other. Accordingly, it is advocated that in general, only one value of each constant should be specified, and that only when absolutely necessary should two values be used, *both for operation at rated voltage*, but one for *high current or saturated conditions*, and the other for rated current conditions. For example, *unless otherwise qualified, the values of direct-axis transient and subtransient reactances and short-circuit time constants will be understood as those values effective for a dead three-phase short circuit, from rated voltage at no load*, these being termed *saturated values*, for convenience.* Unless otherwise qualified, the value of *negative-sequence reactance will be understood as that value effective when negative-sequence fundamental current is circulated, the magnitude of the current being that of the negative-sequence current obtaining in a line-to-line short circuit from rated voltage at no load, this being termed the "saturated" value.* The values of negative-sequence reactance and direct-axis transient and subtransient reactances and short-circuit time constants which are effective at rated current are termed "rated current" values but may be considered generally of secondary importance. Since saturation effects for zero-sequence currents in the range usually encountered are not large, it is sufficient to state that *zero-sequence reactance will be understood as that value effective when zero-sequence fundamental current equal to rated current is circulated*, this being termed the rated current value. Unless otherwise qualified, values of *direct and quadrature axis synchronous reactances will be understood as the unsaturated values*, since these have been defined and used for many years; when qualified as saturated values, they will be understood as those effective at rated load. The views expressed above should be interpreted as definite recommendations as to what value of each constant should ordinarily be specified.

Machine constants are based on fundamental frequency components, and the sequence reactances (positive, negative, and zero sequence) are the ratios of fundamental reactive components of voltage to corresponding *circulated* fundamental currents, (as advocated by Park and Robertson).[5] Harmonics may exist in current and voltage waves, particularly under unbalanced conditions. It is impractical, in testing, to segregate fundamentals from harmonics; fortunately, however, voltages and currents associated with machines found in power systems are such that r. m. s. values of the total waves and of the fundamental components may usually be taken equal, with small error. To illustrate this point, suppose a voltage or current being measured is made up of a fundamental and a third harmonic of 20 per cent the fundamental; now machine constants are based on fundamental components, and if the peak value of the wave were used in computing, say, a certain sequence reactance, the error might be 20 per cent; on the other hand, the r. m. s. value of the fundamental and the total wave are respectively $1/\sqrt{2}$ and $(1/\sqrt{2})\sqrt{(1.00)^2 + (0.20)^2} = (1/\sqrt{2})\ 1.0198$, hence if the r. m. s. value of the total wave is taken as that of the fundamental component, the error is 0.0198 (per unit) or less than 2 per cent.

Resistance is a most important factor concerning time constants of machines. With the armature short-circuited it is usually assumed that the time constant of the field circuit depends on the resistance of the field but not of the armature; similarly, it is usually assumed that the time constant of the armature depends on the resistance of the armature but not of the field. Concerning the impedances to sequence currents, however, resistances are usually unimportant, with the one exception of negative-sequence resistance which may be important due to the rotor loss associated with it. In tests with indicating instrument measurements, a wattmeter measurement, in addition to that of voltage and current, will suffice to determine the importance of the resistance component of any sequence impedance. For simplicity, when an impedance is mainly reactance, no discrimination will be made between numerical values of impedance and reactance. For definiteness it is assumed that the resistances of field and armature circuits are the 75 deg. cent. values.

*In the case of quadrature-axis constants, the corresponding saturated values may, in order to make their test determination as practical as possible, be defined as those values effective when a machine is short-circuited from the slip test (described later) while carrying quadrature-axis current equal to rated current.

Fundamental Basis for Transient Condition Analysis

In transient condition analysis, voltage and current are conveniently divided into certain definite parts. For armature current, the first distinction to be made is that between symmetrical and asymmetrical components. The classification for armature voltage is the

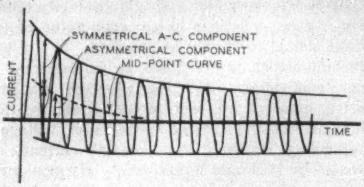

Fig. 2—Typical Wave of Short-Circuit Current

same as that for the symmetrical component of armature current; this classification gives:

1. A sustained component.
2. A slowly-decaying (or changing) component.
3. A rapidly-decaying (or changing) component.

In certain cases, some components are zero; for instance, if a short circuit occurs at such an instant that the current wave is symmetrical, the asymmetrical component is zero.

Consider the current wave in Fig. 2. The symmetrical component is alternating current and is readily determined by drawing two envelopes; half the distance between envelopes, at any instant, is the symmetrical component (envelope value). The asymmetrical component, often called the d-c. component, is determined by drawing a mid-point curve; the distance from the latter to the line of zero current at any instant is the asymmetrical component. This component is in many cases mainly direct current, but may include a large second harmonic as in the case of a three-phase short circuit of a salient-pole machine without dampers; in such a case the second harmonic has positive peak values at the same instants that the symmetrical alternating current has positive and negative peak values, hence the mid-point curve becomes a curve tangent to the total asymmetrical component as in Fig. 3.

Units

In general, it seems unnecessary to use only per unit or only percentage values, one to the exclusion of the other. In analytical work it is often a distinct advantage to use per unit values. In this paper per unit values are generally employed. In calculations of reactances from tests the possibility of error is minimized if the ohmic value is first computed, and this divided by normal ohms;[5] normal ohms, for a given kva. and voltage base, is simply the number of ohms corresponding to 1.00 per unit (or 100 per cent) impedance and is equal to rated line-to-neutral voltage divided by rated line current.

II—Description of Test Methods

Direct-Axis Reactances

Synchronous Reactance X_d. The standard A. I. E. E. definition is, in effect, as follows: Synchronous impedance is the ratio of (field current required to circulate rated current on sustained three-phase short circuit) to (the field current which *would* produce rated voltage at no load if there were no saturation); referring to Fig. 4A, $X_d = a/b$. Another way of defining X_d, one which the author has found results in a clearer conception of the meaning of X_d, is: With a sustained three-phase short circuit, and the field current at any arbitrary value, X_d is (the air-gap line voltage corresponding to this field current) divided by (the sustained short-circuit current). Thus, from Fig. 4B, $X_d = c/d$. The methods in Figs. 4A and 4B give the same result from the same data, giving the unsaturated value.

The slip test[5] affords an alternative means of determining X_d. With the field open, balanced three-phase voltage of about ¼ rated value and rated frequency is applied; either the frequency of applied voltage or the rotor speed is adjusted to give a low value of slip—as evidenced by a very low frequency voltage across the field. Fig. 5 shows the general character of an

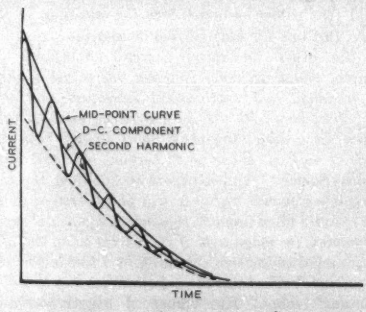

Fig. 3—Asymmetrical Component of Current for a Salient-Pole Machine without Dampers

oscillogram for this test. The slip must be lower than indicated, lest induced rotor currents make X_d appear lower than it really is. When the rotor is magnetized in the direct axis, the generated field voltage is zero, and the ratio of applied voltage to current is X_d.

Transient and Subtransient Reactances X_d' and X_d'' (Saturated Values) Determined from Analysis of a Dead Three-Phase Short Circuit from Rated Voltage at No Load. The procedure, with pertinent details, will be given for the determination of X_d' and X_d'' of any three-phase

synchronous machine, by analysis of an oscillogram of a dead three-phase short circuit from rated voltage at no load.* The machine is driven as a generator, at no load and rated voltage, and suddenly short-circuited. The analysis of the oscillogram reproduced in Fig. 6 will be described. Two envelopes and the mid-point curve are drawn for the wave of each phase. Two factors are computed for each wave, one for the asymmetrical and one for the symmetrical component.

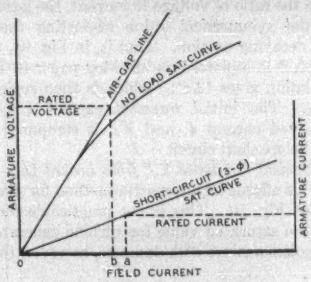

FIG. 4A—SYNCHRONOUS REACTANCE
$X_d = a/b$ (A. I. E. E. DEF.)

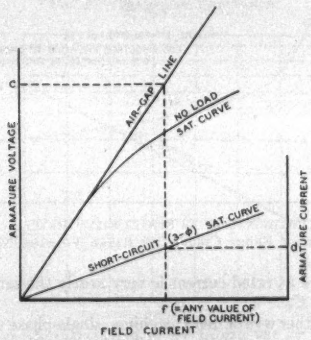

FIG. 4B—SYNCHRONOUS REACTANCE
$X_d = c/d$

Note: If c and d are in volts and amperes, and not per unit values, then e must be line-to-neutral voltage

The factor for the asymmetrical component, times the distance from the mid-point curve to the zero-current line, at any instant, converts this distance to per unit current; it equals (amperes per unit distance of deflection) ÷ (rated r. m. s. current). The factor for the symmetrical component, times the distance between envelopes (usually more conveniently measured than half the distance) converts this distance to per unit r. m. s. current; it equals $1/(2\sqrt{2})$ or 0.354 times the asymmetrical component factor. The asymmetrical components (per unit values) are plotted on semi-log paper (one scale logarithmic, one uniform) as in Fig. 7A;

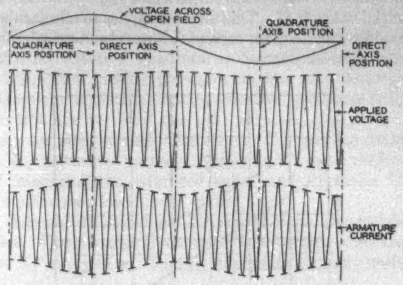

FIG. 5—"SLIP" TEST FOR X_d AND X_q

In practise the slip should be much less than indicated above. If per unit system is not used, voltage must be line-to-neutral value in formulas below

$$X_d = \frac{\text{Applied voltage}}{\text{Line current}} \text{ for direct-axis position}$$

X_q = same for quadrature-axis position

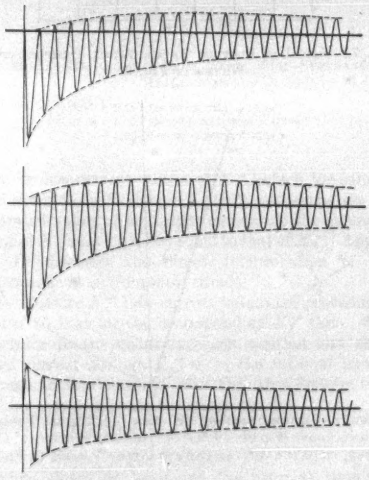

FIG. 6—TYPICAL WAVES FOR THREE-PHASE SHORT CIRCUIT

extrapolation, as shown by dotted lines, gives initial values of the asymmetrical component in each phase. The value of this component in any phase depends upon the particular instant of short circuit. The maximum possible value is obtained by the graphical method in

*If T_d' is determined from this test (it easily can be), there should be no appreciable external impedance in the field circuit, such as field resistors, etc. The ideal condition would be to supply field current directly from an "infinite d-c. bus."

Fig. 7B; a, b, and c are laid off (uniform scale) along three lines 60 deg. apart, the largest value along the middle line; perpendiculars are drawn to each line, through a, b, and c and the point where these three perpendiculars meet determines point M. Then the maximum possible asymmetrical component has an initial value $O\,M$. This method applies to all machines having short-circuit current whose symmetrical component is a sine wave.

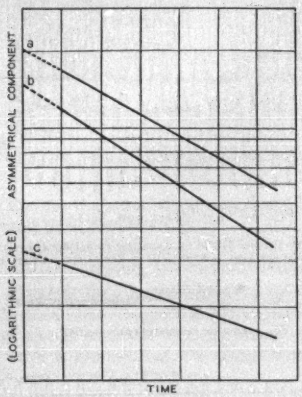

Fig. 7A—Semi-Log Plot of Asymmetrical Components of Waves in Fig. 6

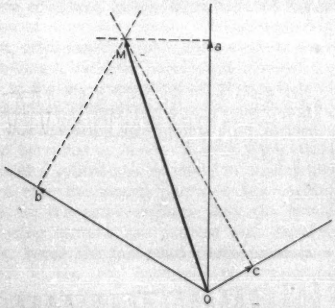

Fig. 7B—Determination of the Maximum Possible Instantaneous Value ($O\,M$) of the Asymmetrical Component, from the Actual Instantaneous Values in Fig. 7A

The average of the per unit r. m. s. values of the symmetrical components is plotted in Fig. 8. (Errors are minimized by taking this average.) Since both envelopes cannot be drawn before one-half cycle after the short circuit, there is no measured average before this time. Subtransient reactance depends on the initial (zero time) current hence the latter must be determined accurately. Most machines have, on three-phase short circuit, a wave of symmetrical current which can be taken as sinusoidal, hence the initial current $I'' = 0.707$ times the initial value of the maximum possible asymmetrical component $O\,M$. X_d'' is then the ratio of voltage before short circuit (rated voltage, hence $= 1.0$) to I''. See Fig. 8. The point determined by I'' may now be joined to the curve by a (dotted) smooth curve.

X_d' is the ratio of voltage to current, the latter being the initial symmetrical value neglecting the initial rapidly decaying portion. That is, in Fig. 9A, the difference $\Delta i'$ is replotted on semi-log paper in Fig. 9B; extrapolation gives $(\Delta i')_0$, which is then replotted in Fig. 9A. The initial *transient* current $I' = (\Delta i')_0 +$ sustained current I, and X_d' is computed as the voltage before short circuit $\div I'$.

Subtransient Reactance X_d'' from Locked Line-to-Line Test. This affords an alternative method for measuring X_d''. See Fig. 10. This is not practicable for determining the saturated value for turbine generators; for salient-pole machines with dampers, however, the value

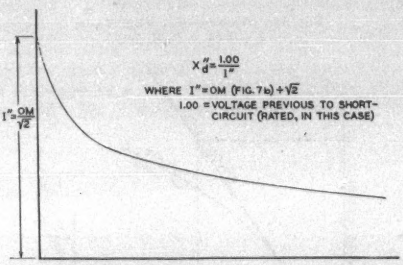

Fig. 8—Determination of Subtransient Reactance from Three-Phase Short Circuit at Rated Voltage No Load

measured at rated current is very nearly the saturated value of X_d''.

For either wye or delta machines, single-phase voltage is applied between any two line leads, with the field winding short-circuited on itself and the rotor at standstill. With about ¼ rated current circulating (rotor heating should be watched, with turbine generators), readings of voltage and field current are taken for various rotor positions, over a pole pitch. Two rotor positions are determined, one for a maximum value of the ratio of field current to applied voltage, called the direct-axis position, and the other for a minimum value, called the quadrature-axis position. With the rotor in the direct-axis position and rated current flowing, $X_d'' = 1/2$ the ratio of applied voltage to line current.

Transient Reactance X_d' (Low Current Value) from Locked Line-to-Line Tests. Only for machines without additional rotor circuits—hence only for salient-pole machines without dampers—does this test measure

X_d'. Made at low currents, it gives the unsaturated value which, when multiplied by the empirical factor 0.88 (See Appendix VIII) gives, approximately, the saturated value. Made at high currents, the saturated value may not result, because saturated conditions are caused by both armature current and field excitation, and the latter is not present in this test.

Transient Reactance X_d' by an Indirect Method. It has been shown that[1]

$$X_d' = \frac{T_d'}{T_{do}'} X_d.$$

On the basis of tests performed, it is stated in Appendix

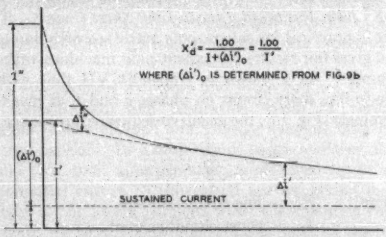

FIG. 9A—TRANSIENT REACTANCE

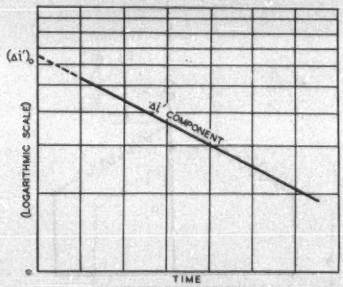

FIG. 9B—EXTRAPOLATION OF $\Delta i'$ COMPONENT OF FIG. 9A

VIII that the ratio of saturated to unsaturated values of X_d' and of T_d' may be taken as 0.88; accordingly,

$$X_d' \begin{pmatrix}\text{saturated}\\ \text{value}\end{pmatrix} = 0.88 \frac{T_d' \begin{pmatrix}\text{unsaturated}\\ \text{value}\end{pmatrix}}{T_{do}'} X_d \quad (1)$$

$$= \frac{T_d' \begin{pmatrix}\text{saturated}\\ \text{value}\end{pmatrix}}{T_{do}'} X_d \quad (2)$$

QUADRATURE-AXIS REACTANCES

Synchronous Reactance X_q. The unsaturated value is obtained from the slip test at the same time that X_d is measured. With the rotor magnetized in the quadrature axis, as evidenced by positive or negative peak voltage across the field, X_q is the ratio of applied voltage to line current. See Fig. 5. A method of determining the saturated value is given in Appendix I. For salient-pole machines without dampers, the locked line-to-line test, with readings taken at low currents in the quadrature-axis position, also determines X_q.

Transient Reactance X_q'. Since salient-pole machines have no effective quadrature-axis rotor circuit, $X_q' = X_q$. The turbine generator is the only common example of a machine having effective quadrature-axis circuits, and for most practical purposes, and particularly for short-circuit calculations, X_q' may be taken $= X_d'$ (saturated values). Appendix II gives a method of measuring the saturated value, and Appendix III a method for a low-current value.

Subtransient Reactance X_q'' from Locked Line-to-Line Test. The remarks given under the corresponding test for X_d'' apply here. That is, this test is normally useful

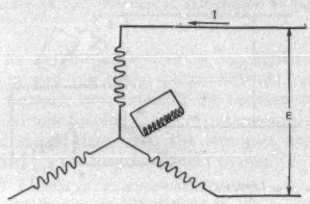

FIG. 10—LOCKED LINE-TO-LINE TEST. FOR SUBTRANSIENT REACTANCES

$X_d'' = E/2\,I$ for direct axis position
$X_q'' = E/2\,I$ for quadrature axis position
E = applied single-phase voltage

only for low-current values of X_q'', which for salient-pole machines with dampers equals approximately the saturated value. For turbine generators the saturated value of X_q'' may be taken equal to that of X_d''. Appendix IV discusses the direct determination of X_q'' (saturated value) from short circuit.

To measure X_q'' (low-current value) the procedure is similar to that for the corresponding X_d'' test. With the rotor in the quadrature-axis position and about rated current flowing, X_q'' is ½ the ratio of applied voltage to line current. See Fig. 10. Serious rotor heating may result with turbine generators.

ZERO-SEQUENCE REACTANCE X_0

Locked Zero-Sequence Reactance Test. With the field winding closed on itself and the rotor at standstill, single-phase voltage is applied to the phases connected either in series (Fig. 11A) or parallel (Fig. 11B); with approximately rated current per phase circulating, zero-sequence impedance is the ratio of voltage per phase to current per phase. If the resistance component is small, this ratio may be taken as X_0. This reactance is a minimum for a ⅔ pitch armature winding, and if

the pitch is near this value, making X_0 low, the series connection has an advantage over the parallel connection since the latter requires a source of relatively high current at low voltage. Results in Part III show that X_0 is little affected by whether the rotor is running or at standstill.

X_0 from Sustained Double Line-to-Neutral Short Circuit. The locked test above may be at a disadvantage due to the requirement of a source of single-phase power. The method to be described, believed

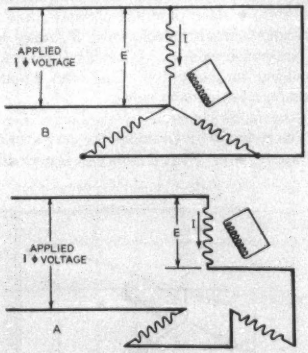

FIG. 11—LOCKED ZERO-SEQUENCE REACTANCE TEST
$X_0 \approx E/I$

A—Series connection
B—Parallel connection

to be presented for the first time, is developed in Appendix V; it has the advantage that no source of single-phase power is required, and the disadvantage that serious rotor heating may result with turbine generators, due to the flow of negative-sequence current. See Fig. 12. It is shown in Appendix V that with a double line-to-neutral short circuit at the terminals,

$$\check{Z}_0 = -\frac{\check{E}_a}{\check{I}_n} \tag{3}$$

where $\check{E}_a$ = voltage on the open phase.
$\check{I}_n$ = neutral current.

If the resistance component is small,

$$X_0 \approx \frac{E_a}{I_n} \tag{4}$$

Since X_0 is the value effective when zero-sequence current equals rated current, the field current should be adjusted to give a value of $I_n = 3$ times rated current, but good results may be obtained at lower current.

NOTE: As in all sustained short circuits, the high currents associated with sudden short circuits are avoided if the short circuit is applied with no field current flowing, and then the field current built up and adjusted to give the desired armature current.

NEGATIVE-SEQUENCE REACTANCE X_2 [16]

The variation of X_2, depending on whether the negative-sequence voltage or current is fundamental, is discussed in Appendix VI. In general, X_2 may be taken as the arithmetical average of X_d'' and X_q'', this being the value obtaining when fundamental current is circulated. For salient-pole machines, X_d'' and X_q'' as determined from a locked line-to-line test can be averaged to give X_2. For turbine generators, X_2 must be the average of certain saturated values of X_d'' and X_q'', which values are usually not obtained from the locked line-to-line test; however, as concluded in a discussion in Part III, it may be said that for turbine generators, $X_d'' = X_q'' = X_2$ (saturated values).

X_2 from Sustained Line-to-Line Short Circuit. This test is practical for salient-pole machines with dampers and gives fair results for salient-pole machines without dampers. It is shown in Appendix VII that with a line-to-line short circuit on phases b and c, at machine terminals (Fig. 13), the negative-sequence impedance

$$\check{Z}_2 = \frac{-j}{\sqrt{3}} \frac{\check{E}_{ab}}{\check{I}_b} \tag{5}$$

where $\check{E}_{ab}$ = line-to-line voltage.
$\check{I}_b$ = short-circuit current.

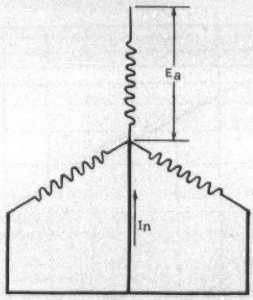

FIG. 12—X_0 FROM DOUBLE LINE-TO-NEUTRAL SHORT CIRCUIT

$$X_0 \sim \frac{E_a}{I_n}$$

If W_2 = watts measured using E_{ab} and I_b,

$$X_2 = \frac{E_{ab}}{\sqrt{3}\, I_b} \cdot \frac{W_2}{E_{ab}\ I_b} \tag{6}$$

If $W_2 \approx E_{ab}\ I_b$ which means a negligible resistance component,

$$X_2 \approx \frac{E_{ab}}{\sqrt{3}\, I_b} \tag{7}$$

There should be no appreciable external resistance or inductance in the field circuit, particularly for salient-pole machines without dampers. The test is made with negative-sequence current equal to rated current, hence

armature current is 1.73 (per unit value). For turbine generators, such a current value may result in serious rotor heating, hence it is usually best to take readings at lower currents, and extrapolate the results.

Time Constants

Consider a coil having constant inductance L (henrys or abhenrys) and constant resistance R (ohms or abohms). If it is short-circuited on itself at any instant at which it carries current I, the current decays according to the familiar exponential expression $i = I/\epsilon^{t/T}$ where i is the current at time t seconds, $T = L/R$ seconds, and $\epsilon = 2.718$ = base of Napierian logarithms. If $t = T$, $i = I/\epsilon = 0.368\,I$ hence the time constant may be defined either as the ratio L/R, or as the time in seconds for a decaying non-sustained current to decrease to 0.368 of its initial value, assuming exponential decay.

In synchronous machines the rotor circuits are not all concentrated, and saturation exists to various degrees, hence current and voltage changes may not be exponential. In a majority of cases, however, it is found that good approximation is obtained by fitting exponential curves to the actual curves, giving more weight to the initial portions when the changing components are large. Convenient use is made of the fact

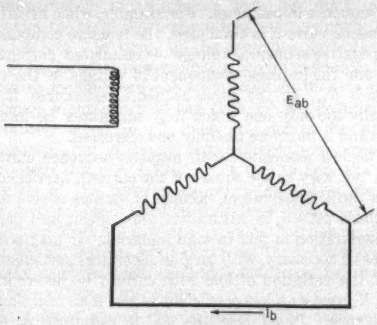

Fig. 13—X_2 from Line-to-Line Short Circuit

$$X_2 = \frac{E_{ab}}{\sqrt{3}\,I_b}$$

that an exponentially decaying quantity is a straight line when plotted on semi-log paper; two points which conveniently determine this line are the value I at zero time and the value 0.368 I at time equal to the time constant. It is a very significant fact that a comparison between actual component and fitted exponential is deceiving if made of their semi-log plots; comparison should always be made of Cartesian coordinate plots of actual and fitted curves.

Armature Short-Circuit Time Constant T_a. This time constant applies to both the d-c. and second-harmonic components of the asymmetrical component of armature current (see Bibliography 4—Part V). As shown in Part III, the asymmetrical component of three-phase short-circuit current may not decay with the same time constant, in all three phases. For simplicity, T_a will be taken as the time constant of that one of the two largest components which has initially the most gradual slope. For example, in Fig. 7A, T_a is taken as the time in seconds for the (asymmetrical) current in phase a to decrease to 0.368 of its initial value.

An alternative means of measuring T_a, not satis-

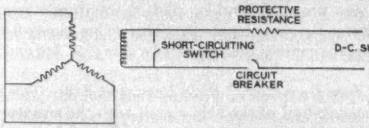

Fig. 14—Connections for Field Decrement Test

Determination of T_d'. (Armature is short-circuited all during test; field is suddenly short-circuited)

factory for turbine generators, is to circulate direct current in two (or three) phases, by applying d-c. voltage between two terminals (or between two terminals tied together and the third); the rotor is driven at rated speed; then when the terminals are short-circuited, T_a is the time for current to decay to 0.368 of its initial value.

Or, knowing the d-c. resistance of the armature r_a, and X_2 (saturated value), T_a may be computed from[5]

$$T_a = \frac{1}{2\pi f}\,\frac{X_2}{r_a} \tag{8}$$

Where f = rated frequency and X_2 and r_a are in the same units.

Direct-Axis Time Constants

Short-Circuit Transient Time Constant T_d'. This is the time constant of the field circuit with the armature short-circuited, hence is the time-constant of the $\Delta\, i'$ component in Fig. 9A, and is readily determined from the plot in Fig. 9B. It is the saturated value that is thus obtained from analysis of a dead three-phase short-circuit from rated voltage at no load. External resistance or inductance in the field circuit affects field-current decrement, hence the ideal condition for determining T_d' directly is to have the field current supplied from an infinite d-c. bus. In practise, it is desirable to have negligible external inductance and resistance in the field circuit, as obtained by use of a compensated exciter.

Theoretically, the variation of $\Delta\, i'$ should be exponential only for a state of no saturation. Tests show, however, that under short circuit, a single exponential curve will usually fit $\Delta\, i'$ out to where it has reached about ¼ of its initial value, and for that reason, and until a more accurate and sufficiently

simple analysis is available, it is proposed that the time constant of the fitted exponential be used.

T_d' from Short-Circuit Field-Decrement Test. This is a simple method of measuring the unsaturated value of T_d' for salient-pole machines but is generally unsatisfactory for turbine generators. See Fig. 14. With a sustained three-phase short circuit at machine terminals and a fraction (about ½) of rated current flowing, the field winding is suddenly short-circuited on itself, and an oscillographic record made of armature and field currents, the latter being recorded because it usually has a sudden initial change which helps in fixing zero time. A semi-log plot of armature current then gives T_d' (unsaturated). This value times 0.88 gives approximately the saturated value (see Appendix VIII).

T_{do}' from Open-Circuit Field-Decrement Test. This is the time constant of the field circuit with the armature open. As with the similar test for T_d', this test is convenient for salient-pole machines but is generally unsatisfactory for turbine generators. See Fig. 15. With the machine running at about ½ rated voltage at no load, the field winding is suddenly short-circuited on itself, and armature voltage and field current recorded by oscillograph. T_{do}' is determined from a semi-log plot of armature voltage.

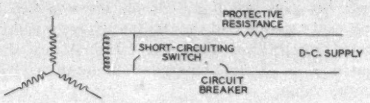

FIG. 15—CONNECTIONS FOR FIELD DECREMENT TEST

Determination of T_{do}'. (Armature is open)

Short-Circuit Subtransient Time Constant T_d''. This, like T_{do}'', has significance only for machines with additional rotor circuits; it is the time constant of the $\Delta i''$ component in Fig. 9A. This component may sometimes be only roughly approximated by a fitted exponential. A semi-log plot is made of $\Delta i''$ and a straight line fitted to it. In Part III, Fig. 21 shows the good accuracy obtained by fitting an exponential to $\Delta i''$ for a large turbine generator.

Open-Circuit Subtransient Time Constant T_{do}''. This can be obtained from a modification of the slip test with the field winding closed on itself; the rotor is magnetized in the direct axis, the applied voltage suddenly disconnected, and terminal voltage recorded by oscillograph. From a semi-log plot of voltage, the slowly decaying component is determined; the difference between the total voltage and the slowly decaying component is plotted on semi-log paper and the time constant of the fitted exponential is T_{do}''.

QUADRATURE-AXIS TIME CONSTANTS

Short-Circuit Transient Time Constant T_q'. This constant, like T_{qo}', has practical significance only in case of turbine generators; it can be determined from the same test used for X_q' (Appendix II). Rated current is circulated as in the slip test, and when the rotor is magnetized in the quadrature axis, a three-phase short circuit is applied. T_q' is obtained from a semi-log plot of the symmetrical component of current, neglecting the rapidly decaying initial portion.

Open-Circuit Transient Time Constant T_{qo}'. This can be determined from the same voltage-decrement test made for the low-current value of X_q' (See Appendix III), as the time constant of voltage decrement, neglecting the initial rapidly-decaying portion.

Short-Circuit Subtransient Time Constant T_q''. This constant, like T_{qo}'', has significance only for turbine generators and salient-pole machines with dampers; it is determined from the same test used for X_q' and T_q' as the time constant of the difference between the total short-circuit current and the slowly-decaying component.

Open-Circuit Subtransient Time Constant T_{qo}''. This is determined from the same voltage-decrement test used for T_{qo}' (See Appendix III) as the time constant of the difference between total voltage and its slowly-decaying component.

RESISTANCES

The zero-, positive-, and negative-sequence resistances are simply the resistance components of the corresponding sequence impedances. For example, when negative-sequence current is circulated, the reactive component of negative-sequence voltage ÷ circulated current is X_2, and the in-phase component of voltage ÷ the current is R_2. Of the three sequence resistances R_2 is usually the only one which may sometimes be important and is therefore the only one discussed.

The loss associated with negative-sequence current may not vary as the square of the current, particularly with turbine generators, because of various effects such as those caused by saturation, skin effect, and depth of penetration of flux in solid material. It may sometimes be necessary, with turbine generators particularly, that the variation of loss with current be determined and R_2 used as a loss coefficient rather than as a simple resistance. *In all cases, R_2 will be considered as that value effective at rated negative-sequence current.* Tests indicate that this loss varies approximately as the 1.8 power of negative-sequence current, for turbine generators, for current of the order of rated current. Unless such a machine is designed for single-phase operation, such a value of current may cause serious rotor heating when maintained for anything more than a short time; results obtained at low-current values can be extrapolated. In the tests below there should be no external resistance or inductance in the field circuit, especially for salient-pole machines without dampers.

R_2 from Loss Measurement. In general, the most accurate and practical means of determining R_2 is by actual measurement of energy loss in a sustained line-to-line short circuit. This may be accomplished by a retardation test; by measurement of shaft input under

two conditions—at no load, and with the machine short-circuited (line-to-line). C. F. Wagner has thoroughly analyzed the distribution and significance of negative-sequence loss and shown,[10] based on certain well warranted assumptions, that

$$R_2 = \frac{\text{Net Shaft Input}}{2\,I_2^2} \tag{9}$$

which can be written

$$R_2 = 1.5\,\frac{\text{Net Shaft Input}}{I_b^2} \tag{10}$$

Where

I_2 = negative-sequence current.
= 0.577 I_b.
I_b = line-to-line short-circuit current.
Net Shaft Input = (Prime-mover input while machine is short-circuited.
– Prime-mover input at no load.
– Prime-mover losses.
– Friction and windage losses).

If practical and not per unit values are used, the input per phase, or ⅓ the net shaft input, is used. If loss is measured by a retardation test, net shaft input becomes the total short-circuit loss minus windage and friction loss. In terms of the sequence networks,[13-14] $I_2^2\,R_2$ is electrical power input when a machine carries negative-sequence current I_2; in addition there is shaft (torque) input which is less than $I_2^2\,R_2$, being more accurately equal to I_2^2 times (R_2 minus the stator resistance), but in general it is sufficiently accurate to take shaft input = $I_2^2\,R_2$. It is well to note that regardless of the latter assumption, in system studies the electrical input must be considered separately from the direct shaft input whenever more than one machine is involved, because while direct shaft input is supplied by the machine itself, electrical input may be divided between different machines.

R_2 from Wattmeter Measurement. The accuracy of this alternative method depends critically upon the accuracy of voltage, current, and watt measurements. Refer to Fig. 13. With a sustained line-to-line short circuit at machine terminals (as in the test above), and with short-circuit current = 1.73 times rated current, we may write—from formulas in Appendix VII,

$$R_2 = \frac{E_{ab}}{\sqrt{3}\,I_b}\sqrt{1-\left(\frac{W_2}{E_{ab}\,I_b}\right)^2} \tag{11}$$

Where

E_{ab} = line-to-line voltage.
I_b = short-circuit current.
W_2 = measurement of watts obtained by using E_{ab} and I_b.

III—Illustrations of Application of Test Methods

This part deals with descriptions of tests made in accordance with methods described in Part II. Illustrations are given not of all the methods previously described, but of those which concern the more important constants—such as direct-axis reactances and time constants.

General Description of Test Facilities and Set-Up. Tests were made with those facilities available in the company with which the author is associated. The usual precautions were taken relative to indicating instruments, and particular care was exercised in oscillographic measurements since many important constants are determined by oscillogram analysis. Appreciating the fact that currents in sudden short circuits contain large asymmetrical components which might saturate current transformers, three non-inductive shunts[15] were employed (Fig. 16) for most sudden short-circuits. Each shunt consists of two brass cylinders

Fig. 16—Non-Inductive Current Shunts

separated by mica and sweated together at one end; the inner cylinder is tapped by two soldered connections which lead to the oscillograph.

Sustained short circuits were made with armature terminals short-circuited with short connecting links bolted in place with the machine at standstill; the condition of sustained short circuit was obtained by driving the machine at rated speed and then applying field current, thus eliminating the high currents associated with sudden short circuits. The latter were effected by a standard three-phase 0-30 circuit breaker (Fig. 17). The shunts were mounted on a platform built on the side of the breaker structure. For the safety of the oscillograph operator the shunts were grounded as shown in Fig. 18.

To eliminate the necessity of a prime mover in the sudden short circuits of larger machines, it was often convenient to run the machine as a motor at unity power factor at or above synchronous speed, then to

suddenly disconnect the power supply and apply the short circuit as the machine reached synchronous speed; this allows a speed decrease, but does not affect the current magnitude appreciably because both generated voltage and reactance decrease with frequency. In all such tests, a constant-frequency timing wave was recorded.

FIG. 17—CIRCUIT BREAKER USED IN SUDDEN SHORT-CIRCUIT TEST

Synchronous Reactances X_d and X_q. Table I gives some values of X_d and X_q as obtained from slip tests, also values of X_d determined from saturation curves.

TABLE I

Machine	X_d From sat. curves	X_d From slip test	X_q From slip test	Voltage applied in slip test
Turbine generator (18,750 kva.)	1.15	1.10	1.10	0.35
Synchronous motor (750 hp.)	1.30	1.35	.97	0.50
Salient-pole generator (100 kva.)	1.25	1.25	.87	0.48

X_d' and X_d'' from a Three-Phase Short-Circuit at Rated Voltage No Load. A test on a 75,000-kva. turbine generator will be described. The oscillogram in Fig. 19 is of a dead three-phase short circuit at rated voltage (11,600 volts). Current in each phase is first separated into symmetrical and asymmetrical components (the latter is d-c. only, since $X_d'' = X_q''$). Fig. 20A is a semi-log plot of the d-c. components and gives the initial values 9.00, 8.20, and 1.80, which when multiplied by averaging factors determined from the symmetrical components, give the corrected values 9.13, 7.79, and 1.85. Fig. 20B gives the initial value of the maximum possible d-c. component = 9.85. The initial value of a-c. (subtransient current) is $I'' = 9.85/\sqrt{2} = 6.96$; speed and voltage just previous to short circuit were normal,* hence

$$X_d'' = \frac{\text{Voltage}}{I''} = \frac{1.00}{6.96} = 0.144 \text{ (or 14.4 per cent)}$$

The symmetrical components are plotted in Fig. 21 and an average curve drawn. Sustained current

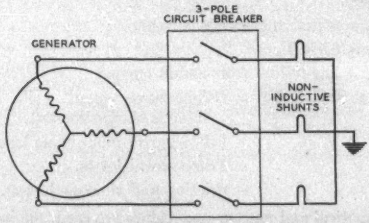

FIG. 18—DIAGRAM FOR THREE-PHASE SHORT-CIRCUIT TESTS

$I = 1.07$, shown as a dotted horizontal line. A semi-log plot of $\Delta i'$ in Fig. 22A gives the extrapolated initial value $(\Delta i')_0$ which is replotted in Fig. 21. The initial transient current $= I' = I + (\Delta i')_0 = 4.20$, hence

$$X_d' = \frac{\text{Voltage}}{I'} = \frac{1.00}{4.20} = 0.238 \text{ (or 23.8 per cent)}$$

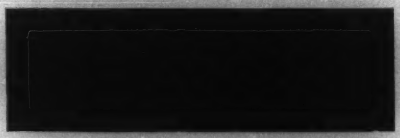

FIG. 19—FIRST 0.5 SEC. OF OSCILLOGRAM OF THREE-PHASE SHORT CIRCUIT AT RATED VOLTAGE, NO LOAD, 75,000-KVA. TURBINE GENERATOR

Highest peak current = 75,000 amperes. Note that the current in the bottom wave does not go through zero until several cycles after the short circuit; this is due to the fact that the asymmetrical component has initially a slower decrement than the symmetrical component

X_d'' from Locked Line-to-Line Test. The important effect of saturation on X_d'' for turbine generators is illustrated by Fig. 23, where reactance values at currents less than 2.0 were obtained from locked line-to-line tests, and those at higher currents from sudden three-phase short circuits. This also illustrates the

*If speed is not synchronous at instant of short circuit, the voltage used in computing X_d' and X_d'' should be read from the no-load saturation curve for the existing field current.

inadequacy of this locked test for the determination of X_d'' for turbine generators.

Fig. 24 shows the small difference between saturated and low-current values of X_d'' for salient-pole machines.

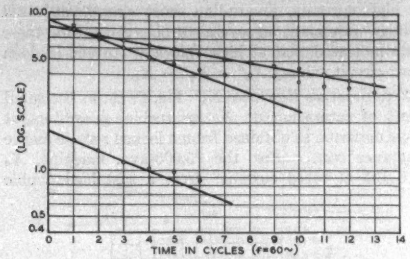

FIG. 20A—ASYMMETRICAL COMPONENTS FROM FIG. 19

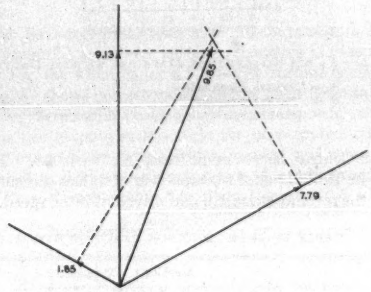

FIG. 20B—MAXIMUM POSSIBLE ASYMMETRICAL COMPONENT

(Instantaneous value)

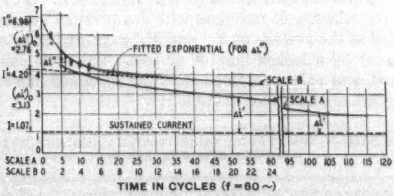

FIG. 21—SYMMETRICAL A-C. COMPONENT TAKEN FROM WAVE IN FIG. 19

$$X_d'' = \frac{1.00}{6.96} = 0.144$$

$$X_d' = \frac{1.00}{4.20} = 0.238$$

It is important to note that the value obtained at about rated current from this locked test is essentially the same as the saturated value obtained from a dead three-phase short circuit from rated voltage at no load.

X_q for Salient-Pole Machines without Dampers. X_q (unsaturated value) is determined from the locked line-to-line test at low currents (¼ or less). Fig. 25

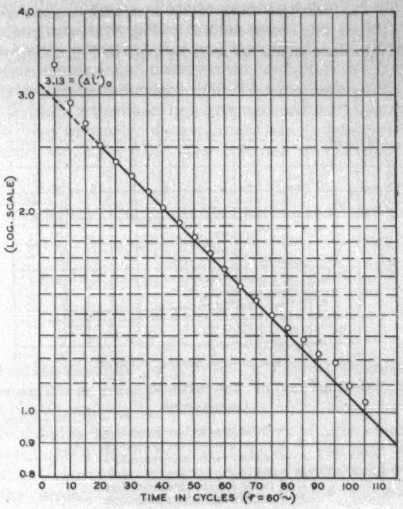

FIG 22A—Δ i' COMPONENT

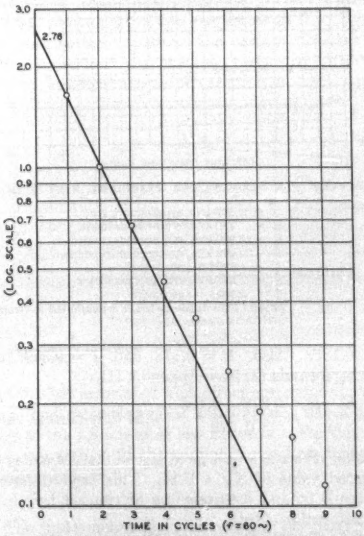

FIG. 22B—Δ i" COMPONENT

shows saturation effects at higher currents. When operating under load, the reduction in X_q may be still greater (Appendix I).

Low-Current Value of X_q' for a Turbine Generator. This was measured on an 18,750-kva. turbine generator by the method in Appendix III. Fig. 26 shows terminal voltage decrement. A semi-log plot in Fig. 27A gives

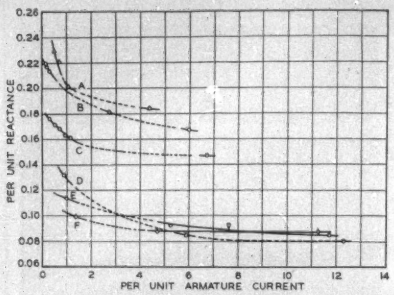

FIG. 23—VARIATION OF X_d'' WITH CURRENT FOR TURBINE GENERATORS

A = 3,125 kva. turbine generators
B = 43,750 kva. turbine generators
C = 75,000 kva. turbine generators
D = 9,375 kva. turbine generators
E = 18,750 kva. turbine generators
F = 12,500 kva. turbine generators

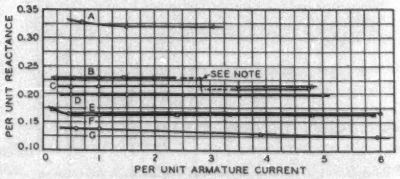

FIG. 24—X_d'' FOR SALIENT-POLE MACHINES WITH DAMPERS

A = 240 kva. synchronous motor
B = 780 kva. synchronous motor
C = 15,000 kva. synchronous condenser
D = 30,000 kva. synchronous condenser
E = 100 kva. generator
F = 331 kva. synchronous motor
G = 5,000 kva. generator
NOTE: This apparent break is attributed to errors

$E' = 0.185$. Also, $E = 0.378$ and $I = 0.379$, and applying formula (2) from Appendix III,

$$X_q' = \frac{E - E'}{I} = \frac{0.378 - 0.185}{0.379} = 0.51 \text{ (low-current value)}$$

A similar direct axis test gave $X_d' = 0.40$, whereas the saturated value of $X_d' = 0.14$. This large difference between 0.14 and 0.40 may be attributed largely to rotor saturation effects.

X_q'' for Turbine Generators. For such machines, the ratio X_q''/X_d'' varies from about 1.0 under saturated conditions to 1.3 for low-current values. Fig. 28A shows X_q'' at low currents, for a 75,000-kva. turbine generator, as found from the locked line-to-line test. In order to study the relative values of X_d'' and X_q'' at high currents, line-to-line short circuits at rated voltage were made on several turbine generators; these tests indicated that at high values of current (sudden short-circuit values), $X_d'' = X_q'' = X_2$.

Zero-Sequence Reactance X_0. Fig. 29 shows the small effect of saturation on X_0 for turbine generators, at lower currents, as obtained from a locked zero-sequence reactance test. For the 75,000-kva. machine, $X_0 = 0.105$ at rated current; from a sustained double

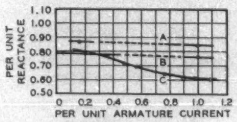

FIG. 25—X_q FOR SALIENT-POLE MACHINES WITHOUT DAMPERS

A = 7,500-kva. generators
B = 750-kva. generator
C = 331-kva. synchronous motors with dampers off

line-to-neutral short-circuit test, $X_0 = 0.098$. The data in Table II afford comparison of values measured with the rotor at standstill and driven at rated speed.

TABLE II—ZERO-SEQUENCE REACTANCE X_0

	Machine 1	Machine 2
Rotor running	0.0290	0.0158
Rotor at standstill	0.0306	0.0163

Negative-Sequence Reactance X_2. As stated in Part II, X_2 for salient-pole machines with dampers can be computed as the average of X_d'' and X_q'' measured at rated current by a locked line-to-line test. Values so computed, and values obtained from the sustained line-to-line test, are tabulated in Table III.

TABLE III—X_2 FOR TYPICAL SALIENT-POLE MACHINES (WITH DAMPERS AND END RINGS)

(1/2) (X_d'' + X_q'') (X_d'' and X_q'' from locked line-to-line test)	$X_2 = E_{ab}/\sqrt{3}\,I_b$ From sustained line-to-line short circuit
0.34	0.34
0.160	0.150
0.23	0.21
0.20	0.20
0.17	0.180
0.135	0.130
0.165	0.160
0.160	0.160
0.24	0.25
0.23	0.23
0.21	0.20
0.195	0.195
0.120	0.120

Armature Short-Circuit Time Constant T_a. In Fig. 20A, that one of the two largest asymmetrical components which has the most gradual initial slope has an initial value of 8.20 and a time constant = 15.5 cycles = 0.26 sec. Hence T_a = 0.26 sec. for this 75,000-kva. turbine generator.

Time Constants T_d' and T_d''. For this same machine, T_d' = 1.53 sec. (saturated value) from Fig. 22A. The fact

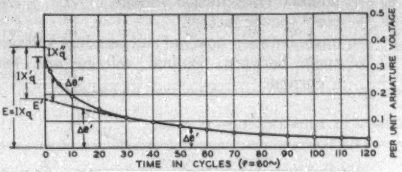

FIG. 26—QUADRATURE-AXIS DECREMENT

18,750-kva. turbine generator

that field-decrement tests give indefinite results (non-exponential decay) for turbine generators is illustrated by Fig. 30, which is for a 9,375-kva. turbine generator. Fig. 31 shows corresponding curves for a large salient-pole generator, illustrating that for salient-pole machines, this test is quite satisfactory; the unsaturated value of T_d' from a short-circuit field-decrement test when multiplied by 0.88 gives the saturated value, approximately.

Short-Circuit Subtransient Time Constant T_d''. Refer

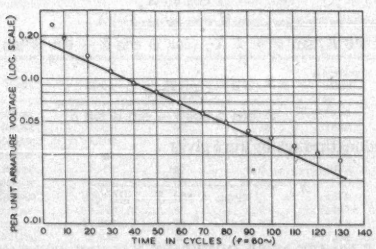

FIG. 27—$\Delta e'$ COMPONENT FROM FIG. 26

T_{qo}' = 60 cycles = 1.00 sec

to Fig. 21 for the test on a 75,000-kva. turbine generator. A semi-log plot of $\Delta i''$ made in Fig. 22B gives the time constant of a fitted exponential as T_d'' = 2.1 cycles = 0.035 sec. Note that the large discrepancy appearing in Fig. 22B, between actual $\Delta i''$ component and the fitted exponential, is really quite negligible when they are replotted in Fig. 21 in Cartesian coordinates.

Transient Time Constant T_{qo}'. (See Appendix III.) Refer to Fig. 26 (for an 18,750-kva. turbine generator); the $\Delta e'$ component is plotted in Fig. 27, and in this particular case an exponential approximates it very closely, giving T_{qo}' = 1.00 sec. for this low voltage. Had this test been made at higher voltage it is doubtful if any fitted exponential would be other than roughly approximate.

Negative-Sequence Resistance R_2. In keeping with a previous statement, all values quoted are those effective when negative-sequence current equals rated current. A 5,000-kva. salient-pole machine gave R_2 = 0.120 by prime-mover input measurement, and R_2 = 0.135 by wattmeter measurement. On the other hand, with a 7,500-kva. waterwheel type generator without dampers, R_2 from wattmeter measurements was in considerable error, but R_2 = 0.033 was obtained from both the

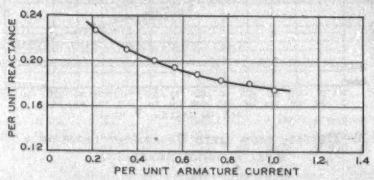

FIG. 28—VARIATION OF X_q'' FOR A TURBINE GENERATOR (75,000 KVA.)

retardation test and a measurement of prime-mover input (for a line-to-line short circuit). For both machines above, loss was found to vary as the square of the current; this is, of course, affected by rotor temperature.

Measurements on turbine generators indicate a variation of negative-sequence loss as the 1.8 power of current. Since part of the loss varies as the 1.5 power and part as the square of the current, this result is quite reasonable. For example, tests on a 43,750-kva. turbine generator showed a variation as the 1.8 power

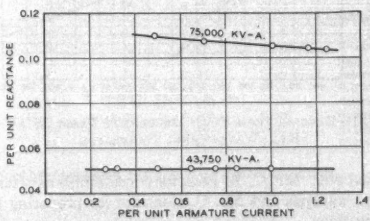

FIG. 29—VARIATION OF X_0 FOR TURBINE GENERATORS

and the loss associated with direct current in the armature (and rotor driven at rated speed) was found to vary as the 1.8 power at low currents. (For this latter loss, practically all the energy is supplied directly by shaft torque.)

SUMMARY

The tables below form an explicit summary of test results. Table IV lists what are considered to be the

more important machine constants. The values of X_d', X_d'', T_d', T_d'' and T_a in Tables V and VI were obtained from three-phase short circuits at rated voltage. Tables VII and VIII give other test values. Table IX gives typical values of various machine constants, based on test results; constants calculated

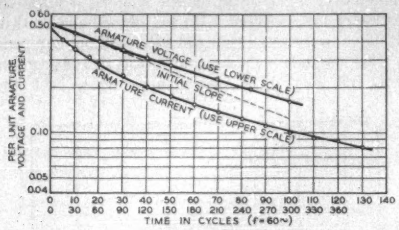

FIG. 30—RESULTS FROM FIELD DECREMENT TESTS ON A 9,375-KVA. TURBINE GENERATOR

from formulas given in the companion paper by L. A. Kilgore[6] were used in arriving at some of the ranges given.

ACKNOWLEDGMENTS

The author wishes to thank Messrs. R. D. Evans and C. F. Wagner for valuable criticisms, Mr. C. J. Fechheimer under whose supervision the work was

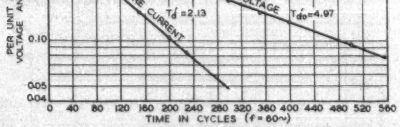

FIG. 31—RESULTS FROM FIELD DECREMENT TESTS ON A 7,500 KVA. WATERWHEEL GENERATOR

carried out; Mr. C. E. Peck for considerable assistance in test analysis and Mr. C. Gentilini for preparing the illustrations.

Appendix I

QUADRATURE-AXIS SYNCHRONOUS REACTANCE X_q (SATURATED VALUE)

This is a practical method for measuring the saturated value of X_q, based on a relation derived from the application of Blondel's two-reaction theory, and involving measurement of relative angle of the rotor and the terminal voltage vector, while the machine is carrying any load (other than a very low power-factor load)—such as normal load for a generator. Fig. 32 is a vector diagram of a generator supplying a lagging power-factor balanced load; machine resistance is neglected. Let

E_t = terminal voltage (line-to-neutral).
I = load current.
I_d = direct-axis component of load current.
I_q = quadrature-axis component of load current.
e_d = nominal or excitation voltage.
ϕ = $\cos^{-1}$ (power factor).
δ = angle between nominal and terminal voltages.
ψ = angle between nominal voltage and load current.

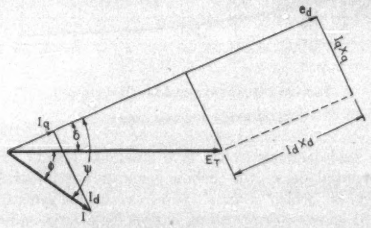

FIG. 32—VECTOR DIAGRAM OF A GENERATOR SUPPLYING A LAGGING POWER-FACTOR LOAD

It is seen from Fig. 32 that

$$E_t \sin\delta = I_q X_q$$
$$= (I \cos\psi) X_q$$
$$= [I \cos(\phi + \delta)] X_q$$

Whence $E_t \sin\delta = I X_q (\cos\phi \cos\delta - \sin\phi \sin\delta)$ and

$$X_q = \frac{E_t \sin\delta}{I(\cos\phi\cos\delta - \sin\phi\sin\delta)}$$

Dividing through by $\sin\delta$ gives

$$X_q = \frac{E_t}{I(\cos\phi\cot\delta - \sin\phi)} \tag{12}$$

TABLE IV—THE MORE IMPORTANT MACHINE CONSTANTS

*X_d	= synchronous reactance } = X_1 = positive-sequence reactance
X_d'	= transient reactance } = X_1 = positive-sequence reactance
X_d''	= subtransient reactance } = X_1 = positive-sequence reactance
$S\,C\,R$	= short-circuit ratio†
X_2	= negative-sequence reactance
X_0	= zero-sequence reactance
T_a	= (short-circuit) armature circuit time constant
*T_d'	= short-circuit transient time constant
*T_d''	= short-circuit subtransient time constant
T_{do}'	= open-circuit transient time constant

NOTE: T_{do}', the open-circuit transient time constant, may readily be computed from the relation

$$T_{do}' = \frac{X_d}{X_d'} T_d'$$

where X_d' and T_d' are both saturated values or are both unsaturated values.

*If a reactance or time constant is not qualified as to direct or quadrature axis, it is ordinarily assumed to be a direct axis value; the subscript d or q always settles this point as far as the *symbol* is concerned.

†Short-circuit ratio is understood as the value defined in the A. I. E. E. Standards; it is well to note that $S\,C\,R$ is the reciprocal of X_d, if there is no saturation (*i. e.*, if the no-load saturation curve is a straight line).

TABLE V—TURBINE-GENERATOR CONSTANTS FROM TEST

Time constants are in seconds. Impedances are per unit values; multiply by 100 to obtain per cent value. The quantity Ω_N, called "normal ohms," is the number of ohms corresponding to 100 per cent impedance; per unit X or R times Ω_N gives the ohmic values of X and R

Kva.	Freq.	Kv. rated	Amps. rated	R. p. m.	Ω_N	X_d	X_d'	X_d''	X_2	X_0	T_a	T_{do}'*	T_d'	T_d''	R_2
1,875	60	0.60	1,805	3,600	0.192	1.00	0.15			Δ Conn.	0.066	2.1	0.31		0.035
3,125	60	0.48	3,750	3,600	0.074	0.99	0.21	0.185	0.185	0.17	0.038	2.9	0.61	0.045	0.046
7,500	60	6.6	657	3,600	5.80	1.20	0.165	0.092	0.092	0.0180	0.043	4.1	0.56	0.026	
8,575	60	4.15	1,195	3,600	2.01	0.96	0.120	0.084	0.084	0.0160	0.110	3.5	0.44	0.036	0.029
9,375	60	6.6	820	3,600	4.65	1.10	0.140	0.075	0.075	0.0160	0.085	3.9	0.49	0.034	0.029
12,500	60	11.0	655	3,600	9.71	1.00	0.135	0.088	0.088	0.0090	0.110	4.0	0.54	0.040	
18,750	60	13.8	785	3,600	10.1	1.15	0.140	0.085	0.085	0.031	0.13	4.8	0.59	0.041	0.024
43,750	60	13.8	1,830	1,800	4.36	1.30	0.25	0.170	0.170	0.051	0.28	5.2	1.00	0.021	0.042
68,750	60	13.2	3,000	1,800	2.54	0.97	0.21	0.120	0.120	0.059	0.25	7.4	1.60	0.035	0.025
75,000	60	11.6	3,740	1,800	1.79	1.10	0.24	0.145	0.145	0.100	0.26	7.0	1.53	0.034	0.030

*These are values computed from the relation given in the note in Table IV.

TABLE VI—SALIENT-POLE SYNCHRONOUS MACHINE CONSTANTS FROM TEST

Type(a)	Kva.	Freq.	Kv. rated	Amperes rated	R. p. m.	Ω_N	X_d	X_d'	X_d''	X_2	X_0	T_a	T_{do}'	T_d'	T_d''	R_2
Sync. motor	240	60	2.2	63	277	20.1	1.05	0.42	0.32	0.33	(b)	0.048		(c)	0.010	0.089
Sync. motor	331	60	0.44	435	1200	0.585	1.60	0.23	0.160	0.160	(b)	0.033			0.028	0.034
Sync. motor	730	60	2.3	183	1200	7.25	1.30	0.30	0.21	0.21	(b)	0.110	3.05	0.70	0.030	0.110
Sync. motor (N. D.)	730	60	2.3	183	1200	7.25	1.30	0.30		0.45	(b)	0.135	3.08	0.69		0.047
Condenser	15,000	60	11.5	753	900	8.82	1.65	0.34	0.21	0.20	0.110	0.180	8.5	1.85	0.033	0.052
Condenser	20,000	60	11.0	1050	720	6.06	1.60	0.45	0.27	0.26		0.21	07.1*	2.0	0.045	0.060
Condenser	30,000	60	10.0	1732	720	3.53	1.50	0.37	0.190	0.180	0.072	0.130	11.3*	2.8	0.030	0.034
Generator	100	60	2.3	25.2	1200	52.7	1.25	0.24	0.165	0.170	0.033	0.023	1.15*	0.22	0.022	0.0170
Generator	5,000	60	7.6	380	900	11.6	0.90	0.22	0.130	0.130	0.063	0.095	3.7*	0.90	0.028	0.0180
H. W. W. Gen. (N. D.)	750	25	6.6	65.6	187.5	58.1	1.20	0.41		0.62	(b)	0.18		(c)		0.032
V. W. W. Gen.	39,000	60	13.8	1632	138.5	4.89	.80						7.07	3.17(d)		0.078

(a) All machines except those marked (N. D.) have dampers and end-rings.
(b) No neutral available.
(c) Value omitted because tests were made with unmeasured value of external resistance in field circuit.
(d) Unsaturated value from field decrement test.
*These are values computed from the relation given in the note in Table IV.

TABLE VII(a)—THREE-PHASE SYNCHRONOUS MACHINE CONSTANTS, FROM TEST*
Salient-Pole Machines with Dampers (and End-Rings)

Type	Kva.	Freq.	Kv. rated	Amperes rated	R. p. m.	Ω_N	X_d	X_d''	X_2	X_0
Condenser	1,000	60	2.3	252	900	5.27	1.60	0.180	0.160	
Condenser	1,250	60	.48	1500	1200	.185	1.90	0 23	0 22	
Condenser	5,000	60	12.47	232	900	31.0	1.70	0.27	0.27	0.044
Condenser	7,500	60	4.8	901	720	3.08	1.75	0 24	0.23	0.120
Condenser	7,500	60	4.0	1085	900	2.13	1.45	0.21	0.190	0.030
Condenser	15,000	60	11.95	725	900	9.52	1.55	0.20	0.190	0.031
Condenser	30,000	60	13.6	1275	720	6.17	1.70	0.23	0.20	0.110
Condenser	30,000	60	5.0	3465	720	.832	1.70	0.20	0.180	0.072
Motor	30,000	50	11.50	1505	600	4.41	.78	0.130	0.120	0.030
W. W. Gen.	3,000	50	6.6	263	750	14.5	1.60	0.165	0.160	0.036
W. W. Gen.	7,000	60	6.6	612	450	6.24	.97	0.26	0.25	0.145
W. W. Gen.	22,500	60	12.5	1040	300	6.95	1.20	0.29	0.28	0.155

*Time constants are in seconds, impedances are per unit values.
(a) X_d is from no-load and short-circuit saturation curves; X_2 from sustained line-to-line short circuit; X_d'' and X_0 from locked tests at rated current.

TABLE VIII(b)
Salient-Pole Machines without Dampers

Type	Kva.	Freq.	Kv. rated	Amperes rated	R. p. m.	Ω_N	X_d	X_d'(c)	X_2	X_0
Motor	730	60	2.3	183	1200	7.25	1.30	0.30	0.45	
W. W. Gen.	750	25	6.6	65.6	187.5	58.1	1.20	0.41	0.62	
W. W. Gen.	2,330	50	6.6	204	125	18.7	1.15	0.45	0.65	0.24
W. W. Gen.	4,250	60	2.3	1066	600	1.25	1.35	0.37	0.62	
W. W. Gen.	7,000	50	6.6	612	94	6.22	1.15	0.44	0.61	0.24
W. W. Gen.	13,750	60	6.6	1200	90	3.17	1.10	0.43	0.58	0.25
Gen.	187	60	.24	450	200	.808	.94	0.41	0.55	
Gen.	30,000	60	11.5	1500	600	4.42	1.15	0.33	0.58	0.093

(b) X_d is from no-load and short-circuit saturation curves; X_2 from sustained line-to-line short-circuit; X_d' and X_0 from locked tests at rated current.
(c) This value of X_d', times 0.88, gives (approx.) the saturated value of X_d'.

TABLE IX—TYPICAL CONSTANTS OF THREE-PHASE SYNCHRONOUS MACHINES

Reactances are per unit values, time constants are in seconds. Values below the lines give a range of values, while those above give an average value

	X_d	X_q	X_d'	X_d''	X_2	X_0*	T_{do}'	T_d'	T_d''†	T_a
2-pole turbine generators	1.10 / .95 −1.45	1.07 / .92 −1.42	0.15 / .12 − .21	0.09 / .07 − .14	(= X_d'')	.01 − .08	4.4 / 2.8 −6.2	0.6 / .35 − .90	0.035 / .02 − .05	0.09 / .04 − .15
4-pole turbine generators	1.10 / 1.00 −1.45	1.08 / .97 −1.42	0.23 / .20 −0.28	0.14 / .12 − .17	(= X_d'')	.015 − .14	6.2 / 4.0 −9.2	1.3 / .8 −1.8	0.035 / .02 − .05	0.2 / .15 − .35
Salient-pole gen. and motors (with dampers)	1.15 / .60 −1.45	0.75 / .40 −1.00	0.37 / .20 − .50	0.24 / .13 − .35	0.24 / .13 − .35	.02 − .20	5.6 / 1.5 −9.5	1.8 / .5 −3.3	0.035 / .01 − .05	0.15 / .03 − .25
Salient-pole gen. (without dampers)	1.15 / .60 −1.45	0.75 / .40 − .95	0.35 / .20 − .45	(= X_d')	0.55 / .30 − .70	.04 − .25	6.6 / 3.0 −10.5	2.0 / 1.0 −3.3		0.30 / .1 − .5
Condensers	1.80 / 1.50 −2.20	1.15 / .95 −1.40	.40 / .30 − .60	.25 / .18 − .38	.24 / .17 − .37	.02 − .15	9.0 / 6.0 −11.5	2.0 / 1.2 −2.8	0.035 / .02 − .05	0.17 / .1 − .3

*X_0 varies so critically with armature winding pitch that an average value can hardly be given. Variation is from about 0.1 to 0.7 of X_d''.
†For single-phase machines (or three-phase machines designed for single-phase operation), T_d'' may have much higher values than listed.

Knowing the terminal voltage, load current and power factor, and δ, X_q (saturated value) is readily computed from equation (**12**). δ can be measured by means of a small auxiliary a-c. generator mounted on the shaft of the machine. This measurement has been described in certain stability studies.[11,12]

The equation expressing δ as a function of the variables listed above is, for the same case of a generator supplying a lagging power-factor load,

$$\delta = \tan^{-1} \frac{I X_q \cos \phi}{E_t + I X_q \sin \phi} \tag{13}$$

Equations (**12**) and (**13**) apply not only to a generator supplying a lagging power-factor load, but also to a motor operating at a leading power factor; for the two converse cases, the sign of the second term in the denominator changes in both equations.

Appendix II

Quadrature-Axis Transient Reactance X_q' (Saturated Value)

To obtain the saturated value of X_q' by the method described, the slip test is made with rated current circulated which puts it at a disadvantage for turbine generators since this means a source of three-phase power of the same order of magnitude as the machine rating. With rated current flowing and the rotor magnetized in the quadrature axis, a dead three-phase short circuit is applied. An oscillogram of short-circuit current in all three phases is analyzed in the manner described (See Figs. 9A and 9B) for the determination of X_d'.

I = armature current just previous to short circuit (= rated value).

I' = initial symmetrical short-circuit current *neglecting* the rapidly decaying (initial) portion of the wave.

E = applied voltage (line-to-neutral value) just previous to short circuit.

The transient internal voltage previous to short circuit is

$$E' = I\,(X_q - X_q') \tag{14}$$

but

$$I' = \frac{E'}{X_q'} \tag{15}$$

hence

$$E' = I' X_q' = I\,(X_q - X_q')$$

whence

$$X_q' = \frac{I X_q}{I + I'}$$

but

$$I X_q = E \tag{16}$$

therefore

$$X_q' = \frac{E}{I + I'} \tag{17}$$

NOTE: Strictly speaking, the values of X_q' before and after the short circuit are different; to account for this difference it is necessary to first determine, at the same current, the low-current value of X_q' as in Appendix III and to use this value in computing E', then to compute the saturated value of X_q' from equation (**15**).

Appendix III

Quadrature-Axis Transient Reactance X_q' (Low-Current Value)

This may be determined by a modification of the slip test; with the rotor magnetized in the quadrature axis, the applied voltage is suddenly disconnected, and terminal voltage recorded by oscillograph. Terminal voltage in Fig. 26 is extrapolated to zero time, *neglecting the rapidly decaying portion*, by the aid of a semi-log plot in Fig. 27.

E' = this (extrapolated) initial value (line-to-neutral).

E = terminal voltage (line-to-neutral) just previous to the removal of applied voltage.

I = armature current just previous to the removal of the applied voltage.

Then

$$E - E' = I X_q'$$

or

$$X_q' = \frac{E - E'}{I} \tag{18}$$

NOTE: When applied voltage is disconnected, the three currents will be opened sequentially thus making zero time, the time at which the extrapolated voltage is to be measured, an indefinite item. The current in one line is first interrupted, then the current in the other two lines is interrupted (simultaneously in these two). There is very little difference, however, between the extrapolated values of voltage at these two instants, hence this matter of defining zero time has no practical importance.

Appendix IV

QUADRATURE-AXIS SUBTRANSIENT REACTANCE X_q'' (SATURATED VALUE)

This may be obtained from the same test used to determine the saturated value of X_q' (Appendix II). Following the same procedure, the *total* symmetrical current is extrapolated to zero time to give the initial symmetrical subtransient current; extrapolation is accomplished as in the analysis for determining X_d'' (Figs. 7, 8, and 9), being based on the maximum possible asymmetrical component (initial value). Let

I'' = initial symmetrical short-circuit current
E = applied voltage (line-to-neutral) just previous to short circuit
I = armature current just previous to short circuit (= rated value)

The subtransient internal voltage previous to short circuit is

$$E'' = I (X_q - X_q'') \tag{19}$$

but

$$I'' = \frac{E''}{X_q''} \tag{20}$$

hence

$$E'' = I'' X_q'' = I (X_q - X_q'')$$

whence

$$X_q'' = \frac{I X_q}{I + I''} \tag{21}$$

but

$$I X_q = E$$

therefore

$$X_q'' = \frac{E}{I + I''} \tag{22}$$

The same note given in Appendix II applies here; that is, the difference in values of X_q'' before and after the short circuit may be accounted for by first measuring the low-current value by another test, similar to that in Appendix III.

Appendix V

X_0 FROM SUSTAINED DOUBLE LINE-TO-NEUTRAL SHORT CIRCUIT

By the method of symmetrical components, it can readily be shown[13,14] that with a double line-to-neutral short circuit on phases b and c at the machine terminals (See Fig. 12), the voltage on the open phase is

$$\check{E}_a = \frac{3 \check{Z}_0 \check{Z}_2 \check{e}}{\check{Z}_0 \check{Z}_1 + \check{Z}_0 \check{Z}_2 + \check{Z}_1 \check{Z}_2}$$

and that the neutral current is

$$\check{I}_n = 3 \check{I}_0$$

$$= \frac{-3 \check{e} \check{Z}_2}{\check{Z}_0 \check{Z}_1 + \check{Z}_0 \check{Z}_2 + \check{Z}_1 \check{Z}_2}$$

Where

$\check{e}$ = generated (line-to-neutral) voltage of phase a.
$\check{I}_0$ = zero-sequence current
$\check{Z}_0, \check{Z}_1, \check{Z}_2$ = sequence impedances of the machine.

Dividing $\check{E}_a$ by $\check{I}_n$ gives

$$\check{Z}_0 = -\frac{\check{E}_a}{\check{I}_n} \tag{3}$$

Appendix VI

NEGATIVE-SEQUENCE REACTANCE X_2

It has been shown[5] that the ratio of voltage to current, both fundamental negative-sequence quantities, depends upon whether (1) the current or (2) the voltage, is maintained fundamental. Condition (1) is approached with large external reactance, and (2) is approached for double line-to-neutral and line-to-line short circuits. In systems where generators are connected to delta-wye transformers they carry no zero-sequence current for line faults, and in view of this and other considerations, more importance is placed on the value of X_2 for line-to-line generator short circuits than for double line-to-neutral short circuits. For condition (1) above, $X_2 = (1/2)(X_d'' + X_q'')$ and for a line-to-line short circuit, $X_2 = \sqrt{X_d'' X_q''}$; the difference between these two values will be computed for a case where X_d'' and X_q'' are quite different, as for a salient-pole machine without dampers where $X_d'' = X_d' = 0.35$ and $X_q'' = X_q = 0.75$. For this case

$$\frac{X_d'' + X_q''}{2} = 0.55 \text{ and } \sqrt{X_d'' X_q''} = 0.51$$

from which it can be concluded that for most practical purposes, the arithmetical average of X_d'' and X_q'' (in this case 0.55) may be taken as the value of X_2.

Appendix VII

Negative-Sequence Impedance from Sustained Line-to-Line Short Circuit

Referring to Fig. 13, it has been shown[13,14] that for a line-to-line short circuit on phases b and c, the short-circuit current

$$\check{I}_b = -j\,\frac{\sqrt{3}\,\check{e}}{\check{Z}_1 + \check{Z}_2}$$

and the line-to-line voltage

$$\check{E}_{ab} = \frac{3\,\check{e}\,\check{Z}_2}{\check{Z}_1 + \check{Z}_2}$$

Where $\check{e}$ = generated voltage of phase a.
$\check{Z}_1, \check{Z}_2$ = sequence impedances of machine.

From which

$$\frac{\check{E}_{ab}}{\check{I}_b} = \frac{\sqrt{3}\,\check{Z}_2}{-j}$$

hence

$$\check{Z}_2 = \frac{-j}{\sqrt{3}} \cdot \frac{\check{E}_{ab}}{\check{I}_b} \qquad (5)$$

Appendix VIII

X_d' and T_d' as Regards Saturation Effects

Since transient reactance is based on an initial value of current (neglecting the initial rapidly decaying portion), the use of different values of X_d' for various degrees of saturation is an accurate treatment. Since time constants define decrement (or increment) only if it is exponential, and since saturation causes decrement to be other than exponential, the use of time constants for short-circuit conditions is not an exact treatment. However, assuming that the most important portion of a decrement is that during the period when the value is relatively large, the approximation of employing a saturated time constant for saturated conditions gives reasonably accurate results.

In order to study the relation between saturated and rated current values of X_d', tests made under various degrees of saturation were analyzed. It was found that in general, the reduction in X_d' could be accounted for in two ways; first, by a factor equal to the ratio of (field amperes at rated voltage on the air gap line) to (field amperes at rated voltage no load)—which ratio is simply the reciprocal of the product of X_d and the short-circuit ratio; second, by an empirical factor = 0.88, based on test results. However, it was also found that the relation between the saturated and unsaturated values of T_d' was better accounted for by the empirical factor 0.88 than by the first factor, and it is on this basis that the factor 0.88 has been adopted.

Three-phase short circuits were made at low voltages as well as at rated voltage, and analysis of results has lead to the conclusion that one of the important causes of reduction of X_d' and T_d' is due, in salient-pole machines, to saturation of the iron in the pole tips. Thus it would be expected that under short-circuit conditions the values of X_d' and T_d' depend not only on armature short-circuit current, but also on the voltage and field current existing before short circuit.

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Discussion

P. L. Alger: This paper presents a thorough summary of test methods of reactance determination, which will be very useful for both designers and operators. The methods of determining zero phase and negative phase sequence reactances from measurement of voltages on the open phases under partial short circuit are particularly valuable.

The paper establishes satisfactory methods for determining the unsaturated values of all the important reactances, but only suggests the determination of saturated reactance by full voltage short-circuit tests. I do not believe that full voltage short circuits are desirable for regular use in factory tests, as they require undue expense and special safety precautions for holding the machine stator in place against the short-circuit forces. Furthermore, the only record of such tests is oscillo-

graphic, and the accuracy obtainable from oscillographic films is not of a high order, so that at best only approximate values of reactance can be so obtained.

For practical purposes, the operator usually requires only saturated values of the transient and subtransient reactances, though unsaturated values may be useful in calculations such as machine oscillation and stability determinations. These are primarily used for determining fault currents and relay settings.

The synchronous reactances are chiefly useful in determining steady state torque angle relations and sustained short-circuit currents, and for these purposes the unsaturated values are desired.

I believe the best way to determine the saturated transient and subtransient reactances is to apply saturation factors to the test values of unsaturated reactance, as mentioned in my discussion of Mr. Kilgore's paper on page 1214. The errors introduced by the use of such factors in my opinion should not be of greater magnitude than allowable with the accuracy requirements for practical use at the present time.

Sherwin H. Wright: The method of evaluating saturated values of synchronous machine constants by applying saturation factors to measured unsaturated values, as commented upon by Mr. Alger, may well be amplified and extended in use in the future. Limited application of this method appears in the paper, wherein the saturation factor for both transient reactance (X_d') and short-circuit time constant (T_d') is given as 0.88; thus in Table VIII (salient pole machines without dampers) the value of transient reactance as measured at rated current by a locked rotor test, when multiplied by 0.88, gives the saturated value. In the case of turbine-generators, transient reactance cannot be determined from this same locked rotor test, but if a sudden three-phase short circuit is made at half voltage and the value of transient reactance thus found is multiplied by 0.88, the result is approximately the saturated value of transient reactance.

Extension of this general method can be accomplished only after considerable additional test analysis is carried out, such as will lead up to a better understanding of the mechanism of saturation effects in machines under short circuit. However, to reach such a state we must first know the actual saturated values of machine constants, and it is for this reason that, in the paper, emphasis has been placed on the determination of certain of the constants by full voltage short circuit.

An A-C. Supervisory Control System

BY OTHMAR K. MARTI[1]

Member, A. I. E. E.

As a result of the rapid extension of electric power systems and the progress of electrification of railroads, the control engineer has been called on to provide solutions for increasingly intricate control operations. Coordination between those operations led to their concentration into the hands of a single operator or dispatcher, and the adoption of automatic and of remotely controlled substations. In order to be able to control equipment located at appreciable distances it was necessary to reduce to a minimum the number of wires required for transmitting a large number of different signals, which led to the adoption of different designs of supervisory control. At the time of the adoption of those systems it was felt that only a d-c. system, operating with a battery as current supply, would be a reliable solution, as a system directly supplied by the power lines to be controlled would be affected by disturbances in these lines, at times when supervision is most urgently required.

Simplified Diagram of System

Alternating current was, however, chosen for operating the present system, whose principle of operation is illustrated diagrammatically in Fig. 1. This figure shows a controlling station *I* and a controlled station *II*, which are connected by five wires. The a-c. supply may be located in the controlling station, or in any one of several controlled substations, and can be made independent of all disturbances by using a fractional horsepower motor-generator set, supplied from the station battery which is usually present. In most instances, however, it will be found possible to dispense with the battery and motor-generator set without sacrificing reliability. Such a possibility exists, for instance, if the house supply of a generating station can be made available for energizing the supervisory control system, as this supply is not affected by any disturbances to the high-tension lines feeding the controlled substations. Another alternative resides in supplying power to the supervisory control system from the power lines at the controlling station and at each controlled substation, if these latter are fed from the same power system. In this case the supervisory system will be dead only if the power fails simultaneously at all feeding points. Disturbances in the low-voltage side of the controlled substations cannot affect the supervisory system.

In networks with a large number of internal connections it may even be felt that the presence of normal voltage on the substation bus can be depended on at all times, so that the stations can be supplied independently of each other, thereby eliminating the necessity for the two supply line wires shown on Fig. 1, which can then be replaced by a single return wire. In some cases an additional degree of safety can be obtained if a supervisory control system of this kind is supplied from one high-voltage network while controlling another independent network. Each individual case will show the best current supply to be used.

Synchronous Selectors

The system described herein was developed with a view to great reliability. It uses equipment of a very

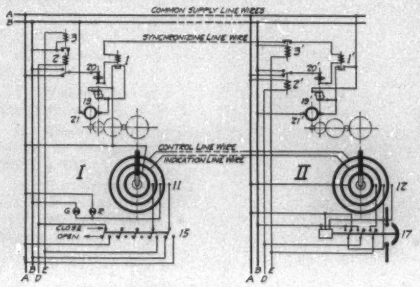

Fig. 1—Simplified Wiring Diagram

I Controlling station
II Controlled station
1, 1' Selector relays
2, 2' Starting relays
3, 3' Stopping relays
11 Controlling station selector
12 Controlled station selector
15 Control switch
17 Circuit Breaker
19, 19', 20, 20' Auxiliary contacts
21, 21' Selector motors

sturdy type, which has been used in the past in relay applications, and is little apt to be disturbed by even severe inductive interference, and can further be protected from disturbances by the insertion of insulating transformers into the lines. It is of the synchronous selector type, each station comprising one selector, started from its rest position when an operation is to be performed, and returned to this position after the operation has been completed. Depending on the complexity of the operations to be performed, and their number, these selectors are provided with a variable number of rows of contacts which are connected to slip rings by means of rotating brushes carried by an arm, and are made with a variable number of operating points.

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Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

Fig. 2 shows a selector having 23 operating positions, but selectors have been made with as many as 67 operating positions. These selectors are driven by a motor which is the heart of the whole installation. This motor is shown in Figs. 3 and 4. Its main component parts are a permanent magnet 1 and a wound armature 2 free to oscillate in the air gap of the magnet. The armature spindle carries a torsion spring fastened to the frame, and so dimensioned that it gives to the armature a natural oscillation frequency equal to the a-c. supply frequency. When alternating current is applied to the armature through flexible leads, the armature therefore oscillates at the system frequency, and, in order to

Fig. 2—Front View of Selector

utilize this oscillatory motion, the armature is connected to the selector arm through a ratchet mechanism and a set of gears. This motor, not possessing any contacts, slip rings, or commutator, is particularly reliable, and it has proved its worth in many years of service in central station and substation relays. The selector spindle carries a gear wheel 5 having as many teeth as operating positions are required, plus one for the rest position. This gear wheel is driven by a pinion 4 having only 2 teeth, driven through gears by the motor. The substantially uniform rotation given by the motor to pinion 4 is transmitted to gear 5 as a step-by-step motion, allowing the brushes of the selector to rest on the contacts for a comparatively long time while a much shorter time is spent in moving them from one contact to the next. One of the intermediate gear spindles carries sets of auxiliary contacts 3, the purpose of which will appear from a description of the

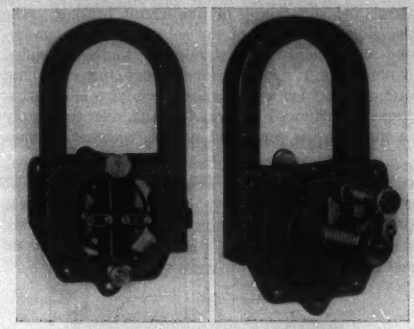

Fig. 3—Front and Rear Views of Selector Motor

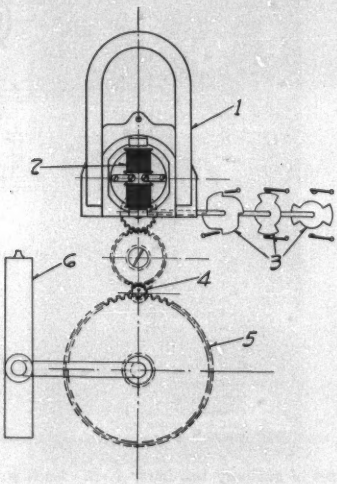

Fig. 4—Diagram of Selector Motor

1 Permanent magnet
2 Armature
3 Auxiliary contacts
4 Pinion
5 Selector gear
6 Selector arm

operation of the system, and makes one-quarter turn each time the selector moves by one step. It has been found that an operating time of ⅓ to ½ sec. per step permits satisfactory operation of all the relays.

Principle of Operation

Fig. 5 shows the complete equipment required for the operation of selectors in two stations and, in order to give a typical example of the operations that can be performed, also shows the equipment required for the control and supervision of one circuit breaker. The principle of operation of the selectors will be understood more readily by first referring to the simplified circuits of Fig. 1. As mentioned above, the controlling station is provided with a source supplying alternating current to auxiliary bus bars, *A*, *B* and, over the supply line wires, to the controlled station, where it supplies also a set of auxiliary bus bars, *A*, *B*. Each station also possesses a starting bus *D* and a stopping bus *E* for controlling the operation of the selectors. If we assume that the selector arms are in the rest position, as shown in Fig. 1, and that it is desired to close the circuit breaker 17, the control switch 15 of the circuit breaker is moved to the "close" position. The control switch, which is illustrated in Fig. 6, is provided with a number of contacts. One of these contacts connects the starting bus *D* to bus bar *A*, thereby energizing starting relay 2. Until the starting relay is energized, the synchronizing wire line is in connection with bus bar *A* in both stations *I* and *II*, and is therefore without current. When the starting relay 2 operates, its contacts connect one end of the synchronizing line to bus bar *B*; the synchronizing line thus receives current, energizing selector relays 1 and 1′. The latter connect to the bus bars the selector motors 21, 21′, which start to rotate. This rotation is transmitted to contacts 19, 19′, which first connect the motors directly to the bus bars, to contacts 20, 20′, which open the synchronizing line and through the two-tooth pinions, to the selector arms which move to the first step. When this step is reached, contacts 20, 20′ first reclose the synchronizing line, and contacts 19, 19′ then disconnect the motors from the bus bars. Starting relay 2 still being closed, another current impulse is sent over the synchronizing line, re-energizing selector relays 1, 1′, before contacts 19, 19′ open, thereby initiating another one step motion of the selector arms through the same sequence of operations. It is apparent from the above procedure that each selector arm can leave a step only if this step has also been reached by the other selector arm, and that the motion of the selectors is obtained independently until they both again reach the following step. If one of the selectors should fail to operate while between two steps,

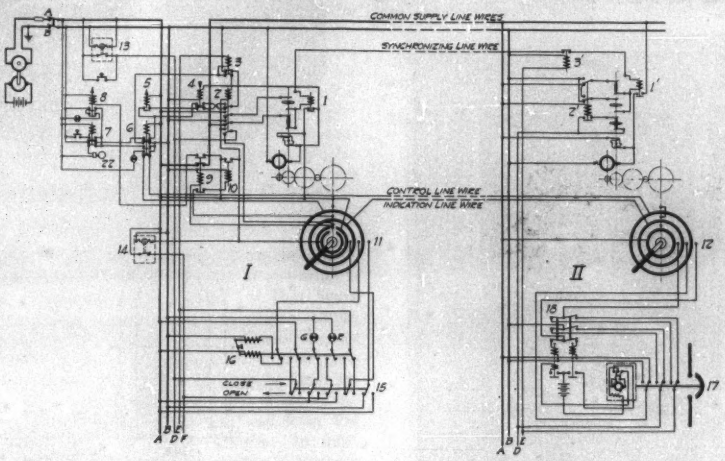

Fig. 5—Wiring Diagram of a Typical Installation

I Controlling station
II Controlled station
1, 1′ Selector relays
2, 2′ Starting relays
3, 3′ Stopping relays
4 Auxiliary selector relay (time delay on opening)
5 Auxiliary signal relay (time delay on closing)
6 Signal relay
7 Alarm relay
8 Trouble relay (time delay on closing)
9 Auxiliary starting relay
10 Synchronism checking relay
11 Controlling station selector
12 Controlled station selector
13 Time switch
14 Blinking relay
15 Control switch
16 Indicating relay
17 Circuit breaker
18 Control relay
19, 20, 19′, 20′ Auxiliary contacts
21, 21′ Selector motors
22 Alarm device

the other selector would be unable to proceed. In normal operation, the two selector arms proceed without stopping over the successive steps until they both reach the step assigned for the control of circuit breaker 17.

One of the brushes of selector 11 then connects bus bar A through the control switch to the stopping bus E. Stopping relay 3 is then energized and opens the circuit of starting relay 2, which reconnects the synchronizing

Fig. 6—Control Switch

line to bus bar A as when the system was at rest. The selector relays 1 and 1′ therefore are deenergized and the selector arms remain at standstill. Another brush of selector 11 then sends the control impulse over the control line wire and over selector 12 to the control circuit of the circuit breaker, causing it to close. The circuit breaker auxiliary contacts then send an indicating signal back over the indicating line wire.

The above operation can be followed in somewhat more detail on Fig. 5, on which the selector arms were displaced from their rest (vertical) position for the sake of clearness. When the control switch 15 is moved to the "close" position, it connects bus bar A to the starting bus D and, through the selector brush, energizes auxiliary starting relay 9. The latter energizes the synchronism checking relay 10, which can close only if both selector arms are in the rest position. If selector 12 happens to be in any other position, the auxiliary starting relay 9 will remain energized and it will operate the time delay trouble relay 8; at the same time, it opens supply line wire A, so as to avoid the transmission of erroneous indications back from station 11. Normally the selectors are in the rest position and permit the synchronism checking relay to close, thereby energizing the starting relay 2, which initiates a rotation of the selector arms as described above. Relay 9 is then deenergized by selector 11 and opens the circuit of trouble relay 8.

As a third protection against the loss of synchronism, the auxiliary contacts of the selectors reverse the polarity of the synchronizing line at each step, so that if one of the selector arms happened to jam on one step, the other selector arm would be unable to proceed further than the next step. Once the selector arms have started to rotate, the starting relay remains energized through the inside slip ring of selector 11. When the circuit breaker has been closed, as explained above, and the return indication transmitted, the stopping relay 3 is deenergized by the indicating relay 16, so that both selector arms again proceed until they reach their rest positions; the starting relay is then deenergized by selector 11 and the system comes to rest. When the circuit breaker trips automatically, it energizes starting relay 2′, which sets the two selectors in motion, in the same

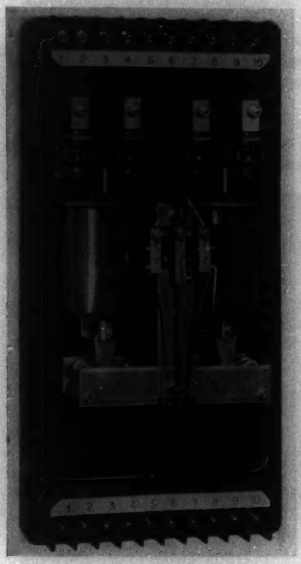

Fig. 7—Control Relay

manner as when starting relay 2 is energized. When selector relay 1 closes its contacts, the auxiliary selector relay 4 is energized, and since the starting relay 2 is not energized, a circuit is completed for energizing the signal relay 6, which lights a signal lamp, and energizes an alarm device 22, such as a bell or a horn. The operator is thereby notified of the tripping of the circuit breaker, and he disconnects the horn by means of the alarm relay 7. The two selectors are stopped by means of

stopping relay 3′, and the return indication is then transmitted over the indicating line wire. The operator then moves the control switch of the circuit breaker to the "open" position, thereby reversing control relay 18 which permits the selectors to return to the rest position. If we now assume that the circuit breaker trips while the selectors are rotating for the purpose of transmitting a control impulse, it will again energize starting relay 2′, but as starting relay 2 is also energized, the two ends of the synchronizing line will be connected to the same bus bar, and both selectors will stop. At the same time, auxiliary indicating relay 5 will be energized, thereby again energizing the signal relay 6, and sounding the alarm. The operator must then cancel the operation he had started by replacing his control switch in the initial position and moving the control switch of the circuit breaker which has tripped. If several breakers have tripped, this operation would be repeated for each one, before the operator can transmit any control impulses.

When the operator wishes to perform a control operation, he is thus positively assured that, while that operation is being completed, the equipment which this operation does not affect will still be in the condition shown by the lights on his switchboard. This condition can only be the result of operations either initiated or acknowledged by himself by means of his control switches. In this manner the danger of the operator becoming confused and performing undesired operations is reduced to a minimum.

An interesting feature of the controlling station is the blinking relay 14. This relay comprises a motor, similar to the selector motors, which rapidly makes and breaks connection between bus bar A and a blinking bus F, and operates only when selector 11 is not in its rest position. A lamp connected between bus bars B and F will then blink instead of giving a steady light. When the operator wants to close breaker 17 and moves control switch 15, the green lamp starts to blink until the circuit breaker has closed, after which the indication relay 16 extinguishes the green lamp and lights the red lamp. Conversely, when the circuit breaker trips automatically, alarm 22 sounds and the green lamp will blink until the operator moves the control switch to the "open" position, when the green lamp becomes steady. This combination of an alarm signal with a blinking light is most effective in bringing the operator's attention to whichever piece of apparatus has performed automatically. In addition, in order to avoid the automatic reclosing of the circuit breaker when it has tripped, it is provided with a control relay 18, shown in Fig. 7, through which the circuit breaker can be reclosed only by first moving the control switch to the "open" position, and then back into the "close" position.

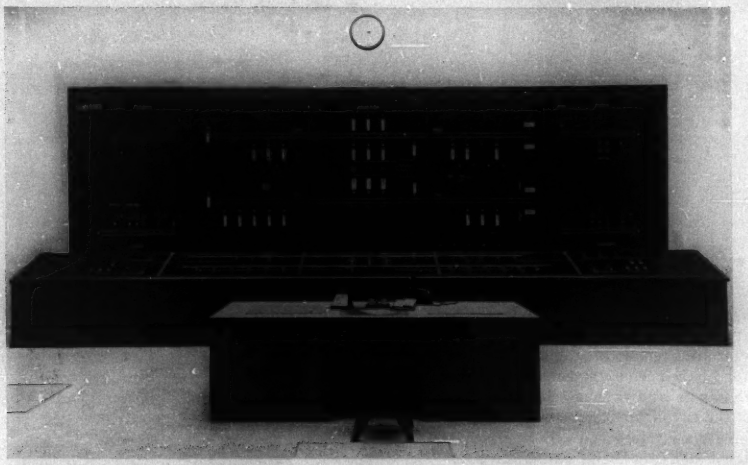

Fig. 8—View of Switchboard and Control Desk

Analogous circuits can be provided for other control and supervision operations such as, for instance, reading of meter indications, automatic position indication of manually operated disconnecting switches, alarm indications to warn of excessive temperatures or other disturbances in the substation, etc.

Operating Details

It may happen, in some instances, that the supervisory system remains idle for hours or even days. In order to give a periodic check of the condition on the

system, station *I* is provided with a time switch 13 which periodically sends an impulse to starting relay 2. This impulse starts both selector arms, which pass over every control step and come back to their rest position, where they automatically stop. Thus every step is checked, and it can therefore be ascertained that no change has occurred in the power system since the last control or supervisory operation. This check operation can also be initiated by means of a push button.

Fig. 9—View of Partially Assembled Luminous Switchboard

It will be noted that, when the operator needs to perform at one time a number of control operations which can be transmitted without any particular sequence, he can move all the corresponding control switches to their new positions, and the selector arms will transmit the control and signal impulses in the order of the corresponding contacts of the selectors.

The installation described above can be considered as typical, but can easily be modified to meet local conditions, as the principle of its operation permits a great amount of flexibility. A single selector in the controlling station can be used for controlling a number of selectors in outlying stations, the controlling selector then being provided with a number of steps equal in number to the total number of steps of the controlled selectors.

The control switches, selectors, control lamps, and various relays, can be mounted in different ways on switchboards or control desks. Fig. 8 shows a disposition applicable both to manual and to supervisory control, which enables the operator to visualize at all times the condition of the system which he is controlling. In this figure the control switches are installed on a switch desk in front of the switchboard, the handles of the control switches being part of a set of dummy buses, so that the positions of all circuit breakers can be visualized. The switchboard, as may be seen in more detail in Fig. 9, is built of sheets of glass, covered by an opaque paint leaving transparencies which form a diagram of the buses and lines duplicating the dummy set of buses on the switch desk. The indicating lamps are disposed behind the switchboard, so as to show, in proper colors, the positions of the circuit breakers. Additional indicating lamps provide for the indication of whether or not the buses and lines are energized, for which purpose they are controlled by suitable relays connected to the circuit breaker controlling circuits. Additional transparencies provided in the lines can be used for indicating the readings of ammeters, voltmeters, or other instruments. Each of these transparencies discloses a meter box having a ground glass front graduated so as to provide a scale, on which the readings are given by the shadow of the moving element of one or of several instruments. In this manner each meter is located so that it is immediately apparent to which circuit the reading refers. Fig. 10 shows a typical arrangement of three feeder ammeters with a three-phase bus voltmeter with suppressed scale.

This method of enabling the operator to visualize at a glance the condition of the network and the changes occurring therein, the position of the switching equipment, and the location of meter readings, greatly facilitates the supervisory operations, and reduces the chances of wrong operations being performed. In addition, the equipment is of a type with which the operators and maintenance crews are familiar, so that no special training is required for the personnel. These two advantages, with the flexibility and reliability of the equipment, should make this system of particular interest to operating engineers.

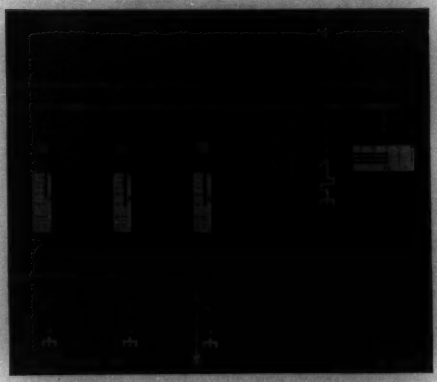

Fig. 10—Detail of Luminous Switchboard

Supervisory Control for A-C. Electrified Railroads

BY C. P. WEST[1] and H. C. GRIFFITH[2]

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THE Pennsylvania Railroad in its electrification in and around Philadelphia has made use of two installations of supervisory control for the remote operation of high-voltage a-c. step-down substations. One installation is on the branch line from Philadelphia to West Chester. It centralizes the control in the signal tower at Wawa, approximately 18 miles from Philadelphia, for four separate substations along the West Chester Branch—one at Morton 10 miles from Philadelphia, one at Lenni 17½ miles from Philadelphia, one

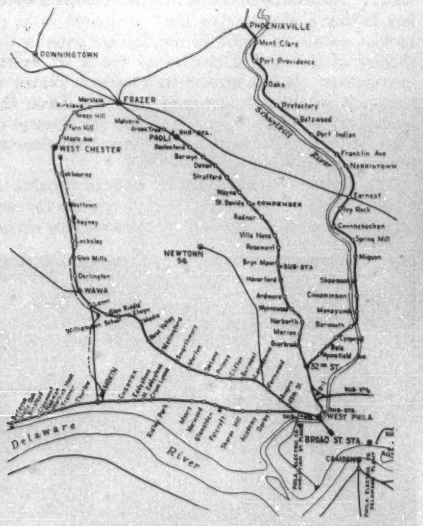

FIG. 1—MAP SHOWING PORTION OF PENNSYLVANIA RAILROAD ELECTRIFIED TERRITORY

at Cheyney 22 miles from Philadelphia, and one at West Chester 27½ miles from Philadelphia. The relative locations of these substations and the control point at Wawa are shown in Fig. 1.

The regular practise of the Pennsylvania Railroad is to install substation control in signal towers adjacent to the individual substations. This can be done in most cases, since the substation locations selected are usually adjacent to interlockings, where signal towers are also located.

On the West Chester branch, however, there is but one signal tower which is manned for the 24-hour period of each day, namely at Wawa. It is, therefore, desirable to concentrate at this point the control of the four stations mentioned above. (Fig. 2.) Supervisory control makes this possible. The operator in the tower at Wawa receives detailed instructions from the system power director at Philadelphia by telephone. This operator then performs by use of supervisory control

FIG. 2—CONTROL BOARD—WAWA TOWER

such operating movements as are desired. In case of trouble developing along the line which involves the automatic opening of circuit breakers in any of the four substations, these indications are brought to the operator at Wawa by supervisory control. The operator in turn advises the power director by telephone of the change in the switching conditions and receives from him suitable instructions as to what movements to make.

The second installation of supervisory control was installed at D-1 substation which is located at 36th Street, Philadelphia, along the main line tracks of the Pennsylvania Railroad to the west and east. (Fig. 3.) This location is directly back of the Philadelphia Zoological Gardens. At the time this substation was initially installed, it was definitely known that the only signal tower near the substation would be abandoned at

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a later date, but the location of the new tower replacing it was not known. It was, therefore, decided that the control of this substation, which is one of the key stations in the Philadelphia district, would be carried directly to the power director's headquarters adjacent to the Pennsylvania Station at West Philadelphia. This permits the power director, who is the operating department man directly in charge of the entire electrification system, to control the operation of the equipment in an outlying substation and to receive direct and prompt indications of trouble involving that station.

In addition to the customary control wires for the manipulation of apparatus, there was also installed as part of the supervisory control, on both the D-1 and the West Chester Branch, voltage wires for the direct and continuous indication at the control point of the

Fig. 3—Typical Supervisory Equipment Installed in Substation Control House

potential on the substation 11,000-volt bus. At the time the supervisory control was built for the West Chester branch, a system known as the synchronous visual type was standard with the Westinghouse company. This equipment was installed in the spring of 1928. The power supply was obtained from 48-volt batteries located at each of the substations and at Wawa tower. Eight wires in an unshielded lead-covered telephone cable were installed between the dispatcher's office and each substation, four for operation of the supervisory equipment and four as spare wires which could be cut into the circuit by operating a push button.

When the electrification was put in service in July 1928, it became apparent that, when the 11,000-volt, 25-cycle trolley circuits were fed from one end only, voltages were induced in the control wires from the substations to the dispatcher's office which interfered with the operation of the supervisory equipment. The first indication of this condition was the blowing of fuses, both in line wires and battery circuits when trolley faults occurred. Later, it was observed that when heavily loaded trolley sections were fed from one end, sufficient voltage was induced in the supervisory line wires to cause some of the relays in these circuits to chatter. Under these conditions the supervisory system was inoperative.

Before the effects of induction were fully realized, many of the supervisory failures were credited to improper adjustments, circuit design or lack of maintenance. The Westinghouse company had numerous installations of synchronous visual supervisory in operation giving excellent service. The railroad company felt that it had done its share in maintaining the equipment and yet the results were most disappointing. Tests were made in January 1930 which definitely proved that the faulty operations were caused by voltages induced in the supervisory wires by the current in the contact lines. Plans were immediately

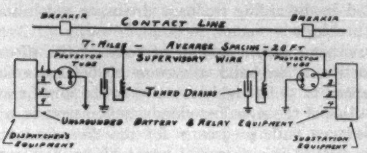

Fig. 4—One Wire of a Supervisory Control Line Between a Substation and Dispatcher's Office

made to change the equipment so that it would not be affected by induction. The obvious method was to arrange some form of drainage circuit to remove the induced 25-cycle energy from the supervisory wires.

A single line wire between a substation and the dispatcher's office is shown in Fig. 4.

High voltage induced in the supervisory wire will cause the protector tubes to break down. This voltage, even on load currents, was sufficient to cause the tubes to flash over and the current through them, being caused by a steady rather than transient condition would continue to flow till the tubes were destroyed. Tuned drainage circuits were connected to the supervisory wires at the substation and dispatching ends. These drains were so designed that they could carry enough current to hold the potential between the supervisory wire and ground below the flashover value of the protector tubes.

Three conditions had to be fulfilled in this design: first, the capacity connected to the line wires must not be so great as to absorb the direct current impulses for relay operation; second, the size of the drainage equipment must be held within reasonable limits; and third, the drains must have sufficient capacity to handle the maximum currents induced by full loads fed from one

end of the trolley section. Investigations showed that not more than 10 microfarads could be connected from the supervisory line wires to ground without interfering with relay operation. Values above this tended to absorb the direct current impulses sent out from one station to the other or to cause incorrect relay operation by condensers discharging through coils. If the drains were to have a sufficiently low 25-cycle impedance (150 ohms) and not use over 10 microfarads of capacity, the

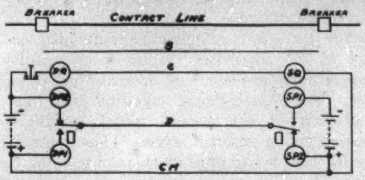

Fig. 5—Schematic Line Circuits for Synchronous Visual Type Supervisory Control

reactor became too large to go into the space available in the dispatcher's office. These considerations resulted in discarding resonant drains as a solution to the problem.

Another common way of overcoming the effects of induction is to build a system of balance coils so arranged as to be highly reactive to the direct current impulses of the signaling circuit and to present a low impedance path to ground for the 25-cycle current. This method is not readily adaptable to synchronous visual type of supervisory control equipment because of the inter-relation of the line circuits. Fig. 5 gives an idea of the connections of the line wires.

The normal condition of the circuits is for C wire to be connected between coils DQ and SQ across the dispatcher's office battery. Thus wires C and CM carry direct current continuously under one condition. When the set is put in operation C wire is dead and wires D and CM carry current first in one direction and then in the other alternately energizing DP-1 and SP-1 coils and then DP-2 and SP-2. This gives the effect of an alternating current of about 6 cycles frequency in wires D and CM. Later other relays (not shown) set up a similar circuit between C and CM wires. During the *pulsing* on D and CM wires, wire S may be energized so that current flows in either direction. This is shown in the following table:

Circuit condition	Wire CM	Wire D	Wire C	Wire S
1. At rest	Energized continuously		Energized continuously	
2. Stepping or selection	Pulsating d-c. reversing direction	Pulsating d-c. reversing direction		Pulsating d-c. may or may not reverse
3. Operation	Pulsating d-c. reversing direction		Pulsating d-c. reversing direction	

A consideration of these circuit conditions shows that a balance coil system could not be worked out with the synchronous visual supervisory system.

In general, it may be said that any satisfactory drainage system for a four-wire supervisory circuit, where induced 25-cycle currents above one ampere must be removed, will be expensive to build, bulky and cumbersome to install, and will require considerable mounting space.

Previous to this time, the Westinghouse company had completed its experimental work on a new type of supervisory control known as the visicode. The fundamental difference is that this system uses only two line wires and operates by means of d-c. unidirectional impulses. This circuit is so ideally suited for eliminating the effects of low frequency, high-voltage and high-current induction that it was decided to rebuild the West Chester equipment. The circuit which was used is shown in Fig. 6.

The reactor shown in each end of the line wires is the key to the successful application of this circuit. The alternating current flows in both line wires in the same direction at the same time. The windings are so connected that the reactance is practically zero, and only about 25 ohms resistance in the reactor and 50 ohms external resistance limit the current to ground. Thus a large induced current can flow continuously in the two

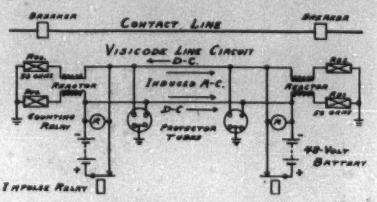

Fig. 6—Schematic Line Circuits for Synchronous Visual Type Supervisory Control

supervisory line wires without flashing the protector tubes or interfering with operation. During tests, the equipment functioned without failure even when short circuits were applied to the contact line circuit while the relay counting chains were operating. This was accomplished with counting relays which were sensitive to less than 0.05 ampere direct current.

Since the d-c. impulses flow in opposite directions in the reactor winding, a maximum of reactance exists. The impulses were timed for approximately eight per second, and the reactor was designed to give an effective resistance of more than 500 ohms at this frequency. This circuit permits paralleling a number of stations on the same pair of wires.

The changes were completed on the four stations on

the West Chester Division about September 1, 1930. Since that time the same high quality of operation has been obtained as with supervisory equipment of standard design. The synchronous visual set which operates between the dispatcher's office and D-1 substation has a rather short line circuit and is not so closely located to contact line circuits. For this reason this set was not rebuilt. The original supervisory equipment on this branch continues to give good operation.

The supervisory cable along the West Chester branch has an induced sheath potential of considerable magnitude, due to the cable being carried adjacent to the track at approximately the same elevation as the catenary construction. This particularly applies at times of trolley short circuits, or when the trolley circuits are fed stub end over a relatively long distance. The lead sheath of the cable carrying the Wawa supervisory conductors has been grounded through resistances such as to limit the current flow to the values allowed by the cable manufacturer. This grounding decreases the induction on the wires inside the cable because the effect of the sheath current opposes the induction from the contact line. The ground connections are applied at the proper intervals so that the potential to ground cannot exceed a reasonable voltage. This prevents any hazard to a lineman if in contact with the sheath when a short circuit occurs on the trolley line.

The control cable for the West Philadelphia D-1 system is approximately one mile in length. The cable installed was of such size as to permit duplication of control conductors in a manner similar to that on the West Chester branch. It had this marked difference in that the control cable was not installed on poles, but in an underground duct system. As a matter of fact, approximately half of this cable is placed in temporary wooden trunking adjacent to the tracks, as the duct system was not available when the cable was installed. It developed that the lead sheath carried rail return current which under conditions of short circuit on the 11,000-volt trolleys, would reach a value sufficient to melt the lead. This difficulty was overcome by segregating the cable from the rail return circuits with a low value of insulation. This trouble was not serious and was readily overcome.

When this type of supervisory control equipment has been properly adjusted it operates with very little maintenance. However, it is essential that the maintainer be familiar with the circuits and have the skill required for adjusting telephone-type relays. A man who lacks this ability may do more harm than good. The relay cases should be sealed so that only authorized persons can get at the relays. It is quite common for this type of supervisory control equipment, after its proper adjustment has been proved by service, to go for many months without removing the relay covers for inspection or maintenance.

Inspection periods, where equipment is in a clean neighborhood and is operating satisfactorily, can be extended to as long as four months. Companies having large installations usually develop a program suited to their own conditions as determined by their experience. More frequent inspections are desirable where the atmosphere is dirty and also while the equipment is new. Frequent operation of the equipment is desirable. It keeps contacts clean and all moving parts operating freely. All supervisory apparatus should be operated at least once a day, and the power apparatus should be operated if conditions permit. While unnecessary experimenting cannot be allowed, it should be remembered that the authorized maintenance men must have frequent enough contact with the apparatus to keep familiar with it. Otherwise, the quality of their work and speed in restoring faulty equipment to service are greatly impaired. The continued success of a supervisory installation depends on proper maintenance and the importance of this cannot be over-emphasized.

The energy for operating the supervisory equipment is supplied from storage batteries. Twenty-four cell groups, nominally 48 volts, are used at each substation and at the dispatcher's office. The batteries are charged by either rectigon or tungar chargers. The substation and supervisory batteries are entirely separate from each other so that the supervisory system will not be affected by grounds on the power control circuits.

The charging rate is adjusted so as to supply the continuously connected load and maintain a terminal voltage between 50.4 and 52.8 volts (2.1 to 2.2 volts per cell). This provides the proper trickle charge for the batteries and keeps them fully charged at all times. Unusual demands on the battery require raising the charging rate for short periods but this is seldom necessary. The condition of the batteries and chargers is checked regularly. This is essential as the whole supervisory system would be out of service if the dispatcher's battery should become discharged.

In conclusion, it may be stated that the two installations of supervisory control in use for a-c. substations operating on the Pennsylvania Railroad on its Philadelphia suburban electrification are in very satisfactory operation. The two major difficulties originally encountered, inductive interference and the training of a properly qualified inspection and maintenance organization, have been overcome. The effects of induction from adjacent conductors which were found to be serious under heavy short-circuit or long stub-end conditions have been taken care of by changing the original synchronous visual type to the visicode type having reactors at each end of the line. The proper maintenance and inspection was accomplished by a careful selection of men for this work and intensively training them.

The results obtained have shown that when the substations are located at points where no control operations are available, or where such points are so

distant that the installation of direct control cables is uneconomical, control can be adequately obtained by the use of proper supervisory equipment.

It permits of centralizing the control of a number of stations at one point, allows greater freedom in the selection of substation sites, eliminates the necessity of special operators when a properly manned adjacent control point is not available.

By centralizing the control of a number of stations at one point, it decreases the delay and possibilities of misunderstandings or mistakes which may occur in the transmission of messages or orders by phone or other methods.

It may be safely stated that supervisory control is destined to play an increasingly important part in future railroad electrifications.

Discussion

For discussion of this paper see page 1374.

Operating Experience With Automatic Stations

Part I—Columbia Engineering and Management Corporation

BY GARLAND STAMPER[1]

Associate, A. I. E. E.

Synopsis.—*The only proper way to test automatic and supervisory control equipment is to submit it to actual service conditions over a period of time. Since no one operating company can possibly serve as a proving ground for all types of automatic and supervisory equipment under all service conditions, the alternative is the interchange of operating experiences and ideas between the several companies using this equipment.*

The Columbia Gas and Electric Corporation has had automatic and semi-automatic stations of various types in operation during the last ten years. It is found that the class of service secured from any type of automatic station is directly dependent upon the excellence of inspection and maintenance. The chief items of trouble experienced in these automatic stations listed in the order of frequency of their occurrence are: graphic meter pens and clocks, circuit breaker auxiliary switches and closing mechanisms, grounds on control circuits, and relays due to obsolescence and loss of adjustment.

The Toronto Hydro-Electric System has long realized the advantages and economies of automatic and supervisory control. As a result, all new stations built since 1920 have been designed for either automatic or supervisory control. The present policy is to design the railway converter stations for automatic control and the distribution stations for supervisory control. The automatic controls have given fairly satisfactory results and the supervisory controls have given excellent results, especially the cable type supervisory. The chief source of trouble experienced with all this equipment has been relay failures. This would be expected naturally since practically all functions are performed by relays of various kinds. However, it is concluded from the results of these experiences that the automatic and supervisory controlled stations when operated properly are equal to and in some respects superior in reliability and flexibility to attended stations.

* * * * *

THE progress in the development and application of the automatic idea in regard to substations is doubtless familiar to all. The automatic principle was first applied to small transformer stations located at a distance from the main source of power and designed to feed suburbs, villages, and adjacent rural districts. These stations generally consist of a step-down transformer bank and one or two outgoing low-voltage feeders equipped with automatic reclosing oil circuit breakers and possibly induction voltage regulators. Since the load on this type of station is comparatively small, the expense of an operating force and the necessary housing for such operators can not be justified. However, the resultant good will and increase in revenue due to decrease in outage time will justify in a short while the small additional expense necessary to install automatic equipment.

As the success of these installations was proved, the application was extended to include larger distribution stations and converter stations for direct current supply to both the Edison and railway systems. As new control schemes and new devices were developed and their reliability improved, the automatic idea has been applied further until at the present time practically all types of stations are being designed for automatic control, either completely or in part. The chief reasons for this wide spread popularity are; first cost is but little more than that of a comparable attended station; reliability and flexibility are as good if not better; operating costs are materially less.

On the Columbia Gas and Electric Corporation System there is installed one or more stations of the several different classes. There are several 1,000- to 3,000-kva. step-down stations for distribution at 4 and 13.2 or 12 kv., with one and two outgoing feeders equipped with automatic reclosing oil circuit breakers and some with automatic induction voltage regulators. There are also several distribution stations having capacities from 3,000 to 8,000 kva. and a maximum of six outgoing automatic feeders. These feeders are all equipped with reclosing oil circuit breakers designed for single pole operation and the transformer bank controls include load response equipment where two banks are installed. Some of these stations have been operating for six years and longer with entirely satisfactory performance. On the supply to the street railway system serving the Northern Kentucky towns, there are completely automatic converter stations equipped with machines having 500 and 1,000 kw. capacity. The control scheme of these machines is designed to function on voltage variation and load demand and includes many other features incorporated in the latest design for this class of equipment. The breakers on the d-c. feeders are designed to open on rate of rise of current and to reclose when the circuit is cleared.

Due to the rapid growth of the d-c. Edison system before the advent of the automatic a-c. network, it was found to be advisable and economical to supplement the larger converter stations with smaller machines located near the centers of concentrated load areas. The control schemes on these machines include automatic starting at predetermined times and automatic stopping determined by the conditions of load and voltage on the system. At the same time in order to increase the

1. Design Engineer, Columbia Engg., & Management Corp., Cincinnati, Ohio.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

flexibility and reliability of the entire Edison system supply, the control on all existing conversion equipment was changed to semi-automatic. This scheme included complete starting operation when initiated by the operator, voltage regulation, load limiting devices, and other improved protective functions.

Around the city of Dayton there is installed a double circuit 33-kv. loop having two sectionalizing stations with step down transformer banks for local distribution at 4 and 12 kv. One of these stations is designed for remote control by means of synchronous selector supervisory equipment from the dispatcher's office over two pairs of telephone wires four miles long. The other station is completely automatic and the control scheme is designed to reclose the 33-kv. oil circuit breakers upon restoration of potential on the lines. The distribution voltage at this station is 4 kv. and the feeders are equipped with regulators and single-pole automatic reclosing circuit breakers. The reclosing cycle consists of three reclosures at intervals of 15, 30, and 75 seconds. If at the end of this cycle the fault still persists, all three phases of the breakers are opened and locked out until the feeder has been cleared of faults. At those stations in which street lighting equipment is installed, the circuits are cut in and out of service by means of time switches equipped with astronomical dials. The operation of any of the automatic equipment transmits a signal over a pair of wires to the dispatcher's office and this signal persists as long as the circuit is de-energized. This pair of wires is leased from the local telephone company and is used also for communication between the station and the dispatcher's office.

The design and construction of these stations have passed through the cycle of improvement and development of apparatus and devices to increase appearance, safety, and reliability of the entire installation. In the earlier types of station the high-voltage bus structure was built up of concrete cells; the low-voltage bus structure was built of pipe or angle iron; the transformers were equipped with bushings and required a bus structure to make connections; and the regulators were installed in concrete cells with little or no protection. The latest design includes high- and low-voltage metal-clad switchgear, pothead type transformers located outside the substation building, and regulators, when located inside, enclosed in individual concrete

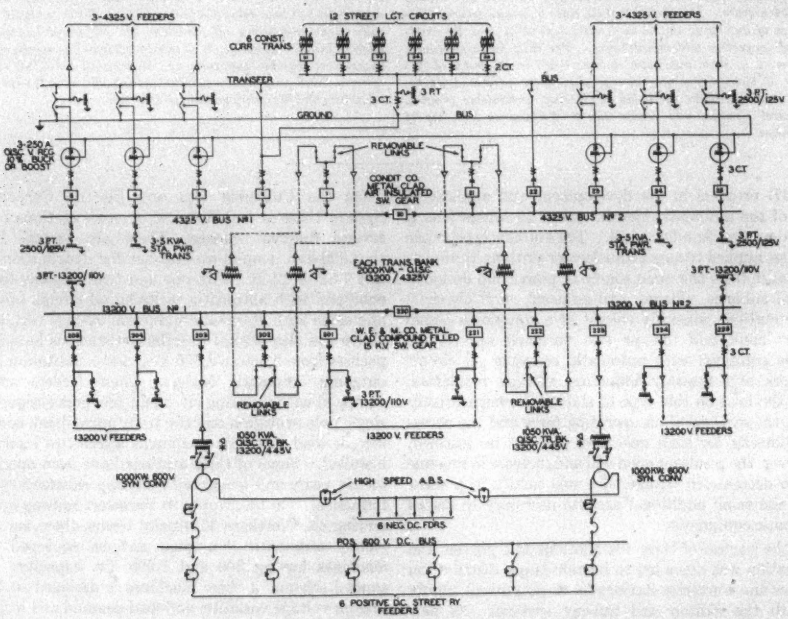

FIG. 1—SINGLE-LINE DIAGRAM OF MADISON SUBSTATION SHOWING THE STANDARD SCHEME OF CONNECTIONS

cells equipped with oil drains and fire extinguishing apparatus.

The type of service secured from an automatic station depends to a great extent on the degree of excellence of operation and maintenance. In the Cincinnati district there are five automatic station operators whose duties are to inspect, test, maintain, and keep clean all equipment in the automatic stations. There are fourteen automatic stations in this district consisting of;

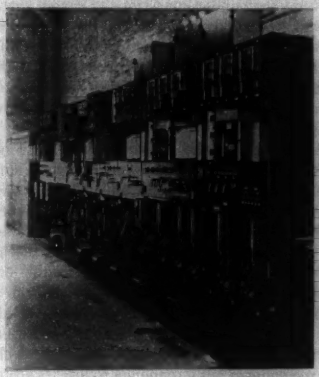

Fig. 2—4,325-Volt Metal-Clad Switchgear Showing Mounting of Relay and Control Equipment

four outlying step down stations having two regulated distribution feeders; five metropolitan step down stations having from two to four regulated distribution feeders; three combination step down and street railway stations having from two to six regulated distribution feeders and one synchronous converter; and two Edison system conversion stations having one 1,875-kw. motor-generator set each. In addition to the care of the above stations, it is the duties of these same operators to inspect, test, and maintain the semi-automatic equipment on the following apparatus: Two 5,000-kva. synchronous condensers, one 1,000-kw. and two 1,500-kw. Edison system motor-generator sets, and seven 3,500-kw. Edison system synchronous converters. This equipment is distributed among six different stations.

The automatic operators are selected as a rule from the operating personnel of the manually controlled stations. They are men who have had some technical training and have shown some special aptitude for the class of work required of such an operator. They are given a special course of training which consists of serving as a helper for a regular operator and other instructions necessary for a complete understanding of all the various types of automatic installations. At the end of the training period they are required to pass an examination in order to become a regular operator.

There is one operator on duty on each of the three shifts in the day and he is given a regular schedule of work for each day in the week. An effort is made to have the operators visit the more important stations at least once in a day and the smaller stations two to three times a week for a regular inspection. This inspection consists of making the necessary meter and operation counter readings, adjusting graphic meters and changing charts, adjusting battery charging equipment, checking temperatures and general operating conditions of all apparatus, checking indicating lamps, fuses, and grounds on control circuits, and in case of rotating equipment, observing the starting and stopping operations. Once each week the rotating apparatus and all its control equipment is cleaned and necessary repairs made, and a complete check is made of the starting and stopping sequence and the proper functioning of all devices. Once every two weeks the equipment on each reclosing feeder is given the same general overhauling. Once each week all the control batteries are given a slight charge and a complete check of their

Fig. 3—Typical Regulator Structure Showing Installation of Lux Fire Extinguisher

condition is made. Finally, twice a year all the apparatus in the station is given a complete overhauling by the regular maintenance crew under the supervision of the automatic men.

It is noted in this connection that the operators are not furnished with any standard form for reporting and recording trouble. Neither are they furnished with any chart or standard method for making inspections. The attitude is taken that any kind of form or chart which

tends to make an inspection a routine duty will tend to destroy the initiative of the operators and to focus their attention on the completion of the form rather than on the inspection of the equipment. It is fully realized, however, that with this system it is necessary to secure operators of a high degree of excellence and efficiency. In order to keep some check on the proper performance of their duties by the operators they are required to turn in to the chief operator at the end of each day a log sheet showing the stations which they visited during the day, what they did outside the usual inspection, and to note any item requiring attention which they could not remedy. It is unfortunate that this condition exists because many items of interest doubtless are never reported. An effort is now being made to keep a more complete record of operating experiences with these stations.

A few of the most common sources of trouble in automatic equipment are poor clocks and poor pen construction on graphic meters, auxiliary switches on circuit breakers, grounds on control circuits, and the master relay, No. 1, losing its calibration. During the past seven years of automatic station operation in the Cincinnati district there has not been an outage of any consequence due to lack of vigilance and maintenance. This is believed to be a good record and a fine testimonial to the efficiency of these five men. It is estimated that they can properly care for a greater number of stations by relieving them of other extraneous duties which they are called upon to perform.

Upon looking back over the operating records of the distribution stations, it is noted that at least 90 per cent of the faults on a circuit are temporary in nature and that the breakers remain closed after the first or second operation. These circuits having automatic reclosing circuit breakers are equipped with instantaneous overcurrent relays set to operate at a predetermined value of current which is slightly below the safe rupturing capacity of the circuit breaker or the thermal capacity of the regulators. The operation of these relays prevents the reclosure of the circuit breaker until the fault has been cleared and the relays reset. To date there has been only one operation of these relays which was caused by a cable failure near the station. By the use of these relays it is considered safe to install equipment having lower capacity or to make the time between circuit breaker reclosures less than advisable otherwise.

The reclosing cycle for manual stations is 20, 45, and 90 seconds, providing the operator is free at the time of fault to perform these functions. Since the operation of the automatic stations is free of the human element, it is found that these functions are performed faster and more accurately than any operator could perform them. Whenever trouble develops in an automatic station, the operator on duty at the time is immediately dispatched to that station to clear up the trouble and to restore the equipment to its normal operating condition. With only one operator on duty and with more than one station in trouble, the outages are sometimes prolonged undesirably. However, during storm periods when trouble is most likely to occur, another operator is pressed into duty for emergency calls and it has not been found necessary to increase the number of regular operators. The application of semi-automatic control to the Edison system conversion equipment has materially increased its reliability and flexibility and decreased the outage time. It is interesting to note that under conditions of severe system disturbances, the automatic stations have performed the necessary functions faster and more accurately than could be expected of any operator. Since the load dispatcher has sufficient confidence to trust them to perform correctly, he can devote his attention to other pressing concerns.

On the system of the Dayton Power and Light Co. there have been for a number of years several automatic distribution stations in operation. These stations are of the outdoor type having reclosing circuit breakers in individual sheet metal houses. The breakers are equipped with a-c. motor-operated closing mechanisms and 24-volt d-c. trip coils. So much trouble has been experienced with this type of control that no similar equipment is being considered for future installations except in special cases. The auxiliary switches are a perpetual source of annoyance and the centrifugal closing mechanism has a tendency to lose its adjustment and cause the breaker to "pump" or else fail to close altogether. It has been decided that the future installations of this nature shall include solenoid-operated breakers with a control battery or some similar source of supply. Since the cost of the solenoid mechanism is less than the motor mechanism, this compensates to some extent for the additional cost of the control battery.

One of these stations was supplied through a tap connection on the double circuit 33,000-volt loop around the city. Approximately two years ago this station was rehabilitated and 33,000-volt sectionalizing equipment was installed. At this time the control scheme was changed from automatic to supervisory which is operated from the dispatcher's office approximately four miles distant from the station. The only source of trouble experienced on the installation to date has been due to interference by the telephone company from which the interconnecting wires are leased. Another point of some interest in this station as well as in automatic stations in general is the use of diverter pole type motor-generator sets for control battery charging. In case of complete outage on the station it was found that this set could be automatically started and reconnected to the battery without any supervision. It is the practise to float the motor-generator set across the battery at all times.

In addition to the above stations this company has

been operating four automatic street railway converter substations for approximately ten years. A great deal of trouble was experienced with these installations as originally designed which was due principally to the types of relays installed. The old plunger and bellows types of relays were not as accurate or reliable as the present induction types and motor-operated timers. Practically all the original relays have been replaced by relays of later design and very little trouble from this source is anticipated in the future. In one station in which two converters were installed for parallel operation considerable difficulty was experienced in securing the proper operation and division of loads. This is an example which shows the necessity for careful and exact specifications and design to secure similar characteristics in machines for this class of service.

It is believed that the automatic station has proved itself thoroughly and that it has a definite application in the distribution of electrical energy on practically all systems. It is to the advantage of all to recognize the economies to be obtained by the application of automatic stations, and there has been enough experience obtained to fully establish their reliability and flexibility. There are a few items which should interest the manufacturers and direct their attentions towards improvement, such as, graphic meters, breaker-operating mechanisms, and some types of relays.

It has been shown conclusively that the key to success in the operation of automatic stations is correct inspection and maintenance of the equipment Recently a proof of this statement was brought very forcibly to the attention of the writer. On the distribution system of one of the subsidiaries of the company there were several small automatic stations which were not giving the type of service expected of them. All manner of trouble was being experienced with this equipment and the frequency and length of outages were becoming a source of great annoyance to all concerned. Upon an investigation of the situation it was found that there were no operators expecially trained to inspect and maintain these stations and that an inspection was made by any employee at whatever time he happened to be near the station. As a result of the long existence of this condition the automatic station was being blamed for the entire situation and was falling into much disfavor. Inspection can be overdone as well as underdone and it is only by experience that the happy and sufficient medium can be attained. Maintenance and inspection result in a saving by minimizing the number of outages and damage due to incorrect operations and failure of equipment. The savings effected by inspection and maintenance beyond a certain point are often overbalanced by the increased operating cost. Finally, since continuity is the keynote of public utility's service to its customers, the automatic station is recommended as a means of securing improved service continuity with a decrease in operating expense.

Part II—Toronto Hydro-Electric System

By F. F. Ambuhl[2]
Member, A. I. E. E.

The rapid growth and expansion of the Toronto Hydro-Electric System from its inception in 1910 resulted in numerous changes and extensions to the stations and distribution system to meet the constantly increasing demand for electric power. Some of the first stations were too small and more or less temporary, making it necessary to rehabilitate these at a comparatively early date.

In 1920, considerable thought was given to automatic control for railway substations and supervisory control for transformer and switching stations. During the years 1921 and 1922, two automatic railway substations and two supervisory control substations were installed on the Toronto system.

The first automatic installation gave a certain amount of trouble when first installed, in that some of the equipment required a considerable amount of attention and necessitated slight changes being made to certain parts of the equipment. After these weak spots had been eliminated, however, the equipment gave satisfactory results.

The first installation of supervisory control, unlike the automatic control, gave satisfactory results from the beginning, there being very little maintenance required on this equipment, which was undoubtedly due to the simplicity of the scheme used.

After the first eighteen months all automatic and supervisory equipments were giving satisfactory operation and these results, together with information on more recent developments in this class of equipment, led the Toronto Hydro-Electric System finally to adopt the policy of locating moderate sized stations near the economical load centers and equipping them for automatic or supervisory control. From this time forward all new substations were equipped for automatic or supervisory control.

All supervisory stations are operated from the nearest manual station, the operators of which are under the jurisdiction of the system load dispatcher.

At present there are installed on the Toronto system, five automatic and nine supervisory equipments, with three more supervisory equipments going in this year. All of the automatic stations were new installations, whereas two of the nine supervisory equipments were installed in old stations rehabilitated and equipped for supervisory control.

Four different types of automatic and four types of supervisory control have been used by the Toronto system.

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2. Asst. to Chief Engr., Toronto Hydro-Electric System, Toronto, Ont., Canada.

Automatic Equipment.

Westinghouse Automatic Railway Equipment.

General Electric Co. Automatic Railway Equipment.

Brown Boveri Automatic Mercury Arc Rectifier Equipment.

Automatic Synchronous Condenser Equipment developed by the T. H. E. S.

Supervisory Equipment.

Westinghouse Synchronous Visual Supervisory Control.

General Electric Distributor Type Supervisory Control.

General Electric Synchronous Selector Type Supervisory Control.

Toronto Hydro-Electric System, Monowire Cable Type Supervisory Control.

In addition to the above automatic and supervisory equipment, there is a Gamewell signaling system which is used for giving signals to the load dispatchers when switches automatically open in small unattended transformer stations where supervisory control has not been provided.

Source of Energy for Supervisory and Control Equipment

Item 1—In general storage batteries and motor-generator sets are installed as the control energy source

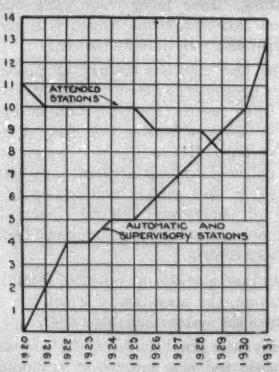

Fig. 4—Curves Show Number of Supervisory and Automatic Stations as Compared to Attended Stations Since 1920

and floating battery operation has been adopted as standard. In some cases, the motor-generator set is supervised, in others it is not.

Item 2—*Supervisory and Control Circuits*—The cable used for supervisory control is lead-covered paper-insulated, impregnated and insulated for 1,500 volts. Each conductor is 16 gage and the cable is laid up in twisted pairs. All cable is underground and is privately owned; also it is used for a number of services such as supervisory control, telephones, telautograph services, remote and totalizing metering.

The installation of supervisory equipment has increased constantly on the Toronto Hydro-Electric System as will be seen by referring to curves of Fig. 4, showing the total installations by years and taking into account the three stations which are now under construction.

Operation

The benefits derived from automatic and supervisory control have in general been quite satisfactory and are accepted today as efficient and economical features of

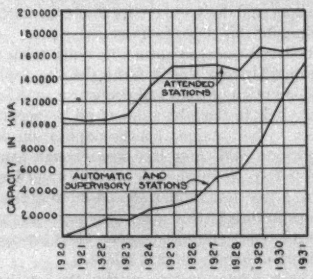

Fig. 5—Curves Showing the Increase of Installed Capacity of Supervisory and Automatic Stations as Compared to Attended Stations by Years.

power distribution. This method of controlling stations has, however, changed the whole aspect of the operating problem and is of such importance that both operating and manufacturing engineers are giving careful attention to the results obtained, with a view to making further improvements in the methods of control, and particularly the apparatus used in connection with such schemes.

A great deal has been done during the last ten years in developing equipment of the desired characteristics to be coordinated into various schemes of operation as required by the apparatus controlled and which may be depended upon to give reliable operation.

The rapid strides that have been made in this class of equipment have not, however, enabled the manufacturer to give all of his equipment a thorough test, particularly where time and certain operating conditions enter into the problem, and it, therefore, devolves upon the operating companies to make these tests over a period of time under the operating conditions for which the apparatus was intended.

The best method of determining the reliability and efficiency of automatic and supervisory equipment is to try it out for a period of time under service conditions.

While there is a number of properly coordinated, well designed systems of automatic and supervisory control

in use, experience has shown that even though great care may have been taken in the manufacture of the equipment, certain weaknesses develop in service which have to be overcome. These are usually minor troubles which very often develop shortly after the equipment goes into service and can generally be rectified without any alteration to the equipment. It is important that proper maintenance be provided as the number of faulty operations may be reduced materially if the maintenance men, looking after the equipment, are thoroughly familiar with the equipment from the beginning and are able to locate and correct the trouble immediately when it occurs.

In order to give some idea as to how this equipment has operated, it is proposed to give a brief description of the various schemes of control used by the Toronto Hydro-Electric System and the performance of each over a period of years.

Automatic Synchronous Converter Stations

Two schemes of automatic control for synchronous converter stations have been used by the Toronto system, the General Electric scheme in which a drum controller initiates the various operations in proper sequence, and the Westinghouse company's scheme, in which these steps take place through the guidance and control of various relays and interlocks which function according to a predetermined plan in a step-by-step fashion. The ultimate results being the same in both cases, namely, that of starting up and shutting down the station as required. In both of these schemes, it is the practise to arrange for starting up on load demand which is determined by a suitable contact making voltmeter, and shutting down when the load is reduced to an uneconomical point. In other words, when the trolley voltage is reduced to a minimum amount by the increased load, a contact making voltmeter energizes a relay which starts the sequence of operations for starting up the station. These will be referred to as Scheme A and Scheme B in the paper.

Scheme A. In Scheme A, a motor-operated drum controller is started through the action of a low-voltage relay as mentioned above. This causes the high-tension breaker to be closed, energizing the transformers and is followed by the closing of the a-c. starting switch, impressing sufficient starting voltage across the rings to bring the machine up to synchronism. When full speed is reached, the converter fields are excited from a small direct current generator, which is driven by the motor on the drum controller. This gives a positive method of obtaining correct polarity. The converter is then connected for self-excitation and the running switch closed, impressing full voltage on the machine and giving normal field excitation. As soon as the machine reaches normal voltage, the d-c. line switches are closed, connecting the machine to the line through the current-limiting resistors in the machine circuit and if the current taken by the machine is not excessive, the current-limiting resistors are immediately cut out. The controller then remains at rest and the converter operates normally on load until the current supplied by the station falls below some predetermined value, when a low-current relay operates, starting the controller toward the off position, thus shutting down the station.

Operating Results of Scheme A. Scheme A was first installed in the North Toronto station in 1922. This station was primarily used to boost the voltage on the north Yonge St. line and supply power to the car barns across the street from the station. A 1,000-kw. converter was installed at first and a second machine was added later. These machines supplied power to the railway over six feeders, protected by automatic carbon-break circuit breakers.

In addition to the current-limiting resistors which are automatically cut in or out as the load demands, these converters are also protected by a type JR high-speed circuit breaker connected in the negative lead to the machine. The equipment in this station has given very satisfactory results for over nine years though a few troubles have been experienced, some of which are enumerated below:

1. When first installed, Nos. 2 and 3 timing relays gave trouble and were replaced by the manufacturer.
2. The No. 30 field relay did not function properly and the coil on this relay was changed, after which no further trouble was experienced.
3. Some trouble has been experienced with the break-bands on the brush lifting mechanism breaking, making it necessary to replace these.
4. An open circuit in the exciter generator, used for correcting polarity, caused considerable operating trouble before it was known that this was the cause. This was an intermittent trouble which was not apparent at all times. This was repaired, after which no further trouble was experienced.
5. In addition to the above, various relays have required adjusting and light repairs, though it cannot be said to be excessive.

The use of a drum controller, through which the various relays are energized in their proper sequence, has probably helped to reduce relay contact trouble, as such a device is very sturdy and the contacts of which are very reliable. The drum controller has also been found very useful in checking trouble as the whole operating sequence can be gone through and inspected in steps by operating the drum controller by hand and in this way each relay circuit can be checked separately. In general this equipment has given very satisfactory operation though no doubt improvements have been made on some of the devices which will still further reduce the troubles which are now being experienced.

Scheme B. In Scheme B the rotary transformers are energized through the operation of a low-voltage relay and subsequent relays, similarly to the other scheme, after which, the starting contactors are closed and the

machine then builds up in speed in the usual manner. During the time interval of starting, the polarity is checked and if found incorrect, is reversed automatically by slipping a pole. This is accomplished by means of a polarized motor relay, driven by a d-c. motor having a permanent magnet field and with the armature connected across the converter commutator. While the converter is accelerating, the voltage across the commutator is alternating and the relay motor armature merely tends to oscillate. As synchronism is approached, the voltage merges into a direct current potential. The relay armature then revolves in one direction or another, dependent upon the polarity of the machine and turns a drum contactor. If correct polarity is obtained, by the order in which the contacts are closed, the starting contactor drops out, the running contactor is closed and the brushes are lowered. If, on the contrary, the converter builds up with incorrect polarity, the relay again operates in such a way as to repeat this operation.

When correct polarity has been obtained, the starting contactor is opened, the running contactor is closed, the brushes are lowered, and the d-c. switches are closed. This connects the machine to the load, through the current-limiting resistors which are then shunted out in steps by contactors which close through the functioning of accelerating relays. The machine remains in operation until cut out of service by light load conditions similarly to that of the previously described scheme.

Operating Results of Scheme B. Scheme B was installed in the first automatic railway station in Toronto, the station being used for boosting the voltage on a long cross town line in the north part of the city. This equipment, to a large extent, consists of standard relays, together with a number of specially designed devices coordinated to give the proper sequence of operation.

This station has been operating satisfactorily now for nearly ten years though numerous troubles were experienced when it was first installed, requiring considerable attention and modification of some of the devices before satisfactory operation was obtained. Some of the principal troubles which occurred when this equipment was first installed are given below:

1. Mica insulation broke down on grid resistors.
2. Trouble was experienced with the latching relay on the *E* switch which was modified to eliminate further trouble.
3. Trouble was experienced with the contacts of pallet switches controlling the main contactors, making it necessary to modify these devices, after which no further trouble was experienced.
4. The contactor coils burnt out repeatedly and were finally rewound with a larger number of turns to avoid over heating.
5. Converter bearing thermostats became inoperative, due to warping of the fiber plunger rods.
6. Brush lifting motor burnt out, also the brush lifting mechanism gave considerable trouble. This mechanism was modified and the original motor replaced with a larger motor, after which no further trouble was experienced.
7. Considerable trouble was experienced with the contacts arcing over on the No. 7 polarized motor relay, which was also modified and the trouble eliminated.
8. There was also a number of other minor troubles, the nature of which is not on record.

Most of the above troubles occurred during the first year, though after they had been eliminated, the equipment gave very satisfactory operation.

Brown Boveri Automatic Mercury Arc Rectifier Station

In 1929 the Ossington Station was increased from 1,000 kw. to 2,200 kw. by installing two mercury arc rectifiers in the station building and placing the transformers and switches outside. In this way, the station could be more than doubled in capacity without increasing the size of the building, and as the rectifiers did not require any special foundation, it became only necessary to move the rotary converter with its automatic equipment into one of the larger stations where more railway machine capacity was required and set the two 1,100-kw. mercury arc rectifiers on the floor of the Ossington Station.

The completed station is a full automatic rectifier plant consisting of two 1,100-kw. nominally rated rectifier units. Energy is taken from a 13,200-volt, three-phase 25-cycle source and converted to 600 volts for traction service. Each rectifier and its transformer is capable of carrying the following load, based on the nominal rating of 1,100 kw. at 600 volts:

Per cent load	Amperes	Duration
100	1,833	Continuously
150	2,750	2 hours
200	3,666	1 minute

Automatic Features. The automatic features can be roughly divided into four groups: Plant control, a-c. closing and reclosing, vacuum pump control, and the protective group.

Plant Control. The rectifier plant is arranged to start and stop according to the demand. Either unit may be made the leading unit, the other one being switched in or out according to the demand on the first. The first feature is known as low-voltage low-current control, the second is the overload control.

Another important automatic feature of the plant control is the lockout control. Should the leading unit be in serious difficulty it will be locked out and automatically transmit all signals received to the lagging unit which then becomes the leading unit.

It is also possible to start and stop the station or any

of its units manually, while retaining all other automatic features.

A-C. Closing and Reclosing. The rectifier is controlled through the oil circuit breaker which connects the unit to the a-c. supply. The circuit breaker is closed according to signals received by the a-c. closing and reclosing controls, all conditions being correct. Should the breaker fail to close, the reclosing control will attempt to close it two more times at definite time intervals. If it does not close the last time, the rectifier will be locked out and the spare unit switched in if desired.

The d-c. breaker is controlled by the oil circuit breaker through electrical interlocks.

Vacuum Pump Control. The vacuum pump is automatically controlled according to signals received from a contact making vacuum meter.

Either or both the a-c. and the vacuum pump controls can be entirely cut out and replaced by purely manual control.

Protective Features. The rectifier is protected against sudden and continuous overloads, high-voltage surges, high temperature, water failure and operation with poor vacuum. The d-c. breaker opens on reverse current.

Operating Results. This rectifier equipment has given very satisfactory operation though some operating trouble was experienced when the apparatus was first installed. These are enumerated as follows:

1. Insulator on excitation anode was found cracked, due to incorrect size and improper packing.
2. Ignition coils burnt out several times, due to relay sticking. This was overcome by modifying the relay.
3. Poor contact on control relay prevented the rectifier from being cut in.
4. Trouble on overload relay contacts prevented the second unit from coming in.

The above defects have all been remedied and no trouble whatever has been experienced with this equipment during the last ten months. It might be mentioned that this equipment has carried loads considerably in excess of 150 per cent load several times, without showing any signs of trouble.

Westinghouse High-Speed Visual Supervisory Control System

The Westinghouse high-speed visual supervisory control system is an all relay system embodying the principal of step-by-step synchronous selection. Apart from the control board, the equipment is mounted in two steel cabinets, one of which is located in the dispatcher's office and the other at the outlying station. At the outlying station there is also a number of interposing relays which serve to relay the signals from the relay cabinets to the units controlled. Four wires are used between the dispatcher's office and the distant station. By means of two of these wires and a synchronous control circuit, selecting relays are kept in step and moved from point to point in a definite sequence, that is, corresponding relays are operated simultaneously at the dispatcher's office and outlying station. The remaining two wires of the trunk line are switched by the operation of these relays from one apparatus control unit to another. The apparatus units at the substation are connected to a set of selecting relays and the dispatcher's control keys and supervisory signaling lamps are connected to a similar set. The selecting relays must operate in synchronism at every step or not at all.

Proper lamp signals notify the dispatcher should the system cease to function. All apparatus is normally at rest. The cycle of synchronous selecting operations takes place when the dispatcher initiates an action or when an apparatus unit changes position at the outlying station.

Operating Results. The synchronous visual supervisory control described above has been installed in four stations of the Toronto Hydro-Electric System, the first being installed six years ago; the second four years ago and the other two, three years ago. This equipment has given satisfactory operation on the Toronto system, though it cannot be said to be entirely free from trouble. The troubles experienced in almost every instance are however of a minor nature and are usually found to be relays requiring slight adjustment, or contact trouble.

Control-key contacts, battery equipment and fuse troubles also contribute to the cause of faulty operations. There are some instances where certain relays cause more trouble than others.

In the case of the Danforth station, as will be seen from the performance chart in Table I, forty-five cases of relay troubles were reported in five years, of which fourteen were due to latching relays sticking. Operating records of the other stations using the same kind of equipment, though more recent, do not show the latching relays to have given any more trouble than the other relays.

There has been a number of faulty operations for which no cause was found, also a number of faulty operations has occurred which was found to be due to main apparatus trouble.

G. E. Distributor Type Supervisory Control

The distributor type supervisory control was installed in the Parkdale station in 1928. This was one of the early developments of supervisory control and uses a modification of the printing telegraph distributor and polarized relays. The purpose of this distributor is to minimize the number of connecting line wires between stations and at the same time allow the transmission of control and indicating impulses in a predetermined sequence, thereby taking the place of two connecting line wires between stations for each switch or unit to be controlled and indicated.

Considerable trouble was experienced with this type of equipment due almost entirely to the distributor mechanism.

TABLE I—PERFORMANCE CHART

Station	Type	Control positions	Months in service	Supervisory relay trouble	Control key trouble	Battery equipment trouble	Fuse Trouble	Human factor	Line troubles	Outages due to supervisory trouble	No. of operations per year
Market	T.H.E.S.	30	114	4	4	2	5	1	None	1	W 100
Keele St.	T.H.E.S.	17	102	3	2	2	1		1	1	175
Carlaw	T.H.E.S.	18	11	5	2	2	2		None	0	400
Danforth	W.E.& M. Co	50	60	45	2	5	2	3	None	8	3,000
Parkdale A.	G.E.Co.	46	21	7* 98†	4	1		3	None	11	300
Parkdale B.	G.E.Co.	46	8	2	0	1		2	None	2	325
Defoe	W.E.& M. Co	30	53	9	1	2	1	2	None	8	175
John St.	T.H.E.S.	30	18	2	1		1	3	1	1	100
Wiltshire A.	W.E.M.Co.	40	47	10	3		3	0	None	8	2,000
Wiltshire B.	W.E.M.Co.	40	47	10	4	4	2	0	None	4	200
Stirling Rd.	W.E.&M.	30	1931								
Glengrove	G.E.Co.	30	1931		Last three not yet in operation						
Duke & Geo.			1931								

*Relays †Distributor

The motor-driven distributors, one being installed at each end of the control system, are operated continuously and must be kept in synchronism if proper operation of the control system is to be obtained. Numerous faulty operations were experienced as a result of this part of the equipment getting out of order, and the matter of keeping this equipment working properly became so difficult that it was finally decided to have the manufacturer change this system to the more recent development known as the synchronous selector type of supervisory control. This change was made during the spring of 1930.

G. E. Synchronous Selector Supervisory

This new system which replaced the distributor type supervisory control in the Parkdale station is essentially a direct system of control and indication, using the familiar type of rotary selector, such as used for automatic telephones, which step in synchronism to transfer the control and indication line wires from one circuit to another as selected by the dispatcher.

The relays, selectors, control keys and indicating lamps are more or less standard devices so coordinated as to insure a proper sending of control impulses and indicating the positions of the various units controlled.

The synchronous selector system is divided essentially into three independent circuits designated as control, indication, and synchronizing. All three use a common return. The equipment at the dispatcher's office is connected to the equipment at the substation by four line wires, all of which must be free from grounds. The dispatcher controls the supervised unit in the substation over the control circuit, and the indications from the supervised units are returned to the dispatcher over the indication circuit. The selector equipment at the outlying station is made to operate in synchronism with the selector equipment in the dispatcher's office by means of current impulses sent over the synchronizing circuit. The principle of this scheme is very similar to that of the distributor type in that all control operations are performed over four wires and distributed by means of synchronized switching devices at the sending and receiving ends.

Results of Operation. This equipment has been in operation since April 1930 and has given very satisfactory operation. By referring to the performance chart, Table I, it will be seen that for the first eight months, only two troubles have been experienced with the relays—both of which were minor troubles, and one with the battery equipment, which was due to the brushes of the motor-generator set sticking in their holders.

T. H. E. S. Monowire Supervisory Control

Four stations of the Toronto system use a cable system of supervisory control which was developed by the Toronto system and is called the "monowire supervisory," since only one wire or channel is used for each unit controlled and two wires or channels being common to all controlled units. The monowire system differs from other cable systems used at present in that only one wire is used instead of two, for each unit controlled and indicated.

Description. The principle of this scheme and mode of operation, will be best understood from the drawings, in which Fig. 6 is an elementary diagram showing the fundamental circuit. Fig. 7 is the circuit of Fig. 6 with indicating features added. Fig. 8 is one form of a complete commercial circuit, with additional alarm and safety devices added.

All of the figures show control for one unit only. It is obvious that one or a plurality of similar controls may be used as required.

In Fig. 6 two common buses, positive and negative (1 and 2) are energized from some source of voltage; a battery 3, is shown. These buses are continuous from controlling station to distant station and serve to energize all units which may be connected to them. Considering the monowire unit as we may term it, a single-pole double-throw switch with terminals 4 and 5 serves to connect the monowire M to either of the two buses as desired, thus in turn energizing either of the coils T and C, depending on which polarity is closed across the switch. Thus if the two coils T and C are actuating coils controlling a circuit breaker it can be opened or closed depending on the position of the switch 4-5. In

practise the coils T and C are usually not continuously energized, and for this reason, and to eliminate a series connection between the coils T and C and the two buses, we insert in the circuits of the coils pallet switches 12 and 13 for opening and closing in accordance with the operation of the breaker being controlled. Switch 12 is closed when the breaker is closed; switch 13 is closed when the breaker is open.

The circuit of Fig. 6 suffices for the mere operation of a circuit breaker, two operations being performed over

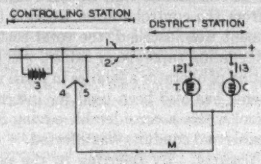

Fig. 6

the monowire M. Indication as to switch position is obtained by the arrangement of Fig. 7 in which like letters and numerals of designation refer to corresponding parts. While the coils T and C of Fig. 7 may control a circuit breaker directly they are here shown as relays which by means of contacts and standard control circuits, not shown, initiate the tripping and closing of a circuit breaker when energized. This arrangement has a number of practical advantages and enables one to adjust the electrical values of the coils T and C to the most favorable point for satisfactory operation, as dealt with below.

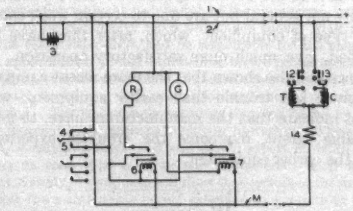

Fig. 7

Further, the control key 4-5 is shown as being of the telephone type.

Relays 6 and 7 and indicating lamps R and G which may be colored red and green respectively are shown in Fig. 7. The manner of their operation is as follows:

Assume that the control key 4-5 has been closed in the 4 position thus energizing the closing relay C and closing in the circuit breaker. On closure, switch 13 opens and 12 closes, thus completing a circuit from positive bus 1 through 12-C and 14 to contact 5 of control key, now open, and preparing the way for the opening of the circuit breaker. A circuit also exists, when the circuit breaker is closed, from positive bus 1 through 12, T, 14 and relay coil 6 back to negative. Relay coil 6 is thus energized and closing its contacts completes a circuit through the red lamp, thus giving an indication that the breaker is closed. While current for the energizing of 6 thus flows through coil T, it is not sufficient in amount to operate the relay T. In other words relay coils 6 and T are chosen of such differing values of resistance that 6 will operate and T will not when placed across the bus potential. This is an essential feature of this control scheme.

Exactly the same sequence is gone through when the breaker is opened or opens by itself, the green light lights through relay 7, operating relay 7 being energized by a circuit from negative through 13, C, 14, and 7 to positive. A resistance 14 can be inserted in the monowire lead if desired so as to adjust line resistance to the proper value for correct relay operation and so enabling

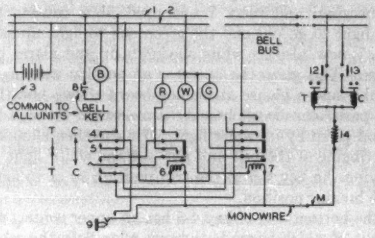

Fig. 8

the use of standard relays over varying distances. The line resistance obviously varies with the distance between stations.

While operation and indication of position is achieved by the arrangement shown in Fig. 7 it is also desirable to provide for the giving of an audible or visible alarm should the circuit breaker or other equipment controlled, change position independently of the monowire control through protective relay action, etc. This is achieved in the arrangement shown in Fig. 8 where an additional indicating lamp W (white) together with additional contacts on the relays 6 and 7 are shown. A bell B and bell key 8 are also shown. The bottom contacts on the control key 4-5 have also been connected up. In this arrangement the control key is of the locking type.

The sequence of this additional alarm circuit is as follows: assuming that the operator desires to close the circuit breaker, he operates the control key 4-5 to place it in the closed position. When this is done the bottom contacts on the key are also closed making two circuits from negative, through relay contacts on 7 to white lamp and to bell bus respectively. Since at the instant of operation of key 4-5, the circuit breaker being open,

relay 7 is still energized together with the bell circuit, showing that key and breaker positions do not agree. The operator may if he so desires, eliminate the ringing of the alarm bell by depressing the bell key 8.

The master key 9 has been added to eliminate malicious or accidental operations through inadvertence and the operator according to the scheme of Fig. 8, is required to operate the master key 9 to complete the operation desired and to send closing energy over the monowire *M*. Master key 9 also clears direct positive or negative potential from the monowire and so permits of the correct functioning of lamp relays 6 and 7.

On the closure of the circuit breaker, relay 7 is de-energized through the pallet switch 13 opening, and the green light is open-circuited together with the alarm light *W* and the bell circuit, the red light circuit also being energized through the operation of its relay 6, this latter being energized through the pallet switch 12.

Should the circuit breaker now open through overload relay action or some other cause, relay 6 becomes deenergized and relay 7 energized, thus causing the red light to go out and the green one to light up as in Fig. 7, now, also the white one lights up and alarm bell rings. This gives the operator an audible warning of trouble and a glance at his key-board shows him that one particular circuit breaker is concerned through the signal given by its white light. He can then eliminate the ringing of the bell and put out the white light by moving the key 4-5 to the trip position so as to agree with breaker position.

The bottom arm of key 4-5 has an upper contact the circuit of which is made through relay 6 to the white lamp, so that when the key is turned to the open position from the closed, with the circuit breaker closed, the white lamp gives an indication that the key in this position also does not agree with the breaker position.

Thus, superimposed on the ordinary green and red light indication we are enabled over the one monowire circuit to provide an additional supervisory indication which shows when the control key does not agree in position with the controlled equipment at the distant substation and which also gives an audible indication of the independent operation of the controlled equipment.

Operating Results. This system of control has given very satisfactory operation during the past ten years. There has been very little trouble with the switch operations though a certain amount of trouble has been experienced with the signal devices. The reason for this is that there are no relays in the control circuit at the dispatcher's office and only one relay, or perhaps two, at the outlying station; also there are only two contacts to be made at the dispatcher's office and one, or in some cases two, at the outlying station for each operation.

The relays used in this system are standard relays which have been tested under severe operating conditions and while they have not been free from trouble they have given very satisfactory operation. The operating results of the cable supervisory installations will be seen from the performance chart, Table I. The very satisfactory operation of this scheme has resulted in its being used where the distance between stations was short and the total cost of installation, including the cost of supervisory cables, was less than other schemes using synchronized switches at the sending and receiving ends for reducing the number of wires.

It might be mentioned that at the Market station, 1¼ miles from the control station where a twenty-five unit equipment was installed, the total cost of the installation, including supervisory cables, was paid for by the elimination of operator's salaries in three years.

This scheme has also been used for internal station switch control where a considerable saving in the cost of control cable and conduits was effected.

General Remarks. By referring to the performance chart, Table I, showing the various kinds and number of faulty operations, of the supervisory equipment it will be seen that most of the trouble was due to faulty relay operation, a condition which one would naturally expect since the apparatus consists mostly of relays. The relay troubles are due to a number of causes such as residual adjustments, contact trouble, loose nuts and screws, dirt, etc., all of which are usually found to be minor troubles and are experienced more in some types of equipment than others.

Trouble has also been experienced with control and contacts, battery equipment, poorly soldered joints, loose fuses, fuses blowing, etc., though troubles from these sources have been very few. It has been the experience of the Toronto system that these troubles are usually not general but are due to certain weak spots in each type of equipment, which, after they have been rectified, give much more satisfactory operation. Experience has also shown that the more recent equipment has given less trouble than earlier equipment, which would indicate that the manufacturers have, to a considerable extent, overcome the troubles experienced with the earlier equipment.

Discussion

SUPERVISORY CONTROL FOR A-C. ELECTRIFIED RAILROADS

(West and Griffith)

OPERATING EXPERIENCE WITH AUTOMATIC STATIONS

(Stamper and Ambuhl)

E. J. Rutan: From these papers one gets a contrast between the opinion of a manufacturer in regard to the maintenance required on this type of apparatus, and the more definite statements of the user. There is no doubt but that a definite program of adequate, possibly frequent maintenance is required in order to have apparatus of the type covered in these papers function satisfactorily day in and day out. The method of developing the program and the costs are of interest.

The cost limits have not been brought out by the authors, but these in general, must not exceed the cost of operating a station with attendants. It would be of advantage if the authors gave some idea of the amount that can be economically expended for maintenance when consideration has been given to the additional capital investment involved in the supervisory and automatic equipment. A partial answer has been given by Mr. Ambuhl through his curve in Fig. 4, which shows that since 1920 there has been a marked increase in automatic and supervisory stations, with a decrease in attended stations. From this it may be concluded that they must have found it economically feasible or such a continuous growth would not have occurred. Specific details in regard to comparative costs, however, would be welcome.

The major difficulties which Mr. Stamper reports as representing the chief troubles in the automatic stations are with graphic meters and circuit breaker, auxiliary switches, and mechanisms. These, I feel, should not be listed as difficulties chargeable to automatic stations, in view of the fact that such equipment has been in service on practically all systems many years; long before automatic stations were attempted. If difficulty is experienced with equipment of this sort, either the available engineering knowledge has not been used in specifying the equipment originally, or the maintenance experience has not been utilized to keep such apparatus in proper condition. It is known that some types of graphic instruments are not as reliable as others, but there are instruments available which will give good performance from the time a new chart is placed in the instrument until it is necessary for the inspector to return to renew the chart. As far as auxiliary switches and circuit breaker mechanisms are concerned, it is felt that the experience in the industry is such that these devices, if of a properly specified design and good manufacture, can be maintained with long intervals between inspections and still be free from troubles so that they should not appear as troubles for automatic stations. The other difficulties which are listed are in line with those to be expected in connection with the operation of supervisory and automatic stations.

In developing a definite maintenance program, the following suggestions are given:

In connection with the installation of the equipment, the men who are to be responsible for maintenance should work along with the installation crew, and be responsible for the checking of the apparatus for proper functioning before initial operation. This insures that the men will have gained experience and knowledge of the equipment while testing for faults before operation is attempted. The maintenance schedule then can be set up on a rather frequent basis, perhaps weekly or bi-weekly. These men should be carefully supervised and should report in detail their findings on each type of relay and equipment. After a few months, a study of their data will show that certain of the devices can have the maintenance period extended to a month or three months. Following the same procedure on the new basis, it is possible to arrive at a program in which only a few of the units may receive attention once a month, other units on a three months' basis and some units may go for six months or longer. While this program is in operation, it would be desirable to have several men working in the crew so that by the time the final program is set up there would be available on the system a number of employees capable of handling the maintenance work and emergency calls when they may arise.

In connection with maintenance programs, I would like to urge on the technical committee a more definite effort to collect the operating experience of the various companies who may have supervisory control systems or automatic stations so that it may be made available more readily to all those in the Institute interested in such problems. Those who have had experience with such equipment know that a period of six months or a year, is required in order to know the weaknesses as well as the advantages of a system. It is then found that when a complaint is made to the manufacturer about the weaknesses, he has developed a new system which he would advance as the answer to the troubles. It is necessary, however, to continue the maintenance of the system already installed. Conditions have been such in recent years that advancement has been so rapid that the maintenance experience which the operating men are obtaining on the various systems is not being made available quickly enough, and in a collective way, so as better to judge the engineering qualities of the newer developments which the manufacturers advise us represent the last word, at least for that current year.

As a definite program, I would urge the Automatic Stations Committee to consider collecting maintenance data and making a report similar to the manner in which information on the operation and performance of protective relays was collected by the Protective Devices Committee some years ago. I think such a program adopted by this committee would be very helpful to users and prospective users of supervisory control systems and automatic substations.

C. E. Stewart: The use of lead-sheathed cable in underground ducts as applied by the Toronto Hydro Electric engineers makes an ideal arrangement for the functioning of any type of supervisory equipment. The line wire troubles as listed by Mr. F. F. Ambuhl in the performance chart attest to this fact. Maximum freedom from interference caused by mechanical, human, or electrical factors is obtained by this arrangement.

In connection with this performance chart on four-wire systems, it would be interesting to know if the relay troubles listed were in the common group of relays or associated with the individual circuit group. Also, were the outages listed as due to supervisory trouble actual outages on individual power circuits or does this represent the time the supervisory equipment was inoperative?

The use of the monowire or direct wire system for short distances over privately owned wires is undoubtedly the most economical for first cost. Whether the sacrifice in flexibility to meet future rearrangement of the supervisory equipment due to changed operating conditions, future growth, etc., offsets this advantage is questionable and must be determined by local conditions.

Usually a reduction in line wires in supervisory systems is accompanied by a sacrifice in operating facilities in one form or another. This, I believe, is true of the monowire system described, in that no indication is given to the operator when a breaker trips instantly on overload, after being closed. Under such conditions an operator would not know, if the supervisory equipment failed to function or if the breaker actually closed and opened.

A sincere attempt has been made in our commercial systems to incorporate this feature for the operator's information. Information relative to line wire and battery voltage conditions is given to the operator in our selective systems. An expression of opinion from operating companies as to the actual value of these points should be interesting and valuable.

The program of maintenance on automatic stations outlined by Mr. Stamper is without doubt one of the major reasons why this type of equipment is making such a fine record for itself. Recognition of the necessity and the acceptance of responsibility for periodic inspection has been followed by a practical and sane program. To keep pace with the march of modern progress, practises in regard to maintenance have changed materially. In many industries there was a time when maintenance consisted chiefly of repair and replacement activities. Today more time is spent on *preventative* maintenance. A point of perfection will never be reached where it can be disregarded on auto-

matic switching equipments. Since the inception of these equipments, however, we have invested heavily in means to reduce this item of expense. Much progress has been and is being made in simplification of design and improved types of devices. Design progress has necessarily been slow. For economic reasons devices and equipment designed for manual operation were first utilized in this new art. These have been modified and replaced as the principles of automatic switching of power apparatus have been learned. There is much yet to be done. By no means has the automatic station reached its highest point of perfection or its widest application.

In reference to the difficulties on graphic meters, would it not be advisable to place these in the dispatcher's office by utilizing the present developed types of telemetering equipment? It would seem that they would be more useful to the power dispatcher in such a location. They would then receive the attention which the present designs are accustomed to receive in manually-operated stations.

Much progress has been made within the past two years on interlock design and motor-operated breaker mechanisms, so that highly satisfactory devices are now available.

In Fig. 2 a view of metal clad equipment is shown with associated relay and control equipment. Has this arrangement of mounting worked out satisfactorily?

M. E. Reagan: Mr. Stamper has advanced the argument that maintenance charts are, in general, not good for getting a complete accurate record of operation. This may largely depend on the type of form used. Some operators have found this to be the one method of obtaining good maintenance records. It would be interesting to get a résumé of experience on different classes of automatic equipments.

Messrs. Stamper and Ambuhl give interesting data on operation. The outstanding fact that equipment gives more trouble when first installed is not entirely explained by making replacements with better equipment. It is true that each year sees an advance in the reliability of automatic equipment. More money is spent on factory test before shipment and the various sources of trouble are "weeded out" as experience is gained. The operating personnel maintaining the equipment is responsible for a large share of credit in getting better equipment performance after becoming familiar with it.

Garland Stamper: It is suggested by Mr. Rutan that the value of this paper would have been enhanced if a cost comparison between automatic and manual stations were included. This question was considered and it was decided that such a comparison was entirely beyond the scope of this paper. However, a recommendation has been made to the chairman of the Committee on Automatic Stations that additional information be accumulated on this subject during the coming year. Such a study is of necessity so complex and involved with a study of the entire distribution system that it merits a separate consideration.

Mr. Rutan objects, also, to the inclusion of graphic meter and circuit breaker mechanism troubles in the list of automatic station troubles. It is true that these problems were present before the advent of the automatic station and they are still present to occupy the time and attention of the operators. Since the trend of present day practise in maintenance is towards prevention rather than repair and replacement, it is felt that all those items which do cause trouble should be included in this list. It is true, however, that the design of this equipment has been improved to such an extent that this trouble is materially less than formerly.

Mr. Stewart suggested that the graphic meters be replaced by telemetering equipment with the receivers in the dispatcher's office. In the majority of cases this scheme is not feasible due to the cost of installing or renting the necessary interconnecting wires.

Mr. Stewart questioned, also, the desirability of mounting relays and control equipment on hinged panels attached to the front of metal clad switching units. This arrangement saves space and is somewhat cheaper in first cost than a separate switchboard, due to the saving in control conduit and cable. The chief objections to this type of construction are:

(1) Crowding of instruments due to the limited space.

(2) Inconvenient mounting height on the larger units.

(3) Danger incurred by the maintenance men, due to the proximity of high-voltage circuit breakers which may fail on the occurrence of a feeder fault.

F. F. Ambuhl: Mr. E. J. Rutan mentioned the necessity of adequate inspection and maintenance on automatic and supervisory control.

On the Toronto System, we have found it essential to have specially trained men to do this work. One man is required to look after inspection of the automatic equipment and any work to be done, is done by maintenance men who are familiar with the automatic equipment. Two men are required for inspection and maintenance of the supervisory equipment. These men are specially trained for inspection and maintenance and also work along with the construction men on all new installations of supervisory control.

As regards the economy effected by using supervisory control, it has been the experience of the Toronto System that after taking into account annual fixed charges on the investment, including the cost of cable between stations, also maintenance and repairs, the supervisory equipment will pay for itself in from three to

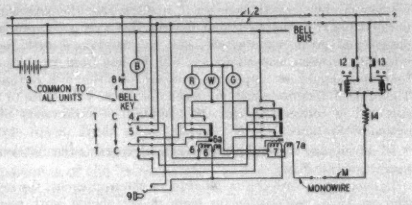

Fig 1

five years, (for stations such as used by the Toronto System) depending on the type and number of positions used; also on the distance between, and location of stations.

In Toronto, two men are required on each shift, which with a relief operator, makes a total of seven men for each manually controlled station. This item alone has justified the installation of supervisory control in Toronto.

Mr. C. E. Stewart has asked if the relay troubles listed were in the common group of relays or associated with the individual group. If I understand Mr. Stewart correctly, no trouble has been experienced with the rotary selectors and some types of relays in the common group of other schemes of supervisory control, though the troubles experienced are not confined to any particular group or circuits but vary in the different schemes used.

On one scheme, the latching relay in the common group has given a great deal of trouble; also trouble has been experienced with residual adjustment and contacts in the individual groups.

The outages, listed on tabulated report as due to supervisory trouble, are assumed as feeders or equipment temporarily out of service due to the inability of the operator to put such equipment in service by supervisory control, and which was found due

to defects in the supervisory equipment, thus making it necessary to send a man to the remote station to put the equipment in service manually or by rectifying the trouble. This sometimes happens when feeders or equipment are automatically tripped out and are ready to go back into service.

Mr. Stewart's comments concerning sacrifices in operating facilities are true in so far as the original monowire equipment is concerned. The lamp indication during switching operations agreed only with key position, irrespective of breaker position, since the operation of closing the control key automatically short-circuited one lamp relay and energized the other with full line potential. However, this objectionable feature has been overcome in the latest development, by providing each lamp relay with a second coil, through which the monowire circuit is carried. (Fig. 1.) When the control key contact is made, the ampere turns on the first lamp relay coil due to operating current, balance and oppose those due to the circuit made from bus to relay, to control key, to position key and thence to bus. Consequently no change in position occurs, until the apparatus controlled has changed its position. The other relay of the pair which before was deenergized, is now excited with monowire operating current and so also remains in its original position. The moment the operating current is broken by switch closure or tripping, the auxiliary coils are no longer effective and the lamps change indication, thus giving an indication of the switch position at all times.

Automatic Combustion Control

BY C. H. SANDERSON[1] and E. B. RICKETTS[1]
Fellow, A. I. E. E. Non-member

Introduction

THE successful application of automatic devices to power plant equipment for control of combustion has been very largely a development of the past ten years. During this period confidence in automatic operation was greatly strengthened by its success in other fields, particularly in that of the electric substations and hydroelectric generating stations. Moreover its development and application has had wholehearted approval and assistance of engineers and operators due to the ever increasing urge for higher efficiencies and lower fuel and labor costs.

Remote control of combustion equipment has become a necessity for the large modern boiler installations for much the same reasons that necessitate its application to the electric equipment. A boiler having a capacity in the order of 1,000,000 pounds of steam per hour may require as many as eight controls; two for induced draft fans, two for secondary fans, one for primary fan, two for excess pressure valves, and one for fuel feeder motors. This equipment is of necessity scattered over so wide an area that direct control would require six or eight men instead of one or two as when all controls are concentrated at one point.

Consider a large modern power plant having a number of boilers, each with centralized hand control, attended by two men; close regulation of combustion will often require their combined effort. It is usually impracticable, because of space requirements and the nature of the control to concentrate it at one point. Therefore, the imagination will be put to little effort in visualizing the release of this small army of men from a tedious and exacting service by an apparently "made to order" application of automatic control. The word "apparently" is used advisedly as there have been and there still exist many problems in the perfecting of the application. A brief review of the historical background, with its changes in apparatus and costs of fuel and labor, which has brought about the extensive modern application of automatic combustion control may be of assistance to a clearer understanding of these problems.

Historical

The typical large boiler plant of thirty years ago presents to memory a multiplicity of small boilers, rarely exceeding 600 hp. in individual capacity. Because of the small size of the apparatus, most of the auxiliary equipment such as dampers, blowers, water gages, etc. were within reasonable reach or view of the operator.

The firing was largely done by hand, requiring a considerable number of men for a given output. Men were normally stationed where they could easily make the few adjustments that were necessary. The fireman was picked for strength rather than intelligence and his qualifications have been aptly stated as being "a strong back and a weak mind."

The boiler room was usually hot, dirty, and generally uncomfortable; a place where one would stay no longer than necessary. Viewing such a boiler room in the light of present day conditions one would be impressed by the almost entire absence of meters, gages, and other aids to good operation. The reaction would be

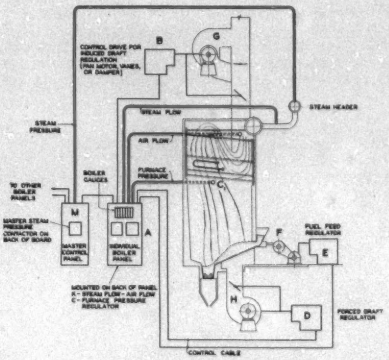

FIG. 1—OUTLINE OF BAILEY SYSTEM

of wonder that good results were ever obtained under such adverse conditions.

When boiler tests were made, meters, gages, thermometers, and apparatus for gas analysis were temporarily installed and a number of test men continually watched conditions and made adjustments. With those aids, efficiencies were obtained which compare very favorably with those of today. When the tests were completed the instruments and test men were removed and the fireman was literally "left to his own devices" which, as previously mentioned were mainly conspicuous by their absence. He could know but little of what was going on in the boiler and could not devote much time to efficiency studies after shoveling 25 or 30 tons of coal during an eight-hour watch. It is therefore no matter for wonder that daily efficiencies obtained in routine operation failed by 10 per cent or more to equal

1. The New York Edison Company.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

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those obtained under test conditions. In spite of these handicaps, due to the prevailing low cost of fuel and labor, steam could be generated for about half of what it costs today. Therefore the present urge for higher efficiencies and lower labor costs was nonexistent.

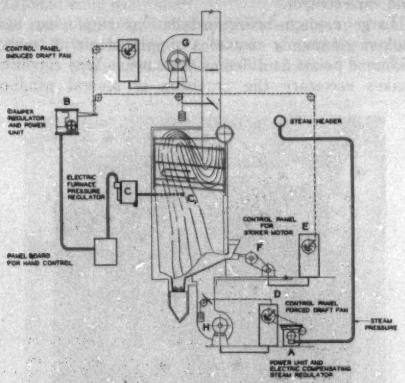

Fig. 2—Outline of Engineer Co. System

The earliest attempts at automatic control employed damper regulators actuated by steam pressure. With the development of automatic stokers and forced draft, regulators were applied to blower engines and to the maintenance of balanced or nearly balanced furnace

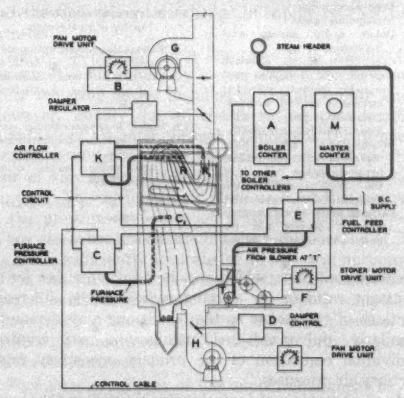

Fig. 3—Outline of Leeds and Northrup System

draft. The earlier apparatus was very crude in its mechanical features, and had the bad habit of being either all "on" or all "off," thus producing a steam pressure chart having frequent and wide fluctuations.

The next step in the development was a departure from these unreliable automatic devices in the application of permanent instruments to indicate draft and rate of coal feed, together with better facilities for hand control. Where hand control of the entire plant was centralized a considerable improvement was realized due to the uniform loading of the various boilers and to the much closer regulation of the steam pressure.

The development of the modern boiler plant necessitates the consideration of large units where in many cases the auxiliary equipment may be located a hundred feet or more from the control point, and the use of large stokers requiring varying air pressure in their different sections, the use of pulverized fuel, gas and oil for which efficient operation requires closely coordinated regulation of fuel and air. The necessary complement of gages and meters for a single boiler has been very greatly increased while of economic necessity the num-

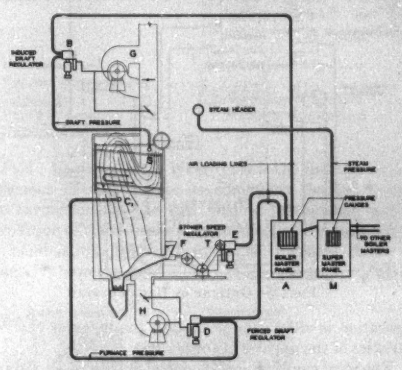

Fig. 4—Outline of Smoot System

ber of men has been reduced. The unit cost of labor and fuel has trebled in the last thirty years and consequently comparatively small savings in these items may justify the expenditure of material sums for new equipment to produce such results.

Reasons for Automatic Control

Because of the usually small amount of steam storage, fluctuations in station load are almost immediately reflected in steam demand; and if maximum efficiency is to be consistently maintained these fluctuations in demand must be met by corresponding changes in fuel and air supply. When a multiplicity of boilers supply the same steam system any change in station load should normally be divided evenly between these boilers. If the operating force is sufficient in numbers, highly skilled and supplied with adequate instruments, results can be obtained with a well planned manual control which compare favorably with the best automatic control. However, the automatic control is ever alert to perform its functions and its chief ad-

vantage lies in its ability to act simultaneously throughout the plant, thus permitting a reduction in the required number of skilled operators.

Large modern boiler installations are commonly equipped with push-button type of remote control which represents probably the most easily and quickly manipulated type of manually operated equipment. The cost of such a system represents a large portion of the cost of a complete automatic control system as it may be incorporated as a part thereof.

Adapting the Control to the Installation

While control systems can be adapted to most any type of boiler auxiliary equipment, the smoothness of

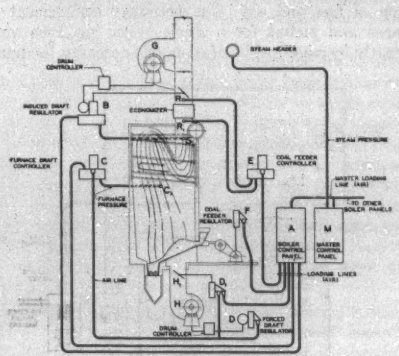

Fig. 5—Outline of Hagan System

operation is often complicated by the inherent characteristics of the apparatus controlled.

The ideal combination of control and auxiliary equipment is one which gives immediate response to a change in demand by delivering to the furnace a proportionately changed amount of fuel and air. Some types of equipment are inherently capable of approaching this ideal more closely than others and an understanding of these characteristics will be helpful in an understanding of what may be accomplished with automatic control.

Air Supply. Air supply control is usually actuated by draft at the boiler outlet. Some types of draft producing equipment can be controlled for smooth changes in small increments; others operate with large increments.

Among the types of draft equipment producing smooth changes are:

1. Chimney draft—damper control.
2. Steam-driven fans. Driver control.
3. D-c. motor-driven fans. Driver control.
4. Wound-rotor a-c. motor-driven fans. Driver control.
5. Single-speed a-c. motor-driven fans. Damper control.

Among the types of draft equipment producing abrupt changes of relatively large magnitude are:

1. Multispeed induction motor-driven fans. Driver control.
2. Multispeed induction motor-driven fans. Driver and damper control.
3. Multispeed induc ion motor-driven fans. Driver and vane control.

Large modern boiler installations employing long stokers commonly require a supply of air at several different points at different pressures. This condition makes necessary the provision of several windbox

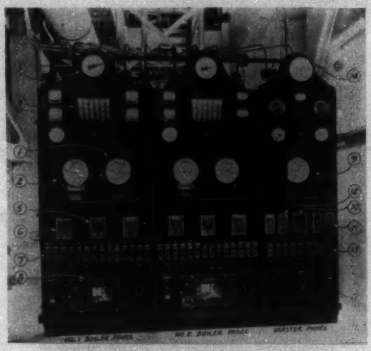

Fig. 6—Example of a Bailey Control Panel

1. Boiler meter with steam-flow —air-flow and furnace-draft contactors
2. Feed-water meter
3. Multipointer gage
4. Boiler drum water level indicator
5. Combustion-control selective switch for complete automatic control, partial automatic control, or push-button control of fuel, forced, and induced draft.
6. Reheat-steam temperature-control switch for automatic or push-button control of the high-pressure steam valve to the steam reheater
7. Fan- and feeder-motor switches
8. Automatic control box for power-control safety valves
9. Steam-pressure and feed-water heater pressure recorder with master pressure contactors
10. Master control switch for combustion control system
11. Reheat steam distribution control switch for automatic or push-button control of the reheat steam distribution valves
12. Boiler feed-water-pump control switches for automatic or push-button control of the pump motor drum controllers
13. Feed-water pump-motor switches
14. Steam header pressure gage

pressure controllers which may be adjusted either for a constant ratio or for varying ratios on the different sections of the stoker as the conditions may demand. Similarly, pulverized fuel installations may require individual regulation of the primary, secondary, and tertiary air pressures.

In most systems of control involving the supply of air for combustion under pressure it is necessary to provide an auxiliary control to prevent the occurrence of a pressure above atmosphere in the furnace. This control is actuated by furnace draft and usually functions as an emergency or readjustment mechanism on the forced draft fan.

Many different types of automatic control are avail-

able for application to the different systems used for producing draft or air supply. For example; in natural draft installations the automatic control is usually applied to the stack damper. For adjustable-speed motors and steam turbines the speed of the driver only is automatically controlled.

In some pulverized fuel systems employing a unit system, it is necessary to introduce a time lag in the air supply equipment to allow for the interval between

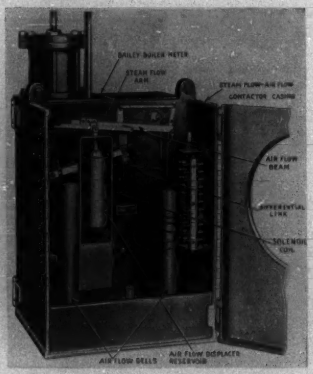

FIG. 7—BAILEY STEAM-FLOW-AIR-FLOW CONTACTOR

the increase or decrease in the coal supply to the mill and the time when this change in supply reaches the furnace.

Air Measurement. In a few cases it is possible to measure the air supply by means of an orifice or some other metering device installed in the air supply line. However, usually due to the fact that considerable quantities of air enter through openings in the furnace setting, it is necessary to base the air control on the flow of the gases of combustion rather than the air supply. The medium most often used for this control is the pressure drop through a bank of boiler tubes. When the tubes are reasonably clean this method of air control is sufficiently accurate for all practical purposes. When the tube bank becomes partially covered with slag, manual readjustment of the air supply controller is necessary unless an additional control connection is made with some form of CO_2 meter to provide automatic readjustment.

Fuel Supply. Types of fuel supply equipment have a very marked influence on the quickness of response to a change in load and on the effect on combustion conditions of large abrupt changes in draft.

A. The following types of supply equipment afford immediate response:

1. Gas
2. Oil
3. Pulverized coal—bin and feeder system
4. Stokers carrying a heavy fuel bed

B. The following types have a slow response (from one to fifteen minutes):

1. Pulverized coal—unit system
2. Stokers carrying a light fuel bed

C. All types of fuel supply are adversely affected by large abrupt changes in air supply, the disturbance in all cases being more marked with water-cooled furnaces than with refractory furnaces. The effect on some of the more widely used systems is as follows:

Conditions	Effect
1. Stokers with deep fuel bed	Smoke and temporary loss in efficiency
2. Stokers with light fuel bed	Blowing bare spots on grates, loss in efficiency and capacity over a considerable period
3. Pulverized coal Bin and feeder system	Possible loss of ignition
4. Oil and gas	Possible loss of ignition
5. Pulverized coal. Unit system	Probable loss of ignition

Fuel Measurement. Where coal is the fuel the measurement of fuel supply is commonly derived from the revolutions of the fuel feeding mechanism, each revolution of which is assumed to inject a given quantity of combustible material into the furnace. Unfortunately, however, coal is one of those natural products

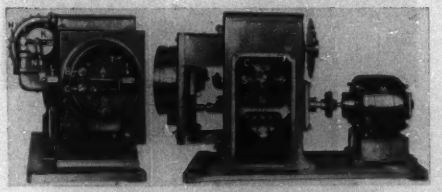

FIG. 8—REGULATOR AND MASTER CONTROLLER—ENGINEER COMPANY

Legend

M	Driving motor	*A*	Contact arm
P	Sprocket wheel	*B* and *C*	Contacts
W	Wheel on ratchet	*H*	Compensating hand screw
C	Center of pawl shaft	*K*	Crank for opening contacts *B* or *C*
S	Solenoids	*N*	Link
T	Bourdon tube	*L*	Lever

which is prone to wide variation in physical and chemical characteristics which are reflected in the fuel value per revolution of the feeding mechanism. Consequently, correct ratio of fuel and air can only be maintained either by occasional manual readjustment of the coal feeding mechanism or by an automatic air adjustment mechanism actuated by a CO_2 meter or a steam-flow—air-flow meter.

Sequence Interlocking of Control. In nearly all large modern boiler installations, it is necessary to provide sequence interlocking between certain of the control points. For example; it is usually necessary to provide that a shut-down of the induced draft fan will simultaneously interrupt the coal feed and air supply and that the coal feeding mechanism will not over travel beyond the ability of the induced draft fan to take away the products of combustion. In many installations employing pulverized fuel firing there is a low limit below which combustion becomes unstable and a further reduction of coal and air feed might cause the fire to blow out.

Wherever automatic combustion control is applied, each boiler unit is provided with a simple and ready means for changing from automatic to hand control as is necessary under emergency conditions such as the interruption or partial interruption of the supply of coal, air or feed water to an individual boiler.

Control-Power Supply. To insure continuity in the operation of the control system, the supply of the medium through which the control impulses are carried to the various regulators must be positively safeguarded by the provision of two or more independent sources of supply and a ready means of changing from one source to another.

Pneumatic and hydraulic systems were originally employed but with the advent of electrically operated remote control, electricity has been adopted by a number of manufacturers as the medium for automatic control.

FIG. 9—ELECTRIC SENDING AND RECEIVING UNIT—ENGINEER COMPANY

C Contacts *L* Lever for group adjustment

The electrical control drives are designed to use either alternating or direct current at any voltage from 110 to 250.

SYSTEMS

Probably no two power plants ever built were exactly alike. Comparison of any one with another as regards the detailed requirements of an automatic combustion control system reveals a surprising lack of standardization. Naturally the same is true of the control systems employed and therefore only a generalized performance specification together with brief descriptions of several typical systems will be given.

In all systems a master controller transmits impulses actuated by steam pressure variations simultaneously to the various individual boiler control panels from which radiate the controls of fuel and draft. The control performance is, in general, as follows:

1. An increase in plant load causes an increased demand for steam and therefore a drop in steam pressure.

FIG. 10—CONTROL PANELS—LEEDS AND NORTHRUP

1. Master controller
2. Fuel-feed controller
3. Air-flow controller
4. Furnace-pressure controller
5. Combustion-rate indicator
6. Combustion-rate rheostat
7. Selector switches
8. Motor-generator field rheostats
9. Motor-generator starting switches
10. Main control switch
11. Fuel control selector switch
12. Fuel control push-button station
13. Voltmeter transfer switch
14. Annunciator
15. Voltmeter

2. Changes in the steam pressure are converted into forces or impulses which actuate a regulating device controlling the rate of fuel and air supply to the boiler in proportion to the load. Forces or impulses which are not linear are adjusted in the regulator.

3. Since the regulator can only control mechanical forces which under normal conditions will supply the correct amount of fuel and air, variations from the assumed normal conditions must be compensated by changing the relation between these mechanical forces which supply either fuel or air so that the correct amount of each is supplied to the boiler. A furnace pressure regulator and a CO_2 recorder will detect and make adjustments for variations from normal conditions. These variations are caused by slagging or damp coal or an irregular supply of coal.

4. Controls can be set so that certain boilers maintain a constant output while other boilers take the variations, and can be provided with maximum and minimum stops to limit the range of output.

5. Control of any or all functions can be accomplished separately or in groups, either directly or indirectly, as required by individual conditions.

6. Some of the available systems are provided with audible and selective visual signals to indicate equipment failure.

Fig. 11—Drive Unit—Leeds and Northrup

Bailey System. The master steam pressure contactor located at M, Fig. 1, is actuated by changes in steam pressure. Each boiler drum switch is energized by a "more" or "less" contactor depending upon the load demands. Each boiler drum switch A, then causes its control units, the induced draft regulator B, the forced draft regulator D, and the fuel feed regulator E, to respond in the direction indicated by the master contactor. A continuously rotating cam causes the impulses to be sent out once each revolution. The speed of the cam is adjusted to suit individual requirements. Each regulator, controlled by the boiler drum switch, is changed in steps until the pressure is again normal. The forced draft regulator D is also readjusted automatically by the furnace pressure regulator C so as to maintain constant furnace pressure or vary it with the load. Static pressure existing inside the furnace is obtained by a small pipe through the boiler wall at C_1. The air or fuel supply to each individual boiler is readjusted automatically to maintain the proper relation between them and is accomplished by using the steam-flow-air-flow relation of the Bailey boiler meter K as an indication of the amount of excess air being used for combustion. The steam-flow-air-flow contactor is illustrated in Fig. 7. Impulses received at the control drive cause the various units to operate in steps and thus eliminate any hunting effects. Response to sudden load changes is inherently slow. A modern control panel is illustrated in Fig. 6.

The control drive (Fig. 1—B, D, and E) consists of a continuously running motor driving a system of gears which advances or retards the equipment to be controlled in response to the impulse sent out by the contactor or push-button.

In pulverized fuel or oil burning installations, the control drives are interlocked, so that when the maximum capacity of the air supply has been reached an interlock prevents any additional fuel from being supplied and when the fuel supply has been reduced to its lowest limit an interlock prevents the air from being further reduced.

Enco System. The master controller shown at A, Fig. 2, is a steam pressure regulator of the weighted diaphram type and controls the amount of air supplied to the boiler by regulating (through regulator D) the forced draft fan speed H or its dampers in steps until normal steam pressure is again restored. This device is illustrated in Fig. 8. Both forced draft and fuel may be directly controlled if required.

Fig. 12—Control Panel—Smoot System

1. Draft gages
2. Air and oil gages
3. Drum water level indicator
4. Position indicator—feed water check valve
5. Control—feed valve
6. Air supply valve—manual control
7. Transfer valves—manual or automatic
8. Gages for equalizing air supply to boiler master when transferring
9. Air supply valve—automatic control
10. Manual master rating control
11. Maximum rating control
12. Minimum rating control
13. Magnetic transfer valve for fuel feed—motor-generator set
14. Boiler master
15. Voltmeters for generators supplying fuel feeder motors
16. Fan speed indicators
17. Transfer switches—manual or automatic control of fans
18. Solenoid operated transfer indicators
19. Manual control for fans
20. Manual control for fuel feed
21. Steam pressure gage
22. Steam pressure gage—forced draft fans
23. Emergency trip—air and fuel supply

The furnace pressure is maintained at a constant value by a balanced draft furnace pressure regulator C which by means of a damper regulator unit B controls

the induced draft fan or its damper G and the fuel feeder speed E. A furnace pressure regulator is installed on each boiler. Static furnace pressure is supplied by a pipe through the furnace wall.

One steam pressure regulator may be used to control any number of boilers or groups of boilers, or two or more steam regulators may be used for large installations.

An electrical sending and receiving unit, Fig. 9, is available for large installations and provides master control for any number of units from a central point. This equipment provides for almost any combination of control for the combustion process. Each boiler can be controlled as a unit or any number of boilers can be controlled automatically as a group from a master switch.

This method of combustion control can be provided with hydraulically or electrically operated equipment and in either case the equipment can be adjusted so as to keep the load on the various boilers within their minimum or maximum limits.

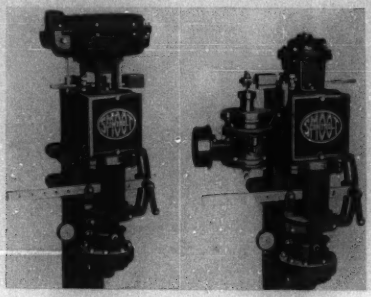

Fig. 13—Regulators—Smoot

A. Standard regulator B. Speed regulator

Leeds and Northrup System. A master electric control current is varied in response to changes in steam pressure by means of a master controller automatic rheostat located at M, Fig. 3. The master control panel is illustrated in Fig. 10. The control current is supplied to the controllers on each of the boilers through the individual boiler panel A where the amount of current supply for each boiler is regulated by a rheostat. The control current is then balanced against the metered rates of fuel and air supply for each boiler at controllers K, C, and E. Contacts on these controllers control the rate of speed of the drive units B, D, F. The drive unit is illustrated in Fig. 11. Differential furnace pressure drop measured at R and R_1 is balanced against the control current in the air-flow controller. Piping through the furnace wall brings the static pressure to the furnace pressure controller where it also is balanced against the control current. However, for the fuel feed controller no natural balancing pressure exists but one is obtained mechanically by means of a blower located at T. The pressure thus produced is proportional to the feeder motor speed.

Irregular action of the fuel feeders or quality of fuel may be corrected by fuel feed compensators controlled

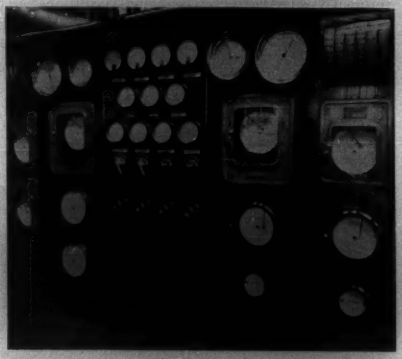

Fig. 14—Control Panel—Hagan

1. Impulse relay between sending and receiving unit for modifying impulse
2. Selector switch (manual or automatic)
3. Impulse indicators—fuel and draft
4. Air, loading and 60-lb. air gages
5. Ammeters for motors on fuel pulverizing equipment

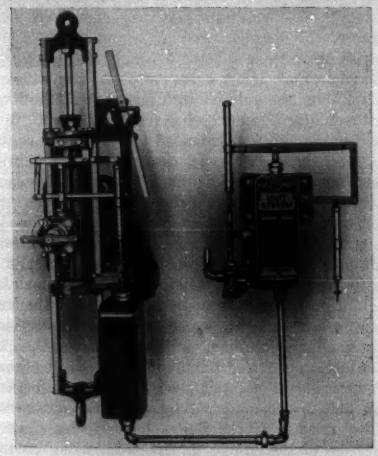

Fig. 15—Primary Controller—Hagan

by a fuel-air ratio rheostat. These devices are not shown. They readjust the fuel feeder speed.

If the control, relay or motor circuit fails, an alarm sounds and the power is automatically cut off the drive units.

Smoot System. The master controller M, Fig. 4, in response to steam demands, regulates the pressure on air loading lines which are connected to boiler masters B on each boiler. From each boiler master there are air loading lines to regulators (at B, D, and E) which directly control the induced draft fan or dampers G, the forced draft fan or dampers H, and the fuel feed motors F. A typical control panel for a large installation is illustrated in Fig. 12. Each regulator (Fig. 13) is balanced against an air loading line and the force which it controls by means of a checking diaphram which is part of the regulator. Pressure at C_1 is a measure of furnace pressure, and at S is a measure of air flow.

The checking diaphram of the regulator controlling the fuel feeder speed is provided with a self-contained speed governor, the speed of which is proportional to the feeder motor speed. Thus, the master loading pressure determines the value of the pressure, draft, or speed which is furnished at any instant. Any changes in steam pressure are rapidly transmitted to all regulators through the master controller and boiler master. By adjusting the valves in the air loading lines from the master controller M the load can be divided among the boilers as desired. The proportion of fuel and air supplied to any boiler can be varied by adjustment of the valves on the air lines from the boiler master. In case constant furnace pressure is desired the furnace pressure regulator is not master loaded but is balanced against a spring.

Hagan System. An element directly responsive to steam pressure is mounted on the panel M, Fig. 5, from which a pneumatic impulse is transmitted to the several boilers under control. The air in the system is maintained at approximately 60 lb. pressure. The proportion between the impulse and the steam pressure may be varied in the field. There is one boiler panel A (illustrated in Fig. 14) per boiler on which are mounted compensating relays by which manual control or a modification of the impulse to (and hence the loading of) that boiler may be obtained. A sending and receiving unit is illustrated in Fig. 15. The impulse is transmitted to the induced draft regulator B and balanced against the draft taken at a given point such as S. The regulator B is of the mercury sealed balanced float type.

A balanced furnace pressure (subject to manual modification or control) is maintained by the regulation of the forced draft fan speed H (and damper H_1). A balanced float controller, C, of the varying impulse type, responds to the furnace pressure and originates an impulse which is sent to the forced draft fan regulator D and to damper regulator D_1, which adjusts the speed of the fans H and damper position H_1. The normal forced draft may be adjusted from the boiler panel as desired.

The gas flow through the boiler is thus established and is measured by means of the pressure drop across the economizer at R and R_1.

In order to determine the rate of coal admission, two pressure connections R and R_1 are taken to a coal feed controller E, of the balanced float type, which is provided with a mechanism for adjusting the ratio of impulse to air flow. This impulse is then taken to the boiler control panel, modified as desired, and then transmitted to the coal feeder regulator F which adjusts the fuel feeder motor F_1.

A control for double boilers, having the following additional features is available:

1. Speed of each induced and forced draft fan can be adjusted separately at the drum controller.
2. Impulses from each coal feeder regulator are totalized before being sent to the coal feeder regulator.
3. Forced draft dampers may be adjusted from the boiler panel independently as furnace conditions require. This is of special value in pulverized fuel installations where the air resistance or flame shape may be abnormal.

Conclusion

The question is often asked, what financial return may be expected from an investment in automatic combustion control? The answer is infinitely more involved than the question.

Savings in operating expense as high as 4 per cent have been reported. Rarely, however, has it been found practicable to make comparative efficiency tests on the same installation, operating with and without automatic control, to determine definitely, the annual saving in operating expense.

As a rule, automatic control is seldom installed on the sole basis of a direct saving in operating expense. In fact its installation is often based mainly on resultant operating advantages otherwise difficult or even impossible of accomplishment. Among the more important of these advantages are:

The uniform loading of all boiler units.

Instant response to changes in load.

The supply of fuel and air in correct proportions at all times.

A ready means of providing for safety of operation through sequence interlocking of auxiliary equipment.

Discussion

I. Melville Stein: Referring to the statement at the bottom of the first column on page 1379, which refers to poor steam pressure regulation obtained with the older forms of "on and off" regulators, it is not altogether correct to charge these older forms of regulators with producing poor pressure regulation because some of them produced very good pressure regulation. They were defective, however, in that the good pressure chart was obtained at the expense of very large changes in the draft, the result being that the air flow was always too large or too small, and either condition resulted in large losses in combustion efficiency. Of course, if these regulators functioned by changing the speed of a steam engine driving a fan instead of being directly connected to a damper, poor pressure regulation may have resulted, due to the lag in the response of the engine, but this defect is not properly chargeable to the regulator.

On page 1380, the authors point out that automatic controls can be arranged satisfactorily to regulate rather complex installations. While this is strictly true, I should like to make a plea that installations be kept as simple as possible, rather than to follow the tendency to make them complicated just because automatic controls can handle complicated installations. In this connection, the station auxiliaries involved in the automatic control of combustion could be designed or selected with the requirements of automatic combustion control fully in mind. It cannot be expected that the best automatic control installation can be made if these auxiliaries are designed just as they were prior to the advent of automatic combustion control. As specific examples of the design of auxiliaries which cause trouble in applying automatic combustion control are the use of drum controllers on the motors of the draft fans which have too few steps and the use of stokers requiring a gear change in order to cover the operating range of capacity.

With reference to the use of interlocks, as discussed on page 1382, it should be mentioned that in installations employing two or more unit mills per boiler, an additional interlock should be provided to prevent increasing the air supply beyond the upper limit of fuel capacity when one or more of the mills is out of service.

In connection with electrical systems of control, I should like to point out that the type of drive unit usually supplied and shown typically in Fig. 11 of the paper is self-locking in the event of the failure of power supply; in other words, an unbalanced damper cannot drift to the full open or full closed position, but is held by the drive unit in the particular position which it has at the moment the power supply fails.

An advantage of automatic combustion control which is not mentioned in the paper is its ability to show up operating difficulties, such as defective baffles, etc., before they would otherwise be detected. Under automatic control, the steaming rate of all boilers is stabilized, and if a particular boiler shows an erratic steaming rate, it will, in most cases, be found that some defective condition of the boiler or firing equipment is developing. The defect is thus anticipated before it causes any real disturbance.

T. G. Robinson: Some very satisfactory experience has been obtained with the Leeds & Northrup Company's system of automatic combustion control (State Line) and also that manufactured by the Bailey Meter Company (Powerton) and the Smoot Engineering Corporation (Waukegan). In two of these installations, automatic control was used in connection with two and three speed induction motors on the draft fans. While satisfactory performance was had with both installations, there has been of late a desire to eliminate multi-speed induction motors with the inherent complications in speed changing switches and damper coordination problems. This led to the application of variable speed Rossman drives for draft fans.[1]

On a recent extension to one station, where three-speed induction motors were formerly used, the general boiler control scheme was simplified considerably, due primarily to employing Rossman drives on the draft fans. The Bailey Meter Company's standard steam-flow–air-flow, and furnace pressure automatic controllers were used in connection with the Rossman simplified scheme of automatic control. With this control the standard Bailey drive units were not employed; instead, the electric impulses from the several automatic controllers were sent direct to the pilot motors on the rheostats for the adjustable-speed fuel feeder motors and speed regulating rheostats for the Rossman drives.

Recently a number of tests was performed on this installation for the purpose of determining the general performance of the automatic control system. The results not only indicated reliable operation, but satisfactory performance as regards its ability to maintain steam pressure, steam production, draft conditions, and over-all combustion efficiency. One very interesting test was made to see if the steam-flow–air-flow automatic controller might not be eliminated by depending on the relation between the time of travel of the speed regulating devices for the induced draft fan and fuel feeder motors. In this test only the furnace pressure automatic controller was used. This was employed to vary the speed of the forced draft fan to maintain automatically the predetermined furnace pressure. Results of the test showed that the steam-flow–air-flow automatic controller was necessary to maintain combustion efficiency, but indicated some promising possibilities in respect to coordinating the time of travel of the several speed regulating devices.

With reference to the authors' mention of interlocking sequences, we would like to point out an interesting application in connection with a recent boiler installation (Philo). Time delay relays were used, in the safety interlocking of the boiler auxiliaries, ranging from thirty seconds up to as high as two minutes. These were employed in the normal shutting down procedure between the pulverizer mill exhausters and the draft fans for the purpose of emptying the pulverizer mill and cleaning out the fuel conduits to the furnace, and to remove the gases of combustion.

We would point out that recent tests and analysis of one boiler installation in the Chicago area (Calumet), indicated an over-all increase in efficiency of approximately 4 per cent which corroborates the statement in the conclusion of the paper. There seems to be a need for more economic studies of this nature on automatic combustion control systems. Of course, as the authors mention, the results will vary with each particular installation. Although it is quite a difficult and tedious matter to integrate the savings due to the complex aggregation of variables in boiler installations and operation, some steps should be taken to tabulate the results of each particular installation in order to secure a clearer economical aspect of the situation.

T. A. Peebles: The statement "That if an operating force is sufficient in numbers, highly skilled, and supplied with adequate instruments results can be obtained with a well planned manual control which compare favorably with the best automatic control," brings to mind a large industrial plant of six units burning two different fuels. This plant is operated under automatic control with two men on the firing floor and securing the same results which the superintendent states could be secured with a manual control system operated by twelve men. It would be possible to recruit and train the required ten extra men per shift or thirty in all, but the turn-over would be large because the training these men would receive would fit them for something better. A plant operated in this manner would be in effect a training school in which young engineers would spend a comparatively short time as a preparation for advancement, but no one employer could hope to absorb into other departments the number of men who would become available.

In a large central station, an eighteen-month study of the relative merits of manual and automatic control convinced the management that in the operation of large boiler units, there are certain adjustments which can be made by machinery to better advantage than by hand; an automatic control was installed.

However, automatic control does not completely take the place of operative skill. Being machines, the control devices can do only those things put into them by the designer, who of necessity makes certain assumptions which are not always correct. As pointed out by the authors, the changing condition of the gas passages and variations in fuel introduce errors requiring manual adjustment. Even the steam-flow meter cannot always be depended upon to show the true condition, because the steam flow is a measure of what a boiler is doing only when the pressure is kept practically uniform and the flow of water to the boiler is proportional to the steam flow. All these variables combine to produce a condition that calls for continuous attention on the

1. *A New System of Speed Control for A-C. Motors*, by A. M. Rossman, Trans., A. I. E. E., March 1931, p. 162.

part of a skilled operator, but since automatic controls take care of all adjustments required to meet varying loads and maintain desired draft conditions, the operator's work is greatly simplified. Experience and judgment are required to interpret the indications of the boiler instruments and make the correct adjustments, but one skilled operator aided by an automatic control can burn a lot of fuel and burn it efficiently.

Engineers considering the use of automatic control should keep in mind the authors' statement that the cost of a remote manual control system represents a large proportion of the cost of a complete automatic control. The manual controls must be grouped on suitable panels with indicators which enable the operator to determine at a glance the settings of the various control devices, and each valve, damper or rheostat requires a power unit for its operation. Such a system represents a large proportion of a complete automatic control. If the cost of manual control and the things it can do are carefully compared with the cost and performance of a complete automatic control, it will be found that substantial operating advantages may be secured at surprisingly low cost.

Three Years' Operating Experience with Miniature Switchboard Supervisory Automatic Control

BY R. M. STANLEY[1]
Fellow, A. I. E. E.

Introduction

THE Ohio Falls Hydroelectric Station at Louisville, Kentucky, contains eight hydroelectric units, each having capacity of 13,500 hp. at 37 ft. head. The generator room is 44 ft. wide and 508 ft. long. The waterwheels are 5-blade propeller type axial flow runner, developing the rated horsepower at a speed of 100 r. p. m. This type of wheel was selected because of its ability to operate under a range of heads varying from the maximum of 37 ft. down to 8 ft. Each generating unit is complete in itself. Waterwheel, generator, exciter, governor, electrical control, oil and water supply, oil circuit breakers, etc., are individual to each generating unit and all related unit auxiliaries are fed entirely from that unit.

A hydraulic control unit and an automatic electrical control unit are located near each generating unit. The electrical control equipment is similar to that used in all stations but in each case is placed inside of a sheet steel cubicle located adjacent to the governor.

There is a centralized supervisory control miniature switchboard located in about the center of the plant with a multi-conductor telephone cable between the miniature board and each generator control unit. The miniature switchboard is illustrated in Fig. 1. The generator switchboard—front view—in its steel cubicle is shown in Fig. 2; rear view—Fig. 3. A one-line diagram of the station generating and transmission buses and the 230-volt unit and station auxiliary buses is shown in Fig. 4.

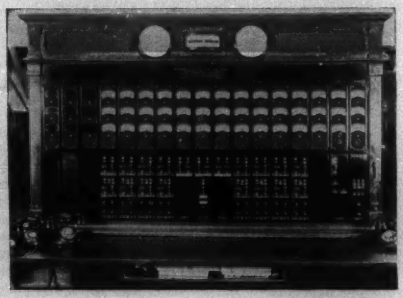

Fig. 1—Miniature Switchboard

A description of the engineering and construction features of this installation may be found in a paper by R. M. Stanley and E. D. Wood, A. I. E. E. Trans., March 1931, page 370.

Miniature Supervisory Switchboard

The miniature supervisory board in some respects resembles a manually operated telephone switchboard. The arrangement of control keys and indicating instruments on 4-inch panels places the control of all generators, transformer banks, etc., within convenient reach. The operator sits at this board with all of the panels, controlling load, reactive voltamperes and voltage conditions of all generators within a distance of 2½ ft. This results in closer supervision of generating apparatus at the hydro, in better loading of machines, and better frequency regulation at such times when regulation is passed to the hydro.

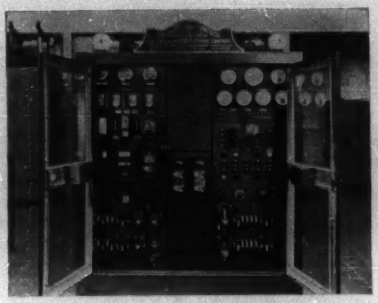

Fig. 2—Generator Switchboard, Front View

Aside from convenience and closer supervision there is another distinct advantage, namely, time is saved in performing operations when the plant is most needed and the item "human error" is controlled to a large extent. There is a considerable saving in labor cost since only one operator is needed per shift, the yearly cost being 9,500 man-hours for three operators per twenty-four hours, including the relief operator. Apt young men of limited experience are entirely satisfactory for supervisory operators in this station.

The time saved in putting machines on the bus is especially important during system disturbances and it is possible, when urgently needed, to bring a 13,500-hp. hydro generator unit up to proper speed and put it on the bus in one minute. Additional units can be put on in approximately thirty-second intervals thereafter.

1. Elec. Engr., Byllesby Engg. & Management Corp., Chicago, Ill.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

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Due to the speed with which units can be put on the bus at the hydro, it has become the practise to carry less reserve steam capacity when units are available at the hydro.

Maintenance

The miniature board is no more costly to maintain than any other type of switchboard. In fact, a comparison of maintenance costs on a benchboard and vertical board with full sized instruments, control switches, etc., at Waterside Steam Station indicates that the maintenance cost and inspection of the miniature switchboard and automatic equipment is less. With the exception of the instruments, the board is made up of equipment similar to telephone equipment. Multi-conductor, lead-covered, telephone cable is used to connect this miniature board with the individual control panels associated with each generator. The performance records of the miniature supervisory board for the years 1928, 1929, and 1930 are given in tables at the end of this paper. This record shows the type of equipment associated with the miniature supervisory board, the approximate number of operations and the number of failures.

There is very little cleaning to be done, the equipment being enclosed in a practically dust-proof cabinet. There is some small maintenance of the board such as cleaning control keys, contacts, etc., and renewing miniature lamps. The major portion of technical service required has been in the nature of checking and testing out new equipment. In general, it may be said that once the equipment has been properly installed and adjusted, it remains so without constant maintenance.

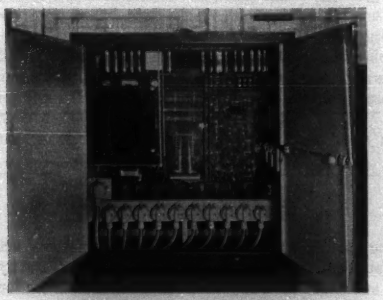


Fig. 3—Generator Switchboard, Rear View

General Scheme of Control

The general scheme of control is that the turning of a miniature control key at the miniature supervisory board sets into operation automatic relays associated with the unit being supervised. These automatic relays function in a predetermined sequence with the starting of generator, shutting down of generator, or making alive a transformer bank and synchronizing same with another system. Remote indication of the kilowatt load, reactive voltamperes and voltage on each generator is given on miniature instruments by direct connection through No. 19 gage, multi-conductor telephone cable to 1/1 insulating transformers and 5/0.5 ampere current transformers. Indication of load, reactive voltamperes and voltage on transformer banks is given by means of Selsyn type miniature instruments on the miniature supervisory board. A brief outline of

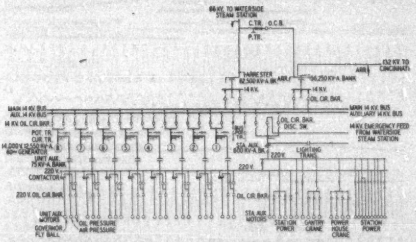


Fig. 4—One-Line Diagram of Generating Station and Transformer Buses and 230-Volt Unit and Station Auxiliary Buses

what takes place when a supervisory operator starts a unit will make clear where time and labor are saved in the control of this station.

1. Supervisory operator turns a key which selects the bus and synchronizing method.
2. Supervisory operator pushes button which gives starting impulse to the following operations:
   - Governor oil pressure automatically maintained at all times.
   - Brakes release automatically.
   - Gates open to predetermined setting.
   - Field applied automatically.
   - Governor fly-ball motor transfers automatically from station supply to generator supply.
   - Generator air cooling water system starts automatically.
   - Machine synchronized automatically to bus selected.
3. Supervisory operator regulates load, voltage, and reactive voltamperes.

Shutting Down a Unit

1. Supervisory operator backs down load.
2. Supervisory operator gives stop impulse.
   - Line breaker opens automatically.
   - Field removed automatically.
   - Gates closed automatically.
   - Brakes applied automatically.

Protective Equipment which Replaces Watch Operator

- Four bearing temperature relays.
- One water-cooling relay.
- One overvoltage relay.
- One governor low oil-pressure relay.
- Generator differential relays.
- Loss-of-field relay.

From the foregoing it is evident that the automatic equipment is used extensively to replace manual oper-

ating duties. During 1928 (the first year the plant was operated) it was customary to test this equipment out completely every week because of the relative novelty of the installation. During 1929 tests were made monthly and now the equipment is tested only at

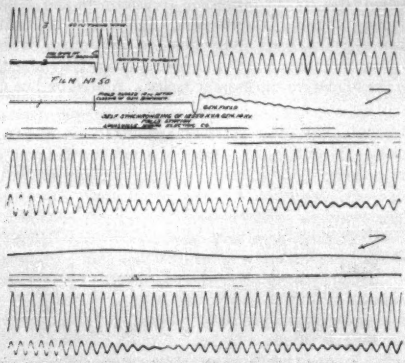


FIG. 5—SELF-SYNCHRONIZING OF A 12,550-KVA. GENERATOR—14 KV.

intervals of three or four months when it appears to be necessary to clean the equipment.

Generators may be synchronized to the bus by two methods; first, through self-synchronizing equipment wherein the machine is brought up to speed, the arma-

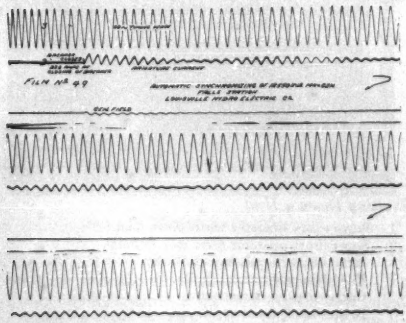


FIG. 6—AUTOMATIC SYNCHRONIZING OF 12,550-KVA. GENERATOR—14 KV.

ture is closed in on the bus and the field applied instantly thereafter. Second, automatic synchronizing relays may be used which closely approximate the manual method of synchronizing, that is, the speed of the incoming machine is regulated to match that of the running machine and, when the phase angle between the voltage of the two machines is satisfactory, the machine is closed in on the bus. Each method has its advantages. The self-synchronizing method is faster, requiring about one minute to bring a machine up to speed and synchronize. This method is generally used when the system capacity is large and the head of the river is above 20 ft. It has a distinct advantage over automatic synchronizing in that, during periods of severe system trouble when the frequency starts going down due to loss of generator capacity, it is possible to self-synchronize a unit and have it pull in step with this lower frequency. From oscillograms made at the plant (see Figs. 5, 6, and 7) the current taken by the armature varies from one and one-half to twice normal. This dies down rapidly and generally reaches normal value in ten cycles and there is a slight dip in system voltage at the Waterside Steam Station equivalent to about

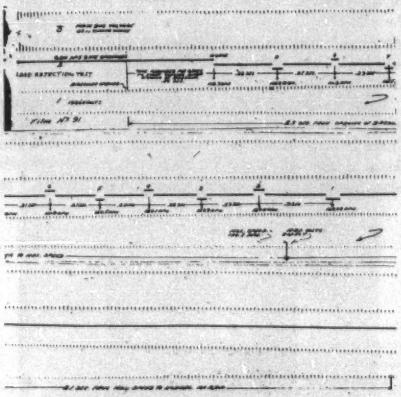


FIG. 7—LOAD REJECTION TEST

one volt on 110-volt basis. The automatic synchronizing equipment is used when the output of the hydro is small due to low head. Synchronizing takes longer but is very exact. The majority of the synchronizing operations with this equipment requires from two to three minutes from the time the unit is started.

At first it was also considered possible that the self-synchronizing method might not be satisfactory due to the "bump" given the system when closing the generator oil circuit breaker and only one machine was thus equipped. However, subsequent operation of the waterwheels has been so satisfactory that the other seven machines have been equipped with self-synchronizing equipment and this method is used whenever it is considered desirable, as outlined in the foregoing, and the self-synchronizing method is now considered preferable.

TABLE I—TABULATION OF DISTURBANCES

	No. 1	No. 2	No. 3	No. 4	No. 5
Cause of disturbances	Lightning tie line	Lightning tie line	Lightning 13-kv. feeder	Lightning tie line	Trans. failure
Outages—tie lines relayed	Yes	Yes	Yes	Yes	High & low O. C. B.
Generators on bus	8	5	1	2	2
Generators shut down by dist.	8	5	1	2	2
Load on station	60,000	27,500	6,000	16,800	8,500
Load dropped	60,000	27,500	6,000	16,800	8,500
Performance of Automatic Equipment					
Generator overvoltage relay	Operated to shut down unit (same as No. 1)				
Master control circuit interrupted	Automatically				
Generator oil circuit breaker	Opened automatically				
Generator field	Removed automatically				
Governor solenoid	Deenergized				
Waterwheel gates	Closed automatically				
Brakes	Applied automatically				
Supervisory indicating lights	O. K.				
Supervisory generator control key	Turned manually				
Station aux. supply transferred	Automatically to emergency source				
Station power and lights	O. K.				
Restoring Service					
Tie lines oil circuit breakers	Closed from supervisory board				
Generator starting impulse	Given by supervisory operator				
Master circuit complete	Through starting impulse				
Governor solenoid	Energized automatically				
Brake released	Automatically				
Waterwheel gates opened	Automatically				
No. of generators syn. manually	4	2			1
No. of generators automatic	4	3	1		
First generator on bus in	2 min.	3 min.		On only ½ min.	
Last generator on bus in	4 min.	4 min.	3½ min.	Trouble still on line	

Reference is made to Table I which gives the performance record of equipment associated with supervisory control. The first item—telephone cable—consists of lead-covered No. 19 gage, double paper insulated telephone cable. Impulses are sent from the supervisory board over this cable to the generator cubicles located near each generator. Relays located in the cubicle receive the impulse and set automatic equipment into service. Supervisory impulse current varies from 0.2 to 0.5 ampere, the voltage impressed being either 60 or 125 volts direct current.

Over a period of three years there have been no

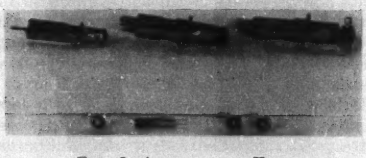


Fig. 8—Appearance of Keys

failures in the cable conductors. Each one of the multi-conductor, lead-covered telephone cables has a few spare conductors so that failures can be taken care of readily and the system can be expanded if desired. Push-buttons used for control of load and voltage are similar in construction to telephone jacks; spring contacts complete the circuit to the generator cubicle where small contactors operate to regulate frequency or voltage. Rotating type keys, similar to telephone equipment, are used for giving the starting impulse, for selecting the bus to which the generator is to be synchronized, either main or auxiliary, for selecting the method of synchronizing, either automatic or self, and for opening or closing oil circuit breakers. Fig. 8 shows the appearance of the keys. The number of operations shown in Table II has been estimated from the total machine hours which are recorded in the station log.

A few cases of trouble were experienced and traced to insufficient spring tension. Thereafter the spring tension was adjusted on all keys and no more failures have occurred. Three telephone lamps give the supervisory operator an indication of the position of the equipment; green for deenergized, red for energized, and white indicating that the position of the control key does not agree with the position of the equipment. Thus, during an outage, the green and white lamps appear, showing that the equipment has taken an open position. The supervisory operator then turns the control key to the open position which extinguishes the white lamp, leaving the green lamp burning. Polarized relays receive the supervisory impulse originating from the rotating type keys on the miniature board. These keys being 2-position keys give an energized signal or a deenergized signal to the polarized relays. The armature of the relay, in turn, moves to the energized or deenergized position. These relays function, (1) to open or close the master control contactor; (2) complete the circuit for the method of synchronizing selected—either automatic or self; (3) give to the synchronizing equipment the control of the generator breaker selected—either main or auxiliary; also complete the circuit for lamp indication.

Throughout the years of service shown on the

tabulation, there has been only one case of trouble which was somewhat unusual. This trouble developed when relays were subjected in summer to an ambient temperature of 50 deg. cent. and the compound with which the coils were impregnated tended to vaporize and deposit on the armature, causing sticking in one position. This type of coil was changed and no further trouble experienced. A glass cover makes this relay practically dust-proof and very little maintenance is required.

meters can be read sufficiently accurately for all practical purposes. The generator meters are calibrated to divisions of 500 kw. while the transformer instruments are calibrated in divisions of 3,000 kw.

Generator Protective Equipment

Overspeed Device No. 12. The overspeed device (Fig. 10) is centrifugally operated and is direct-connected to the shaft of the unit, being mounted above the exciter which in turn is above the generator. This

TABLE II—SUPERVISORY BOARD PERFORMANCE RECORD

No. in service	Equipment	Operations per year	Failures	Trouble chg. to Manufacturer	Trouble chg. to operation	Year
	3,000 ft. of telephone cable			No trouble		1928
						1929
						1930
39	Push-button control load & voltage	80,000	2	None	2	1928
		90,000	2	"	2	1929
		74,000	None	"	None	1930
37	Telephone type control keys	6,022	2	2	None	1928
		5,500	10	None	10	1929
		4,750	None	"	None	1930
78	Polarized telephone relays	9,948	30	30	None	1928
		8,300	5	5	"	1929
		7,590	None	None	"	1930
32	Voltage & load control contactors	80,000	None	None	None	1928
		90,000	"	"	"	1929
		74,000	"	"	"	1930
24	Generator meters, kv., kw. & r.kva	122,676 hr.	% accuracy	2	None	1928
		136,245 hr.	of full	2	"	1929
		101,715 hr.	Scale 1%	None	"	1930
7	Selsyn meter, kv., kw. & r.kva	33,728 hr.	% accuracy	None	None	1928
		25,888 hr.	of full	"	"	1929
		29,712 hr.	Scale 1½%	"	"	1930

Fig. 9—Type of Contactor

Fig. 9 shows the type of contactor used for load and voltage control. These contactors are energized directly from the push-button key on the supervisory board; there has been no trouble experienced. The miniature supervisory switchboard instruments are approximately half the size of typical benchboard instruments, the scale deflection being 3 inches. These

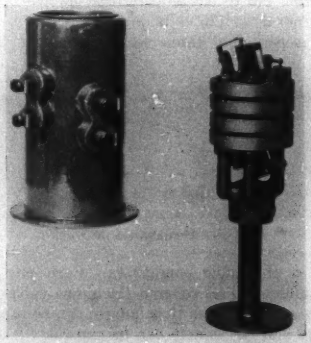


Fig. 10—Overspeed Device No. 12

device is similar to those used in many other hydro plants. A speed of 131½ r. p. m. for operation of this unit is selected as being sufficiently above the speed of 127 r. p. m. which the unit attains when full rated load is rejected. A unit shutdown does not take place in

case of momentary rises in speed but only in case of sustained overspeed due to improper governor action.

Overvoltage Relay—Device No. 59. An induction type relay which functions to shut the machine down on a sustained voltage of 16,500 volts, normal generating voltage being 14,000 volts, functions in 0.8 of a second

FIG. 11—GOVERNOR OIL-PRESSURE RELAYS—DEVICE NO. 63

and, as it operates on an inverse time characteristic curve, voltages of higher magnitude will cause the relay to function in less time. This device only functions to shut the unit down in case the generator is tied in on the bus so that, in case of overvoltage resulting from the generator oil circuit breaker opening and dropping all the load, the unit will not be shut down unnecessarily.

Governor Oil-Pressure Relay—Device No. 63. The governor oil-pressure regulating gages and relays are shown in Fig. 11. The upper or *A* gage holds the oil pressure between 145 and 165 lb. by starting and stopping the oil-pressure pump electric motor while the generating unit is shut down. When the generating unit is started up, a pilot relay causes the oil-pressure pump to run continuously, oil pressure being regulated by means of a by-pass regulating valve which holds the oil pressure between 155 and 175 lb. The *B* gage functions an auxiliary relay to shut the unit down in case the oil pressure should drop to 135 lb. The hand reset on this device will not hold in until the pressure is 140 lb.

Generator Phase Unbalance Relay—Device No. 51 G. The device consists of a low-current, inverse time characteristic induction-type relay connected in the generator current transformer neutral.

Lignum Vitae Bearing Cooling Water Relay. To protect the lignum-vitae thrust bearing from damage due to failure of water supply, a contact closing device—normally held open by flow of water to this bearing—is installed on the water supply line. Water must be flowing to this bearing before the master circuit can be completed and the unit started up and likewise, if the water flow stops while the unit is in operation the master circuit will be interrupted and the unit shut down. Trash collecting in the screen at the entrance to the cooling water supply line has caused this device to function satisfactorily.

Bearing Temperature Relays—Device No. 38. One relay furnished by the generator manufacturer protects the generator bearings. A contact making thermometer (Fig. 12) furnished by the waterwheel manufacturer protects the intermediate guide bearing. Both operate at 55 deg. cent. One operation in 1929 shown in Table IV charged to vibration, was caused by an attendant striking the relay with a dusting mop and causing the relay to jar closed.

Loss of Generator Field Relays Nos. 40, 40-X and 40-Y. This relay is connected in series with the generator field circuit. In case of failure of generator field current, contacts on this relay close, energizing an auxiliary contactor No. 40-Y. This in turn closes contacts of a time delay induction type voltage relay, device No. 40-X, shutting the machine down. Operation of this device in 1928, shown in Table IV, was caused by the operating coil on the main field contactor burning out, interrupting the field circuit.

Ground Protective Relay Device No. 64. An instantaneous hand reset relay with its operating coil completing the connecting circuit between the generator frame and the station ground bus is shown in Fig. 13.

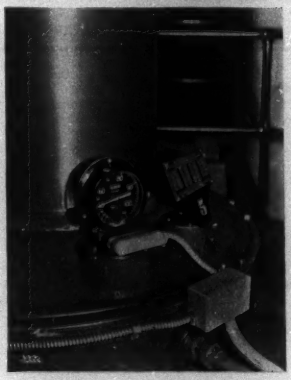


FIG. 12—THERMOMETER TYPE BEARING TEMPERATURE RELAY—DEVICE NO. 38

The relay shuts the machine down in case of a flashover to the generator frame or the grounding of the generator winding. Operations in 1928 of this device charged to vibration were caused by construction work in progress at that time. In 1929 two operations were caused by an electric welder accidentally passing the welding current through the relay coil.

Differential Relay Protective Equipment Device No. 87.

During 1931 all eight generating units will be equipped with differential relays. As the generators are operated either two-circuit wye or single-circuit delta connection, depending on head conditions, two sets of differential relays will be provided per generator. These relays will shut the unit down in case of a phase-to-phase fault in the generator stator winding.

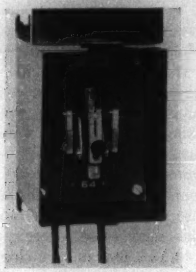


Fig. 13—Grounding Relay—Device No. 64

White and red signal lamps are mounted on top of the generator, the white lamp burning when all temperature relays, ground relays, and loss-of-field relays are normal. If they are not normal, the white lamp is extinguished and the red lamp lights.

protective devices which interrupt this circuit either directly or indirectly.

In starting a unit, this master circuit, provided all protective devices are in a normal position, is completed through the master contactor which is energized during the starting process by the supervisory board operator turning the starting control key to the "start" position. The devices which make up this master circuit are listed in Table III. A lock-out hand reset relay operates to interrupt the master circuit when any of the temperature relays, thermometers, ground relay or loss-of-field relay function.

Fig. 14—Governor Pilot Relay Solenoid—Device No. 65-S

TABLE III—PERFORMANCE RECORD OF GENERATING EQUIPMENT

No. in service	Equipment	Oper. per year	Hours in service	Failures	Trouble chg. to Mfg.	Trouble chg. to Eng.	Trouble chg. to Oper.	Year
8	Governor oil high-pressure gage relay		40,892	4 relays	Coils	None	None	1928
			45,415	None	None	"	"	1929
			33,905	"	"	"	"	1930
8	Gov. solenoid oper. waterwheel gates	2,966	40,892	None	None	None	None	1928
		2,504	45,415	"	"	"	"	1929
		2,180	33,905	"	"	"	"	1930
8	Gen. field contactor oper	2,966	40,892	1	1 coil	None	None	1928
		2,504	45,415	None	None	"	"	1929
		2,180	33,905	"	"	"	"	1930
8	Gov. fly-ball auto. transfer. sta. aux. to unit aux	1,483	40,892	None	None	None	None	1928
		1,252	45,415	"	"	"	"	1929
		1,090	33,905	"	"	"	"	1930
8	Generator cooling water pump	1,483	40,892	None	None	None	None	1928
		1,252	45,415	"	"	"	"	1929
		1,090	33,905	"	"	"	"	1930
8	Generators put on line	1,483	40,892	1	None	1	None	1928
		1,252	45,415	None	"	None	"	1929
		1,090	33,905	"	"	"	"	1930
8	Generator brake valve solenoid	2,966	40,892	8	None	8	None	1928
		2,504	45,415	6	"	6	"	1929
		2,180	33,905	None	"	None	"	1930

Master Control Circuit and Equipment. In order to start and operate a generator, it is necessary that the master control circuit be a complete electrical circuit from the positive to the negative control bus. This circuit is composed of various devices other than the

Protective devices that interrupt the master circuit directly are the overspeed device and the low governor oil-pressure relay previously described. The protective

devices that shut the unit down by indirect operation on the master circuit, other than by means of the lock-out relay, are the overvoltage relay, phase unbalance relay, and differential relays. These last three relays function by short circuiting out the holding-in coil on the master contactor. Deenergizing of the master contactor, in turn, breaks the master circuit, shutting the machine down.

The governor pilot relay solenoid, as shown in Fig. 14 is connected directly in the master control circuit. If the solenoid coil is energized by the completion of the master circuit, a pilot valve is opened on the governor, permitting the governor to function in a normal manner to regulate the waterwheel gates. If this solenoid coil is deenergized, the pilot valve causes the waterwheel gates to close and the generator oil circuit breaker is opened automatically thereafter.

For each generator there is approximately 220 ft. of 19-22 control wire used in making up this master circuit which is protected by the fuses.

A waterwheel gate limit switch (Fig. 15), not a part of the master circuit, functions to complete the tripping circuit to the generator oil circuit breaker, causing this breaker to open when the waterwheel gates have reached a "no load" position and after the master contactor has been deenergized. Differential relays, overvoltage relays, and phase unbalance relays also trip the generator oil circuit breaker and shut the unit

TABLE IV—PERFORMANCE RECORD OF GENERATOR PROTECTIVE EQUIPMENT

No. in service	Equipment	Times oper.	Hr. in service	Operation chg. to: Mfg. defect	Operation chg. to: Protection	Year
8	Overspeed device No. 12	8	40,892	8 vib.	None	1928
		1	45,415	None	1	1929
		1	33,905	"	1	1930
8	Overvoltage relay device No. 59	3	40,892	None	3	1928
		8	45,415	"	8	1929
		6	33,905	"	6	1930
8	Low governor oil-pressure relay device No. 63	4	40,892	None	4	1928
		1	45,415	"	1	1929
		None	33,905	"	None	1930
8	Phase unbalance relay device No. 51G	None	40,892	None	None	1928
		1	45,415	"	1	1929
		None	33,905	"	None	1930
8	Lignum vitae bearing cooling water relay	3	40,892	None	3	1928
		4	45,415	"	4	1929
		1	33,905	"	1	1930
24	Bearing temperature relay device No. 38	1	40,892	None	1	1928
		1	45,415	1 vib.	None	1929
		None	33,905	None	"	1930
8 each	Loss of gen. field relays devices Nos. 40, 40x, 40y	1	40,892	None	1	1928
		None	45,415	"	None	1929
		"	33,905	"	"	1930
8	Generator ground relays device No. 64	5	40,892	None	5 vib.	1928
		2	45,415	"	2	1929
		None	33,905	"	None	1930

MASTER CONTROL CIRCUIT AND EQUIPMENT

No. in service	Equipment	Hr. in service	Normal oper. start stop	Equipment failure	Shut down due to trouble	Year
8	Master contactor device No. 4	40,892	2,966	None	None	1928
		45,415	2,504	"	"	1929
		33,905	2,180	2 def. coils	"	1930
8	Lock out relay hand reset device No. 81	40,892		None	None	1928
		45,415		"	"	1929
		33,905		"	"	1930
8	Protective devices that interrupt master circuit				22	1928
					18	1929
					8	1930
8	Governor pilot relay solenoid	40,892		None	None	1928
		45,415		"	"	1929
		33,905		"	"	1930
16 D-c. control 30 A		40,892			8	1928
		45,415			None	1929
4 D-c. control 200 A		33,905			"	1930

down regardless of position of the waterwheel gates.

Performance During System Disturbances

To date there have been five system disturbances which resulted in the hydro dropping load. Three cases were due to lightning flashing over on a 66-kv. tie line to the steam plant; one to lightning flashing over on a 13-kv. feeder which is connected to the city distribution system; and one to failure of transformers

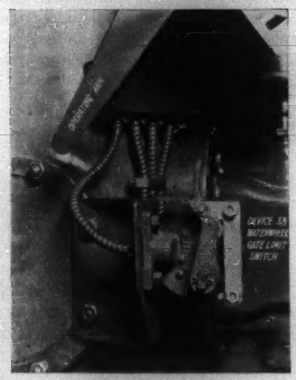


Fig. 15—Waterwheel Gate Limit Switch—Device No. 33

stepping the voltage up to 66 and 138 kv. Table I indicates the manner in which the units were shut down, the manner in which service was restored and that a very short time was required to get the units back into service and on the bus. These disturbances occurred before the generators were all equipped with self-synchronizing equipment and, therefore, in cases where the automatic synchronizing equipment did not function rapidly enough, the operators put the machines on manually in order to save time.

Each disturbance caused a short circuit of considerable magnitude and, when the circuit breakers opened, the overspeed of the generators and consequent rise of voltage was such that the overvoltage protective relays operated, removing generators from the bus and shutting the units down. The bus being dead, the station light and power bank was automatically transferred to the incoming emergency line. It is to be noted that a maximum time of four minutes was required to get the last generator in on the bus in the case of the first two disturbances when, in the first case, eight generators were running and, in the second case, five generators were running. In restoring service, the supervisory operator closes the tie line in on one 13-kv. bus. Thereafter, the starting impulse is given to all generators, automatic means of synchronizing having previously been selected at the miniature supervisory board. As it was necessary to get generators on the bus as soon as possible, it was therefore customary,—previous to the time all units were equipped for self-synchronizing, if the automatic synchronizing equipment had not synchronized the generator promptly enough—for a floor operator to go to each generator cubicle and do so manually. Now that this equipment for self-synchronizing is installed for all generators and, since this method is much quicker, it is being used almost entirely.

Conclusions

Engineering reasons for adoption of miniature supervisory switchboard control and automatic control were to reduce initial capital expenditure and lower fixed charges. The results of more than three years continuous operation of this equipment show the following:

With the miniature switchboard, the operator is at all times closer to indicating instruments, control switches, etc., and can give better and closer supervision to the

TABLE V—OHIO FALLS HYDROELECTRIC STATION—LOUISVILLE, KENTUCKY PERFORMANCE RECORD OF GENERATOR EQUIPMENT

Equipment	Oper. per year	Hours in service	Failures	Trouble charged to: Mfg. defect	Trouble charged to: Operation	Years
Generator oil-pressure built up	1,483	45,000	4 Pilot relay	4 Coils burnt out	None	1928
	1,252	45,400	None	None	"	1929
	1,090	34,700	"	"	"	1930
Gov. solenoid operating waterwheel gates	2,966	45,000	None	None	None	1928
	2,504	45,400	"	"	"	1929
	2,180	34,700	"	"	"	1930
Gen. field automatically operated	2,966	45,000	1	1 def. coil	None	1928
	2,504	45,400	None	None	"	1929
	2,180	34,700	"	"	"	1930
Gov. fly-ball auto. transferred sta. aux. to unit aux.	1,483	45,000	None	None	None	1928
	1,252	45,400	"	"	"	1929
	1,090	34,700	"	"	"	1930
Generator cooling water pump	1,483	45,000	None	None	None	1928
	1,252	45,400	"	"	"	1929
	1,090	34,700	"	"	"	1930

TABLE VI.—MAINTENANCE FOR THREE-YEAR PERIOD

Use	Equipment	Maintenance	Man hours
1—D-c. station control	Two—60-cell batteries	Monthly inspection	468
2—Charging batteries	Two—M. G. sets (25 kw.)	Cleaning windings and commutators	90
3—Control power supply from 13.2-kv. system to sta. aux. equipment	Cubicle and other equipment	Cleaning and adjusting contactors, relays, etc.	72
4—Station power and light supply	Three—200 kva., one—100 kva. regulators	Cleaning and inspection of relays, contactors, etc.	66
5—Station auxiliary motor drive equipment	37—220-volt motors driving oil pumps, unit water pumps, etc.	Usual cleaning and inspection	700
6—Automatic control of gov. oil pressure system	Gage type relays and automatic compensator controlling oil pressure	Cleaning and adjusting contacts	850
7—Interlocking pneumatic brakes on generator & main oil circuit breakers	Unit brake valve solenoid	Periodic inspection and cleaning	216
8—Controlling 66-kv. power transformer banks	Transformer bank cubicle recording meters with Selsyn attachment. R.kva., kw., kv. auto. synch. eqpt.	Calibration, changing charts. Detail changes	518
9—To shut unit down in case of trouble on unit	Bearing thermal relays; water-flow indicator; ground relay; overspeed device; phase unbalance relays; loss of gen. field relays; gov. low-oil pressure relay	Monthly inspection, cleaning, adjusting, etc.	972
10—Automatic control of generator equipment	Contactors in generator cubicles	Inspection, cleaning, etc., replacing burned-out coils; changing and simplifying control circuit	450
11—Control of generator and transformer cubicles from miniature sup. swbd.	Miniature switchboard 39 rotary control keys	One inspection each control panel; removed from cabinet; contacts and spring tensions adjusted	50
12—Ditto	Miniature switchboard 32 push-button control switches	Periodic inspection, cleaning and testing for proper spring tension	20
13—Ditto	111 miniature indicating lamps (24 volt)	Replacing 50 lamps, inspection and cleaning	27
14—Ditto	Miniature instruments on miniature swbd. indicating kv., kw., r.kva.	Adjustment of bearing friction; replacement of defective spring on one meter and recalibration	45
15—Indicating total kw., r.kva. and kv. on 66-kv. power transf. bank	Selsyn control meters on miniature board	Periodic inspection and cleaning	25
16—Various contactors dependent on polarized relays to energize master circuit to start unit; select proper bus; select method of synchronizing	Polarized relays on cubicles 78 for 8 generators; 2 for 66-kv. transfer bank 3 for sta. aux. bank.	Cleaning, adjusting, etc. Replacing a few coils.	165

TABLE VII—INFORMATION ON SYNCHRONIZING OF GENERATORS

Synchronizing methods	Synchronized		Good shots					Poor shots	Year
	Successfully	Failure	2 min.	3 min.	4 min.	5 min.	5 to 8		
Automatic	1,483	None	890	297	148	74	45	29	1928
Automatic	1,252	None	751	251	125	63	37	25	1929
Automatic	440	None	264	88	40	25	12	11	1930
Self	650	None	Oscillogram shows 2 times normal arm. cur. for 2 cycles. To normal in 15 cycles. System voltage dip = 1.4 volts on 110-volt base						1930

operation of the equipment, resulting in better voltage control, frequency control, and division of load among generating units.

There is a great saving in time during regular operation and human error is largely eliminated.

There is a marked saving in labor costs since there is only one shift operator required to operate the entire station.

Maintenance on the miniature supervisory board is no greater than on vertical boards of older types and, as far as comparisons can be made, appears to be less. Very little trouble has been experienced, apparently slightly less trouble than on older types of boards, although this installation was one of the first used in a large generating station. During system disturbances the operation of the miniature switchboard, as compared with a modern switchboard of standard power house type now in service in Waterside Steam Station (92,000-kw. capacity) indicates that the miniature supervisory board is more convenient, saves time, reduces chances for error, and results in quicker restoration of service.

The use of automatic devices controlling the electrical equipment, waterwheel governor, oil pressure, bearing water pressure, etc., has demonstrated its reliability, low cost of maintenance and inspection, saving of time, saving of operating expense, and better and safer supervision and operation of electrical equipment than would be possible by manual system of control.

This operating experience indicates that hydro plants or steam generating stations can be supervised from miniature switchboards with, or without, automatic features. It is believed that, in hydro plants where units are taken off and put on the bus regularly and frequently, this type of control equipment is not only superior to manually operated equipment but the plant will be operated with greater safety to equipment.

Discussion

M. E. Reagan: The statement regarding increased intervals between maintenance periods corresponds to the experience of a great many operating companies. In fact, one similar equipment has just finished a year's service without breaking the seals on the cases. In other words, there has been no maintenance expense in that time. It is gradually becoming evident that, where the equipment is kept in near-dust-proof cases, the less maintenance the better.

The complaint of the maintenance man is that if he does not experiment with it, he cannot know anything and therefore when trouble does eventually come, he is lost. This is, indeed, false reasoning. He is equipped with complete diagrams, a full set of detailed instructions, and he can see the operations through glass covers or doors. Getting his hands on the relays will not help his situation and may mean disaster to the equipment. His practise on relay adjustment would be much better on spare relays which are not in use.

The type of waterwheel used, while ideal in the application described, has a rather unstable speed characteristic at no-load. In other words, with a small amount of water going through, the machine speed is not uniform and it is hard to synchronize. It has been found in similar installations that a synchronizer with an automatic phase advance feature is desirable. It takes a certain definite time for the circuit breaker to close after the synchronizer makes contact. Within some predetermined lockout speed, the synchronizer automatically makes contact ahead of the "12 o'clock" position in proportion to the speed of the "scope" so that the breaker closes its contacts at synchronism. It also means that the synchronizer will not "pass up" chances for closing the breaker because of the adjustable speed of the wheel.

C. E. Stewart: Considerable advance has been made in the design of suitable control keys and meters for the miniature switchboard during the last few years so that the miniature switchboard is now considered as a standard type of board for installation in power and switching stations. There is no question but what the miniature switchboard meets a very important requirement in the present day designs for handling electrical energy. Undoubtedly the conventional long type of standard switchboard will be eventually superseded by the miniature board because of the improved operating conditions and economies effected.

Economics of High-Voltage Cable

BY D. W. ROPER*
Fellow, A. I. E. E.

Synopsis.—The 132-kv. oil-filled cable placed in service in Chicago in 1927 was designed by the manufacturers in 1925 to have the largest carrying capacity commercially feasible for that voltage in the state of the art as it then existed. During the succeeding years the advancement in the art was so rapid that the cable for the same voltage installed in 1930 had a carrying capacity 75 per cent greater and a cost 20 per cent less than the 1927 line. Improved oil, new designs of oil supply tanks and methods of shipping and connecting cable had reduced the cost of installation and the design of the terminals. Successful operating experience with devices and methods for eliminating sheath losses had removed the previous limitation on the economical size of the conductor. Operating data and test results indicated that the insulation on the first line was thicker than necessary and this led to an inquiry regarding the economy of using the oil-filled type of insulation at higher and lower voltages.

Other questions concerned with future developments in the Chicago region indicated the necessity of a careful study of all recent advances in the art, and resulted in extending the investigations so as to include an economic study of transmission cable for all voltages, present and prospective. The investigations confirmed current practise in the use of three-conductor cable at moderate voltages for transmitting load to substations, but indicated that loads much larger than are ordinary can be efficiently transmitted by the use of single-conductor cable with sheath losses eliminated. A brief statement is included of the method adopted for determining the limitations of oil-filled insulation and extending the present voltage range of oil-filled and ordinary types of insulation.

The recent rate of improvement in the art of high-voltage cable practise probably will be continued in succeeding years.

The utilities must improve their standards of design and workmanship in order to keep pace with improvements being made by the manufacturers.

Introduction

THE greatest improvement in high-voltage, underground cable in recent years was realized in 1927[1] upon placing in service in New York and Chicago the commercial installations of 132-kv. oil-filled cable. During the installation of these lines it became apparent that a number of improvements was possible, particularly in connection with the methods of installation and oil supply. Methods were devised of making joints on sections of cable filled with oil, and then the shipment of this cable (with a suitable reservoir inside the cable reel for maintaining positive pressure) eliminated the expensive evacuation and

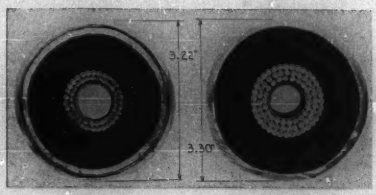


Fig. 1—132-Kv. Oil-Filled Cables in Commercial Use in Chicago

Left: Cable for 103,000-kva. line installed in 1927
Right: Cable for 188,000-kva. line installed in 1930

impregnation processes in the field that were previously required. A number of these improvements was given a field trial during the installation of oil-filled, three-conductor, 33-kv. cable in Chicago in 1928.[2]

The fundamental point in the design of an oil-filled line is that positive pressure be maintained on the oil within all portions of the cable under the most adverse conditions, namely, sudden interruption of full load with minimum ambient temperature. In planning the 1930 line for Chicago it was found that there was available an oil electrically similar to the oil used in

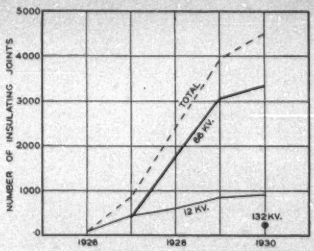


Fig. 2—Insulating Joints in Lead Sheaths Installed on Single-Conductor Cables by Commonwealth Edison Company

the 1927 line, but with lower viscosity and lower flow point. This thinner oil permitted a lower oil pressure which made it feasible to use a single lead-tin alloy sheath instead of the double reinforced sheath used in 1927. The reduction in the thickness of the sheath allowed the use of a much larger conductor with slight increase in over-all diameter (Fig. 1), but it was not commercially feasible to increase the conductor size unless the sheath losses were eliminated. Meanwhile, however, favorable operating experience had been accumulating with insulating joints on single-conductor cables in three-phase lines of lower voltage (Fig. 2), and also with the available methods of bonding the cables so as to keep the induced a-c. sheath voltages within

*Supt., Street Dept., Commonwealth Edison Company, Chicago, Ill.

1. For references see Bibliography.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

safe limits.[3] New designs of the oil-supply tanks and the use of the thinner oil simplified the design of the terminals (Fig. 3) for connection to the overhead line.

By taking advantage of all improvements, a line of 188,000-kva. capacity was installed in Chicago in 1930 at a cost about 23 per cent below the cost of the 103,000-kva. line installed in 1927 (Fig. 4). The cost of the 1930 line compares with the 1927 line as follows:

Fig. 3—Simplification in 132-Kv. Cable Terminals Due to Improvements in Insulating Oil and Reservoirs

Left: Electrically heated terminal and oil reservoir tower of line installed in 1927
Right: Outdoor terminal of line installed in 1930

Cable	14 per cent increase.
Joints, oil reservoirs, and accessories	32 per cent decrease.
Supervision and labor for installation	62 per cent decrease.
Special manholes and towers for stop joints and oil reservoirs	81 per cent decrease.
Conduits and ordinary manholes	No change. Same conduit and manholes.
Total	23 per cent decrease.

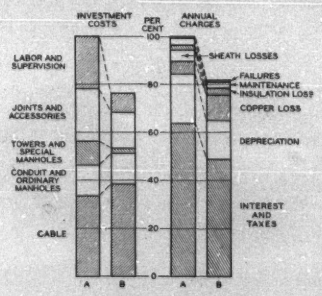
About the same time, other questions arose as follows:

1. What is the maximum economical size of single-conductor cable for connection between the 12-kv. bus and a 60,000-kva. transformer bank?
2. Is there any economy in the use of oil-filled cable for 66-kv. service?
3. What is the maximum load per line that can be carried on 132-kv. oil-filled cable in 4-inch ducts?
4. Can 220-kv. cable be obtained if desired, and what will be the approximate cost?

Therefore it was decided to review for Chicago conditions the entire subject of high-voltage cable so that, in addition to answering all of these questions, it would be possible to determine what changes in existing practise were indicated by recent developments.

Plan of Procedure

Preliminary calculations (Fig. 5) on typical cables in use in Chicago indicated wide differences in the relative amounts of the various items entering into the initial cost and annual charges, and, further, that cable with lowest annual charges might not always be the one with the lowest first cost. Accordingly the plan was adopted of calculating curves showing the variations of the initial costs and annual charges for the entire range of commercial sizes so as to secure data which would indicate the most economical transmission voltage and type of cables for various Chicago conditions.



A. 600,000-cir. mil, 103,000-kva. line installed in 1927. Sheath losses present

B. 1,100,000-cir. mil, 188,000-kva. line installed in 1930. Sheath losses eliminated

Fig. 4—Reduction in Costs of 132-Kv. Oil-Filled Lines in Chicago

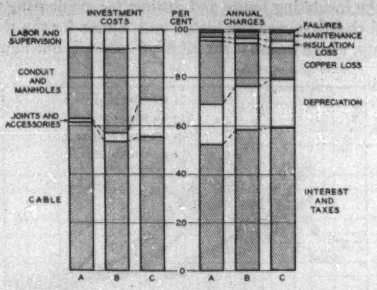
There have been no failures in oil-filled cables, but an allowance is made for repairing failures of all kinds including external injuries

The corresponding reductions in investment cost per kva. and annual charges are about 55 per cent

The calculations were confined to cables for the transmission voltages in use in Chicago except that,

with some assistance from the manufacturers, 220-kv. cable was added. In determining the size, type or voltage of transmission cable, due consideration should be given to the effect on the entire system of generation, transmission, and distribution of which the cable is to be a part.

As the relative initial and annual costs were desired



A. 12 kv., 3-conductor, 500,000 cir. mil
B. 66 kv., 1-conductor, 750,000 cir. mil
C. 132 kv., 1-conductor, 1,100,000 cir. mil, oil-filled

FIG. 5—DIVISION OF COSTS FOR TYPICAL LINES IN CHICAGO

under identical conditions, it was considered more important that the unit cost figures be the same for all calculations than that they should fit any particular case. Accordingly for the purpose of these calculations physical constants, unit costs, and conditions were assumed, as given in Table I and in the following assumptions.

ASSUMPTIONS

1. Impregnated, paper insulated, lead-covered cables installed one per duct.
2. Three-phase, 60-cycle transmission.
3. Engineering details in accordance with Chicago conditions.
4. Three-conductor cables are of shielded type except 12-kv.; conductors are sector-shaped except smaller sizes of 33-kv. and 66-kv. cables.
5. Ducts are round precast concrete, with one-inch concrete between ducts and three-inch envelope.
6. Diameter of cable must not exceed 5/6 of diameter of duct.
7. Pure lead used for ordinary cable sheaths, and 2 per cent tin alloy for oil-filled cable. Sheath thickness for ordinary cables is 5/64 inch for diameters over insulation up to 0.6 inch, and 1/64 inch is added for each increase of 0.6 inch in diameter.
8. Ingot prices of metals per pound: copper, 15 cents; lead, 6 cents; tin, 40 cents.
9. Uniform conductor size used throughout a line. (Actual practise in Chicago is to install larger conductors where cable is in heavily loaded conduits.)
10. No sheath losses on single-conductor cables.
11. Skin effect calculated from data given by Simmons.[4] For three-conductor cables, conductor proximity effect assumed equal to 50 per cent of skin effect and losses due to proximity effect in binding tapes and sheath calculated on basis of formula by Miller.[5]
12. Maximum permissible a-c. sheath voltage to ground is 12 volts.
13. Permittivity of insulation: 3.3 for oil-filled; 3.6 for ordinary.
14. Ambient ground temperature is 15 deg. cent.
15. Annual load factor for 33 kv. and lower voltage cables is 45 per cent as determined by loads found on lines to substations, while for 66 kv. and higher, the load factor is 60 per cent as determined by character of loads on tie lines between stations. Corresponding

TABLE I—TECHNICAL DATA REGARDING THE CABLES ON WHICH CALCULATIONS WERE MADE

Type of cable				Thermal resist.						Duct heating	
Kv.	No. of conds. per cable	Kind of insul.	Insul. thick. 64ths inch*	of insul. deg. cent. per watt per cm.[3]	Max. copper temp. deg. cent.	P. f. of insul. per cent	No. of ducts in conduit	No. of cables in conduit	Average distance between manholes feet†	constant deg. cent./watt/ft. of conduit‡	Length of section between stop-joints feet
12	3	A	(11 + 5) − (12 + 6)	700	78	1.8	12	10	400	0.77 − 0.65	
12	1	A	13 − 15	700	83	1.8	12	9	400	0.99 − 0.84	
22	3	A, S	19	600	77	1.6	9	7	400	0.99 − 0.75	
22	1	A	19	600	77	1.6	12	9	400	0.80 − 0.72	
33	3	A, S	25	600	71	1.4	9	7	400	0.80 − 0.68	
33	1	A	25	600	71	1.1	12	9	400	0.80 − 0.68	
33	3	B, S	14 − 18	550	75	0.4	8	6	400	1.10 − 0.82	2,500
66	3	B, S	26 − 28	550	70	0.4	8	6	550	1.10 − 0.82	2,500
66	1	A	48	600	60	0.7	8	6	550	1.00 − 0.82	
66	1	B	26 − 28	550	70	0.4	8	6	550	1.10 − 0.97	2,500
132	1	B	46	550	70	0.4	8	6	550	1.10 − 0.82	2,500
220	1	B	72	550	65	0.4	8	6	550	0.97 − 0.82	2,500

A—Ordinary type of insulation. B—Oil-filled insulation. S—Shielded type of 3-conductor cable.

*Increased thicknesses are for large conductors.

†For very large single-conductor cables, these lengths would have to be reduced by as much as 30 per cent in order to keep the induced sheath voltage to earth from exceeding 12 volts.

‡Smaller heating constants are for the larger cables requiring larger duct sizes.

daily load factors for winter week day are 55 per cent and 75 per cent.

16. Joints for 33 kv. and above are oil-filled.

17. Oil-filled lines are equipped with signal systems to indicate oil leaks.

18. No credit is given in principal calculations for charging kva. of cables and no allowance for neutralizing charging current of very long high-voltage lines.

19. Transmission line voltage drop not considered.

20. Fixed charges on investment: conduit, 10½ per cent; cable, 11½ per cent; terminals, 12½ per cent.

21. Cost of energy losses, 0.6 cents per kw-hr.

22. Rate of cable failures, 1½ to 3 per 100 mile-years; rate of joint failures, ⅓ to 1⅓ per 1,000 joints per year. Higher rates apply to lower voltages.

Methods and Results of Calculations

Using the carrying capacities of the cables as shown in Fig. 6, corresponding curves were calculated showing the cost of the conduit and manholes (Fig. 7), the cost of the cable installed in conduit (Fig. 8), the cost of the joints and accessories (Fig. 9), and the cost of terminals (Fig. 10). From these data were calculated the initial cost (Fig. 11) and annual charges (Fig. 12) of the completed lines without terminals, also corresponding curves of 10-mile lines with terminals (Figs. 13 and 14).

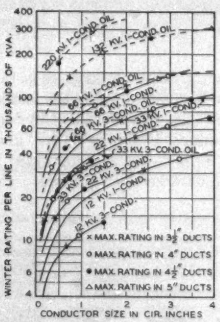


Fig. 6—Carrying Capacities of Lines

No sheath losses on single-conductor cables. Cable diameter not more than five-sixths of duct diameter. Cables are of ordinary type unless otherwise specified.

As single-conductor cables require three ducts per line, the cost curves per kva. for conduit and manholes for these cables (Fig. 7) appear in a separate group considerably higher than the corresponding curves for three-conductor cable.

For the 66-kv., single-conductor cable, it is interesting to note (Fig. 8) that the oil-filled cable costs less than the ordinary type due to less copper, insulation, and lead. The relative cost figures are reversed for the 33-kv., three-conductor cable.

The curves showing the costs of the joints and accessories arrange themselves in three groups, the first containing the three-conductor cables with the ordinary type of insulation; the second group containing single-conductor cable with the ordinary type of insulation having three times the number of joints as for three-conductor cables and, in addition, the cost of the insulating joints and devices for eliminating the lead sheath losses; the higher cost figures for the third group, which includes the oil-filled cables, is due largely

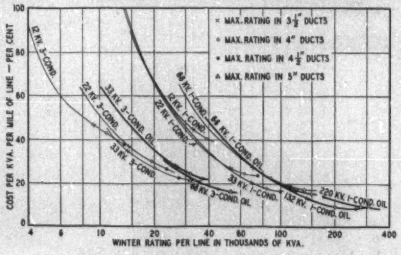


Fig. 7—Relative Costs of Conduit and Manholes per Line

One cable per duct. Vertical scale same as Fig. 8

to the cost of the stop joints, oil reservoirs and accessories required with this type of cable.

A study of the curves leads to the following conclusions:

1. The most economical cable for a given set of conditions should be determined by considering the total annual charges, rather than the first cost.

2. The inclusion of the terminal charges makes but slight difference in the *relative* costs of the various types and voltages of cable. This indicates that the relative costs of the various types and voltages of cable would be only slightly altered by assumptions somewhat different from those used in the calculations.

3. For every load there is one type of cable and one transmission voltage which is more economical than any other combination. (See Table II.)

4. For a given type and voltage of cable, maximum economy is generally found with larger cables than are in ordinary use.

TABLE II—MOST ECONOMICAL TYPES OF CABLE FOR VARIOUS RANGES OF LOAD

Data Taken from Fig. 12

Load range—kva.	Most economical type of cable		
	Voltage—kv.	No. of conductors	Kind of insulation
Up to 8,000	12	3	Ordinary
8,000— 12,000	22	3	Ordinary
12,000— 25,000	33	3	Ordinary
25,000— 48,000	66	3	Oil-filled
48,000—120,000	66	1	Ordinary
120,000—260,000	132	1	Oil-filled

5. Three-conductor cables are the most economical for loads up to about 50,000 kva. For the upper half of this range, three-conductor, 66-kv., oil-filled cable[8] is the most attractive, but it requires larger ducts than are now in ordinary use.

6. For loads above 50,000 kva. per line, single-conductor cable should be used.

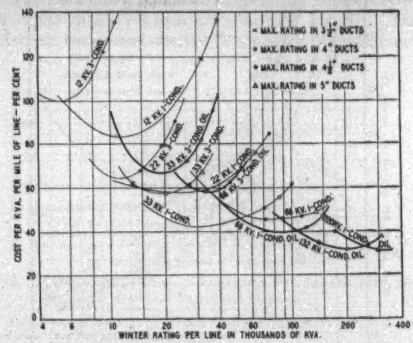


FIG. 8—RELATIVE COSTS OF CABLE INSTALLED IN CONDUIT

Costs include racks and fireproofing but do not include joints, oil reservoirs, and other accessories. Minimum cost for 12-kv., 3-conductor cable assumed as 100 per cent.

7. The elimination of sheath losses permits the efficient use of large single-conductor cables between 12-kv. bus and transformers for high-voltage lines.

8. For cable to operate at 66 kv., a moderate reduction in the cost of oil-filled, single-conductor cable

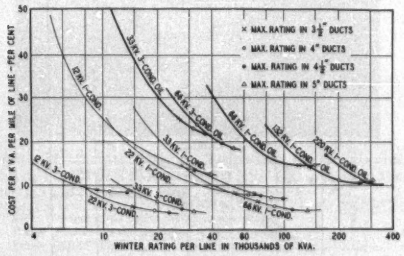


FIG. 9—RELATIVE COST OF JOINTS AND ACCESSORIES INSTALLED

Includes oil reservoirs and apparatus for eliminating sheath losses. Vertical scale same as Fig. 8

and in the cost of the joints and accessories would bring the oil-filled type in competition on a cost basis with cable of the ordinary type.

9. For loads of the order of 100,000 kva. per line, oil-filled, 66-kv., single-conductor cable is preferable to the ordinary type when the over-all diameter of cable is limited by the size of the ducts in existing conduits.

10. There is no economy in using 132 kv. for tie lines between stations in Chicago until the required carrying capacity exceeds 125,000 kva. per line. In

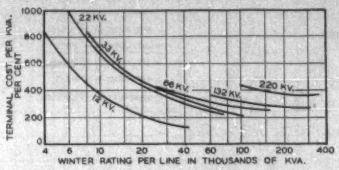


FIG. 10—RELATIVE TERMINAL COSTS

Terminal costs include only a simple connection between cable and bus without high-voltage bus ties or other auxiliary switches as follows:

(a) 12-kv. oil-circuit breaker and portion of switchhouse at generating station

(b) Oil circuit breaker at substation end of 12-, 22-, and 33-kv. lines

(c) Transformers and high-voltage bus structure at generating station for all lines above 12 kv.

(d) Terminal structure and disconnective switches at other end of 66-, 132-, and 220-kv. lines

Vertical scale same as Fig. 8

so far as carrying capacity of the cable is concerned, there is no occasion for increasing the transmission voltage above 132 kv. until the required capacity per line exceeds 250,000 kva.

11. The cost of cable for 132 kv. and higher voltages

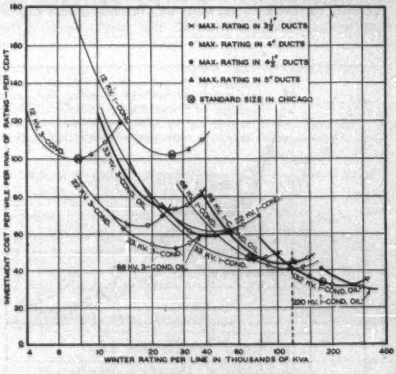


FIG. 11—RELATIVE INVESTMENT COSTS OF UNDERGROUND LINES, EXCLUDING TERMINALS

One cable per duct: 6 to 10 identical cables per conduit. Minimum cost for 12-kv., 3-conductor cable assumed as 100 per cent.

probably will limit its use to underground extensions, within large cities, of long, overhead transmission lines from distant stations.

12. As the fixed charges are about three-fourths of the total annual charges, the best way to reduce annual charges is to reduce first cost.

13. In determining the best transmission cable for a given purpose, the decision should not be restricted by preconceived ideas of the relative importance of transmission voltage, dielectric loss, skin effect, maximum efficiency of insulation, Kelvin's law, the size or number of ducts in the conduit, or other physical laws and constants; but, by taking all such subjects into consideration, the type, size and voltage of cable and conduit details should be determined so as to secure the lowest annual charges for the given conditions.

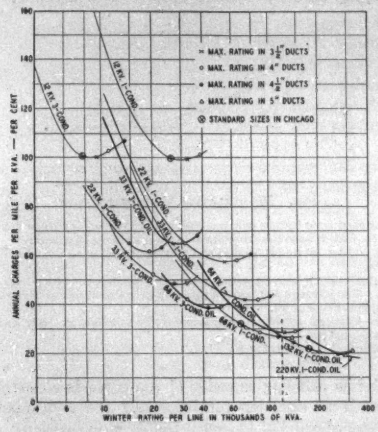


FIG. 12—RELATIVE ANNUAL CHARGES FOR UNDERGROUND LINES, EXCLUDING TERMINALS

Annual load factors: 12 kv. to 33 kv., 45 per cent, 66 kv. to 220 kv., 60 per cent. Minimum charges for 12 kv., 3-conductor cable assumed as 100 per cent.

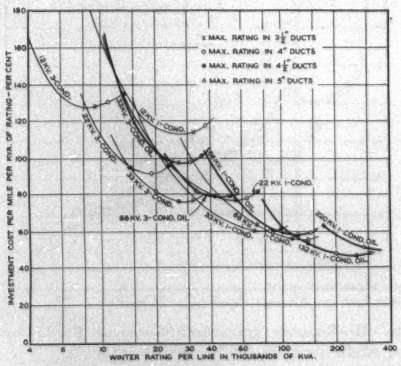


FIG. 13—RELATIVE INVESTMENT COSTS OF TEN-MILE LINES, INCLUDING TERMINALS

Vertical scale same as Fig. 11

DISCUSSION OF FACTORS AFFECTING COST

The elimination of sheath losses on single-conductor cables permits a considerable increase in the rating of the cables and a corresponding reduction in the annual charges. (Figs. 15 and 16.) The devices used for the elimination of the sheath losses on the 132-kv. oil-filled line installed in 1930 are shown in Fig. 17.

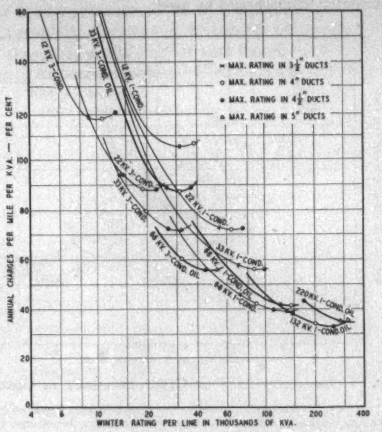


FIG. 14—RELATIVE ANNUAL CHARGES FOR TEN-MILE LINES, INCLUDING TERMINALS

Annual load factors: 12 kv. to 33 kv., 45 per cent, 66 kv. to 220 kv., 60 per cent. Cost of transformer losses included. Vertical scale same as Fig. 12

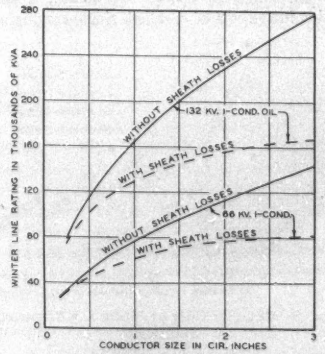


FIG. 15—EFFECT OF SHEATH LOSSES ON CARRYING CAPACITIES OF CABLES

In calculating the carrying capacities, the dielectric

loss was assumed in accordance with recent test data, as shown in Fig. 18, in which there is also shown the effect on the carrying capacity and on the annual charges of different power factors from those assumed. These graphs show the necessity of continually decreasing the power factor with increase in operating voltage.

The maximum permissible operating temperatures of cables of all voltages are shown in Fig. 19. The effect on the annual charges of changing the maximum

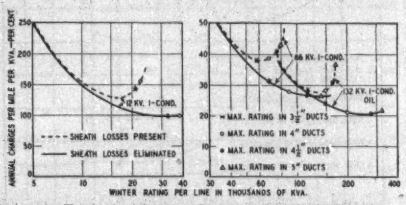


FIG. 16—EFFECT OF SHEATH LOSSES ON ANNUAL CHARGES FOR SINGLE-CONDUCTOR CABLES

Annual load factors: 12 kv. to 33 kv., 45 per cent, 66 kv. to 220 kv., 60 per cent. Vertical scale same as Fig. 12

copper temperature for typical cables is shown in Fig. 20.

It will be noted that the advantage of increased maximum operating temperature is greater for the high-voltage cables, and, further, there is but slight

FIG. 17—INSTALLATION OF EQUIPMENT FOR ELIMINATING SHEATH LOSSES ON 132-KV., 188,000-KVA. UNDERGROUND LINE INSTALLED IN CHICAGO IN 1930

Left: Normal cable joints before fireproofing
Center Right: Insulating joints in cable sheaths
Lower Right: Bonding transformer for reducing sheath voltages

advantage in increasing the temperature for any type beyond 85 deg. cent. A temperature survey of the lines in Chicago shows that but a small portion is operating near the maximum permissible temperature (Fig. 21).

The rating of cables installed in conduit is determined by the total amount of heat to be radiated from the conduit. Increasing the number of cables per conduit beyond rather moderate figures does not reduce the annual charges, as shown in Figs. 22 and 23.

While an increase of load factor increases the amount of energy to be radiated from the conduit and thus reduces the rating of the cables, this reduction is more than offset by the increased energy transmitted as shown in Fig. 24.

For cables below 66 kv., the charging current is not a large item; but for 66 kv. and higher voltage cables,

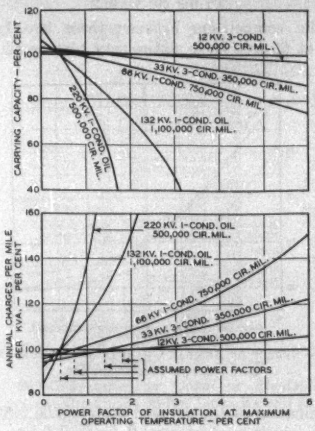


FIG. 18—EFFECT OF CHANGES IN INSULATION LOSSES IN CARRYING CAPACITIES AND ANNUAL CHARGES FOR TYPICAL CABLES

Assumed power factors are in accordance with recent test data as follows:
12 kv., 3-conductor, 1.8 per cent @ 78 deg. cent.
33 kv., 3-conductor, 1.4 per cent @ 71 deg. cent.
66 kv., 1-conductor, 0.7 per cent @ 60 deg. cent.
132 kv., 1-conductor, oil-filled, 0.4 per cent @ 70 deg. cent.
220 kv., 1-conductor, oil-filled, 0.4 per cent @ 70 deg. cent.

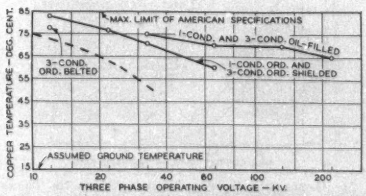


FIG. 19—MAXIMUM COPPER TEMPERATURES ASSUMED IN CALCULATIONS. DASHED LINE SHOWS LIMITS SET BY A. I. E. E. STANDARDS AS OF 1922. AT THAT TIME, THE HIGHEST VOLTAGE CABLE WAS 33 KV., 3-CONDUCTOR, BELTED

the charging current is sufficient to affect appreciably the power factor of the system to which they are connected. If the improvement in power factor obtained in this manner is important, then some credit should be given to the cost of the cables on this account. If the credit is figured on the same basis as the cost of obtaining the same improvement in power factor with

synchronous condensers, the reductions in the annual charges for high-voltage cables would range from 8 per cent for 66-kv., single-conductor cable with the ordinary type of insulation, to 34 per cent for 220-kv. oil-filled insulation, as shown in some further detail in Fig. 25.

Comparison of the Ordinary and the Oil-Filled Types of Cable

As the competition between these two types is closest at 66 kv., their status may best be shown by

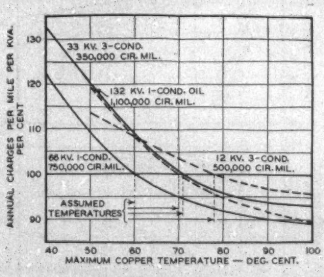


Fig. 20—Effect of Annual Charges of Changes in Maximum Copper Temperatures

an examination of the detailed costs of the two types of 66-kv., single-conductor cable for 65,000-kva. carrying capacity as given in Fig. 26. The comparative sizes of the two cables are indicated by Fig. 27. About

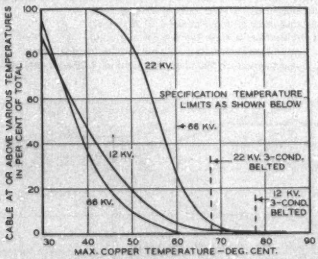


Fig. 21—Operating Copper Temperatures of Transmission Cables in Chicago

Results of survey at time of maximum temperature, which in Chicago occurs in summer-time

260 miles of such cable with the ordinary type of insulation are now in service in Chicago.

In these comparisons it should be noted that the conditions in Chicago are favorable to the oil-filled type of cable. The maximum difference in elevation of portions of the city in which high-tension cables are now installed, as determined from U. S. Geological Survey maps, is less than 30 feet, thus reducing the number of expensive stop joints and oil-reservoir manholes to a minimum.

For 66-kv. lines of 120,000 kva. capacity, contemplated for the future in Chicago, there is an advantage in the oil-filled type as its smaller diameter permits it

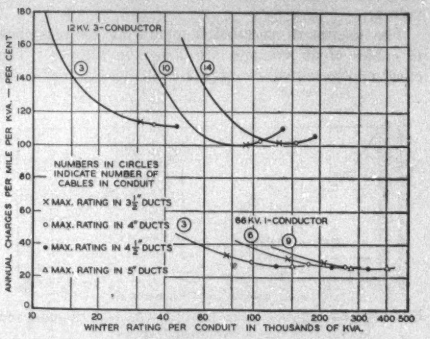


Fig. 22—Effect on Annual Charges of Number of Cables in Conduit.

Vertical scale same as Fig. 12

to be installed in vacant 4-inch ducts of existing conduits along different routes, while the use of the ordinary type would require the construction of special conduits with larger ducts and make it more expensive to secure different routes for the several lines.

For 65,000-kva. capacity at 66 kv., the cost of the joints and accessories of the oil-filled type is nearly four times that of the ordinary type. If the cost of the

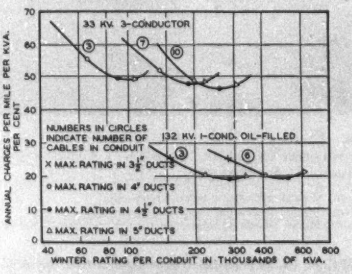


Fig. 23—Effect on Annual Charges of Number of Cables in Conduit.

Vertical scale same as Fig. 12

joints and accessories for the oil-filled type could be reduced by 50 per cent, it would be more than offset by a reduction of 15 per cent in the cost of the ordinary type of cable which might be realized by a reduction in the thickness of insulation. With free competition

such a reduction does not seem impossible in view of the fact that one make of such cable in Chicago with 0.75-inch insulation had no failures of insulation in a service record of 266 mile-years of operation up to March 1, 1931. It also appears that some reductions in the cost of the oil-filled cable might reasonably be

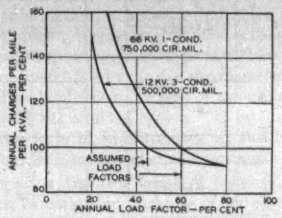


FIG. 24—EFFECT ON ANNUAL CHARGES OF CHANGES IN LOAD FACTORS

expected as the oil-filled type, shown in Fig. 27, uses 19 per cent less copper, 55 per cent less paper, and 32 per cent less lead than the ordinary type, with a consequent reduction of 34 per cent in the weight.

Interesting information in this connection has been

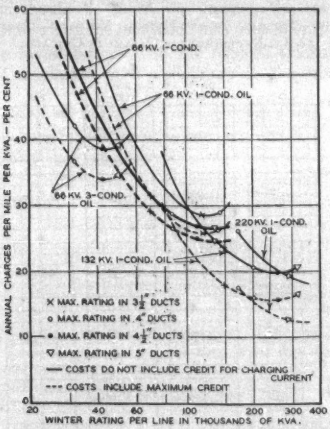


FIG. 25—REDUCTION OF ANNUAL CHARGES FOR HIGHER VOLTAGE CABLES WITHOUT TERMINALS BY ALLOWING MAXIMUM CREDIT FOR CHARGING CURRENT

Vertical scale same as Fig. 12

obtained from an experimental installation in 1927 at 108th Street (Figs. 28 and 29) of several 1,000-ft. lengths of cable with the ordinary type of insulation and one length of oil-filled cable as installed on the commercial line in 1927 (Fig. 1). These cables were operated at first with applied voltage only and without load; later daily load cycles were added and increased at intervals so as to raise the maximum temperature of the cables. In Fig. 30 are shown power-factor measurements of one typical sample of ordinary type of insulation operated to a maximum temperature of 60 deg. cent. and also the similar readings for the oil-filled cable operated to a maximum temperature of 75 deg. cent. From these curves it will be noted that there was a

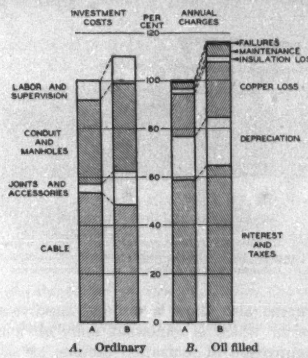


FIG. 26—COMPARISON OF COSTS OF SINGLE-CONDUCTOR ORDINARY AND OIL-FILLED 66-KV. LINES HAVING SAME CARRYING CAPACITY (65,000 KVA.) SHOWN IN FIG. 27

continuous change in the ordinary type of insulation, indicating instability at this voltage, while in the oil-filled type there was practically no change after 19 months' operation. This indication of stability of the oil-filled insulation, together with the perfect operating

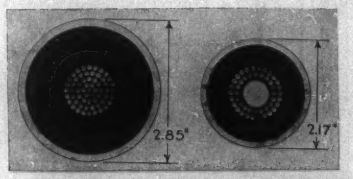


FIG. 27—66-KV. CABLES OF SAME CARRYING CAPACITY

(Left) 750,000-cir. mil, 0.75-in. ordinary insulation. Maximum temperature 60 deg. cent.

(Right) 580,000-cir. mil, 0.406-in. oil-filled insulation. Maximum temperature 70 deg. cent.

record of the oil-filled cable in New York and in Chicago indicated that probably 0.72-inch oil-filled insulation was more than required for 132-kv. operation. Accordingly additional samples of oil-filled cable having approximately 0.50-inch and 0.40-inch insulation have been installed at 108th Street and operated with gradually increasing loads, as above described, with the object of determining the maximum operating

stresses and maximum operating temperatures for oil-filled insulation.

A comparison of typical power-factor curves of cables with normal and reduced oil-filled insulation (Fig. 31) indicates that after daily load cycles for nearly eight months, oil-filled insulation 0.50-inch thick is about as stable at 132 kv. as the 0.72-inch insulation.

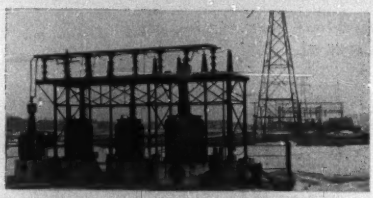


FIG. 28—108TH STREET FIELD LABORATORY FOR 1,000-FT. LENGTHS OF 132-KV. CABLES

Foreground: Circulating current transformers for heating cables
Center: Towers of 132-kv. line to which cables are connected
Right Rear: Laboratory for dielectric loss measurements

These curves also show a curious improvement in power factor which is somewhat more obvious by a direct comparison of ionization factors, as shown in Fig. 32. The diameter of a 132-kv. cable would be reduced almost 20 per cent (Fig. 33) with improvements that do not appear remote.

In considering the feasibility of securing cable of the ordinary type for operation at 132 kv., the most optimistic assumption is that the operating voltage can be increased in proportion to the increase in thickness of the insulation. The maximum thickness of impregnated paper insulation which has been suggested by cable manufacturers is 1.125 inches. The operation of this thickness of ordinary insulation at 132 kv. would then correspond to a thickness of insulation of 0.563 inch for 66 kv. This insulation would operate at approximately the same average stress as the 60-kv., three-conductor cables now in operation in Paris with 13 mm. (0.51-inch) insulation. When successful operating records of 66-kv. cable with 0.55-inch insulation are available, or when 132-kv. experimental cable with less than 1 inch of insulation will withstand tests such as shown in Fig. 29 without signs of instability, then

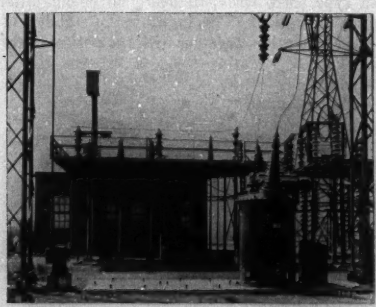


FIG. 29—LABORATORY FOR TESTING 132-KV. EXPERIMENTAL CABLES AND ARRANGEMENTS FOR CONNECTING CABLES TO OVERHEAD LINE

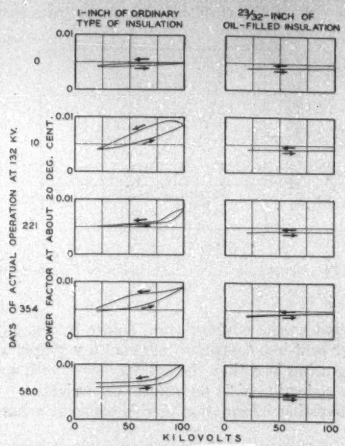


FIG. 30—TYPICAL POWER-FACTOR VOLTAGE CURVES FOR 132-KV. EXPERIMENTAL CABLES IN CHICAGO

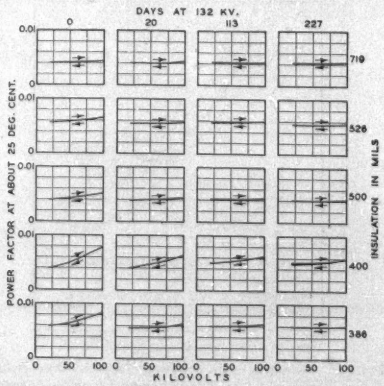


FIG. 31—TYPICAL POWER-FACTOR VOLTAGE CURVES FOR TWO MAKES OF 132-KV. EXPERIMENTAL OIL-FILLED CABLE IN CHICAGO

Cable is subjected to daily load cycles with maximum copper temperature of about 70 deg. cent.

it will be in order to consider the commercial use of the ordinary type of insulation at 132 kv. Calculations indicate that on this basis the ordinary type would compete economically with the oil-filled type at 132 kv.

The use of 1.125-inch oil-filled insulation at 220 kv. would be equivalent to 0.675 inch at 132 kv. on an average stress basis. Taking into consideration all available information, it appears that there would be less risk at present in operating oil-filled cable at 220 kv. than in operating the ordinary type at 132 kv.

Whether the oil-filled type will displace the ordinary type at 66 kv. and lower voltages or the ordinary type compete with the oil-filled type at 132 kv. depends on the relative rates of improvement in the two types of insulation with unrestricted competition.

In comparing the two types of insulation and in considering reductions in thickness of the oil-filled type of insulation, very careful consideration must be given to the effect of transient voltages due to switching or lightning. The best available information is that the short-time breakdown strength of a given thickness of the ordinary type of insulation is about as high as for the oil-filled type. More information on this subject is very much needed; in the meantime when considering reductions in thickness of oil-filled insulation for cables connected to an overhead line, all possible precautions should be taken for limiting the transient voltages to which the insulation may be subjected.

These investigations with the reduced thickness of oil-filled insulation were preliminary to the design of

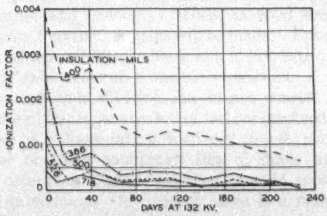


Fig. 32—Variation of Ionization Factor (Increase In Power Factors from 20 to 100 Kv.) with Age for Two Makes of Experimental 132-Kv. Cable in Chicago

Tests at about 25 deg. cent.

220-kv. oil-filled cable. An experimental installation of 220-kv. cable had been planned for the Crawford Avenue Station upon the completion of an overhead 220-kv. line into that station in the fall of 1932. In view of the advances in the art of testing cable as indicated in this and other papers presented at the same meeting, it now appears that the expense of this experimental installation of 220-kv. cable, similar to the installation of 132-kv. cable at 108th Street, is not warranted as all required information can be obtained by means of laboratory facilities now available.

Lead Sheaths

In considering future developments in high-tension cable the lead sheath should not be overlooked. In recent years the number of failures and replacements of high-voltage cable due to defective lead sheaths have been about equal to the number due to defective insulation. One foreign manufacturer claims to have eliminated cable sheath failures by the use of a lead press of a design differing from any in use in this country.[7]

Lead sheath appears to have an "elastic limit" at

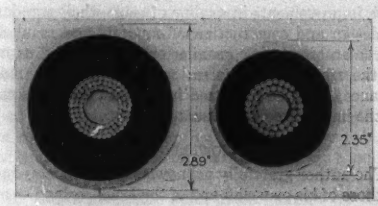


Fig. 33—Reduction in Size of 132-Kv. Cable, if the Insulation Thickness were Reduced from 0.72 to 0.50 In. and the Maximum Temperature Increased from 65 to 80 Deg. Cent.

(Left) 600,000-cir. mil commercial cable
(Right) 475,000-cir. mil experimental cable

about 175 lb. per square inch, that is, there is no stretching at such tensions when long continued. These pressures correspond to about 18 lb. internal pressure on a lead sheath 3 inches in inside diameter. It would reduce the cost of the accessories for the oil-filled type and permit increasing the maximum temperature of the ordinary type if an alloy could be discovered which would raise this elastic limit.

Looking Forward

When all the research investigations now in progress or contemplated by the utilities, universities, and manufacturers have been brought to a successful conclusion:

1. Cables of all types and voltages will be operated at a maximum temperature of 85 to 90 deg. cent. without fear of deterioration.
2. The expensive oil reservoirs and accessories on oil-filled cable will be eliminated due to:
   a. Increase in the elastic limit of the cable sheath.
   b. Reduction in the thermal coefficient of expansion of the oil.
   c. Making the insulation somewhat compressible under moderate pressures existing with the lead sheath.
3. An installation of oil-filled cable will be no more complicated than a present-day installation of ordinary type of cable.

4. Cable of subnormal quality will be eliminated by tests at the factory.

5. The initial cost per kva. of 66-kv. cable completely installed will be from 30 to 40 per cent below the present cost of the ordinary type.

6. Cable failures due to all causes will not exceed one per hundred miles per year.

If all of these results should be achieved within the next 15 years, it would be no more startling than a statement of the present day state of the art would have appeared 15 years ago.

In order that the utilities may realize the benefits of such improvements in quality of cable, their standards of design and construction of conduits and manholes, workmanship during installation and splicing, as well as the standards of operation, inspection and maintenance, must be increased in at least the same ratio as the improvement in insulation.

ACKNOWLEDGMENT

The author gratefully acknowledges his indebtedness to those of his own staff and of the Engineering Department who have contributed so generously to the preparation of this paper. In particular he is indebted to Herman Halperin who had general supervision of the assembly of the data and preparation of curves, and in addition contributed many helpful suggestions.

Acknowledgment is also made to F. O. Wollaston who, assisted by D. D. Nagley, made the calculations for the numerous curves; to G. M. Armbrust and J. B. Jackson of the Engineering Department for information regarding terminals and conduits; and to G. B. Shanklin of General Electric Company and D. M. Simmons of General Cable Corporation for assistance in connection with 220-kv. cable.

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Discussion

For discussion of this paper see page 1447.

Characteristics of Oil-Filled Cable

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Member, A. I. E. E.

and

F. H. BULLER*
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Synopsis.—This paper deals briefly with the general theory and some of the outstanding characteristics of oil-filled cable practise. Oil-filled cable has had a perfect operating record during the past four years and during this time many simplifications and economies have been introduced. Further gains in this direction are indicated. Today, this type of cable is well in line with ordinary paper-insulated cable from an economic standpoint.

The limiting features of oil demand and oil supply are explained and typical methods of oil feed with expansible reservoirs described. Space does not allow the inclusion of many additional things of interest such as a description and drawings of the complete line of cable and accessories that have been developed.

* * * * *

Introduction

RAPID progress is being made in oil-filled cable practise. The space limitations of this paper make it impossible to deal with the many phases of this activity, and the subject matter will be confined to a brief summary of the more important electrical and physical characteristics. The oil-filled cable is now past its infancy. The theory and principles are well understood and the technical soundness of these is fully substantiated. Much of this work has been described in the technical press. If the reader will refer to publications (1) to (9), inclusive, in the Bibliography, and include the contents of the present paper, he will have a fairly complete story of oil-filled cable practise as it exists today.

Large installations of 132-kv. cable, and others of lower voltage rating, have been placed in service during the past four years. All of these have operated with outstanding success, and each successive installation has represented an improvement in the way of simplification and economy. There is good reason for assuming that further gains will be made. Even today, high-voltage, oil-filled cable is well in line with ordinary "solid" cable from an economic standpoint.

The economic dividing line is not sharply defined because of the large number of factors involved. Such factors as voltage rating, load per circuit, single conductor or three conductor, size of ducts available, etc., determine whether oil-filled or solid cable is the more economical. At present there is a rather wide overlap. As further progress is made this overlap will be lowered and narrowed. Even if this expectation is not completely fulfilled, there still remains an overlap from 30,000 to 70,000 volts, grading off at each end, where choice is a matter of local conditions and requirements. Above 70,000 volts rating, oil-filled cable is already in the lead from every standpoint.

Advantages

Some of the advantages of oil-filled cable, as compared with solid cable are:

a. Voids are entirely eliminated, if the installation is properly designed, and insulation deterioration is not a factor to contend with.

b. Maximum safe operating temperature is higher and fluctuations in load and seasonal temperature are not important.

c. The thickness of insulation is about half that for solid cable, resulting in a smaller, lighter cable, or for equal over-all diameters, a cable of larger conductor size and greater current carrying capacity.

d. For a given size duct, much more power can be carried by oil-filled cable, for the reasons outlined.

e. The maximum safe voltage rating for single-conductor, solid cable is about 75 kv.; oil-filled cable has extended this to 132 kv., and 220 kv. appears possible when required.

f. The maximum safe voltage rating for three-conductor, solid cable is about 45 kv., because of physical limitations; oil-filled cable has extended this to 75 kv. and even higher voltage is possible.

g. Positive oil pressure is maintained and because of this, flaws and defects in the lead sheath merely result in slow leakage of oil, which can be located and repaired at any convenient time. Electrical service failure, caused by entrance of moisture and air, is usually the first warning of such defects in solid cable.

Complications

Some of the supposed complications and disadvantages of oil-filled cable have never actually existed. Others were the result of pioneer procedure, where simplicity and economy were either not fully developed or were sacrificed in the interests of surety, until the experimental stage could be passed. Alert engineering and knowledge of a few new principles are necessary. It is in this direction that most of the complications lie, but many engineers now recognize that this new knowledge could also be profitably employed in solid cable practise.

The fact remains, however, that there is a greater number of safeguards and detailed factors in the installation and maintenance of oil-filled cable, requiring careful engineering attention. These are being reduced and simplified as experience is gained. It is largely a matter of education and training of field

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forces in this new practise. Some special equipment is necessary for installation and maintenance. This consists of portable degasifier, vacuum pump and auxiliaries for short-field treatment of filling oil, joints, terminals, and other accessories. The special equipment is not expensive and represents a permanent investment, additional to the regular field equipment.

Those operating companies whose field forces now have the necessary experience and equipment handle oil-filled cable installations in a routine manner and without real difficulty.

Future Progress

The progress that has been made in the way of simplification and economy, will be dealt with by Mr. D. W. Roper.[5] Possible future progress will be briefly dealt with here.

The greatest progress seems to lie in the direction of reduction in insulation thickness. This is indicated by operating experience, laboratory study and field tests. Improved quality and more complete knowledge have recently allowed material reduction in standard thickness with resultant gain in economy. This forward step must be consolidated and confirmed by additional operating experience before the ultimate safe limit in insulation thickness can be determined. From what we know today, it would appear that because of absence of voids and ionization normal working voltage stress will not prove to be the limiting feature, but instead, ability to withstand transient voltages such as switching surges, arcing grounds, and lightning will determine ultimate safe thickness.

Guaranteed maximum temperature has already been increased from 60 to 65 deg. cent. and 70 to 75 deg. cent. Here again it is necessary to consolidate and confirm by operating experience. It would appear that insulation deterioration will not prove a factor in determining ultimate maximum temperature. This will, undoubtedly, be determined by the ability of the sheath to withstand expansional movement of the cable, and internal oil pressure.

Principles

The principles of successful oil-filled cable practise are:

a. A stable, non-freezing oil of superior electrical properties. This oil should have predetermined viscosity characteristics that give assurance against void formation in the cable cross section and excessive loss of pressure along the feed channels in service.

b. A high quality wood pulp paper, compactly and uniformly applied, and of predetermined density.

c. Removal from the cable during manufacture, and from all accessories and fittings during installation, of all impurities such as moisture and gas (both free and in solution).

d. Maintenance of positive pressure inside the entire length of cable, and other parts of the oil system, at all times.

e. Never allow this oil pressure to exceed safe working limits, as determined by the mechanical strength of cable sheath and accessories.

The remaining part of this paper will deal briefly with those electrical, thermal, and physical characteristics that have the most to do with these principles. Scant space can be given to the usual electrical test properties.

The dielectric loss of oil-filled cable is quite low, 60-cycle power factor being in the order of 0.5 per cent at 80 deg. cent. There have been no indications of an increase in loss with time in service. Measurements seem to be of value only in checking uniformity in the factory and in determining reduction in current-carrying capacity at the higher voltage ratings. Dielectric loss causes a reduction of approximately 8 per cent in the current-carrying capacity of 132-kv. cable.

Many engineers feel that routine dielectric strength tests on samples could be dispensed with entirely. They are expensive and difficult to make and disclose very little of value. The breakdown strength of oil-filled cable on the standard 15-minute test is no better than that of solid cable having the same thickness of insulation and about half the voltage rating. Even on the standard 6-hour endurance test the breakdown strength of oil-filled cable is not much better than that of solid cable. It is only when this voltage application is extended into months and years that the real superiority of oil-filled cable is clearly shown.

Determinant Factors and Characteristics

Power cable is subjected to continually changing temperatures in service consisting of daily load cycles superimposed upon seasonal cycles. Expansible oil reservoirs are connected to oil-filled cable at predetermined intervals to maintain pressure and take care of the volumetric changes caused by these fluctuations in temperature.

The contour of the cable run plays a very prominent part in determining the best location of reservoirs and stop joints, and the characteristics of the reservoirs necessary for maintaining oil pressure within prescribed limits. On long or steep hills it is necessary to insert oil stop joints at intervals to break up the hydrostatic head pressure. On level ground the location of stop joints (if any) is determined by maximum pressure drop along the feed channels.

In laying out the most effective and economical design of installation all of the principles and factors previously outlined must be taken into consideration. The primary characteristics that practically control the layout are:

a. Rate of expulsion and absorption of oil by the cable.

b. Oil-pressure drop along the feed channels.

c. Total expulsion or absorption of oil between

5. For references see Bibliography.

the extreme limits of no-load winter temperature and full-load summer temperature.

d. Characteristics of available reservoirs.

Rate of Oil Expulsion and Absorption. When load is suddenly applied or dropped, oil is expelled or absorbed by the cable, the rate being governed by the rate of heating or cooling. Everything else being equal (and if no voids are formed) the expulsion and absorption curves are exactly alike. The rate increases to a maximum in a relatively short time and decreases to zero in a much greater time. Emanueli[1] dealt only with the maximum rate, this being the most important. In a later paper[5] oil expulsion is dealt with in greater detail.

Curves (*a*) and (*b*) in Fig. 5 of this paper[5] are representative of the hollow-core, 132-kv. cable installed in 1927 by our company. It is tentatively suggested on page 350 that the shaded area between curves (*a*) and (*b*) represented a brief, transient void formation. We have not since attempted to duplicate this test. Indirectly and comparatively, however, the evidence since obtained, has been confirmatory in every way. The theoretically calculated oil demand curve (*a*) has been proven correct and there is no reason for questioning the accuracy of the measured supply curve (*b*). In the light of later experience we believe that the void formation shown in Fig. 5 was caused by the following conditions:

a. The viscosity of the oil used at that time was higher than that of the improved oil used since.

b. The oil had been drained from the cable core after the initial impregnation treatment and the cable had stood for several weeks with this core filled with gas under pressure. Before test the cable had been re-impregnated with oil, the gas being removed by a relatively brief application of vacuum.

Subsequent tests and theoretical results indicate that the viscosity of this old oil could not have played anything more than a secondary part in this void formation. The ambient room temperature was 20 deg. cent. and similar tests made since with oil of both lesser and greater viscosity indicate that the ambient temperature would have to be much lower before the viscosity of this old oil would cause voids, *provided the dissolved gas content is of low value.*

The void formation indicated by Fig. 5 in this previous paper is insignificant from a practical standpoint. This is substantiated by the fact that 132-kv. cable of the type dealt with has operated successfully, and without signs of deterioration, for about four years. If progress is to be made, however, in the way of improved quality and reduction in insulation thickness these factors, no matter how insignificant they appear today, must be fully accounted for. We believe that the void formation mentioned was primarily due to dissolved gas content in the inner layers of insulation and that this condition was caused by the previous gas filling of the core. Measurements made since indicate that the dissolved carbon dioxide content of the oil in this cable, after uniform diffusion, was in the order of 8 to 10 per cent by volume at atmospheric pressure, room temperature. This is much below the saturation point of approximately 100 per cent.

Knowledge of the laws of gas absorption and diffusion in cable (and oil) as related to time, pressure, and temperature is still very meager and difficult to obtain. We have a good start in this direction, but there is still a great deal to be done in the way of confirmation and additional research.* It is known that the amount of dissolved gas in oil is a direct function of pressure but increases more slowly with temperature, actually decreasing for some gases. The rate of diffusion is very slow in oil and still slower in cable. Diffusion increases with temperature. Saturation content varies widely with different gases, being low for hydrogen and nitrogen, about average for oxygen and high for carbon dioxide. The type of oil and, more particu-

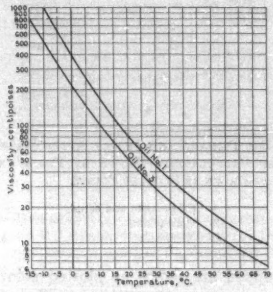

Fig. 1—Viscosity of Oil

larly, its condition affect solubility. When more complete and detailed work is confirmed the results will undoubtedly be published.

However, lack of accurate knowledge in this direction will not hold up progress in oil-filled cable development, since the principle of reducing gas content to a minimum, initially, and preventing further exposure is already firmly established. Real progress is in this direction.

This older cable has been referred to for the purpose of clearly bringing out the improvement that has since been accomplished. As far as can be determined, there is complete absence of void formation in the newer cable. These improvements involve an oil of more desirable characteristics, and methods of shipping and installing the cable while completely filled with oil. Exposure to gas absorption is reduced to a minimum and the dissolved gas content in the finished installation is only a fraction of that formerly experienced.

The viscosity of the old oil (No. 1) is shown in comparison with that of the new oil (No. 3) in Fig. 1. The freezing point of oil No. 3 is approximately − 40 deg. cent. and this safely allows exposure to severe outdoor winter temperature. At the same time, its viscosity is sufficiently high to avoid excessive loss of oil in case of leakage.

Fig. 2 shows typical oil supply and oil demand data for two sizes of the new, hollow-core, single-conductor cable. Space does not allow the inclusion of similar data for other sizes and types of single and three-conductor cable, but Fig. 2 is entirely representative. The measured oil supply and temperature after steady

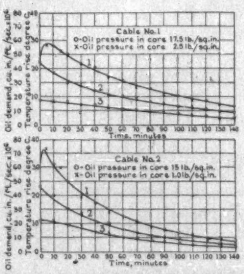

FIG. 2—OIL DEMAND AND SUPPLY AFTER DROPPING LOAD. THEORETICAL CALCULATIONS SHOWN BY FULL LINE CURVES. TEST RESULTS SHOWN BY PLOTTED POINTS

1. Oil demand and supply
2. Copper rise
3. Sheath rise

load is dropped are shown as plotted points. These measurements were made with two different oil pressures maintained in the hollow-core, as shown. Measurements of oil expulsion when the same load is applied were also taken. The two sets of measurements coincided so closely that the expulsion data will not be given. If voids had been formed when load was dropped the expulsion measurements should be greater than the supply during the first few minutes. They were actually either equal to, or slightly less. Also, if voids had been formed the supply measurements with lowest oil pressure should be less than the high-pressure readings. Fig. 2 would not indicate this to be so. The laboratory tests were conducted by Messrs. J. A. Scott and H. W. Bouseman, to whom grateful acknowledgment is made.

The theoretically calculated curves in Fig. 2 furnish a final check of this whole matter. It will be noted that there is almost exact agreement between theory and measurement. Other and similar laboratory measurements have fully confirmed the accuracy of the theoretical work. The calculation of temperature and oil demand is covered by formulas (1) to (7) in the Appendix.

Specifications and measured constants for the two sizes of cable considered in Fig. 2 follow:

Item	Cable No. 1	Cable No. 2
Clear inside diameter of core	0.500	0.500
Inside diameter of stranded conductor	0.600	0.600
Outside diameter of conductor	1.090	1.010
Copper cross section of conductor	600,000 cir. mils	475,000 cir. mils
Thickness of paper insulation	0.720	0.500
Thickness of lead sheath	0.156	0.141
Over-all diameter	2.842	2.300
Ratio oil to copper in core and conductor	0.99	1.15
Ratio oil to fiber in insulation	1.00	1.00
Thermal resistivity of insulation (C.G.S. units)	600	545
Emissivity of sheath (C.G.S. units)	1200	1010*
Thermal capacity copper per ft., (watt-second units)	328	260
Thermal capacity of oil in conductor per ft.	126	116
Specific thermal capacity of insulation (C.G.S. units)	0.360	0.360
Thermal capacity of sheath per ft.	382	275
Coefficient expansion copper	0.00005	0.00005
Coefficient expansion oil	0.00074	0.00074
Coefficient expansion fiber	0.00011	0.00011
Coefficient expansion sheath	0.000085	0.000085

*Note: Tarnished sheath

Fig. 3 shows the calculated temperature distribution of cable No. 1 from conductor to sheath when load is dropped. It will be noted that during the first 10 minutes when voids (if any) have a tendency to form there is little change in temperature of sheath and outer layers of insulation, but a relatively large change in temperature of conductor and inner layers of insulation. Consequently, when voids do form, they must first occur in, or near, this inner zone. It is for this reason that

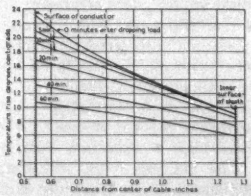

FIG. 3—TEMPERATURE DISTRIBUTION ACROSS THE INSULATION OF CABLE NO. 1 AFTER LOAD IS DROPPED. ISOLATED CABLE IN STILL AIR

viscosity of oil does not play such a prominent part as dissolved gas content in Fig. 5 of the previous paper.[2]

Incidentally, Fig. 3 would indicate that an oil feed channel in the conductor is more effective than channels directly under the sheath. This is true only where there is danger of void formation. When this is eliminated, by methods already discussed, there does not appear to be any difference in the two methods of feed, judging from both theory and test.

Oil-Pressure Drop Along Feed Channels. Oil-pressure drop along feed channels and piping during periods of oil flow can be calculated with acceptable accuracy

only where the inner surface is smooth and cylindrical and the rate of flow is less than turbulent. (See formula (10) in Appendix.) The surface of the hollow-core in cable No. 1 is not smooth, but is ridged, because of the spiraled supporting member. For this reason it is necessary to measure friction factor, b. For cables Nos. 1 and 2

$$b = \frac{0.00283\ Z}{d_1^4},$$

where d_1 is the clear inside diameter of core (0.5 in.). We have found that formula (10) holds with reasonable approximation for the type of core used in cable No. 1 if it is assumed that $d = 1.06\ d_1$. There are similar approximate methods for estimating the pressure drop in channel-type single-conductor and three-conductor cable, but here, it is even more necessary to confirm by actual tests.

Oil supply, (a), is distributed uniformly to the insulation by the feed channels. When factor (a) and the friction factor along the channels, (b) are known, the pressure drop can be determined by formula (11) in the Appendix. This formula was first developed by Emanueli,[1] who considered only the maximum value of (a). As will be shown, the pressure drop P_e is rarely, if ever, greatest when oil supply, (a), is at maximum value. Increase in oil viscosity accounts for this.

Total Oil Required. The total oil required is a direct function of cable length and the two extreme limits of temperature met with in service. These limits (for underground cable) are usually assumed as 0 deg. cent., no-load winter temperature, and 70 deg. or 75 deg. cent., full-load summer temperature. The method of estimating total oil is described in the Appendix. Usually, 40 per cent over and above the theoretical requirement is added to reservoir capacity as an emergency surplus and to safeguard against wrong adjustment of reservoir content and other possible discrepancies. The total oil requirement (theoretical plus 40 per cent) for typical single- and three-conductor cable varies from 0.5 gal. to 2.0 gal. per 100 ft. of cable, as determined by cable cross section, etc.

Reservoir Characteristics. The reader is referred to T. C. Aitchison's article[3] for details of design and performance. A previous article[4] also gives some general information regarding the three different types of reservoirs available and the methods of use. As indicated in these articles, the type CC, gravity-feed reservoir and the type DC, balanced-pressure reservoir have proved the most useful and desirable for oil-filled cable work. The present paper will, consequently, deal exclusively with these two types.

Types CC and DC oil reservoirs are identical in design and construction, with the exception that the outer rigid enclosing tank of type CC is open to the atmosphere through a breather, maintaining practically constant atmospheric pressure on the oil. The type DC reservoir, on the other hand, has a sealed gas-tight outer tank maintained under variable gas pressure. Both types follow the established principle of segregating and sealing off the cable oil system against exposure to gas or other impurities. The inner construction consists of a stack of disk cells, the walls of which are thin, corrugated, flexible metal diaphrams. The design is such that these diaphrams are supported in their fully-extended and fully-collapsed positions. This results in exceptional sturdiness and long life. The cells are connected in parallel by a manifold and are filled with degasified oil.

Type CC is known as an "atmospheric-pressure, gravity-feed reservoir," since it maintains constant

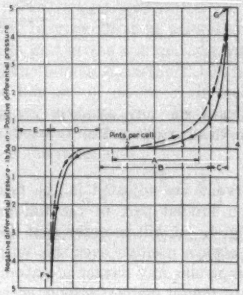

FIG. 4—DIFFERENTIAL OR DIAPHRAM PRESSURE CURVE OF TYPES CC AND DC RESERVOIRS, REPRESENTING DIFFERENCE OF GAS PRESSURE OUTSIDE AND OIL PRESSURE (AT CENTER LINE) INSIDE CELLS

A Operating range from minimum cable temperature to maximum cable temperature
B Full-rated capacity range of reservoir
C Semi-idle range beyond rated capacity
D Semi-idle range below rated capacity
E Idle oil remaining in cells when totally collapsed
F Totally collapsed position of cells
G Totally expanded position of cells

atmospheric pressure and must be elevated above its surroundings for feed purposes. Type DC is used in those locations where elevation is undesirable or impractical and has proved very useful in eliminating expensive towers, sometimes necessary when type CC is used. It is known as a "balanced-pressure reservoir" because both the gas and oil volumes and pressures can be readily adjusted or balanced in such a way that reservoir units at different elevations can be made to work together in harmony and synchronism, each doing its fair share of the work.

The differential pressure or "back pressure" of the CC and DC reservoirs is shown in Fig. 4. This represents the difference in the oil pressure inside the cells (at center line of tank) and the gas pressure in the outer tank. When the cells are put through an

extreme cycle of operation from completely-collapsed position under vacuum to completely-expanded position under six or more pounds positive differential pressure, and returned to their original collapsed position, the curves are in the form of a small hysteresis loop, as shown. In normal service, the cells are subjected to a much smaller total working range and this loop entirely disappears.

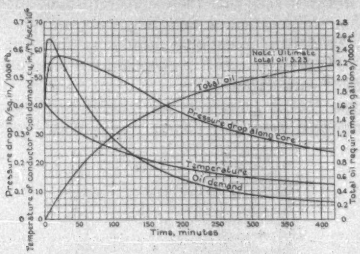

FIG. 5—CABLE NO. 1 DROPPING FULL LOAD IN WINTER WITH 0 DEG. CENT. AMBIENT EARTH, SIX CABLES IN DUCT BANK

The reservoirs are adjusted in the field to always work over a limited part of the full-line curve, as shown in Fig. 4. It will be noted that the differential pressure is always positive over this normal working range. This avoids any danger of entrance of gas or moisture into the oil system through accidental leaks.

The *C C* cell stack fits closely in the outer tank and

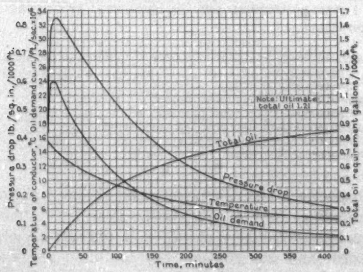

FIG. 6—CABLE NO. 1 DROPPING HALF LOAD FROM SIX CABLES IN DUCT BANK IN WINTER WITH 0 DEG. CENT. AMBIENT EARTH

there is no surplus length to this tank. In the case of the *D C* units, however, there is extra length to the tank to give additional gas volume. This is necessary for maintaining gas pressure within desired limits. The gas volume can be readily adjusted either by adding an auxiliary gas tank for greater volume or introduction of waste oil for lesser volume. The pressure-volume characteristics of *D C* reservoirs can be readily calculated from the known laws of gases. See formula (9). This formula, of course, refers to gas pressure. The oil pressure can be obtained by adding the diaphram pressure given in Fig. 4.

OTHER FACTORS AND CHARACTERISTICS

Thermal and Oil Feed Characteristics. Before an oil-filled installation can be properly laid out it is necessary to know the steady and transient thermal characteristics of the cable and surrounding duct structure. Characteristic curves for cable No. 1 are given in Figs. 5 and 6. It was necessary in calculating these curves to assume a typical cooling curve for the duct bank, as described immediately following formula (5) in the Appendix. If the characteristics of the duct bank are such that the exponential is greater or less than the assumed value 0.0000217 the curves in Figs. 5 and 6 do not hold. However, since we are largely concerned with the maximum values of oil supply and pressure drop, occurring in relatively brief periods of time, possible errors are not of great importance. In Figs. 5 and 6 it is assumed that load is dropped simultaneously from all of the cables in the duct bank. Also, short-circuited sheath-operation with 6½-in.

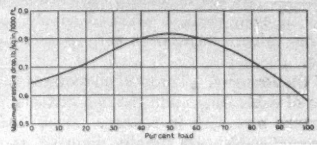

FIG. 7—CABLE NO. 1 SHOWING MAXIMUM PRESSURE DROP ALONG CORE IN WINTER FOR DIFFERENT LOADS

flat spacing between six equally loaded cables and 0 deg. cent. earth ambient temperature are assumed as a "worst" condition.

It will be noted that although the maximum rate of oil supply is less when half load is dropped (Fig. 6) the maximum pressure drop is greater than when full load is dropped (Fig. 5). Higher oil viscosity accounts for this. Fig. 7 shows the relation between percentage load and maximum pressure drop along the hollow core. Zero per cent load refers to dielectric loss only. It will be noted that 50 per cent load represents the worst or limiting condition. This limiting condition varies from 25 to 75 per cent of full load in different cases, depending upon cable size, minimum ambient temperature, and other conditions.

Feeder-Pressure Section. All of the necessary factors for cable No. 1 have been described and we are now in a position to lay out an actual installation. The contour of the duct run, spacing between manholes, dimensions of manholes and other practical considerations will determine the best and most economical layout. Pressure drop along the core, minimum oil pressure at high point, maximum pressure at low

point and other factors will determine the location of stop joints and the type of oil feed for each section between stop joints. Space does not allow us to deal with an entire installation in detail. Instead, two typical sections will be selected for discussion.

First, assume a duct contour as shown in Fig. 8, with stop joints at positions 0 ft. and 3,000 ft. The sheath of cable No. 1 has no reinforcement and steady oil pressure at the lowest point (1,500 ft.) must not exceed 15 lb. per sq. in. Also it is desirable to avoid either a steady or transient oil pressure lower than atmospheric at any point in the oil feed system, consisting of reservoir cells, piping, stop joints and cable core. From the previous work it is known that the total rated oil capacity of the reservoir units for this length of cable is 25 gallons.

Balanced-pressure reservoir feed is obviously out of the question, since the hydrostatic pressure from

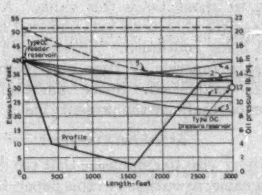

FIG. 8—TYPICAL FEEDER—PRESSURE SECTION CABLE NO. 1

Feeder reservoir rated 25 gal.
Pressure reservoir rated 3 gal. oil, 15 gal. gas
Curve 1—Pressure curve while full load is dropped in winter-time, feeding reservoir only
Curve 2—Same condition as Curve 1 with feeding-pressure reservoir combination
Curve 3—Pressure curve when half load is dropped in winter-time, feeding reservoir only
Curve 4—Same as Curve 3, with feeding-pressure reservoir combination
Curve 5—Same as Curve 3, shows height to which feeding reservoir must be raised to avoid vacuum at the far end under all conditions

position 0 to 1,500 ft. is already the full allowable 15 lb. If gravity-feed units (type $C\,C$) are placed at position 0 ft. and slightly above the stop joint, the minimum pressure in the cable core when full and half-load are dropped in the winter time is shown by curves (1) and (3). Both of these curves intersect the cable contour, and the oil system beyond these points of intersection would be at a pressure below atmospheric for several minutes. If the gravity reservoirs are raised to a sufficient height, as shown by curve (5), this condition could be avoided but this solution is not desirable since, instead of mounting the reservoir units directly in the manhole, it would mean an expensive reservoir housing above ground level. Also, safe working pressure of 15 lb. would be exceeded at lowest point of cable contour.

The real solution is to leave the feeder units at stop-joint level and connect a small pressure reservoir (type $D\,C$) at low end of section. This pressure reservoir functions only during transient periods of temperature change and during these periods assists in maintaining cable core pressure within prescribed limits. Curves (2) and (4) with pressure reservoir connected show the improvement in pressure characteristics accomplished by the addition of this small unit. It will be noted that the limiting condition in this case, curve (2), occurs when full load is dropped, rather than half load. The reason for this is that dropping full load drains the oil from the small pressure reservoir more rapidly, with consequent reduction in its normal level of pressure, as shown.

A pressure reservoir required to meet the conditions in Fig. 8 would have a rated oil capacity of 3.0 gal. and a gas capacity, V_T, of 15 gal. The calculation of curves (2) and (4) involves such complicated differential equations that a step-by-step solution is resorted to.

It is known that, at the first instant, the pressure reservoir feeds half of the section. Assuming that it feeds half of the section for a short, but finite, length of time, the amount of oil which it feeds out and average loss of pressure can be calculated. The average distance fed during the time interval is then obtained from the formula:

$$x = \frac{l}{2}\left(1 - \frac{h - p}{a\,b\,l^2}\right)$$

b and a being respectively the average friction and oil demand during the period under consideration: p is the average pressure in the pressure reservoir. This value of x is less than half the length; accordingly, the output and pressure of the reservoir are corrected to take care of this change and x is re-calculated. Taking the value of x at the end of the period as a starting point, the above process is repeated; and it is continued until x becomes equal to zero. Then the pressure drop from the feeding reservoir over a distance $l - x$ is computed for each time interval.

Balanced-Pressure Section. On level ground, or where the profile of a given section is such that the feeder-pressure combination just described cannot be used without resorting to expensive mounting of reservoirs above ground level, the so-called balanced-pressure feed proves very convenient and economical. Fig. 9 shows a 2,500-ft. section of cable line having a contour that can be most conveniently taken care of by balanced-pressure feed. In all other respects the assumptions are the same as those outlined for Fig. 8. The total rated oil capacity is found to be 20 gal. This should be divided equally, one 10 gal., type $D\,C$ unit being placed at each end, the relative elevations of these reservoirs being as shown in Fig. 9.

The gas capacity of these units and their initial volumetric and pressure settings are so selected that each reservoir takes care of exactly half of the cable section. Also, the pressure at the highest point in the oil system must never drop below 1 lb. above atmospheric, and the pressure at the lowest point must

never exceed 15 lb. Reservoir characteristics that meet these conditions can be determined from formulas (8) and (9) in the Appendix. The calculations call for a gas volume, V_T, of 54 gal. for the lower reservoir and 41.6 gal. for the upper reservoir. The latter can be obtained either by using a shorter tank or by the more practical method of pouring 12.4 gal. of waste oil into the same size tank.

The pressure along the cable core under various limiting conditions is shown in Fig. 9. It will be noted that dropping either half-load or merely dielectric loss in the winter and full-load pressure in the summer represent the extreme limiting conditions.

It should be pointed out that for the type of oil feed considered in Fig. 9 exact balance of the reservoir units is not necessary within reasonable limits, and providing the gas content V_T is greater, but not less, than that called for by exact balance. For instance, instead of injecting 12.4 gal. of waste oil into the upper tank in Fig. 9 the original 54 gal. of gas capacity

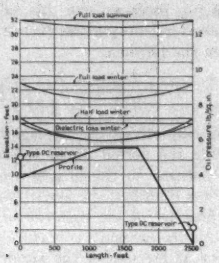

FIG. 9—TYPICAL BALANCED PRESSURE SECTION CABLE NO. 1

Reservoir Data: Oil volume 10 gal. both reservoirs
Gas volume (0 deg. cent. P_{min}) upper reservoir 41.6 gal.
lower reservoir 54.0 gal.

could be retained. The total pressure range would thereby be reduced but the behavior of pressure drop along the core would be complicated and would have to be determined by the step method already discussed.

Other Systems of Oil Feed. Some additional details regarding balanced-pressure feed, as related to three-conductor and channel-type single conductor cable will be found in a previous article.[3] Space does not allow us to deal with other systems of oil feed. Some of these systems are briefly outlined in a previous article.[4] Additional details will be included in an article[6] soon to be published.

There are many factors that determine the best and most economical system of oil feed and general layout of oil-filled cable installations. The best system might vary anywhere from a stop joint and reservoir in every manhole, where very rugged contour is encountered, to complete exclusion of stop joints on level ground. Generally speaking, the use of stop joints should be reduced to a minimum. Stop joints should not be used merely for segregation, since the advantages gained are nullified by the additional expense and the limited amount of reservoir oil available for emergency use in case of accidental leakage. Segregation, where actually required, can be obtained by safer and more economical methods.

Strength of Lead Sheath. The integrity of the sheath is a very important feature of oil-filled cable practise. We have given a great deal of time and thought to this problem. Commercially pure lead and alloys of lead in the form of ordinary sheathing with and without reinforcement and special double sheathing with reinforcement have been investigated and used in service. With ordinary sheathing without reinforcement it has been found that safe continuous oil pressure is limited to approximately 15 lb. per sq. in., and with reinforced sheathing, to about twice this value. A very comprehensive series of tests on lead and lead alloy sheathing is still being continued. Elongation and bursting strength as a function of time and temperature are fairly well established. Space limitations prevent publication of results at the present time.

ACKNOWLEDGMENT

The authors wish to emphasize that the development of oil-filled cable, as outlined in this paper, has involved extensive cooperation of a number of individuals. We wish that each could be given full and merited credit, but this is impossible. Outstanding contributions of thought and effort have been made by Messrs. W. C. Hayman, G. M. J. Mackay, R. H. Park, J. A. Scott, C. A. Piercy, E. D. Eby, E. L. Crandall, T. C. Aitchison, H. W. Bouseman, L. L. Phillips, H. P. Broderson and V. A. Sheals.

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Appendix

In the present appendix, it is proposed to give some of the formulas used in the calculation of oil-filled cable characteristics. Space does not permit the development of these formulas; the reader is referred to an article[6] if such information is required.

Before oil demand calculations can be made, it is necessary to compute the thermal transient:

$$T = \sum A_n F(u_n x)\, \epsilon^{-u_n^2 t/p'q} + T_d\, \epsilon^{-\alpha t} + T_o \tag{1}$$

In the foregoing equation

$$A_n = \frac{\dfrac{p'\,\Delta_1\,\Delta_2}{\pi}\left[\dfrac{W_c}{u_n}\,\dfrac{\Delta_1}{r} - W_s\,\Delta_2\right]}{[\{M_2\,(u_n R)^2 - N\}^2 + (1 + 2\,M_2)\,(u_n R)^2]\,\Delta_2^2 + [2\,M_1 - 1 - M_1^2\,(u_n r)^2]\,\Delta_1^2} \tag{2}$$

$$F(u_n x) = J_o(u_n x) - K_n\, Y_o(u_n x) \tag{3}$$

$J_o(u_n x)$ and $Y_o(u_n x)$ being the Bessel functions of the first and second kinds, and of zero order; see reference (10).

K_n and u_n are given by the following equation:

$$K_n = \frac{M_1\, u_n\, r\, J_o(u_n r) - J_1(u_n r)}{M_1\, u_n\, r\, Y_o(u_n r) - Y_1(u_n r)}$$

$$= \frac{[M_2\,(u_n R)^2 - N]\, J_o(u_n R) + u_n R\, J_1(u_n R)}{[M_2\,(u_n R)^2 - N]\, Y_o(u_n R) + u_n R\, Y_1(u_n R)} \tag{4}$$

The denominators of the two fractions in equation (4) are Δ_2 and Δ_1 respectively (see symbols). The summation in equation (1) is taken over the roots of equation (4).

Also,*

$$M_1 = \frac{Q_1}{2\,\pi\, q\, r^2};\quad M_2 = \frac{Q_2}{2\,\pi\, q\, R^2};\quad N = \frac{p'}{2\,\pi\, R_{the}} \tag{5}$$

α is a quantity dependent on the construction of duct bank, number of loaded cables in it, etc. For average conditions, it may be taken as 0.0000217, if t is measured in seconds.

Equation (1) gives the temperature T at any distance x from the center of the cable at any time t. Equations (1) to (5) are also useful in calculating other thermal effects, such as short-circuit heating and overload duty cycles.

The oil demand of a cable is equal to the rate of contraction of the component parts of the cable when load is dropped. This may be determined by using the following equation:

$$a = 12\,\alpha\, T_d(\eta_i V_i + \eta_o V_{oc} + \eta_c V_c + \eta_o V_{os} - \eta_1 V_s)\, \epsilon^{-\alpha t}$$
$$+ \sum \epsilon^{-u_n^2 t/p'q}\, \frac{12\, u_n^2\, A_n}{p'\, q}\left[\frac{2\,\pi\,\eta_i\, R}{u_n}\left\{\frac{M_2\,(u_n R)^2 - N}{u_n R\,\Delta_1} - \frac{u_n\, r\, M_1}{u_n R\,\Delta_2}\right\} + \frac{\eta_o V_{oc} + \eta_c V_c}{u_n\, r\,\Delta_2} + \frac{\eta_1 V_s - \eta_o V_{os}}{\Delta_1}\right] \tag{6}$$

The total oil requirement of the cable over the temperature range $T_c - T_o$ may be obtained by integrating this equation between the limits $t = O$ to $t = \infty$. It may also be obtained more readily from the following equation:

$$D = 12\left[\begin{array}{l}(\eta_c V_c + \eta_o V_{oc} + \eta_i V_i)\,(T_c - T_o) \\ \qquad + (\eta_o V_{os} - \eta_1 V_s)\,(T_s - T_o) \\ + \dfrac{p'\, W_c\, \eta_i}{2\,\pi}\{(V_i/2) - V_s \log_e (R/r)\}\end{array}\right] \tag{7}$$

If the total oil requirement over the entire yearly temperature range is wanted, it may be obtained from equation (7) by writing T_o equal to the minimum winter earth temperature, T_c equal to the maximum allowable copper temperature, and T_s to the temperature of sheath when copper temperature is T_c.

The reservoir requirement in gallons per 1,000 ft. is equal to $\dfrac{1.4\, l\, D}{231}$, assuming that a 40 per cent surplus is allowed. To this should be added the oil contraction of joints, terminals, reservoirs and piping. The total volume of oil thus computed is called $2\,V_{max}$.

If a pressure reservoir is to be used, the gas volume V_T at temperature T_{min} and pressure P_{min} must also be calculated:

Lower Reservoir

$$P_{min}\, V_{T1} = \frac{P_{max}\,(V_{T1} - V_{max})\,(T_{min} + 273)}{273 + T_{max}} \tag{8}$$

Upper Reservoir

$$(P' - h)(V_{T2} - V') = \frac{(P'' - h)(V_{T2} - V'')(T_{min} + 273)}{273 + T_{max}} \tag{9}$$

Intermediate reservoirs, where such are used, may be calculated as "upper" reservoirs from formula (9), using the proper value of h, provided the reservoir system is so balanced that each reservoir feeds its proportionate length of cable at all times. This will be true of a balanced system, but not of an unbalanced system.

The pressure drop in a pipe with a smooth cylindrical surface may be calculated from the equation:

$$p_1 = 2\,\frac{q'\, f\, l_1}{\pi\, g\, d^4} \tag{10}$$

The pressure drop in the core of a hollow core cable at any distance x_o from the feeding point may be calculated from the equation.

$$p_o = 2\,a\,b\,(l\, x_o - x_o^2/2) \tag{11}$$

*This notation is the same as used by K. W. Miller in his partial solution of this problem. See discussion of paper, Bibliography reference (1).

where,

$$b = \frac{0.8 f}{\pi g d_1^4} \qquad (12)$$

LIST OF SYMBOLS

a = oil demand of cable, cu. in. per ft. per sec.
b = friction factor for hollow core cable.
d = inside diameter of smooth cylindrical pipe, inches.
d_1 = inside diameter of spiral in cable core, inches.
D = total oil requirement of cable, cu. in. per ft.
f = friction coefficient in smooth pipe = 0.00358 Z.
g = gravitational constant = 32.2 ft. per sec. per sec.
h = difference in elevation between upper and lower reservoirs, expressed in lb. per sq. in.
l = length of cable section, feet.
l_1 = length of pipe, feet.
p' = thermal resistance of cable insulation, deg. cent. per watt per in. thickness per 12 sq. in.
p_o = pressure drop in core of cable, lb. per sq. in.
p_1 = pressure drop in a smooth cylindrical pipe, lb. per sq. in.
P' = gas pressure in the lower reservoir at no load, lb. per sq. in. absolute.
P'' = gas pressure in lower reservoir at full load, lb. per sq. in. absolute.
P_{min} = minimum permissible pressure of gas in lower reservoir, lb. per sq. in. absolute.
P_{max} = maximum permissible pressure of gas in lower reservoir, lb. per sq. in. absolute.
Q_1 = thermal capacity of conductor, including oil in strands and core, watt-sec. per ft. per deg. cent.
Q_2 = thermal capacity of sheath, armor, and oil in channel space under sheath, if any, watt sec. per ft. per deg. cent.
q' = rate of flow of oil through pipe, cu. in. per sec.
q = thermal capacity of insulation, watt-seconds per deg. cent. per 12 cu. in.
r = outside radius of cable conductor, inches.
R = inside radius of cable sheath, inches.
R_{ths} = thermal resistance of sheath surface, in thermal ohms per ft.
t = time, in seconds.
T = temperature, rise above duct air in deg. cent. of a point at distance x from center of cable, at time t.
T_c = temperature rise above duct air of cable conductor at beginning of cooling, deg. cent.
T_s = temperature rise above duct air of sheath at beginning of cooling, deg. cent.
T_d = temperature rise above earth ambient of duct air at beginning of cooling, deg. cent.
T_o = ambient temperature, (of earth, air, or water, depending on operating condition).
T_{max} = maximum temperature of gas in reservoir, deg. cent.
T_{min} = minimum temperature of gas in reservoir, deg. cent.
V_c = area of copper in cable conductor, sq. in.
V_{oc} = area of oil in cable core and strand space, sq. in.
V_i = area of cable insulation, sq. in.
V_{os} = area of oil in channel space under sheath, sq. in.
V_s = total area inside cable sheath, sq. in.
V_{max} = one half of the total oil requirement of section of cable, including surplus, gallons.
V' = surplus oil in reservoir at lower end of operating range, gallons. (This refers to working surplus only.)
V'' = V' plus expulsion of oil from cable at maximum full load temperature gallons.
V_{T1} = volume of gas in lower reservoir, at pressure P_{min}, and temperature T_{min}, gallons.
V_{T2} = volume of gas in upper reservoir, at pressure $P_{min} - h$, and temperature T_{min}, gallons.
W_c = sum of copper loss and dielectric loss of cable, watts per ft.
W_s = sum of sheath loss and armor loss of cable, watts per ft.
x = distance from center of cable, inches.
x_o = distance from feeding point on oil-filled cable line, ft.
Z = viscosity of oil, centipoises.
α = multiplying factor in duct cooling exponential; average value 0.0000217.
Δ_1 = $[M_2 (u_n R)^2 - N] Y_o (u_n R) + u_n R Y_1 (u_n R)$ (see equation (4)).
Δ_2 = $M_1 u_n r Y_o (u_n r) - Y_1 (u_n r)$ (see equation (4)).
ϵ = base of Naperian logarithms, = 2.7183 . . .
η_c = coefficient of expansion of copper (volumetric) per deg. cent. = 0.00005
η_i = coefficient of expansion of insulation (volumetric) per deg. cent. = 0.000425.
η_1 = coefficient of expansion of lead (volumetric) per deg. cent. = 0.000085
η_o = coefficient of expansion of oil (volumetric) per deg. cent. = 0.00074.

Discussion

For discussion of this paper see page 1447.

Oil-Filled Cable and Accessories

BY R. W. ATKINSON* and D. M. SIMMONS*
Fellow, A. I. E. E. Fellow, A. I. E. E.

Part I

A Complete Oil-Filled Cable System

Introduction. The term "oil-filled," as applied to impregnated paper cables is ordinarily understood to indicate a cable in which the compound is of such viscosity as to be fluid at all operating temperatures. When such a cable is so constructed as to permit free passage for the oil longitudinally and radially and is connected to a suitable oil supply it will remain completely filled with saturant at all points, throughout all temperature and load cycles.

Under careful attention the modern high-grade solid or hard compound cable may receive a thorough initial impregnation. It is generally agreed, however, that under repeated load and temperature cycles, the expansion of the sheath tends to the formation of "voids" or bubbles in the insulation, leading to ionization, deterioration, and ultimate failure. Provision against this tendency necessitates conservative current ratings. This type of limitation is inherent in the use of saturants of high viscosity and of completely closed lead sheath and joints, and becomes increasingly important with increasing wall thickness, *i. e.*, with increasing voltage rating. The oil-filled cable, by providing for a free ebb and flow of saturant at all times, removes the danger of bubble formation, thus permitting thinner walls, higher temperatures, higher voltages and current ratings.

Cable engineers both in this country and in Europe have long recognized the theoretical advantages of the oil-filled principle, but the first attempt to realize these advantages on a large scale was the extensive installation of General Electric Co. and Pirelli & Co. oil-filled cables in New York and in Chicago about four years ago. The outstanding electrical performance of these installations fully confirmed the early expectations. While the great cost and the complicated operations connected with their installation prevented the immediate wide-spread adoption of oil-filled cable, this performance acted as an intense stimulant to the improvement and simplification of that system, some of the results of which have been and are being described by their makers.[1,2,3,4] It also produced increased concentration on a parallel development which had been proceeding along somewhat different lines, and which has resulted in the system to be described in this paper. This system has been planned from its inception with the aim of approximating the simplicity of the installation of the ordinary type of cable. In its present form, it has made notable progress towards accomplishing this aim, and now also has a background of completely successful practical experience.

General Features. Fundamental features about which this system has been built are:

1. Shipment of cable filled with oil, and without field evacuation and reimpregnation.
2. Stop joints of moderate size and cost, and a rapid and reliable technique of installation, making practicable oil feed lengths as short as desired.
3. Reliable oil feed system, including compact designs of expansion oil reservoir with oil volume indicator, and an automatic signal system designed to give to oil-filled cable the same superiority over solid type cable in freedom from service failures due to mechanical causes that it now enjoys with respect to electrical trouble.
4. For multi-conductor cable provision of oil channels in the filler spaces, making the system equally applicable to single- and three-conductor cable.

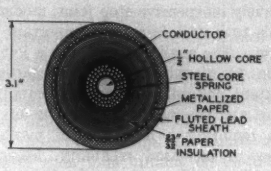

Fig. 1—Cross Section of 132-Kv. Single-Conductor, Hollow Core, Oil-Filled Cable with 23/32-In. Insulation and Fluted Lead

It is the purpose of this paper to give only a very brief outline of the general manner in which these important developments have been accomplished.

The Cable. A cross-section of single-conductor cable for 132-kv. service is shown in Fig. 1. An oil channel is provided in the center of the conductor which is stranded in the usual manner on an open spiral of flat steel. In some cases this is also supplemented by a number of longitudinal slots in the inner face of the sheath. The cable shown is of type *H* construction, having a layer of metal foil between insulation and sheath.

The cable is leaded before drying and impregnation, both of which processes are carried out completely in the factory. The leaded cable is placed in an oven equipped to maintain uniform temperature throughout. Special end seals are supplied which make it possible to apply vacuum or pressure independently to the hollow core and to the space between the paper

*General Cable Corporation, New York, N. Y.

1. For references see Bibliography.

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insulation and the lead sheath, and to admit carbon dioxide or hydrogen after the air is exhausted. Power-factor and insulation resistance measurements are made in the usual way during the drying period. The removal of moisture and gas is accomplished by use of high vacuum pumps capable of producing a vacuum of a few microns absolute pressure. When the electrical measurements and vacuum attained in the cable show the cable to be completely dry, dried, degasified oil is admitted to the hollow core. After the cable is completely filled, a considerable quantity of oil is permitted to flow through. When the fluted lead is used, this procedure makes it possible to flush out completely any gas or vapor occluded by the paper, and to carry it away in the oil as it flows through. This insures the elimination of gas and moisture to the highest possible degree. The possibility of maintaining, throughout shipment, installation, and jointing, the high quality of insulation acquired in the factory, is realized through the use of a transportation reservoir maintaining within the cable sheath a pressure slightly above atmosphere at all times during shipment and installation.

The Stop Joint. The development of a reliable and comparatively inexpensive stop joint, moderate in size and simple to install, is a most important feature of this system. The availability of such a joint makes it practicable to sectionalize the cable as regards oil supply to any extent desired. Independent supply for relatively short sections makes possible the use of smaller reservoirs requiring a minimum of manhole space, contributes to increased reliability, lessens cost of possible cable trouble, and permits lower hydrostatic pressures.

This joint, utilizing the condenser principle, involves radical departures from classical methods of jointing and its strength as indicated by the test results given in Part II, has exceeded anything previously accomplished. A discussion of certain details of theoretical design (which are equally applicable to stop joints and to non-stop joints) will be given separately in Part II.

The advantages of this new type of joint are many. The condenser unit which comprises the complete assembled joint insulation is dried, impregnated, and its casing filled with oil and sealed, and the completed unit tested before leaving the factory. This construction results in great saving of time and of labor in the making of a joint in the field, the operation being practically limited to the assembly of mechanical fittings and the making of wiped joints. The reduction of time and area of exposure of the cable insulation during jointing is also noteworthy. Also of special interest is the moderate space requirement of the completed joint as compared with stop joints of equal rating and earlier design. Stop joints of this type installed on a commercial 132-kv. line in Chicago early in January of last year were approximately 8⅜ in. diameter by 61 in. long. On a later experimental installation at the same voltage, stop joints 5¾ in. diameter by 55¼ in. long were used, and the perfect operation of these joints together with laboratory tests indicate possibility of still further reduction.

The "stop" feature is provided by a simple enclosure (or "stop tube") applied over one or both cable ends. The body of this tube is of rigid insulating material and is cemented into a metal sleeve at each end. A special connector thimble is used which forms an oil-tight cap over the hollow core conductor and which is sealed to the adjacent brass sleeve on the stop tube by means of a gasket joint. A thin solder wipe is made between the sleeve on the other end of the stop tube and the cable sheath, thus completely closing off the end of the cable. Threaded openings are provided through the brass sleeves for the purpose of evacuation and oil filling of the stop tubes. These openings may be sealed off at the time of installation, or may be used for connection to the oil circulating system, thus providing an extremely flexible arrangement by means of which the joint sleeve itself and the adjacent cable sections may be isolated from each other or connected to the oil feed in any desired combination. A joint of similar design is readily adaptable to three-conductor cable.

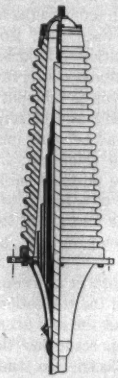

Fig. 2—Barrier Type Terminal Arranged for Oil Immersion

Terminals. Two types of terminal have been developed for systems in the higher range of voltage. Fig. 2 represents an oil-filled terminal of the barrier type arranged for terminating a cable in oil-filled apparatus such as a transformer. The barrier construction which is illustrated and fully described in a previous article has been used for a number of years in connection with cables of the solid type insulation for voltages up to 66 kv. on a large number of installations, and on experimental lines up to 132 kv. In all of these installations no case of electrical failure has been reported.

A terminal of the same general design as the condenser joint is discussed in detail in Part II and illustrated in Fig. 12. For outdoor use the design as shown is supplemented by a protective porcelain bushing.

Oil Supply. Fig. 3 shows an oil reservoir in which the expansible element is a single-unit metallic bellows. This contains the oil and is, in turn, enclosed in an outer gas tight steel casing. The changes in the volume of the oil in the cable during temperature cycles are compensated for by the expansion and contraction of the bellows, thus insuring that the pressure in the cable will at all times be within safe limits. In order to insure prompt and positive return of oil to all parts of the cable when contraction occurs, a small pressure above atmosphere (from 15 to 25 lb.) is maintained. This pressure can be produced by various means such as gas pressure in the outer casing, weights, springs, or combinations of these. With large diameter bellows, gas is found to be the most convenient. Supplementing the gas, a weight or a relatively light compression spring, as shown in Fig. 3, is used to overcome the slight inherent stiffness of the bellows. This insures that the oil pressure will always be higher than the gas pressure

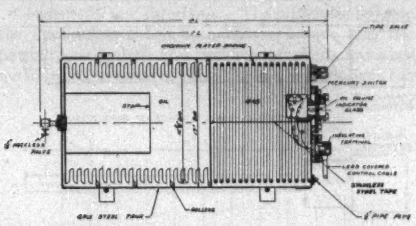

FIG. 3—PRESSURE TYPE VARIABLE VOLUME OIL RESERVOIR

so that in case of a bellows leak, the oil will leak outward into the gas instead of gas passing into the oil, and, in addition, the bellows will be gradually compressed to the point where the low volume contactor will operate.

The advantages of the single bellows type expansible unit in ease of assembly, and in the simplicity of design required for oil volume indicator and signal device which can be operated by the free end of the bellows will be evident, and cyclic life tests on bellows of this type up to 16 in. diameter indicate that they will have indefinitely long life under operating conditions.

Fig. 4 gives a diagrammatic view of the principal elements of the oil feed system. Suitable valves are provided for isolating individual cables or reservoirs, and for manifolding the oil systems of the three individual cables as desired. Under some conditions, an oil pipe line may be extended the entire length of the system and manifolded to individual cables or reservoirs.

In addition to the direct reading gages which are provided to indicate both oil volume and oil pressure, automatic alarm signals may be used to give warning of either low volume or low pressure. The volume indicators are operated by mechanical connection to the bellows and the pressure indicators are connected directly on the oil feed lines (see Figs. 4 and 5). Alarm signals are transmitted through mercury tube contactors which operate an annunciator installed in any convenient central location. All contactors are connected in parallel and in case of low oil volume or low pressure the particular cable section involved is readily determined by opening the switch in the annunciator

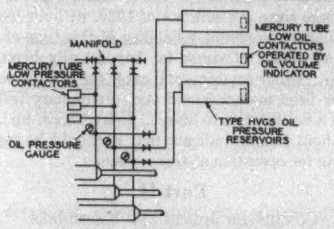

FIG. 4—LAYOUT OF RESERVOIR SYSTEM

circuit and balancing the simple bridge shown in Fig. 5.

This whole oil supply and signal system is very simple, reliable, and sturdy. Whereas, with solid cables, external mechanical causes have resulted in more service failures than have electrical causes, the means here described provide an assurance against most forms of such interruptions on oil-filled cables comparable with the assurance provided against electrical failures by the perfect electrical performance oil-filled cables have so far shown.

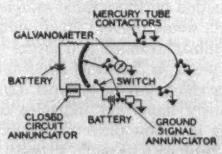

FIG. 5—ELECTRIC SIGNAL SYSTEM AND BRIDGE FOR LOCATING LEAKS

Service Performance. Although the number of complete installations of oil-filled cable systems in commercial operation is still relatively small, these installations nevertheless, in conjunction with experimental studies both in this country and in Europe, have clearly shown a very notable superiority over the older types of cable.

The remarkable performance of the 132-kv. cable system is well known and is indicative of the equal satisfaction which can be expected on cables for lower voltages with reduced insulation thicknesses. Even now in addition to several smaller installations there is in operation about a mile of three-conductor 33-kv.

cable of the oil-filled type installed in Los Angeles, California.

The reduction in insulation wall thickness possible in oil-filled cable is especially noteworthy. The experience with 132-kv. cable is a sufficient example. Cable of the type described here, now in operation in Chicago at this voltage, has a wall thickness of 23/32 in., which is less than used heretofore on any 66-kv. cable having solid insulation. Experimental cables of this type with wall thicknesses of 17/32 in. and even 13/32 in. have been connected to 132-kv. power lines at Chicago since the summer of 1930, without trouble, indicating the apparent reliability of this type of insulation at values of stress far beyond any heretofore possible. Other experimental cable for 220-kv. service has now been manufactured and preliminary tests including a voltage test in 30-sec., 25-kv. steps, up to 725 kv. without failure, indicate that it should be entirely adequate for operation at this voltage.

Part II

Condenser Joints and Terminals

Introduction. With each step in the improvement of cable design, and with the constantly increasing strength of cable insulation, the problem of providing joint and terminal construction which will be as strong as the cable itself has been increasingly difficult. The advent of the oil-filled cable with its large increase in voltage for a given insulation thickness has made the situation still more acute.

It is the purpose of this section to describe very briefly a new form of terminal and joint using the condenser principle which has greatly simplified the problems of joint and terminal design. This type of construction was first developed in connection with tests to destruction on short cable samples to provide terminals which would prevent longitudinal discharges within the terminal structure and insure that ultimate failure would occur in the cable itself and not in the terminal. One characteristic of paper insulation which has an extremely important influence on joint and terminal design is its low dielectric strength longitudinally in comparison with its radial strength. This point, though recognized, has been given very little previous study. Some data recently obtained on the longitudinal strength will therefore be given in this paper.

Problems of Terminal Design. In order to have in mind the problems to be met, we may review the conditions which exist where a cable terminates. Since practically all high-voltage cable now made is either single-conductor or of type *H* multi-conductor construction, and since the individual conductors of a type *H* cable are electrically equivalent to single-conductor cables, for the sake of simplicity, this discussion is confined to cables of the latter type.

Fig. 6 shows diagrammatically a terminal in its simplest form. Here the termination consists in the mere removal of a portion of the cable sheath. The longitudinal distribution of voltage which results in this case is plotted in Fig. 6. The slope of the curves at any point in each case shows the stress at the corresponding point along the surface of the insulation. It is evident that near the end of the lead sheath there is in both the paper insulation and the surrounding air, a high concentration of longitudinal stresses. In addition it is obvious that even greatly increasing the length of the terminal is of little avail in mitigating stress conditions at the sheath edge. The conditions as illustrated in Fig. 6 are well known and result from the elementary relations between the capacities to the conductor and to the sheath of the various points along the surface of the exposed insulation. There are then two difficulties to be taken care of by an adequate terminal design:

The stress in the air surrounding the terminal, which has comparatively low dielectric strength.

The longitudinal stress in the paper which has a low strength in this direction.

Various methods have been devised to relieve the stress in the air, such as building up with insulation

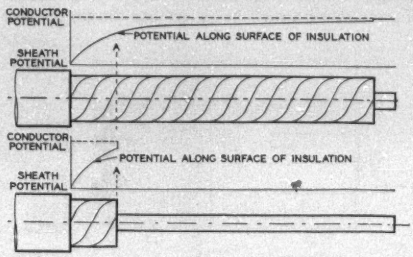

Fig. 6—Cable Terminal in Simplest Form, Showing Longitudinal Distribution of Potential

around the end of the sheath, or surrounding the exposed end of the insulation with insulators or metal cones filled with oil or other insulating compound. But when the difficulty with the breakdown of the air is reduced or overcome, we are immediately confronted by longitudinal discharges in the cable insulation itself.

One partial solution by means of which the longitudinal stresses in the vicinity of the termination of the lead sheath are somewhat redistributed and one which is in extensive use at the present time, consists of a flaring ground shield at the end of the sheath. This principle has been applied in several varied forms. One method which was first used successfully in laboratory tests and later developed for use in commercial terminals employs a spun-copper cone in which are mounted a series of concentric tubes of insulating material which surround the exposed portion of the insulation and which are filled with insulating oil. Sometimes this is accomplished by building up a tapering surface of varnished cloth or paper and the resultant

taper shielded with a metallic braid, or by the method described by Simmons[5] in which a cone is formed during the application of the insulation. Clearly, to solve the problem, there is required some method of controlling the distribution of the longitudinal stresses so as to make them substantially uniform throughout the length of the terminal.

Resistance Type Terminals. A method of accomplishing this which has been used to some extent for test terminals makes use of a semi-conducting material surrounding the end of the cable and having sufficiently low resistance to permit a current flow large enough to control the potential distribution by resistance drop. With direct current or with moderate alternating potentials this can be used satisfactorily. With alternating current, however, in order to control the potential distribution effectively, the current flowing through the resistance must be large compared to the capacity or charging current through the insulation. Consequently, for the higher voltages, say above 100 kv., serious difficulties are encountered in taking care of the heating due to the $I^2 R$ loss, and for the still higher voltages this construction becomes entirely impracticable.

Condenser Type Terminals. The principles by which the final solution has been obtained have long been partially recognized and previous attempts have been made to utilize some of them. Since such designs have

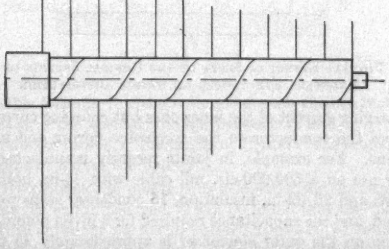

Fig. 7—Terminal with Longitudinal Distribution Controlled by Metal Disks

been only partially successful due to a failure to meet certain fundamental requirements, no detailed description will be made. The final solution may be explained with the aid of an illustration from a very early partial solution illustrated in Fig. 7. This represents a cable end upon which is mounted a series of conducting disks or plates insulated from the conductor and from each other. Consider the potential of the plate adjacent to the one touching the sheath. Following the same reasoning used with reference to Fig. 6, it is evident that this plate will take a potential near to that of the sheath. Also the potential difference between these plates can be brought to any desired small amount by sufficiently increasing their size. The dimensions required to produce an even distribution of potential between these plates may be determined by a fairly simple mathematical analysis. In general, therefore, the arrangement of Fig. 7 can accomplish the desired redistribution of potential both along the surface of the conductor insulation and through the air. This scheme

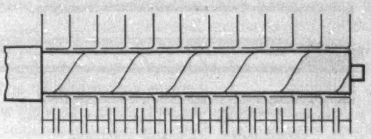

Fig. 8—Disks Supplemented by Properly Proportioned Condensers

was developed independently by several different engineers. It was suggested by C. W. Davis and actually applied usefully in our Pittsburgh laboratory in 1914. It has also been well described in a recent paper by Farmer.[6]

There are two difficulties with the type of terminal illustrated in Fig. 7. In the first place, it is apparent that there is a concentration of stress at the inner edges of the metal plates. In the second place, the physical dimensions required with this type of terminal to produce requisite capacity between plates are prohibitively large except for terminals of only a few inches in length and thus adapted only for relatively low voltages.

The arrangement of Fig. 7 is then, in spite of the difficulties, clearly a step, though only a step in the right direction. An arrangement providing the correct distribution by means of capacitance, and at the same time eliminating the difficulties shown for the device of Fig. 7 gives ideal results. The dimensions are brought within very satisfactory limits by placing between the successive disks, suitably designed and proportioned condensers. Such condensers can easily be produced to stand the required voltage and to give, in very moderate dimensions, all the capacity that is required for the purpose.

Practical Design of Condenser Type Terminal. Fig. 8 shows diagrammatically the construction of such a terminal. The values of capacitance between disks are graded so as to obtain uniform distribution of potential throughout the entire length of the terminal. The capacitances are largest near the lead sheath, the smallest capacitance being near the high-voltage end. (Actually, on account of the increased effect of capacitance to ground of the individual condensers at the points of high potential compared with the effect of their capacitance to the conductor, the condenser of minimum capacitance is slightly removed from the

high-voltage end.) The concentration of stress at the edge of the disks is overcome by supplying them with lips which almost completely enclose the entire cable insulation within the terminal, the condensers being insulated from each other by a wrapping of varnished cloth. A test terminal of this type is shown in Fig. 9. To illustrate actual dimensions, as used for testing cables up to 3 in. in diameter, the individual condensers are about 3 in. inside diameter, about 7 in. outside

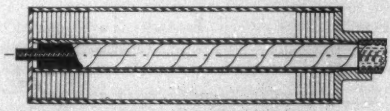

Fig. 9—Practical Design Test Terminal with Disk Type Condensers. (Details Omitted.)

diameter, and ½ in. to 1 in. in thickness. The group of condensers made up in this way is ordinarily enclosed in a suitable tube of insulating material and impregnated in insulating oil. With such arrangement, and with condenser stack less than 4 ft. long, many tests have been made on various sizes of cable at voltages up to 350 kv. One cable having 26/32-in. insulation was tested for over 200 hours at 300 kv.

Condenser Type Joints. The same principles have been applied to joint design. Some new problems are there presented but these were satisfactorily solved and highly efficient joints were produced. However, the much lower cost of the type of construction now to be

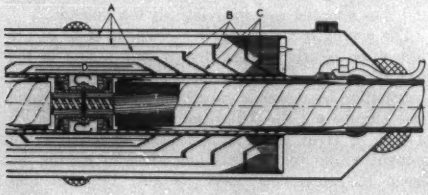

Fig. 10 Condenser Joint with Cylindrical Type Condensers

described has caused this later design to displace entirely the earlier one.

New Form of Condenser Construction. In this design, a schematic cross-section of which is shown in Fig. 10, longitudinal distribution of voltage is likewise controlled by condensers. The condenser electrodes, however, are made up of a nest of conducting cylinders *A* of different diameters arranged coaxially with the conductor and insulated from each other by saturated paper insulation. At the ends of these cylinders are radial sections *B*, which are in effect conducting disks, terminating at their inner edge in short cones *C* by means of which electrical connection is made to various points along the exposed cable insulation, thus controlling the longitudinal stress distribution in this region. The potential of the successive cylinders is controlled so as to produce the desired longitudinal distribution by adjustment of the respective capacitances between them. Two methods of accomplishing this are used. The thickness of the paper insulation between successive cylinders is varied, or multiple layer condensers are produced by connecting alternate cylinders in parallel as shown at *D*. As the first condenser is connected to the conductor and the last condenser to the sheath the potentials of the intermediate points are determined by the series of condensers. For uniform potential distribution the capacitance required for successive condensers increases progressively from the inner part toward the outer due to the fact that the outer condensers carry not only the

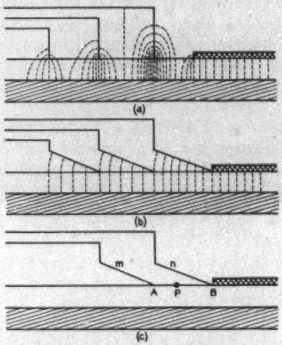

Fig. 11—Effect of Slope of the Conical Section of Condenser Electrodes on Stress Distribution

charging current of the inner ones but charging current from the conductor to the successive flaring end sections. For example, in joints recently manufactured for use on a 500,000-cir. mil cable with ½-in. hollow core and 26/32-in. insulation, 15 condenser steps were used, and the capacitance required for a given potential drop on the outer condenser is approximately 45 per cent greater than that required on the inner.

It has been mentioned that the inner edge of the radial section of the condenser electrodes terminates in a short cone. The slope of the cone varies for the different condensers, becoming longer and with gentler slope as the distance from the center of the joint increases. In that portion of the joint near the end of the lead, there is a large potential difference between the condenser electrode and the conductor. If the radial section were brought down to the inner diameter of the joint and terminated without the cones, a very pronounced concentration of stress would occur, the lines of force being somewhat as shown in Fig. 11a.

It is obvious that by merely causing the inside edge of the disk to terminate in a cone, this concentration may be relieved as is evident from the stress diagram in Fig. 11b. Consider a portion of the joint insulation between the radial sections (or disks) of two adjacent condenser electrodes, m and n Fig. 11c and near their inner edges. This is seen to be under radial stress due to the potential difference between the overlying condenser electrode and the conductor, and under longitudinal stress due to the difference in potential between the condenser electrodes themselves. Consider for the moment only this longitudinal stress along the cylindrical surface of the insulation between the inner edges of these cones, m being the flanged section of the condenser electrode nearer to the conductor potential. If the angle of flare of n is very small, the potential of any point P on the surface between A and B and not very near to A will take a potential near that of B, with a result that the longitudinal stress will be very highly concentrated near A, with correspondingly low stress near B. It will further be seen that the potential at this point P is determined largely by the capacity relations to the conductor and to the overlying surface n.

It will also be evident that a minimum concentration of stress will occur if the distribution longitudinally between A and B is made approximately uniform, and furthermore, that by selection of proper dimensions the radial stress may be utilized to bring about this result. By the following approximate formula, the development of which is omitted on account of space limitations, it is possible to calculate the dimensions of the surface n which will give the desired distribution of potential along the surface between A and B.

$$r'' = r' \left(\frac{r'}{r} \right)^{\frac{v}{V-v}}$$

In this formula V is the voltage between the conductor and n, v the desired voltage between P and n; r the conductor radius, r' the radius at P, and r'' the required radius of n over the point P.

By plotting the points thus calculated for the region near the end of the lead, it will be found that the theoretical curve for the flaring surface to give equal voltage distribution deviates only a little from a straight line within the dimensions used. For the purposes of the joint it is approximated within the limits of practical construction by calculating the value of r'' for the point A, and using a straight line between the point on the conical surface thus determined and B. That is, the slope of the cone n is made such that the potential of the point of origin of the adjacent cone as determined by the radial field tends to be the same as that fixed by the capacity adjustment of the cylindrical condensers. This insures against concentration of radial stress at this point and provides substantially uniform distribution of longitudinal stress between the steps fixed by the condensers. The formula used is substantially correct near the ends of the joint where the radial field is high. In the region approaching the center of the joint, the error of the formula is progressively greater. In this region, however, the radial stress is very low and the importance of maintaining the correct angle is correspondingly less, so that the resultant design is completely adequate at all parts.

With all parts of the insulation of uniform resistivity, correct distribution for alternating current would also produce correct distribution for direct current. In the simplest form of construction, such homogeneity does not occur. Suitable practical means have been found, however, to provide resistance adjustment to produce this uniformity of distribution for direct current as well as for alternating current.

Design and Construction of Commercial Joints. For convenience in manufacture, in the making of practical joints, the condenser construction is not applied directly over the conductor insulation, but is made up as an independent factory built unit. Reference to Fig. 10 will show the construction clearly. There is an inner tube (called the "condenser tube") of rigid insulating material such as cellulak or micarta upon which the joint proper is wound. At the middle of this tube is inserted a short metal ring which forms the contact element between the conductor and the first condenser electrode. The insulation is made up of a continuous wrapping of 5-mil wood pulp paper, of a width equal to that of the total length of the condenser unit, and applied in one continuous length to the condenser tube which is mounted in a lathe especially equipped for this work. The cylindrical condenser electrodes are made of sheets of zinc one or two mils thick, and of suitable dimensions, which are wound in with the paper wrapping at appropriate intervals to provide the desired thickness of dielectric. The first of these plates is applied directly over the condenser tube, and makes contact with the central metal portion. The radial end sections of the condenser electrodes with their flaring terminations are formed by feeding in continuously with the paper wrapping, narrow foil tapes, ⅛ in. wide by 1 mil thick. The tapered or conical section is formed by feeding the spool carrying the metal tape along the tube longitudinally as the tape is wound in. Connection between the foils and the cylinders is made by soldering, and where the length of foil becomes excessive so that the reactance might interfere with the effective potential distribution under high-frequency transients, as in the radial portions where the insulation is quite thick, successive turns are short-circuited by metal pins passing radially through the tapes and intervening paper.

When the winding of the condenser unit is complete, the whole unit is encased in a thin sheet of corrugated copper. After preliminary drying, sheet copper end sections provided with copper tubes for saturation are soldered in place and final drying and impregnation with transformer oil is accomplished in specially con-

structed vacuum tanks arranged with automatically controlled electric heating and special high vacuum pumps. After the copper tubes through the end are sealed off, the condenser units are ready for test. By a very simple temporary assembly, each unit is submitted to a routine high-voltage test before shipment. The possibility of making such a test constitutes a distinct advance over previous jointing practise. The corrugations in the copper casing provide ample capacity for expansion and contraction of the oil due to any changes in temperature either in handling before installation or in service afterwards.

It will be seen that continuous oil films exist in the space between the condenser tube and the stop tube, and between the stop tube and the factory applied insulation. The longitudinal stress in the films in the portion of the joint under the condenser unit is controlled by the condenser construction. In the region near the end of the joint, control of longitudinal stress is provided by tapering down the stop tube where it is fitted into the brass sleeve, this slope being calculated by use of the same formula developed in connection with the slopes in the condenser unit. Even with the longitudinal stress taken care of, however, the large radial stress on the oil film within the stop tube

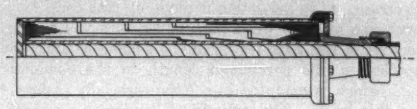

Fig. 12—Simplified Assembly of Test Terminal with Cylindrical Type Condensers

would be excessive and develop failure in this region except for application of the method now to be described. This construction is very simple and now seems quite obvious but has eliminated completely this point of weakness. It consists in building up the conductor insulation inside the stop tube by use of a tube of impregnated paper, which is arranged to be slipped over the end of the cable and tightened up on the factory insulation by twisting in the proper direction. The paper is cut to give the proper taper at the end next the lead and this tapered portion is covered with metal after the paper tube is in place. The diameter to which the insulation is built up in this way may be made sufficient to reduce the stress at the outer surface to a value well below that which will produce ionization in the oil film.

Where the stop feature is not to be used, the stop tubes are omitted and the exposed cable insulation is built up to the desired diameter by the use of a similar tube of impregnated paper. This makes it possible to use a condenser tube large enough to slip back over the lead with ample clearance, and insures against discharges over the surface of the factory insulation.

Cylindrical Type Condenser Terminals. The same general features of condenser design have been applied to high-voltage terminals and have proved equally satisfactory. Fig. 12 shows the cross-section of a test terminal of this type. The use of the impregnated paper tube has also been applied to this and to other types of terminal with gratifying results.

Longitudinal Strength of Oil Films. By this construction it has been possible to obtain substantially uniform longitudinal distribution of stresses and the limiting feature in design has become the actual strength of thin oil films longitudinally and of the impregnated paper insulation in a direction parallel to its laminations. In earlier joints, the high local concentrations of stresses were not easily determined and were usually not known, hence practical methods of reducing these maximum stresses or of introducing constructions giving higher dielectric strength at the points where they occurred were of more importance than information as to the stress which could be sustained by the film if the potential were uniformly distributed. Consequently, very little previous work has been done in determining specific values of longitudinal strength, and so far as we know, there are no published data on the subject. We have found that the longitudinal strength is of the order of 10 per cent of the radial strength. While, in general, factors tending to affect the strength in one direction have a similar effect on the strength in the other, the degree to which they affect these two values is found in some cases to vary enormously. For example, both are affected by the degree of saturation, but the effect on the longitudinal strength is very much more pronounced. On the other hand, the element of time has a very important effect on radial strength while its effect on longitudinal strength is much smaller. As a result, tests involving both radial and longitudinal stresses are more likely to produce longitudinal failure when made at very high voltages for short time and radial failures when made as long-time tests at lower voltages.

A large amount of experimental data has been obtained concerning the longitudinal breakdown strength of oil films similar to those in the cylindrical condenser joints. Tests were made on some 350 specimens constructed by winding strips of 1-mil foil between the layers of a paper tube, using spacings of from ⅜ in. to ¾ in., the samples being impregnated with oil under vacuum and tested under pressures ranging from atmospheric to 55 lb. per sq. in. absolute. No attempt was made to eliminate "edge effect," the intention being to reproduce substantially the conditions in the condenser units. For the conditions of the tests, the results obtained have enabled us to draw the following conclusions:

a. Longitudinal breakdown strength of such an oil film one mil thick with one inch separation of electrodes and tested under 20 lb. per sq. in. gage pressure —40 to 60 kv.

b. No indication of reduction of strength with increase of time of application was found within the limit of the times used in these tests. (1 second to 30 minutes.)

c. Longitudinal breakdown under the conditions of test varies as about the square root of the distance between the electrodes.

d. The effect of pressure is to increase the breakdown strength as something like the eighth root of the absolute pressure.

Test Results on Joints. Typical results of tests on complete joints may be of interest. On the earlier joints having a total of 14 in. longitudinal creepage distance, tests were obtained up to 300 kv. for approximately 8 hours without failure. Further tests on other joints with creepage distances down to 8 in. have given results which are about proportional to the one just cited.

Conclusion

In earlier designs of joints and terminals for very high operating voltage, the difficulty with concentration of stress increases with increased voltage, so that a small increase in voltage requirements necessitates an entirely disproportionate increase in joint and terminal dimensions. As a result, a rather definite limit was established, beyond which further increase in voltage was impracticable. Since the condenser construction provides complete control over the potential distribution, any increase in voltage may be taken care of by a proportional increase in creepage distance and in the number of condenser steps. Consequently, by a simple adjustment of dimensions, the advantages already derived from the use of joints and terminals of this type on the highest voltages now in use become equally available for any operating voltage for which cables may later be built.

These developments have made possible the realization of a unified oil-filled cable system which involves a minimum of complication and which, in the higher-voltage range, has a decided cost advantage over solid type cables. This system not only fully realizes the outstanding advantages inherent in the oil-filled principle, but incorporates additional valuable features. Thus, it is possible to install the cable nearly as readily as solid type cables for lower voltages. The system provides for efficient sectionalizing of the oil feed and for prompt detection of oil leaks or abnormal pressures. As a result, the freedom from electrical failure which is characteristic of oil-filled cables may be expected to be matched by freedom from service failures due to most of those external causes which result in failures on the present solid type system.

So extremely rapid has been the development which has taken place in this field in methods of manufacture and in the improvement in the technique of handling and of installation, that problems which only a comparatively short time ago seemed to be extremely difficult have become of minor importance. An increasing volume of information gives assurance that oil-filled cable will fulfil all theoretical expectations, and as a result, it has opened up entirely new fields of usefulness for insulated cables.

Grateful acknowledgment is made to the many associates of the authors who have contributed largely to the development here described and to the utility engineers who have cooperated in obtaining data on performance of oil-filled cables and accessories in actual operation.

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Discussion

For discussion of this paper see page 1447.

Residual Air and Moisture in Impregnated Paper Insulation—III

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Fellow, A. I. E. E. Associate, A. I. E. E.

Introduction

In an earlier paper[1] we have described a series of studies of the influence of residual air and moisture on the properties of impregnated paper insulation. The power-factor voltage characteristics of impregnated paper, as used in high-voltage cables, are well known. A sharp rise in power factor with increase of voltage has been accepted as an indication of poor quality and generally as indicating large amounts of entrapped air or gas. One of the striking results of our studies has been the relatively small effect of residual air upon the shape of the power-factor voltage curves. The shape of these curves was not appreciably affected in the range of evacuation pressure from one to ten centimeters of Hg., absolute pressure and furthermore the curves were perfectly flat up to 300 volts per mil and 50 deg. cent. Only at impregnation pressures above 25 cm. Hg. did the typical rising power-factor voltage curve become evident. Visual examination always showed dryness and air in samples having rising power-factor voltage curves while evidence of excellent thorough impregnation was associated with samples having flat power-factor voltage characteristics.

In this paper we present the results of further study on the still open question of the influence of residual air upon the life of the finished insulation. Here, as before, we use the value of the pressure of evacuation and impregnation as a measure of the amount of residual air. As in the foregoing studies the present work has also brought out much interesting information on other phases of impregnated paper, notably in the suggestion of a possible correlation between initial changes in power factor and subsequent life.

Accelerated Life Tests

Description of Specimens. The sample used for the accelerated life tests was much the same as that described in our earlier paper. Several modifications were necessary, however, due to the importance of strengthening the ends of the specimens to enable them to withstand the high voltages required for the accelerated life tests. The early tests resulted mostly in flash-overs and punctures of the end of the sample. In order to obtain a true test it is important that the breakdown should occur in a section of the sample in which the electric field is uniform, or in other words, under the test electrode. In the final results this condition has been met, although a tendency to breakdown towards the ends is avoided only with the greatest care in construction.

Each sample consisted of a brass tube one inch in diameter and four feet long. Each tube was cleaned and polished and drilled at each end with thirty-six No. 58 holes spaced uniformly in the region from 3 to 8 in. from each end. These holes were necessary in order to insure complete impregnation of the sample at the ends, where the insulation wall was much thicker. Each tube then received a wrapping of fifteen layers of 1-in. wide, 0.0043-in. thick, manila paper. The paper was fed on spirally in the usual manner, in each layer the successive turns lapped slightly, the lap varying from 1/32 in. to a close butt contact. The laps in successive layers were displaced successively by approximately ¼ in. The insulation wall averaged 65 mils in thickness.

Several different methods of building up the ends were tried before a satisfactory construction was obtained. Various combinations of reenforcing insulation and metal bells were tried. One of these methods met

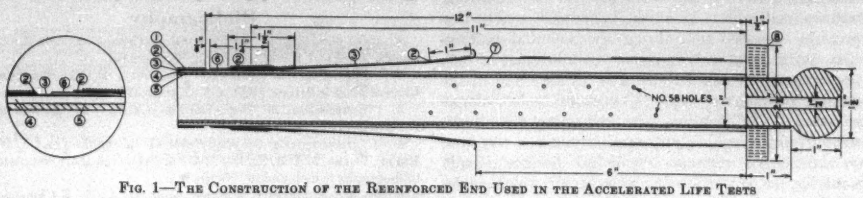

Fig. 1—The Construction of the Reenforced End Used in the Accelerated Life Tests

THICKNESSES

1. Linen thread. 50 turns per in.
2. Lead electrode 1/64 in.
3. Foil extension of electrode 4 mils
4. Paper tape insulation 65 mils
5. Brass tube 1/32 in.
6. Paper insulation 8 mils
7. Paper end reinforcement 0.31 in. max.
8. Bakelite ring

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1. For references see Bibliography.

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31-95

with considerable success but the most satisfactory end construction employed was suggested by R. W. Atkinson. A cross-section of this method is shown in Fig. 1. The end is built up of some 70 turns of kraft paper so cut before applying that when put in place a gentle slope resulted. A lead foil covering of this slope formed the guard electrode of the specimen. The slope was determined by calculation so as to give a uniform longitudinal stress, of a predetermined safe value, along the end of the sample. The guard laps over and is insulated from the end of the test electrode. At the outer edge the lead is turned back and smoothed to avoid sharp edges. This end has been very satisfactory and normally shows no signs of stress. This is true even in cases where a voltage of 50 kv. (800 volts per mil on the insulation wall) has been applied for as long as 180 hours.

Drying, Evacuating and Impregnating. The equipment used for drying, evacuating and impregnating has already been described in the paper referred to above which also gives details of the Schering bridge used for power-factor measurements. One new piece of equipment has been added during the present investigation. This is a chamber into which compound can be admitted under vacuum for the removal of air and other gases. The compound is piped directly from this chamber to the impregnating tank and thus is never in the presence of air after it has once been treated under vacuum.

In view of the importance of residual moisture all the paper samples were first dried to the same standard condition before impregnation. This standard condition was determined by the value of the d-c. conductivity measured at 1,500 volts as described in the first paper. Two methods were used to obtain this standard dry condition. The specimens were either dried in air for 72 hours, or were subjected to a vacuum of ½ mm. Hg. for 48 hours. The temperature for both methods was 105 deg. cent. Check measurements showed that both methods resulted in samples having similar characteristics.

The specimens, having reached the dry state, were evacuated to the various pressures to be studied. For the samples dried in the open this evacuation of 24 hours took place immediately after the air drying period. For the vacuum dried samples a 24-hour period in dry air at atmospheric pressure and 105 deg. cent. was inserted between the drying and evacuating periods in order to offset the effect, on the amount of residual air, of the low vacuum of the vacuum drying treatment.

Following the evacuation treatment at 105 deg. cent. and at the particular pressure being studied, the temperature of the samples was reduced to 60 deg. cent. while the evacuation pressure was maintained. When a constant temperature of 60 deg. cent. was reached the impregnation was started. The samples were impregnated by slowly admitting compound and holding the pressure constant in the impregnating chamber. This was accomplished by regulating properly the opening of the inlet valve and the speed of the vacuum pump. The compound was always evacuated at ½ mm. and 60 deg. cent. before being used for impregnation. After the necessary amount of compound had been admitted to the impregnating chamber the pressure and temperature were held for two hours at the same values used during the impregnation. Following this two-hour interval the pressure in the impregnating chamber was raised to atmospheric by the admission of nitrogen. Direct current measurements of conductivity were then made, after which the samples were allowed to cool to room temperature.

The oil treating equipment consisted of an open reservoir into which the compound was emptied from the containers in which it was shipped. In this reservoir the compound was heated to 60 deg. cent. It was then slowly admitted into the treating chamber which had previously been evacuated to a pressure of ½ mm. Hg. During the admission of the compound this pressure was maintained. The oil passed into the treating chamber through a spray nozzle and was sprayed on the apex of a metal cone. Thus the oil was spread out in a thin layer to facilitate the removal of air and gas. From the top cone the oil dripped on to two other cones and then into the bottom of the chamber where it was stored until needed for impregnation.

After the specimens had been allowed to cool to room temperature in the impregnating chamber they were transferred to the high-voltage test box where they were immersed in compound for the tests. In some cases all three of the samples were transferred at the same time, in other cases they were transferred singly and while one sample of the set was under life test those remaining were kept in the impregnating chamber under compound in an atmosphere of nitrogen.

Test Procedure

Life Tests. For the accelerated life tests it was found desirable to use a value of stress that would give a reasonably long life, say twelve hours, for the poorest specimens and a short enough life for the best specimens so that the testing would not stretch out over too long a period. It was found early that the range of quality of the samples being tested was entirely too wide to permit the use of one given voltage stress for all the tests. After feeling our way in the preliminary experiments the following test procedure, known as "standard test procedure *A*," was adopted:

400 volts per mil for 22 hours
500 volts per mil for 22 hours
600 volts per mil for 112 hours
700 volts per mil to breakdown
Power factor measured two hours after each change of voltage, and then every four hours.

This procedure was used throughout the first group of accelerated life tests. Following this first group a second series of samples was run through in which a newer type of compound was used for impregnation. This new compound resulted in samples having a con-

siderably longer life than those of the first group. Consequently test procedure *A* was found to be too slow and a more rapid procedure was developed. This is known as "standard test procedure *B*," and is as follows:

400 volts per mil for 1 hour
500 volts per mil for 1 hour
600 volts per mil for 10 hours
650 volts per mil for 10 hours
700 volts per mil for 37 hours
750 volts per mil for 40 hours
800 volts per mil to breakdown
Power factor—first measurement at the start and then once at each change of voltage.

The life tests were all conducted at a constant temperature, 40 deg. cent. This value was decided upon as it was well within the operating range of insulation of this type and was sufficiently above the range of ambient values to permit good temperature control. The temperature was maintained, in most cases, to within plus or minus ½ deg. cent. In several tests of very poor specimens the energy dissipated as dielectric loss was so large that it was impossible to hold the temperature down to 40 deg. cent. In one such case the temperature rose to 51 deg. cent. before breakdown. This condition was only encountered in two instances.

The life tests on a group of three specimens were made both singly and three in parallel. Some difficulty was experienced in the use of the parallel connection. This was first noticed by a marked absence of uniformity in the life of the three specimens of one group. The third specimen invariably showed a much longer life than the other two. This was at variance with the results of the tests on single specimens which were very satisfactory from the standpoint of uniformity. There was evidence that the parallel arrangement facilitated the occurrence of oscillations whenever an initial local failure started. These oscillations resulted in the liberation of excessive energy at this point of initial failure, with greatly accelerated approach to breakdown. The first attempt at correction was to insert resistors, of approximately 8,000 ohms each, in the high-tension lead between each outer specimen and the middle one. This improved conditions somewhat but was not entirely satisfactory. A resistance of the same value (8,000 ohms) was then inserted in the high-tension lead in series with each specimen. This connection proved to be fairly satisfactory and enabled us to test some of the specimens in parallel. In view, however, of a continued uncertainty as to the presence of this type of disturbance, the majority of the samples were tested singly.

Power-Factor Measurements. As a preliminary check of the character of the impregnated specimens, power-factor measurements over the voltage range from 60 to 300 volts per mil and at two temperatures, room and 40 deg. cent., were made on most of the specimens before starting the life test. In some cases the power-factor voltage relations were also studied at 60 and 80 deg. cent.

The life test of each specimen, tested by procedure *A*, was interrupted for about six minutes at intervals, usually four hours, in order to measure the power factor. This was only done for those specimens which were tested singly. These measurements permitted us to study the variations of power factor during the life of the specimens. For the samples tested by procedure *B* the power-factor measurements were taken only at each change of voltage. These measurements brought out a number of interesting points which will be discussed later.

Visual Examination. All the specimens tested to destruction were stripped and examined and the condition of the insulation noted. In the early stages of the work the points of breakdown showed considerable evidence of disruptive discharge. The burned areas were large and badly torn. Subsequently the breakdowns took the form of clean punctures of the insulation. These punctures were generally cone shaped holes about 3/16 in. in diameter at the sheath and tapering to a point at the conductor. The area around this type of break was clean and showed the same visual character as unstressed insulation except in some few cases in which the insulation in the vicinity of the break was very dry and contained only a very meager amount of compound.

The disruptive type of breakdown we attribute to oscillations at the instant of failure. Careful grounding and shielding of all circuits eliminated this type of failure. Similar disruptive breakdowns occurred in the parallel test connection before the insertion of damping resistors.

The specimens that were short lived showed, upon examination, evidence of large quantities of entrapped air or gas and incomplete impregnation as was indicated by the dryness of the paper tape. The best specimens were all well impregnated, thoroughly saturated with compound and free of any signs of voids or air spaces. Intermediate specimens were in general well saturated and free of air or gas. These specimens would occasionally show traces of gas at the tape edges.

Results

Voltage-Time Relations. In any study of the life of impregnated paper insulation it is obvious that an accelerated life test is necessary. Furthermore, in order to investigate insulation having a wide range of quality a step test is essential. This immediately raises the question of a basis of comparison as related to the various voltages of the step test.

Very little information is to be found in the literature concerning the voltage-time relations of impregnated paper insulation. In general two types of relationships have been proposed. The first is based on a voltage-time curve which becomes asymptotic to a line parallel to the time axis and gives a law stating that the life varies inversely as some power of the voltage in excess of the value of the asymptote. The second states that the life varies inversely as some power of the voltage.

F. W. Peek, Jr.[2] suggested a law for the first type of relationship. He found that for impregnated paper tested for breakdown in the range from a fraction of a second to sixty seconds the curve fitted the expression

$$G = G_s\left(1 + \frac{a}{\sqrt[n]{t}}\right)$$

where G is the breakdown gradient for a time t and G_s is the gradient which the insulation will stand for an infinite time without failure. The constant a depends on the type of insulation, the temperature and other factors. Peek found that for his experimental results the value for n was four.

J. Delon[3] investigated this relation further and pointed out that the value of n is not a constant when t varies over a wide range. He found in his experimental work that n increased from four to eight when t changed between the limits of one to two thousand hours.

W. Schmidt[4] has given an interesting discussion of the voltage-time relationship of this type. He proposes a method for the determination of the quality of a given sample of insulation on the basis of the instantaneous breakdown strength and the so-called limiting voltage. He takes the value of n as two and criticizes the use in this country of values as high as seven and eight.

From their experiments on three-conductor cable impregnated with heavy oil, D. W. Roper and H. Halperin[5] find that the second type of relationship agrees with their data. They state that "the breakdown voltage varies inversely as the seventh root of the time." F. W. Farmer[6] and F. M. Clark[7] independently check this seventh root relationship. Clark calls attention to the fact that the type of insulation and the temperature as well may vary the slope of the voltage-time curve.

The first type of relationship appears to be more logical but the second seems to be in favor among insulation engineers. It is apparently, based on the average results of extended periods of observations of commercial breakdown tests, the power of the voltage varying from the fourth up to as high as the nineteenth. The power is assumed to depend upon the duration of the test as well as the quality of the insulation. It is obvious that there is little exact knowledge of the subject and that no general law exists. It seems to be agreed, however, that as a method of comparison the simple inverse power relation is roughly reliable, with a considerable range of the power of the voltage. Our tests have all been of comparatively short duration and our insulation of high quality. Discussion with others working in the same field led us to adopt the eighth power law as a basis for comparison in our tests. All our specimens, therefore, were reduced to "hours life at 400 volts per mil" on this basis.

Our results show that this law is only a mode of comparison and in the longer lived specimens it leads to ridiculous results. For example, considering 100 volts per mil as an average operating stress and taking 6,000 hours at 400 volts per mil as the average life of a good quality sample, calculation on the basis of the eighth power law gives an equivalent life at 100 volts per mil of over 50,000 years! Systematic studies of the law of life, as related to the length of accelerated life tests, are badly needed.

As a matter of interest the first type of voltage-time relationship has also been considered in studying our results. This is the law suggested by Peek, Delon, and Schmidt and it assumes that there is some voltage greater than zero for which the insulation life would be infinite. The life of the insulation, then, is inversely proportional to some power of the excess of voltage over this base voltage. The present tests indicate that for the type of sample and compound used a base voltage of 400 volts per mil and the fourth power of the excess of voltage over that base gives reasonable results. The lives of several specimens have been recomputed on this basis and compared with the same lives calculated by the eighth power law. The relative differences between specimens is practically unchanged and the second method of calculation modifies the results not at all, except as to the numerical values of the estimated life.

Life-Pressure Relations. In all, over 135 samples have been built and tested to destruction. Of these 60 samples were used in the development of a reliable method of test and the remainder have been used in the actual results. The results are shown in Figs. 2 and 3. Each point is the average of from five to nine separate tests.

The curve marked A in Fig. 2 shows the variation of life with the pressure of impregnation for samples impregnated with a compound designated as A. The points on the curve are average points and represent all samples except those in which the test was not truly representative of the insulation strength, such as tests in which there were end failures or other troubles. One is struck, upon looking at the curve, by the small difference in life obtained by reducing the air pressure of impregnation from 2.5 cm. to 2 mm. Hg. This is due to the difficulty of obtaining reliable points at these low pressures. It was very difficult to obtain failures in sections of the insulation wall in which the electrostatic field was unquestionably uniform. It seemed desirable therefore to investigate further this particular pressure range and to modify the specimen construction so as to improve the reliability of the results.

The original supply of compound A having run out, it was necessary to obtain a new one. As it was impossible to obtain a duplication of the first supply another compound had to be used. This question will be discussed further in another section of the paper. This second supply of compound has been designated as compound B and the results of an extensive series of observations using it are plotted in the curve marked B of Fig. 2. The results here are more reliable than those of the curve A, within the same pressure range.

We have shown for curve B a much wider difference in life than in the corresponding section of the life curve A. The curve B shows an increase of 50 per cent in life for a decrease in pressure from 2.5 cm. to 2 mm. The data show definitely an increase in life in this pressure range of 10 per cent and indicate, we believe, the increase of 50 per cent shown on the curve.

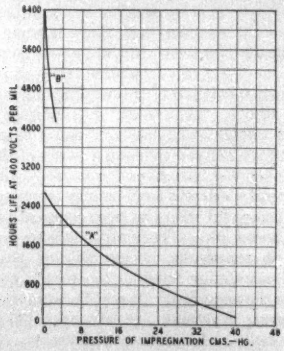

Fig. 2—Relation Between the Pressure of Impregnation and the Life of Impregnated Paper Insulation

Compounds A and B

The scattering of the points obtained at the 2.5 cm. and the 2 mm. pressures is shown in Table I. The average values used in plotting the curves have been based on a critical analysis of all the results. Careful

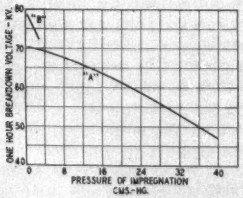

Fig. 3—Relation Between the Pressure of Impregnation and the One-Hour Breakdown Voltage of Impregnated Paper Insulation

study of the test and the examination of each sample led to the selection of those used in computing the average. For example, the first column of Table I shows a range of life from 232 to 3,850 hours. Out of a total of twelve tests five were rejected due to the proximity of the failure to the sheath cut between central electrode and guard.

TABLE I

Impregnating pressures	2 Mm.		2.5 Cm.	
Compound	A	B	A	B
Hours life at 400 volts per mil	1,483	12,290*	2,262*	4,618*
	3,850*	4,100*	896	3,809*
	2,102*	4,213*	668	4,049*
	3,025*	6,189*	2,332*	4,308*
	474	11,614*	251	4,221*
	2,365*	863	2,102*	
	1,345	372	2,442*	
	2,377*	4,141*	1,909*	
	420	4,383*	3,156*	
	232	4,035*		
	3,059			
	2,091			
Average	2,695	6,370	2,367	4,141

*Indicates specimens used in computing average.

The results when examined in detail offer an explanation of the difficulty in obtaining uniform results in the tests in this low-pressure range. When the impregnation pressure is 2.5 cm., or lower, a region of pressure is

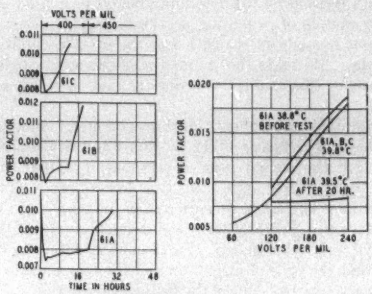

Fig. 4—Power-Factor-Voltage Characteristics and the Variation of Power Factor During the Life Test of Set 61

Impregnated at 40 cm. Hg.

reached in which the normal variations in the structure of the sample become so important that they may overshadow the effect of the reduction in air pressure. This is true despite the most extreme care in the manufacture and preparation to secure uniformity of the samples. One of the most important variations has been found to be the tightness of the assembled structure.

Variation of Power Factor During Life Tests. As already stated, power-factor measurements were made on all specimens before the start of the life tests. The power factor was also measured at 180 volts per mil at intervals during the life of the specimen. These observations brought out several interesting results. The characteristics of samples impregnated with compound A will be discussed first.

Fig. 4 shows the power-factor voltage curves for specimens of set No. 61, which were air dried to our

standard dry condition and then evacuated and impregnated at 40 cm. Hg. The pressure on these specimens was never reduced below 40 cm. so that this set may be considered good from the standpoint of moisture and poor from the point of entrapped air. The power-factor voltage curves are sharply rising as is characteristic of this type of specimen. The life test of 61-A has

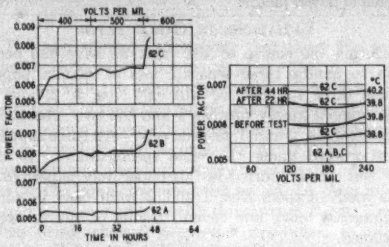

FIG. 5—POWER-FACTOR-VOLTAGE CHARACTERISTICS AND THE VARIATION OF POWER FACTOR DURING THE LIFE TEST OF SET 62

Impregnated 15 cm. Hg.

the immediate effect of bringing the power-factor curve down to an approximately horizontal position after 20 hours. During the test on the *A* specimen, the *B*, and *C* specimens are under compound at about 38 deg. cent., which seems to improve their power factor somewhat, although the curves of power factor are still sharply ascending. This apparent improvement has been noted before and shown to be only temporary (see earlier paper). However, under life test these curves flatten out as did the curve of specimen 61-A. While this indicates an improvement in the ionization factor of the insulation, there is evidently a corresponding deterioration as shown by the increase of power factor with time, also indicated in Fig. 4.

Our experiments show that this initial clean-up of the insulation becomes less noticeable as the amount of residual air in the insulation is decreased. This can readily be seen by referring to Fig. 5 for set No. 62. Here, since the pressure of impregnation was relatively high, 15 cm., the power-factor voltage curves are not as flat as those observed for our best specimens, but have a slightly rising characteristic. The power-factor time curves, Fig. 5, show no evidence of a decrease of power factor upon the application of voltage, but rather tend to an ever increasing power factor as the time of stressing lengthens. Furthermore, power-factor voltage measurements after 22 and 44 hours under stress show curves which lie successively higher.

Specimens impregnated at low pressure and dried to our standard condition show the typical flat power-factor voltage curves discussed in our earlier papers. In addition the power-factor time curves taken during the life of the insulation are of interest. Curves of this general type are shown in Fig. 6 for set No. 65. As a rule a slight rise in power factor is noted during the first two hours following the application of voltage after which the power factor remains constant at this increased value for the next 20 hours. At this point the voltage stress is, according to the standard procedure *A*, stepped up to 500 volts per mil. This increase in stress is also accompanied by an increase in power factor, followed by another 20-hour period without further change. The voltage increase to 600 volts per mil, which then takes place, is accompanied by another increase in power factor. The power factor then remains practically constant up to the breakdown. The large increase of power factor which would be expected to precede the breakdown takes place relatively rapidly, generally within four hours. This point is brought out in Fig. 6 for specimen 65-B. It is interesting to note that the power factor gives no indication of approaching breakdown except within this very short interval before failure. This seems to be strictly a characteristic of good specimens, *i. e.*, specimens impregnated at low pressure, while to the contrary, specimens impregnated at high pressure and known to contain air, show, as a rule, an ever increasing value of power factor from the start of the application of voltage to the point of breakdown. This rise follows the more rapid decrease in power factor which occurs, in these specimens, immediately upon the application of voltage, as noted above.

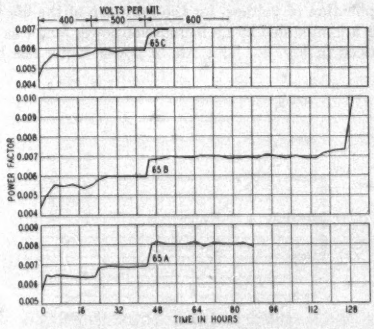

FIG. 6—VARIATION OF POWER FACTOR DURING THE LIFE TEST OF SET 65

Impregnated at 3 mm. Hg., compound *A*

In the studies made with the compound *B* the power-factor time curves have a somewhat different character. (Fig .7.) Fewer power-factor measurements were made during the life tests of this group of samples so that only the general tendency of the power-factor time curves is given. For this type of specimen we find also the initial rise of power factor found with the good samples of group *A*, but following this initial rise the power factor decreases steadily throughout the life of

the sample. The power factor doubtless rises just before breakdown, but it happened that no measurements were made during the short period before failure so that no rises are shown at the ends of the power-factor time curves for this group of samples.

Predetermination of Insulation Life. All through the life test work the data were carefully studied in an effort

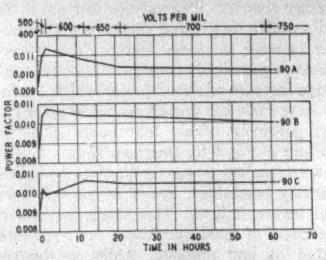

FIG. 7—VARIATION OF POWER FACTOR DURING THE LIFE TEST OF SET 90

Impregnated at 2 mm. Hg., compound *B*

to find some "yardstick" by which the life might be measured. Attention has been called to the fact that in all the life tests in which the power-factor voltage curves showed a flat or fairly flat characteristic the application of voltage, for the first step of the life test, was accompanied by an increase of power factor as was noticed in Fig. 6. This rise in power factor has been

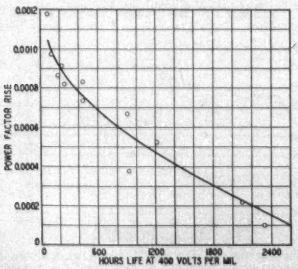

FIG. 8—RELATION BETWEEN INITIAL POWER-FACTOR RISE AND THE LIFE OF IMPREGNATED PAPER INSULATION

Compound *A*, specimens tested in parallel

carefully computed, for all the samples impregnated with compound *A*, and compared with the life of the insulation.

The results of this comparison are shown in Figs. 8 and 9. There is apparently a definite relation between this initial power-factor rise and the life of the insulation. Further study of the question is advisable. The relationship between power-factor rise and life might well be different for different kinds and types of insulation. It is quite possible, too, that under other conditions of test than those used here the effect might not be so evident. However, it appears to be sufficiently striking to warrant calling to the attention of the workers in this field. A similar analysis was not possible for compound *B*, owing to the small number of measurements of power factor.

VARIATIONS IN CABLE PAPER

During the course of the work we have had occasion to handle three separate lots of manila paper. These three supplies we will call Nos. 1, 2, and 3. The original supply was No. 1 and the two subsequent supplies Nos. 2 and 3, respectively. All three supplies were ordered under the same specifications. In the life test work reported in this paper only paper No. 3 was used. Papers Nos. 1 and 2 were used in some preliminary work and several results of interest were obtained.

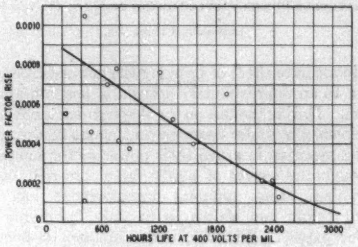

FIG. 9—RELATION BETWEEN INITIAL POWER-FACTOR RISE AND THE LIFE OF IMPREGNATED PAPER INSULATION

Compound *A*, specimens tested singly

In the earlier paper[1] we have described in detail our methods of drying and the use of the d-c. conductivity as a criterion of dryness. The work described in these earlier reports was all carried out with paper No. 1. The dry condition was indicated by a galvanometer deflection of 3.00 cm., twenty minutes after the application of 1,500 volts direct current, representing a resistivity of 6.02×10^{14} ohms per cm.^{3}. This dry condition was normally reached after drying for 72 hours at 105 deg. cent. in the open air or it was invariably obtained following a 48-hour treatment at a pressure of 2 mm. Hg. and 105 deg. cent.

An attempt to dry paper No. 2 to the foregoing standard condition met with failure although paper No. 2 was supposed to be the same as paper No. 1. In order to check the difference in these two papers two identical samples were made up, one with paper No. 1 (50E) and one with paper No. 2 (50D). These two specimens were placed side by side in the drying chamber and subjected to the same drying process. The results are given in Table II.

TABLE II

Specimen	Time in hours at 105 deg. cent.	Galv. defl. cm. 20 min. at 1,500 volts
50 D	24	77.0
Paper No. 2	48	42.5
	72	30.5
50 E	24	10.1
Paper No. 1	48	8.0
	72	7.15

This test showed conclusively that there was some inherent difference in the two papers. The matter was taken up with the manufacturer who informed us that the porosity of these two papers was somewhat different, No. 1 having a porosity corresponding to 322 seconds as compared to 100 seconds for paper No. 2. The manufacturer offered to supply another lot of paper having the same characteristics as No. 1. A group of specimens made up with this shipment of paper, No. 3, when dried, showed properties equivalent to paper No. 1. Table No. III shows the comparative drying data for the three papers.

TABLE III—DRYING DATA FOR THREE SHIPMENTS OF CABLE PAPER

Paper	Time to reach	Conductivity of cm.	Remarks
No. 1	In air 72 hr	3 cm	Standard conductivity 3 cm.
	Vacuum 24 hr	3 cm	Drying temp. 105 deg. cent.
No. 2	In air 5 weeks	Could not get down to 3 cm.	
	24 hr. vacuum	15.3—20.0	Could not get down to 3 cm. in 120 hr.
	48 " "	8.8—11.5	
	72 " "	6.4— 8.5	
	96 " "	4.8— 6.4	
	120 " "	4.5— 5.9	
No. 3	24 hr. vacuum	1.25—2.00	Equivalent to the original paper Deflection lower due to slightly thicker insulation wall

A group of specimens, set 51, made up with paper No. 2, was vacuum dried and impregnated at 2-mm. pressure. The results obtained are shown in Figs. 10, 11, and 12. Fig. 10 gives a comparison of the power-factor voltage curves of this set and those of set 4 which was made under similar conditions but using paper No. 1. The differences are very striking. The power factor of paper No. 2 rises with increasing voltage at all temperatures in contrast to the flat and decreasing power factor with increase of voltage observed for paper No. 1. The values of power factor are also higher, being almost four times as large at 60 deg. cent. Fig. 11 shows the correlation between power factor and absorption which we have discussed in our earlier papers and also brings out most markedly the difference in the two papers. Fig. 12 shows the charge and discharge curves for these two sets. Fig. 13 for set 55 made up with paper No. 3 shows tnat for this paper the power-factor voltage curves are in line with those of paper No. 1. The only difference noticed between papers Nos. 1 and 3 was that paper No. 3 was about 0.0005-in. thicker than paper No. 1.

Summarizing, paper No. 2 was much more difficult to dry than paper No. 1 and even after a prolonged period

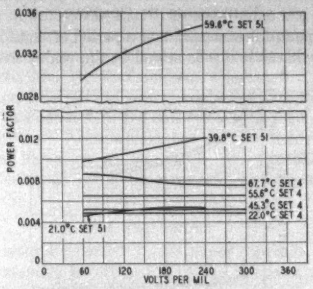

FIG. 10—RELATION BETWEEN POWER FACTOR AND VOLTAGE FOR TWO DIFFERENT CABLE PAPERS

Set 4—paper No. 1
Set 51—paper No. 2

of treatment failed to reach a conductivity as low as the normal value for paper No. 1. Impregnated insulation made up using paper No. 2 showed values of power factor higher than those found for paper No. 1 and the

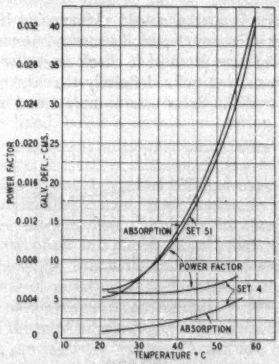

FIG. 11—RELATION BETWEEN ABSORPTION AND TEMPERATURE AND POWER FACTOR AND TEMPERATURE FOR TWO DIFFERENT CABLE PAPERS

Set 4—paper No. 1
Set 51—paper No. 2

increase of power factor with temperature was much more rapid. At 60 deg. cent., for example, numerical values four times as large as those for paper No. 1 were encountered. At the higher temperatures impregnated insulation using paper No. 1 always showed power-

factor voltage curves decreasing with increasing voltage, paper No. 2 shows rising power-factor voltage curves in the same high temperature region. We attribute the differences in the performance of papers 1 and 2 to inherent differences in their composition.

Variations in Compound

Compound *A* was used over a period of eight months. During this time it was stored in the sealed five-gallon

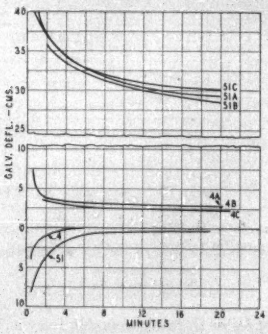

Fig. 12—D-C. Charge and Discharge Characteristics for Two Different Cable Papers

Set 4—paper No. 1
Set 51—paper No. 2

tins in which it was received. After using it for tests throughout a six-month period a marked decrease of life was observed in an attempt to check an earlier run. Investigation showed quite definitely that the compound had deteriorated. Specimens manufactured under con-

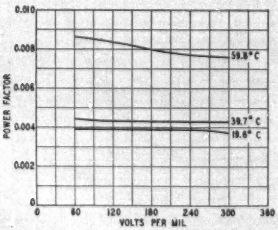

Fig. 13—Relation Between Power Factor and Voltage for Paper No. 3

ditions which earlier gave lives of over 2,000 hours would fail in less than half that time after the compound had been standing several months before using. Study of the data brought out the facts shown in Fig. 14. The d-c. conductivity and the increase in power factor from room temperature to 40 deg. cent. increased steadily with the age of the compound. We attribute this deterioration to the presence of air in the compound, although it was evident that a considerable degree of vacuum existed in the sealed container.

Conference with the manufacturer resulted in a new shipment of compound which was put up in 50-gallon drums in an atmosphere of CO_2. This is the compound *B*. Arrangements were made in the laboratory for

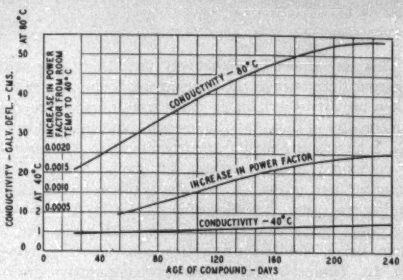

Fig. 14—Curves Showing the Deterioration of Compound with Age Due to the Presence of Air

preventing the exposure of the compound to air. As compound was drawn from the drums CO_2 was admitted. The compound was exploded under vacuum before impregnation, as has been described, in order to remove the CO_2 and subsequently the compound, as well as the impregnated insulation, was held in an atmosphere of nitrogen until actually placed in the high-voltage test box. Throughout the use of this com-

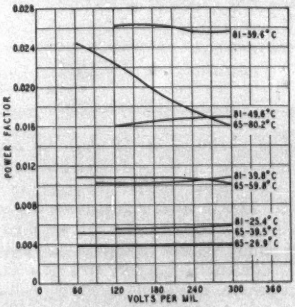

Fig. 15—Relation Between Power Factor and Voltage for Paper Impregnated with Two Different Compounds

Set 65—compound *A*
Set 81—compound *B*

pound, covering a period of ten months, no evidence of deterioration was found.

It has been shown already that this compound gave a much longer life to the impregnated paper than the original compound *A*. A comparison of the two from several other standpoints is of interest. Fig. 15 shows a com-

parison of the power-factor voltage characteristics and Fig. 16 of the power-factor temperature characteristics. It is significant that here the power-factor characteristics give absolutely no clue to the life of the insulation, the insulation with the lower power factor having the shorter life. Figs. 6 and 7 show the variation of power factor during the life of the insulation. Although the power factor increases very slightly with the age of the insulation the increase does not become sharp except very near the time of breakdown. Hence neither the absolute value nor the shape of the power-factor voltage curve provide a certain indication of the condition or life of the insulation.

Variations in Structure of the Insulation

An examination of the results of the life tests, in the pressure range from 2 mm. to 2.5 cm., showed that small variations in material and assembly of the structure of the sample become quite important in this pressure region.

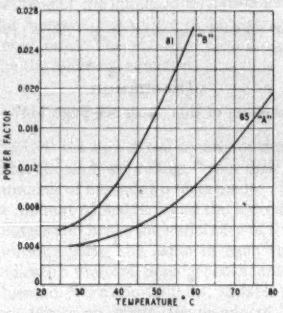

Fig. 16—Variation of Power Factor with Temperature for Paper Impregnated with Two Different Compounds

Set 65—compound *A*
Set 81—compound *B*

Several tests showed quite clearly the importance of the tightness of the structure. The usual electrode of the samples consisted of a sheet of 1/64-in. sheet lead bound on by a layer of linen thread. In every case the lead was carefully smoothed and formed to the insulation and the thread was wrapped on as nearly as possible at the same tension. With this construction the longest lived specimen, impregnated at 2 mm., gave a life of 12,000 hours at 400 volts per mil.

A sample was made up using, in place of the 1/64-in. lead, a sheet of lead foil 0.005-in. thick. This was bound on in the same manner as the 1/64-in. lead using the same thread tension. Due, however, to the thinness of the lead foil a much tighter construction resulted. The samples of this construction, impregnated not at 2 mm. but at 2.5 cm., showed lives of over 20,000 hours at 400 volts per mil, without failure. One specimen tested continuously at 800 volts per mil ran 188 hours in actual time, the equivalent of 48,200 hours at 400 volts per mil, without failure.

Acknowledgment

These studies were carried out in the School of Engineering at Johns Hopkins University with the support of the National Electric Light Association, through its subcommittee on Impregnated Paper Insulated Cable Research, Mr. W. F. Davidson, Chairman. For the continued interest and hearty cooperation of the members of this committee the authors extend their grateful appreciation. Acknowledgment is also made of the skill and devotion of Messrs. E. W. Greenfield and C. O. Newman, whole-time research assistants on the work.

Conclusions

The results reported in this paper conclude a somewhat extended investigation of the influence of small amounts of moisture and air on the properties of impregnated paper. The general conclusions, as related to the main purpose of the work, may be briefly stated:

The influence of residual moisture is universally bad and increasing amounts of such moisture are immediately reflected in higher values of power factor and loss and in other evidence of instability.

The chief importance of residual air is in its effect upon the life of the insulation. The smaller the amount of the residual air the longer the life. The accelerated life tests indicate, that so far as air content is concerned, under properly controlled conditions and with evacuation pressures well within reach in present day manufacturing methods, the life of impregnated paper may be extended to values far beyond those ordinarily supposed to pertain to the best present day cables. On the other hand, below a certain proportion of residual air, as represented say by complete impregnation at 1 cm. Hg. pressure, the further increase in life is not markedly great. This is due to the fact that below this value the increase in life due to reduced air is offset by normal and heretofore uncontrolled variations in the uniformity and quality of oil and paper. Low pressures of evacuation, therefore, are more important for the further withdrawal of moisture than for the extension of ultimate life as dependent on air content.

One of the most interesting conclusions is the evidence that flat power-factor voltage curves are obtained in insulation evacuated and impregnated at pressures up to 10 or 15 cm. Hg. The rising power-factor voltage curve commonly attributed to internal ionization is found only in poorly impregnated samples. There is some evidence that this type of curve may be caused by oil of originally poor quality in the presence of somewhat large quantities of air.

During the course of the work the successive steps in the control of several unforeseen variables have brought out much interesting information on the original and final properties of both paper and oil, the

methods for their control, the stability of each, and other factors bearing on the properties of the completed product. Some of these are reported in the present paper and for others the foregoing papers must be consulted.

The conclusions of the present paper are briefly as follows:

1. The life of impregnated paper insulation increases steadily with a decrease in the amount of residual air.

2. For impregnating pressures above 2.5 cm. the life decreases rapidly with increasing pressure of impregnation. For pressures below 2.5 cm. the results show an increase of life of from 10 to 50 per cent between 2.5 cm. and 2 mm. impregnating pressure.

3. For impregnating pressures below 2.5 cm. normal variations in material and structure, particularly in the tightness of the structure, may offset the increased life due to decreasing the pressure of impregnation.

4. Cable compound deteriorates if it contains only a small amount of air even though it is kept in sealed containers, at room temperature. Air should be entirely excluded to avoid deterioration.

5. Flat power-factor voltage curves and the absolute value of power factor should not be accepted as indications of long life.

6. The power-factor rise due to the initial application of voltage during a life test is indicated as a possible criterion of insulation life.

7. A very satisfactory type of sample and reenforced end for accelerated life tests have been developed.

8. Narrow variations in the quality of both paper and oil may cause wide differences in the electrical characteristics and in the lives of the impregnated products.

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Discussion

For discussion of this paper see page 1447.

Insulation Variability

Its Influence in Determining Breakdown Voltages*

BY M. C. HOLMES†

Associate, A. I. E. E.

Synopsis.—The purpose of the paper is to describe a method of analysis for taking into account the effect of insulation variability in the determination of breakdown voltages and allowable operating stresses. The method is applicable to systems containing large areas of insulation, or large numbers of units of insulation in parallel.

General equations are derived, giving the breakdown voltage and number of failures to be expected, in terms of two parameters; the mean breakdown voltage and the variability of the insulation, both of which are determined from tests made on samples. It is shown how the results may be applied to the special case of electric power cables, for comparing the relative merits of different cables, and for determining the safety factor required to take account of the inherent variability and consequent unreliability of the insulation.

The necessity of considering the variability as well as the so-called strength of insulation is emphasized, and its importance brought out in terms of allowable operating stresses and failures.

A graphical method of analysis is described, which makes use of probability paper, changing the probability scale from "per cent" to "failures per hundred miles per year" in the case of cables.

Predictions of theory are compared with the results of practise, using results from experiments made to determine the effect of electrode area on breakdown voltage. Good agreement is shown between the two.

Introduction

In 1927, in a paper by Messrs. Bush and Moon[1] the importance of the standard deviation as a measure of the inherent variability of insulating materials was pointed out. The standard deviation, sigma, is defined by the following equation:

$$\sigma = \sqrt{\frac{\Sigma\,(\delta)^2}{n}} \qquad \text{(a)}$$

where $\Sigma\,(\delta)^2$ is the sum of the squares of the deviations of individual breakdown voltages from their mean, and n is the number of breakdown tests made. Sigma indicates the amount of scattering of the individual breakdown voltages and hence the variability of the material, test conditions being the same in each case. Small values indicate a uniform, homogeneous material; while large values indicate a non-uniform, variable material.

Curves, based on many thousands of breakdown tests, were given showing the manner in which breakdown voltages are distributed. It was shown that such voltages are distributed about the mean, E_a, approximately in accordance with the normal error curve,

$$y = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\frac{(E-E_a)^2}{\sigma^2}} \qquad \text{(b)}$$

the distribution being completely specified by the two parameters σ and E_a.

These two conceptions,—standard deviation as a measure of insulation variability, and normality of breakdown voltage distributions,—form the starting point and constitute the assumptions of the following theoretical development.

Theoretical Considerations

a. Coefficients of Variability and Uniformity,—Definitions. In specifying the variability of a material by the use of sigma, the mean must also be known. If the standard deviation, sigma, alone is given, no idea of the variability is gained, because the actual magnitudes of both sigma and the mean breakdown voltage E_a depend upon test conditions such as size and shape of electrodes, thickness of insulation, temperature, rate of application of voltage, etc. It happens, however, that both sigma and E_a vary in the same manner, since they both depend upon the same individual readings, so that their ratio remains the same. In other words, it is the ratio σ/E_a, or sigma expressed in per cent of E_a, which is a constant for a given material and which specifies its variability. The ratio σ/E_a, therefore, will be used here to represent variability. It will be called the "coefficient of variability" and will be represented by the symbol S. Its reciprocal, designated by the symbol H, may be taken as a measure of the uniformity of the insulation and called the "coefficient of uniformity."

b. The General Equations. Making use of the foregoing conception of variability and the assumption of normality in the distribution of breakdown voltages the following two general equations have been derived (see Appendix I for their derivation).

$$F = N\left[\frac{1}{2} - erf\left(\frac{1 - E_b/E_a}{S}\right)\right] \qquad (1)$$

$$E_b = E_a\left[1 - S\,.\,erf^{-1}\left(\frac{1}{2} - \frac{1}{2N}\right)\right] \qquad (2)$$

The first equation gives the number of failures, F, to expect in a system consisting of N units of insulation

*Paper based on work done as a graduate student at the Massachusetts Institute of Technology.

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1. For references see Bibliography.

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in parallel at an operating voltage E_o,—the units being removed from the system after failure. F is given in terms of the two parameters E_a and S, which taken together specify the quality of the insulation; E_a being the mean breakdown voltage of the units, and S the coefficient of variability. The second equation gives the breakdown voltage, E_b, to expect for a system of N units of insulation connected in parallel, in terms of the same two parameters E_a and S.

The symbol *erf* is an operator used to designate the Gaussian error function: thus, *erf* (k) is defined by the expression

$$erf(k) = \frac{1}{\sqrt{2\pi}} \int_0^k e^{-\frac{k^2}{2}} dk \qquad (3)$$

The symbol erf^{-1} represents the inverse operation: thus, $erf^{-1}(k)$ means the quantity whose error function is k. These are well-known functions whose values have been computed and tabulated. They are analogous to the two symbols sin and $\sin^{-1}$, and their values may be obtained from tables[2] in a similar manner. They are not always designated by the symbols *erf* and erf^{-1}, however, but often simply by the expression (3) or by an equivalent expression.

In case error function tables are not available, equation (1) may be put in the form of an infinite series. Letting $(1 - E_o/E_a)/S = k$, for small values of k, equation (1) takes the form

$$F = \frac{N}{\sqrt{2\pi}} \left[\sqrt{\frac{\pi}{2}} - k + \frac{k^3}{2 \,.\, 3 \,.\, \underline{|1}} - \frac{k^5}{2^2 \,.\, 5 \,.\, \underline{|2}} + \frac{k^7}{2^3 \,.\, 7 \,.\, \underline{|3}} - \ldots \right] \qquad (4)$$

This series is convergent, and the error is less than the last term retained.

For large values of k the above series converges too slowly for practical use, in which case it can be put in the following form:

$$F = \frac{N}{\sqrt{2\pi}} \frac{e^{-\frac{1}{2}k^2}}{k} \left[1 - k^{-2} + 1 \,.\, 3\, k^{-4} - 1 \,.\, 3 \,.\, 5\, k^{-6} + \ldots\right] \qquad (5)$$

This is an "asymtotic" or semi-convergent series, but the error, again, is less than the last term retained.

c. Discussion. The two equations (1) and (2) are entirely general. They apply to all kinds of insulation, either in the form of continuous areas, as in cables, or in the form of discrete units, as in overhead transmission lines. In the former case N represents length of cable, measured in terms of some standard test length; in the latter case, N represents the number of units of insulation under voltage stress,—the number of strings of insulators along an overhead line, for example:

In the case of thin sheets of insulation, N represents area, and equation (2) gives breakdown voltage as a function of electrode area. Milnor's equation for the effect of electrode area,[3] $E_b = C_1 - C_2 \log A$, is an approximation to the probability expression for the same thing, obtained by considering the elementary areas, making up the total area A, indefinitely small and passing to the limit. It is valid only in cases where E_b is negligibly small compared with the mean breakdown voltage of the elementary areas. The two constants C_1 and C_2 are evaluated in each case by fitting a curve to data obtained from tests made on different size electrodes. The advantage of equation (2) over Milnor's equation is that it allows predictions to be made for any size electrode, from tests made on a single size, and in addition, takes into account the effect of insulation variability. The variability determines how great will be the effect of area. With perfectly uniform insulation, for example, area would have no effect on breakdown voltage.

A useful relation may be obtained by solving equation (1) for E_o; giving

$$E_o = E_a \left[1 - S\, erf^{-1}(1/2 - F/N)\right] \qquad (6)$$

an expression which gives the allowable operating stress corresponding to F failures per N units of insulation. Now, if F is taken as unity and N is taken as 10,000, for example, the operating stress corresponding to one failure per ten thousand units is given by the simple relation $E_o = E_a(1 - 3.72\, S)$, showing clearly the importance of the part played by the insulation variability in determining the allowable operating voltage for a given system. Similar expressions for different specified conditions may be obtained from the general equations, in like manner, by assigning particular values to the parameters. The extremely simple relation

$$E_o = E_a(1 - 4\, S), \qquad (7)$$

in the case of cables, gives the allowable operating voltage corresponding roughly to one failure per 100 miles of cable per year, where E_a is the mean breakdown voltage of standard 10-ft. test lengths reduced to one year basis (See Appendix II). Such a relation may be used for comparing the merits of different cables or made the basis of acceptance requirements and specifications.

In order to avoid misconception as to the scope or applicability of the present paper it should be emphasized that the foregoing theory takes account only of causes inherent in the insulation itself. It takes no account of external causes such as edge effect which might be encountered with different electrodes, nor of abnormal operating conditions such as surges and overloads which might be encountered in practise.

Experimental Considerations

a. Values of S Encountered in Practise. The following values of S have been calculated from the experiments of Hayden and Eddy[4] on the dielectric strength of oil.

Curve No.		S
I	Transil oil, 1 cm. spheres, 2 mm. apart	10.7 per cent
II	Transil oil, 1 in. spheres, 27 mm. apart	10.2 per cent
III	Transil oil, needle and 1 in. sphere, 2 mm. apart	9.85 per cent
IV	Commercial benzol, 1 cm. spheres, 2 mm. apart	6 4 per cent
V	Pure benzol, 1 cm. spheres, 2 mm. apart	4.82 per cent
VI	Air, 1 inch spheres, 1 cm. apart	1.94 per cent

Records of breakdown tests on other insulating materials, and on cable samples, are scattered throughout the literature. In general, breakdown tests made with air as a dielectric show the greatest uniformity and may be expected to result in values for S of 1 or 2 per cent. Such determinations include errors of measurements, as well as those due to irregularities in electrode surfaces, changes in pressure, temperature, etc. For tests made on thin layers of paper, cambric, etc., one may expect values ranging from 6 to 12 per cent. For the present day "solid" type of paper-insulated cable, values ranging all the way from 6 to 20 per cent may be obtained. A value as low as 6 per cent indicates a comparatively uniform insulation; while values as high as 20 per cent should be taken as an indication of poor, non-uniform insulation. Such conclusions also result from theoretical considerations based on the general equations, taking account only of the effect of variability: a value of 6 per cent indicates an allowable operating stress of approximately 75 per cent of the average breakdown stress E_a, whereas, for the same conditions (one failure per 100 miles of cable), a value of 20 per cent gives an allowable operating stress of only 17 per cent.

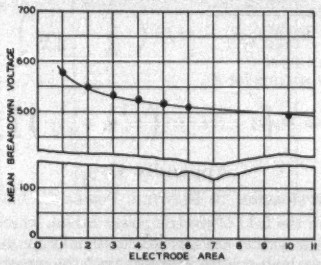

FIG. 1—COMPARISON OF THEORY AND EXPERIMENT (FROM THE RESULTS OF MOON'S EXPERIMENTS ON THE EFFECT OF ELECTRODE AREA)

Tests made on 0.55 mil paper. Electrode No. 1. Run No. 91. $S = 8.25$ per cent

b. Predictions of Theory Compared with Results of Experiment. Figs. 1 and 2 show curves comparing the predictions of theory with the results of experiment. Data for the first curve were obtained from Moon's experiments on the effect of electrode area.[5] The second curve is from Emerson and Webber's experiments.[6] They are both typical. The curves show the predictions of theory; the points show the results of experiments. The curve of Fig. 1 was calculated from the results of 917 breakdown tests represented in the initial point at unit area. In Fig. 2 each point represents the average of only 10 tests. For this reason the points are much more irregular. The theoretical curve fits the points as well as can be expected.

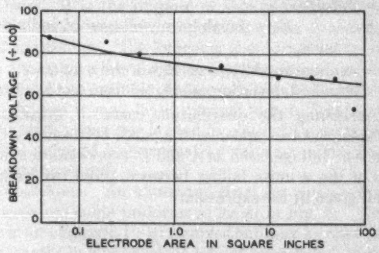

FIG. 2—COMPARISON OF THEORY AND EXPERIMENT ON THE EFFECT OF ELECTRODE AREA. (FROM EMERSON AND WEBBER'S EXPERIMENTS)

Tests made on 5 mil varnished cambric, using flat circular electrodes immersed in oil. Voltage raised 900 volts per second to failure. Average value of $S = 6.9$ per cent.

SUMMARY

1. A general analytic method for the determination of failures, breakdown voltages, and operating stresses, taking account of the mean strength, variability and area of insulation, has been developed.

2. A good agreement is shown between the predictions of theory and the results of experiment.

3. The two factors, mean strength and variability, are equally important in the determination of allowable operating stresses and should always be considered in conjunction.

4. In the Appendix the conditions for accuracy and reliability have been investigated. It is shown that the accuracy depends upon the variability of the insulation itself as well as upon the number of specimens tested, decreasing rapidly with increase in variability. Ordinarily, fairly accurate determinations can be made from samples as small as twenty. Samples as small as ten may be used for obtaining indications of quality, but judgments based on fewer than ten tests would be of questionable validity.

5. A graphical method of analysis, making use of probability paper, is described in the Appendix. It furnishes a test for the normality of distributions and shows how non-normal distributions may be analyzed. A method for dealing with cable problems, where the additional factor of deterioration with time is important, is also outlined.

Appendix I

Derivation of General Equations

a. Number of Failures. Consider a sample of n units of insulation, all of the same size, selected at random from a large lot and subjected to breakdown tests,—all tests being made under the same conditions. (The word "sample" is used here in the collective sense, meaning the entire collection of n units or specimens.)

Let:

n = number of units, or specimens, in the sample.

N = total number of units in the entire lot or system from which the sample has been selected.

E = voltage.

$E_1, E_2, \ldots E_n$ = breakdown voltages of individual units.

E_a = average breakdown voltage of the n units.

σ = standard deviation calculated from test results.

Considering the distribution curve of breakdown voltages, the area under such a curve between any two values of voltage, such as A and B, represents the fraction of the n units failing between those two values, and is given by the expression

$$\frac{f_{AB}}{n} = \frac{1}{\sigma\sqrt{2\pi}} \int_A^B e^{-\frac{1}{2}\frac{(E-E_a)^2}{\sigma^2}} \, . \, dE \tag{8}$$

where f_{AB} is the actual number of failures occurring between A and B. The total area under the curve, therefore, equals unity, and the two areas on either side of E_a each equals ½. The expression for the fractional number of failures between $E = O$ and any voltage E, then, is given by the expression

$$\frac{f_E}{n} = \frac{1}{2} - \frac{1}{\sigma\sqrt{2\pi}} \int_E^{E_a} e^{-\frac{1}{2}\frac{(E-E_a)^2}{\sigma^2}} \, . \, dE \tag{9}$$

where f_E represents the actual number of failures corresponding to the voltage E. For the entire system of N units, and an operating voltage E_o, this will correspond to a number of failures given by the expression

$$F = \frac{N}{n} f = N\left[\frac{1}{2} - \frac{1}{\sigma\sqrt{2\pi}} \int_{E_o}^{E_a} e^{-\frac{1}{2}\frac{(E-E_a)^2}{\sigma^2}} \, . \, dE\right] \tag{10}$$

This expression may be written in a much simpler form by introducing a new variable $k = (E_a - E_o)/(\sigma) = (1 - E_o/E_a)/(S)$, multiplying by $dE/dk = -\sigma$, to take account of the change of variable, interchanging the limits of integration and changing the sign at the same time, giving

$$F = N\left[\frac{1}{2} - \frac{1}{\sqrt{2\pi}} \int_0^k e^{-k^2/2} \, . \, dk\right] \tag{11}$$

and finally, in still more compact form,

$$F = N\left[\frac{1}{2} - erf\,(k)\right] \tag{12}$$

$$= N\left[\frac{1}{2} - erf\left(\frac{1 - E_o/E_a}{S}\right)\right] \tag{13}$$

b. Breakdown Voltage. From the distribution curve of breakdown voltages it is evident that at a voltage corresponding to the mean, E_a, approximately one-half the total number of units considered will have failed. If the size of unit is taken as N, and F in equation (12) is taken as 1/2, the value of voltage, E_o, implicit in the expression will represent the mean breakdown voltage of systems consisting of N of these units. Doing this, there results

$$1/2 = N\,[\,(1/2) - erf\,(k)\,] \tag{14}$$

Transposing,

$$erf\,(k) = (1/2) - (1)/(2\,N) \tag{15}$$

From which,

$$k = erf^{-1}\left(\frac{1}{2} - \frac{1}{2\,N}\right) \tag{16}$$

Putting for k its value $(1 - E_o/E_a)/(S)$, and replacing E_o by the symbol E_b, now, to denote mean breakdown voltage of systems containing N units of insulation, there results the equation

$$(1 - E_b/E_a)/(S) = erf^{-1}\left(\frac{1}{2} - \frac{1}{2\,N}\right) \tag{17}$$

Whence, solving for E_b,

$$E_b = E_a\left[1 - S\,erf^{-1}\left(\frac{1}{2} - \frac{1}{2\,N}\right)\right] \tag{18}$$

Appendix II

Application to Electric Power Cables

It is in the field of electric power cables, where huge areas of insulation are exposed continually for years at a time to high-voltage stresses, that the question of variability is of the greatest importance. Probably it is in this field that the theory may be most advantageously applied.

In applying the theory to cables an additional factor must be taken into account; namely, the influence of time. Experiments on the deterioration of oil-impregnated cable insulation under voltage stress have shown that the life varies approximately inversely as the seventh power of the voltage stress[7] for three-conductor cable. It has been suggested, in the N. E. L. A. report for 1926-27, that all breakdown tests on cable be referred to the same time basis by means of the so-called inverse seventh power relation. Unfortunately the relation has been verified only for times up to about 1,000 hours. The decrease in breakdown voltage from 1,000 hours to one year, indicated by the inverse seventh power

relation, is only 25 per cent (*i.e.* from 100 to 75 per cent). Probably no great error will result, therefore, from assuming it to hold for times up to one year,—at least that is the only way available at present for estimating the effect of time. In the case of cables, then, the breakdown voltages are assumed to be referred to some common time basis before calculating the effect of length and variability. This relation may be written, $E = E_1 t^{-1/7}$, where E is the breakdown voltage for the time considered, E_1 is the breakdown voltage for unit time, and t is the number of units of time to which E_1 is to be referred.

Illustrative Problem. Suppose a number of long-time breakdown tests to have been made on standard 10-ft. samples of cable and referred to some common time

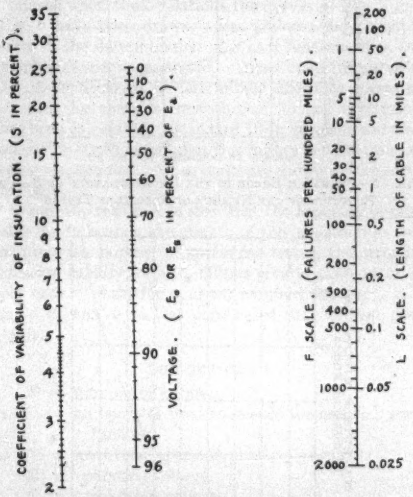

FIG. 3—ALINEMENT CHART FOR ELECTRIC CABLE CALCULATIONS. (BASED ON 10-FT. TEST SAMPLES)

Giving: (*a*) Breakdown voltage as a function of length and variability (*b*) Failures in terms of operating voltage and variability

Voltage scale represents: (*a*) Operating voltage when used with F scale (*b*) Breakdown voltage when used with L scale

basis, say 100 hours, and that the results indicate a mean breakdown voltage E_o of 220 volts per mil and a variability S of 12 per cent. Let it be required to determine the allowable operating stress for such cable, assuming one failure per 100 miles of cable per year.

The reduction in stress, for taking account of insulation variability, is made using equation (7) as follows:

$$E_o = E_o (1 - 4\,S) = 220\,(1 - 4 \times 0.12)$$
$$= 114.4 \text{ volts per mil}$$

A further reduction, for taking account of time, may be made, reducing the 114.4 volts per mil from a 100-hour basis to a 1-year basis. If this is done by means of the inverse seventh power relation the final value of E_o becomes,

$$E_o = 114.4\, t^{-1/7} = 114.4 \times (87.6)^{-1/7} = 60.4 \text{ volts per mil}$$

The final value of E_o thus found considers only the effects of insulation variability and time. It is therefore the upper limit. If it is exceeded, more than the allowed one failure per 100 miles of cable may be expected. It should be noted that the inverse seventh power law is used here merely for an illustration, suggesting how the time factor may be taken care of. If a sixth power or some other law should be established at some future time it can be used in place of the one used here. It is the purpose of this paper merely to show the dependence of the operating stress upon insulation variability. The foregoing should not be taken as an argument either for or against the seventh power relation. As an example of the magnitude of the difference involved it may be of interest to note that if an inverse sixth power law is used in place of the seventh, the final value of E_o becomes 54.4 volts per mil instead of 60.4 volts per mil.

Computations. An alinement chart for convenience in calculating cable problems is given in Fig. 3. The chart is for standard 10-ft. samples but may be used for other lengths by applying the proper correction factors. For example, if test samples of x ft. instead of 10 ft. are used, the number of failures, F, obtained from the chart must be multiplied by the ratio $10/x$.

The use of the alinement chart may be illustrated by the following example. Suppose it is desired to find the per cent decrease in operating voltage to allow for a five-mile length of cable whose insulation has a coefficient of variability, S, of 14 per cent. A straight edge is laid on the chart so that it intersects the S scale at 14 and the L scale at 5. The intersection on the voltage scale then gives 50, indicating a necessary reduction in operating voltage of 50 per cent.

Appendix III

A Graphical Method of Analysis

Probability paper furnishes what is undoubtedly the simplest and most powerful method of analyzing a given set of test data. It was developed in 1914 by Mr. Allen Hazen[8] while studying variations in annual run-off of streams, for the purpose of predetermining the sizes of impounding reservoirs required in municipal water supplies. The rulings on one of the coordinates are spaced in accordance with the normal law of error, so that data which follow the normal law will form a straight line when plotted cumulatively on such paper.

In order to illustrate its use a specific example will be considered. Fig. 4 shows a hypothetical set of data, representing the results of 50 breakdown tests, plotted on such paper. The voltage gradients at breakdown are assumed to have been calculated and tabulated in ascending order of magnitude, and the results plotted cumulatively in the same order. Abscissas represent

breakdown stress, while ordinates represent failures in per cent of the total number of specimens tested. With 50 as the total, the first one will represent 2 per cent, the second 4 per cent, etc. Any point, however, is indeterminate between 2 per cent limits. It is customary, therefore, to plot each point in the middle of its range of indeterminancy. Thus, the first one, which failed at a stress of 45 volts per mil, is plotted on the 1 per cent line, the second one on the 3 per cent line, etc.

The abscissa, at the point where the curve cuts the 50 per cent line, represents the mean, or median, breakdown stress E_s of the samples. The slope is a measure of the coefficient of variability, S, of the insulation. A perfectly uniform material would plot as a vertical line.

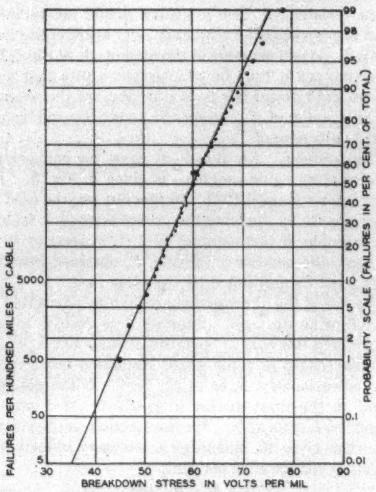

Fig. 4—Hypothetical Set of Breakdown Tests Plotted on Probability Paper

An auxiliary scale, making use of the fact that the most probable number of events is equal to the probability of a single event multiplied by the total number of possible events or trials made, may be constructed. For example, if in the present problem each cable specimen had been 10.5 ft. long, 100 miles of cable would correspond approximately to 50,000 such units and the 10 per cent line would represent 50,000 × 0.10 = 5,000 failures per 100 miles of cable, 1 per cent would correspond to 500 failures, etc. Such a scale is shown at the left margin of Fig. 4.

Some of the essential points of the theory are brought out when the data are plotted thus on probability paper. For example, if the curve is extended downward to the working range, around 5 failures at 35 volts per mil, it is evident that the process is one of extrapolation, as are all sampling processes. It shows the necessity for considering both the mean breakdown voltage *and* the variability. Also, it will be noted that the upper points do not fall on the same straight line formed by the lower set of points. This means that only the lower half of the distribution follows the normal law and shows how data may be analyzed even though they do not follow the normal law as a whole, as was assumed in the

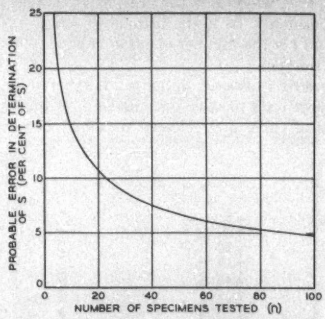

Fig. 5—Probable Error in the Determination of S as a Function of the Number of Specimens Tested

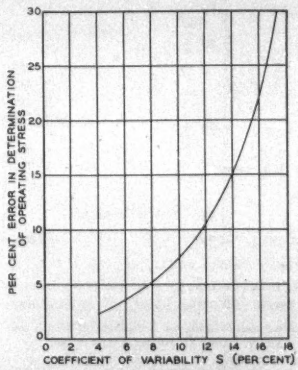

Fig. 6—Error in Determination of Allowable Operating Stress as a Function of S

(For samples consisting of twenty test specimens)

derivation of the equations. It also shows the relative unimportance of samples representing high dielectric strength.

Appendix IV

Accuracy and Reliability

In statistical work, it is customary to employ a quantity known as the probable error to indicate the accuracy of a determination made from a series of ob-

servations. The probable error is defined as the deviation, on either side of the mean, such that half of the observations are included within such a range. Thus, $x = 12.73 \pm 0.27$ means that the quantity x has been determined by statistical processes, such as averaging, to be 12.73 with a probable error of 0.27; or, it is an even bet that the true value of x lies between 12.46 and 13.00, and that the odds are three to one that x is not greater than 13.00.

Error theory considerations give the probable error in determining the mean, by sampling processes, as $0.6745\ \sigma/\sqrt{n}$, where σ is the standard deviation of the lot from which the sample is drawn, and n is the number of specimens in the sample. The probable error in the determination of the standard deviation is $0.6745\ \sigma/\sqrt{2n}$.

Based upon these relations the curves of Fig. 5 and Fig. 6 have been drawn. Fig. 5 shows the probable error in the determination of S as a function of n, the number of specimens tested. Based upon the curve of Fig. 5 the accuracy in determining allowable operating stresses, for samples consisting of 20 test specimens, has been calculated. Standard 10-ft. specimens and an operating stress based upon one failure per 100 miles of cable are assumed. The results are shown by the curve of Fig. 6.

From the results it is seen that the accuracy obtainable depends upon the quality of the insulation as well as upon the number of specimens tested, the error increasing rapidly for variabilities greater than 14 or 15 per cent. Thus, for a given required accuracy, it is more expensive to test poor cable than to test good cable.

List of Symbols

F = number of failures.
erf = an operator used to denote the gaussian error function.
erf^{-1} = an inverse operator, analogous to $\sin^{-1}$.
E_o = operating voltage.
E_b = breakdown voltage.
E_a = mean breakdown voltage of test specimens.
S = coefficient of variability = standard deviation expressed as a decimal fraction of E_a.
H = coefficient of uniformity = $1/S$.
k = $(E_a - E_o)/\sigma = (1 - E_o/E_a)/(S)$.
n = number of test specimens in sample.
N = general symbol used to represent number of units of insulation, the unit being the size of test specimen, = area in the case of sheet insulation, length in the case of cables, and number of insulators in the case of overhead lines.

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Discussion

ECONOMICS OF HIGH-VOLTAGE CABLE
(Roper)

CHARACTERISTICS OF OIL-FILLED CABLE
(Shanklin and Buller)

OIL-FILLED CABLE AND ACCESSORIES
(Atkinson and Simmons)

RESIDUAL AIR AND MOISTURE IN IMPREGNATED PAPER INSULATION—III
(Whitehead and Hamburger, Jr.)

INSULATION VARIABILITY
(Holmes)

G. B. Shanklin: Mr. Holmes' work is of the highest order, theoretically. There are many different factors that might and do cause cable failures in service. Mr. Holmes' theory would be applicable only in the case of inherent insulation failures uniformly produced by overstressing the dielectric under the particular conditions assumed. Few if any failures in modern cable are due to this cause.

The paper by Dr. Whitehead and Mr. Hamburger is a very valuable contribution to our knowledge of the effects of gas and moisture, as well as other impurities, when allowed to exist in high-tension cable. I do not think we need any additional evidence to prove the desirability of eliminating all traces of gas and other impurities, both free and in solution, and preventing their after occurrence in service. The results presented in this paper, as well as other experiences, are too convincing. The whole principle of the oil-filled cable is based on this.

This paper does not, however, show us *how* to eliminate impurities. High vacuum during the drying operations is certainly not the entire answer, as far as initial factory impurities are concerned. Nor do I believe that results obtained in a university laboratory can be used reliably as a practical guide for factory production methods. In removing impurities from cable, each manufacturer has his own distinct problems, depending upon the kind of materials and production equipment used. It would be impossible to have a standardized process equally and safely applicable to all manufacturing plants. Cable quality should be judged by the principles followed and results obtained. Any other criterion would be decidedly dangerous.

The paper by Messrs. Atkinson and Simmons is in good agreement with our own in the conclusion that oil-filled cable is the right answer for high-voltage service. My comments are confined to the condenser principle so cleverly introduced in their joint and terminal designs. There are undoubted possibilities in the use of this condenser principle. This idea is not employed in our present designs of stop joints, normal joints and terminals, and the reliability of these is well proven by several

years of successful service. To meet this competition the condenser designs must prove equally good, or better, electrically and mechanically, and of equal or lower cost. Compactness is also a factor to be considered. In my opinion, the condenser principle is more applicable to stop joints than to normal joints and terminals. In the interests of progress the comparative advantages and disadvantages of these designs should be carefully weighed. Additional experience will indicate the right trend.

We have carefully checked the economic data given by Mr. Roper and have few suggestions or criticisms that would add to their value.

As Mr. Roper ably points out, an economic comparison of ordinary "solid" cable and oil-filled cable reaches its highest point of interest at a voltage rating of 66,000. This might be summed up briefly by saying that oil-filled cable is more economical than solid cable above 66-kv. rating. Also, 66-kv., three-conductor, oil-filled cable is more economical than single-conductor, solid cable up to 50,000-kva. per circuit. With the exception of 45-kv., three-conductor, oil-filled cable, solid cable is the more economical below the limits outlined above. The economic dividing line at the present time, that is of real interest, has to do with 66-kv., single-conductor cable of 50,000-kva. per circuit and higher.

The comparison in Mr. Roper's paper would indicate that within this range single-conductor, solid cable is slightly more economical than oil-filled cable, although he points out that only a small reduction in cost of oil-filled would place the two on an equitable basis. Even since Mr. Roper prepared the data for his paper there have been reductions in cost of 66-kv. oil-filled lines. This has been accomplished as follows:

(a) Reduction in insulation thickness from 0.40 and 0.42 inches to 0.35 and 0.375 inches. This has reduced the comparative cost of cable from 2 to 5 per cent.

(b) A reduction of approximately 25 per cent in cost of oil reservoirs due to simplifications in production.

(c) Increase in guaranteed maximum copper temperature from 70 to 75 deg. cent.

A comparative cost tabulation is given below in which these reductions are included. All other items are as presented by Mr. Roper. His cost of 66-kv., single-conductor cable is used as a standard of reference and the cost of oil-filled is expressed as a percentage of this.

ECONOMIC COMPARISON OF SINGLE-CONDUCTOR, 66-KV., SOLID AND OIL-FILLED CABLE LINES: RELATIVE INVESTMENT COST—NO TERMINAL

Kva. rating	Solid 60° C. copper 15° C. earth D. W. Roper	Oil-filled 70° C. copper 15° C. earth D. W. Roper	Oil-filled 70° C. copper 15° C. earth revised	Oil-filled 75° C. copper 15° C. earth revised
50,000	100%	115.5%	113.5%	113.0%
75,000	100%	106.0%	105.5%	102.0%
100,000	100%	103.5%	100.0%	95.4%
125,000	100%	105.0%	100.0%	96.0%

Because of higher copper loss the annual cost is more favorable to solid cable, but I think this tabulation shows clearly that progress is being made and that eventually the cost of 66-kv. oil-filled cable will in all cases be less than solid. At the present time it is less only for loads below 50,000 kva. and above 100,000 kva.

Mr. Roper, as everyone knows, is a strong advocate of reduction in insulation thickness as a means of further cost reduction. It is an obvious duty of the manufacturers to cooperate as far as judgment dictates. There has been material reduction in insulation thickness of solid cable in recent years and as already pointed out there has been a corresponding reduction in oil-filled cable. In my opinion it would be a mistake to move too rapidly in this direction.

When all possible angles of the situation are considered, it is an open question whether the additional cost reduction that might be accomplished in this way is sufficiently noticeable to compensate for the undoubted sacrifices involved. Reduction in cost of accessories, methods of installation, conduit structure, etc., have greater possibilities.

As an example, assume that the cost of solid cable is reduced 5 per cent by a corresponding reduction in insulation thickness. This means only a saving of about 2.5 to 3 per cent in over-all cost, but it means a far greater reduction in factor of safety. No one would hesitate to pay this much for an insurance policy covering a lesser risk. What I wish to bring out is that a reasonable margin in the way of insulation thickness is a very cheap form of insurance.

It must be considered also that operating requirements are, today, very exacting. More than two or three failures per 100 miles of cable per year is considered a rather poor record and the manufacturer's reputation suffers accordingly. This obviously demands a greater factor of safety than an acceptance of, say, 25 failures per 100 miles per year.

F. A. Brownell: It seems highly improbable that a mathematical solution such as given in Mr. Holmes' paper could be applied to determine the number of failures of a material as temperamental as a power cable. But with more uniform manufacture of cable it seems plausible that the law of probability might be applied where data were based on a relatively large number of test specimens.

We have some very good data on the behavior of cables both under test conditions and in service. The results of the tests of 28 fifteen-foot specimens of cable made by three different manufacturers were used to determine the variability factor and the equivalent volts per mil per year. The variability factor was found to be 4.7 per cent and by using the inverse 6th power law the equivalent volts per mil per hour were 353 or 78 volts per mil per year. This value is somewhat low as compared with the results obtained at other laboratories.

The value of F the number of failures per 100 miles per year was 0. In 1930 the Public Service Electric & Gas Co. of New Jersey had 99.72 mile years of this type of cable in operation with no inherent cable failures.

Using the graphic method on page 1445 with $S = 4.7$ per cent and one failure per 100 miles of cable we find an allowable operating voltage of 80 per cent of E_a or 62.5 volts per mil per year or 37 kv., three-phase. This seems reasonable and in line with Mr. Roper's recommendation of operating our cables at a higher stress than at present.

The cable operates at 44 volts per mil a value which is 56 per cent of E_a the mean breakdown voltage. Assuming one failure per 100 miles of cable, a line through these two points gives a coefficient of variability $S = 11$ per cent which would allow a considerable variation from the mean in our test results.

W. B. Kouwenhoven: Mr. Roper's paper presents a very valuable contribution to the cable art. In Fig. 31 of his paper he shows that there is an improvement in power factor with time. These curves are very interesting and are accompanied by a fall in the ionization factor as shown in Fig. 32.

At the School of Engineering at Johns Hopkins University we are conducting an experimental investigation on cable characteristics, supported by the Utilities Research Commission of Chicago. In this investigation we are determining the electrical characteristics of the oil alone, the paper alone, and then the characteristics of the impregnated paper. In addition to the a-c. measurements we are also taking oscillograms of the charge and discharge curves under d-c. potentials.

In a number of specimens we have noticed an improvement in power factor following impregnation. This improvement has continued over a considerable time before the power factor

reached a constant result. An analysis of the charge and discharge curves taken under d-c. potentials shows that this reduction in power factor and the corresponding reduction in loss is accounted for by a lowering of the initial conductivity of the impregnated material. This reduction in conductivity may be due to the adsorption of ions by the paper or to some other reason not understood. We are continuing our investigation.

K. W. Miller and F. O. Wollaston: Messrs. Shanklin and Buller's solution of the oil flow and temperature transient problem by means of the Heaviside operator is ingenious. However, it would be unfortunate if the impression was left that for complete solution this type of analysis is necessary. The same solution can be obtained by well-known methods of elementary differential equations, which we prefer for two reasons:

(a) The mathematical processes are more familiar to most engineers and at every step the physical interpretation of the equations is evident.

(b) The solution in this case can be made more compact in form without omitting essential steps and without loss of complete generality.

Equation (1) given by Shanklin and Buller is somewhat inaccurate, since the dielectric loss is considered to occur at the conductor (see equation (2)), instead of being distributed throughout the insulation. Miller's solution of this problem includes the correct distribution of the dielectric loss, which introduces an additional logarithmic term in X. For high-voltage cables, especially 132 or 220 kv., incorrect use of the dielectric loss term produces considerable error in the oil demand curves. To correct this would unduly complicate Shanklin and Buller's equation (2), which they obtain by methods of Heaviside.

When the Heaviside method is used for finding the constants of integration d_n, equation (1) will fit the boundary conditions perfectly when, and only when, all values of n between one and infinity are employed. This is quite impractical to do, and usually only about three values of A_n would be calculated. This being so, it is obvious that the boundary conditions would no longer be perfectly satisfied, although the approximation may be very good. Since for practical reasons only about three values of A_n would be used, we found it more convenient, and more accurate, to set up three simultaneous equations in A_n, and force them to fit the boundary conditions at known selected points of chief importance. Incidentally, this method avoids all difficulties in properly providing for the correct distribution of dielectric loss.

The solution of practical problems has convinced us that obtaining the roots of equation (4), which also appears in our form of solution, is a long and laborious task. We have, therefore, made some mathematical transformations, which allowed us to prepare convenient charts from which the roots (K_n) can be obtained in a few minutes.

After obtaining the roots, one proceeds to compute coefficients A_n. Using Shanklin and Buller's method, equation (2) would have to be evaluated three times. Using our method of three simultaneous linear equations, we found the process relatively simple. We have prepared charts from which all the numerical values necessary are obtained directly, and only a few minutes of elementary algebra are required to get the values of A_n.

We find that the superposition principle is valid throughout. Shanklin and Buller have pointed out that the total oil demand is the sum of the conductor, sheath and insulation demands, which are computed separately. We would like to point out that one can compute the effect on temperature or oil demand for a unit amount of copper loss, dielectric loss, or sheath loss separately. Then for ten times as much copper loss, the corresponding part of the answer would be multiplied by ten, and likewise for dielectric and sheath losses. To get the total effect of any combination of losses, one adds the effects of each one acting separately. Once the computations are made on that basis, the results for any specific set of conditions, such as dropping load with or without removal of voltage, effect of sheath losses, etc., are very easily obtained without repeating the labor of solving all the equations.

It might be remarked that the computations for oil demands using Shanklin and Buller's equation (6) generally would be long and tedious. In our form of solution it turns out that the terms which at this point require the most labor to compute also appear in the simultaneous equations for A_n. When we prepared our charts to eliminate the labor from solving the simultaneous equations, we also got rid of the difficulties in solving both for oil demands and cable temperatures.

The paper provides only for load which changes instantaneously and thereafter remains constant at the new value. We have generalized the solution to take care of loads varying in any manner whatever. Church[1] obtained a solution for the generalized case, but the resulting expressions were only approximate, and difficult to evaluate numerically. The additional terms contained in our general solution fortunately are very simple functions of the load curve, and are easily handled.

With the labor saving charts we have prepared, no knowledge whatever of the properties of Bessel functions is required of the computer, and not even a table of their numerical values. Using our methods and aids, the saving in labor of computation is tremendous. At a conservative estimate, it would require at least a week for the uninitiated to compute and verify a complete solution for one cable, using Shanklin and Buller's equations. With our simplified process and graphical helps, this can be reduced to a few hours without any sacrifice in accuracy, a considerable gain in flexibility, and greatly reduced hazard of error.

The results, referred to above, of our development of this problem since Miller solved it in 1927, will be made available to those interested.

Wm. A. Del Mar: This paper, like several others by Mr. Roper, is both a milestone and a direction sign.

As a milestone it gives the present commercial status of various types of cable. Perhaps the most important point brought out in this part of the paper is that, for Chicago conditions, 66 kv. single-conductor oil-filled cables can now compete with cables of the ordinary type for winter ratings between 40,000 and about 115,000 kva. It should not be forgotten, however, that this conclusion applies only to the flat topography of Chicago. Corresponding estimates for more hilly localities would raise the limits of voltage and power up to which cable of the ordinary type would be economical.

Mr. Roper's direction sign points to simplification of the oil-filled cable. Two other papers on this subject in this group bear this out. My own work, along rather different lines, also points in the same direction. An essential feature of any scheme of simplification is, I believe, the use of shipping reel reservoirs for cables impregnated at the factory with liquid oil. This idea, however, was developed many years ago and subsequently patented, in connection with the high-tension cable system of the company with which I am connected.

Another of Mr. Roper's direction signs is toward an elastic sheath to compensate for expansion and contraction. This presumably involves the development of a material or structure which, in addition to having the general characteristics required for a sheath, is elastic circumferentially and plastic longitudinally. Such a sheath is not yet in sight.

Mr. Roper's plea for thinner insulation on cables of ordinary type is based on the fact that the commercial run of such cables is considerably below both laboratory made samples and some factory made lengths of outstanding merit. With careful supervision, he assumes, the general run might be made of this superior quality. This may and probably will be accomplished in time, but I believe that, at the present time, the thicknesses which the American manufacturers approve, are the least safe thicknesses

1. *Temperatures in Electric Power Cables under Variable Loading*, by Elwood A. Church, Trans., A. I. E. E., Sept. 1931, p. 982.

for general American conditions. It is pleasing, however, to know that Mr. Roper is prodding the manufacturers in this matter, because it gives an added zest to our development work, and will help in approaching the ideal cable which is always a thing of the future.

The oil-filled cable has two distinguishing characteristics, of which only one is usually stressed, namely, its "breathing" under varying load. The other property is, in my opinion, of even greater importance, namely, its high dielectric strength over long periods of time, or, in other words, its comparatively flat voltage-time curve combined with a short period dielectric strength at least as high as that of a cable of ordinary type. This, also, as pointed out in my discussion of the Whitehead and Hamburger paper, is due to the mobility of its oil, but we should carefully distinguish between the two properties, interrelated as they are.

The flat voltage-time curve is due to local characteristics of the cable and has nothing to do with hollow cores, reservoirs, stop-joints, etc. When cable cools, especially after the high temperatures of manufacturing, the contracting oil tends to concentrate in the thin passages where the capillary attraction is greatest, and to draw away from the larger passages where the attraction is least. These passages, in the order of their capillary attraction, are as follows:

1. Spaces between fibers of paper.
2. Spaces between flat surfaces of paper tape.
3. Spaces between edges of paper tapes.
4. Spaces between strands of conductor and space between insulation (or shield) and sheath.

There are two opposing sets of forces during contraction, the net capillary attraction and the viscous resistance. In the cable of ordinary type, these forces balance somewhere between items (1) and (3) of the above list; usually between (2) and (3). In the oil-filled cable, on the other hand, the balance is between (3) and (4).

The "breathing" ordinarily discussed is the oil flow in and out of the cable through especially provided passages, and its function is to keep the strand spaces or oil channels filled, these latter being the real reservoirs of the oil-filled cable.

Another feature of the single-conductor oil-filled cable which is more or less taken for granted, is that the very possibility of the use of breathing reservoirs depends on some open passageway being made, at the splices, between the reservoir and the hollow core. The problem is to place this passageway where it will not weaken the joint by promoting tangential leakage. Mr. Shanklin and his associates accomplish this by using pairs of terminators instead of joints of the ordinary type. Messrs. Simmons and Atkinson eliminate tangential stresses in the oil passageway by the use of stress directive condensers.

The following discussion assumes that where the authors (Dr. J. B. Whitehead and F. Hamburger, Jr.) refer to "impregnation pressures" they mean "evacuation pressures." It is the different evacuation pressures which change the air content. The small range of low impregnation pressures always ending with a period at one atmosphere, cannot appreciably affect the air content, as the cable is covered with oil during this phase of the process. There is, of course, no reason why the impregnation pressure should be related in any way to the evacuation pressure. My reasons for calling attention to the matter are both to save readers of the paper from misunderstanding it and to suggest to the authors that they extend their experiments to ascertain the effect of a wide range of impregnation pressures on the residual air content, using a constant evacuation pressure.

Some manufacturers impregnate their cables under a high pressure, say 100 lb. per sq. in. or more, after the vacuum treatment. Such pressures may affect the distribution of free air content by causing the free air to go into solution in the oil, from which it would be partly thrown out, but more evenly distributed, when the impregnating pressure is removed. The authors assume that their impregnation pressures are equal to their evacuation pressures, but this is not strictly true, as with the vertical impregnating tank, which they employ, the impregnating pressure at the bottom of the sample is never less than several additional centimeters of mercury, due to the head of oil.

The reinforced end with logarithmic flare, described by the authors, was suggested several years ago by Mr. C. F. Hanson and further developed by Mr. John H. Palmer, whose discussion gives some design details.

The authors call attention to a very important point in connection with the parallel testing of cables. It has long been known that voltage rises occur due to resonance when two or more reels of cable are tested simultaneously at high voltage. That similar effects could occur due to the multiplication of test samples four feet long, is very surprising. I believe the authors might, with great advantage, follow up the clue suggested by these experiments. It might lead to very important developments.

An investigation was made recently by Mr. John H. Palmer, of the effect of corona on test voltage, by means of Lichtenberg figures.

The high-voltage transformer was rated at 150 kv. and 150 kva. This was connected by a small wire about 6 in. long to a 1¼ in. diameter bus. This bus was approximately 20 ft. long. It passed through a wall with a wall type bushing and had a right angle bend near its end. The ends of the bus were originally rough and unfinished. Before the following tests were made, however, all of these rough corners and ends were rounded and smoothed off with plastic wood and then given a coat of conducting paint. All small conductors were replaced with ones of large diameter. When this was done, no corona was visible at voltages below 90 kv. There was considerable corona, however, visible at voltages above 100 kv.

The apparatus used for obtaining the Lichtenberg figures consisted of a small, light tight wooden box with an electrode projecting through the top. It was electrostatically shielded by lead foil over the outside, which also acted as the ground electrode. Photographic film was placed under the high potential electrode. A pillar type insulator having six petticoats was used as a potentiometer. This was set on a grounded metal plate and had a 2 ft. diameter shielding ring on the top. The top was then connected to the bus by a large diameter lead and the indicator was connected across the lower petticoat.

Tests were made at 60, 80, 100 and 125 kv. on the bus alone, and on the bus plus about 20 ft. of No. 22 A.W.G. copper wire to produce corona. Tests were also made with two cables (15 ft. long under the lead) as load, at 60 and 80 kv. These cables were single conductor No. 4/0 with 220-mil impregnated paper insulation. They were connected to the bus by means of approximately 20 ft. of No. 22 A.W.G. wire.

Typical results of tests are given in Figs. 1 to 4. The following table gives a summary of the complete results:

Voltage kv. load	Figure diameters for various loads and voltages: Bus	Bus + 20 ft. No. 22 wire	Bus + 2 − 15 ft. cables +20 ft. No. 22 wire	Mean
60	0.80 in.		0.88 in.	0.84 in.
63		0.83 in.		0.83 in.
80	1.14 in.	1.09 in.	1.12 in.	1.12 in.
100	1.24 in. 1.28 in.	1.23 in.		1.25 in.
125	1.51 in.	1.48 in.		1.495 in.

These results as a whole show no definite increase in figure diameter with corona. The deviation from a mean has been calculated for each voltage and was a maximum at the 60 and 80 kv. points, where it amounted to 0.04 in. and 0.03 in. respectively, or 3 kv.

If the very considerable amount of corona used in these tests had no appreciable effect on the voltage, it is to be presumed that the shortened cable lives noted by the authors must be due to some kind of resonance effect.

The authors carried all of their life tests to a point of breakdown. This was our practise until the last few months, when we changed to that of stopping the test when the sheath attained a certain temperature. This was ascertained by coating the sheath with wax containing red lead to give it greater visibility. When the melting point of the wax, approximately 60 deg. cent., was attained, the test was discontinued. This enabled us to catch incipient failures in the majority of cases. The paper tapes from such hot spots were removed from the cable and laid out on a board so that they could be studied with ease. One of the results of this study was to find that there are apparently two types of failure which may or may not correspond with the two types described by the authors. We designated these types as the radial and tangential type, respectively. In the radial type of failure, there was a direct puncture through the insulation without formation of dendrites. All tangential failures are radial in their final phase.

A B C

Fig. 1—Tests at 60,000 Volts

(A) Transformer loaded with bus and two cables
(B) Transformer loaded with bus only
(C) Transformer loaded with bus and corona-forming wire

A B C

Fig. 2—Tests at 80,000 Volts

(A) Transformer loaded with bus and two cables
(B) Transformer loaded with bus only
(C) Transformer loaded with bus and corona forming wire

A B

Fig. 3—Tests at 100,000 Volts

(A) Transformer loaded with bus only
(B) Transformer loaded with bus and corona forming wire

The radial failures generally occurred without heating the cable sufficiently to melt the indicating wax, although in one case a partial radial failure was noted, which had not pierced the insulation. In the tangential type dendrites formed on the inner layers of the insulation. In all cases but one the dendrites were on the concave side of the tapes. The only case of a dendrite on the convex side of the tape (except where they were printed by the carbonaceous matter from the concave side of the adjacent tape) was found in a capacity-graded cable, where the hot spot and dendrites developed a little way in from the sheath. In this case the failure progressed from sheath to conductor and all the dendrites were on the convex side.

Dendrites originate opposite channels between tape edges. In the beginning they are isolated spatters of charred oil, but later they link together forming tree patterns which eventually burn into the paper. Each original spatter appears to be preceded by X formation and the X subsequently carbonizes. It is fairly well established that X is the result of ionic or short-wave bombardment of oil. Hence we may infer that the formation of dendrites is preceded by an ionic or short-wave emission from craters in the channels between tape edges.

A B

Fig. 4—Tests at 125,000 Volts

(A) Transformer loaded with bus only
(B) Transformer loaded with bus and corona forming wire

These observations show that the failure of impregnated paper insulation is a progressive event with a definite direction of travel, but they do not reveal the reason for any particular direction.

The authors state that "the specimens that were short-lived showed, upon examination, evidence of large quantities of entrapped air or gas and incomplete impregnation, as was indi-indicated by dryness of the paper tape." It might be inferred from this that the authors regard this entrapped air or gas as the cause of the short life. Without entering upon the question of whether entrapped air does or does not cause short life, we found very strong evidence that cables may have very short life without any considerable amount of entrapped air, the gas which is found at the hot spot being generated by the latter and not being the cause of it.

The temperatures attained at the craters where hot spots originate are above the temperature at which impregnating compounds are converted into gas. Gas which is found in the cable in the neighborhood of a dendritic hot spot consists principally of hydrocarbon gas and not entrapped air.

If tangential type failures originate in craters between tape edges and do not depend upon the pressure of air, what causes them to form? An excellent working theory is that each crater

is a miniature wave or electron-emitting vacuum tube. When the oil in a cable contracts in cooling, it will form vacuous bubbles in the channels if of high viscosity, but will draw oil from a considerable distance if of low viscosity. The dividing line appears to be a viscosity of about 20 minutes Saybolt.

In this connection it is significant that we have been unable to form dendrites in cable impregnated with thin oil (the so-called oil-filled type) even when purposely impregnated very poorly. Such cables, on life test, become hot throughout their length, without developing dendrites. The principal reason for the excellent time-voltage characteristics of oil-filled cable is the absence of dendritic hot spots and this, according to my theory, is due to the absence of minute vacuous bubbles and not to the absence of air.

The authors state in their conclusions that low pressures of evacuation are more important for the further withdrawal of moisture than for the reduction of air content. This is undoubtedly true of the samples having insulation walls of 65 mils. If the thickness of insulation had been ten times as great, say 650 mils, they might have reached a different conclusion.

The principal part of the author's work constitutes a very valuable contribution to our knowledge of the subject. All of the work has been done with such a high degree of experimental skill that the three papers in which it is described will long serve as a reservoir of facts, whatever interpretation may later be placed upon them.

John H. Palmer: Life tests on experimental cables have been in progress on cables with 220 mils of paper insulation and 80,000 volts between conductor and sheath. Considerable difficulty was experienced in testing these samples, due to the fact that ordinary commercial terminals failed to hold the voltage for any appreciable length of time. Several designs of terminals were tried, following an idea of Mr. C. F. Hanson, involving a graded condenser. The terminator as now used is based on a design involving a potential gradient not exceeding 5 volts per mil.

In order to attain a uniform potential gradient along any part of the insulation, it should be shielded with a metal shield, of diameter increasing with the distance from the lead, according to the following law:

$$D = \text{anti-log}\left[\frac{P}{\frac{E}{g\,l} - 1} + Q\right]$$

Where

D = diameter of shield (in.) at distance l (in.) from edge of sheath.
E = kv. conductor to sheath.
g = potential gradient, kv. per in. allowed along surface of paper.

$$P = \frac{K_t}{K_p} \log \frac{d_p}{d_c}.$$

Q = log d_p.
d_p = diameter over cable paper, in.
d_c = diameter of conductor, in.
k_t = Specific inductive capacity of built-up tape.
k_p = Specific inductive capacity of cable insulation.

Example: One-conductor 500,000 cir. mil, 220 mils of paper, to be tested at 80 kv., let us assume

$$\frac{K_t}{K_p} = 1.1$$

$$P = 1.1 \log \frac{1.254}{0.814} = 0.207$$

$$Q = \log \quad 1.254 = 0.0983$$

l	$g\,l$	$\frac{E}{g\,l}$	$\frac{E}{g\,l} - 1$	$\frac{P}{\text{last}}$	Q + last	D anti-log last
0	0	∞	∞	0	0.0983	1.254
1	5	16	15	0.0138	0.1121	1.295
2	10	8	7	0.0296	0.1279	1.342
3	15	5.33	4.33	0.0479	0.1462	1.400
4	20	4.0	3.0	0.069	0.1673	1.470
5	25	3.2	2.2	0.094	0.1923	1.557
6	30	2.67	1.67	0.124	0.2223	1.671

The procedure is to build up to the diameter D with bias cut varnished cambric tape. After the 6 in. of base for the shield have been applied, the tape is built up as steeply as it can be laid for 1½ in. and then tapered in to the end of the cable. The object of the steep taper is to make a bulge beyond the end of the shield as a barrier against flashover. Heavy oil is applied between the layers of varnished cambric. The shield consists of lead foil or copper braid, applied tightly over the varnished cambric for 6 in., *i. e.*, up to the beginning of the bulge. The cable end thus prepared is inserted in a porcelain terminator.

F. M. Clark: The following discussion of the paper by Messrs. Whitehead and Hamburger includes several suggestions presented for consideration.

In drawing conclusions concerning the dielectric behavior of oil-treated insulation, it is necessary that the conclusions be limited to the type of specimen investigated. In the work done at Pittsfield we usually use a flat section of insulation between parallel aluminum electrodes built up to the desired electrical capacity. In some cases the electrodes may be wound cylindrically and later flat pressed to a form resembling a standard commercial capacitor. Our work is based entirely on Kraft and linen paper.

It has been found that in any study of the time voltage relation for fibrous insulation, it is a dangerous procedure to compute low-voltage breakdown from data obtained with breakdown over periods of only a few days or weeks. Experience has been that breakdowns in the laboratory are preceded in every case by corona formation. The factors governing such corona breakdown appear to be coupled with chemical changes in the dielectric solid or dielectric liquid. These changes are largely a function of heat concentration in spots and this in turn owes its source to the value of voltage used. In many cases the absolute value of voltage appears to be of more importance than the voltage stress. The procedure of calculating a breakdown for 2,000 or 3,000 hours from breakdown involving only a few days appears to present to us many obstacles with the limited knowledge at present available.

A rising power factor during the first few days of voltage application has been noted in insulation study at Pittsfield. In all work we have used a voltage as high as 825 volts per mil for dielectric thicknesses up to 8 mils. The life of the insulation may run over a period of several years. In practically every case, this initial increase in power factor is gradually lost and ultimately the original or even a lower power factor is obtained. These tests are carried out in sealed containers. The power factor rise during the initial stages of a life test has not been able to be associated with the total life of the insulation with any degree of reliability. The authors conclude that cable compound will deteriorate even if it is sealed in contact with air. This is a conclusion of considerable value. It should be carefully checked.

In Pittsfield a single stock sample of impregnating liquid of medium viscosity has been used for several years and no deterioration of liquid has been noted either in its chemical, physical or electrical properties, its electrical stability under aging test, or in the electrical characteristics of insulation treated with it. The liquid, however, is protected from the action of light although it is in a standard oil drum, with no special precautions aside from

the usual oil drum seal. I assume that in the work presented by the authors, the treating compound is protected from light.

We can well agree with the conclusions of the present paper that the life of impregnated paper insulation increases steadily with a decrease in the amount of residual air. However, we would restrict this conclusion to experiments of a similar type to those performed by the authors. The effect of residual air in our tests becomes apparent only as the voltage of the test is raised well above those usually met in commercial operation. The effect of residual air appears to be coupled again with corona formation or chemical effects produced in the oil or paper which are strongly accelerated by the higher voltages in the presence of residual air. With low voltage operation, similar to those met in commercial practise, we have found no effect of residual air within the dielectric unless the air be contained in pockets, under which condition of course ionization and early breakdown become possible.

I desire to emphasize the importance of conclusion No. 8 which calls attention to the desirability of carefully controlling both the paper and oil used for electrical insulation. In our tests, we have found that most paper and oil shipments made exactly to the same physical characteristics, from the same source, using the same manufacturing method, and the same basic material may vary widely from shipment to shipment and the variation be of such magnitude as to affect seriously the electrical properties of apparatus involving their use. At the present time much expense is necessarily encountered by the designer of electrical apparatus in order to protect himself from such variations.

J. B. Whitehead: The increases in voltage rating and in carrying capacity and other characteristics of the oil-filled cable have led to the general feeling that it possesses inherently great advantages over cable of earlier, or as described by Mr. Roper, of "ordinary" type. While he does not make a specific statement, Mr. Roper nevertheless reflects a definite feeling to this effect in his comparison of the ordinary and the oil-filled types of cable, notably in his discussion of their relative stability, as indicated from their power factor characteristics, as well as in the obvious hope expressed in one or two places that the oil-filled principle may be extended to a wider economic range of voltage and load rating.

The question arises, therefore, as to whether Mr. Roper has made allowance for a superiority of this character in making his estimate of the relative capital costs and annual charges. Among his assumptions there is no evidence, for example, that any increase in stability or in life is allowed to the oil-filled cables. In item (20) the fixed charges, which presumably include depreciation, are placed at the same figure for all types of cable. In his conclusion (8) and in the results of Table II, Mr. Roper suggests that a moderate reduction in the cost of 66-kv. oil-filled single-conductor cable would cause it to show up much better in comparison with the ordinary type on the basis of economy. I should like to ask Mr. Roper whether a credit for increased stability and life of the oil-filled type might not swing the comparison of the 66-kv. single-conductor cable more definitely to the oil-filled type and perhaps even extend the economic oil-filled range even lower in the scale of load and voltage values.

Referring to Fig 31, it is surprising to find that in the original condition every one of these cables shows a rising power factor—voltage characteristic. This is very disconcerting because one of the principal advantages claimed for this type of cable is improved impregnation and avoidance of this condition. Referring to the improvement of these ionization characteristics with time, as shown in Fig. 31, we have frequently noted this type of change in our laboratory studies of the influence of residual air. In our experiments, however, it indicates usually a relatively poor condition of impregnation and relatively short life, as shown in Fig. 4 of our paper.

Referring to the same question, we have also noticed another type of slow improvement of power factor with time under test, which we have attributed to a slow improvement in the degree of impregnation. This type of change, however, is associated with perfectly flat original and subsequent power factor—voltage curves.

Based on the results of our laboratory studies, therefore, it is my feeling that efforts should be made to obtain flat power factor—voltage curves originally, rather than to rely on the tendency for an original rising characteristic to change to a flat one in the course of time.

R. W. Atkinson: Mr. Holmes has made available for use in the study of cable insulation the laws of probability. He has pointed out and emphasized the importance of the coefficient of variability, S, and has shown how its effect may be determined if sufficient information is available to obtain its value. The above points constitute the important part of Mr. Holmes' paper and these points should be very useful in the study of insulation. Having developed his theory the author then goes on to show how to apply it to practical cases. The writer wishes to emphasize the limitations of the author's method of attack on these practical cases and to point out certain errors in his assumptions.

The author attempts to show how to apply the developed theory to cable operation. He takes certain data which are based on five-minute tests but usually evaluated by means of an arbitrary exponent to a one-hour basis. He then re-evaluates these breakdown values in the same way to a 100-hour basis and assumes further that he can also evaluate it to one year, still on the strength of the five-minute results. Having done all of this extrapolation he applies his mathematics and tells us how much cable probably will fail in a given length of time at a given voltage. He suggests that at some later date cable engineers will determine the correct exponent, apparently failing to recognize the fact that the straight exponential relation between voltage and time applies only for small time differences, if at all, and that there is probably a more important variation, from one cable to another, with respect to this exponent than there is in the property which he is studying. The practical application thus given is, to say the least, definitely misleading.

The application of the developed theory should properly have been confined to the same terms as the observed data. It could then be pointed out how the probabilities so determined could be applied to longer periods if the voltage-time relation were known. For example, if the data from which the coefficient of variability is determined consist of five-minute tests, the method described in the paper would permit us to tell how many failures we could expect in five-minutes if say 100 miles of cable were tested at a given voltage.

The author points out that from 10 to 20 tests are necessary in order to determine the coefficient of variability. I wish to emphasize this point and call attention to the fact that in actual cases in any given lot of commercial cable it is very seldom that sufficient data are obtained to give a fair average, let alone the coefficient of variability. Even in the case of an order of cable large enough to furnish sufficient tests, the cable will be manufactured over a considerable length of time and might introduce a number of variables which would not exist if the cable were made at approximately the same time. The cost of making an adequate number of tests for the determination of variability would ordinarily preclude the possibility of their being made on any given lot of cable.

It should also be borne in mind that even 10 to 20 tests would give the coefficient of variability only for the range in the vicinity of the average breakdown value. If, for example, 10 tests are made, the ordinates must necessarily be in 10 per cent intervals with the lowest point at 5 per cent and the highest 95 per cent. Extrapolation must be resorted to below the 5 per cent value and above the 95 per cent value. Normally, in determining probabilities of failure in operation we would be concerned with probabilities much lower than 5 per cent. As the actual distribution

function approximates the normal law only in the neighborhood of average values, extrapolation can not be expected to give accurate results either in the neighborhood of 0 per cent or 100 per cent. The author recognizes this in his Fig. 4 by plotting points above 80 per cent probability so that they do not follow the curve drawn. One danger of too much extrapolation is indicated in the author's statement under "Experimental Considerations" in the last part of section a. He states that with a coefficient of variability of 6 per cent we will obtain one failure per 100 miles of cable under a stress of 75 per cent of average while with a variability of 20 per cent the same result will be obtained with a stress of only 17 per cent of average. Following out the same mathematics it will also appear that in the case of insulation with 6 per cent coefficient a stress of 125 per cent of average will cause failures in all the cable except one section out of 100 miles while with the 20 per cent coefficient it would appear that we would be able to go to 183 per cent of average stress before the same result is obtained. In other words, the calculation would indicate that the poorer insulation would stand far greater excess stress than the better insulation.

Routine voltage tests made in conformity with cable purchasers' specifications might give important data as to the lower portion of the probability curve were it not for the inconsiderateness of cable manufacturers in making cable so good that it almost never breaks down under ordinary test voltage. By increasing the test voltage sufficiently, some failures could no doubt be obtained, but such a procedure is undesirable both on account of the cost of broken down cable and because of the possible reduction in the life of the remainder of the cable. Some years ago the writer advocated the practise of testing a certain portion of all high-voltage cables at a considerably higher voltage than specified. This practise was adopted three years ago by the A.E.I.C. and now forms a part of that association's cable specifications. Even where these tests do not result in failures, the data may be used in determining a limiting position of the lower part of the probability curve.

It should be recognized that the variability of insulation is more important than the average strength as determined by tests on all or a large fraction of the cable. As has been stated, financial considerations will prevent the destruction of an appreciable amount of cable for the purpose of obtaining the required data for the reliable determination of this coefficient. Attempts are being made by various people to develop a satisfactory procedure for its determination by tests on the entire quantity of cable without destroying any of it. One of the methods being tried, for example, is the discharge or noise bridge.

Cable specifications as now used constitute an attempt to measure insulation variability, but do this very imperfectly. Incidentally, it may be remarked that we are here concerned only with such properties of cable insulation as directly determine the ability of cable to withstand operating conditions. We should not confuse ourselves with a mass of information as to properties unimportant in this connection, the chief reason for the determination of some of which is often only the facility of measurement. Manufacturers' control, inspection, and testing will always be one of the prime factors in giving at least one of the limits of the probability curve. The determination must, however, always be based not on tests of a fraction of a per cent but on tests covering all or a major fraction of the cable.

H. G. Burd: From time to time criticism has been directed at the cable manufacturers for failure to improve their product rapidly enough. The rapid development in simplification of design and resultant economics in oil-filled cable and accessories does credit to the engineers of both the cable makers and the utilities. It also invites a parallel improvement in solid type of insulation. Some engineers are urging a reduction in thickness of insulation for medium voltage cable, as well as for high-voltage cable. Careful study is being given to this feature. Obviously, mechanical considerations affect the minimum walls of insulation permissible. Also, a cable that may be workable on one system may not necessarily be equally successful on a second system. For example, a steady heavy load is less damaging to cable operation than an extremely variable load with wide temperature variations.

Parallel improvement in solid type cable is already noticeable. The research programs started several years ago were planned to develop fundamental conclusions regarding insulation. Professor Dawes of Harvard reported that all occluded air, gas, and moisture could be removed from oil if broken up into thin layers under practically absolute vacuum. Power-factor voltage curves with flat characteristics up to 300 volts per mil were obtained for compounds thus treated. These results held good almost regardless of the type of compound which proved definitely that air, gas, and moisture were the chief causes of trouble. Davis and Eddy indicated that oil at atmospheric pressure will absorb air up to 10 or 15 per cent of its volume. This work connects Professor Dawes' results closely with the work of Emanuelli, Shanklin and Mackay and shows clearly that ionization would increase in direct proportion to the void volume in the cable. Therefore, if ionization is to be prevented, every precaution needs to be exercised to remove air and moisture completely.

The study by Messrs. Whitehead and Hamburger at Johns Hopkins has been directed toward residual air and moisture in impregnated paper insulation. After dealing, in earlier studies, with fairly high pressure of evacuation and impregnation, and a repeated preliminary statement in this paper to the effect that a pressure below 2.5 cm. Hg. is not necessary, we now find that for the studies in paper No. 3 they report (on page 1431) drying of the core under ½ mm. Hg. and spraying of the oil (to break it up) into a chamber evacuated to a pressure of ½ mm. Hg. Finally, keeping the oil under this degree of vacuum.

It is exceedingly gratifying to us to note the use of this extremely high vacuum, for two years ago we had opportunity to build and equip a new mill for cable manufacture and were, naturally, extremely anxious to incorporate in it all the benefits of the latest research developments. Commercial adaptation of these important laboratory advancements was the extremely difficult problem which confronted us. We also made numerous tests on commercial lengths of cable under the vacua available in old impregnating tanks, then with these same tanks improved to hold much lower pressures, namely, higher vacua. And this showed the difference dealing with full lengths of cable, with various degrees of vacua.

The inference seems quite clear from their second and third papers that it is not necessary to go to 1 or 2 mm. Hg. impregnation or oil pressure treatment to secure the best finished cable product. Our disagreement with them is in the same channel as 1928 discussions of the second paper by Messrs. Mackay and Del Mar.[2] The major difference is in university laboratory *vs.* commercial cable mill manufacturing methods. It is quite one thing to handle small amounts of material in miniature apparatus and with miniature samples, and vastly different to treat large masses of strand, paper and compound, bent around a drum, in a cable factory. There we are also dealing with cable from 65 mils wall thickness up to 750 mils. Obviously, to obtain the same results as in a laboratory, large masses would need a higher degree of vacuum.

Even the various results in the laboratory on samples as reported in this latest paper are quite erratic. The conclusions, themselves, on page 1439 are certainly an excellent brief for high vacuum treatment. Finally note that oil-filled cable, with its lesser walls of insulation and perfect operating record, is so treated.

In conclusion No. 5 and other places, reference is made to long life and the inference may too readily be taken that this means long operating cable life in service. The two are vastly different, in fact it should be clearly kept in mind that these are simply

2. Trans. A.I.E.E., July 1928, pp. 839-840.

extremely high-voltage life tests, and on samples. Solid type cables operate at from 30 to 50 volts per mil average stress. Testing at 300 volts per mil and higher can hardly be a completely satisfactory accelerated life test. While continuing to test that way because it is the present specification requirements, in the past several months we have been giving attention to methods which would more nearly simulate the operating conditions of cable in service. We have run cyclic loading tests on both belted and shielded type of cable and also have carried cable through repeated temperature cycles without loading but by oven heating and cooling. Two or three years ago the Detroit Edison and Brooklyn Edison companies tried cyclic loading tests but apparently with results not completely satisfactory. These tests have also been tried abroad. Our tests have been made with much planning and care and have been very gratifying and satisfactory.

We do not propose cyclic loading tests as a satisfactory substitute for the present methods of factory acceptance testing of cable. Obviously, these tests take too much time and are too costly. But it is a significant coincidence that cable made this way and so tested has parallel operating life in service to oil-filled cable.

These two types of test also take the cable to much higher temperatures than the 40-50 deg. cent. mentioned in the papers by Messrs. Whitehead and Hamburger. We feel it is most important to cover this temperature range in an attempt to judge good cable from bad.

Every experience that we had in the past year indicates that while extremely high vacua and separate treatment of sprayed oil and core, keeping the air at all times away, is a more expensive method of cable treatment, its use, combined with continued improvement in the (now quoting from page 1439 of Messrs. Whitehead and Hamburger's paper) "uncontrolled variations in the uniformity and quality of oil and paper," clearly offers a method whereby vastly improved solid type of insulation is secured, paralleling (as mentioned in the first part of this discussion) for solid type of insulation the improvements in oil-filled cable.

E. G. Linder: The influence of the small amounts of moisture and air which reside in the impregnated paper insulation of cables immediately after their manufacture, has been discussed by Messrs. Whitehead and Hamburger, but no mention has been made of the moisture and gases which may be produced in the cable after its installation. The residual moisture and air may serve to initiate the deterioration, but it seems that the reaction products generated by the glow discharges in the voids must play the most important part in the subsequent course of the deterioration.

During the past few years we have been studying at Cornell University, the reactions of organic compounds in the glow discharge, for the purpose of throwing light on the process of decay of cable insulation. The Cornell studies, while specialized and purely scientific in nature, are supplementary to a more general and practical program of cable research being carried out by the Detroit Edison Company.

In spite of the fact that it is now generally believed that ionization in discharges in voids is the principal cause of cable deterioration, a surprisingly small amount of research has been done on the chemical effects of these discharges. In fact, it seems that the Detroit Edison Company's work, together with the supplementary work at Cornell, is the only investigation which has been made along this line. The field is certainly of sufficient importance and breadth to deserve the attention of a much larger number of investigators.

Our work has thus far dealt mainly with the effects of discharges in pure hydrocarbons, rather than in mineral oils, which are usually mixtures of some two hundred different hydrocarbons and therefore too complicated chemically to be suitable for a scientific study aimed at determining the fundamental processes.

About seventy different hydrocarbons, including representatives of all different types, have been investigated, and it has been found that they all react in the glow discharge (without the presence of air or moisture). The discharges have been produced in the vapor phase of the hydrocarbons, as they would occur in the gases in the voids in cables. Solid, liquid, and gaseous products have been found to be produced, in apparently complicated mixtures consisting of many different compounds. The solids are partly waxy, amorphous substances, and partly crystalline. The liquids seem to range from thick, syrupy compounds, to light, volatile ones. However, as yet, only a very meager study of the solids and liquids has been made, and there are but few reliable data as to their nature, the investigation having been largely confined to the gases.

The chemical composition of the gases from the different hydrocarbons does not vary over a wide range. It is about the same for aromatics as for paraffins, and consists of from 40 to 60 per cent hydrogen, 50 to 30 per cent unsaturated gases, and about 10 per cent paraffins. This composition does not correspond with that of gases found in actual cables, which run about 90 per cent hydrogen, but correspondence should not be expected since we are here studying the primary reactions in simple pure hydrocarbons, and not the more complicated series of reactions in complex oils which actually occur in cables. Extensions in that direction logically come later on after the fundamentals have been made clearer.

Under arbitrarily standardized test conditions, it has been found that paraffins produced three or four times as much gas as aromatics, but on the other hand, aromatics produce two or three times as much insoluble wax. The gas-to-wax ratio by weight for aromatics is about 3:1, whereas that for paraffins is about 50:1. There appears to be a rough relation between the degree of saturation and the amount of wax produced, the rate of wax production increasing with decreasing saturation.

Definite relationships between the rate of gas production and molecular structure have been found. The results appear to be in harmony with two rules, namely;

(1) In a series of similar compounds, the rate of production of gas increases with the molecular size.

(2) In a series of similar compounds (all of the same molecular weight) the rate of production of gas decreases with increasing molecular centralization.

As an example of rule (1) the n-paraffin series may be cited, the rate of gas production increasing with the length of the carbon chain; as an example of rule (2) the series, n-butylbenzene, sec-butylbenzene, and tert-butylbenzene, may be given, the rate of gas production decreasing in the order in which the compounds are listed.

We have shown, therefore, that even in the absence of air and moisture, complicated chemical reactions occur in hydrocarbons, which must play an important part in cable deterioration. It is planned to start an investigation immediately on oxygen-containing compounds, such as alcohols, esters and ethers, with the purpose of throwing light on the effect of the presence of cellulose in cables. It is anticipated that the presence of oxygen, along with the carbon and hydrogen in the compounds, will result in the actual formation of large amounts of water.

Some of the above results have been published in the *Physical Review*, 35, 1930, p. 1375. Further data will soon be available in detail in a number of forthcoming papers in scientific journals.

Edwin B. Middleton: In the paper by Messrs. Shanklin and Buller under "Advantages," a statement appears "Positive oil pressure is maintained and because of this, flaws and defects in the lead sheath merely result in slow leakage of oil which can be located and repaired at any convenient time. Electrical service failure caused by entrance of moisture and air is usually the first warning of such defects in solid cable." Also, under "Other Factors and Characteristics—Strength of Lead

Sheath," there is this statement, "The integrity of the sheath is a very important feature of oil-filled cable practise."

Electric power cables for high-tension service consist essentially of a conductor and insulating layer, usually impregnated paper, and an extruded lead sheath. Conductors are universally satisfactory and oil impregnated paper insulation has reached a high state of perfection due to continued study by manufacturers and operators. However, the lead sheath extrusion process has changed but little and is essentially the same as 30 years ago. It is believed that more accurate methods should be developed for the control of the lead covering processes. These should include not only control of the temperature of the material, but also freedom of the molten lead from accumulations of foreign materials that might affect the integrity of the finished product.

After approximately two years use of positive pressure reservoirs in connection with 66-kv. solid type single-conductor cable in the Philadelphia Electric Company system, I feel that the integrity of the lead sheath is equally as important in the case of the solid insulation type of cable as with the oil filled, and for the same reasons in each case. Since replacing the original sylphon installations with positive pressure reservoirs, it has been necessary to withdraw a number of cable sections due to split sheath. These situations were detected by oil leaking from the duct mouth.

An analysis of these troubles seems to indicate that a large proportion of these sheath failures are undoubtedly the result of inherent manufacturing defects. By their nature they could not be readily disclosed by tests. It appears, therefore, that these defects only become apparent as the result of many repeated movements of the cable closely following the load cycle, combined with the pressures imposed by the reservoirs. The former factor was probably a very large contributor to the ultimate failure since laboratory tests on cable samples at sustained pressures well above those which it is reasonable to believe exist in service, did not disclose discernible changes in the sheath.

There also appears in the Shanklin and Buller paper, under "Other Systems of Oil Feed," the statement "Segregation where actually required can be obtained by safer and more economical means." This refers to what the authors might call the elimination of unnecessary stop joints. I would like to learn just how the authors propose to eliminate or minimize the present form of stop joint and still secure adequate segregation.

Howard S. Phelps: Referring to Fig. 3 of the paper by Messrs. Atkinson and Simmons, there is an operating problem to which attention should be called. The authors make no mention of studies they may have made of materials other than galvanized steel for the outer casings of the oil reservoirs. It would seem, however, serious consideration should be given this feature of the system they propose.

Observations in Philadelphia of both aerial and underground transformers housed in steel tanks, even the so-called copper-bearing steels, have led to investigations of corrosion resistant materials and finishes that might be used advantageously for transformer tanks and similar applications in underground construction.

The condition of some 150 samples, after approximately 18 months' installation in manholes of the Philadelphia Electric Company's underground transmission system indicates that for sheet materials, such as used in the outer shell of the oil reservoir, there are but few commercially available that may be expected to have a life anything like approaching that which is expected of lead-sheathed cable.

It would be reasonable to presume that the outer shell of the pressure type variable volume oil reservoir would require a number of welding operations in its manufacture. It has also been found, and the Philadelphia tests confirm the point, that even the "stainless steels" have very poor resistance to corrosion in the event that any welding processes have been applied to them.

In lieu of using materials concerning which there may be questions as to their ability to resist corrosion, but which are used because of lesser cost, it might be found economically advantageous to use other materials concerning which there is little, if any, question of ability to withstand corrosion and thus obviate routine inspections of equipment at intervals of two or three years to determine if corrosion has set in, together with a program of protective painting should corrosion be found.

T. N. Riley: Direct-current control of drying makes it difficult to obtain comparative and reproducible results with any high degree of accuracy. If a short time reading is taken the conductivity is rapidly changing owing to the absorption effect, and if a long time reading is taken the real conductivity of fibrous hygroscopic materials is liable to be obscured by polarization effects. We have preferred to make control measurements of a-c. leakance and conductivity at 800 cycles alternating current because at this frequency the power-factor temperature curve is not so steep as at 50 cycles and small temperature errors are less likely to obscure any changes in power factor due to other causes. This seems to me particularly important in dealing with papers of quite different characteristics as in the tests reported in the paper by Messrs. Whitehead and Hamburger.

Arising out of this difference of paper supplies I think too much weight is attached to porosity tests alone. It would be quite possible to get two totally different papers with equal porosities one having, for example, few relatively large spaces between fibers and the other a large number of relatively small spaces giving equal air flow on test. Assuming an equal condition of the fiber surface the latter would probably take up oil more rapidly owing to a greater capillary pull. The fiber surfaces themselves may differ widely according to the beating received in manufacture. The No. 2 paper, for example, appears to have been highly beaten resulting in a considerably greater hydration of the cellulose than occurred in Nos. 1 or 3. The conductivity is therefore intrinsically greater, since a large part of the water is chemically combined and cannot be removed without breaking up the paper structure. Such excessive beating would tend to give a paper of lighter weight for a given thickness, particularly if the porosity is high. It seems to me therefore that to obtain comparative results, paper of known and uniform absorbency, density and porosity must be used.

It appears quite possible that the increase in power factor with increase of voltage shown, for example, in Fig. 6 may really be due to a local rise of temperature in the insulation caused by the increased loss at the higher stress (which temperature control of the oil bath could hardly prevent) and is not a voltage change at all. I am the more inclined to this view since the change appears to be slow and to take two hours or so to establish, whereas a change really due to voltage would probably take place very quickly.

The next point is that the small differences obtained in life for impregnation at pressures between 2 mm. and 2.5 cm. are probably due largely to the conditions of the test, that is to say that when the thinly insulated sample is exposed to oil penetration along the whole length, the oil, whatever its condition after impregnation rapidly takes up gas from the oil bath exposed to nitrogen at atmospheric pressure. The result of the low-pressure treatment is therefore largely nullified. A tighter structure would slow down oil movements and by retarding the re-gassification give longer life as is reported.

With 40-cm. impregnation on the other hand there is a tendency for an opposite effect, *i. e.*, for oil penetration into the body of the paper to continue slowly while the cable is in the bath, the improvement in filling counterbalancing any increased gassification of the oil. There seems good reason to believe that the real differences produced by impregnation at different pressures are greater than the test results indicate.

With actual high-voltage cables employing a much thicker insulation and having an impervious lead sheathing preventing

easy access of gassified oil after impregnation, the improvement obtained by high vacuum impregnation would be much more marked.

With good impregnation and gas free oil there is no initial rise of power factor as shown in Fig. 7. Such rise is definite evidence of defective impregnation. It is therefore to be expected that longer life is obtained with a smaller initial rise of power factor, but it is somewhat surprising that any quantitative relationship should exist between the two factors such as is shown in Fig. 8. The erratic distribution of the single tests in Fig. 9 is more easily appreciated.

There is another effect occurring in actual cables which tests in an oil bath do not show up. Suppose a cable is impregnated in relatively poor vacuum and is then lead sheathed. A test immediately after manufacture may give a flat power factor—voltage characteristic. If now the cable is allowed to stand for a few weeks a repeat test may show a characteristic rising with voltage. The explanation probably is that in impregnation the separating layers between papers are first filled leaving the papers themselves isolated. Such trapped air or gaseous spaces as exist at first are largely in the body of the paper, and being finely divided, do not ionize under voltages up to values considerably above normal. On standing, however, the paper continues to take up compound at the expense of the layers between papers, in which case what gas there is tends to collect in larger aggregates. These spaces now ionize at a lower voltage so that the characteristic of the cable is changed on standing. If oil is available from a surrounding bath, it is not to be expected that this effect would show up so markedly. It may be, however, that the initial rise of power factor found is really due to this time factor alone and is not dependent on continuous application of voltage. It would be interesting to know whether any tests have been made which would clear up this point.

Robert J. Wiseman: Mr. Roper's paper is a very fine treatise on the economics of cable operation in comparing the oil-filled type of cable with the solid type of cable. It is true that a positive pressure must be maintained at all times on the oil-filled cable. I think it is equally true that we should have positive pressure on solid cable. With present day compounds used in solid cables, this is not obtainable at all times. Studies are now being made to determine how fluid an oil must be to accomplish this end. We are learning that other factors beside fluidity of an oil have an important bearing on the successful operation of a cable and that the use of a low viscosity oil in solid type cables may be undesirable. However, we have found by accelerated aging tests that solid type 132-kv. cable can be made to operate successfully.

It is interesting to note that the 1930 oil-filled cables used a lower viscosity oil than the 1927 cables. Mr. Roper gives this as the reason why the single lead sheath was used for the 1930 cables as compared to the double reinforced sheath of the 1927 cables. As the 1927 cables have not been loaded as high as permissible, they have not yet been subject to the effects of sudden changes in temperature due to changing load. I am more inclined to believe that the lead sheaths on the 1927 cables were too thin to start with and each by itself was not able to withstand the positive pressure necessary. If a single heavy lead sheath was used, they would not have experienced the sheath failures they did. It is believed also that equally as good operation can be obtained with a slightly heavier oil than even used in the 1927 cables and also obtain a more stable oil.

The paper by Messrs. Shanklin and Buller is a very interesting treatise on the theoretical aspects of oil-filled cable. Some of the advantages over the solid cable are being met by changes in types of oil and paper in the latter, but from tests which we have recently made, we are still not able to duplicate entirely the oil-filled cable, although the solid 132-kv. cable is a feasible design.

It is possible to obtain on solid type cable, power factor—voltage curves similar to those shown by Mr. Roper in Fig. 31. We have under test, a 132-kv. solid cable which has been subject to double voltage for over 500 hours and many load cycle tests up to 78 deg. cent. on the copper. The power factor is decreasing with time and compares favorably with the oil-filled cable. We are planning to increase the voltage to three times operating voltage in order to speed up the test. With our changing views on solid cable, it is possible that three-conductor, 66-kv. cable may be furnished in the near future.

We have found when studying the absorption of gases by oils that the more easily the oil is absorbed, the harder it is to extract the gas again. Carbon dioxide was very easily absorbed and hard to extract, whereas nitrogen would not be easily absorbed and very easy to extract. Air was intermediate between the two gases.

The paper by Messrs. Atkinson and Simmons opens up a new field of study and application of the principles of electrostatics to cable joints and potheads. I believe that the condenser type of joint will be adopted for all high-voltage, single-conductor cables and also three-conductor for 33 kv. and over. The advantages of reduction in length, the making of the unit in the factory, the reduction in the time of making of a joint in the field and, therefore, the time of exposure of the cable to the air are all important enough to consider for the future cable.

The difference in the results obtained by the two oils A and B in the Whitehead, Hamburger paper indicates what we have all found, namely, that the oils during the last year or so have been far superior to previous oils and it is not possible to make a direct comparison of results because of the changes in the characteristics of the oils.

The reduction in power factor which is shown in Fig. 7, Section 90-A has been found on some cables we have tested with current loading and overvoltage, our time extending over several hundred hours.

The influence of the type of cable paper on the electrical characteristics of the combination of oil and paper is most important. We have tested many samples of paper with the same oil and obtained widely varying results. The effect of porosity (air density) is quite important. Also, the weight density, fiber formation and manner in which the paper was made, that is, the boiling of the pulp, washing, beating, and manner of calendering.

The ideas advanced in the paper by Mr. Holmes would have possibilities if a large number of tests could be collected on the same cables to obtain the proper constants. However, each reel of cable manufactured is slightly different than all other reels and a group of cables of one manufacture will have different constants than cables of another manufacture. Considering the low number of high-voltage tests which are made on a given order (one short time test and one long time test, per 15,000 ft. of cable), there are not a great deal of test data to analyze. However, this should not deter us from trying to apply Mr. Holmes' ideas to our laboratory test data. As his predictions are based on the ratio of sigma to $E\alpha$, or coefficient of variability, the smaller sigma becomes, the more nearly we can operate a cable to its average breakdown stress $E\alpha$. To date, we have had quite a variation in sigma, due equally as much to end failures of the samples as variations of failures under the sheath, perhaps more so by end failures. It is true that higher and more uniform values of breakdown voltage are being obtained today, but further improvement is desirable and all of it cannot be obtained by improvements in cable manufacture; improvement in test methods is necessary.

Too much must not be expected from this method of predicting cable failures in service. Only about 20 to 25 per cent of all cable failures are chargeable as inherent cable defects. Therefore, we are only guessing as to the possibility of some of these failures. Also, our predictions are based on laboratory voltage tests. Cable failures are caused from the effects of the current load cycles a cable is subjected to in service. As the cable goes

through the temperature effects due to current loading, the compound migrates due to the increase in its volume and pressure created. This movement can be both longitudinal and radial as the cable heats up. On cooling if the compound does not return to its original position in the cable, the uniformity of saturation is upset and we have the beginnings of the deterioration of a cable. It may take years for it to develop in service to a condition which is serious. The time as to when the cable will fail cannot be predicted ahead as the influencing factors—variation of current loading, suddenness of change and range of temperature from cold to hot to cold—vary from day to day.

The best that can be obtained from these new ideas is that considering the laboratory tests only, the cable having a low value of coefficient of variability may give a better account of itself than a cable having a high coefficient of variability. Note, however, that as the breakdown voltage goes up, a higher value of sigma for the same coefficients is permissible. Is this not a talking point for the highest breakdown voltage possible?

The law of probability appears to have been substantiated in this paper. The writer recalls studying the effect of size of electrode area on the breakdown voltage of varnished cambric about fifteen years ago. Tests were made with a single electrode and also with a large number of small electrodes to give the same area. In each case the same breakdown was obtained for the same total area. The law of probability did not apply. Instead it appeared that the effect of electrode area on the dielectric strength could be expressed as a function of the reciprocal of the square root of the radius of the electrode.

G. B. Shanklin: I fully agree with Dr. Whitehead's statement that a flat power factor—voltage characteristic is a very important criterion of cable quality. We have never accounted for the slightly rising characteristic in the test cable furnished Mr. Roper. Apparently, this particular cable is "the exception that proves the rule."

Mr. E. G. Linder rightfully points out that the chemical changes that occur when ionization exists have a very important bearing on cable life. We do not fully agree with him, however, in concluding that knowledge of this effect is even more important than ways and means of removing initial impurities. Ionization is merely a symptom or result of disease. Prevention of the disease (voids) is the correct treatment.

Mr. Middleton wishes to know how segregation of lengths of oil-filled cable can be obtained without resorting to stop joints. This is accomplished by use of simple semistop joints as described in a separate publication, *Gen. Elec. Review*, July, 1931.

Although Mr. R. J. Wiseman seems to agree with us regarding the relative merits of oil-filled cable for 132-kv. service, he also feels that solid cable is, or can be made feasible for 132-kv. service by using amongst other things, a comparatively thin impregnating compound. We have had a good deal of experience along these lines and agree, as far as improvement in endurance quality of solid cable is concerned, but our research and experience have never indicated that this improvement will carry solid cable into the 132 kv. class.

F. H. Buller: There are two points in the discussion by Messrs. Miller and Wollaston with which we are in whole-hearted agreement. First, the equations for the temperature transients and oil demand are decidedly cumbersome; and second, we are fully aware of the fact that the principle of superposition applies to this problem.

We cannot, however, accept the estimate of one week as the time required to obtain numerical results from our equations. Experience indicates that from one and one-half to two days are actually required. The most tedious part of the process is the evaluation of the root of equation (4), which feature is also essential to Mr. Miller's solution.

We wish to point out that a complete theoretical solution is always the first and most necessary step in the handling of any problem, even though it may be cumbersome. After such a solution is developed, it gives an exact understanding of the problem, and furnishes a basis for further progress in the way of simplifications and short cuts, the accuracy of which can be determined by comparison with the complete solution.

There are two methods of reaching a simplified short-cut solution of this particular problem. First, by the use of charts, as proposed by Messrs. Miller and Wollaston. Second, by the use of a mechanical model, as proposed and used by Emanueli. We are yet undecided as to which of these two methods will be adopted for future routine calculations. We would not be willing to adopt either of these approximate methods if the complete solution were not available as a final check and guide. A set of charts, such as Mr. Miller proposed for the rapid evaluation of the roots of equation (4) would be very useful indeed.

Since Messrs. Miller and Wollaston have drawn attention to the principle of superposition, it may be pointed out that it was used in deriving equation (2) for A_n; and that it can be used readily, in conjunction with our equations, to obtain the solution for loads varying in any manner whatever.

It is, of course, unnecessary to solve equation (2) for A_n for each change of load. A simple multiplication is all that is required, so long as copper loss, sheath loss, and dielectric loss maintain a constant ratio. Even if these ratios are not constant, the labor of solution is only increased very slightly.

One of the criticisms presented by Messrs. Miller and Wollaston concerns the manner in which the distribution of dielectric loss has been handled. In this connection, it would be well to note that one of the authors, working in collaboration with Mr. R. H. Park, to whom grateful acknowledgment is hereby made, developed a formula for A_n which takes into account with full precision almost any conceivable distribution of losses, dielectric or otherwise. This formula is more complicated than the one which appears in the present paper. It is, however, available, and can be used to check the oil demand and temperature transients whenever such a check is warranted. For most practical cases, however, the more simple formulas presented in our paper hold with good accuracy.

Messrs. Miller and Wollaston's method of evaluating the A_n coefficients is perfectly practical. It will give a close approximation to the right answer providing that all the preceding theoretical and numerical work has been correctly performed; but it is not self-checking. The solutions developed by the present writers involve the calculation of the coefficients without direct reference to the numerical values of the boundary condition mentioned by Messrs. Miller and Wollaston. When the coefficients have been thus evaluated, the resultant expression will be found to check this boundary condition closely, providing a sufficient number of terms is used, unless some numerical error has been made. The method is thus self-checking. Usually, three terms will give a result close enough for all practical purposes.

R. W. Atkinson: We have been very much gratified by the favorable reception of the condenser type joints and terminals. One or two points brought out may be developed a little further.

Mr. Shanklin, agreeing with the correctness of the condenser principle as applied to high-voltage joints and terminals, points out that, in competition with the non-condenser construction which has been proved by use, they must be shown by experience to be equal mechanically and electrically, and equal or lower in cost. This is, of course, entirely true, and considerable operating experience is now available to confirm very satisfactorily their mechanical and electrical reliability. It is believed that all of the expectations regarding these are well justified.

It has been customary to assume that a "feeder" joint (that is a joint at which the cable is connected to the oil supply) needs to be a "stop" joint, and this requirement has been the controlling factor in determining the number and location of stop joints in single-conductor lines having comparatively level contour. With the earlier types of stop joint and hand-wrapped normal

joints this was necessary, but the condenser construction lends itself readily to use as a non-stop, feeder joint. When so used the stop tubes may be eliminated entirely, resulting in a joint much simpler than the hand-wrapped type, and an actually lower installation cost. This feature, combined with the reduction in manhole space required below that required by earlier designs of stop joint having comparable electrical strength, is of great importance in its effect on the overall cost of the installation.

In his discussion Mr. John H. Palmer gives the formula which he has used in determining the contour of metal shields for cable terminals.

In our paper the following formula is given for the determination of the slopes of electrodes in the cylindrical condenser joint:

$$r'' = r'\left(\frac{r'}{r}\right)^{\frac{v}{V-v}} \tag{1}$$

in which r is the radius of the conductor
r' the radius over the conductor insulation
r'' the radius at a given point on the shield
V the cable voltage
and v the voltage at a point on surface of the conductor insulation radially below the given point on the shield.

If the added insulating material were different than that in the cable and if we represent by K_1 and K_2 the respective specific inductive capacities of the cable insulation and the joint insulation, the formula is as follows:

$$r'' = r'\left(\frac{r'}{r}\right)^{\frac{K_2}{K_1}\frac{v}{V-v}} \tag{2}$$

The formula given by Mr. Palmer is mathematically identical with formula (2).

Lack of space in our paper caused us to limit our discussion of the formula to its use for terminations of condenser plates in cylindrical condenser joints. However, we have recognized that the formula is equally applicable to shields for terminals and have employed it for that purpose.

The above formula has the interesting property that the longitudinal stress along the surface of any cylinder concentric with the conductor is uniform. The value of this stress is different for each cylinder, increasing as the diameter of the cylinder increases, so that at the end of the shield the longitudinal stress may be considerably higher than at the beginning. We derived a formula for shields which gives uniform longitudinal stress along the shield. This formula is as follows:

$$r'' = r'\left(\frac{r'}{r}\right)^{\frac{K_2}{K_1}\left(e^{\frac{gl}{v}}-1\right)} \tag{3}$$

in which g is the longitudinal stress at any point on the shield
l the distance from the edge of the sheath to the given point on the shield
e the base of Naperian logarithms
and the other symbols have the same significance as for (1) and (2).

In various applications we have found it of advantage to construct terminal shields in accordance with formula (3) rather than formula (2). The shields used by Whitehead and Hamburger, for which they gave credit in their paper to one of the authors, were designed in accordance with formula (3).

As was stated in our paper, the matter of using distributed capacity and shields so shaped as to relieve stress concentration has been given a great deal of thought by engineers throughout the world for some time. Various formulas have been derived by various people under certain assumed conditions. Under the same assumptions formulas derived by different engineers will, of course, be mathematically the same. For example, German Patent 383346 issued to the Pirelli Company in October 1923 mentions a formula for terminal shields which is mathematically identical with formula (3) when $K_1 = K_2$.

The importance of such mathematical analysis, and the utilization of means of control of potential distribution, such as is offered by the condenser construction, is progressively enhanced as operating and test voltages are still further increased. With earlier types of construction, the size and cost of joints and terminals bear an exponential relation to the electrical strength, so that at the highest operating voltages, any increase in voltage requires an enormously greater proportional increase in size. With the condenser construction, on the other hand, the necessary increase in dimensions is almost in direct proportion to the increase in electrical strength required. Whereas only a few years ago we seemed to have almost reached the economic limit in voltage for cable operation, the new developments which have here been discussed have again pushed the practical limit safely beyond the present demands. The remarkable operating record of 132-kv. oil-filled cables; the even more remarkable results of tests at high stresses; the actual making and testing of normal commercial lengths of cable adapted for operation at 220 kv.; and the development of accessories and of joints and terminals of the type described, make the use of cables at this voltage immediately practicable.

M. C. Holmes: Referring to Mr. Atkinson's discussion, it seemed obvious, but perhaps it should be stated explicitly, that the equations do not apply in the case of operating voltages higher than the mean breakdown voltage of the samples, so that arguments as to what they predict for operating voltages of 125 per cent are meaningless as well as trivial. If Mr. Atkinson understands a long-time test to mean one of five-minutes duration then he is justified in his criticism of the method of application. The author believes, however, that he has applied it in exactly the manner which Mr. Atkinson suggests.

Mr. Shanklin discusses the limitation of the theory on the ground that few if any failures in modern cable are due to causes inherent in the insulation. Mr. Wiseman objects to it for the same reason but states that only 20 to 25 per cent of all cable failures are chargeable as inherent defects. As a matter of fact, neither view is entirely correct. The facts are easily ascertainable. They are contained in the annual report for 1928 of the subcommittee on cable operation of the N.E.L.A. The results on the 10,000 miles of cable in the United States for the period 1923 to 1928 show 33 to 45 per cent of all failures to be attributable to "Inherent or Unknown" causes, and for the same period, 25 to 31 per cent "known to be due to causes inherent in the insulation."

In considering the applicability of the laws of probability to such problems as the present one it should be emphasized that while we cannot predict with any degree of certainty the results of a single test on a piece of cable, we can predict with certainty and exactness what will happen "in the long run" or "in the aggregate." The quantities dealt with in this paper should be considered from the same viewpoint. The laws of probability apply in the case of the chaotic motions of molecules, in determining gaseous pressures, as well as for the life insurance companies in predicting the behavior of aggregates of such variable units as human individuals. The subject is an extremely practical one as may be seen from a consideration of the many different fields in which it has been successfully applied.

Mr. Wiseman speaks of experiments made to determine the effect of electrode area on breakdown voltage and concludes that the law of probability did not apply. The author would be interested to know which one of the laws of probability did not apply, and by what chain of reasoning the conclusion was arrived at. It should also be obvious that end failures of samples and failures in pot-heads should not enter into the calculation of sigma.

In conclusion, it is felt that the main objectives of the paper should be restated. They are two-fold; first, to provide a measuring stick for gaging the hitherto vaguely expressed term, variability; and second, to show quantitatively, the part played by the variability in determining breakdown voltages.

D. W. Roper: It is very gratifying to have Mr. Shanklin's confirmation of my economic data on cable with the two types of insulation. Mr. Shanklin's comparison of the relative future prospect of the two types, however, assumes that no reduction will be made in the cost of cable with the ordinary type of insulation. If such is the case, the progress and experience abroad indicates that it will be because the American manufacturers have lost interest in the improvement in insulation of the ordinary type. In Chicago there have been only seven failures in 238 miles of 66-kv. cable that was shipped 1927-1930, inclusive, and in each case careful examination subsequent to failure indicated that the cause of the trouble was due to a local deficiency in impregnation. While such deficiencies might have been eliminated by a longer time of impregnation, it is also pointed out that the elimination of such locally deficient spots is facilitated by the use of thinner insulation. In considering reductions in thickness of insulation, it is suggested that the manufacturers might very profitably give more careful consideration to the actual causes of the failures which have occurred with the present thicknesses.

In paragraph (a) of his discussion, Mr. Shanklin claims a reduction in thickness of oil-filled insulation for 66-kv. cable from 0.40 and 0.42 inch to 0.35 and 0.375 inch. In considering this claim, it should be noted that the thickness of oil-filled insulation on the cable installed a year or two ago by the Central Electricity Board in England was 0.355 inch.

Mr. Shanklin lays great stress on the reduction in the factor of safety which results from a reduction in the thickness of either type of insulation. It is not the percentage reduction in factor of safety that is important, but rather the factor of safety of the reduced insulation. All of our information indicates a high factor of safety with the reduced thicknesses of insulation which have been advocated and with the high quality of insulation now obtainable.

Referring to Mr. Shanklin's comparison of the two types of insulation for various operating voltages, apparently the best reply is to reiterate a paragraph from the paper, as follows:

"Whether the oil-filled type will displace the ordinary type of 66 kv. and lower voltages or the ordinary type compete with the oil-filled type at 132 kv. depends on the relative rates of improvement in the two types of insulation with unrestricted competition."

Mr. Shanklin's concluding paragraph hardly gives the American manufacturers due credit. N.E.L.A. records show that all high-voltage cable made in the past three years averages well below two insulation failures per 100 miles per year.

Mr. Del Mar is in error in stating that my paper shows that 66-kv. single-conductor, oil-filled cables can now compete with cables of the ordinary type between 40,000 and 115,000 kva. He seems to have drawn his conclusions from Fig. 8 of the paper, but this includes only the cost of the cable installed in the conduit, without conduit, joints or accessories. Fig. 12, however, shows that the annual charges for 66-kv. cables of the ordinary type are below those of the oil-filled cable for the entire range mentioned by Mr. Del Mar.

Mr. Del Mar also intimates that my recommendations for reduced thickness of insulation were obtained from "laboratory-made samples and from factory-made lengths of outstanding merit." If the results of the tests on the few laboratory-made samples were entirely neglected, the conclusion would not be altered. I cannot object to Mr. Del Mar's statement that the factory lengths tested were of outstanding merit, but I will add that the lengths tested were sufficient in number to insure that they were typical and representative of entire shipments on commercial orders, and the conclusions from these tests have been confirmed by a perfect operating record of the same shipments, which now amounts to over 400 mile-years, without any insulation failures. As this cable has withstood not only heavy continuous loads, but also variable loads without appreciable deterioration, this record appears to be a sufficient answer to Mr. Burd's comment at the end of the first paragraph of his discussion.

In reply to Dr. Whitehead's question, it does not appear proper to allow an increased credit to the oil-filled type of insulation because of its lack of deterioration, as it appears perfectly possible to secure at the present time the ordinary type of insulation that will not deteriorate in service.

Dr. Whitehead's comments on the gradual rising of power-factor-voltage characteristic of the oil-filled cables with reduced insulation are very interesting. Theoretical considerations indicate that a horizontal characteristic within the operating range would be advantageous, but if a cable that initially has a slightly increasing characteristic will acquire a horizontal characteristic under operating conditions in about one year's time, it is questionable if the added expense of obtaining an initially horizontal characteristic is warranted.

It is interesting to note Dr. Wiseman's prediction that better results can be obtained with the ordinary type of insulation by using a more fluid oil for the impregnating compound. This is in accordance with my suggestions in meetings with the cable manufacturers for many years. It is also interesting to note that he hopes to be able to produce a successful 132-kv. cable with solid type of insulation. This would be quite within the prediction in my paper. With reference to his prediction regarding three-conductor 66-kv. cable, it may be noted that three-conductor 60-kv. "solid" type cable has been in operation in Paris for a few years.

Extinction of Short A-C. Arcs

BY T. E. BROWNE, Jr.*

Associate, A. I. E. E.

Synopsis.—This paper presents the results of a study made with short, stationary a-c. arcs between metallic electrodes, in order to determine the effect of arc length, electrode material, and current magnitude on the rate of recovery of dielectric strength of the arc space following a current zero.

The experimental results show (1) that the arc space recovers the ability to withstand a hundred volts or more within a few microseconds after a current zero, as predicted by Slepian's theory, (2) that the maximum allowable rate of rise of voltage across the arc space which will just permit arc extinction is much greater for very short arcs (1 cm. or less) than for longer arcs, (3) that the critical rate of rise of voltage depends largely on the electrode material used and tends to vary as a decreasing linear function of the boiling point, and (4) that the higher current arcs have a much lower critical rate of rise of voltage than the lower current arcs.

* * * * *

I—Introduction

THE extinction of an a-c. arc, which normally occurs at a current zero, has been shown[1] to depend upon the outcome of "a kind of race" between two opposing processes taking place within a very short period embracing the moment of zero current. The first is the building up of voltage across the arc space tending to cause the arc to reignite, and the second is the diminishing of the conductivity of the arc space, or the increasing of its dielectric strength, tending to prevent the arc from restriking. The first depends on the constants of the circuit and so can theoretically be calculated. In practise, it may be approximately determined if the circuit involved is sufficiently simple. The second process, with which this paper will deal, depends on the characteristics of the arc itself, which are exceedingly complex and as yet understood to only a very limited extent.

II—Short Arcs in Electrical Engineering

Short arcs are employed in all circuit interrupters, such as knife switches, snap switches, and fuses, operating in a-c. circuits of less than 150 volts. They may also occur as the result of faults in low-voltage circuits and machinery, and in this connection are of very great importance in the case of the modern secondary network used for distribution of low-voltage power in congested city districts. Here the working voltage is limited by the necessity for self-extinction of arcs following "burning clear" of faults, without interrupting the power supply. The extinction characteristics of short a-c. arcs are also made use of in the deion air circuit breaker, where a large number of short arcs is placed in series for operation at higher voltages.[2]

III—Review of Theory of Extinction of Short A-C. Arcs

According to the modern theory of conduction in gases, the rapid recovery of dielectric strength by an arc space immediately following a current zero must be due to the disappearance of the ions which are required for conduction of the arc current. These ions are lost from an arc space in air at atmospheric pressure in three principal ways: first, by recombination of positive and negative ions throughout the space; second, by diffusion from the space to the electrode surfaces and into the surrounding cooler air; and third, by formation of an almost completely deionized layer adjacent to the cathode by the action upon the ions of the growing impressed electric field.

Simplified calculations made by Slepian,[1] assuming ions to be lost from the arc space only by direct ion recombination, predicted a manner of recovery of dielectric strength substantially the same as that actually revealed by experiments made on cold-electrode arcs. These experiments were made by controlling the rate of recovery of voltage across the arc space following a current zero by shunting the arc with resistance. The maximum allowable rate of rise of voltage was determined from the maximum value of shunting resistance at which the arc would just be extinguished after one half cycle. It seemed desirable to repeat these experiments on the more common type of stationary arcs with molten electrodes such as occur in electrical engineering, and to determine the influence upon the arc characteristics of such parameters as electrode material, current strength, and arc length.

IV—Experimental Results

In general, the manner of dielectric recovery following a current zero, of short, stationary a-c. arcs with molten electrodes, was found to conform to the predictions of Slepian's theory and to be similar to the recovery of rapidly moving cold-electrode arcs previously investigated. That is, for electrodes whose boiling temperatures are below that required for appreciable thermionic emission, the ability to withstand a hundred volts or more was recovered within a few microseconds after a current zero, and further dielectric strength was regained at a very much lower rate. This is shown by the curves of Fig. 1 which are noticeably similar to the curve first obtained and published by Slepian[3] of dielectric recovery with time of the cold-electrode arc.

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1. *Extinction of an A-C. Arc*, J. Slepian, Trans. A. I. E. E., 47, 1928, page 1398.

2. *Theory of the Deion Circuit Breaker*, J. Slepian, Trans. A. I. E. E., 48, 1929, page 523.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

3. *Extinction of a Long A-C. Arc*, J. Slepian, Journal A. I. E. E., April 1930, page 310.

As predicted, the initial very rapidly recovered dielectric strength was found to depend solely on the electrode material and gas medium, and not at all on any other conditions within the limits investigated. The further more slowly recovered dielectric strength did not, however, prove to be independent of such conditions as arc length and current magnitude, as Slepian's approximate theory predicts that it should.

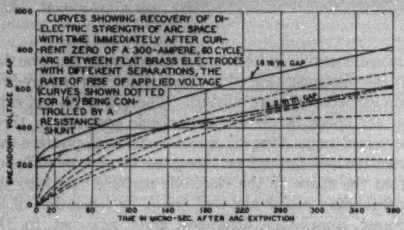

Fig. 1

This simply indicates, as expected, that factors neglected in the simplified theory, such as diffusion of ions to the electrodes and to the surrounding air, the electric field, and temperature variations, are not actually negligible.

IVA—Effect of Arc Length

The effect of length of the arc on recovered dielectric strength was found to depend on the electrode material used. In the first series of tests made with brass electrodes, the effect of arc length was very striking and at first sight seemed paradoxical. As shown in Fig. 1, the dielectric recovery of a 0.158-cm. arc was considerably more rapid than the recovery of an arc of twice that length under otherwise similar conditions,

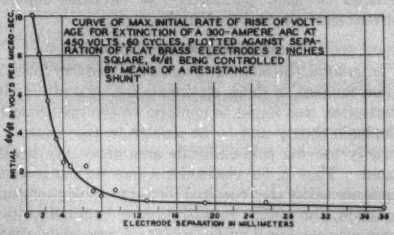

Fig. 2

indicating that the shorter arc is easier to extinguish than the longer. This effect is shown in a still more striking manner by plotting the initial slopes of critical applied voltage curves, like those shown dotted in Fig. 1, against arc length, as in Fig. 2. This shows that the effect extends up to five cm. or more, but is most pronounced at lengths of less than one cm. A similar but still more pronounced effect was observed with metals of lower boiling point than that of brass. With electrodes of the higher boiling metals, however, such as copper, iron, and tungsten, the effect was very much less and reversed its direction at the shorter lengths. This is illustrated by Fig. 3, which is the curve of Fig. 2 for brass, redrawn to a different scale and compared with similar curves for mercury, zinc, copper, iron, and tungsten-electrode arcs. The curves for copper and iron are seen to coincide practically with that for brass near five mm. and above and to exhibit the same falling characteristic above about four mm.; but below this length, instead of rising rapidly with diminishing length as with the lower boiling metals, they fall off again; the curve for iron rapidly approaching zero and the curve for copper falling and then apparently tending again to turn upward at the shortest lengths investigated. With the very refractory metal tungsten, the curve is

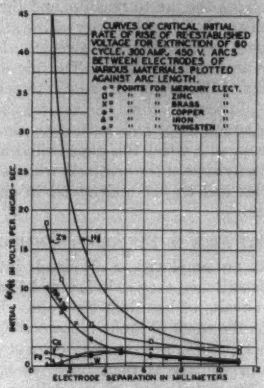

Fig. 3

much lower even than with copper and iron, but still appears to reach a maximum at some arc length less than one centimeter and to fall off with increasing length above that.

IVB—Effect of Electrode Material

As already shown, the behavior of a short a-c. arc depends very largely on the material of the electrodes. Fig. 4 shows the results of a series of experiments on 300-ampere, 60-cycle, 450-volt arcs, 0.079-cm., 0.158-cm., and 0.316-cm. long, using various metallic elements as electrodes. The critical initial rate of rise of voltage as measured by the critical shunting resistance for arc extinction is plotted in each case against the boiling point of the element. The interesting and surprising feature of this graph is the definite tendency for the points at each arc length to fall on intersecting straight lines. This approximately linear manner of variation with boiling point applies to all the metals for which

results were obtained except mercury, which has values at about twice those required to fit the straight lines, and magnesium, aluminum, manganese, molybdenum, and tungsten (not shown), which either form stable refractory oxides or themselves boil at high enough temperatures for copious thermionic emission, thus preventing the immediate recovery of dielectric strength which characterizes the non-refractory metal arcs.

In Fig. 5, the performance of aluminum, copper, and

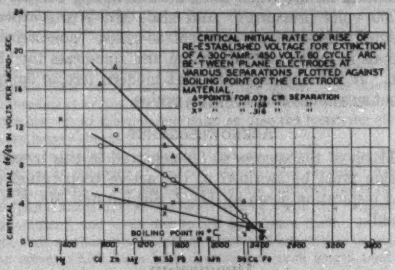

FIG. 4

brass-electrode arcs is shown comparatively in the form of curves of critical initial rate of rise of re-established voltage plotted against the effective circuit voltage at which the values were obtained, other conditions than voltage remaining the same. Remembering that the ordinates of any one of these curves may be considered as a measure of the speed with which the arc space regains the ability to withstand voltages equal to the

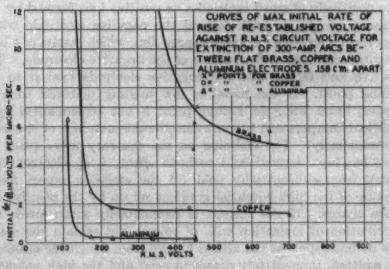

FIG. 5

maxima of sine waves having the effective values given by the abscissas, it may be seen that 0.158-cm. arcs with aluminum electrodes require a very much longer time to recover ability to withstand a given voltage than do arcs of similar length, between copper electrodes, and that 0.158-cm. copper-electrode arcs in turn recover dielectric strength very much more slowly than do 0.158-cm. brass-electrode arcs, other conditions being equal. The rapid rise of all of these curves at the lower voltages is attributed to the approach of the maximum voltage values to the minimum dielectric strength of the cathode space charge sheath, which is recovered in practically zero time. This initially-recovered dielectric strength is approximately 160 volts (maximum) for both copper and aluminum arcs, but is nearly 230 volts for brass-electrode arcs. Because of the scale used, the upper portions of the curves for copper and brass are not shown in this figure.

IVc—Effect of Current Magnitude

The simplified theory, already referred to, predicted that the dielectric recovery of a short arc space should also be independent of magnitude of the arc current, but this prediction depended on the assumption that ionic diffusion to the surrounding air is negligible, which can be true only if the diameter of the arc is large in proportion to its length. Even for arcs only 0.158 cm. long, this condition is approached only at currents of more than a thousand amperes or so; therefore independence of current magnitude may be expected only

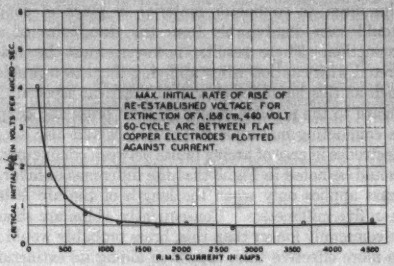

FIG. 6

with currents of this order. Fig. 6 shows this to be the case for a 0.158-cm. copper arc at 450 volts; current variations produce little change in the rate of dielectric recovery above 1,500 amperes, but below this value, decreasing the current causes a rapid increase in speed of dielectric recovery. Below about 150 amperes, the behavior of the arc becomes increasingly erratic so that definitely consistent values of a critical initial de/dt were very difficult or quite impossible to determine. Tests made by F. C. Todd[4] with the cathode-ray oscilloscope show that the restriking voltage of continually restriking arcs is extremely variable when the current is as low as 90 amperes.

V—Application to Engineering Practise

In view of the results here presented, some suggestions may be made with regard to the construction of low-voltage circuit-interrupting apparatus or of circuits where extinction of low-voltage a-c. arcs resulting from

4. "Restriking of Short A-C. Arcs," F. C. Todd and T. E. Browne, Jr., *Phys. Rev.*, pages 732-737, Vol. 36, Aug. 15, 1930.

faults is important. Because of the marked superiority of short brass-electrode over short copper-electrode arcs with regard to speed of recovery of dielectric strength at current zero, it would seem advisable to use brass for the contact breaking parts of low-voltage switches working near their limit. Even where final dependence is placed on drawing out to sufficient length to insure extinction of the arc, use of brass might conceivably result in quicker opening of the circuit with less resulting flame and burning of the contacts than when copper is used. Similarly, aluminum is evidently inferior to copper as a contact breaking material, from the standpoint of extinguishing the arc while it is still short.

Since, as already mentioned, the problem of extinguishing short a-c. arcs is of particular importance in the operation of secondary distribution networks, the above suggestions in regard to use of brass, copper, and aluminum may seem to apply with even more force here than they do to switch construction. Thus the possibility is suggested that the permissible operating voltage of secondary networks might be raised if brass conductors were used instead of copper conductors. Again, use of aluminum as conductor material in place of copper would seem inadvisable from the standpoint of arc extinction. These results, however, must be interpreted with caution when applying them to a problem such as this having such varied practical aspects. In regard to the possible choice between brass and copper as conducting materials where they would also act as electrodes in case of arcing faults between conductors, it should be noted in Fig. 3 that the difference in favor of brass applies only when the length of the arc is less than five millimeters. Consequently, if the arc should be commonly longer than this, brass probably would be no more effective than copper in aiding its extinction. Sufficient data for a similar curve for the aluminum arc have not yet been obtained, but it can be predicted with reasonable certainty that the inferiority of aluminum to copper will be at least greatly reduced at arc lengths of a centimeter or more.

It may be noticed, also, that there is a discrepancy between practical experience with faults in secondary networks and the results given here, in that faults are commonly cleared on networks at as high as 230 volts (r. m. s.) whereas in the laboratory high-current copper-electrode arcs 0.158 cm. long could be easily maintained at as low as 113 volts. This may be due to less severe circuit conditions in the case of the practical network caused by a fairly high power factor on short circuit and the presence of distributed capacitance of the cables and of shunting resistance in the form of connected lighting load, all of which tend to decrease the rate of recovery of voltage across the arc space at a current zero and so allow time for deionization of the arc and recovery of dielectric strength. Probably of more importance, however, is the deionizing action of gases resulting from burning of the cable insulation, the action of which is similar to that of gases from the decomposing oil in an oil circuit breaker.[3]

Before closing, mention must be made of the fact that some of the principles described in this paper have been known in a general way for a long time, and moreover, have been applied to commercially successful devices. The writer refers to the lightning arresters developed by Mr. A. J. Wurts and described by him as long ago as 1892.[5] In this remarkable paper the author states, "The smaller the air gap (with zinc, brass and antimony electrodes) the less the tendency there is to maintain an arc when using an alternating current at 1,000 volts pressure." And also, Mr. Wurts experimentally determined a list of so-called "non-arcing" metals in which zinc, cadmium and zinc-brass top the list. It is to be regretted that modern testing equipment was not available to this genius.

ACKNOWLEDGMENT

Much of the success so far achieved in this investigation must be ascribed to the active cooperation of and inspiration given by Dr. J. Slepian and Mr. F. C. Todd.

Discussion

D. C. Prince: It is rather surprising that Mr. Browne's paper and other previous papers to which it refers should ignore some very prominent arc characteristics which cannot help but be an important factor in circuit interruption. No mention has been made of the rectifying effect of arcs in general and especially metal vapor arcs.

When an arc is drawn between metal electrodes, the mechanism of conducting the current is considered to be substantially as follows:

Electrons proceed from the negative electrode striking, after a short distance, neutral vapor or gas molecules which are ionized. The resultant electrons are added to the electrons initially emitted and progress toward the positive electrode or anode. The positively charged molecules or positive ions formed by the collision are drawn back toward the negative electrode where they form a positive ion space charge film, which results in a high potential gradient at the surface of the negative electrode. The positive ions falling into the negative electrode heat it to the boiling point. If the metal is low melting material such as copper, brass, etc., the boiling point is not accompanied by a sufficient temperature to emit considerable numbers of electrons thermionically. The emission on the contrary takes place by what has been termed cold cathode effect, *i. e.*, electrons are drawn from substantially cold (in the emission sense) metal because of the high potential gradient set up by the positive ion space charge. If the current is momentarily interrupted, the distribution of ionization will, of course, not remain the same. The cathode spot, as the area is known from which the electrons are emitted, immediately ceases to exist and the high potential gradient produced by the positive ions falls off rapidly. Therefore, if the arc is to continue, the impressed voltage in the same direction must be somewhat higher than the voltage across the arc while it was being maintained. These considerations would lead one to expect that the voltage required to reestablish an arc in the same direction would increase more or less continuously with time from the voltage required to maintain the arc.

On the other hand, if the test procedure involves the use of alternating current, the condition after a current zero is that a cathode spot must be formed on the opposite electrode to that on which it was prior to current zero. Immediately after the

5. "Lightning Arresters," A. J. Wurts, TRANS. A. I. E. E. Vol. 9, 1892, page 102.

current zero, the distribution of charges is substantially identical with that immediately before the current zero. There is thus no high potential gradient present at the surface of the new cathode, and therefore an impressed potential different from and greater than the drop in the arc preceding the current zero must be provided if an arc is to start. This arc is really a new arc and not a continuation of the old arc. It flows in the other direction. It has a cathode spot located on an electrode where there was none before and its starting is only facilitated by the fact that there are ions present which can be rearranged so as to produce a high potential gradient close to the surface. This condition, which has received a great deal of study over a number of years, is essentially the same as that which exists in rectifiers.

In the case of high melting material, such as tungsten, etc., the positive ion bombardment of the cathode tends to bring the latter up to the emission point where it may remain for a considerable time due to heat stored in the metal. The electron bombardment of the anode or positive electrode will heat the latter also up to the emission point so that the rectifying effect is very largely absent.

These considerations should tend to explain the marked difference in behavior of low and high melting materials noted by Mr. Browne. The arc phenomena are naturally made up of two parts, those due to the electrodes of which the cathode is predominant, and those due to the arc stream itself. In short arcs, such as those under consideration, the arc stream deionization effects which have been discussed at such length tend to become secondary. The cathode spot phenomena assume a predominant role in explaining the behavior of short a-c. arcs.

T. E. Browne, Jr.: The reference made by Mr. Prince to the so-called "rectifying effect" of a short metal vapor arc as a factor better able to explain the observed short-arc extinction phenomena than existing theory[1,2] is rather puzzling. It is true undoubtedly that the metal vapor rectifier is essentially a circuit-interrupting arc device and therefore that the same general theory of electric conduction in gases applies to it as to ordinary a-c. arc devices, but, since the rectifier is a more complex and highly specialized piece of apparatus, it would seem to be more natural to use analogies based on the older and simpler devices in describing rectifier phenomena rather than to reverse the process. Also, the rectifier approach of Mr. Prince may be actually misleading, since it would seem to suggest that the reignition of an a-c. arc in the reverse direction after a current zero should be essentially different from reignition in the same direction, and therefore require a different minimum voltage. However, the results of experiments conducted by Attwood, Dow, and Krausnick,[3] using a cathode-ray oscillograph, indicate that no such condition exists in the case of short arcs between similar electrodes symmetrically placed.

The importance of thermionic emission from refractory electrode materials, mentioned by Mr. Prince, has been pointed out in a previous paper,[2] but the large effect of electrode boiling point in the case of the low-boiling metals, as illustrated by Figs. 3, 4, and 5 of the paper, probably cannot be explained on this basis. Although the phenomena that determine the extinction or reignition of a short a-c. arc at current zero do occur, as is generally recognized, at or immediately adjacent to the electrode surfaces, they still take place *in* the gaseous arc space and so must, in general, depend very largely upon the magnitude and rate of change of ionization density in this region. When thermionic emission is lacking, then, the principal influence of the electrode material upon short-arc reignition must be through the medium of its vapor, of which the gas adjacent to the electrodes is mainly composed. The Langmuir high-field theory (or "cold cathode effect") obviously cannot be applied to the process of arc reignition after an appreciable current zero period, since it depends essentially upon the existence of a high current density at the cathode, inasmuch as high current density is essential to the maintenance of the required space charge.

1. *Extinction of an A-C. Arc*, J. Slepian, TRANS. A.I.E.E., V. 47, p. 1398 (1928).
2. "Restriking of Short A-C. Arcs," F. C. Todd and T. E. Browne, Jr., *Phys. Rev.*, V. 36, pp. 732-7, (1930).
3. *Reignition of Metallic A-C. Arcs in Air*, S. S. Attwood, W. C. Dow, and W. Krausnick, TRANS. A.I.E.E., Vol. 50, September 1931, p. 854. (Figs. 15 and 16.)

A Corona Tube Voltage Regulator

BY H. W. DODGE* and C. H. WILLIS†
Non-member Member, A. I. E. E.

Synopsis.—A circuit is described whereby a corona tube may be used to regulate the output of a full-wave rectifier and therefore to control the excitation of a generator to give constant voltage. Performance curves are given for a five-kva. generator whose voltage is controlled by such a regulator.

* * * * *

A VOLTAGE regulator consists essentially of a constant element; an element which varies with the voltage and which is balanced against the constant element; and an arrangement for using this balance to control the excitation of the machine whose voltage is to be regulated.

In the familiar vibrating contact regulator the modulus of elasticity of a spring is the constant element. Acting against the spring is the pull of a solenoid which is dependent upon the voltage. The forces due to the solenoid and the spring act on a plunger which operates a contact by which resistance is put into or taken out of the generator field circuit, thereby regulating the generator voltage.[1]

The most serious disadvantages of the vibrating contact regulator are:

1. Wear and sticking of the contacts.
2. Periodic fluctuations of the voltage with opening and closing of the contacts.
3. Slow response of the regulator to fluctuations of the voltage, due chiefly to the time lag of the field circuits of the exciter and the generator.

Recently a number of new types of regulators has been devised. Almost any phenomenon which varies with voltage may be made the basis of a voltage regulator, such as the saturation of an iron core reactor, the resistance of a conductor heated by the voltage, or the internal resistance of a thermionic tube whose filament is heated by the voltage.[2,3] The regulator described below employs the critical voltage of a corona tube as the basic phenomenon.

The corona discharge tube employed consists of two symmetrical electrodes sealed into a glass tube containing an inert gas at a suitable pressure. Due to the fact that the tube contains a definite quantity of gas constrained to a fixed volume, the mean free path of an ion and therefore the corona voltage are practically independent of the temperature. Since ionization builds up very quickly at a voltage above the critical value, the breakdown voltage is also independent of the frequency for frequencies within the commercial range. With the electrodes surrounded by an inert gas such as argon there is negligible disintegration of the electrodes during discharge; the breakdown voltage of the tube is constant and the life very long.

These characteristics of the corona discharge tube make its critical voltage suitable for the constant element in a regulator. The fact that it regulates to a constant peak voltage rather than a constant effective voltage is not a serious disadvantage, for the variation of wave form with load is not appreciable and there will therefore be a constant form factor.

In order to control the excitation of a generator we may use the charge passing through the corona tube above the corona voltage to bias the grid of a rectifying tube and thereby regulate the amount of rectified current which will be supplied to the generator field.[4] Any type of rectifying tube may be used. However the high efficiency and large amplification factor of a gaseous tube, such as the thyratron, make this type of tube very desirable.

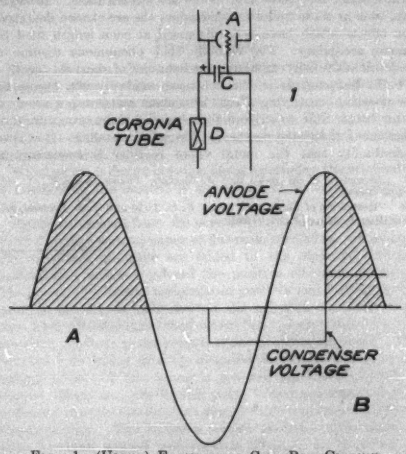

FIG. 1—(UPPER) ELEMENTARY GRID BIAS CIRCUIT
FIG. 2—(LOWER) RELATION OF ANODE VOLTAGE TO GRID VOLTAGE

Referring to Fig. 1, *A* is a thyratron with a negative grid characteristic, and it therefore normally conducts current into the field of the generator during the half wave when the anode voltage is positive as shown in Fig. 2A.

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1. For references see Bibliography.

Presented at the Summer Convention of the A. I. E. E., Asheville, N. C., June 22-26, 1931.

During the half wave when the anode voltage is negative the condenser C will be charged as shown in Fig. 1, provided the circuit is connected with the proper polarity and provided the voltage rises above the critical value for the corona tube. During the first part of the next positive half wave the thyratron cannot conduct current because the grid is negative (see Fig. 2B). However the grid bias will be reversed by the corona tube at the peak of the voltage wave, and

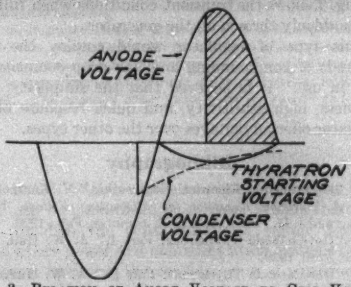

Fig. 3—Relation of Anode Voltage to Grid Voltage Using a Grid Leak

the thyratron will conduct current during the latter half of the positive voltage wave as shown by the cross-hatched area. In this way the average value of the rectified current will be reduced to one-half the full half wave or maximum value when the voltage is above the breakdown value for the corona tube.

If a voltage is impressed on the corona tube which lags in phase behind the rectifier voltage, the average current may be reduced to less than 50 per cent of its

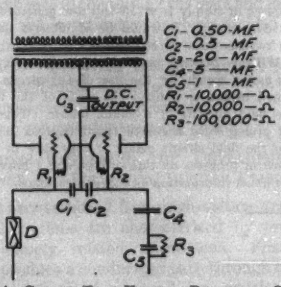

Fig. 4—Corona Tube Voltage Regulator Circuit

maximum value because the grid bias will then be reversed after the peak of the voltage wave. When the corona tube is supplied from a voltage 60 degrees behind the rectifier voltage as in a three-phase machine, the corona tube will reduce the average current to 7 per cent of the maximum value.

The circuit as described above will vary the excitation by definite steps and will give fluctuations in the excitation just as are produced by opening and closing the contacts of a vibrating regulator. However if a leak is placed across the grid biasing condenser of a value such that the time constant $C\,R$ of this condenser and leak is about equal to the period of one-half wave, then the grid condenser voltage may be

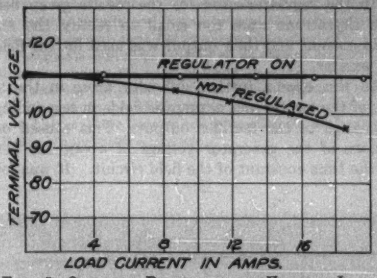

Fig. 5—Operation Regulator with Variable Load

reduced by leakage to a value where the thyratron will start. The time it will take to reduce the grid voltage to the starting value will depend on the initial charge of the grid condenser, and this is determined by the excess of the voltage above the critical voltage of the corona tube. Therefore the time of starting for the thyratron rectifier in any half wave will be delayed by an amount which is determined by the excess of the voltage above the breakdown voltage of the corona tube (see Fig. 3). A small rise in total voltage above the critical value will

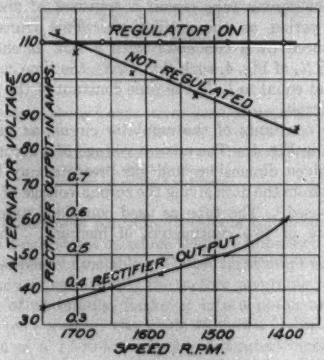

Fig. 6—Operation of the Regulator with Variable Frequency

serve to reduce the rectified current to about 7 per cent of its maximum value. However this change is continuous.

Fig. 4 shows the circuit used for controlling a full wave rectifier by means of a corona tube. It was found quite difficult to make corona tubes so symmetri-

cal that they would break down at the same voltage on the positive and negative half waves. However by adding the condenser C_4 shown in Fig. 4 it is possible to eliminate any unidirectional components of current and cause the corona tube to have the same positive and negative critical values.

In the regulator used for the results given below, the thyratrons were too small to rectify the entire field current and their output was used for the field of an exciter.

In this case a further difficulty arose in the time lag of the exciter and alternator fields in responding to a change in the rectifier output. This caused oscillations of the generator voltage of about the period of the time constant of the field circuit. It was found

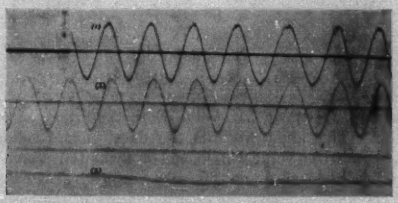

Fig. 7—(1) Load Current
(2) Alternator Voltage
(3) Regulator Output

possible to suppress these oscillations by introducing into the corona tube circuit a transient of about the same period as the oscillations. This transient is introduced by a condenser with a leak as shown at C_5 and R_3 of Fig. 4, such that $C_5 R_3$, the time constant, is about equal to L/R, the time constant of the slowest field circuit.

The constants of the regulator circuit as used are shown in Fig. 4. The critical voltage of a corona tube with given dimensions and gas pressure may be predicted from the data giving the corona voltage of the gas to be used.[5] The tube as used contained argon at a pressure of two centimeters of mercury. The electrodes were parallel wires 0.25 mm. in diameter and separated 1 cm. The critical voltage of this tube was approximately 350 volts peak value.

The results shown in the curves of Figs. 5 and 6 were taken on a 5-kva. three-phase motor-driven alternator excited from a 110-volt motor-driven exciter. In the curves of Fig. 5 are shown the effects of load variation while Fig. 6 shows the regulation when the speed of the driving motor is varied. The oscillograms shown in Fig. 7 shows the transient conditions when full load was suddenly thrown on the generator.

This type of regulator would employ the same methods of compounding and line drop compensation now in use. It is believed that the simplicity, compactness, high sensitivity, and quick response of this regulator offer advantages over the other types.

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Discussion

W. B. Kouwenhoven: The authors of this paper have presented in condensed fashion a description of a voltage regulator that is apparently quite simple and at the same time possesses no moving parts. In our work on dielectrics at Johns Hopkins University the accurate control of frequency is essential. I should like to ask the authors if it is possible to apply their system to the control of frequency. I should also like to have them give a complete diagram of the circuits involved, assuming that the source of supply is a 115-volt a-c. generator driven by a 230-volt d-c. motor. It would be of interest to know how closely the frequency may be controlled.

C. H. Willis: Several methods have been developed for controlling motor speeds by the use of thyratrons. We have used a tuned bridge to control the grids of the tubes which supply the field current of a shunt motor. This method is capable of giving very accurate speed control, but the time constant of the field circuit makes it very slow in responding. Work is now in progress on this subject and I hope to be able to present further information on this subject later.

Experimental Studies of Arcing Faults on a 75-Kv. Transmission System

BY J. R. EATON,* J. K. PECK,† and J. M. DUNHAM‡

Associate, A. I. E. E. Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*Qualitative results of an investigation, in which high-voltage impulses were applied to an energized transmission line, indicate that the probability of a lightning flashover developing into a power arc is increased when current is flowing past the flashover point, and when the normal frequency voltage is increased or the insulation decreased.*

An oscillographic study of power arcs ranging in magnitude from 8 to 800 amperes indicated that the resistance of an arc varies over the cycle and over the duration of the arc. The effective arc resistance is approximated by the ratio of the peak voltage to the peak current. The apparent power factor of the arc current and voltage is practically unity in most cases. The tests indicated that arcs formed in a capacitive circuit are more persistent than those in inductive or resistive circuits, and that the average peak volts per foot of arc length, for currents above 100 amperes peak, is about 300.

The maximum overvoltage recorded on an isolated neutral 75-kv. transmission system, 134 miles in length, as a result of 63 arcing ground tests was 3.0 times normal on a sound phase. The overvoltage is essentially a fundamental frequency phenomenon in which the sound phase voltage takes its new value without going through an oscillation of appreciable magnitude.

* * * * *

Introduction

A DISCUSSION of the continuity of service which can be maintained on a power system frequently involves the consideration of power arcs. In cases where power and communication systems occupy the same territory, the consideration of power arcs to ground is also important from the standpoint of their reaction on communication systems. In spite of the importance of arcs in the operation of power systems, there is but little available experimental information on the causes of power arc follow-ups due to surges, characteristics of arcs, or the resulting effects of arcs.

This paper presents the results of cooperative experimental investigations made by the Consumers Power Company, the General Electric Company and the Joint D. and R. Subcommittee of the N. E. L. A. and Bell System, to study the character of arcing faults. For convenience in conducting the tests and in preparing the paper, the investigation has been divided into three parts, namely:

1. Factors governing the development of a power arc following impulse flashover.
2. Characteristics of power arcs.
3. Overvoltages due to arcing grounds.

I. Factors Governing the Development of a Power Arc Following Impulse Flashover

Power arcs resulting from high-voltage transients in electrical systems are characterized by two distinct though closely related phenomena. First, voltage stresses produce a conducting path through the insulating material (ionization by collision) which permits the flow of current commonly known as spark discharge. If in addition to the transient, there is a continuous supply of energy available, certain changes take place resulting in heating of the electrodes (thermal ionization of the dielectric) and a continued flow of current known as an arc. Although the spark and the arc are quite different in nature, there is no sharp line of demarcation between the two, and the transition from one to the other is but little understood.

Operating data presented in the past few years have indicated that frequently lightning may cause sparkover of transmission line insulators without producing a power arc. This fact does not seem particularly surprising when consideration is given to the extremely short duration of lightning transients and to the nature of the a-c. systems on which they are impressed. With the equipment available at Croton an attempt was made to study experimentally the factors governing the transition of an impulse flashover into a power arc. For these tests a setup was made as shown in Fig. 1. By means of a 1,500-kv. impulse generator[1] high-voltage surges were applied to a transmission line conductor which was in turn energized from the transformer bank. Line insulation was so correlated that flashover took place at a predetermined test tower at which spacing D was varied at will. During part of the tests, connections were made equivalent to closing switch A, thereby

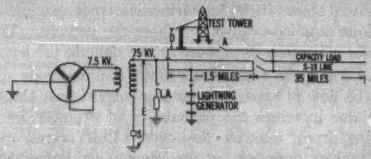

Fig. 1—Diagram of Connections Used in Impulse Flashover Tests

D—Variable distance between conductor and tower
With line off, switch *A* was open
With line on, switch *A* was closed

*Consumers Power Co., Jackson, Michigan.

†National Electric Light Association, New York City.

‡American Telephone and Telegraph Co., New York City.

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1. For references see Bibliography.

causing a normal flow of power current past the test insulator due to the capacity load of 35 miles of the three-conductor transmission line. As no attempt was made to synchronize the impulse generator discharge with the 30-cycle excitation of the line, the point on the 30-cycle voltage and current waves at which flashover occurred was governed entirely by chance. During certain tests, this point was determined by oscillographic measurements of voltage at E and by current measurements at S.

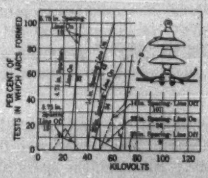

Fig. 2.—Per Cent of Impulse Flashovers which Developed Into Power Arcs at Various Line Voltages and Electrode Spacing, D with and without Capacity of 8-19 Line as Load

Numbers under spacing designation indicate number of tests in group

A total of 278 tests was made under a variety of test conditions. The curves of Fig. 2 show the per cent of sparkovers which resulted in power arcs (as determined by visual observation) for various electrode spacings and various applied values of 30-cycle voltage. These curves indicate several more or less definite tendencies:

1. For given excitation voltage and electrode spacing the flow of capacity charging current past the test insulator increases the probability of a power arc resulting from impulse flashover. This seems quite reasonable inasmuch as arc current is immediately available by the shift of current flow from the line to the arc, without the necessity of overcoming the inductance of the circuit, as would be the case if no initial current were flowing.

2. For a given electrode spacing, the possibility of power arc follow-up is dependent on the value of the normal frequency excitation voltage. Below certain values of voltage, a given insulator assembly apparently is almost immune from power arc follow-up. This critical value of power frequency voltage appears to be considerably higher in test than would be borne out by practical experience on operating lines.

During 66 of the 278 tests conducted, oscillograms of voltage and current were obtained as described above. From these records it is possible to determine the point on the 30-cycle voltage wave at which flashover occurred. Fig. 3 shows diagrammatically the results of these tests plotted according to line voltage and point on the wave at which the flashover occurred, but without regard to electrode spacing or whether current was flowing past the test insulator. This diagram shows that a high percentage of the flashovers occurring near the 90 deg. point of the voltage wave, resulted in power arcs. From a further analysis which is not shown on this diagram, it was found that this grouping of tests near the 90 deg. point was very definite for tests in which current was flowing past the sparkover point, but for the case when there was no current in the line (switch A open) the points were fairly well scattered.

In many of these tests, oscillograms indicated that the sparkover caused a considerable transient in the 30-cycle voltage wave, and in some cases even caused this voltage to fall to zero for a period of 5 or 10 elec. deg., without developing into a power arc. It appears that the development of a power arc is dependent on conditions existing within the first few thousandths of a second after sparkover. It is possible that on systems having large connected generating capacity carrying heavy loads, in which case large arc currents are immediately available, the per cent of sparkovers developing into power arcs may be much higher than is indicated by these tests. It is felt that the conclusions drawn from these experiments must be interpreted as being qualitative rather than quantitative.

II. Characteristics of Power Arcs

In connection with relaying and short-circuit computations, it is desirable to know, under certain fault conditions, the characteristics of power arcs. The difficulties of investigating experimentally under practical conditions the effect of each of the many variables determining a long arc in air, are manifold. Experimental investigations[2] of arc characteristics have been made from time to time under practical conditions

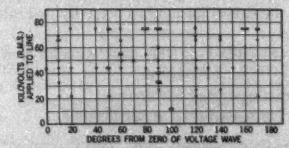

Fig. 3—Point on 30-Cycle Wave at which Impulse Flashover Occurred

X Arc formed
O Arc did not form

which add considerably to the present knowledge of the behavior of arcs on transmission systems. The tests which are described here deal chiefly with a discussion of the variations in arc resistance, wave shape, persistence of the arc, and arc length, obtained experimentally from 30 tests made under various conditions.

Test Setup. The connections for setting up the arc circuit are shown on Fig. 4. In general, the arc was formed in three types of circuits energized at 75,000 or 7,500 volts by pulling a 2-ampere fuse wire between copper electrodes. The major component of the im-

pedance of each of these three circuits was, respectively, resistance, capacitance, and inductance. It should be noted that the phase angle of the impedance of the inductive circuit shown in Fig. 4c was +30 deg. while that of the inductive circuits shown in Figs. 4d and e was nearly +90 deg. The initial arc current for these three types of circuits ranged from 8 to 60 amperes (peak). Arc currents ranging from 200 to 800 amperes (peak) were obtained in an inductive circuit energized at 7,500 volts (r. m. s.). This circuit is shown in Fig. 4e.

Measuring Equipment. The measurements of arc voltage and current were made with a magnetic oscillograph. Shunts were used for current measurements and resistance potentiometers for voltage measurements. Each of these potentiometers consisted of a 25-ft. length of doubled garden hose through which water was circulated by a pump. A metal coupling at one end of this hose was connected to the line, and the voltage drop taken from a short length adjacent to the grounded end. The resistance of this potentiometer was approximately 500,000 ohms. The difficulty of measuring the arc voltage, which varied from the normal line voltage to a very small value, was overcome by the use of two oscillograph elements of different sensitivities. To prevent damaging the high sensitivity element, a spark gap was shunted across a section of the hose which included the section for the high sensitivity element. Breakdown of this gap was recorded by an element connected in series with it. Two moving picture cameras placed at right angles to each other were set up to photograph the arc.

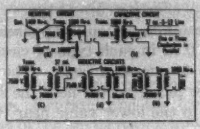

Fig. 4—Connections for Tests on Arc Characteristics

Arc Resistance. Since the resistance of an arc is not constant but is dependent on current, length, wind, etc., some discussion of the variations of resistance, based on experimental data, will be of value. It is proposed here to discuss the experimental data from the standpoint of the variation of resistance over the cycle and throughout the duration of the fault. Typical oscillograms for the three types of circuits employed, that is, resistive, inductive, and capacitive, are shown in Fig. 5 for tests 101, 112, 117, and 123.

Considering first the variation of instantaneous resistance of the arc over a cycle, that is, the instantaneous voltage divided by the instantaneous current, an analysis was made for a cycle near the start of the arc and one just previous to arc extinction. The curve of resistance of an arc in a resistive circuit (test 101), for a

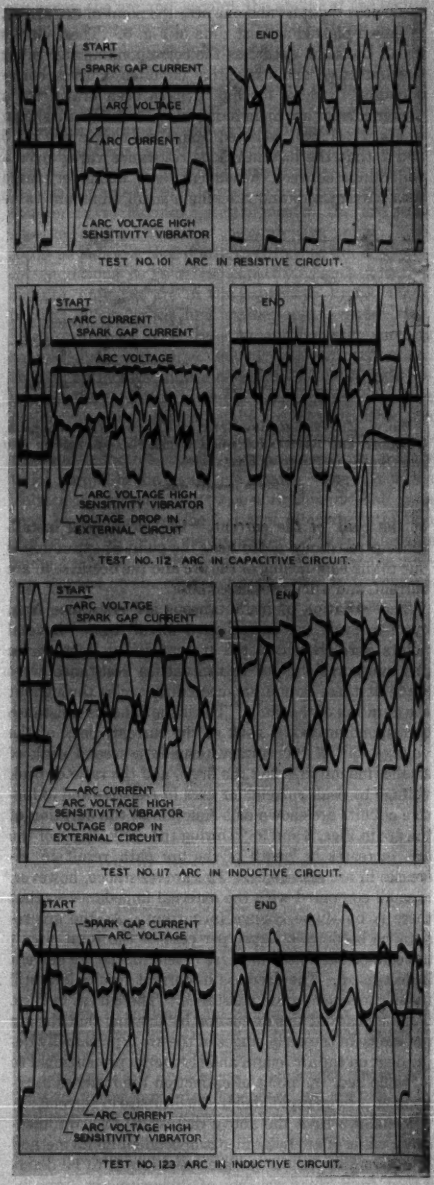

Fig. 5—Typical Oscillograms for Arc Characteristic Tests

cycle near the start and one near the point of extinction, is shown plotted against time in Fig. 6. These curves are of the same nature as the corresponding curves for the inductive and capacitive circuits. Curves corresponding to those for Fig. 6 have been drawn for a higher current arc in an inductive circuit and are shown in Fig. 7. It will be noted from these curves in Figs. 6 and 7 that the instantaneous resistance varies over a wide range between the current zeros, being very large near current zero and reaching a small minimum value

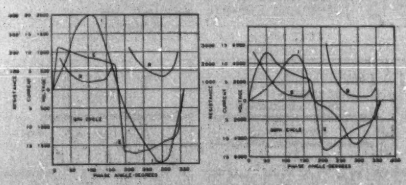

Fig. 6—Curves Showing Relations Between Arc Voltage, Current, and Resistance for a Cycle Near Start of Arc and One Near Point of Extinction

Arc in resistive circuit

at the peak of the current. The increase in instantaneous resistance noted near the point of extinction is due to the lengthening of the arc and the decrease in arc current, and will be discussed later.

The variation of instantaneous arc resistance from cycle to cycle was investigated for all tests. Several points were chosen on the current and voltage waves and measurements of instantaneous resistance were made at these points on succeeding cycles throughout the duration of the fault. The instantaneous resistance corresponding to the peak of the current, peak of the voltage, tail of the voltage (usually a second peak of smaller magnitude than the first), and the ratio of peak voltage to peak current for arcs in resistive and inductive circuits are shown as a function of the duration of the arc in Figs. 8 and 9. During the early stages of the arc, increases in length of the arc path result in increases in arc resistance. This arc resistance, however, is small compared with the external impedance so that there is but little change in current. As the arc resistance becomes a larger per cent of the total circuit impedance due to the increase in length, the arc current is decreased. The decrease in current results in a still further increase in arc resistance. The curves in Figs. 8 and 9 bear out this discussion by showing that the arc resistance in the early stages of the arc is principally dependent on the arc length and is roughly proportional to it, while near the point of extinction the increase in resistance is more rapid, due to the decrease in current.

It follows from the above discussion that the measured value of arc resistance is dependent on its method of determination. If the arc resistance is to be determined from measurements of dynamometer type instruments, which measure r. m. s. values, the effective resistance of the arc will be obtained and it is defined as the ratio of the r. m. s. voltage to the r. m. s. current. The assumption that the arc impedance is practically all resistive is borne out by a subsequent discussion.

A very close approximation of the effective arc resistance was found to be the ratio of the peak voltage to the peak current. This provides a simple method for obtaining the effective resistance of the arc when the current or voltage waves are not sine waves. It should be noted in connection with this approximation for effective resistance that it was found to be true within a few per cent for arc resistances above 10 ohms and the error varied from 15 to 25 per cent for lower values of arc resistance.

Wave Shape of Arc Current and Voltage. In the past there has been some question as to the phase relation between the current in an arc and the voltage across the arc. The oscillograms of current and voltage of an arc shown in Fig. 5 indicate that in these tests the current and voltage waves have simultaneous zeros. However, if the variations in current and voltage are not sinusoidal, the apparent power factor may not be unity. The apparent power factor of the arc was determined by analyzing typical cycles to obtain the ratio of the average of the instantaneous products of current and voltage to the product of effective current times voltage. These analyses indicate a variation in apparent power factor from unity to 0.75 in each type of circuit for current varying up to 60 amperes peak. The high value of apparent power factor occurs near the start of the arc and decreases to the lower value near the point of extinction. In the so-called high-current tests in which

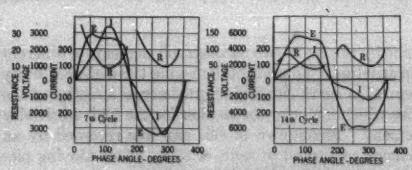

Fig. 7—Curves Showing Relations Between Arc Voltage, Current, and Resistance for a Cycle Near Start of Arc and One Near Point of Extinction

Arc in inductive circuit

the current varied from 200 to 800 amperes the power factor was very nearly unity throughout the duration of the arc. The apparent power factor of the arc current and the voltage at the source, was found to be practically unity for all tests.

In discussing the wave shape of arc voltage and current qualitatively, use will be made of the relationship referred to above, namely that the arc resistance varies inversely with the current, and directly with arc lengths The wave shape of the arc voltage and current is dependent on the type of circuit in which the arc is formed,

the characteristics of the arc itself, and the presence of harmonics in the supply voltage. The effects on wave shape of the factors mentioned above are shown on the oscillograms in Fig. 5.

Consider the oscillogram for test 101 in which the circuit impedance exclusive of the arc was principally resistance. For this case the arc resistance at the start of the arc was small compared with the external resistance so that the current is not affected appreciably by the arc and is practically a sine wave. The arc voltage, however, is not a sine wave. This is to be expected

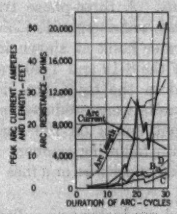

FIG. 8—CURVES OF RESISTANCE, CURRENT, AND LENGTH FOR TEST 101

A Instantaneous resistance for peak voltage
B Instantaneous resistance for tail voltage
C Instantaneous resistance for peak current
D Ratio of peak voltage to peak current

since the arc resistance varies over the cycle, being high for small values of current and low for large currents. As the duration of the arc increases, its length increases, resulting in an increase in resistance. As the arc resistance, still varying over the cycle, becomes a larger per cent of the circuit impedance, the current wave becomes distorted and the arc voltage approaches a sine wave.

The tests which were made with an arc formed in a capacitive circuit indicated the presence of a very large third harmonic current, as shown on oscillogram 112, in Fig. 5. The reason for this large triple harmonic current is apparent from the nature of the circuit. Any harmonics which appear in the voltage across the arc circuit result in large harmonic currents, since the capacitive circuit affords a lower impedance path for the harmonic currents than forthe fundamental current.

Effect of External Circuit on Persistence of an Arc. In forming arcs in three different types of circuits, it was hoped that some information would be obtained on the relation between the persistence of the arc and the type of circuit in which the arc was formed. In spite of the fact that it was impossible to duplicate exactly conditions of wind, current, and wave shape for the different types of circuits, certain conclusions as to the persistence of an arc can be drawn from the experimental data.

The factors determining whether an a-c. arc will be extinguished at current zero have been described by Dr. J. Slepian[3] as, first, the rate of recovery of the voltage across the arc and, second, the ability of the arcing medium to recover dielectric strength. If the rate of recovery of the voltage across the arc exceeds the rate at which the arcing medium recovers dielectric strength, the arc will persist. In this discussion it will be assumed that the recovery of dielectric strength of the arcing medium is the same for each type of circuit.

A comparison, based on whether or not the arc restruck following extinction at current zero, was made of the oscillograms for arcs formed in resistive, capacitive, and inductive circuits. Restrikings of the arc were observed only in the case of the capacitive circuit. This seems to indicate, that if arcs were formed under identical conditions in resistive, capacitive, and inductive circuits, only the arc in the capacitive circuit would persist, following the first extinction at current zero. The arcs formed in the resistive and inductive circuits showed about the same tendency toward extinction, that is, an increase in arc resistance followed by a decrease in arc current to the point of extinction. The oscillograms show that after the arc is extinguished the voltage across the electrodes rises to normal in the resistive and inductive circuits, but rises to approximately twice normal in the capacitive circuit. The rise of this voltage to approximately twice normal in one-half cycle may be explained by the fact that after arc extinction the voltage across the electrodes tends to rise to a value equal to the sum of the generated voltage and the voltage trapped on the equivalent condenser in the circuit.

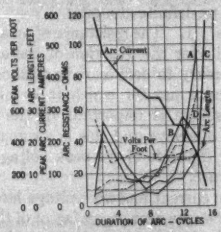

FIG. 9—CURVES OF RESISTANCE, CURRENT, LENGTH, AND VOLTS PER FT. FOR TEST 123

A Instantaneous resistance for peak voltage
B Instantaneous resistance for tail voltage
C Instantaneous resistance for peak current
D Ratio of peak voltage to peak current

The greater persistence of the arc in the capacitive circuit seems to indicate that not only is the rate of the recovery of arc voltage important but also the maximum value to which this voltage rises.

Arc Length and Arc Voltage. Arc length, arc current, and arc voltage are inter-dependent and are very much influenced by the direction and velocity of the wind and by electrode spacing. Arc length has been determined from measurements of moving pictures taken with two cameras at right angles. The variations of arc length

have been plotted against the duration of the arc as shown in Figs. 8 and 9.

Other experimental investigations[2] have shown that a fairly definite relation exists between the voltage across an arc and the length of the arc. This relation may be expressed in peak volts per foot of length. To observe the effect of variation of arc current on the peak voltage per foot of arc length, a "shot gun" diagram was made and is shown in Fig. 10. The points plotted on

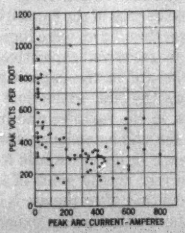

Fig. 10—Plot Showing Arc Voltage Per Foot of Arc Length for Various Currents

this diagram represent the peak voltage per foot, computed for various tests and for several cycles of each test. This diagram indicates a very definite grouping at 300 volts per foot for currents greater than 100 amperes peak. For currents below this value, there appears to be no definite grouping.

Conclusions. 1. The resistance of an arc varies over the cycle and over the duration of the arc, so that it becomes necessary to define arc resistance with regard to the particular use for which it is intended. The effective resistance is important in the operation of certain types of relays and this is approximated fairly accurately by the ratio of the peak voltage to the peak current of the arc.

2. The arc current and arc voltage have simultaneous zeros. The apparent power factor of the arc current and voltage was found to be as low as 0.75 under certain conditions, but in general it can be considered as unity. Due to the nature of the arc, either the arc current, the arc voltage or both may be distorted.

3. The arcs formed in the capacitive circuit showed less tendency to extinguish than those in the inductive or resistive circuits. Arcs formed in either the resistive or inductive circuit showed about the same tendency to be self-extinguishing.

4. The average peak volts per foot, for arcs ranging in current from 100 to 800 amperes peak was found to be 300 volts per foot.

III. Overvoltages Due to Arcing Grounds

Opinions vary rather widely, regarding the maximum overvoltages which can be developed on an isolated-neutral transmission system at the time of an arcing ground, according to whether they are based on theoretical considerations, laboratory tests or limited operating data. From the tests on an actual transmission system herein described, information on the nature and magnitude of such overvoltages was sought. The term "arcing ground" as used in this paper refers simply to the condition of a transmission line fault to ground through an arc, and the term "sound phase" refers to either one of the two unfaulted phases. Overvoltages are expressed in terms of the ratio of the maximum peak voltage to ground during a fault, to the same peak voltage just previous to the fault, the latter quantity being referred to as normal voltage.

Nature of Tests. A three-phase transmission line, open at the far end, was energized at 75 kv. by a generator and transformer bank as shown in Fig. 11. The characteristics of the S-19 and T-20 transmission lines, which were used in the tests, are given in Fig. 11. Arcing ground faults were produced on phase *M* conductor at different points on the line. These faults were initiated in the following manner:

1. A grounded copper wire or a fuse wire was pulled into the line.

2. A grounded conductor was pulled into and away

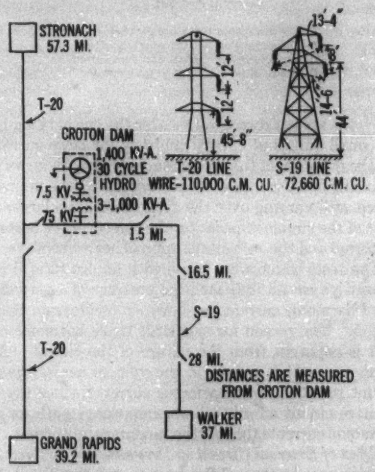

Fig. 11—Layout and Configuration of Transmission Line

from the line, or held within arcing distance of the line.

3. A grounded green oak stick was swung close to the line.

4. A line conductor was swung above the sandy soil.

Oscillographic measurements of phase voltages to ground and of arc current were made at the time of the fault. Magnetic type oscillographs were employed for all measurements of current and voltage, with the aid

of a shunt for measuring arc current, and water hose resistance potentiometers, previously described in Section II, for measuring voltages to ground. Surge-voltage recorders were connected to the three phases at the generating end and at the open end of the transmission line to supplement the oscillographic measurements of voltage. These surge recorders were calibrated with high-voltage impulses supplied by the lightning generator. Conditions which were varied included the length of line, as shown in Fig. 11, transformer connections, and the use of several neutral impedances, as indicated in Table I, and the location of the fault.

missed by the latter. The maximum values of voltage from transformer neutral to ground recorded for the arcing ground tests on the longer lengths of 75-kv. isolated systems, ranged from 42 to 135 kv. (peak).

Nature of the Transient. Fig. 12, which shows the beginning and end of the fault for test 79, illustrates the nature of the transient changes at the point of fault as they took place in a large number of the tests. For this test an arcing ground was produced with a swinging wire on phase *M* at Croton, the 37-mile line being connected and energized through Δ-Δ transformers.

An analysis of faulted phase voltage indicates that

TABLE I—SUMMARY OF MAXIMUM AND AVERAGE OVERVOLTAGES ON SOUND PHASES FOR VARIOUS INDICATED SETUPS

Length of line (miles)	Fault location	Transformer connections at Croton 75 kv.	Neutral impedance (ohms)	Number of tests in each setup	Maximum[3] overvoltage[1] (times normal) From max. surge-recorder reading	Measured by oscillographs At Croton Phase T	At Croton Phase B	At Walker Phase T	At Walker Phase B	Average[2] overvoltage[1] Measured by oscillographs At Croton Phase T	At Croton Phase B	At Walker Phase T	At Walker Phase B
1.5	Croton	(5)Y Y	Isolated	3	2.3	2.1	2.6			2.0	2.4		
16.5	"	"	"	6	2.1	2.5	2.6			1.9	2.2		
26	"	"	"	8	2.9	2.1	2.6			2.0	2.4		
37	"	"	"	11	1.8	2.3	2.4	2.3	2.5	2.1	2.2	2.1	2.3
37	Walker	"	"	4	1.4		2.1	2.1	2.4			2.0	2.3
37	Croton	"	"	2[4]	1.4		2.2	1.9	2.1		2.2		
37	Walker	"	"	2[4]	1.4		2.1	2.1	2.5		1.9	1.9	2.3
28	Croton	Y Y	"	4	2.5	2.1	2.2			1.9	2.1		
28	"	Δ Δ	"	4	2.5	2.1	2.3			2.1	2.3		
37	"	"	"	5	2.1	2.1	2.5			2.0	2.2		
94	Croton	Δ Δ	"	3	2.7	2.3	2.5	2.2	2.4	2.3	2.5	1.9	2.3
"	Walker	"	"	2	2.5	2.3	2.6	2.2	2.6	2.3	2.5	2.2	2.5
"	Stronach	"	"	2	2.8	2.4	2.7	2.5	2.7	2.4	2.6		
134	Croton	"	"	3	2.8	2.5	2.6	2.5	2.6	2.5	2.5	2.5	2.5
"	Walker	"	"	2	3.0	2.4	2.7	2.5	2.6				
"	Stronach	"	"	2	2.9	2.4	2.8	2.3	2.5	2.2	2.7		
37	Croton	(5)Y Y(6)	0	2	1.0	2.8	1.9			2.5	1.9		
"	"	"	1800/0°	2	1.0	2.2	1.6			2.2	1.6		
"	"	"	3350/73°	2	1.5		2.1				1.9		
"	"	"	6070/54°	3	1.9		2.1				2.0		
"	"	Δ Y(6)	3350/73°	2	1.6		1.6				1.5		
"	"	"	6070/54°	4	1.8		2.1				2.1		

Notes:
1. "Overvoltage" is expressed as a ratio of maximum peak during fault to normal peak just previous to fault.
2. "Maximum overvoltage" refers to highest voltage recorded for all tests in each setup.
3. "Average overvoltage" refers to the average of the maximum voltages for all similar tests of each setup.
4. For these tests 75-kv. aluminum cell lightning arresters were connected to each phase at Croton.
5. Low-voltage neutral grounded solidly.
6. High-voltage neutral grounded through impedance

Magnitude of Recorded Overvoltages. It can be seen from Table I that the highest sound phase overvoltage recorded by the oscillographs in the 63 arcing ground tests on the isolated system was 2.8 times normal, and that the maximum surge-recorder measurement was 3.0 times normal. The faulted phase voltage in no case exceeded the sound phase voltages. In no case were the surge-recorder measurements more than 15 per cent higher than the corresponding oscillographic records. Inasmuch as the surge recorder is inherently a less accurate instrument than the oscillograph, its measurements are chiefly valuable as a check on the order of magnitude of the oscillographic measurements, and as assurance that no high magnitude transients were

at the fault location the voltage is reduced from the normal value of the 30-cycle wave just before the fault, to zero just after the fault starts. A traveling wave condition is set up, which becomes a standing wave as it is maintained for a short period of time by successive reflections back and forth from the open ends of the line to the grounded point. This standing wave is evidenced by damped oscillations in the wave of arc current and by similar oscillations in the voltage wave recorded at the open end of the line. An inspection of the transient oscillations in oscillograms 82-*A* and 82-*C*, shown in Fig. 12, illustrates this point. Oscillogram 82-*A* was taken at the fault location (Croton) while 82-*C* shows the voltages at one open end of the 94-mile line. When

the fault was located at one end or the other of a single line system which was varied in length up to 94 miles, other oscillograms showed that the frequency of these

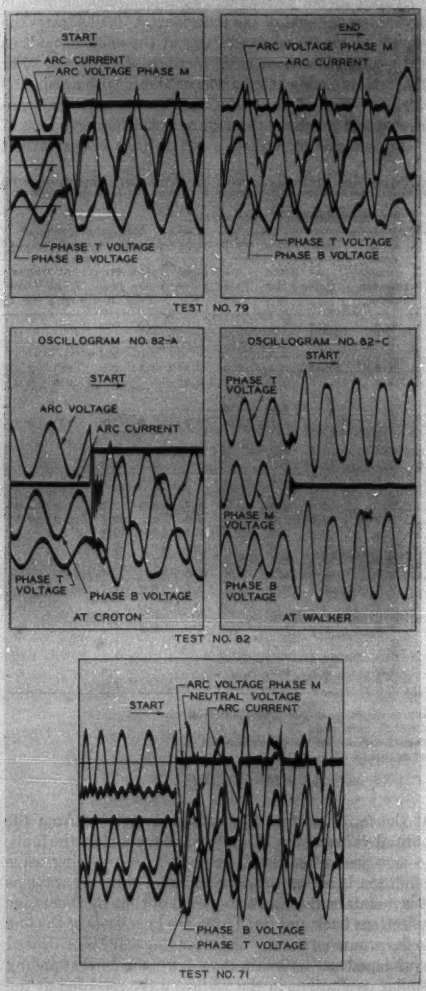

Fig. 12—Typical Oscillograms of Arcing Ground Tests

oscillations (called oscillatory frequency) corresponded to the free or natural period of oscillation of the line. In the tests on the 94- and 134-mile lines, arcs at Croton resulted in arc currents of quite complicated wave shape due to the several frequencies produced by the unequal lengths of line from Croton to the ends.

It can be seen in test 82 shown in Fig. 12, that the standing wave oscillations on the faulted conductor are accompanied by similar oscillations of relatively low amplitude in the voltage waves of the sound phases B and T. It is possible that these small voltage oscillations are caused by the oscillations in the faulted phase current or voltage, and not by reflection of a traveling wave initiated by a sudden change in the sound phase voltages themselves. Their small amplitude is apparently due to a low mutual coupling between phases and to the relatively high impedance at oscillatory frequency offered by the transformer windings, through which very little of the wave can be propagated. The barely noticeable oscillations do not occur at the highest crest of the sound phase voltage waves and add nothing to their maximum values. After undergoing shifts in phase relations, these voltages reach their crest values as waves of fundamental frequency on which a small d-c. transient, which shifts the zero of the normal frequency wave, is superimposed. A noticeable third harmonic voltage appears in the sound phase voltage during the fault.

In the majority of the tests the maximum overvoltage occurred during the first cycle of the fault, and was from 10 to 15 per cent higher on phase B than on phase T. Assume that the fault occurs on phase M when the voltage is at its peak value. As a result of the order of phase rotation $B\ M\ T$, V_B is advanced and V_T retarded about 30 deg. in their phase relations, at the instant V_M falls to zero. V_B, which at that instant is increasing in magnitude, is thus hastened to its crest value; while V_T, which has just passed its crest, is sent through a reverse curve because of its retardation in phase. It is probable that this difference in the behavior of V_B and V_T, when the fault occurs, accounts for the rather consistent superiority of V_B in the magnitude of its first cycle overvoltage.

Unstable Arcing Ground. The oscillogram for test 71, shown in Fig. 12, represents conditions during one of the most unstable arcing grounds that was produced. In this case the current usually restruck from zero once every half cycle, which is typical of a condition often assumed in constructing a theory for high arcing ground overvoltages. The high-frequency voltage oscillations are again very small, however, and the maximum sound phase overvoltage for this test, as recorded by the oscillograph and surge recorders, was respectively 2.4 and 2.6 times normal. It is furthermore assumed in theories that an unstable arc waits each time to restrike until the voltage across it attains successively higher values. Oscillogram 71 showed that the arc restruck when the voltage was equal to or less than its normal crest value and in most of the tests the restriking took place at less than normal crest voltage.

Effect of Length of System on Overvoltages. As the

length of the isolated system was increased, there was a gradual, though quite definite trend toward higher overvoltages due to arcing grounds. This can be seen from the curves of maximum and average overvoltages shown in Fig. 13, in which the average overvoltage on phase *T* with a 134-mile line is 28 per cent higher than that for a 16.5-mile line. The data for these curves were selected considering only the system length, and neglecting the effects of other variables.

Effect of Point on the Wave of Faulted Phase Voltage Where Arc Strikes. There were 46 arcing ground tests with the neutral isolated in which the point on the wave of faulted phase voltage where the arc struck could be determined. For these tests the point of striking varied from 15 to 160 deg. In the majority of tests the arc struck in the portion of the cycle between 45 and 90 deg., but there appears to be no definite relation between the magnitude of overvoltage and the point on the wave where the arc strikes.

For most of the tests in which the arc struck between 100 and 160 deg. the overvoltages are all greater than 2.5. However, the majority of these tests were made

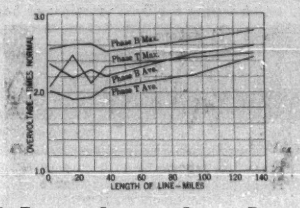

FIG. 13—EFFECT OF LENGTH OF LINE ON RECORDED OVERVOLTAGES DUE TO ARCING GROUNDS ON PHASE M 76-KV. ISOLATED-NEUTRAL SYSTEM

on the 94- and 134-mile systems. The fact that the maximum overvoltages were recorded for these tests may be due either to the increased length of system or to the point on the voltage wave where the arc struck.

Effect of Type of Fault. The various types of arcing ground faults investigated may be classified as arcs formed through high external resistance, and those formed through low resistance. It was found that a high-resistance ground fault, such as an intermittent tree ground, even if it persists for some time, produces a negligible rise in voltage on the sound phases. It is, therefore, concluded from these tests that an arc to a low-resistance ground, which allows maximum fault current to flow when the arc resistance is low, is more conducive to overvoltages than one to a high-resistance ground.

Effects of Other Changes in System Conditions. No important changes in the magnitude or nature of the overvoltages could be detected for different transformer connections, different locations of measuring equipment, whether at the source or the open end of the line, or for the few tests which were made on a short line when lightning arresters were connected to the system at Croton.

Overvoltages with System Neutral Grounded Through an Impedance. The essential results of the tests made with a neutral impedance can be found in the lower six lines of Table I. With the exception of two tests made with the neutral grounded solidly, in which the sound phase voltage was raised considerably higher than the normal value by an exaggerated third harmonic, the maximum voltages recorded during the tests varied from 1.6 to 2.2 times normal. These are somewhat less than the maximum values obtained with the system neutral isolated. Due to the fact that dangerous overvoltages were not developed in the tests under the isolated conditions, however, it is impossible from these tests to evaluate the minor changes resulting from the insertion of impedance in the neutral connection to ground.

Existing Theories of Arcing Ground Overvoltages. In general, three theories have been proposed to explain the development of the supposed overvoltages resulting from an arcing ground on an isolated system. These theories have been reviewed by others[4,5] and will not be discussed here. They may be briefly differentiated according to important assumptions made, that is, (1) both extinguishing and restriking of the arc controlled by oscillatory high frequency; (2) arc extinguishing controlled by oscillatory frequency and restriking by normal frequency; and (3) both extinguishing and restriking controlled by normal frequency. In all of these theories the maximum overvoltage occurs on a sound phase when its voltage is at the peak of a transient oscillation occasioned by the restriking of the arc, and can attain respective values of 19, 7.5, or 3.5 times normal neglecting damping. If factors for damping, leakage, and mutual capacitance are assumed, the maximum overvoltage for the second theory is about 5.5 times normal.

Analysis of Results of Tests from Standpoint of Above Theories. 1. In all the tests of this investigation, the oscillograms showed the striking and extinguishing of the arc to take place according to normal (30-cycle) frequency, in accordance with the third theory. This fact automatically prevents the tests from supporting the first two theories, since in both of these theories, the assumption is made that the arc is extinguished while the arc current is going through zero in an oscillation. This conclusion was also reached by Messrs. Peters and Slepian in their laboratory tests.

2. At the striking or restriking of the arc, the two sound phase voltages take their new value after going through an oscillation, the magnitude of which is only a very small fraction of the normal frequency peak of the wave. In no case was the magnitude of the maximum overvoltage affected by the amplitude of the oscillation. In this respect the tests fail to substantiate any of the theories.

3. If, in the normal frequency theory, a damping factor is taken into account, which damps the oscillation

in the sound phase voltage, the character of the phenomenon as observed in these tests agrees in certain respects with the normal frequency theory.

Conclusions from Arcing Ground Tests. The tests indicate that for the range of conditions investigated on this isolated neutral transmission system, faults involving arcs to ground produce overvoltages which may be characterized as follows:

1. The maximum overvoltage is approximately 3.0 times normal peak voltage from line to ground, and occurs on a sound phase, most frequently during the first cycle of the fault.

2. The overvoltage is essentially a fundamental frequency phenomenon, and is made up of the following components: A change in the impressed voltage from that of line to neutral to that of line to line without going through an oscillation of appreciable amplitude, a d-c. transient of several cycles duration which shifts the zero of the normal frequency wave, and in some cases a harmonic of lower order. Oscillograms showed that high frequencies within the range of the magnetic oscillograph did not affect the magnitude of the maximum overvoltage, while surge recorders testified that no higher magnitude transients were present.

3. Variations in the transmission system and in the test conditions which could be made with equipment available did not change the nature of the overvoltages, and caused changes in their magnitude of not more than 28 per cent. Conditions which were varied are as follows:

Length of system (1.5 to 134 miles).

Point on wave of voltage where arc strikes.

Transformer connections (delta and star, see Table I).

Fault location (at end or middle of line)

Method of starting arc (fuse, swinging wire, etc.).

4. An arc to a high-resistance ground limits the shift of the system neutral and hence tends to reduce the magnitude of overvoltages produced.

5. At the striking of the arc, a highly damped standing wave follows the traveling wave set up on the faulted conductor. The frequency of the resulting oscillation is determined by the distance from the arc to the ends of the line.

ACKNOWLEDGMENTS

The authors wish to acknowledge the cooperation of Messrs. K. B. McEachron and J. G. Hemstreet in arranging the general conduct of the tests. They also wish to acknowledge the personal assistance of Messrs. E. J. Wade, W. J. Rudge, and M. F. Beavers of the General Electric Company, who aided in the impulse and arcing ground tests, and of Messrs. C. C. Tanner, F. D. Blackwell, D. E. Hackett and others of the Consumers Power Company, who took part in all tests.

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Discussion

J. K. Peck: As a supplement to the tests which were made at Croton, Michigan, in the study of overvoltages due to arcing grounds, which have been described in the paper by Messrs. Eaton, Dunham and myself, an automatic oscillograph was installed this year on the 140-kv. 60-cycle system of the Consumers Power Company at its Blackstone substation, which is located at Jackson. This oscillograph is a three-element string type instrument. It is connected through potential transformers to record phase-to-ground voltages, and is operated by residual voltage to begin operation within one-half cycle of the start of a disturbance on the system. The date and time of occurrence of the fault are recorded automatically by a camera which photographs a clock.

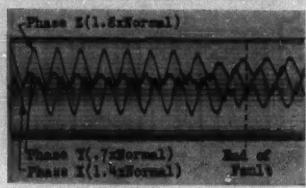

FIG. 1—FAULT ON 140-KV. ISOLATED SYSTEM VOLTAGES AT BLACKSTONE

At the present time this 140-kv. system, which operates with its neutral isolated, usually has a connected length of about 670 miles, and is fed by a generating capacity of from 150,000 to 170,000 kva. It would therefore seem to offer good possibilities for the development of overvoltages at the time of arcs to ground on the system.

Between March 1st and May 23rd there were 18 oscillograph operations at the time of disturbances on the system, most of which resulted from insulator arcovers. The highest overvoltage recorded by the oscillograph during each of these faults varied from 1.2 to 2.6 times normal peak voltage to ground. The duration of the faults ranged from 4 to 112 cycles. In most of these oscillograms evidences of arcing were observed in several cycles just prior to clearance of the fault, but in no case did cumulative oscillations appear in the voltages during the fault.

The oscillogram shown in Fig. 1 is typical of a number of operations. This particular fault, lasting for 32 cycles, was an arc to ground at the time of a lightning storm occurring on May 18th. The final 10 cycles of the disturbance are shown here, with 2 cycles of succeeding normal conditions. The first 22 cycles of the oscillogram exhibited no marked difference from the waves which appear at the beginning of this figure. Evidence of arcing is seen in the last 6 cycles of the fault in all phase voltages. There is, however, no building up of voltage. The maximum voltage to ground was 2.0 times normal on phase Z near the

beginning of the fault, and gradually decreased to 1.8 times normal at the point shown.

In order to obtain still more complete data on this system at the time of disturbances, a three-phase automatic surge recorder is also being installed at the Blackstone 140-kv. substation. Records from this instrument should materially substantiate the automatic oscillograms, since the oscillograph cannot record transients which occur during the first half cycle of the fault, or surges of a very high frequency.

The nature of the phenomena and order of magnitude of the overvoltages recorded by the automatic oscillograph at the time of phase-to-ground faults, agree very well with the results of the experimental investigation at Croton, inasmuch as the magnitude of the maximum overvoltage was not affected by high frequencies within the range of the oscillograph, and was in all cases less than 3.0 times normal. Moreover, the system length and generating capacity for the 140-kv. system faults are considerably more than those which were available for the Croton tests, and thus these records from an operating system are a very valuable supplement to the tests described in the paper.

R. H. Park: In the tests described in the paper, it was found that the probability of a power arc resulting from an impulse flashover was increased when a capacitance load consisting of 35 miles of transmission line was connected in parallel with the point of flashover. Apparently this effect may be attributed to the discharge through the arc of the capacitative charge stored on the line. Thus in the tests the stored charge on the line had the effect of supplementing the charge stored in the lightning generator, and consequently causing a greater degree of initial ionization in the arc path, with a resultant greater likelihood of power arc follow-up. This is substantiated by the fact that in the case of tests made with parallel capacitance the power arc follow-up was found to be closely correlated with the point on the voltage wave at which flashover occurred, follow-up occurring most frequently when the flashover occurred at the maximum of the 30-cycle voltage wave. This effect would increase as the connected capacitance increased and in the case of long lines, or of an interconnected network of lines involving a large total length of transmission, might be very pronounced. It should be noted that this particular effect should depend on the connected miles of transmission line and should be independent of the connected generating capacity.

With regard to the method of measuring voltage, it might be well to point out that with a potentiometer resistance of 500,000 ohms the potentiometer itself may have an appreciable effect on the oscillations which develop during the arcing period, and it is possible that the presence of the potentiometer has prevented the occurrence of some of the phenomena which are under investigation.

J. M. Dunham: Mr. Park has suggested that the probability of a power arc follow-up resulting from an impulse flashover is increased when the connected capacity of the system is increased. Any effect which increases the ionization resulting from an impulse flashover will, other conditions remaining constant, increase the probability of power arc follow-up. There are several factors, however, which influence the effect of increased system length on the ionization of the arc path following an impulse flashover. Considering first the initial current due to the impulse flashover, and the initial current due to the discharge of the line capacitance through the arc path, it is evident that the influence on the ionized arc path of the discharge current may be great or small depending on the relative magnitudes of these initial currents. Another important factor to be considered is the duration of the discharge current compared with the duration of the impulse surge.

From the above it appears that a general statement regarding the effect of increasing the extent of a power system on the probability of a power arc resulting from a flashover, can not be made without qualification. It is unfortunate that because of the limitation of time and equipment more extensive tests could not be carried out at Croton to separate the various factors affecting power arc follow-up.

Mr. Park has brought up a very interesting point regarding the effect of the resistance potentiometers, used for measuring line voltages-to-ground, on the occurrence of overvoltage phenomena. Although, as stated in Section III of our paper the presence of a high damping factor would explain an agreement in certain respects between the normal frequency theory and the observed phenomena, the damping due to the resistance potentiometers cannot be considered as a high damping factor.

If the circuit conditions are simulated by a series circuit composed of the potentiometer resistance (R) from one phase wire to ground and the direct capacitance (C) of one phase wire to ground, the following formula can be used to determine the voltage on one of the unfaulted lines following arc extinction, expressed as a per cent of the initial voltage

$$E - E_o \epsilon \frac{t}{R\ C}$$

Calculations based on the preceding formula indicate that two cycles after the initial time the voltage on one of the unfaulted phases of the 135-mile system has decreased only 12 per cent due to leakage current through the potentiometer. For the 37.5-mile system the decrease in voltage after two cycles is 28 per cent. This leakage of charge, as evidenced by the calculated drop in voltage, was not sufficiently rapid to have prevented the building up, or at least an indication of the building up, of large voltages-to-ground on the unfaulted phases in accordance with any one of the three existing theories outlined in Section III of our paper.

Automatic Stations

ANNUAL REPORT OF THE COMMITTEE ON AUTOMATIC STATIONS*

CONSIDERABLE data on many phases of the automatic station art have been collected by subcommittees and are made part of this report, which is treated under headings as follows:

Technical Papers	Wire Designations
Standards	Research
New Subjects	Operating Data
Unfinished Business	Telemetering
Fire Protection	Bibliography

Technical Papers

The following papers are being presented under the auspices of the Committee at the Summer Convention, Asheville, N. C., 1931.

1. *A-C. Supervisory Control System.* By Othmar K. Marti.
2. *Supervisory Control for A-C. Electrified Railroads.* By C. P. West and H. C. Griffith.
3. *Automatic Combustion Control.* By C. H. Sanderson and E. B. Ricketts.
4. *Operating Experience with Automatic Stations.* By Garland Stamper and F. F. Ambuhl.
5. *Three Years' Operating Experience with Miniature Switchboard Supervisory Automatic Control.* By R. M. Stanley.

Standards

Standards No. 26, Automatic Stations, has again been reviewed by the Committee. Recommendations for revisions in the list of device function numbers and in the table of minimum protection for power apparatus have been forwarded to the Standards Committee.

New Subjects

Automatic combustion control and automatic alternating current networks have been studied this year by the Committee. A paper on Automatic Combustion Control, covering this subject extensively for the first time, is being presented at the annual convention.

Unfinished Business

Of the items listed last year, preliminary reports on Fire Protection, Load Dispatching and Wire Designations have been completed and are contained herein.

The following topics have been under consideration but no final reports have been rendered:

1. Economical Construction.
2. Noise Mitigation.
3. Automatic Alternating Current Networks.
4. General Telemetering.

Subcommittee Reports

To the Subcommittees contributing the following reports generous acknowledgment is given. It is desired to make known that the efforts so prominently reflected in these reports are appreciated in full measure.

Fire Protection

Contributed by I. E. Moultrop and D. W. Taylor.

The methods of accomplishing suitable fire protection from the points of view of safety to personnel, safety to apparatus, and continuity of service have long been the subject of discussion and investigation by the electrical engineering profession. The size and importance of electrical installations make adequate fire protection a matter of utmost importance. The increasing use of automatically operated stations brings into prominence the special problems confronted by the designers of these usually unattended installations, which may in some instances be so far removed from the nearest aid in case of fire as to make automatic fire protection a corollary of automatic operation. It is the purpose of this report to point out some of these special problems.

When the subject of fire protection is considered, it is well to make a study of the causes of fires to obtain a perspective of the various elements entering into the problem. Consideration can then be given to design, and apparatus and material specifications, which will eliminate as much as possible the trouble at the source. Studies may be made to determine if oil circuit breakers have the necessary interrupting capacity; if all load carrying apparatus has the proper thermal capacity; if relay protection can be provided to clear short circuits before damage may occur, etc. Many apparatus characteristics, such as oil circuit breaker interrupting capacities, have been proven by tests so that greater reliability may be placed in their performance than was the case a few years ago.

Means to limit the fire to the vicinity of its source and prevent involving other apparatus in an attended or unattended substation, may take any one or more of

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three forms, *viz.*: automatic means of fire detection and alarm; structural means of fire isolation; and means of fire fighting, either manual or automatic.

Automatic means of fire detection, as at present commercially developed, take the form of thermocouples and fusible links to convert the heat energy released by combustion into electrical energy, or of the more complicated apparatus working as a detector of the resultant smoke. Although these automatic means of fire detection have occasionally been installed in attended stations merely to alarm the operators, the recent mechanical improvement of the devices commercially obtainable for this service has resulted in considerable application of such devices in unattended stations to give alarm at some central attended installation and/or to operate fire-fighting equipment adjacent to the source of trouble. Unattended stations whose operations are reported to a central office through some means of supervisory control are particularly adapted for the installation of an alarm system between the station and the central office.

The second form of fire protection, structural means of fire isolation, is applied for the same reason and with equal effect in both attended and unattended stations. The large quantity of oil used in modern electrical apparatus makes fire isolation an important problem both indoors and outdoors. Generally, if possible, it is desirable to install large oil-filled apparatus outdoors and eliminate as much indoor oil-filled apparatus as is possible.

Indoors, apparatus of different types and different degrees of importance may be placed in different rooms, with fire doors between, either normally closed or closed by gravity upon the release of a fusible link. Some companies go so far as to install oil-immersed transformers in one room, large oil circuit breakers in a long narrow room with windows only at the ends, and oil-immersed voltage regulators in still another room; only the smaller breakers being installed in the same room with the switchboard and major bus structure. In connection with the location of windows, many stations are so arranged that any outdoor apparatus liable to catch fire, if installed adjacent to the building wall, are located at considerable distance from the nearest opening into the building. Another precaution taken as a means of indoor fire isolation comprises the use of cell doors and other trim made of treated wood or other fireproofed material. Current transformers, on which the relay protection is dependent, have been sometimes placed in separate compartments so they will not be involved in an arc from other equipment before they have actuated the tripping relays. Proper grounding of apparatus often lessens the chance of stray currents damaging other equipment during a short circuit. Elimination, as much as possible, of unprotected apparatus connected to the main bus is often desirable. Protection of apparatus, such as bus potential transformers, by means of resistance and fuses and careful isolation of bus arresters, should be considered.

Outdoors, transformers may be installed with fire walls between banks of two or three units or, if larger and more important transformers, between individual units. One company has recently adopted the policy, on large outdoor transformers, of eliminating the column-type oil gage from the conservator and of installing fuse-releasing gravity-closing valves on all connections between conservator and main tank. It is intended by this policy to prevent the spreading of possible transformer fires by the release of the large quantity of oil contained in the conservator, as sometimes happens through the cracking of the gage glass on the occurrence of fire in the immediately adjacent transformer and through the feeding out of burning oil from a rupture in the cover of a faulty transformer.

Both indoors and outdoors, fire isolation may be augmented by the installation of pits and drainage systems properly disposed with respect to oil-immersed transformers and regulators and oil circuit breakers.

The third form of fire protection, means of fighting the actual fire, divides naturally between the use of liquids and the use of gases. The most obvious liquid for this purpose is water, which is often piped to the end-bells of large rotating machinery and may be used elsewhere in connection with one of the commercial sprinkler systems; there is, however, considerable question as to the efficacy of water as a means of fighting oil fires. Other liquids take the form of special preparations, often generated on the premises at the time of fire, either by the mixture of two liquids or of water and a powder. Gaseous means are limited commercially to the use of carbon dioxide, which is heavier than oxygen and serves to smother the fire when applied in sufficient concentration; it is therefore particularly suitable in confined spaces where liquid might damage the apparatus and/or where the gases emanating from some of the commercial liquid preparations would be obnoxious. The efficacy of all these fire-fighting fluids may be considerably increased by the installation of permanent piping, designed to permit the direction of the fluid at the most advantageous point in case of fire. Whether or not the valves of this piping should be automatically operated depends not only upon the importance of the installation and the liability of a large fire, but also upon the proximity to a central attended installation and the attendant costs of automatic equipment for fire alarm as compared with the costs of automatic equipment for fire fighting in the unattended station.

It is believed that the art of fire protection is developing so rapidly, particularly in connection with unattended automatic stations, that no standards should be promulgated at this time by the American Institute of Electrical Engineers or others. It is thought that only by giving comparatively free rein can it be assured that progress will continue to be made in accordance with the best economic dictates.

The attached Appendix A is a preliminary attempt to classify for further investigation or study, I. the causes

of fires, II. the spreading of fires, III. preventive measures, and IV. the extinguishing of fires.

Appendix A

I. Causes of Fires. II. Spreading of Fires. III. Preventive Measures. IV. Extinguishing of Fires.

I—Causes of Fires

A. Faulty Design
1. Insufficient spacing.
2. No allowance for temperature changes.
3. Reduced clearances from distortion due to short circuits.
4. Conductors or equipment improperly or insecurely supported.
5. Weakness of mechanical structure or supports.
6. Insufficient provision against heaving by frost, entrance of rain, or accumulation of sleet.
7. Cannon construction (by cannon construction is meant the arrangement of barriers or compartments in such a manner as to direct an arc outward into other equipment or into operating aisles).

B. Faulty Equipment
1. Improper design.
2. Poor material.
3. Defective materials or parts.
4. Insufficient insulation, spacing of parts, strength or thermal capacity.
5. Improper assembly.
6. Improper and insufficient testing.
7. Unnoticed failures as a result of testing.

C. Improper Application of Equipment
1. Wrong material—moisture absorbing, subject to chemical change.
2. Wrong apparatus—type, size, rating.

D. Aging of Apparatus and Material
1. Deterioration of insulating material on conductors, supports, windings.
2. Corrosion of metals from action of air, moisture, electrolysis.
3. Effect of moisture, gases and corona in hastening deterioration.

E. Static Stresses
1. Causing destruction of insulation.
2. Collecting conducting material.

F. Magnetic Stresses
1. Mechanical movement.
2. Collecting conducting material.

G. Surges or Lightning
1. Atmospheric conditions.
2. Switching operations.
3. Improper grounding.

H. Arcs (other than flashovers)
1. Switching operations.
2. Contactors.
3. Commutators.

I. Exceeded Ratings
1. Exceeded current rating, overload.
2. Overvoltage.
3. Exceeded temperature rating.
4. Exceeded time.

J. Errors in Operation

K. Failure of Apparatus Containing Oil
(This includes oil circuit breakers, oil switches, transformers, pot-heads, etc.)
1. Insufficient amount of oil.
2. Surplus amount of oil.
3. Slopping of oil (mufflers, discharges).
4. Dripping of oil on other apparatus.
5. Lowering of oil level due to leaks, syphoning or intense cold.
6. Freezing of oil.
7. Carbonization of oil.
8. Presence of moisture.
9. Explosions due to arcs following short circuits.
10. Poor contact.
11. Puncture of bushing.
12. Internal corona discharges.
13. Internal earth fault.
14. Internal short circuit.
15. Phase interruptions due to mechanical failures.
16. Short-circuited laminations.
(Note: See other sections for failure of apparatus in general).

L. Exposure to External Attack
1. Animals—birds, rats, nests, etc.
2. Foreign material thrown on equipment—stones, kite strings, airplanes.

M. Dirty Apparatus
1. Dirt from coal piles, or adjacent industries.
2. Boring action of metallic slivers.
3. Causing reduced radiation.
4. Causing flashovers.

N. Spontaneous Combustion
1. Oily waste or rags.
2. Coal.

O. Friction
1. Lack of lubrication.
2. Improper alinement.
3. Dirt.

P. Condensation
1. Unheated conditions.
2. Temperature changes in conduits.
3. Unventilated conditions.
4. Wrong materials.

Q. Heating Equipment and Piping
1. Failure of electrical heaters.
2. Defective flues.
3. Explosions of oil burners.
4. Escaping steam or water from pipes.
5. Broken fuel lines.

R. Carelessness
1. Improper tagging or operation (disconnects, etc.).
2. Cigarettes, matches.

S. Atmosphperic Conditions
1. Flashovers due to fogs, especially salt water fogs.
2. Sleet or wind.

T. Exterior Fires
1. Adjacent buildings.
2. Grass and forest fires.

II—Spreading of Fires

A. Inflammable Material in Apparatus
1. Insulating material in conductors, such as rubber and varnished cambric.
2. Insulating supports or braces, such as wood.
3. Oil or inflammable compounds.

B. Inflammable Material in Construction
1. Wooden buildings, compartments, or structures.
2. Forms used during construction.

C. Insufficient Segregation
1. Insufficient barriers, compartments, cells or rooms.
2. Insufficient spacing between types or classes of apparatus.
3. Improper grouping of equipment, wiring, etc.
4. Improper drainage of oil, etc.

D. Poor Housekeeping
1. Rubbish during construction.
2. Accumulation of rubbish in operation.
3. Storage of inflammable material.

E. Fanning Action of Ventilating Equipment.
F. Lack of fire Extinguishing Apparatus.
G. Failure to Announce Start of Fire, Smoke or Heat.

III—Preventive Measures

A. Use of fire-proof or fire-resisting material in construction.
B. Elimination of oil so far as possible.
C. Keeping premises and apparatus clean.
D. Segregation of apparatus.
E. Correct design or application.
F. Constant expert supervision.
G. Careful inspection before and during installation.
H. Periodic testing.

IV—Extinguishing of Fires

A. Detection.
B. Alarms.
C. Accessibility.
D. Automatic fire extinguishers—classification and suitability of each type.
E. Hand-operated extinguishers—classification and suitability of each type.
F. Drainage of burning oil.

Wire Designations

Contributed by O. J. Rotty and M. E. Reagan.

The development of a uniform scheme of wire designations for station control wiring has been given some study with the idea that if a satisfactory scheme could be devised it should be offered for standardization.

The general use of such a scheme would produce uniform marking of control wiring at all stations regardless of whether the panels were wired by different manufacturers or by the user himself. This seems desirable for many reasons. The engineer laying out a new station or an extension to an old one would have to be familiar with only one scheme of wire designations and if the apparatus for a particular station was supplied by a number of manufacturers the wire designations would be alike for all the apparatus, resulting in simplification of interconnecting wires and common circuits. These points are also of importance to the construction or installation men, to the testing men, maintenance men, trouble shooters, etc.

At present there are many schemes in use, with each manufacturer and many utilities and engineering organizations having their own schemes all of which are different although some may have certain features in common. There seem to be many reasons why each scheme is preferred by its sponsor and why other schemes are objectionable. However, it is felt that a scheme can be devised which will fulfill practically all requirements and overcome the more important objections. The standard scheme must be of a general nature and must be applicable to all kinds of stations; *viz.*, railway, Edison, mining, hydro, a-c. distribution, and even steam plants.

There is a number of points in the designation of wires on which there is fairly general agreement. These are briefly as follows: 1. The scheme should be simple, easily understood and applied and with few special cases or exceptions; 2. it should be flexible so as to permit of additions or changes to the control without completely revising the existing designations; 3. it should provide the "tie-in" with or reference between the actual wire and the diagrams (either the wiring diagram, the schematic diagram, or both); 4. the wire designations should appear on the schematic (or elementary) diagram.

It also seems desirable that certain other ideas be worked into the standard scheme, which are briefly as follows: 1. The wire designation should be made up of as few digits as possible; 2. it should not be a complicated combination of letters and numbers; 3. it should preferably have some significance as to the apparatus or panel the wire connects to, although this appears to be difficult to devise and very likely an arbitrary scheme will be finally adopted; 4. operating and control buses and circuits of the nature of buses should be included in the standard scheme.

This subject has been taken up by this committee only recently and the time has been insufficient for the development of a finished scheme. However, it is felt that the work should be continued by the succeeding committee.

Several closely allied subjects have presented themselves for investigation but lack of time has prevented work upon them. They are: 1 Wire group or conduit designations or numbers, and 2, wire color codes. It is felt that these should also be given some study with a view to suggesting standards for them.

Research

Contributed by M. E. Reagan.

Research items to be studied and followed by the committee are divided under four main divisions. One object of investigation is to assemble data on operation while another object is to establish the trend of automatic expansion in operation. For instance, in the use of vacuum gas filled tubes in power and control systems, it will be the function of this committee to carefully record the various applications and keep experience charts showing the growth of usefulness and their performance.

The four general classifications are as follows:

I. Automatic Control of Rotating Machines (including mercury arc rectifiers).
   a. Synchronous converters.
   b. Motor-generator sets.
   c. Mercury-arc rectifiers.
   d. Hydro-electric generators.
   e. Frequency-changers.
   f. Synchronous condensers.
   g. Balancer sets.
   h. Battery chargers
      1. Automatic end cell switches.

II. Automatic Control of Static Machines.
   a. A-c. transformer substations.
   b. Periodic reclosing feeders.
   c. Static condensers.

III. Network Protectors.

IV. Supervisory Control.
   a. Terminal apparatus.
   b. Channels.
      1. Fused lines.
      2. Private.
      3. Carrier, etc

Suggested Items.
   a. Curve of total installations by years.
   b. Curve of total kva. by years.
   c. Curve of per cent automatic to manual installation by years.
   d. Curve showing rehabilitation of old units for automatic service.
   e. Curve showing per cent availability by years.
   f. Data on primary causes of outages, such as
      1. Line trouble.
      2. Main apparatus trouble.
      3. Control trouble.
      4. Due to testers or maintenance man.
   g. Performance data on fuses in control circuits.
   h. Ventilation as it affects "sweating" of apparatus.
   i. General field testing of equipment expand last year's report
   j. Data on vacuum tube applications of all types in control work

Telemetering in Connection with Load Dispatching

Contributed by A. M. Garrett and J. J. Samson.

In order that electrical systems, particularly those of the public utility and traction companies, function with the highest degree of service standard and efficiency, it is necessary that the control of certain system operations be centralized.

Such operating knowledge and duties as restoration of the service to normal during or following a case of system trouble; the interchange of energy between systems or parts of the same system; switching in and out of service of generating station or substation units in accordance with load demands; the load quantities

existing in different parts of the system; the maintenance of proper voltage, frequency, temperature of equipment; adjusting the system to take care of construction work in connection with system extensions, maintenance and repairs, are some of the functions that must come under the guidance of this group.

The majority of these functions is under the control of the load dispatcher or system operator who depends upon his knowledge of the operating condition of the system through the forces which actually operate it. This information comes to him in the nature of a periodic check, generally by telephone.

In recent years as the system developed through growth and expansion, and in some instances through combination, the problems associated with the handling of these systems become more involved, so that the need for quicker and more accurate information as to the operating status of the system became apparent. In order to handle the situation intelligently, a continuous record or "running account" so to speak was needed instead of a periodic sampling of conditions. To meet these requirements telemetering or remote indication equipment has been developed and placed in service in many systems in this country in the past five years. To take care of these problems under the old method would mean quite a material increase in the load dispatchers' force.

Telemetering is a means of transmitting to a central control point over a communicating circuit or special conductors, information necessary for the operation of the equipment, station or system. This may be electrical quantities (volts, amperes, watts, power factor), temperature, position of breakers, regulators, or any other information that it may be desired to obtain from the system.

It consists essentially of a transmitter at the remote end connected to the equipment from which information is desired, a receiver at the control end and the connecting circuit. The quantity of energy to be measured or position of equipment to be indicated is converted to a voltage and current (suitable for telephone cable) whose value is proportional and is transmitted to the receiver which is calibrated to record the information.

With a view to determining the trend of telemetering devices and methods, the committee decided that a survey in the form of a questionnaire should be made of those companies now operating this class of equipment.

The questionnaire covered the following points:

1. What of the following do you use telemetering for?
   - Totalizing load.
   - Current and voltage conditions.
   - Temperature of equipment.
   - Position of switch gear.
   - Tap changer and regulator positions.
   - Uses other than above.
2. State briefly the type of system and the principles of operation.
3. What channels are used?
   - Telephone lines.
   - Transmission lines.
   - Special conductors.

   State approximate distances of transmission.
4. Is this system of aid in load dispatching? In what manner?
5. What criticism do you have of telemetering?
6. What, in your opinion, is the trend of development?

The term "load dispatching" is understood to cover broadly the function of load control and switching operation which is dependent upon telemetering information.

This survey has just been completed and a total of 35 companies canvassed. The following information has been compiled:

Under uses or application we found approximately 40 per cent of the companies use the equipment for determination of current and voltage conditions, while 22 per cent of the installations were for totalizing load and 18 per cent were for position indication of equipment.

Under types used, there seems to be a general use of direct current either in the form of balanced current or rectified current.

As the connecting circuit between the remote end and the receiving end, about 30 per cent used active telephone circuits, while the remaining 60 per cent used special conductors. The distance of transmission varies from 500 feet to 15 miles with no special average.

In answer to the direct question of whether the system was an aid in load dispatching, 60 per cent stated that it was, while the rest of the replies gave no opinion or indicated lack of experience due to the short time the equipment had been in service.

In reply to the inquiry on criticism of the equipment, the survey showed little or no adverse expression. One point emphasized was the lack of high speed response with accuracy comparable to the usual line of switchboard indicating instruments. There is a tendency where high speed is desired to sacrifice somewhat the matter of accuracy. Another criticism was on the cost basis, the installed cost of the equipment running into "considerable money." Especially is this true where special conductors are required over long transmitting distances.

Improvements toward simplification of the present complicated equipment, as means to greater reliability, was offered as one criticism.

On trend of development, practically all replies indicated a favorable reaction to future development, especially where extension to the system was in prospect. This applies particularly to the situation where interconnection is concerned. One reply mentioned there would be no future development for telemetering in this company as the type of distribution was being changed to an a-c. network whereby the substation would be eliminated.

In general, it can be stated that telemetering has established itself, and with extension of load control and switching operations made necessary through an increase in the size of systems, in interconnection between systems, and constantly increasing number of unattended stations, telemetering is proving a distinct aid in the solution of those problems of handling and operating an electric system.

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Communication

ANNUAL REPORT OF THE COMMITTEE ON COMMUNICATION*

IN accordance with the usual practise, this committee submits herewith a brief review of the developments during 1930 in electrical communication.

Telephone Service Improvements

Telephone service was improved during the year by the application of new developments and methods and improved maintenance. The average time to complete toll calls was lowered by 12 seconds to an average of one minute. Improved maintenance methods have reduced the frequency of troubles affecting subscribers' lines so that a trouble now occurs on the average but once in 22 months, a reduction in trouble frequency of about 11 per cent as compared with 1929.

Steady advancement in the quality of speech transmission was accomplished. On the longer haul toll business, unsatisfactory transmission conditions were experienced on only 1.5 per cent of the messages, compared with 1.7 per cent in 1929. One of the important factors contributing to this improvement was the institution of a more effective method for controlling the amplification of speech on toll calls involving two or more long circuits. An improved general plan for the design and rearrangement of plant to handle calls between remote points not connected by direct circuits was adopted. Under this plan, the number of intermediate switches is minimized with a consequent improvement in speed, accuracy, directness of routing, and transmission. A paper describing this plan, by Mr. H. S. Osborne, was presented at the 1930 Summer Convention. (A. I. E. E. Trans., October 1930.)

Subscriber's Equipment and Services

During 1930 the number of hand telephone sets in service increased to approximately 2,000,000 as against about 1,000,000 at the beginning of the year. The development of a type of hand set which would meet the electric and acoustic requirements of telephone practise in this country has involved the solution of difficult technical problems. These problems have been concerned primarily with the suppression of singing, for which the hand set has a tendency because of the rigid connection of the transmitter and receiver, and with the efficient operation of the transmitter in a wide range of angular positions. These difficulties have been successfully overcome through the application of a large amount of scientific work in the design of the hand set handle, the transmitting and receiving elements, and the electrical circuit connecting them. These achievements have made it possible to use the hand set without any sacrifice in performance as compared with the desk stand set.

For subscribers requiring a large number of attendants to take messages or orders, there has been made available an automatic call distributing arrangement associated with a simple type of attendant's position, whereby calls are immediately connected to idle attendants or, if none are available, are held and subsequently connected in the order in which they arrived.

A new type of private branch exchange suitable for larger residences and business offices of moderate size was made available during the year. No operator or attendant is required. Calls may be originated, answered, and transferred from any of the telephones. Intercommunicating calls are made by means of the dial. Complete privacy for both outside and intercommunicating connections is provided.

While arrangements have been available for some time for private branch exchanges whereby a number of telephones could be connected together simultaneously in a conference connection, these arrangements have been extended during the past year so as to enable the interconnection of a larger number of stations and to include more than one trunk circuit out of the private branch exchange.

Central Office Systems and Apparatus

During 1930 there was an increase of about 1,000,000 dial stations, bringing the total to approximately 5,565,000, or more than one-fourth of the total number of stations in this country. Besides the conversion to dial of additional portions of large cities, increasing use has been made of dial equipment in small communities where services requiring the assistance of operators can economically be handled from near-by centers.

Further extensions have been made in the mechanical tandem facilities used primarily for service between the larger areas and neighboring suburban points. Arrangements are now in service for automatically inserting voice repeaters when required on calls between

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the more distant points. Facilities are also included for completing calls to a variety of types of offices, either mechanically or by key sets at call distributing tandem positions. An installation of call announcer apparatus, described in last year's report, has been made in New York. The mechanical tandem also includes a trouble recorder which provides a continuous count of calls affected by various classes of irregularities and prints, by means of a teletypewriter, a complete record of the channels and equipment used by such calls.

The panel system has been provided with a new type of equipment known as the decoder for translating the digits dialed by the subscriber into the electrical information required for routing the call to various types of offices. Panel equipment in certain areas is also being arranged to permit subscribers to dial calls directly to distant zones which previously required the supervision of an operator because of the additional charges involved. The equipment is arranged to determine the number of message units corresponding to the charge to the zone in which the call terminates and to cause a corresponding number of operations of the message register.

Time announcement systems for use in the larger areas have been modified by the addition of facilities for the distribution of announcements over a trunk network, equipped with repeaters to insure adequate volume and quality, so that one operator may furnish time announcements for a network of offices.

An improved low-speed interrupter, utilizing mercury make and break contacts, has been developed for use in giving flashing signals to operators and in timing sequence operations. For producing tones used as signals to operators and subscribers, there has been developed an alternator which enables close control of the frequency, magnitude, and harmonic content of the tones. The field windings and the tone output windings of this alternator are formed on the stator, and the tone is produced by an unwound, toothed rotor which varies the magnetic flux in the tone windings.

Privacy apparatus was put into service during the year on nearly all of the intercontinental radio telephone circuits operated out of this country. By means of this apparatus the radio waves transmitting conversations are so modified in their characteristics that if listened to with an ordinary radio set, speech is not intelligible. At the receiving end this process is reversed and intelligible speech restored. While this does not guarantee complete secrecy, it does furnish a considerable degree of privacy and represents an important step in advance.

Another important toll equipment development was a new type of toll switchboard in which the use of smaller jacks makes possible a substantial increase in the number of trunks and toll lines appearing at each toll operator's position. Tandem positions are also provided through which an operator may secure access to any circuit not appearing in the toll line multiple at her position.

Outside Plant Construction and Methods

To facilitate the placing of tape armored toll telephone cables, a method has been developed whereby the trenching, laying of the cable, and the filling of the trench are simultaneously accomplished in one operation. This method utilizes a tractor behind which are pulled the trailer supporting the cable reel, the plow, and the drag which fills in the trench. From the reel, the cable is threaded through a pipe which leads it to the trench at a point directly in back of the plow. Under ordinary conditions it has been practicable, on the average, to lay more than one mile of cable per day with one outfit using this method.

A new method which utilizes a single tube sleeve has been developed for making joints in aerial line wire. A hand operated rolling tool compresses the sleeve onto the wire in such a way as to provide a joint with no greater electrical resistance than a corresponding length of line wire and with a tensile strength at least equal to that of the wire. This new joint is expected to avert the electrical troubles now experienced with open-wire joints of the twisted sleeve type.

An important step has been taken by the communication interests cooperating with other pole using utilities, and others connected with the pole industry, in arriving at uniform classification dimensions for wood poles of southern yellow pine, western red cedar, northern white cedar, and chestnut. This work was carried on by the American Standards Association. Standard fiber strengths lb. per sq. in. have been established as follows: 7,400 for southern yellow pine, 5,600 for western red cedar, 3,600 for northern white cedar, and 6,000 for chestnut. These values have been accepted and made American Standards.

Under the new classification, a pole of a given class designation will have approximately the same breaking load regardless of pole length and species of timber. The adoption of the new grouping by all users of wood poles is expected to result in economies in the production and stocking of poles. The problems introduced by joint use will also be simplified as both parties to the joint use agreement will be using the same classification, whereas in the past this was seldom the case. Cost comparisons between the various species can be more easily made under the new classifications as prices may be directly compared class for class, whereas under the old tables calculations had to be made to determine the class of equivalent poles of other species of timber.

Long Distance Toll Cables

About 5,000 miles of cable for toll use were added to the telephone system of this country in 1930, bringing the total toll cable mileage to approximately 24,000 miles. Including the toll cable network on the Pacific Coast, which is connected with the toll cable network of the rest of the country by several individual and widely separated open-wire toll routes, there are now 129 out of 176 cities of the country having populations

of 50,000 or more connected directly into the toll cable network. The fourth telephone cable between Key West and Havana was completed in January 1931. The new cable is non-loaded and employs the recently developed paragutta insulation. It is equipped with carrier telephone apparatus which now provides three telephone message circuits, as well as one d-c. telegraph circuit. Provision has been made for the addition of other telephone circuits when needed. The telephone facilities now afforded by the single new cable are equal to those furnished by the three older cables in total. This gain in efficiency is brought about by improvements in the art of cable construction, including employment of paragutta, as well as advances in the application of carrier methods using high frequencies for multiplexing.

About 35,000 miles of telephone circuit are now being regularly utilized for radio program transmission service and almost 200 radio broadcasting stations receive programs from these chains of wire circuits. Heretofore practically all of this service has been furnished by means of open wires using voice-frequency channels. However, a new type of circuit for toll cables has recently been developed which permits of transmission over long distances without material distortion of a frequency range from 50 to 8,000 cycles, at the same time transmitting a volume range of about 40 db. (energy ratio 10,000 to 1). These characteristics are desirable for the satisfactory transmission of music where the frequency and volume range requirements are much greater than in the case of speech. This new type of cable circuit is loaded at 3,000-ft. intervals and special repeaters are placed at approximately 50-mile intervals. Associated with each repeater is an attenuation equalizer and a delay equalizer, which correct for the attenuation and delay differences at different frequencies. A paper on this subject, by Messrs. A. B. Clark and C. W. Green, was presented at the 1930 Summer Convention (A. I. E. E. TRANS., October 1930, p. 1514.)

CARRIER TELEPHONE AND TELEGRAPH

During 1930 about 160,000 channel miles of carrier telephone facilities and 400,000 channel miles of carrier telegraph facilities were installed in the United States. Recent improvements of carrier circuits involve improved transposition arrangements and the introduction of 8-in. spacing of wires in place of the 12-in. spacing. This latter development permits an increase in the number of facilities obtainable from a given number of wires on a pole line.

The operation of voice-frequency carrier telegraph over the channels of high-frequency carrier telephone systems has been successfully applied. For example, each of the three two-way channels of a carrier telephone system has been filled with 12 carrier telegraph channels. Such a pair of wires accommodates a duplex d-c. telegraph circuit, an ordinary two-way message telephone circuit and 36 duplex carrier telegraph channels which if multiplexed would afford means for the transmission of about 7,500 words per minute for the telegraph facilities alone.

A paper by Mr. E. I. Green, discussing the transmission characteristics of open-wire telephone lines, and a paper by Mr. L. T. Wilson, discussing telephone line insulators, were presented at the 1930 Summer Convention. (A. I. E. E. TRANS., October 1930.)

INTERCONTINENTAL TELEPHONY

Most notable of the advances made during 1930 in the intercontinental telephone field was the opening on April 3 of service to South America. This service is provided by a short-wave radio circuit operating between the transmitting and receiving stations at Lawrenceville and Netcong, New Jersey, and corresponding stations near Buenos Aires, Argentina. The ratio portion of this circuit is 5,300 miles in length and it is therefore, the longest circuit now operating from the North American continent. The transmission characteristics of this circuit are, however, somewhat better than those of the short-wave circuits between the United States and England, due principally to the fact that the transmission path of the South American circuit is further removed than is that of the North Atlantic circuit from the polar regions and the adverse influences of magnetic storms. Furthermore, the South American circuit, running nearly along the meridian, experiences a more uniform condition as to daylight or darkness. As a result of these factors, extreme variations in transmission, which form an important problem on the east- and west-bound radio circuits, are materially reduced on this circuit.

At Buenos Aires the radio circuit connects with wire and cable circuits to more than 325,000 telephones in the Argentine Republic, Chile, and Uruguay, including the cities of Santiago and Montevideo.

Another feature of interest during the year was the establishment of radio communication between the United States and Australia by way of London. At London the transatlantic circuits are connected to a London-Sydney short-wave radio circuit, which at Sydney connects in turn with the wire network of the Australian Telephone Administration. This combined circuit is approximately 14,000 miles in length.

Four papers, discussing transoceanic telephone service in its various aspects, were presented at the 1930 Winter Convention by Messrs. T. G. Miller, Ralph Bown, A. A. Oswald, and F. A. Cowan.

By the end of 1930, many parts of the world had been linked together by radio-telephone circuits in addition to those mentioned above. Most of these circuits have one end in Europe, far ends in such distant places as Brazil, Argentina, Australia, New Zealand, Siam, Indo-China, Java, and Algiers. Not all of these circuits are yet connected with the telephone systems at their terminals, and they do not all, therefore, possess to the same degree the universal accessibility and utility of those radiating from North America.

Ship-to-Shore Radio-Telephone Systems

A novel type of ship-to-shore service was inaugurated in the middle of the year through the equipment with radio-telephone apparatus of a New York City fire-boat. This equipment is arranged to enable continuous two-way telephone service between the master of the fire-boat and his land headquarters when the boat is under way. By means of this close communication, it has been possible to coordinate the efforts of the land and marine fire forces more effectively. As a result of the demand for telephone service of this kind in important harbors, both for harbor craft and ocean-going vessels, plans have been made for opening to such service in 1931, radio-telephone stations at Los Angeles, San Francisco, Puget Sound, and New York.

Commercial telephone service between ships at sea and the telephone system of the United States was extended during 1930 so that this service is now available to five ships: the Leviathan, Majestic, Homeric, Olympic, and Belgenland. Several organizations are involved in the provision of this service: the International Marine Radio Company, a subsidiary of the International Telephone and Telegraph Company, the Marconi Company, and the American Telephone and Telegraph Company. The shore stations are operated by the American Telephone and Telegraph Company on this side and by the British Post Office on the European side. During 1930, construction of a new land station at Ocean Gate, New Jersey, to care for transmission from shore to ship was completed. As in all radio-telephone transmitting stations, the antennas at this new station have a marked directional characteristic, the transmitted waves in this case being directed so as to traverse the North Atlantic shipping lanes. Four different wavelengths are used for transmission, ranging between 17.5 and 63 meters. Generally speaking, reliable transmission has been obtainable at distances from New York up to about 1,000 miles, corresponding with two days' sail. While satisfactory talks have been made at distances of 2,500 miles or greater, transmission at such distances has not been uniformly good.

A paper by Messrs. William Wilson and Lloyd Espenschied, discussing radio-telephone service to ships at sea was presented at the Northeastern District Meeting in May 1930.

Telegraphy

During 1930 all Western Union tickers handling New York Stock Exchange service in the United States were converted to the high-speed type. This permitted increasing the output of stock quotations from 300 to 500 characters per minute. In the field of telegraphic distribution of quotations and other market information, many improvements in apparatus and methods of operation were developed and applied during the year which resulted in a reduction of manual handling, greater output and extended operation into territory not heretofore reached, including quotation service to Cuba.

There has been a considerable expansion during the past year in the use of automatic quotation boards for electrically indicating market prices and this service has been extended to a number of other cities in addition to New York, where the first boards were installed.

Numerous additional installations have been made of concentration equipment for printing telegraph circuits. A description of the apparatus used for this purpose is contained in the paper entitled *A Printing Telegraph Concentrator* presented by Mr. W. B. Blanton at the Winter Convention in January 1931.

The Western Union Telegraph Company has developed a method whereby channels of a multiplex system circuit may be automatically repeated and extended individually over single wires by means of simplex printers to offices. Circuits from several offices using simplex printers may be grouped at a central point and combined into a multiplex circuit to a distant central office. This system will give many small offices direct outlets to the large centers which could not have been economically provided with previous methods.

During the year there was a marked increase in the tendency to employ, in place of direct current telegraph to outlying points superposed on telephone circuits, a method of operation which is called "two-path polar operation." This involves the use of independent one-way polar circuits for the two directions and affords improved service with a lower maintenance cost as compared to the usual polar duplex method.

Several methods of measuring the quality of telegraph transmission have been developed. One form of apparatus which has been found very effective both in the laboratory and the field was described by Mr. F. B. Bramhall in a paper entitled *Telegraph Transmission Testing Machine*, presented at the 1931 Winter Convention.

A paper describing various technical features in connection with the submarine high-speed duplex telegraph cable between Bay Roberts, Newfoundland and Horta, Azores was presented at the Winter Convention in 1931 by Messrs. J. W. Milnor and G. A. Randall. At the same convention, Mr. Milnor presented a paper on the influence of interference in submarine cables.

In addition to the statewide police teletypewriter systems in Connecticut and Pennsylvania, an extensive system of about 30 stations was installed last year in New Jersey for the New Jersey State Police. There is an increasing number of large city police networks, such as those of New York City, Boston, Buffalo, and St. Louis. The United States Coast Guard also has such a system in use along the New England Coast.

Progress has been made in the development of switchboards for teletypewriter service to permit the interconnection of these machines in a manner similar to that by which telephone instruments are interconnected.

A new type of belt conveyer has been developed by

the Western Union Telegraph Company for carrying telegrams between the various parts of the central office. In this conveyer messages are carried along between a steel channel and a moving flat belt about 1¾ in. wide, the motion of the messages being due to their sliding along the steel channel with practically no friction while adhering quite firmly to the belt. The messages can be taken around vertical or horizontal turns as well as through vertical and horizontal twists.

Municipal and Protective Signaling

There has been a steady increase in the use of electrical traffic signals. In the larger cities there have been several installations of flexible progressive systems, while in smaller places and in outlying sections of the larger cities, the detached intersection type of signal is still largely used. For important intersections and through traffic highways, vehicle controlled signals have been fairly successful but are expensive to install and maintain, as compared with the predetermined period signals. Vehicle control of signals obtained otherwise than by switches imbedded in the street surface, such as light beam control, has been tried out.

There has been a considerable use of electrical caution flashers in place of earlier types employing compressed gas, and practically all new installations are electrical. More illuminated caution signs are being installed and cheaper methods of illumination than by the neon tube method are being investigated

An important development of the past year concerning municipal fire alarms, has been the production of signal boxes and central office control apparatus, making effective use of ground return to send in alarms at such times as the circuit is in abnormal condition. Previous attempts to accomplish this introduced some abnormal condition on an otherwise normal circuit. This defect has now been practically overcome, and signal boxes have been developed that will operate in the usual manner on a normal circuit while at the same time they will transmit signals over a grounded, short-circuited or broken circuit without the necessity of making any temporary alterations at the signal box. While this development is too recent to be able to judge of its effects, it may permit of more leeway in circuit construction than is now considered safe, such as the installation of both sides of a circuit in the same aerial cable. The importance of this improvement may be estimated when it is considered that fires often occur at times when circuit conditions are apt to be disturbed by storm, explosion, or accident.

The use of single battery with trickle charger in place of duplicate batteries charged alternately by motor-generators is increasing with consequent increase in battery life.

The use of rectifiers and floating batteries or of transformers and a-c. circuits in connection with institutional fire alarms has progressed to such an extent that practically all new installations make use of public electric supply for the operating current in preference to primary batteries or to storage batteries charged from motor-generators. The practise of connecting automatic sprinkler alarm valves to municipal fire alarm boxes so that an alarm will be sounded automatically for any appreciable flow of water, has spread.

Experiments have been made on improved flash-light mechanisms for calling patrolmen to signal boxes. One such system has been installed lately in Boston. This permits any citizen desiring to call a policeman to operate an exposed lever which starts the flash-lights blinking the number of the signal box from which the call is made.

Television

Further advance was made in television as an accessory of telephone conversations by the development of means whereby two people at a distance are enabled both to converse and to see one another as if seated face to face in the same room. One of the interesting problems encountered in this development was that of sufficiently illuminating the faces of the parties engaged so that their images could be transmitted without, at the same time, dazzling their eyes to such an extent that they would be unable to see before them the image of the other party. This difficulty is met by employing blue light for scanning the faces of the parties. While the eyes are relatively insensitive to this color, the photoelectric cells used to pick up the light reflected from the face and generate the television signals are highly sensitive to it. Concealed microphones and loud speakers connected by four-wire telephone circuits are used in place of conventional telephone instruments to provide the talking portion of the system. The problem of operating a loud speaker and a sensitive microphone in the same small booth in such a manner that a self-sustaining singing circuit would not be created has been solved by specially treating the walls of the booth to prevent the reflection of sounds into the transmitter and by so locating both transmitter and loud speaker that the transfer of sound between them is minimized. While the experimental system is only two miles long, the television apparatus will function satisfactorily over distances of hundreds of miles when connected by suitable circuits as was shown by the one-way demonstration between New York and Washington.

Three papers dealing with television, by Messrs. H. E. Ives, F. Gray, M. W. Baldwin, H. M. Stoller, D. G. Blattner, and L. G. Bostwick, were presented at the 1930 Summer Convention. (A. I. E. E. Trans., Oct. 1930.)

Airplane Radio-Telephony

Development of telephone communication with airplanes continues to progress, and it has been found to be of very great importance for communication between transport planes and ground stations. Two-way radio-telephone apparatus for communication between planes

and ground has been developed and supplied for equipping a comparatively large number of planes. The value of such equipment may be indicated by the fact that legislation provides for increased revenue to air mail contractors flying planes equipped for two-way radio communication

Two papers on this subject were presented during the year, one by Mr. R. H. Freeman at the Pacific Coast Convention, the other by Mr. Eugene Sibley at the Middle Eastern District Meeting in October.

Synchronizing Radio Broadcasting Stations

There has been considerable experimental work carried on during the past year by the prominent radio organizations along the line of synchronizing broadcasting stations so that the same radio program may be broadcast from different stations operating at the same frequency. Best results are obtained only with a high degree of synchronism between the carrier frequencies of the different radio stations. In certain of the more successful experiments the control of the radio station frequencies has been carried out by sending over wire circuits standard frequencies of 4,000 or 5,000 cycles which, at the radio stations, are stepped up and used either directly to provide the radio carrier or indirectly to control the frequency of a local oscillator. While it is evident from the tests which were made that line circuits can be used very satisfactorily for transmitting control frequencies, certain requirements are imposed on the radio station frequency step-up equipment to iron out minor line fluctuations, which are, of course, greatly exaggerated when the frequency is raised from the relatively low frequency transmitted over the lines to the radio frequency of from 500 to 1,000 kilocycles.

Standardization of Radio Terms

During the past two years the Standardization Committee of the Institute of Radio Engineers has had under way a complete revision of definitions of terms, abbreviations, symbols, and methods of measurements and tests. A preliminary report of this work was issued in December, 1930.

Foreign Communication Matters of Interest

During the year international service was extended in Central America by the interconnection of Guatemala and El Salvador. In Mexico the subscribers of a second large network were given access to the United States and all points reached therefrom.

Conference toll service or multiple telephone service was established in the Netherlands. By means of this system from three to six subscribers in the same or different cities may be interconnected at the same time so that any one of the subscribers may be heard by all of the others.

Picture transmission service was extended in a number of countries, particularly in Japan.

In Europe the toll cable network was increased during the year by 6,000 to 8,000 kilometers. Two additional submarine cables were placed across the English Channel and one was placed between Sweden and Germany.

A complete rotary automatic exchange system was installed in Vatican City and connections were established with the international network.

Direct dialing over long distance telephone lines was successfully demonstrated in Europe and installations were made in several countries. Toll dialing has been employed in the United States to a limited extent for a number of years.

Transatlantic radio-telephone service was extended to include a number of new countries in Europe and to reach additional points in countries to which service had previously been given. Connections to points within the Arctic Circle became possible for the first time when transatlantic radio telephone service was extended to include all of Sweden.

During the year the International Telephone and Telegraph Corporation, in cooperation with the government administrations concerned, opened direct radio-telephone links connecting Buenos Aires with Paris, Berlin, and London. These additional links between South America and Europe not only decrease the use of long land lines formerly required to serve points distant from the radio terminal, but greatly increase the reliability of the international connections as a whole. When, because of atmospheric disturbances or for other reasons, one link is unable to give commercial service, the traffic may be routed over one of the other links. The question of allotting the traffic over the different circuits and adjusting rates so that the rate between two points will not depend upon the routing of the call has been by no means a simple one, but with the cooperation of the various government administrations much progress has been made.

Commercial service was made available between telephone subscribers in Chile, Uruguay, and Argentina, and subscribers in Belgium, Denmark, Switzerland, Austria, Lithuania, Poland, Esthonia, Norway, Sweden, Czechoslovakia, and Jugoslavia through one or another of the four direct links from Buenos Aires which, of course, also serve Spain, France, Germany, and Great Britain.

Commercial telephone service was opened to some of the Canadian National Railway express trains and equipment for the reception of broadcast programs on moving trains was provided in a number of countries, including Jugoslavia, Italy, and Austria.

During the year technical development work continued to prepare for improvements in existing apparatus and methods as well as to explore new fields, such as signaling and dialing over radio-telephone circuits, and the use of the so-called ultra short waves, those of under five meters.

Cooperation Between Wire Using Companies

The many problems arising in coordinating the wire plants of the power and telephone companies led to the formation of the Joint General Committee of the National Electric Light Association and Bell Telephone System in 1921. Since that time an extensive program of investigation has been in progress and many important results of this work were published in 1930 as Volume 1 of Engineering Reports of the Joint Subcommittee on Development and Research. A review of the work of this subcommittee was given in a group of four papers presented at the 1931 Winter Convention as a symposium on the coordination of power and telephone plants.

During the last two or three years three other Joint General Committees have been organized with the general objective of working out methods of procedure for the coordination of wire plants. These are the Joint General Committee of the Western Union Telegraph Company and the National Electrical Light Association, the Joint General Committee of the American Railway Association and National Electric Light Association and the Joint General Committee of the American Railway Association and the Bell Telephone System. These committees are proceeding with the formation of the fundamental bases of procedure for the cooperative handling of mutual problems between the utilities concerned and the carrying out of such development and research work as appears to be necessary in connection with the problems involved.

Applications of Probability Theory in Telephone Practise

The importance of the probability theory and the related theory of statistics in telephone practise are, in a general way, well known by the engineering profession. However, a short résumé of the matter together with a brief statement of additional applications made during the last year may be welcome.

The oldest and best known application of the theory in the art of communication is its use in the determination of the loads which can be placed on various manual and dial trunking arrangements in order that efficient service be rendered. Unbalance effects on loaded circuits and other transmission lines of periodic structure also give rise to linear and other types of error function problems for the solution of which recourse to probability theory must be had.

The scheduling of adequate sampling plans for determining the quality of telephone equipment at various stages of its manufacture, after delivery to the operating companies and after installation, make heavy demands on probability and statistical theory. Within the last year extensive sampling schedules have been made for determining the extent to which the human factor in the telephone business is performing its assigned functions. For example, sampling theory is now being applied with reference to the performance of the repair clerk and test desk man, the quality of work done by installers and repairmen, the quality of subscribers' contacts in commercial offices and the quality of directory printing and delivery service.

Electric Welding

ANNUAL REPORT OF THE COMMITTEE ON ELECTRIC WELDING*

THIS report consists of two sections; the first dealing with the Organization and Policies, and the second with the Progress of the Art.

I. Organization and Policies

During its first three years, this newly created committee has gradually extended its field of action until the field overlapped that of various technical committees of the American Welding Society. It was necessary, therefore, to establish a clear understanding with the A. W. S. in order to create an exchange of information and to avoid duplication of effort. It became apparent that it would be more profitable to leave such problems as the metallurgy of the welding processes, methods of testing the completed welds and organization of the procedure control, to the committees of the A. W. S.

Our efforts have been concentrated, therefore, on sponsoring the advancement of the theory of the electric arc, the design of the electric welding apparatus and the use of electric welding processes in the manufacture of electrical machinery.

The gathering of all available information on the progress of the art and the presentation of these facts through the medium of our annual report to the whole membership of the Institute is another important function of this committee.

As the activities of our committee have increased, it was felt at the beginning of the present fiscal year that the time had come to organize several subcommittees composed of physicists and engineers especially interested in one of the above specific problems. The committee as a whole has been concerned with the general survey of the advance in the progress of the art.

The present organization of subcommittees is as follows:

Research (Phenomenon of the Electric Arc) Dr. S. Dushman, Chairman, Dr. J. Slepian, Mr. H. M. Hobart, Mr. W. Spraragen, Mr. A. M. Candy.

Electric Welding Machinery (Designs of) Prof. F. Creedy, Chairman, Mr. J. C. Lincoln, Mr. A. M. Candy, Mr. A. Churchward, Mr. H. M. Hobart, Mr. K. L. Hansen.

Resistance Welding. The subcommittee on Resistance Welding has only been partly organized with Mr. E. Lunn and Mr. H. W. Tobey gathering preliminary information.

Papers

Through the efforts of the members of our committee, a symposium on welding was presented during the Winter Convention of the Institute. A paper by Dr. G. M. Shrum and Mr. H. G. Wiest, Jr. was presented by Dr. S. Dushman which related to the new experiments of the phenomenon of the electric arc.

Three papers were presented by Mr. J. H. Blankenbuehler, Mr. S. R. Bergman and Professor F. Creedy on the subject of the design of electric welding generators.

Two papers by Mr. M. Thomson and Mr. S. Martin, Jr. were on the subject of the resistance welding as applied in the manufacture of electrical apparatus.

Another paper on the design of the electric welding generator was published in the Journal of the A. I. E. E. prior to the above Convention, by Mr. C. J. Holslag.

II. Progress of the Art

General

The past year was characterized, first, by an outstanding improvement in the quality of the welds produced by various methods, secondly, by the refinement in the design of electric welding equipment, and thirdly, by the unprecedented expansion in the use of the resistance welding process.

The methods of testing and the procedure control attained such refinements that the quality of the welds produced on pressure vessels can be checked with accuracy.

The most outstanding accomplishment in the field of application of the electric arc in the past year, is the fabrication of 24 all-welded steam boilers by the Babcock & Wilcox Company for the United States Navy for installation on the new scout cruisers.

In the past, the arc welded oil stills for high pressure were manufactured successfully for a number of years by the A. O. Smith Corp., yet it is the first time that high-pressure steam boilers were fabricated by the electric arc and installed on the fighting ships.

Improvement in the Quality of Arc Welds

Arc welding was introduced into general use in this country during the World war. The first attempts to adopt the European method of welding with heavily fluxed electrodes, did not meet with general approval. Instead, the technique of welding with bare wire electrodes was developed to such an extent that practically

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C. A. Adams,	K. L. Hansen,	Ernest Lunn,
A. M. Candy,	H. M. Hobart,	B. T. Mottinger,
Alexander Churchward,	C. J. Holslag,	J. W. Owens,
F. Creedy,	J. E. Kearns,	J. Slepian,
S. Dushman,	J. C. Lincoln,	William Spraragen,
F. M. Farmer,		H. W. Tobey.

all the arc welding in this country was done, until recently, by that method.

Since the tensile strength of the welds produced with such electrodes is about the same as that made by the most expensive heavily fluxed electrodes of European make, for many applications the bare wire is entirely satisfactory. It should be noted, however, that both bare wire electrodes and heavily fluxed electrodes of European make produced welds lacking in ductility.

With the extension of the field of application of arc welding, it soon became apparent that for the use of arc welding on such structures as pressure vessels, electrodes of different types would be needed. Over ten years ago, work in this direction was started by the Electric Arc Cutting & Welding Co., Wilson Welder & Metals Co., and some other concerns and individual investigators, each of whom produced a new type of heavy or lightly covered electrode. These electrodes had certain advantages over the bare wire electrodes.

Decided advance in the improvement of the quality of the welds was made a few years ago by the General Electric Co. with the hydrogen gas shielding of the weld, by A. O. Smith Corp. with special wood pulp covered electrodes, and by several other individual investigators and concerns, each of whom developed in the last year or two, a very efficient type of fluxed electrode giving welds of high ductility, a quality which is absolutely essential for work on steam boilers and many other types of pressure vessels.

At the present time, several manufacturers of pressure vessels, in advertising their welds guarantee a ductility unheard of only a few years ago. The General Electric Co., A. O. Smith Corp., Blaw-Knox Co., Struthers-Wells Co., Kellog Company, Fusion Welding Corp., The Lincoln Electric Co., and Babcock & Wilcox Co. are probably only a part of the list of concerns now producing highly ductile welds. This advance in the quality of the produced welds opened new fields for arc welding, of which the steam boilers for the United States Navy is an outstanding example.

Resistance Welding

The welds produced by this process are of the highest quality with respect to strength, ductility and resistance to fatigue. The development, therefore, consisted mostly in the improvements in the design of the welding machines and application of such machines in new fields.

The most outstanding feature of the past year was the great development of welding pipes by the resistance process. Several machines up to 6,500 kva. each have been built for welding, by the flash welded process, longitudinal seams 40 ft. long. By the use of such machines, a weld 40 ft. long is produced simultaneously all along the seam and in less than one minute on pipe some 24 inches in diameter and of a plate up to ½ inch in thickness. Similar resistance welding machines of 1,200-kva. capacity weld circumferential seams of similar pipe. All the electrical parts for the above machines have been built by the Swift Electric Welder Company. A spot welder of 800 kva. with electrodes of 3½ inches in diameter, which probably is the largest machine ever built in this country, has been successfully built by the same Company for special heavy welding work.

Other concerns (amongst which the Thomson-Gibb Electric Welding Co. is the pioneer), developed the continuous method of seam welding. This last method had previously been used on a large scale for welding thin walled tubing. Within the last year, this method has been applied to the continuous seam welding of pipe of considerable diameter and wall thickness. At the present time, several manufacturing concerns are producing welded pipe up to 40 ft. long and it is expected that by butt welding two of such lengths in the factory, a pipe of 80 ft. long will be available for construction work.

This great development in the welding of industrial pipe by the resistance process was due to the decision of several large concerns to extend the pipe lines for transportation of natural gas and gasoline from the producing areas to the consuming districts over 1,000 miles away.

This development should be of especial interest to engineers since it indicates the possibility of transmission of power over longer distances by gas than have been accomplished by the electric current.

As was remarked by a well-known consulting engineer, the electrical engineer should rise to the occasion and surpass gas with respect to distance and amount of power transmitted. He has plenty of as yet undeveloped alternatives to the 60-cycle synchronous system as at present employed.

The resistance welding process is being further developed in the application to the automobile industry where it greatly outdistances all the other welding processes. Another important field of application of resistance welding is the manufacture of wire goods which comprises even such fields as the welding of wire netting for reinforced concrete. Machines making up to 24 welds simultaneously have been built and are in successful use. In the fabrication of electrical machinery, the resistance process has already been applied on a large scale to the manufacture of small transformer tanks. The application of this process has also extended to the fabrication of motor magnet frames.

Structural Steel Welding

In the field of structural steel, arc welding has already proved to be entirely satisfactory, and therefore, received further important applications. All arc welding of this type of construction is done with entire satisfaction with the bare wire electrodes.

Amongst the high buildings erected during the past year, (partly by welding and partly by riveting) are the buildings of the Southern California Edison Co., the Edison Electric & Illuminating Co. of Boston, and

the DuPont Corporation at Wilmington, Delaware. The highest building entirely welded was erected by the Dallas Power & Light Co. This building has 19 stories and measures 246 feet above the foundations. The Westinghouse company erected several industrial and office buildings, all entirely welded. The highest of these buildings measures 190 feet above the foundations. It should be noted that in the erection of the buildings, only very few welding machines are necessary. In the erection of the 19 story building at Dallas, only four arc welding machines were used.

Shipbuilding

In the shipbuilding field, arc welding is being extended rapidly and all our shipbuilding companies such as the Newport News Shipbuilding & Drydock Co., The Bethlehem Shipbuilding Co., and many others as well as the U. S. Navy Yards, are using arc welding quite extensively for fabricating various internal structures on ships. On smaller ships, the electric arc has been used successfully even in welding the hulls.

The Electric Boat Company, at Croton, launched during the past year, an all welded barge, 118 ft. long, of Ewertz patented design. The Federal Shipbuilding & Drydock Co., at Curney, N. J., welded a number of barges, and the Standard Steel Shipbuilding Corp., of Los Angeles, completed an all-welded 65-ft. yacht.

The application of arc welding in shipbuilding requires a careful study of the design of the welds and the details of the technique of welding. The long welded seams characteristic in this type of construction may be subject to high internal stresses and therefore should be produced by the best known methods and electrodes. The amount of welding per ton of steel in shipbuilding is very much greater than in the erection of buildings where only the short welds are necessary. It may be expected that in the construction of barges and small vessels for internal navigation, arc welding will soon outdistance riveting.

Automatic Machines, Alloy Electrodes and Special Arc Torches

The design of automatic arc welding machines advanced still further and several concerns are now manufacturing either full automatic or semi-automatic machines of a very efficient design for use with metallic electrodes. The Lincoln Company made further improvements in automatic carbon arc welding machines which produce welds of high ductility, and the General Electric Company applied the atomic hydrogen process to automatic welding as well as for hand welding.

The development of special alloy electrodes for hand welding as illustrated by the Stoody process, made considerable progress. The surfacing of various parts of machines subject to wear and abrasion with hard surfaces in certain cases can be successfully accomplished.

The development of special torches for use of indirect arc (similar to the flaming arc) extended the field of arc welding to the low-melting-point alloys. A special torch developed by Hansen and utilizing the d-c. arc can now be used for brazing, soldering, and welding low-melting-point extruded alloys. Another torch utilizing the a-c. arc has also been put on the market by the Warner Co., of Los Angeles. It can be used successfully on light work of brazing, soldering, and welding steel sheets of light gage.

Design of Electric Arc Welding Generators

The advances and refinements in the design of the electric welding generators have been carefully followed by the subcommittee on Electric Welding Machinery, which reports as follows:

An outstanding feature of the year's work in the development of direct-current arc welding generators, is the attention which has been devoted to their transient characteristics. It has been realized that the momentary shortening of the arc produces a large momentary transient current, which tends to cause explosions or "sputterings" detrimental to welding. Means have, therefore, been sought by several workers by which these momentary transient currents may be eliminated and papers on the subject have been presented by Blankenbuehler, Bergman, Holslag and Creedy.

The direct-current welding generators contain a main exciting coil either shunt, separately excited, or a combination of the two, and a demagnetizing series circuit whose function is to reduce the voltage with increasing current. One important type of transient is caused by the voltage induced in the main exciting circuit by transformer action from the series demagnetizing circuit and this type of transient may be eliminated by reducing this voltage to a small value. There is no need to eliminate this voltage altogether because it is only necessary to reduce it to such a degree that the solutions of the differential equations which determine the current approximate to the logarithmic instead of the oscillatory type. Blankenbuehler discussed a machine with separately exciting coil and reverse series winding only, and injects into the separately excited circuit a voltage proportional to the current flowing through the reverse series winding (the welding current) by means of a series transformer. This voltage is opposite in direction to that induced in the same coil by the reverse series winding so that the two tend to cancel.

The machine treated by Bergman has two distinct poles whose flux cuts the conductors between positive and negative brushes. One has constant flux and the other is shunt excited from a third brush intermediate between the main brushes, armature reaction being relied upon to reduce its voltage with increasing current. In this case a series transformer is used to inject into this shunt circuit a voltage proportional to, but opposite in direction to that due to induction from the armature.

In Creedy's machine in which the effect of shunt and

separately excited windings is produced by a single coil, no transformer is employed but the same result is produced by reducing the reluctance of the main magnetic circuit so that the number of turns on the main exciting coil is very much reduced and consequently the e. m. f. induced in it by the series demagnetizing circuit becomes small. With this latter device a neutralizing winding becomes necessary to avoid sparking and excessive reactance of the armature winding.

Testing of Welds

Several non-destructive methods of testing of welds have been developed among which the X-ray method is already being applied as a regular procedure control on welds made on important structures. The magnetic and the stethoscope methods are also available. It should be noted also that a series of experiments has been conducted with the gamma rays produced by radium emanation for testing of welds on heavy plates and steel castings.

Further Research

Since the electric arc is the foundation on which the entire arc welding industry has been built, our subcommittee on Research recommends further study of this phenomenon. The outlined problems are as follows:

1. Probe measurements in arcs between iron electrodes, using a technique somewhat similar to that adopted by Nottingham in his investigations on the arc between copper electrodes in air.

2. Arc characteristics should be taken as a function of the pressure for different gases, such as carbon dioxide, air, and argon.

3. The question of the part played by fluxes, or additions of various "dopes" to the welding rod.

4. Determinations of the energy consumption at the electrodes, using a calorimetric method.

5. Accurate determination of cathode drop and its variations under varying conditions. This is an important factor in all theories of the electric arc and more accurate information on this point is certainly desirable.

6. Accurate determination of anode drop under varying conditions.

7. Study of small current arcs and particularly the glow to arc transition.

8. Mathematical study of some of the newer ideas on stability of discharges as developed by Barkhausen and others.

9. Cathode-ray study of stability of arcs such as used in electric welding. The spontaneous variations in current and voltage which are always taking place will probably be frequently beyond the capacity of the ordinary oscillograph. The influence of high frequency characteristics of the external circuit could be studied in this connection.

10. Stability of arc in magnetic field. Since arcs used in welding usually have quite strong magnetic fields, the influence of this factor in the stability of an arc is quite important.

11. Study of influence of nature of cathode surface on stability.

12. Study of influence of nature of anode surface on stability.

13. Study of passage of the deposited metal from the electrode to the work. Although a large part of the metal is deposited by liquid drops, it may be that a considerable portion passes in the form of vapor.

14. Study of the influence of superimposed alternating current of moderate and high frequency. The use of high frequency has been proposed for actual welding, but the influence upon the arc has not been studied in much detail.

It has also been suggested that the carbon welding arc be further studied, especially with respect to the blast of carbon vapor from the cathode spot.

University Activities

Through the initiative of the American Bureau of Welding, twelve engineering colleges and universities have become interested in various problems relating to welding. In addition to the research work conducted by professors, the students in several colleges write their theses on welding in preparation for the degree of B. S. or M. S.

It should be noted that welding, and especially electric welding, is receiving considerable attention in various universities abroad. Germany is probably leading in that respect because electric welding is considered to be of such importance that some colleges in that country confer the degree of Doctor of Engineering for any adequate research work on problems of welding.

In the Soviet Union, welding has received during the past year, an unprecedented expansion. To deal more adequately with the problem of a highly trained personnel, a special Welding Institute is being organized which will cost over $1,500,000, to train future welding engineers.

Electrical Machinery

ANNUAL REPORT OF COMMITTEE ON ELECTRICAL MACHINERY*

I. Organization and Activities

THE committee is organized with five permanent subcommittees, and its work has consisted of the review of papers, arrangement of programs for technical sessions, and the preparation of Standards and Test Codes.

Forty papers on electrical machinery were presented during the calendar year 1930. Others could have been made available if facilities for presentation and publication had permitted. The committee believes that the preparation of short and informal papers designed to bring out discussion at conventions should be encouraged, and that highly technical papers whose chief value is for reference should be presented before small groups in parallel sessions.

Standards

The Subcommittee on Transformers has been very active and during 1930 prepared "Recommendations for the Operation of Transformers" and brought to conclusion a revision of the A. I. E. E. Standards for Transformers.

The Subcommittee on Synchronous Machines, besides completing A. I. E. E. Standards for Capacitors, has initiated a number of revisions for the A. I. E. E. Standards for Synchronous Machines.

The Subcommittee on Mercury-Arc Rectifiers is working in close cooperation with the Sectional Committee on Rectifiers, which has under preparation Standards for Mercury-Arc Rectifiers.

Test Codes

Preliminary drafts of test codes for transformers, induction motors, direct-current machines and synchronous machines have been prepared. These codes are intended to provide definite instructions for the more generally applicable field and factory methods of conducting and reporting acceptance and other tests of general commercial value.

II. Resume of Progress of the Art

SYNCHRONOUS MACHINES

Steam-Turbine Driven Alternators

The size of the single-shaft units has again been increased, two 200,000-kva., 0.8 power-factor, 1,800-r. p. m., single-shaft, tandem-compound, turbine-generator units (GE)† now being built for the Hudson Avenue Station of the Brooklyn Edison Company. They have double windings rated at 16.5 kv., and double-winding auto-transformers will step up the voltage to 27.6 kv.

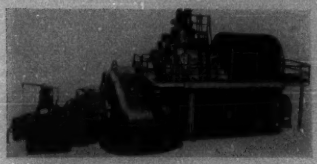

FIG. 1—50,000-KW., 3,600/1,800-R. P. M., VERTICAL-COMPOUND, TURBINE-GENERATOR UNIT (GE)—PACIFIC GAS & ELECTRIC CO.

A 110,000-kw., 1,200-lb., vertical-compound, double-winding turbine-generator unit (GE), with duplicate generators on the high- and low-pressure turbines, has been constructed for the Ford Motor Company.

A 50,000-kw., 3,600/1,800-r. p. m., 1,200-lb. unit, in which the high- and low-pressure elements were combined to form a vertical-compound turbine-generator set (GE) was constructed for the Pacific Gas & Electric Company and a duplicate unit is nearing completion. (Fig. 1.)

Three 18,750 kva., 3,600-r. p. m., turbine-generator units (W), the largest machines of this speed yet manufactured in this country, were put in operation at Baton Rouge by the Louisiana Steam Products Company. An outstanding feature of these machines is the internal propeller fan used, the success of which has created a new interest in internal fans for large machines.

High-voltage turbine generators rated 121,000 kva., 18 kv.; 147,000 kva., 22 kv.; and 94,000 kva., 22 kv. are now in the first stages of construction (AC).

The satisfactory completion of tests on a 9,375 kva.

†Legend for Manufacturer Designation:

In order to conserve space and avoid numerous repetitions of the names of manufacturers, the following designation has been used throughout the report:

AC—Allis-Chalmers Manufacturing Co.
BB—American Brown Boveri Co.
CGE—Canadian General Electric Company
CW—Canadian Westinghouse Co., Ltd.
EM—Electric Machinery Mfg. Co.
GE—General Electric Co.
W—Westinghouse Electric & Mfg. Co.

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hydrogen cooled, turbine generator (W) (Fig. 2) have encouraged the serious consideration of hydrogen cooled turbine generators for ratings above 30,000 kva.

Waterwheel-Driven Alternators

The year 1930 has been notable for the number of large hydro-electric developments which have either been contemplated or put into operation. Table I is a list of some of the more interesting hydraulic-driven generators installed or under construction during the past year.

The 77,500-kva. generators (GE) for the Dnieper River Development in Russia, operating under a hydraulic head of 123 feet are the largest waterwheel generators yet constructed. (Fig. 5.) The total weight of each generator will be approximately 880 tons, the weight of the rotor and shaft will approach 445 tons, and the shaft itself, which is 36 ft. long and 40 in. in diameter, with a 70-in. flange on each end, will weigh about 68 tons. The stator is fabricated in six sections with punchings and windings assembled in each section before shipment.

TABLE I—WATERWHEEL-DRIVEN ALTERNATORS

Purchaser—Location	No.	Kva.	R. p. m.	Type	Fig.	Mfr.
Amtorg Trading Co.	4*	77,500	88.25	Vert	4	GE
Dnieper River (Russia)	1	77,500	88.25	Vert		GE
City of Seattle Skagit River	2	66,700	171.5	Umbrella		W
Inland Power & Light Co. Ariel Development	1	56,250	120	Overhung		GE
Alcoa Power Co. Chute-à-Caron	4	50,000	120	Vert		CW
New York Power & Light Corp. Spier Falls	1*	47,000	81.8	Overhung		GE
Beauharnois Power & Light Co. St. Lawrence River	2	46,625	75	Umbrella		CGE
St. Lawrence River	2	43,883	75	Umbrella		CGE
Western Power Co. of Can. Ltd. Ruskin Generating Station B. C.	1	44,000	120	Umbrella		CW
Lexington Water Power Co. Saluda Development	4*	40,625	138.5	Umbrella	3	W
New England Power Construction Co. Fifteen Mile Falls	4*	39,000	138.5	Umbrella		W
Shawinigan Engineering Co. Rapid Blanc Development	4	36,000	109.1	Vert		CW
Pennsylvania Water & Power Co. Safe Harbor, Susquehanna River	4	31,111	109	Overhung		GE
	2	31,111	109	Umbrella		W
New Kanawha Power Co. New Kanawha River	4	30,000	150	Vert		W
Pacific Gas & Electric Co. Tiger Creek	2	30,000	225	Horiz		W
Gatineau Power Co. Paugan Falls P. Q.	1	28,500	125	Vert		CW
James MacLaren Co. Masson Power Development	4	28,000	166.7	Vert		CW
Shawinigan Water & Power Co. Grand Mere, P. Q.	1	25,000	112.5	Vert		CW
Toccoa Electric Power Co. Blue Ridge	1	25,000	164	Umbrella		W
Montana Power Co. Morony	2*	25,000	81.8	Umbrella		W
Hydro Electrica Espanola Spain	2	25,000	375	Vert		GE
Union Electric Lt. & Pr. Co. Osage River	6	23,888	112.5	Umbrella		W
Chats Falls Executive Board	8	23,500	125	Vert		CW
San Joaquin Light & Power Co. Merced Falls	1	4,000	128	Vert	4	W

*Included in report of last year.

The 31,111-kva., 109-r. p. m., umbrella-type generators (GE and W) for the Safe Harbor Development of the Pennsylvania Water & Power Company are to be driven by Kaplan-type turbines, and will operate as motors in a reversed direction to pump water back into the fore-bay for use during the peak-load periods.

High Frequency Generator

A 500-kva., 960-cycle generator (W) built for the American Brass Company will supply power for an

Fig. 2—9,375 Kva., Hydrogen-Cooled, Turbine-Generator (W)

electric furnace. The rotor required special construction to retain the field coils in place at the high peripheral speed of 20,000 ft. per minute.

A 666-kva., 960-cycle single-phase generator (GE) was placed in operation at Watertown (Mass.) Arsenal in November, 1930, and a 750-kva., 960-cycle, three-phase generator (GE) is being built for the American Brass Company.

Fig. 3—Four 40,625-Kva., 138.5-R. P. M., Umbrella-Type, Waterwheel-Generator Units (W)—Saluda Development, Lexington Water Power Co.

Synchronous Condensers

A 50,000-kva., 13.2-kv., 600-r. p. m., 50-cycle hydrogen-cooled condenser (GE), equal in capacity to the largest air-cooled machines, has been constructed for the Southern California Edison Company. (Fig. 6.) The total capacity of hydrogen-cooled condensers now in service is 137,500 kva. with 70,000 kva. additional on order.

Two 15,000-kva., outdoor hydrogen-cooled condensers (W) have been constructed; one rated at 50 cycles, 750 r. p. m. for the Southern California Edison Company, and the other rated at 60 cycles, 720 r. p. m. for the Indiana & Michigan Electric Company.

A 7,500-kva., 6.6-kv., 900-r. p. m., 60-cycle air-cooled, outdoor condenser (AC) has been constructed for the Central Illinois Public Service Company. (Fig. 7.)

Industrial Synchronous Motors

The application of synchronous motors to industrial purposes is being extended as the knowledge of the starting, acceleration, and pull-in requirements is increased and as the prediction of these characteristics becomes more exact. A number of unique designs has been developed during the year.

A 500-hp., 72-r. p. m., vertical-shaft motor (GE) has been applied to a Fuller Lehigh coal pulverizer, in which the motor frame forms the base on which the pulverizer is built. The motor develops considerably more than normal torque from start to pull-in in order to obtain successful starting with a choked mill. Over-

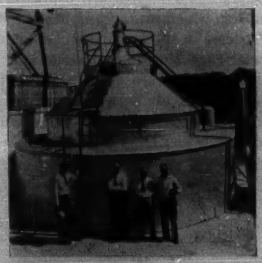

Fig. 4—4,000-Kva., 128-R. P. M., Vertical-Type, Outdoor, Waterwheel Generator (W)—Merced Falls, San Joaquin Light & Power Co.

voltage for starting is supplied by a step-up auto-transformer.

A 700/350-hp., 2.3-kv., 277/138-r. p. m., three-phase, 60-cycle, two-speed synchronous motor (W) has been applied to a vertical centrifugal pump for the Ford Motor Company.

A new method for securing reduced starting kva. has been introduced, which utilizes the inherent reactance of the motor itself, a 3,000-hp. (GE), five 3,500-hp. (EM) and several smaller motors having been built. The stator is wound with two or more parallel circuits so arranged within the core as to obtain increased reactance when only one of the circuits is energized. The starting characteristics obtained are somewhat similar to those secured by using external reactors in series with the lines.

A 600-hp., 440-volt, 300-r. p. m., three-phase, 60-cycle, synchronous motor (EM) was applied for the first time for continuous reversing duty on a copper

rolling mill, synchronous motors having been generally considered unsuitable for constant reversing duty on account of their inferior starting characteristics. The operating requirement that the mill be reversed from full speed forward to full speed reverse in five seconds necessitated special design of the stator and rotor to

FIG. 5—77,500-KVA., 88.25-R.P.M., VERTICAL-TYPE, WATER-WHEEL GENERATOR (GE)—DNIEPER RIVER DEVELOPMENT (RUSSIA) AMTORG TRADING COMPANY

dissipate heating, which was accomplished by the use of ventilating spacers in the armature core and field poles and by special cage end-ring construction. (Fig. 8.) Three parallel circuits were used in the stator, which were switched successively both for forward and reverse acceleration in order to reduce the increment starting kva. demand.

A slow-speed, vertical-shaft motor (EM) has been incorporated in a pulp hydrator base. The motor was

FIG. 6—50,000-KVA., 13.2-KV., 600-R. P. M. 50-CYCLE, HYDROGEN-COOLED SYNCHRONOUS CONDENSER (GE)—SOUTHERN CALIFORNIA EDISON CO.

provided with a thrust bearing which carries the weight of the rotor and hydrator rotating member. To permit adjustment of the clearance between the rotating and stationary members of the hydrator, the motor was provided with a micrometer raising and lowering device, the field poles being slightly longer than normal to permit a moderate movement of the rotor axially with respect to the stator.

Marine Synchronous Motors

Several electric-propelled vessels of interesting design were placed in service or are under consideration.

TABLE II—MARINE SYNCHRONOUS MOTORS

Vessel (or purchaser)	No.	Hp.	R. p. m.	Mfr.
S. S. Santa Clara	†2	6,300	120	GE
S. S. Morro Castle	†2	8,000	143	GE
S. S. Oriente	†2	8,000	143	GE
S. S. President Hoover*	†2	13,250	133	GE
S. S. President Coolidge	†2	13,250	133	W
United Mail S. S. Company (6 vessels)‡	12	5,500	125	GE

*Launched December 1930, Transoceanic service 1931.
†Included in report of last year.
‡Under construction

The 13,250-hp., 133-r. p. m. ship-propulsion motors (GE, W) are the largest marine synchronous motors yet built. These motors are completely fabricated.

FIG. 7—7,500-KVA., 6.6-KV., 900-R. P. M. 60-CYCLE, AIR-COOLED, OUTDOOR, SYNCHRONOUS CONDENSER (AC)—CENTRAL ILLINOIS LIGHT CO.

Frequency Converters

There is a growing tendency to install large machines outdoors in order to reduce building costs. The first frequency-converter units to be so installed have been built during 1930 and will be placed in operation during 1931.

Two such units, the generators being rated at 21,000 kva., 0.7 power factor, 300 r. p. m., 25 cycle, one-phase, and the motors at 18,000 kva., 0.9 power factor, 300 r. p. m., 60 cycle, three-phase (W) will be installed at the Wayne Junction Substation of the Philadelphia Electric Company to supply power for the new Reading Railroad Electrification (Fig. 9). Conventional indoor design was followed, with spring mountings to absorb the single-phase torque pulsations, the machine being adapted for outdoor service by enclosing in a sectionalized sheet steel housing approximately 64 ft. long by 25 ft. wide.

INDUCTION MACHINES

A major activity of the manufacturers during the year has been the development of new lines of induction

motors to conform with the frame sizes and mounting dimensions standardized by the National Electrical Manufacturers Association as announced in last year's report. In many cases this program has included a change in electrical design to improve the noise and torque characteristics. Much attention has also been given to the appearance in order to meet the ever more exacting aesthetic tastes of the public.

Fig. 8—600-Hp., 440-Volt, 300-R. P. M. 60-Cycle, Continuous-Reversing-Duty, Synchronous Motor (EM)—Copper Rolling Mill

Several manufacturers now furnish lines of motors in integral up to 100 hp. as well as fractional horsepower sizes for class I and class II hazardous locations (inflammable gases and combustible dust) as defined in the National Electrical Code.

Complete lines of totally-enclosed, fan-cooled continuous duty 55-deg. cent. motors as large as 250 hp., particularly suited for use in harmful dust conditions, have now been developed by many manufacturers and the use of this type is steadily growing.

An extensive application of a novel induction motor (GE) has been made to conveyor tables in steel mills. (Fig. 10.) The conveyor roller is a hollow cylinder mounted on ball or roller bearings rotating around a stationary supporting shaft on which the stator is mounted. The cylindrical convey roll, which surrounds the stator, serves as a secondary, the induced currents flowing in the solid steel inner surface of the roll.

D-C. Machines

Motors and Generators

An 8,000-hp., 800-volt, 40- to 100-r. p. m., motor, having a peak capacity of 22,000-hp., 2,900,000-lb. ft. torque, has been built (GE) for a blooming mill of the Illinois Steel Company. This is the largest single-armature motor employing fabricated construction for the major parts.

Two 5,000-hp., 40-r. p. m., double-armature motors were built (W) for operating the main rolls of the new 10,000-hp. blooming mills at the South Chicago Plant of the U. S. Steel Corporation, which is the most powerful mill yet built. Each of the two main rolls is driven by one of the motors, the two being kept at the same speed by electrical control instead of being geared together as in former installations. Each double motor has a peak-torque capacity of two million lb. ft. Power is supplied from three 3,000-kw. induction motor-generator sets. The series fields of the three generators are interconnected in such a way that parallel operation is secured with satisfactory division of load.

Planer motors up to 75 hp. rating with speed ranges by field control of 6 to 1 have been developed (GE and W), and further increases in size of this class of motor are under consideration.

A 50-hp., 1,600-r. p. m., 75-deg. cent. one-hour motor has been developed (W) for application on a coal cutter to meet a limiting over-all height of 12 in.

Six 3,500-hp., 330-r. p. m,, motors have been constructed (AC) for a 96-in. plate mill of the Illinois Steel Company. Power is supplied by two 6,000-kw. synchronous motor-generator sets.

Two motor-generator sets, consisting of two 3,000-kw., 600-volt, compensated generators driven by one 6,500-kva. synchronous motor, have been constructed (GE). These sets will deliver 6,000 kw. continuously with 9,000 kw. for two hours or 7,500 kw. continuously without overload.

Exciters

Further developments in "rapid response" exciters have occurred during the year, with the objects of

Fig. 9—21,000/(18,000)-Kva., 300-R. P. M. 25-(60)-Cycle, One (Three) Phase, Frequency-Converter Outdoor Housing (W)—Wayne Junction Substation, Philadelphia Electric Co.

securing higher efficiency and improved control. The use of a special Wheatstone Bridge type of rheostat (GE) enables the exciter to operate down to zero or even reversed voltage.

A 90-kw., 3,600-r. p. m., direct-connected, turbine-generator exciter has been developed (W) which is one of the largest exciters ever built for this speed. (Fig. 11.)

Synchronous Converters

Four 5,440-kw., 167-r. p. m., 16,000-ampere, 25-cycle, synchronous converters were constructed (2 GE and 2 W) for a chemical plant in the Niagara Falls District. The converters are enclosed in volute housings, and the

outgoing air is discharged outside the building. Direct-current voltage adjustment is obtained by means of tap-changing equipment on the supply transformers. A novel feature of construction used on two of these machines (GE) is a "tandem" commutator, which consists of two sets of shorter copper segments, together with insulation and clamping rings, mounted on a common support so arranged that air can pass between the adjacent ends of the two commutators where their segments are connected together. This construction (Fig. 12) results in lower temperature rise and smaller expansion strains and, consequently, a smoother running surface than is usually obtained with an equivalent single-unit commutator.

Transformers

Power Transformers

Extensive theoretical and experimental study of transient voltage phenomena was continued during the year, and the development of means for protection against lightning surges advanced rapidly. One indica-

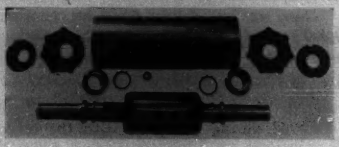

Fig. 10—40-Lb.-Ft., 250-Volt, 125-R.P.M., 25-Cycle, Conveyor-Roller Induction Motor (GE)

tion of the progress made during the year is given by the successful outcome of full voltage flashover tests on a large transformer.

A 13,000-kva., 230-kv., wye-grounded, non-resonating transformer built for commercial use (GE) was subjected to a series of artificial lightning voltage tests. The transformer was connected to a lightning generator by a short transmission line insulated with fourteen 10-in. units, spaced 5¾ in. in suspension, which is used quite generally for 230-kv. service. The insulator strings were repeatedly flashed over without apparent damage to the transformer. The line insulation was then increased until the transformer bushings were flashed over. The transformer was subjected to repeated waves both above and below the bushing flash-over voltage, after which it withstood all A. I. E. E. standard tests.

A study of transient voltage phenomena within auto-transformer windings resulted in the development of the non-resonating auto-transformer (GE), of which several have now been built for 230-kv. service.

Through the development of the grounding neutral impedor (GE), the use of the non-resonating transformer was extended to include banks with neutrals isolated or grounded through resistance or reactance.

Investigation (W) has shown that a comparatively high resistance can be used in the neutral of a bank of power transformers and the bank will perform practically the same as though it were solidly grounded. The same result can be obtained with reactance in the neutral paralleled by a protective device, such as a lightning arrester. The internal voltages at points within the winding have been found to be practically identical for a bank with a solidly grounded neutral and for one with a reactance paralleled by a lightning arrester.

Fig. 11—90-Kw., 3,600 R.P.M., Direct-Connected, Turbine-Generator Exciter (W)

In order to extend the benefits of load-ratio control to small transformers, a lighter type of equipment has been developed for 15-kv. service (GE). This consists of a multi-point switch which permits the load current to leave each tap point over two paths; *i. e.*, over two switch arms which connect to the bridging reactor. A typical application of this equipment is represented by a 3,333-kva., 69.7/115 Y-13.8-kv., one-phase transformer, in which thirteen operating positions are

Fig. 12—5,440-Kw., 340/270-Volt, 16,000-Ampere, 25-Cycle, Shunt-Wound, Synchronous Converter with Tandem Commutator and Brushes (GE)

provided by the installation of the control device in the low-voltage winding. In this type of construction all moving parts, that is, the ratio adjuster and two contactors, are mounted outside the main transformer tank.

Several 5,000-kva., 13.2-kv., three-phase, furnace transformers, with a new type of tap-changing equipment, have been built (W) for the Ford Motor Company. These transformers deliver a normal low-voltage current of 12,800 amperes at 225 volts, with a maximum current

of 19,200 amperes at 150 volts. A special motor-operated tap changer provides six different delta low voltages and a change from delta to star on the high-voltage winding gives six additional delta low voltages. Two individual tap changers are operated by one motor and all the voltages and connections are obtained by the operation of a single control switch at the switchboard.

Fig. 13—4,200 Kva., 11,000,-915-46 Volts, 25-Cycle, Air-Blast, Locomotive Transformer (GE) (W)—Pennsylvania R. R.

A 4,200-kva., 11,000-915-46-volt, air-blast transformer of special construction was built (coordinated design of GE and W) for installation in a locomotive where space is at a premium. (Fig. 13.) It was, accordingly, designed for minimum dimensions and weights, the floor space occupied being 6 ft. 8 in. by

Fig. 14—Outside-Edge-Welded Transformer Radiator (W)

7 ft. 3 in. and the total weight being approximately 26,000 lb. A 2,800-kva. transformer of the same type has also been developed. These transformers are suction air-cooled and of exceptional mechanical strength, being braced and insulated for the severe operating conditions incident to locomotive service.

Power-Transformer Auxiliaries

While the method of cooling transformers by means of "banked" radiators was not used to any great extent following its development several years ago, it was extensively applied during the past year. About 200,000 kva. in transformers of this construction are in service and more than 600,000 kva. are now under construction (GE).

A new type of transformer radiator has been developed to replace the tubular type on all Westinghouse large self-cooled power transformers. (Fig. 14.) All of its joints are formed with edge welds on the outside of the radiator, and there are no crevices or flat surfaces where water can accumulate. The open construction facilitates cleaning and painting.

A 20,000-kva., 66-22-kv., load-ratio-control transformer, equipped with unusual pothead disconnecting switches has been built (W) for the State Line Generat-

Fig. 15—100-Kva., 2,080-115/230-Volt, 60-Cycle, Distribution Transformer Equipped with Detachable Bushing (AC)—Commonwealth Edison Co.

ing Company. These high-voltage switches disconnect the transformer from the line and connect test bushings to the 66-kv. cable lines, which can then be tested with 330 kv., direct current. The pothead equipment also included oil reservoirs used to maintain pressure on the oil-filled cables to which the transformer is connected.

A new no-load ratio adjuster has been developed (BB) which is operated through the side of the tank wall instead of the cover, making it more convenient to operate and permitting the tap position to be safely checked without deenergizing the transformer. A novel form of connection is used between the internal and external parts which permits engagement or disconnection of the two parts, when the transformer core is tanked or untanked, without the necessity of crawling inside to remove pins, bolts, etc.

Distribution Transformers

A new single-bushing, single-phase transformer has been designed (GE) for 110-kv. rural service. It is arranged for connection from line to neutral on a 110-

kv. circuit, which is insulated for 132-kv. service. In order to secure a low over-all height, the interior was so constructed as to give the equivalent of an inverted winding. The bushing was then connected to the bottom of the coil stack, thereby effecting a low height in spite of the large size of the bushing and the amount of insulation required for operation on a 132-kv. insulated line.

A detachable bushing has been developed (AC) for distribution transformers, particularly for installation where lightning conditions are abnormally severe, which can be replaced without removing the case from the pole. (Fig. 15.)

Rectifiers

The installation of rectifiers has continued to increase at a rapid rate, particularly when measured in terms of the aggregate kw. capacity.

	No.	Kw.
Put into operation during 1930	33	75,225
Put into operation during 1929	30	48,125
Increase	10%	58%
Put into operation during 1930	33	75,225
Being erected during 1930	26*	71,400*
On order at end of 1930	29	79,275
Total for 1930	88*	225,900*
In service at end of 1929	90	102,309
Grand total	178*	328,209*

*In addition one spare 500-kw. rectifier tank (without transformer) was supplied.

The mercury-arc rectifiers put into operation and being erected during 1930 and on order at the end of the year are shown in Table III.

The placing in operation of fourteen units with an aggregate rating of 40,000 kw. of 3,000-volt rectifiers

TABLE III—MERCURY-ARC RECTIFIER UNITS PUT INTO OPERATION DURING 1930 OR ON ORDER DECEMBER 31, 1930

Purchaser	No. of sets	D-c. volts	Kw. per set	Total kw.	Control	Service	Put in service	Manufacturer
American Gas & Electric, New York	2	610	500	1,000	Automatic	Railway	On order	Brown-Boveri
Boston Elevated Railway	2	600	3,000	6,000	Automatic remote control	Railway	Being erected	General Electric
Chile Exploration Co	1	650	1,000	1,000	Automatic	Heavy mine haulage	1930	General Electric
Clinton, Davenport & Muscatine Railway	1*	700	500	500*	Automatic	Railway	Being erected	Brown-Boveri
Commonwealth Edison Co	2	1,500	1,500	3,000	Manual	Railway	1930	Brown-Boveri
Commonwealth Edison Co	4	625	3,000	12,000	Manual	Railway	1930	Brown-Boveri
Commonwealth Edison Co	3	625	3,000	9,000	Manual	Railway	Being erected	Brown-Boveri
Commonwealth Edison Co	3	625	3,125	9,375	Manual	Railway	Being erected	General Electric
Consolidated Mining & Smelting Co. of Canada	2	650	6,500	13,000	Manual	Electrolytic	On order	Brown-Boveri
Consolidated Mining & Smelting Co. of Canada	1	650	6,500	6,500	Manual	Electrolytic	On order	General Electric
Delaware, Lackawanna & Western R. R. Co.	12	3,000	3,000	36,000	Manual	R. R. electrif'n	1930	General Electric
Delaware, Lackawanna & Western R. R. Co.	2	3,000	2,000	4,000	Automatic remote control	R. R. electrif'n	1930	General Electric
Detroit, City of	1	600	2,000	2,000	Automatic	Railway	On order	General Electric
Edmonton, City of	1	575	1,325	1,325	Manual	Railway	1930	Brown-Boveri
Hershey Chocolate Company	1	600	500	500	Manual	Interurban-railway	1930	General Electric
I. G. Farbenindustrie for Standard Oil Co. of Louisiana	1	3,500/9,600	2,200	2,200	Manual	Electro-chemical	Being erected	Brown-Boveri
Iowa Nebraska Light & Power Co	1	600	1,000	1,000	Manual	Railway	1930	Brown-Boveri
Italian State Railways	2	2,900	2,000	4,000	Automatic	R. R. electrif'n	1930	General Electric
Long Island Railway Co	5	650	3,000	15,000	Automatic	Railway	Being erected	Brown-Boveri
Los Angeles Railway Corp	2	600	1,500	3,000	Automatic remote control	Railway	On order	General Electric
Montreal Tramways Co	2	600	1,500	3,000	Automatic remote control	Railway	Being erected	General Electric
Montreal Tramways Co	2	600	1,500	3,000	Automatic remote control	Railway	1930	General Electric
New York Board of Transportation	10	625	3,000	30,000	Automatic remote control	Subway	Being erected	General Electric
New York Board of Transportation	13	625	3,000	39,000	Automatic remote control	Subway	On order	General Electric
Northern Indiana Public Service Co	1	1,500	3,000	3,000	Automatic	Railway	Being erected	Brown-Boveri
Paris Orleans Railway, France	1	1,500	1,500	1,500	Automatic	R. R. electrif'n	On order	General Electric
Philadelphia, City of	2	630	2,500	5,000	Manual	Subway	1930	Brown-Boveri
Public Service Co. of No. Ill., Chicago	1	600	1,900	1,900	Manual	Railway	1930	Brown-Boveri
Public Service Co. of No. Ill., Chicago	1	600	1,900	1,900	Automatic	Railway	1930	Brown-Boveri
Quebec Power Co	1	550	1,200	1,200	Automatic	Railway	Being erected	Brown-Boveri
Regina, City of	1	575	1,200	1,200	Manual	Railway	On order	Brown-Boveri
Saskatoon, City of	1	575	600	600	Automatic	Railway	1930	Brown-Boveri
Trenton Transit Co	3	600	1,400	4,200	Manual	Railway	On order	Brown-Boveri

Totals:

Put in service	33	75,225
Being installed	26*	71,400*
On order	29	79,275
Grand total for year	88*	225,900*

*In addition one spare 500-kw. rectifier tank (without transformer) was supplied.

on the Delaware, Lackawanna & Western Railroad Electrification (Fig. 16) was the outstanding event of this year. This is the first installation of 3,000-volt rectifiers in the United States and the first use anywhere of so great a capacity of such units. It is also the first instance of a major railroad electrification relying

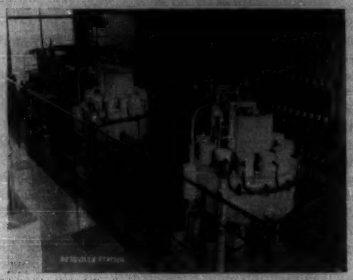

Fig. 16—3,000-Kw., 3,000-Volt, Mercury Arc Rectifiers (GE)—Roseville Substation, Delaware, Lackawanna & Western R. R. Electrification

entirely on rectifiers for the supply of power. These rectifiers are equipped with excited grids and with automatic compounding.

Another outstanding feature is the very large number of units ranging in voltage from 550 to 650 volts with a current rating in excess of 3,000 amperes, notable among which are 23 sets (GE) with an aggregate capacity of 69,000 kw. on order by the New York Board

Fig. 17—3,000-Kw., 650-Volts, Mercury Arc Rectifier (BB)—Long Island R. R.

of Transportation. (The first seven units of this order were reported last year under the heading "City Subways of New York.")

Five units (BB), having a normal rating of 4,650 amperes at 650 volts, with overload ratings up to 15,000 amperes for twenty seconds, are being erected for the Long Island Railroad (Fig. 17). These rectifiers have eighteen anodes and normally operate from a 25-cycle source of supply but are arranged for immediate throw-over to a 60-cycle source of supply in the event of failure. The transformers are also suitable for operation at either 11 or 33 kv.

A 2,200-kw., high-voltage electric furnace set is also being erected (BB). The d-c. voltage may be varied from 3,500 to 9,600 volts by means of load-ratio control on the rectifier transformer. This is the highest voltage for commercial rectifiers which has been reported in this country. Smaller units of higher voltage have, however, been used for large radio broadcasting stations in Europe.

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Electrochemistry and Electrometallurgy

ANNUAL REPORT OF COMMITTEE ON ELECTROCHEMISTRY AND ELECTROMETALLURGY*

THIS report covers the period 1929 and 1930, inasmuch as no report was made in 1929.

It is believed worthwhile to indicate the magnitude of the electrochemical and electrometallurgical industries in terms of installed capacity and yearly power consumption. The figures for the United States are as follows:

Approximate installed operating capacity, kw	7,000,000
Approximate percentage of National total generating capacity	18
Approximate annual power consumption, kw-years	4,900,000
Approximate percentage of total	36
Number of workers	1,000,000
Value of product	$12,000,000,000

Quantities and percentages such as these would seem to warrant the liveliest interest on the part of men having electrical engineering training, particularly the younger men who find themselves faced with limited opportunities for advancement in those more standardized fields of the electrical art involving the design and manufacture of apparatus for the production and distribution of electrical energy. The development of new materials or processes based upon the utilization of this power is a field requiring talent of high order and one which it seems should be very attractive to men of electrical training who can and will superpose on an electrical background a knowledge of the fundamentals of physical chemistry.

Below is given a list of the more important industries whose basic processes are dependent upon the utilization of electricity for other than mechanical purposes:

Production of Metals by Wet Electrolysis. Copper, Nickel, Sodium, Zinc.

Production of Metals from Fused Electrolytes. Aluminum, Barium, Beryllium, Calcium, Magnesium.

Non-Metallic Electrolytic Products. Caustic, Chlorine, Hydrogen, Oxygen.

Electrothermic Products. Abrasives, Carbides, Carbon, Ferro-Alloys, Graphite, Refractories.

Miscellaneous. Cleaning and Surface Treatment of Metals, Dry Cells, Electric Steel Melting, Electrolytic Synthesis Organic Compounds, Electroplating, Electrostatic Precipitation, Inductive Melting and Heating, Nitrogen Fixation, Pasteurization and Sterilization, Secondary Batteries, Water Purification, Welding.

When the process problems of these industries are analyzed it will be found that they should be attacked by men who have been trained to deal with intangibles, and that most of the phenomena may be resolved into elements among which an electrical engineer who has been soundly trained in the basic science of his art should feel at home. Pursuit of these problems will probably lead him far from consideration of the newest thing in circle diagrams or the latest refinement in slot design but he will have broader scope for his imagination and the mental technique bred by electrical training.

The foregoing comments have been made because there are many who feel that the electrical industry is more in need of increased outlet for power than of further small gains in the efficiency of electrical apparatus *per se*, and that the great body of expectant young electrical engineers must look for adequate opportunities for advancement more to the fields of utilization and electrical processes than to the conventional ones of apparatus and generation.

Below is given an outline of the more important developments in electrochemical and electrometallurgical art that have become known during the past two years.

Copper

Although copper refining is probably the largest of the electrochemical industries in point of tonnage, no important modifications of electrochemical process have been reported although operating economies have been improved by advances in mechanical equipment and materials handling facilities.

Sodium

Metallic sodium is now being produced on such a large scale and so cheaply by electrolytic methods that it is the cheapest pure metal in terms of cost per unit volume. Its electrical conductivity is high enough so that its cost on a conductivity basis is lower than that of any other metal. These facts have led to a proposal that electrical conductors be formed of metallic sodium enclosed in a thin-walled tube of some non-magnetic alloy steel.

Metallic sodium is finding increasing application in connection with certain synthetic organic reactions

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whose nature is not disclosed and certain electrochemical interests are said to be preparing to manufacture sodium on an unprecedented scale.

Zinc

The success of electrolytic zinc refining plants was noted in the 1928 report of this committee. This development has since been subject to severe competition because of improvements in thermochemical methods. Electrothermal methods have also received the attention of investigators and it is probable that another year will see marked changes in zinc technology due directly to the application of electricity in one way or another.

Aluminum

The Hoopes process of producing very pure aluminum has become of increasing commercial importance, particularly for containers for various articles in the drug and pharmaceutical field. Although the electrical conductivity is appreciably higher than that of ordinary commercial aluminum it has not been commercially applied as electrical conductor material. Accurate alloy investigations have been made possible by this very pure material and the knowledge thus gained has assisted materially in the development of the strong aluminum alloys.

The usual Hall process rarely produces aluminum purer than 99.7 per cent, while the Hoopes process is regularly producing metal better than 99.98 per cent pure and some metal has been produced by this process having a purity of 99.99 per cent. The process is carried out in a three-layer cell, wherein the metal to be purified exists as a molten anode alloy and rests on the bottom of the cell. The electrolyte consists of a mixture of cryolite, aluminum fluoride and barium fluoride. The purified aluminum is electrolytically deposited in a layer of pure molten aluminum floating on top of the electrolyte. Graphite electrodes are employed to lead the current out of the aluminum layer. The cell operates at a voltage varying from 5 to 7 volts and normally a current of 20,000 amperes is employed.

By anodic treatment in wet electrolytic cells aluminum may be given a tough, adherent and corrosion resistant surface of oxide which serves as a base for color and which is also finding use for protection of aluminum alloy aircraft structure from severe corrosion attack such as that of salt spray and sea-water. This development is still going forward rapidly and is expected to become of much greater importance in aluminum technology.

Electro-plating with aluminum on copper and other base metals has been accomplished by the use of a fused electrolyte of low melting point comprising the chlorides of aluminum and sodium. Tough, adherent, and non-porous deposits of pure aluminum are produced and many commercial uses will no doubt be found as the technique is improved.

Barium

Barium is one of the alkaline-earth elements and is characterized by great reactivity and interesting thermionic properties. It is produced by the electrolysis of fused barium salts and has been receiving considerable attention because certain of its alloys have useful thermionic properties. It has found commercial application in a nickel-base alloy for spark plug electrodes and has been of interest as an essential constituent of alloys intended for use as cathodes in thermionic devices. Such barium alloys are, in general, electric furnace products.

Beryllium

During the past few years the metal beryllium has received a great deal of attention, which has been primarily due to the results of systematic research both in this country and abroad. Beryllium may be produced by an electrolytic process which consists of the electrolysis of mixed beryllium oxyfluoride and barium fluoride. The only commercial ore of beryllium is a silicate of beryllium and aluminum. The beryllium oxide content of available ores lies between 6 and 12 per cent. The production of beryllium has been carried out both in this country and abroad.

The future of beryllium does not appear to lie in its use as a basic material, but rather as a minor constituent in more or less complex alloys.

About the only use thus far developed for the pure metal is as windows in X-ray tubes, inasmuch as beryllium is extremely transparent to X-rays. Experimental use in pistons and piston rings has been reported. Continued research will doubtlessly develop new uses for this interesting metal, inasmuch as it possesses a combination of high elastic modulus, low expansivity and lightness which is very attractive to many who are interested in mechanical developments. Its electrical resistivity is approximately 6.7 microhms per cu. cm. at room temperature.

Magnesium

This metal is produced by the electrolysis of molten magnesium chloride and more recently by means of processes in which magnesium oxide is dissolved in a bath of fused fluorides and decomposed by electrolysis to give the metal. The production of magnesium in the United States has increased from approximately 50,000 lb. in 1921 to more than 1,000,000 lb. in 1930. During 1930 magnesium forgings have appeared on the American market and with continued research it is expected that increased quantities of strong magnesium alloys in the form of castings and forgings will be rather extensively employed particularly in aircraft. With improved production technique purer metal is being produced and the susceptibility to corrosion lessened in consequence.

The outstanding difficulty in the way of increasing the use of magnesium has been the development of

suitable alloys and fabricating processes and great strides are being made in this direction.

Electrotechnology of Gases

The treatment of gaseous mixtures by electrical discharges in order to effect chemical reactions that would not ordinarily occur has received a great deal of attention. The work of Fischer and Peters in Germany is particularly interesting in that they find it possible to produce acetylene from methane with an energy consumption that compares favorably with that required by the conventional calcium carbide process. The cracking of heavy hydrocarbon vapors by corona discharge also shows some possibilities. A great deal of research is going on in connection with the chlorination of hydrocarbons such as methane, pentane, etc., and this promises a considerable expansion in the use of chlorine, which is an electrolytic product. Research investigations of the chemical effects of bombardment by high-speed electrons should also be mentioned.

Abrasives—Carborundum

Tungsten carbide cutting tools have found increasing application and this has necessitated the development of improved abrasive wheels for shaping them. As a consequence special grades of carborundum are being made, apparently of unusual purity, from which grinding wheels specially adapted to the shaping of carbide tools are made. Carborundum base resistors for high temperature electric heating furnaces have been considerably improved, and several manufacturers have designed about this type of resistor commercial furnaces for operating temperatures between 1,000 deg. and 1,400 deg. cent.

Aluminum silicate refractories of very high melting point are being fused in electric furnaces and cast into blocks and shapes for various purposes, particularly for parts for glass-melting furnaces. The melting and casting of such highly refractory substances is a new and interesting departure in industrial ceramic technology.

Cleaning of Metals

A wet electrolytic process for the removal of scale from metals which is said to be competitive with pickling and free from many of its objections has been put on a commercial basis. Hydrogen embrittlement is prevented by the automatic deposition of a thin film of lead on any area of clean metal as soon as it is exposed by scale removal. This lead film may be allowed to remain as a protection against corrosion or removed by anodic oxidation by reversal of the electrolyzing current.

Electrochemical Standards

The interest in our electrochemical standards for the international ampere and the international volt has been continued and during the coming summer cooperative measurements on these standards will be carried out at the Physikalisch-Technische Reichsanstalt. For this purpose Dr. G. W. Vinal, Chief, Section of Electrochemistry, U. S. Bureau of Standards and a member of this Committee is being sent to Germany. He will also make measurements at the Laboratoire Central d'Electricite in Paris, and the National Physical Laboratory in England.

Primary and Secondary Batteries

During the past year the American standard for dry cells has been revised and a new standard approved by the American Standards Association. This standard has been published by the Association and also by the Bureau of Standards as Circular No. 390. The technical requirements of the Government specifications are identical with those of the American standard, but the Government specification is being revised to accord with the form prescribed by the Federal Specifications Board for all Government specifications.

The Federal specifications for automotive storage batteries have been approved and published during the year. These are designated as W-B-131 and are contained in section 4, part 5 of the Federal Standard Stock Catalog.

The past twelve months have witnessed a considerable decline in the demand for primary battery products in general due to the business depression and the reduction in the number of battery operated radio sets in active use. This has naturally affected most the production of B batteries, though the production of 6-in. dry cells, many of which were also used for radio, has been seriously affected as well. The demand for flashlight batteries seems actually to be on the increase due perhaps to the appearance on the market of a rather wide variety and a considerable number of inexpensive flashlight cases which have apparently been widely bought.

The general decrease in demand for battery products has resulted in a tendency to reduce prices and on the part of at least the larger manufacturers to strengthen quality. Many of the smaller manufacturers have been forced out of the business as a natural consequence.

The year has brought the development of the industrial flashlight battery intended for use by public service corporations, railroads, theaters, and manufacturing organizations where the demand on the battery is considerably heavier than in ordinary flashlight use. At a recent conference at the Bureau of Standards two tests were standardized as representative of the industrial types of service and this has greatly facilitated and intelligently guided the development of satisfactory cells for this specific application.

Special radio B batteries of superior performance have been developed in response to demand created by radio-equipped automobiles for police squad use as well as for privately owned machines.

In the 6-in. dry cell group there has been a further advance in the life of the better telephone cells. Certain makes have reached the remarkably high figure of 360 days on the standard light intermittent test which

compares with a maximum of about 180 days ten years ago.

An interesting new development which has occurred in the primary battery field during the past year is the introduction by one manufacturer of an air depolarized portable caustic soda battery designed primarily for use as an A battery for specially designed radio receivers using the new two-volt tubes and operated entirely by batteries. It is hoped that radio receivers of this type will develop wide application in rural districts where power is not available for operating electric radio sets and where storage batteries cannot be conveniently charged. This so-called air cell radio A battery is said to be good for a year's average service with one of these receivers. The basis of its operation is the absorption by a porous carbon electrode of the oxygen from the air which is carried to the surface of the electrolyte where it acts as a depolarizer for the cell. While the principle employed is not a new one, its application to a portable practical battery is believed to be new.

During the year, the revised American Standard for Dry Cells has been published, both by the American Standards Association of 29 West 39th Street, New York City, and by the Bureau of Standards in its Circular No. 390.

In the field of storage batteries the recent publication of the Federal Specification for Automotive Batteries, designated as W-B-131, may be mentioned.

A new storage battery, the Drumm alkaline cell, which involves the use of an electrolyte of potassium hydroxide with electrodes of nickel and other substances not disclosed, is said to provide reduced weight, increased efficiency, and the possibility of recharging in a 30 minute period. The e. m. f. is 1.5 volts. Successful trials in heavy trucking service are reported from England.

Electric Melting Furnaces

The demand for better metallurgical performance of arc furnaces is bringing about refinements in control gear among which are the use of tap-changing transformers to enable more precise voltage control and a tendency toward the adoption of hydraulic drive for electrodes, in which pilot valves responsive to furnace current, voltage or kva. operate to control fluid flow to the electrode driving cylinders. More prompt response, improved sensitivity and absence of hunting are said to be provided by hydraulic operation.

Induction furnaces of the submerged ring type are finding somewhat wider application owing to improvements in refractories and in a few places abroad have been used on high-nickel alloys and for superheating cast iron for high-strength castings.

Coreless induction furnaces operating at high frequency have found commercial application in sizes up to two tons, with power ratings up to 600 kw. High-frequency generating units of 1,250 kw. are being built to supply 3½ ton furnaces. A common frequency is 980 cycles, but the trend is downward, particularly for large units. Commercial frequencies have been successfully applied and one manufacturer has successfully operated experimental four-ton units at 60 cycles, the entire kva. requirement being supplied by a motor-generator set. Considerable attention has been given to the technique of refining in these induction furnaces and some success has been attained.

Oxygen-free copper from large inductively heated furnaces is a commercial possibility of the near future. Such a material would avoid some of the weaknesses of present commercial copper and in particular its tendency to embrittlement when subject to hot reducing gases. Unusually high ductility may make it interesting for some mechanical uses.

The heat treating of steel and other alloys in controlled atmospheres by means of electric resistor furnaces has become a well established and rapidly growing practise. Initially forced by severe requirements of steel users it appears that economies may be realized which will offer further incentives to advance in this direction. Such processes are being applied to copper, brass, low-carbon and high-carbon steels, nickel, corrosion-resistant alloys, and magnetic materials over a range of temperatures extending from 500 deg. cent. to 1,200 deg. cent.

Water Purification

Electrolytic purification of water has been receiving considerable attention following the announcement, in Germany, of a process and apparatus in which water is passed through a succession of diaphram type electrolytic cells which cause concentration of ionogens in the electrode compartments. It is susceptible of large-scale operation but so far has been used mainly to complete the purification of distilled water.

Sterilization of Milk

The electropure process of milk conditioning effects pasteurization by passing the raw milk rapidly between carbon electrodes where it is heated by the passage of 60-cycle current. This scheme enables refinements in the control of the time and temperature variables that have hitherto been impracticable and the quality and uniformity of the product are thereby enhanced.

Electrostatic Precipitation

The results of test of improved hot-cathode vacuum rectifiers indicate that it will be possible to improve the operation and decrease the physical dimensions of the electrical end of precipitation plant. Increased use of powdered coal firing has caused the ash which is carried up the stack to be troublesome and electrostatic precipitator equipment is finding application in this connection. The same fundamental ideas are being applied to the separation of suspended solids from liquids. The purification of insulating oils is a case in point and it is stated that by electrostatic cleaning higher breakdown values can be obtained than by methods now in general use.

Metal Powders

Finely divided metals find numerous uses in chemical technology and as pigment in the decorative arts. Electrolytic means have been developed for producing copper, tin, nickel, silver, and lead in crystalline form in sizes as fine as 500 mesh.

Miscellaneous Process and Plant

Mercury Arc Rectifiers

There are under construction for a Canadian Plant mercury arc rectifiers rated at 6,500 kw. each (10,000 amperes, 650 volts) to be used for the electrolytic production of hydrogen.

Rotary Converters

Another electro-chemical plant has installed synchronous converters with load ratio control type transformers to furnish direct current for an electrolytic process. These machines are rated at 5,440 kw. each (16,000 amperes, 340/370 volts). These machines are also of interest because of their use of volute housings as part of the ventilation system.

Electrodeposition of Tungsten

The electrodeposition of tungsten in the wet way from alkaline baths has been accomplished on a laboratory scale. Tungsten possesses such marked resistance to acid attack and to wear that this development may prove to be of considerable industrial importance.

Heat Treating Furnaces

A marked interest was shown by the steel industry in the use of electric furnaces for heat treatment service. One plant installed two 1,100-kw. resistor type furnaces 50 tons capacity each, one 1,000-kw. similar furnace, and other furnaces, making a total of 4,500 kw.

The use of artificial atmospheres in electric furnaces for heat treatment processes is making considerable progress. One steel plant has installed 2,000 kw. in furnace capacity for bright annealing of steel strip.

Electrically heated and controlled apparatus for controlling the composition of the gases used are coming into evidence.

The special requirements of the nitriding process which involves heating to approximately 550 deg. cent. articles of special alloy steel (containing small amounts of Cr, Al, Mo, and Ni) in an atmosphere of ammonia has led to the development of furnaces with special provisions for circulating the ammonia. Heat transfer is effected by forced convection instead of by radiation. It appears possible that the application of this principle will find extended application in special-purpose furnaces in which control of atmosphere is important.

In both the heat-treating and melting fields the refinements of metallurgical technique and the growing recognition of the value of high quality and uniformity of product have brought about such profound changes in the design and operation of furnaces that they are coming to resemble machine tools rather than the piles of brick that the word usually calls to mind.

Conclusions

It is believed that this report reflects the more important trends in electrical process technology and hoped that it may draw the attention of electrically trained men to the very interesting possibilities in the field of the development of processes and materials dependent upon the forces that only electricity can bring into play.

The Chairman wishes to acknowledge the assistance given by members of the Committee and by numerous individuals in the preparation of this report which is respectfully submitted on behalf of the Committee on Electrochemistry and Electrometallurgy.

Discussion

Lawrence Addicks: There is no need to confine the list of metals produced (by electrochemical means) to copper, nickel, sodium and zinc. Most of the silver and gold and a large minority of the lead is so produced.

An important new use of calcium is well under way, namely its use to displace bismuth from lead. Much work is being done on a cheap method of producing the lead-calcium alloy used as a reagent and one of the most promising starts with calcium carbide, an electrochemical product. One of the more expensive methods was by way of Frary metal, another electrochemical product.

Electrochemistry is about one-third each, chemistry, physics, and electrical engineering. Very few are proficient in all three.

J. Lester Woodbridge: I have been able to obtain considerably more detailed information in regard to the Drum alkaline storage battery than is contained in this report. It appears that the positive plate used in the tests in electric traction service conducted under the supervision of the Irish Free State last summer was of the standard nickel oxide type. The negative plate consisted of the standard nickel plated negative grid used in the nickel alkaline battery into which metallic zinc was plated from the electrolyte which consisted of a solution of potassium hydroxide and a soluble compound of zinc.

I have this information on quite reliable authority and believe it to be correct.

Magnus Unger: Mention should be made of the development of fused quartz mirrors for observatories, work being done by the Thompson Research Laboratory of the General Electric Company.

Electrophysics

ANNUAL REPORT OF THE COMMITTEE ON ELECTROPHYSICS*

THE annual report of the Committee on Electrophysics has generally taken the form of an account of the progress in the field of electrophysics during the preceding year. In view of the steady increase of the activity in this field the matter to be covered has increased greatly, and the report for the year 1929-1930 was a lengthy one in spite of all efforts to keep its size down to a minimum.

At a meeting of the committee held January 30, 1931 its field of activity and in particular its annual report were discussed at considerable length. It was the sense of the committee that it was one of the most important of its functions to bring to the attention of electrical engineers the progress being made in electrophysics. It was felt very strongly, however, that this cannot be done in the best possible fashion by reporting the progress for a year at one time and publishing it as a unit. Accordingly, the committee decided that it could be of much greater service to the engineering profession if its reviews of the progress in electrophysics were published from time to time by subjects instead of annually as a unit. As a result the plan was adopted of publishing reviews of the progress of electrophysics by subjects in ELECTRICAL ENGINEERING. Each contribution will aim to cover the progress in its particular field from the date of the preceding report as nearly up to the date of publication as is possible. A tentative schedule of the proposed subjects is as follows:

Magnetism and Magnetic Materials.
Propagation of Electric Waves.
Electric Discharges in Gases.
Dielectrics.
Thermionics and Photoelectricity.
Vacuum Gas Discharge and Photoelectric Tubes.
Electric Conduction in Solids.
Electromagnetic Theory.

In addition the Committee plans to sponsor from time to time for publication in ELECTRICAL ENGINEERING papers by recognized authorities on various subjects in electrophysics of interest to electrical engineers. One such paper is published herewith. After all the reviews for any one year have been published, they may be collected and published together in the TRANSACTIONS if it should be considered desirable.

*COMMITTEE ON ELECTROPHYSICS:
Oliver E. Buckley, Chairman.

V. Bush,	W. B. Kouwenhoven,	Leigh Page,
W. G. Cady,	G. M. J. Mackay,	W. S. Rodman,
W. F. Davidson,	K. B. McEachron,	J. Slepian,
C. L. Fortescue,	L. W. McKeehan,	W. F. G. Swann.
W. S. Gorton,	H. Nyquist,	

Magnetism and Magnetic Materials

The following review of progress in magnetism and magnetic materials covers the period since the last report of the Electrophysics Committee. The progress in other phases of electrophysics will be covered similarly in succeeding reviews. The Committee also may offer from time to time papers on subjects in electrophysics of interest to electrical engineers.

MAGNETISM

Our understanding of ferromagnetism has advanced during the period since our last report (JL. A. I. E. E. **49**, 721, 1930) in several important respects. Heisenberg's interaction theory, which refers the stability of magnetization to the low potential energy of codirected electron spins in neighboring atoms, has been applied by Powell (*Proc. Phys. Soc.* **42**, 390, 1930) to explain the change in dimensions which take place when iron and nickel lose their ferromagnetism on heating. This change, a decrease in the case of iron, an increase in the case of nickel, is isotropic, being thus wholly unlike the principal part of magnetostriction which differs greatly in different crystallographic directions. It is concluded from the fact that both signs of change occur that some account must be taken, at least in the case of iron and probably in all cases, of atoms more remote than the immediate neighbors. Becker and Kirsten (*Zeitsch. f. Physik* **64**, 660, 1930) who identify the stabilizing forces with internal elastic strains, have succeeded in getting rather good agreement between the permeability of highly stressed nickel as measured and as calculated from limiting magnetostriction and the elastic moduli. Rankin (*J. Roy. Tech. Coll.* Glasgow **2**, 385, 1931) has continued his experiments on magnetostriction in overstrained metals. Akulov has continued his discussions of the relations between magnetostriction, magneto-resistance and elasticity concluding in his latest papers (*Zeitsch. f. Physik* **67**, 794, 1931; *Phys. Zeits.* **32**, 107, 1931) that the magnetic behavior of single crystals is fully explained by a dual process of reversals and rotations of the magnetic elements and that the magnetic properties of polycrystalline metal *without large scale anisotropy* can be calculated from data obtained for monocrystals. McKeehan (*Rev. Mod. Phys.* **2**, 477, 1930; **3**, 190, 1931) has reviewed the content and relations between the current theories of ferromagnetism and some of the recent experimental evidence pertinent thereto.

Several papers, in addition to that of Powell, already referred to, deal with the experimental and theoretical

aspects of the Curie point. Tyler (*Phil. Mag.* [7] **11**, 596, 1931) shows that the decrease of apparent saturation values with rise of temperature is practically according to the same law for iron, cobalt, and nickel, and that this is consistent with the idea that in all three metals single spinning electrons are free to change their axial direction, although in all these cases the average number of such magnetizable electrons per atom is greater than one. According to these data, therefore the ferromagnetic unit is sub-atomic rather than atomic. Above the Curie point it appears (Stoner, *Phil. Mag.* [7] **10**, 27, 1930) that atoms begin to act as units. Forrer and Hoffmann (*Comptes Rendus* **191**, 1046, 1930) present new evidence for a doubling of the (ferromagnetic) Curie point in nickel, the coercive force first decreasing, then increasing, and finally decreasing again as the temperature rises. An apparent disagreement between two experimenters, Weiss and Bates, in regard to the temperature at which the specific heat anomaly becomes greatest as the Curie point is approached has been dissolved by new evidence. Bates (*Proc. Phys. Soc.* **42**, 441, 1930; **43**, 87, 1931) now also finds this temperature to be where $-d\,I^2/d\,T$ is maximum; his earlier experiments were vitiated by an irreversible change in the properties of his material (manganese arsenide) produced by the small rise in temperature (magneto-caloric effect) associated with applying the magnetizing field. Incidentally, the specific heat anomaly is here far too large to be ascribed to an elementary magnet like that in iron or nickel.

The anomalous temperature coefficient of electrical resistance for ferromagnetics near their Curie points is closely associated with their magnetizability. Gerlach (*Zeitsch. f. Physik* **59**, 847, 1930; *Ann. der Physik* [5] **6**, 772, 1930) and Borelius (*Ann. der Physik* [5] **8**, 261, 1931) present opposing opinions regarding the anomaly. The former in agreement with earlier work of Cabrera regards the high-temperature state (above the Curie point) as normal and the low-temperature state as possessing an abnormally low resistance, the latter supports the contrary view that it is the low-temperature or completely magnetized state which has normal resistance. The transition as the Curie Point is approached from below is the same whichever view is taken.

The closely related subject of heat production during magnetization and demagnetization has been examined in a preliminary way by Gilbert (*Comptes Rendus* **191**, 1309, 1930) who finds no cooling in cobalt steel at H up to 18,000 except that attributable to the magneto-caloric effect of Weiss. The more complete report of Ellwood's results (*Phys. Rev.* [2] **36**, 1066, 1930) confirms his earlier finding of a cooling in carbon steel at low values of H on the ascending branch of a loop, and announces a second stage of cooling at moderate values of H. It should be noted that the magneto-caloric effect would account only for a minute heating in this part of the loop.

An effort has been made by Bitter (*Phys. Rev.* [2] **37**, 91, 1931) to coordinate ferromagnetic behavior with the small scale imperfection of metallic crystals recently predicted by Zwicky (*Helvetica Physica Acta* **3**, 269, 466, 1930; **4**, 49, 1931). Frenkel and Dorfman (*Nature* **126**, 274, 1930) deal with the same problem on purely thermodynamic principles, deciding that a large crystal cannot act as a magnetic unit.

Experiments on the Barkhausen effect in a rotating magnetic field (Cisman, *Ann. der Physik* [5] **6**, 825, 1930), make it more plausible than ever to suppose that the occurrence of a discontinuity in magnetization requires a definite degree of divergence between the applied field and the local magnetization. There is, in fact, a decided parallelism between Cisman's Barkhausen effects and hysteresis in rotating fields, most recently studied by Seiyama (*Mem. Kyushu Univ. Coll. Engg.* **5**, 227, 1930). Heaps and Bryan (*Phys. Rev.* [2] **36**, 326, 1930) have shown that large changes in magnetization and in magnetostriction are simultaneous. Bozorth has just discovered (*Nature* **127**, xxx, 1931) that in a very slowly changing magnetic field the Barkhausen discontinuities which can be detected by a high-speed oscillograph occur in closely grouped sets with quiet intervals between them. These compound events, in proximate analysis, can easily be mistaken for single events. This apparently explains the wide disagreement between published statements as to the volume of metal affected in a single discontinuity. The time required for the spreading of a major magnetic disturbance has been studied by Sixtus and Tonks (*Phys. Rev.* [2] **36**, 1441, 1930).

The gyromagnetic effect, hitherto only observed in ferromagnetic materials—most recently by Barnett (*Phys. Rev.* [2] **36**, 789, 1930)—has now been measured for a paramagnetic material by Sucksmith (*Proc. Roy. Soc.* [*A*] **128**, 276, 1930; *Proc. Phys. Soc.* **42**, 385, 1930). The phenomenon is merely easier to observe in ferromagnetics on account of their easier magnetization.

In studies on diamagnetic and weakly paramagnetic metals the theory has been improved by Landau (*Zeitsch. f. Physik* **64**, 629, 1930) and Teller (*Zeitsch. f. Physik* **67**, 311, 1931) so that better predictions are now possible as to the diamagnetism to be expected from conduction electrons. On the experimental side relatively enormous changes in susceptibility—much larger than the changes in conductivity make probable—have been reported by Honda and Shimizu (*Nature* **126**, 990, 1927) and by Bitter (*Phys. Rev.* [2] **36**, 978, 1930), both of whom measured the susceptibility of copper as affected by cold work and annealing. Banta (*Phys. Rev.* [2] **37**, 634, 1931) has attempted without success to duplicate Bitter's findings.

Spectroscopic evidence that some atomic nuclei have a small magnetic moment continues to accumulate (Fermi, *Zeitsch. f. Physik* **60**, 320, 1930; Goudsmit and Young, *Phys. Rev.* [2] **35**, 1418, 1930; Frisch and Ferchmin, *Naturwiss*, **18**, 866, 1930). The effect upon gross magnetic properties of such nuclear magnetic

moments as have yet been shown to exist must be very small.

Effects of magnetic fields on electrical resistance and electric potential with or without the presence of thermal currents and temperature differences have been reviewed on the basis of the new electron theory of metals by Sommerfeld and Frank (*Rev. Mod. Phys.* **3**, 1, 1931). The proper mode of description of these galvanoelectric, thermoelectric, galvanomagnetic and thermomagnetic effects have been discussed by Perrier (*Helvetica Physica Acta* **3**, 317, 400, 1930). The conditions are especially complicated in the case of ferromagnetics. Pugh (*Phys. Rev.* [2] **36**, 1503, 1930) has shown for iron that the magnetization, I, or the ferromagnetic induction, B-H, is more important than either H or B in fixing the magnitude of the Hall e. m. f. Stierstadt (*Zeitsch. f. Physik* **65**, 575, 1930; **67**, 725, 1931) has studied magneto-resistance in electrolytic iron for both longitudinal and transverse magnetizations. Williams and Sanderson (*Phys. Rev.* [2] **37**, 309, 1931) present new data on magneto-resistance in nickel strips of different mechanical history.

The magneto-resistance of bismuth single crystals at extremely low temperatures, below 20 deg. K, has been studied in detail by Schubnikow and de Haas (*Nature* **126**, 500, 1930; *Proc. Amsterdam Acad.* **33**, 363, 418, 1930), who find a whole series of unexpected phenomena. Unlike the magneto-resistance at higher temperatures the relative change in resistance is no longer a smoothly varying function of the applied field, H, nor of the azimuth in any crystallographic plane yet examined. At higher temperatures, however, theory has caught up with experiment, for Frank (*Zeitsch. f. Physik* **64**, 650, 1930) has shown that the straightening out of $\Delta R/R$ *vs.* H curves at high H values, first observed by Kapitza, is completely accounted for by the quantum theory of electronic conduction. The parabolic increase of $\Delta R/R$, formerly supposed to be called for by the theory, is only possible for relatively limited ranges in H.

One of the most puzzling experimental results recently published is that of Allison and Murphy (*Am. Chem. Soc. Jl.*, **52**, 3796, 1930), who report that the magnetic double refraction (Faraday Effect) in certain solutions is characteristic, not only of the elements present but even of the number of isotopes of each, and furnishes a highly sensitive means for chemical analysis.

In the domain of magnetometry Sanford has been investigating magnetic permeameters, particularly those for use at high inductions (*Bu. Std. J. Res.* **4**, 177, 703, 1930; **6**, 355, 1931). A new level of precision in magnetic curve tracing has been attained by Haworth (*Bell System Tech. Jl.* **10**, 20, 1931; *Rev. Scient. Instr.* [N. S.] **2**, 125, 1931) who compensates the torque of a fluxmeter suspension by a photoelectric current depending upon the momentary deflection. Brüche (*Zeitsch. f. tech. Physik* **12**, 94, 1931) has considerably refined the methods for exploring magnetic fields by narrow beams of cathode rays. Freed and Kasper (*Phys. Rev.* [2] **36**, 1002, 1930) have improved the accuracy of Gouy's method for measuring the susceptibility of weakly magnetic solutions.

A stimulating book by S. R. Williams[1] gives a readable account of the experimental side of the subject of magnetism, with especial emphasis on ferromagnetism.

An important announcement regarding magnetic units as adopted by the International Electrotechnical Commission was made by Dr. Kennelly at the Winter Convention in New York. The electrophysics committee is by no means unanimously in accord with the suggestions of the international body. Some of our members feel that the already existing confusion in regard to the physical significance of B and H is increased rather than diminished by giving physical dimensions to their ratio. It is interesting to notice that a body of German engineers went even farther than the I. E. C. in the effort to rationalize empirical relations (*Elektrotech. u. Maschinenbau* **48**, 950, 1930).

Magnetic Materials

The results of many investigations carried out in the past three or four years confirm the view that crystal grain size is one of the important factors in magnetic materials. Yensen (*Metals and Alloys*, **1**, 493-495; 1930) has proposed an expression for the hysteresis loss in iron in terms of the crystal grain size and impurities in the metal, leading to the conclusion that for perfect single crystals, free from impurities and internal strain, the hysteresis loss vanishes. Gerlach has more recently arrived at a similar conclusion. (*Zeitsch. f. Physik*, **64**, 502-506; 1930). On the other hand Von Auwers and Sizoo (*Zeitsch. f. Physik*, **60**, 576-580; 1930) conclude that crystal grain size has much less influence on remanence than on hysteresis loss and coercive force.

What promises to be of great importance is the announcement by Cioffi (*Nature*, **126**, 200-201; 1930) that ordinary polycrystalline iron may be made to acquire very high permeability by heat treating at high temperature in hydrogen. The reported initial and maximum permeabilities are 6,000 and 130,000, respectively, and the coercive force and hysteresis loss are respectively 0.05 gauss and 300 ergs/cu. cm./cycle, for $B_m = 14{,}000$. Later unpublished values give 180,000 for the maximum permeability, 0.025 gauss for the coercive force and 190 ergs/cu. cm./cycle for the hysteresis loss for the same maximum induction. These properties are ascribed partly to purification of the iron and partly to the hydrogen actually absorbed in the metal. Further correlation appears to be necessary between crystal grain size and magnetic properties in view of these results, since the grain size of this material appears to be of little importance.

The existence of the A_2 point in carefully purified iron occurring reversibly between 907 deg. cent. and

1. "Magnetic Phenomena," McGraw-Hill, 1931.

910 deg. cent. has been confirmed by Roberts and Davey (*Metals and Alloys* **1**, 648-654; 1930), by X-ray crystallographic determinations. This fact has recently been a matter of some dispute (Yensen, A. I. M. & M. E., *Inst. Metals Div.*, 320-332; disc. 332-349; 1929).

The increasing importance of nitrided steels has led to further investigations of the iron-nitrogen system. Qualitative and quantitative measurements by Lehrer (*Zeitschr. f. Electrochem.*, **36**, 460-473; 1930) by means of magnetization-temperature curves have been used to construct the equilibrium diagram for the iron-nitrogen system. The results show that up to 11 per cent nitrogen there are four phases, in agreement with the X-ray determinations of Hägg (*Zeitsch. f. Physik. Chem.* **8**, 455-474; 1930) and Eisenhut and Kaupp (*Zeitschr. f. Electrochem.* **36**, 392-404; 1930). Köster (*Archiv. f. Eisenhüttenwesen* **3**, 637-648; 1930) has shown that nitrogen in α-iron remains in supersaturated solution even after slow cooling, requiring prolonged heating at low temperatures for complete separation, with a resultant increase in conductivity and increase in coercive force from 4 to 7 gauss. The segregation of nitrogen in the supersaturated solution is further accelerated by cold working (*Archiv f. Eisenhüttenwesen*, **3**, 649-658; 1930).

A number of alloys of iron and non-magnetic elements has been investigated. Pure electrolytic iron having a trace of phosphorus has been found to have better magnetic properties than commercial silicon steel. (*Gayler, Metallwirtschaft*, **9**, 677-679; 1930.) The iron-silicon system has been studied by Haughton and Becker, (*Iron and Steel Inst. Jl.*, **121**, 315-335; 1930) using metals of higher purity than heretofore. The existence of phases corresponding to the compounds FeSi, and Fe_3Si_2 have been confirmed, and in addition a phase which is probably the compound Fe_2Si_3. The addition of 14.5 per cent silicon lowers the Curie point of iron to 490 deg. cent. A magnetic transformation point occurring at 82 deg. cent. in the Fe_3Si_2 phase confirms the earlier work of Murakami (*Tohoku Imp. Univ. Sci. Rep.*, **10**, 79-92, 1921). Oya (*Tohoku Imp. Univ. Sci. Rep.* **19**, 235-245; 1930) has found that additions of vanadium to iron first raise the Curie point then depress it, so that it is room temperature for 35 per cent vanadium. The alloys of iron and chromium with approximately 18 per cent chromium have been found to have zero magnetostriction (Dean, *Rensselaer Polytech. Inst. Bull. No. 26*, Eng. and Sci. Series; 1930). Fischer (*Rensselaer Polytech. Inst. Bull. No. 28*, Eng. & Sci. Series; 1930) has found that this range of alloys has higher permeability and is less sensitive to quenching strains than other alloys of this group. This behavior has already been observed in permalloy (Buckley and McKeehan, **26**, 261-273; 1925) and is in accordance with McKeehan's theory of magnetostriction (*Phys. Rev.* **28**, 158-166; 1926). Kussmann, Scharnow, and Messkin (*Stahl u. Eisen*, **50**, 1194-1197; 1930) have found that additions of copper up to 0.7 per cent increase the resistance of iron to corrosion without impairing its magnetic and electrical properties. A general deterioration of these properties is obtained for higher copper concentrations.

The relation of magnetic properties to mechanical hardness has been investigated by Messkin (*Stahl u. Eisen*, **50**, 105-106; 1930) who has found that cold rolling increases the remanence and coercive force of carbon steel; and by Herbert (*Proc. Roy. Soc.*, **130**, 514-523; 1930) who has observed age hardening of steel and non-magnetic metals subjected to a changing magnetic field.

Sachse and Hasse have found that (*Zeitsch. f. Physik Chem.*, [A] **148**, 401-412; 1930) regular crystals of ferromagnetic ferric oxide, Fe_2O_3, are unstable at all temperatures, the susceptibility falling by 60 per cent in 4 years at room temperature. The fact that ferric oxide has 10^{-10} the conductivity of the ferromagnetic metals, leads Sachse (*Zeitsch. f. Phys. Chem.*, [B] **9**, 83-91; 1930) to conclude that free electrons cannot be concerned in its magnetic properties. The ferrites of magnesium, lead, copper, and nickel are also ferromagnetic at ordinary temperatures, cadmium ferrite being sometimes ferromagnetic and sometimes paramagnetic (Holgersson and Serres, *Comptes Rendus*, **191**, 35-37; 1930). In mixtures of Fe_2O_3, NiO, and C_0O, Veil (*Comptes Rendus*, **190**, 181-183; 1930) has found a higher coefficient of magnetization for the composition represented by $(Fe_2O_3)_2$ $NiOC_0O$.

A somewhat higher Curie point has been obtained for nickel by Jordan and Swanger (*Bur. Std. Jl. of Res.* **5**, 1291-1307; 1930) which is believed to be due to the greater purity of their nickel specimen. Sadron has found that (*Comptes Rendus*, **190**, 1339-1340; 1930) small additions of chromium to nickel, causes the saturation value at absolute zero to diminish linearly with chromium content, the Curie point for an alloy with 12 per cent Cr being absolute zero.

Widespread interest in the nickel-iron alloys coming under the collective name of permalloy is indicated by its numerous modifications to meet specific needs. Elmen has obtained very high initial permeability and resistivity in nickel-iron alloys by additions of molybdenum, chromium or tungsten (U. S. Pat. 1,757,178; *Jl. Frank. Inst.* **207**, 583-618; 1929). Stäblein (U. S. Pat. 1,760,326) has obtained high resistivity by additions of silicon. Bandur (U. S. Pat. 1,743,089) and Yensen (*Elec. Jl.* **27**, 214-218; 1930) report that with suitable heat treatment some nickel-iron alloys may be made to acquire sensibly constant permeability over a limited range of induction. Similarly, Smith and Garnett (U. S. Pat. 1,746,500) have obtained substantially constant permeability in nickel-iron alloys by additions of copper. Also, Gumlich, Steinhaus, Kussmann and Scharnow (*E. N. T.*, **7**, 231-235; 1930) report that high resistivity and small variation of permeability with magnetizing force are obtained in nickel-iron alloys by small additions of manganese.

Von Auwers, (*Wiss. Veroffentlich Siemens-Konzern* **9**, 262-293; 1930) has studied the effect of strain on the

magnetic properties of permalloy. The results of numerous experiments indicate that the properties of permalloy cannot be accounted for on the basis of strain alone.

Methods of design and testing of permanent magnets have kept abreast with the improvements in magnetic properties. Elenbaas (*Physica*, **10**, 273-286; 1930) has derived a formula for calculating the field intensity in a toroid with an air-gap, based on the assumption that the demagnetization curve is an ellipse; this should prove useful in the design of permanent magnets. It is well known that the magnetization of a ferromagnetic material by a steady field is considerably increased by the superposition of an alternating field. Schrankow and Janowsky (*Zeitsch. f. tech. Physik* **11**, 429-432, 1930) apply this method of bringing permanent magnets to high inductions. An accurate method of testing bent permanent magnets has been described by Webb and Ford (*I. E. E. Jl.*, **68**, 773-778; 1930) whereby both B and H are measured by means of search coils and a ballistic galvanometer. The superior properties of cobalt-steel as permanent magnets have found application in the magnetic compass for aircraft, disturbances from stray magnetic fields being avoided by shielding with hipernik, a nickel-iron alloy of high permeability (J. R. Gier, *Elec. Jl.*, **27**, 114-115; 1930).

The need for accurate and constant substandards of frequency has stimulated further work on vacuum tube oscillators controlled by the resonant magnetostrictive vibrations of magnetic materials. This method of control requires the use of materials which have good magnetostrictive properties and small temperature coefficients of permeability and expansion. Vincent (*Proc. Phys. Soc.* **43**, 157-165; 1931) has extended the range of the magnetostriction oscillator of Pierce, (*Proc. Am. Acad. Sci.*, **63**, 1-47; 1928) to radio frequencies by using corronil, a nickel-copper-manganese alloy, and glowray, a nickel-iron-chromium alloy (*Proc. Phys. Soc.* **41**, 476-486; 1929); the latter, particularly, has a high degree of stability. Smith (*Proc. Phys. Soc.* **42**, 181-189; 1930) has determined the nature and magnitude of the coupling between the mechanical vibrator and the electric circuit for a toroidal specimen of a nickel-iron alloy vibrating radially in its fundamental mode. Muzzey (*Phys. Rev.* **36**, 935-947; 1930) has used the magnetostriction oscillator as a means of exciting cylinders of stainless steel in studying the dependency of frequency of vibration on length and diameter of the cylinders.

ACKNOWLEDGMENT

The Committee is indebted to Professor L. W. McKeehan, of its membership, and to Mr. P. P. Cioffi for material for this report.

General Power Applications

ANNUAL REPORT OF COMMITTEE ON GENERAL POWER APPLICATIONS*

THE principal activity of the Committee during the past year has been in arranging and carrying out of plans for the Industrial Session held at the Winter Convention. As this Committee is interested in the application of electricity to all industries which are not specifically covered by other committees of the Institute, it was agreed that papers to be of interest to the greatest number of engineers should be of a general nature and not confined to a specific industry or application. The papers presented were consequently arranged and prepared with this thought in mind. During this period of industrial depression an increasing amount of attention is being given to the subject of reducing manufacturing costs and it was hoped that industrial engineers would find in these papers various ideas which could be applied to their own problems.

Industrial Papers Presented

Besides those papers presented at the Winter Convention several other papers of an industrial nature were presented at district meetings. Following is a list of those papers which have come to our attention:

Philadelphia Meeting Oct. 13 to 15, 1930:

Air Conditioning in Industry, by A. H. Clogston of the Cooling and Air Conditioning Corp.

A New System of Speed Control—A-C. Motor-Driven Power Station Auxiliaries, by A. M. Rossman of Sargent and Lundy, Inc.

Louisville Meeting, Nov. 19 to 22, 1930:

Electricity in the Cement Industry, by R. H. Rogers of the General Electric Company.

Electric Power in the Lumber Industry, by A. H. Onstad of Weyerhaeuser Timber Co.

Winter Convention, January 26 to 30, 1931:

Electron Tubes in Industry, by W. R. King of General Electric Co.

Automatic Regulators in Industry, by J. H. Ashbaugh, Westinghouse Electric and Manufacturing Co.

Electrical Distribution Systems for Industrial Plants, by W. J. McClain, of Louis T. Klauder Co.

The Synchronous Motor with Phase-Connected Damper Winding as a Drive for High-Torque Loads, by M. A. Hyde, Jr., Westinghouse Electric and Manufacturing Co.

The Design and Application of Synchronous Motors to Meet Special Requirements, by D. W. McLenegan and A. G. Ferriss of General Electric Co.

Pittsburgh Meeting, March 11 to 13, 1931:

Conversion and Distribution of General Purpose D-C. Power in Large Industrial Plants, by R. D. Abbiss, Carnegie Steel Co. and D. C. West, Westinghouse Electric and Manufacturing Co.

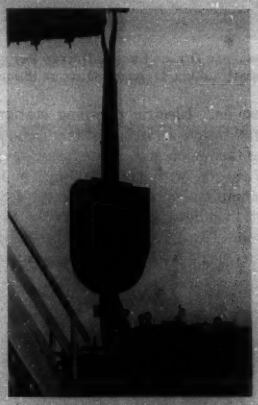

Fig. 1—Combination Pumping Unit for Oil Wells Including Motor, Gears, and Control

Symposium on *Interconnection Between Utilities and Industries*. Several papers.

The Use of Electricity in Large Annealing Furnaces, by J. C. Woodson, Westinghouse Electric and Manufacturing Co.

A Modern Electrified Dairy Plant, by A. J. Dreux, Reick-McJunkin Dairy Co. and H. C. Brunner, Westinghouse Electric and Manufacturing Co.

Review of Industrial Progress

Since the past year has not been one of great industrial development or activity, it is felt that this report

*COMMITTEE ON GENERAL POWER APPLICATIONS:

C. W. Drake, Chairman.

R. A. Armstrong,	Fraser Jeffrey,	D. M. Petty,
James Clark, Jr.,	A. M. MacCutcheon,	F. O. Prior,
J. F. Gaskill,	H. A. Maxfield,	H. W. Rogers,
Clyde D. Gray,	John Morse,	L. D. Rowell,
John Grotzinger,	N. L. Mortensen,	M. R. Woodward,
	A. M. Perry,	

should include not only a review of the major industrial developments but also should indicate the trend of development along both application and apparatus lines in industry.

Petroleum Industry

During the past year pipe-line transportation of liquid fuel over long distances has been extended to

Fig. 2—Portable Diesel Engine-Driven Set for Variable Voltage Oil Well Drilling Outfits (Shop View)

include gasoline. Electric pumping stations supplied with central station service, are increasing in number and proving their economy. The latest development is an electric station with three 800-hp. pumping units which is completely controlled from a dispatching office about 15 miles away. This is the Sand Springs Station of the Texas Empire Pipe Line that connects Oklahoma oil fields with the Chicago refineries. The

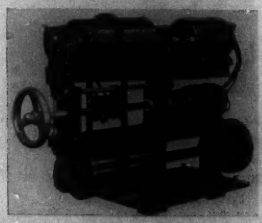

Fig. 3—Selsyn Generator and Cone Pulleys for Sectional Paper Machine Drive

supervisory control permits the dispatcher to exercise complete control over the station and gives a visual indication of its operation. The dispatcher can tell at a glance what units are operating, what the intake and discharge pressures are and as much other pertinent information as could be obtained if the operator were in the station. In case of trouble the equipment is shut down automatically and the dispatcher advised of the nature of the trouble.

Development in electrical oil field pumping and drilling equipment has been towards the use of combination, or factory assembled apparatus which eliminates many parts and reduces the installation expense in the field. One of the latest pumping units consists of a combination of gear, motor, and control built in one unit. The equipment is weatherproof and requires no housing. The unit is also provided with change gears for obtaining several speeds as required for various pumping conditions.

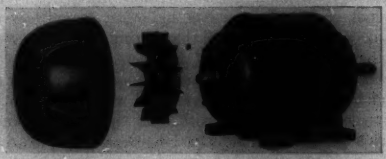

Fig. 4—Totally Enclosed Fan-Cooled Motor with Fan and Fan Casing Removed

During the past year several variable voltage d-c. oil well rotary drilling outfits have been built and put in service, using Diesel engines as prime movers. These outfits are being used primarily on locations where central station power is not available. The ease and simplicity of control obtainable with the variable voltage d-c. system, its low energy requirement and flexible operating characteristics, make this system very desirable for this type of oil service.

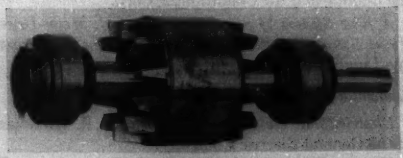

Fig. 5—Induction Motor Rotor Showing Cartridge Type Ball-Bearing Housings

Two installations are also reported where standard a-c. drilling equipments were installed and supplied with power from Diesel engine-driven generators, and, although this system may lack some of the desirable features of the variable voltage system, it makes it possible to use standard a-c. motors for pumping and other uses.

Pulp and Paper Industry

Sectional paper machine drives were installed during the year on a wide variety of machines from low-speed cylinder to high-speed fourdrinier and the use of section regulators of the carbon pile type instead of the contact

making type was quite noticeable. A description of some of these carbon pile regulators is given in Mr. Ashbaugh's paper, presented at the Winter Convention.

A regulator for sectional drives using Selsyn equipment so that the main regulating equipment may be mounted remote from the paper machine has been placed in operation during the year. At the paper machine, in connection with each sectional motor, is a Selsyn generator driven through suitable cone pulleys which provide necessary speed adjustment or "draw" between the various sections. The resistance elements of the regulator are of the carbon pile type.

Rural Electrification

The use of electricity on the farm has increased rapidly during the past year and reports from central stations indicate that approximately 90,000 farms were

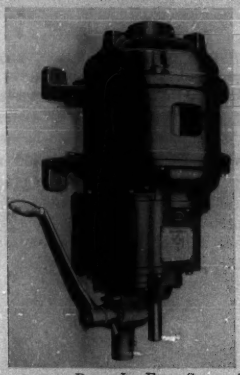

Fig. 6—Motor with Built-In Four-Speed Transmission Showing Speed Changing Lever

connected to their lines during the year. This represents an increase of 16 per cent over the total number previously connected. This growth is accounted for partly by the interest of the power companies in the farm load and partly by improved rate structures and power schedules. But of fully as great importance however is the increasing list of electrical equipment which is available and specially adapted for farm service. The attention being given to the production of milk with a low bacteria count has done much to force the installation of dairy sterilizing and dairy refrigerating equipment and these devices in connection with other dairy and household equipment make a very desirable electrical load.

Engineering research and investigation have proven the possibility of using 5-hp. electric motors for practically all stationary work such as silo filling, feed grinding, wood sawing, etc., instead of using 15- or 20-hp. tractors. Numerous improvements have recently been made in the design and construction of portable motor equipment for such service which includes automatic linestarters with thermal overload protection, effective methods of belt tightening, and other desirable features.

Food Products Industry

Although this industry has received but little attention from electrical engineers, it is the fourth greatest user of electric motors on the basis of connected horse-

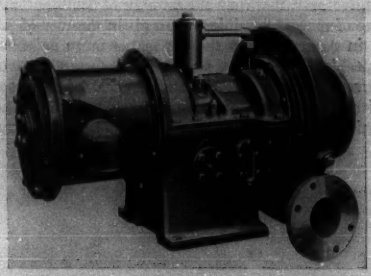

Fig. 7—Geared Centrifugal Refrigeration Compressor Driven by Totally-Enclosed Induction Motor

Fig. 8—300-Hp. 164-R. P. M. Synchronous Motor of the Phase-Connected Damper Type as Installed in a Cement Plant

power. In addition to motors, electric energy is used for various other purposes as the production of ultraviolet rays to help preserve or increase the vitamin content of foods; photoelectric tubes for actuating operations of wrapping, sorting, and counting; cooking of foods by various electrical processes, and the pasteurization of milk.

A paper on the subject of *A Modern Electrified Dairy Plant* was presented at the Pittsburgh District Meeting of the Institute March 11-13, 1931, by Mr. A. J. Dreux and Mr. H. C. Brunner. This paper

illustrates how a modern electrified dairy offers a very desirable type of load for central stations, particularly when using the electrical conductivity method of pasteurization.

Induction and D-C. Motors

During the past year several new lines of induction motors have been placed on the market and the trend is toward simplification and standardization of design. The greater use of standardized parts reduces the stocks which must be carried both by the user and manufacturer, increases the number of combinations or types of motors possible with a given number of parts and in general makes the apparatus more universally adaptable.

Several new lines of totally enclosed fan-cooled squirrel-cage induction motors have been announced and in practically all cases these motors are on the

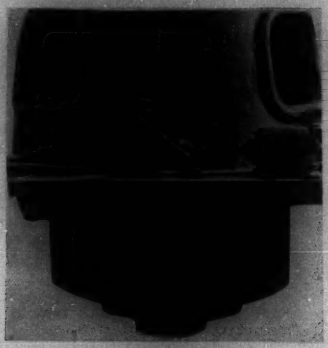

Fig. 9—Special Synchronous Motor with External Revolving Field, Direct Connected to Gyratory Crusher

same frame size as corresponding ratings of open motors. In these fan-cooled motors ball or roller bearings are used almost without exception and the cartridge type of bearing housing is quite generally adopted. This construction permits the dismantling of the motor and the removal of the rotor without exposing the bearings.

A number of manufacturers has also brought out lines of totally enclosed fan-cooled motors for use in locations where explosive gases are present. These motors are approved by the Underwriters Laboratory for use in gasoline refining, garages, gasoline handling, and filling stations, dry cleaning establishments, chemical plants, paint factories, and in similar locations.

To meet the requirements of testing large vertical pumps and turbine wheels a vertical dynamometer rated at 300 hp. has been developed. The stator is supported in such a manner that it is free to rotate and the torque developed is measured on a beam and dial scale. The direct-current armature is especially constructed and makes use of a shrink-ring type commutator for high-speed operation.

Fractional horsepower motors of the universal commutator type have a wide speed range from no load to full load and consequently are not generally applicable for the constant speed service. A novel type of clutch

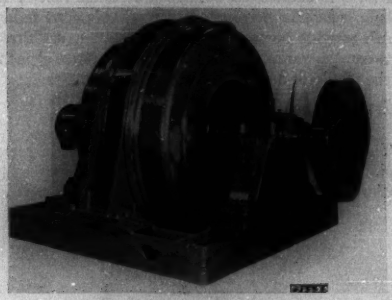

Fig. 10—250-Hp., 450-R.P.M. Totally-Enclosed Fan-Cooled, Synchronous Motor

has been developed which when installed in these fractional horsepower motors maintains a practically constant speed at the coupling regardless of how much higher the motor speed may go. This device consequently makes it possible to use universal motors for such applications as cash registers, calculating machines, small motor-generator sets, etc.

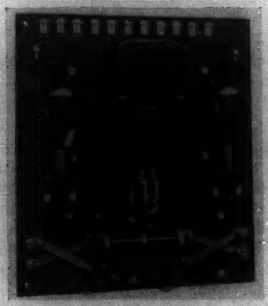

Fig. 11—Electronic Tube Control for Small D-C. Motors

Although speed reducers or gear reducers are now extensively used there is a decided trend towards the combination of various types of these gear reducers with industrial motors in order to form a more compact and self-contained unit. Some of these are of the fixed ratio type with either single, double, or triple reduction while

others have a variable ratio. A recent addition consists of a standard constant speed motor in combination with a 4-speed gear forming a self-contained and very compact unit. With this combination various gear ratios are available and it is possible to change from one speed to another under full load conditions.

Another example of a combined unit is a centrifugal type compressor for refrigerating service in which the compressor, gears, and motor are combined in one unit.

Fig. 12—Sample Chart Obtained with Smoke Recorder

This is a high-speed single-stage compressor of approximately 25 tons refrigeration capacity and by means of the unit type construction it is possible to eliminate the usual shaft seals.

A paper presented by Mr. A. M. Rossman at the Philadelphia meeting *A New System of Speed Control for A-C. Motors*, gives a detailed account of a new method of speed control, its possibilities, and savings. The system consists essentially of a constant speed a-c.

Fig. 13—Console Type Board for Electronic Tube Control of Theater Lighting and Dimming

motor of either the synchronous or induction type, supplemented by an adjustable speed d-c. machine of much smaller size and a motor-generator set of similar capacity. The frame of the a-c. motor is mounted on bearings so that the frame as well as the rotor may rotate.

Synchronous Motors

At the Winter Convention two papers on the subject of synchronous motors were presented. The motor described by Mr. M. A. Hyde, Jr., was developed especially for low-speed applications requiring high starting and high pull-in torques with a low-current inrush. The damper construction instead of being of the usual squirrel-cage type is phase wound and in starting an external resistance is inserted in this circuit and the motor accelerates in a manner quite similar to that of the ordinary wound rotor induction motor.

The paper presented by Messrs. McLenegan and Ferriss discusses the design features of normal synchronous motors and shows how these features are varied to adapt synchronous motors to the requirements of various special applications. Among the various modifications discussed are the use of part-winding starting for applications where low-starting torques and low-starting inrush are desirable, also various methods of obtaining high-starting torque such as by means of over-sized motors, over-voltage, delta-Y starting,

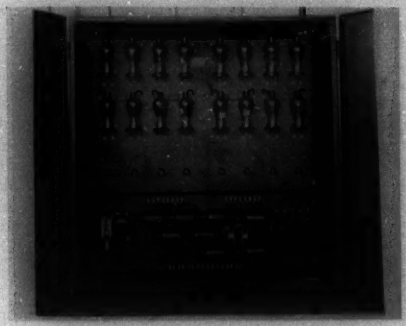

Fig. 14—Electronic Tube Control of Polychromatic Flood-Lighting System

tapped windings, phase-connected damper windings for both salient and non-salient pole motors, and a rotable stator scheme.

It is reported that the first application of a synchronous motor directly connected to a grain elevator leg was made during 1930. Ordinarily the starting torque required by these legs or bucket elevators is comparatively light but in case of emergency such as starting after a power failure, with the buckets loaded, a high-starting torque and a high pull-in torque are required.

Although several high-torque synchronous motors have been applied to primary or large gyratory crushers most of these have been of the standard mechanical construction and usually belted to the crusher pulley. A unique application has been made which utilizes a vertical motor of the umbrella type in which the revolving field poles are external to the stator which in turn is bolted rigidly to the frame of the crusher.

Another interesting and rather unusual application of a synchronous motor is for driving a hot copper rolling mill which requires continuous reversing service. The motor is guaranteed to reverse in five seconds and a special type of damper winding construction is used to increase the radiating surface and dissipate heat caused by the frequent starting and stopping.

For use in the Chilean Nitrate Industry a number of synchronous motors rated at 250 hp., 450 r. p. m., was built of the totally enclosed fan-cooled type. This construction is necessary to protect the motors against the saline solutions and vapors and the construction details are along very much the same lines as carried out on the smaller industrial motors.

When protected type motors are required either on account of injurious vapors or dust, it is sometimes undesirable or impossible to locate the control equipment in separate rooms and to take care of those cases

Fig. 15—Motor for Lower Elevator of Dual System with Combination Motor-Generator Set and Control Foreground

in which the control must be mounted in the same location as the motor, synchronous motor starters have been developed with the entire control immersed in a tank of oil.

Electronic Tubes

During the past year the electronic tube has become an accepted industrial device. Its original use was confined to the arts of communication and science and not until recently has any concerted effort been made to apply it to industrial purposes. Electronic tubes possess characteristics not available in older forms of electrical apparatus but offer many possibilities for industrial applications. These characteristics may be listed briefly as follows: speed of operation, small actuating energy requirement, amplification properties, accurate response, quietness of operation, and freedom from moving parts and contacts. In addition, the actuation of electronic tubes may be brought about by a large number of different physical effects including changes in resistance, inductance, capacitance, phase angle, frequency, temperature, sound, light, color, radio or carrier current.

There has developed a demand for electronic tube units which can be adapted to many applications for experimental purposes. Such units have been brought out by several manufacturers. The more common of these are the photoelectric units and electrostatically controlled arc discharge units. The tubes used in the latter are better known by the trade names of grid glow or thyratron.

In line with this increasing interest a paper entitled *Electron Tubes in Industry*, was presented by Mr. W. R. King at the Winter Convention. This paper gives a general discussion of the theory and characteristics of electron tubes without going into design details or calculations. Several of the more common control schemes involving tubes are explained for the purpose of showing how tube characteristics can be adapted to various applications.

To eliminate the human element in determination of smoke density a device utilizing a light source and photoelectric tube in connection with a recording instrument has been developed and gives a continuous accurate record of smoke density. The light source is mounted on one side of the stack and throws a beam of light through an aperture on the photoelectric tube mounted on the opposite side. The amount of light reaching the photoelectric tube is determined by the smoke density, and indicating or recording instruments may be installed at any convenient location.

One of the most interesting and largest electron tube applications is the control of lighting for stages and auditoriums. Installations of this type involve the use of between 300 and 600 electron tubes. Three installations of this type are in the Civic Opera House in Chicago, the Severance Memorial Hall in Cleveland, and a theatre in Los Angeles. The latest of these units, the one at Cleveland, has all of the control circuits operated from an organ type console. With this control it is possible to pre-set scenes, gradually fade from scene to scene with either manual control or automatically at a pre-determined rate, and vary the intensity of any light circuit in the hall individually or in combination with any other circuit or circuits. In the old types of stage lighting control the intensity was varied by means of rheostats. This new method uses three-legged reactors located conveniently for load distribution and the degree of saturation and consequent change in light intensity is varied by rectified direct current from tubes. Another application for the same type of equipment as used for theatre dimming is the control of building flood lights. On a large number of new buildings the lighting is so arranged that the shading and colors vary in some regular cycle.

Electron tubes are also being used for speed control

of d-c. motors in a number of various applications. At the present time for smaller motors, that is 3 hp. or below the control can be accomplished directly by control of armature voltage and current supplied from tubes, while for larger motors the field current only is controlled and supplied by tubes. One of the better known applications of this type is the maintaining of proper tension in the wire for a wire drawing machine and reel.

Photoelectric units have also been adapted to any number of production line applications where the objects are too light or fragile to operate ordinary control making or breaking devices by physical contact. On any application where an object can be made to intercept or otherwise vary a light beam this action can be used to actuate an operation. Some of the more common applications have been for counting, wrapping of packages such as foods, gum, etc., flag switches for reversing rolls as in steel mills, flashover protection of rotary converters, cutting of paper sheets to the proper length as required in connection with paper bag machines, paper break indicators in connection with paper machines, turning on and off lights in factories or offices to give proper illumination without wasting of power, and determining the transparency of paper.

Elevator Equipment

During the past year numerous improvements and refinements in elevator electrical equipment were made in keeping with the tendency toward higher speeds, automatic landing control, and quieter operation. The common trend toward combination units is also illustrated in elevator apparatus by building of motor-generator sets with exciter and control in one frame.

One of the completely new and interesting developments of the past year was the building of an elevator with two cars operating in one shaftway. Obviously the object of this arrangement is the saving of floor space in tall buildings where the space occupied by elevator shaftways is both valuable and large, inasmuch as the shaftway area must be subtracted from each floor. This elevator embodies all of the latest features in elevator design including automatic floor landing, door opening, and futuristically decorated cars. The cars use the floor bottom system of control and in addition a set of signals are displayed in each car indicating the relative position of both cars in the shaftway. The cars are prevented from approaching each other closer than two floors by an automatic block system of control which brings the car to a normal stop when this limiting distance is reached.

Conclusion

The Committee feels that there is a large field open for the electrical engineer in industry and that the Institute can be of material assistance to its members by bringing before them the important developments and trends in the various industries. In the preparation of this report we desire to acknowledge the assistance given by the various members of the committee and also by the Electric Machinery Manufacturing Co.

Instruments and Measurements

ANNUAL REPORT OF COMMITTEE ON INSTRUMENTS AND MEASUREMENTS*

THE Committee on Instruments and Measurements has been active during the past year on the following subjects:

1. Standard definitions for telemetering.
2. Standards for recording instruments.
3. Standards for indicating instruments.
4. Revision of Standards No. 14—Instrument Transformers.
5. Revision of Electrical Units.
6. Symposium on precision measurements.
7. Technique of temperature measurements.
8. Measurement of reactive power.
9. Method of measuring distortion factor.
10. Review of proposed papers.
11. Conclusion.

Standard Definitions for Telemetering

The subcommittee devoted to telemetering has been active for a number of years. Last year in this report there were published for comment standard definitions on this relatively new subject. It has been felt that, with the interest in the field of telemetering, it would be desirable to have some standard definitions so that the terms which are used would not be results from a hit and miss selection, as often occurs in newly developed fields. With comments received this year a set of standard definitions was approved by the membership of the Instruments and Measurements Committee, and is now ready for transmittal to the Standards Committee for adoption.

Standards for Recording Instruments

As a result of several years work, a subcommittee has completed a draft of proposed standards for recording instruments. These standards have been approved by the Instruments and Measurements Committee, and are now ready for submission to the Standards Committee.

The work of the subcommittee is under the chairmanship of Mr. Kinnard. A large amount of work has been involved in the development of these standards so as to cover this field of measuring instruments. The Committee has considered many new items which have shown that the Standards for Indicating Instruments should also be revised.

Standards for Indicating Instruments

Since 1927, Instrument Standards No. 23 has been used by the industry. Since that time considerable progress has been made, and experience with the Standards has indicated that a revision would be desirable. A subcommittee was appointed at the beginning of this year to start the revision.

Revision of Standards No. 14—Instrument Transformers

A subcommittee has been active during the past year revising Standards No. 14 covering instrument transformers. This standard was approved in 1925. It was the feeling of the members of the Instruments and Measurements Committee that progress in the art warranted revision at this time. The committee is under the chairmanship of Mr. Gibbs. Several tentative drafts have been prepared and revised so that these standards will soon be in form to be submitted to the Committee membership for approval.

Revision of Electrical Units

The report of the Committee on Instruments and Measurements for the Institute year ending July 31, 1929, contained a brief account of the participation of this Committee in the events which led up to the first meeting of the (international) Advisory Committee on Electricity at Paris in 1928. At that meeting resolutions of far-reaching importance were adopted, to the effect that the absolute system of electrical units, derived from the c. g. s. system, may be substituted with advantage for the present international system of units which is based upon arbitrary legalized material standards, namely, the mercury ohm and the silver voltameter.

The second meeting of the Advisory Committee on Electricity was held in Paris in June 1930. The American representative on the Committee, Dr. G. K. Burgess, Director of the Bureau of Standards, was prevented from attending, and sent Dr. H. B. Brooks as his substitute.

Representatives were sent by the national standardizing laboratories of England, Germany, Japan, and the Soviet Republics; by the Laboratoire Central d'Electricite of Paris, and the International Bureau of Weights

*COMMITTEE ON INSTRUMENTS AND MEASUREMENTS:

E. J. Rutan, Chairman,

H. S. Baker,	M. Eppley,	H. C. Koenig,
P. A. Borden,	R. C. Fryer,	W. B. Kouwenhoven,
H. B. Brooks,	J. B. Gibbs,	F. A. Laws,
O. J. Bushnell,	W. N. Goodwin, Jr.,	E. S. Lee,
A. L. Cook,	I. F. Kinnard,	Paul MacGahan,
E. D. Doyle,	O. A. Knopp,	R. T. Pierce,
Melville Eastham,	A. E. Knowlton,	W. J. Shackelton,

and Measures at Sevres. Italy was represented by Professor L. Lombardi of the Royal School of Engineers at Rome. Other experts were present as invited guests. A large part of the sessions was taken up with the discussion of a new duty which the International Committee of Weights and Measures had recently laid upon the Advisory Committee on Electricity, namely, that of advising the International Committee on all questions relating to the methods of measurement and to the units and standards of light.

Concerning the primary standard of light, resolutions were adopted supporting the view that a black-body radiator should be adopted; requesting the national laboratories to examine the specifications for the Waidner-Burgess primary standard of light and to give their views regarding the practicability of its adoption; and urging the desirability of making additional determinations of the brightness of the black-body radiator, especially under the conditions prescribed by the Bureau of Standards' specifications, or at least under conditions comparable with them.

Concerning the unit of light and its maintenance, the Advisory Committee adopted a resolution to the effect that it is inexpedient to change the unit (*i. e.*, the international candle) which has been in use since 1909. Other resolutions adopted relate to the exchange and comparison of groups of lamps by the national standardizing laboratories, and to the adjustment of the values of the practical secondary standards, as may be necessary in the future as a result of reference to a primary standard.

Concerning the electrical units and standards, two resolutions were adopted, as follows:

1. With regard to the unit of resistance, the ohm, considering that methods of determining the absolute ohm are sufficiently advanced and that the agreement between the measurements of the coils (secondary standards) of the different laboratories remains within the limits of precision of the measurements, it is not necessary at present to undertake further comparisons of the resistance coils with mercury ohms.

2. As to the units of electromotive force, on the contrary, the international comparisons of standard cells show differences, exceeding the limits of possible precision, between the values of the electromotive force of the standards of different countries. The committee considers that it is absolutely necessary to make new determinations of the electromotive force of the international Weston cells in each national laboratory by means of the silver voltameter.

The conditions under which the silver voltameter should be used are specified sufficiently in the report of the international committee which met in Washington in 1910. The Advisory Committee recommends, however, that the national laboratories use the Smith or the Kohlrausch voltameter, avoiding organic material, which is detrimental to the electrolytic deposit.

The groups of Weston cells prepared in the several laboratories can be considered as sufficiently constant until the absolute unit of current shall be established.

It should be noted that these recommendations concerning standard cells and voltameters do not in any way modify the Advisory Committee's recommendation of 1928 looking to the ultimate legalization of the absolute ohm, volt, and ampere on a basis free from all arbitrary character. The recommendations merely recognize that the standard cell and the silver voltameter are useful tools, when properly used, and are essential to the maintenance and dissemination of the international electrical units under the existing limitations.

The Advisory Committee also made recommendations regarding the equipment of the International Bureau of Weights and Measures and plans for cooperation between that bureau and the national laboratories in work on electric and photometric standards. Since an international General Conference on Weights and Measures is to be held in 1933, the Advisory Committee decided that it should meet again before that time to study the comparisons of standards which shall have been made and to assign values in absolute units for the standards of resistance and of electromotive force.

Symposium of Precision Measurements

In view of the recent activity of the International Advisory Committee on Electricity, it was felt desirable to have some outstanding papers written concerning the fundamental electrical units. The subject is being treated in a symposium which is being presented at the 1931 Summer Convention and which includes four papers. These are:

1. *International Standard of Electromotive Force and Its Low-Temperature Coefficient Form*, by Marion Eppley, the Eppley Laboratory, Inc.
2. *The Unit of Electrical Resistance; Past History and Impending Change*, by H. B. Brooks, National Bureau of Standards.
3. *Design of Potentiometers*, by I. Melville Stein, Leeds and Northrup Company.
4. *Electrical Units and Their Application*, by L. T. Robinson, General Electric Company.

Technique of Temperature Measurements

In order to set up a standard code for temperature measurements, a subcommittee has been appointed. This committee has already analyzed all of the temperature requirements included in the various standards of the Institute, and is now summarizing these data preparatory to setting up the necessary measurement technique.

Measurement of Reactive Power

Considerable interest is being displayed in the matter of measurement of reactive power. A subcommittee has been considering this subject, and is cooperating with a committee appointed by the

Standards Committee under the chairmanship of Mr. A. E. Knowlton.

Method of Measuring Distortion Factor

In line with the recommendation of this Committee, made in last year's annual report, a subcommittee has been following the performance of Belfils Bridge which is being used by one of the manufacturing companies. The work has progressed to the point where a report has been prepared covering the performance of this apparatus. At the present time the report is in circulation among the members of the Instruments and Measurements Committee in order that their comments and recommendations may be obtained in regard to the suitability of this device for the measurement of distortion factor.

Papers

In addition to the four papers mentioned above in connection with the symposium of precision measurements, nine papers have been submitted to this Committee for review. Some of these have already been approved for publication and presentation and the remainder are in circulation and will be reported on shortly. These papers are listed below:

Measurements of Cable Insulation Characteristics

A High-Sensitive Power-Factor Bridge, by W. B. Kouwenhoven and A. Banos.

High-Voltage Bridge for Measurement of Cables with Grounded Sheaths, by C. F. Dawes and A. F. Daniel.

The Elimination of an Inherent Error in the Usual Form of Capacitance Bridge for Capacitance and Power-Factor Measurements by the Substitution Method, by R. P. Siskind.

Measurements of Magnetic Flux

Core Loss Measurements at High-Flux Densities, by B. M. Smith and C. Concordia.

This paper was presented at the Northeastern District Meeting, Rochester, N. Y., April 1931.

Miscellaneous Subjects

A Direct-Current Bushing Transformer, by A. S. Fitzgerald.

A New Overcurrent Relay with Straight Line Characteristics, by S. L. Goldsborough and R. M. Smith.

A Thermionic Type Automatic Synchronizer, by F. H. Gulliksen.

The Characteristics of the Oscillograph-Galvanometer, by V. S. Thomander.

The Amplifier-Oscillograph, by S. K. Waldorf.

Conclusion

Active interest has been shown by the members of the Committee on Instruments and Measurements through the year. The meetings held were well attended and subcommittee reports indicated considerable progress was being made in each of the assigned activities, and definite steps have been taken on several as indicated above. During the coming year it is expected to complete some of the other projects.

Applications to Iron and Steel Production

ANNUAL REPORT OF THE COMMITTEE ON APPLICATIONS TO IRON AND STEEL PRODUCTION*

THE functions of this committee have, in the past, been largely confined to (1) encouraging the presentation of papers before the Institute dealing with those phases of electrical engineering which are major factors in the utilization of electricity in the steel industry. (2) To preparing an annual report which briefly summarizes such phases of the application of electricity to the production of iron and steel as are considered new or of special interest to the membership.

During the past year, the committee's work has followed the previous precedent, in procuring papers. It also actively cooperated in arranging a joint session of the Institute with the Association of Iron and Steel Electrical Engineers at the meeting of the Middle Eastern District held in Pittsburgh on March 11th to 13th inclusive, 1931. The papers presented before this meeting which the members of this committee were active in procuring are as follows:

1. "Conversion and Distribution of General Purpose D-C. Power in Large Industrial Plants," by R. D. Abbiss, Carnegie Steel Co., and D. C. West, Westinghouse Elec. & Mfg. Co.
2. Symposium on Interconnection Between Utilities and Industries: Davison Coke and Iron Co. and Duquesne Light Co. Interconnection, by G. E. Dignan, Davison Coal and Iron Co. and R. L. Kirk, Duquesne Light Co.
3. "Interconnection of Power Supply Between Public Utilities and Large Industrial Users," by F. O. Schnure, Bethlehem Steel Co.
4. "Absorption of By-Product Power," by A. Hoeffle, Toledo Edison Co. and W. T. Woodmancy, Interlake Iron Corp.
5. "The Use of Electricity in Large Annealing Furnaces," by J. C. Woodson, Westinghouse Elec. & Mfg. Co.

The Committee wishes to report that this joint meeting of the Institute and the Association of Iron and Steel Electrical Engineers was a very successful part of the Middle Eastern District Meeting Program, and suggests that joint meetings with the Association of Iron and Steel Electrical Engineers be encouraged in the future.

*COMMITTEE ON APPLICATIONS TO IRON AND STEEL PRODUCTION:

A. C. Cummins, Chairman,
F. B. Crosby, A. G. Pierce, G. E. Stoltz,
M. M. Fowler, F. O. Schnure, Wilfred Sykes,
S. L. Henderson, W. B. Shirk, T. S. Towle,
A. M. MacCutcheon, H. A. Winne.

Electrical Developments in the Iron and Steel Industry During 1930

During the year 1930 there has been a general recession from the unusually high operating rates of the preceding year, and an accompanying reduction in expenditure for improvements and new plant facilities. However, an impressive amount of electrical equipment has been purchased and many interesting developments have taken place. Undoubtedly, the very successful electrification of rolling mills and their auxiliaries has been an important factor in the steel industry.

The iron and steel industry during 1930, purchased 163,490 total horsepower of main drive motors, rated 300 hp. or over, as compared to a total of approximately 350,000 hp. during 1929. The fact that the rate of increase in installed horsepower for main drives is greater than the rate of increase for ingot capacity, indicates the tendency to modernize old mills.

Direct-Current Motors

Many important installations have been completed during 1930, with others rapidly nearing completion. From various viewpoints, a new 10,000-hp., 54-inch blooming mill in the Chicago District is memorable. It is the most powerful mill yet installed by a margin of 2,000 hp., an increase in power which will be useful in rolling thicker and heavier sections. This tends toward greater output of the whole plant, the blooming mill being the first of all the rolling operations.

Even more than for its size, this mill is unique in that the two main rolls, usually geared together, are driven individually, each by its own motor. This is a great improvement over the former method of a single motor coupled to a pinion stand. The ordinary construction of mill pinions and housings permits wear and misalinement from which develop accelerated wear, noise, vibration, and occasional breakage. Often the mill must continue in operation for several days after trouble has been developed before it can be conveniently shut down.

Each roll of the new 54-inch mill is driven by a double armature motor rated at 5,000 hp., 700 volts, direct current, at 40/80 r. p. m., both motors being under electrical control which keeps the speed of the two rolls precisely the same. The torque developed is 4,000,000 lb-ft., from standstill up to normal speed. The motors are of minimum diameter to avoid angularity of the

mill spindles between motor and rolls, an incidental advantage being smaller inertia.

Immediately following the 54-inch blooming mill just referred to is the 52-inch beam mill. The roughing mill is driven by a 7,000-hp. reversing motor, and a 2,000-hp. reversing motor for the edging stand. Likewise the intermediate mill will be equipped with reversing motors rated at 6,000 hp. and 2,000 hp.

There is now under construction by this same company, a 44-inch universal slabbing mill, the driving motors of which will have a continuous capacity of 12,500 hp., and a maximum emergency capacity of 35,000 hp. The upper and lower horizontal rolls are to be individually driven by two 5,000-hp., 40/80-r. p. m. reversing motors, constituting a twin motor drive of 10,000 hp., 4,000,000 lb-ft. maximum torque capacity, duplicate of the 54-inch blooming mill now in operation. The vertical edging rolls are to be driven by a third reversing motor, rated 2,500 hp., 79/225 r. p. m., 500,000 lb-ft. maximum torque capacity. Power for the three reversing motors will be furnished by a 10,500-kw. flywheel motor-generator set consisting of a 180,000-lb. flywheel, a 6,500-hp., 25-cycle, 6,600-volt, 370-r. p. m. induction motor, and three 3,500-kw., 700-volt generators connected in parallel. This installation will be of unusual interest to steel mill engineers because of the large size of the mill and the electrical equipment, and also because of the method of drive, utilizing three reversing motors with control to provide adjustment of relative speeds, thus permitting variations in drafting and the use of rolls of unequal diameter.

To secure the necessary refinement for the accurate automatic control of the screwdown mechanisms on a 32-inch three-high universal plate mill and on roughing and intermediate 52-inch universal structural mills, seven variable-voltage screwdown equipments have been installed. The variable-voltage control permits much more accurate settings than could be obtained with rheostatic control, either automatic or manually operated, and thus speeds up the operation of the entire mill. A further advantage is the elimination of the large capacity reversing and accelerating contacts and starting resistor, with accompanying lowered maintenance.

In addition to the 54-inch and 44-inch mills referred to above, this company is also completing a 96-inch continuous plate mill, a 10-inch alloy bar mill, a large open-hearth department, and a 20,000-kw. addition to the existing steam turbine power station.

The 10-inch alloy bar mill will be driven by nine direct current motors, totaling 7,627 hp. Some of the motors have a speed range of approximately 5 to 1, which is obtained partly by motor field adjustments and partly by armature voltage control through the use of auxiliary bucking generators, one for each of two motors which have this large speed range. The use of these bucking generators permits operating the three main 1,750-kw., 600-volt generators in parallel at constant bus voltage. The motors will be equipped throughout with speed regulators of the vibrating type, designed to hold the speed regulation within very close limits.

An 8,000-hp., 40/100-r. p. m. reversing motor is now being built to drive a 44-inch blooming mill at Gary. This motor will be capable of developing a maximum torque of 2,900,000 lb-ft. The mechanical parts of this motor, including the armature spider, will be of fabricated steel construction throughout. Several reversing motors with fabricated and electrically welded armature spiders have been in service for some time, demonstrating beyond doubt the reliability of this type of construction. Power for the 8,000-hp. motor will be supplied by a 375-r. p. m. motor-generator set, consisting of two 3,500-kw. generators, one 6,000-hp., 6,600-volt, 25-cycle induction motor, and one 200,000-hp-sec. flywheel.

A southern steel company has placed in operation at Alabama City, a new 40-inch blooming mill which replaces an engine-driven 36-inch mill. The 40-inch mill is driven by a 7,000-hp., 700-volt, 50/120-r. p. m., single-armature reversing motor supplied with power from a 6,000-kw. flywheel motor-generator set. This company has also installed a new 32-inch three-high universal plate mill driven by a 4,000-hp., 80/160-r. p. m., d-c. motor, similar to a reversing mill motor, with variable-voltage power supplied by a 3,200-kw. flywheel motor-generator. The major part of the mill output is light gage sheets. The type of drive used permits the operator conveniently to select low speeds for the roughing passes, and high speeds for the finishing passes, with very rapid acceleration to the desired rolling speeds. An important feature of this mill is the automatic screwdown. When the operator moves the pass master switch to each successive position, the desired screwdown setting for each pass is obtained by automatic control.

A 20-inch continuous hot strip mill in the Detroit District was put into operation in 1930. This mill has ten stands, each driven by an individual d-c motor. The drive for this mill has one very interesting feature. When rolling from slabs of such length that the piece will reach from stand 4 to stand 5, the section of the bus to which the first four motors and one 5,000-kw. generator set are connected, is segregated from the bus to which the remainder of the motors and the other generators are connected. This is necessary because the minimum speeds of the first four motors when operating at normal rated voltage are such that the metal would be delivered from stand 4 at a very much higher speed than it enters stand 5. Consequently, when the piece is continuous between stands 4 and 5, the speeds of the first four stands must be lowered by reducing the voltage of the generators on the section of bus to which the first four motors are connected. A similar arrangement was incorporated in the control of the first two motors on the 60-inch strip mill in a plant at Steubenville, Ohio.

Motor drive has for the first time been successfully applied on a large scale to the operation of a 900-ton ingot pressing application that was formerly monopolized almost entirely by steam or hydraulic operation. This press is in the new plant of a wrought iron manufacturer near the Pittsburgh District and compresses wrought iron sponge balls, about five feet in diameter, into rectangular ingots, measuring about 6 ft. by 18 inches square, preparatory to rolling in the blooming mill. On the final squeeze, it is necessary to exert a pressure of at least 1,000 pounds per square inch. Maintenance of factory production at full speed, requires operating the rams up to 75 feet per minute. Still the motors and press must be protected from the enormous stress which would result if the ram was brought up suddenly against a solidly compressed ingot. The solution of these and other problems involved a coordination of generator, exciter, and motor characteristics with manual and automatic control facilities, so that, as the pressure increases, the ram speed is slowed up towards the end of its stroke and the current will never exceed the allowable limit.

The main 900-ton ram is driven, through four racks and eight pinions, by a double-armature, 1,200-hp., d-c. reversing motor, having a normal full-load speed of about 125 r. p. m., and a maximum light-load speed of 275 r. p. m. The motor was built with two armatures in order to reduce the $W R^2$ and to halve the current to be carried by each commutator. The auxiliary 200-ton end rams are driven by a 325-hp. single-armature motor, having a nominal full-load speed of 250 r. p. m. and a maximum light-load speed of 550 r. p. m. The motors are of sturdy mechanical construction, with cast steel armature spiders, special armature coil bracing, and thrust bearings at the coupling end. Rear enclosing end bells are provided so that the motors may be ventilated. This is necessary as the motors carry heavy loads at very low speeds, and are therefore incapable of self-ventilation.

Variable-voltage d-c. power for the two press motors is furnished by a four-unit synchronous motor-generator set, consisting of a 1,000-kw. generator for the large motor, a 300-kw. generator for the small motor, a 1,500-hp., 80 per cent power factor, 6,600-volt, three-phase, 60-cycle, synchronous driving motor, and a 50-kw., 250-volt exciter.

In addition to the electrical features enumerated, the mechanical design of the press incorporates a number of features intended to provide safe and reliable operation. The faces of the main ram head and stationary side stop are not smooth but have corrugations vertically and horizontally across them, so that on the first impact, only about 10 per cent of the total face of the ram strikes the ingot and the ram has to move in about 1½ inches before the entire face strikes.

The electrically driven press has been in operation several months, and its success has demonstrated the practicability of electrical drive for such machines.

A mill in the Youngstown District has purchased the electrical equipment for a 10-inch bar mill having 12 tandem stands. Ten d-c. motors totaling 7,350 hp. with variable-voltage control from two 2,000-kw. motor-generator sets will be used with this mill. The mill will have two unique features, one being the method of speed control and the other is the use of vertical motors to drive the vertical roll main stands.

A speed regulator of the carbon pile type is used with each motor and with a master frequency generator to maintain a desired speed relation between stands. The customary bevel gear drive to the rolls has been eliminated by the use of vertical motors. The vertical motor is mounted above the mill on an overhung support. This support also carries the rolls and pinion housing. The rolls can be moved sideways or up and down with respect to the bar being rolled so that the vertical rolls can be lined up with the grooves of the horizontal rolls.

Synchronous Motors

Among the more important synchronous motors which have been purchased during the year may be mentioned two 4,500-hp., 156.5-r. p. m., 6,600-volt, 60-cycle units which will drive two rolling mills in the plant of a Pennsylvania manufacturer. This company also purchased one 5,000-hp., 360-r. p. m., 6,600-volt synchronous motor to drive a 24-inch billet mill and one 4,000-hp., 360-r. p. m., 6,600-volt motor to drive a 28-inch billet mill at the same plant early this year. This company put into operation a 3,000-hp., 100-r. p. m., 11,000-volt, three-phase, 60-cycle synchronous motor, driving a three-high billet mill. This motor is of interest not only because it is one of the few main roll motors operating at over 6,600 volts, but also because it is started directly from the 11,000-volt circuit.

A West Virginia manufacturer has purchased two synchronous motors, one 2,000-hp., 300-r. p. m. and one 1,000-hp., 360-r. p. m., which will drive the hot tin mills. This is believed to be the first installation of synchronous motors on mills of this character, previous practise having been to use induction motors with flywheels. Synchronous motors were chosen because of the necessity of maintaining a good power factor, and also because of the gain in efficiency. Not only is the synchronous motor itself more efficient than the corresponding induction motor, but the friction and windage losses of the usual flywheel and the power losses in the secondary resistor or liquid slip regulator are eliminated. It is estimated that the power per ton of tin plate rolled will be at least 5 to 10 per cent less than if an induction motor flywheel drive had been used.

Switchgear

All the modern improvements in switchgear equipment are rapidly being adopted by the iron and steel industry. The metal clad type of switchgear with its

live parts completely enclosed, excluding dust and dirt and at the same time protecting the operator from possible injury, is ideally suited to steel mill service.

Auxiliaries

Automatic screwdown equipment has been developed to the point where it will rapidly become more generally used. Two recent applications, one in a universal plate mill and the other in a structural mill have been mentioned in this article. The use of variable-voltage equipment for these screwdown drives has proved very desirable.

There is increasing industrial activity in the application of vacuum tubes and related devices. The crane operator on the soaking pit cranes for a 54-inch mill in the Chicago District will control the opening and closing of the soaking pit covers through the action of photoelectric cells. In another plant, a photoelectric cell acts as a "flag" switch to make the necessary set-up for registering the temperature of each billet as it approaches the rail mill. Undoubtedly, the field for such applications will rapidly increase.

The majority of the motor-driven heavy shears, such as bloom and slab shears, which are now in operation, are driven through clutches by induction motors equipped with flywheels. The mechanical clutches are a source of more or less trouble, and to eliminate these and also the losses in the continuously running motor and flywheel, several shears have recently been installed with d-c. motor drives, arranged to start and stop with each cut.

Two of these are in a new plant near Detroit. The larger of the two is an 800-ton slab shear, designed to cut a slab 8 inches thick by 25 inches wide. Another midwestern steel plant has recently placed in operation eight shears, cutting cold stock up to a maximum of ¼ inch thick by 42 inches wide. Some idea of the high rate of acceleration and retardation may be obtained from the fact that the shear makes a complete cycle in less than one second, and during this time the motor starts from rest, attains a speed of over 450 r. p. m., cuts the plate and stops.

A Pittsburgh tube mill has installed two scrap shears for crushing and shearing pipe up to 24 inches. They are of the new type with the shear motors starting and stopping for each cycle.

What is believed to be the first application of synchronous motors to flying shear drive is in connection with a 20-inch hot strip mill in the Detroit District. The shear is of the rotary flying type, and, to insure that the metal shall be cut into equal lengths, it is necessary that the speed of shear be synchronized with the delivery speed of the metal from the mill. To accomplish this, an a-c. generator is provided on each stand, from which the metal may be delivered. The shear is driven by a synchronous motor, the power for which may be obtained from any of the a-c. generators.

Three ore unloaders recently placed in operation at one of the Lake Erie Docks are interesting on account of their large size. The combined capacity of the three unloaders is 3,600 tons per hour, and the actual time for unloading a 10,000-ton boat is four or five hours. The bucket capacity for each unloader is 17 tons. Each unloader uses eight motors totaling 1,060 hp. The beam hoist motor is a 350-hp., 500-r. p. m., shunt-wound mill motor and the trolley travel motor is a 170-hp. motor, these two motors having variable voltage control. The bucket closing and opening motor is a two-speed wound-rotor mill type motor, and all other motors are a-c. motors. The motor-generator set for the variable-voltage control is mounted inside the beam and serves to help balance the stiff-leg weight.

As electrical improvements in the iron and steel industry are reviewed each year, the opportunities that exist for improvements in steel mill drives and auxiliary equipment become increasingly evident. These improvements will result from a close cooperation between the steel plant engineer, the mill builders, and the engineers of the electrical manufacturer.

Production and Application of Light

ANNUAL REPORT OF THE COMMITTEE ON PRODUCTION AND APPLICATION OF LIGHT*

Production of Light

Statistics on Incandescent Lamp Sales

THE sale of incandescent lamps held up remarkably well during the past year, large lamps showing a decrease of about 1 per cent as compared with 1929. The Central Station revenue derived from the two classes of service in which light is a large factor (domestic and commercial, small power and light) showed an increase of about 7.5 per cent as compared with the previous year, offsetting the reduction from other uses of electricity, and leaving a net increase of about 3 per cent.

The average lumens, watts, and efficiency of the large lamps sold last year all increased as compared with the year before. The average lumens went from 797 to 814; the average watts from 60.7 to 61.2 and the average lumens per watt 13.1 to 13.3.

Of the sixteen standard wattages from 10 to 1,000 inclusive, used in multiple service, three sizes 60, 100, and 200 watts, represent nearly half (46.6 per cent) of the total wattage demand.

The percentage of 110-volt and 115-volt lamps sold decreased slightly, while that of 120-volt gained as compared with the previous year. The percentage distribution for last year for 110-, 115-, and 120-volt lamps was 5.2 per cent, 48.9 per cent, and 41.3 per cent respectively. Lamps for 200-250-volt service decreased from 3.3 per cent to 2.6 per cent of the total quantity sold for multiple service.

The demand for street series lamps is concentrating on the 6.6 ampere rating. This now represents 71.2 per cent of the total; the next largest demand being 14.6 per cent which is for the 15- and 20-ampere compensator type lamps.

New Sources of Ultraviolet Radiation

A new type of lamp (called a glow lamp) has been developed which produces ultraviolet light by virtue of a glow discharge through mercury vapor between two hot cathodes of the oxide coated type. The two heater filaments for the cathodes are connected in series and their resistance produces a sufficient difference in potential to cause a discharge between the two cathodes, once they have become heated. The lamp is rated at 2 amperes and 18 volts. This lamp, designed for operation on alternating current requires a starting potential of some 35 volts, after which 18 volts is sufficient to maintain the discharge. One, two or three lamps may be operated on the ordinary 115-volt lighting circuit in connection with a suitable current-limiting resistance, reactance, or transformer. The glass bulb has been selected for its transmission characteristics so that the lamp will produce practically no radiation below 2,800 Angstrom units. In addition to producing the desirable radiations of ultraviolet light, it also produces blue and green light in the visible spectrum which may possibly be used to advantage in the future to supplement the light of Mazda lamps, to secure a nearer approach to a light subjectively white in color as well as to obtain the healthful ultraviolet radiation.

In last years report a new tungsten-mercury arc lamp was referred to. This lamp consumed about 315 watts. A lamp of the same general characteristics, but of about forty per cent of this wattage has recently been announced. The bulb is smaller, and is fitted with a metal cap which raises the operating temperature and pressure of mercury vapor in the bulb, and increases the efficiency of production of ultraviolet radiation. The ultraviolet output of the new lamp is approximately one-half of that of the older one. The transformer to be used with the new lamp weighs about 7½ pounds and is small enough so that it can easily be embodied in the design of fixtures, to permit a wider use of this type of lamp, especially a source for dual lighting service.

Still another type of lamp employing a tungsten filament, operated at a high temperature has been found to produce ultraviolet radiation in the biologically active range in sufficient quantity to be of value at least to poultry and animals where exposure times are not too short. Lamps for such service must, of course, be made with special bulbs which will transmit this radiation, and two sizes, 60 and 500 watts for operation on standard lighting circuits have been made available.

A powerful, long-burning type of carbon arc has been developed for the production of ultraviolet radiation which was described in an illumination item in the December issue of the A. I. E. E. Journal, page 1031.

Gaseous Conductor Sources of Light

An arc, in an atmosphere of krypton and sodium, has been described by Dr. M. Pirani.* This operates in a

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tube about one inch in diameter and 35 inches long which has to be maintained at a temperature of 350 deg. cent. (somewhat above the melting point of lead). A very high efficiency, about 70 per cent of the theoretical maximum, is obtained based on the input to the arc alone. With the supplementary heating required, the over-all efficiency would, of course, be reduced. Previous to this time sodium arcs with efficiencies of from 50 to 70 lumens per watt have been described. The light from such lamps, however, is nearly monochromatic and would, alone, be of limited utility for ordinary lighting service.

Dr. Pirani has also obtained great increases in brilliancy in mercury and in neon discharge tubes.†

The combination of hot cathode neon and mercury vapor lamps to produce light apparently white in color, at a high efficiency, (mentioned in our report for 1928) is again attracting interest, as is also the combination of the mercury arc with incandescent lamps.

Small Gaseous Conductor Lamps

These small negative glow lamps used for night lamps, indicative and current detectors, are now designed to operate on line voltages from 90 to 130, and to consume from 0.01 to 2 watts,. The former is merely an indicator, the latter gives a maximum normal candlepower of 0.2.

Photoflash Lamp

A lamp has been designed to produce a high intensity flash for use in photographic work, consisting essentially of crumpled aluminum foil in an atmosphere of oxygen, which is ignited by a small filament that can be operated on any voltage from 2 to 130. It is made in a bulb of the size and shape of the 100 watt lamp used for general lighting service.

The flash lasts approximately 0.02 second, the radiation is about such as would be obtained from a black body at a temperature of 4,000 deg. K. The flash is equivalent in candlepower to about 250 ordinary 500-watt lamps. It is noiseless and fumeless, the products of combustion being retained within the bulb. The lamp is finding wide use in the photographic field.

40-Watt, Gas-filled Lamp

The 40-watt lamp for ordinary lighting service is being changed from a vacuum to a gas-filled type, and, at the same time put in a slightly smaller bulb. The initial efficiency of the new lamp is slightly higher, and the light output is maintained better throughout life.

50-Watt Lamp

The 50-watt incandescent lamp, for general lighting service, has been removed from the regular schedules of the manufacturers of Mazda lamps. Its demand can apparently be shifted largely to the 60-watt size which is gaining steadily in popularity. The three sizes, 40, 50, and 60 represented unnecessarily close steps.

*Zeitschrift fur Techneache Physik—Nov. 1930.

†E. T. Z.,—June 1930.

New Incandescent Lamps

Three new lamps for 110-, 115-, and 120-volt lighting service have recently appeared on the market. These are, a 5-watt lamp in a bulb ¾ inch in diameter, a 100-watt in a tubular bulb 34 inches long, and a 150-watt in a tubular bulb about 12 inches long.

Application of Light

Ultraviolet

The utility of ultraviolet radiation for purposes, other than merely the prevention of rickets, has been attracting wide attention and has been the subject of much experimental work. Its most promising physiological benefits seem to lie in the direction of prevention of colds and relief of anaemia. In the industrial field powerful carbon arcs are used for making accelerated tests (which must however be carefully evaluated) and for the treatment of food, tobacco, and other products. Small incandescent sources can be used in poultry houses with good effect.

The use of screens with arc sources to absorb the ultraviolet radiation shorter than that which it is desired to use, has grown during the past year, particularly in outfits intended for use in the home. A time limit switch is also incorporated with some outfits to reduce the probability of over-exposure. The development of very powerful arc sources with long-burning carbons, has extended the possible use of ultraviolet radiation in both industrial service, and in treating large groups in hospitals and sanatoriums.

Trends in Lighting Practise—1930

The interest of architects and decorators in the use of artificial light as a component part of their decorative scheme, is becoming more and more widespread. At first the new ideas were accepted in only the larger cities, now in most of the smaller cities and larger towns excellent examples of the new practise can be found.

Instead of trying to imitate flame sources, candles and oil lamps in fixtures intended for rooms of classical treatment, the skillful designer produces a luminaire which serves the modern light sources in an effective manner and then incorporates in the detail of decoration elements which are inherent to the decorative period under consideration.

The more commercial or stock fixtures which have appeared on the market during the last few months exhibit a tendency to new forms, characterized in general by simplicity of line and pleasing proportions.

If satisfactory illumination is to be obtained from side wall outlets, these must be fitted with some sort of device quite different from the ordinary candlestick bracket. Manufacturers are beginning to realize the possibility of so-called semi-indirect, side wall pockets or urns and a number of interesting varieties have been placed on the market during the period under consideration.

Only a few years ago it was the practise to finish the building and then choose the lighting fixtures. As a

result they often tended to appear as an incongruous after-thought, and not a component part of the structure. We are now seeing many instances of what has been termed "built-in" lighting, but these arrangements cannot be carried out unless suitable provision is made in the initial plans for the placement of lighting equipment.

There is a growing appreciation of the value of light to look at (as well as to see by) and in the more modern buildings we see interesting installations of luminous panels, niches, and touches of color which give no useful illumination but are pleasing to the eye.

Naval architects and ship owners realize that their vessels must be made more interesting from a decorative standpoint if they are to attract patronage under conditions of severe competition, and decorative lighting is receiving more attention than ever before in marine service.

There seems to be a trend in commercial lighting practise toward the greater use of indirect lighting equipment, both semi-indirect and totally indirect. Such forms of lighting which are desirable for general use in commercial interiors have been rather slow in being adopted probably because they require better maintenance than the enclosing globes.

Technical progress has been made in practically every field of lighting. In some of the newer ones, such as lighting for outdoor sports and for aviation, there has, in addition, been a very rapid growth. There have been no revolutionary changes, however, in equipment and methods generally employed.

Windowless Buildings

Perhaps the most outstanding development in connection with the use of daylight or artificial light for buildings that has taken place to date was started during the past year. The new factory of the Simonds Saw & Steel Company, Fitchburg, Mass., which is now nearing completion is built entirely without window openings. This pioneering project is receiving much publicity and the results will unquestionably be watched with interest and have a distinct bearing on the future building construction. In this building lighting units consisting of a combination of mercury vapor tubes and tungsten filament lamps will be used to secure a subjectively white color of light. Some of the tubes will be of ultraviolet transmitting glass to provide the desired amount of such radiation. Another notable example of this nature is the Parker Bridget Department Store in Washington, D. C. This store depends entirely on artificial illumination for its operation. It is felt that by eliminating windows lighting conditions are constant at all times. It is much easier to accurately control temperature and ventilation. Then, too, from a merchandising standpoint the larger percentage of wall area is available for display space.

With the record for continuity of service which has been attained by the central stations, and the development of effective means of lighting, ventilation and air conditioning, windows are no longer essential. There are so many advantages to be gained by their elimination in certain classes of use, that it appears as though this subject will receive increasing attention in the future.

Proper Operating Voltage

The N. E. L. A. Lamp Committee gives in its report this year, the results of further surveys of socket voltages, which show some improvement in the average value as compared with the sample cities studied the year before. The spread in voltage, however, shows no improvement. The results given in these surveys apply particularly to residential service. In large office buildings and industrial plants the average socket voltage is probably relatively lower than in residential service because of the greater voltage drop in interior wiring.

The N. E. L. A. Lamp Committee Report for 1928 contained a summary of data on operating voltages as reported in use for some 18,000 communities. This summary showed that 5.4 per cent of the population considered was in communities served at more than one standard voltage. Five years earlier the percentage had been 13.2. This trend toward the use of one standard operating voltage in each community aided in putting lamps of correct voltage in service, because, it is obviously difficult, if not impossible, to avoid mixing voltages in service where two or more standards are in use. The maintenance of but one standard service voltage throughout a community makes it much easier to be sure that lamps of only the correct voltage are used, and this is important from the standpoint of lighting service in general.

Minimum Wiring Specifications

In addition to the minimum specifications for the adequate wiring of lighting circuits in commercial and public structures, mentioned in last years report, specifications covering industrial and residential service will soon be made available as a result of further work by N. E. L. A. Committees. All of these specifications are in such simple form that they can be used with the greatest ease. They provide minimum standards to which all new buildings should conform in the interests of good lighting service. We again recommend that every electrical engineer who is brought into contact with the design or operation of lighting installations should be familiar with these specifications.

Stroboscopic Developments

A new type of stroboscope has been developed in which the discharge characteristics of a three-element grid glow tube are utilized in conjunction with a condenser to produce the brilliant flashes of light of exceedingly short duration and high frequency needed in such work. Another method, described in German technical literature produces flashes of the desired character by the combination of motion of three disks carrying different numbers of radial slits. A more complete description

of these devices has been submitted for publication as an Illumination Item.

Group Replacements

The replacement of incandescent lamps in street lighting service in complete groups rather than individually as failures occur, offers a possible means of securing a higher general level of illumination and reducing the number of outages at no increase in operating cost. It is being used, or tried out, in a number of localities. If the burning hours of all the lamps in a large group are identical, and if the cost of replacing lamps individually as they burn out is relatively high, it may be advantageous to replace the entire lot as soon as the burnouts begin to occur with some frequency, which will ordinarily be at the time the lamps have attained about 70 per cent of their average life. If this general replacement is made to coincide with one of the periodic cleanings of the lighting equipment the replacement cost per lamp is relatively small. This same practise might well be applied in other classes of service, where the cost of making individual replacements is relatively high, and where all the lamps in a large group burn the same length of time.

Fixed Focus Automobile Headlights

It has long been recognized that one of the greatest factors contributing toward glare from automobile headlights has been the fact that the lamp filaments were not properly focused in the reflectors. Although the system of approval of headlight equipment by the motor vehicle authorities of the various states after laboratory test has become quite general and all cars are equipped with head lamps capable of producing adequate road light with minimum glare, it has been impossible to keep these equipments in proper adjustment in the hands of the public. The motorists may learn to aim the headlight beams properly, but the more intricate operation of focusing the filament in the reflector has proved a stumbling block.

Because improvements in lamp manufacture now permit more accurate location of the filaments with respect to the lamp base, and because headlight reflector and lens combinations have been designed that are less sensitive to focal adjustment, it has become possible for the motor vehicle authorities to approve the use of so-called "fixed focus" headlamps from which the focusing mechanisms are entirely eliminated. The sockets in these headlamps are rigidly fastened in the reflectors and the filaments in the lamps of any reputable manufacturer in this country are accurately designed to fit them so that an acceptable beam can be secured by simply inserting the lamp in the socket.

Headlamps of this type are already being used by several of the motor car manufacturers and improvements in the general headlight situation are to be expected as their use becomes more widespread and cars with the old types of equipment are gradually replaced on the highways.

General

Present Status of Candlepower Standards

While the needs of practical photometry have been met reasonably well for many years by the expedient of establishing units of candlepower maintained by means of electric incandescent lamps, this procedure does not give a permanent solution for the problem of photometric standards. There remain unsettled three major difficulties: 1. Several countries, particularly Germany and Austria, still use the Hefner unit, which is 0.9 of the unit recognized by the International Commission on Illumination; 2. no reproducible primary standard has been accepted as representing the international unit which was established for carbon-filament lamps; 3. no specific method has been agreed upon for passing from the carbon-filament lamps to those giving light of other colors.

These difficulties have resulted in discrepancies between measurements made in different countries. The differences are large enough to be very serious in such precise measurements as are made in rating and testing incandescent lamps. Consequently efforts have been made for several years to find a basis for a comprehensive international agreement on this subject. These efforts have been furthered by discussion at several sessions of the International Commission on Illumination. In order to give a more definite legal status to the negotiations the International Committee on Weights and Measures decided in 1929 to take up the problem of photometric units and standards, collaborating with the Commission on Illumination. Both of these organizations meet in 1931, and it is hoped that some definite progress can be made toward the solution of the problem.

As a basis for action, an international Advisory Committee on Electricity, meeting in June, 1930, has prepared recommendations which may be summarized briefly as follows:

1. As a *primary standard of light* a black-body radiator is definitely recommended; other laboratories are asked to consider the specific form of standard which has been developed by the Bureau of Standards, and to make determinations on that form or on others which will be comparable with it.

2. As a *unit* for general use the present international candle is to be retained.

3. Present discrepancies between basic units at carbon-filament color as maintained in the different national laboratories are to be reconciled by comparisons now in progress, and thereafter periodic comparisons and necessary adjustments are to be made under the auspices of the International Committee on Weights and Measures.

4. Through comparisons of colored filters now under way between the national laboratories it is hoped that agreement can be reached on a standard method for measuring lights of various colors. This method can then be used in setting up practical standards for lamps of various types, all based upon the primary standard and all consistent with each other.

Architects and Lighting

During the past year the Illuminating Engineering Society has given great attention to securing better

cooperation between the architects and illuminating engineers. As one approach to this problem, three very well attended schools on architectural problems have been held, one at Columbia University, one at Chicago, and one at the Massachusetts Institute of Technology. Such schools for illuminating engineers serve to acquaint them with architects' viewpoints and nomenclature, thus helping to bridge the gap between the two professions. On the part of practising architects greater attention than ever before has been given to coordinating lighting with building design.

Important steps have been taken in fostering better lighting in the homes and schools of the United States. In this manner the best principles of lighting have been made available to school boards and home owners. This work is spreading rapidly.

A code of principles of Street Lighting has been prepared by the Street Lighting Committee of the Illuminating Engineering Society, which is likely to take a very important part in guiding the practise in this field. For the first time experts and engineers representing diverse interests and viewpoints have been able to agree upon a statement of some of the fundamental factors involved.

The Lamp Committee of the National Electric Light Association presents each year to that association a report in which the statistics relating to incandescent lamp production are analyzed in detail, and in which important developments in the production and application of light are reviewed. Those who wish more detailed statistics, as well as those who are concerned with the commercial and engineering problems of lighting, will find valuable the reports of various committees of the National Electric Light Association and the Illuminating Engineering Society.

The Committee wishes to record its appreciation for the assistance rendered in the preparation of this report by Dr. E. C. Crittenden (who contributed the section on the Present Status of Candlepower Standards), Mr. A. L. Powell, and Mr. A. B. Oday.

Applications to Mining Work

ANNUAL REPORT OF COMMITTEE ON APPLICATIONS TO MINING WORK*

THERE has been a minimum of new installations made this year owing to the general depression.

Hoists. Probably fewer hoists have been installed than in many past years. It is interesting to note that two manufacturing companies alone report that combined they have sold to date nearly 800 induction motor hoists from 200 to 1,800 hp. in size, and over 100 variable control d-c. hoists from 200 to 5,000 hp., the latter two motors in series.

Cleaning Plants. There is a continued trend toward larger and more complete preparation plants especially at mines using mechanical loading instead of hand loading. One plant, Hazle Brook Coal Co., has installed two 200-hp. Westinghouse synchronous motor drives.

Shovels. Electrification of 4- to 10-yard shovels has continued in the metal mines.

Mining Machines. The manufacturers of this specialized electrically driven equipment have been very active in producing new types of mobile high-powered machines. These machines now undercut from 600 to 1,200 tons per day in contrast with older types which did well to undercut 200 tons per day. See Fig. 1.

Loading Machines. Rapid increase of the amount of coal mechanically loaded indicates that these labor saving devices are permanently established in the industry. Various makes have sustained records of 350 to 500 tons loaded per 8 hours. Probably there are 500 such machines now in operation.

Welding. In addition to miscellaneous repair work by electric arc welding several companies have installed full automatic welding heads to be used for filling up the worn tread of locomotive tires and wheels. This has been quite successful and undoubtedly will be very generally used.

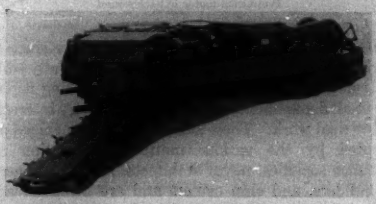

FIG. 1—JEFFREY 29L TRACK-MOUNTED CUTTER

Safety Work. The Bureau of Mines has continued to test and approve many new devices for mining work.

NOTE: Many references to miscellaneous improvements and applications are given in the *Coal Age*, Vol. 36, No. 2, Feb. 1931, p. 76.

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J. H. Edwards, J. E. Kearns, B. F. Tillson,
F. E. Fisher, W. H. Lesser, J. F. Wiggert.
D. E. Renshaw,

Applications to Marine Work

ANNUAL REPORT OF COMMITTEE ON MARINE WORK*

THE major items of activity of the committee for the current term have been:

a. Reprinting Standards No. 45—Recommended Practise for Electrical Installations on Shipboard.

b. Promoting licensing of the electrical operating personnel on shipboard.

c. Keeping in touch with electric propulsion of ships and the electrification of auxiliaries on shipboard.

Standards No. 45

During the last term of this committee these standards were completely revised to incorporate developments and expansion in the marine practise since the previous issue of June 1927. They were carefully edited and reprinted in October 1930 and are being rapidly sold to all departments of the marine industry interested in electrical work. These standards are recognized and accepted by the various marine classification and insurance societies, naval architects, marine engineers, shipbuilders, and ship owners and are being incorporated in many of the specifications regulating the construction and repair of ships. The increasing use of them testifies to the need of such rules in order to standardize the electrical installations on shipboard and to stimulate the use and proper care of electrical machinery in the marine field.

Fig. 1—View in Lower Engine Room, *S. S. Morro Castle*

Propulsion motor in background

They are referred to in the various rules of classification societies not as a requirement but as representing good practise. This regard for these rules is justly merited due to the many years of hard work by experienced representatives of the diversified interests composing the marine field.

The fire alarm rules in the present Steamboat Inspection Service are consistent with the Standards No. 45 and were referred to this committee during their preparation.

Operating Personnel

Our efforts were continued with the Steamboat Inspection Service to obtain classification and rating for electrical operating engineers on shipboard.

Fig. 2—View in Upper Engine Room, *S. S. Morro Castle*

Propulsion control at left and generators at right

A review of the work of the subcommittee in charge of this item carries one back to the year 1922 when overtures were made to the Steamboat Inspection Service to require the operating personnel on vessels with electric plants to qualify and obtain electrical licenses.

The committee's efforts to date have been rewarded by a slight raising in the standard of the electrical

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questions in the examination for license for steam and combustion engineers.

The Board of Supervising Inspectors has been very courteous and granted a number of hearings. The necessity for the requested action, is recognized by a few of the supervising inspectors and it is hoped that there will soon be a sufficient number to put the movement into effect.

The cause for this movement was the improper maintenance of electrical equipment on shipboard and is today perhaps more prominent than when the movement was started owing to the increase in the use of electrical apparatus and auxiliaries, with the exception of vessels equipped with electric propulsion and all of these are not immune.

On electrically propelled vessels the standard of the electrical force is somewhat higher than on other vessels. Consequently, the electrical apparatus and auxiliaries receive the benefit of more expert attention.

The committee has been in touch with the President of the American Steamship Owners Association soliciting its good offices. Our committee was cordially received and has been requested to submit its proposal relative to the absence of knowledge and training on the part of the operating personnel. This proposal is almost complete and it is hoped it will be submitted at an early date.

The subcommittee is keeping in close touch with this situation and is optimistic in feeling that favorable action will be taken by the Steamboat Inspection Service before long and their regulations modified accordingly.

Electric Propulsion and Electric Auxiliaries

The year 1930 takes its place in marine history of the United States with a record of activity, progress, and accomplishment in shipbuilding and marine engineering, which has not been approached since the busy days of the World War. This is, of course, due largely to the stimulation resulting from the Jones-White Merchant Marine act of 1928, which has made available to American ship operators, large mail contracts and loans at low interest rates for construction of new vessels. Thus, our merchant marine is to receive a much needed strengthening by the addition of a fleet of fine ships now under construction or in process of design.

In the design of these last words in naval architecture, electricity has taken an outstanding place. Some will be propelled by means of electricity. All will have electrically driven auxiliaries which are so necessary to the operation of the ship. Electrical appliances will add to the comfort and convenience of the passengers.

In order to illustrate the progress of electric propulsion especially during the past year the following data covering vessels completed or contracted for to January 1, 1931, are listed below.

	Hp.	No. vessels
Total turbine electric in U. S. A.	867,600	62
Total turbine electric outside U. S. A.	127,900	9
Total	995,500	71
Commissioned or under construction during 1930 in U. S. A.	196,900	20
Total diesel electric in U. S. A.	88,280	121
Total diesel electric outside U. S. A.	16,700	8
Total	104,980	129
Commissioned or under construction during 1930 in U. S. A.	15,195	25

It may be interesting to know the individual vessels and their horsepowers involved in the above figures.

Name		Horsepower
Turbine Electric Drive—Ships Commissioned During 1930 in U. S. A.		
Santa Clara		12,600
Morro Castle		16,000
Oriente		16,000
City of Flint (Great Lakes Car Ferry)		7,200
Corsair		6,000
4—Coast Guard Cutters		12,880
2—River towboats		4,000
Total		74,680
Turbine Electric Drive Equipments—Under Construction During 1930		
President Hoover		26,500
President Coolidge		26,500
United Fruit Co.—6 ships 11,000 hp. each		66,000
One Coast Guard Cutter		3,220
Total		122,220
Diesel Electric Drive—Installed During 1930		
Frying Pan	Lightship	350
Fenwick Island	Lightship	350
Nantucket	Lightship	350
Tidewater	Tanker	1,000
Ward Island	Ferry	400
Tenkenas	Ferry	400
Veedol No. 2	Tanker	1,000
L. T. C. No. 1	Tanker	500
L. T. C. No. 2	Tanker	500
L. T. C. No. 3	Tanker	500
Liston	Survey Boat	350
Scott	Towboat	150
Ft. Armstrong	Towboat	150
Cleveland	Tug	800
Rochester	Tug	800
Scranton	Tug	800
Oleon	Tug	800
Venus	Tug	515
Luna	Tug	515
Prescottont	Tug	800
One tug *Vandyke No. 4.*		425
Total		11,455
Diesel Electric Drive—Under Construction		
One yacht		660
One tanker		200
One fireboat		2,130
One ferryboat		750
Total		3,740

Prior to the commissioning of the *S. S. California* in 1928 the total turbine-electric horsepower installed on merchant ships was but 38,800. Out of a total of 356,200 horsepower completed and in course of construction as of the end of 1930, 317,400 horsepower, or

89 per cent, was undertaken during the past three years.

Slightly apart from the marine field, but with equal

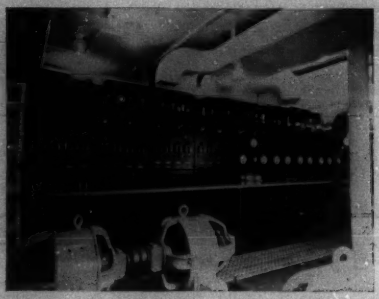

Fig. 3—Auxiliary D-C. Switchboard, *S. S. Morro Castle*

interest to it, was the installation of a central station in the *Jacona*, which is to be used as a mobile and subsidiary power plant along the shores of the New England Coast. Two turbine generators were installed, each rated 10,000 kilowatts for delivering power ashore at 6,600 volts or 11,400 volts three-phase 60 cycles.

During 1930, a 96-in. (wheel diameter) gyroscopic stabilizer complete with electrical equipment, was constructed. This unit will be installed in a large yacht. A second unit with 11-ft. diameter wheel, complete with electrical driving motors, control and turbine generator set for power supply, is nearing completion for installation on one of the largest yachts ever built. The casing of this stabilizer will be of welded construction.

During the year 1930 great strides were made in shipboard installations of modern watertight door control systems. This system provides that in the event of collision electric operation of the watertight bulkhead doors is instantly available. The system provides remote control of the watertight doors from the wheel house as well as at the door itself.

It is the intention of this committee to keep Standards No. 45 up to date and assist in any practicable way the various classification societies and other bodies or interests in the marine field in the application of electricity to marine use.

We further are ready to cooperate with the American Marine Standards Committee in establishing electrical marine standards such as watertight receptacles, etc.

Power Generation

ANNUAL REPORT OF COMMITTEE ON POWER GENERATION*

TWO meetings of the entire committee were held during the year, one at the Philadelphia District Meeting on October 14, 1930, and the second at the Winter Convention in New York on January 28, 1931. In addition to the regular survey of immediate developments and problems in the scope of the committee, a continuing organization or method was adopted through which news items on the subject of power generation will be reported for publication in ELECTRICAL ENGINEERING. Each member of the committee was asked to appoint some one of his business associates as a permanent news correspondent to represent his particular affiliation irrespective of membership in the future on the Power Generation Committee.

The subject of interconnection continued to occupy the major attention of the committee, and especially of the Joint Interconnection Subcommittee on which Messrs. F. C. Hanker, F. H. Hollister, and A. E. Silver represented the Power Generation Committee. The formation of this subcommittee composed of members from the Power Generation, Power Transmission and Distribution and Protective Devices Committees, was predicted in last year's report, and the subcommittee has functioned throughout the year under Mr. Hanker's chairmanship not only to the fullest extent, but has also promoted a close working arrangement among the three main committees in dealing with a subject that has many common elements.

The Joint Interconnection Subcommittee has directed the preparation of a second report on "Grounding," which was presented at the Pittsburgh meeting in March 1931 under the title *Present Day Practise in Grounding of Transmission Systems*, and while of technical import to this committee, is also of interest because the original report on *Grounding* was prepared in 1923 by Mr. E. C. Stone now of the Power Generation Committee.

The Interconnection subcommittee has also sponsored a session in this summer's convention, consisting of four papers that treat the subject of interconnection primarily from the viewpoint of geographical, as contrasted to urban territory, interconnection. The introductory paper, prepared by Mr. Alex. E. Bauhan, is a comprehensive summary of the various services rendered by interconnection, and an exposition of the methods of evaluating the economies to be secured. Two papers that illustrate the plans and operating procedures adopted in different regions, are as follows:

Interconnection in New England, by E. W. Dillard and W. R. Bell; *The Pennsylvania-Ohio-West Virginia Interconnection*, by Howard S. Fitch.

The fourth paper, by G. M. Keenan, is on *Load and Frequency Control on Interconnected Systems* and summarizes the methods in use and the experience on the subject that has accumulated in the past two to three years on practically all the major system interconnections in this country.

The committee reviewed during the year and recommended as of great interest on the subject of interconnection other papers as follows:

Governor Performance During System Disturbances, by R. C. Buell, R. J. Caughey, E. M. Hunter, and V. M. Marquis.

Reestablishing Excitation of a Loaded Alternator in Parallel with Others, by D. D. Higgins and E. Wild.

Other papers describing the design of modern power plants that have been reviewed by the committee include the following:

The Ohio Falls Hydroelectric Station at Louisville, Kentucky, by R. M. Stanley and E. D. Wood.

Modern Steam Stations of Duke Power Company, by Marshall E. Lake.

Steam Power Development of the Pacific Gas and Electric Company, by Richard C. Powell.

Two papers with an introductory summary have been initiated and are now in preparation on the subject of steam and electric drives. These papers are being written in collaboration for the purpose of uniform presentation, and are scheduled for one of the fall meetings this year.

Another activity initiated that will result in a symposium during the 1932 Winter Convention, is suggested by the lack of information reported to the committee on the viewpoint of system operation from the viewpoint of economy and reliability in carrying load schedules. There is considerable literature relating to load division among boilers and generating units

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in single stations, but new problems in load allocation have resulted from the extensive interconnection of systems previously operating as separate units. It is planned to secure a series of papers describing the practise of at least three of the larger operating systems on which steam generation is exclusively employed, and to reserve the subject of combined steam and hydroelectric sources for later discussion.

The Progress Report which follows, covers the developments of the past two years. Together with a bibliography of the more salient literature on power generation.

Recent Developments and Present Status of Power Generation

Volume and Distribution of Generation

The combination of a two-year drought and the business depression had a marked effect upon the distribution of the total electric generation in the United States between fuel burning and hydroelectric plants. From 1921 and 1928 the average yearly increase in total generation was quite uniform at 11.5 per cent, in fuel generation at 10.9 per cent, and in hydroelectric generation at 12.7 per cent. Applying these average rates to the figures for 1928 there is a calculated deficit at the end of 1930 of 12.1 per cent in total generation, 4.0 per cent in fuel generation, and 32.5 per cent in hydroelectric generation. These figures show that the loss in hydroelectric generation caused by the drought allowed fuel generation almost to hold its normal increase notwithstanding a decrease of 1.5 per cent in total generation in 1930. The total generation in 1930 was 96 billions, of which 34.6 per cent was generated in hydroelectric plants in contrast to 40.4 per cent in 1928.

Generating Plant Building Programs

The added steam generating capacity in the United States in 1929 was secured by about 57 per cent in new plant construction and 43 per cent in additions to existing plants. The total added capacity was nearly 2,250,000 kw. It is interesting to note that in 1930, in spite of the fact that the total electric production for the year was about 1.5 per cent below that of 1929, there was nearly as much steam generating plant capacity added as in 1929, a total of slightly over 2,000,000 kw. This was divided 47 per cent in new plants and 53 per cent in additions to existing plants. Apparently the depression of the past year is to be reflected in the amount of capacity contemplated for 1931 additions, which is about 1,250,000 kw. These additions are mostly in existing plants where the added capacity is 75 per cent of the total, a condition approaching that of 1928, when practically all of the new steam capacity was in additions to existing plants.

Compared to 1928, the number of new hydroelectric plants and the capacity installed in 1929, was strikingly small; the new installations in 1930 were over three times greater than in 1929, but they still were appreciably less than in 1928. The plants now under construction that will be completed in 1931 will total about 50 per cent of the additions to capacity in 1928, while present indications are that the new capacity developed in 1932 will be only a fraction of the 1931 installations. Contrasted to the schedule of immediate construction are the probabilities within the next few years of the super-capacity plants that are being considered on the Columbia, Colorado, and St. Lawrence rivers.

Major Influences in Current Design of Steam-Electric Plants

The fundamental factors influencing the design of modern steam-electric generating plants are about the same as those which dictated design in the past. They are only modified by new and better apparatus now available, such as improved and alloy steels which make possible the use of higher pressures and temperatures, improved fuel burning equipment and furnaces which permit of much higher boiler ratings and greater efficiences, together with numerous other improvements in auxiliary apparatus.

For a number of years the tendency of steam plant investment costs was upward and these increased costs were justified by the considerable improvement in economy. The point has now been reached where improved efficiencies are becoming increasingly difficult to secure, and any increase in capital cost is difficult to justify. It would seem, therefore, that the effort to lower production costs must lie in reductions in capital investment through simplified design, raising the output of the equipment, and increasing the capacity for a given unit of space.

Probably the most attractive feature in a program of simplification is the possibility of reducing the number of boiler units for a given turbo generator output. Reports published within the last year indicate that the modern boiler has a reliability factor practically equal to that of the turbo generator. It would, therefore, seem that spare boiler capacity should be considerably reduced over past practise up to the point where boiler and turbine capacity would be equal. In other words, the boiler and turbine might be considered as one complete unit. This would be particularly practical where there are several units in the same station, in which case a certain degree of flexibility could be secured where a boiler and turbine forming separate units might be out of service, in the one case for boiler inspection or repairs and in the other case for turbine inspection or repairs.

The greater cleanliness in the operation of fuel preparation and feeding apparatus will now permit the installation of turbines, boilers, and fuel equipment in single buildings without the use of the dividing walls which have up to this time been considered necessary. There has been a number of instances where plants have been constructed without these walls and are

giving very satisfactory operating results. The omission of dividing walls not only saves the cost of the wall but reduces the amount of building steel and permits of a more compact arrangement of apparatus, thus bringing about a natural reduction in the cubic contents of the building. In certain cases a reduction in operating labor has also been secured.

Outdoor installations of boilers, turbines, and electrical equipment are advocated by some engineers and many attractive studies have been brought forth. To date, however, no large outdoor installations have been projected. While such installations are not in entire favor with operating engineers the reduced investment has made the designs of interest.

In practically all of the older designs there was a great deal of duplication in the piping layouts. Numerous cross connections, loops and sectionalizing valves were used, partly with the idea of protecting against failure of any part of the piping system and partly due to the desire to protect against failure of apparatus used in the feed-water heating cycle. Experience with operation of piping, valves, and equipment has shown that their factor of reliability is, in general, as great as that of the turbines and boilers. The opinion has

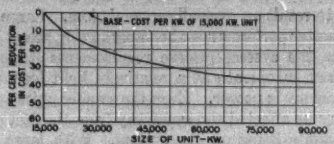

Fig. 1—Per Cent Reduction in Unit Cost of Single Shaft Turbo-Generator Units Designed for 450 Lb. 775 Deg. Fahr. Steam

frequently been expressed that provisions made for cross-connecting and sectionalizing have never been used even though they have been in the plant for a considerable number of years. Simplification of piping design and reduction in number of valves, joints, etc., would reduce both the capital expenditure and the operating cost.

Considerable thought has been given to reducing the number of duplications of auxiliary equipment, such as condensate, air and circulating pumps for turbo generator units. In designs where several spare boilers were provided it was the practise to install single auxiliaries such as stoker drives, forced and induced draft fans, whereas on the turbo generator units it was customary to duplicate the auxiliary equipment. With the present consideration being given to reduction and possible elimination of spare boilers, the question of duplication of auxiliaries should be carefully considered. The usual argument that it is good insurance to protect the large unit by the installation of duplicate and comparatively cheap auxiliaries, carries some weight. Against this, however, must be balanced the lower capital and operating costs by minimizing the number of pieces of apparatus and simplifying the piping design.

The use of larger units, both boilers and turbines, has resulted in a lower unit cost of equipment and in a marked reduction in the cost of the building per kilowatt of capacity. There is a number of installations either already made or pending, where the capacity of the new turbo-generator unit is from two to three times that of the unit for which the building was originally planned. It is evident, therefore, that the

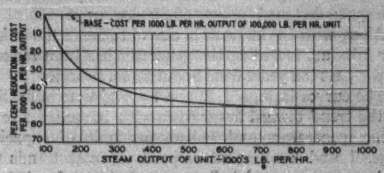

Fig. 2—Per Cent Reduction in Cost of Boiler Units Designed for 450 Lb. 775 Deg. Fahr. Steam

Prices used in plotting the curve include cost of boiler, superheater, soot blowers, air heater economizer, and pulverized fuel equipment

cost of this portion of the building per kilowatt would be reduced to one-half or one-third of the original cost. A few years ago the cost of buildings per kilowatt of capacity was roughly $30. It is, therefore, evident that the possible savings in building alone due to use of larger equipment results in a material reduction in cost per kilowatt. This would have a particular application where equipment is to be installed in a vacant space in an existing building or where it is desired to increase the capacity by replacing equipment in an old plant which is strategically located in the distribution system.

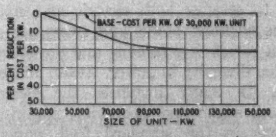

Fig. 3—Per Cent Reduction in Cost of Single Shaft Turbo-Generator Units Designed for 1,000 Lb. 775 Deg. Fahr. Steam Reheated to 775 Deg. Fahr.

The reduction in floor area has assumed importance not only from the standpoint of building and land value but from the resulting reduction in cost of much of the piping and auxiliary equipment. In order to reduce the floor area some of the modern designs resorted to the vertical compounding of units as typified by the Jersey Central Power and Light Company and the Pacific Gas and Electric Company where a high pressure turbine and generator are mounted on top of a low-pressure generator. By such means and by the

increase in the maximum size of turbines from 50,000 kw. in 1920 to 200,000 kw. at present, the floor area has been reduced from an average of 42 sq. ft. per installed kilowatt to less than half that area.

Typical of this trend are the Ford Company in which a 110,000-kw. unit is to be installed where one 12,500-kw. unit is being removed; the Hell Gate Station of The United Electric Light and Power Company where two 160,000-kw. units have been installed in the space designed for two 35,000-kw. units and the Hudson Avenue Station of The Brooklyn Edison Company, Inc., where two 160,000-kw. units have been purchased for the space designed for two 50,000-kw. units.

Along with the increase in size of turbines the weight per kilowatt including the generator has been reduced in the past ten years from about 28 lb. to 15 lb.

The curves in Figs. 1, 2, and 3 will give an indication of the trend toward lower costs with increased size of turbo generators and boilers per unit of capacity. Figs. 1 and 2 represent respectively cost trends for turbo generator and boiler units designed for 450 lb. pressure and 775 deg. fahr. total temperature. Fig. 3 is representative of a compound single-shaft turbo generator where the initial steam condition would be 1,200 kw. and 775 deg. fahr. total temperature, with steam bled at 400 lb. and reheated to the initial temperature.

Along a line totally different from the selection and arrangement of equipment, and also for the purpose of reducing the cost of power generation, has been the cooperative interconnections adopted in the last few years between steam-electric stations and industrial plants. The latter in some cases require large quantities of steam under conditions that may or may not be suitable for the supply of power necessities; in other cases the industrial plants may have the disposal of byproduct fuel either solid or gaseous. It has been found advantageous to both parties in a variety of industries to arrange both electric and steam interconnections whereby power and steam are generated in the most economical fashion, and the fullest benefit derived from diversity in power, steam, and byproduct conditions. In many instances the electric plant supplies both steam and electricity, requiring certain modifications in the standard feed water system. Recent literature has described such arrangements at Deepwater, New Jersey, Rochester, New York, Toledo, Ohio, Baton Rouge, Louisiana, Mobile, Alabama, and Cedar Rapids, Iowa.

Factors at Present Determining Maximum Size of Boilers and Turbines

While large unit size turbo generators reduce the cost per installed kilowatt they sometimes represent a greater capital charge for reserve capacity. A check of available published information regarding peak loads and installed capacity in fourteen large central station systems indicates the following reserve capacity in per cent of the total installed:

Four systems were between 30 and 36 per cent.
Seven systems were between 20 and 30 per cent.
One system was about 17 per cent.
Two were less than 10 per cent.

The same published information indicates that the two systems having the smallest reserve capacity have a substantial percentage of purchased power, so that it is very probable that actual reserve capacity would be materially increased by available spare capacity through interconnection. In one of these systems, where the per cent of reserve capacity is the greatest, it is known that the generator capacity reported on was not all available at the time the tabulation was made, since the boiler capacity in one of the principal plants limited its capacity to a point considerably below the generator name plate ratings. The selection of the very large units in some of the recent developments may have been due to a program of interconnection or expansion which it was estimated would occur within a few years following the installation of the new machine.

Allowance must be made for the loss of the largest unit during operation and also for the possibility of its being out for repairs. If, therefore, units are purchased out of proportion to the size of the system there is likely to result an over-all economic loss. Large units show better performance at comparatively heavy loads. Due to the system reserve requirements it may be necessary to operate at a relatively light load where the economy is poorer. Even the smaller units operating may be affected the same way from the fact of the larger units being on the system. With large units the total amount of auxiliary power requirements for any given system output will also be increased at low system loads. With a large unit carrying heavy load a greater system disturbance would result if the unit should suddenly trip out. Other things being equal the time required for starting a large unit is greater than for one of smaller size.

Transportation difficulties for damaged parts, a longer time necessary for repairs, and larger floor area necessary for dismantling are some of the factors which must be weighed against the lower unit investment and lower labor charges.

Turbine manufacturers have indicated that up to 200,000-kw. capacity in single shaft machines would be feasible. By compounding on multiple shafts the limits, of course, would be considerably higher. One of the limiting features in size would be in connection with the generator and here the use of double windings or raising the voltage will offer a partial solution of the electrical problems.

The limitations to the building of large boilers are more structural than economic. Since at present larger capacity is concentrated in single turbines than in single boilers the problem of boiler reserve is of less importance. Also the experience with pulverized fuel boilers having capacities in the neighborhood of

1,000,000 lb. of steam per hour indicates an availability factor during demand periods approaching 100 per cent.

Large boilers have an exceedingly flat efficiency curve and can be operated at relatively low loads at efficiencies equal to that obtainable from boilers of smaller size. The parts involved in boiler construction are small and do not involve any transportation problems.

Rehabilitation of Existing Steam Plants

Many studies are still being made by steam station designers to increase station capacity by means of improved thermal cycles using higher temperatures and pressures, but not much actual revamping has taken place in the past year or so because of the falling off in power station demand and construction. Then again, while many studies of rehabilitation have been made, all recent improvements in steam station design have been quite broad in their application involving considerable major equipment. Generally speaking, where there has been a demand for increased capacity it has been for such large increments that invariably more consideration is given to the building of new stations with units of large capacity than to some modification of existing generating facilities.

Some steps looking to improved steaming have been taken, such as the installation of water-cooled walls frequently combined with pulverized fuel firing with resultant greater output from the same boiler; also, the installation of air-heaters resulting in a reduction of stack losses. Recent studies made on binary cycles indicate that improvements in efficiency can be made in old plants employing old boilers.

The important recent developments of interest to steam station designers both in the United States and abroad which will, no doubt, in the future have a great influence on rehabilitation, have been:

a. The installation of a 10,000-kw. mercury turbine in the South Meadow Station, Hartford, Connecticut.

b. The installation of high-pressure steam (750 lb.-2,800 lb.) units at Langerbrugge Power Station in Belgium.

c. The construction of new peak load plants in Germany of low pressures.

d. The construction of heat storage plants in Germany.

Rather than follow these lines of securing an increase in capacity and improvements of economy by changes in old stations, the trend in American practise is, with a few exceptions, more along the lines of building new efficient high-pressure steam plants and utilizing interconnections to a greater degree for permitting the operation of the new stations at high capacity factors on base load, and retaining the older less efficient stations for peak loads. With ample interconnections, addition of new capacity at old load centers is not often thought to be as economically advantageous, everything considered, as establishing new generating points where ample land, rail, and water facilities are available.

Reduction in Operating Costs of Steam-Electric Plants

The production of electricity from steam central stations has developed so remarkably in the last decade that it seems fitting in this report to review the progress that has been made during this time.

The average unit production cost has decreased about 50 per cent. The heat consumption per kilowatt hour has been reduced from about 30,000 B. t. u. in 1920 with a best record of 18,000, to an average of 18,500 B. t. u. in 1930 with the best record approximating 12,500 B. t. u. This is for stations using steam only. The mercury steam cycle installation at the South Meadow Station of the Hartford Electric Light and Power Company reports an average heat consumption of around 10,500 B. t. u. per kw-hr.

The progressive increase in the size of turbo generator and boiler units together with the improvements that have resulted in the reduction of supervisory requirements have brought about a constant decrease in labor costs. Great improvement has been made in control equipment with the consequent extension of the scope of supervision of individual operators.

Boiler maintenance costs have been reduced by the use of cooled furnace walls and from the studies by engineers of the effect of operating conditions on maintenance, with a consequent increase in availability of equipment.

Allocation of the load to various stations of a system on a basis which gives a minimum combined cost for each momentary change in load demand and at the same time taking into consideration the effect of loading on the maintenance of equipment, losses incidental to starting and labor costs, has brought about a great reduction in operating costs.

For example, for any particular load demand on the system there is one combination of units and loads at the various stations which will give the lowest combined cost for the system. The determination of this equipment set-up is made by grouping various combinations of units at the different stations through the use of cost input-load output and incremental cost characteristics of the units.

These cost input-load output and incremental cost characteristics are computed from tests on the equipment and it is very important that these tests be obtained with a high degree of accuracy. An operating factor based on a knowledge of the performance of the unit under normal conditions of operation is applied in order to obtain the true normal operating characteristic of the unit. The amount of equipment and location of equipment operating at the various stations must in all cases conform to the standard system operation reliability, which takes into consideration such factors as physical layout, storms, voltage regulation, and protection of the load.

With the determination of the amount of equipment operating at the various stations any change in system demand will be made on the basis of the station incre-

mental cost curves for the equipment operating to a point at which it becomes more economical to operate on another set-up. This method of loading for every momentary change in demand gives a minimum cost for all ranges of load, limits the number of units operating to those required for economy and reliability, and results in a greater utilization of the more efficient units.

In the case of one large power system during the last year the generating capacity of the system was increased about 2 per cent whereas the output delivered by the newer units was raised 13 per cent. As a matter of fact on that system during the year 1930 considerably more kilowatt hours were generated than in 1929 at a total production cost lower than that of 1929.

Table I shows the actual operating results obtained from several different types of stations. The low heat rate obtained by Station A indicates the possibility of a greater adoption of the mercury steam turbine in the near future.

River Rouge plant starts operation this year of the very interesting steeple compound 110,000-kw. turbo generator. The initial steam condition is 1,200 lb. at 750 deg. fahr. Live steam reheaters will raise the temperature of exhaust steam from the high-pressure element to 550 deg. fahr. at 80 lb. absolute pressure before it enters the low pressure element. Of particular interest is the fact that this new machine is installed in space originally occupied by two 12,500-kw. turbines and sufficient space will remain for the addition of a second 110,000-kw. unit of the same type at a later date. Two boilers, each having a maximum steam capacity of 700,000 lb. per hour, will serve this turbine.

The Port Washington plant will have one boiler of 690,000 lb. of steam per hr. to operate at 1,300 lb. pressure and initial and reheat steam temperatures of 825 deg. fahr.

In the realm of boilers for pressures above 1,400 lb. there has been no development except in that of size.

TABLE I—RECENT OPERATING RESULTS

Station	Pressure lb./sq. in.	Temperature deg. fahr.	Cycle	Load factor per cent	Period	Heat rate B. t. u./net kw-hr.
A	70	884	Mercury		Dec.-Feb. 1930	10,180
B	1200 350	750	Reheat regenerative	72.9	Oct. 1930	13,910
C	600	725	Reheat regenerative	65.5	Dec. 1929	12,455
D	1250	750	Reheat regenerative	52.0	Oct. 1930	14,091
E	600	725	Reheat regenerative	71.5	Oct. 1930	14,581
F	600	730	Reheat regenerative	62.7	Feb. 1930	13,368
G	400 250	725	Regenerative	54.8	Oct. 1930	16,290
H	400	700	Regenerative	59.3	Oct. 1930	15,712

Technical Developments in Steam-Electric Plant Design

The seven high-pressure stations described in the last report have been in operation long enough to confirm the results expected from them. Excepting continuing trouble with boiler feed pumps in some of the plants, no unsolved difficulties have been encountered. The more prominent additions to the list of high-pressure plants within the past two years have been the State Line plant near Chicago, the River Rouge plant of the Ford Motor Company in Detroit, and the Port Washington plant of the Milwaukee Railway and Light Company.

State Line plant, described in previous reports, was placed in operation in 1929 with a single three-element turbo-generator unit of 208,000-kw. capacity. The initial steam conditions were 630-lb. pressure and 730 deg. fahr. The extension to the plant now under construction includes two single shaft units of 150,000- and 125,000-kw. capacity respectively. The steam conditions will be 1,200 lb. and 825 deg. fahr., using boilers fired with pulverized coal from unit type mills. An interesting feature in connection with the turbines will be the reheating of steam from the high-pressure cylinder to 825 deg. fahr. at 400 lb. pressure, and its reintroduction into the high-pressure cylinder for expansion to the cross-over pressure of 15 lb. absolute. Reheating will be done in boilers of the reheat type.

The Benson and Loeffler boilers are now being built in Europe in capacities above 100,000 lb. of steam per hour; three Loeffler boilers, each with a designed output of 150,000 lb. of steam per hour at 1,850-lb. pressure and 932 deg. fahr. have recently been ordered for a plant in Czechoslovakia. The Benson boiler in use at the Langerbrugge plant in Belgium produces 220,000 lb. of steam per hour at 2,270-lb. pressure. An experimental boiler to operate at 3,200 lb. pressure, being installed at Purdue University, will be the first at that pressure in America. Boilers for 1,800 lb. pressure to supply steam to two 6,400-hp. triple expansion engines are now in course of erection at the Philip Carey plant in Cincinnati. The design of European types of high-pressure boilers was reviewed in the 1929 report.

There is some evidence that the average steam temperature in large power plants in this country has increased slightly during the past two years, and the maximum temperature in use is now considered to be approaching 800 deg. fahr. The Langerbrugge plant in Belgium has operated at a temperature of 850 deg. fahr., for 4½ years and in general there is more of a tendency to use extremely high temperatures in Europe than in this country. In certain respects modern European boiler designs lend themselves to the generation of high temperature steam, and the practise of multi-barrel turbines, common in Europe, is also

adaptable to its use. In the United States the Detroit Edison Company has recently put into operation a 10,000-kw. turbo generator at its Delray No. 3 plant, designed to use steam at 1,000 deg. fahr. At the present time the turbine is operating on steam from the station main under 400 lb. pressure and at 750 deg. fahr. For high temperature operation the supply steam will be passed through a separately fired superheater using oil for fuel. The steam main between the superheater and turbine is provided with several types of pipe joints, and other arrangements have been made to note the effect of the high temperature upon the piping, turbine, and superheater, so that the installation constitutes in reality a full scale experimental investigation.

During the past two years the mercury vapor boiler and 10,000-kw. turbine installation of the Hartford Electric Light Company has been perfected to the point where it was available for 82.3 per cent of a period covering 7,520 hours. Excluding causes arising from the use of mercury, the installation would have been available 95.1 per cent of the time. The average thermal rate was 10,310 B. t. u. per net kw-hr. Of the total energy generated, 42.5 per cent was produced by the mercury turbo generator, and the remainder by the steam formed in the mercury condenser.

As a result of the experience at Hartford, mercury vapor units of 20,000-kw. capacity each are being planned for the Kearney plant of the Public Service Corporation of New Jersey and for the power plant of the General Electric Company at Schenectady. The latter installation and an associated steam boiler will be one of the first outdoor steam plants. The mercury vapor pressure will be increased to 125 lb. per sq. in. in these new units, and it is expected that a thermal rate of 8,800 B. t. u. per net kw-hr. will be secured at a load of 20,000 kilowatts on the mercury turbine and with the power developed from the 240,000 lb. of steam per hr. produced at 400 lb. pressure in the mercury condenser.

Higher steam pressures and investigations of troubles resulting from caustic embrittlement have indicated the desirability of boiler drums without seams. Forged drums have been used exclusively for the 1,400-lb. boilers, and lately hammer-welded drums for medium pressure boilers such as the four drum boilers in the James H. Reed Station of the Duquesne Light Company in Pittsburgh, that operate at 400 lb. pressure. The possibilities of fusion welded boiler drums have been studied intensively in the past two years, and "Proposed Specifications for Fusion Welding of Drums or Shells of Power Boilers" were published in the March 1931 issue of *Mechanical Engineering*. The Boiler Construction Code does not at present permit fusion welding of parts where the safety of the structure is dependent upon the strength of the weld.

While no boiler installations have been made of larger capacity than those in the East River and Hell Gate plants in New York that were described in the last report, operating experience at the East River plant has shown that loads can be carried on the boilers there considerably in excess of the expected capacity. A unit generation of 1,250,000 lb. of steam per hour has been maintained, the actual operating limit being met in the capacity of the steam piping leading from the boiler unit. A boiler of 1,000,000 lb. per hr. capacity was recently installed in the Kips Bay plant of the New York Steam Corporation on a floor area originally allotted in 1926 for a boiler of half the output. These large boilers are in reality twin-set boilers having two independent water circulating systems and steam uptake connections, located over a common furnace. The largest single boiler to date has a capacity of about 600,000 lb. of steam per hour.

The installation of boilers of large capacity has focused the attention of operating and designing engineers upon the problem of boiler availability. Although in most plants the number of boilers still exceeds the number of turbo generators, boiler outage in many cases impairs to some extent a plant's generating capacity. Protection of the furnace walls with water cooling surface, improved reliability of the coal feeding and burning equipment, as well as of the ash handling apparatus, increased attention to the details of the auxiliary apparatus, and correct boiler water conditions, have resulted in numerous instances of boiler availability equal to that of turbo generators. The first statistical study of this subject on a large scale was directed in 1929 and 1930 by Mr. C. F. Hirshfeld of this Committee, and was published in *Transactions*, A. S. M. E., Vol. 52, No. 27, p. 265. The data compiled covered only a year and there were several other reasons why the statistics were necessarily inconclusive, but they seemed to indicate that the details of design and construction and operating skill are more important in obtaining high boiler availability than such matters as rate of output per unit of surface, type of fuel, and method of firing. The average availability factor appeared to be between 84 and 90 per cent.

During the period being reviewed, the point of major technical interest in connection with the pulverized coal method of combustion has been the long-time comparisons in the same plant of the unit and bin systems. No generally accepted opinion regarding their merits has been reached. Both systems were installed in the Deepwater plant, New Jersey, where it has been reported that the unit method of firing gave a higher efficiency but a lower reliability. Other comparative studies have been recently published for central stations in Chicago and St. Louis.

Experimentation has continued with the slag tap type of furnace for pulverized coal firing, and its use has been extended. The major difficulty has been the gradual expansion in a horizontal plane of the furnace floor, with consequent bulging of the furnace walls. This results from penetration of the hearth bed by the molten slag, through cracks formed when the furnace is cooled during boiler shutdowns.

The size and total fuel burning capacity of underfeed

stokers continue to increase, and steam outputs per boiler of 500,000 lb. per hr. are now possible with stoker firing. There has been only a foot increase in the maximum width of stokers in the last ten years, but the length has been increased from about 13 ft. to 26 ft. 7 in. The largest underfeed stoker in this country is now being installed in the Hudson Avenue plant of the Brooklyn Edison Company, and will be 26 ft. 7 in. long by 26 ft. wide, with a grate area of 694 sq. ft. Stokers of this size have been made possible by extensive cooling of the furnace walls by water cooled surface, improved coal feeding and distributing mechanism, and better control of the air distribution to the several parts of the grate surface and ash pit. In come cases automatic control of the air distribution has been installed. Rotary ash dischargers are almost standard for the larger size of stokers.

Although the fuel burning capacity of stokers per foot of width has been greatly increased, there has been no increase in the rate of combustion per square foot of total grate surface, which is limited by the possibility of clinkering and the blowing of coal from the firebed. Neither has there been any modification of the fact that the combustion rate depends upon the type or burning characteristics of the coal burned. Maximum rates of 60 to 70 lb. per sq. ft. for 10 hour periods have been maintained with good quality eastern coals; 50 to 60 lb. with coals from the central field. Slack coal reduces the maximum rate to the neighborhood of 40 lb. per sq. ft. With all coals an increase in the combustion rate results in an increased loss from incomplete combustion, cinders, soot, ash pit, radiation and unaccounted for losses, in some reported tests these losses reaching a value of 20 per cent.

The limiting temperature for preheated air with underfeed stokers seems to be in the neighborhood of 350 to 400 deg. fahr. Although some installations have attempted to use higher temperatures, it has been found that the stoker maintenance has been adversely affected.

The installation of large turbo generators has continued although no units have been projected larger than those mentioned in the last report. The largest machines now on order are two 160,000-kw. units for the Hudson Avenue plant of the Brooklyn Edison Company. It is noteworthy that all of the turbo generators except one, that are now on order in sizes greater than 100,000 kilowatts, are of the tandem compound type. The exception is the 110,000-kw. high-pressure vertical-compound machine for the Ford Motor Company. The development of generators of large capacity was an essential to the use of tandem-compound turbines.

The record in capacity for single cylinder turbines operating at 1,800 rev. per min. is now carried by two 80,000-kw. turbo generators in the Charles R. Huntley station of the Buffalo General Electric Company. Steam supply to these units is at 435 lb. and 750 deg. fahr.

In the field of high-speed turbo generators, single cylinder condensing units are now in operation in the United States up to 10,000-kw. at 3,600 r. p. m., and tandem-compound units at the same speed up to 15,000 kw. A 15,000-kw. single-cylinder condensing unit of English manufacture has recently been installed in Canada and the C. A. Parsons & Company are willing to build single-cylinder units of 25,000-kw. capacity to operate at 3,600 r. p. m.

Intensive study in turbine design is being made in connection with the use of higher steam temperatures. This problem as well as that of blade erosion is a matter of metallurgy, and the developments to date indicate that it will soon be possible to operate turbines at initial steam pressures above 500 lb. without the necessity of reheating. At the present time a 60,000-kw. unit has been built to use steam at 650 lb. and 825 deg. fahr., and illustrates the simplification and turbine arrangements that will be possible when higher steam temperatures are available.

High-Voltage Generators

There has been a significant demand for generator units of larger and larger capacities, and while this has possibly been the condition since the beginning of power station construction it appears to advance by periods with the most recent one occurring within the past five years. With an increase in generator capacity the designing engineers have been faced with the problem of providing suitable and reliable insulation, conductors, supports, and terminals both internal and external to the machines. Machines of larger capacity for a given voltage mean greater currents. Greater currents mean larger conductors, and consequently greater weights to support normally and greater stresses to provide for at times of short circuits. With the greater currents other external difficulties also arise, such as the provision for the installation of large cables if the power is taken away in underground cables, the reduced carrying capacity of the cables if many are required in restricted spaces, and the provisions necessary to nullify the effect of the sheath voltages. Consequently there has been a very natural trend to higher generator voltage and to multiple-winding machines to reduce the amount of current per conductor.

Unless it is necessary to consider generation at a pre-established voltage and the distribution problems connected therewith, in other words, if generation alone, is to be considered, the most economical voltage usually increases slightly with the increase in the size of the unit. From the generator manufacturer's standpoint, for generators of 100,000 kw. or more, voltages in the neighborhood of 16,000 to 18,000 volts are reported to result in the most economical design. In some instances, economics dictate machines within this range of voltage with transformers for stepping up to the desired transmission voltage, so that, where the generator voltages are not well established, it might be said that for machines above 100,000 kw., the present

trend is toward the use of multiple-winding machines with transformers or auto-transformers, rather than to higher voltage machines. The trend will no doubt continue toward multiple-winding machines until the point in capacity is reached when it will prove more economical to increase the voltage.

Some attempts have been made to standardize on generator voltages above 13,800 volts, but since the introduction of successful multiple-winding machines, the study and demand for higher standard voltages has not been so insistent, and may be said to be dormant for the time being. In other words, the double-winding design has made it possible to increase the capacity without increasing the voltage, and to postpone the day when it will be necessary to go to higher voltage. It is not at all unlikely that in the future, generators will be built with more than two windings. The principal difficulty with more than one winding, is in finding sufficient space at the terminals of the machine windings, to install potheads and make connections with satisfactory clearances.

Within the past three years there have been installed in the territory adjacent to Chicago, and there are now in operation, seven 22,000-volt generators with a combined generating capacity of 474,000 kw., ranging in size from 52,500 kw. to 105,000 kw. Additional 22,000-volt generators totaling 432,000 kw., and one 18,000 volt, 115,000-kw. generator are on order for this territory, to be installed during the years 1931-1932.

Outside of the above there have been but four steam-driven generators placed on order or installed recently in this country for voltage ratings above 13,800 volts, namely:

Two 160,000-kw., 16,500-volt units for the Brooklyn Edison Co., and

Two 94,000-kw., 16,500-volt units for the Southern California Edison Company.

Abroad, the outstanding recent high-voltage installations which will be watched with considerable interest are the two 33,000-volt, 25,000-kw., 3,000-r. p. m. generators at Brimsdown, England, and the 36,000-volt, 25,000-kw., 3,000-r. p. m. generator at Langerbrugge, Belgium.

Switch Structures, Switching, and Control

The housing of major switching in both congested and open localities is receiving much study, particularly new installations, because of the advent of metal-clad switchgear. There has been a marked increase in and trend toward the use of metal-clad switchgear in place of indoor cell type and outdoor open type bus structures. Many hundred indoor and outdoor type metal-clad combined switch and bus units of American design are now in service, some with air and some with oil-filled metal enclosed bus compartments.

The first installation in this country, and possibly in the world, of high-voltage outdoor metal-clad switchgear was the 33-kv. substation of the Public Service Company of Northern Illinois, at Wheaton, Illinois. This has been in service since March 1928. Since then there have been many other installations which embody improvements in design and manufacture. This type of switchgear can now be obtained in practically all capacities for which circuit breakers have been developed, including 132 kv. One 132-kv. installation is now being made. The earlier large units involved a considerable amount of field assembly. The latest designs permit complete factory assembly and testing of very large capacity units, such as a full double bus section for one circuit, complete with instrument transformers, buses, etc., for breakers of 22-kv., 3,000-amperes, 2,500,000-kva. interrupting capacity. These units are shipped completely assembled and oil filled thereby entailing a minimum of field assembly and installation work, and avoiding the necessity of drying out, reconditioning, etc., by purchaser.

In the case of several generating stations it has been found that an outdoor oil-filled metal-clad switchgear structure could be installed cheaper than an isolated or separated phase indoor cell structure if the cost of the building is taken into consideration; in some cases a saving as high as 20 per cent has been indicated. Other companies report the reverse where installation conditions are different. One company has proved by actual cost data that for 33-kv. substations the total over-all cost of metal-clad outdoor switchgear erected and ready for service, compares favorably with that of the conventional open type formerly used. This latter comparison, however, would not yet hold for all conditions and localities.

Additional advantages to be gained from complete factory built equipment are the saving in space, the salvage value in case of replacement or rearrangement, and the much greater safeguards to operators and service.

There is a growing demand for faster operation of circuit breakers in case of fault. The manufacturers have made considerable progress toward meeting this condition in the design of circuit breaker operating mechanisms and contacts and methods of extinguishing the arc as well as in the high-speed relay designs. Tests have shown repeated operations where the arcing time was as low as one to two cycles. Large breakers are now available with a conservative over-all time rating of eight cycles. In view of these developments it is quite possible a more severe duty cycle than the 2-OCO may be considered for the rating of oil circuit breakers.

The interest indicated in locating equipment outdoors is not confined entirely to switchgear. There are a few instances where large rotating units have been installed outdoors, and the subject is being given much study.

There has been an increase in the tendency toward the use of steel switchboard panels and the use of hinged panels or cubicals. The desirability of smaller switchboards in the larger power stations and substations is also receiving attention. Various attempts

have been made to use miniature equipment, and a smaller number of items, and combination units of instruments and relays. At the same time there is a growing demand for more relays and instruments for one purpose or another. The supervisory system of control seems to be arousing more and more interest as a means of condensing or consolidating the control of a large number of circuits within the convenient reach of a single operator.

The so-called distance relays are becoming more popular. The automatic synchronizing, automatic frequency, and automatic load division control are receiving more attention. Although the first has been available for a number of years, the latter are more recent developments and it is quite possible that the demand for both may grow.

If the predictions indicated in recent announcements with reference to vacuum tube developments, such as the "thyratron," may be anticipated, we are about to see some very wonderful and radical changes in the entire scheme of station control, switching, transformation, transmission and control station design in general. To what extent and how rapidly this may take place is at present problematical. The possibilities we must admit, however, are tremendous. Already there have been some novel commercial applications and very promising laboratory tests extending over a considerable period.

A new type of adjustable-speed control for a-c. motor drive is now available, permitting practically a straight line speed curve with infinitely small steps from zero to maximum. The drive can be installed at nearly the same cost as the drives previously used. The method of control permits high efficiency which in some cases shows a considerable saving over any other type of control. The drive will maintain constant speed regardless of load, is simple, permits the use of standard electrical design features, and to a considerable extent, standard machine parts. To date the service applications have been confined to boiler draft fans and boiler feed pumps, but the drive is adaptable to many other services where fine speed adjustments, and simplicity of operation with high efficiency, are important factors. Thirty-five of these equipments were ordered in 1930, some of which have now been in service for several months. For those who are not familiar with the "Rossman Drive" system it may be of interest to state it consists primarily of an induction motor of standard design, except that the mechanical construction is modified to permit what would be the stator to revolve. This revolving stator is connected by texrope or other means, to a smaller standard d-c. machine fed from a standard motor-generator set or other d-c. source. The speed of the small d-c. machine is controlled by adjusting the voltage impressed on its armature by the generator of the motor-generator set. This may or may not be supplemented by field control. By varying the speed and direction of rotation of the d-c. machine, the load may be driven at any speed within the design range.

Factors at Present Affecting Hydroelectric Development

The increasing attention directed to the large-scale hydro developments on the Colorado and St. Lawrence rivers by governmental agencies is perhaps a new factor influencing the trend of hydroelectric installations. At the same time the federal waterpower licensing policies are being questioned and subject to criticism by both private and political interests, and both have been coincident with severe drought during the past two years in many parts of the country. The complementary value of hydro and steam power sources on systems supplied mainly by hydro has been fully realized, and the principal hydroelectric systems on the Pacific Coast, in the southeastern states, in New England, and in the vicinity of Niagara Falls, have either put into operation during the last two years steam plants of large ultimate capacity or have affected large capacity ties with existing steam plants in adjacent territory. The recent availability of low cost gas and oil fuels in the West, and the advances made in the thermal efficiency of large boilers and turbines, combined with their attractive investment costs, are causing a pronounced predominance there of steam-electric installations that will continue for an unknown period. In the East and South the necessity of steam reserves for low-flow periods has accelerated steam expansion programs. At Niagara the entire water diversion permitted under the existing treaty is utilized and the load growth necessarily must be assumed by steam generation.

Type of Steam Plants on Hydroelectric Systems

A marked similarity in certain design features is noticeable in all of the new steam plants supplementing power systems hitherto dependent upon hydro generation. Gas and oil are the outstanding fuels in the West, while in the East pulverized coal exclusively is fired. Boiler capacities range from 400,000 to 550,000 lb. of steam per hr. at pressures between 425 to 475 lb. per sq. in. with the exception of one plant on the Pacific Coast operating at 1,300 lb. per sq. in. Turbine capacities vary from 40,000 kw. up to 100,000 kw., and one to three boilers are installed per turbine, the usual number being two. The selection of pulverized coal firing appears to be made for reasons other than the emergency starting of boilers; and in fact extra facilities for standby operation of turbines and boilers are provided in only one of the recent plants, which operates on a system where steam generation will be a minimum yet service demands require continuous availability of the steam-electric generator.

General Features of Recent Hydroelectric Development

No hydroelectric plants of unusual total capacities have been put into operation during the past two years, although there have been one or two new records for

turbines of large physical dimensions, and the three 49,000-hp. Francis units in the Waterville plant in North Carolina surpass in capacity all previous units of that type operating under high heads. An unusual construction procedure was adopted for the Chute-a-Caron plant on the Saguenay River in Canada. The preliminary drawings for the Boulder Dam plant on the Colorado River reveal no unusual features except the height of the dam and the contemplated power capacity. The Spier Falls plant on the Hudson River is an interesting example of rehabilitation and design investigation by means of model tests. In fact the increasing use of model testing, and its application to the design of the propeller type turbine wheel, constitute the outstanding technical advance in this country in hydroelectric practise during the past two years.

Propeller Turbine Development and Model Testing

Notable installations of propeller turbines of the fixed and manually adjustable blade types were in operation or approaching completion in 1928, and one small capacity wheel of the automatically adjustable type, the Kaplan turbine, was in the course of erection. Since 1928 there have been numerous installations in America of small capacity Kaplan type propeller turbines, that have been designed and installed under European licenses. The Kaplan turbine was first developed in Europe where there are now some large capacity wheels in operation. It is essentially suitable for low-head developments, being used for heads up to about 70 ft. European designs of the fixed blade and Kaplan turbines have been based upon extensive model testing of the turbine wheels as well as of the associated inlet casings and draft tubes.

One of the principal objects of testing models of propeller type wheels has been to determine the limiting conditions of cavitation, for which adjustable draft head conditions are required in the test laboratory. Cavitation, or the formation of bubbles of vapor and air in low-pressure areas adjacent to the runner surfaces, causes a drop of wheel efficiency, but its most prominent effect is the rapid disintegration of the runner material. Cavitation pitting had been encountered in this country with Francis type wheels under certain conditions of specific speed and draft head, and an amount of field experience has been accumulated by which proper operating conditions can be selected for Francis wheel installations. One of the chief advantages of the propeller turbine is the higher specific speed that is possible, but as field data were lacking for such operation, it was considered essential in Europe to determine the optimum installation conditions in advance of construction by means of cavitation testing. Following such investigations propeller turbines of record dimensions, horsepower, and water discharge capacity have recently been installed in the Ryborg-Schworstadt and Kembs plants on the Rhine River in Europe. Pertinent data about these plants are as follows:

Plant	Ryborg-Schworstadt	Kembs
Type of runner	Kaplan	Fixed blade propeller
Number of units	4	5
Head in feet	35	54
Turbine rating-horsepower	35,000	36,000
Water discharge-cu. ft. per sec	10,500	6,800
Rev. per min	75	94
Runner diameter-inches	276	220.8

Cavitation laboratories have been built during the past year, one or two in the United States and one in Canada, for the investigation of design problems concerning propeller type wheels. The construction of a national Federal hydraulic laboratory has also been recently authorized; model testing has been done extensively at one of the technical colleges in the United States for the design of dam spillways, water courses, and turbine settings, and two or three other colleges and individual companies have done a certain amount of such investigation. The first large plant in this country to install automatically adjustable blade propeller turbines is now being built at Safe Harbor on the Susquehanna River in accordance with data obtained from cavitation and wheel setting tests. Six turbines having a rating of 42,500 hp. each at a gross-head of 55 ft. are being installed initially. The wheels will have a diameter of 220 in., a water discharge of 8,000 cu. ft. per sec. at rated head and output, and will operate at 109.1 rev. per min.

Briefly described the Kaplan turbine consists of a propeller, or axial flow turbine, whose blade angle is controlled by the speed responsive governor that controls the turbine wicket gates. Separate servo-motors are provided for operating the blades and gates, and a cam is used in the governing mechanism to direct the movement of the runner blades according to the movement or position of the wicket gates. The servo-motor for operating the runner blades forms part of the turbine shaft, and is usually located just under the generator coupling. The links and crank arms for moving the blades are enclosed in the runner hub.

By automatically maintaining the correct relation of blade position to gate opening, the peak efficiencies of a series of propeller wheels each having the proper blade angle for a particular turbine load, are secured, with the result that the efficiency-horsepower curve of such a turbine is quite flat compared to similar curves for fixed blade propeller and Francis turbines, the efficiency at low loads being materially higher. Another feature of the Kaplan turbine is the overload capacity possible by adjustment of the blades, which is of particular value in times of reduced head during high river stages. It is expected that the efficiency of the Safe Harbor turbines will exceed 85 per cent over two-thirds of the load range, and to equal 90 per cent or better for the third quarter of the unit capacity.

Rehabilitation of Hydro Developments

The 57,000-hp. unit recently added to the Spier Falls plant on the Hudson River, New York, exemplifies two distinct tendencies in hydroelectric practise, illus-

trating in addition to the use of model testing as a design adjunct, the recognition of the value of an over-developed plant for supplying peak-load capacity and the benefit derived from a storage resource by a run-of-river development. At full load the wheel discharge is 7,000 cu. ft. per sec., making it the largest Francis-type runner to date in this respect as well as in physical dimensions. The minimum regulated flow of the Hudson River at Spier Falls is 3,000 sec.-ft., and 5,000 sec.-ft. for 50 per cent of the time. Both the turbine and generator approach record sizes in physical dimensions, the total weight of the rotating element being about 450 tons. The generator stator is 37 ft. in diameter, and the diameter of the scroll-case entrance of the turbine is 26 ft. The unit operates at a speed of 81.8 rev. per min. under a head of 81 ft. Tests of a model of the forebay and intake structures enabled the best forebay outline to be determined with regard to amount of necessary excavation balanced against the head loss in forebay and intakes. The tests revealed that the area of the intake gates could be economically reduced 20 per cent, and this was of special value because the water passages are so large that they gave rise to structural design difficulties. It is reported that the tests resulted in a saving equal to ten times the cost of the tests.

High Head Plants

The Waterville plant of the Carolina Power and Light Company on the Pigeon River in North Carolina has an unusually high operating head, 840 to 750 ft., for a hydro plant in the eastern part of this country. There are three 49,000-hp. reaction wheels in the plant, operating at 400 rev. per min. and these are of record note for their capacity at such a head. The six-mile tunnel from the reservoir created by the dam to the power house is unprecedented even in the notable high head installations on the Pacific Coast. The use of cast steel butterfly valves of 9 ft. diameter under a total head of 840 ft. located in the penstocks near the power house, has been reported quite successful, and is an advance in the size of such valves under high pressure. Johnson valves are also provided in the penstocks where they enter the plant.

The record for large capacity high head Francis wheels will shortly be established in the Diablo plant of the City of Seattle, where two turbines of 90,700-hp. capacity each, to operate under a head of 327 ft. are being installed. The turbines will have cast steel scroll cases and concrete elbow draft tubes.

The record head, however, for Francis wheels occurs in a European plant, the Bringhausen plant in Germany, where four 40,500-hp. horizontal-shaft Francis turbines are being installed to operate under a head of from 950 to 1,000 ft. This is a pumped storage development mentioned later.

Large Scale Hydro Developments

The Beauharnois development on the St. Lawrence River in Canada is notable for the ultimate capacity being planned when the entire flow of the river passes to the power house through the combined navigation and power canal connecting two of the lakes on the flow line of the river. The initial section of the plant will contain ten 50,000-hp. units, to operate under a head of 83 ft. The final capacity will be 2,000,000 horsepower. The construction of the canal between lakes St. Francis and St. Louis, which will be 15 miles in length with a width of 3,000 ft. and a depth of 27 ft. is outstanding because of its magnitude rather than any engineering difficulties.

Another plant that illustrates the scale of development possible on some of the rivers in Canada is the Chute-a-Caron plant on the Saguenay River where four 65,000-hp. units have been installed as the initial step of the future Shipshaw plant of 1,000,000 horsepower. The Chute-a-Caron dam will serve to divert the river for later construction of the Shipshaw plant, but because the head at Chute-a-Caron is only 151 ft. as contrasted to that of 205 ft. at the parallel site, the Chute-a-Caron plant will be operated in the future only during river flows sufficient to supply both plants. The difficulty of construction in the gorge of the Saguenay River is thus being solved by the step-by-step development at the two neighboring plant locations. The river was diverted in a unique manner for the construction of the Chute-a-Caron dam by toppling a concrete cofferdam 92 ft. long into the swiftly flowing current, which caused the flow to pass into an artificial channel previously prepared. The action of the concrete monolith weighing 11,500 tons was accurately predicted by means of model tests.

Storage Developments

The largest hydroelectric plant in New England has been recently completed at Fifteen Mile Falls on the upper reaches of the Connecticut River, in conjunction with the first 220-kv. transmission line in New England. The plant is designed for four 53,000-hp. units at 175-ft. head, and is the first development at two nearby sites which will eventually have a total combined rating of 300,000 horsepower. The completion of the second plant will create a reservoir for regulating the river flow not only for the plants at Fifteen Mile Falls, but also for the plants at Bellows Falls and Vernon, farther down the river. The 220-kv. line carries the output of the plant to a distributing point in the vicinity of Boston.

The Osage plant at Bagnell, Missouri, now approaching completion, will contain six 33,500-hp. units operating under an average head of 90 ft. Without storage this plant would not be feasible since the river flow varies from 324 to 110,000 cu. ft. per sec. with an average of 10,500 sec-ft. The dam will create a reservoir 95 sq. mi. in area having an impounded volume of 87 billion cu. ft., so that the flow of the river will be completely equalized with a draw-down of 30 feet. The average yearly energy available will be about 400 million kw-hr. The Osage plant is also a late example of the omission of a superstructure over the

generator bay, the station crane having one gantry leg while the other end of the crane rests on top of the outer wall of the electrical bay which is placed over the penstocks between the dam bulkhead and the generator bay. Low metal housings are provided over the tops of the generators.

The Saluda storage development in South Carolina, described in the last report, was placed in operation last year and is notable for its reservoir capacity of 102 billion cu. ft.

Outdoor Type Plants

The omission of the generator room superstructure continues both for small plants of a single unit and for plants of large installed capacity. Generator designers are predicting that the flexibility of design now possible with fabricated construction, and the latitude permissible in the layout of auxiliary power and excitation systems, will lead to the arrangement of generators to conform to the design of the power house, and that the logical outcome may be the wider use of outdoor generator designs.

Pumped Storage Plants

No installations of pumped storage plants in America have followed the completion of the Rocky River plant in Connecticut, although the use of pumped storage continues popular in Europe where electric system peaks are of shorter duration than in this country. One illustration is the Bringhausen plant on the Weser River in Germany, now under construction, which will contain four combination pump and waterwheel units, each consisting of a 40,500-hp. Francis type horizontal turbine coupled directly to a 36,000-kva. generator, on the other side of which is a clutch for coupling to a 100,000-gal. per min. pump requiring 29,000 hp. for pumping. The plant is connected to an upper reservoir at an elevation 1,000 ft. higher by two penstocks 3,250 ft. in length. The water to be pumped to the reservoir amounts to 26,800,000 cu. ft., from which 500,000 kw-hr. will be generated. Power for pumping during off-peak hours is obtained from interconnected steam plants and other hydro plants on the system.

The horizontal coupling of a pump and a turbine with a generator and/or motor seems to be the preferred arrangement for pumping plants in Europe, and is in contrast to the Rocky River plant layout of pumping units with individual drive motors separate from the main turbine and generator, all of the equipment being of vertical type. The Rocky River pumps which are the largest high-head pumps in this country, have a volute discharge casing without diffusion vanes, and are of the single inlet type. On test the Rocky River pumps showed a maximum efficiency of 91.7 per cent; the efficiency expected by the designers of the German pumps is approximately 80 per cent.

Hydroelectric Generator Developments

There has been a widespread adoption during the past two years of fabricated steel plate construction of both stator and rotor of vertical waterwheel generators, in place of the use of castings formerly standard. Such generator construction of welded steel plate is lighter in weight, is considered to be more reliable mechanically, and is more adaptable to design arrangements. There has been an increased use of the umbrella, or overhung type of generator rotor, in which by means of downwardly curved rotor arms the centerline of the rotor rim or pole faces is brought below the horizontal centerline of the hub. By locating a combined guide and thrust bearing, sometimes of the spherical type, just below the rotor hub, a very material reduction in over-all generator height is secured. The use of rolled steel plates and fabricated construction for the stator has also facilitated the arrangement of ventilating ductwork in connection with the water tube air coolers that are now used very largely for generator cooling. Other advantages of fabricated plate construction are quicker manufacture and possibility of design for shipment without restriction by permissible weights or clearances.

Acknowledgments

This report has been compiled by a subcommittee, the members of which respectively prepared the several sections as follows:

F. A. Allner, Hydroelectric Topics; N. E. Funk, Major Influences in Current Design of Steam-Electric Plants, and Factors at Present Determining Maximum Size of Boilers and Turbines; F. H. Hollister, Rehabilitation of Existing Steam Plants, High-Voltage Generators, Switch Structures, Switching, and Control; H. W. Leitch, Reduction in Operating Costs of Steam-Electric Plants, and Technical Developments in Steam-Electric Plant Design.

The subcommittee wishes to express its appreciation of the helpful suggestions received from the main body of the Power Generation Committee.

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Power Transmission and Distribution

ANNUAL REPORT OF COMMITTEE ON POWER TRANSMISSION AND DISTRIBUTION*

THE Committee has continued with its work during the past year through a number of subcommittees organized as in preceding years. All of these subcommittees have been very active in studying the problems in their respective fields, and in promoting investigation work and the preparation of papers for presentation before the Institute. The reports of the work of the several subcommittees follow.

Subcommittee on Distribution

One of the most interesting developments in connection with the distribution of electrical energy during the past year has been the practical application of the network principle to primary distribution. The economies and improvements to service which are made possible through the adoption of the low-voltage network have been so clearly demonstrated by the generally satisfactory performance of the many recent installations, that a wider application of the principle is not surprising.

At the present time there is being installed a 4-kv. primary network in Pittsburgh, Pa. This will serve a residential district of comparatively low-load density which was previously served by an overhead radial system.

The application of the network principle to 4-kv. service for such an area requires certain radical departures from the designs which have become fairly well established for distribution. Since underground construction in the areas of low-load density is not usually justifiable from an economic viewpoint, overhead construction must be used. The primary network deserves careful consideration from the standpoint of economy, reliability, and system planning in handling loads in areas not readily adaptable to low-voltage network distribution.

The past year has seen a continued application of vertical network distribution in large office buildings. This type of distribution has been used in a number of large cities including New York, Pittsburgh, and San Francisco. At the present time several features of design have not been settled. These features are being given consideration and it is expected that with the extended application of the vertical network principle a better understanding of them will result.

The question of secondary faults on low-voltage a-c. networks is still being discussed at length. There is a feeling among some engineers that the problem is not radically different from that encountered on the d-c. network. Others feel that short-circuit currents available are generally less and the condition is therefore more difficult. The use of buried cable has also been studied and tests made which will be reported in the near future.

Subcommittee on Lightning and Insulators

Past Year's Activities in Investigation Work

The activities in the field of lightning investigation during the past year have been carried on by means of:

1. Field investigation of natural lightning.
2. Field investigation with artificial lightning.
3. Collection of operating data on lightning performance of lines.
4. Theoretical analysis and coordination with the past lightning data.

In the field investigations, use was made of cathode-ray oscillographs, klydonographs (and surge recorders), surge indicators, lightning intensity recorders, direct-stroke recorders, and artificial lightning generators. This year the field investigation was extended to include a wood-pole line on which was recorded lightning voltages in the order of 5,000,000 volts.

By concentrated efforts in the field with the artificial lightning generator, a vast amount of knowledge has been gained on the effect of tower footing ground resistance, counterpoises, effect of normal voltage in sustaining a power arc in the presence of lightning flashover of a line, and particularly on the operation of the lightning arrester.

The behavior of traveling waves at the junction of cables with overhead lines was further studied by the use of the artificial lightning generator with particular attention to the application of lightning arresters.

Extensive use was made of the lightning (direct) stroke recorder and our knowledge in regard to the

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Harold C. Dean,
L. L. Elden,
H. S. Evans,
R. D. Evans,
F. M. Farmer,
J. H. Foote,
K. A. Hawley,
L. F. Hickernell,
J. P. Jollyman,
A. H. Lawton,
L. L. Perry,
T. F. Peterson,
D. W. Roper,
A. E. Silver,
D. M. Simmons,
C. T. Sinclair,
L. G. Smith,
Philip Sporn,
W. K. Vanderpoel,
C. F. Wagner,
H. S Warren,
T. A. Worcester.

current in direct strokes amplified, and made more definite.

The cathode-ray oscillograph application was modified to trace the history of a natural lightning wave over a period of some 2,000 microseconds, in addition to recording the initial characteristics of the waves.

Surge indicators were used for the first time, although mentioned in last year's report. These definitely established the fact that a lightning flashover of a transmission line can take place without sufficient follow-up current flowing to trip out the line.

The past year has brought out some intensive study of a theoretical nature on the lightning problem dealing with the mechanism of the lightning stroke and the subsequent surges which may appear on a transmission line. Both the direct stroke and induced stroke have been analyzed and the conclusion pointed out that only direct strokes appear to have consequential effect on the highly insulated lines, while medium and lower insulated lines will be affected in an increasing order by induced strokes.

Results of Last Year's Work

It is impossible here to point out in detail all the results of last year's investigation. There are, however, some outstanding accomplishments, among which the following are cited.

1. The effectiveness of ground wires in reducing lightning voltages and line outages has been further demonstrated.

2. The value of low tower footing resistance in lessening the effects of lightning voltages has been shown.

3. Currents in the lightning stroke in the order of from 50,000 amperes and upwards have been indicated.

4. Counterpoises at a tower footing, even although of comparatively short length are beneficial in reducing the lightning voltage to which a line may be subjected.

5. Lightning sparkover of a line can take place without power follow current.

6. Lightning voltages of both direct and induced stroke origin appear to be present on transmission lines. It is not known in what normal insulation and voltage range the induced voltage ceases to be a factor in line flashover.

Insulation Coordination

In order to coordinate insulation on a lightning basis, its lightning characteristics must be known. Whether these are secured on a single impulse wave or voltage time lag basis, a standard test method must be agreed upon by all who supply this type of data. The Committee has this matter well in hand at the present time, with the probability that an agreement will soon be reached by all actively engaged in this work whereby a group of two or more preferred waves will be used for securing this type of data.

Insulator Flashover Values.

It has been known for some time past that discrepancies have existed in 60-cycle flashover values assigned to insulators of practically the same physical dimensions. The cause for these different flashover values has been traced to a variation in humidity conditions at the time and place of test. A great deal of experimental work has been done on this problem, and it is expected the information obtained will soon be made public through a group of papers before the Institute.

Subcommittee on Interconnection and Stability Factors

In order to coordinate the work of the Power Generation Committee, the Protective Devices Committee and the Power Transmission and Distribution Committee in the field of interconnection and system stability, there has been formed a Joint Interconnection Subcommittee. This Subcommittee was organized during the year and has arranged several sessions of interest in the field of transmission and distribution. These include the session on System Grounding at the Pittsburgh Meeting in March 1931 and the session on System Interconnection for the Summer Convention of 1931. In addition, plans are under way for a symposium at the Winter Convention of 1932 on the subject of Machine Characteristics and System Stability. In last year's report mention was made of the organization of a Subject Committee on Definitions of Terms Used in Stability Investigations. The work of this Subject Committee is now substantially complete and it is planned to present it as a paper at the Winter Convention in 1932.

Subcommittee on Cable Developments

This year has witnessed the continued practically universal use of shielded cable (type H) for all three-conductor high-voltage cables. Other tendencies are the increased use of oil-filled cables, the use of super-calendered paper, a great increase in the use of non-metallic armored cables for direct burial in the ground for low-voltage work, and the use of steel-wire armored, non-leaded cables for submarine service and for vertical distribution cables in high buildings.

The Following Cable Installations May Be Noted:

1. A second circuit, 175,000 kva. (winter rating), of single-conductor 132-kv. oil-filled cable about 6 miles long by Commonwealth Edison Co. in the same conduit bank as their first circuit. The new circuit has 1,100,000 cir. mils conductors as compared with the 600,000 cir. mils previously used. The larger conductor size is made commercially feasible by insulating sleeves and grounding devices to eliminate sheath losses. Due to the many improvements in accessories and technique, the net result was that the investment cost per kva. of capacity is less than one-half the cost of the first line.

2. Two lines of single-conductor 600,000-cir. mil 132-kv. cables each three-quarters of a mile long installed by the Commonwealth Edison Co. one (90,000 kva. rating) similar to the cable of paragraph 1 above, and the other (95,000 kva. rating) with novel features of oil channels cut in the interior of the lead sheath as well as an oil channel in the conductor, a simple and small stop-joint, making it possible to have every joint a stop-joint, which joint is practically a factory-made and factory-tested joint.

3. The Deepwater installation of the United Gas Improvement Co. across the Delaware River consisting of two 60,000-kva. circuits (and two spare cables) of single-conductor 750,000-cir. mil 75-kv. solid type cable manufactured and installed in single lengths each about 4,000 ft. long. These are the first 75-kv. submarine cables and longest lengths of power cable ever manufactured.

4. Two circuits rated at 55,000 kva. each of single-conductor 750,000-cir. mil 75-kv. solid type submarine cable of the Union Electric Light and Power Co. of St. Louis across the Mississippi. The cables were made and installed in single lengths of about 2,800 ft. each.

5. An installation of five three-conductor 48,000-kva. 66-kv. oil-filled cables by the Union Gas and Electric Co. of Cincinnati. This is the first three-conductor 66-kv. oil-filled cable, and is buried direct in the ground. The conductor size is 500,000 cir. mils sector, insulated with 0.40 in. impregnated paper and shielded. The cables are made in single lengths 1,300 ft. long, thus avoiding joints, and are steel-tape armored cables buried directly in the ground.

6. Three commercial installations have been made during the year of three-conductor 33-kv. to 35-kv. oil-filled cable, although the amounts were rather limited. These installations were made by the city of Los Angeles, the Staten Island Edison Co., and the Syracuse Lighting Co. All three installations were made with three-conductor type H cable with oil channels in the filler spaces. A novel feature was the three-conductor positive stop-joint by the city of Los Angeles, and also the fact that a large installation of cable with oil channels in the filler spaces including a more viscous oil was installed by this company, who after installation replaced the oil in the channels by oil of the type used in oil-filled cables.

7. An experimental installation of single-conductor 132-kv. oil-filled cable by the Commonwealth Edison Co. of Chicago in cooperation with two manufacturers, each of whom supplied 1,000 ft. of 450,000-cir. mil or 500,000-cir. mil cable insulated with 0.40 in. paper and 1,000 ft. with 0.50 in. paper. The purpose of this experiment is to study the limiting stresses and temperatures in oil-filled cable for present and future voltages.

8. An experimental installation of single-conductor 132-kv. cable by the Public Service Electric and Gas Co. as a continuation of their 1927 experiment studying the possibility of "solid type" insulation for 132-kv. service. The installation consists of three cables each of 500,000 cir. mils and 1,000 ft. long, two supplied by one manufacturer and one by another. Each manufacturer supplied a length of solid-type cable. The third cable is of an intermediate or hybrid type, using oil intermediate in viscosity between that of an oil-filled cable and of the usual type of cable, thus greatly facilitating installation methods while sharing in part the operating advantages of an oil-filled cable.

The Subcommittee on Conduit and Manhole Temperature of the N. E. L. A. Underground Systems Committee has standardized on conditions and constants for calculating current-carrying capacity. This standardization has made it possible to calculate current-carrying capacities of cables covering a large range of sizes and conditions. These tables of carrying capacities will be published by the Underground Systems Committee, and should prove of great value to the industry.

Subcommittee on Steel Transmission Towers and Conductors

The Subcommittee on Steel Transmission Towers and Conductors was organized to assist in the standardization of steel transmission towers and conductors by clarifying the problems associated with their design and application, by papers prepared by recognized authorities on these subjects and presented before the Institute.

The work of the Subcommittee has been divided into three groups, each of which is responsible for one of the following subjects:

I—Towers.

II—Clearances and Electrical Characteristics.

III—Conductors.

During the past year, four meetings of the Subcommittee have been held, together with some additional meetings of groups within the Subcommittee.

An addition to the clause of the N. E. L. A. specifications on uplift of tower foundations and a recommendation for a straight line compression formula for the N. E. L. A. specifications is in course of preparation. Consideration is being given to elimination of the term "factor of safety" by substituting therefore "per cent overload." Contact is being maintained with the A. S. T. M. on its investigation of embrittlement of hot dipped, galvanized steel.

It is proposed to recommend that the 60-cycle flashover be the basis for clearance of the tower under winter or heavy loading conditions and that lightning flashover be the limit for summer or light loading conditions. The width of the right-of-way is to be limited by similar conditions.

Intensive study is being given to the vibration of conductors. Efforts are being directed principally to the coordination of experimental work being done by widely separated investigators, including California, Texas, Illinois, Michigan, Ontario, England, and Australia. A bibliography is being established as an adjunct to this work.

The present membership of this committee, including

the chairman, is now twenty, with five men in Group I, six men in Group II and eight men in Group III.

It is felt that the work of this subcommittee is now progressing smoothly and during the next year, concrete results can be shown as the result of its activities.

Conclusion

For the sake of brevity, this year's report excludes the bibliographies of papers which appeared in last year's report. It is planned to include such bibliographies in the Annual Report at appropriate intervals, rather than every year.

In conclusion, it is desired to stress the conscientious and enthusiastic work of the chairmen and members of the subcommittees to whose liberal contributions of time the progress made is due.

Transportation

ANNUAL REPORT OF THE COMMITTEE ON TRANSPORTATION*

YOUR Committee on Transportation submits a brief review of the recent developments of importance in the application of electricity to transportation.

Steam Railroad Electrification

Pennsylvania Railroad

The Pennsylvania Railroad has been actively continuing its electrification program. On June 29, 1930 electric operation was inaugurated between Philadelphia and Trenton, N. J., a distance of about thirty miles. On July 20th, electric operation was started between Philadelphia and Norristown, Pa., on the Schuylkill Division, seventeen miles. Fifty-nine multiple unit cars have been recently acquired. This installation completes the suburban electrification about Philadelphia. Distribution of power is at 11,000 volts, 25-cycle, single-phase. The equipment for suburban service is at present all motor cars, with multiple unit control.

The new suburban terminal at Broad Street was opened on September 28th. This is of necessity, entirely electrically operated, for the tracks are, like those of the passenger terminal at New York, under cover. The new suburban station at Thirtieth Street on the west bank of the Schuylkill River was also placed in service at this time. This will ultimately be an integral part of the Pennsylvania Terminal at West Philadelphia now under construction.

The Pennsylvania Railroad now has in electrified operation about Philadelphia, 132 miles of route and 437 miles of track; 345 multiple unit cars are in service.

Work is actively progressing in the New York Terminal zone preparatory to changing from the 600-volt third rail to 11,000-volt, single-phase trolley wire between Sunnyside Yard and Manhattan Transfer. This installation, which will among other things allow New Haven locomotives to enter the Manhattan Terminal, should be completed during the present year.

Construction is progressing between New Brunswick and Manhattan Transfer and will shortly be started between New Brunswick and Trenton, and between Wilmington and Washington.

Four types of locomotives are being developed for the complete electrification. Two passenger types, one freight and one yard switching. The light passenger type (designated as 0-1) is of 2-B-2 arrangement, and two of these units will be used on the heavy passenger trains. The heavy passenger type (P-5) is of the 2-C-2 arrangement. The freight type (L-6) is of the 1-D-1 arrangement. The passenger locomotives will use twin motors rated at 500 hp. per armature, or 1,000 hp. per axle. The freight locomotive will be equipped with four motors of single armature type, and will thus have a rated capacity of about 2,000 hp.

Long Island Railroad

The multiple unit suburban service of the Long Island portion of the Pennsylvania Railroad, operated by third rail distribution, is rapidly growing. Eighty-five multiple unit cars now being added to the equipment, bring the total number of cars to 825. To provide power for the increasing load, the Floral Park, substation is being remodeled and will be equipped with two rectifiers of 3,000-kw. capacity. Four new substations—Hempstead, Manhasset, Rockaway Park, and Goose Creek—are being built, in addition to four planned for installation next year. Each of the new substations will contain one 3,000-kw. mercury arc rectifier, except at Goose Creek where portable synchronous converter equipment will be installed. The rectifiers will be adapted for 60 or 25 cycle operation, an external reactance being added for the lower frequency.

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*COMMITTEE ON TRANSPORTATION:

Sidney Withington, Chairman,

P. P. Ash,	K. T. Healy,	A. S. Richey,
R. Beeuwkes,	D. C. Jackson,	S. A. Spalding,
A. E. Bettis,	J. Murphy,	N. W. Storer,
J. V. B. Duer,	I. Otverholm,	W. M. Vanderslula,
H. H. Field,	H. Parodi,	R. H. Wheeler,
I. W. Fisk,	W. B. Potter,	R. P. Winton,
W. A. Giger,	R. H. Rice,	O. I. Wright.

Delaware, Lackawanna & Western Railroad

The suburban electrification of the D. L. & W. R. R. was inaugurated between Hoboken and Montclair, N. J. on September 3, 1930, the service to South Orange, N. J. September 22, 1930 and that to Morristown, N. J. December 18th. The present electric installation was completed with the electric operation in January 1931 to Gladstone and Dover, N. J. This electric operation comprises about sixty-eight route miles and one hundred and fifty-eight miles of track; and includes 282 multiple unit cars. These cars operate in pairs, the units consisting of one motor car and one trailer each. Five substations, each containing mercury arc rectifiers, furnish the distribution power at 3,000 volts direct current. Two electric three-power locomotives (oil, storage battery, and trolley operated) have been acquired and are used for switching and transfer service between Jersey City and Secaucus freight yards. The D. L. & W. is the first heavy traction service in this country to utilize 3,000-volt direct current on motor cars, and the first to install exclusively mercury arc rectifiers.

New York Central Railroad

The Cleveland Union Terminal electric operation was inaugurated June 28, 1930. This project comprises 17 miles of route and about 60 miles of track, between Collinwood and Linndale, Ohio, and employs twenty-two locomotives. These locomotives are 2-C + C-2, weighing about 204 tons, of which 150 tons is on drivers. They are designed for a maximum speed of 70 mi. per hr. The locomotives are used interchangeably for switching or road service. Train heating boilers, oil-fired, are provided on the locomotives. The power distribution is by means of 3,000-volt direct current, using a catenary system. This is supplied from two substations, one 3.5 miles west, and the other 7.2 miles east, of the Union Station; each equipped with motor-generators, remotely controlled from the terminal building by a supervisory system of the synchronous selective type. The installed capacity is 9,000 kw and 6,000 kw. respectively.

The New York City electrification of the New York Central is progressing. The "West Side" tracks will be electrified by the middle of the current year as far south as Seventy-second Street, the tracks north of that point being on a viaduct, or on private right-of-way. Forty-two freight locomotives have been purchased for this service. Distribution of power is by direct current at 650 volts using a third rail. South of Seventy-second Street the tracks are on city streets, and the operation will be accomplished by three-power (oil-electric-storage battery) locomotives, of which the railroad has purchased thirty-five.

Reading Railroad

The electrification of the Reading Railroad at Philadelphia is progressing, and operation should be started about the middle of the current year. This project includes about 69 route miles and 161 miles of track; seventy multiple unit cars will be operated initially. The distribution system is at 11,000 volts, single-phase, 25 cycles.

Great Northern Railway

The Great Northern Railway has placed in service four new motor-generator locomotives with axle arrangement 1-C + C-1, essentially similar to those of that type already in operation. Each locomotive weighs 263 tons, of which 210 tons is on drivers. The distribution of power is at 11,000 volts, 25 cycles, single-phase.

Illinois Central

The Illinois Central Railroad placed in service in 1930, four 100-ton straight-electric locomotives for switching service, and also five 600-hp. oil-electric locomotives of the Diesel type. The motive power now in suburban service consists of 280 multiple unit cars, 140 of which are motor cars and 140 are trailers. In the freight switching service, there are now four straight-electric locomotives and six 600-hp. oil-electric. The power distribution is 1,500 volts, direct current. About 154 track miles are electrified.

New York, New Haven and Hartford Railroad

The final extension of the New York, Westchester and Boston Railway to Port Chester, N. Y. was completed late in 1929. The New York, Westchester and Boston—an integral part of the New Haven electric operation, now operates from Harlem River to Mount Vernon, N. Y., with branches to White Plains, N. Y. and to Port Chester, N. Y. The total New Haven a-c. electrified mileage is now about 577 track miles, over approximately 116 miles of route. The road operated, as of the end of 1930, 141 electric locomotives and 267 multiple-unit cars. The New Haven last year placed in service nine multiple-unit cars. During the current year there will be added to the equipment, ten 200-ton passenger locomotives and 33 multiple-unit cars. The power distribution of the New Haven is at 11,000 volts single-phase, 25 cycles, but the passenger motive power is designed to take power also from the New York Central third rail at 650 volts, direct current.

Substations for Electrified Railroads

The conversion of power for distribution on the Delaware, Lackawanna and Western electrification is, as previously noted, entirely by means of mercury arc rectifiers. This installation consists of 18 units in 5 substations with a total capacity of 40,000 kw. The rectifiers are also provided with a means of compounding, so that voltage can be maintained over a fairly wide load range at substations. Reference is made to the report of the Technical Committee on Automatic Stations covering this feature. Four of the substations are manually operated but are provided with many of the usual automatic protective features. One sub-

station is completely automatic. Remote and supervisory control are provided for the automatic substation and the tie stations.

In its electrification the Pennsylvania Railroad has continued with the 11,000-volt distribution system now in use on other parts of its system involving step-down transformer substations installed at intervals of 7 to 8 miles. The Reading electrification consists of the three-wire distribution system with 11,000 volts on the trolley and a feeder of 33,000 volts. Auto transformer stations are used at intervals, to transfer energy from the 33,000-volt feeder to the 11,000-volt trolley. Some of the substations of the Pennsylvania Railroad electrification are remotely controlled from adjacent signal towers, and those of the Reading are controlled by supervisory control so that the power director indirectly or directly has control of all circuit breakers within his jurisdiction.

The Cleveland Union Terminal Substations contain two and three 3,000-kw. motor-generator sets respectively, with complete automatic control. A synchronous selective supervisory control system with remote telemetering provides the dispatcher, located in the Terminal Building, with complete supervision and indication of the substation, feeders and tie station.

High-speed air circuit breakers have been employed in all substations of the d-c. systems mentioned above, and the Pennsylvania Railroad and Reading Railroad a-c. systems have employed high-speed a-c. breakers of the air and oil type. Reference is made to the report of the Committee on Protective Devices for data regarding the various types of high-speed circuit breakers available for a-c. and d-c. service.

Electric Locomotives

A novel development in the field of a-c. locomotive design has occurred abroad. In this design the low-voltage terminals of the transformer are permanently brought out, and all voltage control for the motors is by means of an oil-immersed tap changer and air switches from the high-voltage windings. The arrangement is a development of the growing use of the scheme of "tap-changing under load" and if it is found to be practicable under service conditions, should materially simplify the design of a-c locomotives.

A novel development in electric locomotive design has been accomplished in the new locomotive of the Illinois Central Railroad. This compensates the tendency of reduced weight upon the forward drivers at high drawbar loads, especially at starting, on account of the moment of the drawbar pull and rail reaction. The arrangement consists of shunts for weakening the field of the leading motor of each truck, which reduces the tractive effort of the leading motor and increases that of the rear, in proportion to the field strengths of the two motors. The forward drivers thus show less tendency to slip and the drawbar capacity is therefore increased.

There has been a number of notable improvements recently in the design and construction of the single-phase motor which have materially decreased its weight per horsepower and have improved efficiency and power factor. Attention is called to the report of the Committee on Electrical Machinery for descriptions of these developments.

Oil-Electric Locomotives

In addition to 35 oil-electric-storage battery locomotives acquired by the New York Central Railroad for the "West Side" operation in New York City, seven similar units have been placed in service in Chicago, and as above stated, two on the Delaware, Lackawanna & Western Railroad. The Canadian National last year acquired a Diesel locomotive, as have various industrial concerns.

A novel arrangement of Diesel locomotives has been developed, wherein the engineer's driving position is raised to a sort of mezzanine floor, so the engineer may look over the top of the train he is moving. In this design, controllers, air brake handles, etc., are provided at both sides of the cab so the driver may move from side to side as desired to observe signals from the switching crew.

Rail Cars

Fifty-six rail cars of gasoline and oil electric design were produced in this country during the past year. As an indication of the increasing use of this type of equipment in heavier service, the average power of these cars was about 450 hp. as compared with about 350 hp. the average in 1928.

Marine Transportation

A number of turbo-electric and Diesel-electric vessels of various types was produced in 1930. The *S.S. President Hoover* is the largest electrically driven liner produced thus far. The propelling equipment consists of two 14,000 hp. turbines with three-phase 10,100-kw. generators, and two 13,250-hp. synchronous induction motors. Auxiliary electric apparatus on steam and Diesel propelled craft has developed in a number of different directions, replacing steam auxiliaries. Attention is called to the report of the Technical Committee on Application of Electricity to Marine work.

Aviation

A number of electrical devices has added to the art of aviation during the past year. Among these is the "turn compensator," an electrically driven device to correct the error experienced in all types of "earth inductor" compasses when the direction of the ship is suddenly changed. This device may also be used as a "turn indicator" to show quickly and with reasonable accuracy the angular change in direction.

Recently developed engine-performance indicators, electrically operated, showing oil-pressure, oil-temperature, speed, etc., are of value especially on multiple-engine ships, by permitting remote indication.

In the field of illumination for aviation, wing-lights set into the leading edge of plane wings increase efficiency by reducing non-productive air resistance. Airways are being illuminated by increasingly powerful and efficient beacons. The illumination of landing fields by means of flood lights and marking lights is keeping pace with other developments.

Railroad Signaling

Centralized traffic control by means of coding systems is rapidly being extended on railroads. This enables a dispatcher or tower operator to remotely control switches and signals over a large territory and thus direct trains quickly and effectively without the necessity of train orders.

Urban Transportation

The chief developments in urban transportation are efforts to obtain faster car acceleration, higher speeds, and better braking, to compete with increasingly difficult traffic conditions. The increased use of double-reduction and worm gears has made the operation of higher speed motors practicable with marked decrease in weight (especially in unsprung weight). Lower floors, with greater speed in loading and unloading, are features of some of the new drives.

The modern trolley bus is being used for many services where street cars or gasoline buses were applied previously. Economy of operation and low investment cost insure a widespread use of trolley buses, especially where it is possible to utilize existing power plant, substation, and power distribution facilities. In Chicago, for instance, over fifty-seven miles of line have been equipped for trolley bus operation, and seventy-five buses are in use.

The gas-electric coach, which has hitherto shown its chief advantages in congested urban traffic where frequent and rapid acceleration is necessary, is invading the interurban field.

Attention is called to the expansion of the subway mileage in Philadelphia, New York, and Brooklyn. Plans for a subway in Chicago have been approved.

Rail Fissure Detector

The Sperry device for detecting fissures in track rails has been operated intensively throughout the country with valuable results. Continued use of this device should materially reduce rail failures.

Welding

Electric welding is increasingly used not only in railroad shops (as in other industries), but for building up rails and "special work" at frogs, crossings, etc. Rail bonds for traction and signal purposes are in many instances welded to the head of the rails.

Vertical Transportation

The ever increasing height of buildings necessitates constantly increasing efficiency and speed of elevators. Speeds of 900 ft. per min. are now operated. A recent development to increase space efficiency in high buildings, is the operation of two or more elevators in a single shaft, so synchronized that one elevator may be started and run as "express" for a certain number of floors before the car next below it starts. Needless to say the operation is interlocked by means of limit switches and kindred devices to prevent the cars from coming together.

General

Attention is called to the development of various types of apparatus which apply to transportation as well as to other forms of utilization of electricity as described in various technical committee reports. Included in this category are turbo-generators and other power plant facilities, motors, circuit breakers, insulators, transformers, lightning arresters, cables, relays, substations, etc. Many of these types of apparatus are applicable to transportation, only by accomplishing some more or less important modifications, and the attention of various technical committees has been called to special modifications of this kind.

Attention is called to the work which is being carried on under the auspices of the American Standards Association on Definitions of Electrical Terms (C-42) for which the A. I. E. E. is sponsor. To Subcommittee No. 6 is assigned the field of "Horizontal Transportation." It is hoped that when this very important subject is complete, and published, general use will be made of the definitions and that some confusion which now exists will thus be avoided.

Protective Devices

ANNUAL REPORT OF COMMITTEE ON PROTECTIVE DEVICES*

THE work of the Committee on Protective Devices during the past year was accomplished largely through subcommittees, which are listed below with their chairmen:

Relays, W. W. Edson; Oil Circuit Breakers, Switches and Fuses, L. F. Hickernell; Lightning Arresters, H. K. Sels; Fault Current Limiting Devices, H. B. Wood; Joint Interconnection Subcommittee,† F. C. Hanker.

The activities of the Protective Devices Committee may be divided into the following three classes:

1. Preparation of standards
2. Review of research and development
3. Fostering the preparation and presentation of papers.

General

During the year the work of the committee in co-operation with the National Electric Light Association in the preparation of a supplement to the Relay Handbook was completed.

Revised standards for lightning arresters have been prepared and issued in report form by the Standards Committee.

Progress has been made in the preparation of standards for fuses and revised standards for disconnecting, horn gap and knife switches. It was hoped that the standard for fuses could be sent to the Standards Committee this year, but due to difficulties in obtaining complete agreement as to the subject matter of this standard there has been some delay. These standards should be forwarded to the Standards Committee in the latter part of 1931.

The committee has maintained close contact with research and development relating particularly to lightning protection, high-speed oil circuit breakers, and stability of operation, which greatly facilitated its work in fostering the preparation of suitable papers relating to these subjects.

The work of the committee during the past year has emphasized more than ever the following points. First, the desirability of planning the presentation of papers sufficiently in advance so that they may be grouped for presentation in the most suitable manner. Second, the necessity, if maximum advantage is to be realized from Institute sessions, of having papers available for reading sufficient time in advance of meetings to permit of well considered discussion. The importance of this latter point is so great that every effort should be made to see that it is carried out effectively.

On account of the fact that in a number of cases other technical committees of the Institute have a very definite interest in the application or use of devices coming under the jurisdiction of the Protective Devices Committee, very close coordination of activities is required. This is particularly true in matters relating to interconnection and stability of operation handled at the present time by the joint subcommittee having representatives from the Power Generation and Power Transmission and Distribution Committees, in addition to the Protective Devices Committee. It may be that there will be sufficient important matters which affect a number of committees to warrant selecting some method of handling them that will permit of direct effective action without the complication and delays that are bound to happen in coordinating work with several large committees.

Relays

Report of Subcommittee

Further development of high-voltage transmission and interconnection with rather severe requirements as to stability and continuity of service, have focused attention on the necessity of rapid clearing with extreme precision, and the development of the art of relay protection has advanced as a consequence. Relays have been developed including directional, balanced, and differential relays capable of operating in one-sixtieth of a second.

These are being tried out in service and their operation is being watched with much interest. Apparently, additional work should be done to simplify these relays and their mechanical construction details, so as to insure among other things high-speed operation under all conditions, and to prevent shifting of the balance point. Also, further development will apparently have to be gone ahead with to eliminate certain difficul-

*COMMITTEE ON PROTECTIVE DEVICES:

Raymond Bailey, Chairman,

J. E. Allen,	F. C. Hanker,	A. H. Schirmer,
A. C. Cummins,	E. A. Hester,	H. K. Sels,
H. W. Drake,	L. F. Hickernell,	H. P. Sleeper,
W. S. Edsall,	Dr. M. G. Lloyd,	E. R. Stauffacher,
W. W. Edson,	J. P. McKearin,	H. R. Summerhayes,
L. E. Frost,	H. A. McLaughlin,	E. M. Wood,
E. E. George,	A. M. Rossman,	H. B. Wood.

Herman Halperin.

†This is a joint subcommittee of the committee on Power Generation, Power Transmission and Distribution and the Protective Devices Committee.

ties in the application of these relays to short line sections or to ground protection.

The most important work of the subcommittee for the year is the completion of the revised edition of the Relay Handbook, in cooperation with the corresponding committee of the N. E. L. A.

The following papers, prepared under the auspices of this subcommittee, have been presented before the Institute:

Relay Developments on Southern California Edison System, E. R. Stauffacher, Southern California Edison Company.

Operating Experience with Reactance Type Distance Relays, E. E. George, the Tennessee Electric Power Company.

Protection of Three-Winding Power Transformers, R. E. Cordray, General Electric Company.

Fundamental Basis of Distance Relaying, W. A. Lewis, Westinghouse Elec. & Mfg. Company.

Oil Circuit Breakers, Switches, and Fuses

Report of Subcommittee

A prominent feature of oil circuit breaker development is the number of large installations of metal-clad switchgear. Among these may be mentioned the installation of 33-kv. oil-filled gear on the system of the Public Service Company of Northern Illinois, the 13-kv. gum-filled gear at the Leaside Substation of the Hydro-Electric Power Commission, and air-filled gear on the system of the Jersey Central Power & Light Company.

Since last year's development of the deion circuit breaker, further results have followed the discovery of this method of arc extinction. With the addition of deion grids, ordinary oil circuit breakers have opened 3,500 amperes at 110,000 volts with 1.5 cycles of arcing; and 3,000 amperes at 200,000 volts with 3.5 cycles of arcing or less. Decreased arcing assures more satisfactory breaker performance, less deterioration of oil and contacts and lower maintenance. The stability of the whole power system is improved, since short circuits are more likely to be cleared before they can affect synchronous machines.

The deion oil-less breaker in the meanwhile has been developed for a greater range of ratings. At one extreme is a one million kva. 23 kv. circuit breaker which has undergone interrupting capacity tests including some shots as high as 800,000 kva. (It will be recalled that among last year's achievements were reported successful interruptions of short circuits in the order of 60,000 kva.) On the other extreme is the breaker (using 2.5-inch plates) which repeatedly interrupted 10,000 amperes at 600 volts alternating current. This small deion breaker shows great promise in a wide field of the moderate and smaller powers; such, perhaps, as industrial service.

Efforts to improve rating standards for oil circuit breakers continue.

Announcements by the manufacturers indicate healthy activity in the development of new forms of circuit interrupters, such as the oil-blast oil-circuit breakers, multiple break and others.

The following papers, prepared under the auspices of this subcommittee, have been presented before the Institute:

The Mechanical Performance of Oil Circuit Breakers, A. C. Schwager, Pacific Electric Mfg. Co.

Outdoor Switching Equipment at Northwest Station, W. F. Sims and C. G. Axell, Commonwealth Edison Co.

Magnitudes and Rates of Rise of Circuit Breaker Recovery Voltages, R. H. Park and W. F. Skeats, General Electric Co.

Trend in Development of Modern Circuit Interrupters, J. B. MacNeill, Westinghouse Elec. & Mfg. Co.

Oil Circuit Breaker Tests—Philo 1930; Aims, Set-Up and Results from a System and Operating Point of View, Philip Sporn and H. P. St. Clair, American Gas & Elec. Co.

The Oil-Blast Circuit Breaker, D. C. Prince and W. F. Skeats, General Electric Co.

Field Tests on Standard and Oil-Blast Explosion Chamber Oil Circuit Breakers, R. M. Spurck and H. E. Strang, General Electric Company.

Lightning Arresters

Report of Subcommittee

The Subcommittee is advised that studies of lightning and the effects of lightning were continued throughout the year by means of field investigations on transmission systems in several widely separated localities. The knowledge of the character and behavior of high-voltage surges originating from lightning on transmission lines was extended by the use of cathode-ray oscillographs at field stations located in Arkansas, Illinois, Michigan, New Jersey, Ohio, Pennsylvania, and West Virginia. In two cases, Michigan and New Jersey, the behavior of waves on transmission lines was studied in detail by the use of both portable cathode-ray oscillographs and impulse generators. Practically a continuous investigation was maintained also of the performance of lightning arresters and many other associated phases of the lightning protection problem. About 3,000 oscillograms were taken which have already proved valuable in the study of problems related to lightning protection.

Better coordination of records in the 1930 lightning investigations has indicated the growing importance of direct strokes. Records obtained on high-voltage lines tend to confirm the thought that overvoltage due to direct strokes are more important than those due to the induced effect from lightning strokes primarily on account of their greater magnitude.

Impulse tests made in the field tend to confirm the conclusion that best protection with lightning arresters is obtained when the arrester is located close to the equipment to be protected and the leads as short as possible with low ground resistance. If the arrester cannot be located close to the equipment and at the

same time retain short leads, keeping the leads short appears to be the more important. Also, the arrester and equipment should be connected to the same ground but if this is not convenient, more attention must be given to the arrester ground to be sure that its resistance is as low as possible.

The highest voltage lightning arresters ever built were recently installed by the Hydro-Electric Power Commission of Ontario. Under emergency conditions of generator overspeed and abnormal line voltage, they will not be damaged by voltages as high as 268,400 volts. Normally, four of the twenty-two arrester units in each phase leg are short circuited by a high speed switch which opens automatically in less than half a second when waterwheel overspeed and overvoltage occurs. This is also the largest arrester installation for voltages over 200,000. One three-phase arrester is installed for the protection of each of four banks of transformers.

The commercial development of the porous block autovalve arrester has made available a new arrester which is considerably smaller and has much better performance characteristics than the earlier autovalve. The arrester is constructed in fewer porcelain-cased units with a totally enclosed gap structure which in the lower voltage sizes results in the complete arrester being housed in a single casing or two casings with gaps and elements separate. The protective action of the arrester is similar to the old one but it has practically no time lag in its operation and a low ratio of discharge voltage to normal voltage. A small and compact form of line arrester has been developed which can be applied to prevent the flashover of insulator strings. A number of installations of these line arresters has been under observation for the past two years in order to gather data on the effectiveness of such protection for transmission line insulators.

With the commercial development of thyrite, a new form of lightning arrester became available. The arrester is constructed in the form of self-contained units, it only being necessary for any application to mount an appropriate number of units one upon another. A gap structure has been provided in each unit so constructed that substantially no time lag occurs in the operation of the gap. This arrangement, coupled with the characteristics of the material, insures an arrester which has practically no time lag in its operation. A number of installations of these arresters has been made, the highest voltage arrester in service being of 230 kv. rating. Another form of thyrite arrester was developed which is incorporated in a string of suspension insulators and used to prevent the flashover of the insulator string for all but the most severe conditions.

A new protective device operating on a deion principle in a small bore tube for use in the prevention of insulator flashover by lightning has been developed in certain voltage ratings.

The revision of the proposed Standards for Lightning Arresters for alternating-current power circuits was completed and accepted by the Committee on Protective Devices for printing in report form. The subcommittee hopes that there will be widespread use of these standards in order that they may be put in final shape for adoption by the Institute. It is expected that development in the art of protection from lightning will probably make it necessary to revise these standards from time to time as additional technical information becomes available.

The work of the subcommittee indicates that the following matters should receive considerable attention in the future: (1) the determination of more representative test surges, (2) the development of tests to determine operating condition of arresters in service, (3) investigation of present operating practise on transmission and distribution systems relative to arrester installation, operation and performance as affecting suitable standards, and (4) the coordination of line and station type arrester application in the field with the activities of other committees interested in the grading of system insulation.

The following papers, prepared under the auspices of this subcommittee, have been presented before the Institute:

An Experimental Lightning Protector for Insulators, J. J. Torok, Westinghouse Elec. & Mfg. Co.

Field Tests on Thyrite Lightning Arresters Using Artificial Lightning of 1,500,000 Volts, K. B. McEachron and E. J. Wade, General Electric Company.

Fault Current Limiting Devices

Report of Subcommittee

The reactor installations made during the year are characterized by high electrical ratings and greater physical size, particularly in bus-sectionalizing applications which seem to be on the increase.

Among the largest ever built in physical size is an application of 4,000-kva., 161-kv. bus sectionalizing reactors giving 10 per cent reactive drop when 120,000 kva. is transferred across the reactor.

Another outstanding installation from the standpoint of physical size consists of three 5,400-kva. oil-immersed reactors on the 110-kv. system of the Pacific Gas & Electric Co.

Noteworthy from the standpoint of electrical rating are eighteen bus-sectionalizing reactors rated 7,340 kva. giving a 16½ per cent reactive drop in a 133,000-kva., 27,600-volt circuit now on order.

Current-limiting reactors of the dry type installed in metal housings and rated 833 kva., 693 volts, 1,200 amperes, 60 cycles, were used to sectionalize a 24,000-volt bus. They introduce 5 per cent reactive drop between the sections when 50,000 kva. are being transferred. The housings are supported on insulators and are connected to a ground bus.

Bus-sectionalizing reactors were in some instances provided with disconnecting devices which automatically break the circuit when the reactor is removed from its operating location.

Representative opinion has been secured on the

desirability of including in the new A. I. E. E. Standard for Fault Current Limiting Devices the material on reactors now in A. I. E. E. Standards No. 13 and also information on grounding transformers, generator neutral reactors, and Petersen coils. As yet no definite decision has been reached.

INTERCONNECTIONS

Report of Joint Subcommittee

The work of the Joint Interconnections Subcommittee has related almost entirely to the preparation of papers. A comprehensive paper on *Present Day Practise in Grounding of System Neutrals* was prepared by the subcommittee and presented at the Pittsburgh District Meeting.

A paper entitled *Impulse Test on Substations*, by Messrs. Brookes, Southgate, Roman, and Whitehead, was presented at the Winter Convention.

The following papers were presented at the Pittsburgh Meeting:

Committee Report on Present Day Grounding Practises, C. A. Powel, Chairman.

Reactance of Transmission Lines with Ground Return, J. E. Clem, General Electric Company.

Power System Voltages and Currents Under Fault Conditions, R. D. Evans and S. H. Wright, Westinghouse Elec. & Mfg. Company.

Simultaneous Faults and Six-Phase Transmission, Edith Clarke, General Electric Company.

Power System Voltages and Currents Under Fault Conditions, R. D. Evans and S. H. Wright, Westinghouse Elec. & Mfg. Co.

Series Resistance Method of Increasing Transient Stability Limit, R. C. Bergvall, Westinghouse Elec. & Mfg. Co.

Research

ANNUAL REPORT OF COMMITTEE ON RESEARCH*

DURING the year the Committee members have reviewed twenty-six papers and recommendations have been made to Headquarters for their disposition. Each paper has been reviewed by three or more members and outside experts. The Committee held one session at the Winter Convention at which eight of the approved papers were presented. Additional papers, obtained through the Committee on Research, are scheduled for the Summer Convention and several papers made available for district meetings. All business was carried on through correspondence as the only Committee meeting scheduled, at the Winter Convention, had to be abandoned because of conflicts.

The following topics are of sufficient importance to be called to the attention of the electrical engineering profession. The subjects reported have all been suggested by the Committee members, special assistance having been given by Messrs. W. G. Cady, V. Karapetoff, H. H. Race, D. W. Roper, T. Spooner, and J. B. Whitehead.

PHYSICS

In the field of physics there has been a number of theoretical and experimental accomplishments of some interest during the past year.

The knowledge of cosmic rays has been advanced somewhat with two schools of thought: certain investigators, as a result of certain experimental evidence, are convinced that cosmic rays are actually high velocity particles. However, Dr. Millikan, from intensity measurement of the cosmic rays in northern latitudes, feels quite certain that they are actually rays, since they are not affected by the earth's magnetic field. Dr. Millikan concludes that the cosmic rays have their origin not in the stars, but in interstellar space and that they are due to the building up of the commoner heavy elements from hydrogen, it being thoroughly established by spectroscopy that hydrogen is widely distributed through space. Several of the more common elements and also helium exists in interstellar space.

After seven years of intensive work, Dr. Paul Heyl of the Bureau of Standards has remeasured the constant of gravitation, or as the newspapers would put it, the weight of the earth. This figure is 6.670×10^{-8} c. g. s. units.

Eddington has developed a theory which makes possible the calculation of the ratio of masses of an electron and proton as well as the sizes of the electron and of the universe.

The generation of very short radio waves is possible by a number of means, among them the Barkhausen tube. This tube is a three electrode tube with suitable voltages applied to the grid and plate and provided with parallel Lecher wires. The frequency is dependent on the speed of the electrons as affected by the voltages

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E. H. Colpitts,	V. Karapetoff,	D. W. Roper,
E. C. Crittenden,	A. E. Kennelly,	T. Spooner,
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and spacing of the electrodes and is independent of the external characteristics. Shuster has applied quantum mechanics to the problem of the Barkhausen tube and has calculated correctly the observed frequencies.

As a result of the work of Grondahl it has been known for some years that copper oxide on copper had a photoelectric effect, making possible the generation of electric energy from sunlight. Dr. B. Lange of the Kaiser Wilhelm Institute for silicate investigation has obtained some interesting results with the copper oxide cell. According to newspaper reports he has, with the use of silver selenide, obtained similar results, but of one hundred or more times as much magnitude. However, for the generation of power from sunlight, the development is very far from being commercial owing to the high cost and low output.

Medicine and Physiology

In the realm of medicine and physiology there has been a number of developments of interest to the electrical engineer. The recent paper on "Experiments on High Voltage Tubes," by Tuve, Hafstad, and Dahl of the Department of Terrestrial Magnetism, Carnegie Institution of Washington, describes an X-ray tube, the voltage of which can be as high as 2,000 kilovolts. At 1,300 kilovolts the absorption coefficient, after filtering through one inch of lead is the same as that for the gamma rays from radium in equilibrium measured under the same conditions with the same instrument. This result has been attained by the so-called heat working of the pyrex glass. This high-voltage tube should serve as a new tool to the medical profession.

For the treatment of cancer there has been developed by Dr. W. D. Coolidge a 900,000-volt X-ray tube to be installed in the New York Memorial Hospital.

It is well known that fever assists in destroying certain disease germs. It is possible, by the use of high frequency electrostatic fields, to produce artificial fever, thus assisting nature. Considerable success has been obtained by this method during the past year.

Dr. George Sperti of the University of Cincinnati has succeeded in using selected regions in the ultraviolet spectrum for the irradiation of foods; for example, he has been able to create Vitamin D in milk without spoiling the taste.

Dr. H. C. Rentschler has developed a method for measuring ultraviolet radiation by the use of a suitably designed photocell and a condenser. The condenser is charged slowly through the photocell and discharges through a counter at a rate dependent upon the ultraviolet radiation received by the photocell.

Dielectrics

Considerable progress has been made in insulation and dielectric research during the past year. Much work is being done to improve the processing in the manufacture of high-voltage cable insulation and also to detect a faulty impregnated cable before it goes into service. At Johns Hopkins University the value of thorough drying by the use of vacuum and air pressure during impregnation and the use of gas free oils have been studied further. To detect poor insulation, research workers of the University of Illinois are measuring high frequency discharges which constitute internal ionization.

Studies on the oils themselves are under way at the Detroit Edison Company, Cornell University, and the Brooklyn Edison Company, with special emphasis on the life characteristics and products of such oils under electrical stress, particularly ionic and electronic bombardment. At the Detroit Edison Company, it has been found that by bombardment of the oil vapors in a vacuum, the composition of the gas produced varies from one hydrocarbon to another and is quite complex, but consists roughly of 50 per cent hydrogen and 50 per cent unsaturated hydrocarbons. Liquids and wax-like solids are also produced. The solid formed is a duplicate of the compound "X" found in cables which have been in service. The formation of this substance in cables indicates internal corona. Five normal paraffin hydrocarbons of simple chemical structure and of very high purity have been prepared at the Bureau of Standards and studies are being made of their electrical characteristics.

Further studies have been made of polar compounds in liquid dielectrics. The frequency-temperature-loss characteristics have been studied in a number of laboratories with most interesting results. In addition, the polar characteristics of certain commercial insulating liquids are being studied, particularly the change and polar characteristics during service. These fundamental studies should aid in a choice of better insulating oils.

At Johns Hopkins University correlation between d-c. absorption and a-c. dielectric loss has been verified for several oils, dried paper and impregnated paper consisting of the materials previously studied alone. The relatively high d-c. conductivity of oil obtained up to about one second after electrification was found to account for the a-c. losses. The electrical characteristics of the separate and combined materials of impregnated paper insulation have been studied. This is a very important step in the fundamental side of dielectric research.

Another field of investigation which is just getting under way is the use of X-rays to study the molecular structure of dielectrics. The work of Dr. Zwicky at the California Institute of Technology on mosaics and on sodium chloride crystals, and the work in Chicago by Dr. Bennett on the effect of an electric field on the arrangement of paraffin crystals are worthy of notice.

There have been numerous important and interesting engineering developments during the past year.

Lightning Research

As the result of the investigations of natural and artificial lightning carried on at a number of field stations in various parts of the country, it has been pretty definitely established by the use of the cathode ray oscillograph that most outages and damage to transmission lines are the result of direct strokes rather than induced lightning. Two records during 1930 were obtained of crest values of 4,500 and 5,000 kv. The latter value was recorded at a distance of several miles from the stroke. At the place of flashover it is estimated that the potential was at least 10,000 kv. It has been concluded that a lightning stroke to a ground wire, if flashover does not take place, does not result in a surge over the line. By using proper precautions it is believed, as a result of these lightning investigations, that it will be possible to build transmission lines which will be practically immune from lightning.

There have been developed and placed on the market during the year, two new types of lightning arresters, namely the thyrite arrester by the General Electric Company and the new porous block autovalve arrester by the Westinghouse Elec. & Mfg. Co. These arresters have characteristics which are considerably superior to their predecessors.

The commercial development and application of new types of discharge tubes have been very active recently. It seems evident that these devices are going to occupy a very large place in the control and conversion of power.

Radio

In connection with radio there have been developed and put into commercial operation, 200 kw. transmitting tubes for broadcast stations.

Mercury-glass three-phase rectifiers have been developed to supply plate current for these high power radio tubes. These rectifiers are rated at 10,000 volts and 30 amperes. Additional grid-controlled gaseous discharge tubes, namely, the Westinghouse grid glow tubes and the General Electric thyratrons which are similar in their characteristics, have been developed during the year, the larger sizes being able to handle 75 amperes at voltages up to 1,000 or even 2,000. A grid energy of ½ watt or less is required.

Many new applications for grid glow tubes have been made recently. Among them is the theater light dimmer system which is controlled by very small rheostats which regulate the voltage applied to the grids of the tubes. These tubes supply rectified alternating current to choke coils which are saturated by the pulsating current, thus changing the inductance and controlling the lights. The small control wires can be led to any desired location, thus concentrating the control and simplifying the operation of the system. There are no moving parts and contacts, except for the small control rheostats. A very great advantage of this type of control is that the whole lighting program for the evening may be preset.

Miscellaneous

A new stroboscope called a stroboglow has been developed, which illuminates a moving object for a period of only 3/10 of a microsecond or less. This interval is so short that an object moving at seven miles a minute appears to be perfectly stationary with no appreciable blur. No moving parts are necessary and no connection between the stroboglow and the moving object is required. The instrument is calibrated for frequency and may be used to determine the speed of rotation of the illuminated object.

A very small tube rectifier has been developed, rated at 3/10 of an ampere and 5,000 volts. This rectifier is 4½ in. long and 1¼ in. in diameter. It can be operated satisfactorily up to 10,000 volts and under special conditions, has been carried as high as 30,000 volts. These tubes are particularly suited for airplane radio transmitters.

A new oscilloscope with a linear time axis has been developed which employs a new type of cathode ray tube for its essential element. This oscilloscope has the advantage over previous types in that it has a larger field and the illumination is very much greater, making it possible to observe recurrent phenomena in a well lighted room. Non-recurrent phenomena lasting for a few microseconds, can also be observed readily for qualitative results. For recurrent phenomena the cathode beam may be synchronized so that the image appears stationary.

An automatic traffic control system has been developed and put into experimental use on main highways, using a beam of light crossing the side streets. When a car or other vehicle approaches the main highway on the side street, the beam is intercepted, thus causing a photocell to act giving the minor street the green light.

There has been developed in Germany another application of the light sensitive devices for the purpose of stopping railroad trains. Light from the headlight of the locomotive is reflected from a mirror on the semaphore and returns to photocells on the locomotive, which when illuminated operate the automatic stop mechanism.

There has recently come into general use in this country the electric photo-flash lamp which was developed in Germany and is used as a substitute for flash-light powder. The bulb contains a quantity of aluminum foil in an atmosphere of oxygen. A small filament, when heated electrically, causes the aluminum foil and oxygen to combine with an intense illumination. The elimination of noise and smoke is greatly appreciated by the average audience.

Studies and applications of piezo electric quartz crystals have been continued at Wesleyan University. By means of the cathode ray oscillograph, the rate of

dying away of the oscillations of these crystal resonators has been studied. It has been found that a 600-kilocycle crystal started vibrating and left to itself, makes 60,000 vibrations before its amplitude falls to 1 per cent of its initial value. Various types of mountings for the crystals as well as the effect of the shape, cut and material of the crystal itself are being studied. A determination of the viscosity of quartz as a function of frequency is yielded by the above measurements. The coefficient of viscosity or quartz is approximately five at 600 kilocycles as contrasted with 500 for aluminum at 60 kilocycles and with values of the order of 10^8 ordinarily quoted for the low-frequency viscosities of metals.

A very accurate clock developed by the Bell Telephone Laboratories was exhibited at Cleveland last December, which is unique in that it is controlled electrically by a spherical quartz crystal.

The Stevens Institute of Technology reports the development of a clock which is automatically corrected by the standard radio time signals.

The limitations and advantages of commutation and current collection in hydrogen have been determined by laboratory and commercial experiments. In general, in the hydrogen atmosphere there results a lower contact drop. Satisfactory operation and long life are obtained if sparking is kept low. If appreciable sparking is present the humidity must be kept low.

The application of the helical groove to slip rings and commutators has been developed and applied commercially. Under severe conditions of commutation and current collection this apparently reduces brush wear and improves the electrical operation by reducing the formation of local hot spots; also better distribution of current between parallel brushes results.

A sample of commercial hipernik (a 50 per cent nickel-iron alloy) has been obtained having a maximum permeability of 167,000. Test samples from other castings have given as high as 13,000 for initial permeability and upwards of 400,000 for maximum.

Work during the past year on the fundamental nature of arcs and their extinction has given a greater insight into the problem of circuit interruption and is having a very definite effect on the design of circuit breakers.

There has been developed a non-destructive magnetic weld test meter for use with butt welds, which shows some promise commercially. For a given type of weld and a definite kind of steel, comparative results on welds are quite reliable.

At the Naval Research Laboratories interesting work has been done using the radiographic method for detecting imperfections in large steel forgings. Ordinary X-rays can penetrate only three or four inches of steel, but by the use of radium it has been possible to obtain photographs through twelve inches of steel. The radium has the advantage that it can readily be transported to the piece to be inspected and requires no complicated auxiliary equipment. If thick pieces of steel are to be photographed, however, either a long time must be used or a considerable quantity of radium or radium emanation must be available.